

Enforcing Domestic Quality Dominance through Quotas

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Abstract

We study duopoly competition between a domestic and a foreign firm who first choose their quality and then compete in prices in the domestic market. As is well known, the free-trade equilibrium exhibits quality differentiation and indeterminacy of the quality leader. We show that an import quota can enforce, as the unique subgame-perfect equilibrium outcome, the quality ranking that favors the domestic producer and thereby can increase domestic welfare.

1. Introduction

The impact of trade policies on products' quality has been a recurrent research topic since the early contribution of Falvey (1979). Recent theoretical contributions stress the strategic implications of trade barriers on the quality selection operated by oligopolistic firms. Two key papers in this area are Das and Donnenfeld (1989) and Herguera et al. (2000). The former studies quantity and quality restrictions in a game where quantities and qualities are chosen simultaneously. Arguing that quality is best viewed as a long-run variable, the latter article studies the impact of quantitative restrictions on quality choice when quality and quantities are chosen in sequence.

These two papers share several features. First, the presence of a quota significantly affects the equilibrium quality selection of the firms. Secondly, as acknowledged by the authors, the impact of the quota depends to a large extent on the position of the foreign product in the quality ladder. Quoting both papers' abstracts, we are told that "the change in quality depends on whether the foreign firm is of high or low quality" (Herguera et al., 2000, p. 1259) or that "the effects of trade policy hinge on the location of the quality produced by the firms in the quality spectrum" (Das and Donnenfeld, 1989, p. 299). Thirdly, firms are assumed to compete in a Cournot mode.¹ A more normative approach may be used to study the strategic trade policies that governments could try to implement in order to improve the quality of exports (Zhou et al., 2000). In this paper, the optimal policy depends also on the position of the national product in the quality ladder.

The fact that most results in this literature depend on the quality hierarchy is somewhat problematic. Indeed, standard duopoly models of vertical differentiation display *two* equilibria in pure strategies: one sees the domestic firm selling the high quality product and the other displays the reverse quality ranking. These two equilibria coexist even under relatively large cost asymmetries. Therefore, in the long run, it seems fair to argue that no hierarchy should be imposed *a priori*. The literature we referred to above does not address the issue of whether trade policies could actually *select prod-*

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ucts' ranking in the quality spectrum. Typically, scholars perform their analysis under the strong assumption that “no firm has an incentive to leapfrog its rival” (Herguera et al., 2000, p. 1268). Clearly enough, however, the two hierarchies are not equivalent from a domestic welfare viewpoint. Indeed, even if costs are identical (so that equilibrium prices, sales, and quality levels are the same in both equilibria), the domestic welfare is larger if the domestic producer sells the high quality good, simply because the high quality firm is also the high profit firm (Lehmann-Grube, 1997). As recently argued, however, in trade models where quality is endogenous, “the resulting asymmetry in profits creates powerful incentives for lagging industries as well as their national governments to reverse [the] situation to their advantage, i.e. to induce leapfrogging” (Herguera and Lutz, 1998, p. 77).

In this context, a trade policy whose main effect would be to select the domestic firm as the unique possible quality leader in equilibrium seems particularly desirable. In the long run or in the context of an emerging industry, such a policy could be viewed as *enforcing a specific quality ranking* by removing from the set of possible equilibrium outcomes the configuration where the foreign firm acts as a quality leader, thereby increasing domestic welfare. In this article we theoretically establish that a quota has precisely this “equilibrium selection” property under Bertrand competition. Such an effect is likely to take place in the context of an emerging industry where, by protecting the domestic industry with a quota, the government allows it to develop products in the high quality range while inducing entry through low quality foreign products.

As we further demonstrate, this policy also tends to increase average quality. This theoretical finding is well in line with the empirical evidence reported for several industries (Aw and Roberts, 1986; Feenstra, 1988; Boorstein and Feenstra, 1991). Notice, however, that these studies cover cases where the focus is put on the increased quality of *imports*. In the present article, the key effect of the quota is to increase the *domestic* quality, and, as a possible byproduct, that of the foreign one.

We establish our result using a two-stage game that starts after the (domestic) government has decided on a quota level. Firms choose quality in the first stage and compete in price in the second stage. Under free trade, firms never find it profitable to choose identical qualities because they end up selling at marginal costs. Accordingly, if one firm has selected a high quality, the other one has no other choice than to adopt a low quality, irrespective of whether this firm is the domestic or the foreign one. Hence, two subgame-perfect equilibria exist under relatively general conditions. In order to obtain a unique equilibrium under free trade, one essentially needs highly asymmetric quality costs. As shown hereafter, a quota dramatically alters the previous argument because, being protected by the quota at the price competition stage, the domestic firm may credibly commit to mimic the quality of the foreign firm, even if the latter sells the best available quality. More generally, it allows the domestic firm to be much more aggressive at the quality stage, which in turn leaves no other choice to the foreigner than to accommodate with a low quality.

2. The Model

A domestic firm, indexed by d , competes with a foreign one, indexed by f , on the domestic market. The good with label $i = f, d$ is identified by a quality index $s_i \in [0, 1]$. Consumers exhibit unit demand for the good and are characterized by a “taste for quality” x , which is uniformly distributed in the unit density interval $[0, 1]$. The indirect utility function is $u(i, x) = xs_i - p_i$ for $i = d, f$. Not consuming yields a utility normalized to 0.²

We consider a two-stage game $\Gamma(q)$ which starts after the domestic government has chosen the level of the quota, which we denote by q . In the first stage, firms choose quality levels $s_i \in [0, 1]$ at zero cost.³ In the second stage, they compete in prices by setting p_d and p_f . Production takes place at a marginal cost c , which we assume to be equal to 0 for simplicity. At the price stage, we assume that firms must produce to satisfy demand.⁴ We denote price subgames by $\Gamma(s_d, s_f, q)$.

In order to complete the specification of the model, it then remains to explain how the quota is implemented. Assuming that firms produce to satisfy demand essentially implies that they do not ration consumers. This is at odds with the mere idea of a quota, which is supposed to place an upper bound on the quantity the foreign firm may actually dump on the domestic market. In this respect, however, empirical studies report that for the case of voluntary export restraint on automobiles, Japanese firms were in fact allowed to sell beyond the quota but had to incur a penalty apparently set at around 10% of the price (Berry et al., 2000). As documented in the Market Access Database (see <http://mkacddb.eu.int>) maintained by the European Commission, import restraints may also be combined with differentiated tariffs for in-quota and out-quota imports. Selling beyond the quota is thus allowed but an extra cost (a higher tariff) has to be paid for units beyond the quota.

The key point here is to realize that for the quota to be effective, we need either that the foreign firm cannot sell beyond the quota or that it does not find it profitable to do so, in which case it prefers to reduce demand up to the quota level by raising its price. In order to simplify the exposition, we will assume that the foreign producer can legally sell beyond the quota but has to pay a penalty $\theta \geq 1$ per out-quota units.⁵

3. Analysis of the Price Competition Stage

Let us start the analysis with the specification of demands in the general model of vertical differentiation.⁶ We denote by s_h (resp. s_l) the high (resp. low) quality product. It is then straightforward, using the utility function, to define the following demand functions:

$$D_l(p_l, p_h) = \begin{cases} 1 - \frac{p_l}{s_l} & \text{if } p_l \leq p_h - s_h + s_l \\ \frac{p_h s_l - p_l s_h}{s_l(s_h - s_l)} & \text{if } p_h - s_h + s_l \leq p_l \leq p_h \frac{s_l}{s_h} \\ 0 & \text{if } p_l \geq p_h \frac{s_l}{s_h} \end{cases} \quad (1)$$

$$D_h(p_l, p_h) = \begin{cases} 1 - \frac{p_h}{s_h} & \text{if } p_h \leq \frac{s_h}{s_l} p_l \\ 1 - \frac{p_h - p_l}{s_h - s_l} & \text{if } \frac{s_h}{s_l} p_l \leq p_h \leq p_l + s_h - s_l \\ 0 & \text{if } p_h \geq p_l + s_h - s_l. \end{cases} \quad (2)$$

Notice that in the limiting case where products are homogeneous, i.e. $s_i = s_j = s$, demand functions are discontinuous and defined as:

$$D_i(p_i, p_j) = \begin{cases} 1 - \frac{p_i}{s} & \text{if } p_i < p_j \\ \frac{1}{2} \left(1 - \frac{p_i}{s} \right) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j. \end{cases} \quad (3)$$

In the price subgames, firms' payoffs are defined as $\pi_i = p_i D_i(p_h, p_l)$ with $i = h, l$. We now partition the set of subgames $\Gamma(q, s_d, s_f)$ into three classes according to whether $s_d > s_f$, $s_d < s_f$, or $s_d = s_f$ holds. We shall identify the first class with the superscript $d > f$, the second with $d < f$, and the third with $d = f$.

Consider first subgames $\Gamma^{d>f}(q, s_d, s_f)$. Demands addressed to the domestic and foreign firms can be derived using (1) and (2) by relabeling the indices ($h = d$ and $l = f$). Best replies are easy to construct using first-order conditions on profits.

Assume first that $q = 1$, i.e. the free-trade analysis applies. Notice that the first and third branches of (1) and (2) cannot be relevant in equilibrium since one of the two firms enjoys zero demand while it is always possible for her to secure a positive demand, and thus positive profits, along the second branch. Accordingly, under free trade, firms' best replies can be safely derived using only the second branches of (1) and (2). Straightforward computations yield the following best-reply functions:

$$\phi_d^{d>f}(p_f) = \frac{s_d - s_f + p_f}{2}, \quad (4)$$

$$\phi_f^{d>f}(p_d) = \frac{p_d s_f}{2s_d}. \quad (5)$$

When $q < 1$ there exist price constellations such that the demand addressed to the foreign firm exceeds the quota. In the game $\Gamma^{d>f}$, we have

$$D_f^{d>f}(p_d, p_f) = \frac{p_d s_f - p_f s_d}{s_f(s_d - s_f)}.$$

Thus, prices lower than the solution $\hat{p}_f^{d>f}$ of $D_f^{d>f} = q$ generate a demand which is binding with the quota. Formally,

$$\hat{p}_f^{d>f} = \frac{s_f}{s_d} (p_d - q(s_d - s_f)). \quad (6)$$

Because of the penalty, the foreign firm never finds it profitable to meet demand beyond the level of the quota. Accordingly, the best reply of the foreign firm is given by (5) only if the corresponding demand does not exceed the quota and by the limit price $\hat{p}_f^{d>f}$ otherwise. Formally, we obtain:

$$\psi_f^{d>f} = \begin{cases} \phi_f^{d>f}(p_d) & \text{if } p_d \leq 2q(s_d - s_f) \\ \hat{p}_f^{d>f}(p_d) & \text{if } p_d \geq 2q(s_d - s_f). \end{cases} \quad (7)$$

The best reply of the foreign firm is thus continuous and kinked. The critical level of p_d where the kink occurs is increasing in the level of the quota.

As for the domestic optimal pricing policy, notice that since the foreign producer never rations consumers, the domestic payoff is totally independent of the quota. Accordingly, domestic best reply in $\Gamma^{d>f}$ is defined by equation (4) whatever the level of the quota.

Combining (4) and (7), we can identify two equilibrium candidates, among which we select the feasible one depending on the level of the quota. Straightforward computations lead to the characterization of Nash equilibrium in $\Gamma^{d>f}$.

LEMMA 1. *For $s_d > s_f$, the equilibrium in a subgame $\Gamma(s_d, s_f, q)$ is defined as follows:*

$$\begin{aligned} \text{if } q \geq \frac{s_d}{4s_d - s_f}, & \quad \begin{cases} p_d^* = \frac{2s_d(s_d - s_f)}{4s_d - s_f} & \pi_d^* = \frac{4s_d^2(s_d - s_f)}{(4s_d - s_f)^2} \\ p_f^* = \frac{s_f(s_d - s_f)}{4s_d - s_f} & \pi_f^* = \frac{s_f s_d(s_d - s_f)}{(4s_d - s_f)^2} \end{cases} \\ \text{if } q \leq \frac{s_d}{4s_d - s_f}, & \quad \begin{cases} \hat{p}_d^* = \frac{(s_d - s_f)(s_d - s_f q)}{2s_d - s_f} & \hat{\pi}_d^* = \frac{(s_d - s_f)(s_d - s_f q)^2}{(2s_d - s_f)^2} \\ \hat{p}_f^* = \frac{s_f(1 - 2q)(s_d - s_f)}{2s_d - s_f} & \hat{\pi}_f^* = q \frac{s_f(1 - 2q)(s_d - s_f)}{2s_d - s_f} \end{cases} \end{aligned}$$

We may then develop a similar analysis for the class of subgames where the foreign producer sells the high quality product, i.e. subgames $\Gamma^{d<f}(\cdot)$. Computing a Nash equilibrium in this class of subgames involves no specific difficulty as compared to the previous case. Relabeling equations (1) and (2) with $h = f$ and $l = d$, we derive firms' best replies. Because of the quota, the foreign firm will prefer to sell the quota at the limit price $\hat{p}_f^{d<f}$ against high p_d , where $\hat{p}_f^{d<f}$ is the solution of the equation

$$\frac{s_f - s_d - p_f + p_d}{s_f - s_d} = q.$$

Straightforward computations enable us to characterize the equilibrium in subgame $\Gamma^{d<f}(\cdot)$ in the following lemma.

LEMMA 2. *For $s_d < s_f$ the equilibrium in a subgame $\Gamma(s_d, s_f, q)$ is defined as follows:*

$$\begin{aligned} \text{if } q \geq \frac{2s_f}{4s_f - s_d}, & \quad \begin{cases} p_f^* = \frac{4s_f(s_f - s_d)}{(4s_f - s_d)^2} & \pi_f^* = \frac{4s_f^2(s_f - s_d)}{(4s_f - s_d)^2} \\ p_d^* = \frac{s_d(s_f - s_d)}{(4s_f - s_d)} & \pi_d^* = \frac{s_d s_f(s_f - s_d)}{(4s_f - s_d)^2} \end{cases} \\ \text{if } q \leq \frac{2s_f}{4s_f - s_d}, & \quad \begin{cases} \hat{p}_f^* = \frac{2s_f(s_f - s_d)(1 - q)}{2s_f - s_d} & \hat{\pi}_f^* = q \frac{2s_f(s_f - s_d)(1 - q)}{2s_f - s_d} \\ \hat{p}_d^* = \frac{s_d(1 - q)(s_f - s_d)}{2s_f - s_d} & \hat{\pi}_d^* = \frac{s_d s_f(1 - q)^2(s_f - s_d)}{(2s_f - s_d)^2} \end{cases} \end{aligned}$$

We now turn to the analysis of the third class of subgames, in which firms sell homogeneous products. Characterizing equilibria in subgames $\Gamma^{d=f}$ is less straightforward because, even though products are homogeneous, the standard Bertrand analysis does not apply. The nature of price competition may be intuitively captured as follows. Recall first that under our assumptions, the foreign producer must meet demand, even if this implies losses for units sold beyond the quota. Now assume that prices are equal and such that the foreign demand is exactly equal to the quota, i.e. $p_d = p_s = \tilde{p}$ where

$\frac{1}{2}(1 - \tilde{p}/s) = q$; in such a situation the foreign producer has no incentive to undercut its rival. Indeed this would imply a discontinuous upward jump in demand and all of these extra sales generate losses due to the application of the penalty θ . Thus, the foreign producer will cease to undercut the domestic price well above the level of the marginal cost. This, by itself, relaxes price competition. However, the domestic producer still has an incentive to undercut $\tilde{p}$ in order to grab the whole market. On the other hand, when facing $p'_d = \tilde{p} - \varepsilon$ the foreign producer will match p'_d because it only induces losses on a small number of units sold beyond the quota, while units sold within the quota remain profitable.

The logic of price competition is thus the following: for high prices, the standard cut-throat war takes place, down to the price level at which the foreign producer sells exactly the quota when matching the domestic price. Once we reach this level, the foreign producer matches the domestic price but the domestic firm keeps undercutting the foreign price. This “matching–undercutting” sequence ends when prices are so low that the foreign firm makes zero profits at equal prices (what is gained on in-quota units exactly offsets what is lost on out-quota units). From this limit on, the domestic firm undercuts and the foreign one is better off keeping its price above the domestic price because matching would yield negative profits.

Formally, we characterize the equilibria of subgames $\Gamma^{f=d}$ in Lemma 3. Note that we focus here on the particular subgames where $s_d = s_f = 1$. As shown later, this is indeed the only case which will be relevant when considering quality choices.

LEMMA 3. *Assume $s_d = s_f = 1$ and $q > \frac{1}{2}$, then there exists a continuum of asymmetric equilibria in $\Gamma(s_d, s_f, q)$. The set of equilibria is any pair (p_d, p_f) such that $p_d \in [0, \max\{0, 1 - \sqrt{2q}\}]$ and $p_f > p_d$. If $q \leq \frac{1}{2}$, $(p_d, p_f) = (0, 0)$ is the unique equilibrium.*

A formal proof of Lemma 3 is provided in the Appendix. Note that, for quotas above $\frac{1}{2}$, the equilibrium is unique and sees both firms selling at marginal cost. In order to study quality competition, we will select the Pareto correspondence⁷ from the Nash correspondence.

LEMMA 4. *A Pareto selection in the set of equilibria of $\Gamma(1, 1, q)$, with $q > \frac{1}{2}$, is $p_d = 1 - \sqrt{2q}$ and any $p_f > p_d$. Under this Pareto selection, $\pi_d = \sqrt{2q}(1 - \sqrt{2q})$ and $\pi_f = 0$.*

The key intuition here is to note that, in the presence of the quota, the domestic firm can secure positive profits even when products are homogeneous, whereas the foreign firm can only secure zero profits. The presence of multiple equilibria may be problematic when going backward in the game tree. This is why we have to rely on equilibrium selection. Notice that in the present case, the Pareto selection is quite intuitive since the foreign firm is totally indifferent to which equilibrium is selected while the domestic one unambiguously prefers the high price equilibrium.

4. Enforcing Domestic Quality Dominance

Under free trade, no firm would ever choose a quality which is identical to that of her opponent since this would yield the zero-profit Bertrand equilibrium in the price subgame. Direct computations (using Lemma 1 or 2) show that the payoff of the high quality firm is strictly increasing in own quality. Accordingly, the high quality firm selects the highest available quality, $s_h^* = 1$, whereas the best reply of the low quality

firm is defined as a fixed proportion of the high quality one: $s_i^* = \frac{4}{7}s_h^*$ as we recall in Lemma 5.⁸

LEMMA 5. *In the free-trade game $\Gamma(1)$ with no cost for quality, there are two symmetric subgame-perfect equilibria, $(s_d^* = 1, s_f^* = \frac{4}{7})$ and $(s_d' = \frac{4}{7}, s_f' = 1)$.*

On the equilibrium path, the prices are $p_h = \frac{1}{4}$ and $p_l = \frac{1}{14}$, leading to market shares $D_h = \frac{7}{12}$ and $D_l = \frac{7}{24}$ and a consumer surplus (CS) of

$$CS = \int_{1-\frac{7}{12}}^1 (x - \frac{1}{4})dx + \int_{1-\frac{7}{12}-\frac{7}{24}}^{1-\frac{7}{12}} (\frac{4}{7}x - \frac{1}{14})dx = \frac{7}{24}.$$

Profits are $\pi_h^* = \frac{7}{48}$ and $\pi_l^* = \frac{1}{48}$ so that the domestic surplus is either $W_h \equiv \frac{7}{24} + \frac{7}{48} = \frac{7}{16}$ when the domestic firm is the high quality firm or $W_l \equiv \frac{7}{24} + \frac{1}{48} = \frac{5}{16}$, a reduction of 28.5% when the domestic firm sells the low quality. This difference in profits creates an incentive for the domestic government to intervene in order to guarantee the better result. By introducing a nontrivial quota into the analysis, the following proposition states our main result.

PROPOSITION 1. *When quality is not costly, any game $\Gamma(q)$ where $q < \tilde{q} \approx 0.48$ has a **unique** subgame-perfect equilibrium in which the domestic firm selects the best available quality whereas the foreign producer selects a lower quality.*

PROOF. First we show that, irrespective of the quota, at least one of the two firms must select $s_i = 1$ in a subgame-perfect equilibrium. Then we focus on the best reply of the low quality firm against $s = 1$, depending on whether this firm is a domestic or a foreign one. Thirdly, we identify the conditions under which only $s_d = 1$ can be part of a subgame-perfect equilibrium.

STEP 1. At least one firm selects $s_i = 1$.

We first need to show, using Lemmas 1 and 2, that the payoff of the high quality firm, be it the domestic or the foreign one, is increasing in own quality, irrespective of the quota value. Straightforward computations indicate that this is indeed the case. Accordingly, provided *firms choose different qualities* in a subgame-perfect equilibrium (so that Lemmas 1 and 2 are indeed the relevant pricing subgames), the high quality firm must choose $s_i = 1$.

Then we have to show that there cannot be a subgame-perfect equilibrium where firms choose identical qualities s , with $s < 1$. This follows immediately from Lemma 3, since the foreign firm nets zero profits in any possible equilibrium of $\Gamma(s, s, q)$, whereas it can always secure positive profits by deviating, either to a higher or a lower quality and thereby inducing a price subgame where products are differentiated.

STEP 2. Derivation of the best reply of a low quality firm against $s = 1$.

Assume the domestic firm has chosen $s_d = 1$. We thus focus on $s_f < 1$. The foreign best reply depends on whether the quota is binding or not. Using Lemma 1, we may define the domain of quality where the quota is nonbinding in the price equilibrium, as a function of the quota level. Computations yield the condition $s_f < 4 - (1/q)$ (non-binding quota). The best reply in quality, conditional on this region, is therefore either in the interior of the region or at the corner, depending on the level of the quota. We know from Lemma 5 that when the quota is not binding, the interior best reply is

$s_f^a \equiv \frac{4}{7}$ with $\pi_f^a = 0.0208$. Solving $s_f^a < 4 - (1/q)$ yields the critical level $q \geq 0.2917$. For more restrictive quotas ($q < 0.2917$), the interior best reply is not feasible and the corner solution applies: $s_f^a \equiv \max\{0, 4 - (1/q)\}$ and $\pi_f = (4q - 1)(1 - 3q)$.

A similar analysis can be conducted in the quota-binding domain, i.e. for $s_f \geq 4 - (1/q)$. The best reply is

$$s_f^b \equiv 2 - \sqrt{2} \quad \text{and} \quad \pi_f^b = \frac{q(1 - 2q)(2 - \sqrt{2})(\sqrt{2} - 1)\sqrt{2}}{2} \quad \text{if } q \leq 0.2929$$

while $s_f^b \equiv \min\{1, 4 - (1/q)\}$ and $\pi_f = (4q - 1)(1 - 3q)$ if $q > 0.2929$.

It then remains to compare payoffs in these two cases and the only comparison we need to perform is between π_f^a and π_f^b over $[0.2917; 0.2929]$. It is a matter of computation to show that the best-reply candidate is s_f^b whenever $q \leq \hat{q}$ and s_f^a otherwise, with $\hat{q} \equiv (6 + \sqrt{12} - \sqrt{6})/24 = 0.2923$.

$$\sigma_f(1) = \begin{cases} \frac{4}{7} \approx 0.571 & \text{if } q \geq \hat{q} \\ 2 - \sqrt{2} \approx 0.586 & \text{if } q \leq \hat{q}. \end{cases} \quad (8)$$

Assume now that the foreign firm has chosen $s_f = 1$. In the region where $s_d < 1$, we may use Lemma 2 to show that the best reply is $s_d^a \equiv \frac{4}{7}$ with associated payoff $\pi_d^a \equiv \frac{1}{48}$ if the foreign firm is not constrained by the quota in the ensuing price subgame, and $s_d^b \equiv \frac{2}{3}$ with associated payoff $\pi_d^b = \frac{1}{8}(1 - q)^2$ otherwise. Note then that, contrary to the previous case, we also have to consider $s_d = 1$ with associated payoff $\pi_d = \sqrt{2q}(1 - \sqrt{2q})$ as a valid best-reply candidate since, according to Lemma 3, the domestic firm may secure positive profits in the corresponding price subgames. The solution of $\pi_d^a = \pi_d$ is

$$\tilde{q} \equiv \frac{23}{96} + \frac{\sqrt{33}}{24} \approx 0.48$$

while the solution of $\pi_d^a = \pi_d^b$ is

$$1 - \frac{1}{\sqrt{6}} \approx 0.59.$$

Comparing profits associated to these three best-reply profiles, computations indicate that $s_d = 1$ dominates for low quotas, while s_d^b is optimal for intermediate ones, and s_d^a for large ones.⁹ Formally:

$$\sigma_d(1) = \begin{cases} 1 & \text{if } q < 0.48 \\ \frac{2}{3} & \text{if } q \in [0.48, 0.59] \\ \frac{4}{7} & \text{if } q > 0.59. \end{cases} \quad (9)$$

STEP 3. When $q < \tilde{q} \approx 0.48$, only $s_d = 1$ can be part of a subgame-perfect equilibrium.

We know from Step 1 that at least one firm selects $s_i = 1$ in a subgame-perfect equilibrium. Suppose the foreign firm has selected $s_f = 1$, then the best reply of the domestic firm is to match s_f since $q < \tilde{q}$. But identical qualities cannot be part of a subgame-perfect equilibrium because the foreign firm always gains by deviating downward. Suppose then that $s_d = 1$. Depending on whether q is above or below $\hat{q} \approx 0.2923$, the best reply of the foreign firm is s_f^a or s_f^b , i.e. the foreign producer accommodates with a lower quality. But, against either of these two qualities, the best reply of the domestic firm is indeed to stick to the best available quality. Accordingly, we have shown that when $q < \tilde{q}$, the unique subgame-perfect equilibrium sees the domestic firm selling the best available quality. $\square$

5. Conclusion

The presence of the quota does not directly alter the willingness of the foreign firm to be the quality leader. Indeed, irrespective of the quota level, and whatever the quality level of the domestic firm, it remains true that in case it is located at the high end of the quality spectrum, the foreigner tends to choose the best available quality. However, *the presence of the quota allows the domestic firm to be more aggressive at the quality stage*. Indeed, the threat of matching the foreign quality is credible in the presence of a quota. Facing this threat, the foreign firm is better off accommodating by downgrading its quality. It is therefore sufficient for the government to impose a quota at, or below, the level that makes the “quality matching” threat credible as a domestic best reply to enforce the desired quality ranking.

It is noteworthy that the quota can be set well above the foreign sales level in the equilibrium (which are also its free-trade sales if acting as a quality follower). This is well in line with previous findings (Krishna, 1989) according to which a quota may be very effective even if set at an apparently nonrestrictive level. The difference with her analysis is that, here, the effects of the quota spill over to the quality stage. Our analysis is a simple example of how the mere existence of a *policy instrument*, the quota, *can change the outcomes of the game without ever being used*. The sole market conditions (prices and quantities) would not be enough for the foreign firm to sustain a case against the quota at the World Trade Organization; a more involved and dynamic analysis would be called for.

The quota may thus be used as a selection device. This qualitative implication of the quota is the key message of the present note. Since most of the literature has focused on quantity competition (Das and Donnenfeld, 1989; Herguera et al., 2000), it is worth stressing here that our result is deeply rooted in the presence of price competition: the “matching quality” threat is quite effective because the foreign producer cannot afford *Bertrand* competition with homogeneous products, whereas the domestic one can. Under Cournot competition, no such threat would exist because selling identical qualities secures positive profits to the two firms. This does not imply that quotas are not able to enforce a specific quality ranking under Cournot competition. However, if they do, this will result from the way quotas alter the *magnitude* of relative payoffs, and not from a basic change in the behavior of the domestic firm.

Our central result has been established under quite specific assumptions. The most restrictive assumption lies in the fact that the foreign producer does not have the possibility to retaliate by imposing a quota in the foreign market. Accordingly, the normative implications of our result are unclear since they should take the retaliation possibility into account in order to correctly assess the desirability of the policy. A second key assumption is that quality is not costly. It should be clear by now, however, that the presence of a positive quality cost, sunk at the quality stage, will not alter the nature of our findings. Qualitatively, it leaves untouched the possibility for the domestic firm to credibly commit to matching the foreign quality. If anything, the presence of a positive cost for quality will affect the magnitude of the domain where q acts as a selecting device. More generally, any cost structure such that the high quality firm is the high profit firm will yield the same conclusion. Any reasonably convex quality cost functions have this property (Lehmann-Grube, 1997).

Incidentally, it should be observed that, regarding the quality range that is effectively offered in a subgame-perfect equilibrium, the conclusions are quite clear: either equilibrium qualities are unaffected by the presence of the quota, as compared to free trade

($q \geq \hat{q}$) or the low quality is slightly above the free-trade one ($q \leq \hat{q}$) so that, on average, the quality of the products available in the market is higher.¹⁰

Appendix

Proof of Lemmas 3 and 4

Let us first define the profit function of the foreign firm in subgame $\Gamma(s, s, q)$, assuming $\theta = 1$.

$$\Pi_f(p_f, p_d) = \begin{cases} p_f \left(1 - \frac{p_f}{s}\right) & \text{if } (1-q)s \leq p_f < p_d \\ p_f q + (p_f - 1) \left(1 - q - \frac{p_f}{s}\right) & \text{if } p_f < p_d \text{ and } p_f < (1-q)s \\ \frac{p_f}{2} \left(1 - \frac{p_f}{s}\right) & \text{if } p_f = p_d \geq (1-2q)s \\ p_f q + (p_f - 1) \left(\frac{1 - (p_f/s)}{2} - q\right) & \text{if } p_f = p_d < (1-2q)s \\ 0 & \text{if } p_f > p_d. \end{cases} \quad (\text{A1})$$

In a first step we analyze firms' optimal behavior as a function of the other's price and then we identify the set of equilibria in a second step.

Step (i) Best-reply characterization Assume $q < 1$ and focus on the price best reply of the foreign firm. Using the specification of a firm's payoffs retained above, we may observe that $p_f q + (p_f - 1)(1 - q - p_f/s) = q - (1 - p_f)(1 - p_f/s)$, hence undercutting a price below

$$\frac{1 + s - \sqrt{(1-s)^2 + 4qs}}{2}$$

yields negative profits. Likewise

$$p_f q + (p_f - 1) \left(\frac{1 - p_f/s}{2} - q\right) = q - \frac{(1 - p_f)(1 - p_f/s)}{2}$$

so that matching the other's price yields negative profits whenever

$$p_d < \phi(q) \equiv \max \left\{ 0, \frac{1 + s - \sqrt{(1-s)^2 + 8qs}}{2} \right\}.$$

Hence, when $p_d < \phi(q)$, both undercutting and matching yield negative profits so that the best reply is any higher price which we denote by p_d^+ .

When $\phi(q) < p_d < (1 - 2q)s$ (if this interval is non-void), both matching and undercutting p_d leads firm f to sell beyond the quota, thus matching is better, and since it yields a positive payoff it is also better than "uppercutting." Hence over this range the best reply is matching.

The price that leaves firm f indifferent between (matching and not selling above capacity) and (undercutting while being constrained) is the smallest root of equation

$$\frac{(1-p_f/s)p_f}{2} = q - (1-p_f)(1-p_f/s)$$

which is

$$\rho(q) \equiv \frac{s+2-\sqrt{(2-s)^2+8qs}}{2}.$$

We check that $\rho(q) > 1-2q$ so that firm f is indeed not constrained when playing this price. Noticing finally that $\rho(q) > \phi(q)$, we conclude that

$$BR_f(p_d) = \begin{cases} p_d^+ & \text{if } p_d < \phi(q) \\ p_d & \text{if } \phi(q) \leq p_d \leq \rho(q) \\ p_d^- & \text{if } \rho(q) < p_d \end{cases}$$

Turning now to the domestic firm, we notice that the shape of the domestic profit is not affected by the presence of the quota. Accordingly, the best reply of firm d to any $e \in]0; s/2[$ is to undercut this price, i.e. the domestic firm behaves in a traditional Bertrand fashion.

Step (ii) Equilibrium characterization Whenever $q \geq \frac{1}{2}$, $\phi(q) < 0$, so that the Bertrand equilibrium $(0, 0)$ prevails: firms undercut each other throughout the relevant range of prices. Consider now $q < \frac{1}{2}$. Using the above characterization of best replies, one immediately notices that there cannot be symmetric equilibria. Indeed, the domestic firm always undercuts, whereas for $p_d \in]0; \phi(q)[$, the foreign firm secures zero sales by “uppercutting” the foreign one.

We can now see that no $p_d > \phi(q)$ can be part of an equilibrium since it would be matched by the foreign one, defining a symmetric price candidate. Thus, the set of asymmetric equilibria is $[0; \phi(q)]$ and since the foreign firm gets zero profits, the Pareto-dominant equilibrium is the upper bound as it pleases the domestic firm who remains as a monopolist. If we restrict attention to the case where $s = 1$ then $\phi(q) = 1 - \sqrt{2q}$. $\square$

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Notes

1. We may suspect that the above results do not easily generalize to the Bertrand case since it has been shown that quotas tend to have very specific implications under price competition, as compared to quantity competition (Krishna, 1989).
2. This framework, initiated by Mussa and Rosen (1978) and popularized by Tirole (1988) is now standard in the literature.
3. As discussed in the conclusion, our analysis qualitatively holds for convex costs such as s_i^2/F , with $F > 0$.
4. We follow in this respect the standard definition of Bertrand competition: firms make a commitment with customers to supply the forthcoming demand (Vives, 2000).
5. Let us stress that this assumption, if instrumental in keeping the analysis of the price competition stage reasonably short and simple, is not necessary at all. Recall also that since we are assuming zero marginal cost, p essentially measures the unit margin on sales. The level of the penalty should thus be compared with unit margins, not with absolute prices.
6. Since the construction of demands in the present model is fairly standard, we do not derive them in detail. The reader is referred to Aoki and Prusa (1997) or Lehmann-Grube (1997) for a detailed exposition.
7. The Pareto correspondence associates to a set $A \subset \mathbb{R}^n$, the set $\mathcal{P}(A)$ of points in A that are not Pareto-dominated by other points of A .
8. We refer again the reader to Aoki and Prusa (1997) or Lehmann-Grube (1997) for a detailed analysis.
9. Note that the critical values defining the intervals are numerical approximations of exact (but rather long) analytical solutions. They are available upon request.
10. Again, this result is robust to the introduction of cost for quality because the quota, if effective, increases both firms' profits in the pricing game.