

SURFACE GRAVITY MODELLING AND SPACE GRAVIMETER DEVELOPMENT IN THE CONTEXT OF SOLAR SYSTEM SMALL BODIES

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Pour Maman

The exploration and use of outer space, including the Moon and other celestial bodies, shall be carried out for the benefit and in the interests of all countries, irrespective of their degree of economic or scientific development, and shall be the province of all mankind.

Outer space, including the Moon and other celestial bodies, shall be free for exploration and use by all States without discrimination of any kind, on a basis of equality and in accordance with international law, and there shall be free access to all areas of celestial bodies.

There shall be freedom of scientific investigation in outer space, including the Moon and other celestial bodies, and States shall facilitate and encourage international co-operation in such investigation.

Article I
of the United Nations
1967 Outer Space Treaty

ABSTRACT

Small bodies in the Solar System hold an exciting position in the family of celestial bodies, as they might play a key role in understanding the formation of our heliocentric system, and, ultimately, life. Unlike (terrestrial) planets and larger natural satellites (moons), small bodies have experienced far less transformation, comparing today's state with theirs state in the early Solar System. These objects are therefore sometimes referred to as *time capsules of the Solar System*. It is hence little surprising that various space exploration missions target these bodies. Here, it is not solely the sciences that drive all missions, but likewise a new discipline referred to as *planetary defense*, where capabilities to deflect asteroids from an apocalyptical collision course with the Earth are being developed.

Two planetary defense missions are the ESA Hera and the NASA DART mission. The former will follow the latter to the binary asteroid system Didymos. After the kinetic impact of the DART spacecraft into the secondary asteroid Dimorphos in 2022, altering its orbital period, the European mission will embark for detailed characterisation of the binary system. One payload of this mission is the GRAvimeter for small Solar System bodies (GRASS) to land on Dimorphos, for in-situ surface gravimetry. This will be the first-ever extraterrestrial surface gravimeter experiment on a small body. The presented PhD thesis contributed to this instrument development as is laid out in Part I of this document. After a literature review of various gravimeter working principles, the choice of a spring-based gravimeter type is justified. Following this, the instrument type is adapted to a space exploration mission, considering the harsh deep-space and small gravity environment, and the strong need to minimise the system. With the design in place, an analytical electromechanical gravimeter model has been developed, allowing to predict the general instrument behaviour in small gravity environments. This model did not only prove valuable in the early instrument electromechanical design and sizing, but remains a valuable tool for possible future gravimeter missions, directly allowing to change the design parameters and the targeted gravity.

The preparation of gravimeters to any celestial target, but moreso the interpretation of the finally returned data to Earth demand excellent (surface) gravity simulations. Especially on small, generally non-spherical, bodies, this is non-trivial and addressed in Part II of this work. Firstly, the different contributions to gravity and the different existing gravitation modelling methods are introduced. This is followed by a comparison of three of these methods and the subsequent computation of surface gravity for different cases, including Hera's target. Finally, zooming in from the global to the local scale, the novel Wedge-Pentahedra Method (WPM) is presented, allowing to take into account local topography surrounding a gravimeter measurement location and influencing the measurement.

RÉSUMÉ

Les petits corps du système solaire occupent une position extrêmement intéressante dans la famille de corps céleste, car ils jouent un rôle clé dans la compréhension de la formation de notre système héliocentrique, et, finalement, de la vie. Contrairement aux planètes (telluriques) et aux plus grands satellites naturels (lunes), les petits corps ont connu beaucoup moins de transformation, quand on compare l'état actuel à celui du début du système solaire. Ces corps sont désormais nommés « capsule temporelle » du système solaire. Par conséquent, il n'est pas surprenant que diverses missions spatiales prennent ces corps pour cible. De plus, la science n'est pas la seule motivation : une nouvelle discipline, nommée « défense planétaire », voit le jour, s'intéressant aussi à la capacité à dévier de sa trajectoire un astéroïde en cours de collision apocalyptique avec la Terre.

En particulier, les agences spatiales lancent deux missions de défense planétaire : la mission Hera de l'ESA, qui va suivre la mission DART NASA vers le système binaire d'astéroïdes Didymos. Après l'impact cinétique de la sonde spatiale DART sur l'astéroïde Dimorphos en 2022, qui a changé la période de son orbite, la mission européenne sera lancée pour une caractérisation détaillée du système binaire. Une des expériences de la mission est le gravimètre pour les petits corps du système solaire (GRASS), prévu d'atterrir sur Dimorphos pour mesurer l'accélération de la gravitation sur la surface. Ce sera la toute première expérience gravitationnelle de surface extraterrestre sur un petit corps. Cette thèse doctorale a contribué au développement de cet instrument, comme présenté dans la première Partie de ce document. Après l'étude de la littérature concernant les divers types de gravimètres, nous avons justifié et sélectionné le type de gravimètre utilisant un ressort. Par la suite, ce type d'instrument a été adapté pour une mission spatiale, en considérant l'environnement difficile, la gravité petite, et la nécessité absolue de miniaturiser l'instrument. Après avoir réalisé sa conception, un modèle analytique électromécanique du gravimètre est présenté, permettant de présager du comportement général de l'instrument dans un environnement de faible gravité. Ce modèle n'était pas seulement valable pour le design préliminaire électromécanique de l'instrument, mais reste, toute de même, un outil valable pour les missions futures éventuelles, en permettant de changer directement les paramètres de design et la gravité envisagée.

La préparation du gravimètre pour une quelconque cible céleste, mais plus important, le traitement et l'interprétation des données ultimes retransmises vers la Terre, demande des simulations fines de la gravité. Celles-ci sont particulièrement non-triviales dans le cas de corps petits et non-sphériques, ce qui fait l'objet de la Partie deux de ce travail. Premièrement, nous détaillons les différentes contributions à la gravité et les différentes techniques de simulations utilisées. Nous comparons ensuite trois de ces méthodes et effectuons les simulations de la gravité de surface pour différents cas, et en particulier celle de cible de Hera. Finalement,

en zoomant de l'échelle globale à l'échelle locale, la nouvelle méthode de coin-pentaèdres (WPM) est présentée. Cette méthode permet de prendre en considération le terrain local à proximité de l'endroit de la mesure gravitationnelle et de calculer, au bout du compte, son influence.

ZUSAMMENFASSUNG

Kleine Körper im Sonnensystem nehmen eine spannende Stellung in der Familie der Himmelskörper ein, da sie eine Schlüsselrolle im Verständnis der Entstehung unseres heliozentrischen Systems, und schlussendlich des Lebens spielen können. Anders als bei (terrestrischen) Planeten und größeren natürlichen Satelliten (Monden), haben kleine Körper deutlich weniger Transformation erfahren, wenn man den heutigen Zustand mit dem im frühen Sonnensystem vergleicht. Diese Körper werden daher auch gelegentlich als *Zeitkapseln des Sonnensystems* bezeichnet. Es ist daher wenig überraschend, dass verschiedene Sonnensystemexplorationsmissionen diese Körper anvisieren. Hier ist nicht nur die Wissenschaft die einzig treibende Kraft, sondern auch eine neue Disziplin, die sogenannte *Planetenverteidigung*, welche die Befähigung zur Ablenkung eines Asteroiden auf apokalyptischem Kollisionskurs mit der Erde herstellen soll.

Zwei Planetenverteidigungsmissionen sind die ESA Hera Mission, welche der NASA DART Mission zum binären Asteroidensystem Didymos folgen wird. Nach dem kinetischen Einschlag der DART Raumsonde in den sekundären Asteroiden Dimorphos im Jahr 2022, welches einer Veränderung der Umlaufperiode hervorgerufen hat, bricht die europäische Mission für eine detaillierte Charakterisierung des binären Systems auf. Eine Nutzlast dieser Mission ist das Gravimeter für kleine Körper im Sonnensystem (GRASS), welches auf Dimorphos landen wird, um Oberflächenmessungen durchzuführen. Dies wird die erste jemals durchgeführte Gravitationsmessung auf einer extraterrestrischen Oberfläche eines kleinen Körpers im Sonnensystem darstellen. Die vorliegende Dissertationsschrift trägt zur Entwicklung dieses Instruments bei, welches im Teil I dargestellt ist. Nach einer Literaturstudie der verschiedenen Gravimeterfunktionsprinzipien wird die Wahl einer federgestützten Variante begründet. Hiernach wird der Instrumententyp für eine Weltraumexplorationsmission angepasst, wobei insbesondere die rauen Umwelteinflüsse des Weltraums, die kleine Gravitation, sowie der herausragende Bedarf der Miniaturisierung des Systems bedacht werden. Mit diesem Design wurde ein analytisches elektromechanisches Gravimetermodell entwickelt, welches ermöglicht, das allgemeine Instrumentenverhalten unter kleiner Gravitation zu simulieren. Diese Methode hat sich nicht nur während der frühen Instrumentenentwicklung für das Design und die Auslegung als wertvoll erwiesen, sondern bleibt ein wertvolles Werkzeug für mögliche zukünftige Gravimetermissionen, da es eine direkte Anpassung der Designparameter und der anvisierten Gravitation erlaubt.

Die Planung eines Gravimeters für jegliches überirdisches Ziel, aber insbesondere die Interpretation der schließlich zur Erde zurückgesandten Daten, erfordern exzellente Gravitationssimulationen. Insbesondere bei kleinen, generell nicht kugelförmigen Körpern, ist dies nicht trivial und im Teil II dieser Arbeit adressiert. Zuerst werden die verschiedenen Komponenten der Gravitation eingeführt und

ein Überblick über die verschiedenen Methoden zur Berechnung ebendieser wird gegeben. Hiernach wird ein Vergleich zwischen drei dieser Methoden gemacht und die Oberflächengravitation von verschiedenen Körpern, inklusive des Ziels von Hera, wird präsentiert. Letztlich wird von der globalen auf die lokale Ebene hereingezoomt, und die neue Keil-Pentaeder Methode (WPM) wird vorgestellt, welche es erlaubt, den Einfluss der lokalen Topographie im direkten Umfeld einer Gravimetermessung auf ebendiese zu berücksichtigen.

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Matthias
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LIST OF ABBREVIATIONS

1D	one-dimensional
2D	two-dimensional
3D	three-dimensional
A17	Apollo 17
AFC	asteroid framing camera
AGEX	Asteroid Geophysical Explorer
AIDA	Asteroid Impact Deflection Assessment
AIM	Asteroid Impact Mission
AQG	Absolute Quantum Gravimeter
areo-	Mars-prefix [analogous to geo-]
ASI	Agenzia Spaziale Italiana
AU	Astronomical Unit
BDU	bulk density uniformity
BEE	Backend electronics
B-Grave	Belgium-Astro-Gravimeter
BS	Beam splitter
ca.	circa
CAD	computer-aided design
CCM	retro-reflectors
CE-3	Chang'E 3
cf.	confer [compare]
CDR	critical design review
CNES	Centre national d'études spatiales
CO-DATA	Committee on Data for Science and Technology
CoM	Centre of Mass
CONOPS	concept of operations
CPU	central processing unit
CS	coordinate system
CS'	rotated coordinate system
d	day
DART	Double Asteroid Redirection Test
DEM	digital elevation model
DIN	Deutsches Institut für Normung
DLR	Deutsches Zentrum für Luft- und Raumfahrt e. V.
DPVI	delta percentage volume increase
Dr.	Doctor
DRACO	Didymos Reconnaissance and Asteroid Camera for Op-nav
DRI	delta radius increase
EES	Earth-equivalent spring

e.g.	exempli gratia [for example]
EMC	electromagnetic compatibility
EOM	equations of motion
eq.	equivalent
ESA	European Space Agency
et al.	et alia [and others]
eva.	evaluation
EVA	extra-vehicular activity
FEE	front-end electronics
FEM	finite element method
FM	Flight Model
g	gravity
GOCE	Gravity Field and Steady-State Ocean Circulation Explorer
GRACE	Gravity Recovery and Climate Experiment
GRAIL	Gravity Recovery and Interior Laboratory
GRASB	GRAvity of Small Bodies
GRASS	GRAvimeter for small Solar System bodies
IAU	International Astronomical Union
Ibid.	ibidem [in the same place]
i.e.	id est [that is]
IMU	internal measurement unit
interp.	interpolated
IPA	isopropyl alcohol
ir.	ingenieur [engineer]
ISRU	In-situ Resource Utilisation
ISS	International Space Station
JAXA	Japan Aerospace Exploration Agency
JMARS	Java Mission-planning and Analysis for Remote Sensing
JPL	Jet Propulsion Laboratory
JuRa	Juventas Radar
JUV-GRASS	GRASS for Juventas
L	Lenses
LICIA	Light Italian CubeSat for Imaging of Asteroids
LM	Lunar module
LOLA	Lunar Orbiter Laser Altimeter
LRO	Lunar Reconnaissance Orbiter
LRV	Lunar Roving Vehicle
LS	Landing Site
M	Mirrors
MASC	mascon spherical coordinates
mascon	mass concentration
MATLAB	MATrix LABoratory
max.	maximum
meas.	measured
MEMS	Micro-Electro-Mechanical System

MEX	Mars Express
min.	minimum
MMX	Martian moon eXplorer
MSP	mascon sphere packing
n/a	not applicable
NaN	not a number
NASA	National Aeronautics and Space Administration
N.B.	Nota Bene [note well]
nd	no date
NEA	near-Earth asteroid
NEAR	Near Earth Asteroid Rendezvous
nom.	nominal
OBC	on-board computer
opt.	optimised
orig.	original
OSIRIS	Optical, Spectroscopic and Infrared Remote Imaging System
PALT	Planetary ALTimeter
PDR	preliminary design review
PhD	Philosophiae Doctor [Doctor of Philosophy]
PM	polyhedron/polyhedral method
PNA	phosphoric-nitric-acetic
pp.	pages
ppd	pixel per degree
ppmeter	pixel per meter
Prof.	Professor
QSAM	Quasi-Steady Acceleration Measurement
res.	resolution
SB	small body
SBG	small body gravimeter
SEIS	Seismic Experiment for Interior Structure
SELENE	Selenological and Engineering Explorer
SOTA	state-of-the-art
SRP	solar radiation pressure
surf.	surface
TAGS	Traverse and Gravity Simulator
TGE	Traverse Gravimeter Experiment
TID	Total Ionizing Dose
TIRI	Thermal Infra Red Imager
TVAC	thermal-vacuum
URL	Uniform Resource Locator
VBA	Vibrating Beam Accelerometer
VEGA	VEctor Gravimeter/Accelerometer
Vol.	volume
VP	volume proportional
VSA	vibrating string accelerometer
WPM	Wedge-Pentahedra Method

w.r.t. with respect to

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1

INTRODUCTION

Following successfully completed small body missions, such as NEAR to Eros, Hayabusa to Itokawa or Rosetta to 67P/Churyumov-Gerasimenko, the story of small body (SB) exploration has just begun. The catch-all term *small bodies* unites *small Solar System bodies*, e.g. asteroids and comets, with the smaller, non-spherical natural satellites, such as the Martian moon Phobos. This illustrates well how diverse this celestial body family is, and so is the motivation exploring these bodies. Sometimes referred to as *time capsules of the Solar System*, SBs indeed experienced less changing evolution since their formation early in the Solar System, compared to planets and larger moons. And so, the scientific interest in these bodies can be explained, which, more recently, has been augmented by attention to planetary defense and asteroid mining. Actively avoiding SBs to collide with Earth, maybe the only natural disasters that can be actively mitigated next to climate change, is referred to as *planetary defense*. Asteroid mining is the concept of harvesting natural resources on SBs, and to either return them to Earth or to use them in space, e.g. to supply a (human) space exploratory mission.

In the field of SB exploration, the present work is focused on *small body surface gravimetry*, i.e. the measurement of surface gravity on extraterrestrial surfaces. This will be addressed twofold, firstly by contributing to the GRAvimeter for small Solar System bodies (GRASS) instrument, and secondly by simulating the surface gravity environment of SBs and the possible response of GRASS to it. These two parts go hand in hand with each other. With preliminary simulations of the surface gravity to be expected, GRASS can be developed accordingly, knowing the absolute magnitude to be measured, but also the variations in gravity, e.g. due to dynamic effects or depending on the landing location. At the same time, once the instrument returns measurements to Earth, the interpretation will rely on gravity simulations, as a first step to data interpretation. For example, the minimum GRASS target accuracy is 1 % of the local gravity. This instrument target would be little beneficial, if the errors in gravity modelling would exceed this value.

This introductory Chapter first lays out the motivation for gravimetric research on small bodies (Section 1.1). Next, the GRASS gravimeter with its development history is introduced in Section 1.2. Two current and one possible future space exploration missions related to the instrument are discussed in Section 1.3, alongside with the relevant target SBs (Section 1.4). Following this, the research questions of this PhD thesis are formulated in Section 1.5 and the document structure is described in Section 1.6. Finally, the author's main contributions and publications

related to this PhD work are presented in Section 1.7.

1.1 MOTIVATION OF SMALL BODY SURFACE GRAVIMETRY

As mentioned above, three main pillars (science, planetary defense and resource utilisation) motivate small body exploration throughout the Solar System. This Section introduces specifically the motivation to include surface gravimetry in any of the three exploration drivers, as basis for this work's motivation.

First, an improved understanding of SB interiors can provide valuable information about the evolution of the formation and history of the bodies. As mentioned above, both is crucial to improve today's understanding of the Solar System's evolution, eventually leading to harbour life on at least one of the formed celestial bodies. Unlike for the surface, the interior cannot as easily be investigated, e.g. by remote observation or surface sampling (Scheeres et al., 2015). Remote spectral analysis of the surface might provide misleading results of the internal composition of a body, as space weathering can alter the exposed surface (Ballouz et al., 2019), while the internal composition is not affected by space weathering (Matsumoto et al., 2020). Next to the composition of the bodies, it is likewise necessary to constrain the interior structure. Small bodies can consist of solid rock (monoliths), or be composed of loose material, so-called rubble piles (Walsh, 2018). While monolithic bodies have internal strength keeping them together, rubble piles are an accumulate only held together by its own gravitation. Clearly, the latter can be expected to have a much larger porosity, i.e. the ratio of volume occupied by voids and the overall body volume (i.e. zero voids lead to zero porosity). Carry (2012) plots that the porosity of SBs with diameter smaller than 50 km (for those objects with relative precision better than 20 %) ranges from about 1 % to 72 %, with an estimated mean of about 37 %, which is in agreement with Walsh (2018), who reported an average porosity of $35^{+38}_{-35}\%$ for SBs with diameter smaller than 10 km, based on the same study. In a forward-thinking way, known structure and (material) composition can be used to model the (surface) gravitational field. Therefore, taking one or more surface gravitational measurements, valuable information can be obtained regarding the interior of the target. Clearly, this inverse problem is non-unique, and a measurement can only aid in constraining the interior structure, e.g. by obtaining the overall mass. For a better understanding, additional measurements, such as seismic and radar measurements can improve the understanding of the inside (Scheeres et al., 2015). Additional assistance can come here from gravitational field reconstruction from radio-science experiments, tracking an orbiter using the Doppler shift. However, orbits around SBs are generally less stable and the tracking accuracy can be considered less accurate than surface gravimetry measurements. It is difficult or impossible to propagate the measurements from orbit down to the surface, while, on the contrary, the global coverage might be a benefit, which is not achievable for a (stationary) surface lander.

Next to the scientific motivation of this research, planetary defense against an imminent asteroid impact threat can, in an undesired scenario, become a crucial mo-

tivation to understand an asteroid interior. The above mentioned scientific motivations are likewise valid for preparing any action to hinder an impact on the Earth. Two relevant impact mitigation techniques are the usage of a gravity tractor or a kinetic impactor. The former would use the pure gravitational attraction of a spacecraft to slowly deviate the asteroid from its initial trajectory. The latter would use the kinetic energy of a spacecraft to deviate the orbit of a hazardous object by impacting it at full throttle. This will first be tested by the Asteroid Impact Deflection Assessment (AIDA), consisting of the NASA Double Asteroid And Deflection test (DART) and the ESA Hera missions (Subsection 1.3.2). Without question, total mass, and internal structure and composition will be crucial inputs in planning a deviation mission. While it might not always be possible to send a precursor mission ahead of a planetary defense mission, any improvements in SB knowledge will aid in eventually planning to save the Earth. This is specifically true when the result of a technology demonstration mission (DART) can be combined with detailed asteroid characterisation of the same target (Hera).

Lastly, for completeness, asteroid mining both for returning resources to Earth and supplying (human) space exploration missions could pose a possible application for surface gravimetry on small bodies. Depending on the application, water, precious materials, or other materials could be searched. A possible target could be initially visited by a small, even CubeSat, precursor mission, performing different geophysical experiments. For the same reasons as mentioned earlier, with a specific interest in the body materials to be mined, a gravimeter is here an important asset. While space mining attracted a lot of public attention and even raised legal questions on space law, this industry has not yet established any track record in exploration. Thinking long-term, the mining industry will, like on Earth, benefit from geophysical studies and instrumentation developments for extraterrestrial gravimetry.

The main motivation of this work, as presented in this Section, however, lies in science and planetary defense. The next Section introduces the GRASS gravimeter, developed to become the world-first small body gravimeter for surface measurements.

1.2 THE GRASS GRAVIMETER

The present PhD work has been conducted in the framework of the development of the GRAvimeter for small Solar System bodies (GRASS) gravimeter. Originally, GRASS was derived from the AGEX (Asteroid Geophysical Explorer) CubeSat Gravimeter instrument that was proposed for the ESA AIM (Asteroid Impact Mission) mission, targeting Didymos (Karatekin, 2016). The gravimeter was then proposed for the DLR/CNES Phobos lander as part of the JAXA MMX mission (Subsection 1.3.1) by the Royal Observatory of Belgium (ROB) with engineering support of the Royal Belgian Institute for Space Aeronomy (BISA) and the VRE Consultancy. This PhD work commenced in the framework of developing a first prototype for this mission. This prototype is described in Appendix A and formed the basis of the advanced GRASS instrument development. It was specifically characterised by

a large degree of adaptability to facilitate adaptations of design parameters. Next to the 3D-design of the prototype, the author performed various trade-offs between purchase components (e.g. gear-motor, slip ring) and discussed initial test results with the suppliers. Further, the test setup for testing under Earth gravity (Subsection 4.1.4) was implemented based on the present work.

Finally, the rover mission was not able to provide enough mass and volume budgets to accommodate this payload (Ulamiec et al., 2019). Nevertheless, the then well advanced development allowed to propose the instrument for the ESA Hera planetary defense mission (Subsection 1.3.2) to land with the Juventas CubeSat on the asteroid moon Dimorphos (Subsection 1.4.2). For this, the instrument had to be significantly miniaturised to fit inside the 6U-XL-CubeSat. After the author and Emiel Van Ransbeeck had adapted the design to meet the volumetric and mass budgets, the payload was indeed included to the satellite design, initially as an optional payload. With this, the instrument development was continued in a ROB-lead consortium with the above mentioned partners and additionally Embedded Instruments & Systems (Emxys). Following a successful preliminary design review (PDR), the optional instrument was promoted as baseline instrument to the Juventas spacecraft, and so was the landing operation in the final mission phase of the CubeSat. After successful completion of the instrument critical design review (CDR) in late 2021, the scientific payload is under preparation for the flight model (FM) delivery in 2023. At the time of finalisation and typesetting of the work in September 2022, the GRASS instrument mechanical design can be considered as nearly finalised. The space-qualification test is ongoing, and expected to be closed before the end of 2022, kicking off the production of the FM. Regarding the mechanical and mechanism design presented in this work, no significant changes are to be expected at this development stage. Final improvements will, if any, concern the electronics and software design of the instrument, which, in detail, is beyond the goal and scope of this work. The GRASS instrument electromechanical working principle as well as the contributions of this work to this gravimeter development form part in this PhD thesis.

To provide a first overview of the GRASS gravimeter, the key instrument parameters of the FM instrument version are presented in Table 1.1. Clearly, this data sheet does not provide a complete overview, with the detailed instrument characterisation and testing ongoing. Still, it provides a basis to be extended in the future with additional figures. The key development challenge is to maximise the gravimeter spring deflection, simulated to be in the order of tenths of one micron to one micron, while measuring precisely this displacement. Using means of capacitance, the displacement will be measured, with base capacitances in the order of 8 pF. To measure the minimum instrument target of 1 % of the local Dimorphos gravity ($4.5 \cdot 10^{-5} m/s^2$), changes in gravity in the order of hundreds of atto-Farad (aF) will have to be measured.

Table 1.1: GRASS instrument data with key figures. Clearly, the gear-motor selected in the frame of this thesis work is the key driver for the overall instrument power consumption. The ongoing instrument testing and characterisation will allow to extend this Table in the future, especially regarding the final determined performance. The data was compiled from the instrument interface control document (ICD, from GRASS team (2022a)), and the instrument motor data sheet (Micromotion GmbH, 2020). The tested cold motor torque was tested as reported in GRASS team (2022b).

Quantity	Value
Mass	319 gram
Volume	$99\text{mm} \cdot 99\text{mm} \cdot 49\text{mm}^a$
Power consumption	$< 1\text{ W}$ (nominal) ^b
Rated current of motor	250 mA
Rated voltage of motor	2 V
Rated motor torque	15 mNm
Tested cold motor torque ^c	4.37 mNm
Min. required torque	1.2 mNm
Min. non-operational T	-30°C
Max. non-operational T	$+50^\circ\text{C}$
Min. operational T	-30°C
Max. operational T	$+20^\circ\text{C}$
Min. target accuracy	1 % of local g

^aOverall volume envelope, the L-shape has a thickness of 20.9 mm, leaving the remaining volume available.

^bThe final power consumption will be determined during the instrument characterisation.

^cThe working of the motor at -40°C , below the rated min. motor operating temperature (-10°C), has been verified by test.

1.3 RELEVANT SMALL BODY EXPLORATION MISSIONS

There is a large number of past, current and future small body exploration missions, each contributing to a better understanding of SBs. In this Section, the JAXA-led Martian Moons eXploration (MMX, Subsection 1.3.1) and the Asteroid Impact Deflection Assessment (AIDA, Subsection 1.3.2) will be introduced, as they are relevant to this work. On top of this, one possible future mission opportunity to Apophis is briefly discussed (Subsection 1.3.3).

1.3.1 MARTIAN MOONS EXPLORATION MISSION

The JAXA MMX mission¹ is initially planned to be launched in September 2024 for an arrival in areocentric orbit in 2025 (Ulamec et al., 2019). According to Kuramoto et al. (2022), the main mission objectives are to shed light on the origin of the two Martian moons Phobos (Subsection 1.4.1) and Deimos², to improve the understanding of the early Solar System evolution and to observe surface evolutions not only of the two moons, but also the Red Planet. It is added that this will be achieved not only by long-term close-up observations of Phobos and several flybys to Deimos, but also by a sample return of the Phobian surface back to Earth for detailed study. Here, it is foreseen to sample two different surface locations on Phobos (Fujiya et al., 2021).

As part of this mission, a rover developed by DLR/CNES is to land on the surface of Phobos, with a baseline surface operation of 100 days and a traverse of at least 100 m over the surface. The selected payloads are two kinds of cameras (NavCAMs and WheelCAMs), a Raman Spectrometer RAX, and a miniRAD radiometer. As partially stated in the previous Section, the GRASS gravimeter and a ground-penetrating radar (GRAMM) were initially considered, but had to be removed from the instrument baseline due to limited accommodation options inside the rover. (Ulamec et al., 2019)

This work did not only accompany the early GRASS development towards a space mission gravimeter prototype as stated above, but also considered Phobos as a target for (near-) surface gravity simulations. This will likewise be presented in this document, and some of the work might still prove valuable in the future to the MMX mission, for example for landing or rover traverse planning.

¹<https://www.mmx.jaxa.jp/en/>, accessed on 16.08.2022.

²The IAU definition of *Small Solar System Bodies* formally excludes natural satellites, such as the two Martian moons (IAU General Assembly, 2006). However, due to their irregular shape, and their possible ex-situ origin, we consider these bodies in this work (and for the GRASS development) and refer more generally to *small bodies*. One definition of SB reads: “The small bodies in the Solar System include comets, asteroids, the objects in the Kuiper Belt and the Oort cloud, small planetary satellites, Triton, Pluto, Charon, and interplanetary dust.” Cf. <https://science.nasa.gov/solar-system/focus-areas/small-bodies-solar-system>, accessed on 24.01.2022.

1.3.2 ASTEROID IMPACT DEFLECTION ASSESSMENT

The Asteroid Impact Deflection Assessment (AIDA) consists of the two individual missions, NASA DART³ and ESA Hera⁴ both targeting the binary asteroid system Didymos (Cheng et al., 2015). Before its approval at the ESA ministerial council in November 2019, the HERA asteroid mission had been previously proposed by ESA as the Asteroid Impact Mission (AIM). From this heritage, a lot of elements were kept in the HERA mission.

Launched in late-2021, the DART spacecraft is designed as a kinetic impactor to the asteroid moon Dimorphos (*informally and used here interchangeably*: Didymoon). The kinetic impact on 26 September 2022 (US time)⁵ aims at altering the orbital period of the secondary around the primary Didymos (*informally*: Didymain) to an extend of at least 1 % of the orbital period that should be observable from Earth. Moreover, the Italian ASI CubeSat LICIACube will separate from the main spacecraft prior to impact, designed to observe the impact and subsequent plume, while missing DART's target, allowing the survivor to send first imagery back to Earth (Dotto et al., 2021). As the DART spacecraft is doomed to crash, no science payloads except the Didymos Reconnaissance and Asteroid Camera for Optical navigation (DRACO) instrument are on-board the spacecraft.

Following this mission, the ESA Hera mission will embark in 2024 to follow DART towards the binary asteroid system. Upon arrival, a detailed characterisation of the asteroid system will take place with special attention to the artificial DART impact crater, and any subsequent observable changes to the asteroid system.

The main science objectives of this mission are the detailed characterisation of the binary asteroid system. To this, the orbital state, gravitational field, shape and volume, internal structure and density, and many more belong. To achieve this, the main spacecraft carries two asteroid framing cameras (AFCs), the Hyperscout-H spectral imager, the microLIDAR (PALT), a thermal infrared imager called TIRI, and a radio science experiment (Michel et al., 2022). Moreover, two deep space CubeSats, Milani (Ferrari et al., 2021) and Juventas (Goldberg et al., 2019) will be released to aid the binary asteroid system investigation. The latter CubeSat, Juventas, will carry the Juventas Radar (JuRa) experiment (Herique et al., 2020) and the above-introduced GRASS gravimeter. The main high level product to which GRASS will contribute are the orbital state of Dimorphos, the gravitational acceleration, the total mass and density, and potentially the mechanical surface properties (Michel et al., 2022).

1.3.3 APOPHIS MISSION OPPORTUNITY

Today, no definitive mission plan has been fixed for encounter with the (99942) Apophis near-Earth asteroid (NEA). This “potentially hazardous objects” (PHO) (Shuvalov et al., 2017) provides an interesting mission opportunity due to its upcoming close-encounter with the Earth. In 2029, it is predicted to miss the Earth by

³<https://www.nasa.gov/planetarydefense/dart/dart-news>, accessed on 16.08.2022.

⁴<https://www.heramission.space/>, accessed on 16.08.2022.

⁵Note that this PhD thesis has been finalised and typeset in September 2022 before the impact, despite the PhD work being defended post impact.

about 32,000 km (Kim et al., 2021), a distance comparable to geostationary satellite orbits(!). Different mission concepts have been formulated, e.g. Kim et al. (2021) show a conceptual mission scenario to either rendezvous with the asteroid prior to closest Earth approach, or to simply directly follow it after Earth encounter. Such a close encounter certainly is a prime example why the above introduced planetary defense is no science-fiction, but a preparation on the question when, and not if, a hazardous asteroid will hit the Earth.

With regards to the science and in the context of this work, sending a landing element to the surface of Apophis could be achieved relatively easy by adding such element to a mission architecture. Here, surface gravimetric measurements could be performed, allowing to shed more light on the internal composition of the body, including the total mass, its density and the composing material. On top of this, and especially for a mission arriving before the closet encounter, a gravimeter with sufficient performance could aid in observing any Earth-induced dynamical changes to the object, leading to variable surface gravitation (Noiset et al., 2022). Therefore, in addition to the MMX and Hera mission with their respective target, Apophis is briefly included in this work to illustrate *one* possible future application of both the GRASS gravimeter and the results presented in this work.

Next to the science, however, it should be stressed that interacting with a PHO using a spacecraft could potentially (theoretically) alter the orbit such that the near-Earth-missing trajectory is changed to a catastrophic orbit, causing the PHO to hit the Earth. It is needless to say that mission designers will consider both risks, that of an undesired collision of the spacecraft with the PHO, as well as the gravitational pull of the spacecraft itself, changing the orbit of the target asteroid. The latter will have a very small, yet existing effect, being effectively a gravity tractor as introduced in Section 1.1. In any case, not only the upcoming very close approach of Apophis to Earth in 2029 should be studied, but likewise the possible (very) long-term effects on the future trajectory, where smallest alternations to the orbit today might result in large variations in the long-term. This responsibility towards future generations has been proclaimed in Article 4 (Preservation of life on Earth) of the UNESCO *Declaration on the Responsibilities of the Present Generations Towards Future Generations* (1997)⁶. It is therefore urged to consider potential long-term risks and suggested to discuss such from additional points of view, including ethics, philosophies, politics and legals. Regarding the latter, it is regretfully recognised that no relevant binding international agreement is in place today, which would be a vital basis for any such discussions.

⁶“The present generations have the responsibility to bequeath to future generations an Earth which will not one day be irreversibly damaged by human activity. Each generation inheriting the Earth temporarily should take care to [...] ensure that life is not prejudiced by harmful modifications of the ecosystems and that scientific and technological progress in all fields does not harm life on Earth.” Cf. <https://en.unesco.org/about-us/legal-affairs/declaration-responsibilities-present-generations-towards-future-generations>, accessed on 21.09.2022.

Table 1.2: Overview of small gravity reference values for Phobos (Scheeres et al., 2019), Deimos (Pratt and Hopkins, 2011), Dimorphos (Murdoch et al., 2017) and Apophis (Subsection 6.4.1). The listed references formed the basis for the here-assumed general reference values.

Dimorphos	Apophis	Phobos	Deimos
$4.5 \cdot 10^{-5} m/s^2$	$6 \cdot 10^{-4} m/s^2$	$5.7 \cdot 10^{-3} m/s^2$	$4 \cdot 10^{-3} m/s^2$

1.4 RELEVANT SMALL BODIES

While a review of the diverse family of small bodies could fill many pages, this introduction is limited to the three relevant mission targets as introduced in the previous Section. These are Phobos (Subsection 1.4.1), Dimorphos (Subsection 1.4.2) and Apophis (Subsection 1.4.3). With these three bodies, being a natural satellite, an asteroid moon and an PHO in heliocentric orbit, respectively, the diversity of small bodies is well represented in this work. To illustrate the small gravity environment, Table 1.2 provides the three reference gravity values assumed for these three bodies, and for Deimos (Pratt and Hopkins, 2011) for reference. The diversity of the gravity of small bodies is well visible, with the gravities varying by two orders of magnitude.

1.4.1 PHOBOS AND DEIMOS

The Mars system consists, next to the Red Planet, of two Martian moons, Phobos and Deimos, with mean diameters of 22.2 km and 12.4 km, respectively. Phobos had been previously the target of dedicated but non-successful exploration missions, being the Sovjet Phobos programme (Sagdeev and Zakharov, 1989) and the Russian Fobos-Grunt mission (Marov et al., 2004). Yet, there have been several close fly-by encounters of missions in Mars orbit, e.g. by ESA Mars Express (Witasse et al., 2014). These contributed to today's relatively good knowledge of this object, represented e.g. by a very good available shape file (Willner et al., 2014).

Still, the origin of the moons is subject to ongoing scientific debate. The similarity of the density to C-type asteroids, alongside with its shape would indicate that Phobos was captured by Mars, thus coming from an external (ex-situ) origin (Jones et al., 1990). However, other theories, such as the accretion from a circum-Martian disk (Rosenblatt and Charnoz, 2012) or from ejected material following a giant impact (Craddock, 2011) also exist. Therefore, an improved understanding of Phobos' composition and structure can contribute to understanding the history and origin of the Martian moon(s). A first overview of the physical parameters of Phobos is provided in Table 1.3.

Figure 1.1 shows a projected map of the Phobian surface. The most predominant feature is the ca. 10 km large Stickney impact crater. Another feature clearly visible on the moon's map are the grooves, for whose origin different theories exist. Murray and Heggie (2014) suggest the formation by impact ejecta, whereas Hurford et al. (2016) identified tidal stress to cause the surface features.

Finally, the orbital parameters of Phobos and his companion Deimos are listed in

Table 1.3: Physical parameters of Phobos overview as listed by the NASA JPL Solar System Dynamics list, adapted and extended with further literature. From https://ssd.jpl.nasa.gov/?sat_phys_par, accessed on 03.04.2020. References (Ref.) according to code: [1] Jacobson (2010), [2] Thomas (1989), [3] Willner et al. (2010), [4] Zellner and Capen (1974).

Parameter	Notation	Value Phobos	Ref.
GM	μ	$0.0007112 \pm 0.0000010 \text{ km}^3/\text{s}^2$	[1]
Mean radius	r	$11.1 \pm 0.15 \text{ km}$	[2]
Radii	$a \times b \times c$	$12.61 \times 11.33 \times 9.07 \text{ km}^3$	[3]
Mean density	ρ	$1860 \pm 13 \text{ kg/m}^3$	[3]
Volume	V	$5741 \pm 0.6\% \text{ km}^3$	[3]
Magnitude	V_0	11.4 ± 0.2	[4]
Geometric Albedo		0.071 ± 0.012	[4]
Forced libration amplitude		1.14°	[3]

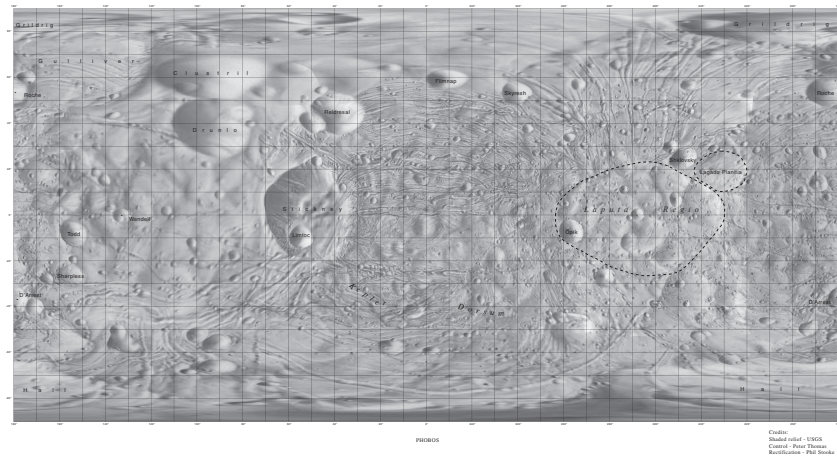


Figure 1.1: Simple cylindrical projection of Phobos surface with International Astronomical Union (IAU) nomenclature of craters and regions. Most predominant feature is Stickney crater with Limtoc crater in itself. Several groove features as discussed in this Subsection are also visible. From <https://astrogeology.usgs.gov/search/map/Phobos/Nomenclature/phobos-cylindrical-grid>, accessed on 02.04.2020.

Table 1.4: Orbital parameters of the Martian moons Phobos and Deimos. From Wakker (2015).

Orbital parameter	Notation	Phobos	Deimos
Semi-major axis	a ($10^3 km$)	9.376	23.458
Eccentricity	e	0.0151	0.0002
Inclination	i ($^\circ$)	1.075	1.788
Orbital period	T (msd) ^a	0.3189	1.2624
Orbital period	T (h)	7.6536	30.2976
Tidally locked		yes	yes

^amean solar day, i.e. 24 hours = 86,000 s.

Table 1.4. It becomes clear that both moons have a nearly circular and nearly equatorial orbit around Mars. Both moons are in synchronous rotation around Mars (Lissauer and de Pater, 2013) which can be explained by the relative strong tidal forces (Schmedemann et al., 2014). In addition, Phobos moves inside the Martian Roche limit (Subsection 5.2.2) according to Wakker (2015), which is calculated at $3.19R_{Mars}$ (Hurford et al., 2016).

1.4.2 DIMORPHOS

Today's current knowledge, that is, before the DART (Subsection 1.3.2) approach to the binary Didymos system, is solely based on Earth-based observations, since no mission has yet visited the system. In preparation of the mission, the European Space Agency (ESA) has released a Didymos Reference Model⁷ for mission planning of the Hera mission giving an overview of the literature providing physical parameters. The relevant parameters are extracted and presented below in Table 1.5 with the relevant references. From the listed uncertainty errors it is clearly visible that the uncertainties for Didymos are much larger than for Phobos (Table 1.3). Regarding the rotation, the system is assumed to have both spin pole and binary orbit pole aligned (Naidu et al., 2020).

Therefore, the shape file of the two asteroids can only be assumed. While it is possible to obtain some information from the observations on the primary, the ESA reference shape of the secondary Dimorphos was limited to a simple triaxial ellipsoid for initial mission planning. Here, the equivalent diameter was set to 150 m, compared to 780 m for the primary Didymos. A proposed more realistic representation is provided in the software shapeViewer⁸ with a dedicated version for the Hera mission and sizing of existing shape files to the observed sizes given in Table 1.5 (Vincent et al., 2018). Taking the scaled shape file of Ryugu as Didymos, the two shape files of the secondary are rendered in Figure 1.2. While the scaling of a realistic shape has various advantages, e.g. for thermal observation simulations, the bound volume is much smaller, yielding a much smaller surface gravity. Therefore, for the preparation of the GRASS gravimeter and the corresponding gravity simulations, this work generally considers the initial ellipsoidal shape (Figure 1.2

⁷Date 03/02/2020 Issue 5 Rev 2.

⁸<https://www.comet-toolbox.com/shapeViewer.html>, accessed on 17.08.2022.

Table 1.5: Physical parameters of Didymos derived from observations and modelling. Uncertainties correspond to 1σ -uncertainty, unless otherwise stated. The subscripts P and S of the notation indicates the primary and secondary value, respectively; no subscript refers to the binary system.

Parameter	Notation	Value	Reference
Asteroid dynamical class		Amor	Lange et al. (2018)
Asteroid type		S-type	Lange et al. (2018)
Volume Equivalent Diameter	D_p	$780 \pm 30 m$	Naidu et al. (2020)
Principle axes	x,y,z	$832m \pm 3\%$, $838m \pm 3\%$, $786m \pm 5\%$	Naidu et al. (2020)
Diameter Secondary	D_s	$150 \pm 30 m$	Naidu et al. (2020)
Binary orbit semi-major axis	a	$1190 \pm 30 m$	Naidu et al. (2020)
Binary orbit eccentricity	e	<0.05	Naidu et al. (2020)
Orbital period	T	$11.93 \pm 0.01 h$	Naidu et al. (2020)
System mass	m	$(5.4 \pm 0.44) \cdot 10^{11} kg$	Naidu et al. (2020)
Bulk density	ρ	$2170 \pm 350 kg/m^3$	Naidu et al. (2020)
Geometric ^a albedo	A_0	0.15 ± 0.04	Naidu et al. (2020)

^aIn the abstract of the reference referred to as "optical albedo".

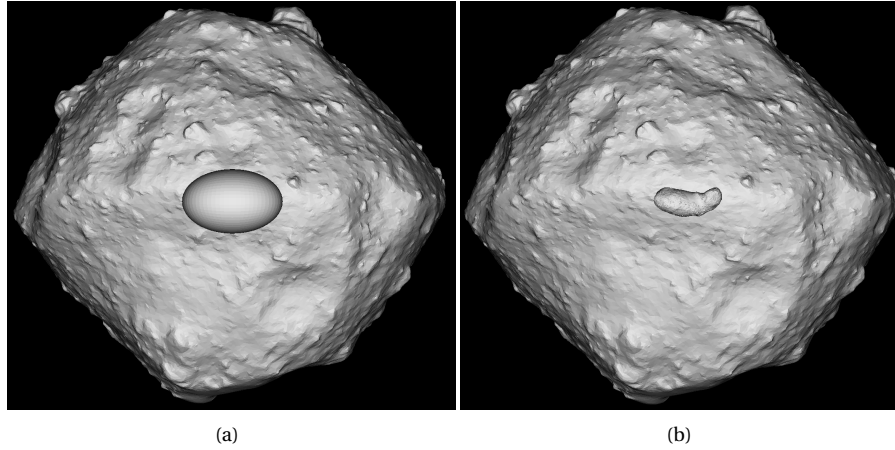


Figure 1.2: Dimorphos shape files considered in this work. (a) ESA reference shape file triaxial ellipsoid. (b) Itokawa shape file from ShapeViewer, scaled to Dimorphos size. Both views rendered in shapeViewer (Vincent et al., 2018): This research has made use of the scientific software shapeViewer (www.comet-toolbox.com). Note that the orientation of the secondary is for size comparison only.

(a)).

1.4.3 APOPHIS

As mentioned above, the asteroid (99942) Apophis (2004 MN4) is a potentially hazardous asteroid with an assumed diameter of 340 m. The name of the body originates from ancient Egypt god Apophis who embodied *chaos*. Table 1.6 lists the main known parameters, well noting that this will most certainly change or become more precise with more observations (or a rendezvous mission) becoming available in the future. Supplementary to the table, the shape file developed by Pravec et al. (2014) is a valuable information for a first global gravity assessment. For this, the shape file is obtained from the 3D Asteroid Catalogue⁹. A 3D-representation is provided in Figure 6.16 (a), where the simulated gravity is plotted over the surface.

1.5 RESEARCH QUESTIONS

While Solar System gravimetry is not completely new discipline, having been performed before once on the Moon (Subsection 2.3.1) and once on Mars (Subsection 2.3.3), surface gravimetry on small bodies has never been performed before. In preparation of this endeavour, two fields are of equally great importance. On the one hand, surface gravimetry demands a gravimeter instrument measuring the surface gravity, and on the other hand, surface gravity needs to be modelled accu-

⁹<https://3d-asteroids.space/asteroids/99942-Apophis>, accessed on 22.03.2021.

Table 1.6: Assumed Physical Parameters of Apophis based on observations or assumptions.

Parameter	Notation	Value	Reference
Diameter	d	340 m	Brozović et al. (2018)
Mass	m	$4.1 \cdot 10^{10} \text{ kg}$	Center for NEO Studies ^a
Density	ρ	$3,190 \text{ kg/m}^3$	assumed, Brozović et al. (2018)
Albedo	A_0	0.33	T. Mueller (2013) ^b
Rotation period	T	30.4 h	Behrend (2005) ^c

^a<https://cneos.jpl.nasa.gov/sentry/details.html#des=99942>, accessed on 22.03.2021.

^bhttp://www.esa.int/Science_Exploration/Space_Science/Herschel_intercepts_asteroid_Apophis, accessed on 22.03.2021.

^c<http://obswww.unige.ch/~behrend/r099942a.png>, accessed on 22.03.2021.

rately in order to enable science return, as laid out in the motivation of this research (Section 1.1). The research questions are therefore formulated separately for these two field in the following.

1.5.1 RESEARCH QUESTIONS: SMALL BODY GRAVIMETER

In essence, the development of a small body gravimeter (SBG) can be seen as a technology transfer, from Earth to SBs. However, all design choices are explicitly motivated by the (very) low-gravity environment, and have to be reinvestigated when testing the instrument on Earth, or transferring the technology to e.g. the Moon or Mars (non-small bodies). For example, the travel from terrestrial gravity of 9.81 m/s^2 to the surface from Dimorphos (with assumed gravity $4.5 \cdot 10^{-5} \text{ m/s}^2$) corresponds to a change of five(!) orders of magnitude in gravity. In this context, the first research question can be formulated as:

(i) Which of the existing gravimeter working principles is best suited for space gravimetry in the low-gravity environment, when specifically considering a robotic, non-human Solar System exploration mission with tight constraints on power, volume and mass, and which adaptations are still required for the selected principle?

Terrestrial gravimeter are generally one-dimensional due to the “vertical” orientation of the gravity, often equipped with small range levelling mechanisms for perfect alignment of the instrument axis and the local gravity vector. Unlike on spherical planets and moons, the gravity vector on small bodies is unknown and additionally the robotic lander can be expected to lack a (precise) levelling mechanism. Therefore, the second research questions is formulated as:

(ii) How can the classical 1D-gravity measurement be augmented for a 3D-measurement in the most compact way, assuming the lack of a levelling mechanism on the lander, no prior precise knowledge of the surface gravity vector on small bodies and the special requirements of a space exploration mission?

Under the assumption that indeed at least some modifications of an existing gravimeter type will be required for a small body exploration mission, the third re-

search question concerning the instrument can be formulated as:

(iii) What is the expected electromechanical behaviour of the space gravimeter concept, and how can the sensitivity be optimised for the (very) small gravity environment?

1.5.2 RESEARCH QUESTIONS: SMALL BODY SURFACE GRAVITY

In the context of the SBG, it is essential to simulate the surface gravity of SBs. This is not only required for the instrument development and sizing, but likewise for the science return upon reception of in-situ measurement data returned from an exploration mission. In the modelling context, the first research question can be addressed as:

(I) What are the contributions to surface gravity and which methods to model gravitation of small bodies exist that are specifically well suited in the context of surface gravimetry and how do they compare?

Taking a selection of SBs of differing size will allow to gain knowledge of the surface gravity environment, which is important for seizing a gravimeter. Here, it is important to consider the range of surface gravity of the global surface, but also dynamical effects (tides, rotation libration) contributing, next to gravitation, to gravity. Therefore, the following second research question in this field is formulated as:

(II) What is the expected surface gravity and its variation on possible small body gravimeter targets on a global level?

Next to the global gravitation modelling, the local topography surrounding the gravimeter measurement location might be important for the data analysis. Therefore, the last research question reads as:

(III) How can local topography be included in the gravitation modelling of small bodies and how large are the topographic corrections on the resulting surface gravity?

1.6 THESIS OUTLINE

As indicated in this Chapter, the overall thesis work is comprised of the two main contributions being development work of the GRASS instrument and the gravity simulation of small bodies. The latter is not only required for early sizing of the instrument, but also the scientific interpretation of the instrument measurements. While this connection is represented in the entirety of this PhD work, the thesis is structured in two parts. Part I is dedicated to the small body gravimeter and Part II presents the gravity simulation work. While the research questions formulated in the previous Section are always discussed in the corresponding chapters dealing with them, the future work and applications (Chapter 8) and conclusions (Chapter 9) concern the entire work, and are therefore placed in a concluding Part III, which helps to clearly mark the break to the previous Part. In the following, the structure of the two main parts is briefly laid out.

Part I begins with a review of the state-of-the-art (SOTA) of gravimeters (Chapter 2). Here, the different working principles are presented, commercial gravimeters are shown, and proposed and flown space gravimeters are introduced. With this, a discussion on the best suited gravimeter type for the GRASS development is presented. Based on this, Chapter 3 is then concerned with the adaptation and development of the GRASS gravimeter. This is clearly focused on the mechanical design and development, but also includes the electromechanical design of the gravimeter spring displacement transducer. The theoretical behaviour of the designed instrument is then described with an analytical instrument model presented in Chapter 4, which closes the first part of this thesis. The analytical description depicts the measurement chain from gravity-induced spring deflection, to a differential capacitance signal, which is then translated into an output signal.

Following this, Part II composes of a first introductory chapter presenting the theory of the different gravity contributions and the SOTA of various gravitation modelling techniques (Chapter 5). Three of these techniques are then systematically compared in Chapter 6, followed by global computations of dynamical and gravitational contributions on different SBs. Finally, zooming in on the local surrounding of a possible surface gravity measurement, the novel Wedge-Pentahedra Method (WPM) is presented extensively in Chapter 7. The WPM is finally implemented in a traverse and gravity simulator (TAGS) and an adaptation from the small body environment to the Lunar surface is presented.

1.7 MAIN CONTRIBUTIONS AND RELATED PUBLICATIONS

Based on the thesis outline sketched in the previous Section, the author's main contributions and related publications are laid out in this Section. Publications here refer to both peer-reviewed papers and contributions to international conferences. Regarding the latter, only those are mentioned that were first-authored by the author and that are directly relevant to the presented thesis. Where parts of this thesis are based on the below-mentioned publications, a brief note repeats the links in the relevant parts of the text.

In Part I, the contributions to the space gravimeter part have been presented. Chapter 2 is mostly concerned with the literature study regarding gravimeters, in a commercial and scientific context, and for terrestrial and space applications. This Chapter is the sole contribution of the author. Regarding the GRASS gravimeter development presented in Chapter 3, the overall instrument development is attributed to the entire GRASS team to which the author belongs, and therefore the Chapter aims at focusing on the author's contribution. This is mostly concerned with the mechanical and mechanism engineering of the instrument. This was done together with Emiel Van Ransbeeck as a joint effort, but with different centres of attention. Here, the author's focus was on the 3D-CAD modelling, and Emiel Van Ransbeeck's focus was the hardware integration and assembly. Some work from other's had to be included in the text to best put the author's contribution into context, which has been clearly marked. The mechanism development lead to the pub-

lications of Noeker et al. (2021c) and Noeker et al. (2022c). The electromechanical analytical description of the instrument presented in Chapter 4, is again the sole contribution of the author. Here the final simulation output is eventually compared to laboratory measurements, which have been performed by other GRASS team members, as acknowledged in the text.

The subsequent Part II is again opened with a literature review regarding gravity contributions and different modelling techniques. This is the sole work of the author. Chapter 6 starts and ends with gravitation modelling method comparisons, regarding the homogeneous and inhomogeneous case, respectively. These comparisons of three methods have been done with two other teams from the University of Bremen and the Bundeswehr University Munich. By motivation of this comparison, each team was responsible for providing results for one method, where the author contributed by using the polyhedral method. This led to the publications of Meißenhelter et al. (2022) and Noeker et al. (2022a), for the homogeneous and (preliminary) inhomogeneous results, respectively. The remaining contributions in the Chapter are the sole contribution of the author. Following this, the novel Wedge-Pentahedra Method has been developed as a sole contribution of the author (Chapter 7). This led to the publications of Noeker et al. (2021b), Noeker and Karatekin (2022b), Noeker et al. (2022b) and Noeker and Karatekin (2022a).

Lastly, one possible future application, among others, was the proposal to explore intact subsurface lunar lava tubes with the GRASS gravimeter, for *differential* gravity measurements from above and below the lava tube roof (i.e. the material above the lava tube and below the surface). The associated LAVA-TUBE iNvestigAtion (LA-TUNA) mission concept has been described in Noeker et al. (2021a). Below, the here-references publications are listed to provide an overall overview.

Peer-Reviewed Publications

1. **Noeker, M.**, and Karatekin, Ö. The Wedge-Pentahedra Method (WPM): Topographic Reduction of local Terrain in the context of Solar System Surface Gravimetry and Robotic Exploration. In *Frontiers in Space Technologies*, 3:982873. (2022).
2. Meißenhelter, H., **Noeker, M.**, Andert, T., Weller, R., Haser, B., Karatekin, Ö., Ritter, B., Hofacker, M., Machado, L. Zachmann, G. Efficient and Accurate Methods for Computing the Gravitational Field of Irregular-Shaped Bodies. In *2022 IEEE Aerospace Conference (AERO)* (pp. 1-17). IEEE. (2022).
Best Paper in Track Award.

Conference Contributions

3. **Noeker, M.**, Tasev, E., Ritter, B., Karatekin, Ö., Application of the Wedge-Pentahedra Method (WPM) to the Apollo 17 Landing Site and Comparison to the Traverse Gravimeter Experiment Measurements. In *53rd Lunar and Planetary Science Conference 2022 (LPI Contributions, 2678, 1442)*. (2022).
4. **Noeker, M.**, and Karatekin, Ö., Gravity Traverse Simulations for Small Bodies and Moons of the Solar System using the Wedge-Pentahedra Method. In *EPSC 2022, Granada, Spain, Vol. 16, EPSC2022-361*. (2022).

5. **Noeker, M.**, Meißenhelter, H., Andert, T., Weller, R., Karatekin, Ö, Haser, B., Comparing Methods for Gravitational Computation: Studying the Effect of Inhomogeneities. In EPSC 2022, Granada, Spain, Vol. 16, EPSC2022-562. (2022).
6. **Noeker, M.**, Van Ransbeeck, E., Ritter, B., Karatekin, Ö. The GRASS Gravimeter Rotation Mechanism for ESA Hera Mission On-Board Juventas Deep Space CubeSat. In 46th Aerospace Mechanisms Symposium (pp. 159-172). (2022).
7. **Noeker, M.**, Karatekin Ö., Ritter B. Accessing the Lunar Underground: The LAva-TUbe iNvestigAtion (LA-TUNA) Mission Concept. In 52nd Lunar and Planetary Science Conference 2021 (LPI Contrib. No. 2548), online, 1574, (2021).
8. **Noeker, M.**, Karatekin Ö., Ritter B., Tasev, E., Artificial terrain on Phobos: Assessing the influence on local gravity using the Wedge-Pentahedra Method. In EPSC 2021, online, Vol. 15, EPSC2021-370. (2022).
9. **Noeker, M.**, Van Ransbeeck, E., Karatekin Ö., Ritter B., 2021, Development of a compact Payload Mechanism enabling continuous motorized Sensor Head rotation and Signal Transfer. In 19th European Space Mechanism and Tribology Symposium (ESMATS), online, (2021).

I

SMALL BODY GRAVIMETER

2

GRAVIMETER STATE-OF-THE-ART

When developing a science instrument such as the introduced GRASS gravimeter instrument, various aspects need to be addressed. The contributions of this research work to the instrument development are presented in this Part I of the thesis, and where applicable, credit is given to work attributed to (parts of) the GRASS team. Starting with the review and discussion of the gravimeter working principle in this Chapter, the electromechanical and mechanism design of GRASS will be subsequently presented in Chapter 3. With the instrument layout and design parameters in place, an analytical gravimeter model is presented at last in Chapter 4.



The development of a completely novel, unique gravity sensing method would be beyond the goal and scope of this work, and such development without commercial backing would prove infeasible. Rather, to realise a Solar System gravimeter, an existing gravity measuring method should be selected and adapted as necessary. This Chapter presents and compares the different sensing methods and the state-of-the-art (SOTA) of the different instrument types, allowing a trade-off between these sensing methods for subsequent adaptation for a SBG.

To this aim, firstly, the different gravimeter working principles and sensing methods are explained in Section 2.1. Next, a market overview of commercial (terrestrial) gravimeters is given (Section 2.2) and space gravimeter, both proposed and heritage, are presented (Section 2.3). A summary of the associated figures of merits of the different working principles is provided in Section 2.4. Based on this, a selection trade-off for small body application is performed in Section 2.5. Finally, this Chapter is summarised in Section 2.6.

2.1 GRAVITY MEASUREMENT PRINCIPLES

In this Section, various measurement principles are presented that can be used to measure gravity. The introduced types allow a better understanding of the commercially available gravimeters (Section 2.2) and the space gravimeters (Section

2.3). The figures of merit of the here-introduced gravimeter types are summarised in Section 2.4. This overview also forms the basis for the trade-off between the different types (Section 2.5) for the best application to small body gravimetry measurements. Indeed, as will be shown, a manifold of different techniques to measure the same single quantity, *gravity*, exists. Spontaneously, it is hard to name another quantity that can likewise be sensed by so many different measurement principles. Where supportive, the measurement principle description is supplemented by an instrument example based on the respective working principle. The sensing methods will be examined in the context of the envisaged space application.

2.1.1 PENDULUM

Simple Gravity Pendulum

Pendulum gravimeter have been used for gravity measurements for a long time, also together with other gravimeters such as the LaCoste-Romberg gravimeter introduced in Subsection 2.1.2 (Morelli et al., 1972). The basic measurement principle comes from a small-angle approximation where for the deviation from the vertical θ the approximation $\sin \theta \approx \theta$ can be made, and the period T_0 of the pendulum is calculated as shown in Equation (2.1) (Kidd and Fogg, 2002).

$$T_0 = 2\pi \sqrt{\frac{l}{g}} \quad (2.1)$$

Where l/g is the ratio of the pendulum length and the gravity. Measuring the pendulum period for small amplitudes, i.e. deviations from the vertical θ , and neglecting the friction in the pivot and (if present) the atmosphere, it is possible to solve for the acting gravity.

One technical implementation of a pendulum to sense gravity was the *Gulf Gravity Pendulum Apparatus* described by e.g. Morelli et al. (1972). The instrument (Figure 2.1 (a)) consists of two isochronous¹ quartz miniaturised pendulums (Figure 2.1 (b)) swinging in antiphase. The pivots of the pendulums are realised as knife-edges made of fused quartz and these swinging parts lay on Pyrex (glass) surfaces. It is possible to clamp the two pendulums, e.g. for transportation. It can be added that the quartz design minimised temperature effects in the instrument². Overall, the pendulum can be considered as an *absolute* gravimeter. The instrument precision was reported by Gay (1940) to be in the order of $10^{-7}g$.

Conical Pendulum Gravimeter

For the above described simple gravity pendulum, the suspended mass is swinging. An alternative measurement principle is the conical pendulum, where a suspended mass is rotated at a (constant) rotation speed. Like this, the mass follows the trajectory of a circle, and the centrifugal force is balanced by the gravitational force.

¹Occurring at the same time.

²Cf. https://www.si.edu/object/gulf-gravity-pendulum:nmah_892807, accessed on 31.01.2022.

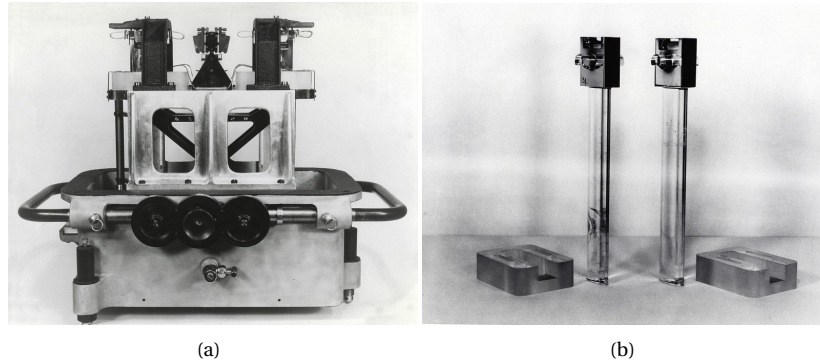


Figure 2.1: (a) Gulf Pendulum Apparatus consisting of two quartz pendulums shown individually in (b). From https://www.si.edu/object/gulf-gravity-pendulum:nmah_892807, accessed on 31.01.2022.

At known rotational speed, the (measured) deflection angle of the mass from the vertical yields the gravity acting on the rotated mass.

A technical implementation of this concept in a gravimeter to measure the Earth tides was developed by Rasson (1989). Here, a pair of two masses would be rotated and their displacement as a function of the acting gravity would be precisely measured by means of two individual capacitance transducers with fixed electrodes. A schematic of this setup is shown in Figure 2.2. For precise measurements, a very smooth rotation should be ensured. One limitation of this setup is that it should align the rotation axis with the gravity to ensure a smooth and uniform motion of the test masses. The instrument is reported to be setup for relative measurements and for non-optimal temperature conditions the instrument noise is in the order of $30\mu\text{Gal}$.

2.1.2 SPRING-BASED GRAVIMETER

The deflection of a spring is directly proportional to the (gravity) force deforming it. *Relative* changes in gravity are directly visible in a change of the spring deflection. In principle, the *absolute* deflection of the spring also yields the absolute gravity, however, this requires precise knowledge of the spring characteristic or in-situ calibration.

In this Subsection, first the zero-length-spring concept is introduced, which forms a core element in the subsequently introduced LaCoste-Romberg and Worden gravimeter, both representing two classical spring-based gravimeters that have been used for generations on Earth (Morelli et al., 1972). Following this, the Gulf gravimeter and optomechanical resonator concepts are introduced. All of them have in common that the core element of the instruments is a spring that deflects differently under variable gravity.

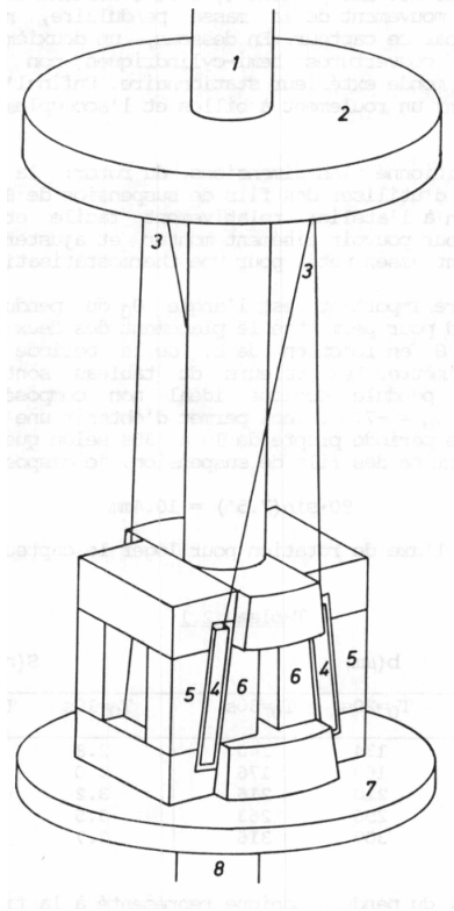


Figure 2.2: First layout of the conical pendulum gravimeter. Supported by two bearings (1, 8), the sensor head is rotated. Two test masses (4) are suspended bifilarly (3) and their displacement is measured by capacitance transducers, with pairs of fixed electrodes (5, 6). From Rasson (1989).

Zero-length Spring

In advance, the zero-length spring concept is introduced, as it is an element in the two following gravimeter types. The element was developed by LaCoste (1934), specifically to form part in the LaCoste-Romberg Gravimeter, originally designed as a vertical seismography of theoretically infinite period³.

Harding and Shepherd (2011) explain that while having a (theoretical) natural length of zero, a zero-length spring can still be described by Hooke's law, stating that the elongation from the equilibrium position is proportional to the force acting on the spring. With the zero-length spring having an equilibrium at zero-length, the elongation becomes equal to the length.

Obviously, a real spring fixed between two points cannot have a (physical) length of zero. Therefore, a zero-length spring will be created such that its physical length becomes the elongation. Like this, the initial length, i.e. under no (fixing) force, and thus without elongation, would be zero. Again, the zero initial length is only a theoretical concept, as the spring could not obtain this state. A special winding technique exists to achieve even a negative initial length. By then adding (straight) wire to the ends of the winded spring the initial length is increased to zero. While the physical length is measured in a straightforward way, the elongation of a spring is obtained by measuring the frequency of a (zero-length-) spring-mass system. (LaCoste, 1934)

How did this zero-length spring revolutionise gravimetric measurements? The next two gravimeter setups discussed below both show the combination of a beam and a test mass, suspended with (one or more) zero-length spring(s). As was proven, this setup has an (theoretical) infinite period (LaCoste, 1934) (generalised for variable setup geometries by LaCoste (1935)) at all mass positions, or beam inclinations. According to Bradner (1972), the infinite period, meaning an equilibrium of the spring force and the weight force, corresponds to an infinite sensitivity of the gravity sensing setup. In other words, this corresponds to a long period pendulum (Bradner, 1972), which can also be explained with Equation (2.1) where (small) gravity changes become increasingly influential to the pendulum period the longer the (base) period is. And this finally leads to comparably large (measurable) change in the equilibrium position for small changes in gravity using the zero-length spring.

Obviously, the sensitivity of this described setup depends on the ability to accurately measure changes in the equilibrium position. Two gravimeters demonstrating this are introduced in the following.

LaCoste-Romberg Gravimeter

The LaCoste-Romberg Gravimeter can be considered revolutionary in gravimeter development for the time of its development. The version shown in Figure 2.3 was a commercialisation based on the work by LaCoste (1934, 1935). As described above,

³The actual period was not infinite, but still longer than 1 minute, which still allows to measure seismic ground motion, if the period of this is sufficiently shorter than the period of the pendulum. Cf. <https://www.britannica.com/science/seismograph/Basic-principles-of-the-modern-seismograph>, accessed on 19.08.2022.

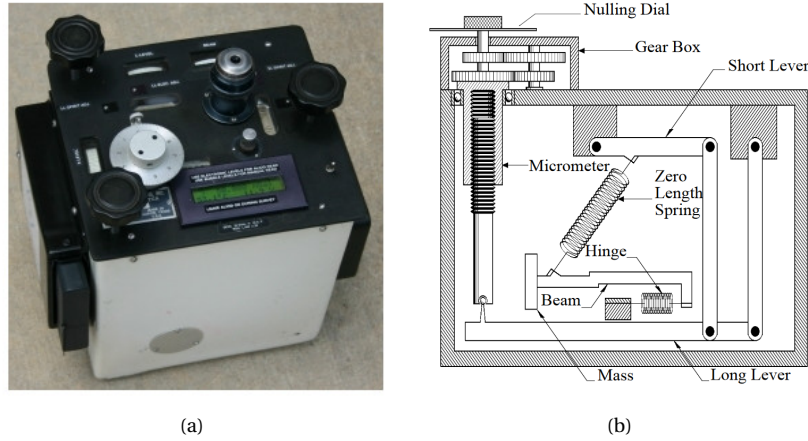


Figure 2.3: LaCoste-Romberg relative gravimeter. (a) Aliod G instrument type. From

<https://www.exiusa.com/item/gravity-meters/landr-aliod-g-meter>, accessed 01.02.2022. (b) Working principle of LaCoste-Romberg gravimeter. From http://userpage.fu-berlin.de/~geodyn/instruments/Manual_Lacoste_GD1.pdf, accessed on 31.10.2019.

a change in gravity causes a deflection in a beam and test mass setting, expressed by an elongation of the zero-length spring. Then, using a nulling dial, the setup is moved such that the test mass goes back to its initial position. The correction caused by the user with the nulling dial then yields a precise measure of the relative change in gravity with respect to a reference value, i.e. the null-position. The accuracy after calibration is about 1 part in 20,000, and a later added electronic feedback systems provides a precision of 0.01 mGal^4 . One Gal is equal to 0.01 m/s^2 , and thus one milligal is $1 \text{ mGal} = 10^{-5} \text{ m/s}^2$.

Worden Gravimeter

The Worden gravimeter works similarly to the LaCoste-Romberg gravimeter, however, as can be seen in Figure 2.4, the nulling dial and gear box is replaced by the combination of calibrated dial and a magnifying eyepiece. As detailed by Burger et al. (1992), changes in the zero-length spring suspended-mass-beam setup are directly observed by the beam moving out of the null position. The off-centred beam is then restored in its null position and the required restoring force for this gives the relative change in gravity that caused the off-centre movement of the beam that is directly connected to the moving test mass. Here, it is noted that the schematic in Figure 2.4 shows additionally a levelling mechanism for the instrument. Bonini and Woollard (1957) reported a relative accuracy (i.e. precision) of 0.3 mGal .

Both spring-based gravimeter can therefore be considered as *relative* gravimeters.

⁴Cf. http://userpage.fu-berlin.de/geodyn/instruments/Manual_Lacoste_GD1.pdf, accessed on 02.12.2022.

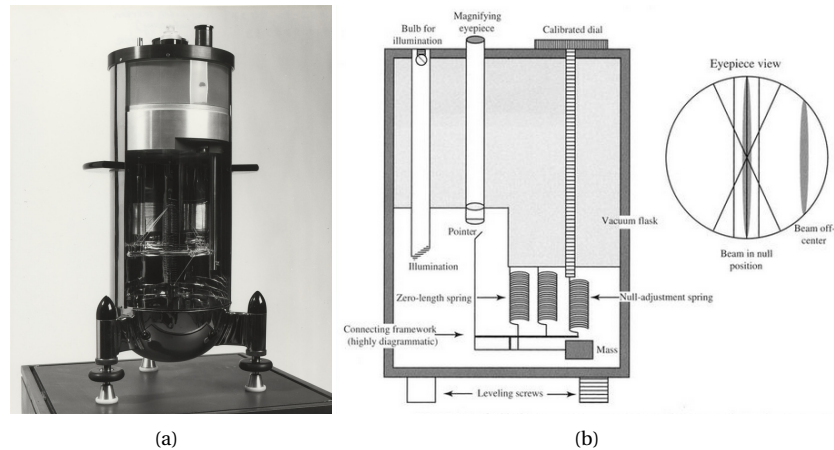


Figure 2.4: (a) Worden gravimeter display model with central element (fused quartz) enlarged by a factor 5 and case and dials slightly smaller than the original. From https://www.si.edu/object/nmah_865076, accessed on 31.01.2022. (b) Schematics of Worden gravimeter. From Burger et al. (1992).

Absolute measurements, however, can be obtained by calibration of the spring or with respect to an absolute gravimeter at the same measurement location.

Gulf Gravimeter

Another version of the spring-based gravimeter is the Gulf gravimeter, not to be confused with the Gulf Pendulum Apparatus (Subsection 2.1.1). Both were developed by Gulf Research & Development Corp. and the spring-based gravimeter was developed as improvement to the pendulum equipment. One example is at display at the National Museum of American History⁵ which is further described to use a helical spring (Figure 2.5) of which the (un-) winding is measured rather than the elongation with a precise rotation measurement. The instrument is stated to be designed by Archer Hoyt who also managed to allow subsurface operations off-shore for oil exploitation. It is stated that an relative accuracy of up to $0.05mGal$ was achieved with this displayed instrument, again being a relative gravimeter.

Optomechanical Resonator

The microscopic sensor developed by Hines et al. (2020) uses a suspended test mass similar to the above spring type gravimeter, however, using flat beams as springs (Figure 2.6 (a)). The achieved performance is reported to be suitable for gravity monitoring on Earth.

Here, use is made of the resonant frequency of this spring-mass system. Excited by a force or vibration, the deflection from the equilibrium position is ob-

⁵https://americanhistory.si.edu/collections/search/object/nmah_865070, accessed on 14.04.2020.

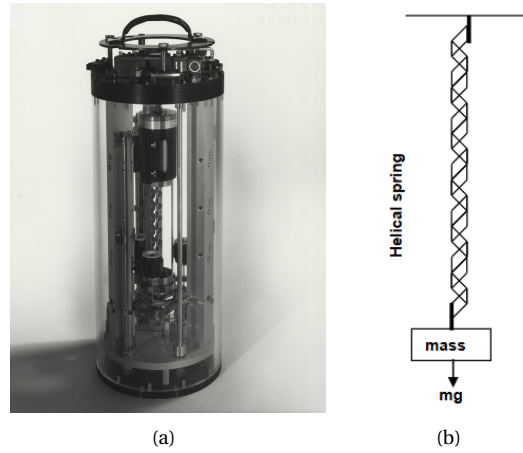


Figure 2.5: (a) Gulf gravimeter display model with central element being the helical spring clearly visible. From https://americanhistory.si.edu/collections/search/object/nmah_865070, accessed on 31.01.2022. (b) Schematics of Gulf Gravimeter. From Alsadi and Baban (2014).

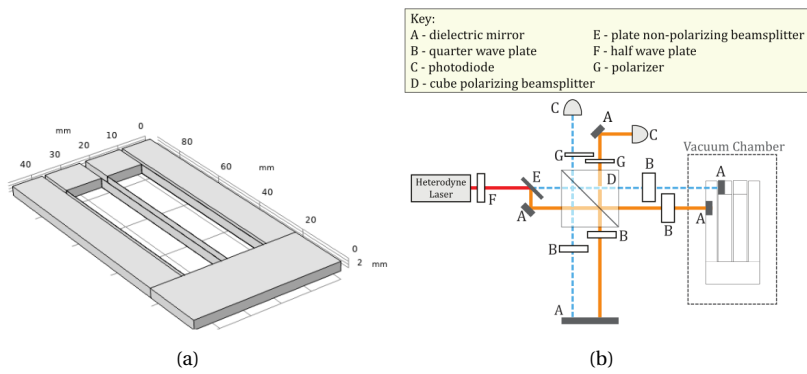


Figure 2.6: (a) Mechanical layout of optomechanical resonator (b) Laser interferometer setup to monitor the test mass displacement. Both from Hines et al. (2020).

served with laser interferometry (Figure 2.6 (b)), from which the acceleration can be computed. Overall, the sensor is lightweight (26 gram) and the volume is $92\text{mm} \times 48\text{mm} \times 3\text{mm}$. One reported issue was the gas damping, which, obviously, is not problematic in space. The noise floor is anticipated to be better than $10^{-11}\text{m/s}^2/\sqrt{\text{Hz}}$ ⁶ in the sub-Hertz sampling rate, where the thermal effect is the main noise contribution. No information on measurement bias and its mitigation was reported. Overall, this instrument is still under development, e.g. demanding an integration of the laser interferometer into the optomechanical sensor, potentially increasing the current compact sensor design. Likewise, without precise calibration of the springs, the instrument will return relative measurements.

2.1.3 FREE-FALL GRAVIMETER

The story of Galileo Galilei actually dropping two different masses from the leaning tower of Pisa around 1590 might be disputed regarding its actual happening (Segre, 1989), but the core principle that the motion of objects in gravity fields is independent of the mass of the object is a fundamental principle in gravity science. While on Earth the atmospheric drag of the test objects influences the flight time, the experiment was repeated by Apollo 15 astronaut David Scott on the Moon, demonstrating the same drop time for a hammer and feather⁷.

This principle is utilised in free-fall gravimeters. Dropping a test mass vertically and timing the drop time over a defined distance returns the gravity. Integrating the gravity g_0 (acceleration) twice over the fall time t_f , we obtain the travel distance s by a free-falling test mass in vacuum, under the assumption that the gravity is constant along the drop distance:

$$s = \iint_0^{t_f} g \, d^2t \quad (2.2)$$

$$s = \int_0^{t_f} (g t_f + v_0) \, dt \quad (2.3)$$

$$s = \frac{1}{2} g t_f^2 + v_0 t_f + s_0 \quad (2.4)$$

where v_0 and s_0 are the velocity and position at the beginning of the measurement, respectively. These can either be set to zero if the measurement is initiated with the freefall of the instrument, or adapted accordingly if the measurement is initiated with some offset below the separation to avoid error sources due to the release mechanism. Equation (2.4) can be rearranged for the gravity if the fall distance in a free-fall gravimeter is known and the fall time is measured by the instrument.

⁶The unit of “Signal” (here acceleration m/s^2) per square-root of frequency (Hz) refers to the (noise) amplitude spectral density. This is the square root of the (noise) power spectral density, which is measured per frequency (i.e. $1/\text{Hz}$). Cf. <https://www.ap.com/blog/fft-spectrum-and-spectral-densities-same-data-different-scaling/>, accessed on 19.08.2022.

⁷https://upload.wikimedia.org/wikipedia/commons/e/e8/Apollo_15_feather_and_hammer_drop.ogv, accessed on 03.02.2022.

Equation (2.4) can be extended to a polynomial of fourth order (Robertson, 2001), taking into account the vertical gravity gradient γ along the free fall distance (Equation (2.5)).

$$s = \frac{1}{2}gt_f^2 + v_0 t_f + s_0 + \gamma \left(\frac{1}{2}s_0 t_f^2 + \frac{1}{6}v_0 t_f^3 + \frac{1}{24}g_0 t_f^4 \right) \quad (2.5)$$

The free-air gradient γ , i.e. the gravity gradient due to larger distances from the centre of the spherical gravity-inducing body, is the derivative of gravity with respect to the distance r between the centre of the body (Equation (2.6), e.g. Talwani et al. (1973)).

$$\frac{\partial g}{\partial r} = -2 \frac{g}{r} \quad (2.6)$$

Free-fall timing is usually done by reflecting a laser beam from the falling test mass and comparing it by interferometry to a reference laser beam. On Earth, it is required to obtain a 1 nm precision to obtain an accuracy of 10 nm/s^2 (Van Camp et al., 2017). Rothleitner et al. (2009) present two implementations of free-fall absolute gravimeters using laser interferometry to track the falling object. Like on the Moon, the free-fall takes place in vacuum and the free fall lengths are 50 cm and 15 cm for the stationary and portable instrument versions, respectively. The test mass is repeatedly raised by a lift setup. The laser interferometry between test mass and instrument is done with two retro reflectors (CCM), while the retroreflector of the instrument is decoupled from the experiment using a SuperSpring. The concept is illustrated in Figure 2.7 for the stationary free-fall gravimeter. Generally, it is possible to double the freefall time by using a catapult-like mechanism that launches the test mass upwards from the bottom of the free-fall distance, adding the rise and fall time of the test mass.

A commercial implementation of a free-fall gravimeter is presented in Subsection 2.2.1. Overall, the free-fall gravimeter is an absolute gravimeter, but requires leveling to align the falling direction with the gravity vector.

2.1.4 COLD ATOM GRAVIMETER

The cold atom gravimeter, sometimes referred to as *Quantum Gravimeter* is an advanced form of the free-fall gravimeter (Subsection 2.1.3), where not a test mass, but rather a cloud of cold atoms are dropped to determine the acceleration (gravity) acting upon the atoms. A detailed overview of the functionality is provided by dos Santos and Bonvalot (2016), stating that the instrument creates an atom interferometer that is sensitive to acceleration using the wave property of matter. Here, the main steps of the measurement are described as listed below:

1. A cloud of atoms (order of magnitude of 10^8) of cold atoms (temperature of few μK) is concentrated in a magneto-optical trap.
2. Free fall-phase: The atoms are dropped vertically (or launched upwards to double the free fall time, improving the sensitivity).

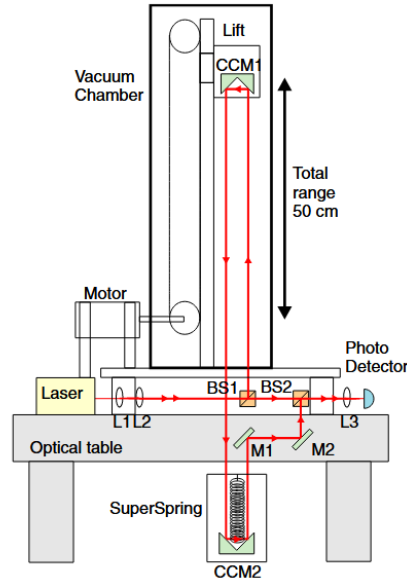


Figure 2.7: Free-fall gravimeter concept tracking the falling test mass with laser interferometry. This setup is realised by a He-Ne laser, lenses (L), mirrors (M), beam-splitter (BS), and retroreflectors (CCM). From Rothleitner et al. (2009).

3. An atom interferometer is created using three laser pulses to split the population in two, redirect and recombine the waves of the atomic matter.
4. The two-wave interferometer then compares the populations in its two outputs ports. The population, in turn, depends on the phase difference between the split atomic sub-population. Finally, this phase difference is related to the gravity acting on the atoms.

Like the “classical” free-fall gravimeter, the cold atom gravimeter is an *absolute* gravimeter. The first commercially available cold atom gravimeter is described in 2.2.3, also picturing the instrument. A proposed first step towards an application in space is presented in Subsection 2.3.6.

2.1.5 SUPERCONDUCTING GRAVIMETER

A test mass in form of a sphere, comparable to the test mass of the spring-based gravimeter (2.1.2), also forms the core element of the superconducting gravimeter. Rather than dispensing the test mass from a spring, however, a superconducting sphere is magnetically levitated with superconducting coils (Figure 2.8 (a)). Any displacement of this sphere can be used as a measure of changes in gravity. The sphere-coil is placed inside a Dewar vessel providing vacuum, which in turn is placed in liquid Helium at operating temperature of 4.2 K to allow the instrument operation at cryogenic temperatures (Goodkind, 1999). The liquid Helium is effectively the only part of the instrument demanding regular maintenance, a superconducting gravimeter during such refill is shown in Figure 2.8 (b).

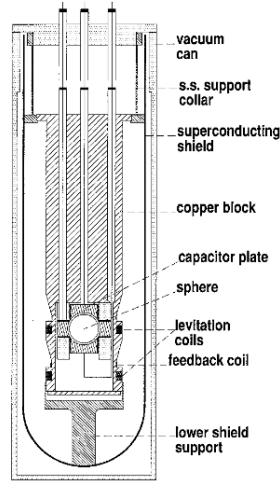


Figure 2.8: (a) Concept sketch of the core element of the superconducting gravimeter, that is the part inside the liquid Helium with the gravity sensing element. From Goodkind (1999). (b) Superconducting gravimeter of the Royal Observatory of Belgium in the underground Membach station during a Helium refill. Picture taken by the author.

Instead of measuring the displacement directly, it is also possible to introduce a feedback mechanism to hold the sphere in a constant position, deriving the changes in gravity from the changes in force required to hold the levitated sphere in place (Prothero Jr and Goodkind, 1968). Goodkind (1999) states that this type of instrument has very good long-term stability, while it is comparably difficult and resource-intensive to operate the instrument. Further, it is stated that it requires levelling for a 1D-measurement using two pendulum type tiltmeters on top of the Dewar vessel inside the liquid helium.

Overall, the instrument is, like classical spring-based gravimeter, a relative instrument. Using precise calibration for absolute measurements and the drift, however, it is possible to obtain absolute measurement with great precision (Merlet et al., 2021). A commercial version of this instrument type is introduced in Subsection 2.2.4.

2.1.6 VIBRATING STRING ACCELEROMETER

A concept fundamentally differing to those introduced in this Section so far is the vibrating string accelerometer (VSA). For the measurement, a mass is suspended from a conductive string (wire) or bar (*hereafter: string*) as shown in Figure 2.9 (a). Then, a current is applied to the conductive suspension piece, generating a Lorentz force and therefore causing a motion of the string perpendicular to the suspension direction at its natural frequency. (Talwani et al., 1973)

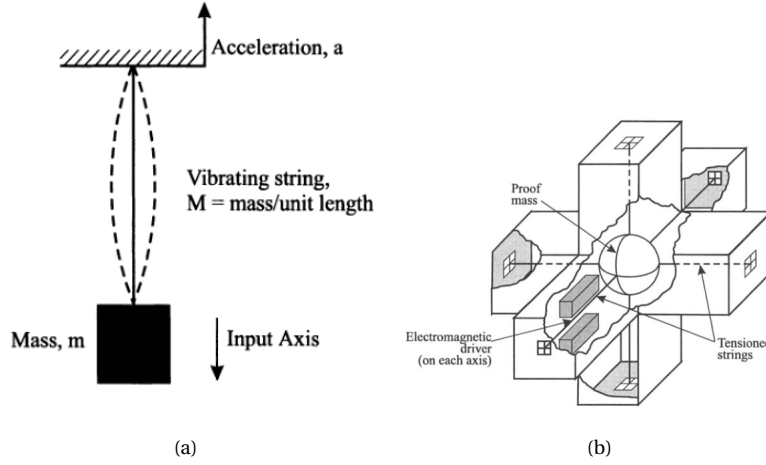


Figure 2.9: (a) Vibrating string accelerometer principle with suspended test mass to control the tensile strength as a function of acceleration (gravity). (b) Schematic of 3D-VSA with a test mass being fixed in position by 3 pairs of string allowing a 3D-measurement. Both from Lawrence (1998).

According to Lawrence (1998), this transverse resonant frequency can be measured as a function of the tensile stress of a string or bar. The tensile stress in the string, in turn, can be a function of the acceleration acting on the test mass and the natural frequency of the string is calculated as shown in Equation (2.7) (Lawrence, 1998).

$$f_0 = \frac{1}{2} \pi \sqrt{ma/M} \quad (2.7)$$

where m is the suspended mass, a , the acceleration (gravity) and M the string mass per unit length. Therefore, the instrument can be classified as an *absolute* gravimeter. Another version of this instrument is the double-string VSA, that has been deployed to the lunar surface and is discussed in Subsection 2.3.1. In principle, it is also possible to extend the design for a 3D-measurement as shown in Figure 2.9 (b), however, early developments in the 1950s lead to a reduced stability of the measurement (Lawrence, 1998).

Using this working principle, a MEMS microscopic version of a differential vibrating beam accelerometer (VBA) has been developed by Mustafazade et al. (2020) using the same functioning principle as the macroscopic instrument. The differential measurement allows to compensate for instrument drift, and it is reported that using this technique, this accelerometer is the first to have a long-term stability good enough to allow usage as a gravimeter. Initially designed for terrestrial application to monitor tides and seismic events with $\pm 1g_0$, it has a very fine resolution better than $10\mu Gal$. This accelerometer is, analogue to the double-string VSA (Subsection 2.3.1), suspending the test mass with two beams, and in principle

a 3D-measurement seems feasible by 3D-arrangement of three sensors.

2.1.7 MEMS GRAVIMETER

In principle, the terms accelerometer and gravimeter can be used interchangeably, as both instruments measure an acceleration. A distinction is proposed by Middlemiss et al. (2016), saying that an accelerometer only qualifies as a gravimeter, if the performance is good and stable enough to be called a gravimeter, i.e. by allowing to measure Earth tides. Thus, every gravimeter can be referred to as an accelerometer, but the inverse case is not always true. Extending this Section to all variable accelerometer types would be beyond the goal and scope of this work. Rather, two different Micro-Electro-Mechanical Systems (MEMS) accelerometer are presented, that shall be representative not only for accelerometers in general, but likewise of MEMS that stand example for the possible miniaturisation of gravimeters. This will be extended by the accelerometers used on Mars for gravimetric measurements (Subsection 2.3.3).

Generally, one disadvantage of MEMS is the bad signal-to-noise-ratio (SNR) however, recent advancements in the technology allowed e.g. to measure the Earth tides with a MEMS gravimeter. This MEMS device has a sensitivity of $40\mu Gal/\sqrt{Hz}$. The linear drift, however, is an additional problem with MEMS devices, even for this high performance MEMS gravimeter the linear drift of (reported) less than $150\mu Gal/day$ had to be characterised and removed. This can be done rather easily for long term measurements, but it is more challenging for shorter measurement periods. (Middlemiss et al., 2016)

Figure 2.10 (a) shows the spring-test mass MEMS device, which can generally be classified as a relative spring type accelerometer (gravimeter). A proof mass is suspended from a cantilever flexure (beam). To keep the measurement axis one-dimensional, two anti-springs are added to the bottom of the mass.

Anti-springs also have an additional benefit regarding the sensitivity of accelerometers with test masses. Independent of the sensing technique, there is a limitation to sense the test mass position, and therefore gravity, due to the instrument noise floor. To improve the reading, the accelerometer (gravimeter) natural frequency ω_0 can be reduced, as the device sensitivity is proportional to $1/\omega_0^2 = m/k$, the latter being the ratio of test mass m and the spring constant k . As for a mechanical setup, neither m can be increased ad infinitum, nor k decreased to zero, the anti-springs can aid in increasing the sensitivity. This can be best explained using Figure 2.11, where the two anti-springs compensate a (defined) proportion of the restoring force of the main accelerometer (gravimeter) spring, lowering its effective spring constant and henceforth increasing the instrument sensitivity. (Boom et al., 2017)

It should be noted that Middlemiss et al. (2016) reported some non-linearities of the setup with the anti-springs compared to linear force-displacement behaviour for the single cantilever. Finally, the test mass deflection is measured by means of a light-emitting diode (LED), paired with an optical shadow sensor.

Tang et al. (2019) present another high sensitivity MEMS gravimeter. The main

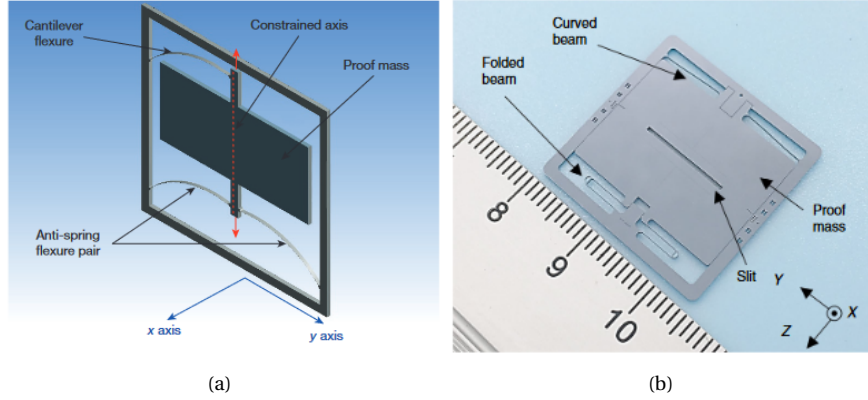


Figure 2.10: MEMS gravimeter spring-mass setups, the sizes refer only to the shown spring-mass setups, and therefore the full sensor with the optical displacement transducer will be larger. (a) From Middlemiss et al. (2016) with size ca. $10\text{mm} \times 10\text{mm} \times 0.2\text{mm}$. (b) MEMS gravimeter from Tang et al. (2019) with size of $25\text{mm} \times 25\text{mm} \times 0.4\text{mm}$.

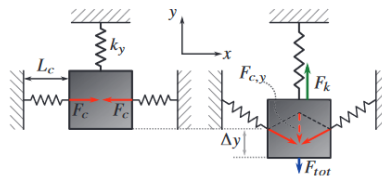


Figure 2.11: (a) Accelerometer proof mass in null-position, suspended from (vertical) spring with spring constant k_y and with two anti-spring shown horizontally and with preload F_c . (b) Test mass in displaced position, where the preloaded anti-spring compensate for some of the accelerometer restoring force F_k , effectively improving the sensitivity of the main spring. From Boom et al. (2017).

principle is comparable to the MEMS device described above, but as is shown in Figure 2.10 (b), the test mass is stabilised by a total of four beams, two curved and two folded beams. The position measurement here is done using again an optical displacement transducer, using a LED, the test mass slit (width of 0.5 mm) and a quadrant photodiode.

2.1.8 CLOCK

According to Einstein's general theory of relativity (Einstein, 1923), time, and therefore clocks, runs differently fast under the influence of gravitational *potential*, with massive bodies slowing them down. This effect, however, is already very small, yet measurable for the Earth, so that for example the oscillators onboard the GPS satellites are set to 10.229999999543 MHz on the Terrestrial surface so that they have the (faster) nominal frequency of 10.23 MHz on orbit at a larger distance from the Earth. (Schutz et al., 2004)

Therefore, the usage of very precise (atomic) clocks, can serve to absolutely measure the gravitational potential, making the “timing” approach a gravitation potential sensing method, but not directly a gravimeter. The option to use clocks for geodesy was studied by Mehlstäubler et al. (2018), concluding that the potential is less sensitive to local mass anomalies than gravimeters measuring the potential gradient, and therefore making clocks a good supplementary measurement to gravimeters. A single clock alone, however, can only provide absolute information on the absolute gravitational potential, but more measurement points would be needed to measure the potential field gradient, i.e. the gravity. In the next Subsection, the gravitational potential measured from orbit using spacecraft tracking is discussed, where a manifold of points allow computing the gradient of the potential, thus the gravity.

2.1.9 GRAVITATION FROM ORBIT

The last option introduced in this Section is the option to sense gravitation from orbit. Without perturbations, a satellite in orbit around a spherical celestial body will follow a Keplerian orbit. However, perturbations such as atmospheric drag (if applicable), solar radiation pressure etc. are generally present. To remove them, high performance accelerometers onboard the satellites are used, leaving only the influence of the gravitational field. In reality, no perfectly spherical celestial body exists and therefore, the satellite will be subject to inhomogeneities in the gravitational potential.

In fact, the satellites then can serve as test masses, which requires precise tracking of them. This can be either done from Earth (Andert et al., 2010), or from another spacecraft. The latter is foreseen as radio experiment of the Hera main spacecraft with the Juventas CubeSat to be deployed by it (Goldberg et al., 2019). Alternatively, two spacecraft following each other, such as the GRACE (Gravity Recovery and Climate Experiment) mission around Earth (Tapley and Reigber, 2002) and the GRAIL (Gravity Recovery and Interior Laboratory) mission around the Moon (Zuber et al., 2013) can be used to track gravity anomalies. For example, simply speaking, imagine that the leading satellite approaches a larger gravitational influence due to a positive (i.e. denser than the surrounding) mass anomaly. The leading spacecraft

will then be accelerated towards this mass anomaly earlier than its following companion. By (very) precisely measuring the distance between the two spacecraft, this (temporal) increase in distance can be sensed and information on the gravitational field and specifically the presence of the exemplary anomaly can be derived. An alternative tracking approach was proposed by Woodward et al. (2020), using tracking satellites in Geostationary or medium-Earth Orbit (GEO / MEO) to track spacecraft in low-Earth Orbit (LEO).

Like for the clocks (Subsection 2.1.8), this technique only returns the gravitational potential, and therefore the gradient of the field needs to be computed to obtain the gravity. Other than for the clock, the tracking data provides generally a larger number of data points (measurement locations), making this easier to achieve than for a single point. However, the difficulty of the inverse problem of the potential theory remains, meaning that the solution space of possible conditions causing the gravitational potential is non-unique (Michel and Fokas, 2008). In other words, measurements taken from orbit cannot trivially be propagated downwards to the surface or provide unique solutions on the interior structure of the orbited body, yet it can provide valuable constraints to it (Le Maistre et al., 2019). Therefore, gravitation from orbit can well be used to obtain a wealth of information on the orbited body, however, it generally cannot be used to mathematically replace a surface gravity reading.

2.2 COMMERCIAL GRAVIMETER OVERVIEW

Following the working principle overview in the previous Subsection, a brief overview of some commercial gravimeters is provided in this Subsection, making use of the introduced instrument types. A commercial implementation of the LaCoste-Romberg Gravimeter was already introduced in Subsection 2.1.2. A general overview of the figures of merit is provided in Section 2.4. For more details on the detailed figures of merit of the here-presented individual instruments, the interested reader is referred to the product websites (cf. caption of Figure 2.12).

2.2.1 MICRO-G LACOSTE: FG5-X

Van Camp (2003) referred to the most accurate available absolute gravimeter to be the FG5 from Micro-g LaCoste (Niebauer et al., 1995), a further development of the same company is the FG5-X instrument⁸ shown in Figure 2.12 (a). Clearly, the publication date of this statement has to be considered. The instrument is a free-fall gravimeter (Subsection 2.1.3) initially designed with a free-fall length of 20 cm, whereas the FG5-X has an extended freefall length of 33 cm, corresponding to a free-fall time of 0.260 ms (Equation (2.4)). The freefall time is measured using laser interferometer. The precision is $15\mu\text{Gal}/\sqrt{Hz}$ and the (delivered) mass is 150 kg.

⁸Cf. <http://microglacoste.com/product/fg5-x-absolute-gravimeter/>, accessed on 16.10.2019.



Figure 2.12: Overview of commercial gravimeters. (a) FG5-X. From <http://microglacoste.com/product/fg5-x-absolute-gravimeter/>. (b) CG-6 Autograv. From <https://scintrexltd.com/product/cg-6-autograv-gravity-meter/>. (c) Absolute Quantum Gravimeter. Form <https://www.muquans.com/product/absolute-quantum-gravimeter/>. (d) iGRAV Gravity Sensors. From <https://www.gwrinstruments.com/igrav-gravity-sensors.html>. All accessed on 14.02.2022.

2.2.2 SCINTREX: CG-5 AUTOGRAV

The next identified commercially available instrument is the Scintrex CG-5 Auto-Grav gravimeter (Figure 2.12 (b)), the successor of the CG-5. The instrument is a relative spring-based gravimeter (Subsection 2.1.2), where deviations from the zero-position due to variable gravity are compensated by electrostatic force, which provides the relative difference in gravitation (Aly et al., 2020). The cost is above US\$ 100,000 (Middlemiss et al., 2016).

2.2.3 ABSOLUTE QUANTUM GRAVIMETER

The reported first commercially available (dos Santos and Bonvalot, 2016) cold atom gravimeter (Subsection 2.1.4), is the Absolute Quantum Gravimeter (AQG, note the different names) from Muquans (μ quands)⁹ pictured in Figure 2.12 (c). It is described that continuous data acquisition can last from a few seconds up to several months. Moreover, the instrument has no moving parts, making it more reliable and less intensive with maintenance. The long term stability is better than $1\mu\text{Gal}$ (Ménoret et al., 2018). Additionally, the instrument can well cope with ground vibrations, using active damping, yet without the need of a mechanical isolation mechanism. The accuracy is reported to be a “few μGals ”.

2.2.4 IGRAV GRAVITY SENSOR

The iGRAV Gravity Sensor is the fourth and last introduced commercially available instrument, developed by GWR Instruments¹⁰. The superconducting gravimeter (Subsection 2.1.5) uses a Niobium (Nb) sphere and the Dewar has a volume of 16 litres for the liquid helium. Regarding the performance, it is worth to highlight the reported precision in the order of $0.05\mu\text{Gal}$ and a drift better than $0.5\mu\text{Gal/month}$.

2.2.5 EXCURSUS: BLUESEIS3A

While not being a gravimeter, the blueSeis-3A by iXblue is a novel fibre-optic gyroscope, developed for rotational seismography. Other than gravimeters, the gyroscope would not measure acceleration, but 3-axes rotation rates at the surface location (Bernauer et al., 2018). The instrument makes use of the Sagnac effect (Sagnac, 1913), where two counter propagating beams in a rotating optical fibre loop are used (Bernauer et al., 2020). The difference in the optical path lengths is then measured by the phase difference of the two beams, yielding the angular velocity of the rotation. While therefore not measuring gravity (acceleration), such a gyroscope would allow to precisely measure the dynamical (rotational) effects on a surface and the variation thereof (Bernauer et al., 2020), allowing to better understand dynamical contributions in a gravity measurement. Currently, a gyroscope prototype, paired with acceleration sensors for space exploration is under development (Bernauer et al., 2020).

⁹Cf. https://www.muquans.com/wp-content/uploads/2019/03/muquans_aqg.pdf, accessed on 15.06.2020.

¹⁰Cf. <https://www.gwrinstruments.com/igrav-gravity-sensors.html>, accessed on 14.02.2022.

2.3 PROPOSED AND FLOWN SPACE GRAVIMETER

Complementary to the overview of gravimeter sensing principles (Section 2.1) and commercially available gravimeters (Section 2.2), the limited number of space gravimeters is reviewed in this Section. This is done in two steps, firstly presenting heritage instrument that measured gravity on an extraterrestrial surface, namely the two Apollo 17 gravimeter experiments (Subsections 2.3.1 and 2.3.2) and the accelerometers used on Mars for a gravimetric study (Subsection 2.3.3). Secondly, a total of four proposed concepts for space gravimetry are presented in Subsections 2.3.4 through 2.3.7.

2.3.1 APOLLO 17: TRAVERSE GRAVIMETER EXPERIMENT

The last Apollo mission carried the first two gravimeters to the surface of an extraterrestrial body. While the *Lunar Surface Gravimeter* (Subsection 2.3.2) did not function as planned (Chapin, 2000), the *Traverse Gravimeter Experiment* (TGE) described by Buck (1973) worked very well, providing relative gravity measurements at the Apollo 17 landing site, located in the valley of Taurus-Littrow as part of the Mare Serenitatis region¹¹. The measurement principle used a double-stringed vibrating string accelerometer, where the second string can be considered as a modification compared to the single-string instrument introduced in Section 2.1.6. The Lunar gravity was also measured relative to an absolute gravity measurement on Earth with an accuracy of the relative measurement between Earth and the Moon of 5 mGal reported by Talwani et al. (1973). Here, the relatively measured gravity at the landing site is presented with an absolute value of 162,694 mGal, nearly identical to the value 162,694.6 mGal reported by Lyndon (1973). During a total of 26 readings (thereof 22 usable, including 2 debiasing measurements) during three extra-vehicular activities (EVAs) traverses, the gravity deviated from +4.7 mGal to -44.8 mGal (-1.3 mGal to -50.8 mGal with empirical correction of -6.0 mGal to all measurement taken on the Lunar Rover Vehicle (LRV), rather than the lunar surface) relative to the landing site (Lyndon, 1973). The debiasing was performed by astronauts, setting the instrument up in the inverted (upside-down) position.

The gravimeter drift during the EVA activities, lasting ca. 7 hours, was determined preflight to be quasi-zero, which can also be explained by the temperature control within 0.01 K realised with two warm ovens stacked around the instrument (Lyndon, 1973). The design layout of the instrument is shown in Figure 2.13 with a weight of 14.6 kg and a volume of $50.8 \times 27.9 \times 24.8 \text{ cm}^3$ ¹². The TGE uses a levelling mechanism consisting of motorised gimbal assemblies and this mechanism works in a cone with an opening angle of 15° around the vertical.

2.3.2 APOLLO 17: LUNAR SURFACE GRAVIMETER EXPERIMENT

Next to the TGE described in the previous Subsection, the Apollo 17 astronauts also deployed the Lunar Surface Gravimeter Experiment (LSG) on the Lunar Surface as described in the following according to Giganti et al. (1971). The prime science objective of the instrument was to detect gravitational waves and the secondary

¹¹Cf. <https://www.hq.nasa.gov/alsj/a17/a17.site.html>, accessed on 02.03.2020.

¹²Cf. <https://www.hq.nasa.gov/alsj/a17/a17-TGE.html>, accessed on 02.03.2020.

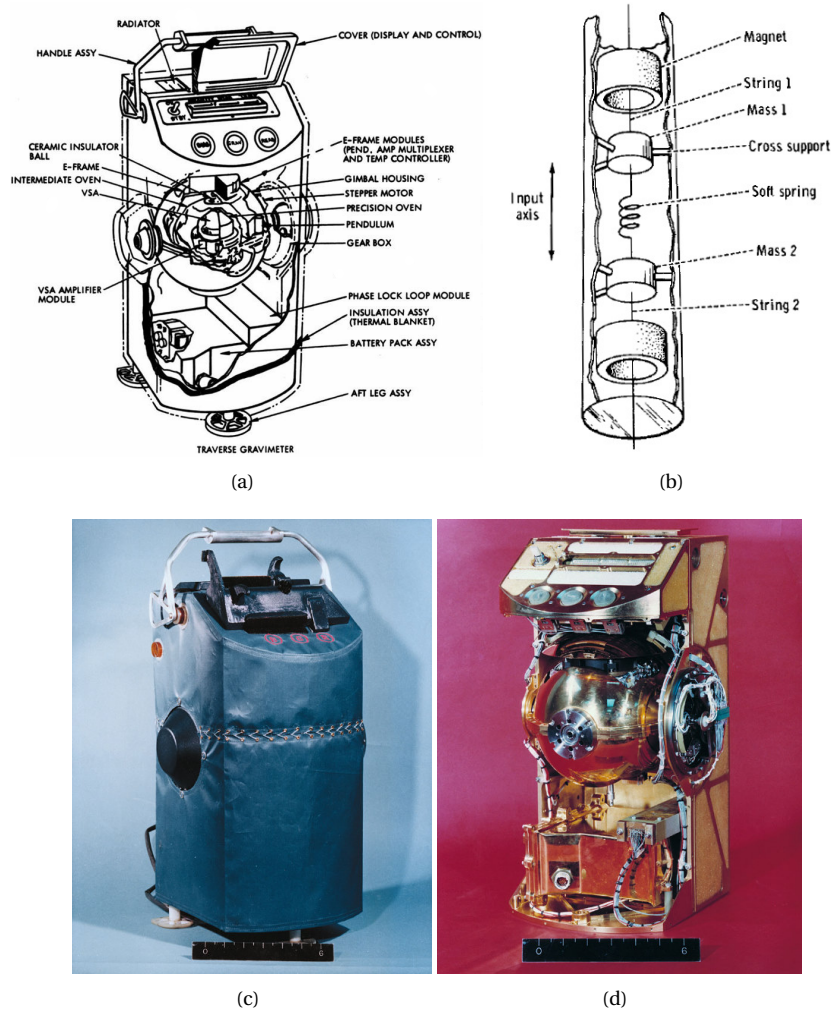


Figure 2.13: The Traverse Gravimeter Experiment as successfully flown on the Apollo 17 mission. (a) Detailed cutaway sketch of the inside of the gravimeter showing i.e. the double-string VSA, the levelling mechanism and the thermal insulation. (b) Detailed working principle of double-string VSA. Both from Lyndon (1973). (c) and (d) show the instrument closed and opened, respectively, the scale is in inch. From <https://www.hq.nasa.gov/alsj/a17/a17-TGE.html>, accessed on 02.03.2020.

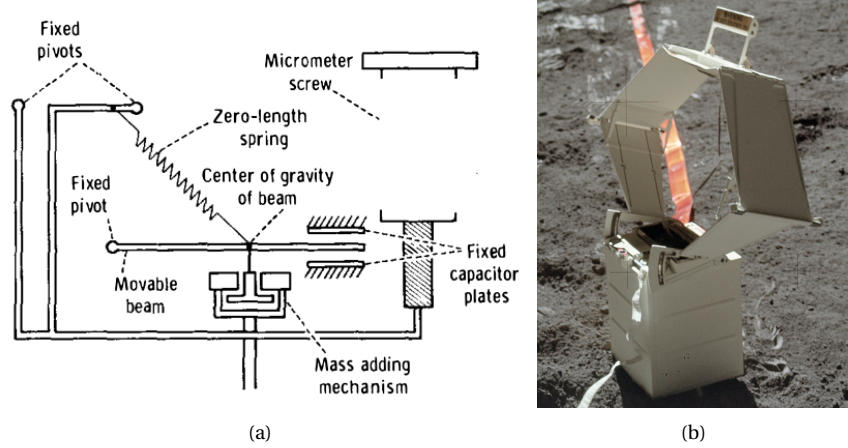


Figure 2.14: (a) Working principle of lunar surface gravimeter. The setup of movable beam (balance) with zero-length spring and micrometer screw resembles the design of the LaCoste-Romberg design (Figure 2.3), with both instruments being developed by the same persons. From Lyndon (1973). (b) LSG deployed to the lunar surface with a total mass of 12.7 kg. The sunshield and ribbon cable for power supply are clearly visible. From <https://nssdc.gsfc.nasa.gov/nmc/experiment/display.action?id=1972-096C-09>, accessed on 14.02.2022.

objective was to monitor the lunar tides. As indicated in the previous Section, this instrument malfunctioned because the beam balancing mechanism did not work as planned. As the mass spring and lever sensor was developed by LaCoste and Romberg, there are clear similarities visible to the spring-based LaCoste-Romberg gravimeter described in Section 2.1.2.

The instrument (Figure 2.14) measures accelerations in the frequency range from 0 – 16 Hz using a sensitive balance equipped with a mass, zero-length spring and lever system. The balance deflection is measured with two fixed capacitor plates. The balancing is done with the above mentioned mass adding mechanism. The capacitor plates are parallel to the undistorted, i.e. the horizontal beam. The gravimeter can operate with and without an implemented feedback mechanism. If the balanced beam leaves its centred position due to changes in the gravity, an error voltage is created from the changes in the two capacitances between plate and beam, and beam and plate, respectively. This error voltages can directly be read as a change in gravity, or, it is used to control a restoring electrostatic force to centre the beam.

The sensitivity of the instrument is stated to be ca. 1 part in 10^{11} of the lunar gravity. Considering the reported values of the found lunar gravity in the previous Subsection, this corresponds to a sensitivity of ca. $1.6 \text{ nGal} = 16 \text{ pm/s}^2$. In order to ensure this precision, there is a strong need for precise temperature control as the spring behaviour generally depends on the temperature. The thermal control system was sized to maintain the spring temperature to 1 millidegree throughout the long lunar

Table 2.1: Performance specifications of Northrop Grumman LN-200S IMU as flown on Mars Curiosity Rover with diverse heritage from other space missions, including the Spirit and Opportunity rovers. Data sheet obtained from <https://www.yumpu.com/en/document/read/36659626/ln-200s-brochure-northrop-grumman-corporation>, accessed on 02.06.2020.

Quantity	Value
Bias Repeatability	$300\mu\text{g}, 1\sigma$
Noise	$35\mu\text{g}/\sqrt{\text{Hz}}$
Scale Factor Accuracy	300 ppm, 1σ
Input Axis Alignment	0.1 mrad
Max. Input Acceleration	40 g

day and night cycle.

2.3.3 CURIOSITY ROVER: ACCELEROMETER

The (conditional) interchangeability of the terms gravimeter and accelerometer has no better prime example than the work performed by Lewis et al. (2019). The on-board inertial-measurement unit (IMU) of the Mars Curiosity rover equipped with a three-axis gyroscope and a solid-state silicon MEMS accelerometer, originally designed for navigation purposes, was re-calibrated to obtain local gravity measurements. The relevant specifications of the Northrop Grumman LN-200S IMU are listed in Table 2.1. It is stated that the IMU accelerometer and gyros have the lowest white noise in the medium accuracy IMU class. The alignment between the axis is stated to be better than 0.1 mrad.

The calibration is well described in the supplementary material¹³ that uses a non-linear least squares regression to model the Bouguer anomaly g_B (Section 7.1) to characterise the subsurface density from the Curiosity Bradbury landing site to the lower slopes of Mount Sharp. The best fitting result indicates a density of $1680 \pm 180 \text{ kg/m}^3$, indicating high porosity of the sedimentary rocks in Gale crater.

2.3.4 AGEX GRAVIMETER/B-GRAVE

Leaving the above-introduced past experiments that have flown to space, there have been some proposed instruments for previously considered and future missions. The “Belgium-Astro-Gravimeter” (B-Grave) for the in-situ surface gravity measurements of an asteroid has been described in detail by van Ruymbeke et al. (2016). The target accuracy of the local gravity is stated to be 1 %. This conceptual instrument was proposed to be part of the geophysical instrument package of the Asteroid Geophysical Explorer (AGEX), proposed by a Royal Observatory of Belgium-lead consortium as part of the precursor of the Hera mission, the Asteroid Impact Mission (AIM) (Karatekin, 2016). AGEX was planned to consist of two 3U CubeSats with geophysical packages. One of the CubeSats would land, carrying

¹³Cf. https://science.sciencemag.org/content/sci/suppl/2019/01/30/363.6426.535.DC1/aat0738_Lewis_SM.pdf, accessed on 02.06.2020.

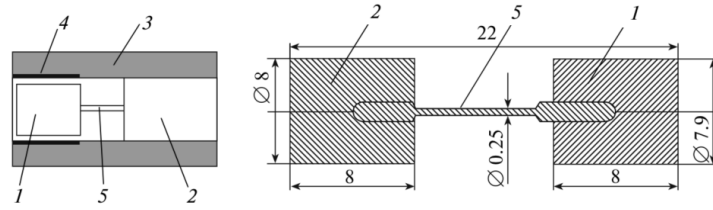


Figure 2.15: GRAS-F seismogravimeter layout (left) and technical drawing (right) of seismogravimeter. Subcomponents are (1) cylindrical proof mass, (2) cylindrical mounting structure, (3) tube structure, (4) four electrodes forming two pairs of electrodes for differential capacitance measurement and (5) round bending spring (rod). Figure adapted from Manukin et al. (2010b).

the B-Grave gravimeter¹⁴. The gravimeter is reported to be designed for a gravity environment of $50\mu m/s^2$ and to be tested on Earth in near-vertical position.

This proposed instrument is a spring-based gravimeter, using a cantilever beam (spring). To measure its deformation under gravity, a capacitive transducer is used comparable to the LSG (Subsection 2.3.2). The capacitance of a parallel plate capacitor is well-known (Equation (4.5)) which is inversely proportional to the distance between the two plates. If the beam is considered as one of the plates, a change in the distance, or, deflection, can be measured by measuring the capacitance. Positioning the spring between two capacitor plates allows to take a differential capacitance measurement. It is proposed further that the blade shall be rotated along its long axis to remove systematic bias errors and to eliminate the need of levelling of the instrument. To increase the deformation, a tip mass is added to the blade to increase the deformation under low gravity. Simulated low-gravity tests were done by setting up the blade vertically on a tilt table and deflecting the blade with small angles from the vertical. (Van Ruymbeke et al., 2018)

2.3.5 GRAS-F BARBELL SEISMOGRAVIMETER DESIGN

The *Compact High-Sensitivity Accelerometer-Seismometer* was first introduced by Manukin et al. (2010a), a three-axis instrument for “gravi-inertial” measurements. Here, “gravi-inertial” does not only include the general definition of gravity used in this work (i.e. gravitational field including dynamical accelerations and variations thereof), but it explicitly also includes seismology. Generally, this type can be classified as a spring-based gravimeter.

Similar to the concept described in the previous Subsection, the GRAS-F proposal consists of a cylindrical test mass connected with a circular beam. Again, the movement of the test mass is monitored by means of a capacitive transducer (Figure 2.15). The capacitance measurement is therefore not only done in one axis but in two axes, with two perpendicular pairs of sensing electrodes.

¹⁴Cf. https://www.esa.int/Space_Safety/Hera/AGEX, accessed on 13.12.2022.

2.3.6 SPACE-BASED COLD ATOM GRAVIMETER

In Subsection 2.1.4, the cold atom gravity sensing technique has been introduced. Regarding future applications in space, some early proposals have been made, as the application of cold atom gravimeters in space could provide better results, while the measurement principle will be hard to implement for exploration missions. A first step towards such a goal has been proposed by Devani et al. (2020) that would demonstrate cold atom trapping onboard a 6U CubeSat in Earth-orbit. Similarly, using two cold-atom gravity measurements for gradiometry, i.e. measuring the gravity gradient, has been proposed by Trimeche et al. (2019) for measuring the Earth gravity field from orbit (Subsection 2.1.9) that would use two different atom interferometers. With the instrument performance being modelled to a sensitivity of $5mE/\sqrt{Hz}$ ¹⁵, it is stated that the gravity field recovery could be improved compared to the GOCE mission data, yet requiring a realistic mission scenario. Moreover, it is required to improve the volume, mass and power budgets for this concept to be well implementable for a space flight mission.

These concepts, however, address mainly gravity measurements from orbit, e.g. around the Earth. Yet, the mentioned technology demonstration could become a first technological step towards also performing cold atom gravimetry on a landed instrument on an extraterrestrial surface.

2.3.7 VEGA

Another early concept of a space gravimeter proposed by Carroll et al. (2016) is the VECtor Gravimeter/Accelerometer (VEGA) instrument. This instrument uses a not nearer defined “accelerometer” for the gravity measurement (Carroll, 2015). A first mechanical breadboard of the instrument has the volume of $10cm \times 10cm \times 15cm$ at a mass of about 1.5 kg. It is claimed further that the accuracy is approaching 1 nano-g and that the vector directional reconstruction is better than 1 arcmin, making it suitable for asteroid gravimetry. A measurement is approximated to take about 10 minutes. While the amount of publicly available information is very limited, an additional source¹⁶ states that the instrument measures without bias and that a deployment to the Moon could be possible.

2.4 FIGURES OF MERIT

Based on the thus far presented wide range of gravimeter types in this Section, it is now possible to provide a quantitative overview. The different figures of merit have been compiled in Table 2.2, based on the different instrument types (Section 2.1), the commercial gravimeters (Section 2.2), and the space gravimeters (Section 2.3). The main figures of merit presented are the precision, accuracy and noise. Moreover, the need for levelling and the type (absolute vs. relative) are listed. Clearly,

¹⁵E = Eotvos, the unit of gravity gradient over a distance. One Eotvos corresponds to $10^{-9}m/s^2/m = 10^{-9}1/s^2$.

¹⁶Cf. <https://lunar-landing.arc.nasa.gov/sites/default/files/presentations/2018.01%20LSLMW%20Lunar%20Surface%20Gravimetry%20Science%20opportunities%20poster.pdf> accessed on 14.02.2022.

this Table remains incomplete, as often the provided information on an instrument does not provide all key numbers. Also, the values presented should serve as a first orientation, as it cannot be claimed exhaustively that no better-performing gravimeter of the respective type exists.

The presented numbers can be compared to the minimum GRASS gravimeter accuracy target of 1 % of the surface gravity of Dimorphos, thus $1\% \cdot 4.5mGal = 0.045mGal$. The precisions of all instrument except the Worden gravimeter are better than the target accuracy, but precise in-situ calibration will be required to also obtain such levels of accuracy. The accuracy of the VSA would not be sufficient to meet the target accuracy. Regarding the cold atom gravimeter, the accuracy is even one order of magnitude better than the GRASS target, but implementing this instrument type in the mission with the given constraints will be difficult, as discussed further in the next Section.

2.5 GRAVIMETER CONCEPT DISCUSSION FOR SMALL BODY APPLICATION

Recalling the first research question (i), regarding the best suited small gravity sensing principle in the context of Solar System exploration, it is now possible to compare and recommend a working principle for future study in the context of Solar System gravimetry following the review of different concepts in this Chapter. Here, the nine principles introduced in Section 2.1 will be considered for this discussion. While these represent the most common gravimeter principles, there is no claim to completely cover all existing principles.

In a first step, the following hard criteria are formulated, where a non-fulfilment directly leads to an exclusion from the further discussion. Hence, the criteria are formulated such, that a realisation demands fulfilment of the following:

1. No human (astronaut) intervention shall be needed to operate, level, or maintain the instrument working.
2. The working principle shall be undistorted from the landing attitude, meaning that misalignment with the gravity vector still allows measurement.

Criterion 1: Within the goal and scope of this work, the gravimeter shall form part of *robotic* exploration missions, leading to this requirement. Even during later use in human missions, e.g. to the Moon and Mars, astronaut time will be limited, and intervention should be minimised for gravimetric surveys. Overall, all concepts fulfil this criterion, as no technical aspect cannot be automated.

Criterion 2: While all principles demand a levelling to measure the full gravity, a difference has to be made between the concepts that can still measure some fraction of the gravity, if not aligned, and those that will not work given the nature of the principle. The latter have to be excluded, as explained in the following. A *simple gravity pendulum* (Subsection 2.1.1) gravimeter is designed for swinging in one direction, allowing to minimise the friction losses in the pivot of the pendulum. Tilting the pendulum is therefore not feasible. Likewise, the rotation axis of the *conical*

Table 2.2: Figures of merit of different gravimeter types. The measurement type being absolute (abs.) or relative (rel.) is provided, as well as the need for levelling (level.).

Type	Abs./Rel.	Level.?	Precision	Accuracy	Noise
Gravity Pendulum	Abs.	Yes	0.01mGal		
Conical Pendulum	Rel.	Yes			30μGal
LaCoste-Romberg	Rel.	No	0.01mGal	1 part in 20000	
Worden	Rel.	No	0.3mGal		
Gulf	Rel.	Yes	0.05mGal		
Optomechanical Resonator	Rel.	No			$10^{-11} m/s^2/\sqrt{Hz}$
Free-fall	Abs.	Yes	$15\mu Gal/\sqrt{Hz}$		
Cold Atom/Absolute Quantum	Abs.	Yes		few μGals	
Superconducting	Rel.	Yes	0.05μGal		
Vibrating String	Abs.	(Yes) ^a		5mGal	
MEMS	Rel.	No	Only sensitivity	provided:	$40\mu Gal/\sqrt{Hz}$

^a3D-setup also possible.

pendulum gravimeter should be aligned with the gravity vector to ensure a uniform motion of the rotated test masses. The *free-fall gravimeter* (Subsection 2.1.3) and likewise the related *cold atom* gravimeter test objects fall, by design, in the direction of the gravity. Again, a tilt is not possible, excluding all of these principles.



This leads to a total of six remaining principles, that will be discussed below qualitatively with a focus on applicability to an exploration mission with tight constraints and the degree of required adaptation.

Spring-based gravimeter: Generally, a spring-based gravimeter is a good choice for Solar System gravimetry due to the, comparably, simple design. However, the applicability strongly depends on the spring and instrument design. For example, the Gulf gravimeter with helical spring requires vertical alignment, whereas the other presented gravimeters (Subsection 2.1.2) will also work with inclinations relative to the gravity vector, only returning the corresponding fraction of the gravity. This generally requires the spring setup to be designed for a one-dimensional sensitivity. However, if placed upside down, the presented zero-length spring setup will no longer work, one could refer to a half-dimensional operational range. Speaking more generally, there is therefore a need for a spring design and gravimeter setup that allow a measurement in 3 dimensions, working completely independent of the instruments attitude. Here, the optomechanical resonator provides a good robustness, allowing to measure in all three dimensions and both directions. The laser interferometric displacement transducer as presented appears still to be the design driver compared to the compact spring-mass system. Likewise, the GRAS-F-Gravimeter (Subsection 2.3.5) is a generally good concept working independent of measurement orientation and direction. Here, a potential problem might be the cross-coupling between the two measurement directions. Overall, the “classical” three spring-based gravimeters as presented in this Chapter are henceforth not directly suitable, whereas the optomechanical resonator and GRAS-F principles are promising solutions, especially considering the need to comply with arbitrary measurement attitudes and 3D-vector measurements. One challenge of the spring-based designs to be addressed is the measurement bias and drift.

Superconducting gravimeter: In theory, it should be possible to extend the one-dimensional levitation of the superconducting sphere to a 3D-setup, with three pairs of coils, which is why this instrument type was not discarded above (Criterion 2). However, the one-axis instrument alone is a complex system and one can expect a more complex instrument that has to be adapted both to a 3D-measurement and to space. Furthermore, one must not make a mistake as in “space is cold”. While this is true, this is not necessarily true neither for extraterrestrial surfaces nor the interior of spacecraft. The possible argument that the required cryogenic temperatures are easier to achieve in space therefore disqualifies.

Vibrating String Accelerometer: As was shown, this type generally qualifies for a 3D-measurement, as has been laid out with some early development works on this. One point of concern that can be identified here is the possible cross-coupling between the three pairs of strings attached to the single central tip mass. If a single-

axis acceleration moves the tip mass one-dimensionally, the two sensing elements in both orthogonal directions will be affected and the tension in these pairs will change. Separating the cross-coupling from possible accelerations adds complexity to the instrument and sensing unit, but it should, in theory, be possible to distinguish these effects. This should be achievable, as an acceleration in one direction would have the same effect on all other four strings that are orthogonal to that one direction.

MEMS gravimeter: The MEMS systems presented here can be regarded as specific (miniaturised) types of spring-based gravimeter. Generally, these systems should be likewise applicable as the spring-based gravimeters. Again, the challenge for measurement bias and drift persists. A general challenge is the production of MEMS devices, which is complex to setup in the first place, and generally, only large production numbers can decrease the cost per piece. Additionally, metallic microscopic structures needed for MEMS gravimeter are prone to internal stresses, strongly depending on the deposition conditions and material combination (Korlyakov et al., 2018). Furthermore, MEMS are fragile (Yunas et al., 2020), which poses an additional challenge for Solar System exploration, e.g. due to vibration and shock loads.

Clock: To measure relativistic effects on time, very precise and thus complex clocks will be required, making the use difficult for exploration missions. However, the concept is interesting and should be considered further for Solar System exploration, e.g. due to the limited sensitivity of local mass anomalies. Regardless, the focus should be for large objects such as (spherical) moons and planets, as the relativistic effect is obviously (much) larger than for small Solar System bodies. To measure the gravitational potential of small bodies, the required precision of clocks forming part in a space mission can be expected to be currently beyond reach.

Gravitation from orbit: This concept should certainly be considered as part of Solar System gravimetry, especially as any delivered landing element often is deployed by an orbiting spacecraft from orbit that can be used for this purpose. Here, precise reconstruction of the landing trajectory can possibly even improve the gravitation field reconstruction. As discussed, this element alone cannot serve as replacement to surface gravimetry, but it can well be used as complement, especially in the context of small bodies, to constrain the gravitational potential field and the total mass of the body.



In conclusion of this Chapter and considering the requirement to obtain a 3D-surface gravimeter that can operate independently of measurement (landing) attitude and the local surface gravity vector orientation, a spring-based gravimeter appears to be the best suited sensing type. The superconducting and vibrating-string gravimeter will introduce additional challenges on the adaptation for a 3D-measurement and especially the former type is technologically challenging. Using clocks to measure the gravitational field should be used for larger gravitational fields, making them less suitable for small bodies. Gravitation from orbit will play an important role, but should be regarded as supplement to the surface gravime-

ter, not as alternative. Another possible supplement is the introduced commercial fibre-optic gyroscope, which might aid in measuring gravity, i.e. by allowing to understand the dynamical contributions forming part in the gravity measurement. For the spring-based gravimeter, from a development perspective, a macroscopic design promises a more straightforward development path and it can be expected that ensuring the structural integrity of the gravimeter to be developed will be easier for a macroscopic system compared to a microscopic MEMS system. However, this does not mean that based on the macroscopic developed system no MEMS system can be developed from this as a future continuation of the development.

The design and development of a macroscopic spring-based gravimeter will be presented in the next Chapter. Considering the only successful flown space gravimeter, the TGE (Subsection 2.3.1), the reported accuracy of the *absolute* Lunar gravity relative to the calibration performed on Earth was $5mGal$. For the GRASS gravimeter, the target absolute accuracy is 1 % of the surface gravity, being $4.5mGal$ and thus an increase in absolute accuracy of a factor 100 is required for the new instrument. Inversely, the *relative* accuracy w.r.t. the surface gravity of 1 % for Dimorphos is not as demanding as was the case for the Moon. On the lunar soil, the relative accuracy was about $5mGal/162,694mGal = 0.003\%$.

Some of the elements forming part in the spring-based gravimeters presented in this Chapter will form part in the development, while the overall instrument can be considered as new instrument type, a gravimeter designed for Solar System gravimetry. One limitation of the spring-based gravimeter is that it requires calibration of the spring, as discussed above for the TGE to obtain an absolute gravity measurement. Next to detailed characterisation on Earth, in-situ calibration should be performed during, or just prior, the instrument operation on the extraterrestrial surface. This is because the spring might change their properties during the long ground storage and spaceflight in the harsh space environment. A first option for such calibration has been presented in Subsection 2.3.2, where the lunar surface gravimeter experiment was equipped with two fixed capacitor plates around the gravimeter spring setup. Gravity-induced changes of the beam between the electrodes created an error voltage, either directly interpretable for changes in gravity, or controlling a restoring electrostatic force for a feedback mechanism (Giganti et al., 1971). An alternative approach for calibration could be to use the known centrifugal force for a given rotation speed. Here, the rotated spring would require to be offset from the rotation axis.

2.6 SUMMARY CHAPTER 2

In this Chapter, various gravimeter concepts have been introduced and discussed with respect to the framework of this research. First, a variety of different gravity measurement concepts was introduced in Section 2.1, followed by a brief overview of commercially available (terrestrial) gravimeters (Section 2.2). After this, gravimeter experiments that have been performed on the Moon and Mars were introduced, followed by some proposed gravimeter concepts that do not yet have spaceflight

heritage (Section 2.3).

Based on these information, a qualitative discussion of the different concepts was made in Section 2.5 to identify the most promising concept for a surface gravimeter for Solar System exploration missions, specifically considering the absence of a complex levelling mechanism and hence the need to comply with arbitrary measurement attitude. Overall, the most promising technology is the spring-based gravimeter, due to its comparably less complex technology. To facilitate the development, a focus will be laid upon macroscopic setups, while the possibility to transfer the functioning to a MEMS device later on is maintained. It is ascertained that additional measures will be required to ensure obtaining an absolute gravity measurement, rather than solely a relative gravity measurement.

The next Chapter will present the development of the novel spring-based gravimeter for extraterrestrial surfaces. This development will partially use and combine different individual aspects of the spring-based concepts presented in this Chapter to comply with the here-relevant requirements.

3

GRAVIMETER ADAPTATION FOR SOLAR SYSTEM GRAVIMETRY

The trade-off between different gravity sensing concepts made in the previous Chapter concluded that a spring-based gravimeter appears to be best suited for Solar System gravimetry within the goal and scope of this work. This appears to be due to the basic possibility to measure without levelling and the, comparably, less complex implementation possibilities. It was also pointed out, however, that while use can be made of the general spring-based concept, none of the yet existing concepts is directly applicable to small body gravimetry due to the tight mission constraints, and therefore, a new gravimeter instrument needs to be developed.

This gravimeter development work was performed in the context of the ESA Hera mission (Subsection 1.3.2), which allowed to perform an end-to-end instrument development, going from the initial design up to the implementation of the GRAVimeter for small Solar System bodies (GRASS) flight model (FM) for the deep-space CubeSat Juventas, foreseen to land on Dimorphos for surface gravimetry (Section 1.2).

Clearly, a science payload for Solar System exploration cannot be the sole contribution of one author. Therefore, this Chapter covers the instrument development focusing on the mechanical and mechanism aspects of the instrument. The 3D-instrument design and mechanical GRASS development was the main responsibility of the author, which was strongly supported by guidance and consultation from Emiel Van Ransbeeck. Among other tasks, he led the instrument integration, and any relevant aspects concerning the assembly directly fed back into the design. Additionally, the here presented design was verified by tests performed by further GRASS team members, which will be credited accordingly in the relevant sections. Overall, the instrument development as presented in this Chapter would not have been possible without the contributions of the entire GRASS team, as well as the Juventas and Hera teams. The starting point of the here presented work was the B-Grave gravimeter (Subsection 2.3.4) concept developed by Michel van Ruymbeke and a subsequent first breadboard mechanically designed by Sabrina Bonnewijn and Emiel Van Ransbeeck.

Following the formulation of the space gravimeter design requirements (Section 3.1), the design is laid out at three levels. Starting with the gravimeter spring design (Section 3.2), subsequently the displacement transducer is presented in Section 3.3, needed to observe the gravity-induced deflection of the spring. Following this, the

GRASS rotation mechanism is presented (Section 3.4) and the overall design is verified by showing instrument test results in Section 3.5. In addition, to show the adaptability to different future mission concepts and targets, possible design adaptations of the instrument are discussed (Section 3.6). Finally, the new design will be discussed in the frame of this work (Section 3.7) and this Chapter is summarised in Section 3.8.

3.1 SPACE GRAVIMETER DESIGN REQUIREMENTS

At basic level, the gravimeter concept selection done in Section 2.5 already formulated qualitative selection requirements. Most importantly, the overall constraints in instrument complexity for a robotic exploration mission, and the need for a 3D-measurement in absence of a global levelling mechanism, lead to favouring a spring-based gravimeter. Here, a macroscopic design was favoured over a microscopic (MEMS) design, which indeed allowed a faster development advancement, while keeping the option to transform the working principle to a MEMS application open for future work. In this Section, the qualitative requirements will be supplemented by quantitative requirements driving the gravimeter design. Fortunately, the development within the frame of a ‘real’ space mission allowed to utilise first-hand real numerical requirements derived together with the Juventas satellite lander contractor, making the values much more valuable as design drivers. The requirements listed here are only those concerning parts of the GRASS gravimeter development discussed in this work. The instrument electronics design is beyond the goal and scope of this work, and therefore requirements only concerned with this part of the development are omitted here, which, consequently, also includes the achievable gravity precision of the spring displacement transducer. However, a target of 1 % of the surface gravity with an orientation reconstruction better than 0.5 degrees can serve the reader as reference target (as formulated in the initial instrument science requirements).

Overall, the numerical requirements did not put any limitations on how to develop the space gravimeter, yet they demanded an extremely compact design over a large temperature range. The overall requirements were formulated in the instrument requirement document (GRASS team, 2020). The top-level design requirements, designated with GRASS-00X, considered for flight onboard the 6U-XL Juventas CubeSat were:

- GRASS-001: The instrument volume shall not exceed one CubeSat unit ($1U = (100mm)^3$).
- GRASS-002: The total instrument mass shall be smaller than 380 g (gram).
- GRASS-003: The thermal contact between the instrument and the satellite bus shall be minimised.
- GRASS-004: The gravimeter shall operate in the temperature range from $-30^{\circ}C - +20^{\circ}C$.
- GRASS-005: The instrument shall have a non-operational temperature range

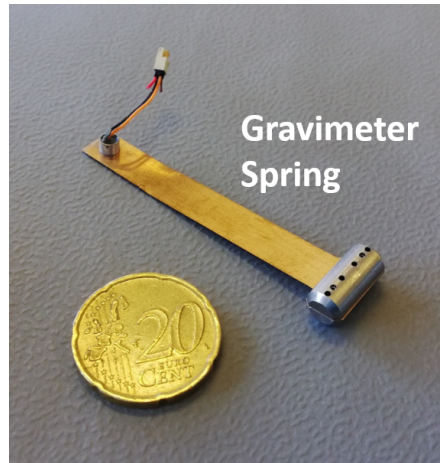


Figure 3.1: GRASS gravimeter spring assembly with 20-euro-cent coin for size reference. The base spring material is CuBe and the tip mass is made in tungsten.

from -30°C – $+50^{\circ}\text{C}$.

- GRASS-006: The payload shall withstand (launch) vibrational and (separation/landing) shock loads as defined in the environmental requirements.
- GRASS-007: The minimum gravimeter lifetime shall be $2 \times 12h = 24h$ after surface deployment.

The above repeated qualitative requirements together with the here-listed numerical requirements formed the basis of the GRASS gravimeter development. Clearly, no direct implications on any design details were given, allowing to come up with an indeed novel design, partially incorporating aspects of previously proposed space gravimeters presented in the literature review (Section 2.3). This design will be presented in the following sections, and, where applicable, design choices between different options will be elaborated upon.

3.2 GRAVIMETER SPRING

Key element to construct a spring-based typed gravimeter is, not surprisingly, the gravimeter spring. Dealing first with the spring type selection (Subsection 3.2.1), this Section is continued with the description of the developed reference spring design (Subsection 3.2.2) and the production of the spring system (Subsection 3.2.3). The eigenfrequency of the spring is computed analytically in Subsection 3.2.4. The detailed analytical description of the spring behaviour is done in the next Chapter (Section 4.1). A first impression of the gravimeter spring assembly is shown in Figure 3.1 with a 20-euro-cent coin for size reference.



Figure 3.2: Evenly distributed load q on a one-sided clamped free spring (cantilever). From: Weinberg and Fritzen (2013).

3.2.1 SPRING TYPE SELECTION

As mentioned above, the spring-based gravimeter shall be designed such that no levelling mechanism of the gravimeter is required. Additionally, the measurement must work in all orientations.

The helical spring as used in the Gulf gravimeter (Subsection 2.1.2) had been previously excluded, as the measurement principle demands the spring to align with the gravity vector, allowing to monitor the gravity-induced (un-) winding of the spring. More generally speaking, helical springs as used in the Worden and LaCoste-Romberg gravimeters should generally be adaptable to inclined measurement, i.e. measurements where the mass moving direction does not align with the gravity vector. If the measurement direction, however, is flipped “upside-down”, the working principle would break down, as the pre-tensioned zero-length springs would not be able to absorb compression. This is the reason why the classical helical springs were not selected for the GRASS gravimeter. It might be well possible to circumvent this problem, e.g. by adding another zero-length spring symmetrically to the “bottom”. Such solution, however, can be expected to create a manifold of new problems, for example, different thermal behaviour and expansion due to differential temperatures on both sides of the test beam, intrinsic (minimal) different spring properties between two individual pieces, an overall increase in instrument volume etc.

The other option coming from various examples in the literature study is the cantilever spring, i.e. a single-sided clamped beam often with a mass at the free-hanging end. This was applied in the optomechanical resonator (Subsection 2.1.2), the microscopic MEMS (Subsection 2.1.7) and were proposed for the B-Grave gravimeter (Subsection 2.3.4) and GRAS-F seismogravimeter (Subsection 2.3.5). The advantage here is that the measurement will work, regardless of the measurement orientation. In the simplest case, without additional tip mass, the deflection of the beam solely depends on the distributed gravity load q on the bending spring as shown in Figure 3.2.

The evenly distributed load q acting on the spring (cantilever beam) is calculated as (Equation (3.1))

$$q = \frac{m}{l} g \quad (3.1)$$

where m is the free-mass of the beam, l is its free-length and g the local effective gravity acting on the probe. Here, the free-mass refers to the non-clamped, i.e.

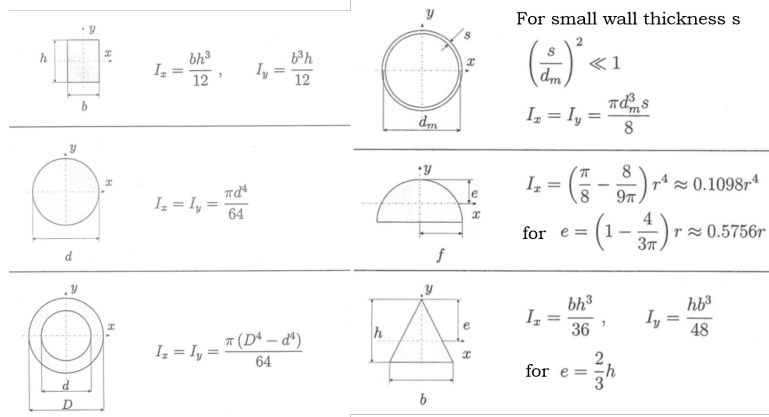


Figure 3.3: Calculation of the second moments of area with respect to the x - and y -axes, respectively. From: Weinberg and Fritzen (2013).

flexible, part of the spring that is subject to deformation under the influence of gravity.

For a cantilever beam with constant cross section dimension and material properties, the theoretical deflection under q can be calculated analytically with Equation (3.2). It is important to note that the analysis presented here is valid for small deflections only, while nonlinearities can occur for larger deflections (Ohtsuki, 2005, 2010). A direct scaling from the tested deflection under (full) Earth gravity to a small body gravity environment is therefore generally not feasible.

$$w_q = \frac{q}{24EI} (6l^2x^2 - 4lx^3 + x^4), (0 \leq x \leq l) \quad (3.2)$$

where E is the Young's modulus of the material and x the coordinate of the spring at which the deformation needs to be calculated (running from the clamping point $x = 0$ to the tip of the spring $x = l$). The *second moment of area* I is dependent on both the cross section and bending direction. The calculation for various cross sections is calculated as shown in Figure 3.3.

From this overview we find that radially symmetric cross sections will deform independently of the bending direction, as can be expected. These shapes are the circular hollow and solid rods, as well as the rectangular cross section (including a square as a special case of the rectangle with $b = h$).

Using a solid circular bending spring for a high-sensitivity accelerometer-seismometer was done for the GRAS-F seismogravimeter (Subsection 2.3.5). Indeed, such a shape does not require any levelling. One problem with this occurs, when thinking ahead to the spring deflection measurement (Section 3.3) that will require two independent, orthogonal measurements to monitor the spring deformation in 2D. The problem here is the non-trivial capacitance computation for a non-concentric cylindrical capacitance transducer as described by Manukin

et al. (2010a,b). Equation (3.3) shows the capacitance between two non-concentric cylinders with radii $r_1 < r_2$ of length L , with a (one-dimensional) offset D (Morales et al., 2015). Thinking this further in 2D, the capacitance computation becomes more complex, and deciphering the contributions to two overlaying accelerations in orthogonal direction will be highly non-trivial.

$$C = \frac{2\pi\epsilon_0 L}{\operatorname{arccosh} \frac{r_1^2 + r_2^2 - D^2}{2r_1 r_2}} \quad (3.3)$$

Alternatively, it is possible to investigate the rectangular cross section as used in the optomechanical resonator (Subsection 2.1.2), the microscopic MEMS (Subsection 2.1.7) and as proposed for the B-Grave gravimeter (Subsection 2.3.4). As shown in Figure 3.3, the second moment of area I is calculated as

$$I = \frac{bh^3}{12} \quad (3.4)$$

with h the thickness and b the width of the blade. It is calculated with respect to the axis of deformation and therefore the thickness has a cubic influence on the deformation while it simply scales with the width. Considering now a bending spring with $h \ll b$, very large deflections parallel to the thickness and a much smaller, negligible, deflection in the perpendicular direction can be realised. For example, for the spring with width of 7.5 mm and thickness of 75 micron (Figure 3.4), the sensitive deflection will be exactly 4 orders of magnitude larger than the insensitive deflection in perpendicular direction. Such a spring therefore requires the deflection measurement only in one direction, namely the direction of largest deflection.

Other than for the radially symmetric circular cross section, however, this comes at the cost that the sensitive direction needs to be aligned with the gravity. While no global instrument levelling was possible, this problem will be bi-passed together with the bias rejection realised with the gravimeter rotation mechanism (Section 3.4).

Regarding the, so far omitted, third dimension along the long axis of the spring, it can be shown that, if (partially) aligned with the gravity vector, the elongation of the spring is negligibly small compared to the bending in the sensitive direction. Considering again the spring setup shown in Figure 3.4, the deflection on Dimorphos would be about 375 nm (Table 4.3), and the elongation Δl due to the tip mass under gravity can be computed as shown in Equation (3.5) (Young et al., 2002)

$$\Delta l = \frac{Fl_0}{AE} \quad (3.5)$$

where the Young's modulus E is the ratio of stress $\sigma = F/A$ to elongation $\epsilon = \Delta l/l_0$ in the elastic regime, l_0 is the original specimen length, $F = mg$ is the (weight) force of the tip mass causing the elongation, and A the cross section of the spring. Substitution yields an elongation of

$$\Delta l = \frac{0.0078 \text{ kg} \cdot 4.5 \cdot 10^{-5} \text{ m/s}^2 \cdot 0.0437 \text{ m}}{75 \cdot 10^{-6} \text{ m} \cdot 0.0075 \text{ m} \cdot 131 \cdot 10^9 \text{ Pa}} = 2.082 \cdot 10^{-13} \text{ m} \quad (3.6)$$

This elongation is therefore indeed six orders of magnitude smaller compared to the desired deflection of the bending spring. This order of magnitude has been confirmed using a finite element method simulation. The associated stress in this spring under elongation is

$$\sigma = 0.0078 \text{ kg} \cdot 4.5 \cdot 10^{-5} \text{ m/s}^2 / (75 \cdot 10^{-6} \text{ m} \cdot 0.0075 \text{ m}) = 0.624 \text{ Pa} \quad (3.7)$$

Therefore, in conclusion, a bending spring with rectangular cross section is selected that will deform in one direction under the influence of gravity. Generally, a larger deflection will improve the measurement performance since it is directly proportional to the gravity (Equation (3.1)). Considering Equations (3.1), (3.2) and (3.4), the maximum deflection can be improved by the following spring design parameters:

- larger blade mass m (density ρ)
- larger overall length l
- smaller Young's modulus E
- smaller second moment of area I
 - smaller blade width b
 - smaller blade thickness h

However, it should be noted that for the here-discussed simple case, without added mass at the tip of the spring ("tip mass") a change in spring width b does not influence the deflection, as it inversely influences the second moment of area I and the spring mass m . As will be shown, an additional tip mass will be considered for the GRASS spring and therefore this compensation of opposite effects disappears. Clearly, all but the thickness have a linear influence on the deflection, making it the most powerful parameter to increase the deflection with its cubic influence.

One influence which was not studied in detail in this work is the torsional rigidity. Despite the rotation of the spring, the symmetric design of the spring, paired with the alignment of the long axis of the spring with the rotation axis, leads to a theoretical absence of any torque T . In practise, small deviations from the symmetry and rotation axis will exist, yet the induced torque will remain small, due to the small gravity environment. The angle of twist θ in radian of a bar can be computed as shown in Equation (3.8) (Young et al., 2002)

$$\theta = \frac{Tl}{JG} \quad (3.8)$$

where l is the length of twisted bar, J is the polar moment of inertia of the cross section, which can be approximated with $J \approx 0.333bh^3$ for large ratios of cross section

width b and thickness h (Ugural and Fenster, 1987), and G is the modulus of rigidity (shear modulus) of the material, being 50.0 GPa for CuBe¹. If we now assume that 10 % of the mass of the tip mass would induce a torque under Dimorphos gravity at the maximum distance from the spring rotation axis (7 mm), the angle of twist of the JUV-GRASS spring (Figure 3.4) can exemplarily be calculated as:

$$\theta = \frac{0.1 \cdot 0.0078 \text{ kg} \cdot 4.5 \cdot 10^{-5} \text{ m/s}^2 \cdot 0.007 \text{ m} \cdot 0.0437 \text{ m}}{0.333 \cdot 0.0075 \text{ m} \cdot (75 \cdot 10^{-6} \text{ m})^3 \cdot 50 \cdot 10^9 \text{ Pa}} = 2.038 \cdot 10^{-8} [\text{rad}] \quad (3.9)$$

which is indeed negligible on Dimorphos. Replacing the gravity by Earth gravity, the angle of twist is increased to about $4.443 \text{ mrad} = 0.0255^\circ$, a value that should be reassessed during the instrument characterisation on Earth.

The above-mentioned tip mass will be introduced in the next Subsection forming part in the GRASS gravimeter spring reference design. The precise influence on the deflection is analysed further in the next Chapter (Subsection 4.1.2).

3.2.2 GRAVIMETER REFERENCE DESIGN

In the previous Subsection, the case was made for a flat bending spring with rectangular cross section, clamped single-sided. The developed spring design for the GRASS instrument for the asteroid moon Dimorphos is presented here as a reference design. This reference spring design will also be studied analytically in the next Chapter. Overall, this final design converged from two sides. Firstly, the overall instrument requirements, most notably GRASS-001 on the volume, drove the overall instrument mechanism design (Section 3.4), in turn influencing the displacement transducer (Section 3.3), finally also driving and constraining the spring design. Secondly, considering the expected (very) small reference gravity on Dimorphos of $4.5 \cdot 10^{-5} \text{ m/s}^2$, the deflection and thus gravimeter sensitivity had to be maximised (optimised). The latter is described analytically in Section 4.1, whereas the here-presented design can serve as a reference for the GRASS instrument.

Key element of the *spring assembly*, shown in Figure 3.4, is the above introduced bending spring, also referred to as *blade*. One requirement to the blade coming from the electrical transducer design was that the gravimeter spring shall be electrically insulated from the instrument and that it shall be grounded. This was achieved by realising the single-sided clamping of the blade in the insulating ceramic Macor (glass-ceramic) material. Clamped between two screw-connected pieces, the final position is fixed with glue added to dedicated pockets in the Macor parts.

As mentioned above, a tip mass was added to the free-hanging end of the blade to increase significantly the deformation. Taking two half-cylinders, they are added above and below the blade, clamping it using two slotted spring pins (DIN 7343), and again fixing the final setup with glue filled in pockets of the tip mass halves. To protect this free-hanging setup and the displacement transducer (Section 3.3),

¹Cf. <https://www.matweb.com/search/datasheet.aspx?matguid=ae0103ab776344b9bf4b6b4bd6eb6f8b&ckck=1>, accessed on 23.11.2022.

inserts are added to the tip masses that serve as *dampers*. These stick (just) out of the tip masses, avoiding both a contact between cradle (Figure 3.10) and tip mass, and blade and sensing electrodes (Figures 4.6 and 4.7). This does not only avoid cold welding as a consequence of metal-metal contact, but also protects the sensitive setup in vibration. Of course, a small risk remains here that the dampers could also stick to the titanium cradle. This risk is reduced by the applied blue anodisation surface treatment of the cradle. The tip mass material used was tungsten with a nominal density of $\rho_{Tungsten} = 19,300 \text{ kg/m}^3$. Using platinum would result in an additional slight increase of the sensitivity, with a nominal density of $\rho_{Platinum} = 21,450 \text{ kg/m}^3$, but had to be discarded for GRASS due to the very high material cost of the precious metal.

Coming back to the now fully equipped spring, its dimensions can be explained as follows. The overall length was limited by the transducer development, important for the deflection is the free-length, i.e. the length of 43.7 mm between the tip mass and the Macor clamps. This free, un-clamped part of the blade is subject to deformation under gravity. The width b of the blade was set to 7.5 mm. As discussed in Subsection 3.2.1, reducing the width increases the deformation, while a sufficiently large width needs to be maintained to ensure the stability of the entire bending spring, e.g. against buckling or torsion. Further, it was shown that the thickness h has the most important influence on the deformation. Here, the nominal thicknesses of $50 \mu\text{m}$ and $75 \mu\text{m}^2$ were considered. The final choice between the two thicknesses will be based on latest instrument characterisation tests in future work.

3.2.3 GRAVIMETER SPRING MAKING

The concept of the spring assembly has been shown and explained in the previous Subsection. Here, the making of the gravimeter spring, from raw material to the finished probe, is described in the four following steps.

1. Raw Material

For the flat bending spring design, the Copper Beryllium (CuBe) alloy has been selected for very good spring properties. Elasticity is here combined with high strength and a high resistance against stress relaxation³.

We obtained the foil raw material Cu98/Be 2 with temper half hard. Nominally with a thickness of 75 micron, the raw foil material had a constant thickness of 80 micron, which was within the specified tolerance of $\pm 10 \%$. The key material properties are tabulated in Table 3.1 as provided by the supplier. Clearly, some properties are provided with rather large ranges, demanding additional calibration steps for the (gravimeter) spring application. As the material was rolled, the supplier was asked to clearly mark the direction of rolling on the sheet material, as discussed in the next step.

2. Laser Cutting

²The raw material had an actual thickness of $80 \mu\text{m}$ within the supplier tolerance.

³Cf. https://www.matthey.ch/fileadmin/user_upload/downloads/Fichiers_PDF/GuideToCopperBeryllium_EN.pdf, accessed on 03.02.2022.

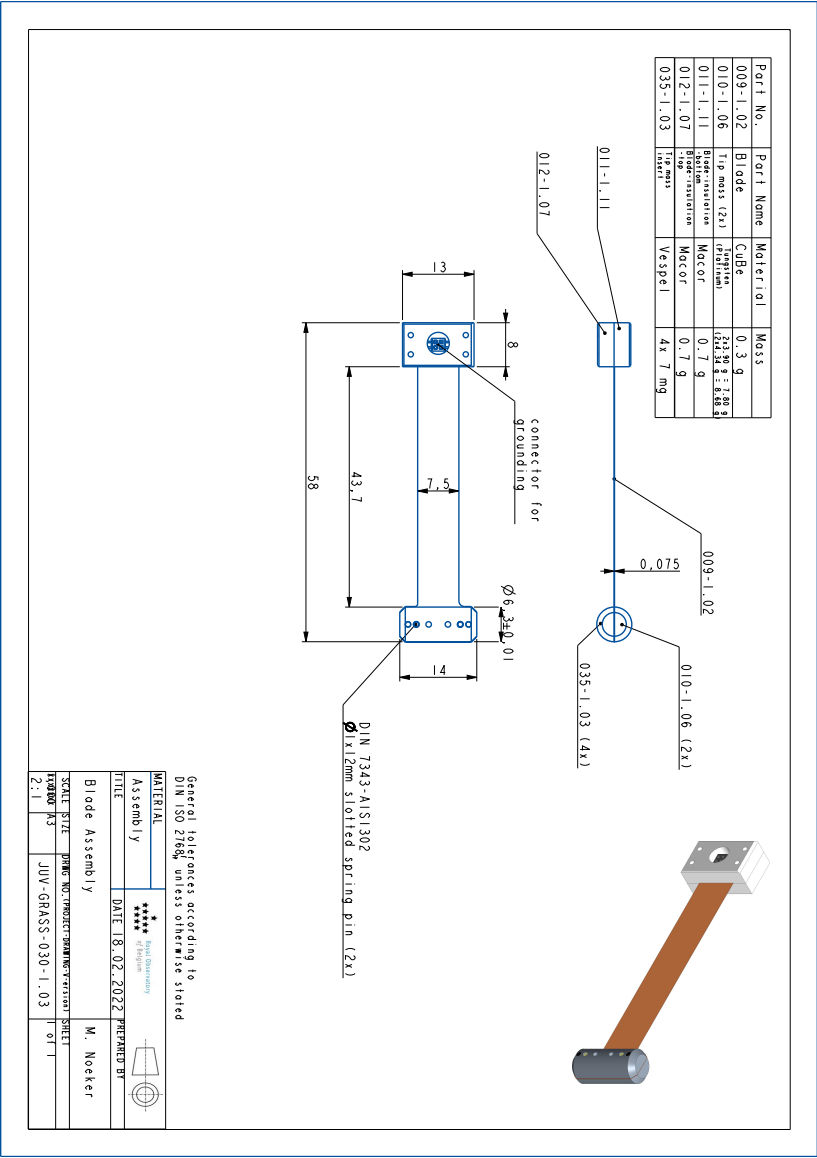


Figure 3.4: Technical drawing of the original and optimised JUV-GRASS spring designed for the HERA mission. The design parameters are, analogue to Figure 3.4, the free blade length between clamp and tip mass (43.7 mm), the width of blade (7.5 mm), the thickness (0.075 mm), and the tip mass dimensions (2 half-cylinder with diameter 6.3 mm and height 14 mm, with 1 mm × 1 mm chamfer to not exceed the 15 mm diameter cradle) and the high-density tip mass material.

Table 3.1: Raw material specification of the CuBe spring material according to the supplier.

Properties	Value	Unit
Young's modulus	120 – 160	GPa
Tensile strength	500 – 1300	GPa
Density	8,250	kg/m^3
CTE ^a	17 ^b	$\times 10^{-6}/K$
Melting point	860 – 1,000	°C

^aCoefficient of thermal expansion.

^bAt 25 - 300°C.

This raw material was then send to laser cutting. This method was selected to minimise any stress-induction to the material. Alternatively, spark erosion cutting was considered, however, to obtain a larger quantity of springs allowing a selection, laser cutting was the faster and more economic solution. As described in the previous Subsection, the long axis of the spring was aligned with the rolling direction of the raw material.

One additional aspect was altering the material properties due to the heating of the laser. However, the supplier explained that only the edge will be heated with a maximum extend of two times the material thickness, and that this might cause a local change in material colour but not its properties.

Overall, the result was of very good quality meeting all specified dimensions and tolerances. The cut springs are shown in Figure 3.5 (a).

3. Heat Treatment

After the laser cutting, the “raw” springs are age hardened, not only to increase the yield strength but also to maximise the probes flatness. According to the supplier, the precipitation hardening should be done at a temperature of 315°C. This is indicated for a duration of 2 hours by the supplier, but slight improvements can be expected for a duration of 3 hours⁴, which was applied. Following the heat treatment, the probe is air-cooled. To ensure the flatness improvement of the raw springs, they are clamped between two massive stainless steel blocks bolted together during both oven heating and air cooling. These two blocks have been specifically grinded on their contact surfaces to ensure the best flatness of the springs. Finally, the springs are cleaned with phosphoric-nitric-acetic (PNA) (38 % H₃PO₄, 2 % HNO₃, 60 % acetic acid) etch. After this treatment, the blades are rinsed with distilled water, dried and ultrasonic cleaned with isopropyl alcohol (IPA). The probes are then stored in nitrogen until the gold sputtering takes place that is described below.

4. Gold Sputtering

To protect the blades from aging, e.g. due to oxidation, they are protected with a gold layer. This is applied by physical vapour deposition (PVD) with plasma etch-

⁴Ibid., accessed on 16.02.2022.

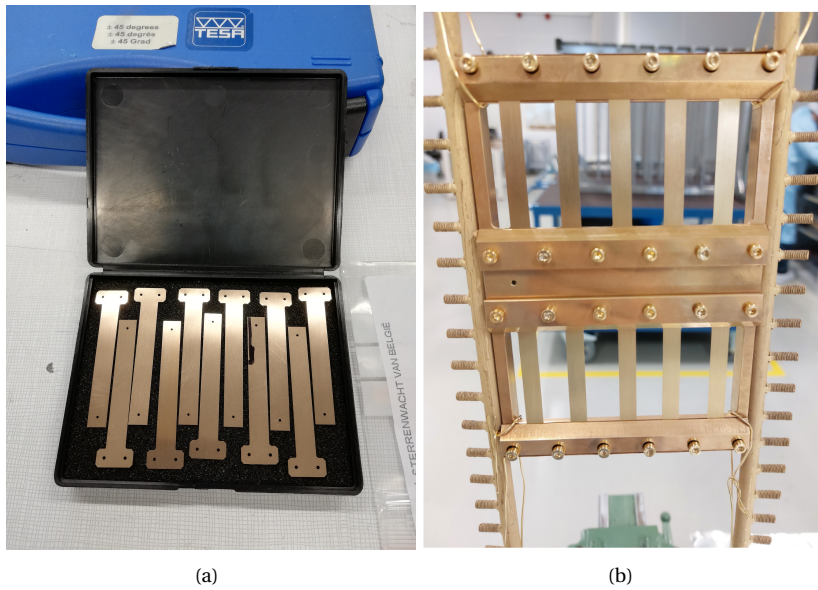


Figure 3.5: (a) Laser cut gravimeter springs. (b) Gold sputtered springs in support frame designed specifically for the GRASS spring. The setup allows simultaneous treatment of two thicknesses (i.e. 50 and 80 micron) in two batches of fives pieces. The gold deposited layer thickness is nominally 30-40 nm which was confirmed by a sample witness.

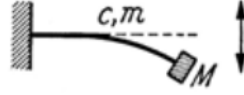


Figure 3.6: Sketch of parameters driving the first eigenfrequency of a cantilever, where M is the tip mass, m the mass of the beam and c the spring constant. The sketch also indicates the movement of the spring when excited at the first eigenfrequency, i.e. at the first mode. From Kormann et al. (1952).

ing prior to starting the process, improving and renewing the cleaning after the heat treatment and removing oxidation. The layer deposition thickness, controlled by a sample witness, is 30 - 40 nm, and therefore not influencing the spring behaviour.

To handle the blades, they are clamped in a dedicated frame, clamped on the two ends (Figure 3.5 (b)). These two ends correspond to the clamping portion of the Macor insulation pieces and the tip masses. While these regions will not be coated, this is less critical than the free blade part, i.e. the length of the spring that deforms.

3.2.4 SPRING EIGENFREQUENCY

With respect to the two considered spring thicknesses of 80 (nominal 75) and 50 micron under consideration, being the main driver for the spring deflection, the first eigenfrequency of the spring-mass setup can be computed analytically. The first mode, i.e. the kind and direction of the deformation of a cantilever with tip mass is shown in Figure 3.6. This setup is exactly a model of the here considered spring with tip mass, assuming a constant cross section, thus $EI = \text{const.}$

The first eigenfrequency f_1 (in cycles per second) considering both the spring mass m and the tip mass M can be calculated with the formula reported by Kormann et al. (1952):

$$f_1 = \frac{\sqrt{3}}{2\pi} \sqrt{\frac{EI}{l^3(M + 0.23m)}} \quad (3.10)$$

where l is the *free* length of the spring.

Alternatively, Young et al. (2002) provide a similar expression for the first eigenfrequency that assumes the beam own-weight m to be negligible:

$$f_1 = \frac{\sqrt{3}}{2\pi} \sqrt{\frac{EI}{l^3 M}} \quad (3.11)$$

With these two analytical expressions introduced, it is possible to compute the first eigenfrequency for the two spring thicknesses. The results are listed in Table 3.2. The difference between the two concepts is indeed small, showing that in this setup

Table 3.2: Analytically computed eigenfrequencies f_1 for the JUV-GRASS spring using the two methods introduced in Equations (3.10) and (3.11), and compared to numerical computations by Berk Senel (2021). For the analytical computation of the JUV-GRASS spring, the considered tip mass is $M = 7.8 \text{ gram}$, the free spring mass is $m = 201.6$ milligram, the Young's modulus $E = 131 \text{ GPa}$, and the second moment of area $I = 2.637 \cdot 10^{-16} \text{ m}^4$.

Thickness	Kormann et al. (1952)	Young et al. (2002)	Berk Senel (2021)
50 micron	1.091 Hz	1.093 Hz	0.987 Hz
80 micron	2.205 Hz	2.212 Hz	2.003 Hz

the mass of the spring is negligible compared to the tip mass with respect to the eigenfrequency calculation.

These results are important for e.g. a possible application for seismic measurements, as will be discussed in Subsection 5.2.4. The analytical results were independently confirmed by Berk Senel (2021) with a numerical COMSOL simulation shown in Figure 3.7 and also listed in Table 3.2. The analytical solutions were both slightly larger than the numerical solution computed in COMSOL. The colour coding indicates the direction of the deformation at the first mode, which is in accordance with Figure 3.6.

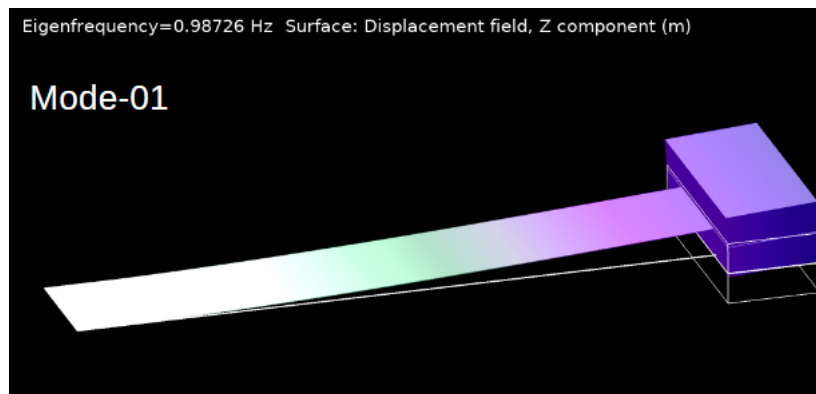
3.3 SPRING DISPLACEMENT TRANSDUCER

In general, transducers convert a movement (displacement) into an electrical signal. In Chapter 2, different kinds of transducer principles have been presented, i.e. using laser interferometer (optomechanical resonator, Subsection 2.1.2), optical transducer (shadow sensor/quadrant photodiode, Subsection 2.1.7), and capacitance transducer (B-GRAVE gravimeter, parallel plate, Subsection 2.3.4; Lunar surface gravimeter, parallel plates, Subsection 2.3.2; and GRAS-F, concentric cylinder, Subsection 2.3.5).

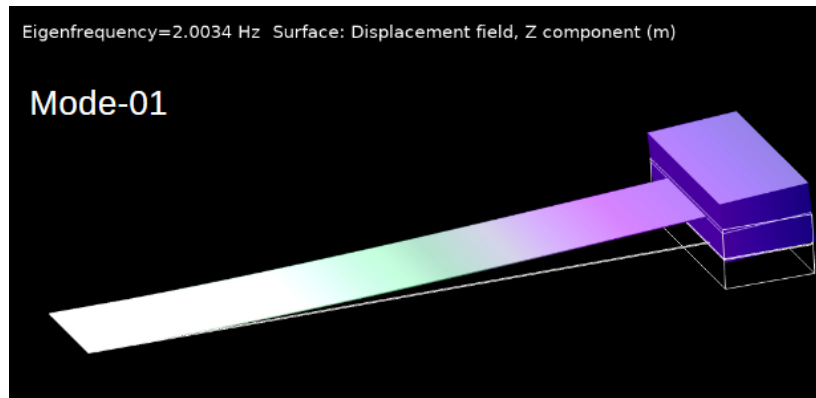
This Section therefore begins with a discussion on the selection of the transducer working principle (Subsection 3.3.1). Following this, additional details on the selected working principle are provided in Subsection 3.3.2. Finally, the electromechanical implementation of the transducer is detailed in Subsection 3.3.3, without going into details of the sensing electronics design, which would be beyond the goal and scope of this work (and could fill a good part of another PhD thesis).

3.3.1 DISPLACEMENT TRANSDUCER PRINCIPLE SELECTION

While there was indeed some preliminary decision towards a capacitance transducer based on this work's starting point being the B-GRAVE (Subsections 2.3.4) gravimeter, it was important to reassess this decision, similarly to the spring type selection, before starting with the transducer design. Therefore, this Subsection provides a brief qualitative discussion about possible alternatives, and their advantages and disadvantages.



(a)



(b)

Figure 3.7: First modes of the 50 and 80 micron springs. (a) Thickness 50 micron, eigenfrequency of 0.987 Hz, (b) thickness 80 micron eigenfrequency 2.003 Hz. From Berk Senel (2021).

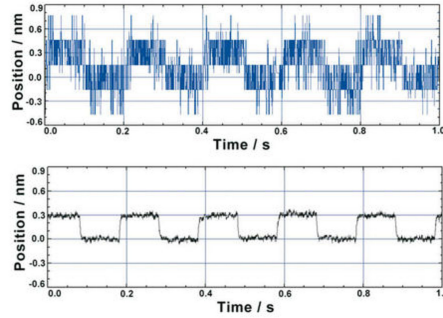


Figure 3.8: Case study comparing laser interferometer (top) with capacitance transducer (bottom). Clearly, the noise level of the latter is much lower compared to the former. From <https://www.physikinstrumente.com/en/expertise/technology/sensor-technologies/capacitive-sensors/>, accessed on 21.02.2022.

As will be shown in Chapter 4, the spring deflection to be measured is in the order of tenths of a micron, i.e. 300 – 400 nm. As the deflection is directly proportional to the gravity, the minimum GRASS target of measuring changes of at least 1 % translates into changes of 3 to 4 nanometers (!).

The first identified option was laser interferometry. Here, a case study from Physik Instrumente (PI) GmbH Co. KG⁵ compared the performance of a capacitive positioning sensor with a laser interferometer (Figure 3.8). Clearly, the noise level of the laser interferometer exceeded the noise of the capacitance sensor. While this comparison can only be understood qualitatively, the tendency is clear. Moreover, observing Figure 2.6 (b), the complexity of the geometric setup of the laser interferometer is clearly visible and it can be well expected that such setup would be a significant design driver affecting the overall instrument volume (GRASS-001).

The second concept to be considered is an optical transducer. Here, the displacement can be observed by different concepts as mentioned above, i.e. measuring the (changing) position of a light beam/shadow corresponding to the movement of the gravimeter test mass. An alternatively studied idea was to position a mirror on the (flat) bending spring and observe the movement of a light source reflected from the spring, providing information on the angle of deflection of the blade. For a small gravity (i.e. $4.5 \cdot 10^{-5} \text{ m/s}^2 = 4.5 \text{ mGal}$) gravimeter, as the developed GRASS gravimeter, the movement of the tip mass and the deflection angle are expected to be small. The movement of the light beam can therefore be increased by increasing the distances, e.g. between the blade and both light source and positional receiver. Again, maximising the angular deflections was expected to provide problems to the overall instrument volume and was therefore not studied further.

A third studied solution in this work, not found in the literature so far, were strain gauges for the displacement of the (bending) spring or the usage in a gravimeter.

⁵Cf. <https://www.physikinstrumente.com/en/expertise/technology/sensor-technologies/capacitive-sensors/>, accessed on 25.01.2022.

While this option is theoretically promising using strain gauges with large strain factors, such as semi-conductor gauges, there was a problem with altering the properties of the delicate spring. Here, it was found infeasible to reliably mount the strain gauges on the gravimeter spring with little stress on the gauges, while not altering the spring properties of the gravimeter spring.

The fourth and last considered option is the capacitance transducer that had been previously proposed for displacement transducer settings. Here, the disadvantage of the significant volume increase to the instrument will not appear, as generally the sensing electrodes should be positioned as close as possible to each other. Further, it is reaffirmed, e.g. by Manukin et al. (2010a), that capacitive transducer are an electrical transducer type with highest sensitivity. The concentric complex cylinder capacitor was discussed above during the spring type selection, regarding the 2D-case (Equation (3.3)). The parallel plate setup, however, is much more simple and shows an inversely proportional relation to the displacement (Equation (4.5)). Furthermore, from an integration point of view, the parallel plate setup will be easier to assemble and control compared to the concentric-cylindrical setup.

Therefore, overall, the capacitive transducer appears to be indeed the best choice among the compared options. Next to the expected best sensitivity, the suitability for a compact design is here best given. Moreover, the spring is not influenced nor altered, as this transducer method works contactless. In the next Subsection, the working principle of the parallel plate capacitance transducer is explained in detail.

3.3.2 CAPACITIVE TRANSDUCER

In this work, thus far, the case was made for a spring-based gravimeter with flat bending-spring of rectangular cross section. Above, the gravity-induced spring deflection was found to be best monitored with a capacitance transducer. This Subsection describes the working principle of this transducer type in the context of the gravimeter development.

With the spring being clamped on one side, the deflection of the spring becomes a conventional cantilever problem. In the capacitance transducer, the spring becomes one plate in the well-known parallel plate capacitor, which is complemented by an electrode. With the inverse relation of capacitance and plate separation (Equation (4.5)), sensing a change in the capacitance can be directly related to a movement of the (flexible) gravimeter spring. As will be discussed in Section 4.3, a differential capacitance measurement is advantageous, provided the (very) small changes in absolute capacitance. Therefore, adding two electrodes, one above and one below the gravimeter spring forms two plate capacitors, each using the spring. In this differential setting, one capacitance increases, while the other decreases as a result from the movement of the spring towards one of the two electrodes.

The described working principle is shown in Figure 3.9. To illustrate the development history of the transducer, here the first breadboard developed by Emiel Van Ransbeeck and Sabrina Bonnewijn is pictured and the final development of the transducer is detailed in the next Subsection. It is noted that initially the tip mass added to the gravimeter spring was placed in-between the electrodes, which was

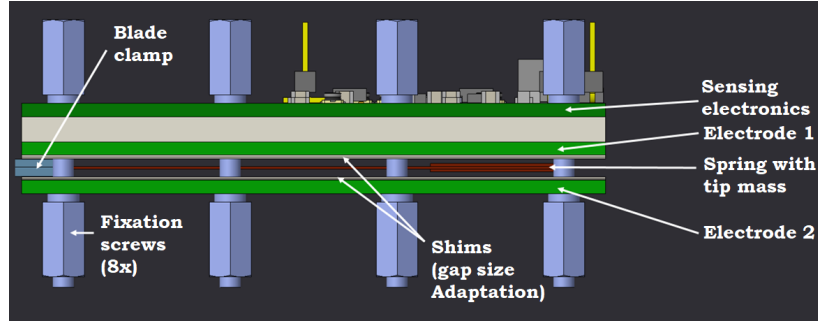


Figure 3.9: Working principle of the spring-based gravimeter capacitance transducer. The shown CAD design from Emiel Van Ransbeeck and Sabrina Bonnewijn represents the first GRASS breadboard, forming the starting point of the final transducer design. This initial design still considered the electrodes to run along the entire spring, and the tip mass. Clearly, the separation between the electrodes was therefore controlled by the tip mass thickness.

changed during the course of the development as will be discussed. The next Subsection will also include a discussion regarding the possibility to perform relative or absolute gravity measurements using the here presented working principle.

3.3.3 DISPLACEMENT TRANSDUCER DESIGN

Following the capacitance transducer working principle description in the previous Subsection, the detailed electromechanical design is described in this Subsection. Here, electromechanical refers to all mechanical aspects, as well as the design of the capacitors with the sensing electronics. The detailed electronical design to sense the change of the capacitance (and therefore gravity), is not discussed in this work.

The entire capacitance transducer is also referred to as sensor head. As will be discussed in the next Section, this sensor head will be rotated along its long axis to account for both the bias rejections and the 3D-gravity measurement. The design is therefore compliant to this and based on its core element, the *cradle*. This shaft element (diameter of 15 mm) is mounted on two bearings, allowing the rotation by a two-gear gear train. The interior of the cradle is machined with maximum precision using multiple-step spark erosion to allow mounting of the clamped bending spring, (non-contact) sandwiched between two sensing electrodes. Differing from the early breadboard design (Figure 3.9), the tip mass has been moved outside of the electrode setup. This does not only allow a much smaller electrode separation and a larger tip mass, but also the addition of the tip mass dampers, protecting both the spring and the electrodes.

This transducer has different design parameters, e.g. the separation of electrodes to the spring, and the start and end points of the electrodes, that are discussed in Subsection 4.2.2. This gravimeter spring displacement transducer forms the central element of the gravimeter instrument with its rotational mechanism being intro-

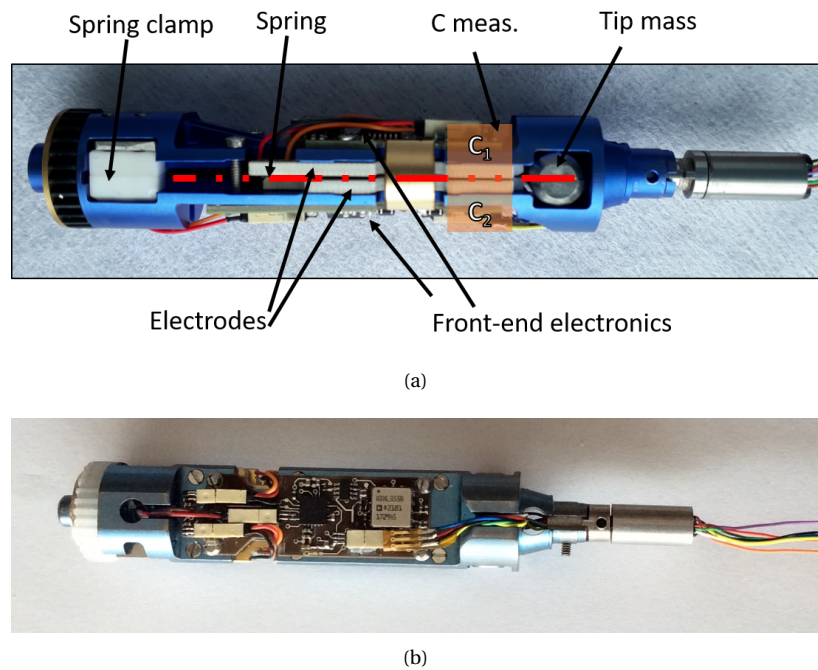


Figure 3.10: Displacement transducer design with the cradle piece as core element. (a) Annotated side view with the gravimeter spring and displacement transducer components shown. The region of capacitance measurements C_1 and C_2 is highlighted in orange. (b) Cradle in-line assembly with the slip ring and early sensing electronics, top view.

duced in the next Subsection.

3.4 GRAVIMETER ROTATION MECHANISMS

This Section 3.4 is an adapted excerpt from Noeker et al. (2022c). The reader is referred to this reference for greater technical details, which cannot be represent here in the same detail.

Thus far, the gravimeter spring (Section 3.2) and the deflection transducer (Section 3.3) have been introduced in this Chapter. The gravity measurements will intrinsically contain a measurement bias. A method to remove a systematic bias in measuring acceleration is described by Hamacher et al. (1994) as was applied for the QSAM (Quasi-Steady Acceleration Measurement) accelerometer onboard the Spacelab. The output signal a_r can be described under negligence of certain error sources by the expansion shown in Equation (3.12).

$$a_r = b_0 + a + K_2 a^2 + K_3 a^3 + \dots \quad (3.12)$$

With a being the true acceleration, b_0 is the measurement bias and the K_2 and K_3 coefficients are second and third order non-linearity coefficients, respectively.

The proposed solution to modulate the signal is by turning the sensor by 180° with respect to (w.r.t.) the sensitive axis. Taking the difference of two measurements $a_{r,0^\circ}$ and $a_{r,180^\circ}$ yields then the true acceleration free of the bias and the non-linear term up to second order (Equation (3.12)). One has to note, however, that the noise is generally not affected by the modulation as it is generated internally (Hamacher et al., 1994). The rotation of the sensor head had been proposed for the B-GRAVE instrument (Van Ruymbeke et al., 2018), for a MEMS accelerometer (Du et al., 2016), and for the conical pendulum gravimeter (Subsection 2.1.1).

Additionally to the bias rejection, there is a need for a 3D-gravity measurement. Extending the rotation to a continuous rotation while recording the phase allows a full 3D-gravity vector reconstruction. While generally a 3D-vector requires three independent measurements with orthogonal basis, it has been shown by Van Hove (2019) that two orthogonal instruments suffice due to the rotation around the long axis of the spring⁶. The 3D-vector reconstruction from two orthogonal instruments can also be explained with spherical coordinates. The rotation of two orthogonal springs will return two angular orientations (e.g. azimuth and elevation in spherical coordinates) of the largest deflection of the two springs. This yields the orientation of the gravity vector. Then, the two deflection magnitudes, measured orthogonally, can be used to obtain the total gravity vector magnitude (radius in spherical coordinates). This can be achieved, as both spring deflections contain two of the three

⁶The measurement can be taken either continuously during rotation, or in-between incremental rotations. Considering the fine mass-spring setup, a continuous measurement during rotation of the gravimeter spring and transducer might induce vibrations and oscillations to the spring. Future instrument testing will be required to identify the best measurement mode for GRASS.

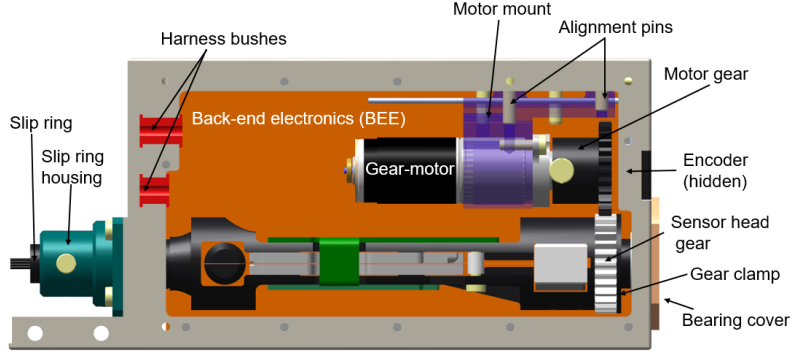


Figure 3.11: JUV-GRASS one-axis payload rotation mechanism with annotations.

Clearly, the slip ring with its housing are not exceeding half of the instrument height, allowing to orthogonally combining two instrument axes by flipping one axis upside down.

(Cartesian) gravity vector components, e.g. X-Z and Y-Z. With this information, the measured 3D-gravity vector can be obtained. For this gravity reconstruction, as part of this work, a rotation mechanism has been developed as described below.

The one-axis GRASS gravimeter rotation mechanism is shown in Figure 3.11. The instrument sensor head is mounted in two flanged bearings inside the mechanism housing. The rotation and pointing of the instrument is realised with the gear train powered by a high-performance gear-motor. The gear-motor consists of a stepper motor with an in-line planetary gear box of reduction ratio 1:80 and nominal torque of $M_{nom} = 15mNm$. The gear train has a gear ratio i equal to the ratio of the gear reference diameters (d_{in} and d_{out}) which is, with both gears having a module of 0.50, equal to the ratio of the number of teeth z (Equation (3.13)).

$$i = \frac{d_{out}}{d_{in}} = \frac{z_{out}}{z_{in}} = \frac{14mm}{14mm} = \frac{28}{28} = 1 \quad (3.13)$$

Assuming conservatively a gear efficient of $\eta = 0.9$ and having a measured rotational torque of the sensor head of $M_{sensor} = 1.2mNm$, the safety factor S on the torque can be computed (Equation (3.14)) as 11.25, which fulfils the minimum requirement of a factor 3, generally demanded for space mechanisms.

$$S = \frac{M_{in}}{M_{sensor}} = \frac{M_{nom}i\eta}{M_{sensor}} = \frac{15mNm \cdot 1 \cdot 0.9}{1.2mNm} = 11.25 \quad (3.14)$$

A contactless magnetic encoder records the rotational movement of the motor gear, which is, given the gear ratio $i = 1$, identical to the rotation of the gravimeter spring, yet with inverse rotational direction. Best alignment and therefore minimisation of the mechanical noise is not only ensured between the two bearings drilled in the same piece (housing), but likewise the motor rotational axis is very well aligned

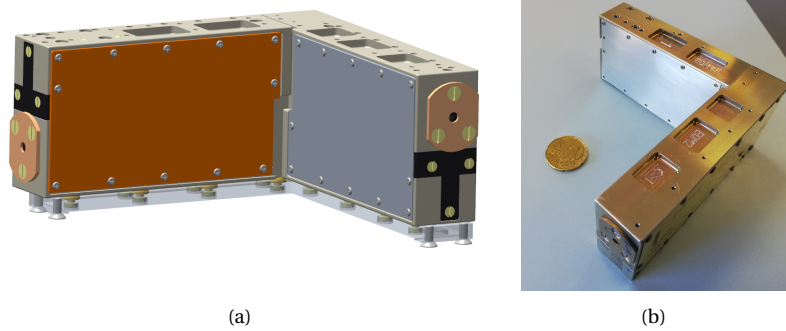


Figure 3.12: Assembly of two one-axis gravimeters (Figure 3.11) combined orthogonally, forming the full two-axes GRASS instrument. (a) CAD model. (b) Assembled instrument with 20-euro-cent coin for size reference.

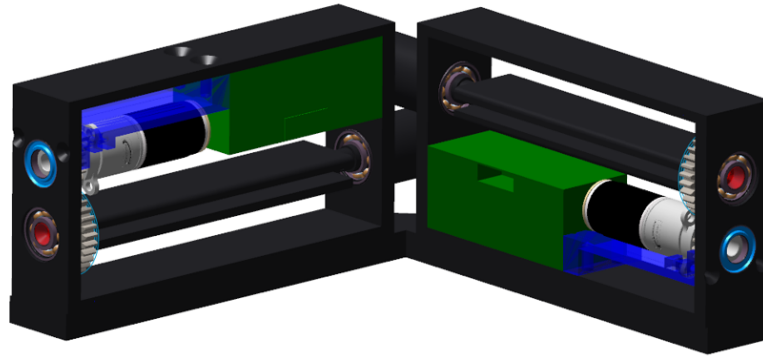
with the rotational axis of the sensor. This is achieved twofold, using iterative machining and alignment pins. Moreover, to avoid additional friction in the gear train (or even the jamming of the mechanism), due to differential thermal expansion, the nominal axis distance of 14 mm has been increased to $14.02 \pm 0.005\text{ mm}$.

Finally, the communication with and the power supply of the front-end electronics (FEE) of the capacitive displacement transducer (Section 3.3) are realised by means of a slip ring, arranged in-line with the sensor head and supported by a dedicated slip ring housing. Other than the gravimeter parts interior to the housing (Figure 3.11), the part of the slip ring does not exceed half of the total instrument height. This was done to allow a trick when combining two single-axis gravimeters to form the GRASS instrument. By stacking two quasi-identical instruments upside down, it is possible to combine them orthogonally as shown in Figure 3.12. Like this, both axes can make use of the full instrument length, limited to 100 mm and finally designed at 99 mm . The instrument development from first concept to FM design is illustrated in Figure 3.13 showing the interior of the two combined gravimeter rotation mechanisms.

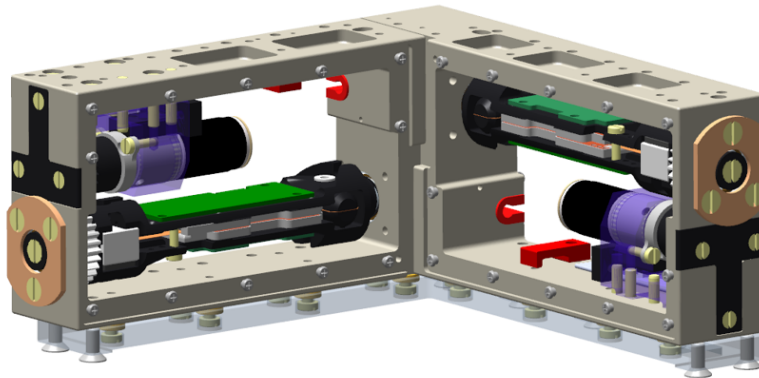
3.5 DESIGN VERIFICATION: TESTS

In order to complement the mechanical design of the GRASS gravimeter, the here presented design should be verified by tests. Sponsored by the fact that the here presented payload design forms part of the ESA Hera mission, it was possible to perform different tests of the hardware. While the test preparation was partially supported by the author, e.g. regarding the mechanical test interfaces, the test campaigns were under the responsibility of other GRASS team members, who will be credited accordingly. Nevertheless, reporting the test results here is an important component to show the design value and integrity of the here-presented development.

While further instrument qualification tests are ongoing towards the GRASS flight



(a)



(b)

Figure 3.13: (a) First concept design of the GRASS gravimeter instrument. Clearly visible, many key features, such as the upside-down combination, or the mounting-L-plate have been maintained throughout the instrument development. (b) Interior view of the GRASS instrument, with the (inside) backend electronics (BEE) and the (outside) cover hidden.

model (FM) instrument delivery that are not reported here, a set of three tests relevant to the mechanical design has been performed as a first design verification for the instrument critical design review (CDR). Starting with the thermal-vacuum (TVAC) test results (Subsection 3.5.1), a cold motor torque test of the gear-motor will be discussed in Subsection 3.5.2. Finally, pre-vibration test results will be presented in Subsection 3.5.3. These tests are discussed in Subsection 3.5.4.

For the full instrument space-qualification, a vibration and shock test, an electromagnetic compatibility (EMC) test, a thermal-vacuum (TVAC) test, and a total ionizing dose (TID) radiation test are being performed. Regarding the latter test, it is worth mentioning that it was performed at UCLouvain for GRASS, in the Centre de Ressources du Cyclotron.

3.5.1 THERMAL-VACUUM TEST

A so-called pre-TVAC testing campaign was performed by other GRASS team members (GRASS team, 2021b) at the Royal Belgian Institute of Space Aeronomy in preparation of the GRASS CDR, with special attention to the working of gear-motor and gravimeter mechanism (Section 3.4) in the relevant environment. The test was performed at a pressure smaller than $10^{-5} hPa$ and at the acceptance test temperature range was planned for $-35^{\circ}C$ to $+25^{\circ}C$ (operational) and $-35^{\circ}C$ to $+55^{\circ}C$ (non-operational), thus $5^{\circ}C$ warmer and colder than the maximum and minimum temperatures (respectively) formulated in GRASS-004 and GRASS-005. For the test, a one-axis gravimeter was mounted vertically on a dedicated mechanical interface to simulate performance in (near) zero-gravity⁷. This is pictured in Figure 3.14 together with the thermo-couples attached to the gravimeter to monitor the temperature throughout the instrument.

While the thermal-vacuum chamber (Figure 3.15) reached the targeted cold temperature of $-35^{\circ}C$, the holding temperature of about 2.5 hours was not long enough for the gear-motor to reach this steady temperature, cooling to about $-29^{\circ}C$. This can be explained by the mechanical design of the instrument, that was adapted such that the gear-motor shall be thermally insulated as good as possible from the Juventas spacecraft (here: test interface) in order to keep the gear-motor as warm as possible during surface operations (GRASS-003).

This thermal insulation was achieved by different measures that were implemented into the design. The thermal conductivity k of the motor mount was reduced by changing the material from aluminium ($k = 237 W/(m \cdot K)$) to titanium ($k = 24.5 W/(m \cdot K)$). A lower thermal conductivity reduces the heat transfer by conduction. In addition, the contact surface with the gravimeter housing as visible in Figure 3.11 was reduced from initially $207 mm^2$ to $67.3 mm^2$.

From the temperature of $-29^{\circ}C$, the motor with nominal minimum operational temperature of only $T_{min,op} = -10^{\circ}C$ was “cold-started” without problems, rotating the gravimeter sensor head without anomalies. Likewise, no problems occurred with the slip ring ($T_{min,op} = -30^{\circ}C$) and, as expected, with the dry self-lubricating

⁷The vertical mounting allows to test the instrument in (near) zero-gravity under the condition that the sensor head axis aligns with the local Earth gravity vector. Small misalignments with the vector are used on purpose to simulate the small-gravity environment of small bodies.

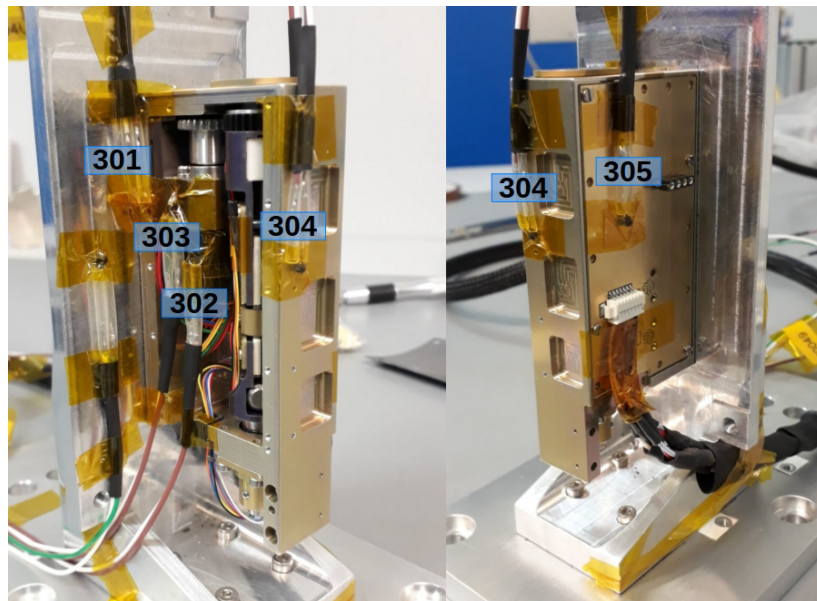


Figure 3.14: Test setup of pre-TVAC test for design verification. Special attention was given to the mechanism and gear-motor functionality, especially at cold temperatures. The thermo-couples are housing motor-near side (301), gear motor (302), motor mount (303), housing motor-far side (304), backend electronics (BEE, 305). From GRASS team (2021b).

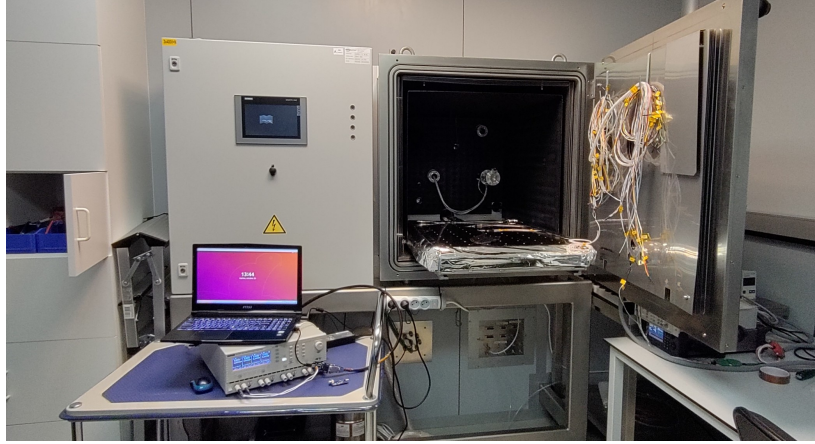


Figure 3.15: Vacuum chamber of pre-TVAC test on the right. The GRASS test setup is installed on the left for functional tests in TVAC conditions. From GRASS team (2022b).

BarTemp bearings ($T_{min,op} = -200^{\circ}C$). This was confirmed by both visual observation of the instrument rotation through the vacuum chamber window, and the recorded encoder data.

This test was a good demonstration of the gravimeter mechanism working at cold temperature. An additional test described in the next Subsection had been performed that ensured the full qualification at $-35^{\circ}C$. A first additional measure developed together with the motor supplier was to pre-heat the motor by powering the stepper motor coils of the gear-motor without turning the motor to dissipate heat. This measure, however, was not tested in absence of a need. Still, this method can be implemented in the flight software of the gravimeter as one backup measure, were the motor to fail to rotate in a cold scenario.

3.5.2 COLD MOTOR TORQUE TEST

Based on the first pre-TVAC test (Subsection 3.5.1) it was decided to perform an additional cold motor test. This was to ensure the motor working at a cold temperature of $-40^{\circ}C$, representative of an assumed cold Juventas spacecraft scenario on the surface of Dimorphos. Generally, the reduced viscosity of the motor-internal lubricant will yield reduced performance at lower temperatures, demanding a test. For this, GRASS team members including the author (GRASS team, 2022b) conceptualised the following test setup to test the motor torque after the ground storage.

The test setup is effectively a crane (“GRASS crane”) where a mass has to be lifted by the mechanism at a defined radius under the influence of Earth gravity, thus demanding a certain torque of the gear-motor. The option to directly lift the mass with the gear-motor output shaft was discarded to limit the radial loads on the shaft and to account for the gear efficiency η of the gear-train. Rather, the mass was hence suspended from the (unpopulated) sensor head cradle at radius $r = 7.5mm$

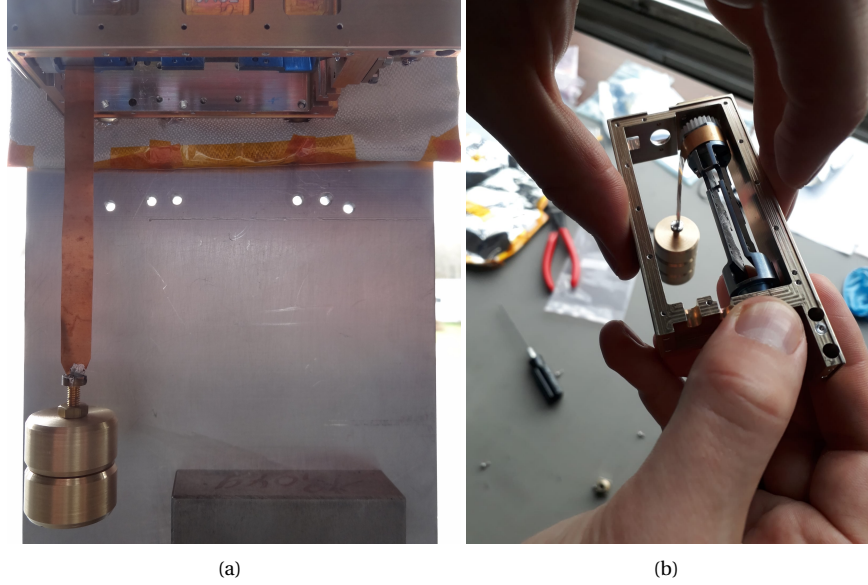


Figure 3.16: GRASS crane cold motor torque experiment. (a) Suspended test mass of 59.4 gram. (b) Top view of GRASS crane setup, with the motor still missing.

as pictured in Figure 3.16.

Regarding the mass to be lifted, the rotational torque of the sensor head was 1.2 mNm (Equation (3.14)) and the selected tested mass to be lifted was 59.4 gram. Multiplication of the mass m with the (Earth) gravity g yields the weight force F_w and further with the radius r yields the minimum motor torque M_T to pass the test (Equation (3.15)).

$$M_T = m \cdot g \cdot r = F_w \cdot r = 0.0594 \text{ kg} \cdot 0.0075 \text{ m} \cdot 9.81 \text{ m/s}^2 = 4.37 \text{ mNm} \quad (3.15)$$

With this torque, and the minimum required torque M_{sensor} the safety factor S is computed to be

$$S = \frac{M_T}{M_{\text{sensor}}} = \frac{4.37 \text{ mNm}}{1.2 \text{ m}} = 3.64 > 3 \quad (3.16)$$

The test was finally performed by Brigit Ritter and proved successful at the tested load, thus confirming a safety factor of $S = 3.64$. This fulfils the general requirement of at least $S = 3$ for space mechanisms. Generally, this demonstrated well the functioning of the motor and the developed mechanism, even in the tested harsh conditions. Clearly, this qualification test only concerns the rotation of the GRASS mechanism, but not the nominal 15 mNm of the gear-motor. If needed, this torque would demand additional tests with a corresponding larger test mass.

Despite this satisfactory test result, additional work was performed by the GRASS team regarding an external heater on the gear-motor as an addition to the above introduced option to pre-heating the stepper motor coils. The advantage here is that the heat is applied close to the planetary gear box, being the most sensitive element of the gear-motor to cold temperatures due to the decreasing viscosity of the there-applied lubricant.

The cruise phase of the gravimeter can be as long as four years for the Hera mission. During space flight, other effects on the lubricant, such as creep or separation, might occur. Therefore, as an additional measure to ensure the best functionality upon arrival at the destined extraterrestrial surface, an additional maintenance has been proposed as part of this work. Here, the gravimeter mechanism shall be exercised regularly, both during ground storage and cruise phase, ideally once per month with the minimum requirement being bi-monthly. Like this, the lubricant in the gear-motor can be redistributed to limit the performance losses of the torque driver over time. Lubricant build-ups and separation can be mitigated like this. Furthermore, the critical surfaces of the gear-motor over which the lubricant could leave the part are treated with epilamination. (Noeker et al., 2022c)

During the course of the instrument development, one ordered spare gear-motor had been in storage for now more than three years. This work therefore proposes the repetition of the above described cold motor torque test. Possible lubricant build-ups might result in reduced observable performance, strengthening the case for the maintenance procedure. Given the long storage, such a test would be a single-shot opportunity. On the contrary, a still successful outcome does not solely justify removal of the mechanism maintenance, recalling the different storage conditions (i.e. atmosphere vs. vacuum).

3.5.3 VIBRATION TEST

To show the structural integrity of the gravimeter presented in this Chapter, a preliminary vibration test has been performed by other GRASS team members (GRASS team, 2021c) at the vibration acceptance level at ALTER Technology TÜV Nord Group (Spain). This first test was done with a two-axes instrument, close to, but not fully corresponding to the here described (near-final) design. Most notably, the overall instrument length had been increased by 2.3 mm to allow a larger bending radius of the slip ring output harness. The overall deviations, however, are small and not expected to influence the structural integrity compared to the final design.

The vibration test requirements, motivated by the considered launcher, were provided by Gomspace as the Juventas satellite contractor. The success criteria have been applied as formulated in the European Cooperation for Space Standardization (ECSS) standard on space engineering - testing (ECSS-E-ST-10-03C). Most notably, check out tests (here: instrument functional test after each vibration test) were performed, and additionally visual inspections were performed to ensure that no damages (cracks, loose screws, delamination etc.) occurred. This led to the following acceptance test levels as implemented by the test facility:

Quasi-static test:

- Levels:
 - $\pm 9.6g$ at 20 Hz
- Tolerances: standard $\pm 10\%$
- Duration: 1 s

Sinusoidal vibration test:

- Levels:
 - $0.8g$ at 5 – 18 Hz
 - $9.6g$ at 18 – 100 Hz
- Tolerances: standard $\pm 10\%$
- Resonant search: 4 octaves/min

Random vibration test:

- Levels:
 - 20 Hz: $0.013g^2/Hz$
 - 20 – 50 Hz: +6 dB/octave
 - 50 – 800 Hz: $0.08g^2/Hz$
 - 800 – 2,000 Hz: -6 dB/octave
 - 2000 Hz: $0.013g^2/Hz$
 - Total gRMS = 10g
- Tolerances: standard $\pm 10\%$
- Duration: 60 s

Next to verifying any potential damages visually, a low-resonant profile has been performed to identify the structural resonances (resonance search) of the instrument after each of the tests. Here, the success criterion (sinusoidal vibration test and random vibration test, no “success criteria” are formulated for the static test) as formulated in the ECSS-E-ST-10-03C is that:

1. The frequency shift is smaller than 5 % for modes with an effective mass > 10%.
2. The amplitude shift is smaller than 20 % for modes with an effective mass > 10%.

For this, a total of four accelerometers have been attached to the instrument, as shown in Figure 3.17. Of course, these accelerometers can only determine the resonant frequencies of the entire instrument, and not of individual pieces, such as the spring (Subsection 3.2.4). Therefore, the eigenfrequency of the two springs is not visible in the vibration profiles.

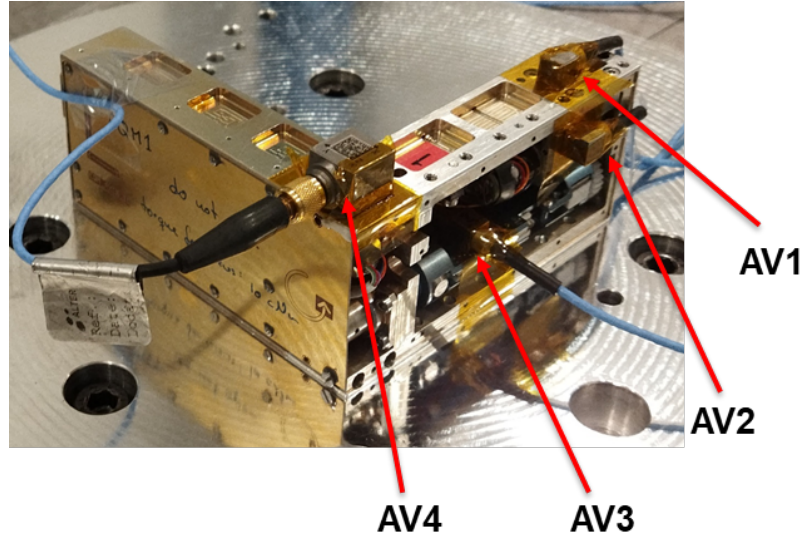
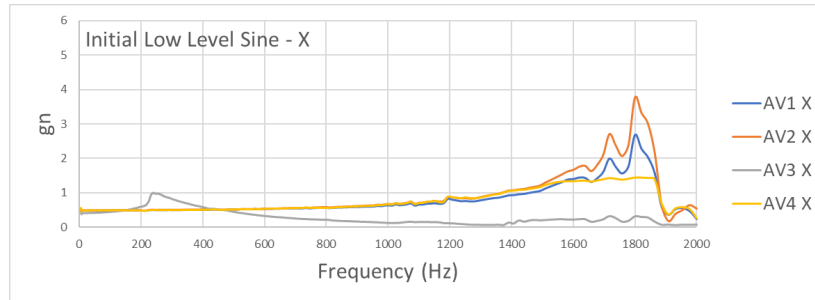


Figure 3.17: Enumeration of the four accelerometers of the pre-vibration test attached to the GRASS instrument (AV1 – AV4). The right and left instrument axes align with the accelerometer X- and Y-axis, respectively. The Z-axis is vertical, accordingly. From GRASS team (2021c).

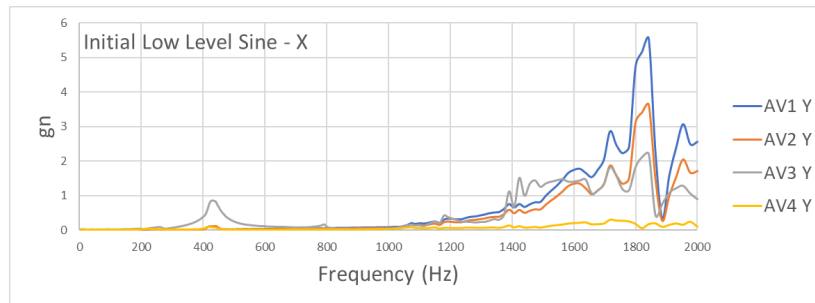
The test results of the initial low-level sine vibration test (X-axis vibration) are plotted in Figure 3.18, where the instrument was mounted to the vibration interface plate as shown in Figure 3.17. It can be observed that all critical modes are higher than 1,500 Hz. The resonant frequency levels were not shifted in the subsequent tests beyond the above-mentioned thresholds, confirming that the structural integrity of the instrument was maintained. The vibration test was therefore passed successfully. (GRASS team, 2021b)

Overall, the vibration test was therefore passed successfully and the presented gravimeter mechanism and sensor head remained intact for both axes. Regarding the gravimeter spring, the last design choice was the bending spring thickness, being the key parameter regarding the deflection, and therefore the gravimeter sensitivity (Equation (4.9)). Decreasing the thickness results in a better instrument performance in the low-gravity environment, however, the structural stability likewise decreases, finally resulting in destruction of the spring, e.g. under vibrational loads. Therefore, one axis was equipped with a spring of thickness $80\mu\text{m}$ (nominal $75\mu\text{m}$) and the other axis was equipped with a spring of identical design but thickness $50\mu\text{m}$.

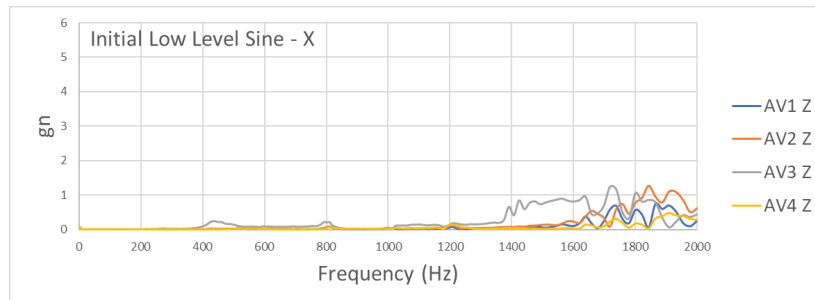
Regarding the two springs, the thinner, but not the thicker tested gravimeter spring was damaged as visible in Figure 3.19. The damage is visible in the gold-sputtered surface, and lead to some plastic deformation of the initially straight probe. The damage can be sourced to be caused by contact with the edge of the sensing electrode. Moreover, it had been observed that the damaged blade was positioned



(a)



(b)



(c)

Figure 3.18: Low-level sine vibration test results at the beginning of the pre-vibration test for the X-axis vibration test. Clearly, the main resonant frequencies appear above 1500 Hz. The results of the four accelerometers (Figure 3.17) are plotted for the 3 axes (a) X, (b) Y, and (c) Z as described in Figure 3.17. From GRASS team (2021b).

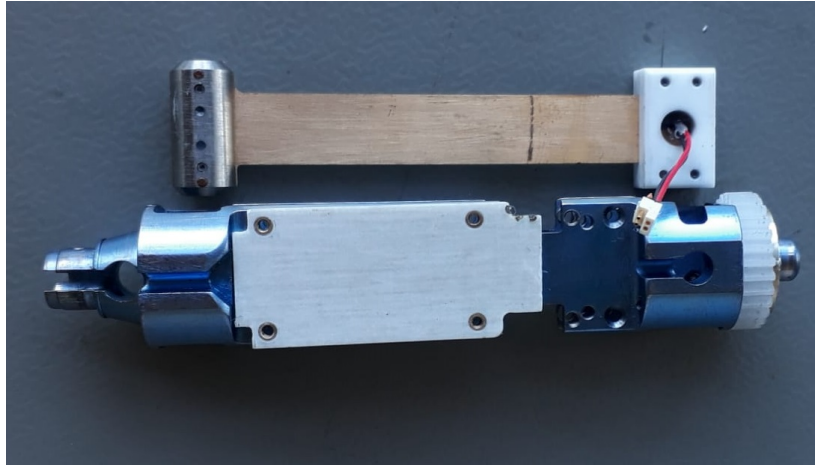


Figure 3.19: Damaged 50 micron bending spring. The internal electrode is placed on the exterior of the cradle, showing the agreement between damage and electrode edge. From GRASS team (2021c).

slightly off-centre between the two electrodes, reducing the distance between the damaged surface and the electrode edge. To obviate such damages in the future and at higher vibration qualification level loads, the following remedial measures have been applied to the design:

- The centring of the blade has been improved by ordering centring shims with smallest thickness of 5 micron, increasing in steps of 5 micron. Like this, the maximum off-centre value is limited to 2.5 micron.
- The sharp edges of the electrodes pointing towards the springs are broken by applying a chamfer.

The GRASS vibration qualification tests were performed in August 2022 with the two-axes instrument, again with the two different spring thicknesses at ALTER Technology TÜV Nord Group (Spain). The vibration qualification loads have been applied to the engineering-qualification model (EQM), which are larger than the above-mentioned acceptance loads that will be (solely) applied to the FM. The tests results are not discussed here, but will be essential in selecting the final gravimeter spring version. If both springs remain undamaged, the final choice of the spring thickness as the last unknown in the spring design will therefore depend on the final instrument characterisation and testing. The selection will be reported in future publications.

3.5.4 DESIGN TEST CONCLUSION

Towards the delivery of the FM of the GRASS instrument, further tests of the EQM are foreseen to fully space-qualify the instrument. The here presented three tests alone, however, already represent relevant results in the context of this work developing a small body gravimeter. Clearly, the design has been validated, not only

regarding the structural stability, but also for the (very) harsh temperature environment, specifically considering the cold case. The here-presented design can therefore be considered mature beyond reasonable doubt, with final tests to be done to completely confirm the suitability for space flight. Should any final problems occur, the design still allows final modifications to account for these.

3.6 DESIGN ADAPTATION FOR FUTURE MISSIONS

The development of the spring-based GRASS gravimeter has been laid out in this Chapter to a level of detail appropriate for this thesis. The aim of the here-presented instrument development, however, is not limited to the one-time Hera mission. Rather, the idea is to be able to use this concept throughout the Solar System for various extraterrestrial surfaces. For this, adaptations to the instrument will be required as listed below:

- Adaptation to different gravity environments, most notably affecting the spring.
- Selection between a 3D- and 1D-measurement. The latter might be sufficient for the terrestrial planets and other spherical Solar System objects, while the former is more favourable for non-spherical bodies.
- Volume and mass payload limitations. A longer bending spring can improve the instrument sensitivity. For example, a (human) rover on the Moon might allow accommodating a larger instrument compared to a robotic rover.

In this regard, a second mechanical gravimeter version has been developed, initially proposed to form part in the JAXA Martian-Moon-eXploration (MMX) mission that includes a DLR/CNES rover to be deployed to the surface of Phobos. The main working principle is identical to the one described in this Chapter, but the instrument is overall larger, allowing a larger degree of adaptability, e.g. regarding the gravimeter spring and displacement transducer design. More details on this second developed design are presented in Appendix A and in Noeker et al. (2021c).

3.7 GRAVIMETER DESIGN DISCUSSION

In the context of the second formulated research question (ii), the conventional spring-based gravimeter principle has been adapted for a 3D-gravity measurement in this Chapter. As formulated in the research question, the design was specifically tailored for a space science payload for operation on extraterrestrial surfaces. The instrument adaptations presented in this work that mark clear differences to the “classical” spring-based gravimeters are briefly discussed in this Section.

Starting with the instrument core element, the spring, the classical helical spring design has been replaced by a flat bending spring of rectangular cross section. While this cantilever design had been previously proposed, the spring design was optimised for arbitrary measurement orientations to provide the best sensitivity in one direction while minimising the sensitivity in the other two dimensions. A ref-

erence design for the GRASS gravimeter spring and the concept of the gravimeter spring making has been presented.

One of the novel aspects, differing to previous designs, was that the tip mass was moved outside the capacitance displacement transducer. Like this, it is possible to provide damping to the tip mass and therefore protect the spring, increase the tip mass, and reduce the separation between the capacitor electrodes.

This setup as a combination of flat bending spring and capacitance transducer needs to be rotated along the spring long axis. This rotation inverts the measurement direction, rejecting any measurement biases in the spring or transducer. Furthermore, the combination of two orthogonal axes enables a 3D-vector measurement, independent of the gravity vector orientation and instrument orientation on the extraterrestrial surface. For this, a novel rotation mechanism has been developed, that allows unrestricted, bi-directional rotation of the gravimeter spring while recording the angular position of the sensor head. One general limitation, applying to any developed gravimeter, is that attitude information (e.g. from context cameras or star trackers) is still required in order to orient the measured gravity vector with respect to the SB shape file.

Regarding the here-presented mechanical design, one risk is that the tip mass might not move upon arrival. While metal-metal contact has been avoided in the design by the addition of (vespel) dampers, there is a theoretical risk that this material could stick to the titanium cradle. The blue anodisation of the metal reduces this risk. Moreover, the proposed mechanism maintenance procedure suggested to perform a half extra rotation on Earth, to ensure the spring orientation in storage being regularly inverted. This maintenance procedure generally reduced the risk of lubricant creep and separation inside the gear-motor, supported by the application of epilamination. The thin gravimeter spring has been shown to be a very sensitive part within the instrument. While measures to avoid the observed damages have been implemented, a risk remains here that the spring could be damaged. For completeness, there is a general risk that the instrument electronics might fail. This risk could have been minimised by solely relying on space-qualified components, however, this would have created a different order of magnitude for the project budget.

Overall, a compact novel design has been developed that includes different (conceptual) aspects previously described in literature as presented in Chapter 2. Various design challenges, such as the dampers, harsh environmental conditions or differential thermal expansion have been addressed. The here presented *mechanical* gravimeter design can be considered as nearly-identical to the final FM design. One key aspect regarding future missions to other extraterrestrial surfaces is the high degree of adaptability that has been clearly represented with a second design, initially designed for the inner Martian moon Phobos (Appendix A, Noeker et al. (2021c)).

In the next Chapter, an analytical electromechanical gravimeter model is developed based on the instrument design described in this Chapter. The order of presenting these two chapters is not straightforward, as the overall instrument development was an iterative process between design choices (this Chapter) and modelling (next

Chapter). While some of the simulations were performed before commencing the design, other simulations are based on the (near) final design as presented here. It has therefore been decided to firstly describe the overall instrument here, so that the reader develops an understanding of what is analytically described in the next Chapter, together with the various introduced design parameters that serve as variables to the following simulations.

3.8 SUMMARY CHAPTER 3

This Chapter was motivated by adapting the conventional 1D-gravimeter for vertical measurements to a space gravimeter capable to measure the 3D-gravity vector at arbitrary orientation. For this, first gravimeter design requirements have been formulated in Section 3.1. Then the space gravimeter design has been detailed in three steps, starting with the gravimeter spring (Section 3.2), continuing with the displacement transducer design (Section 3.3) and closing with the gravimeter rotation mechanism design (Section 3.4). This overall design was validated by various preliminary tests performed by different GRASS team members and reported in Section 3.5. Lastly, the possibility to adapt the design to other mission targets (Section 3.6) and the overall gravimeter design (Section 3.7) are discussed, before the overall Chapter is summarised.

4

ANALYTICAL ELECTROMECHANICAL GRAVIMETER MODEL

In the previous Chapter, the mechanical gravimeter design has been presented. This was done mainly on three levels, i.e. starting at the gravimeter spring (Section 3.2), continuing with the capacitive displacement transducer¹ (Section 3.3) and concluding with the gravimeter rotation mechanism (Section 3.4). The presented design corresponds to the GRASS gravimeter design for the Juventas CubeSat, forming part in the ESA Hera mission. In order to understand the GRASS gravimeter behaviour and as prime input to the seizing of the instruments electronics, it is helpful to develop a model describing the spring and transducer behaviour.

This Chapter is therefore targeting the research question (iii) laid out in Subsection 1.5.2, describing the electromechanical behaviour of the gravimeter. For this, an analytical gravimeter model is developed, consisting of three main building blocks (Figure 4.1). Starting from the gravity, the gravity-induced spring deformation (Section 4.1), and subsequently the transducer capacitances (Section 4.2) will be described. This differential signal is then transformed to an output signal (Section 4.3) in the form of a duty cycle. It is noted that the output signal generation stands exemplarily for a multitude of possible solutions and it is recalled that the detailed electronics design, creating the final output signal, is beyond the goal and scope of this work.

In the here-sketched context, the sensitivity of the instrument has been optimised in two regards. The addition of the tip mass to the spring shows a clear increase in the instruments deflection (Subsection 4.1.2), and therefore the capacitance sensitivity (Subsection 4.2.1). Moreover, an optimisation of the lengths split-up between free-spring length and tip mass length will be presented (Subsection 4.1.3). On the other hand, a variation in the sensing electrode geometry as discussed in Subsection 4.2.2 will be shown to provide an additional leverage in improving the output sensitivity of the gravimeter.

¹ *Capacitance* and *capacitive* are used interchangeably to describe the displacement transducer.



Figure 4.1: Logical chain of the analytical electromechanical GRASS simulation. Overall, the gravity is transformed to an output signal.

Table 4.1: Reference gravities assumed for gravimeter spring deflection for celestial bodies of interest. The gravity for Earth is the standard value, and the values for Dimorphos and Phobos were introduced in Table 1.2.

Dimorphos	Phobos	Earth
$4.5 \cdot 10^{-5} m/s^2$	$5.7 \cdot 10^{-3} m/s^2$	$9.81 m/s^2$

4.1 GRAVIMETER SPRING DESCRIPTION

Initial discussions on the bending spring were introduced regarding the spring-type selection in Subsection 3.2.1. Starting from there, the deflections for different gravity environments, including Phobos and Dimorphos, are firstly simulated in Subsection 4.1.1 for a single spring. Building on this, the improvement of the deflection using the previously introduced and tacitly added tip mass is described in Subsection 4.1.2, forming the basis of the analytical gravimeter model. Directly applying this model, it is possible to perform an optimisation between the tip mass and spring lengths split-up for a defined maximum overall mass-spring length (Subsection 4.1.3). Finally, using the implemented spring deflection tool, an Earth-equivalent blade is sized, aiming at creating the deflection expected on Dimorphos for the JUV-GRASS blade under Earth gravity for laboratory testing.

4.1.1 BASE SPRING DEFLECTION

For the purpose of the spring type selection, the analytical description of the bending spring was introduced in Subsection 3.2.1. This forms the starting point of the analytical gravimeter model and has been implemented accordingly. To be able to assess the increase in gravimeter spring sensitivity due to the tip mass described in the next Subsection, the maximum deflection of the different gravimeter springs is computed in this Subsection.

While precise gravitation computations will be presented in Part II, reference values for the Earth, Phobos and Dimorphos gravity (Table 4.1) will be assumed here for the electromechanical gravimeter computation. The Earth gravity has been added, as it is interesting in the context gravimeter characterisation in the laboratory environment when reproducing the microscopic deflection with a thicker spring (Subsection 4.1.4).

To show the variability of the different spring designs presented in this work, the deflection analysis will be done for the GRASS spring for Juventus (denoted with the JUV-prefix as JUV-GRASS, Figure 3.4). This will be done for the two initially

Table 4.2: Max. deflection w_{max} at the tip of the blade, calculated analytically for different reference gravities without tip mass. The spring reference density of $\rho = 8,200 \text{ kg/m}^3$ and $E = 131 \text{ GPa}$ are used for the calculations. The (free) length l , width b and thickness h is given accordingly for each example.

h [mm]	b [mm]	l [mm]	$w_{max,Earth}$	$w_{max,Phobos}$	$w_{max,Dimorphos}$
0.100	15	65.5	1.695 mm	985.085 nm	7.777 nm
0.075	7.5	50.0	1.023 mm	594.656 nm	4.695 nm
0.050	7.5	50.0	2.303 mm	1337.977 nm	10.563 nm

considered thicknesses of 75 and 50 micron. This offers a nice demonstration of the bending spring thickness influence on the maximum deflection. In addition, the spring design originally developed for Phobos (“Rover Class”, Appendix A.1) is studied.

The free length of the spring without tip mass is set to the overall spring length with their tip masses, i.e. 50 mm for JUV-GRASS and 65.5 mm for GRASS. For consistency, the maximum deflection is always reported at the very tip of the springs, i.e. at these mentioned distances from the clamping, irrespectively of the presence of a tip mass. The resulting deflections are presented in Table 4.2.

The maximum deflection (Equation (3.2)) is directly proportional to the ratio of the distributed load (Equation (3.1)) and the second moment of area (Equation (3.4)). It can therefore be shown that reducing the thickness of 75 micron to $\frac{2}{3}$ (50 micron), results in an increase in maximum deflection of $\frac{9}{4}$, which corresponds to the here-presented values.

As can be clearly seen, the deflection on Dimorphos in the range of nanometers is microscopic. Moreso, this only concerns the maximum deflection for a given reference gravity, deviations from this gravity will therefore result in sub-nanometer deflection variations. Without detailed study, it can be stated that this would pose an extreme challenge to the displacement transducer electronics. The addition of a tip mass for small body gravity environments is therefore inevitable and the influence on the deflection is discussed in the next Subsection.

4.1.2 IMPROVED SENSITIVITY USING TIP MASS

One option to increase the deflection of any spring is by adding a mass (*sometimes called: seismic or test mass*). In the case of the flat bending spring, the mass has obviously to be added to the free end to maximise the deflection as shown in Subsection 3.2.2. Due to its position, the mass is here referred to as *tip mass*. Provided the tight constraints on the overall space gravimeter volume, it is obvious that the tip mass should be of high-density material, i.e. using tungsten with a reference density of $19,300 \text{ kg/m}^3$. Therefore, the added gravity force of the tip mass $F = g_0 m$ acts on the spring tip as sketched in Figure 4.2. Here, the tip mass had been approximated with a cylinder, while the final design contains additional features such as holes, chamfers (both reducing the mass), glue and dampers (both increasing the mass). Moreover, the above-mentioned density is only an assumption. Final weighting of the tip mass will allow to repeat the here presented computations with

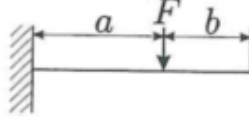


Figure 4.2: Schematic of a cantilever beam with point force, i.e. tip mass, acting on it. The gravity force acting on the tip mass added to the tip of the spring will be acting in its centre of gravity, i.e. not at the very tip of the spring. From Weinberg and Fritzen (2013).

the actual added gravity force F .

The analytical equation to compute the deflection of the cantilever beam due to a point force acting at distance a from the clamping is given by e.g. Weinberg and Fritzen (2013) as:

$$w_F = \frac{F}{6EI} (3ax^2 - x^3), (0 \leq x \leq a) \quad (4.1)$$

where a is the free, i.e. flexible, spring length between clamp and tip mass, and x is the coordinate at which the deflection is computed, starting from the clamping point, where $x = w_F = 0$.

For the total deflection $w_{sup.}$, both effects of the distributed load of the spring and the added tip mass have to be superimposed to obtain the total deflection. Compared to the spring, the added tip mass with mass m can be considered as rigid. Therefore, the deflection of the tip mass part of the spring is negligible compared to the spring deflection. Starting from the tip mass, the overall deflection is only based on the inclination at the transition spring-mass, which remains constant over the length of the mass.

The inclination ϕ of the spring with respect to the vertical is the negative derivative of Equation (4.1) with respect to the length coordinate x . Taking the derivative with respect to x and subsequently inserting $x = a$ for the spring-mass transition yields Equation (4.2).

$$\phi(a) = -\frac{Fa^2}{2EI} \quad (4.2)$$

With this, it is possible to compute the deflection w at the part of the tip mass ($a \leq x \leq l$). For this, the position of the transition $x = a$ is substituted in Equation (4.1) and for the constant inclination $\phi(a)$ a tangents term is introduced accounting for the linear increase over the linear coordinate $x = x - a$. Adding both terms leads to the deflection Equation (4.3). Here, use is made of the small angle approximation $\tan \phi \approx \phi$ as the gravimeter spring inclination will be small for SB gravity environments. Clearly, this will not be the case for deflection measurements of the small body gravimeter springs on Earth.

$$w_F = \frac{F}{6EI}(3a^2x - a^3), (a \leq x \leq l) \quad (4.3)$$

Adding the deflections from Equations (3.2) and (4.1) yields the deflection for the free spring length ($0 \leq x \leq a$). Similarly, for the part of the (rigid) tip mass ($a \leq x \leq l$), this is done with substitution of $x = a$ in Equation (3.2) and the addition of Equation (4.3). Here it is noted that only Equation (4.3) considers the effect of the (much larger) inclination (i.e. the angle with respect to the horizontal) at the intersection with the tip mass. The inclination due to the distributed load acting on the spring is much smaller and therefore neglected here.

$$\begin{aligned} w_{sup.} &= w_q + w_F \\ w_{sup.} &= \frac{q}{24EI}(6a^2x^2 - 4ax^3 + x^4) + \frac{F}{6EI}(3ax^2 - x^3), (0 \leq x \leq a) \\ w_{sup.} &= \frac{qa^4}{8EI} + \frac{F}{6EI}(3a^2x - a^3), (a \leq x \leq l) \end{aligned} \quad (4.4)$$

Using this analytical description, the MATLAB gravimeter model has been extended to account for the addition of the tip mass. Provided the different spring designs with cylindrical and cuboid tip mass, both geometries have been implemented. For verification, a comparison between the analytical tool and a finite element method (FEM) implemented in PTC Creo Simulate 5.0 has been performed (Figure 4.3). For this, the JUV-GRASS blade with thickness of 75 micron has been simulated with the added cylindrical tip mass and the deflections have been plotted in Figure 4.4. For technical reasons, when exporting the deflection data from Creo, only the deflection of the free spring part is plotted. Clearly, the agreement between the deflection shape is good, but it becomes visible that the absolute deflection magnitude in the FEM method is larger than the analytical solution (350.9 nm and 326.1 nm). It was not possible to identify the cause for this deviation, but it was observed that this deviation decreased for longer springs, and that the agreement for the cuboid tip mass was generally better than for the cylindrical tip mass. Nevertheless, the overall agreement is still sufficient for the purpose of describing the global instrument behaviour. Regarding the absolute deflection magnitude, neither solution can be considered as the truth. Rather, at a later stage, it will be required to improve the input parameters, including the spring and tip mass densities, as well as the spring Young's modulus, using additional measurements and further calibration.

In the following, the developed and verified analytical MATLAB tool will be used to update the deflection numbers presented in Table 4.2, using the same springs with now added tip masses. Despite some limitations of the MATLAB tool as discussed above, it is powerful to directly assess and present the influence of the measures taken to improve the spring deflection and, therefore, the gravimeter performance. The results are shown in Table 4.3. It should be noted that the deflections under Earth gravity are purely mathematical. The analytical tool is not developed for macroscopic deflections, and the deflections unrealistically exceed partially the spring dimensions.

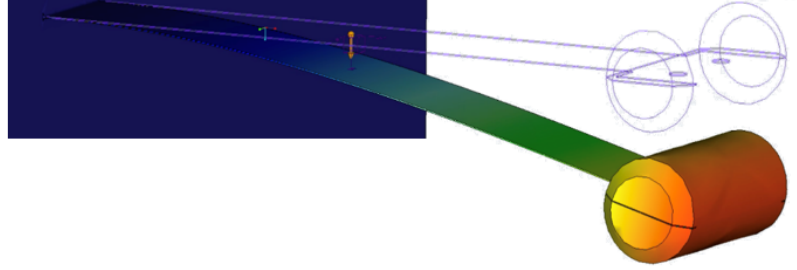


Figure 4.3: Deflection simulation of gravimeter spring using FEM in PTC Creo Simulate 5.0. The undeflected spring is shown with a wire frame, and the colour code represented the (graphically exaggerated) deflection, with the largest deflection at the tip of the tip mass.

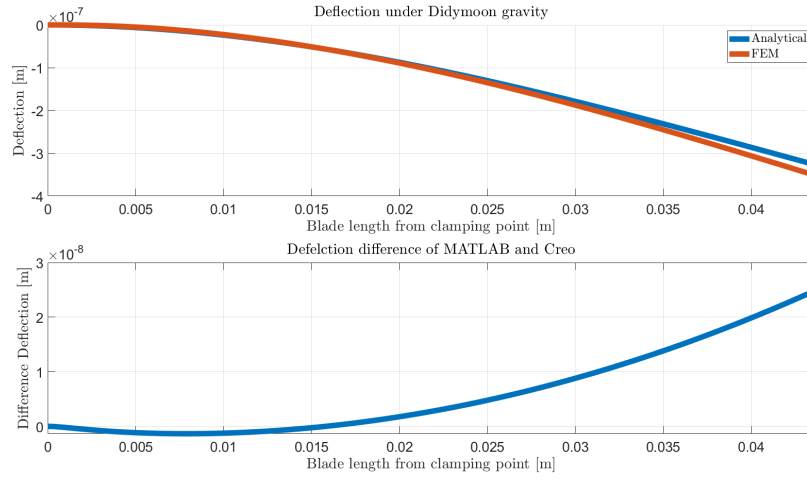


Figure 4.4: Deflection analysis of the gravimeter spring as defined in Figure 3.4 simulated with FEM (PTC Creo Simulate 5.0) and with the analytical model (MATLAB). The FEM method here is used as a verification of the generic analytical MATLAB code.

Table 4.3: Max. deflection w_{max} at the tip of the added tip mass. The 100 micron blade is the GRASS blade and the other two blades correspond to the JUV-GRASS blade, all with the respective cuboid and cylindrical tip masses, respectively. The spring reference density of $\rho = 8,200 \text{ kg/m}^3$ and $E = 131 \text{ GPa}$ are used for the calculations. The free length l , width w and thickness d is given accordingly for each example.

d [mm]	w [mm]	l [mm]	$w_{max,Earth}$	$w_{max,Phobos}$	$w_{max,Dimorphos}$
0.100	15.0	63.0	39.904 mm	23.186 μm	183.045 nm
0.075	7.5	43.7	81.811 mm	47.536 μm	375.281 nm
0.050	7.5	43.7	275.127 mm	159.860 μm	1262.049 nm

Table 4.4: Maximised (max.) blade deflection w resulting from the ideal setting of optimised (opt.) free blade and tip mass lengths. The results are here shown for the commonly investigated GRASS (thickness 0.1 mm) and JUV-GRASS (thicknesses 0.075 mm and 0.050 mm) blades. The maximum deflection is shown for all springs using Didymoon reference gravity.

Blade thickness	opt. free length	opt. mass length.	opt. max. w
0.100 mm	41.6 mm	23.9 mm	0.854 μm
0.075 mm	23.8 mm	26.2 mm	2.263 μm
0.050 mm	23.8 mm	26.2 mm	7.635 μm

Clearly, the influence of the tip mass is significant comparing Table 4.3 with Table 4.2. For example, the nominal gravimeter spring for the Hera mission has an increased deflection from about 4.7 nm to 375.3 nm, corresponding to an increase of nearly a factor 80. Given the defined (limited) overall spring length, however, it has to be noted that while the added tip mass increases the overall deflection, the available free length of the spring shortens due to the added rigid tip mass. These two counteracting effects will be investigated further in the next Subsection.

4.1.3 SPRING AND MASS LENGTHS SPLIT-UP

In the previous Subsection, the option to add a tip mass to the tip of the blade was introduced. Looking beyond the spring designs presented in this work, and bearing in mind future developments, an optimisation can be performed between the free length of the spring and the length of the tip mass for a given overall mass-spring length. As stated above, an increase in tip mass length increases the mass of the tip mass, while it reduced the free spring length, and thus the flexible length available for deformation. These two effects are therefore counteracting.

Considering the three spring designs as in the previous Subsections, an optimisation between these two can be obtained, providing a valuable starting point for future developments. For the GRASS spring, the cuboid tip mass will be studied (Figure A.1) and for the JUV-GRASS spring, cylindrical tip mass shapes (Figure 3.4) are studied.

Using the above introduced MATLAB deflection tool, a simple loop over the free length can be built around the deflection analysis script. For a given overall length, i.e. the sum of the free length and the tip mass length, the iteration is done over the free length in steps of 0.1 mm, and the mass length (equal to the diameter for the cylindrical mass) results directly from the difference of overall length and free length. All other parameters, including the spring and tip masses thickness and widths are kept constant, but can directly be varied in future studies. The geometry with the largest deflection is stored, where this is independent of the acting gravity, which only scales the deflection. For reference, the Dimorphos gravity is applied to illustrate the effect on the deflection. The optimisation results are tabulated in Table 4.4.

Considering the 75 micron blade, the maximum deflection could be increased further by a factor 6 by applying the length split-up optimisation. However, taking this

best-deflection case will not necessarily be practical. For example, the optimised cylindrical tip mass length (which is equal to the “height” due to the circular cross section) of 26.2 mm, would directly drive the gravimeter transducer diameter, currently limited to 14 mm. The design of the sensor head would therefore require adaptation, potentially leaving less room for the front-end electronics. Moreover, with increasing tip mass the structural stability of the mass-spring system will decrease and additional tests will be required, especially for larger tip masses. Still, this study is a valuable addition to the gravimeter design and these results should be included in future gravimeter developments. It is observed that the ratio for the two thicknesses of the JUV-GRASS blade (75 and 50 micron) appears identical, which can be explained with Equation (4.4). The term $\frac{g}{EI}$ can be factored out, and therefore the deflection only depends on the free spring mass per length m/l , the free-length a and the mass of the tip mass m . Therefore, indeed the ratio will not be perfectly identical, but is very close to each other for variable spring thickness, as the spring mass per length is much smaller than the mass of the added tip mass, and therefore much less affecting the deflection than the combination of tip mass mass and free spring length.

4.1.4 SMALL GRAVITY SIMULATION

The core of this Chapter is based on the fact that the spring deflection is directly proportional to the acting gravitation. Therefore, testing of the thin springs on Earth requires a special test setup. For example, it is possible to mount the instrument perfectly vertically (i.e. aligning the long axis of the spring with the gravity vector) on a rotating table and simulate (small) gravity by the rotation-induced centrifugal force. The moving setup and possible tilt variations in the course of one rotating table revolution are, however, not ideal, e.g. for static instrument measurement. An off-Earth solution to this problem was proposed by Noeker et al. (2020), proposing to test the GRASS gravimeter onboard the International Space Station (ISS) sensing the accelerations induced by the atmospheric drag constantly decelerating the ISS and requiring regular orbit maintenance of the human outpost. Alternatively, the instrument can be mounted again vertically and an effective small gravity is realised by small inclinations from the vertical.

For the latter, the required inclinations for absolute gravity and a 1 % change thereof are listed in Table 4.5. For example, considering the available Newport BGS50PP motorised goniometer², with an absolute accuracy (typical) of ± 35 mdeg = 126 arcsec = $610.865 \mu rad$, an uni-directional repeatability (typical) of ± 0.5 mdeg = 1.8 arcsec = $8.727 \mu rad$ and a minimum incremental motion is 0.20 mdeg = 0.72 arcsec = $3.491 \mu rad$, it is not possible to meet the inclination requirements for Dimorphos. These requirements are beyond reach for such laboratory equipment and would need a more advanced tilting mechanisms. Even for the Phobos reference gravity, only the order of magnitude could be achieved by precise tilting, and in all cases additional precise tiltmeters are required in combination with any inclination setup.

As the implementation of the high-performance tilt mechanism and tilt reading was found to be non-trivial, an alternative solution has been developed as part of

²Cf. <https://www.newport.com/p/BGS50PP>, accessed on 03.03.2020.

Table 4.5: Overview of necessary inclination from vertical α , changes in inclination to realise a 1 % change in local gravity $\Delta\alpha$ (same for both signs due to small angle approximation).

Body	α [arcsec]	α [μrad]	$\Delta\alpha$ [arcsec]	$\Delta\alpha$ [μrad]
Didymoon	0.946	4.587	$9.462 \cdot 10^{-3}$	$45.872 \cdot 10^{-3}$
Phobos	119.848	581.040	1.198	5.810

this work, allowing to facilitate early instrument testing and electronics development.

In order to achieve spring deflections under (full) Earth gravity in the same order of magnitude as expected on Dimorphos, an Earth-equivalent spring (EES) was sized, using the analytical spring deflection model introduced in this Section. The aim of this is that by increasing the thickness of the bending spring, and by omitting the tip mass, a deflection under (full) Earth gravity comparable to the absolute deflection of the reference spring as expected under Dimorphos (reference) gravity g_{ref} (Table 4.5) is achieved. The JUV-GRASS blade was simulated to have a maximum deflection of 375 nm (Table 4.3). The EES is not equipped with a tip mass to limit the overall deflection. Therefore, the deflection is not simulated at the end of the spring (i.e. the end of the tip mass), but rather at the end of the spring without tip mass. Therefore, the overall length is set equal to the free length of the studied springs, i. e. 43.7 mm for the JUV-GRASS design (Figure 3.4).

The deflection value at the intersection of spring and tip mass has been computed with the deflection tool (Figure 4.4), being 326.1 nm. This value is therefore the target value for the maximum deflection of the (shorter) EES.

With the here-developed deflection analysis tool, it is therefore easily possible to determine the required thickness that realises a comparable deflection. Rather than solving for the very precise value to exactly match the deflection, a typical raw material thickness is searched, to facilitate the machining of the equivalence blade. Due to in-house availability, this equivalent blade was machined in titanium, considering a Young's modulus of $E = 116 GPa$ and a density of $\rho = 4,500 kg/m^3$. Applying this, by trial and error, a good representation of the deflection is achieved with a thickness of 3.0 mm, leading to a maximum equivalent (eq.) deflection of

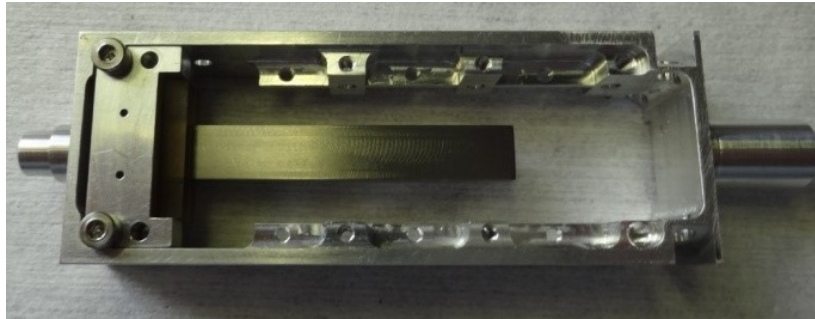
$$w_{max,eq.} = 229.2 nm$$

which is smaller than the target deflection of 326.1 nm and therefore provides a conservative solution, representative of a gravity value smaller than the reference gravity, while maintaining the same order of magnitude for testing the gravimeter sensing electronics. Figure 4.5 shows the machined 3 mm thick equivalence blade, with width of 7.5 mm and free length of 44.1 mm³ [footnote, not cube]. The larger EES blade was mounted in the GRASS-designed sensor head (Appendix A.1),

³Note that following an increase in the tip mass diameter, the free length had been reduced from the here-shown 44.1 mm to 43.7 mm in the FM design.



(a)



(b)

Figure 4.5: Machined Earth-equivalence blade. (a) Earth-equivalent blade assembly. (b) Blade assembly clamped in GRASS cradle. Machined and pictured by Emiel Van Ransbeeck.

showing the large adaptability of this design for various springs, and likewise the accordingly smaller sensing electrodes.

4.2 CAPACITANCE TRANSDUCER

In the previous Section, the gravimeter spring design has been studied and the bending spring deformation in the low-gravity environment was simulated. As for all spring-based gravimeters, the spring deformation needs to be measured in order to infer the acting gravity. The selected working principle is a capacitive deflection transducer (Section 3.3). With the deflection simulation in place, the next element in the measurement chain and therefore in the instrument simulations is the capacitance simulation.

This Section introduces firstly the theory of the capacitance transducer as the basis for the simulation (Subsection 4.2.1). The classical capacitive transducer assumes parallel surfaces (plates). In the context of the gravimeter development, but also as input to the instrument simulator, the transducer design parameters are in-

troduced in Subsection 4.2.2. After this, the capacitance of the undeflected spring (Subsection 4.2.3) and the deflected spring (Subsection 4.2.4) are computed. This is followed by likewise computing the capacitance gradient (Subsection 4.2.5) and a trade-off between absolute capacitance and sensitivity (gradient) in Subsection 4.2.6. Finally, the computations are applied to the setup of the GRASS instrument as designed for Juventas in the Hera mission (JUV-GRASS).

4.2.1 CAPACITANCE: PHYSICAL PRINCIPLE

The GRASS physical principle baseline is measuring the spring deflection under gravity by means of capacitance. For this, basically *two* capacitances between (1) the top electrode and the blade and (2) the bottom electrode and the blade are measured. A measurement device that translates movement (deflection) by means of capacitance is generally referred to as *capacitance/capacitive transducer*. The dependency of capacitance C and distance between the blade and the electrode is a basic physical relation that is commonly used to electrically measure displacements. The formula for the capacitance C of a parallel plate capacitor is given in Equation (4.5).

$$C = \epsilon_0 \epsilon_r \frac{A}{d} \quad (4.5)$$

where $\epsilon_0 = (8.854187817...) \cdot 10^{-12} C/(Vm)$ is the permittivity of vacuum, ϵ_r is the permittivity of the medium in-between the plates, using exactly 1 for vacuum (space) and being 1,00059 for air (laboratory), A the area of the plates and d the distance between the surfaces of the capacitor plates. The latter is here also referred to as *gap size*, the distance between the gravimeter spring and the electrodes *surfaces*.

Considering Equation (4.5), using a dielectricum having a permittivity $\epsilon_r > 1$ could therefore be used as a scale factor for both the absolute capacitance and the gradient of it. While liquids and gases would obviously disqualify for the space instrument, some other materials could be introduced to increase the electric permittivity between the electrodes and the blade. This option, however, of adding additional material in the very small gaps of no more than 100 micron will introduce mechanical problems regarding the tolerances and the gap size realisation. Alternatively, the gap can be increased to introduce a dielectric material, but a larger gap size d will work in opposite direction than the increase in capacitance induced by a larger permittivity ϵ_r .

In the previous Chapter, the spring deflection has been analysed and plotted. It is clear that the blade and the electrodes will not be parallel, as the deflected blade has a non-constant deformation with changing angle w.r.t. the horizontal. As the deflection and the gap size change along the blade length, the integral of the function needs to be taken (Equation (4.6)).

$$C = \int_a^b C((d(x)) dx \quad (4.6)$$

where C is calculated for the variable gap size d which is non-constant due to the deflection of the blade. The bounds of the integral are simply the start- and end-points $[a; b]$ of the sensing electrodes that will be discussed in Subsection 4.2.2. The total capacitance is numerically calculated using the trapezium method rule as shown in Equation (4.7).

$$C = h[\frac{1}{2}f(x_0) + f(x_1) + f(x_2) + \dots + \frac{1}{2}f(x_n)] \quad (4.7)$$

For this, an equidistant division h (Equation (4.8)) over the integration range, i.e. equal to the electrode length is simply done for n number of increments, where here generally 10,000 increments were used with little influence on the computation time.

$$h = \frac{b - a}{n} \quad (4.8)$$

In the context of this work, not only the absolute capacitance is of interest but also its derivative with respect to the gap size. The change of capacitance with respect to changes in the distance follows from the derivative of Equation (4.5) as:

$$\frac{dC}{dd} = -\epsilon_0\epsilon_r \frac{A}{d^2} \quad (4.9)$$

This ratio is also representative of the sensitivity of the capacitance transducer, as it describes the change in C per unit length of displacement. It is obvious that a smaller gap size d will increase the sensitivity. Recalling Equation (4.4), the deflection and thus the gap size d is directly proportional to the gravity. The capacitance gradient, with respect to the gap size d , is non-linear due to its scaling with $1/d^2$. However, for small changes in the deflection, e.g. due to variations of the local gravity, due to tides (Subsection 5.1.4), libration (Subsection 5.1.3), or changes in the measurement position (rover traverse), this gradient can be linearised.

Without question, the here-provided analytical capacitance computations consider a parallel plate setting, which is not the case for the bending spring as plotted in the previous Section. Patla (2012) presents an approximate analytical formula for small angle approximation for non-parallel plate capacitors and shows that significant influences on the resulting capacitance occurred at plate inclinations larger than 1 mrad. In this context, an inclination assessment had been performed (Equation (4.2)) for the GRASS spring on Phobos and the JUV-GRASS spring on Dimorphos, leading to maximum inclinations of 0.6 mrad and $12\mu\text{rad}$, respectively. Hence, the parallel plate description can be considered valid for the small body gravimeter, but needs to be reconsidered for much larger deflections, e.g. using the same springs with their full deflection on Earth, Mars, and the Moon.

4.2.2 DESIGN PARAMETERS

In Subsection 3.2.1, the design parameters have been listed for the gravimeter spring. Likewise, before performing simulation and optimisation, the design parameters of the capacitance transducer are introduced in this Subsection.

Gap size

The gap size has been already discussed in Subsection 4.2.1. From Equations (4.5) and (4.9), it is directly visible that both total capacitance and sensitivity, respectively, benefit from a separation distance as small as possible. This is physically limited by machining and geometric tolerances. For example, the flatness of the electrodes and the parallelism of the contact areas between cradle and electrodes will have some machining tolerances. Moreover, the spring has to be centred between the two electrodes. While an off-centre position can be accounted for in the data treatment (Section 4.3), it is an additional limitation to decreasing the gap size, as the centring position error could become larger than the gap size. If the machining of all parts is of superior quality, a gap size of not smaller than 50 micron can be achieved. Hereof, a deviation of the central blade from better than 5 micron (improvable to ± 2.5 micron with 5 micron shims) has been measured during instrument integrations by Emiel Van Ransbeeck.

In the early instrument design, the gap size was further limited by the inclusion of the tip mass in-between the two electrodes. This was resolved by moving the tip mass outside the electrode setup. Of course, it could also be possible to monitor the capacitance only over the tip mass to increase the sensitivity, however, this makes the damping of the seismic mass more difficult.

Sensing electrode start and end points

The beginning and the end of the electrodes is equivalent to the integration bounds $[a, b]$ for calculating the capacitance (Figure 4.6). For the shown JUV-GRASS geometry, representative of the maximum sensing electrode length achievable, the start $a = 1.15\text{ mm}$ and the end $b = 41.65\text{ mm}$ are both measured from the clamping point. Recalling the free blade length of 43.7 mm, the sensing electrode only covers 40.5 mm due to the guard ring (to minimise the parallel plate capacitor fringe effects⁴) and design margins. The same applies for the GRASS electrodes, where $a = 2.9\text{ mm}$ and $b = 61.75\text{ mm}$. Here, the sensing electrode covers 58.85 mm of the total free blade length of 63.0 mm.

It is obvious that for cantilever setup, the deflection on the clamp-far side is most sensitive to changes in gravity, while the deflection closest to the clamp will be most insensitive. Therefore, an adaptation of the start a towards the tip will make the instrument more sensitive, yet decreasing the absolute capacitance. This will be studied in the capacitance trade-off in Subsection 4.2.6. One example of this more sensitive electrode-pair was used in the final version of the JUV-GRASS, as shown in Figure 4.7. Here, the shown starting point measured from the clamp is $a = 31.95\text{ mm}$

⁴Fringe effect is referred to as outward bending of the electric field lines at the edges of capacitor electrodes. Other than inside the capacitor, the field lines are hence no longer parallel.

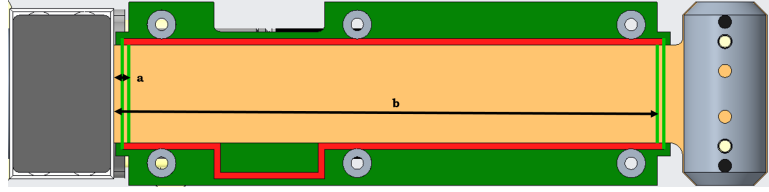


Figure 4.6: Definition of start a and end b of the electrode here highlighted in green and shown below the JUV-GRASS blade. Counting from the clamping side (left) the second and third vertical green bars, showing the hidden lines of the gap between electrode and guard ring, indicate the start and end points of the electrode, respectively, as indicated by the arrows. The here shown setting $a = 1.15\text{ mm}$ and $b = 41.65\text{ mm}$ is the maximum achievable size that will effectively be a few millimetres shorter than the free blade length, due to the required distance to clamp and tip mass.

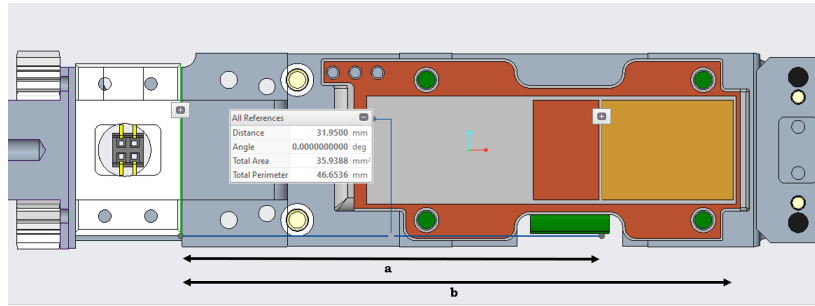


Figure 4.7: Final design of JUV-GRASS sensitive electrode highlighted in brass-color. The integration bounds for the sensing capacitance are $a = 31.95\text{ mm}$ and $b = 41.95\text{ mm}$. The pad left of the sensing pad (copper-color) is for calibration purposes using electro-static forcing. Electrode design from GRASS-team.

and the length is 10 mm, thus $b = 41.95\text{ mm}$. This design is referred to subsequently as the optimised (opt.) JUV-GRASS electrode design.

Regarding the possible calibration using electrostatic forcing with the secondary pad on the electrode, the electrostatic force should counteract the weight force on the spring with the tip mass. While this has not been studied here in detail, the order of magnitude of required electrostatic force will be comparable to the weight force of the tip mass ($0.351\mu\text{N}$). A first numerical simulation in COMSOL confirmed this order of magnitude, calculating a required force load of ca. 8 mN/m^2 , which yields a total force of $1.08\mu\text{N}$ for a possible calibration electrode pad of 18 mm. As the lever arm of the electrostatic force is shorter than that of the tip mass, the required force is larger accordingly. For a gap size of 80 micron, the numerical simulation returned a demand of a voltage of 2.41 V to create this electrostatic force. (GRASS team, 2022c)

4.2.3 BASE CAPACITANCE

For the development of the sensing electronics a good knowledge of the expected capacitance and changes thereof is required. While the detailed electronics design was not part of this work, the simulation of the capacitance values were included in the here-developed analytical gravimeter instrument model. This analytical capacitance computation is based on the spring deflection computation (Section 4.1) and can be considered as prime input when starting the transducer sensing electronics design, while additional numerical simulations might be performed.

The capacitance simulation presented in this Section will be divided in three steps. Firstly, in this Subsection, the base capacitance is calculated for the two gravimeter models GRASS and JUV-GRASS, i.e. the capacitance without any spring deflection. Following this, in the next Subsection, the deflection expected under different gravities will be used to compute the deflected spring capacitances, and finally, the resulting capacitance gradient, as important measure of the overall instrument sensitivity, is computed in Subsection 4.2.5.

The base capacitance is a first measure of the absolute capacitance that the two capacitor settings *spring-electrode* and *electrode-spring* will create. These values will only occur in a zero-acceleration regime, since they assume no deflection of the blade. Moreover, the first values shown are an idealisation that assume that the blade is perfectly centred (two identical gap sizes) and perfectly straight, and that the electrodes are both mounted perfectly parallel. The deviations from this ideal setting were discussed in Subsection 4.2.2. Regardless, the absolute capacitance of the setup is an important first figure when sizing the electronics design.

Using Equation (4.5), the base capacitance will be calculated for the two instrument versions introduced in this thesis, being the GRASS (Appendix A.1) and JUV-GRASS (Section 3.3) displacement transducers. Since the initial instrument testing was done in the laboratory under atmosphere, rather than vacuum, the relative permittivity $\epsilon_r = 1.0006$ is used in all calculations to allow later comparison of simulation and laboratory measurements (Section 4.4). This value must be adapted to $\epsilon_r = 1$ for vacuum and space conditions.

In Table 4.6, the capacitance is calculated for the two instruments for the following cases, the nominal (nom.) design gap size and the gap size as measured (meas.) after the first complete assembly, as well as the optimised (opt.) case with the more sensitive electrodes (Subsection 4.2.6). On top of this, to assess the sensitivity of the gap size to the machining tolerances, the measured gap size of 82 micron has been tested with the specified tolerance of $\pm 0.005\text{ mm}$, resulting in one gap of 87 micron and in one gap of 77 micron (opt. tol.). All cases consider the blade thickness of 75 micron. The final gap size, e.g. of the final JUV-GRASS flight model (FM) will have slightly different gap sizes, determined by the centring of the spring, and the tolerances of the cradle and the electrode material thickness.

The values presented here provide a first overview of the capacitance regime of the two instruments. Inspection of Equation (4.5) allows directly to make the following statements regarding the capacitance displacement measurement:

- The total capacitance directly scales with the common surface area of blade

Table 4.6: Baseline capacitance of GRASS and JUV-GRASS instrument for nominal (nom.) and measured (meas.) gap sizes for the two gaps. JUV-GRASS optimised (opt.) considers measured gap size and short sensing pad of length 10 mm (Figure 4.7). The sensitivity to tolerances are assessed for the machining tolerance of $\pm 0.005\text{ mm}$ (opt. tol). Instrument dimensions as described in Chapter 3 (JUV-GRASS) and Appendix A (GRASS). Obviously, two identical gap sizes lead to identical capacitances $C(A)$ and $C(B)$.

Instrument	Gap size (measured/designed)		Calculated C	
	small gap (A)	large gap (B)	$C(A)$	$C(B)$
GRASS nom.	0.1 mm	0.1 mm	78.141 pF	78.141 pF
GRASS meas.	0.056 mm	0.057 mm	139.54 pF	137.09 pF
JUV-GRASS nom.	0.1 mm	0.1 mm	26.911 pF	26.911 pF
JUV-GRASS meas.	0.082 mm	0.082 mm	32.818 pF	32.818 pF
JUV-GRASS opt.	0.082 mm	0.082 mm	8.103 pF	8.103 pF
JUV-GRASS opt. tol.	0.077 mm	0.087 mm	8.629 pF	7.638 pF

and electrode A , halving the area halves the capacitance.

- The capacitance is inversely proportional to the gap size d , halving the gap size doubles the capacitance.
- The base capacitance is, in principle, independent of the blade thickness, as the gap size is only measured from electrode *surface* to blade *surface*. Of course, changing the blade thickness, i.e. between 75 and 50 micron as in JUV-GRASS, without further adaptations of the cradle, will influence both gap sizes.

4.2.4 DEFLECTED SPRING CAPACITANCE

Building upon the spring deflection simulation (Subsection 4.1), it is possible to calculate the resulting capacitance due to the deflection as was described in Subsection 4.2.1. Like in the previous Subsection, the two displacement transducer models with the nominal and measured gap sizes will be investigating regarding the two resulting capacitances. As GRASS was designed for Phobos and JUV-GRASS for Didymoon, the respective gravity is applied for computing the deflection. The results shown in Table 4.7 provide a first indication of the differential capacitance to be sensed when comparing them to the base capacitance (Table 4.6). Regarding this, the capacitance gradient will be computed in the next Subsection. By definition, here, the smaller gap is decreased further by the deflection, whereas the larger gap is increased by the deflection. This can be adapted in the simulation tool, as required. As can be expected, the capacitance of the small gap (decreased by the spring deflection) is always larger than the base capacitance, which in turn is always larger than the large gap (increased by spring deflection) capacitance.

4.2.5 DEFLECTED SPRING CAPACITANCE GRADIENT

In the same fashion as for the absolute capacitance due to the deflection, the gradient (Equation (4.9)) is evaluated. As stated, linearising the gradient at the deflected

Table 4.7: Resulting capacitances in the GRASS and JUV-GRASS instrument on Phobos and Didymoon, respectively. Gap sizes and abbreviations as in Table 4.6.

Instrument	C gap small (A)	C gap large (B)
GRASS nom.	78.195 pF	78.085 pF
GRASS meas.	139.71 pF	136.92 pF
JUV-GRASS nom.	26.950 pF	26.871 pF
JUV-GRASS meas.	32.877 pF	32.759 pF
JUV-GRASS opt.	8.134 pF	8.072 pF
JUV-GRASS opt. tol.	8.667 pF	7.610 pF

Table 4.8: Resulting capacitances gradient in the GRASS and JUV-GRASS instrument on Phobos and Didymoon, respectively. Gap sizes and abbreviations as in Table 4.6.

Instrument	C gap small (A)	C gap large (B)
GRASS nom.	782.50 nF/m	780.30 nF/m
GRASS meas.	2498.0 nF/m	2399.1 nF/m
JUV-GRASS nom.	269.90 nF/m	268.31 nF/m
JUV-GRASS meas.	401.66 nF/m	398.78 nF/m
JUV-GRASS opt.	99.577 nF/m	98.060 nF/m
JUV-GRASS opt. tol.	112.985 nF/m	87.151 nF/m

setup allows to make a statement of the displacement transducer sensitivity, i.e. capacitance per distance (Farad per meter). The gradients that are only true for the gap sizes reported in Table 4.6, are listed in Table 4.8. As the deflection is directly proportional to the gravity, the here-computed gradient can be used to obtain the expected change in capacitance, e.g. resulting from a 1 % change in gravity (deflection). The gradient computation is included in the here-developed analytical gravimeter tool.

As can be expected, the smaller gaps are more sensitive than the larger gaps. It is important to repeat that the gradient evaluation is a linearisation that is only valid for small changes of the absolute deflection. For example, given that the deflection under the nominal gravity would change slightly, e.g. by 1 %, the gradient would not change significantly in this small portion of the gap size d .

4.2.6 CAPACITANCE TRADE-OFF

As worked out in this Section, there is no simple optimum between absolute capacitance and sensitivity. Larger absolute capacitance will result in smaller sensitivity, and vice versa. Therefore, an optimising trade-off between the absolute capacitance (C) and the *relative* sensitivity (dC/dd) is developed. In this Subsection, on the example of the JUV-GRASS design, the connection between absolute capacitance and the relative sensitivity will be assessed. In this trade-off, it is clear from Equations (4.5) and (4.9) that both the absolute capacitance and “absolute sensitivity” directly scale with the common electrode surface area and therefore both decrease with a decreasing electrode size towards the tip mass. Likewise, the abso-

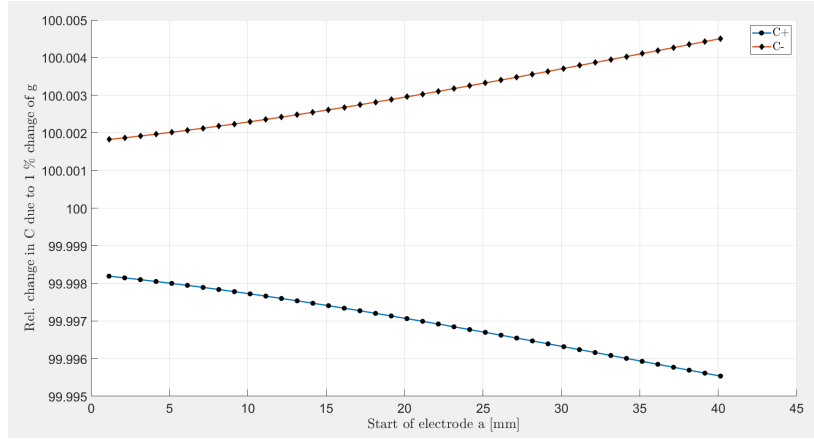


Figure 4.8: Relative capacitance change for variable starting point of the sensing electrodes, shown for a change in Didymoon gravity of +1 %. C+ denotes the by deflection increased gap and C- denotes the by deflection decreased gap.

lute difference due to a change in gravity will decrease. However, the spring portion towards the tip mass is most sensitive to changes in the gravity, therefore it can be expected that the *relative* changes in *total* capacitance due to a change in gravity will increase with the electrode starting point moving towards the tip mass. This is here referred to as *relative sensitivity*.

The relative capacitance change is simulated for a 1 % change of the Dimorphos reference gravity and is plotted in Figure 4.8. The capacitances changes about 0.002 % for the full-length electrode and nearly double (0.0039 %) for an electrode of length 9.8 mm (comparable to current JUV-GRASS design of 10 mm). So, indeed, the developed tool can be used to optimise the relative sensitivity for the respective gravity environment, and spring and electrode geometries. Clearly, the electrical design will demand a minimum (base) absolute capacitance, therefore, the start of the electrode cannot be moved just next to the end point, to maintain a finite electrode surface area and base capacitance. This trade-off can be performed using the here-developed simulation tool for the current and future electrode layouts and electronics design. For example, if the minimum needed *absolute* capacitance is known, the connected sensitivity can be obtained and the required sensitivity regarding the local change can be assessed. In all cases, the gap size should be as small as possible, so an analysis in this regard is not needed, as the smallest achievable gap is solely limited by mechanical constraints (tolerances and integration).

4.2.7 CAPACITANCE SIMULATION JUV-GRASS

With the capacitance simulation in place, it is now possible to simulate the change of capacitance by rotating the spring. This will be demonstrated here for the reference design of the JUV-GRASS spring with the electrodes of 10 mm length. This is representative of the improved relative sensitivity (Subsection 4.2.6).

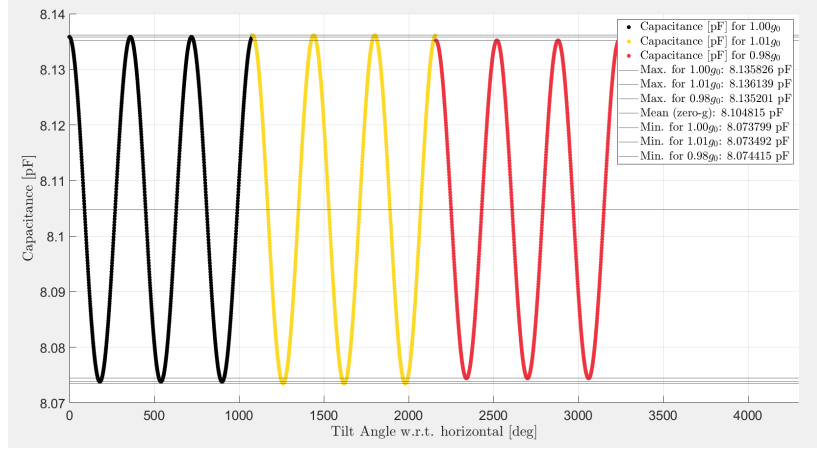


Figure 4.9: Simulated capacitance in the optimised JUV-GRASS setting. Given that here both gaps are of equal size, the capacitance is identical, yet inverted in phase for both gaps.

Starting with the Dimorphos reference gravity $g_0 = 4.5 \cdot 10^{-5} m/s^2$ for three gravity values, $1g_0$, $1.01g_0$, and $0.98g_0$, the simulation tool, so far consisting of the deflection and capacitance simulation, is executed. To simulate the mechanism rotation of the spring, the tool rotates the gravity vector in angular steps, such that a tilt $\alpha = 0^\circ = 180^\circ$ corresponds to the full gravity (deflection), and that the gravity (deflection) at $\alpha = 90^\circ = 270^\circ$ is zero (Equation (4.10)). This rotation can be performed for any number of turns, and it is possible to vary the base gravity g_0 after a defined number of turns.

$$g = g_0 \cdot \cos \alpha \quad (4.10)$$

In the first presented case, the measured identical gap sizes of 82 micron for both gaps are taken as simulation input. In this case, the capacitances for both gaps will be identical in magnitude, but their phases are inverted by 180 degrees, as one gap is increased by the deflection while the other is decreased and vice versa. Figure 4.9 shows the capacitance simulation starting at $\alpha = 0^\circ$, with the largest capacitance values of 8.135826 pF, which then decreases towards $\alpha = 180^\circ$ (8.073799 pF) and increases again towards the same maximum at $\alpha = 360^\circ$. This means, that this plotted capacitance corresponds to the gap towards which the spring bends at $\alpha = 0^\circ = 360^\circ$ and from which it bends away at $\alpha = 180^\circ$.

In addition, as for the capacitance simulations, a machining tolerance of ± 5 micron on the gap size, together with the worst case of one negative tolerance one gap and one positive tolerance deviation on the other gap. In this case, one gap will be $(82 - 5)\mu m = 77\mu m$ and the other gap $(82 + 5)\mu m = 87\mu m$ large. Such error is not only possible due to measurement errors, but likewise the worst case scenario of the machining tolerances, generally specified to $\pm 0.005 mm$ for the cradle

manufacturing. The simulation can be easily adapted to an asymmetry, yielding a different capacitance signal per gap.

The same scenario as in Figure 4.9 is simulated with the adapted, unequal gap sizes. The result for both gaps is shown in Figure 4.10, as expected, the capacitance signal varies in overall magnitude due to the different gap sizes. Likewise, the signal amplitude over one spring rotation and the amplitude change due to a gravity change of 1 % is larger for the smaller gap.

From this capacitance simulations, it becomes clear that directly reading the individual capacitance is not ideal, as the individual signals are prone to systematic errors. These errors include the exemplary inequality of the gap sizes, but can be extended to slight offsets of the centred gravimeter spring, the spring straightness, or the mounting of the sensing electrodes. While the 3D-transducer design is designed perfectly symmetrical, the machined and integrated hardware will never perfectly resemble this design. Therefore, rather than working directly with capacitance, an output signal is created based on the *differential* capacitance signal, making it easier to infer the gravity from the output signal. One possible option for this output signal generation is introduced and discussed in the next Section.

4.2.8 SENSING ELECTRONICS REQUIREMENTS

As previously stated, the detailed electronics design of the GRASS gravimeter was performed by other GRASS team members, and was beyond the goal and scope of this work. Here, the capacitance simulations performed in this Section can serve as *input* to the displacement transducer electronics design. For this, the latest design for the Hera mission, the JUV-GRASS instrument setup is considered. Here, the optimised (opt.) setting and the optimised setting with machining tolerances (opt. tol.) provide not only the required capabilities of the electronics for the nominal case, but also take into account naturally occurring deviations due to deviations from the nominal. The mechanical tolerance on the gap size is ± 5 micron, and the worst case assumes a positive tolerance to one gap, and a negative tolerance to the other gap, resulting in two gaps of 77 micron and of 87 micro, thus with a difference of 10 micron. Like this, the range in capacitance is well illustrated, rather than only the nominal capacitance.

Table 4.9 provides a total of four key quantitative figures for the opt. and opt. tol. case, always for both gaps and a spring thickness of 75 micron. First, the nominal (nom.) capacitances (no deflection) and the capacitances under deflection are reported. Second, the capacitance gradient and the absolute capacitance difference ΔC for a change of 1 % of Dimorphos gravity are computed. Regarding the latter, the spring deflection of the 75 micron blade of 375 nm (Table 4.3) is considered. The change in capacitance due to 1 % in gravity, and thus in deflection, is then computed using the linearised capacitance gradient for small changes of the capacitance C . Multiplication of a change in deflection of 3.75 nm yields the change in capacitance. The changes in capacitance due to the variation in gravity have been found to be in the order of hundreds of atto-Farad, i.e. $10^{-18} F$, clearly showing the high demands to the gravimeter electronics, especially in the deep space environment. Moreover, the electronics have to be adaptable over a range of capacitances,

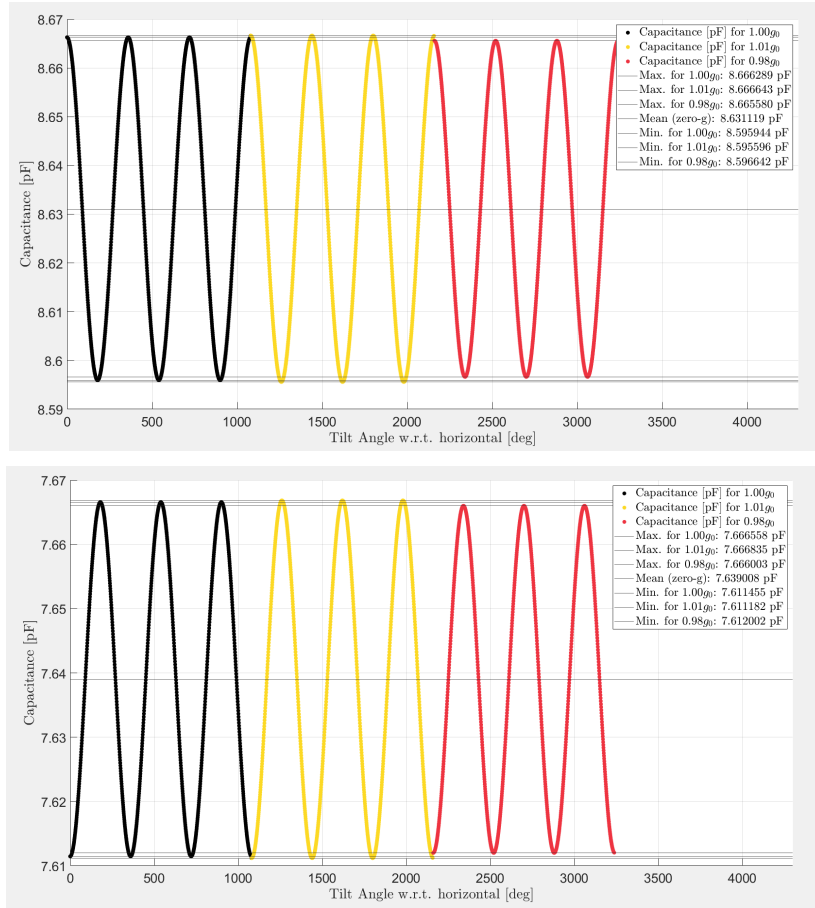


Figure 4.10: Simulated capacitance in the optimised JUV-GRASS setting as in Figure 4.9, but with application of the assumed machining tolerance of $\pm 5\mu m$. For the negative tolerance, this leads to a (top) small gap and for the positive tolerance to a (bottom) large gap.

Table 4.9: Overview of sensing electronics requirements for capacitance sensing. Considered are the optimised case (opt.) and the optimised case considering the machining tolerance $\pm 0.005\text{ mm}$ (opt. tol.), both for the JUV-GRASS design. The deflection is simulated under Dimorphos gravity $g = 4.5 \cdot 10^{-5} \text{ m/s}^2$. Gap sizes and abbreviations as in Table 4.6.

Case	$C_{nom.}$	$C_{deflect}$	$\frac{dC}{dd}$	ΔC
Opt. gap small (A)	8.103 pF	8.134 pF	99.577 nF/m	373.414 aF
Opt. gap large (B)	8.103 pF	8.072 pF	98.060 nF/m	367.725 aF
Opt. tol. gap small (A)	8.629 pF	8.667 pF	112.985 nF/m	423.670 aF
Opt. tol. gap large (B)	7.638 pF	7.610 pF	87.151 nF/m	326.816 aF

as presented. These values only illustrate one possible variation of the capacitance, and the electronics development should take into account other scenarios, e.g. the (significant) variation of the here-assumed reference gravity.

4.3 OUTPUT SIGNAL GENERATION

So far, the blade deflection (Section 4.1) and the capacitance of the two transducer gaps (Section 4.2), being two elements of the measurement chain (Figure 4.1), have been investigated. In this Part, gravity was only considered using reference values and relative changes thereof, while the detailed gravity modelling will be done in Part II of this PhD thesis. The last missing piece in the quadripartite chain is the output signal generation, as a more practical (simulated) instrument reading than a direct capacitance signal.

Still, the detailed design of the gravimeter electronics was not part of this research. However, to quickly judge and compare the influence of both design parameters (both mechanical (Subsection 3.2.1) and electrical (Subsection 4.2.3)), but also variations in gravity (Section 7.7), it is worthwhile to transform the simulated differential capacitance signal to a more accessible readout signal.

For this, the gravimeter electronics that were implemented in the GRASS gravimeter version have been implemented in the analytical gravimeter model. The idealised principle was proposed by Michel Van Ruymbeke and is introduced in this Section. The detailed electronics development using discrete elements, relevant for the comparison of simulation and instrument measurement (Section 4.4), was implemented by Sophie Berkenbosch, Michel Van Ruymbeke, Bram Beeckman and Ertan Ümit, and will not be reported in this work. This here presented solution included in the simulation should be understood as a “proxy”, that can later be simply replaced, e.g. by the final transfer function that will be implemented in the GRASS electronics.

The instrument working principle is based on two capacitance measurement, i.e. between electrode 1 – spring and spring – electrode 2. A straightforward method to translate this signal to an output reading is to consider a *differential* measurement. Rather than considering the capacitance directly, the spring deflection is measured

by means of differential capacitance, transforming the two capacitances into a duty cycle reading using a 555-timer. The transfer function from differential capacitance to the duty cycle reading will be introduced in the following.

The measurement setup consists of the two introduced “plate”-capacitors, consisting each of one of the two electrodes and the centre-spring. In the measurement circuit, each of these two capacitors is included in a RC-circuit with a resistance of $R = 100\text{k}\Omega$. While one capacitor charges up towards the supply voltage $V_S = 5\text{V}$, the other one discharges from $V_S = 5\text{V}$ towards 0V . This is continued and inverted constantly. One then measures how long the charging takes from 0V to $\frac{2}{3}V_S = 3.333\text{V}$ and the discharging from 5V to $\frac{1}{3}V_S = 1.667\text{V}$ takes. For this, a 555-timer chip is used. The functioning can be explained when considering the pin definition⁵ especially regarding pin 2 (trigger), pin 3 (output), and pin 6 (threshold).

At the above mentioned thresholds, 1.667V and 3.333V , the 555 output signal is changed from the low state 0V to the high-state 3.3V (1.7V lower than the supply voltage, by definition), and vice versa, from which the duty cycle can be measured. If the RC-voltage at the *trigger* drops below one third of the supply voltage (1.666V), the output voltage (pin 3) is “triggered” from zero to the high output state. Likewise, the *threshold* (pin 6) sets the output voltage from high to low once the “threshold value” of 3.333V (see above) is reached.

The capacitor voltage V_C in the RC-circuit for charging and discharging as a function of time is described by Equations (4.11) and (4.12), respectively.

$$V_C = V_S \cdot (1 - e^{-\frac{t}{RC}}) \quad (4.11)$$

$$V_C = V_S \cdot e^{-\frac{t}{RC}} \quad (4.12)$$

where t is the time elapsed since the supply voltage V_S has been applied to the capacitor (charge) or disconnected (discharge). Both equations can easily be rearranged for the charge/discharge time, respectively. It is obvious that the charging and discharging times from 0V to 3.333V and 5V to 1.666V , respectively, will be equal if

$$R_1 C_1 = R_2 C_2 \quad (4.13)$$

for the same supply voltage. Any deviations in the capacitance, both systematic and introduced by the blade deflection under gravity, will make the two times differ. At the same time this *differential* measure eliminates e.g. thermal effects assuming it affects $R_1 C_1$ and $R_2 C_2$ in the same way.

By definition, the duty cycle is the time where the output signal is “on”, in this setup $5\text{V} - 1.7\text{V} = 3.3\text{V}$, that is, relative to the full duration of one period. This can be measured and displayed by simple timing of the different states. Repeating this

⁵Cf. <https://www.jameco.com/Jameco/workshop/TechTip/555-timer-tutorial.html>, accessed on 29.08.2019.

measurement at large numbers allows to average the output signal, reducing the measurement errors.

Figure 4.11 gives an illustration of the connection between charge and discharge times and the output signal. The plots are intended qualitatively, but the numbers are kept close to reality. The resistances were kept at $R = 100k\Omega$, and the capacitances are in the same order of magnitude as shown in Table 4.7, but the difference has been significantly increased to 6 pF and 10 pF to visualise the difference in charge and discharge times.

Figure 4.14 (a) shows the case where the charging capacitor C_{charge} is smaller than the discharging capacitor $C_{discharge}$ and thus the charge time t_{charge} to the threshold 3.333 V is shorter than the discharge time $t_{discharge}$ to 1.667 V. Considering that one measurement period is $T = t_{charge} + t_{discharge}$, the duty cycle (DTC) results as shown in Equation (4.14) and as visible in Figure 4.14 (c).

$$DTC = \frac{t_{charge}}{t_{charge} + t_{discharge}} \cdot 100\% < 50\% \quad (4.14)$$

Inversely, if the capacitance of the charged capacitor is larger than the discharged capacitor, the duty cycle will be larger than 50 %. This is shown in Equation (4.15) and Figure 4.14 (b) and (d).

$$DTC = \frac{t_{charge}}{t_{charge} + t_{discharge}} \cdot 100\% > 50\% \quad (4.15)$$

A big advantage of the duty cycle measurement is that it is a digital signal, insusceptible to the noise level induced by the slip ring as connection between sensor head and instrument electronics. On the contrary, if an analog signal, such as the capacitor voltages, were transmitted through the slip ring for treatment outside the sensor head, the noise level could make the reading unusable.

With the duty cycle computation defined, it is possible to create an output signal of the gravimeter and to complete the analytical instrument model. Before doing so, we have to consider possible error sources in the duty cycle. In Equation (4.13), it was defined that at zero-gravity, both resistors and both capacitances are equal. Due to tolerances on the resistor resistance, and the discussed mechanical tolerances on the two gap sizes, Equation (4.13) will not be true, which will shift the signal mean from the theoretical 50 % up or down. A further error source is on the 555 timer and the precision of the trigger and threshold values, which can be expected to improve by implementing a solution with discrete components, rather than using the integrated circuit.

Finally, to create the output signal, the spring (or rather, the gravity), will be rotated around the long spring axis as shown for the capacitance. Like this, the spring will deflect in both directions, periodically and inversely increasing and decreasing both gaps. Considering the JUV-GRASS transducer with equal measured gaps of two times 82 micron, and Dimorphos gravity, the output signal of the simulation

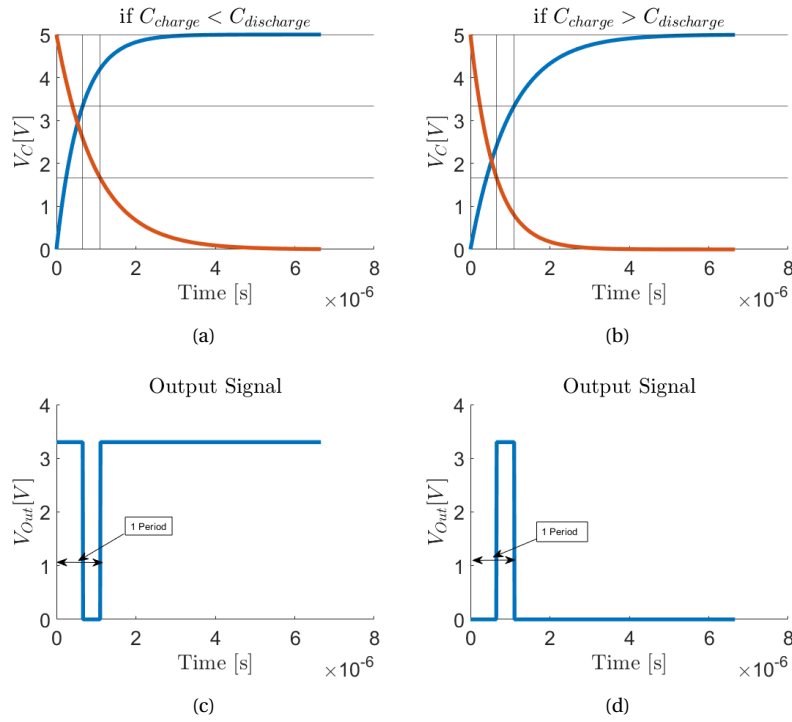


Figure 4.11: (a+c): Duty cycle calculation for the case that the charged capacitor C_{charge} is smaller than the discharged capacitor $C_{discharge}$, i.e. the charged capacitor has an increased gap and the discharged capacitor has a decreased gap due to gravity, (b+d) and vice versa. N.B. The charging and discharging takes place sequentially and continuously, but is shown here simultaneously for better comparability. It becomes also clear that, in theory, the duty cycle of a perfectly symmetrical instrument under no gravitational influence should be constant at 50 %.

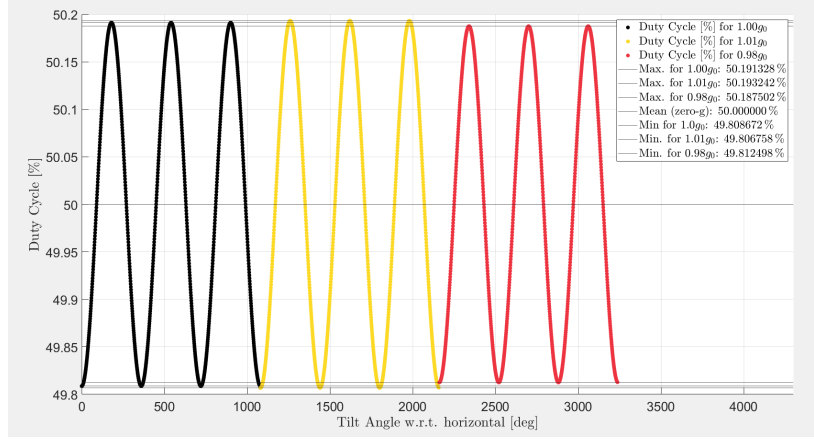


Figure 4.12: Simulated duty cycle output signal for equal gap sizes of 82 micron under Dimorphos reference gravity and for two small variations of gravity.

tool is shown in Figure 4.12. As expected, the mean of the duty cycle for the equal gap sizes (for equal resistances R) is exactly 50 %.

As studied before for the capacitance (Subsection 4.2.7), the influence of two different gap sizes (77 and 87 micron) has to be investigated regarding the DTC. With this respect, it is therefore necessary, to assign one gap, e.g. the larger gap to the charging time (Equation (4.11)) and the smaller gap to the discharging time (Equation (4.12)). As mentioned above, this has only an effect on the mean signal moving up or down from the 50 % duty cycle. For the signal interpretation, only the signals amplitude is of interest, not the mean value.

So, the rise time is connected, arbitrarily, to the larger gap A (87 micron) and the small gap connected to the smaller gap B (77 micron). Like this, the capacitance of the large gap C_A will be smaller than the capacitance of the smaller gap C_B :

$$C_A < C_B \quad (4.16)$$

Rearranging of Equations (4.11) and Equation (4.12) shows that the charge and discharge time is directly proportional to $C \cdot R$. Therefore, the charging time t_A will be smaller than the discharging time t_B

$$t_A < t_B \quad (4.17)$$

and therefore, the duty cycle will be smaller than 50 %, by definition, in Equation (4.15), and the DTC signal mean is moved from 50 % down. Of course, it is possible to connect the rise time to the smaller gap, inversely leading to the duty cycle signal to increase. Again, this does not provide a problem to the measurement, as the gravity signal now is only depending on the signal amplitude. The simulation result is shown in Figure 4.13, again for 3 different gravity signals to show the different

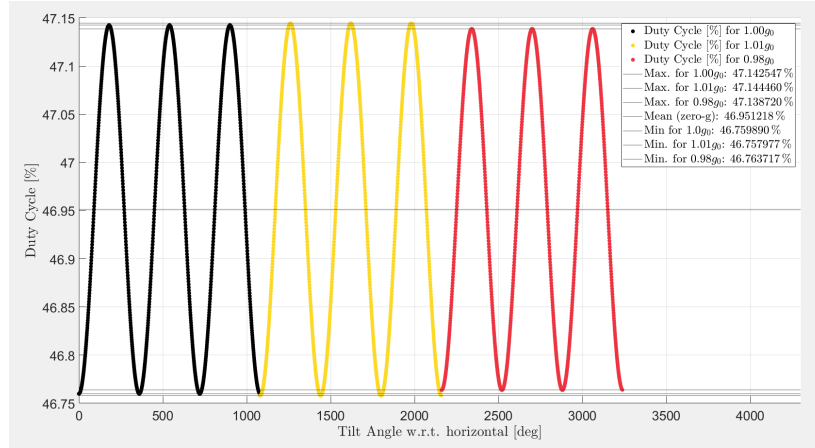


Figure 4.13: Simulated duty cycle output signal for non-equal gap sizes of 77 and 87 micron under Dimorphos reference gravity.

magnitudes. Like discussed above, considering different gap sizes is only one possible error sources, next to the variation in the reference resistances, the thresholds in the 555 timer, or even the capacitance in the harness wiring forming the circuits, and others. All these have an offsetting effect on the mean of the duty cycle, but not the amplitude, due to the differential measurement. As is visible in the plot, the mean value is offset by more than 3 % due to a variation of 10 micron in the gap size. While, from the machining perspective, this is a small deviation within the tolerances specified, the relative difference is large. Regarding the duty cycle amplitude, this is 0.38266 % for $1.00g_0$, 0.38457 % for $1.01g_0$, and 0.37883 % for $0.98g_0$. So, using the simulation tool, we find that the variation in the duty cycle, due to a change of one percent in gravity, is in the order of 0.002 %. These numbers also proof the concept of the gradient linearisation for small variations (Subsection 4.2.5), given that the deviation in magnitude for -2 % is close to double the deviation than for +1 %. This change, however, is clearly not linear for larger variations.

4.4 SIMULATION AND MEASUREMENT COMPARISON

With the gravimeter simulation completed in the previous Section, this developed tool can be complemented with a comparison of the simulated and actually measured instrument output. For this comparison, the developed Earth-equivalent blade (Section 4.1.4) has been implemented in the GRASS instrument design with the sensing electronics credited in the previous Section.

The equivalence spring was introduced in Section 4.1.4 to replicate the same deflection under Earth gravity as is expected on Didymoon. While adapting the blade thickness, the other dimensions, including the design parameters relevant to the capacitance sensing, i.e. gap size (82 and 82 microns, Table 4.6) are kept constant. One exception to this is the electrode length, which was not set to the final layout of 10 mm, but to 20 mm at the top, which was a first interim optimisation step starting

from the initial full electrode length of 40 mm.

With this in place, a laboratory test series had been recorded (GRASS team, 2021a). The test results, performed for different motor rotation speeds and directions are then compared for the output signal of the instrument simulation in Figure 4.14.

As discussed above, there are multiple explanation for the offset in the mean value, for the comparison, the signal amplitude is of interest. The simulation yields a duty cycle amplitude of about 0.091 %, whereas the measurement duty cycle amplitude is about 0.08 %. Therefore, the simulation was already in good agreement with early laboratory measurements. This overall agreement is a good result for both, the implemented simulation *and* the implemented instrument electronics, confirming the working principle of each other. Yet, the deviation is still relatively large. Overall, this will be caused by a multitude of reasons. For example, it was found by the GRASS team that the here-tested electronics had an uncertainty of about 10 % caused by the grounding principle (*Birgit Ritter, personal communication. (2022)*).

Without question, further testing and comparisons will be required to verify both, the here-presented simulation tool and the electronics to be used in the GRASS gravimeter. Regarding the presented duty cycle data, however, the available data for comparison is limited, as the GRASS team decided to adapt the electronics design to a more robust design, using a frequency output solution. In future work, and to enable more detailed comparisons, it will therefore be required to replace the duty cycle building block in the simulation with one that incorporates the latest transfer function between differential capacitance and output frequency signal.

The here-presented Earth-equivalent data can also be used to verify the equivalence approach by comparison with the JUV-GRASS duty cycle simulation (Figure 4.12). As the maximum deflection for the equivalent blade was found a bit smaller than the reference design on Dimorphos, the amplitude of the signal can be expected to be likewise smaller, here about 0.091 %, compared to 0.191328 % for the simulation on Dimorphos. So overall, the order of magnitude in the output signal is the same, while the Earth-equivalence blade is a bit more than twice as “demanding” to the read-out electronics, due to the smaller signal amplitude. This, however, is a good case, provided that the final gravity can be smaller than the reference gravity, and taking into account that the surface gravity varies over the surface of small bodies, as will be studied in Part II.

4.5 DISCUSSION ANALYTICAL INSTRUMENT MODEL

In this Chapter, an analytical instrument description has been developed, following the chain starting from gravity, via the spring deflection and the differential capacitance, and arriving at an instrument output signal. Rather than performing a detailed numerical description of the final instrument layout, the aim of the here-developed instrument model is the flexibility, adaptability, and extendibility.

Starting with the spring and the deflection under gravity, it has been shown that the

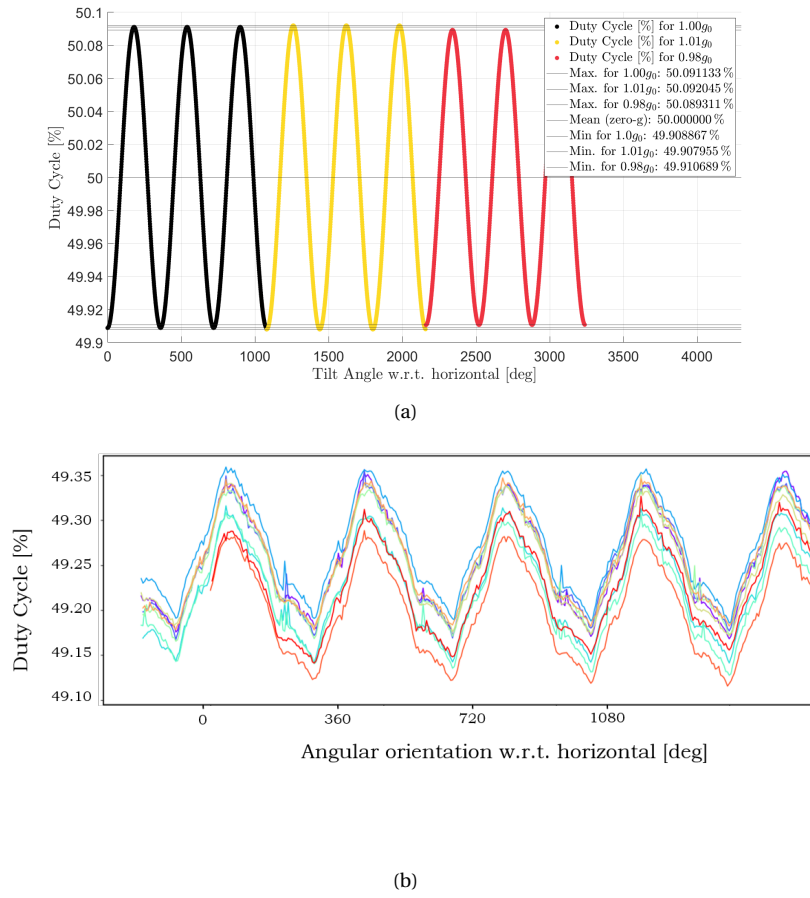


Figure 4.14: Comparison of duty cycle for Earth-equivalent blade. (a) Simulated duty cycle using the analytical gravimeter model (this work). (b) Duty cycle recorded in the lab for different rotation speeds and directions from GRASS team (2021a).

maximum deflection to be expected, e.g. on Dimorphos, is in the order of nanometers (4.7 nm and 10.6 nm for 75 micron and 50 micron thickness, respectively). The addition of the tip mass to the gravimeter spring has been demonstrated to be indispensable in the small gravity environment, providing a deflection increase to 375 nm and 1262 nm, respectively. In the analytical spring deflection simulations, different spring design parameters were tested alongside with three different reference gravities, being Dimorphos, Phobos, and the Earth. Regarding the latter gravity, the spring deflections cannot be considered microscopic, but rather macroscopic, and non-linearities will occur in the deflection. This limits the here-developed simulation tool to either small gravity environments *or* thicker springs with microscopic deflections. Such thicker spring has been sized using the deflection simulations, providing a deflection under Earth gravity comparable to the original JUV-GRASS spring deflection on Dimorphos. This is a powerful solution to be able to simulate small gravity deflections on Earth without a complex vertical inclination setup. The deflection simulations have been compared to a finite-element method (FEM) simulation. Overall, the agreement was good, but not perfect, with possible uncertainties in both FEM and the analytical simulation. With regards to the simulation tool, the aim is an overall and adaptable instrument behaviour description, easily configurable to a large number of design parameters, e.g. to perform optimisations, such as the performed optimisation for the length split-up between spring and tip mass lengths. Therefore, some uncertainty in the analytical solution is acceptable, and can be mitigated, if necessary, by additional (numerical) simulations.

Similarly, the electromechanical simulation has been performed for the capacitance simulations of the two capacitances over the two transducer gaps. Like for the deflection, the analytical description is only valid for small deflections, e.g. on small bodies, as the sensing surfaces are assumed parallel (while the deflection is considered). Various numbers regarding the capacitance and the gradient have been computed using the tool. Again, while future numerical simulations might be performed in future work, the here-presented numbers form the prime input to the sensing electronics development. In the same fashion, the tool allows simple iteration over different design parameters, and so it was found that the relative sensitivity of the instrument can be increased by reducing the overall sensing electrode length towards the top mass, being the most sensitive portion of the sensing device. This work's finding has been included in the baseline design of the JUV-GRASS instrument for the ESA Hera mission.

Lastly, the duty cycle output signal has been introduced as one possible option considered by the GRASS team to generate an output signal taking into account the differential capacitance measurement. The advantage here is that the differential signal allows to account for the measured gravity, while other error sources, such as thermal effects, are widely excluded. Detailed modelling of the thermal instrument behaviour cannot be done here, and will require extensive instrument characterisation under thermal-vacuum conditions. By implementing the duty cycle transfer function to complement the simulation, it was possible to make a first comparison of the simulated instrument output and a laboratory instrument reading. The overall agreement, considering that the measured data came from early and on-

going instrument test, was satisfactory, showing that the here-developed code is a simple and powerful tool for quick instrument planning, adaptation, and transfer to different gravity environments.

Regarding improvements to the instrument sensitivity, the following limitations exist in the context of this work:

- The spring thickness needs a finite thickness (structural stability).
- The spring length and tip mass volume is limited (volume budget).
- The capacitance gap sizes have a minimum dimension (machining tolerances and assemblability).
- The absolute capacitance cannot approach zero to improve the relative sensitivity (Subsection 4.2.6).

Like this, research question *(iii)* has been answered in this Chapter. The analytical electromechanical behaviour of the space gravimeter has been described and, where possible, optimisations have been performed, i.e. regarding the length split-up of free spring and tip mass, and the sensing electrodes length.

4.6 SUMMARY CHAPTER 4

In this Chapter, the electromechanical behaviour of the spring-based GRASS gravimeter has been described. For this, analytical models of the spring deflection (Section 4.1), the displacement transducer capacitances (Section 4.2) and the output signal (Section 4.3) have been implemented and interconnected.

For the spring, the improvement of the spring deflection without (Subsection 4.1.1) and with (Subsection 4.1.2) tip mass has been computed. In addition, an optimisation between the length split up of spring and mass has been performed (Subsection 4.1.3) and the simulation tool was used to create an Earth-equivalent blade (EEB), capable to resemble microscopic spring deflections under full Earth gravity (Subsection 4.1.4).

Regarding the capacitance transducer, the physical principle was introduced in Subsection 4.2.1, followed by an introduction to the design parameters in Subsection 4.2.2. After this, the base capacitance (Subsection 4.2.3), the capacitance under deflection for different springs (Subsection 4.2.4), and the (linearised) capacitance gradient have been computed (Subsection 4.2.5). Finally, an optimisation study for the electrode design (Subsection 4.2.6) and the capacitances of the JUV-GRASS transducer measurements under Dimorphos gravity (Subsection 4.2.7) have been presented.

Based on this, one possible solution to transfer the differential capacitance to an output signal in form of a duty cycle has been presented and implemented in the simulation chain (Section 4.3). With this in place, a comparison of simulation output and laboratory reading was presented in Section 4.4. Lastly, the work of this Chapter has been discussed in Section 4.5, before this Section summarises and closes this Chapter.



With this, this first Part of the thesis regarding the small body gravimeter development is completed. After a discussion and selection of the gravimeter working principle (Chapter 2), the instrument development was presented in Chapter 3. Lastly, this Chapter presented the developed analytical gravimeter model. The chapters were discussed individually, while future work and applications (Chapter 8), and the conclusions (Chapter 9) will be presented for this entire work. In this Part, gravity was considered only as (constant) reference values. In the following next Part, gravity of non-spherical small bodies will be studied and simulated in detail with a strong emphasis on *surface* gravity, to complement the here-presented instrument development for extraterrestrial *surfaces*.

II

SURFACE GRAVITY OF SMALL BODIES

5

CONTRIBUTIONS TO GRAVITY AND MODELLING TECHNIQUES

The small body gravimeter (SBG) presented in Part I is of limited scientific avail without appurtenant gravity simulation. The eventually measured surface gravity readings can only be understood and interpreted with appropriate gravity modelling of the small, usually non-spherical celestial bodies. This Part II of the presented work is therefore focused on numerical gravity modelling of small bodies. It begins with this Chapter by introducing contributions to gravity and the state-of-the-art (SOTA) of gravity modelling techniques. Next, gravity modelling is performed on a global scale, considering entire surfaces of small bodies (Chapter 6). Lastly, a novel method to account for the local topography directly surrounding a surface gravimetric measurement location is developed in Chapter 7.



Today, extensive research work has been performed in forward modelling both in the case of gravitational attraction and of gravity of *small bodies (SB)*. This Chapter, introducing Part II, will firstly provide the theoretical basis for all significant contributions to surface gravity (Section 5.1). Using this theoretical basis, the gravity environments relevant to this work will be described analytically on a global level in Chapter 6. Related aspects and further contributions to gravity are briefly discussed in Section 5.2. Generally, the largest contribution to gravity is the gravitational attraction induced by the inter-attraction of all mass. In the context of surface gravimetry, the existing methods that are suitable to forward model surface gravitation are presented in Section 5.3. Where available, implementations of these methods used for the numerical part of this work are likewise introduced and referenced. Finally, research question (I) is discussed in Section 5.4 and this Chapter is summarised in Section 5.5.

5.1 INTRODUCTION TO GRAVITY CONTRIBUTIONS

Often used interchangeably, *gravity* and *gravitation* must not be confused. Both gravitational force and gravitational attraction is caused solely by the attraction between mass as formulated by Newton's law of universal *gravitation* (Subsection

5.1.1). This is generally the largest, but not the sole contribution to *gravity* that denotes the *resulting* force or acceleration, thus including all further influences (Subsections 5.1.2 through 5.1.4), e.g. centrifugal influences on a rotating body. In some cases, however, for fast rotating objects (“fast rotators”) the rotational acceleration can even overcome the weak gravitational field, leading to material lift-off (Drahus et al., 2015). If the cohesive force of the body is weak, indeed a very high rotation speed can lead to the destruction of the body. On a side note, in the context of the binary asteroid (Subsection 1.4.2), it is interesting to note that this mass liftoff can lead to the formation of a secondary asteroid, forming a binary system (Čuk, 2007).

This Section introduces the theory underlying the different contributions from literature. Based on this, different gravity environments will be investigated in Chapter 6 as a global description of the SBG measurement environment.

5.1.1 GRAVITATION

Within this work, it is inescapable to firstly introduce gravitation. Newton formulated in his *Philosophiæ Naturalis Principia Mathematica* (Newton, 1687) the law of gravitation that two particles are attracting each other with a force F proportional to the two masses and inversely proportional to the square of the separation between these two particles. For the two point masses m_1 and m_2 at distance r this can be expressed mathematically (e.g. Wakker, 2015) as shown in Equation (5.1).

$$F = G \frac{m_1 m_2}{r^2} \quad (5.1)$$

where G is the universal gravitational constant $G = 6.67408(31) \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^2$. Wakker (2015) warns that G is hard to measure precisely and that this natural constant is only known with limited accuracy. The determination is subject to ongoing research, which can also be followed from the fact that Wakker (2015) himself used the (old) value $G = 6.67428 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^2$ in 2015 which was updated in the recommendations from the Committee on Data for Science and Technology (CODATA) (Mohr and Taylor, 2016) dated only one calendar year later. It is well noted that the uncertainty bracket of the latest value includes the previously accepted value. In this work, the value of $G = 6.674 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^2$ will be used consistently. Regarding comparisons with other works (Section 6.1), the precisely used value of G can become a pitfall and due care has to be taken.

Based on Equation (5.1) above, Wakker (2015) introduces the definition of the gravitational potential as described in the following. The gravitational acceleration $\vec{g}_2$ of point mass m_2 induced by point mass m_1 can be expressed as shown in Equation (5.2) below:

$$\vec{g}_2 = -G \frac{m_1}{r_2^3} \vec{r}_2 \quad (5.2)$$

Then, the scalar quantity of the potential can be expressed as shown in Equation (5.3),

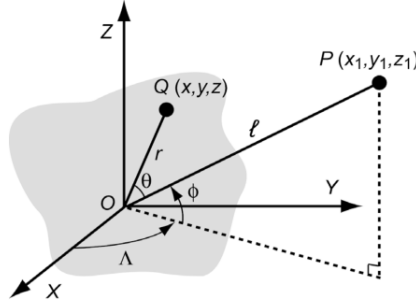


Figure 5.1: Arbitrary body L and an external point P . The origin of reference frame XYZ coincides with the centre of mass of L . Figure and caption from Wakker (2015).

$$U_2 = -G \frac{m_1}{r_2} + U_{2,0} \quad (5.3)$$

where $U_{2,0}$ is an arbitrary constant. From both equations above it follows directly that

$$\vec{g}_2 = -\vec{\nabla}_2 U_2 \quad (5.4)$$

where $\vec{\nabla}_2$ is the nabla operator indicating the gradient of U with respect to the reference frame of body m_2 . It is stated further that a function can be called potential if the local field strength follows directly from partial differentiation. Moreover, it is commonly adopted to define the potential to be zero at an infinite distance from where it directly follows that the arbitrary constant $U_{2,0} = 0$ and the potential is negative at distances smaller than infinity (Equation (5.5)).

$$U = -G \frac{m_1}{r} \quad (5.5)$$

It is added that this (gravitational) potential is conservative, as it is not explicitly depending on time, meaning that the sum of potential and kinetic energy of a body in this field is constant. For spherical bodies, the gravitational force as described above for point masses is identical for spherical bodies, as long as the interior mass distribution is radially symmetric. Therefore, this allows a good approximation of the gravitation of planets and larger natural satellites, such as the Moon. At the same time, however, non-spherical bodies with non-symmetric mass distribution can clearly not be approximated like this. (Wakker, 2015)

Regarding this, the gravitation of bodies with both arbitrary shape and mass distribution are considered. For this, Wakker (2015) provides a good description using a setting shown in Figure 5.1 to arrive at the potential of this arbitrary body L . Within this body, a mass element Q with mass dm at location (x, y, z) is considered. At

distance l from the origin (placed in the centre of mass of L), there is a particle P at (x_1, y_1, z_1) with mass m_1 . Here, Wakker (2015) demands the particle to be outside the Brillouin sphere (Figure 5.4). With Equation (5.1) the force acting between particle P and mass element Q can be described with Equation (5.6).

$$\vec{F} = G \frac{m_1 dm}{\|\vec{PQ}\|^3} \vec{PQ} \quad (5.6)$$

with $\vec{PQ}$ being the vector between mass element and particle. Integration over the full body, i.e. all mass elements dm , gives us the gravitational potential

$$U = -G \int \frac{dm}{\|\vec{PQ}\|}. \quad (5.7)$$

Equations (5.7) and (5.4) have been formulated by Laplace (1799) (expressed differently), and with them the gravitational field can be obtained by numerical quadrature (Fukushima, 2017).

While different methods to solve this have been implemented, they do not always converge, e.g. inside the Brillouin sphere (Figure 5.4) as mentioned above. Overall, this Subsection has introduced the concept of gravitation and the dependency on both, mass (distribution) and the location. In essence, regardless of the method used to arrive at the gravitational field, we have to highlight that integrating over mass elements at a defined location will provide the resulting gravitational field.

5.1.2 ROTATIONAL INFLUENCE

As stated above, resulting gravity is not only composed of gravitational attraction, but also other contributions that will be introduced in this and the following Subsections. Firstly, the influence of the rotation of a body will be presented hereunder. As is generally known, the rotation of a body creates a centrifugal force that is maximum at the equator, or, more general, at the location farthest away from the rotation axis. This influence disappears at the (rotation) poles, or, more generally, at the intersection of the body rotation axis and its surface. For tidally locked natural satellites, such as the Moon, Phobos and (assumed) Dimorphos, the rotation period is equal to the orbital period around the respective primaries.

The centrifugal potential is added to the gravitational potential (Equation (5.5)) as shown in Equation (5.8) (Wakker, 2015).

$$U = -G \frac{m_1}{r} - \frac{1}{2} r_{rot}^2 \omega^2 \quad (5.8)$$

Where ω is the angular velocity computed with orbital period T (Equation (5.9)) of the rotating body and r_{rot} is the perpendicular distance from the rotation axis.

$$\omega = \frac{2\pi}{T} \frac{[rad]}{s} \quad (5.9)$$

In the same fashion as for the gravitational acceleration, the centrifugal acceleration on the surface $a_{cen,sur}$ can be obtained by differentiation giving the centrifugal acceleration on the surface (Equation (5.10)).

$$a_{cen,sur} = \omega^2 r \quad (5.10)$$

The here-presented is, of course, a simplification of the problem. The dynamics and rotation of small bodies is a complex research field in itself (e.g. Rambaux et al., 2012), far beyond the goal and scope of this work. In the context of surface gravimetry, the motivation is to estimate the order of magnitude of dynamic effects and to put them into perspective with the gravitational attraction. Like this, with the basic formulations, it is possible to state, if and how well the surface gravimeter can provide information on the dynamical state and variations thereof.

With that being said, the rotational speed and also the distance from the rotational axis have to be understood as approximations. Regarding the latter, the shape files of the small bodies are generally provided in the planetocentric coordinate system, where the positive Z-axis passes through the north pole¹. While, especially for SBs, this will not align with the rotation axis, we will use the Z-axis for the approximate calculations presented in Subsection 6.2.1.

5.1.3 FORCED PHYSICAL LIBRATION

In the previous Subsection, the term *dynamical state* was introduced en passant. In reality, the dynamical state can be understood as an umbrella term for all influences concerning the rotation of the body but also the interaction within a binary (or trinary Mars) system. Additionally to the rotation, the libration and tides will be introduced in this Chapter and included in the analysis of dynamical contributions to gravity (Section 6.2). This overview of the dynamical state is not complete, but other influences, such as nutation and precession, are neglected in this work due to their long periods.

In this Subsection, the theory of libration will be introduced. There are two types of libration, optical and physical libration. Optical libration is an observational effect where a body appears to have libration as described by Włodarczyk (2011). Due to the eccentricity e of the Moon's orbit around Earth, the observed angular velocity of the rotation does not appear to be constant. This causes an optical libration in longitude and, for the Moon, can have a magnitude of $\pm 7.9^\circ$. Secondly, the libration in latitude is caused by a tilt of the Moon's rotation axis and the orbital plane, reaching a magnitude of $\pm 6.8^\circ$.

For this study, it will be assumed that the satellite spin axis is normal to the orbital plane with eccentricity $e \ll 1$. Moreover, the satellite rotation is assumed tidally locked and the satellite mass m_s is much smaller than the primary body

¹E.g. <https://3d-asteroids.space/moons/M1-Phobos>, accessed on 10.06.2021.

mass m_p , thus $m_s \ll m_p$ (Caudal, 2017). These conditions are all true for Phobos and the Martian system. Regarding the mass ratio of secondary and primary, this condition must be questioned for the Didymos system, with a ratio of $m_s/m_p = 0.0093 \pm 0.0013$ (ESA, 2020) and the results must be considered with extra care.

Next to the optical libration, there is the physical libration, which can be split into forced physical libration and free physical libration. Here, the free libration can be neglected due to the very low amplitude in combination with the very long period. For example, the Moon's free libration in longitude was calculated by Rambaux and Williams (2011) to have an amplitude of only ca. 1.3 arcseconds with a period of ca. 1056 days. Therefore, in this Subsection, the *Forced Physical Libration*, often just referred to as *forced libration* will be introduced, and the contribution to the surface gravity is computed in Subsection 6.2.2.

The equations of forced libration are presented by Murray and Dermott (1999): Considering a satellite in 1:1 spin-orbit resonance, as the Moon, Phobos and (assumed) Dimorphos, it is stated that basically the long axis of the satellite is pointing exactly to the planet during the pericentre passage.

Figure 5.2 shows a reference frame centred at the satellite and rotating with the satellite mean motion n . This rotation can be explained by the fact that for Keplerian orbits, the line $F'F$ (connecting the satellite and the empty focus) of the elliptical orbit rotates with the uniform angular velocity n (Murray and Dermott, 1999).

For this rotating frame, the primary moves about the shown 2:1 ellipse about its guiding centre and creates a torque due to the angle between the long axis and the direction towards the planet Ψ representing the optical libration. This misalignment creates a torque creating librations around the equilibrium position where the long axis aligns with the satellite-planet line.

In other words, libration is caused by a periodic torque acting on the orbiting body (i.e. moon, planet). This torque is induced by a misalignment of the longest axis of the body (i.e. with the smallest moment of inertia) and the vector connecting the centres of the satellite and primary (asteroid, planet, star). The term *forced* libration follows from the fact that this perturbation is induced by the gravitational *force*. It is stated further that small, irregular bodies allow to verify forced libration models with observations. (Caudal, 2017)

Next, we can consider Figure 5.3 and introduce the angle Ψ between the long satellite axis and the satellite-planet connecting line:

$$\Psi = f - \theta \quad (5.11)$$

where f is the true anomaly. Neglecting the tidal torques, the equation of motion (EOM) of the satellite can be written as (Murray and Dermott, 1999):

$$C\ddot{\gamma} - 1.5(B - A) \frac{GM_p}{r^3} \sin 2\psi = 0 \quad (5.12)$$

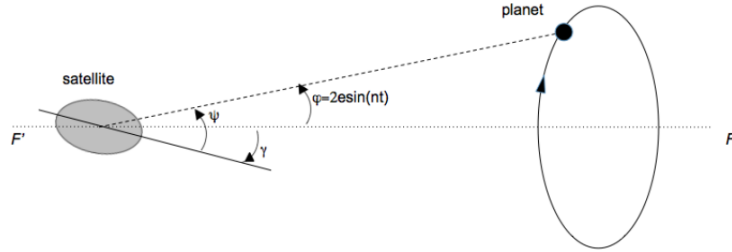


Figure 5.2: Rotation of a satellite in a rotating reference frame centred on the satellite and rotating with the satellite mean motion. The planet moves in a 2:1 ellipse about its guiding centre, which, in this frame, is stationary. The angles are physical libration γ , optical libration Ψ , and $\phi = 2e \sin nt$. Figure from Makarov et al. (2015) based on Murray and Dermott (1999), caption merged from both.

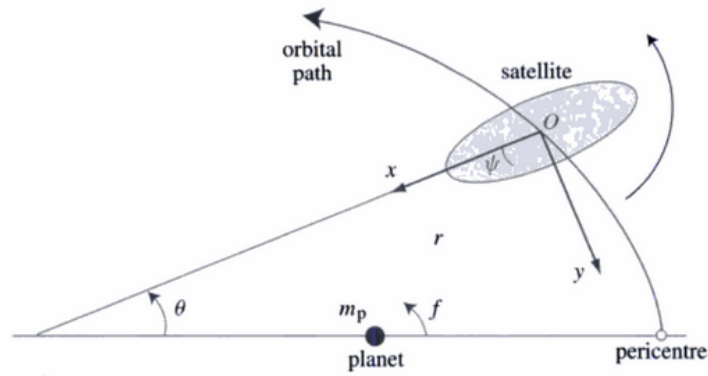


Figure 5.3: The long axis of the satellite makes an angle θ with a reference axis that is fixed in inertial space, which we take to be the major axis of the satellite fixed orbit. Figure and caption from Murray and Dermott (1999).

To lowest order of eccentricity e and ψ , and for small deviations from the equilibrium, Equation (5.12) can be reduced and rewritten to Equation (5.13) (Murray and Dermott, 1999).

$$\ddot{\gamma} = -\omega_0^2 \gamma + 2\omega_0^2 e \sin nt \quad (5.13)$$

The mean motion n is calculated as:

$$n^2 = \frac{GM_P}{a^3} \quad (5.14)$$

where $GM_P = \mu$ the product of gravitational constant and primary mass is the gravitational parameter of the orbited body. One parameter introduced by Makarov

et al. (2015) is the triaxiality σ , which is defined as

$$\sigma = \frac{B - A}{C} \quad (5.15)$$

where A , B , and C are the three principal moments of inertia, normalised by $1/(Mr_0^2)$ of the body with $C > B > A$, by definition. Here, r_0 is the mean radius. For example, the values for the Moon and Io are given as 2.279×10^{-4} and 6.4×10^{-3} , respectively. For Phobos, this value can be computed as 0.1325, based on the moments of inertia provided by Willner et al. (2014), and for Dimorphos, it can be computed as 0.256 for the ellipsoidal shape file with the density and mean radius listed in Table 1.5. Thirdly, ω_0 is the libration frequency, computed with Equation (5.16) (Murray and Dermott, 1999).

$$\omega_0^2 = 3\sigma n^2 \quad (5.16)$$

This equation considers the leading frequency of libration only, which, again, can well be implemented in the context of this work, but does not provide a perfect description of the dynamical state. Equation (5.13) is describing a forced harmonic oscillator (Makarov et al., 2015), of which the solution is shown in Equation (5.17) (Murray and Dermott, 1999).

$$\gamma = -\frac{2\omega_0^2 e}{n^2 - \omega_0^2} \sin nt \quad (5.17)$$

Finally, substitution of Equation (5.16) in Equation (5.17) yields Equation (5.18), which is in accordance with Makarov et al. (2015).

$$\gamma = \frac{2e \sin nt}{1 - \frac{1}{3\sigma}} \quad (5.18)$$

In summary, with Equations (5.13) and its solution (Equation (5.18)) it is possible to calculate the angular acceleration $\ddot{\gamma}$ and with the distance from the libration rotation axis, the resulting acceleration due to the libration can be calculated similarly to the rotational acceleration (Subsection 5.1.2). This will be done in Subsection 6.2.2 with the angular velocity $\dot{\gamma}$ and like for the rotation, the Z-axis will be assumed as the axis of rotation for the purposes here. Regarding the time, from looking at Equation (5.18), it is clear that the libration amplitude will be zero in the peri- and apocentres, while the maximum libration amplitude is generally computed with the $\sin nt$ term equal to 1 (Willner et al., 2014).

5.1.4 TIDAL SURFACE ACCELERATION

Generally, the Sun and the Moon are the largest contributors of tidal effects on Earth and the contributions of other bodies in the Solar System can be neglected (Schrama, 2016). Moreover, the period of these tides is determined by the spin rate of the Earth ($359.02^\circ/d$), and the longer period movements of the Moon ($\approx 13^\circ/d$)

and the Sun ($\approx 1^\circ/d$). A difference has to be made for representing the gravity field. For representing the geopotential of the Earth, it is convenient to describe the gravitational potential as static, i.e. to neglect time depend effects such as the solid-Earth, ocean and polar tides (Wakker, 2015).

The tidal surface acceleration can be calculated to first order by calculating the difference in gravitational pull by the tide inducing body (planet, moon, star) between the surface point and the centre of gravity as has been done by e.g. Lissauer and de Pater (2013) as shown below. It is important to note that the assumptions were made that (a) it is assumed that the body influenced by the tides with radius R is spherical and (b) that the distance to the tide inducing body r_0 is much larger than the influenced body itself so that $r_0 \gg R$. It is clear that both Phobos and Didymos have a deviation from a spherical shape and that the orbital separation in the Martian and Didymos system are smaller than, e.g. in the Earth-Moon system. This limits the assumption $r_0 \gg R$ as prerequisite to assume a point mass as tide inducing source, and again the analytical computation should be considered with care, while they are sufficient for the here-presented orders of magnitude. For solar tides, however, this assumption holds true.

The tidal force $F_{Ti}(\vec{r})$ at location $\vec{r} = (x, y, z)$, induced by the inducing body i , assumed as point mass m_i at the position $\vec{r}_i$, is calculated as the difference between the gravitational pull g_m at location $\vec{r}$ on the surface $g_m(\vec{r})$ and at the centre of mass (CoM) $g_m(\vec{r}_0)$. The latter is placed at the origin of our reference frame ($\vec{r}_0 = 0$), of the body experiencing the tides. This can be formulated as:

$$a_{Ti}(\vec{r}) = g_m(\vec{r}_i - \vec{r}) - g_m(\vec{r}_i - \vec{r}_0) \quad (5.19)$$

Substitution of Equation (5.2) in Equation (5.19) then gives Equation (5.20).

$$a_{Ti}(\vec{r}) = \frac{Gm_i}{\|\vec{r}_i - \vec{r}\|^3}(\vec{r}_i - \vec{r}) - \frac{Gm_i}{\|\vec{r}_i - \vec{r}_0\|^3}(\vec{r}_i - \vec{r}_0) = \frac{Gm_i}{\|\vec{r}_i - \vec{r}\|^3}(\vec{r}_i - \vec{r}) - \frac{Gm_i}{\|\vec{r}_i\|^3}(\vec{r}_i) \quad (5.20)$$

In order to now characterise the gravity environment relevant to this work further, it is possible to compute the amplitude of the tides and the temporal variation over one orbit of a secondary. For this, a simplified orbit integrator is implemented and applied to the Martian and Didymos systems. In this model, the tide-induced gravity field of the primary is assumed to be uniform, while the position of the secondary changes throughout its (non-circular, elliptical) orbit.

The orbit is propagated using an Euler integration, and the initial state vector $\vec{x}_0$, containing initial position and velocities, is updated by adding the derivative of the vector $\vec{x}_0$ that contains the velocity and acceleration, i.e. the gravitational attraction multiplied with the integration step Δt (Equations (5.21) and (5.22)).

$$\vec{x}_1 = \vec{x}_0 + \vec{\dot{x}}_0 \Delta t \quad (5.21)$$

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ \dot{x}_1 \\ \dot{y}_1 \\ \dot{z}_1 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \\ \dot{x}_0 \\ \dot{y}_0 \\ \dot{z}_0 \end{pmatrix} + \begin{pmatrix} \ddot{x}_0 \\ \ddot{y}_0 \\ \ddot{z}_0 \end{pmatrix} \Delta t \quad (5.22)$$

Using this, an analysis of the tidal contributions to gravity is presented in Subsection 6.2.3. The orbit propagation was verified by propagating the orbit over one orbital period T (Equation (5.23)) and assessing the deviation of expected identical start and end positions and velocities. Here, a time step of $\Delta t = 0.5s$ was found to be an adequate trade-off between computation time and accuracy.

$$T = 2\pi \sqrt{\frac{a^3}{\mu}} \quad (5.23)$$

5.2 RELATED ASPECTS AND FURTHER CONTRIBUTIONS

Supplementary to the introduction of the four main contributions to gravity, being gravitation (Subsection 5.1.1), rotation (Subsection 5.1.2), forced physical libration (Subsection 5.1.3), and tides (Subsection 5.1.4), this Section provides the theoretical background on additional concepts and influences. Firstly, the Hill sphere (Subsection 5.2.1) and the Roche limit (Subsection 5.2.2) concepts will be introduced. Other than for the contributions introduced in the previous Section, these concepts are directly accompanied with basic calculations, as they will not be elaborated upon in the next Chapter. This is the same for the solar radiation pressure (SRP, Subsection 5.2.3). While effectively contributing to the gravity, i.e. creating an acceleration that contributes to the resulting gravity, basic calculations will show that the influence on the surface gravity is negligibly small compared to the gravity. It will be shown, however, that an acceleration measurement due to the SRP on the non-landed spacecraft might be an interesting option. In a final step, complementary to the SRP, further influences not studied in this work will be listed in Subsection 5.2.5.

5.2.1 HILL SPHERE

The hill sphere with radius R_H represents the region within which the gravity of the secondary is dominating over the gravity of the primary. The sphere radius is computed as shown in Equation (5.24) (Lissauer and de Pater, 2013).

$$R_H = \sqrt[3]{\frac{m_S}{3(m_P + m_S)}} a \quad (5.24)$$

where a is the semi-major axis and m_S and m_P are the masses of the secondary and primary, respectively. The Hill spheres are calculated for three small bodies

Table 5.1: Calculated Hill spheres radius R_H with references for input parameters for Phobos (Wakker, 2015), Dimorphos (ESA, 2020), and Apophis (Wakker, 2015; Müller et al., 2014).

Secondary	$m_S[kg]$	$m_P[kg]$	$a[km]$	$R_H[km]$
Phobos	$1.0596 \cdot 10^{16}$	$6.4180 \cdot 10^{23}$	9,376	16.55355
Dimorphos	$4.8633 \cdot 10^9$	$5.2293 \cdot 10^{11}$	1.19	0.17298
Apophis	$6.1 \cdot 10^{10}$	$1.9886 \cdot 10^{30}$	$1.37995 \cdot 10^8$	29.951

(Phobos, Dimorphos and Apophis) in Table 5.1. All three bodies can be considered as secondaries (i.e. satellites). Phobos is a secondary of Mars, Dimorphos is a secondary of Didymos and Apophis with a heliocentric orbit is a secondary to the Sun.

The Hill sphere is larger than the dimensions of the three bodies. For Didymoon and Phobos, however, the difference of the largest surface elevation to the Hill sphere is not very large. It is an interesting mind experiment to imagine that while being on a surface of a secondary, the effective gravity would be induced by the primary rather than the secondary. This would, of course, lead to loss of material of rubble piles asteroids outside the Hill sphere, and would therefore only be imaginable for monolithic bodies with sufficient internal strength. Similar considerations can be made for the Roche limit, introduced in the next Subsection.

5.2.2 ROCHE LIMIT

Each system of primary and secondary has a Roche limit, within which the tidal acceleration (Subsection 5.1.4) induced by the primary is dominating over the self-gravitation of the secondary. In absence of other, e.g. cohesive forces (widely absent in a rubble pile asteroid), the secondary disintegrates. A clear derivation of the Roche limit can be found in Wakker (e.g. 2015), the transition from stable to unstable occurs when the tidal force is equal to the (self-) gravity of the secondary. This is the case at the Roche limit r_{RO} computed with Equation (5.25).

$$r_{RO} = \sqrt[3]{3 \frac{\rho_P}{\rho_S}} R_P \quad (5.25)$$

where ρ_P and ρ_S are the densities of the primary and secondary, respectively, and R_P is the orbital radius.

Lissauer and de Pater (2013) provide the same formula, pointing out that some simplifications have been made for this expression:

- A small moon/large planet approximation has been used: This is well valid for Phobos/Mars, but questionable for Didymoon/Didymain.
- Circular orbits with eccentricity $e = 0$ have been assumed with synchronous rotation and low obliquity: This is true for Phobos and assumed for Didymoon, but not for Apophis.

Table 5.2: Calculated Roche limits r_{RO} with Equation (5.25) for the Didymos and Martian system.

System	$\rho_P [kg/m^3]$	$\rho_S [kg/m^3]$	$R_P [km]$	$r_{RO} [km]$	Reference
Martian	3934	1872	3396.2	6274	Wakker (2015)
Didymos	2170	2170	0.390	0.5625	ESA (2020) ^a

^aEspecially Naidu et al. (2020).

- Both bodies have been assumed to be spherical, which is not the case for Phobos or Didymos (triaxial ellipsoids).

The Roche limits are computed for the Didymos and Maritan system (Table 5.2). Due to the high eccentricity of Apophis ($e = 0.191$) (Souchay et al., 2014), the Roche limit is not computed here using the simplified equation. It is noted, however, that the Roche limit is not only limited with regards to tidal force induced by the primary, but can also be studied for possible disruption during flybys, such as the Apophis Earth flyby (Souchay et al., 2014). The assumption of a same bulk density for both primary and secondary in the Didymos system shall be understood as a first estimate and might change with better knowledge of the shape (Naidu et al., 2020) and will further improve with surface gravimetric measurements by the GRASS gravimeter (Part I).

The semi-major axes of Phobos is $a = 9,376 km$ (Wakker, 2015) and 1.19 km (Naidu et al., 2020) for Didymoon. It has been shown that both satellites are orbiting outside the Roche limit. However, these values have still to be considered with caution considering the assumptions, i.e. the spherical shape assumption, the large-small body assumption and assumed best knowledge values will influence the actual Roche limit. It shall be well noted here, that, in principle, satellites might be able to exist within the Roche limit. The mere fact that small bodies are non-spherical can be explained by the internal strength that hinders the body to form a spherical, energetically optimised shape. Therefore, this internal strength allows secondaries to exist within the Roche limit.

The presence outside the Roche limit has another advantage regarding loose surface objects and therefore landers: Loose objects will remain on the surface and will not have the tendency to move away from the surface. This allows the possibility to have loose material such as regolith or rocks on the surface as well as robotic landers, e.g. the Juventas CubeSat with the GRASS gravimeter. As a final note, disintegrated bodies within the Roche limit might form a ring, due to different orbital speeds of the fragments. A prominent comet that was observed after tidal disruption and before collision with Jupiter was Comet Shoemaker–Levy 9.

5.2.3 SOLAR RADIATION PRESSURE

Of specific interest for small body application and especially the binary Didymos asteroid system are the non-gravitational forces. In the absence of an atmosphere, and therefore, drag and lift, solar radiation pressure (SRP) will be the dominant perturbing force.

In this Subsection, the SRP on Dimorphos will be estimated, so that it can be judged whether this contribution should be considered in the context of gravity. Firstly, the calculation of the solar radiation force F_R is introduced as derived by e.g. Wakker (2015). Generally, modelling the detailed radiation force is stated to be rather complex, but an approximation can be made by considering the effective cross section A^* , that is the cross section perpendicular to the Sun-pointing vector. The solar radiation force F_{SRP} is therefore calculated as

$$F_{SRP} = (1 + \mathbb{R}^*) \frac{WA^*}{c} \quad (5.26)$$

where W is the solar flux per area (W/m^2) and $c = 299,792,500 m/s$ is the speed of light. The surface reflectivity coefficient C_R is mathematically described by Equation (5.27).

$$C_R = 1 + \mathbb{R}^* \quad (5.27)$$

where $\mathbb{R}^*$ is the mean reflection coefficient which is defined as the ratio of the reflected radiation flux W^* to the incoming radiation flux W :

$$\mathbb{R}^* = \frac{W^*}{W} \quad (5.28)$$

Therefore, for a perfect black body with full absorption $\mathbb{R}^* = 0$ and for an ideal reflector $\mathbb{R}^* = 1$, hence $1 \leq C_R \leq 2$. This parameter is also the reason why the concept of “painting” asteroids white to increase the reflectivity has been proposed as a planetary protection concept to deviate hazardous asteroids (Ivashkin and Smirnov, 1995).

The same definition, i.e. the ratio of incident, and reflected and scattered light, is given by Lissauer and de Pater (2013) for the albedo, more specifically, the geometric albedo is defined as the albedo at phase angle $\phi = 0$. The phase angle is the angle between the two lines connecting the Sun and the object, and the observer and the object. Therefore, we can assume the surface reflectivity coefficient C_R from its geometric albedo for asteroids. Substitution of C_R into Equation (5.26) yields Equation (5.29).

$$F = C_R \frac{WA^*}{c} \quad (5.29)$$

Wakker (2015) states further, that a value of $C_R = 1.2 - 1.4$ is assumed, if no other information is available.

In a final step, it is stated that the solar power density is (like gravity) inversely proportional to the square of the distance from the Sun (as the Sun is radiating over a sphere). Therefore, taking the average solar flux at 1 astronomical unit (AU) for the Earth being $W_E = 1361 W/m^2$ (Wakker, 2015), this value can be substituted and

Table 5.3: Estimation of the order of magnitude of accelerations acting on Didymoon and Hera s/c caused by SRP.

Body	C_R	$A^* [m^2]$	$m [kg]$	$a_{SRP} [m/s^2]$
Didymoon	1.15	2112.4 ^a	$4.86 \cdot 10^9$	$2.21 \cdot 10^{-11}$
Hera s/c	1.5	12	400	$1.53 \cdot 10^{-7}$
Juventas CubeSat	1.5	0.463830	10.62	$2.90 \cdot 10^{-7}$

$$^a A = \pi \cdot (82m)^2$$

corrected for by multiplying with the inverse square of the heliocentric distance in AU as shown in Equation (5.30).

$$F = C_R \frac{1361 W/m^2}{r_{AU}^2} \frac{A^*}{c} \quad (5.30)$$

For the acceleration, the force is simply divided by the spacecraft or small body mass, giving the above introduced ballistic coefficient.

$$a_{SRP} = C_R \frac{1361 W/m^2}{r_{AU}^2} \frac{A^*}{mc} \quad (5.31)$$

With Equation (5.31), the solar radiation pressure acceleration acting on the asteroid Didymoon is computed. In addition, the SRP acting on the main Hera and Juventas CubeSat spacecraft is computed. This is interesting, as the GRASS instrument being effectively an accelerometer could potentially be used in-flight to measure the SRP-accelerations. The input parameters for Didymoon are taken from Naidu et al. (2020) and the Hera spacecraft geometry is taken from the Hera Proximity Operations Guidelines²; the Juventas numbers were kindly provided by the Juventas team. All evaluations are made at the perihelion of the Didymos system at the heliocentric distance of $r_{AU} = (1 - e) \cdot a = (1 - 0.3838) \cdot 1.644 AU = 1.013 AU$ to compute the upper limit of the order of magnitude to be expected. The results with the respective input parameters are reported in Table 5.3.

The value for the Hera spacecraft is in accordance with calculations presented in the Hera Proximity Operations Guidelines. While the acceleration caused by the SRP acting on the asteroid is about 6 orders of magnitude smaller than the surface gravity, the acceleration on the Hera spacecraft is only two to three orders of magnitude smaller than the target surface gravity and therefore in the same order of magnitude as the target measurement accuracy (1 % of $g = 4.5 \cdot 10^{-7} m/s^2$).

Regarding the surface gravimetry experiment, the SRP does not have to be included in the modelling of this work. Still, the GRASS gravimeter could serve as an accelerometer on Juventas, providing a valuable asset to the inter-satellite link (ISL) experiment between the Hera spacecraft and the Juventas CubeSat. Moreover, for calibration of the GRASS instrument in-flight, the accelerometer data could be

²Date 10/09/2020 Issue 6 Rev 3, Table 2-2.

compared to spacecraft tracking data. Here the non-gravitational forces measured by GRASS would also disturb the orbit of the spacecraft. It is therefore recommended to study these options further, which would include the gravimeter in the mission prior to landing and might improve the in-situ calibration.

5.2.4 EXCURSUS: SEISMIC INFLUENCES

Another influence on the surface acceleration are seismic events. Effectively, seismic accelerations could indeed be considered as (temporal) contributions to gravity, and so they are briefly discussed in this Subsection. Seismometer, such as the Seismic Experiment for Internal Structure (SEIS) instrument (Lognonné et al., 2019) as part of the InSight mission to Mars, are very high performance accelerometers. Theoretically, it would therefore be possible to use the GRASS instrument for monitoring seismic events. Such events could be artificial events, e.g. using explosives (Watkins and Kovach, 1972) or even, at very high risk, a spacecraft impact comparable to the DART impact. Next to this, natural seismic events might be observed, such as seismic shaking following impacts on an asteroid surface (Murdoch et al., 2015).

Studying seismic events on the surface of Dimorphos has been suggested by Murdoch et al. (2016) with the AGEX study (Subsection 2.3.4) to carry, next to a gravimeter, three seismometers and (commercial) geophones. The baseline of the GRASS gravimeter development, however, does not account actively to detect seismic measurements. On the one hand, the (considered) GRASS measurement is not continuous, but distributed to a limited number of measurements over one orbit of the asteroid moon. A seismometer would ideally measure continuously at high frequency, which, however, is not possible due to limitations in the power and data budgets. One possible solution here is to process the data on-board and only transmit relevant data, or to have the instrument triggered by events from a low-power mode.

For a spring-based seismometer (gravimeter), the modes of the spring (Subsection 3.5.3) are very important as they determine the instrument response to signals. Below the first eigenfrequency, there is a linear relation between spring deflection and the acting acceleration, meaning that increasing the gravimeter sampling frequency is limited by the first resonant frequency (Middlemiss et al., 2016). For the springs of 50 and 80 micron, this frequency was computed to be 1.09 Hz (numerical: 0.99 Hz) and 2.21 Hz (numerical 2.00 Hz), respectively. Therefore, it is very important to consider the spring-mass-system eigenfrequencies and modes, as they determine the performance regarding non-static accelerations.

On the other hand, the GRASS instrument performs a sensor head rotation (Section 3.4) so that the seismic signal could be lost in the raw data, that will be treated further, e.g. considering the rotation, stacking data for an improved reading etc. Moreover, the seismic acceleration could be acting on the insensitive direction of the bending spring. One solution to this, demanding an adapted instrument design and operations, could be to initially identify the “vertical” direction with the rotating spring-instrument and to then position the spring accordingly in the permanent position so that any seismic signal would affect the spring in the sensitive

direction (Subsection 3.2.1). By then measuring continuously with a measurement frequency below the spring resonant frequency, seismic events could be recorded, provided that they are observable at that measurement frequency.

5.2.5 FURTHER INFLUENCES

For completeness, this Subsection briefly lists further influences on gravity. These additional effects³ will not be studied further in this work as they are negligible for the surface gravity measurements. These influences are, not claiming exhaustiveness:

- Solar tides
- Attraction of other planets (e.g. Jupiter)
- Relativistic effects
- Thermal re-emission radiation pressure (Jarkowski effect)
- YORP and Binary YORP (BYORP) effect, influencing the rotation of a body, therefore influencing the gravity
- Electromagnetic force, e.g. of the Sun's interplanetary magnetic field acting on a magnetic body, a body with proper magnetic field, or an electrically charged body.
- ...

5.3 GRAVITATION MODELLING METHODS

In Section 5.1, the theory of gravitation and the main further contributions to gravity have been introduced. Detailed dynamical studies are beyond the goal and scope of this work, and so, the basic theory has been presented allowing to *approximate* the dynamical contributions to gravity. The gravitational attraction is a key element of this work and will be simulated in great detail in this Part II in the following two Chapters. In this regard, this Section presents a literature study of gravitation modelling techniques with an emphasis on both, (small) non-spherical bodies with irregular shape, as well as the gravity evaluation on the surface in the context of the GRASS surface gravimeter (Part I).

The methods presented are spherical harmonic expansion (Subsection 5.3.1) and diverse mass concentration (mascon) methods (Subsection 5.3.2 through Subsection 5.3.5). Moreover the polyhedron gravitation method (Subsection 5.3.6), and the polyhedron sliver approach (Subsection 5.3.7) are introduced.

5.3.1 SPHERICAL/ELLIPSOIDAL HARMONIC EXPANSION

The representation of the gravitational potential of spherical bodies, such as planets, is generally done using spherical harmonics (Kaula, 2013). For irregularly

³Some of the listed aspects were added to the list thanks to the Wikipedia article on Bahnstörung, cf. https://de.wikipedia.org/wiki/Bahnst%C3%B6rung#Einteilung_nach_den_Ursachen, accessed on 11.06.2021.

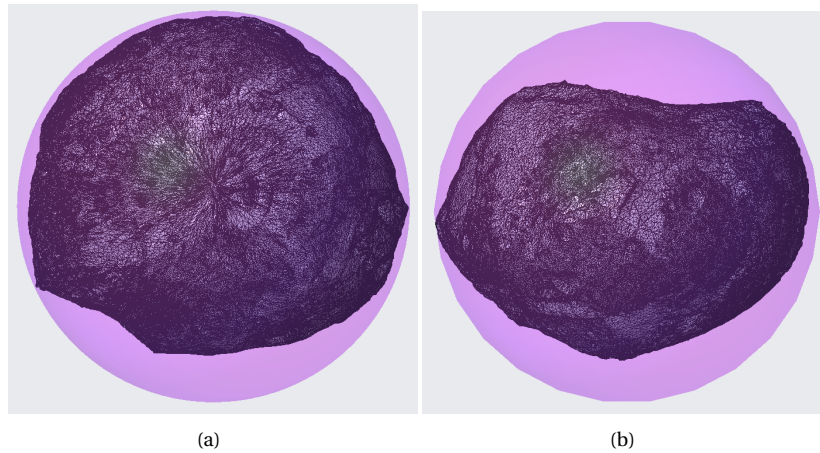


Figure 5.4: Brillouin sphere around Phobos from two different perspectives. Generally, for small bodies this circumscribing sphere will touch the bodies surface only in two points. Own figure using Phobos shape file from Willner et al. (2014).

shaped bodies, sometimes ellipsoidal harmonics (Romain and Jean-Pierre, 2001) are used. Still, these representations can be problematic for SBs, yielding very large errors, especially for highly non-spherical bodies (Takahashi and Scheeres, 2014b), as it only converges outside the Brillouin sphere (Brillouin, 1933). This sphere is defined as the smallest sphere that contains all of the celestial body (Figure 5.4). This outside gravity field is referred to as exterior gravity field, which has generally two contact points with the surface, thus it is not suitable to analyse the surface gravity generically.

To represent the gravity field inside the Brillouin sphere, the so-called *interior gravity field* is reported by Takahashi and Scheeres (2014b) to be derived by Werner (2010)⁴. The interior gravity field is stated to be converging in the *Interior Brillouin sphere*, which is different to the “inside” of the exterior Brillouin sphere. Rather, the interior Brillouin sphere is described to be a sphere of arbitrary size with the condition that it only touches the surface tangentially in exactly one point. The example given by Takahashi and Scheeres (2014b) is that this could represent the landing point on the surface of this body. While a single computation can be sufficient in case of a stationary lander, multiple gravity field computations with different interior Brillouin spheres become necessary in case of different measurement points over the surface (Takahashi and Scheeres, 2014b).

Takahashi and Scheeres (2014b) perform a bridging between the null space of exterior and interior gravity field expressions by solving Poisson’s equation, yet the problem with the different locations of the interior gravity field evaluations persists, requiring numerous evaluations of the interior gravity fields for a full mapping of

⁴Original reference currently only available for download to US Citizens, <https://www.techbriefs.com/component/content/article/tb/pub/techbriefs/information-sciences/8726>, accessed on 27.08.2020.

the surface. Takahashi and Scheeres (2014b) concluded further that the exterior gravity field should never be used to model surface gravity of small bodies. The interior gravity field is tainted with errors in the order of a few percent compared to the polyhedron method (Subsection 5.3.6). The advantage, however, is that the interior method is stated to be the most efficient regarding computation time for surface gravity.

Furthermore, it is in the nature of spherical harmonics that they have to be truncated at some degree and order, making it always an approximation to the true gravity field (Werner and Scheeres, 1997). Nevertheless, one of the advantages of the spherical harmonics expansion is that non-uniform density distribution can be expressed. Overall, despite various above-mentioned advantages, this method is not ideal to represent surface gravity, making it less suitable for this work in the context of the GRASS surface gravimeter development.

5.3.2 MASCON METHOD (1): CUBE AND SPHERE

An alternative approach to compute the gravitational attraction of arbitrarily shaped bodies is to discretise them into small elements and with individually assigned density. Calculating the gravity of each individual element and integrating over all elements provides the gravity. For example, this method was performed by Geissler et al. (1996) for the asteroid Ida using mascons over a grid.

The most basic mascon methods are using cubes (voxels⁵) and spheres. These two methods have been widely used and a comparison has been done by Park et al. (2010).

Regarding the cubes, an analytical solution exists, (Chappell et al., 2012), which is computationally intensive. Likewise, it is also possible to apply the polyhedral method (Subsection 5.3.6) to the cube, as will be presented in Subsection 6.1.2. This representation is advantageous, as the interior of the body is entirely filled, only leading to a less precise “Minecraft”-surface representation, which, however, is less ideal for this work regarding surface gravimetry.

Alternatively, regarding the sphere, the gravitational attraction of a uniform sphere with homogeneous mass distribution is identical to the attraction of a point mass in which the entire mass is concentrated (Equation 5.2). Doing this, another error source is introduced. Firstly, relatively large voids within the modelled body are created between the spheres, increasing the inaccuracy of the gravity calculation. This can be reduced by different stacking methods, however, the maximum filled volume of monodisperse (i.e. same-sized), spheres was conjectured by Kepler in 1611 (Kepler, 1611) and formally proven by Hales et al. (2017) to be, non-exclusively, face-centred cubic packing, but in no case exceeding the density of filled volume of $\pi / \sqrt{18} \approx 0.740$.

Secondly, as these spheres cannot interfere, but only touch, the densities of the spheres need to be larger to cope for the empty volume between the stacked spheres. For example, considering both simple cubic-sphere-packing, the centres of the sphere being on a regular straight grid, the ratio of the volumes of sphere to

⁵A voxel is the 3D-equivalent to a 2D-pixel.

cube with the diameter being equal to the side length of the cube $2r$ can be computed (Equation (5.32)). The density of the spherical mascons will thus need to be increased by a factor of $(\pi/6)^{-1} \approx 1.910$.

$$\frac{V_{sphere}}{V_{cube}} = \frac{\frac{4}{3}\pi r^3}{(2r)^3} = \frac{\pi}{6} \approx 0.524 \quad (5.32)$$

Like this, the mass distribution inside the body is not homogeneous, but concentrated in the spheres and leaving empty voids in-between. Again, a perfect surface representation is not possible. As the gravity depends on the distance between evaluation point to mass element, the mascon approach is only an approximation. This error vanishes, when the distance to the mascons approaches infinity, and can likewise be expected to be largest on the surface (Takahashi and Scheeres, 2014b), being most relevant for a surface gravimeter. Likewise, increasing the number of mascons to infinity will reduce the error to 0, but the number of mascons is limited by the computation time. Indeed, it is stated by Werner and Scheeres (1997) that for a given computation time, the spherical harmonics approach is more accurate, however, the above discussed limitation to convergence close to the surface still applies, which is a strong advantage for the mascon method.

A combination of the discretisation of the body in mass elements and the spherical harmonics (Subsection 5.3.1) was presented by Le Maistre et al. (2019) not only to compute the gravity field, but also the moment of inertia and the libration amplitude of the Martian moon Phobos. Here, the body was divided in cubes of side length 500 m. Then, the spherical harmonics are computed per cube individually.

5.3.3 MASCON METHOD (2): SPHERE PACKING

An innovative method building upon the mascon method described in the previous Subsection uses polydisperse (i.e. variable size) mascon sphere packing (MSP) developed by Srinivas et al. (2017). The algorithm is reported to be adaptable to any body and to have a computation time two orders of magnitude smaller than other gravity forward modelling tools such as the polyhedral method (Subsection 5.3.6). Like all mascon methods, the density distribution can be varied throughout the body. The spheres are generated with the ProtoSphere sphere packing algorithm (Weller and Zachmann, 2010) and an illustration of both polyhedral (Subsection 5.3.6) and polydisperse sphere shape representation is shown in Figure 5.5.

In the previous Subsection, the mass assignment to the individual mascons has been already discussed. Srinivas et al. (2017) studied this problem in greater detail, presenting four different approaches to account for the voids inside the body:

- Volume proportional method (VP)
- Bulk density uniformity method (BDU)
- Delta radius increase method (DRI)
- Delta percentage volume increase method (DPVI)

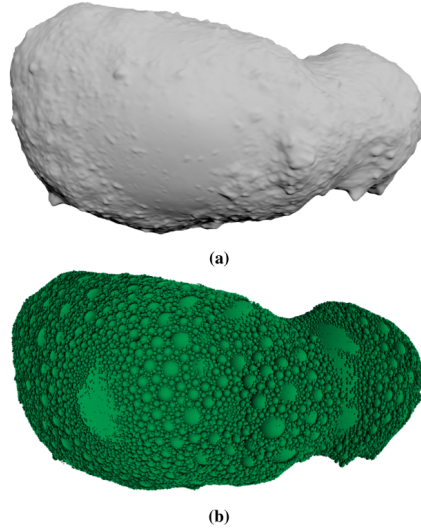


Figure 5.5: (a) Polygonal mesh of Itokawa with 200,000 faces and (b) respective sphere-packings (100,000) of the above mesh generated using the Protosphere algorithm with default parameters. Figure and caption from Srinivas et al. (2017).

In the **VP method**, the mass portion of each Mascon corresponds to the portion of the sphere in relation to the volume of all spheres. The total mass is thus equal to the initial total asteroid mass M .

For the **BDU method**, a focus is laid on the packing fraction of the spheres. In a region where large spheres are placed, the bulk density will be larger than in regions where a larger number of smaller spheres is present. The method adapts the masses of all spheres such that the bulk density is as homogeneous as possible. For this, the mass of larger spheres is reduced and the mass of the smaller spheres is increased. All masses are finally normalised in order to maintain the total mass M .

In the third method, the **DRI method**, the volumes of all spheres are increased by a constant δ so that the sum of all sphere volumes is equal to the polygonal volume (Appendix B.1). This method is powerful to remove inaccuracies due to the voids between the spheres, physically the increase of the volumes means, however, that the spheres interfere somewhat with each other.

Lastly, the **DPVI method** is a further development of the DRI method, with the difference that the volume is increased by a variable inversely proportional to the individual spheres radii. Again, this means that smaller spheres are increased more in volume to cope for the voids between smaller spheres, improving the uniformity of the bulk density.

A comparison between all four methods has been performed to the “true” solution being the polyhedron method (Subsection 5.3.6). The plot shown in Figure 5.6 shows that, as can be expected, a larger number of spheres decreases the error asymptotically. The difference between DRI and DPVI is hard to identify with the

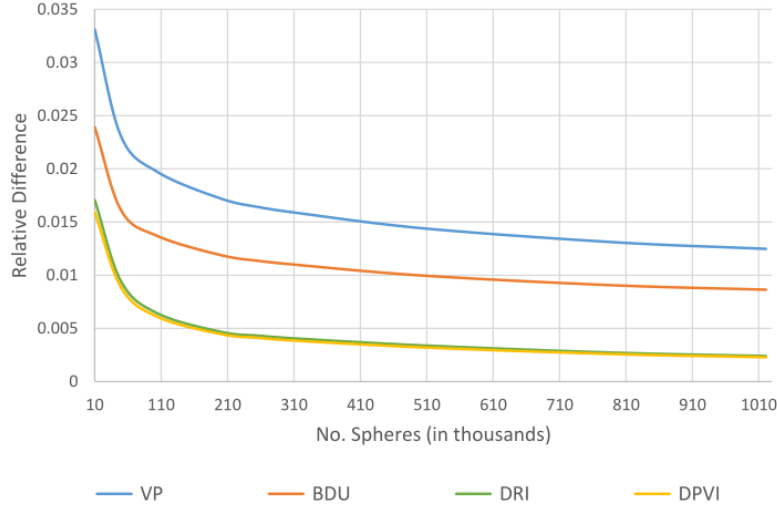


Figure 5.6: Plots of RMS relative differences of the estimated gravitational accelerations at altitude 1 m from the Itokawa surface w.r.t. different numbers of sphere packings for different methods [compared to polyhedral method results].
Figure and caption from Srinivas et al. (2017).

DPVI having a smaller error. Both methods are performing better than the BDU, which in turn has a smaller difference than the VP. Srinivas et al. (2017) also performed an analysis 100 m above the surface, and as can be expected, the relative differences are smaller. The 1 m analysis, closer to the surface, can achieve relative errors in the order of 2 – 3 ‰. This, in combination with the reported much faster computation time being more than two orders of magnitude faster than the polyhedron method makes it a promising candidate for gravimeter data interpretation and inversion, where a large number of computations is required. For this, the performance closer to and on the surface needs to be investigated further.⁶

5.3.4 MASCON METHOD (3): SPHERICAL COORDINATES

Instead of using spheres or cubes, an alternative mascon approach is by creating tesserooids, i.e. spherical prisms. This approach has been explained in Meißenhelter et al. (2022) in 3 steps and is referred to as *Mascons - Spherical coordinates (MASC)*. Firstly, the small body is subdivided in spherical coordinates, i.e. in latitude and longitude. Secondly, the radius as third spherical coordinate is subdivided. These two steps allow forming the individual tesserooids, as sketched in Figure 5.7. For

⁶Starting with the here-presented literature study, a contact to the research group behind this work has been established. This lead to an international cooperation aiming at comparing three different gravitation computation techniques, with special attention to the surface, also motivated by the GRASS gravimeter (Part I). The results of this comparison are presented in Section 6.1. The second computation method, forming the contribution of this work to the comparison is the polyhedron method (PM, Subsection 5.3.6). The third method of the comparison is using spherical coordinates as described in Subsection 5.3.4.

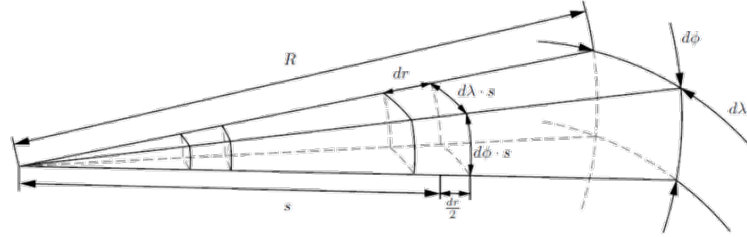


Figure 5.7: Mascon subdivision in tesseroïd mascons, using spherical coordinates. From Meißenhelter et al. (2022).

each tesseroïd, a density is applied and the centre of mass (volume) is computed. Thirdly, the gravitational attraction per tesseroïd is computed by concentrating the tesseroïd mass in the centre of the prism. Again, the resulting gravitation results from summing over all mascons.

5.3.5 MASCON METHOD (4): TETRAHEDRON MASCON

A solution of similar philosophy to the presented spherical coordinates mascon (3) approach (Subsection 5.3.4) was developed by Chanut et al. (2015). Here, the basis forms a tetrahedra polyhedron shape file of the body as also used for the polyhedron method (Subsection 5.3.6). This method takes the triangular surface facet of the shape file and forms a tetrahedron by connecting the three (surface) vertices with the vertex 0 at origin (inside the body). Then, the total mass of this tetrahedron is concentrated in the centre point of this shape and the gravitational field is computed for each tetrahedron (Figure 5.8 (a)) and summed up to obtained the resulting gravitational field. This method is referred to as *Mascon 1* and is supplemented by the *Mascon 3* method. Here, the addition is that the tetrahedron is subdivided further into three parts, each with their centre having the corresponding mass of the subdivided part attached to it as visible in Figure 5.8 (b). Obviously, the number of subdivisions might be increased to a larger number by changing the computation implementation.

The results of the *Mascon 1* and *Mascon 3* were also compared by Chanut et al. (2015) with the polyhedron gravitation method introduced in the next Subsection. The difference for *Mascon 1* inside the body was generally larger than 10 %. This method generally appears worse in the comparison than *Mascon 3* method, as one would expect. Regarding the surface gravitation, the deviation is in the order of 1 – 10 percent as read from the comparison plot, again improving with larger distance from the surface.

5.3.6 POLYHEDRON METHOD

The *Polyhedron Method* (PM, also: *Polyhedral Method*) was developed by Werner and Scheeres (1997) that provides a close-form gravity calculation for any polyhedron body, even on the surface, which is most valuable to this work in the context of surface gravimetry. The authors name the two limitations of this approach, namely

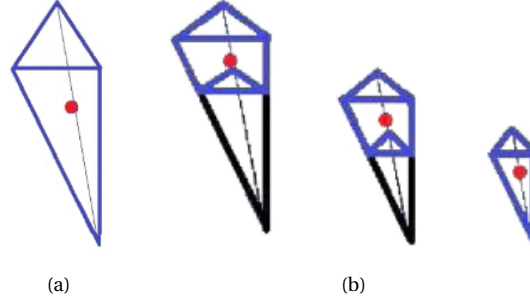


Figure 5.8: The tetrahedron mascon approach can either concentrate all mass in the centre of the tetrahedron (a) or further subdivide the tetrahedron in three (multiple) regions (b). From Chanut et al. (2015).

that the polyhedron shape can always only be an approximation of the true shape and that this approach assumes a constant density throughout the body. Another disadvantage is the relatively large computation time, especially for real-time navigation (Takahashi and Scheeres, 2014b), which, however, is less of a problem in the context of this work. In sum, this method is therefore very powerful with only few limitations of which a first solution approach to the constant density is presented in Subsection 5.3.7. This solution can still be considered as state-of-the-art (SOTA) and the algorithm used in this work was implemented by Tasev (2019) focusing on tetrahedral polyhedra, i.e. shapes with triangular surface facets, always bound by three vertices. This Subsection briefly introduces the method while referring the interested reader to the original paper giving the full derivation of the method. The following summary of the method described by Werner and Scheeres (1997) is also based on the well-written summaries provided by Van wal (2018) and Tasev (2019).

Firstly, the field point (that can also lie *on* the surface) is defined in a right-handed Cartesian coordinate system (CS) with the coordinates (x, y, z) . The vector $\vec{r}$ is defined as the vector pointing from that field point towards the subscript referenced point on the asteroid surface.

Next, the concept of common edges and dyads has to be introduced by referencing Figure 5.9, where we will stick to the nomenclature of Figure 5.9 (b).

The edge dyads of the entire body E_{ij} (Equation (5.33)) are computed with the outward-pointing normal vectors $\hat{N}$ and the vectors $\hat{M}$ that are orthogonal to the edge while going through the remaining third point of the respective facet.

$$E_{ij} = \hat{N}_{ijk} \hat{M}_l^T + \hat{N}_{ijl} \hat{M}_k^T \quad (5.33)$$

In the same fashion, the facet dyad is computed for every facet with Equation (5.34) based on the normal vectors:

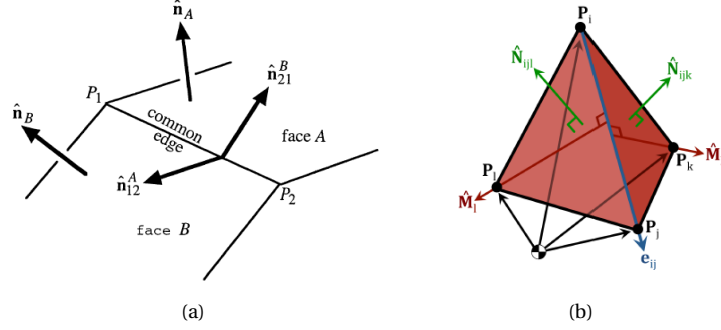


Figure 5.9: (a) Two face-normal and two edge-normal vectors are associated with each edge. Caption and Figure from Werner and Scheeres (1997). (b) Vectors used in the evaluation of the constant-density polyhedron model. Caption and Figure from Van wal (2018).

$$F_{ijk} = \hat{N}_{ijk} \hat{N}_{ijk}^T \quad (5.34)$$

with the edge length $e_{ij} = \|\overrightarrow{P_i P_j}\|$ and the above introduced vector is complemented by its length $r = \|\vec{r}\|$, the per-edge factors L_{ij} are computed in Equation (5.35):

$$L_{ij} = \ln \frac{r_i + r_j + e_{ij}}{r_i + r_j - e_{ij}} \quad (5.35)$$

Here, we note that while the polyhedron method (PM) works on the surface of the polyhedron, it is not defined on a polyhedron vertex or edge, as the summed distance from the field point to the two vertices (edge-end point) are equal to the length of that edge in the denominator of Equation (5.35).

The per-facet factors ω_{ijk} are evaluated as (Equation (5.36)):

$$\omega_{ijk} = 2 \arctan \frac{\vec{r}_i \cdot (\vec{r}_j \times \vec{r}_k)}{r_i r_j r_k + r_i (r_j \cdot r_k) + r_j (r_k \cdot r_i) + r_k (r_i \cdot r_j)} \quad (5.36)$$

Finally, introducing the vectors $\vec{r}_{ij}$ as the vector connecting the field point with any point on the edge, spanning from vertex i to j , and $\vec{r}_{ijk}$ going from the field point to any surface point of the facet, spanned by the vertices i, j , and k , allows to compute the gravitational potential U (Equation (5.37)) and the gravitational attraction $\vec{g}$ (Equation (5.38)). This is done by summing over all edges and facets of the polyhedron.

$$U(x) = \frac{1}{2} G \rho \sum_{e \in \text{edges}} \vec{r}_{ij} \cdot \vec{e}_{ij} \cdot \vec{r}_{ij} \cdot L_{ij} - \frac{1}{2} G \rho \sum_{f \in \text{facets}} \vec{r}_{ijk} \cdot \vec{F}_{ijk} \cdot \vec{r}_{ijk} \cdot \omega_{ijk} \quad (5.37)$$

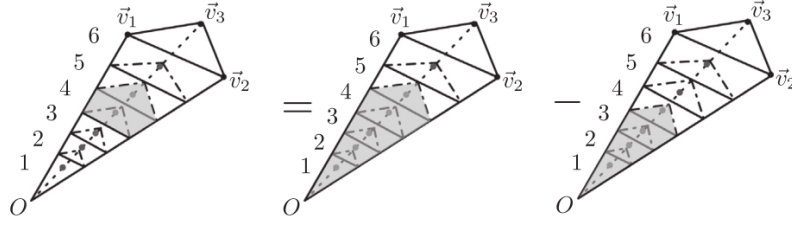


Figure 5.10: Gravitation by a sliver. There are a total of six slivers in the tetrahedron. The gravitation caused by sliver 4 is the gravitation caused by slivers 1 through 4 minus that created by slivers 1 through 3. This operation is repeated for each sliver. For faster computation, the slivers with the same density value can be grouped. Figure and caption from Takahashi and Scheeres (2014a).

$$\vec{g}(\vec{x}) = -G\rho \sum_{e \in \text{edges}} E_{ij} \cdot \vec{r}_{ij} \cdot L_{ij} + G\rho \sum_{f \in \text{facets}} F_{ijk} \cdot \vec{r}_{ijk} \cdot \omega_{ijk} \quad (5.38)$$

where again G is the gravitational constant and ρ the (constant) density of the body. The sums are taken over the polygon edges e and faces f , respectively.

5.3.7 POLYHEDRON SLIVER APPROACH

As discussed in the previous Subsection, one limitation of the polyhedron method (PM) is the assumption of homogeneous density throughout the body. One possibility to adapt the PM is the sliver approach developed by Takahashi and Scheeres (2014a). For this, each individual tetrahedron, spanning from the surface triangle to the origin, is dissected into multiple slivers. Counting the slivers from the inside, the gravity of the n^{th} sliver alone is computed as the difference of the tetrahedron spanned up to the outer sliver surface and the tetrahedron spanned to the inner sliver surface as shown in Figure 5.10.

The individual assignment of the sliver densities can be done with different approaches, for example, Takahashi and Scheeres (2014a) use density maps. Using only two slivers is referred to the surface layer model, where a surface layer, e.g. regolith, would be allowed a different density than the remaining body being the core. This is illustrated in Figure 5.11. The principle of adding and subtracting gravitation can also be used to add a core to a body, e.g. adding a simple spherical core and only considering the *differential* density.

5.4 DISCUSSION GRAVITY CONTRIBUTIONS AND MODELLING

In this Chapter, research question (I) has been addressed with regards to the surface gravity contributions and different methods of modelling gravitation. Generally, the main contribution is the gravitation, and the underlying theory has been introduced. However, gravity consists of more than that, additional contributions

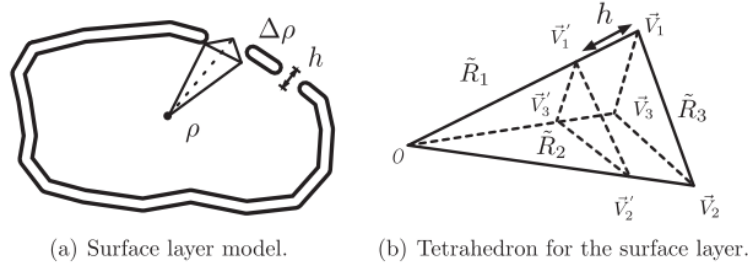


Figure 5.11: Surface layer model and its tetrahedron. Figure and caption from Takahashi and Scheeres (2014a).

are, most notably, dynamical contributions, including the rotational influence, the tidal surface acceleration, and further the forced physical libration. The precise rotation dynamics of small bodies is an entire research topic in itself, and so only analytical descriptions were presented allowing to *approximate* the dynamical contributions and to compare them to gravitation. Using the presented theory on the dynamics, the contributions to surface gravity will be computed in Section 6.2. Further contributions to gravity have been introduced and discussed (Section 5.2), and it has been shown e.g. for the solar radiation pressure that the contribution can be neglected in the context of surface gravimetry.

Regarding the gravity modelling, three fundamentally different existing approaches have been identified and presented. Spherical/elliptical harmonic expansion (Subsection 5.3.1) is a powerful tool, also for arbitrary internal mass distribution, but it can break down close to or on the surface, which is a specific problem for small bodies, oftentimes being irregularly shaped. The second approach is the mascon approach (Subsections 5.3.2 to 5.3.5), where a body of arbitrary shape is subdivided into small volumetric elements, to which any density can be assigned, again allowing for an arbitrary mass distribution inside the body. A total of four different implementations has been presented, differing by the shape of the mascons. Here, the problem is that the discretisation into mass elements either does not allow a perfect shape representation, especially regarding the surface, or that material is concentrated as a point mass in the centre of a volume, introducing an error regarding the location of the mass, or introducing voids between the elements (especially for spherical mascons). Theoretically, these problems can be removed by taking an infinite number of mass elements, practically, however, this is impossible due to the computation time. Lastly, the polyhedron method (PM, Subsection 5.3.6) provides an analytical closed-form solution to the gravitation computation, yet the density inside the body cannot be varied without limitation. One solution here is the sliver approach (Subsection 5.3.7), but generally, the variation in internal density is still limited by the shape file and cannot be done completely arbitrarily.

Regarding the last part of the research question, the comparison of different gravitation computation methods, the above qualitative discussion is supplemented by a quantitative discussion between two mascon implementations and the PM. This

comparison is presented in the next Chapter (Section 6.1).

5.5 SUMMARY CHAPTER 5

This Chapter opens Part II of this work, focused on surface gravity of small bodies. For this, firstly, the different contributions to gravity, are introduced (Section 5.1). The main contributions are gravitation (Subsection 5.1.1), rotational influence (Subsection 5.1.2), forced (physical) libration (Subsection 5.1.3), and tidal influences (Subsection 5.1.4). Related aspects and further contributions are discussed in Section 5.2 to draw a complete picture. Here, the hill sphere (Subsection 5.2.1), the Roche limit (Subsection 5.2.2) and the solar radiation pressure (SRP, Subsection 5.2.3) are introduced and computed for different settings. Moreover, a brief discussion on seismic influences in the context of gravimetry is presented (Subsection 5.2.4) and further, less relevant, influences are listed in Subsection 5.2.5.

With the basic theory introduced, different gravitation modelling methods are presented (Section 5.3) with a specific focus on non-spherical (small body) shapes and their surfaces as evaluation point for the GRASS space gravimeter and the JUV-GRASS version. For this, spherical/ellipsoidal harmonics (Subsection 5.3.1), four mascon methods (Subsections 5.3.2 through 5.3.5), and the polyhedron method (PM, Subsection 5.3.6) with the sliver approach as an extension of the PM to vary the internal density (Subsection 5.3.7) are presented. All of the presented content is discussed in Section 5.4 with regards to research question (I), before this Chapter is summarised in this Section. The here-presented concepts and theory lay the foundation of a thorough analysis of the small body gravity environment for surface gravimetry on a “global” scale. The SB gravity environment is modelled in the next Chapter, together with a novel comparison of different gravitation modelling techniques.

6

GLOBAL SMALL BODY SURFACE GRAVITY AND GRAVITATION METHODS COMPARISON

This second Part of this thesis is focused on the gravitation modelling of small bodies (SBs) to which we here also count natural satellites, such as the Martian moons. The previous Chapter 5 has introduced both the different contributions to the net gravity and different methods found in literature that allow to compute the gravitation. In direct continuation of the previous Chapter, and to complement research question (I) with regards to the comparison of different gravitation computation methods, the comparison results of a total of three different gravitation computation methods is presented in Section 6.1. This comparisons comprises of three test cases, being a sphere, a cube and the Martian moon Phobos as the initially foreseen target of the GRASS gravimeter proposed for the MMX rover. Building upon the latter case, and in order to provide a concise example, the other gravity contributions and the variations throughout one orbit are studied in Section 6.2, using the theory introduced in Section 5.1 next to gravitation. Returning to the current context and in view of one possible future mission, the simulations are repeated for Dimorphos and Apophis, respectively (Sections 6.3 and 6.4). With this in place, and related to the gravimeter development, it is then possible to focus on gravitation and the signatures in the surface gravity caused by possible inhomogeneities inside small bodies (Section 6.5). Finally, research question (I), regarding the comparison, and research question (II), regarding the surface gravity, are discussed in Section 6.6, before the Chapter is summarised in Section 6.7.

6.1 COMPARISON OF GRAVITATION MODELLING METHODS

This Section is an adapted excerpt from Meißenhelter et al. (2022), where the author contributed as second author.

As discussed in Chapter 5, different approaches to compute the gravitational field of irregularly shaped bodies exist. To compare different methods, with special attention to *surface* gravitation, a collaboration between three different research groups has been established by the author to compare different simulations. In

this Section, the comparison of the polyhedron method (PM) with the analytical and/or two different mascon methods is presented. This is done for three different shapes, being a sphere (analytical comparison, Subsection 6.1.1), a cube (analytical and method comparison, Subsection 6.1.2) and Phobos (method comparison, Subsection 6.1.3). The two mascon methods forming part in the comparison are the MSP tested by Hermann Meißenhelter, René Weller and Gabriel Zachmann from the University of Bremen (Subsection 5.3.3), and the MASC tested by Tom Andert, Benjamin Haser, Max Hofacker and Larissa Balestrero Machado from the Bundeswehr University Munich (Subsection 5.3.4).

This work contributed to the comparison by applying the PM (Subsection 5.3.6) to the different test cases. Here, an algorithm implementation developed by Tasev (2019) with minor adaptations has been used. This code was verified with an independent PM implementation, provided by Van wal (2018). This Section thus forms the basis for more complex studies, e.g. regarding inhomogeneities as studied in Section 6.5 or future work.

6.1.1 COMPARISON: SPHERE

A (perfect) sphere can only be approximated by a polyhedron, and hence the polyhedron method (PM) cannot reach a perfect representation of the analytical surface gravity of a sphere. Still, a sphere can be approximated increasingly well by a polyhedron with increasing number of facets.

In a first step, the agreement of the surface gravity obtained by the PM for different spherical polyhedra is compared to the analytical solution in this Subsection. For this, two types of spheres, the Icosphere and the UV-sphere, are used with variable and increasing numbers of facets, where the perfect sphere would be achieved for an infinite number of facets. Starting from a icosahedron (a polyhedron with 20 faces), each triangular facet is subdivided into four triangular facets as shown in Figure 6.1 (a). The UV-sphere (Figure 6.1 (b)) is subdivided in latitudinal and longitudinal direction using spherical coordinates, hence offering more control regarding the resulting number of facets. In the comparison, the ratio of latitude and longitude for the UV-sphere was kept constant. Overall, the number of facets of the UV-sphere were set such that they were comparable (not identical) to the number of facets of the Icosphere.

For the comparison between PM surface gravity and analytical solution, the agreement of numerical surface gravity g_{num} with respect to the analytical solution g_{ana} is computed as the ratio:

$$\tau = \frac{g_{num}}{g_{ana}} \cdot 100\% \quad (6.1)$$

The agreement for both kinds of spheres for different number of facets is listed in Table 6.1 and the deviation is plotted over one hemisphere each in Figure 6.2. As can be expected, $\tau < 100\%$ for all test cases, as the bound volume of the polyhedron is always smaller than the perfect sphere, whereas the agreement increases towards 100 % with increasing number of facets. However, as also reported in Table 6.1, the

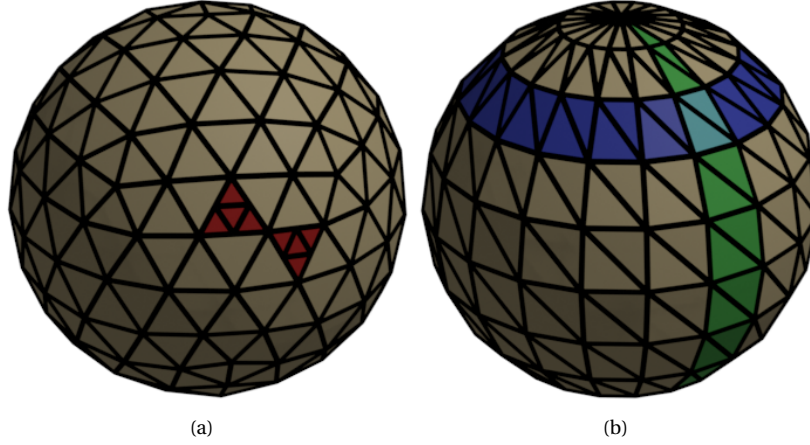


Figure 6.1: (a) Icosphere (b) and UV-sphere. The number of facets for the Icosphere grows by a factor 4 per subdivision, thus the precise number of facets cannot be chosen arbitrarily. On the contrary, the UV-sphere subdivision is controlled by spherical coordinates, and thus the latitudinal and longitudinal subdivision has a larger adaptability. From Meißenhelter et al. (2022).

computation time increases significantly, always demanding a trade-off between accuracy and computation cost.

The relation of the achieved accuracy τ as a function of the computation time T_{comp} is plotted in Figure 6.3. Clearly, the increased number of facets, resulting in the larger computation time, increase the computational accuracy. For example, for the Icosphere, increasing the number of facets from 1,280 ($T_{comp} = 73.2s$) to 5,120 facets ($T_{comp} = 277.1s$), the accuracy is increased from 98.9 % to 99.7 %. What is also visible for both spheres, however, is that the increase in accuracy saturates for larger computation times when approaching 100 %. Therefore, it might be well possible to trade small improvements in accuracy against significant savings in computation cost.

Next to the number of facets, the sphere type influences the deviation, not only considering the numerical agreement regarding τ , but also observing the plotted global deviation patterns. This becomes best visible when comparing the coarsest resolutions of only 320 (Icosphere) and 360 (UV-sphere) facets (Figure 6.2 (a-a) and (b-a), respectively).

For the Icosphere, compared to the analytical solution of a perfect sphere, there are hexagonal regions of maxima and minima of the gravitation, clearly visible for the lowest resolution (Figure 6.2 (a-a)). When increasing the resolution, two observations can be made, while repeating that the range of differences decreases: Firstly, the individual regions of minima and maxima decrease in size while the number of these regions increases. Secondly, these regions begin to group in larger regions (Figure 6.2 (a-b) – (a-e)), eventually forming very large regions of smallest and

Table 6.1: Comparison of gravitation computed for different sphere shape models using the PM, compared to the analytical solution. Due to the approximated polyhedron lying within the perfect sphere, all values are below 100 %. All values give the ratio of the PM to the analytical solution τ . Additionally, we provide the corresponding computation times T_{comp} and the standard deviation over the full surface σ . Table and caption from Meißenhelter et al. (2022).

Shape	Min. [%]	Mean [%]	Max. [%]	σ [%]	T_{comp} [s]
ico:320	95.8599	96.6437	97.9965	0.3968	27.3
ico:1280	98.9134	99.1447	99.4826	0.1121	73.2
ico:5120	99.7232	99.7836	99.8566	2.7918E-02	277.1
ico:20480	99.9302	99.9442	99.9578	5.4776E-03	345.3
ico:81920	99.9825	99.9844	99.9878	1.2096E-03	2,710.5
ico:327680	99.9955	99.9960	99.9970	3.0489E-04	12,749.2
uv:360	94.6775	95.9555	97.8482	0.6550	28.2
uv:1224	98.2785	98.7374	99.3877	0.2242	68.1
uv:5040	99.5589	99.6832	99.8505	5.9350E-02	266.0
uv:20448	99.8887	99.9209	99.9632	1.5308E-02	642.7
uv:82368	99.9702	99.9793	99.9905	4.0139E-03	2,597.6
uv:328328	99.9931	99.9954	99.9979	9.4118E-04	11,233.3

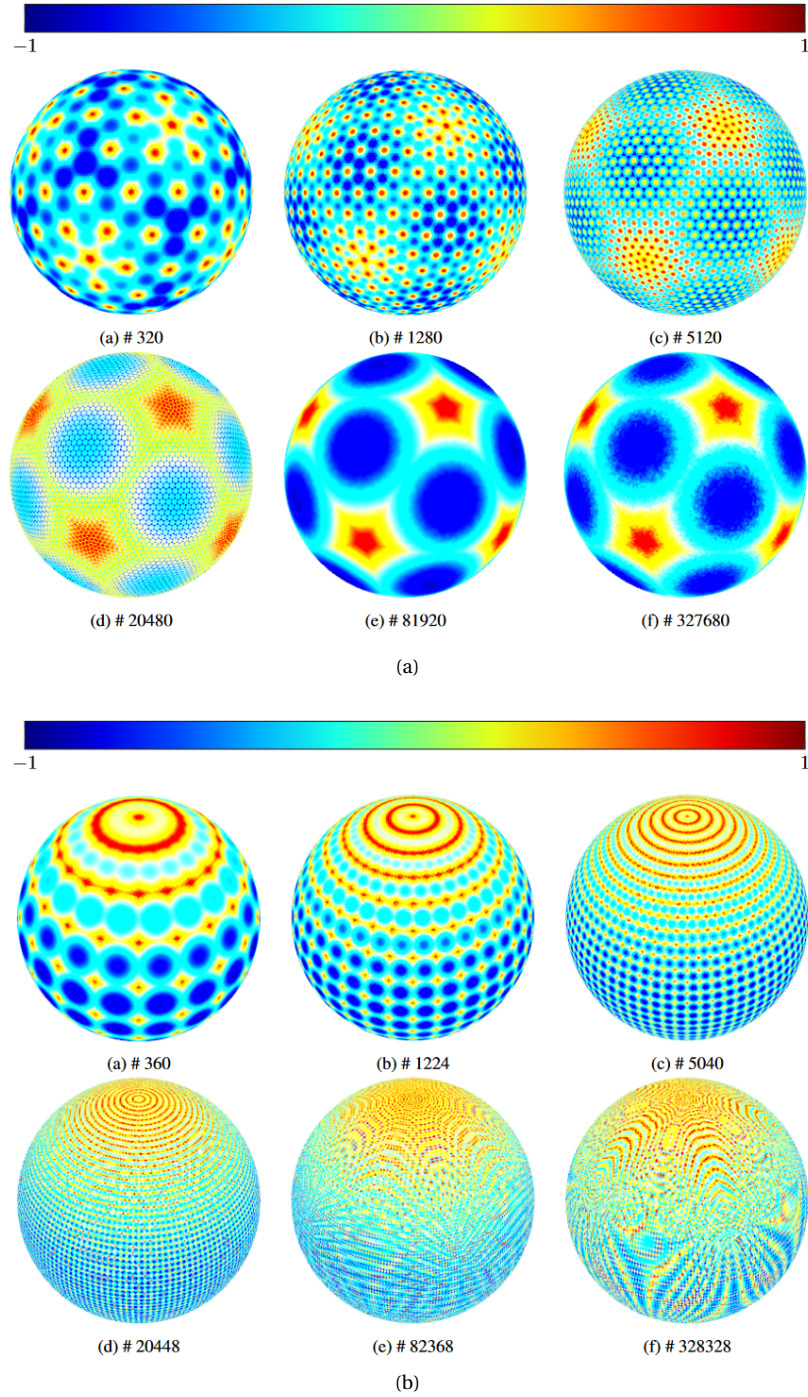


Figure 6.2: Results Icosphere (a) and UV-sphere (b). The uniform colour bar corresponds to the respective min. and max. values in Table 6.1 and are interpolated over the corresponding range. From Meißenhelger et al. (2022).

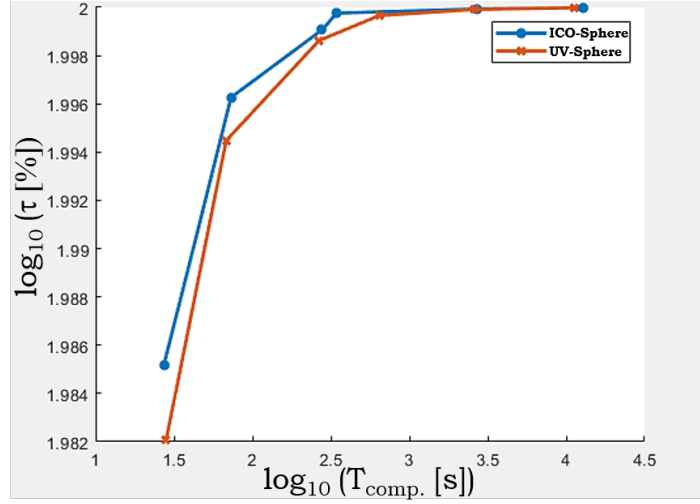


Figure 6.3: Comparison of the accuracy τ with the computation time T_{comp} as computed in Table 6.1. Note the double-logarithmic scale. From Meißenhelther et al. (2022).

largest agreement between computed and analytical solution τ (Equation (6.1)). Finally, the number of these regions is decreased to 4 regions of maximum agreement and eight regions of largest underestimation for the plotted visible hemispheres (Figure 6.2 (a-f)).

In contrast, the results for the UV-sphere form a quite different pattern. The (dark blue) regions of the minima form circular regions, while lesser underestimates form diamond-like shapes (Figure 6.2 (b-a) and (b-b)). At the “equator”, these overshoots are individual regions, while they grow together forming latitudinal bands towards the “poles”. Other than for the Icosphere, for the UV-sphere, these features become continuously smaller, not forming visible global patterns (Figure 6.2 (b-d) – (b-f)).

This well-observable and interesting difference can be explained by the difference in the way the two sphere types are created. For the Icosphere, most vertices border six triangular facets with the exception of 12 vertices bordering only five triangular facets. Closer to these 12 vertices, the triangles become smaller, leading to more triangles per surface area and therefore to a smaller error in gravitation, as observable in Figure 6.2 (b-f). For the UV-sphere, the spherical coordinates lead to smallest triangles towards the “poles” and largest triangles at the “equator”. Hence, the triangle density at the poles is largest, leading to generally smallest errors there, which is well visible with the red bands (e.g. Figure 6.2 (b-b)). Here, more facets lead to finer bands of alternating larger and smaller errors.

6.1.2 COMPARISON: CUBE

The previous Subsection provided a comparison of PM results with the analytical solution of a sphere. In this Subsection, the analytical comparison is extended by a comparison of the mascon methods with the PM, here applied to a cube. Other than a sphere, a cube can be perfectly described by a tetrahedral polyhedron, simply by dissecting diagonally the 6 faces into 12 triangles. The comparison is done with the analytical solution from Okabe (1979) for the surface gravitation of a cube, for an (arbitrary) density of $\rho = 1,000 \text{ kg/m}^3$ and cube side length of 2 km (Figure 6.4 (a)). As can be expected, the resulting gravitation using the PM is quasi-perfect, only leaving the numerical noise of the computation as shown in Figure 6.4 (b). Here, it is important to note that the PM is not defined at the polyhedron edge and vertices (Equation (5.35)), so an evaluation exactly there is not possible, returning *Not a Number (NaN)*, which is partly visible in black on the face-cutting edges. The relative comparison between the PM solution (being quasi-identical to the analytical solution) and the two mascon methods is visible in Figure 6.4 (c) for the MSP and Figure 6.4 (c) for the MASC.

Regarding the singularity of the PM at the shape edges, it was further investigated how close the evaluation point can approach the edge. For this, the above described cube was used. In a first step, it was investigated how close the evaluation point can (numerically) approach the surface to still obtain a gravitation result. For the surface lying at $Z = +1,000 \text{ m}$, the evaluation worked, independently of the X- and Y-coordinates, down to $Z = (1,000 + 10^{-4}) \text{ m}$. It returned *NaN* for $Z = (1,000 + 10^{-5}) \text{ m}$ and below. Thus, in this example, the evaluation point requires a separation of seven orders below the size of the body. In a second step, the cube was scaled up by a factor 1,000, by curiosity. Again, the evaluation worked without problems till $Z = (10^6 + 10^{-1}) \text{ m}$, so the relative separation of 7 orders of magnitude remained. Here, another interesting observation was made. For separations from $Z = (10^6 + 10^{-2}) \text{ m}$ down to $Z = (10^6 + 0) \text{ m}$, the computations only partly returned *NaN*, depending on the distance from the edge (dissecting the square face of the cube). To assess the influence not only of the distance from the plane, but also from the edge, the distance from the edge was varied. While in the first case of the unscaled cube, this did not make an observable difference (apart from the expected small gravitation variations), this was the case for the upscaled cube. The edge crosses the point $X = Y = 0$. The evaluation points were placed at $X = Y = 10^n$, with $n = +1, 0, -1, \dots, -30$. In addition, the point $X = Y = 0$ was tested. At the upscaled cube, results in the plane were obtained for $n = +1, -3, -4, -5, -7, -8, -9, -10$. So, there is clearly a dependency from the distance between evaluation point and shape file edge. What was surprising is that after *NaN* for small distances was obtained, results were re-obtained twice for even smaller distances. Future investigations will be required to explain this phenomenon, but it has been clearly shown that numerical effects appear close to the singularities of the PM.

Overall, it is clearly visible that both mascon approaches have some problems with the orthogonal shape. The MSP has one centralised very large sphere, leading to an overestimation of the computed gravitation just at the six face centres, surrounded by an underestimation. The mass assignment of the individual spheres is adapted similarly to the BDU method (Subsection 5.3.3), where the spheres are

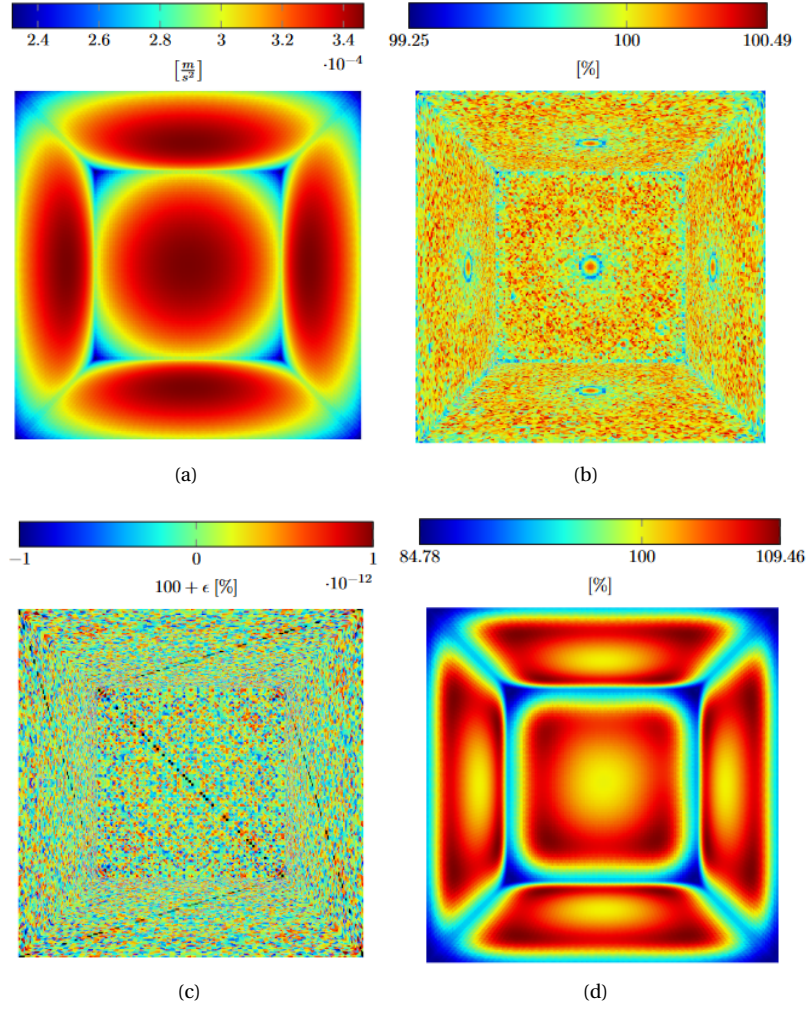


Figure 6.4: All plots show an inside-view into the cube, thus looking from one face into the cube with the remaining five faces being visible. (a) Analytical surface gravitational acceleration of the studied cube. (b) Relative difference between the MSP and the PM. (c) Relative difference between PM and analytical solution. (d) Relative difference between the MASC and the PM. From Meißenhelter et al. (2022).

assigned proportionally more mass to account for the empty volume in the cube. Like this, the reddish regions of overshooting appear, where a large number of smaller spheres appear, as these spheres are assigned relatively more mass than the larger spheres to maintain a homogeneous bulk density. This is also the reason why around the face centres (with an overshoot), there is a region of undershoot as some small gap appears around the contact points of central mascon sphere and cube. Similarly, the cube edges are regions of undershoot, as the orthogonal edges and corners are difficult to fill with (spherical) mascons. Still, the agreement is better than $\pm 1\%$. Similarly, the MASC overshoots at the centres of the face centres, while it underestimates the surface gravitation at the edges. Compared to the MSP, this effect is much stronger at the cube corners. The relative difference for the MASC ranges from about minus 15 % to about plus 10 %, disqualifying the spherical coordinate approach for sharp (orthogonal) edges. Clearly, the PM is best suited for a cube, and the comparison with the mascon methods visualises well the effects of different internal mascon distributions and their mass assignments.

6.1.3 PHOBOS: GLOBAL GRAVITATION AND COMPARISON

While sphere and cube allowed a comparison of modelled and analytical results, this is no longer the case for small bodies (SBs) of irregular shape. As final step of the comparison of the mascon methods are compared with the PM, which provides an analytical solution considering a constant density of here $\rho = 1,860 \text{ kg/m}^3$ for Phobos (Willner et al., 2014). In a first step, the surface gravitation is computed, which is an important simulation with regards to the space gravimeter (Part I). As visible in Figure 6.5, the surface gravitation varies significantly over the shape of the irregularly shaped body, and both landing location (e.g. Juventas lander, Subsection 1.3.2) and (rover) traverse (e.g. MMX mission, Subsection 1.3.1) have an important influence of the surface gravitation and the variation thereof.

Considering the PM solution as the truth for a homogeneous density, the method comparison with the two mascon methods was extended to the Martian moon. Here, the MSP was applied with 800k mascons (Figure 6.6 (a-c)) and the MASC with 64,800 mascons (Figure 6.6 (a-c)) inside Phobos for the comparison with the PM. While both methods use mascons, the patterns are very different. Regarding the MSP method, a similar effect than for the cube (Figure 6.4 (b)) appears. Assigning proportionally more mass to the smaller spheres leads to a relative error $\tau > 100\%$, whereas edges, e.g. the rim of the Stickney crater (Figure 6.5 (b)) lead to empty spaces inside Phobos, resulting in an undershoot ($\tau < 100\%$). Overall, the agreement of MSP and PM is better than $\pm 0.5\%$.

A completely different deviation pattern appears for the MASC. Globally, the agreement is very good and the majority of the surface evaluation points have an agreement of close to $\tau = 100\%$. However, individual local features, such as the Stickney crater exhibit large deviations, leading an agreement range of 96.9 % to 104.6 %. As has been shown for the cube, sharp edges such as crater rims are more difficult to represent by spherical harmonic mascons (tesseroids).

Overall, the here-presented method comparison has shown that for small bodies all three methods are generally well suited. While the best result for *homogeneous*

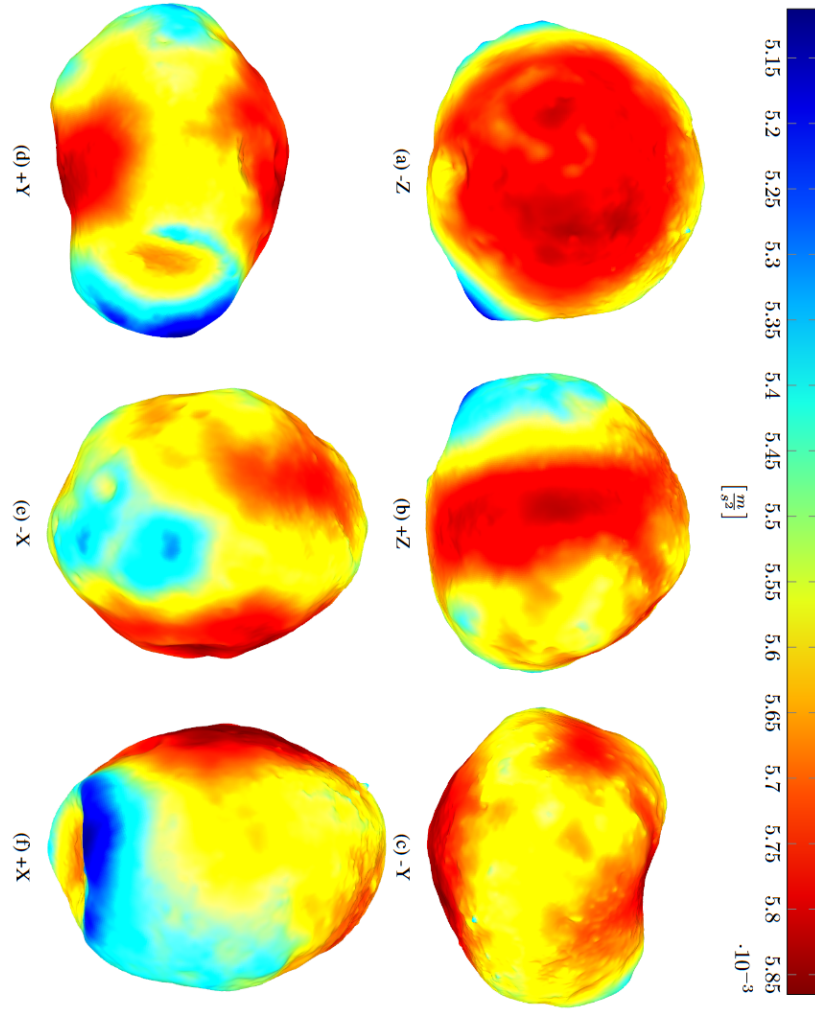


Figure 6.5: Surface gravity computed with the PM for Phobos: min. (blue) and max. (red). Axis indicates the viewing direction. Larger gravitation for $\pm Z$ and smaller for $\pm X$ viewing direction. Also increased gravitation at giant crater Stickney (visible in $+Y$ direction, on the right side). Figure and caption from Meißenhelter et al. (2022).

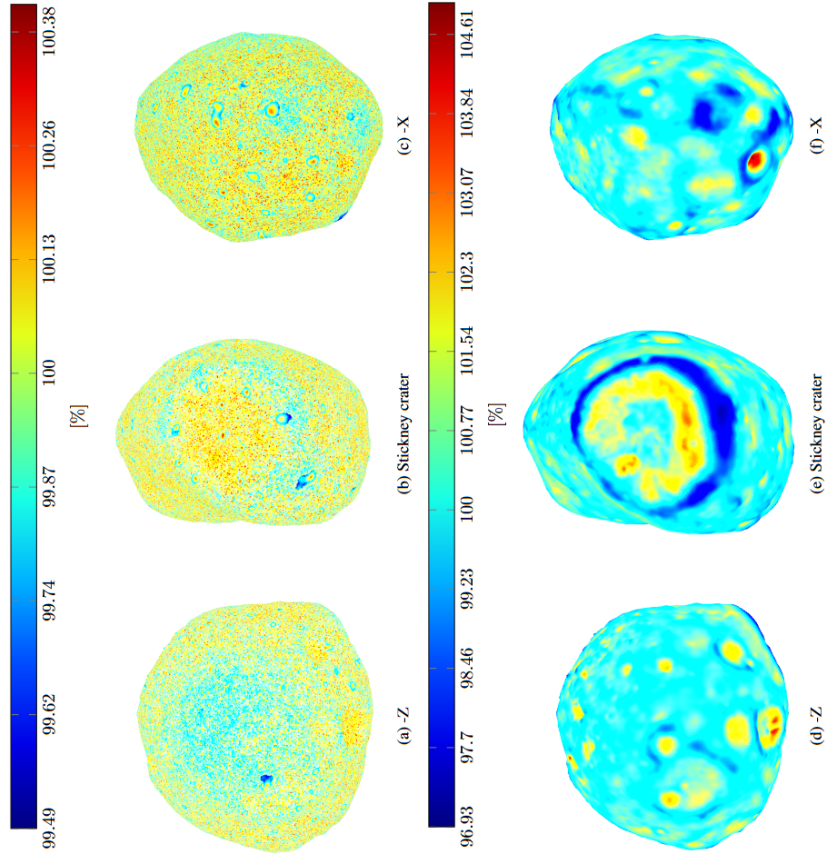


Figure 6.6: Surface gravity comparison of mascon methods to polygon method as percentage reference: min. (blue) and max. (red). Axis is showing view direction.

First row (a, b, c) is using MSP 800k and second row (d, e, f) MASC with 64.8k mascons. Figure and caption from Meißenhelter et al. (2022).

density distribution can be obtained from the PM, this likewise poses a limitation to study internal heterogeneous mass distributions, as will be studied in Section 6.5. A clear downside of the PM compared to both mascon methods is the computation time, being one and two orders of magnitude faster for MASC and MSP, respectively. This is an important aspect when considering on-line mission planning, or a large number of computations, e.g. for Monte Carlo or fitting analysis.

In order to close the loop from the here-studied *gravitation* to *gravity*, being the measurand of the surface gravimeter, the here-presented Phobos surface gravitation is complemented with the main other contributions to gravity, analytically studied in the next Section.

6.2 DYNAMICAL CONTRIBUTIONS TO GRAVITY

Generally, (astro-) dynamics are concerned with the motion of objects in space, and more specifically, their dynamics also concern the rotational state of the body, influencing the, variable, resulting gravity. While detailed studies of the gravitational dynamics, e.g. of the binary Didymos (Subsection 1.4.2) asteroid system (Agrusa et al., 2020), and the spin state, e.g. after the (artificial) DART (Subsection 1.3.2) impact, are beyond the goal and scope of this work, it is possible to approximate the influence on the resulting gravity analytically.

Using the theory introduced in Section 5.1, the dynamical gravity contributions from rotation (Subsection 6.2.1), libration (Subsection 6.2.2) and tides (Subsection 6.2.3) are studied as contributions to the surface gravity on Phobos that have to be added to the surface gravitation presented in Subsection 6.1.3.

Generally, these computations have been done before, partially in more complex setups, e.g. for Phobos¹. Still, the analytical toolkit developed and presented here for natural satellites (e.g. Martian moons) or binary asteroid system (e.g. Didymos) is valuable for the GRASS gravimeter and possible future missions. It allows to describe heads-on the global dynamical gravity environment, and most importantly, the temporal variations thereof. Consider a stationary surface measurement, the measured gravity will vary, which has to be distinguished from instrumental effects, such as (thermal) instrument drift. Moreover, to the best of our knowledge, we did not identify a (forced) libration study that focused on the librational effect on the surface gravity, while previous work considered the effect on the rotation rate (Bernauer et al., 2020).

6.2.1 ROTATIONAL ACCELERATION

Phobos is tidally locked to Mars, meaning that it completes one rotation around itself in the same time as it orbits Mars, that is 27,425.4 s (Wakker, 2015). From this, the angular velocity ω follows with the rotational (orbital) period T as

$$\omega = \frac{2\pi}{27425.4} = 2.291 \cdot 10^{-4} \frac{[rad]}{s} \quad (6.2)$$

The reference surface centrifugal acceleration $a_{cen,sur}$ follows considering the mean radius of Phobos being 10,993 m (Willner et al., 2014) as

$$a_{cen,sur} = \omega^2 r = \left(2.291 \cdot 10^{-4} \frac{[rad]}{s} \right)^2 \cdot 10.993 m = 5.770 \cdot 10^{-4} \frac{m}{s^2} \quad (6.3)$$

Considering the range of surface gravitation of Phobos (Figure 6.5), the centrifugal acceleration at the equator amounts to about 10 % of the gravitational attraction. It is obvious that the local surface centrifugal acceleration varies with the variable moon radius, and the latitude, i.e. the distance from the rotational axis. Therefore, it is possible to compute the centrifugal acceleration globally, computing the local

¹Cf. http://www.issibern.ch/teams/phobosdeimos/downloads/2011/ShiXian_Phobos_ISSI.pdf, slide 14, accessed on 18.01.2022.

Table 6.2: Rotational acceleration on Phobos minimum (min.), mean and maximum (max.) values as computed in this work (computed for every hundredth surface point). These values are compared to the values computed by Scheeres et al. (2019) at the poles, the leading/trailing edges and the sub/anti-Mars points. It has to be noted that these two sets of values are not identical, by definition. Still, the comparison show the agreement of the order of magnitude of both results.

This work		
Min.	Mean	Max.
0	$5.1 \cdot 10^{-4} m/s^2$	$7.2 \cdot 10^{-4} m/s^2$
Scheeres et al. (2019)		
Poles	Leading/trailing	Sub/anti-Mars
0	$6 \cdot 10^{-4} m/s^2$	$6 \cdot 10^{-4} m/s^2$

radius r_n at point n (Equation 6.4), while simplistically assuming the Z-axis to be identical to the rotation axis.

$$r_n = \sqrt{x_n^2 + y_n^2} \quad (6.4)$$

The magnitude of the centrifugal surface acceleration is shown in Figure 6.7, here using a 2D-projection with latitude and longitude. The min., mean and max. values are listed in Table 6.2 and the order of magnitude is compared to values provided by Scheeres et al. (2019) for the poles, the leading/trailing edges and the sub/anti-Mars points. Those two sets are not identical, but the comparison shows the agreement between the computations.

As can be expected, the maximum accelerations appear towards the equator and reach zero at the (spin) poles. The maximum value plotted, computed for every hundredth² surface point is $7.204 \cdot 10^{-4} \frac{m}{s^2}$, thus larger than the above computed reference centrifugal acceleration. This analytical approach is here only concerned with the mean rotation of the moon being tidally locked in areocentric orbit. Variations of this mean rotational acceleration are caused by the libration, whose influence is approximated analytically in the next Subsection.

6.2.2 LIBRATION INFLUENCE

The analytical description of the forced physical libration, giving the libration as a deviation from the uniform rotation, was introduced in Subsection 5.1.3. For Phobos, we compute firstly the forced libration angular acceleration $\ddot{\gamma}$ (Equation (5.13)), using the eccentricity $e=0.01511$ (Jacobson, 2010) and the normalised moments of inertia $A = 0.3615$, $B = 0.4265$, and $C = 0.5024$ (Willner et al., 2010). With this, it is again possible to infer the influence on the surface acceleration globally, by computing the radius distance as for the centrifugal acceleration (Equation (6.4)), likewise approximating the Z-axis as rotation axis of the forced libration. The libration angle is plotted in Figure 6.8 and the amplitude of this angle is 1.085 degrees,

²For a better readability of the triangular plot and to limit the computation time of the dynamical acceleration computations, the number of global evaluation points has been reduced.

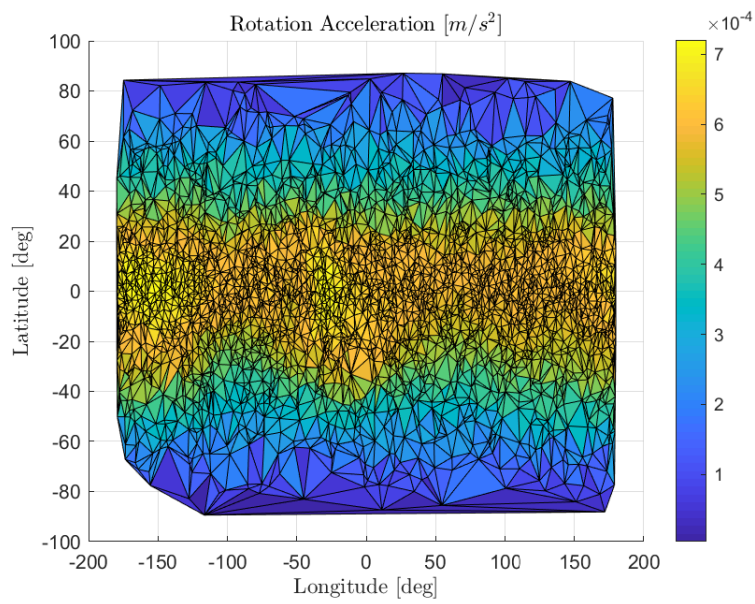


Figure 6.7: Rotational acceleration on the surface of Phobos, projected onto a 2D-map with longitude and latitude. By reducing the surface evaluation points to every hundredths point, the visibility of the tri-plot is maintained while limiting the computation time.

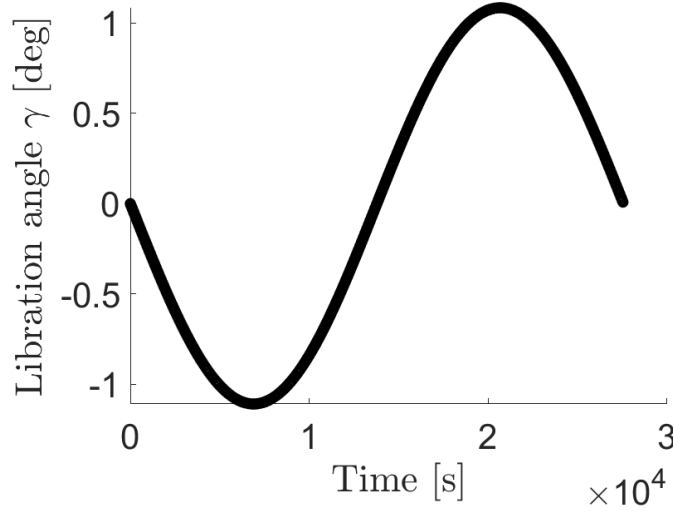


Figure 6.8: Forced libration angle over one Phobos orbit around Mars.

which is in accordance with Willner et al. (2010) reporting the amplitude as 1.1 degrees and with the libration angle plotted in Rambaux et al. (2012).

With the libration angle computation, it is possible to obtain the instantaneous angular velocity by differentiating Equation (5.18) with respect to time, resulting in Equation (6.5). It can be shown that differentiating Equation (6.5), leading to the angular libration acceleration, is equal to the expression in Equation (5.13).

$$\dot{\gamma} = \frac{n \cdot 2e \cdot \cos nt}{1 - \frac{1}{3\sigma}} \quad (6.5)$$

In the same fashion as for the rotation, the surface acceleration due to the librational angular motion can be approximated by assuming the libration rotation axis being equal to the Z-axis of Phobos (Equation (6.6)).

$$a_{libration,n} = \dot{\gamma}^2 \cdot r_n \quad (6.6)$$

The angular speed is maximum at the mean anomaly $M = 0^\circ$ and $M = 180^\circ$ of the orbit, with opposite signs. To illustrate the maximum surface acceleration caused by the libration, Figure 6.9 snapshots the maximum effect for the Phobian surface, while it is also possible to visualise the acceleration over one orbit, e.g. using GIF-animations. For this maximum overall state, again the numerical values are provided in Table 6.3 for the entire surface.

Overall, the maximum surface acceleration due to libration is again largest in the equatorial region, and zero at the poles as for the rotational acceleration. Here, the largest value (only appearing twice per orbit) is about $2.5 \cdot 10^{-7} m/s^2$ and thus four

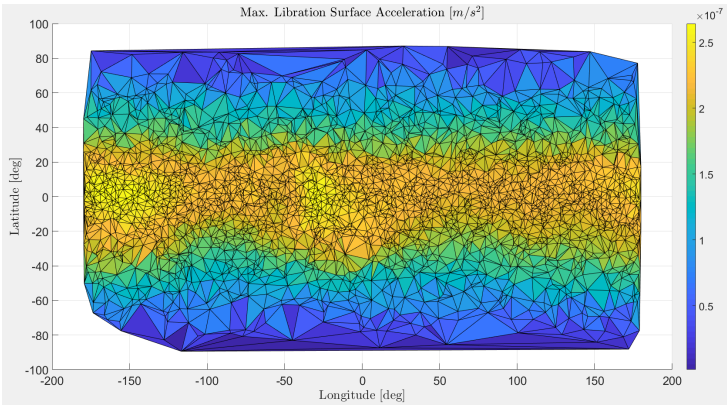


Figure 6.9: Maximum forced libration surface acceleration for every hundredth point on Phobos.

Table 6.3: Librational acceleration on Phobos minimum (min.), mean and maximum (max.) values as computed in this work (computed for every hundredth surface point). This is done for the case where the maximum librations are occuring at the mean anomaly $M = 0^\circ$ and $M = 180^\circ$. No work was identified to which the here presented values could be compared.

Min.	Mean	Max.
0	$1.8 \cdot 10^{-7} m/s^2$	$2.6 \cdot 10^{-7} m/s^2$

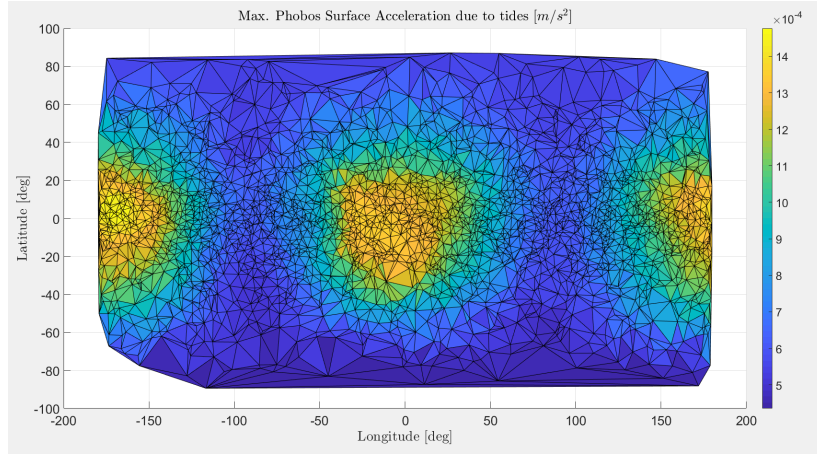


Figure 6.10: Maximum tidal acceleration on Phobos during one orbit around Mars.

orders of magnitude smaller than the surface gravitation. Therefore, this contribution to gravity is indeed small, and potentially not detectable by GRASS when recalling the target accuracy to be better than 1 % of the surface gravity.

6.2.3 TIDES

The third dynamic influence assessed here as contribution to surface gravity are the Mars-induced tides acting on the surface of Phobos. The concept was introduced in Subsection 5.1.4, along with the orbit propagation of Phobos around Mars. This was done under the assumption that the secondary body is spherical, which is not the case for Phobos and therefore provides a first approximation and a limitation to the here-presented analysis. The position of Phobos on its orbit determines the magnitude of the tidal acceleration due to the eccentricity. As can be expected from Equation (5.20), the tidal influence is the largest at periapsis and the smallest at apoapsis. Inversely, the gravity that includes the tidal influence is therefore the largest at apoapsis and the smallest at periapsis, as the tidal acceleration points away from the gravitational attraction of Phobos.

Again, like for the libration, the analysis is extended over one Phobian orbit and over the complete surface. From this, the maximum tidal acceleration appearing at periareion is plotted for the full Phobian surface in Figure 6.10. The min., mean and max. values are listed in Table 6.4 and again compared to Scheeres et al. (2019). As can be clearly observed in the sub-Mars and anti-Mars points, the surface acceleration due to tides is reduced by more than about 20 % of the local surface acceleration!

The tidal acceleration, is not a “static” contribution the surface gravity, such as the rotational acceleration. Therefore, it is interesting to not only consider the *maximum* contribution, but also the variation in tidal acceleration over one orbit. To illustrate the variation, the difference from the minimum to maximum tidal ac-

Table 6.4: Max. tidal acceleration on Phobos minimum (min.), mean and maximum (max.) values as computed in this work (computed for every hundredth surface point). These values are compared to the values computed by Scheeres et al. (2019) at the poles, the leading/trailing edges and the sub/anti-Mars points. Here, it is not specified if these values also correspond to the maximum tidal case. It has to be noted that these two sets of values are not identical, by definition. Still, the comparison show the agreement of the order of magnitude of both results.

This work		
Min.	Mean	Max.
$4.4 \cdot 10^{-4} m/s^2$	$8.9 \cdot 10^{-4} m/s^2$	$1.5 \cdot 10^{-3} m/s^2$
Scheeres et al. (2019)		
Poles	Leading/trailing	Sub/anti-Mars
$4 \cdot 10^{-4} m/s^2$	$6 \cdot 10^{-4} m/s^2$	$1.3 \cdot 10^{-3} m/s^2$

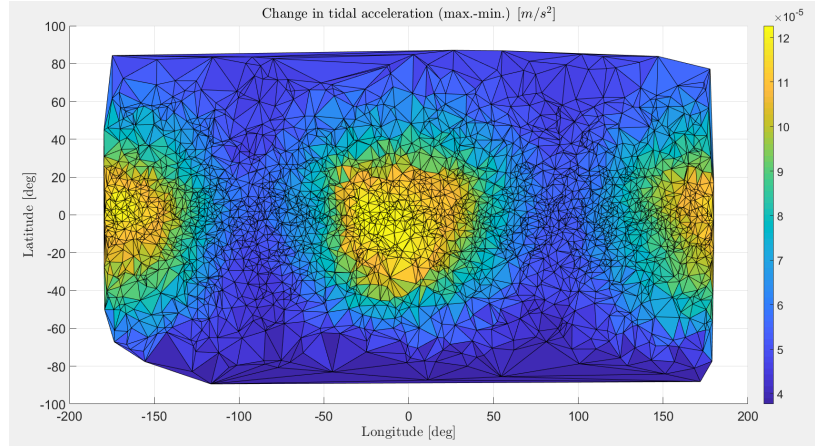


Figure 6.11: Range (max. - min.) Tidal acceleration on Phobos during one orbit around Mars to show the amplitude of the variation of the tides.

celeration is plotted in Figure 6.11. Let us consider the maximum variation of $1.226 \cdot 10^{-4} m/s^2$. Relative to the reference surface gravity, set to $5.7 \cdot 10^{-3} m/s^2$, the *variation* in surface gravity due to tides amounts to 2.15 %, and is henceforth certainly measurable by GRASS on Phobos' surface. This also shows that it is not possible to perform a “static” measurement over longer periods without variation due to the change in tides. As visible, this variation is largest in the sub- and anti-Mars points, but does not disappear nowhere.

6.3 APPLICATION TO THE HERA MISSION

In the context of the current Hera mission, for which the GRASS gravimeter is designed, but also in view of a possible future mission to close-approaching Apophis, it is worthwhile to simulate the gravity environment for both bodies in this and the

next Section, respectively. Clearly, more precise simulations will be possible in the future with better shape files, e.g. following the DART impact (Subsection 1.3.2), improving the gravitation computations (Subsection 6.3.1). Likewise, as discussed in Section 5.1, the dynamical gravity contributions (6.3.2) within the goal and scope of this work are simplified and described analytically. An overview of the computed accelerations for Dimorphos is presented in Subsection 6.3.3. At this mission phase, the here-presented analysis is valuable, not only to size the GRASS instrument (Part I), but likewise to plan the instruments the CONcept of OPERATIONs (CONOPS). Some recommendations for this are provided in Subsection 6.3.4.

6.3.1 DIMORPHOS: GLOBAL GRAVITATION

For the *mechanical* GRASS gravimeter development and spring sizing (Chapter 3), it was sufficient to initially consider the reference gravity value for Dimorphos of $g_{ref} = 4.5 \cdot 10^{-5} m/s^2$. Considering the ongoing mission preparation, including the fine-tuning of the displacement transducer electronics and the instrument characterisation, it is helpful to assess the *range* of surface gravity to be expected. In this Subsection, the surface gravitation will be computed for two shape files discussed in the following.

At the time of finalising the present manuscript, the DART spacecraft (Subsection 1.3.2) did not yet reach the binary asteroid system Didymos. While this mission will return images, improving our knowledge of its target shape (at least on the side of impact), the current shape files are solely based on ground-based observations. Therefore, future shape files might update the here-presented results, yet, the current shape files allow a first assessment of the gravity environment for GRASS. The first shape file is the reference triaxial ellipsoid³ used by ESA for mission planning, with the three semi-axes $a = 104m$, $b = 80m$, and $c = 66.5m$. Using the assumed density of $\rho = 2,170 kg/m^3$ (Table 1.5), the surface gravitation can be computed globally as plotted in Figure 6.12. This simple shape consists of only 3072 facets, resulting in a fast computation time and allowing for a good global representation using a tri-plot.

It is clearly visible that the chosen reference gravity is just below the minimum simulated surface gravitation. This is, however, no problem for instrument sizing, as larger gravity measurements are easier to account for than even smaller spring deflections (Section 4.1). An alternative shape file has been distributed as part of shapeViewer⁴, where the detailed shape file of Itokawa has been scaled to fit the current knowledge of Dimorphos. Overall, the extend of this shape is a bit smaller, namely $-76.15m \leq X \leq +64.27m$, $-40.31m \leq Y \leq +36.09m$, $-32.56m \leq Z \leq +28.22m$, but the detail with 100,000 facets is much larger than the reference ellipsoid. Using the identical density, the resulting surface gravitation is plotted in Figure 6.13. Due to the very large number of facets, a tri-plot representation is not feasible, so the 3D-surface is plotted in 2D using the latitude and longitude of the surface points.

³Filename `g_08438mm_lgt_obj_didb_0000n00000_v002.obj`

⁴This research has made use of the scientific software shapeViewer (www.comet-toolbox.com). Vincent et al. shapeViewer, a software for the scientific mapping and morphological analysis of small bodies, LPSC, (2018).

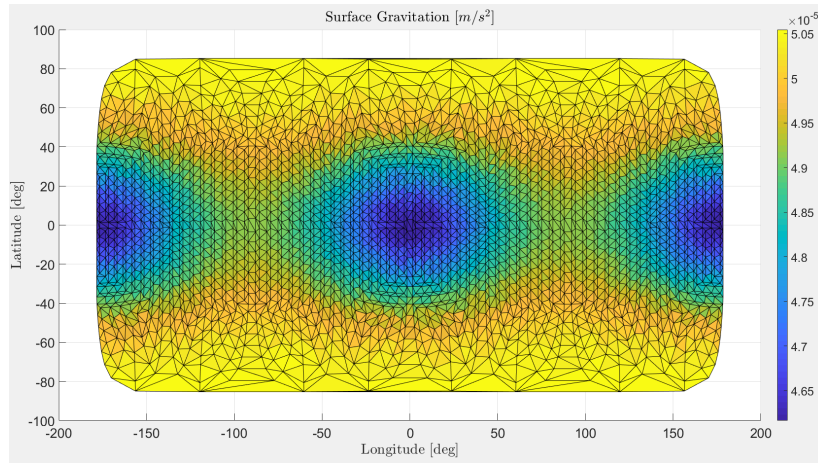


Figure 6.12: Surface gravitation computed with PM on the surface of the ellipsoid shape model initially assumed for Dimorphos.

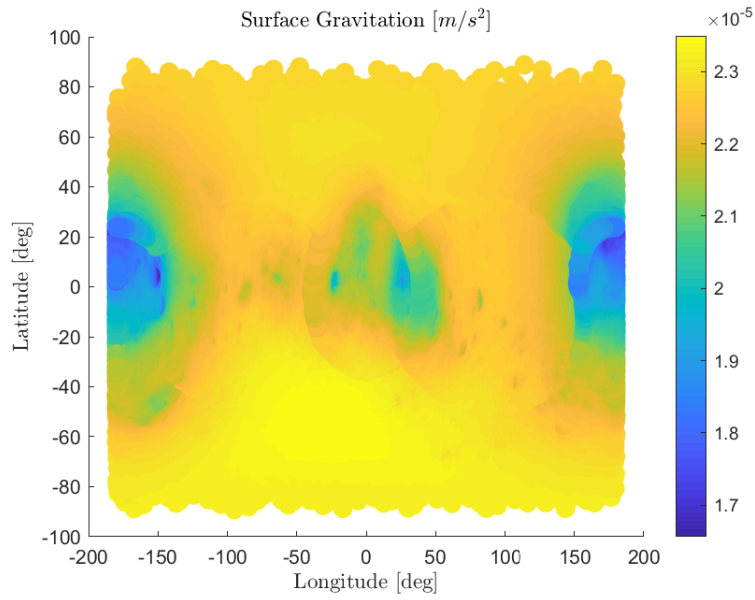


Figure 6.13: Surface gravitation computed with PM on the surface of the Itokawa shape model scaled to fit the Dimorphos size.

Overall, the surface gravitation maximum here is about half the minimum surface gravitation of the ellipsoid (Figure 6.12). Of course, this can be expected, provided the smaller overall size of the asteroid. At the same time, this alternative scenario is a good example illustrating that significant deviations from the here-assumed reference gravitation might occur. Clearly, this deviation can be bi-directional, leading to a smaller or larger surface gravity. Not only the final shape and its composition (density), but clearly the final landing (measurement) location will influence this. The ongoing fine-tuning of the gravimeter sensing electronics should take this option into account, potentially accounting for in-situ measurement range adaptation.

6.3.2 DIMORPHOS: DYNAMICAL GRAVITY CONTRIBUTIONS

In the previous Section, the dynamical influences were directly approximated for Phobos. The application of the same computations can likewise be done for the Didymos binary system in preparation of the GRASS deployment on Dimorphos. Clearly, the analytical approximations will have larger errors in the binary asteroid systems with more complex dynamics. Taking the ratio of the centre distances between the primary and secondary, to the secondary radius, the Didymos system has a value of $1.19\text{km}/0.082\text{km} = 14.5$, compared to $9376\text{km}/11.2667\text{km} = 832.2$ for Phobos and $384,400\text{km}/1738.1\text{km} = 221.1$ for the Moon. Due to the much smaller orbital separation, and the assumed non-spherical shape of the secondary, the first order approximation used for the tidal computation (Subsection 5.1.4) has to be treated with care. The same is true for the librations. For both cases, the actual mass distribution within the binary system must be considered. The rotational acceleration is less affected. Regardless, in the context of instrument development, the here-presented approximations provide a first understanding of surface gravity variations to be expected.

Figure 6.15 provides a summarising overview of the approximated dynamical contributions to the surface gravity of the ellipsoidal shape file. For consistency, these results are compared to the surface gravitation of the identical shape (Figure 6.12). For the rotational acceleration (Figure 6.15 (a)), the tidally locked rotation causes centrifugal accelerations, with the largest values being at the equator at longitudes 0 and 180 degrees, under the assumption that the long axis points towards the primary asteroid. When relating this to the surface gravitation, it becomes clear that the maximum rotational acceleration appears where the minimum surface gravitation occurs, i.e. at latitude 0, and longitudes of 0 and 180 degrees. At these locations, the rotation amounts to about $2.2 \cdot 10^{-6} \text{m/s}^2 / (4.65 \cdot 10^{-5} \text{m/s}^2) = 4.7\%$ of the local gravitation, a value clearly non-negligible in the instrument design, which must be considered in the measurement data analysis.

Regarding the tidal approximation (Figure 6.15 (b)–(c)), the maximum acceleration appears at the periapsis at the sub-Didymos point. For the computation shown here, the upper limit of the orbit eccentricity of $e = 0.05$ (Table 1.5) is considered for a maximum variation in the tidal acceleration. While this analysis is an approximation as stated above, it is clearly visible that the two tidal acceleration peaks are not identical. The acceleration on the point facing the primary is always larger than the acceleration on the primary-far side. When commonly referring to tidal

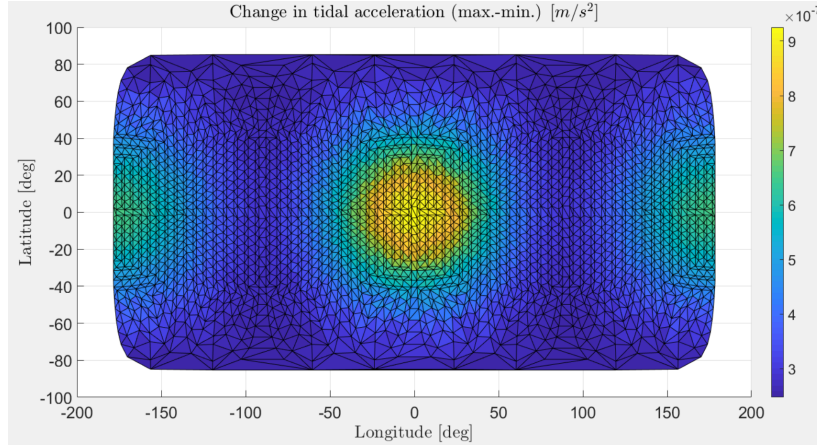


Figure 6.14: Surface tidal acceleration range, represented as difference of maximum minus minimum acceleration on Dimorphos (ellipsoid).

bulges, it is generally accepted that both tidal bulges are equally large. However, this is only true to first order and it can be shown that, especially for the small orbital separation and while maintaining the analytical approximation (Subsection 5.1.4), the inner tidal bulge is larger than the outer. In accordance with that, the *range* between the minimum (apoapsis) and maximum (periapsis) tidal acceleration is likewise the largest at longitude of 0 degrees (Figure 6.14).

Considering the maximum acceleration due to tides, this amounts to about $5.3 \cdot 10^{-6} \text{ m/s}^2 / (4.65 \cdot 10^{-5} \text{ m/s}^2) = 11.4 \%$. Again this contribution to the surface gravity is significant. Moreover, the *variation* in tidal acceleration over one half-orbit has to be considered. In the maximum case (periapsis) it is about $5.3 \cdot 10^{-6} \text{ m/s}^2$, whereas, in the same location (i.e. with a shift of 180 degrees in longitude), at the minimum case (apoapsis), the tidal acceleration is only $4.4 \cdot 10^{-6} \text{ m/s}^2$. This difference ($9 \cdot 10^{-7} \text{ m/s}^2$) amounts to about 2 % of the local gravitation. Therefore, significant variations in the order of a few percent of the surface gravity signal will appear over the course of the asteroid moon orbit, i.e. in the course of less than six hours(!).

Lastly, Figure 6.15 (d) shows the *maximum* acceleration induced by libration, which are again the largest at the equator, at the largest distance from the (assumed) rotation axis. Here, the magnitude is of about $8.4 \cdot 10^{-8} \text{ m/s}^2 / 4.65 \cdot 10^{-5} \text{ m/s}^2 = 0.18 \%$. Again, it is recalled that the presented number follows from a simplified analytical description of the libration. Nevertheless, this variable contribution might become the litmus test for the GRASS instrument. The here-presented simulation shows that the instrument target accuracy of (initially) 1 % does not demand a precise measurement of the librational surface acceleration and the variation thereof. Yet, the order of magnitude of the maximum acceleration contribution is clearly within reach of the instrument, and careful instrument calibration and post mission data analysis might allow to sense the periodical increase of the libration acceleration in the surface gravity data.

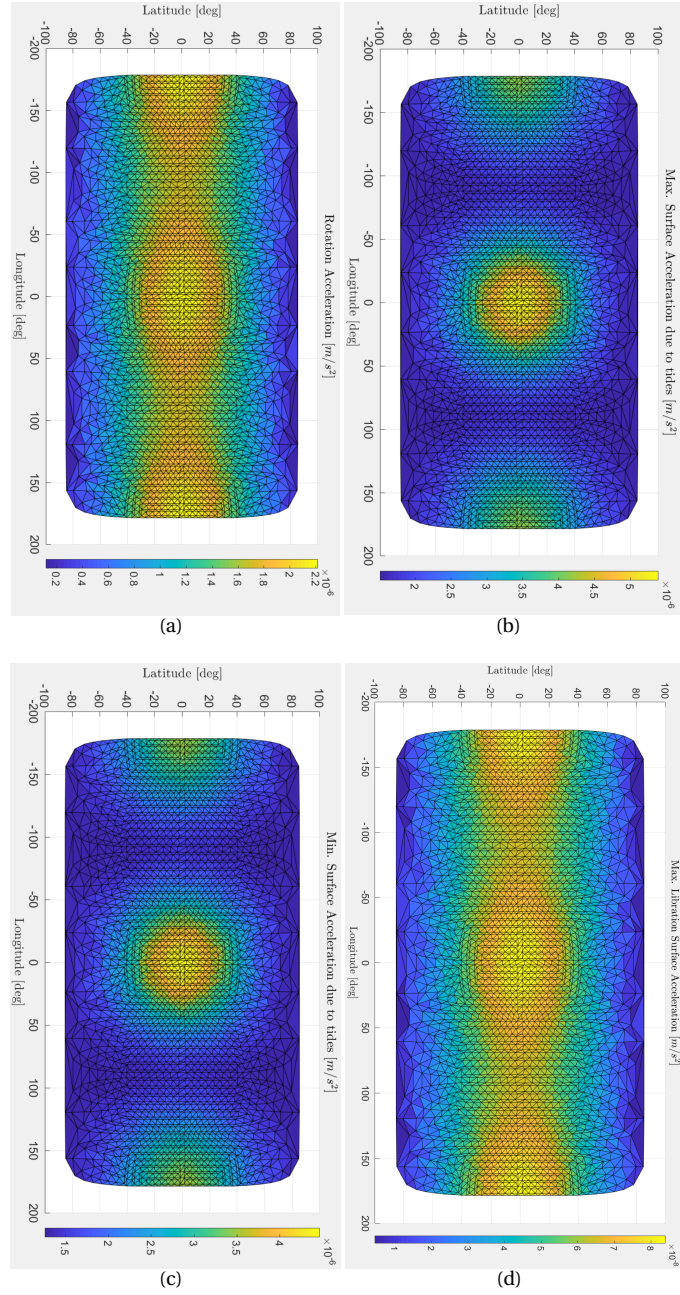


Figure 6.15: Approximation of dynamical contributions to the surface gravity of Dimorphos applied to the ellipsoidal shape file. (a) Rotational acceleration. (b)–(c) Maximum and minimum surface acceleration due to tides at periapsis and apoapsis, respectively. (d) Maximum surface acceleration due to libration.

6.3.3 DIMORPHOS ACCELERATION OVERVIEW

The resulting accelerations, as computed in the previous two Subsections, are summarised in Table 6.5. Here, the values computed for the ESA reference shape being the ellipsoid are being presented. The presented values can be compared with the plots in Figure 6.12 (gravitation) and in Figure 6.15 (dynamics).

Table 6.5: Overview of min., mean, and max. accelerations acting on the surface of Dimorphos, considering the ESA reference shape (ellipsoid). For the tides, the maximum and minimum case are listed.

Acceleration	Min.	Mean	Max.
Gravitation	$4.62 \cdot 10^{-5} m/s^2$	$4.88 \cdot 10^{-5} m/s^2$	$5.05 \cdot 10^{-5} m/s^2$
Rotation	0	$1.48 \cdot 10^{-6} m/s^2$	$2.22 \cdot 10^{-6} m/s^2$
Tides (min.)	$1.26 \cdot 10^{-6} m/s^2$	$2.29 \cdot 10^{-6} m/s^2$	$4.46 \cdot 10^{-6} m/s^2$
Tides (max.)	$1.51 \cdot 10^{-6} m/s^2$	$2.74 \cdot 10^{-6} m/s^2$	$5.39 \cdot 10^{-6} m/s^2$
Libration (max.)	0	$5.58 \cdot 10^{-8} m/s^2$	$8.37 \cdot 10^{-8} m/s^2$

6.3.4 IMPLICATIONS ON GRASS CONOPS

With the simulation of the variable dynamical effects on the resulting surface gravity, first recommendations can be made for the GRASS concept of operations (CONOPS). This has practical implications for the Hera mission, including the ideal time of landing and the operations of the landed surface gravimeter. Clearly, to measure the largest variations of the surface gravity, it would be ideal to land in the equatorial region, furthest away from the rotational axis of the secondary. Considering that the assumption of the tidally locked rotation is not disproved upon arrival, landing on the sub- or anti-Dimorphos points would best allow to measure tidal variations in the surface acceleration. This is because in the tidally locked rotation, the long axis of the ellipsoidal shape will align towards the primary, Didymos. Today, it is not clear, if and to what extent the landing region will be controllable, but if a landing site selection will be possible, this should be taken into account from the GRASS science perspective.

The minimum surface operation time of GRASS is sized for two orbits, if solely depending on the battery power of the Juventas CubeSat. Thus, it has to be assumed that the measurement time on the surface is limited, because of the limited available power, here assuming the worst case that the solar panels will not provide any power after the uncontrolled landing. A first allocation plans with twelve measurements per orbit, and thus a total of 24 measurements on the Dimorphian surface.

A simplistic approach would be to equally space the measurements temporally. Alternatively, and considering the dynamic contributions simulated in the previous Subsection, the following approach is proposed:

After the arrival of the Hera spacecraft in the Didymos binary system, observations will be available to infer the time of the periapsis and the apoapsis, paired with

more detailed observation of the secondary orbital period T and eccentricity e . Using this information, it is then suggested to plan the first five of the total twelve measurements in orbit such that they can be taken at and around periapsis. For this, the landing time could be adapted accordingly. For example, the measurements would take place at times $t_1 = t_{\text{periapsis}} - 1.5h$, $t_2 = t_{\text{periapsis}} - 0.5h$, $t_3 = t_{\text{periapsis}}$, $t_4 = t_{\text{periapsis}} + 0.5h$, $t_5 = t_{\text{periapsis}} + 1.5h$. As was shown, the smallest net gravity can be measured at periapsis, where the largest librational and tidal accelerations will be reducing the surface gravitation. By taking these five measurements, it will be possible to interpolate a curve for the smallest acceleration, ideally allowing to isolate the tidal and librational signals. Moreover, the execution of five measurements early during the surface operations aids in securing a science return from a reliability perspective. Once operational on the surface, both the (landed) spacecraft solely relying on battery power, i.e. in another power operational state, and the gravimeter can “secure” early one five measurements (more than 20 % of the target) within the first three hours of operation. Considering the possibility of “early infant mortality failure” with regards to reliability engineering, this strategy helps to ensure maximising the science return, even in case of operational anomalies. Obviously, the timing of the landing described here is solely advocating for GRASS measurements, and additional consideration, such as the illumination conditions on the surface, will play a role in an overall trade-off.

The same could be performed around the apoapsis, with additional five measurements, allowing to observe the largest surface gravitation with the smallest tidal acceleration, which reduces the net gravity. The remaining two measurements can then be placed in-between apoapsis and periapsis to ensure covering both arcs of the elliptical orbit. Repeating this measurement pattern during the second and potentially subsequent orbits will allow stacking of the measurements. Of course, the inverse approach, starting with measurements around apoapsis instead of periapsis, can likewise be considered.

On a final note, starting the orbit with five measurements aids to reach a potentially larger number of measurements compared to equally spaced measurements, considered that for example after “two orbits and a bit” the battery will die. Lastly, it should be noted that the gravity intrinsically cannot be considered perfectly constant during one measurement, as tides and libration will vary, even in the course of (initially considered) ten-minute-long GRASS measurements. This has to be taken into account in the instrument measurement design and data interpretation.

6.4 APPLICATION TO FUTURE APOPHIS MISSION

Complementary to the initially Phobian exploration MMX mission with the surface rover and the ongoing Hera mission, it is worthwhile to consider one exemplary future missions that could include a surface gravimeter element. The potential hazardous NEA (99942) Apophis was introduced in Subsection 1.4.3, motivated by its very close flyby with the Earth in 2029 at about 6 Earth radii (Hirabayashi et al., 2021). Due to the proximity to any Terrestrial launch pad, a rendezvous mis-

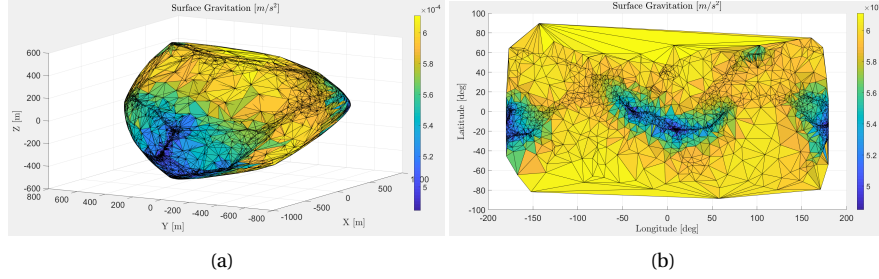


Figure 6.16: Surface gravity modelled for Apophis (a) plotted on the 3D-shape file and (b) projected in 2D using spherical coordinates.

sion is of great avail, and currently being discussed in the scientific community. The imperative to carefully consider negative (catastrophic) consequences of artificially interacting with the motion of a PHO was critically reflected in Subsection 1.3.3.

For the aim of a possible Apophis mission, and in a first step to enable proposing the GRASS gravimeter (Part I) as a payload for a potential Apophis lander, the surface gravity environment is simulated in this Section. Here, the contributions are limited to the surface gravitation (Subsection 6.4.1) and the rotation (Subsection 6.4.2). Additional work is performed by e.g. Noiset et al. (2022), considering Earth-induced tidal accelerations during the close Earth-flyby.

6.4.1 APOPHIS: GLOBAL GRAVITATION

For modelling the surface gravitation of Apophis, the derived shape file⁵ is used (Pravec et al., 2014) with the density of $3,190 \text{ kg/m}^3$ as listed in Table 1.6. The surface gravitation plotted on the shape file in 3D and projected on a 2D-map is shown in Figure 6.16.

As the variation with reduced gravitation only occurs locally in the lobes, the reference gravity for any future GRASS gravimeter adaptations shall be set to

$$g_{ref,Apophis} = 6 \cdot 10^{-4} \text{ m/s}^2$$

This is interesting insofar, as this reference value is one magnitude *smaller* than Phobos and one order of magnitude *larger* than Dimorphos. Clearly, the shape file and density are only derived information based on observations, and this reference value should be considered accordingly.

6.4.2 APOPHIS: ROTATION

Table 1.6 further lists the (assumed) rotation period of 30.4 hours. With this, the surface acceleration due to the rotation is computed as in the previous Subsec-

⁵From <https://3d-asteroids.space/asteroids/99942-Apophis>, 3D-shape viewer available online, accessed on 20.01.2020.

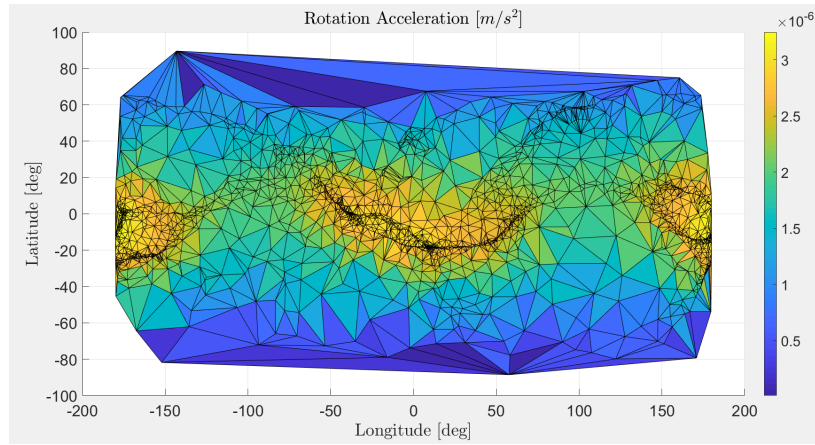


Figure 6.17: Surface acceleration on Apophis due to rotation.

tions. The rotational acceleration is shown in Figure 6.17. Compared to the gravitation of Apophis (Figure 6.16), the colouring appears to be inverted, with the largest rotational acceleration in the region of lowest gravitation. At these lobes, the rotational acceleration is simulated to amount to somewhat more than 0.5 % of the surface gravitation. Therefore, the contribution is significant, and should not be neglected in any future work regarding a possible and valuable inclusion of the GRASS gravimeter in any Apophis mission including a landing element.

6.5 SENSITIVITY TO INHOMOGENEITIES

In this Chapter, thus far, only the homogeneous case of the different bodies (sphere (Subsection 6.1.1), cube (Subsection 6.1.2), Phobos (Subsection 6.1.3), Dimorphos (Subsection 6.3.1), and Apophis (Subsection 6.4.1)), have been studied. This was done in this work with the polyhedral method (PM) and, in collaboration with partners in comparison with their mascon methods. For completion of this Chapter, and to lay out the path for ongoing and future work, two approaches in introducing inhomogeneities in small bodies are presented in this Section, allowing to circumvent the homogeneous density limitation. First, a spherical inhomogeneity (“core”) is added to a UV-sphere and the PM result is compared to the mascon methods (Subsection 6.5.1). Second, a regolith layer is added to Phobos, while keeping the total body gravitational parameter $\mu = GM$ constant (Subsection 6.5.2).

6.5.1 INHOMOGENEOUS SPHERES

This Subsection is an adapted excerpt from Noeker et al. (2022a).

In continuation of the comparison work regarding a homogeneous density distribution (Section 6.1.3), it is possible to extend the results by studying the effect of inhomogeneities on the different gravitation computation method results. For this, two first test cases have been computed and the results are summarised in this Subsection. Again, the mascon computations were performed by the partners from

the University of Bremen and the Bundeswehr University Munich. We started with the simplest case of an inhomogeneity, being a spherical region of different density.

This could be implemented by a simple point mass, as has been discussed for the mascon methods (Chapter 5). However, for a larger degree of adaptability in future work, both the exterior shape as well as the inside inhomogeneity are described by shape files, here using the UV:328,328 sphere with very high resolution, approximating a perfect sphere with a polyhedral shape. Like this, any irregularity can be modelled with an appropriate shape file of the corresponding (assumed) inhomogeneity. Clearly, there is theoretically no limit of the number of such additions, but here only a single inhomogeneity is tested. Finally, the superposition of the different density regions is performed by first computing the “main body” with its homogeneous density ρ_0 and then computing the inhomogeneity with the differential density $\Delta\rho$, i.e. the difference between the two density fields inside the body $\Delta\rho$; simple addition of the two gravitational fields returns the resulting gravitation. Obviously, a negative differential density will yield to a negative contribution to the overall gravitation. Like this, the computation time doubles for the PM, which evaluates the two different shapes.

Geophysically, this could be interpreted by e.g. a denser core (positive differential density $\Delta\rho > 0$), a void with zero density ($\Delta\rho = -\rho_0$) or a region of high porosity (negative differential density $\Delta\rho < 0$).

In both experiments, a main UV-sphere with radius $r = 1,000$ m and a density of $\rho_0 = 1,000 \text{ kg/m}^3$ is initialised at origin. Inside, a mass deficit in form of a sphere of radius $r = 100$ m and $\rho_{inhom} = 500 \text{ kg/m}^3$ is placed (thus, $\Delta\rho = \rho_{inhom} - \rho_0 = -500 \text{ kg/m}^3$). The difference between the two test cases lies in the position of the introduced inhomogeneity. While in Case I the inner sphere is placed concentric with the outer sphere, the centre of the inhomogeneity is translated to $X = -100 \text{ m}$ and $Y = -100 \text{ m}$ for Case II.

With the UV-sphere approximating the mathematical sphere, a comparison similar to the homogeneous case (Subsection 6.1.1) of the different methods, with the perfect sphere, describable by a point mass (Equation (5.2)), can be formulated. As for the previous gravitation modelling method comparison, the ratio τ (Equation (6.1)), giving the agreement of modelled to analytical solution, is reported in Table 6.6. Here, it is interesting to observe for both cases that, while the mascon sphere packing (MSP) has a mean value of τ closer to 100 % than the polyhedral method (PM), the standard deviation over the surface of the (perfect) sphere is much larger, coming from the inhomogeneous sphere packing. The mascon spherical coordinates (MASC) method is nearly as good as the PM regarding the mean value, and the spreading represented by the standard deviation is about 27.5 % larger. Most interesting regarding the comparison between the two cases with different positions of the inhomogeneity, however, is the observation that the PM method is not influenced on the position of the added inconsistency. The overall undershoot again depends solely on the two UV-spheres being only an approximation of the perfect sphere. Conversely, both mascon methods return locations with under- or overestimations of the surface gravitation, where the MASC has a much smaller, nearly

Table 6.6: Comparison of all methods with the analytic solution. Case (I) is the core at centre, and case (II) is the translated core. The number in the name indicates the number of mascons. Gravitational constant used: $6.674 \times 10^{-11} m^3 kg^{-1} s^{-2}$. Table and caption from Noeker et al. (2022a).

Case	Method	Min. [%]	Mean [%]	Max. [%]	σ [%]
I	PM	99.993143	99.995461	99.787864	0.000942
	MSP 800k	99.542132	99.996047	100.345711	0.117449
	MASC 64.8k	99.994138	99.995459	100.004137	0.001201
II	PM	99.993143	99.995461	99.997864	0.000942
	MSP 800k	99.590773	99.995998	100.307859	0.111008
	MASC 64.8k	99.994137	99.995459	100.004158	0.001201

negligible, variation than the MSP between the two tested cases.

6.5.2 REGOLITH LAYER

The second experiment regarding the effect of inhomogeneity presented in this Section considers a surface regolith layer with reduced density, and was carried out solely by the author. While it is foreseen to extend the gravitation comparison methods, here, this Subsection only reports on the effect computed with the PM. The here-tested shape is again the Martian moon Phobos, allowing to better assess the layer influence as compared to the homogeneous density case of Phobos (Figure 6.5).

For a sound comparison, the total mass M of the body, and therefore the gravitational parameter $\mu = GM$ is kept constant. For the regolith layer a rather simple approach can be followed by assuming the “core” having the exact same, but smaller shape than the body. This approach is comparable to the one shown in Figure 5.11 by Takahashi and Scheeres (2014a). For this, it is simply possible to scale all the vertices coordinates V as given in Equation (6.7).

$$V_{core} = \frac{r_{mean} - d_{layer}}{r_{mean}} V \quad (6.7)$$

for a(n) (assumed) thickness of 10 meters, the downscaling ratio is 0.9991 ratio, using the mean radius $r_{mean} = 10,993m$ (Willner et al., 2014).

For the density, the overall density of the homogeneous case of $\rho = 1,860 kg/m^3$ is used for the overall body. For the outer regolith layer, the density is assumed to be $\rho_{regolith} = 850 kg/m^3$, based on the estimate of the surface density at the Chinese Chang'E-3 (CE-3) mission landing site on the Moon (Fa, 2020). Here, it is noted that this density is reported to increase to a steady state at $2,250 kg/m^3$ at 5 m depth, but to best illustrate a possible density contrast due to less dense regolith on Phobos, the lower end of the range is assumed.

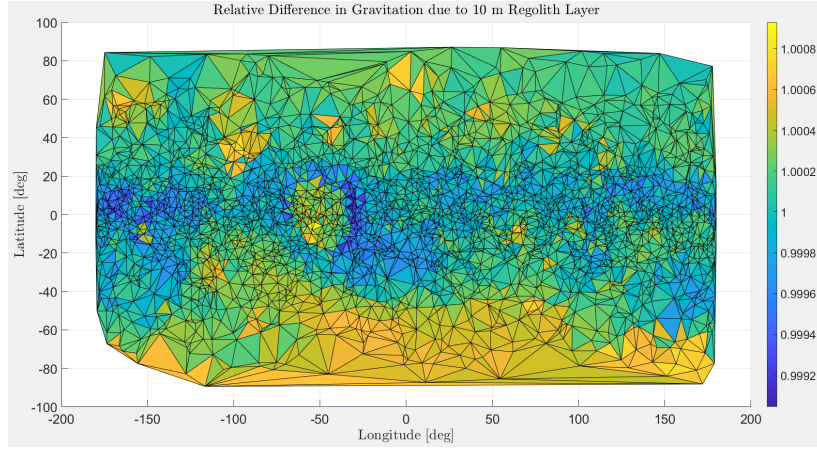


Figure 6.18: Resulting relative difference of the surface gravitation including the 10 m regolith layer to the homogeneous surface gravitation (Figure 6.5). Surface computation for every hundredth surface point for better visualisation and reduced computation time.

For the gravitation computation, first, the entire body is computed with the regolith density $\rho_{regolith}$. Secondly, the interior “core”, i.e. the part below the regolith, scaled by Equation (6.7), is computed with the difference of the adapted density ρ_{core} and the regolith density $\rho_{regolith}$. This differential approach has already been discussed in the previous Subsection. The inner density is readapted, i.e. increased, to maintain the total mass of the body and account for the reduce regolith density. This is done by computing the volume of the two considered polyhedra as shown in Appendix B.1. Here, the adapted core density is $\rho_{core} = 1,862.8 kg/m^3$. The resulting *relative* difference due to the added regolith surface layer is plotted in Figure 6.18.

Clearly, both regions of increased and decreased surface gravitation appear. Regarding the magnitude, the relative change is always smaller than one per mille (‰). To assess the sensitivity to the surface layer thickness, the computation is repeated for a thickness of 50 m, leading to $\rho_{core} = 1.873.9 kg/m^3$ (Figure 6.19). Here, the sensitivity is increased to 4 per mille (‰), again with regions of increased and reduced surface gravitation. Recalling the target requirement of 1 % for the GRASS gravimeter, indeed the detection of a distinct surface regolith layer will still be a challenge, even for the increased thickness. However, combination with different high-level products, including surface imagery, radar measurements and/or thermal inertia measurements could aid in constraining such hypothetical surface layer. Clearly, a 10 m thickness on a much smaller body, such as Dimorphos, would have a different signature compared to the homogeneous case, and the here-presented approach can be used for future investigations.

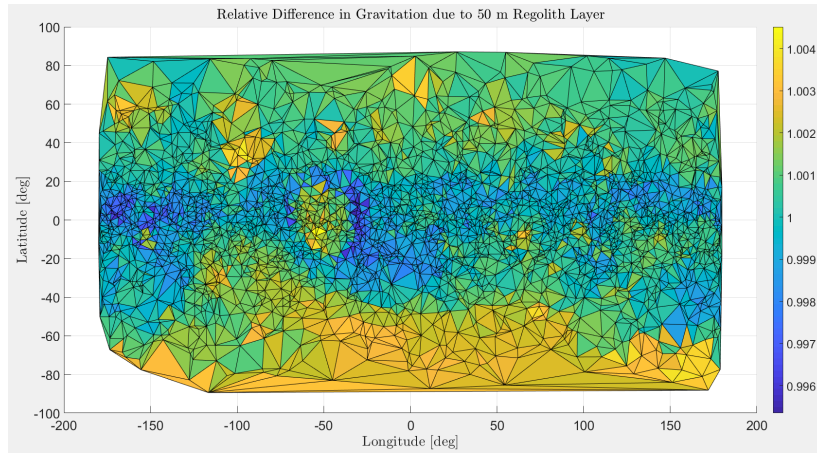


Figure 6.19: Idem as for Figure 6.18, but for a 50 m regolith layer.

6.6 DISCUSSION

In this Chapter, two research questions were addressed. For the last part of research question (I), the comparison of different surface gravitation methods was investigated. Comparing the resulting surface gravitation computed in this work using the polyhedral method (PM), with two other teams using mascon methods, it has been shown that the PM provides the most accurate results. This is true for the homogeneous case (Section 6.1), as well as for simple case inhomogeneities (Section 6.5). The PM, however, has also some disadvantages compared to the mascon methods. First, the computation time of the PM is one (MASC) to two orders (MSP) of magnitude slower than the mascon approaches. The computationally heavy PM is therefore less suited for (semi-) autonomous navigation (i.e. using an onboard gravitational model), for a large number of computation runs (e.g. performing forward propagation of Monte Carlo landing analysis), or for time-critical mission operation planning. Here, mascon methods should be favoured for such application cases. Implementations of the PM using parallelisation will help in resolving this problem, but this has not been investigated further in this work. Second, as has been shown for the homogeneous and inhomogeneous spheres, such shape can never be assessed perfectly by the PM, qua definition, demanding a polyhedral shape in approximation of “round” shapes. Third, it can be expected that highly inhomogeneous density distributions will put a limit to the here-shown superposition approach and other existing solutions (sliver approach, Subsection 5.3.7), circumventing the natural limitation of the PM demanding a homogeneous internal density distribution. Nevertheless, being the most precise method, the PM has also been demonstrated to be a powerful tool in the context of the GRASS gravimeter development (Part I). It is possible to obtain global surface gravitation maps at the highest precision, giving it an advantage over the mascon methods. This is specifically helpful for designing the dynamical range of the instrument.

The gravimeter development also builds the bridge to research question (II), which

does not only ask for the expected surface gravity, but likewise for possible variations due to dynamical effects influencing the net surface acceleration. For the initially considered (Phobos), current (Dimorphos) and possible future (Apophis) target, the surface gravity has been simulated, returning a measurement regime of three different orders of magnitude. Clearly, this underlines the importance of both, the large degree of adaptability in the developed GRASS gravimeter design (Chapter 3), as well as the developed analytical electromechanical gravimeter model (Chapter 4). Like this, it is directly possible to perform instrument simulations for different target bodies (gravities) and adapt the electromechanical design, as required. Next to this, not only the absolute magnitude has been developed, but likewise the variation in surface gravity was assessed. For example, the variation in surface gravity due to the tidal acceleration on Phobos was simulated to vary about 2.15 %, a non-negligible variation for both sizing of the GRASS instrument and the planning of measurements. Recalling the target accuracy of the developed GRASS gravimeter for Dimorphos, being $1\% \cdot 4.5 \cdot 10^{-5} \text{ m/s}^2 = 4.5 \cdot 10^{-7} \text{ m/s}^2$, the variations in tides will be measurable in some, but not all surface locations. The minimum variation is $2.5 \cdot 10^{-7} \text{ m/s}^2$ and therefore not measurable. Considering the variation of the mean tidal signal, this is exactly $4.5 \cdot 10^{-7} \text{ m/s}^2$ and should be measurable, and so would the tidal variations in locations with a variation larger than the variation of the mean over the full surface. The librations, however, are below the target accuracy and might not be detectable by the instrument.

Generally, the presented results in this Chapter should be considered with care. While they certainly aid in the gravimeter development and mission planning, they are based on various assumptions and simplifications. The surface gravitation considered homogeneous density distributions inside the body, and the (analytical) dynamical analysis is a simplification in the context of Solar System surface gravimetry. While a successful gravimeter mission is required to improve the here-presented results, the tools used and presented in this Chapter do not only aid in preparation and sizing of GRASS, but can likewise form the basis for more advanced post-mission (gravimeter) data analysis.

6.7 SUMMARY CHAPTER 6

The present Chapter addressed the global surface gravity environment of selected small bodies, and three different methods to compute (surface) gravitation have been compared. In Section 6.1, three methods (PM, MASC, MSP) to compute gravitation of arbitrarily shaped bodies have been compared for the case of homogeneous internal density distributions. This was done using different spheres (Subsection 6.1.1), cubes (Subsection 6.1.2), and the Martian moon Phobos (Subsection 6.1.3). Supplementary to the gravitation as (generally) main contributor to the surface gravity, the dynamical contributions have been studied analytically with respect to the net surface gravity (Section 6.2). While for the rotational acceleration (Subsection 6.2.1), the rotation and hence the acceleration has been assumed to be uniform, the librational (Subsection 6.2.2) and tidal influences (Subsection 6.2.3) vary, resulting in a non-constant surface gravity during one orbit. The tidal variation on Phobos would be measurable with the GRASS gravimeter with a target

accuracy of 1 % of the Phobian gravity, while the librational influence is below this threshold.

Supplementary to studying the Phobian gravity environment, the same tools were applied to the Hera mission, currently under preparation, in Section 6.3. Not only the global gravitation field (Subsection 6.3.1), but also the dynamical contributions (Subsection 6.3.2) have been simulated to aid the ongoing GRASS gravimeter development. Clearly, for the binary asteroid system, the dynamical contributions shall only be understood as a first approximation, but it has been shown that tidal variations will be measurable in some locations on Dimorphos (where the variation is as large as the mean variation or larger), and that the librations can be expected to be too small to be sensed by GRASS. These results were then fed into first considerations regarding the JUV-GRASS gravimeter CONOPS in Subsection 6.3.4. In the same fashion, the surface gravity and rotation has been simulated for a possible future mission target, being Apophis (Section 6.4). In continuation of the ongoing work for gravity comparisons, the PM has been adapted such that inhomogeneities can be studied, either in the form of spherical cores (Subsection 6.5.1), or in the form of a surface regolith layer (Subsection 6.5.2). Finally, the results of this Chapter are discussed in Section 6.6, before this summary concludes the present Chapter.

7

DEVELOPMENT OF THE WEDGE-PENTAHEDRA METHOD (WPM)

This Chapter, with the exception of Section 7.7, is an adapted excerpt from Noeker et al. (2021b) and Noeker and Karatekin (2022b).

In the previous Chapter, SB surface gravity, with a specific focus on gravitation has been studied in the context of inter-comparison between different computation methods. Here, the focus was laid on the surface acceleration which will be measured by the GRASS gravimeter (Part I). This was done on a global level. Zooming in on a more regional level, this Chapter presents the development of the novel Wedge-Pentahedra Method (WPM) that has been invented to allow the assessment of the local terrain in direct surrounding of the gravimeter measurement location (*also*: “evaluation point”).

In this Chapter, the development of the WPM will be introduced in detail in Sections 7.1 through 7.6. Following this, the method will be incorporated in a traverse simulator in the context of surface rover traverses carrying a gravimeter (Section 7.7). Finally, this work is discussed in Section 7.8 and the Chapter is summarised in Section 7.9.

7.1 LOCAL TOPOGRAPHIC CORRECTIONS

In classical (terrestrial) gravimetry, different corrections, sometimes also referred to as *reductions*, are applied to gravimetric measurements (Hammer, 1939). A generally used concept is the Bouguer anomaly, accounting for (1) the Bouguer correction, (2) the free-air gradient and (3) the terrain (topography) correction. More types of corrections exist, but those are not discussed here. The Bouguer correction considers the gravitational effect of material between the measurement point (i.e. the gravimeter location), and a (lower) reference datum (Vaníc et al., 2001), e.g. with respect to the geoid or to any arbitrary datum. The free-air gradient accounts for the increased distance to the centre of the central body with respect to the reference datum (e.g. Talwani et al., 1973). Therefore, the Bouguer correction *increases* the resulting gravitation depending on the subsurface material density, while the free-air gradient *reduces* the resulting gravitation. The third contribution

is the topographic reduction, where the topography in the surrounding of the gravity measurement point is considered. Imagine a measurement at half height of a hillside: The material above the measurement point up to the peak lies above the gravimeter and “pulls up”, the resulting gravity is reduced. Likewise, from the measurement location down to the valley floor, material is “missing”, decreasing the resulting gravity.

On Earth, the topographic effect is largest in mountainous regions, e.g. up 40 arc-seconds in the Alps (Barzaghi et al., 2016), and elsewhere much smaller; often this correction has even been neglected (Heck and Seitz, 2007). When applied, the topographic, or mass, reduction can generally be done using tesseroids, prisms or point masses (Heck and Seitz, 2007). On the local scale, e.g. considering local surface gravimetry, tesseroids appear not suitable for a flat-Earth assumption. Point masses, or mass concentrations (mascons), result in largest computation uncertainties on the surface (Park et al., 2010; Meißenhelter et al., 2022), making it less suitable for surface measurements compared to air-/water-/spacecraft-based measurements, providing some separation between the approximated surface and the gravimeter. Regarding prisms, it is possible to obtain an analytical and therefore precise solution for right rectangular 2D- (Talwani et al., 1973) or 3D-prisms (e.g. Heck and Seitz, 2007). However, this assumes a flat-topped rectangular top surface, which again is only an approximation of the real surface (Urbancic et al., 2017). The prism approximation therefore leads to a manifold of plateaux, giving a discontinuous surface and neglecting the surface slopes.

While the topographic correction might be oftentimes negligible on the Earth, the relative contribution will increase with decreasing gravity, e.g. on Mars and the Moon. For example, the Mars science laboratory (MSL) onboard gravity model differed from the local Martian gravity by -4.4 mm/s^2 (Brugarolas, 2017), leading to vertical touchdown velocity errors roughly equal to the 3σ error and resulting in the recommendation to include corrections for local gravity (including topography!) to avoid damages to a (planetary) lander in the final landing phase¹. Moreso, the effect will be even larger on SBs such as the Martian moons Phobos and Deimos. To date, one gravimetric study was performed on the Moon (Subsection 2.3.1) and one on Mars (Subsection 2.3.3). The GRAvimeter for small Solar System bodies (Part I) is being developed for surface gravimetry on the secondary of the asteroid system Dimorphos as discussed in Part I. Solar System *surface* gravimetry therefore demands a method well-suitable to account for local topography reduction in the proximity of the surface, avoiding the above mentioned top-surface approximation.

We will therefore address two main problems in this Chapter: (1) the computation of a local high-resolution terrain shape and (2) the computation of its contribution to gravity. After introducing the shape file and topography file used in this work (Section 7.2), we provide a geometric method to merge high-resolution 2D-terrain data with a lower-resolution global shape file, exemplary demonstrating this on the Martian moon Phobos (Section 7.3). The validity and value of this method can be proven when an existing shape file can be locally augmented by high-resolution local terrain observations. With this, we further compute the local high-resolution

¹Cf. <https://llis.nasa.gov/lesson/27901> accessed on 28.04.2022.

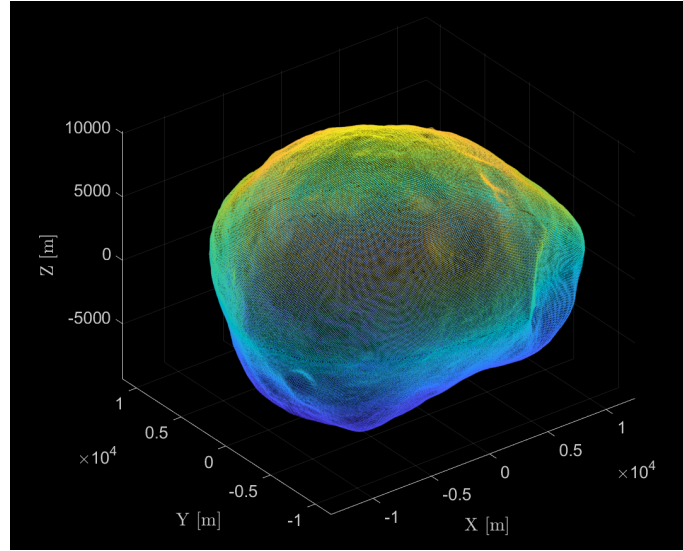


Figure 7.1: Vertices of Wavefront (extension .OBJ) file of Phobos by Willner et al. (2014). Figure from Meißenhelter et al. (2022).

surface by creating wedge-pentahedra (Section 7.4) and we evaluate in mathematically closed-form the gravitation contribution (Section 7.5) wedge-wise using the polyhedral method (PM) (Subsection 5.3.6) applied per-wedge. To show that the newly developed wedge-pentahedra method (WPM) is working, we further provide here a verification by comparing the resulting gravitation with a single, same-sized cube (Subsection 7.5.3). Lastly, the gravitational influence of the surface with artificial topography is compared to the corresponding smooth surface of identical extend in Section 7.1. This change in gravitation is also computed relative to the absolute resulting surface gravitation.

7.2 SHAPE FILE AND DEM

7.2.1 SHAPE FILE

For demonstration of the newly developed WPM, we use the Martian moon Phobos and the latest shape file by Willner et al. (2014) shown in Figure 7.1. The there reported extend of this shape is represented with its radii, being $13.03\text{km} \times 11.40\text{km} \times 9.14\text{km}$, with a volume of $5742 \pm 35\text{km}^3$ and a density of $1,860\text{kg/m}^3$. The shape model with a resolution of 100 m/pixel was developed based on imagery data from the Mars Express Mission (MEX) and the Viking orbiter cameras.

7.2.2 DIGITAL ELEVATION MODEL

While the above introduced shape file of Phobos is one of the most detailed in the Solar System, the resolution is still limited to 1/100 ppmeter (pixel-per-meter, comparable to pixel-per-degree, ppd). Therefore, in order to have a high-resolution sur-

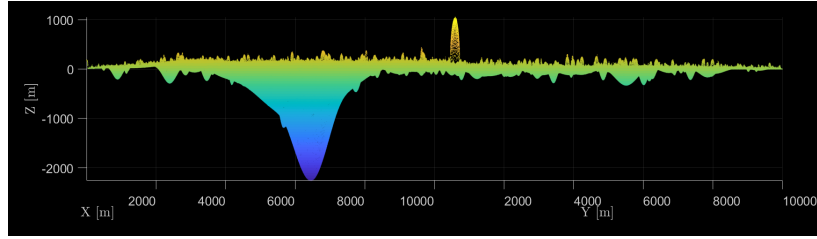


Figure 7.2: Original DEM file imported to MATLAB and plotted as a point cloud with every fifth point shown for array reduction and vertical elevation scaled by a factor 100 for better visualisation. This DEM is fully artificial, and not representative of a real surface.

Table 7.1: Key values of unscaled artificial DEM (Miyamoto, 2021), at full (original) resolution. Shown is the full data set of size $10,001^2$ and the used subset spanned as a square between points (1000|1000) and (2000|2000). The first line shows the values of the full data set and the second line shows the same figures for a subset of the DEM used in this work.

Data min. (X Y)	Data max. (X Y)	min. Z	mean Z	max. Z	std. Z
(1 1)	(10001 10001)	-22.590	-0.107	+10.586	1.947
(1000 1000)	(2000 2000)	-0.220	+1.029	+3.354	0.453

rounding surface, an artificial digital elevation model (DEM) will be applied to the Phobian surface, kindly provided by Miyamoto (2021). Obviously, any 2D-DEM, being artificial or real, can be used for this method, meaning that for example gravity data analysis combined with the observed high-resolution topography can be performed following a successful mission.

The original, dimensionless data set contains $10,001$ points squared, thus it is spanned in the X-Y-plane from point (1|1) to (10,001|10,001). To quantify the DEM, visualised in Figure 7.2, the minimum, maximum, mean and standard deviation (std.) of the (original) DEM Z-coordinates is provided in Table 7.1. Given the very large amplitude of the full DEM (Table 7.1), and to exclude individual features, such as the very large and deep crater, we take a subset of the full DEM (Figure 7.2), that is quadratically spanned from data points (1000|1000) to (2000|2000). This subset is plotted in Figure 7.6.

7.3 MERGING TOPOGRAPHIC DATA AND SHAPE FILE

For Small Bodies, the knowledge of their shapes, and thus the available associated shape file resolution is limited. Especially for objects that have not been visited by spacecraft, the shape file is usually solely based on Earth-based observations. Even for bodies that were a target of a fly-by such as Phobos, the shape file resolution is limited despite the good quality of the data. Generally, the presence of a lower resolution global shape (e.g. 0.01 ppmeter) file and high resolution local

shape file (e.g. better than 0.1 ppmeter) is a realistic combination. For example, the NASA DART spacecraft (Subsection 1.3.2) will kinetically impact Dimorphos to demonstrate planetary defense technology (Cheng et al., 2012). This will return high-resolution information on the shape file, but obviously only on the side/region of impact (while LICIACube might return some images from the impact-far side). The same is true for the Juventas CubeSat landing on Dimorphos (Goldberg et al., 2019), and the MMX mission with a planned rover deployment (Ulamec et al., 2019) and a surface-sample-acquisition-manoeuve (Kuramoto et al., 2022). All these examples have in common that they will return detailed surface information on the surface surrounding the landing point/rover operation region. Likewise, in preparation of surface operations/landing, more detailed topographic observations of possible landing regions are obtained just-in-time, allowing additional planning using the here-developed method.

Different solutions from observation acquired during mission and proposals of robots for perception systems exist that consider (local) precise surface observations in the context of larger scale mapping. For example, Haase et al. (2019) used Apollo 17 astronaut imagery for angular measurements to precisely determine coordinates of landmarks and equipment. Details on the topography, and more specifically the surface roughness, of Itokawa were derived by Abe et al. (2006) using LIDAR observations. However, both approaches were very specific to individual features/locations, and did not return systematic improvements in the surrounding mapping of the surface. On the contrary, a prime example of obtaining an efficient high-resolution terrain model on a local scale is the surrounding of the Huygens landing site on Titan. Using (stereo-) imagery from the descent, Soderblom et al. (2007) derived an unprecedented DEM of the Titanian surface, “only” surrounding the landing Huygens landing site. In the frame of surface exploration, a legged surface vehicle with a ranging sensor was presented by Kweon et al. (1989) together with a method to construct and combine elevation maps. Overall, it is therefore reasonable to assume that a high-resolution DEM will be available for the surrounding proximity of landing sites on extraterrestrial surfaces, without going further into detail on the (2D-) mapping techniques.

In order to allow for good topographic reduction, here we firstly developed a method to *merge* a global 3D- (small body) shape file with a local 2D-high(er)-resolution digital elevation model (DEM). Alternatively, the merging method can be adapted for large spherical bodies (e.g. Moon, Mars, Titan), where locally a flat surface might be assumed; this will be discussed in Subsection 7.7.2 and in future work. In absence of a suitable readymade ground-truth data set, we use the exemplary artificial DEM introduced in Subsection 7.2.2. In a first step, the local surrounding of the evaluation point is isolated (Subsection 7.3.1) and the (artificial) high-resolution digital elevation model (DEM) surface is combined with the lower-resolution shape file (Subsection 7.3.2). This merging process described in this current Section is an optional preparatory step to the WPM described in the subsequent Sections, and can be skipped if the local topography is readily available.

Table 7.2: Coordinates of evaluation point in original and rotated coordinate system on Phobos, being centre points of their facets on the shape file by Willner et al. (2014).

Point	X [m]	Y [m]	Z [m]
Original	-945.67	-145.67	-8044.33
Rotated	308.39	1259.79	7996.54

Table 7.3: Reference gravitations components and norm, computed with the polyhedral method for the (original) surface point listed in Table 7.2.

$g_x[10^{-3} m/s^2]$	$g_y[10^{-3} m/s^2]$	$g_z[10^{-3} m/s^2]$	$g[10^{-3} m/s^2]$
-0.1957	-0.2193	5.7880	5.7955

7.3.1 LOCALISING THE REGION OF INTEREST

Initially, the region of interest, i.e. the local surrounding of the location of interest, is isolated. For our evaluation (landing) region, we select one exemplary location ("Evaluation Point") on Phobos to illustrate the localisation of the problem. To prove the stability of the method for all macroterrains, we selected a surface point inside a depression. Rather than using the vertices of the original Wavefront (extension .OBJ) file to identify surface points, we have decided to take the central points as evaluation (eva.) point $(x_{eva.}, y_{eva.}, z_{eva.})^T$ of each face by taking the average coordinates of the three boundary vertices i, j, k :

$$\begin{bmatrix} x_{eva.} \\ y_{eva.} \\ z_{eva.} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} x_i + x_j + x_k \\ y_i + y_j + y_k \\ z_i + z_j + z_k \end{bmatrix} \quad (7.1)$$

The original coordinates of the studied evaluation point at the centre of its facets is listed in Table 7.2 and the point on the Phobian surface is indicated in the global context in Figure 7.3.

In a first step, the overall shape model is used to compute the reference gravitation g_{ref} at the surface evaluation point with the well-known polyhedral method (PM) by Werner and Scheeres (1996), using a verified implementation by Tasev (2019). The reference gravitation is computed with uniform density of $1,860 kg/m^3$ (Willner et al., 2014) and listed in Table 7.3 for the evaluation point.

With the gravitation vector in place, the first localisation step is the dissection of the body. For this, a dissection plane is simply constructed by taking the evaluation point and the gravity vector as plane normal, and translating the plane by a defined distance, here 1,000 m, along the plane normal. The evaluation point, the gravity vector as plane normal, and the translational distance define the dissection plane as shown in Figure 7.3.

We then dissect the body along this plane and remove the points on the side away from our local point. Figure 7.4 shows the reduced point set with the dissecting

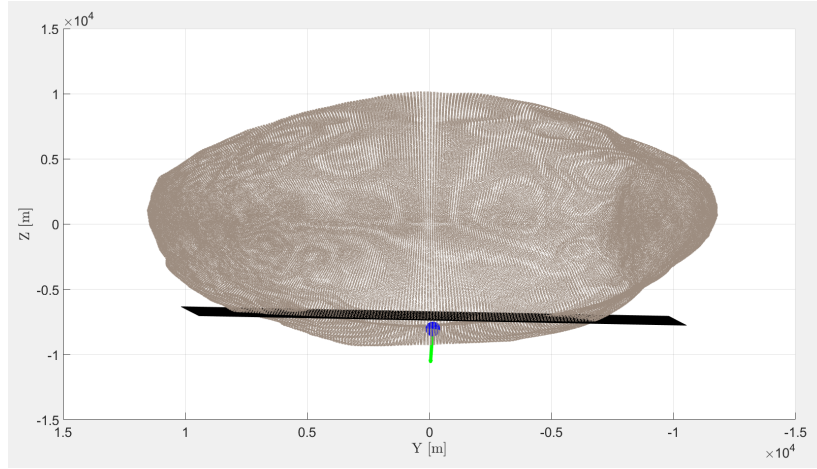


Figure 7.3: Phobos shape file with dissection plane offset by 1,000m and evaluation point with (flipped) gravity vector (green). View in +X-direction.

plane and the (outward-flipped) gravity vector.

Due to the arbitrary orientation of the reference gravitation g_{ref} in the shape file coordinate system (CS), the forthright geometric manipulation is facilitated by coordinate system rotation, such that the (outward-pointing) gravity vector aligns with the +Z'-axis in the rotated coordinate system CS'. This rotation is described in Appendix B.2. The rotated evaluation point coordinates are listed in Table 7.2. The rotated reference gravitation vector has hence the magnitude as its Z-component and the X- and Y-components are quasi-zero (leaving numerical artefacts).

Alternatively to using the (normalised) reference gravitation vector in the above described steps, the usage of the normal to the surface shape was studied. However, this was found to work less reliably for bodies such as Phobos. As will be shown in the next step, the method to reduce the global shape to a local shape demands sufficient surrounding material (or: surface) in all directions. The relatively large detail in the shape file of Phobos can orient the surface normal locally such that it is not helpful for the here-shown implementation, while the gravitation vector is generally sufficiently pointing towards the body centre. This is usually true for small bodies larger than Phobos, but not necessarily for even smaller bodies. On a smooth shape, such as an ellipsoid, e.g. initially assumed for Dimorphos, the gravity vector will cause a skewed dissection, making the surface normal on the smooth surface of an elongated shape preferable. Generally, the gravitational field and shape file (quality) will determine if the rotation should be done with the reference gravitation (this work) or surface normal, which will be investigated further in future work, especially for smaller bodies, or highly-elongated (e.g. dog-bone) shapes.

Finally, following the above introduced “vertical” translation offset, we now add a “horizontal” offset, here again exemplary $\Delta h = 1,000m$, but this can well vary from

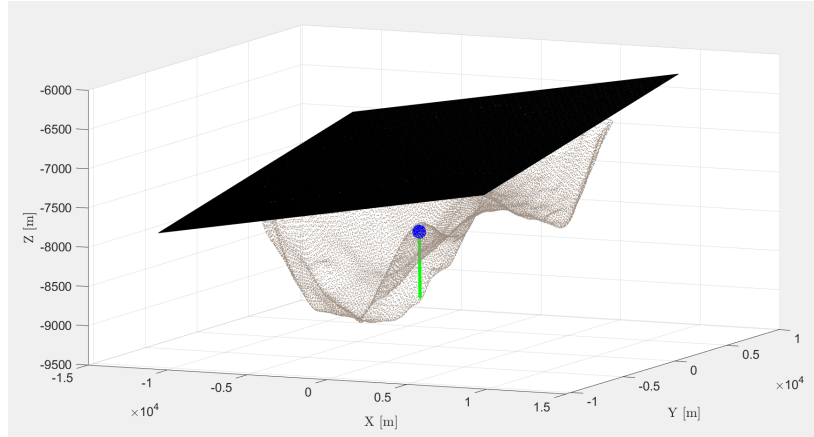


Figure 7.4: Local points on the evaluation point side of the dissection plane only. The evaluation point is visible in blue and the (inverted) gravity vector is displayed in green. Note the different scaling compared to Figure 7.3.

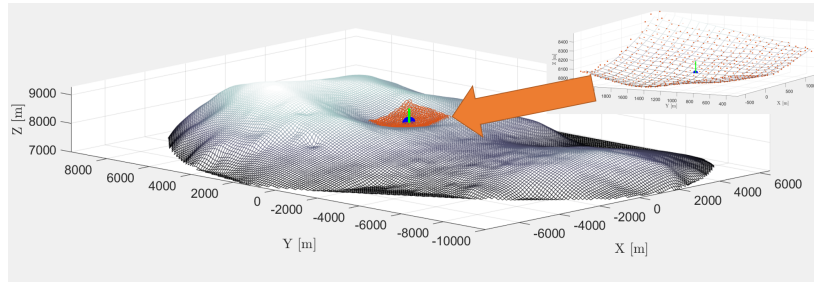


Figure 7.5: Local environment above dissection plane rotated with local points in proximity of the evaluation point highlighted. For better context, axes are set equal.

the vertical value and be set much larger. The terms horizontal, above/below, are artificial and only corresponds to the rotated reference frame CS' . With the “horizontal” offset Δh , we only keep the points inside a square in the X-Y-plane of side length $a = 2 \times \Delta h = 2 \times 1000m$, centred at the evaluation point, which gives us the local surrounding for our further analysis. The rotated points above the dissection plane with the local points highlighted and isolated are shown in Figure 7.5. These remaining points alone form the basis for the local analysis and the superposition of the digital elevation model (DEM) that will be introduced in the next Subsection.

7.3.2 SUPERPOSITION OF THE DEM

The localised points in the area spanning 2,000-m-squared (Figure 7.5) consist of 624 points of the original shape file. Therefore, we have here a one-dimensional resolution of about 25 points over 2000 m, meaning 1/80 ppmeter (thus a bit better

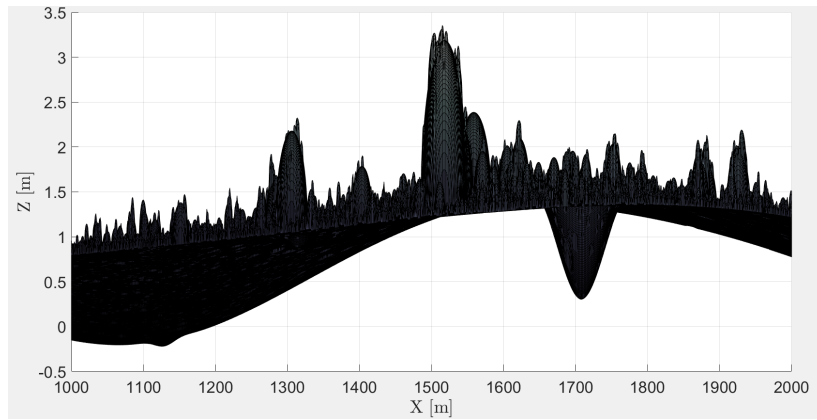


Figure 7.6: Side-view (in +Y-direction) on the subset spanned from (1000|1000) and (2000|2000) at the original data set resolution where all features appear smooth with a continuous surface representation.

than the nominal 1/100 ppmeter). While this is a good value for the global shape representation, the aim here is to artificially improve this resolution locally for the proximity terrain analysis.

The DEM data set, consisting of 10,001 points squared, thus just more than 100 million points, corresponds to a resolution of 5 ppmeter, if projected onto the 2000-m-squared area. However, we have to limit the number of points used for different reasons:

- The computation time of the algorithm, also with respect to gravity evaluation (Section 7.5) needs to be limited.
- The triangulation (Subsection 7.4.2) fails, if the ratio of artificial to smooth resolution becomes too large.

The limitation in used points is introduced two-fold, by taking a square subset of the full DEM and by taking only every n^{th} point of the data set.

For example, we take the data subset spanning from 1000 to 2000, meaning that the points (1000|1000) and (2000|2000) span up the square for the subset. This subset is shown in Figure 7.6. Hence, we have increased the surface resolution to 1000 pixel/2000m = 1/2 ppmeter, thus we achieve a resolution 40 times better than on the original data file (1/80 ppmeter).

The number of surface points can then be further reduced by only taking every n^{th} point of the subset, reducing the number of used pixels. This is shown in Figure 7.7 for every 20^{th} point. The above calculated ground resolution therefore decreases from 1/2 ppmeter to 1/40 ppmeter. Yet, while the ground resolution now mathematically is only twice as large as the original data set, the added terrain does not completely disappear. Clearly visible comparing Figures 7.6 and 7.7, the base terrain deviation, above and below the zero-elevation plane, is only little influenced by the ground resolution at the resolution reduction. Moreover, the main features,

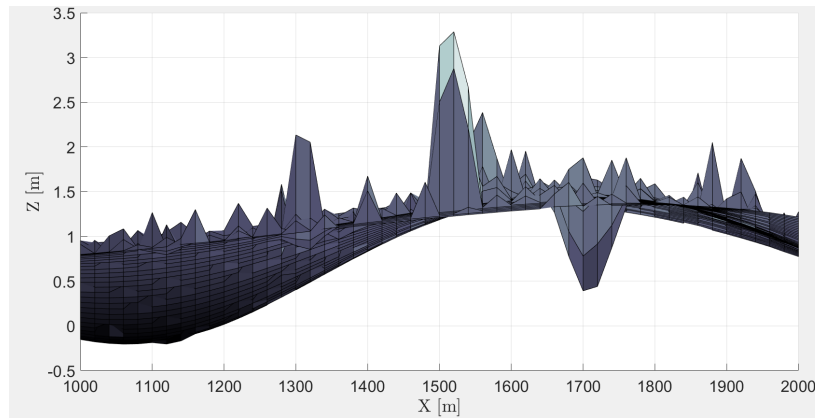


Figure 7.7: Same as Figure 7.6, but reduced resolution by taking only every twentieth point.

i.e. elevations and depressions (e.g. craters) are also visible in both resolutions. Best visible when inspecting these features, however, is the difference in surface smoothness. While the original data set contains an appearing continuous, (microscopic) smooth surface, the reduced data set clearly shows the (flat) facets between the individual points. Moreover, the reduction in resolution leads to a reduction in surface features, meaning that smaller features, i.e. lying between the reduced number of points, will not appear. Likewise, the shape of surface features might be altered for this reason. The pure quantitative comparison of ground resolutions might therefore be misleading, as the DEM with lower resolution still provides much more terrain and features than the original surface. An analysis of the influence of the ground resolution is therefore included in this work. This is also important with the above-mentioned computation time/algorithm for gravity and the formulation of observation requirements.

As a next step, the selected subset of the DEM with variable ground resolution (Figure 7.7) needs to be combined with the smooth original surface (Figure 7.5). This process is described step-wise in the following.

Firstly, the DEM is translated and scaled such that it lies above the reduced original surface (Figure 7.8). The vertical position “above” the original surface is only temporary and therefore arbitrary. At this stage, the DEM still has its original scaling thus within the range of -0.220 to +3.354 (Table 7.1). This arbitrary value is here treated as meter, while this was not specifically stated for the original data set.

By definition, the resolution of the two data sets to be merged differs. In order to project the higher- onto the lower-resolution data grid, we perform an interpolation of original data set to match the specified (variable) resolution of the used data set. The interpolation uses the MATLAB function *griddata* (option ‘natural’) that allows interpolation of the scattered data (original surface). For this, firstly a mesh is initialised with the size of the original surface (here 2000-m-squared) with a step

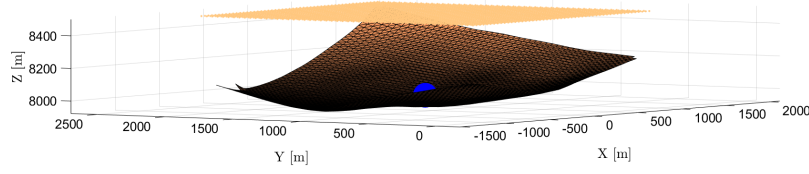


Figure 7.8: The DEM with original scaling translated and scaled to lie over the original surface in preparation of the superposition. The unscaled DEM appears flat, but has the magnitude listed in Table 7.1.

size equal to the (specified) DEM resolution. We then use the X- and Y- coordinates of this mesh for the DEM-superposition, by taking the Z-coordinate of the interpolated surface $Z_{local,surface}$ and adding the corresponding Z-value of the DEM $Z_{DEM,original}$ multiplied by a scale factor as:

$$Z_{DEM,included} = Z_{local,surface} + Z_{DEM,original} \cdot S_T \quad (7.2)$$

where S_T is the terrain scale factor. A value of $S_T = 0$ effectively removes the added DEM, only leaving the interpolated surface, a value of 1 is representative of the DEM plotted in Figures 7.6 and 7.7, showing the original and resolution-reduced data, respectively. Any other value for S_T will scale the DEM accordingly, where a negative scale factor inverts the topographic profile.

In a last step, we perform a final vertical adaptation such that our evaluation point and the (newly added) artificial DEM coincide in the central evaluation point. This adaptation is arbitrary, but necessary, e.g. for the gravity evaluation. Otherwise, the evaluation point on the smooth surface could end up below the DEM surface (if the DEM locally is positive) or floating in empty space (if the DEM locally is negative).

Finally, the complete superposition of the DEM with a scale factor of $S_T = 250$ on the local interpolated original surface is shown in Figure 7.9. It is clearly visible that some of the (scaled) DEM terrain appears above and some below the original surface. This was found to be the best way of including the terrain, instead of systematically lying above or below the original surface, also with respect to the common boundary, introduced in Subsection 7.4.1.

7.4 WPM: SOLIDIFICATION USING WEDGE-PENTAHEDRA

To obtain inter-comparable results between original and DEM surface, we have to ensure a common boundary for all tested surfaces, which is done by creating a boundary frame (Subsection 7.4.1). In addition, to be able to calculate the gravitational influence of the local region, the surface has to be solidified, forming a solid volume (Subsection 7.4.2). This Section therefore describes the transition from the surface to the solid volume.

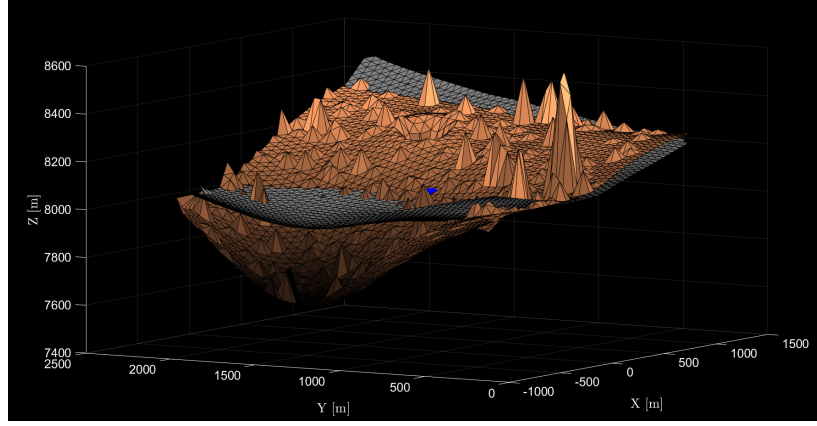


Figure 7.9: Original (smooth), interpolated surface (grey) and superpositioned DEM surface in copper. Both surfaces touch in the evaluation point (blue).

7.4.1 COMMON REFERENCE BOUNDARY

As indicated above, the geometric operations performed here need special attention towards a common boundary of the localised area. To compute the influence of the topography on the surface gravitation, we need a comparable environment, independent of the terrain scaling and realised by a common boundary frame.

For this, we start at the (central) evaluation point $(X_0|Y_0)$, and span a quadrangle bound by four points. The X-Y coordinates of these four points are simply calculated as

$$X = X_0 \pm S_f \Delta h \quad (7.3)$$

$$Y = Y_0 \pm S_f \Delta h \quad (7.4)$$

where S_f is an arbitrary scale factor, chosen here as 1.001, to ensure that the frame is always outside the treated surface. Δh was above defined as the horizontal offset in the localisation (here 1,000 m). The vertical (Z-) coordinate of these four points is set to be identical to the respective closest point of the DEM surface, by finding the nearest neighbour.

Starting from these four points, the frame is built in four legs, comparable to a launching gantry. Starting from one point, either the X- or Y- coordinate is kept constant. The other coordinate is varied stepwise (e.g. 10 m) in one direction. The Z-value of the framing points is then again set equal to the Z-value of the closest neighbouring point of the interpolated (smooth) DEM surface, with scale factor $S_T = 0$ (Figure 7.9). By taking the smooth surface high-resolution surface, it is ensured that the frame points always find a close-by surface point for the Z-coordinate adaptation. This means, that the boundary will slightly vary for different ground

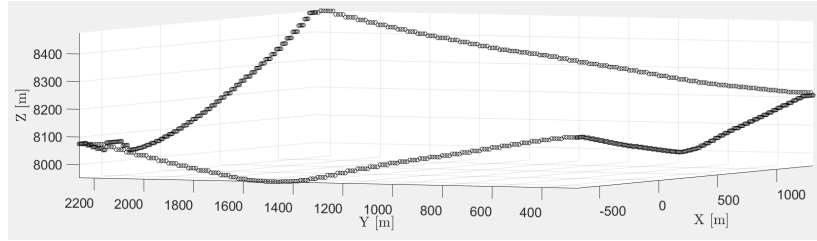


Figure 7.10: The common frame is built around the DEM with an offset factor of 1.001, i. e. for the horizontal offset of 1000 m, 1 m outside the original surface range.

resolutions, while the frame for the (interpolated) flat surface and the DEM-applied surface are always identical, which is important for a sound comparison of the DEM at that scale factor S_T .

The built frame is shown in Figure 7.10. On the local scale, it can be observed that the frame is build with small (staircase) steps of identical Z-value. This, however, does not pose a problem as no points overlap in the vertical direction. Using always the same boundary for both the (interpolated) flat surface and the surface with applied topography allows for the topographic reduction where the two cases are compared with identical boundary. This ensures to focus the comparison to assess the influence of the local topography without variable influence from the boundary. Finally, these frame points are added to the respective surface point sets. This boundary plays therefore an important role in the solidification of the surface, described in the next Subsection.

7.4.2 BUILDING THE LOCAL SOLID VOLUME

Above, the evaluation point has been localised and rotated (Subsection 7.3.1), the artificial DEM was superpositioned on the original surface (Subsection 7.3.2), and a reference boundary frame was built to generate a boundary of the studied surface (Subsection 7.4.1).

The last step in the geometric computation is to solidify the newly created surface, i.e. creating a 3D-volume from the 2D-surface. Creating a solid volume is absolutely required to compute the gravitational effect of the surface-forming topography (Section 7.5).

In order to create a solid volume from the triangulated surface, each of the surface triangles is considered individually. The three triangle vertices are projected downwards to a common reference plane, i.e. the X-Y-coordinates are kept constant while the Z-coordinate is adapted. These total of six points form a wedge, i.e. a pentahedron, consisting of five facets, where the top triangle forms part in the DEM-surface and the bottom triangle lies in the X-Y-plane (Figure 7.11 (left)). Repeating this for all surface triangles, a solid volume, consisting of a multitude of pentahedra-wedges is created.

For a generic algorithm, we define the projection elevation Z_{bottom} here as

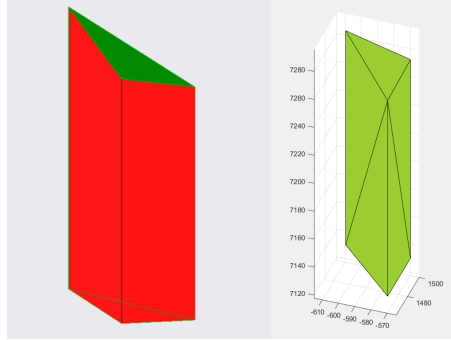


Figure 7.11: (Left) Geometry of a wedge as a pentahedron with the top surface forming part of the local surface highlighted in green. (Right) AlphaShape of wedge-pentahedron with 8 triangular facets.

$$Z_{bottom} = 0.9 \cdot Z_{local,min} \quad (7.5)$$

where $Z_{local,min}$ is the minimum Z-value of the surface. In general, the exact value is not important, as long as it is kept constant between the smooth and rough surface in the topographic reduction (Section 7.5), similarly to the common boundary (Subsection 7.4.1). However, the down-projection distance should be large enough to always be below the surface, including the DEM-surface with negative terrain such as craters. Otherwise, the superpositioned surface will appear under the projection elevation, causing the “downward” projection to fail.

Each of the wedges, bound with $3+3=6$ vertices is now transformed to an *alphaShape* (α – Shape). Due to the simplicity of the (individual) wedge-pentahedra shapes, this is straight-forward. From this alphaShape, the (five) outside facets are triangulated, thus automatically splitting the (three) quadrangle facets in two triangles, giving a total of eight facets (Figure 7.11 (right)). The advantage of this triangulation of the enclosed volume is that this format directly allows us to evaluate the gravity using the polyhedron method (PM). This evaluation is discussed further in the next Section. Showing the full created WPM is of little avail to illustrate the interior and therefore, we truncate the WPM-solidification process after (arbitrarily) 500 out of (here) 7998 constructed wedges, showing indeed the full volume is built of individual wedges (Figure 7.12). As will be discussed, the advantage to evaluate the gravity pentahedron-wedge-wise is that the density can be varied per-wedge, and that the density does not have to be kept homogeneous throughout the entire volume as would be the case applying the PM to the bounding volume.

Finally, we apply the solidification to the example evaluation point (Table 7.2) once using the terrain scale factor $S_T = 0$ and once using a large terrain scale factor $S_T = 500$ to clearly visualise the added DEM. Evaluating the gravitational influence of the latter case and comparing it to the interpolated and smooth former case allows to account for the topographic reduction presented in the next Section. This is shown with the evaluation point centred (yellow) in Figure 7.13. Here, it is clearly

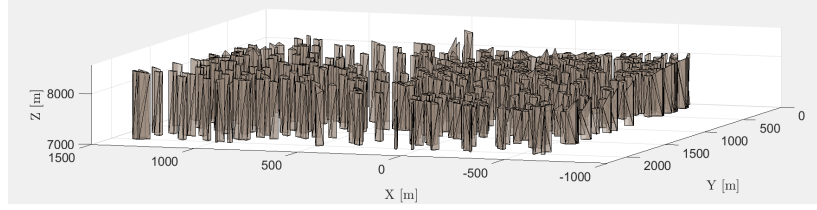


Figure 7.12: Surface solidified by WPM, truncated after 500 built wedges to show the interior of the volume.

visible that the common boundary frame closes the volumes off, e.g. closing of the depression on the left for $S_T = 500$.

7.5 WPM: TOPOGRAPHIC REDUCTION

With the solid in place, we can now proceed with the gravity evaluation of the local topography. In simple terms, this evaluation is done in three steps:

1. Firstly, we compute the resulting gravity in our reference point for the entire body (Phobos). For this, the homogeneous density ($1,860 \text{ kg/m}^3$, Willner et al. (2014)) is assumed. We call this gravity *reference gravity*: g_{ref} .
2. Second, we subtract the gravity of the volume bound by the local, quasi-original surface. For this, we set the DEM scale factor to $S_T = 0$. This means that the surface triangulation and resolution is identical to the one in step 3, yet no DEM is applied. This returns the gravity of the smooth surface, to which the one from step 3 is compared: $g_{DEM=0}$. In other words, this is representative of the original surface with the addition of the original surface being interpolated between the (lower-resolution) original surface points.
3. Third, we then add the gravity of the same bounding volume, now with the DEM applied with the respective scale factor S_T . This gives: $g_{DEM=S_T}$.

The then resulting gravity g_{res} is simply calculated with a cut-and-paste approach as

$$g_{res} = g_{ref} - g_{DEM=0} + g_{DEM=S_T} \quad (7.6)$$

This can be compared to the reference gravity g_{ref} , e.g. to assess the relative change in new resulting surface gravity, compared to the smooth surface, solely due to the local terrain. Generally, the relative change κ

$$\kappa = \frac{g_{res}}{g_{ref}} \quad (7.7)$$

is a good measure to understand the influence on the local resulting surface gravitation due to terrain.

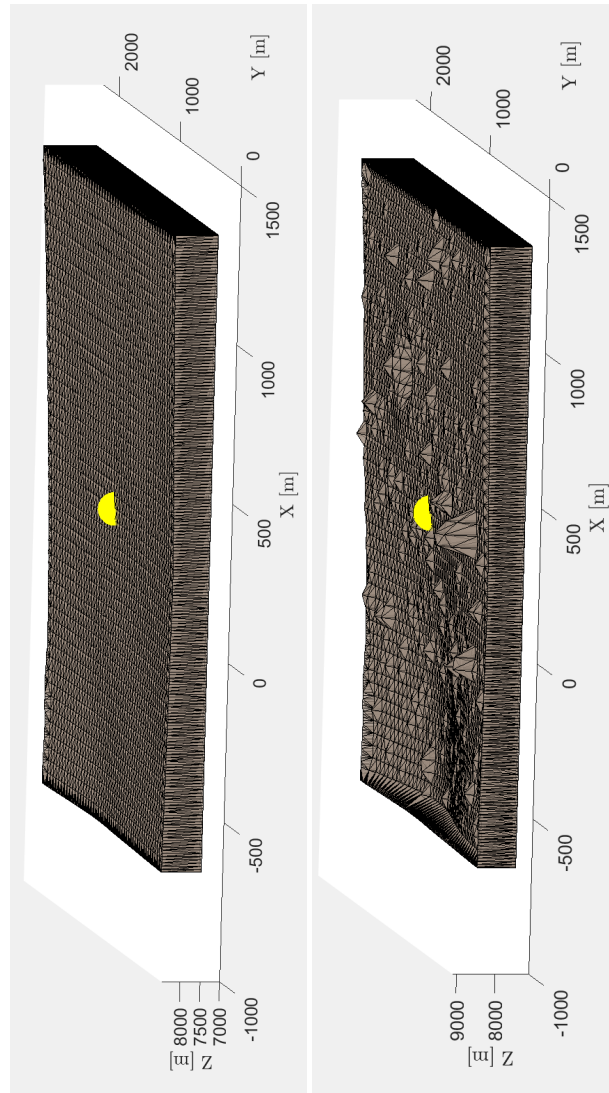


Figure 7.13: Solidified local surrounding of evaluation point (yellow) “in-crater” with local region of $(2000m)^2$ for scale factors $S_T = 0$ (left) and $S_T = 500$ (right).

Next, the gravitation per pentahedron-wedge will be computed (Subsection 7.5.1). The gravitational computation per-pentahedron-wedge is then verified using a cube in Subsection 7.5.3.

7.5.1 REFERENCE AND PENTAHEDRON-WEDGE GRAVITATION

The polyhedral method (PM) by Werner and Scheeres (1996) was already used tacitly in Subsection 7.3.1 to perform the initial rotation of the larger surrounding area. Here, we use an implementation developed by Tasev (2019). This code was verified with another implementation of the PM, by Van wal (2018), and further verified by comparing it to two different mascon implementations and to analytical solutions for a sphere and cube, as presented in Meißenhelter et al. (2022) and Section 6.1.

The gravitational attraction of any polyhedron shape with constant density ρ can be expressed as

$$\vec{g}(\vec{x}) = -G\rho \sum_{e \in \text{edges}} E_{ij} \cdot \vec{r}_{ij} \cdot L_{ij} + G\rho \sum_{f \in \text{facets}} F_{ijk} \cdot \vec{r}_{ijk} \cdot \omega_{ijk} \quad (7.8)$$

where E_{ij} , F_{ijk} , L_{ij} , ω_{ijk} , have been defined in Chapter 5, with Equations (5.33), (5.34), (5.35), and (5.36), respectively, and where Werner and Scheeres (1996) give the full derivation and we adapted the notation used by Van wal (2018). Here, the number of polygon edges is $e = 12$ and faces is $f = 8$ per pentahedra-wedge.

As was introduced in Subsection 7.4.2, the solid volume consists of a multitude of individual pentahedra-wedges. With the surface triangulation shown in Figure 7.13, the gravitation of each of these wedges can be computed using Equation (7.8), where the number of (triangular) facets amounts to 8 and the number of edges is 12. Without question, the density of each wedge can be assigned anew, allowing a local density variation in lateral direction (and vertically in future work). Therefore, for the local gravity of the DEM, we can extend Equation (7.8) by summing over z number of pentahedra-wedges, each with density ρ_z as

$$\vec{g}(\vec{x}) = G \sum_{z \in \text{wedges}} (-\rho_z \sum_{e \in \text{edges}=12} E_{ij} \cdot \vec{r}_{ij} \cdot L_{ij} + \rho_z \sum_{f \in \text{facets}=8} F_{ijk} \cdot \vec{r}_{ijk} \cdot \omega_{ijk}) \quad (7.9)$$

It is noted that the PM is not defined on vertices or edges of the polyhedron (here: pentahedra-wedges) (Meißenhelter et al., 2022). Therefore, to avoid problems in the gravitational computation, we systematically increase the Z-component of the evaluation point generally by 0.1 m, to avoid any interference with the solidified volume. At the same time, such offset is a realistic addition, e.g. accounting for a rover/lander surface offset carrying a gravimeter that can be expected to be (at least) some 10 cm above the surface ground for technical reasons. By applying this offset to all gravitation evaluations (g_{ref} , $g_{DEM=0}$, and $g_{DEM=S_T}$) the comparison remains consistently in the same location.

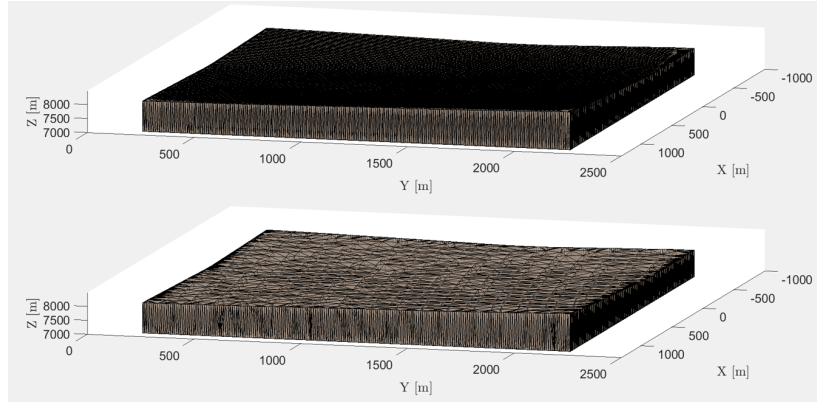


Figure 7.14: Comparison of the high-resolution (interpolated) and original surface at top and bottom, respectively. Clearly, the surrounding frame boundaries and centre points are identical.

7.5.2 COMPARING SMOOTH ORIGINAL AND INTERPOLATED SURFACE

As stated above, the here-presented WPM performs initially an interpolation on the lower-resolution original (regional) surface to be able to accommodate the DEM projection (Subsection 7.3.2). In this Subsection, the influence of this interpolation will be studied to assess how close the interpolated surface corresponds to the original surface coming from the global shape file.

For this, we compare the original surface with lower resolution (here 624 surface points coming from the original surface) and we have the (generally) higher resolution interpolated surface, created in preparation of the DEM application (i.e. scale factor $S_T = 0$). These two surfaces are shown in Figure 7.14. By visual judgement, both surfaces appear identical, yet with different resolution. Therefore, additional analysis is required to assess the influence of the interpolation in detail.

To compare the two surfaces, two different assessments are performed to indirectly judge the likeness of the two surfaces. First, the WPM gravitational evaluation is applied to both surfaces. As identical surfaces would yield identical surface gravitation, some expected variation serves as a measure of the similarity between the two surfaces. This analysis can be made for different resolution reduction values. Table 7.4 shows the resulting gravitation evaluated with the WPM on both the original and the interpolated surface. In addition, the ratio between both is provided.

Second, the volume of the created solid volume as visible in Figure 7.14 is computed and compared.

Regarding these two Tables, it is important to pause for a moment. The careful observer will have realised that the gravitation (Table 7.4) and the volume (Table 7.5) of the smooth (original) surface is not perfectly constant with a small yet distinct variation. Generally, one would expect this to be constant, as the WPM method

Table 7.4: Absolute WPM gravitation for local surrounding for different interpolation (interp.) resolutions compared to the WPM of the original (orig.) surface (surf.) to assess the influence of the DEM interpolation at scale factor 0. The ratio refers to the quotient of resulting WPM gravity g from original to interpolated surface. Note that due to the very high resolution, the original DEM resolution of 1/2 ppmer produced an graphics error. Resolution in ppmer.

Res.	Surf.	g_x [$10^{-5} m/s^2$]	g_y [$10^{-5} m/s^2$]	g_z [$10^{-4} m/s^2$]	g [$10^{-4} m/s^2$]	Ratio [%]
1/10	orig.	9.0973	1.6387	-4.2445	4.3439	
1/10	interp.	8.7912	1.5833	-4.2975	4.3893	101.05
1/20	orig.	9.0664	1.6445	-4.2439	4.3427	
1/20	interp.	8.8075	1.6369	-4.2895	4.3821	100.91
1/40	orig.	9.1043	1.6146	-4.2428	4.3424	
1/40	interp.	8.8837	1.6759	-4.2640	4.3578	100.35
1/80	orig.	9.0999	1.6468	-4.2444	4.3440	
1/80	interp.	8.8795	1.7324	-4.1303	4.2282	97.33
1/200	orig.	9.0739	1.6283	-4.2496	4.3484	
1/200	interp.	10.1451	1.6487	-4.0546	4.1828	96.19

Table 7.5: Overview of bounded solid volume for different surface resolutions. This allows a comparison of the original (orig.) and interpolated (interp.) surface.

Res. [ppmeter]	orig./DEM	V [$10^9 m^3$]	Ratio [%]
1/10	orig.	3.76145	
1/10	interp.	3.82236	104.57
1/20	orig.	3.75786	
1/20	interp.	3.81328	101.47
1/40	orig.	3.75308	
1/40	interp.	3.79108	101.01
1/80	orig.	3.75845	
1/80	interp.	3.79542	100.98
1/200	orig.	3.76455	
1/200	interp.	3.56247	94.63

is applied to the same (original) surface points. However, due to the variation in the resolution parameter, the building of the common boundary frame (Subsection 7.4.1) varies slightly. This is due to the fact that here the Z-coordinates depend on the nearest neighbouring point, which varies slightly for different resolutions. It is recalled that here the interpolated DEM grid is used with $S_T = 0$, of which the resolution depends on the resolution parameter.

Nonetheless, even if the variations in gravity and volume were larger, this does not pose a problem, since the WPM topographic reduction always is concerned by a comparison between smooth surface and surface with applied DEM at the same resolution.

In the same way, one must not make a mistake regarding the provided ratios. The agreement regarding both, the gravitation of the WPM solid volume alone and the volumes are generally in good agreement, especially for 1/40 ppmeter. Here, the gravitation varies only by 0.35 % and the volume by about 1 %. This is a very good agreement, showing that the surface interpolation in preparation of the DEM application is of very good quality. Therefore, the interpolated surface is well representative of the original surface, while some deviation due to the interpolation has been shown, as can be expected. These percentages must not, by no means, be compared to the percentages presented in Section 7.5, where the effect of the local topography on the resulting surface gravitation is assessed! While the variations are in the same order of magnitude, they must not be confused.

7.5.3 WPM GRAVITATION VERIFICATION

Before presenting results of the gravitation computation, we have to verify the working of the method. For this, we apply the WPM to a flat surface with DEM scaling $S_T = 0$ and create a cube out of individual pentahedra-wedges. The resulting gravitation is then compared to the gravitation of a simple continuous cube, computed with the PM. The validity of the PM applied to a cube was shown in Subsection 6.1.2 (Meißenhelter et al., 2022). For the WPM, a simple Wavefront shape file can be created with side length $a = 10,000m$ to represent the cube.

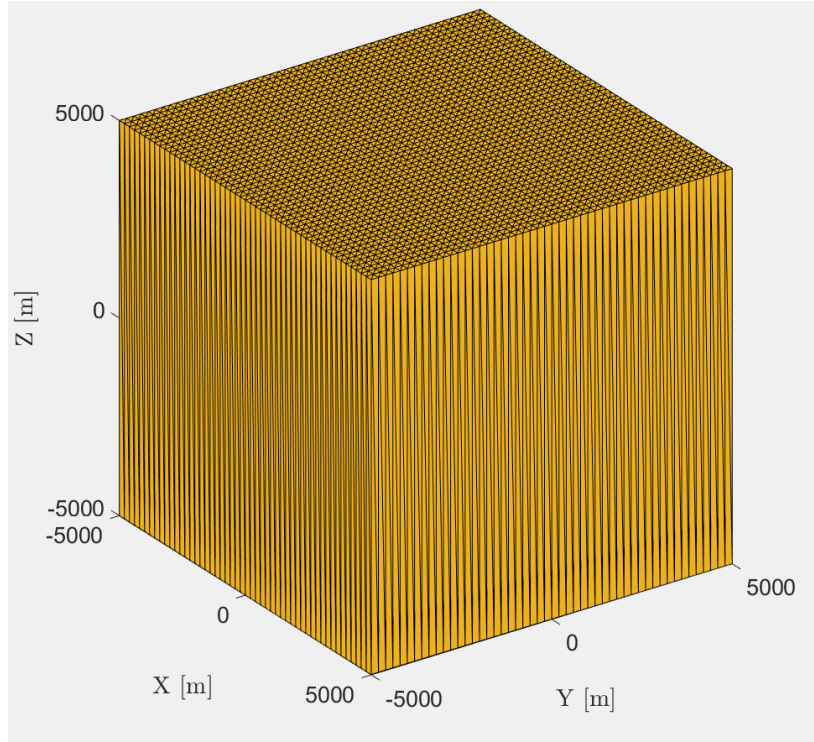


Figure 7.15: WPM applied to a created flat surface of $(10,000m)^2$ projected from $Z = +5,000m$ down to $Z = -5,000m$, creating a cube.

For the WPM cube, a surface is spanned from $-5,000m$ to $+5,000m$ in both X- and Y-direction. The surface is created and triangulated at $Z = +5,000m$ and the wedges are projected downwards to $Z_{bottom} = -5,000m$. This is shown in Figure 7.15. Here, the cube with top and bottom triangular facets parallel provide a special case, where the wedges are all likewise right prisms. The WPM cube surface is created by points every 200 m, thus creating $(10,000m/200m)^2 \cdot 2 = 5,000$ wedges. For the purpose of the cube, no framing using the common frame of the shape has been performed, since the flat cube surface ensures a perfect cut-off of the top surface (Subsection 7.4.1).

With both in place, it is possible to compare the gravity calculated by the PM, which is a closed-form analytical solution and therefore considered as the truth, and the here-developed and presented WPM, computing the gravitation as summation of the gravitation of all individual wedges. The evaluation point is done 0.1 m above the centre of the top facets, i.e. at $(0,0,5000.1)^T m$ and the density is arbitrarily kept at $1,860 kg/m^3$, like for Phobos. The resulting 3D-gravitation vector components of both methods are listed in Table 7.6.

Obviously, the X- and Y-components of the gravitation is expected to be zero. Indeed, both methods return near-zero values, which can be explained to be non-

Table 7.6: Gravity verification between PM and WPM using a cube. The theoretical X- and Y-components are zero and we provide the numerically created non-zero order of magnitude.

	$g_x[m/s^2]$	$g_y[m/s^2]$	$g_z[m/s^2]$
PM (truth)	0	$\mathcal{O}(10^{-19})$	-0.003223625925733
WPM (to verify)	$\mathcal{O}(10^{-17})$	$\mathcal{O}(10^{-17})$	-0.003223625925733
Ratio PM/WPM [%]	0	0.6052581804426	$100 - \mathcal{O}(10^{-14})$

zero due to numerical noise. Still, the non-zero values are larger for the WPM compared to the PM, which can be expected given that within the WPM the PM is executed (here) 5,000 times, i.e. individually per pentahedron-wedge, and thus accumulating numerical noise. Clearly, the Z-component computed in both methods is in agreement, verifying the WPM gravitation.

7.6 GRAVITATIONAL INFLUENCE OF THE LOCAL TOPOGRAPHY

With the gravitational computation (Subsection 7.5.3) of the WPM being verified and the quality of the surface interpolation assessed (Subsection 7.5.2), the new Wedge-Pentahedra Method (WPM) can now be used to test the influence of the local topography on the surface gravity. For consistency, this will be done for the Phobian surface point used throughout this work (Table 7.2).

Regarding the gravitational influence, we present here the assessment of two parameters, the scale factor S_T and the DEM resolution discussed in the previous Subsection. Firstly, the resulting gravitation at the evaluation point (Equation (7.6)) and the relative change due to the added topography (Equation (7.7)) is shown in Table 7.7 for different terrain scale factors S_T . Here, the ground resolution is kept constant at 10 ppmeter.

For the scale factor 0, the X- and Y-components are near 0 due to the applied rotation. The components are still non-zero due to the applied interpolation and numerical artifacts of the applied rotation. The scale factor 0 approximates, with some approximation error (Subsection 7.5.2), the original surface. As can be expected, the changes in gravity due to the topography increase with increasing scale factor in all three dimensions and magnitude. In this location, the ratio decreases, making the DEM gravitation smaller than the original gravitation. Here, this is due to the shape of the DEM, and, obviously, the increasing change can be inverted for a different DEM or a different evaluation location. At a scale factor of 750 and 1000, no result was obtained, which is why the scale factor $S_T = 600$ was added. It was observed that for some other tested evaluation point on Phobos, it is still possible to obtain results for $S_T = 1000$ in some locations. This can be explained by the fact that the here-studied “in-crater” location has a negative global terrain profile (depression), and a DEM of too large amplitude will go beyond (“below”) the lower wedge projection elevation, causing the method to.

Table 7.7: Absolute gravitation for different terrain scale factors S_T for the WPM.

The ratio κ provides insights on the resulting gravitation as a function of the increased amplification of the local artificial terrain, compared to the original, smooth surface gravitation.

S_T	$g_x [m/s^2]$	$g_y [m/s^2]$	$g_z [10^{-3} m/s^2]$	$g [10^{-3} m/s^2]$	$\kappa [\%]$
0	-1.4472E-08	2.6931E-09	-5.7955	5.7955	100
1	3.1993E-07	-1.4756E-07	-5.7955	5.7954	99.9995
10	3.5863E-06	-1.3576E-06	-5.7950	5.7950	99.9915
25	9.0795E-06	-3.2298E-06	-5.7939	5.7939	99.9719
50	1.8070E-05	-6.2595E-06	-5.7914	5.7914	99.9289
100	3.5117E-05	-1.0273E-05	-5.7819	5.7821	99.7678
250	8.2717E-05	-1.9609E-05	-5.7537	5.7544	99.2901
500	1.3694E-04	-3.0724E-05	-5.6926	5.6944	98.2547
600	1.5268E-04	-3.3594E-05	-5.6696	5.6718	97.8654
750	no result				
1000	no result				

In the same fashion, we investigate the influence of the resolution at which the DEM is applied (Table 7.8). This is similar to the study presented in Table 7.4, however, there the DEM scale was kept constantly at $S_T = 0$. Here, we chose to fix the terrain scaling at $S_T = 250$, with a deviation of about 0.7 % of the gravitation due to the added DEM (Table 7.7). Additionally, we track the computation time of the WPM evaluation only (thus excluding the initialisation), to demonstrate the influence of the resolution on the computation time, driven by the number of pentahedron-wedges. For example, increasing the ground resolution from 1/200 ppmeter (19.8 s) to 1/20 ppmeter (119.2 s) lead to an increase in computation time of a factor 6. Regarding the computation time tracked in MATLAB, the used hardware (CPU) is an Intel® Core™ i7-8850H.

For the resolution, other than for the scale factor, the result is less unambiguous. Generally, one could expect that with lower resolution, and thus less surface detail, we will approximate the original surface with fewer surface points spanning the DEM surface. Looking at the two highest resolutions, the gravitation ratio κ indeed increases from 1/10 ppmeter to 1/20 ppmeter towards 100 %. However, going then to 1/40 ppmeter, the ratio decreases again, which can be explained by the fact that some topographic features that have an decreasing influence on the gravitation, e.g. material “above” the evaluation point such as hillocks, might appear smaller or even not be included at all at the smaller resolution. In other words, these small features lie between the considered DEM points and therefore disappear. Then, with further decreased ground DEM resolution, indeed the gravitation ratio increases again towards 100 %, meaning that fewer details, here decreasing the resulting gravitation in sum, are represented on the lower-resolution surfaces. The second coarsest resolution of 1/80 ppmeter is indeed equal to the original shape file resolution of Phobos. Yet, while having the same resolution as the original shape, the surfaces are not identical and thus $\kappa \neq 100\%$, as even with this resolution, the DEM is adding local terrain, and thus altering the local topography. This is even the

Table 7.8: Absolute gravitation for different DEM resolutions compared to the WPM of the original surface to assess the influence of the DEM interpolation at scale factor 250. Computation time T for comparison of different resolutions.

Res. [ppmeter]	g_x [m/s^2]	g_y [m/s^2]	g_z [$10^{-3} m/s^2$]	g [$10^{-3} m/s^2$]	κ [%]	T [s]
1/10	8.2717E-05	-1.9609E-05	-5.7537	5.7544	99.2901 ^a	454.7
1/20	8.2255E-05	-2.1151E-05	-5.7548	5.7552	99.3046	119.2
1/40	7.5848E-05	-1.8113E-05	-5.7500	5.7505	99.2227	34.4
1/80	7.8634E-05	-1.5219E-05	-5.7634	5.7639	99.4552	21.6
1/200	5.4792E-05	-1.5634E-05	-0.5769	5.7700	99.5545	19.8

^aCompare to Table 7.7, for reference.

case for 1/200 ppm, having a resolution lower than the original file, yet presenting added DEM features. Generally, while here $\kappa < 100\%$ it is well possible that the added DEM results in $\kappa > 100\%$. This solely depend on the macroscopic (shape file) and microscopic (DEM) regional circumstances.

7.7 GRAVITY TRAVERSE SIMULATIONS USING THE WPM

In the previous Sections, the novel Wedge-Pentahedra Method (WPM) has been developed around a single, centralised evaluation point. In the context of current and future Solar System exploration missions that consider a rover in the mission architecture (e.g. Subsection 1.3.1), a Traverse And Gravity Simulator (TAGS) has been developed. By moving the evaluation point along a possible rover traverse, it is possible to assess the variation in surface gravity along the traverse. This is presented in Subsection 7.7.1. The application of this simulator is not only limited to SBs, and so the WPM and TAGS has been extended to work for a “flat-Moon”, considering the actual topography around the Apollo 17 landing site and comparing the results to in-situ data (Subsection 7.7.2).

7.7.1 TRAVERSE AND GRAVITY SIMULATOR

This Subsection is an adapted excerpt from Noeker and Karatekin (2022a).

Starting with the WPM, the simulator has been developed such that the evaluation point (“rover position”) is varied per simulation step, each repeating the WPM gravity evaluation (Section 7.5) in the new location. Again, the 3D-orientation of the gravitation vector is computed, and based on this the variation of the vector orientation with respect to the initial reference gravitation along the traverse is plotted in the simulator. Likewise, the resulting gravity g_{res} and the “vertical” component g_z along the rover traverse is plotted, illustrating the gravity variations over the non-spherical body surface, including the effects of the local (artificial) topography.

An exemplary traverse simulation on Phobos for the terrain scale $S_T = 500$ is shown in Figure 7.16. Starting in the centre of the terrain, the traverse steps between the individual evaluation points are $+50m$ in $+X$ -direction, and $-50m$ in $-Y$ -direction for a total of 12 evaluation points.

By varying the terrain scale factor S_T , the influences of the terrain with variable amplification on the resulting surface gravity can be assessed. While the WPM is only concerned with *gravitation*, the TAGS is purposely referring to gravity, as the simulator will allow to include additional (dynamical) effects to the resulting gravity (Section 5.1), such as the contributions from rotation, libration and tides in future work, as required.

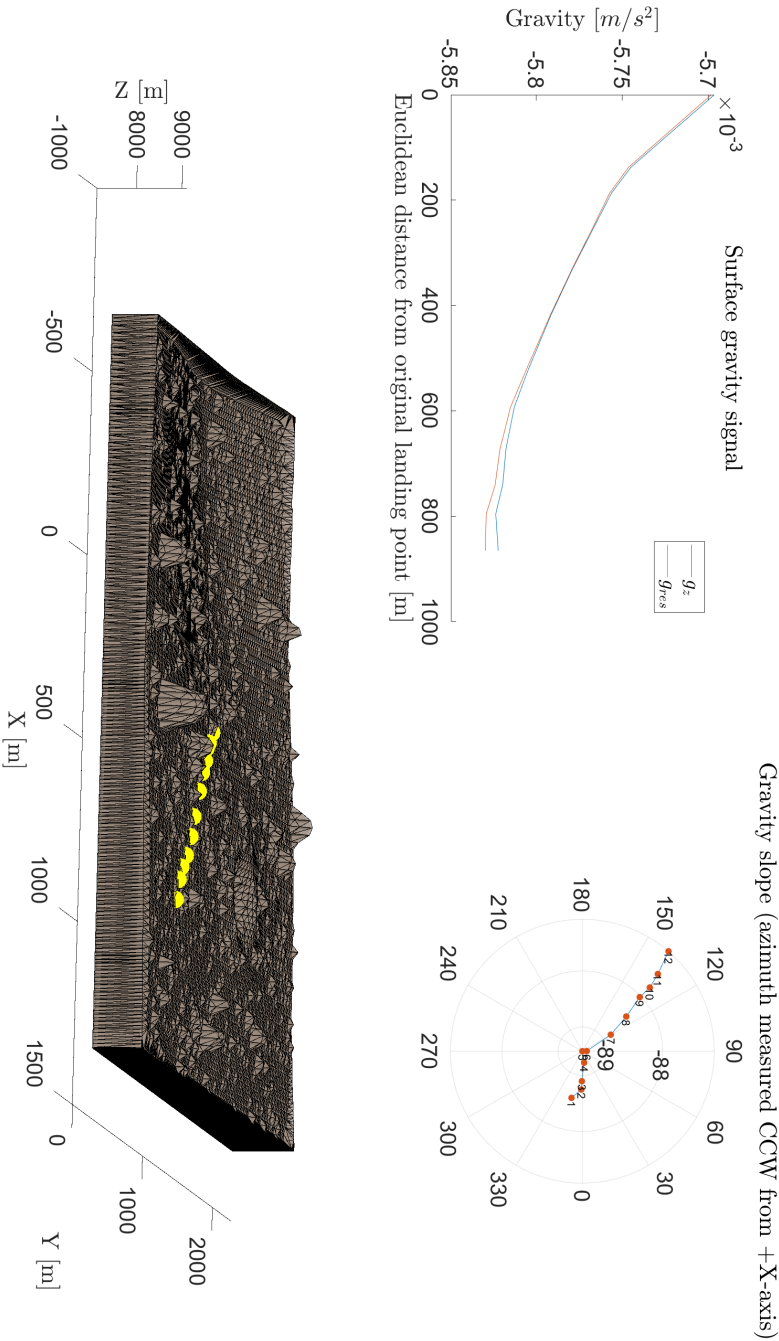


Figure 7.16: TAGS output with $S_T = 500$. The general gravity slope direction flips along the traverse. While at the starting point the slope direction points to an azimuth of about 345 degrees, the azimuth changes to about 140 degrees at the end of the traverse.

Table 7.9: Apollo 17 TGE relative gravity readings with corresponding measurement coordinates on the Moon.

Station	Δg [mGal]	Lon. [$^{\circ}E$]	Lat. [$^{\circ}N$]	Radius [m]
LM	0	30.77168	20.19091	1,734,774
Station 1	-4.6	30.78653	20.15690	1,734,765
Station 2	-50.8	30.53033	20.09916	1,734,953
Station 2A	-40.5			
Station 3	-16.5	30.56556	20.17402	1,734,901
Station 4	-4.5	30.62427	20.21532	1,734,854
Station 5	-10.6	30.72611	20.18572	1,734,809
Station 6	-36	30.79997	20.28993	1,734,835
Station 8	-31.6	30.87753	20.27663	1,734,777
Station 8	-32.5	30.87753	20.27663	1,734,777
Station 9	-12.7	30.83264	20.23141	1,734,749
Station 9	-12.9	30.83264	20.23141	1,734,749

7.7.2 EXCURSUS: TAGS TRANSFER TO THE MOON (APOLLO 17)

This Subsection is an adapted excerpt from Noeker et al. (2022b).

As has been discussed regarding the WPM, in principle, it is possible to apply the here newly developed method to spherical bodies, such as Mars, the Earth, and the Moon. In this Subsection, a Lunar traverse is simulated, following the skid marks of the Apollo 17 (A17) rover, using the TAGS introduced in the previous Subsection.

For this, we have obtained the topographic data from JMARS (Christensen et al., 2009) taking the Blended LRO/LOLA and SELENE/Kaguya DEM data set². The spherical coordinates of the measurement stations reported in Talwani et al. (1973) are obtained from Haase et al. (2019) and converted by simple transformation into Cartesian coordinates. The measurement stations and the relative gravity difference as measured by the Traverse Gravimeter Experiment (Subsection 2.3.1) are presented in Table 7.9³.

For the WPM gravitation evaluation, the application of the topography (Section 7.3) for small bodies (SBs) has been adapted to the Moon. Here, a flat initial surface is considered on which the lunar topography is applied (i.e. flat-“Earth” assumption). The coordinate system rotation is changed such that the initial Apollo 17 landing site (LS) vector, i.e. pointing from the centre of the Moon to the surface point, is used. The applied topography, with a surrounding of $40 \times 40 km^2$, after the rotation is shown in Figure 7.17 (a). Following this initialisation, the solidification of the WPM in preparation of the gravitation evaluation is shown in Figure 7.17 (b). As is well visible, the TAGS (Subsection 7.7.1) is not limited to a linear traverse, but can be adapted easily to specific coordinates (Table 7.9).

²From https://astrogeology.usgs.gov/search/details/Moon/LRO/LOLA/Lunar_LRO_, accessed on 17.12.2021.

³Station 2A is not reported by Haase et al. (2019) and therefore, excluded to avoid inconsistencies in the evaluation location.

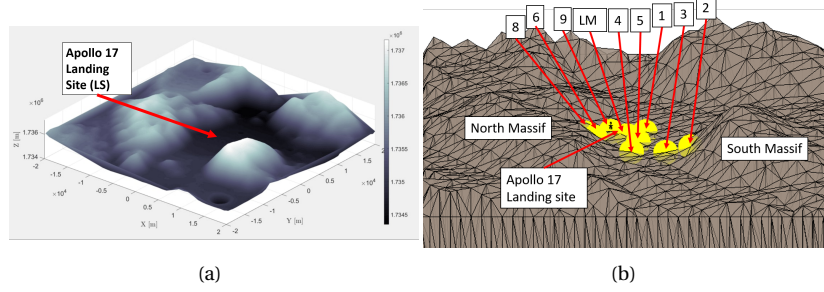


Figure 7.17: (a) Imported A17-LS-surrounding rotated surface, with reduced resolution of every 20th point and a total area of $(40\text{km})^2$. Note that here the Y-axis aligns with the Taurus-Littrow Valley, the North Massif is on the -X and the South Massif on the +X side. (b) WPM volume from solidified surface, with measurement stations (yellow). Compare to (a) for different view angle.

Finally, the surface gravity can be computed. Here, the reference gravity is computed differently compared to the SB case. The starting point is the (positive) reference gravitation g_{ref} of the spherical Moon, to which the (negative) free-air correction g_{FA} and the positive Bouguer reduction g_B is added. Further, the (negative) topography g_{topo} contribution is added, which comes from the WPM but here only considers the material (terrain) *above* the measurement location. Finally, the (negative) dynamic influences due to (Earth) tides a_{tidal} and rotation a_{rot} is added, yielding Equation (7.10).

$$g_{res} = g_{ref} + g_{FA} + g_B - g_{topo} - a_{tidal} - a_{rot} \quad (7.10)$$

In a first experiment, only the Bouguer anomaly, consisting of free-air gradient, Bouguer correction (plate) and topographic correction (here: WPM), are considered for a *relative* comparison. The relative changes have been reported by the TGE and can therefore be directly translated to the surface gravity difference between the measurement locations. Therefore, the reference gravity g_{ref} can be neglected for this relative comparison, as well as the dynamical effects being tides and rotation. Using initially a homogeneous density for both terrain and Bouguer plate of $\rho = 2,000\text{kg/m}^3$ (Talwani et al., 1973), the adapted TAGS result is shown in Figure 7.18. Clearly, a misfit between the simulated and measured surface gravity data is visible. By varying the constant density using a simple least square approach, it was not possible to entirely remove this misfit. A, geophysically little meaningful, better result was obtained for $\rho = 750\text{kg/m}^3$. However, this misfit is supportive of the conclusions drawn by Talwani et al. (1973); Urbancic et al. (2017) indicative of a subsurface anomaly. So far, this subsurface anomaly has been only approximated with a flat topped surface and in 2D-along the valley floor. Therefore, future work is required to extend the here-presented TAGS with the option to include a, 2D- or 3D-, subsurface anomaly. Like for the inhomogeneities (Section 6.5), this anomaly could be added again with differential density. Using the WPM also for this, it will

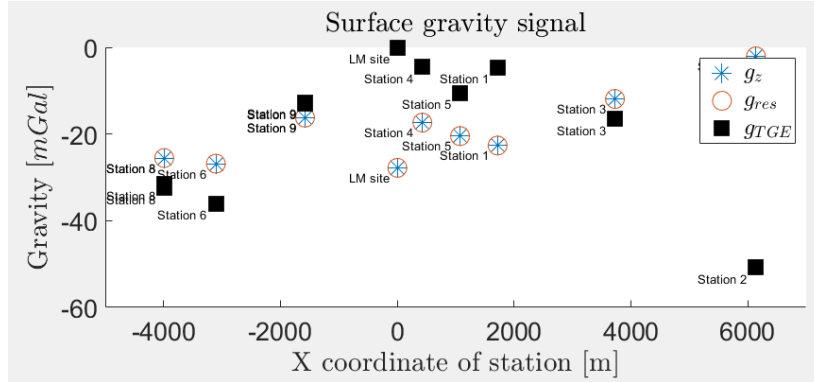


Figure 7.18: Simulated relative gravitation results for constant density $2,000 \text{ kg/m}^3$ with respect to LM site measurement. TGE denotes Apollo 17 data, g_{res} and g_z resulting and vertical gravitation, respectively.

be possible to better constrain the anomaly, e.g. by allowing to also represent the anomaly with the actual surface on its top, rather than a flat surface approximation.

7.8 DISCUSSION LOCAL GRAVITY

In this Chapter, research question (III) has been discussed. The Wedge-Pentahedra Method developed and presented in this work provides an innovative and novel approach to include local topography in gravitation modelling of small bodies. To date, topographic corrections on (surface) gravimetric measurements only used approximating topographic corrections, if applied at all. The here-developed Wedge-Pentahedra Method (WPM) provides a new technique for topographic reduction. Possible applications range through the entire family of SBs and are not only limited to Phobos. One possible future application target of the WPM will be Dimorphos, where the highest terrain resolution can be expected around the DART impact zone (Subsection 1.3.2). Here, an improved understanding of the local topography will be valuable information in the context of the GRASS surface gravity data interpretation.

Moreso, as has been shown in the previous Section, this can even be extended to larger spherical bodies such as the Moon or Mars. On the example of Phobos, it has been clearly shown that indeed local topography can have a relative influence on the surface gravity that cannot be neglected. Depending on the terrain amplification, the sole effect of the local surrounding topography has a non-negligible effect on the surface gravitation, important for surface gravimeter (Part I) or landing/surface operations. For example, in the studied test case, the deviation in gravitation due to the terrain amounts to 0.7 % for a terrain scale factor $S_T = 250$. So, this topographic effect would be comparable to the minimum relative GRASS target of 1 % of the local gravity. The extend of the topographic effect can henceforth be computed with the WPM, allowing to compare it to the gravimeter accuracy.

Without question, the above-mentioned amplification of the DEM terrain is arbitrary, and larger scaling factors might, in some locations, be unrealistically large. With very good observational knowledge of the inner moon in areocentric orbit, the presented topography might be exaggerated for larger scale factors. On the contrary, considering a similar celestial body with much less known detail about the shape, large topographic variations as shown in this Chapter cannot be ruled out. In any case, it is well possible to limit individual, unrealistically large features in the (artificial) DEM data set, by introducing cut-off-criteria or proportional scaling in future work. A clear statement of what is still realistic in terms of terrain amplification, and thus gravity, will only be possible once real terrain data becomes available, e.g. from the MMX (Subsection 1.3.1) and DART/Hera (Subsection 1.3.2) missions.

Therefore, in absence of high-resolution ground truth data, the results on local topography should be understood as a priori estimates of possible topographic influences on the surface gravity. Like this, mission planning could be aided, e.g. in estimating uncertainties in the gravity for landing trajectories or rover stability. If high resolution terrain data of the selected/considered landing sites becomes available just before descent, final corrections might be applied and post-mission data analysis will certainly be able to include WPM corrections, which is valuable not only for surface gravimeters. Here, the analysis and variability of the DEM resolution (res.) parameter is an interesting asset, e.g. when formulating terrain/shape observation resolution requirements. The effect of the ground resolution on the computation time was also studied, showing that for example an increase in ground resolution by a factor 10 increases the computation time by a factor 6. In all studied cases, the overall computation time was in the order of minutes for a single run, not imposing constraints here.

In addition, with the presented modification, the WPM has been implemented in the Traverse And Gravity Simulator (TAGS). Here, the WPM is executed per traverse step, and the computation time will become more important, and future work might improve the computation time, or perform a priori computations of the gravity before starting the traverse. The traverse is not only important for the above mentioned rover surface operations on small bodies, but likewise interesting for post processing of in-situ data taken by a mobile surface gravimeter such as GRASS. This has been well illustrated for the analysis of the Apollo 17 TGE gravimetric data. Here, again, the need for an option to include subsurface inhomogeneities has been underlined. Future work can either implement this by locally varying the density per-wedge, or by superposition of subsurface anomaly with differential density to which the WPM is applied separately, before combining the two resulting surface gravitations.

In Subsection 6.5.2, the problem of altering the total body mass by introducing inhomogeneities has been discussed. Strictly speaking, the WPM with the addition of topography will also alter the total body mass, as topography exceeding the original shape file boundary will increase the mass and “negative” topography will reduce it. Nevertheless, the alteration of the total mass can be neglected in the here-shown case for Phobos, whereas this aspect should not be neglected in future work, especially regarding smaller bodies, where this effect might become significant and

adaptations of the density will again be required.

7.9 SUMMARY CHAPTER 6

This Chapter focused on local contributions due to terrain on a local scale, e.g. in the direct proximity of a surface gravimeter, such as GRASS. In a first step, the existing concepts for topography corrections have been reviewed (Section 7.1), showing that to date only approximations exist, and hence a perfect surface representation is not possible using the identified tools. Based on this identified lack, a novel concept being the Wedge-Pentahedra Method (WPM) has been developed. Starting with a global shape file of Phobos and an artificial digital elevation model (DEM) introduced in Section 7.2, the latter has been merged with the former in Section 7.3. This can be considered as a hierarchical approach, where locally a much higher resolution terrain knowledge is combined with a less precise but global shape file. With the superpositioned high-resolution surface in place, a distinct volume is created in Section 7.4, creating a multitude of wedges (i.e. pentahedra). Subsequently (Section 7.5), the resulting surface gravitation is computed per-wedge using the polyhedral-method (PM) for the DEM surface and a comparable original surface with identical boundary conditions. Comparing both allows to assess the change in surface gravitation to be expected by the local topography. With the method in place, the actual influence on an exemplary surface point on Phobos is assessed in Section 7.6, clearly showing that the influence of local terrain cannot be neglected on small bodies per se. The WPM is carried forward by integrating it in a (rover) traverse simulator, which is tested on both Phobos and the Moon (Section 7.7). Finally, the local gravitational environment is discussed in the previous Section before this summary closes this Chapter.



This third Chapter of Part II concludes the work regarding the numerical gravity simulation in the context of Solar System gravimetry with a special focus on small body gravimetry, as is planned for the ESA Hera mission. In Chapter 5, different gravitation modelling methods have been reviewed and presented, and the fundamental concepts of the main contributions to the (resulting) gravity were discussed. A selected number of these methods were then compared in Chapter 6, alongside with an analysis of surface gravity to be expected on possible GRASS gravimeter targets. Finally, this Chapter developed the novel WPM, specifically aiming at the local influence of topography surrounding an extraterrestrial measurement location. In the following final Part III, future work and applications (Chapter 8) are discussed, before this thesis is concluded in Chapter 9.

III

CONCLUDING PART

8

FUTURE WORK AND APPLICATIONS

This penultimate Chapter looks forward on future work based on, and in continuation of, this PhD thesis. First, future work regarding the GRASS gravimeter, including the developed rotation mechanism is discussed in Section 8.1. Second, a possible continuation of the here-presented gravity modelling work is proposed in Section 8.2. The synergies between the two parts in future work are finally presented in Section 8.3.

8.1 GRASS SPRING-BASED GRAVIMETER AND ROTATION MECHANISM

In the context of small body exploration and planetary defense, the spring-based gravimeter has been identified as the best suited sensing principle in Chapter 2. The literature reviewed in that Chapter provides a solid basis also for future trade-offs. For example, future landing missions, with much larger mass, volume and power budgets, or even the presence of astronauts, might result in the selection of another gravimeter principle out of the types studied and presented in this work.

The technical development of the GRASS gravimeter (Chapter 3) forming part in this work, was characterised by a very large degree of adaptability to different gravity environments. Together with the analytical electromechanical gravimeter model (Chapter 4), this lays out the avenue for future application cases of the GRAvimeter for small Solar System bodies (GRASS). The analytical instrument model can be understood as a sizing-tool, allowing to directly assess different instrument configurations for any assumed reference gravitation. Like this, it will be possible to directly adapt the identified design parameters such that e.g. the requirements from the sensing electronics are met. With this, it will be for example achievable to directly estimate the overall required instrument length based on the required (bending) spring length. This information can directly allow a first estimation, if the adapted GRASS payload could be accommodated in a lander under study, without going into too much detail. In the same fashion, a first orientation on the demands to the displacement transducer (Section 4.2) electronics can be obtained, and the accuracy of sensing gravitation can be directly estimated, based on the ongoing

sensing electronics characterisation and testing. Furthermore, the analytical tool might also be extended in future work to return additional instrument figures of merits, such as the dynamic range.

Not only the (adaptable) gravimeter spring and transducer were developed for the GRASS gravimeter, but also the GRASS rotation mechanism for the sensor head rotation. While it has been developed for the GRASS gravimeter and the Juventas CubeSat as part of the ESA Hera mission, it can be adapted to other use cases in space. For example, payloads likewise requiring sensor head rotation or pointing can make use of the here-presented system. In principle, this could even allow to replace/augment the current gravity sensor head by a different working principle, while maintaining the rotation mechanism. Together with the larger prototype design (Appendix A), a wide range of sensor head sizes is covered, and the presented mechanism can be adapted or scaled for other sensor head dimensions. Further miniaturisation of the presented CubeSat mechanism will be challenging, but achievable when identifying corresponding smaller solutions for dimension-drivers, e.g., the gear-motor or slip ring. As discussed above, regarding the gravimeter spring, the total mechanism length might be changed for other sensor head lengths. While in the gravimeter design, a two-axes setup allows the 3D-vector reconstruction, other applications might only require a single rotation mechanism, or a three-axis mechanism. Both is well achievable using the presented rotation mechanism, with minor modifications (Noeker et al., 2022c).

8.2 GRAVITY MODELLING

For the gravity modelling, there are three main roads opening up for future work, pathed by the present thesis. Generally, the (top-level) gravity modelling is a first key step before adapting the GRASS gravimeter to any other target. This does not only demand the definition of a reference gravity, but also possible variations, e.g. due to dynamical effects (Section 6.2) or a rover traverse (Section 7.7).

The first continuation concerns the global gravity modelling (Chapter 6). The here-developed tools are not only valuable for future missions as mentioned above, but will likewise form the basis for GRASS data analysis. While the (surface) gravity inversion was beyond the goal and scope of this work, first data interpretation could be done by fitting forward modelling results to the measured surface gravity. By simple least-square method or Monte Carlo analysis, the homogeneous density best fitting the surface gravity measurement in a specific location can be directly identified. If two regions of distinct dimensions and different density were found, i.e. from the JuRa radar onboard the Juventas CubeSat (Herique et al., 2020), the inversion of a GRASS reading could still be implemented, under the condition that the total mass of the body, resulting from the mass of the two regions, is kept constant.

As has been shown in Section 6.5, the comparative work between different gravitation computation methods is already being continued, and first results regarding the inhomogeneities have been presented. Here, many more test cases will be developed and compared in future work. One most interesting result will be to push

the limit so far that the added inhomogeneities can no longer be handled appropriately with the polyhedral model, e.g. due to prohibitively long computation times or other constraints. In this case, possible solutions, such as the combination of the polyhedral method and e.g. the superposition of mascons, again using the introduced *differential* density, can be implemented and tested.

Given the entire novelty of the developed Wedge-Pentahedra Method (WPM), this concept might provide the largest number of opportunities for future work and applications. Clearly, the results presented in this thesis (Chapter 7) are just one snapshot of a multitude of possible test cases. The presented analysis shows only the results for one exemplary surface location on one exemplary body, one artificial DEM and one local subset thereof. All these parameters can be studied further by varying them. Without question, the WPM therefore opens up a manifold of future applications across the Solar System. For example, the WPM can be used further to assess the gravity slopes on the surface of a body, i.e. the orientation of the gravity vector with respect to the local surface normal. A location on the anti-Mars side could be studied as a possible landing region (facilitating the direct communication with the Earth).

Likewise, while kept constant in this work, the density per-wedge can be varied to assess the influence of variation of the density. This is not limited to the “horizontal” variation, but can be extended vertically in future work. The variation of the local density is solely limited by the DEM resolution in lateral direction (determining the number of wedges) and by the computation time in the subsurface direction (“vertical”). This vertical variation, stacking different right prisms or wedges above each other, will be done in future work. One interesting example here could be an evaluation inside the Stickney crater, where a denser compaction layer could be tested above less dense material. Moreover, the created DEM surface solid can be exported and used for illumination assessments, that will be affected by shadows of the local terrain.

Possible applications of the here-developed method include, but are not limited to, small body landing (bouncing) analysis, surface operation planning, illumination assessment (shadowing, solar power etc.), or gravity science preparation. Initially targeted specifically at small, non-spherical bodies, the scope of the WPM application can be extended to larger, spherical bodies, as has been already done for the Moon in Subsection 7.7.2. Without problem, this can likewise be adapted to Mars, and the results could initially be compared to the findings by Lewis et al. (2019), which are based on in-situ surface acceleration data. Likewise, the WPM could be applied to the DEM surrounding the Huygens landing site on the Saturnian moon Titan. Generally, good surface gravity interpretation will not only aid the science, but can be equally important in preparation of in-situ resource utilisation (ISRU) or the detection of intact subsurface lava tubes, especially on the Moon and Mars.

In a more forward looking way, the same principle of the WPM could also be extended to the 3D-space. For example a small body could be represented perfectly by a multitude of small polyhedra. Each polyhedron could be assigned an individual density, and the gravitational field per polyhedron could be computed.

Like this, the advantages of the polyhedral method (PM) with highest accuracy, and of the mascon method allowing to vary the internal density, could be combined.

Finally, regarding the Traverse and Gravity Simulator (TAGS), this tool will also benefit from the above mentioned variations in density to be implemented in the WPM. On the Moon, the tidal and rotational influences were included, but solely on a static level. In future developments of TAGS, it is of course possible, to also consider the temporal variations of the surface gravity due to the variable dynamical contributions to gravity (Section 5.1), and to include this in the gravity simulator. This is especially interesting for small bodies, where the dynamical effects would be relatively large compared to the resulting gravity and the variation would be measurable by the GRASS surface gravimeter. Without question, it will be possible to replace the analytical approximations of the dynamical effects with more advanced studies beyond the goal and scope of this work, and to include them in the resulting surface gravity signal computed by TAGS. Obviously, the simulator could then also be used for a static lander such as the Juventas CubeSat, by solely simulating the temporal gravity variations in one (fixed) location.

Next to roving vehicles, stationary landers such as the discussed Juventas CubeSat can carry a gravimeter in future missions to increase the science return. As already discussed in Subsection 1.3.3, one possible target is the close-approaching NEA Apophis, where a possible mission could include a landing element. Likewise, other small landers as part of Solar System exploration missions could be considered to include a surface gravimeter.

Finally, disconnected from ground, it will in principle be possible to use the GRASS gravimeter as an accelerometer. With sufficient sensitivity, non-gravitational forces could be measured in flight. In deep-space, this could be the solar radiation pressure (SRP), or atmospheric pressure, e.g. on spacecraft or space stations around atmosphere-holding planets, including Mars, Venus and the Earth. Regarding the latter, it had been proposed to test GRASS onboard the International Space Station (ISS) to measure the atmospheric drag in Earth orbit (Noeker et al., 2020).

8.3 SYNERGY

In the overall frame of this work, future work will also benefit from bridging the two presented parts. In this work, the instrument sizing has been done using (fixed) reference gravity values. Likewise, it has been shown that specifically for non-spherical bodies, the surface gravity varies significantly over the surface. Future work could build a synergy by taking the minimum and maximum (expected) surface gravity as input to the analytical electromechanical instrument model. This could generate an output-band, based on the smallest and largest spring deflections to be expected. Like this, the sensing electronics design could be better prepared to measure over a larger, yet realistic range, rather than only considering a nominal (reference) gravity.

This synergy could be expanded further, such that the electromechanical instru-

ment simulation (Chapter 4) could be implemented into the Traverse And Gravity Simulator (TAGS, Subsection 7.7.1). So not only the gravity would be simulated along a rover traverse, but likewise the direct instrument responds could be displayed in the TAGS with the simulated instrument output signal.

Another clear synergy regarding possible future missions, and especially regarding rover-based gravimetric surveys, is the proposition and the preparation of future GRASS instrument adaptations for other celestial targets. For example, using and advancing the presented Wedge-Pentahedra Method (WPM, Chapter 7), it will be possible to better understand the small, yet distinct gravity variations along possible rover traverses. Taking this information, and combining it with the capabilities of the sensing electronics, it will be possible to directly feed this into the electromechanical instrument model (Chapter 4) and to adapt (e.g. upscale) the design parameters to arrive at the required sensitivity. Next to SBs, possible targets of such adapted GRASS gravimeter versions could be the Moon and Mars. Here, the Lava-Tube iNvestigAtion (LA-TUNA) mission concept (Noeker et al., 2021a) was proposed, where differential gravity measurements from above and below the lava tube roof (i.e. the material above the lava tube and below the surface) would yield a differential measurement, providing specific insights in the Lunar top-surface layer only. This could of course be extended to lava tubes on Mars (and the Earth).

9

CONCLUSIONS

The presented PhD thesis was focused on Solar System gravimetry, and more specifically on small body surface gravimetry in the context of the GRASS gravimeter development for the ESA Hera mission. This instrument will be the first-ever extraterrestrial surface gravimeter experiment on a small body. The presented PhD thesis contributed to this novel instrument development, from early prototyping to the final instrument flight model (FM), scheduled for launch in 2024. By approaching the problem from two sides, the technical and the science sides, this work provides a very broad scope. A total of six research questions has been answered in a structured way, where both aforementioned sides concern three questions each. The first three research questions concerning the payload were addressed in Part I and read:

- (i) *Which of the existing gravimeter working principles is best suited for space gravimetry in the low-gravity environment, when specifically considering a robotic, non-human Solar System exploration mission with tight constraints on power, volume and mass, and which adaptations are still required for the selected principle?*
- (ii) *How can the classical 1D-gravity measurement be augmented for a 3D-measurement in the most compact way, assuming the lack of a levelling mechanism on the lander, no prior precise knowledge of the surface gravity vector on small bodies and the special requirements of a space exploration mission?*
- (iii) *What is the expected electromechanical behaviour of the space gravimeter concept, and how can the sensitivity be optimised for the (very) small gravity environment?*

Overall, in Part I, the contributions made to the GRASS gravimeter development have been presented. In the context of research question (i), the different gravity sensing methods have been introduced and discussed. Here, a very wide range of different measurement techniques has been found and their figures of merit were presented. The identified precisions ranged from 0.01 mGal down to $0.05\mu\text{Gal}$, and the accuracies from 5 mGal down to a few μGals . This was put into context of the GRASS gravimeter minimum accuracy target of 1 % of the (assumed) Dimorphos gravity, thus 0.045 mGal . Finally, for the Hera mission and this work, a spring-based gravimeter has been identified to be best suited, while other principles might be chosen for future missions with different constraints and/or science requirements. As part of the instrument development, a mechanical design with a very

strong demand to minimise the overall payload (i.e. mass smaller than 380 grams and volume smaller than $(100\text{mm})^3$), while allowing to obtain a 3D-measurement has been worked upon in the context of research question (ii). The gravimeter spring was optimised for small gravity environments (i.e. 4.5mGal), by maximising the deflection while maintaining structural stability. The deflection of the spring (in the order of hundreds of nanometers), being proportional to the present gravity, will be measured by a displacement transducer, entirely built around the gravimeter spring with an extremely-compact design, still offering room for sensing electrodes and electronics. The absolute capacitance has been computed to range from 7.6 to 8.6 pF (depending on the machining tolerances), and the expected changes to be sensed for a 1 % change in gravity are in the order of 300 – 400 aF. A novel rotation mechanism for 3D-measurements and bias rejection complements the triad, offering reliable rotation and pointing in the harsh deep space environment, making the space mechanism a possible basis for future payloads in space, demanding sensor head rotation and/or pointing. Using the rotation mechanism, the classical 1D-measurement is augmented to the 3D-vector measurement. For research question (iii), the various design parameters of the electromechanical design have been implemented into an analytical model, taking into account the spring-deflection, the differential capacitance signal, and one possible output signal in the form of a duty cycle. The computed deflections and capacitance (changes) have been repeated above. It was found that the present analytical model provides a valuable, top-level description of the GRASS instrument. Here, one specific advantage is to use the tool directly for future adaptations of the current gravimeter layout, which will be required for different gravimeter targets.

The main contributions of this first Part can be summarised as:

- The extensive quantitative and qualitative overview of various gravimeter types will facilitate future instrument-type selection trade-offs, e.g. for Solar System gravimetry on other asteroids, moons, including the Moon, or Mars (Chapter 2).
- This thesis has significantly contributed to the GRASS gravimeter mechanical design, the tests, and the realisation of the overall development: A novel space instrument, scheduled to be launched in October 2024 onboard the ESA Hera spacecraft to arrive at the binary asteroid system Didymos in December 2026 (Chapter 3).
- The developed analytical GRASS model was used to aid the instrument development, it can be used for the ongoing and future instrument characterisation, and for future mission studies (Chapter 4).

Regarding the figures of merit of various gravimeters, some numbers, e.g. regarding the noise, remained incomplete. Obtaining these numbers, potentially from (comparative) testing of these instruments, could further improve the presented overview. The analytical instrument simulations used only normative material properties, such as the densities, or the Young's modulus of the spring. The determination of the actual spring properties could further improve the simulations of the actually integrated instrument (versions).



Part II is entirely focused on gravity simulations, and most specifically on the computation of surface gravitation. The second set of research questions addressed in this part were:

- (I) *What are the contributions to surface gravity and which methods to model gravitation of small bodies exist that are specifically well suited in the context of surface gravimetry and how do they compare?*
- (II) *What is the expected surface gravity and its variation on possible small body gravimeter targets on a global level?*
- (III) *How can local topography be included in the gravitation modelling of small bodies and how large are the topographic corrections on the resulting surface gravity?*

Motivated by research question (I), a thorough literature study has been presented not only focusing on the different (main) contributions to surface gravitation, but also on the various numerical techniques to model gravitation. Regarding the latter, individual methods exhibit different advantages and disadvantages, and so, three methods have been compared with a focus on surface gravitation, to which this work has contributed by performing the computations using the polyhedral method (PM). This method was indeed found to be the most accurate when comparing to analytical solutions of simple shapes, but the clear disadvantages was the computation time. For the Martian moon Phobos, the agreement of the gravitation computation over the entire surface was ranging from 99.488 % to 100.415 % (MSP 800 k mascon method), and from 96.927 % to 104.916 % (MASC mascon approach), both compared to the PM. On the contrary, the computation time for the MSP was two, and the MASC was one order of magnitude faster than the PM. In addition, the comparison was extended by simple inhomogeneities. Here, the same result was drawn, but with an even more expensive computation time for the PM, as it effectively doubled for the evaluation of two shapes. At some point, however, more complex inhomogeneities will not be representable by the PM, providing an advantage to the mascon methods, or demanding further work in adapting the PM accordingly. Research question (II) then required to apply the identified methods and analytical concepts in order to analyse the gravity environment of possible gravimeter targets, including Phobos, Apophis and Dimorphos. Here, two main conclusions can be drawn. Firstly, different targets will require, without questions, adaptations of the above-mentioned instrument design parameters, as the three different bodies surface gravities each vary by one order of magnitude. Phobos has a surface gravity in the order of $10^{-3} m/s^2$, Apophis in the order of $10^{-4} m/s^2$, and Dimorphos in the of $10^{-5} m/s^2$.

Secondly, and specifically interesting for the GRASS gravimeter, the dynamical gravity contributions as well as the variations thereof have been approximated analytically. On Phobos, the maximum variation in tidal acceleration was found to be 2.15 % of the reference gravity, thus clearly detectable with a 1 % accuracy. Here, the libration was below this threshold, being four orders of magnitude smaller than the reference gravity. The same was the case for the variations on Dimorphos. The

librations are out of reach for the 1% accuracy target. For the tides, it was interestingly found that the detectability with this target will depend on the landing location. While the smallest variation in tides is not detectable, it is detectable in locations with the mean variation of tides and larger variations.

Zooming in from the global level to the regional level, as asked by research question (III), the local topographic effects on surface gravity were assessed. For this, the new Wedge-Pentahedra Method (WPM) has been developed. Using the Polyhedral Method (PM) prevalently, it provides a closed-form analytical description of the topographic influence of the local surface, while allowing a triangulated description of the surface, rather than approximating it using (flat topped) prisms or mascons. In addition, it is possible to vary the density per wedge, and further vertically, by stacking wedges or right prisms. The method has been successfully verified and it was shown for the tested example that indeed the local topography can have a significant influence that cannot be neglected. With the (artificial) terrain amplified by a factor of $S_T = 250$, the surface gravitation on Phobos changed by about 0.7 %. On top of this, a Traverse and Gravity Simulator (TAGS) has been developed, that allows simulating rover locomotion over surfaces. Here, it was shown that the application is not only limited to small bodies, but that traversing the Moon, Mars or Earth can likewise work. This does not only benefit gravimeter data post processing, but likewise mission planning, landing, and further aspects. The WPM is therefore a powerful and innovative tool, offering a large number of possible use cases in future research.

Part II therefore achieved the following:

- The overview of different gravitation modelling techniques and their discussion with a focus on surface gravity (Chapter 5) lead to a novel systematic comparison between three methods, aiding future applications with an appropriate selection (Chapter 6).
- The gravity modelling of Dimorphos as the target body of GRASS will aid in optimizing the final instrument settings, i.e. considering the potential range in surface gravity and the variations thereof to be expected (Chapter 6).
- The gravitational influence of local topography can now be better studied and corrected for with the newly developed Wedge-Pentahedra Method (WPM – Chapter 7).

Clearly, the formulated goals concerning the gravity modelling have therefore also been achieved. The gravitation method comparison could of course always be extended by further methods. The general limitation of the small body gravity modelling was the assumption of a homogeneous density, in absence of better geophysical knowledge. With the future GRASS gravimeter measurements, potential inhomogeneities will have to be included in more detail in the modelling. The WPM has thus far only been tested with an artificial DEM and terrain scaling. Using real elevation data, e.g. surrounding the GRASS landing site, will allow to understand and remove the actual influence from the local terrain. The quality of the (real) terrain assessment will finally also depend on the observational resolution obtained.



While the two parts express a clear difference in themselves, being dedicated to the space gravimeter development and the surface gravity modelling, the common framework is the umbrella term *Solar System Gravimetry*, as embodied in this thesis' title: *Surface Gravity Modelling and Space Gravimeter Development in the Context of Solar System Small Bodies*. Gravimetry cannot be effectively performed without addressing both sides. Without a high-performance science instrument, no gravimetric surveys can be performed. By the same token, the gravimetric measurements are of little avail without appropriate modelling techniques allowing to interpret the obtained data. Clearly, forward modelling alone, as done in this work, has its limits that will demand further work regarding inversion.

In addition, possible continuations of this work have been proposed, also with regards to synergies between the two parts of the thesis. Clearly, the GRASS gravimeter is only at its dawn of the development stage, and extensive future testing and developments will allow increasing its performance further. Using the generic electromechanical design and the analytical gravimeter model, it will be possible to quickly assess the feasibility for future missions, and if favourable, adaptation of the instrument to the novel target gravity. Likewise, the gravitation simulation work has opened up a manifold of possible future applications. Most notably, additional comparisons of the gravitation methods will provide more insights in advantages and limitations. The novel WPM is just in its infancy, and various studies, either standalone or in the TAGS rover simulator, can be performed using this novel method on all solid surfaces in the Solar System.

This work is finally closed “in the hopes that this work may in some way contribute to [the] exploration of space”¹ and to the gravimetric survey of our Solar System, unravelling what lies below . . .

¹Edwin Eugene (Buzz) Aldrin, Jr., Doctoral Thesis Dedication, January 1963.

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A

ROVER CLASS GRAVIMETER DESIGN

In this Appendix, the mechanical design of the alternative space gravimeter mentioned in Section 3.6 is described. Like for the flight version of the Hera mission, the 3D-instrument design and mechanical GRASS development was the main responsibility of the author, which was strongly supported by guidance and consultation from Emiel Van Ransbeeck. Again, for the full instrument development beyond the here presented mechanical aspects, full credit is given to the entire GRASS development team.

The here described instrument version has the same working principle as the one described in Chapter 3, but allows for a large degree of adaptability, making it not only useful for sensing electronics testing as part of the GRASS development, but also a good alternative concept for different exploration targets, i.e. with a different (order of magnitude of) surface gravity. Generally, this larger instrument type allows to incorporate larger tip masses for even more sensitivity, or thicker blades to reduce the bending for larger gravities such as on the Moon or Mars. The additional room also provides more space for the instrument electronics. This bigger instrument is less suitable for a CubeSat, but could be deployed for example on a, robotic or human, rover and is therefore referred to as *rover class*.

In the same fashion as for the other presented gravimeter design, the description is done in three steps, regarding the reference spring (Section A.1), the displacement transducer (Section A.2), and the gravimeter mechanism (Section A.3).

A.1 ROVER CLASS: REFERENCE SPRING

In this Section, the reference spring design will be introduced. Figure A.1 shows the spring layout. This design was initially considered for Phobos, with a larger thickness of $100\ \mu\text{m}$. Inside the larger overall instrument volume, the overall spring length was longer, being 65.5 mm. Hereof, 2.5 mm were attributed to the tip mass length. The (cubic) tip mass spring has been included in the analytical instrument model (Chapter 4), supplementary to the later used cylindrical tip mass.

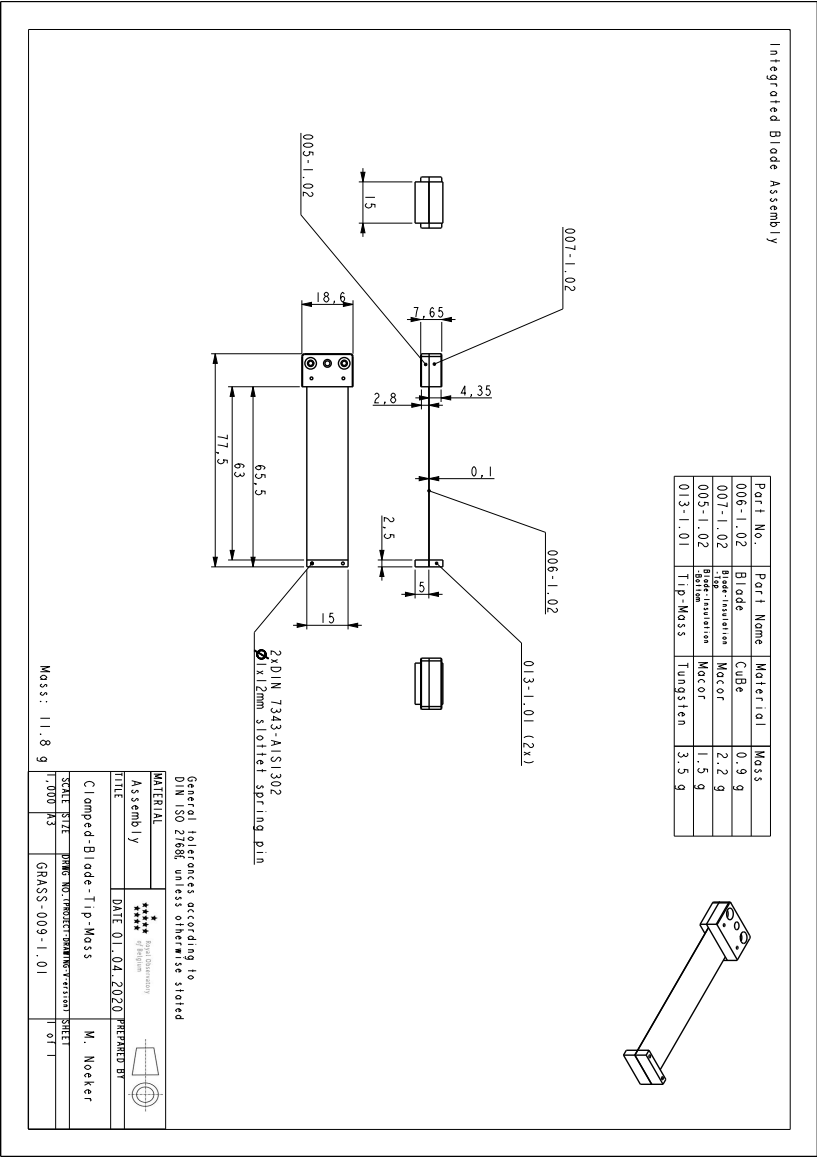


Figure A.1: Technical drawing of the original GRASS spring designed for operation on Phobos. Here, the first design parameters concerning the sensing elements are introduced, i.e. the free blade length between clamp and tip mass (63 mm), the width of blade (15 mm), the thickness (0.1 mm), and the tip mass dimensions ($2 \times 15\text{ mm} \times 5\text{ mm} \times 2.5\text{ mm}$) and the high-density tip mass material (tungsten).

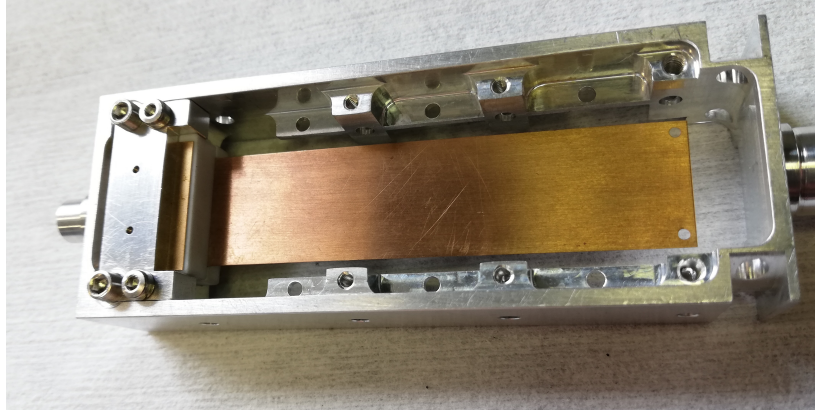


Figure A.2: Cradle with blade assembly (without tip mass) mounted. The Macor pieces holding the blade are fixed in the cradle with the visible clamp.

A.2 ROVER CLASS: DISPLACEMENT TRANSDUCER DESIGN

The sensor-head main structure is the single-piece cradle that can be placed at the intersection of the mechanism and the sensor. The cradle resembles a continuous shaft that allows mounting and rotation with the two mechanism bearings. At the same time, it is the base structure for the sensor head, mounting the gravimeter spring assembly (Appendix A.1), the front-end electronics (FEE) and provides an interface to the slip ring. Moreover, it mounts the pair of sensing electrodes and therefore is key in driving the capacitance gap size as described in Subsection 4.2.1. Lastly, the spring assembly with the tip masses again requires a damping mechanism which is also mounted onto the cradle.

In this Section, the cradle version of the *Rover Class* is described with the pieces mounted in and on the piece being briefly introduced. This cradle is characterised by a large adaptability to support the instrument development, and instrument testing and characterisation. For example, the cradle allows, with no practical restriction, to change the thickness of the spring used: nominally designed for a thickness of 0.1 mm (Appendix A.1), it was possible to mount the Earth-equivalent blade of 3 mm thickness, simulating equivalence deflection under Earth gravity (Section 4.1.4). The same was the case for the gap size, i.e. the position of the electrodes within the cradle that have to be easily adaptable without removing the (centred) spring. The cradle is shown in Figure A.2 and is machined in aluminium Al 7075 T7351. The surface treatment is Alodine 1200.

A.3 GRASS MECHANISM DESIGN: ROVER CLASS

This Subsection A.3 is an adapted excerpt from Noeker et al. (2021c).

The Rover Class design was the first stand-alone prototype developed beyond

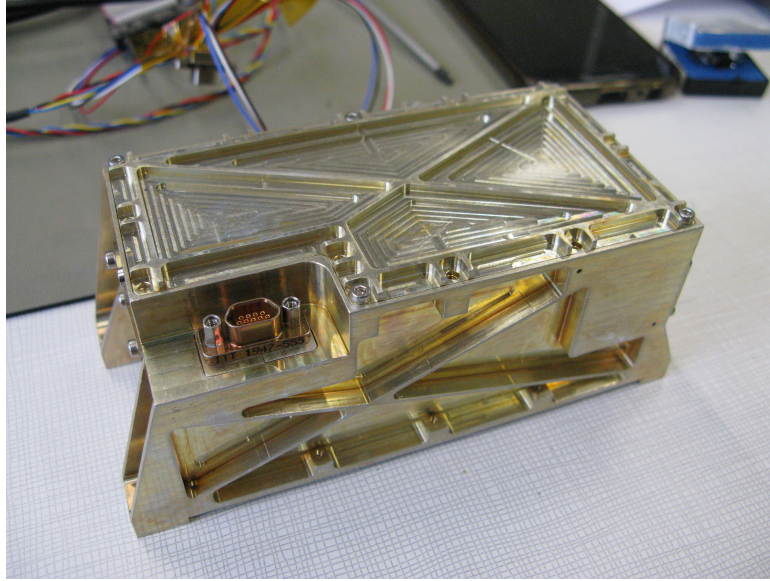


Figure A.3: The stand-alone GRASS mechanism of the Rover Class. The volume envelope is $129 \times 60 \times 52 \text{ mm}^3$.

breadboard state and originally intended as equipment on exploration rovers, such as the MMX rover (Subsection 1.3.1). This version, with the working title GRASS (rather than JUV-GRASS, see Subsection 3.4), is characterised by a compact design with a relatively large sensor head design. This allows a larger degree of flexibility for the instrument development, as discussed in the previous Section. The designed mechanism is a one-spring design, so for a full 3D-vector reconstruction, at least two instruments, arranged orthogonally are required. This one-axis instrument might be considered for surface gravity measurements on the Moon or Mars.

To ensure the desired functioning of the gravimeter, a mechanism providing continuous motorised sensor head rotation and signal transfer was developed. The instrument is pictured in Figure A.3 with a total enclosing volume of $129 \times 60 \times 52 \text{ mm}^3$. The reference sensor head (Appendix A.1) contributes ca. 74 gram to the total instrument mass of 288.3 gram, thus about 214.3 gram can be attributed to the mechanism.

The bi-directional motorised rotation is realised by a gear motor and a gear train, where the gear motor is aligned in-parallel with the sensor head to limit the overall instrument length. For a good measurement, the instrument noise, hence also the mechanical mechanism noise to the prone spring design (Subsection A.1), needs to be minimised. This is achieved by increasing the nominal axis distance of the gear motor from 25 mm to $25^{+0.03}_{+0.02} \text{ mm}$ to account for thermal expansion, and by best colinear alignment of the two axes (gear motor and sensor head). This is realised by machining the motor mounting piece and the bearing holes in the housing as

one single piece (Figure A.4). The two gear wheels are of stainless steel and Delrin, respectively, to avoid cold welding in the space environment. The resulting gear ratio i is computed as (Equation (A.1)):

$$i = \frac{30mm}{20mm} = \frac{60teeth}{40teeth} = 1.5 \quad (A.1)$$

With a measured starting torque of $M_{T,start} = 3.1mNm$ of the sensor head, the minimum starting torque $M_{T,min}$ of the gear motor was calculated using the gear ratio (Equation (A.1)) and an assumed gear efficiency of $\eta = 0.9$ as (Equation (A.2)):

$$M_{T,min} = \frac{M_{T,start}}{i \cdot \eta} = \frac{3.1mNm}{1.5 \cdot 0.9} = 2.296mNm \quad (A.2)$$

With the nominal output torque of the used Micromotion GmbH CoograDrive Space 10 mm - Type 6 gear motor ($M_{T,nom} = 15mNm$), the safety factor is calculated as $S = 15mNm/2.296mNm = 6.532$, which is more than twice than the generally demanded safety factor of 3 for space applications.

Additional elements of the mechanism are sketched in Figure A.5. The signal transfer is done with a separable slip ring, allowing unrestrained rotation, and data and power transfer. This type of data transfer, however, does not allow undisturbed analog signal transfer, due to induced noise, therefore demanding the capacitance measurement to be transformed to a digital signal (Subsection 4.3) on the sensor head using front-end electronics. The slip ring output is passed to the back-end electronics (BEE) that do not only perform the output signal treatment but also control the GRASS instrument; the BEE are also connected to the instrument connector as an interface to the respective onboard computer (OBC).

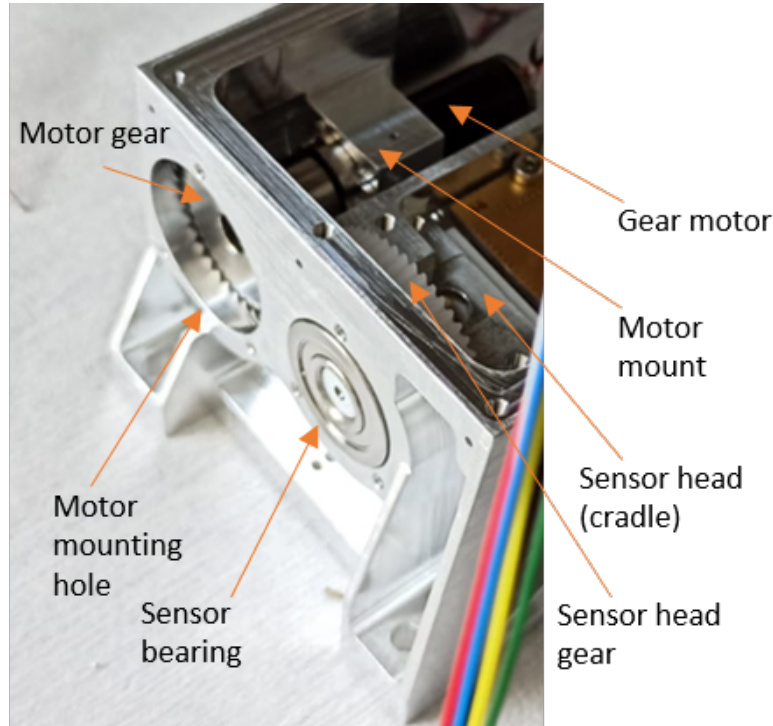


Figure A.4: Gear train of the Rover Class GRASS mechanism. The motor and the sensor head bearings are mounted in the housing. As this piece is machined in one piece, the orientation of the two axes will have the best achievable colinear alignment.

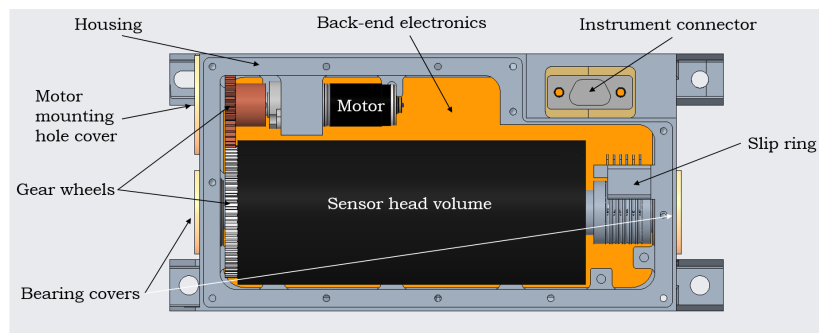


Figure A.5: Annotated working principle of the Rover Class GRASS mechanism. The sensor head volume is shown as solid of revolution with diameter of 33.5 mm and length of 89.9 mm.

B

SUPPLEMENTARY MATHEMATICAL METHODS

This Appendix provides mathematical background implemented in the numerical gravity simulation tools. Used oftentimes, the volume of any polyhedron was computed as described in Appendix B.1. The coordinate frame rotation forming part in the WPM method (Chapter 7) is subsequently described in Appendix B.2.

B.1 POLYHEDRON VOLUME

This Section shows how the volume of polyhedra is computed, which is an important aspect when altering internally a SB density distribution while demanding the total mass to remain constant. Dobrovolskis (1996) provides the formulae of computing the area, volume, centre and moment of inertia of a polyhedron as will be presented below. It is here applied to triangular facets, therefore making it directly applicable to the triangular Wavefront file format used in this work.

The computation of each facet area with vertex positions $\vec{D}$, $\vec{E}$ and $\vec{F}$ is done using the two edges $\vec{G} = \vec{E} - \vec{D}$ and $\vec{H} = \vec{F} - \vec{D}$. Next, the outward pointing normal $\vec{N}$ is calculated as the cross product of $\vec{G}$ and $\vec{H}$

$$\vec{N} = \vec{G} \times \vec{H} \quad (\text{B.1})$$

The incremental area of that facet ΔA is then calculated with the norm of the normal as

$$\Delta A = \|\vec{N}\|/2 \quad (\text{B.2})$$

The implementation of this is straight forward and was simply verified with an imported cube of known side length a and therefore known surface area of $6a^2$.

Correspondingly, the incremental volume of the tetrahedron spun between the surface vertex and the origin ΔV is calculated using the scalar triple product $(\vec{D}\vec{E}\vec{F})$ as

$$\Delta V = (\vec{D}\vec{E}\vec{F})/6 = ((\vec{D} \times \vec{E}) \cdot \vec{F})/6. \quad (\text{B.3})$$

While Willner et al. (2014) do not provide a value for the surface area of the used shape file of Phobos, it does provide the moon volume being 5741 km^3 (value abstract) and $5742 \pm 35 \text{ km}^3$ (value Table 3 in Willner et al. (2014)). The computation of the volume for the shape file in MATLAB returns 5741.2 km^3 , verifying the here presented and implemented volume computation. With known volume, it is well possible to readapt the uniform density of a body, using inhomogeneities (Subsection 6.5.1) or a surface (regolith) layer (Subsection 6.5.2), for a given total mass following local mass changes to keep the total mass constant.

B.2 COORDINATE SYSTEM ROTATION

An initial rotation implementation with three rotation matrices around the the axis was found to be not fully reliable. While the rotation worked for some cases, it did not fulfil the requirement to work reliably for all surface points and gravity vectors in all octants. Therefore, we implemented and verified the method proposed by Kuba hasn't forgotten Monica (Stackexchange User) (2014) to rotate one unit vector (normalised g-vector) A to align with another vector B , here, generally the Z-axis, thus $B = (0, 0, 1)^T$.

The algorithm postulates that any rotation from one vector to the other can be performed as a 2D-rotation within the plane of normal $A \times B$ using the rotation matrix G :

$$G = \begin{pmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\text{B.4})$$

Then, the $\cos \psi = A \cdot B$ term is replaced with the dot-product term. Moreover, the $\sin \psi$ is replaced as follows starting with the definition of the cross-product:

$$A \times B = \|A\| \|B\| \sin \psi \vec{n} \quad (\text{B.5})$$

where the $\vec{n}$ is the unit vector perpendicular to the plane of A and B , and thus collinear to the vector of the cross product $A \times B$. With the norm of the unit vectors A and B being 1 we arrive at

$$\sin \psi = \frac{A \times B}{\vec{n}} \quad (\text{B.6})$$

Finally, the quotient of a vector and its unit vector is equal to its norm $\|A \times B\| = \frac{A \times B}{\vec{n}}$ and therefore we have

$$\sin \psi = \|A \times B\| \quad (\text{B.7})$$

The matrix G therefore reads as:

$$G = \begin{pmatrix} A \cdot B & -\|A \times B\| & 0 \\ \|A \times B\| & A \cdot B & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\text{B.8})$$

Next, the orthogonal basis of the rotation of the rotation matrix G is spanned by the three *normalised* orthogonal vectors: The vector projection of B onto A is, due to the same length (normalised) equal to A .

$$\vec{u} = \|B\| \cos \psi \hat{A} = (B \cdot A) A \quad (\text{B.9})$$

where use can be made by the fact that the vectors A and B are normalised and therefore the normalisation is equal to these vectors ($\hat{A} = \frac{A}{\|A\|}$).

The normalisation of u can be then done as:

$$\hat{u} = \frac{(B \cdot A) A}{\|(B \cdot A) A\|} = A \quad (\text{B.10})$$

The term is equal to A as the dot products cancels and A is equal to $\hat{A}$.

Secondly, we take the vector rejection of B onto A , which, by definition, is the difference of B and the projection of B onto A (above u):

$$v = B - u \quad (\text{B.11})$$

noting that we substitute the non-normalised vector for u and then normalise the full vector v , we obtain:

$$\hat{u} = \frac{B - (B \cdot A) A}{\|B - (B \cdot A) A\|} = A \quad (\text{B.12})$$

these two vectors are complemented with the third vector w , normal to the rotation plane of A onto B

$$w = B \times A \quad (\text{B.13})$$

where we note that Kuba hasn't forgotten Monica (Stackexchange User) (2014) did not normalise this vector which we are doing in this work.

$$w = \frac{B \times A}{\|B \times A\|} \quad (\text{B.14})$$

So, we obtain a orthonormal basis rather than an orthogonal basis.

The orthogonal matrix is then formulated as

$$F = \begin{pmatrix} u & v & w \end{pmatrix} \quad (\text{B.15})$$

and substitution yields

$$F = \left(A \quad \frac{B - (B \cdot A)A}{\|B - (B \cdot A)A\|} \quad B \times A \right) \quad (\text{B.16})$$

Note that we do not invert F as is done by Kuba hasn't forgotten Monica (Stackexchange User) (2014), which will be accounted for in Equation (B.18).

For this matrix, because of the normalisation of all three column vectors forming an orthonormal (other than in Kuba hasn't forgotten Monica (Stackexchange User) (2014)) it can be shown that

$$F^{-1}F = F^T F = I_3 \quad (\text{B.17})$$

which is true for orthogonal matrices.

Now, the final step is the change of the basis of the rotation matrix G from the standard basis to the orthonormal basis formulated by F , which gives us the rotation matrix U from A to B

$$U = FGF^{-1} \quad (\text{B.18})$$

and with this rotation matrix U , we can rotate vector A to align with vector B as

$$A_{rot} = B = UA \quad (\text{B.19})$$

which works very stable for all surface points to be rotated in all eight octants.

C

SURFACE GRAVIMETRY ARTIST'S IMPRESSIONS

You have reached the end of this work and found two Easter eggs! Below you can see two Artist's impression created by the author illustrating the beauty of extraterrestrial surface gravimetry.

Figure C.1 shows the landed Juventas CubeSat (Structure from Gomspace) with the JUV-GRASS two-axes instrument. Background Artist's impression of Didymos taken from <https://www.iau.org/news/pressreleases/detail/iau2007/>, accessed on 06.05.2021.

In Figure C.2, we look over the shoulders of a Phobos rover with two-axes GRASS instrument looking down on the Red Planet. Background picture credit: True color image of Mars taken by the OSIRIS instrument on the ESA Rosetta spacecraft during its February 2007 flyby of the planet. ESA & MPS for OSIRIS Team MPS/UPD/LAM/IAA/RSSD/INTA/UPM/DASP/IDA, CC BY-SA IGO 3.0.

Both pictures use the shape file from Willner et al. (2014) in the foreground.

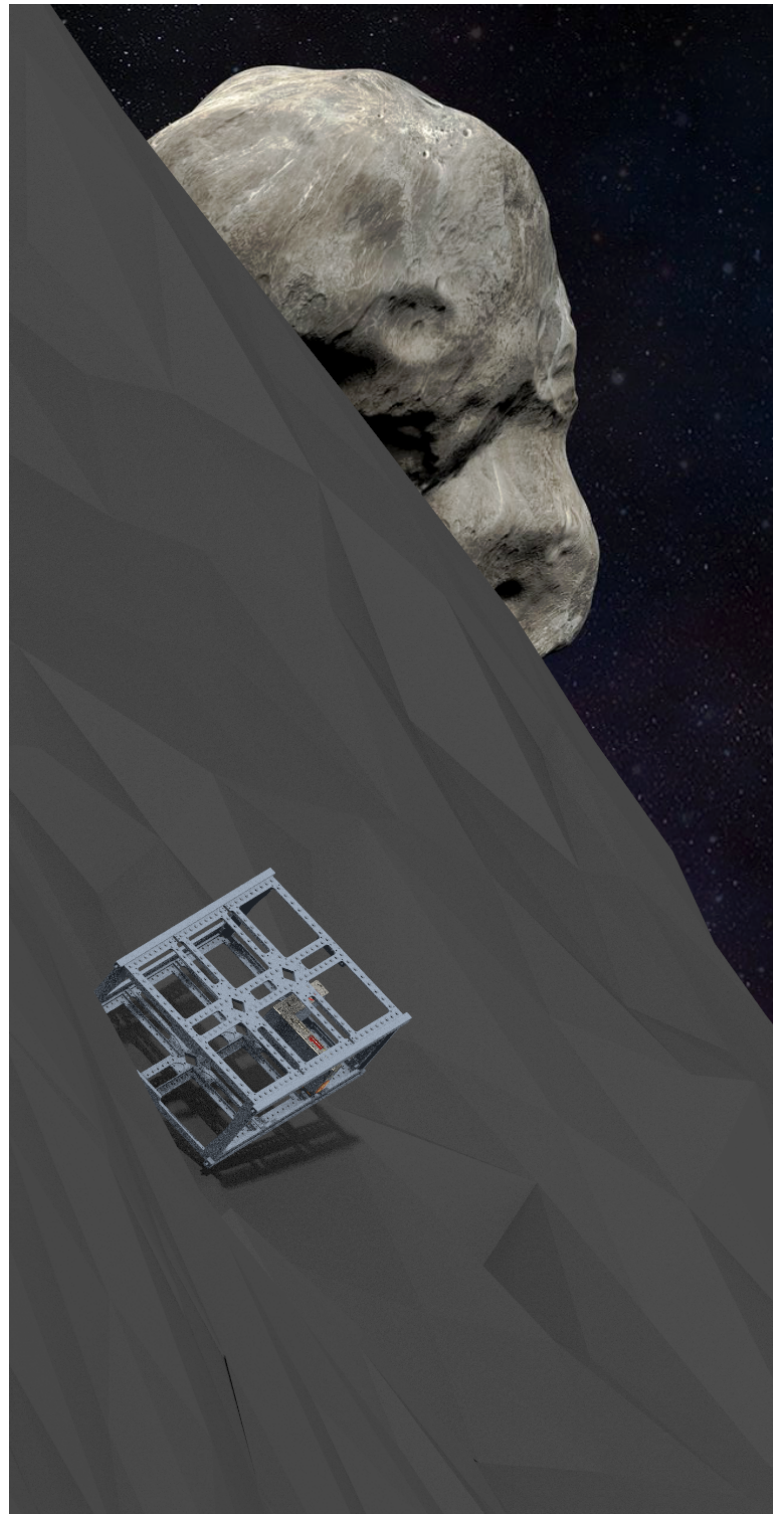


Figure C.1: Hopefully never to return to Earth...

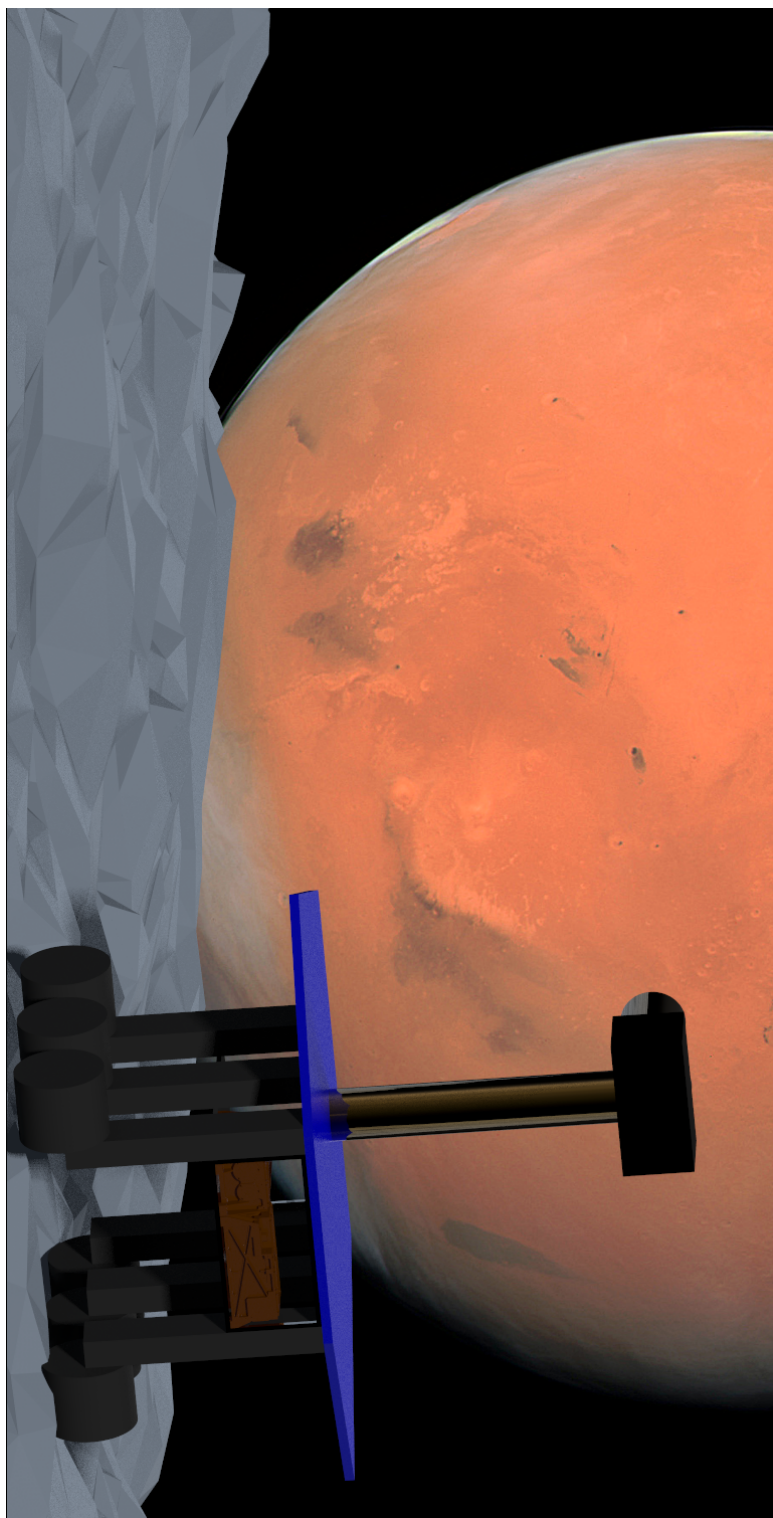


Figure C.2: Seat in a box for the next Mars landing(s).