Specific and general cognitive factors underlying numerical development

Specific and general cognitive factors underlying numerical development

How training these factors impacts arithmetic in preschoolers?

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Thèse présentée en vue de l'obtention du grade de Docteur en sciences psychologiques et de l'éducation

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Illustration de couverture:
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Abstract

The development of numerical cognition relies on specific and general cognitive factors. The objective of this PhD thesis is to understand better the relations between these different factors and numerical as well as arithmetical capacities in 5- to 6-year-old children. The present work starts with a literature review about the associations between specific numerical factors and one general cognitive factor, working memory, and numerical development. The following chapters present four studies which took an interest in the impact of training these factors on numerical and arithmetical competences. In the first study, the effects of two numerical training conditions (symbolic and non-symbolic) were compared; the second study compared the impact of two Arabic number magnitude processing (number comparison and number line positioning) training programs; in the third and fourth studies, participants received a working memory training program. Finally, a longitudinal study was conducted to assess the predictive value of these different factors on calculation skills one and three years later.

Résumé

Le développement de la cognition numérique est sous-tendu par des facteurs cognitifs spécifiques, mais également par des facteurs généraux. L'objectif général de cette thèse est de mieux comprendre les relations entre ces différents facteurs et les capacités numériques et arithmétiques d'enfants âgés entre 5 et 6 ans. Ce travail débute par une revue de la littérature concernant l'association entre les facteurs spécifiques ainsi qu'un facteur général, la mémoire de travail, et le développement numérique. Les chapitres suivant présentent quatre études qui se sont intéressées à l'impact d'un entrainement de ces différents facteurs sur les compétences numériques et arithmétiques. Dans la première étude, les effets de deux entrainements numériques (symbolique et non-symbolique) ont été comparés, la deuxième étude a comparé l'impact de deux entrainements de la magnitude des nombres arabes (comparaison de nombre et positionnement sur une ligne numérique), et dans les troisième et quatrième études, les participants ont reçu un entrainement de la mémoire de travail. Finalement, une étude longitudinale a été menée pour évaluer la valeur prédictive de ces différents facteurs sur les capacités de calcul un et trois ans plus tard.

Chapter 1

Introduction

4. Objectives of the research

As there is a large body of evidence showing that numerical and arithmetical development is related both to basic number processes and to more general cognitive factors, including WM, the objective of the present work was to examine the nature of this relationship. To that end, four studies were run to evaluate whether numerical and arithmetical development could be enhanced through training. Then, a fifth study took a particular interest in the predictive power of basic numerical and WM skills on later arithmetic. The studies were conducted with preschool children who are not learning mathematics at school, which allowed us to study the foundational skills of arithmetic before any formal mathematical education had taken place. However, as the main objective of the present work was to assess the relationship between basic number magnitude processing and arithmetic, the participants were supposed to be able to solve simple arithmetical tasks. Therefore, we decided to focus on children who were in their last year of preschool, just before entering primary school (aged from 5 to 6 years old). All the participants in the five studies were typically developing children who did not suffer from any known developmental or learning deficit.

The second chapter aimed to observe the possibility of improving number magnitude processing and calculation trough numerical training. Furthermore, we wanted to disentangle the existing debate about the basis of numerical development relying on symbolic versus non-symbolic numerical magnitude processing. Therefore, we investigated the impact of two basic number magnitude training programs; one using symbolic (Arabic) numbers and one using non-symbolic (dot collections) numbers, on numerical and arithmetical skills, compared to a control condition. The non-symbolic training program had a significant effect on non-symbolic skills and the symbolic condition yielded improvement in symbolic and non-symbolic skills as well as in an addition task, whereas the control group did not show any significant change, indicating that the effects were induced by training and not related to spontaneous evolution.

In the third chapter, we wanted to evaluate the relationship of the two commonly used tasks to assess magnitude processing, i.e. number comparison and number line estimation, with numerical and arithmetical development. Therefore, we assessed the differential effects of magnitude comparison and number line training on numerical and arithmetical competence, compared to a control condition. Training young children in number magnitude comparison increased their ability to compare Arabic numbers and to position Arabic numbers on a number line while training them in number line positioning led to higher precision when positioning an Arabic number on a number line only. The control group did not show any improvement, controlling thus for spontaneous development.

The fourth and fifth chapters examined the efficiency of WM training on both WM and numerical abilities. In the third chapter, we used the computerized WM training program, Cogmed, which mainly trains the VSSP and also a little the visuospatial CE as young children's arithmetic performance would be more associated to the VSSP than the PL (De Smedt, Janssen, et al., 2009; Holmes & Adams, 2006; McKenzie et al., 2003; Rasmussen & Bisanz, 2005). Results indicated an effect of training on the trained processes, but almost no transfer to numerical abilities. Therefore, in the fourth chapter, we made a second attempt to study the impact of improved WM on numerical and arithmetical skills by developing a non-computerized training program tapping the verbal and visuospatial CE. As strategy teaching has been shown to enhance WM abilities and to transfer to arithmetic (St Clair-Thompson et al., 2010; Witt, 2011), our training program also aimed to teach and encourage children to use a visual-encoding strategy. This program led to short-term improvement in the VSSP and a long-term effect appeared on the visuospatial CE but no impact was observed on numerical abilities.

The sixth chapter focuses on the links that arithmetic has with general versus specific numerical factors, with symbolic versus non-symbolic magnitude processing, and with number comparison versus number line positioning by adopting a longitudinal design. In three different experiments, we investigated the predictive role of preschool WM versus numerical skills on performance in addition one year later and of symbolic versus non-symbolic magnitude processing in kindergarten on addition skills one or three years later. The results showed first that when WM and numerical skills are considered together, only magnitude processing skills significantly predict addition skills one year later and second that symbolic but not non-symbolic magnitude processing skills significantly predict addition performance one and even three years later.

Chapter 2

Improving Preschoolers' Arithmetic Through Number Magnitude Training: The Impact of Non-Symbolic and Symbolic Training

The numerical cognition literature offers two views to explain numerical and arithmetical development. The unique-representation view considers the approximate number system (ANS) to represent the magnitude of both symbolic and non-symbolic numbers and to be the basis of numerical learning. In contrast, the dual-representation view suggests that symbolic and non-symbolic skills rely on different magnitude representations and that it is the ability to build an exact representation of symbolic numbers that underlies math learning. Support for these hypotheses has come mainly from correlative studies with inconsistent results. In this study, we developed two training programs aiming at enhancing the magnitude processing of either non-symbolic numbers or symbolic numbers and compared their effects on arithmetic skills. Fifty-six preschoolers were randomly assigned to one of three 10-session-training conditions: (1) non-symbolic training (2) symbolic training and (3) control training working on story understanding. Both numerical training conditions were significantly more efficient than the control condition in improving magnitude processing. Moreover, symbolic training led to a significantly larger improvement in arithmetic than did non-symbolic training and the control condition. These results support the dual-representation view.

Reference

Honoré, N. & Noël, M. P. (2016). Improving Preschoolers' Arithmetic Through Number Magnitude Training: The Impact of Non-Symblic and Symbolic Training. *PLoS ONE*, 11(11): e0166685. doi:10.1371/journal.pone.0166685.

Chapter 3

How are the number comparison and the number line positioning tasks related to each other and to arithmetic?

Number comparison and number line positioning (tasks commonly used to assess number magnitude representations) correlate with arithmetic and training one or both of them impacts numerical and mathematical development. However, few studies compared the relation between math abilities and performance in both tasks.
Here, we first compared the relation of magnitude comparison and number line positioning with arithmetic, in preschoolers, and compared how training them impacts arithmetic. Second, we examined whether they rely on a common magnitude representation by analyzing their correlations and the impact of training each task on performance in the other.
Arithmetic correlated with both tasks but, although each numerical training condition improved the trained skill, arithmetic was not enhanced after training. Furthermore, the correlations between the two tasks were inconsistent, precision measures correlated whereas measures of representation linearity did (almost) not. Finally, number comparison training enhanced number line positioning but not the reverse.

Article (revised version) submitted to the journal "Trends in Neuroscience and Education".

Chapter 4

Can working memory training improve preschoolers' numerical abilities?

A large number of studies have pointed out the role of working memory throughout numerical development. Working memory capacities seem to be improved after training and some studies have observed an impact of working memory training on academic performance. In our study, we examined whether training visuospatial working memory (with Cogmed) enhances working memory abilities and numerical development in the short and middle term in 5-6 year-old children. Fourty six children were randomly assigned to the experimental condition (adaptive working memory training) or the control condition (non-adaptive, demo version). The program was implemented daily for a period of five weeks in both groups. We observed an immediate impact of the adaptive version on VSSP and visuospatial CE abilities and a small impact on Arabic number comparison. No training effect was observed in verbal working memory, in counting, collection comparison and addition. Furthermore, the observed effects were not sustained ten weeks later. These results are discussed in the context of specific and general cognitive factors that support numerical development and we argue against the idea of developing general cognitive factors to efficiently boost numerical development.

Reference

Honoré, N. & Noël, M. P. (in press). Can working memory training improve preschoolers' numerical abilities? *Journal of Numerical Cognition*, 3(2)

Chapter 5

Impact of working memory training targeting the central executive on preschoolers' numerical skills

Working memory capacities are associated with mathematical development. More and more studies have tried to improve working memory abilities through training. Furthermore, the central executive has been shown to be the component of working memory which is the most strongly related to numerical and arithmetical skills. Therefore, we developed a training program that specifically targeted the central executive to examine the possible impact on preschoolers' working memory and numerical abilities. Thirty four 5-6 year olds were randomly assigned to the experimental or the control conditions. Both groups received 16 30-minute sessions of their respective training program. The experimental group was trained with games tapping the verbal and the visuospatial central executive and the control group received a vocabulary-learning program. The results showed an immediate training effect on one verbal central executive task and an effect of training appeared on a visuospatial central executive task at follow-up, 6 months later. However, no impact was observed on numerical and arithmetical skills

Article (revised version) submitted to the “Journal of Education and Training Studies”.

Chapter 6

Preschoolers' working memory, symbolic and non-symbolic magnitude processing abilities as predictors of later addition skills

Arithmetical development is assumed to be based upon both general cognitive factors, such as working memory, and on specific numerical processes, such as the ability to process the magnitude of symbolic or non-symbolic numbers. However, it is unclear whether these processes are equally important or if one of them is more strongly related to later math achievement. Therefore, the first aim of the present study was to compare the predictive value of working memory and magnitude processing skills measured in preschool on addition performance one year later. The second aim was to assess the role of symbolic and non-symbolic magnitude processing skills in later math abilities. To this end, three longitudinal experiments were conducted. In the first experiment, preschoolers were tested in working memory as well as symbolic and non-symbolic magnitude processing and then in an addition task one year later. In the second experiment, preschoolers' symbolic and non-symbolic skills were assessed and their performance in addition was evaluated one year later. In the last experiment, participants were also tested in symbolic and non-symbolic magnitude processing in preschool but the addition test was administered three years later. Correlational and regression analyses revealed the predominant role of number magnitude processes in math achievement compared to working memory. More specifically, it is the processing of symbolic, and not non-symbolic, magnitude that predicted performance in addition one and even three year(s) later.

Chapter 7

General discussion

General discussion

Numerical and arithmetical development builds on many cognitive factors. Some of these factors are very specific to numerical cognition, such as magnitude processing, Arabic number knowledge… Other more general factors are involved in other cognitive activities: intelligence, attention, inhibition, processing speed, WM.

In the present work, we took a particular interest in the relationships of specific numerical factors and one general factor, WM, on the one hand and, with numerical development, on the other hand. More precisely, we set up different training programs, aiming at improving either specific numerical factors or WM and assessed the impact of these programs on numerical and arithmetical competence. The use of such a methodological design allowed us to examine the causal nature of the relations between each cognitive factor and numerical development. In an additional study, we used a longitudinal design to evaluate the predictive power of WM and numerical magnitude processing on future arithmetic performance. We studied a population of young children, aged between 5 and 6 years old to investigate these relations before any formal mathematical learning had taken place.

1. Summary of the research

First of all, we compared the effects of two numerical training programs to a non-numerical training condition on numerical (non-symbolic comparison, non-symbolic number line, symbolic; i.e. Arabic and verbal numbers, comparison, symbolic number line) and arithmetical (additions) competence. The numerical training programs consisted of both a magnitude comparison task and a number line positioning task; one of the programs involved only non-symbolic numerosities (non-symbolic training) while the other involved only symbolic (Arabic) numbers (symbolic training). In the control condition, participants received a story-understanding stimulation. In addition to testing the causal nature of symbolic versus non-symbolic magnitude processing with numerical and arithmetical development, this study allowed us to confront two theoretical approaches concerning the basis of numerical development: the unique-representation view, which considers the ANS to be the root of any further numerical and mathematical development; and the dual-representation view, according to which mathematics rely on the ability to process symbolic numbers. Our data showed that training preschoolers on non-symbolic magnitude processing tasks (magnitude comparison and number line positioning) improved their ability to compare non-symbolic numerosities and to position collections of 10 to 19 dots on a non-symbolic number line (1-20). Moreover, playing the same games (magnitude comparison and number line positioning) with symbolic numbers yielded enhancement in verbal number comparison, symbolic number line positioning and addition and, to a lesser extent, in non-symbolic comparison (ratio 1/2) and non-symbolic number line positioning (numerosities 2 to 9).

The relation between magnitude processing and arithmetic was further examined by distinguishing the two typically used magnitude processing tasks, i.e. number comparison and number line positioning. The impact of training, consisting of either number comparison or number line positioning, compared to an early literacy training condition, serving as control, was assessed on Arabic number comparison, symbolic number line positioning and addition. In this study, we analyzed, on the one hand, the causal nature of the specific relations of number comparison and number line positioning with arithmetic and, on the other hand, the relation between these two magnitude processing tasks. The results indicated improvement on the trained skill, i.e. number comparison training increased performance in number comparison and number line training enhanced abilities in number line positioning. Moreover, a cross-over effect was observed on the positioning of Arabic numbers after number comparison training. The reverse was not true; number line training did not enhance the ability to compare numbers. The correlational analyses between the precision measures of number comparison and number line positioning revealed that accuracy in number comparison highly correlated with the precision of number line positioning. In addition, the correlations between the linearity measures were inconsistent; the linear R^2 and the size effect significantly correlated at a given moment (T1 of the second study) but not 3 to 5 weeks later (T2 of the same study); and the number line linear R^2 did not correlated with number comparison distance effect. The association between these measures and arithmetic revealed different profiles; both precision measures were significantly associated with performance in addition and the linear R^2 was highly predictive of addition skills whereas the relations between the number comparison size and distance effects were not significant. Finally, neither the number comparison nor the number line training programs impacted arithmetic.

Next, the impact of WM training on WM (PL, VSSP and CE) as well as on numerical (counting, non-symbolic comparison, symbolic comparison and symbolic number line) and arithmetical (additions) abilities was investigated in two different studies. First, a computerized program composed of VSSP and visuospatial CE tasks was used both as training and as control. In the training version, the program was adaptive with the difficulty level increasing as the participant's performance increased. In the control condition, a demo version was used; the program was non-adaptive and stayed at a low level. Second, we developed a program consisting of eight verbal and eight visuospatial CE games and compared its effects to a vocabulary-learning training condition. The analyses of the training effects were intended to understand better the nature of the relationship between WM and numerical development. Results indicated immediate close-transfer effects; i.e. improvement in VSSP and visuospatial CE abilities; but these effects were not sustained 10 weeks or 6 months later. However, a central-executive training effect appeared on the verbal CE 6 months later. Finally, there was almost no transfer to numerical abilities, with only a slight immediate effect observed on Arabic number comparison after the visuospatial program. WM training, then, mainly yielded short-term and specific effects on WM.

In these four studies, the control groups did not show any significant change of performance from pre-test to post-test in any of the assessment tasks, demonstrating thus the training-related character of the observed improvements.

In the last study, we examined the predictive value of numerical and WM abilities on later arithmetical performance. So, in a first experiment, participants were tested for their numerical magnitude processing (non-symbolic comparison, Arabic number comparison and symbolic number line) and WM (PL, VSSP and verbal and visuospatial CE) abilities at the end of kindergarten. We tested the predictive power of these skills on addition performance one year later. In a second experiment, we compared the predictive power of symbolic (Arabic number comparison and number line) and non-symbolic (comparison) number magnitude processing skills in explaining arithmetical abilities one year later. Finally, in a third experiment, we studied the predictive value of symbolic (Arabic and verbal number comparison and number line) and non-symbolic (comparison and number line) number magnitude processing, measured at the end of preschool, on addition skills three years later. The results showed, first, that compared to WM, numerical factors are better predictors of addition skills one year later. Indeed, none of the WM measures was associated to later performance in additions; Arabic number comparison was the only unique predictor of addition skills one year later. Next, our data indicated that preschoolers' performance in Arabic number comparison and in symbolic number line, but not in non-symbolic comparison, significantly predicted performance in addition one year later. Finally, Arabic and verbal number comparison were significant predictors of addition skills three years later. Nevertheless, when considered together, only verbal number comparison remained a significant predictor of addition.

2. Symbolic or non-symbolic processing as precursor of arithmetical development?

The relation between symbolic and non-symbolic magnitude processing and arithmetic has been extensively explored in this research. We analyzed its causal nature by examining the effects of symbolic and non-symbolic training on arithmetic. We then computed correlational analyses to test the associations between preschoolers' symbolic and non-symbolic processing skills and first- or third-graders' arithmetical abilities. Finally, we conducted regression analyses to assess the respective predictive power of symbolic and non-symbolic magnitude processing skills on arithmetical competences one or three years later.

All the data from these various analyses converge towards the idea that symbolic, and not non-symbolic, magnitude processing skills underlie arithmetical abilities. These findings are totally in line with the growing body of evidence demonstrating a robust association between the processing of symbolic numbers and mathematics. Indeed, the relationship between symbolic magnitude comparison and mathematics is very consistent across studies whereas it is not for non-symbolic comparison (see De Smedt et al., 2013 for a review). In addition, in a recent meta-analysis, Schneider et al. (2016) concluded that this association was stronger for symbolic than for non-symbolic skills. Our results support the assumption according to which different representations underlie symbolic and non-symbolic magnitude processing. Indeed, if there was only one type of magnitude representation, produced by the ANS, then symbolic and non-symbolic training should have had a similar impact on basic number processing skills and on arithmetic. Furthermore, symbolic and non-symbolic processing skills should have been significantly associated to each other and should have similarly predicted future addition skills. However, this was not the case: symbolic and non-symbolic training differently affected magnitude processing skills and non-symbolic training did not improve performance in addition; non-symbolic and symbolic skills were not systematically correlated; and non-symbolic skills did not predict later performance in additions. Our data are thus in line with numerous studies showing that symbolic and non-symbolic numbers activate different magnitude representations (e.g. Bulthé et al., 2014; Holloway & Ansari, 2009; Lyons & Ansari, 2015; Matejko & Ansari, 2016; Noël & Rousselle, 2011; Reynvoet & Sasanguie, 2016; Sasanguie et al., 2014).

Furthermore, our results suggest that non-symbolic magnitude processing is not even related to mathematics; a finding that questions the role of the ANS. We are not the first to call into questions the theory of the ANS. For example, some authors have demonstrated that performance in symbolic comparison fully mediates the relation between non-symbolic comparison skills and math ability (Price & Fuchs, 2016). Other authors have shown that general processes are responsible for the association between non-symbolic comparison and mathematics. First of all, it has been shown that performance in non-symbolic comparison was associated to mathematical achievement when incongruent trials alone (visual cues do not positively correlate with numerical value) were considered; and this association became non-significant after controlling for inhibition (Fuhs & McNeil, 2013; Gilmore et al., 2013). When performing incongruent trials of non-symbolic comparison, individuals must inhibit irrelevant information coming from the perceptual variables of the stimuli (surface occupied by the items, perimeter if the items, density… do not positively correlate with numerical value). And it is the ability to inhibit a response based on the visual cues that would determine performance on incongruent non-symbolic comparison trials. (Gilmore et al., 2013). Furthermore, Bugden and Ansari (2016) revealed that the worst performance of children with developmental dyscalculia on incongruent trials of a non-symbolic comparison task, compared to typically developing children, was mediated by visuospatial WM abilities.

In a recent review, Reynvoet and Sasanguie (2016) evaluated the arguments supporting one assumption of the unique-representation view: the ANS mapping account, according to which the numerical meaning of number symbols is acquired by mapping them on the ANS (Feigenson et al., 2013; Piazza, 2010). The authors proposed an alternative approach to explain how symbolic numbers acquire their meaning: the symbol-symbol association account. This alternative account suggests that symbolic and non-symbolic numbers activate distinct magnitude representations and that, consequently, number symbols are not mapped onto the ANS. Instead, this account relies on Carey's developmental model (2001, 2004, 2009) suggesting that symbolic numbers acquire their meaning first by associating them to an individuation system allowing us to keep track of up to four objects. Then, for numbers beyond 3 or 4, the increasing knowledge of the counting list combined with the fact that adding one element to a set leads to a cardinal that just follows the one of the previous set (in the counting list). In this way, children infer critical principles of the numerical system; i.e. the order relation of numbers (the numbers are organized in a sequence) and the successor function (each number is exactly one more than the previous one). Finally, Reynvoet and Sasanguie (2016) argue that the foundation of mathematical abilities relies on the knowledge of symbolic numbers, culture- and language-dependent, i.e. in the form of a counting list. Thus, the association between the knowledge of ordinality, and not only cardinality (comparison tasks), and math achievement should be assessed. In this line, more and more studies have used order judgment tasks, in which participants are presented with pairs or triplets of symbolic or non-symbolic numbers and asked if they are presented in the correct (ascending) order (see Lyons, Vogel, & Ansari, 2016 for a review). These studies suggest that performance in Arabic number order judgment tasks is associated with mathematics (Goffin & Ansari, 2016), even after controlling for performance in Arabic number comparison (Lyons & Beilock, 2011; Lyons, Price, Vaessen, Blomert, & Ansari, 2014).

To conclude, although the unique-representation view, considering that numerical and arithmetical development relies on the ANS, has long dominated in the domain of numerical cognition, more and more studies propose a different theory of numerical development. Indeed, there is now a large body of evidence showing that symbolic and non-symbolic numbers activate different numerical representations and that it is the processing of symbolic numbers that underlies mathematics achievement. This dual-representation view does not necessarily refute the existence of the ANS but it does not give the ANS much importance in the development of mathematics. However, a couple of studies have suggested that measures of ANS actually reflect more general cognitive abilities, such as inhibition (Fuhs & McNeil, 2013; Gilmore et al., 2013) or WM (Bugden & Ansari, 2016). Although more research is needed to investigate exactly how symbolic processing is related to mathematics, i.e. cardinal or ordinal knowledge of numbers (see Lyons et al., 2016; Reynvoet & Sasanguie, 2016) or both, it seems reasonable to conclude that symbolic, rather than non-symbolic, magnitude processing is the basis of numerical and arithmetical development. In this line, Vanbinst et al. (2016) claim that “symbolic numerical magnitude processing is as important to mathematics as phonological awareness is to reading”.

3. Number magnitude processing tasks

The relation between number comparison and number line positioning has been examined in two ways in this work. First, we investigated whether training preschoolers on one task could positively impact performance in the other task. If the two tasks rely on the same magnitude representations or if they share common mechanisms, we expected to observe a transfer from increased abilities in one task to increased performance in the other.

Second, we analyzed the relations between the two tasks by means of correlational analyses between the different measures (precision and linearity measures) of each task. We also tried to understand better their relationship by analyzing their respective association with arithmetic, by analyzing the impact of training on arithmetic and by examining their relation with later arithmetical performance.

3.1. What do they measure?

The processes involved in number line positioning are supposed to be various and to develop with age (Friso-van den Bos, Kroesbergen, et al., 2015b; Link et al., 2014; Petitto, 1990). Indeed, Link et al. (2014) suggested that positioning a number on a number line requires proportion judgment, addition and subtraction as well as comparison skills. In addition, Friso-van den Bos, Kroesbergen, et al. (2015b) showed that young children do not make use of the end point as a reference but scale their response based on the start point of the line only and that they start using more reference points with age. This result is in agreement with Petitto (1990)'s work which showed that young children use a counting strategy to perform number line positioning tasks and progressively use proportional reasoning as they get older. Moreover, knowledge of Arabic numbers and numerical order is associated with accuracy in number line positioning (Berteletti et al., 2010; C. Xu & LeFevre, 2016). C. Xu and LeFevre (2016) even showed that increased knowledge of the sequential relations among numbers helps young children to make use of reference points on a number line, hence to increase precision in this task. So it could be hypothesized that the maturity of the strategies used by children to position a number on the number line depends on their knowledge of numerical ordinality.

Thus number line positioning is a complex measure comprising several processes, involving number comparison. Therefore, number comparison appears as a simpler and purer measure of magnitude processing in that it underpins number line positioning and it does not involve processes other than comparing numbers.

3.2. How are they related to each other?

The analysis of the training-related effects of each magnitude processing task on the other showed that both training conditions were efficient in improving the trained process (enhanced number comparison skills after number comparison training and increased precision to position a number on a number line after number line training). However, only number comparison training led to improvement in the non-trained magnitude processing task (i.e. number line positioning). These results suggest that some processes are shared by the two tasks, making possible a transfer from increased number comparison skills to the ability to position a number on a number line. This is perfectly in line with the findings about the processes underlying performance in number line positioning, e.g. involvement of comparison skills and Arabic number knowledge. However, they do not tell us anything about the underlying magnitude representations. Indeed, it could be hypothesized that the same representations are activated by the two tasks and are thus improved through both kinds of training, but that the unshared processes give rise to different results. Therefore, the correlational analyses between the number comparison and the number line positioning tasks shed more light on this issue. First, they revealed that the two tasks highly correlated when the precision measures were considered; however, the linearity measures, which are supposed to reflect the underlying magnitude representation (Sasanguie & Reynvoet, 2013), did not systematically correlate. These results suggest either that number comparison and number line positioning do not activate the same magnitude representation or that the measures used to reflect these magnitude representations (number line R^2 and number comparison size and distance effects) do not actually measure the underlying magnitude representations.

So, number comparison and number line positioning do seem to be related to each other. However, it is still unclear whether they operate on the same magnitude representation or if their relation reflects common cognitive processes, such as Arabic number recognition.

3.3. How are they related to mathematics?

In this work, we showed that the relation between arithmetic and number line positioning was stable, regardless of the measure used, precision (Link et al., 2014; Sasanguie, De Smedt, et al., 2012; Sasanguie, et al., 2013) or linearity (Booth & Siegler, 2006; Friso-van den Bos, Kroesbergen, et al., 2015; Friso-van den Bos, Van Luit, et al., 2015; Siegler & Booth, 2004). In contrast, the association between arithmetic and number comparison varied according to the measure. Indeed, only the precision, and not the linearity (i.e. size and distance effects), measure correlated with addition skills, which is often observed (see De Smedt et al., 2013 for a review). Thus, the linearity measures are not similarly associated with arithmetic, undermining the assumption that number comparison and number line positioning tasks activate the same magnitude representations (Dehaene, 1997; Gallistel & Gelman, 1992; Laski & Siegler, 2007). Indeed, we would have expected the measures supposed to reflect the linearity of numerical representations (linear R^2 and size and distance effects) to be similarly associated to arithmetic. It may thus be that these measures assess different things. It has been suggested that the relation of mathematics with number comparison and number line positioning is mediated by the processes (proportion judgement, decisional processes…) involved in the tasks (Sasanguie and Reynvoet, 2013).

We also showed that number comparison skills in preschool were related to addition performance one and three years later. In contrast, number line positioning did not consistently predict later arithmetical competences. Symbolic comparison thus appears to be strongly associated with arithmetic (see De Smedt et al., 2013) and to be an important predictor of later performance in arithmetic (e.g. De Smedt, Verschaffel, et al., 2009; Vanbinst et al., 2016; Vanbinst et al., 2015). As regards number line positioning, although it correlates with arithmetic (Booth & Siegler, 2006; Friso-van den Bos, Kroesbergen, et al., 2015b; Kolkman et al., 2013; Link et al., 2014; Sasanguie et al., 2013), its predictive power to explain later performance in addition is unclear. Only a few studies have examined this predictive association (Friso-van den Bos, Kroesbergen, et al., 2015b; Friso-van den Bos, Van Luit, et al., 2015; Kolkman et al., 2013) and, unlike the present work, they showed that number line positioning predicted later mathematical skills. However, these studies used a general math test whereas we specifically assessed children's addition skills. This difference might give rise to very different results. Indeed, as number line positioning involves many complex processes (i.e. proportion judgment, addition/subtraction…) and general math tests include a large variety of processes as well, Link et al. (2014) suggested that the relation between number line positioning and arithmetic was driven by these complex processes. Therefore, the relation found with general math tests, compared to specific tests, might be overestimated. According to Friso-van den Bos, Kroesbergen, et al. (2015b) number line positioning should not be seen as a predictor of mathematics achievement. Indeed, these authors showed that number line positioning and math achievement mutually influence each other.

Finally, despite the strong correlations found between magnitude processing and addition skills, we failed to find any impact of number comparison or number line positioning on arithmetic. In contrast, previous studies showed that training children on magnitude comparison (Hyde et al., 2014; Maertens et al., 2016; Praet & Desoete, 2014) or number line positioning (Link et al., 2014; Maertens et al., 2016) tasks or on both (Honoré & Noël, 2016) had a positive impact on arithmetic.

Number comparison and number line positioning, then, do correlate with arithmetic. However, these associations vary according to the measure used for the magnitude processing task; both number comparison and number line precision measures are related to addition performance whereas only number line positioning correlates with addition when the linearity measure is considered. These results, together with previous findings, suggest that the relation of number comparison and number line positioning with mathematics is driven by non-numerical cognitive processes (Defever, Sasanguie, Vandewaetere, & Reynvoet, 2012; Sasanguie & Reynvoet, 2013). Further, both tasks do not similarly predict later math abilities. Indeed, while number comparison has largely been shown to be a strong predictor of later arithmetical skills, few studies showed that number line positioning predicted future mathematical competences but none have investigated its predictive power on future arithmetical performance. Finally, the causal nature of these relations is still unknown and therefore needs more investigation.

3.4. Conclusions

Although there is a semblance of debate concerning the processes/representations underlying number comparison and number line positioning, very few studies have taken an interest in the relationship between these two magnitude processing tasks (see Laski & Siegler, 2007; Maertens et al., 2016; Sasanguie & Reynvoet, 2013). Moreover, the findings from these few studies are very difficult to compare because different number formats (symbolic or non-symbolic) and measures (precision and linearity) were used. Laski and Siegler (2007) showed that the linearity measure of a symbolic number line positioning task significantly correlated with the precision measure of a symbolic comparison task, with 5-

to 8-year olds. Sasanguie and Reynvoet (2013) found that the linearity measures of non-symbolic number line positioning and non-symbolic comparison did not correlate, with 6- to 8-year-old children. Maertens et al. (2016) showed that training preschoolers, aged 5 years old, in symbolic and non-symbolic number line or in symbolic and non-symbolic comparison did not impact their performance in the non-trained tasks. So, it is very difficult to combine the data from these different studies or to link them to the data from the present research. Like us, Laski and Siegler (2007) used symbolic numbers, but they analyzed the relation between one linearity (for number line positioning) and one precision (for number comparison) measures. Sasanguie and Reynvoet (2013) showed that the linearity measures did not correlate, which is in line with our results; however, they used non-symbolic numerosities. Finally, the training study conducted by Maertens et al. (2016) revealed no cross-over effect whereas we did find an impact of number comparison training on number line positioning. Nevertheless, the training programs differed in terms of numerosity formats, i.e. symbolic and non-symbolic in Maertens et al. (2016)'s study and symbolic only in the present study, and in terms of duration, i.e. 60 minutes (Maertens et al., 2016) versus 100 minutes (present research).

However, despite the difficulty in bringing together the available data to draw a conclusion, it seems important to clarify the existing "debate". Some authors (Laski & Siegler, 2007) claim that number comparison and number line positioning activate the same numerical representation while, according to others (Maertens et al., 2016; Sasanguie & Reynvoet, 2013), the two tasks rely on different processes. These two assumptions are nevertheless not exclusive. Indeed, number line positioning involves several processes (Link et al., 2014), including number comparison, whereas number comparison is a more basic task which does not involve complex mechanisms. Thus, as suggested by Sasanguie and Reynvoet (2013) and Maertens et al. (2016), the two tasks do not rely on the exact same mechanisms, number line positioning involving additional processes. The numerical representations activated by the tasks might depend on the strategy, hence the process, used to perform it. It has been shown that the nature of the magnitude representations is different according to the type of numerical processing required by a given task. Indeed, cardinality and ordinality seem to operate on different numerical representations as demonstrated by their reversed distance effects. In number comparison (i.e. processing of cardinality), it is easier to compare pairs of digits that are numerically distant, e.g. 3-8, rather than close, e.g. 3-5, whereas in symbolic numbers order judgment tasks (i.e. ordinal processing), reaction times are shorter for consecutive ascending pairs, e.g. 3-4, than for non-consecutive pairs, e.g. 3-6. As the distance effect is supposed to be a manifestation of the underlying numerical representations (Dehaene, 1997; Gallistel & Gelman, 1992), these findings suggest that different representations are activated according to the type of numerical processing, i.e. cardinality or ordinality. In support for this hypothesis, single case studies of patients with Gerstmann syndrome have shown a dissociation between these two processes, with altered ordinal processing abilities and preserved cardinality processing skills (Chen, Xu, Shang, Peng, & Luo, 2014; Turconi & Seron, 2002). Moreover, in an event-related-potential (ERP) study, Turconi, Jemel, Rossion, and Seron (2004) observed different spatio-temporal patterns of the distance effect for quantity and order processing.

Therefore, the way number line positioning is performed would lead to the activation of a specific numerical representation. For instance, young children perform number line positioning tasks mainly with sequential strategies (Petitto, 1990) thus requiring the processing of ordinality, but not cardinality. Consequently, they might activate different representations for number comparison (cardinality processing) and number line positioning. In contrast, participants using comparison to perform number line positioning might activate the same representation as in number comparison. Some participants might also use different strategies, e.g. comparison and counting, proportion judgment and comparison…, leading to the activation of both cardinal and ordinal numerical representations.

To conclude, the correlations observed between number comparison and number line positioning might reflect their common processes, hence the same numerical representations. However, depending on the strategy used to perform the number line task, different processes, hence different representations, can be activated which might lead to non-significant correlations between number comparison and number line positioning. So, to further understand the relation between the two tasks, future studies should investigate the strategies used to perform them. In addition, they should also consider numerical order processing to determine whether it also correlates with number line positioning and whether this relation depends on the strategy used and on age. Finally, number line positioning is often considered as a foundational ability for mathematical learning, in the same way as number comparison. However, number line positioning involves many complex processes whereas number comparison and the processing of ordinality seem to be more basic number skills. Therefore, future studies should focus on number comparison and order judgment abilities as basic number skills and examine how they influence number line positioning and mathematical skills.

4. Working memory and numerical development

As already shown in several studies, we demonstrated that the CE was highly correlated with numerical and arithmetical skills (Holmes & Adams, 2006; Kroesbergen et al., 2009; Noël, 2009; Rasmussen & Bisanz, 2005) in preschoolers. Our data also showed significant but weaker correlations between the PL and numerical and arithmetical as well as between the VSSP and performance in addition. The data we obtained concerning the slave systems illustrate the inconsistencies found in the literature, leading to the absence of consensus as regards the role of the PL and the VSSP in numerical development. There is a substantial heterogeneity in terms of mathematical measures used in the studies examining the link between the slave systems and math achievement, making comparison between them very difficult. However, even when the results are analyzed according to the type of math skills investigated, no clear conclusion emerges. Indeed, among the studies taking an interest in young children’s arithmetical abilities, some found a significant correlation with VSSP abilities in young children (Bull et al., 2008; McKenzie et al., 2003; Rasmussen & Bisanz, 2005) and others with the PL (Bull et al., 2008; Noël, 2009). When broader mathematical abilities (i.e. measures combining numbers, measurement, arithmetic) are considered, a significant correlation was found with the VSSP (De Smedt, Janssen, et al., 2009).

In addition, our results indicated that increased WM (visuospatial CE and VSSP or verbal CE) abilities hardly transferred to the numerical field. This suggests that WM training does not lead to far-transfer effects on academic achievement skills, which is the result that seems to emerge from most WM training studies (Melby-Lervag & Hulme, 2013; Shipstead, Redick, et al., 2012).

Finally, none of the WM measures in preschool predicted performance in addition one year later. This result is in opposition to several studies showing that WM was a significant predictor of future mathematical skills (Attout et al., 2014; Bull et al., 2008; Passolunghi & Lanfranchi, 2012; Toll & Van Luit, 2013). However, when Passolunghi and Lanfranchi (2012) examined math performance two years later, WM was no longer a significant predictor. Moreover, Bull et al. (2008) used a general math achievement test and Passolunghi and Lanfranchi (2012) and Toll et al. (2016) measured early numeracy skills. Therefore, only one study assessed the predictive power of WM on future arithmetical abilities (Attout et al., 2014) and showed that the PL was a significant predictor of calculation skills one and three years later.

So, although there is a relationship between WM, especially the CE, and numerical development, the results from training studies suggest that the nature of this relation is not causal. In addition, it is still unclear if WM abilities, measured at a young age, can reliably predict future performance in arithmetic.

5. Is training effective in improving numerical and arithmetical development?

5.1 Working memory training

The literature about the impact of WM training on numerical abilities with preschoolers is still weak and the results are inconsistent. A first concern about this literature is to evaluate the effect of training on WM. Indeed, if WM training fails to improve WM, it is not possible to measure the impact on numerical or arithmetical abilities. Possible increase of performance in this field would therefore be associated to spontaneous development (in the case of absence of a control group) or to type I error. As raised by Melby-Lervag and Hulme (2013), this pattern of results is uninterpretable as any far-transfer effect induced by WM training is supposed to be associated to increased WM abilities.

Most studies assessing the impact of WM training on numerical abilities with preschoolers did find improvement in WM (Kroesbergen et al., 2014; Passolunghi & Costa, 2014; St Clair-Thompson et al., 2010; Thorell et al., 2009), although generally specific to the trained components. In accordance with the existing literature, the two types of training implemented within this research were effective in improving the trained component(s) of WM.

These increased WM abilities are expected to transfer to numerical and arithmetical skills as a strong association exists between WM and mathematics (Espy et al., 2004; Kroesbergen et al., 2009; Noël, 2009; Rasmussen & Bisanz, 2005) and several studies even showed that WM abilities in preschool predicted later math competences (Attout et al., 2014; Bull et al., 2008; Passolunghi & Lanfranchi, 2012). In addition to the possible practical and educational applications, such effects would demonstrate the causal nature of this relationship. However, there is little evidence supporting the idea that WM training impacts numerical and arithmetical development in young children. Indeed, concerning numerical skills, increased WM abilities have been shown to transfer to non-symbolic comparison, as well as to counting when the training program focused on the processing of numerical information (Kroesbergen et al., 2014); Passolunghi and Costa (2014) observed training gains in counting; Honoré and Noël (in press) showed a slight improvement in Arabic number comparison which did not last 10 weeks later, and no effect was found in counting, non-symbolic comparison and number line positioning. As regards arithmetic, St Clair-Thompson et al. (2010) observed a significant training impact while no effect was found in Honoré and Noël (in press). Finally, no impact of WM training was found on problem solving in Thorell et al. (2009)'s study and the program we developed to increase CE abilities failed to improve preschoolers' numerical and arithmetical skills.

So results are inconsistent, not only across but also within studies. Indeed, when different numerical processes are assessed, the impact of WM training, if any, concerns only a part of these numerical abilities. In this line, a meta-analysis which synthesized the size effects from 23 WM training studies (Melby-Lervag & Hulme, 2013) concluded that "there is no evidence that WM training produces generalized gains to the other skills that have been investigated (verbal ability, word decoding or arithmetic), even when assessments take place immediately after training" (p. 281). These authors identified an important methodological issue in studies of WM training: passive control groups are often used. This is indeed the case in the studies of Kroesbergen et al. (2014), Passolunghi and Costa (2014) and St Clair-Thompson et al. (2010). Thus, it is possible that these studies overestimated the training effects. Results from studies using passive control groups cannot be interpreted with confidence because of the many confounds (all the possible causes of improvement that are of no interest such as motivation, self-esteem, feeling considered…) that are not controlled for. Moreover, a meaningful evaluation of the possible far-transfer effects of WM training requires the presence of a close-transfer effect (Melby-Lervag & Hulme, 2013). Close-transfer effects were indeed found in the WM training studies discussed here, except in Kanerva and Kyttala (2016) and Kyttälä et al. (2015), in which no impact was observed on numerical development either. Furthermore, close-transfer effects should be assessed with multiple WM measures, instead of a single measure which includes error variance (Hulme & Melby-Lervag, 2012), and these measures must be different from the training task (Shipstead, Redick, et al., 2012). However, none of the discussed studies used multiple tasks to assess the different WM constructs. Hence, it is not possible to reach a conclusion about a real improvement of the tested ability. In our WM studies, we used several tasks (3 and 4) to measure CE capacities; but improvement was observed in only one of these tasks, suggesting that the effects were not large. Indeed, if training was successful in increasing the capacities of the CE, then the increase in performance should have been observed in most tasks tapping the CE.

So for then, there is no evidence that WM training is effective in impacting preschoolers' numerical and arithmetical development. Several reasons might be considered. First, WM training does not sufficiently improve WM capacities. Indeed, the impact of training on WM measures is quite weak. As WM is constantly solicited during everyday activities, hence naturally trained, the possible effect of training might be masked by this natural kind of WM training. Then, span measures do not allow large changes; indeed, Nichelli, Bulgheroni, and Diva, (2001) examined the developmental patterns of verbal and visuospatial spans in children and showed that, from 5-6 to 7-8 years old, the verbal span (repetition of words) evolves from 4.4 to 4.5 in girls and from 4.1 to 3.8 in boys and the visuospatial span increases from 3.4 to 3.8, regardless of sex. Therefore, it would be interesting to use other measures, such as the total number of correct responses to increase the variance, hence the possibility of observing an impact of training. Then, other types of training, inspired by theoretical WM models, should be developed. For example, Barouillet and Camos (2007) suggest that maintenance and processing are dependent systems that require attention, which is a limited resource. When attention is turned away from items to remember, their memory trace fades over time and a new attentional focus is necessary to refresh them. According to their functional model of WM (compared to Baddeley and Hitch (1974)' structural model), the time-based resource-sharing model, a WM task implies constant shifts of attention between processing and maintenance; the processing is interrupted by micro pauses of attention in order to refresh the memory traces. Hence, performance in a WM task depends on the proportion of time during which the processing task captures attention, which the authors call the cognitive cost. This model offers a new perspective of how to WM works and, thus, how to improve it. Second, preschoolers are too young and are not able to transfer their increased WM abilities to the numerical field. Implementing additional sessions helping the children to use their "new" abilities in the mathematical domain after WM training could be more effective, especially with school-age children who are learning mathematics and for whom the transfer would therefore be more direct and concrete. Similarly, training programs aiming at improving school-age children's WM capacities by teaching strategies that will then be transferred to the mathematical field should be developed. This is, by the way, perfectly in line with the time-based resource-sharing model according to which performance in WM depends on the ability to perform the processing task, the more automatized the task is, the least it takes time to perform it and the most attention can be allocated to the maintenance of traces in memory. Third, the numerical testing tasks used to assess WM far-transfer effects might prevent training effects from appearing. Indeed, there is no evidence that WM is involved in symbolic, non-symbolic comparison or in number line positioning. Furthermore, when WM is likely to play a role, i.e. counting, addition solving, there is no evidence that young children make use of increased WM skills; instead they might just keep using their compensation method, e.g. use of tokens, fingers…

5.2 Numerical training

Results from numerical training studies so far are promising. Indeed, a large number of studies showed that numerical training improves children's counting skills (Fischer et al., 2011; Räsänen et al., 2009; Sella et al., 2016; Whyte & Bull, 2008; Wilson et al., 2009), numerical magnitude processing skills (Fischer et al., 2011; Kucian et al., 2011; Link et al., 2013; Maertens et al., 2016; Obersteiner et al., 2012; Räsänen et al., 2009; Sella et al., 2016; Whyte & Bull, 2008; Wilson et al., 2009) and even arithmetical competence (Hyde et al., 2014; Kucian et al., 2011; Link et al., 2013; Maertens et al., 2016; Obersteiner et al., 2012; Praet & Desoete, 2014; Räsänen et al., 2009; Sella et al., 2016; Vilette et al., 2010; Wilson et al., 2009). These findings are very important and encouraging for clinical applications (e.g. rehabilitation of children with math learning disability or prevention with young children). However, they do not allow us to draw theoretical conclusions about which specific processes enhance numerical and arithmetical development. Indeed, these studies differ on many aspects (training tasks, testing tasks, duration…). For instance, some training programs include arithmetical tasks (Kucian et al., 2011; Obersteiner et al., 2012; Räsänen et al., 2009; Sella et al., 2016; Vilette et al., 2010; Wilson et al., 2009), others train counting strategies (Siegler & Ramani, 2009) or mix different numerosity formats (Praet & Desoete, 2014; Wilson et al., 2009).

Therefore, in the present research, we showed that training preschoolers on symbolic magnitude processing tasks, during ten 30-minute sessions, was effective in improving magnitude processing and arithmetical skills. Training preschoolers on Arabic number comparison or number line positioning, during ten 15-minute sessions, increased magnitude processing skills, but not performance in additions. As no other study has examined the exact same kind of training on the same numerical abilities, it is not possible to directly compare our results to previous ones. Nevertheless, the absence of number comparison or number line training effect on arithmetic suggests that it is the association of both training tasks that leads to improvement in arithmetic or that more intensive training is needed. Nevertheless, it is worth noting that the addition tasks used in the two studies (chapters 2 and 3) differed in terms of format and presentation. In both studies, the first term of the addition was presented in a non-symbolic format (Easter eggs in the second chapter and apples in the third one) and the second term was presented in a symbolic format, although Arabic numbers in the second chapter and verbal numbers in the third one. Then, in the second chapter, the children had to choose the answer between two possibilities while, in the third chapter, they had to verbally produce the answer. So, it is possible that requiring the children to produce the answer is more difficult than selecting the correct answer among two possibilities. In addition, it could be argued that the multiple-choice format is similar to the number comparison task and, therefore, that a transfer effect appeared in chapter 2 only because of the similarity of the addition task to one of the training tasks. However, the effect of training observed after symbolic number comparison and number line training (chapter 2) cannot entirely be attributed to this task similarity as they were still quite different, i.e. selection of one Arabic number among 3 in the number comparison training task and selection of one numerosity (presented in both symbolic and non-symbolic formats) among 2 for the addition testing task. Moreover, to correctly solve the addition task, the children had to perform exact calculation, and not simply magnitude comparison, as the distractor was smaller/larger than the correct response in half of the trials. Nonetheless, it remains possible that the absence of transfer from number comparison training to arithmetic was due to the greater difficulty of the production addition task compared to the multiple-choice one.

Thus, although it is clear that numerical training does yield improvement in numerical and arithmetical development; from a theoretical point of view it is important to differentiate the studies according to whether or not they trained arithmetical skills. This leaves us with six studies, run with preschoolers, that did not include calculation in the training program (Fischer et al., 2011; Hyde et al., 2014; Link et al., 2013; Maertens et al., 2016; Praet & Desoete, 2014; Whyte & Bull, 2008). Fischer et al. (2011) did not find an effect on calculation and Whyte and Bull (2008) did not assess arithmetical skills. The four others observed a positive impact on calculation (Hyde et al., 2014; Link et al., 2013; Maertens et al., 2016; Praet & Desoete, 2014). However, Hyde et al. (2014) did not test arithmetical skills before training. Maertens et al. (2016) as well as Praet and Desoete (2014) used a pictured arithmetic task, which might have been performed by simply counting instead of calculating. Finally, Link et al. (2014) showed that embodied number line training (i.e. walking on a number line until the estimated position of a given number) significantly improves performance in addition.

To date, many studies have shown that numerical and arithmetical competences can be improved through training. However, only two studies indicated that training number magnitude processing, without calculation, significantly impacts arithmetical skills (Honoré & Noël, 2016; Link et al., 2014). Further studies are therefore needed to replicate these findings.

5.3 Conclusions

Numerical training seems more efficient than WM training in impacting arithmetical development. Although WM abilities are significantly associated with arithmetical performance, there is little evidence that increased WM skills yield improvement in arithmetic.

However, it is important to keep in mind that the aim of our research was to address theoretical questions about the roots of numerical and arithmetical development. Accordingly, we developed training programs specifically tapping number magnitude or WM and examine their impact on calculation. In contrast, if the objective of training is to improve numerical and arithmetical skills in a clinical context, e.g. helping a child with math learning disability, the training program would involve as many mathematics-related factors as possible rather than one very specific factor.

Indeed, as noted by Maertens et al. (2016), arithmetical development takes place under the influence of many factors; thus, focusing on the manipulation of one specific factor cannot lead to very large effects. For instance, a recent meta-analysis (Schneider et al., 2016) showed that the association between symbolic comparison and mental arithmetic was *r*=.38, meaning that symbolic comparison explains at most 14% of arithmetical skills. Thus, when training is effective, it will impact 14% of arithmetical performance at best. So, the higher the correlation between the trained task and mathematics is, the greater the improvement that can be expected, if these two skills have a causal relationship. This might be a reason why a training program combining number comparison and number line positioning (chapter 2) is more effective in improving arithmetic than a program in which only one magnitude processing skill is trained (chapter 3). Thus, the key to effectiveness could be including more variables that are strongly associated with the target ability to increase the power of the association between the training program and the target ability, hence to increase the possibility of improving this ability.

As regards WM, the correlations commonly observed between the CE and math vary between *r*=.20 and *r*=.50. So training tapping the CE can impact between 4 and 25% of the mathematical ability. However, in the CE training study of the present work (chapter 5), the correlation between the CE factor and addition performance was quite high, *r*=.68. So, if this relation was causal, a considerable impact should have been observed on addition skills; unless working memory abilities themselves were not sufficiently enhanced.

6. Limitations of the present research and future perspectives

The present research has some limitations. First, the four training studies were run on small samples of participants, i.e. between 16 and 23 children per condition, probably making the data not very powerful. Moreover, these small samples prevented us from computing valid additional analyses. For instance, it would have been interesting to split the experimental groups according to their abilities (in addition, symbolic magnitude processing, WM) before training, leading to the creation of two subgroups; the high-achieving and the low-achieving groups. We could then have examined whether the low achievers benefitted more from training than the high-achievers. We did compute these analyses to explore whether a trend would appear, but no significant result emerged; probably due to the very small size of the groups, making the results unreliable.

Next, although we used three and four measures to test CE abilities in the WM training studies (chapters 4 and 5), the PL and the VSSP were assessed with one task only. However, as suggested by Hulme and Melby-Lervag (2012), it is important to use multiple tasks to measure a given construct, to be sure to assess the target variable and reduce error variance.

The training studies were carefully designed in order to avoid uninterpretable effects. For instance, active control groups were included in each training study. However, they differed from the experimental training conditions not only in terms of content (except for the Cogmed training study, in which the same games were used in the experimental and control conditions) but also in terms of challenge. Indeed, children in the control groups did not have to do their very best to continue training; whatever their performance or their involvement in the activity, they would not be disadvantaged. In contrast, in the experimental groups, children had to perform at their best to go on to the next level, to be rewarded… According to Shipstead, Hicks, et al. (2012), control training that is not challenging enough entails the risk that the participants of the experimental and control groups do not approach the post-test with the same level of motivation. Finally, the experimenter was blind to the condition the child belonged to for post-testing in one study only, i.e. CE training (chapter 5). We would have liked to use this blind testing design for the other studies as well, but we lacked experimenters. These last two limitations must be kept in mind and must remind us to be careful not to overestimate the effects.

The present research opens doors for future studies.

First, the hypothesis according to which the relation between non-symbolic magnitude processing and mathematics relies on inhibition could be causally tested. For example, the effects of non-symbolic comparison training on mathematics could be examined while controlling for inhibition skills. This would allow the experimenter to analyze whether a possible effect on mathematics is related to higher inhibition skills. Furthermore, these effects could be compared to those of an inhibition training program to assess the relative association of mathematics with non-symbolic comparison and inhibition skills.

Second, the relation between number comparison and number line positioning should be further examined. Indeed, the very few existing studies on this issue do not allow conclusions to be drawn. The available data suggest that the two tasks have both shared and unshared processes that are determined by the strategy used to perform the task. Moreover, the type of numerical process leads to the activation of specific numerical representations. Accordingly, the numerical representations activated in the tasks depend on the strategy used. Therefore, future studies should focus on the strategy used to perform number line positioning and its relation should be examined with both number comparison and order judgment tasks, which reflects quantity and ordinality processing skills, respectively. Correlations that vary according to the strategy used (e.g. higher correlation between number line positioning and number comparison when comparison, proportion judgment or addition skills are used and higher correlation with order judgment when counting is used) would demonstrate, first, the importance of the type of numerical process in explaining these relations and, second, that number comparison and order judgment are more basic number skills that underlie the development of number line positioning abilities. Then, more research is needed to understand better the relation between number line positioning and both current and later arithmetical skills. To this end, tests assessing specific mathematical competences (e.g. number knowledge, calculation, measurement…) instead of general math tests should be used. Indeed, it has been suggested that the relation between number line positioning and mathematics is driven by the complex processes involved in number line positioning (Link et al., 2014) and, as general math includes a large variety of processes as well, the relations found between these two variables might be overestimated. It is therefore important to clarify this relation by using specific tests, which will then allow us to make reliable comparisons between each mathematical competence and each magnitude processing task (i.e. number comparison and number line positioning).

Fourth, the issue about the predictive power of preschoolers' WM skills on future mathematical abilities should be further clarified by conducting longitudinal studies. As it has been shown that tests assessing a large range of mathematical abilities are more likely to correlate with WM (Friso-van den Bos et al., 2013), there is a crucial need for studies using systematic measures of specific numerical and arithmetical abilities as well as multiple and valid measures of WM. Results from these studies would shed light on the specific predictive relations of each WM component with different mathematical skills, thus better targeting children at risk of developing math learning difficulties and, more importantly, offering them the most appropriate prevention methods.

Fifth, WM training studies should adopt a very different approach in order to assess the impact on WM capacities but also on academic achievement. First, the training program should be developed on the basis of functional rather than structural WM models. Based on the time-based resource-sharing model (Barouillet & Camos, 2007), presented earlier, WM training should, for example, teach the participants how to allocate attentional resources at best, i.e. frequent switches between the maintenance of information and the processing of a simultaneous task. As performance in WM would be completely dependent on the ability to perform the processing task itself, it seems very important to develop programs in which the WM tasks are domain-specific in order to improve the target ability (i.e. numerical/arithmetical WM tasks if mathematics is the target ability).

as already mentioned, the effects of WM training might not be observable in most of the typical numerical and arithmetical tasks. Special attention should be paid to the development of numerical tasks that allow more use of higher WM abilities or strategies. For example, more concrete, school-based, tasks could be used and the participant could be incited to make use of their trained abilities. The analysis of training effect when instruction about using the trained skill is given compared to when it is not would be informative of how to help children transfer from rehabilitation to classroom.

7. Conclusions

This work aimed at understanding better the relations between preschoolers' very specific numerical factors, one general cognitive factor (WM) and numerical as well as arithmetical development. In four out of five studies, we used training as the methodological design to investigate the possibility of improving numerical and arithmetical development through numerical or WM training. This method allowed us to test for the causal nature of these relations. Indeed, if a given ability is causally related to arithmetic, then improving the ability should lead to increased arithmetical skills. Our data showed that symbolic magnitude processing, combining number comparison and number line positioning, underlies numerical and arithmetical development. However, we failed to show that each of these magnitude processing tasks, number comparison or number line positioning, underpinned the development of arithmetical abilities. This absence of result could be due to too little intensive training (i.e. too short a training period) or to the fact that it is the combination of the two training tasks that leads to improvement in arithmetic.

Indeed, multiplying the number of arithmetic-related skills to be trained increases the power of training and therefore the possibility of training effects. We also showed that WM training is effective in impacting trained skills but that it does not transfer to mathematical abilities.

Finally, the longitudinal study evaluating the predictive power of these specific and general cognitive factors on later arithmetical competence indicated that numerical processing skills significantly predict addition skills one year later whereas WM does not. Moreover, symbolic magnitude processing is a significant predictor of arithmetic one and even three years later. In contrast, the processing of non-symbolic magnitude is not related to future addition performance. More precisely, symbolic comparison appears to be a strong predictor of addition performance one and three years later. In contrast, the relation of symbolic number line positioning with arithmetic is, once again, unclear: it predicts addition skills one but not three years later.

To conclude, both specific numerical factors and WM are associated to numerical and arithmetical development in young children. However, numerical magnitude processing skills alone are causally associated to arithmetic and predict later performance in arithmetic. Among numerical factors, it is the ability to process symbolic, and not non-symbolic, magnitude that underlies numerical and arithmetical development and that predicts later performance in addition. Finally, symbolic comparison skills are the most important predictor of arithmetic and, although symbolic number line positioning plays an important role in arithmetical development, the exact nature of this relation is still to be discovered.