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Model based extremum-seeking controller via modelling-error compensation approach

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ABSTRACT

This paper is focused on the design of a robust model based extremum-seeking controller (ESC) aimed at the online optimization of a class of uncertain nonlinear systems. The ESC scheme is based on the modeling-error compensation approach using a robust input–output linearizing control law coupled with a first-order gradient estimator. The feedback control scheme is able to drive the closed-loop system into a vicinity of the optimal operating set-point. In contrast with perturbation-based ESC algorithms, the control scheme includes an observer-based uncertain estimator for computing unknown components related to model uncertainties and a continuous gradient estimator, while avoiding the use of a perturbation signal for achieving closed-loop convergence to the optimal neighborhood. The convergence of the closed-loop system to the unknown optimal set-point is analyzed. Numerical simulations illustrate the effectiveness of the proposed ESC scheme in two different nonlinear systems.

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1. Introduction

The mainstream feedback control schemes are developed for regulating dynamical systems at well-known set-points or for tracking specific reference trajectories. However, the control objective in many applications in engineering systems is to optimize a prescribed objective function in order to keep the controlled system at its optimal operating conditions [1,2]. The extremum seeking control (ESC) or self-optimizing control, is a feedback control method that has been designed to solve this class of on-line optimization problems [3–6]. The main task of the ESC is to find operating set-points, a priori unknown, such that the objective function reaches its extremum or optimum value [4,5]. Nowadays, the ESC methods have been an active research area with distinct application issues including radio telescope antennas to maximize the received signal, water and wind turbines to achieve maximum power generation, antilock braking system (ABS), photovoltaic systems, bioprocesses, among many others [7–10]. The extremum-seeking control problem was popular from the 1950s and 1960s. In the recent two decades, different approaches have been considered for addressing suitable solutions, from classical free-model perturbation-based methods to adaptive schemes and sliding-mode techniques, among others [2]. The perturbation-based ESC methods became an active and popular field of research in the control community after the formal proof of asymptotic convergence by Krstic and Wang [11]. The perturbation based approaches considered one dynamic feedback composed of a persistent perturbation signal combined with an adaptive extremum search to find the unknown optimal operating condition of the plant [2]. A characteristic of such schemes is that the estimation of the objective function gradient is performed via a combination of linear filters that does not require specific knowledge of the process dynamics and the objective function. However, the main limitations of perturbation-based methods are: (i) the requirement of a multiple time-scale separation between the process dynamics, the perturbation frequency and the adaptation rate, and (ii) the oscillatory behavior in the control signal due to the continuous external perturbation (dither signal) [2]. With respect to the time-scale separation, to avoid possible instabilities in the closed-loop systems the perturbation frequency must be slow (about two order of magnitude slower than the process dynamics), causing an extremely slow convergence at the optimum. Sluggish convergence can be unacceptable for processes in which the dynamics are slow of the order of hours or days like some chemical and biochemical systems [1,3,4]. Different approaches have been proposed to improve the convergence rate of the perturbation-based ESC. A dual-mode ESC and a phase compensator to correct the phase shift introduced by the system dynamics at the perturbation frequency was proposed

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[12]. More recently, Guay and Dochain [13] proposed a different approach by posing a time-varying estimation problem to minimize the impact of time-scale separation. The same authors proposed a modification of the gradient estimation based on recursive least square and one proportional-integral ESC approach that improve the convergence rate of the controlled systems [14]. On the other hand, the extremum-seeking control problem has been studied in the sliding-mode control framework [7,8,15,10]. Applications of sliding-mode concepts in ESC can be find in [10]. More recently, ESC schemes based on sliding-mode techniques ware developed for different automotive applications in [7,8,15]. The main idea of the sliding mode-based ESC is to ensure that the performance function follows an increasing time signal as close as possible to its extremal value via discontinuous control and sliding mode motion [9]. The main drawback of the sliding-mode based ESC methods is that the control performance can be very poor, exhibiting high-amplitude oscillations about of the optimal operation conditions [8,16].

In the present work we propose a model-based extremum-seeking control method based on the modeling-error compensation approach [17] for the on-line optimization of a class of uncertain nonlinear systems. By using the input–output linearizing method and an adaptive learning technique based on modeling error estimation coupled with a first-order gradient estimator, we show how the ESC scheme is able to self-optimize the closed-loop system. Unlike previous ESC approaches, the proposed control approach comprises a dynamic uncertain estimation scheme that computes unknown terms for providing robustness of the controller despite unmodeled dynamics and external disturbances and, does not make use of a continuous perturbation signals for achieving closed-loop convergence to the optimal conditions. The rest of the paper is organized as follows: A brief description of ESC problem considered in this work is given in Section 2. The proposed ESC is described in Section 3. Two simulation examples to illustrate the performance of the ESC approach and compare it with other schemes are presented in Section 4.

2. Problem statement

Consider the following nonlinear system

$$\dot{x} = f(x) + g(x)u \tag{1}$$

$$y = h(x)$$

where $x \in \mathbb{R}^n$ is the internal dynamical state, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}$ is a measurable signal that affects directly the global performance criterium (defined below) and $h : \mathbb{R}^n \to \mathbb{R}$ is the output map. The functions $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^n$ are smooth in a domain $\Omega \subset \mathbb{R}^n$ and are partially know, *i.e.*, the system (1) is corrupted by modelling errors and external disturbances. In order to formulate the specific control problem we consider the following:

Assumption 1. There exists a static objective function $F : \mathbb{R} \to \mathbb{R}$ from the output signal y to the global performance criterion J, given by

$$J = F(y) \tag{2}$$

that is locally smooth and has an unique maximum (or minimum) at $y = y^*$, such that $F(y^*) = J^*$ with J^* as the maximum (or minimum) static global performance criterion.

Assumption 2. The global performance criterion J and the output signal y are available for online measurement; However, the static objective function F is not known, and consequently the optimum set-point y^* is unknown.

Assumption 3. The system (1) has a well-posed relative degree one with respect to the output signal y, and is minimum-phase in the usual sense of the stability of the zero dynamics [18].

Definition 1. The gradient of the objective function F is defined as $\sigma = \frac{dJ}{dy} = \frac{dF}{dy}$

The control objective is to design a robust extremum-seeking controller to achieve the convergence of the system (1) around to the optimum set-point y^* (a priori unknown) while maximize the global performance criterion J, without requiring the knowledge of either F, σ, f and g.

3. Controller design

The design of the extremum seeking controller will be carried out as follows. First, based on the input–output linearizing methods for minimum-phase systems, an ideal ESC control law assuming perfect knowledge of the mathematical model, the objective function and its gradient, is proposed. In the second step, an observer-based uncertain estimator that computes the unknown terms related to model uncertainties and unmeasured disturbances is developed. And finally, a first-order gradient estimator is coupled to guarantee the robustness of the ESC scheme.

Since (1) has a well-defined relative degree one and is minimum-phase (Assumption 1), it is possible to rewrite the system (1) in the canonical form [18].

$$\dot{z}_1 = L_f h(x) + L_g h(x) u \tag{3}$$

$$\dot{\varsigma} = \zeta(z, \varsigma)$$

where $z_1 = h(x)$ and $\dot{\varsigma} = \zeta(z, \varsigma)$ represents the zero dynamics. The terms $L_f h(x)$ and $L_g h(x)$ are the first Lie derivatives of $h(x)$ with respect to the vector fields $f(x)$ and $g(x)$, respectively.

Now, the control objective can be achieved if the feedback system is regulated in the state where the gradient is zero $\sigma = 0$ (see Assumptions 1–3). Before to present the feedback control law it is necessary to do the following assumption about the control input.

Assumption 4. The control input u and its time-derivative $\dot{u}$ are continuous and bounded, i.e., for all $t > 0$, $|u| < \bar{u}$ and $|\dot{u}| < \delta u$ with $\bar{u}, \delta u < \infty$.

Now, the following input–output linearizing ESC law is proposed.

Proposition 1. *Consider the system (1) under Assumptions 1–4 with the input–output linearizing control law*

$$u = \frac{1}{L_g h(x)} \left[-L_f h(x) + k_c \sigma \right]; \tag{4}$$

where σ is the gradient defined in Section 2. Then there exists a control gain $k_c > 0$ (or $k_c < 0$ for minimization problems) such that the closed-loop system has an asymptotically stable equilibrium at $y = y^*$.

Proof: Replacing u from (4) in (3), the dynamics of the controlled system in terms of the output signal can be written as

$$\dot{y} = k_c \sigma \tag{5}$$

Because the zero dynamics is stable (Assumption 3) from Assumption 1 and the Definition of the gradient of the objective function σ, it is verified the following: First, $\forall y < y^*$ the gradient is $\sigma > 0$ in consequence by (5) $\dot{y} > 0$, *i.e.*, the output signal y will be increased; on the other hand, $\forall y > y^*$ the gradient is $\sigma < 0$ in consequence $\dot{y} < 0$, *i.e.*, the output signal y will be decreased; and if $y = y^*$ then $\sigma = 0$ (Assumption 1) consequently $\dot{y} = 0$, *i.e.*, $y = y^*$ is an asymptotically stable equilibrium for the closed-loop system. □

The above linearizing ESC law cannot be implemented directly, because (4) requires perfect knowledge of the uncertain terms that are associated with modelling errors and external disturbances of the system. In what follows, the ideal control law (4) will be modified to take account the uncertainties in the mathematical model of the plant. In the spirit of Alvarez-Ramírez [17] about of modelling error compensation techniques, let us define a function for lumping the uncertain terms as follows: $\eta \equiv \Theta(z_1, \varsigma, u) = a(z_1, \varsigma) + b(z_1, \varsigma)u$. We make the following assumption with respect to the uncertain function.

Assumption 5. The function $\eta \equiv \Theta(z_1, \varsigma, u)$ is locally smooth, bounded and its time derivative denoted by $\Xi(z_1, \eta, \varsigma, u)$ is also bounded.

Then once the uncertain function η is defined it is possible rewrite the system (3) in the following extended state-space representation

$$\begin{aligned} \dot{z}_1 &= \eta + u \\ \dot{\eta} &= \Xi(z_1, \eta, \varsigma, u) \\ \dot{\varsigma} &= \zeta(z, \varsigma) \end{aligned} \tag{6}$$

where η is interpreted as an augmented state whose dynamics can be reconstructed from measurements of the input and the output signals [17]. The controller designer can choose the uncertain function as $\eta = u - k\sigma$ as a guidance to do that. In this way we can highlight the following: (a) It can be proved that the solution of the system (3) is a projection of the solution of system (6); (b) A feature of system (6) is that the uncertainties have been lumped into an uncertain function $\Theta(z_1, \varsigma, u)$ that can be estimated by an unmeasurable but observable state η [17]. Thus, if the system (6) is stabilized then, as a consequence, the system (3) will be also stabilized as well as its equivalent system (1). Some works have been focused on robust stabilization of nonlinear systems via output feedback (see e.g., [17,19,20]), where the main idea consists in design an extended high-gain like observer as an uncertainty estimator. Thus the dynamics of the states (z_1, η) can be reconstructed from the measurements of the output signal $y = h(x) = z_1$ in the following way [17]

$$\dot{\hat{z}}_1 = \hat{\eta} + u + \Gamma\kappa_1(z_1 - \hat{z}_1) \tag{7}$$

$$\dot{\hat{\eta}} = \Gamma^2\kappa_2(z_1 - \hat{z}_1)$$

where $(\hat{z}_1, \hat{\eta})$ denotes the estimate for z_1 and the lumping uncertainty state η, respectively. In order to ensure the convergence of the estimation method the observer parameters κ_1 and κ_2 should be chosen such that the polynomial $\lambda^2 + \kappa_1\lambda + \kappa_2$ has its roots on the left side of the complex plane (Hurwitz), and $\Gamma > 0$ is the observer gain. Then, the following ESC controller can drive the trajectories of the system (1) arbitrarily close to the optimal set-point y^*.

$$u = \frac{1}{\gamma}\left[-\hat{\eta} + k_c\sigma\right] \tag{8}$$

where γ represent the well-known terms in the Lie derivatives defined in (4). In this work we assume that γ is bounded away from zero.

The following result is based on the convergence analysis of feedback linearizable uncertain systems under the action of an extended high-gain observer based uncertain estimator [19,20]. Let us consider before it the following definition related with the practical stability concept [19].

Definition 2. [19] One system is practically stabilizable at x^*, if for any set $\Sigma \subset M$ containing x^* and such that its closure $cl(\Sigma)$ is compact, there exists a control law (practical stabilizer) which renders this set the stable attractor of the system within some neighborhood $\delta(\Sigma)$ of Σ, and moreover any system trajectory beginning in $\delta(\Sigma)$ enters Σ in a finite time interval.

Proposition 2. *Consider the uncertain nonlinear system (1) and suppose: (i) the Assumptions 1–5 are satisfied, (ii) κ_1 and κ_2 are chosen such that the polynomial $\lambda^2 + \kappa_1\lambda + \kappa_2$ is Hurwitz, (iii) $\Gamma > 0$ is a positive parameter (high-gain observer). Then the ESC scheme (7–8) achieves the practical stabilization around of the optimal set-point y^*.*

Proof: From Proposition 1, it follows that the control law (4) achieves the asymptotic convergence of output signal y at the optimal set-point y^*. Now with respect to the high-gain observer-based uncertain estimator (7) we will define the estimation error vector $e \in \mathbb{R}^2$ as

$$e_1(t) = \Gamma(z_1 - \hat{z}_1)$$

$$e_2(t) = \eta - \hat{\eta}$$

By replacing $(\dot{z}, \dot{\eta})$ and $(\hat{z}, \hat{\eta})$ from (6) and (7), respectively, and from algebraic manipulation, it follows that the dynamics of the estimation error is governed by

$$\dot{e}(t) = \Gamma A(\kappa)e(t) + B\Phi(z, N(\Gamma^{-1})e, \Xi, u)$$

where $A(\kappa) = \begin{pmatrix} -\kappa_1 & 1 \\ -\kappa_2 & 0 \end{pmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \in \mathbb{R}^2$, $A(\kappa)$ is a square matrix, $N(\Gamma^{-1}) = diag\left[\Gamma^{-1}, 1\right]$ and $\Phi(z_1, N(\Gamma^{-1})e, \Xi, u)$ is a continuous and bounded function. Note that if the function $\Phi(\cdot)$=0 then the estimation error system is reduced to a nominal dynamics of the estimation error

$$\dot{e}(t) = \Gamma A(\kappa)e(t)$$

For $\Gamma > 0$ and since κ_1, κ_2 are chosen such that $A(\kappa)$ is Hurwitz, the system $\dot{e}(t) = \Gamma A(\kappa)e(t)$ is asymptotically stable ($\lim_{t\to\infty} e(t) = 0$). By Assumption 5 and from the robust observer Lemma about the stability of feedback linearizable uncertain systems under the action of a high-gain observer-based uncertain estimator [19,20], it is known that there exists a positive number Γ^*, depending mainly on the magnitude of Ξ such that the nonlinear term Φ will be negligible with respect to the nominal dynamics of the estimation error. Then for a given compact set of the initial condition $I \subset \mathbb{R}^n$ and for all $\Gamma > \Gamma^*$ there exists a region of attraction $Y_\delta^* = \{y \in \mathbb{R} | y^* - \delta \le y \le y^* + \delta\}$, with $\delta > 0$ for the closed-loop system. Hence, the controller (7-8) drives the trajectories of the system (1) to the region of attraction Y_δ^*. □

Now with respect to the gradient of the performance function, we consider that the static performance function F is unknown (Assumption 2) and, consequently the gradient $\sigma = \frac{dF}{dy}$ is not available for feedback in (8). A practical solution to this issue is to obtain an estimate of the gradient σ from the measurable signals. Now considering that the gradient is defined as $\sigma = \frac{dF}{dy} = \frac{dJ}{dy}$, then we can obtain a good approximation as $\sigma = \frac{\dot{J}}{\dot{y}}$. From the technique proposed in [21–23] to obtain an estimate of the velocity using only position measurements in electromechanical systems, the following gradient estimation scheme based on first-order compensators is proposed. In this sense, let us denote the time derivatives of y and J by δy and δJ, respectively. Then in the spirit of the works [21–23] approximate values of the time derivatives for y and J can be obtained as the output of the proper filtering as:

$$\delta J = \left(\frac{D_t}{\tau_J D_t + 1}\right) J \tag{9}$$

$$\delta y = \left(\frac{D_t}{\tau_y D_t + 1}\right) y$$

where $D_t = \frac{d}{dt}$ is the time-derivative operator and τ_y, τ_J are the filter time constants. The time-domain realization of the above estimation expressions can be carried out via use the auxiliary states ω_J and ω_y as:

$$\delta J = \frac{\omega_J + J}{\tau_J} \tag{10}$$

$$\delta y = \frac{\omega_y + y}{\tau_y}$$

where

$$\dot{\omega}_J = -\frac{\omega_J + J}{\tau_J} \tag{11}$$

$$\dot{\omega}_y = -\frac{\omega_y + y}{\tau_y}$$

In this way the estimate value for the gradient is given by

$$\hat{\sigma} = \frac{\tau_y}{\tau_J}\frac{\omega_J + J}{\omega_y + y} \tag{12}$$

Remark 1. With respect to the convergence of the gradient estimator ((11)-(12)) we can highlight the following: By developing the time-derivative operator D_t in (9), we get to $\tau_J \dot{\delta J} + \delta J = \dot{J}$. This expression can be written as $\dot{\delta J} = -\tau_J^{-1}(\delta J - \dot{J})$; and similarly we have $\dot{\delta y} = -\tau_y^{-1}(\delta y - \dot{y})$. If we define the estimation errors as $\varepsilon_J = \delta J - \dot{J}$ and $\varepsilon_y = \delta y - \dot{y}$. Then, if $\dot{J}$ and $\dot{y}$ reach a constant values it is follows that the dynamics for the estimation errors are given by $\dot{\varepsilon}_i(t) = -\tau_i^{-1}\varepsilon_i(t)$ for $i = J, y$, and consequently ε_J and ε_y converge asymptotically to zero with a convergence rate proportional to τ_J^{-1} and τ_y^{-1}, respectively.

Hence the resulting robust ESC scheme is given by the feedback control

$$u = \frac{1}{\gamma}\left[-\hat{\eta} + \lambda\hat{\sigma}\right] \tag{13}$$

where the uncertain function $\hat{\eta}$ and the estimation of the gradient $\hat{\sigma}$ are computed by (7) and ((11)-(12)), respectively.

We can highlight the following points about the guidelines for tuning the parameters of the overall ESC scheme. The proposed ESC scheme is composed by the linearizing feedback control law (13), the extended high-gain observer (7) and the gradient estimator ((11)-(12)). In this way we have to choose the control and observer parameters in order to ensure the global convergence of the ESC scheme towards to the optimal set-point y^*. More specifically, the feedback control law (13) should be slower than the observer and estimation dynamics. Then, since the convergence rate of the gradient estimator is proportional to τ_J^{-1} and τ_y^{-1} (see Remark 1), it is necessary to choose sufficiently small values for τ_J and τ_y. On the other hand, from the proof of Proposition 2, we know that it is necessary to choose a large value for the observer gain $\Gamma > \Gamma^*$ to ensure fast convergence of uncertain function error $\eta - \hat{\eta}$ to zero. However, it is important not to forget that the high-gain observers are very sensitive to measurement noise, then the controller designer should find a good balance between fast convergence (high value for Γ) and measurement noise attenuation (low value of Γ).

4. Numerical verification

In this section numerical simulations for two nonlinear systems illustrate the performance of the ESC control scheme proposed.

4.1. Example 1

As first example we consider the following two-dimensional system

$$\begin{aligned} \dot{x}_1 &= x_1^2 + x_2 + u \\ \dot{x}_2 &= x_1^2 - x_2 \\ y &= x_1 \end{aligned} \tag{14}$$

where $x = (x_1, x_2) \in \mathbb{R}^2$ is the dynamical state, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}$ is the output signal and the objective function is given by

$$J = F(x) = 1 - x_1 + x_1^2 \tag{15}$$

with J is a cost measurable criterion to minimize. The above system has a minimum value (extremum) at $x_1^* = 0.5$, $x_2^* = 0.25$, $J^* = 0.75$ and $u^* = -0.5$. Applying the methodology described in Section 3, and defining the function for lumping the uncertainties of the system as $\eta = -x_1^2 - x_2$, we have the following ESC scheme

$$\begin{aligned} u &= \hat{\eta} + k_c\hat{\sigma} \\ \dot{\hat{x}}_1 &= \hat{\eta} + u + \Gamma\kappa_1(x_1 - \hat{x}_1) \\ \dot{\hat{\eta}} &= \Gamma^2\kappa_2(x_1 - \hat{x}_1) \end{aligned} \tag{16}$$

where the estimation value for the gradient $\hat{\sigma}$ is given by ((11)-(12)).

Numerical simulation results are shown in Fig. 1. The controller parameters are set to the following values: $k_c = -0.5$, $\kappa_1 = 1$, $\kappa_2 = 2$, $\Gamma = 2.5$, $\tau_y = 0.1$ and $\tau_J = 0.1$. As we can see in Fig. 1 the ESC scheme (16) stabilizes the uncertain system at the unknown optimal conditions in $x_1^* = 0.5$ with $J^* = 0.75$. Unlike to perturbation-based and sliding-mode based ESC [16] the control action is very smooth and the controlled system reach the optimum in short time interval. In order to have an insight about the effect of the tuning parameters on ESC performance, numerical simulation with changes in the controller and observer parameters are shown in Figs. 2, 3 and 4 . The effect of the control gain k_c is directly related with the convergence rate at the optimal set-point (Fig. 2). However we can see also more oscillations of the controlled signal and major control effort on the control input as k_c is increased (see Fig. 2). The effect of the high-gain value Γ on the convergence at the optimal set-point for the controlled signal is illustrated in Fig. 3. As it can be seen, as Γ is increased the controlled system gets better performance. However high values of Γ is also related with the amplification of the measurement noise, as we will show below. With respect to the gradient estimator parameters it is clear that as we choose smaller values for τ_y and τ_J the gradient estimation is improved as well as the global ESC performance (see Fig. 4). Numerical simulations show how the values of $\tau_{y,J}$ directly affect the performance of gradient estimator and that is the reason of the spike in the control signal around of $t = 4$ in Fig. 1 (see the gradient estimation error for $\tau_J = \tau_y = 0.1$). Finally, numerical simulation considering measurement noise in the signals J and y have been shown in Fig. 5. In Fig. 5, it is shown poor performance by measurement noise effect, that behavior is agreed with a common drawback of the high-gain observers. However one practical solution for the implementation of high-gain observers can be the design of low-pass filter for the measurement signals or redefine the observers parameters in order to attenuate it as has been proposed in [24].

4.2. Example 2

As second example we consider a more complex system that have been studied in previous ESC schemes [27]. The system consists in a continuous Anaerobic Digestion process (AD) modeled in

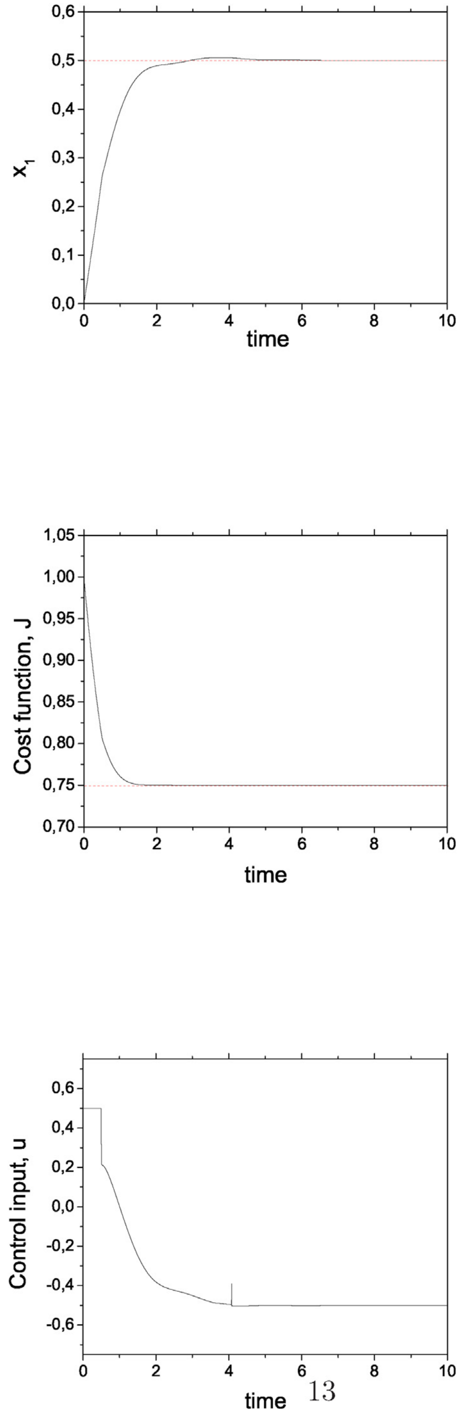

Fig. 1. Closed-loop performance for the system (14) the optimal condition are represent in dashed line.

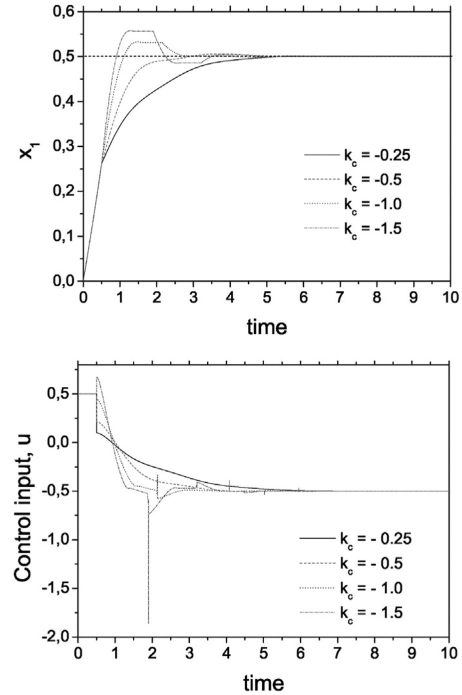

Fig. 2. Effect of the control gain k_c on the ESC performance for the system (14).

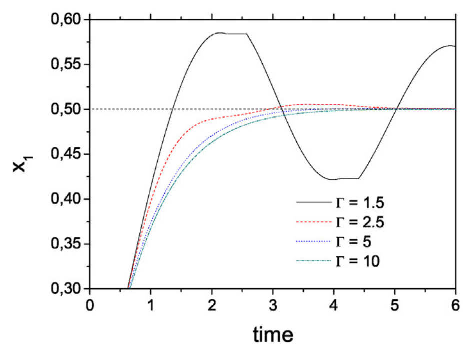

Fig. 3. Effect of the high-gain value Γ on the ESC performance for the system (14).

[25]. The mathematical model assumes two main bacterial populations, the first one, called acidogenic bacterial X_1, consumes organic substrate S_1 (total soluble Chemical Oxygen Demand (COD) except Volatile Fatty Acids (VFA)) and produces VFA, that is considered as secondary substrate S_2 through an acidogenesis stage. The second population, known as methanogenic bacteria X_2, uses VFA as substrate in a methanogenesis stage to growth and produces methane and carbon dioxide. Thus, the global AD process can be written as the reduced biochemical reaction network

$$k_1 S_1 \overset{\mu_1(\cdot)X_1}{\looparrowright} X_1 + k_2 S_2 \tag{17}$$

$$k_3 S_2 \overset{\mu_2(\cdot)X_2}{\looparrowright} X_2 + k_4 CH_4$$

The growth rates are denoted by $\mu_i(\cdot)X_i$, for $i = 1, 2$, where each $\mu_i(\cdot)$ is a smooth function of the respective substrate, and the symbol $\looparrowright$ denotes the corresponding bacterial population X_i is an autocatalyst (*i.e.* the microorganisms are at the same time products and catalysts). From a mass balance in a Continuous Stirred Tank Reactor (CSTR), the system dynamics described by the reaction network (17) is described by the four-dimensional dynamical system

$$\begin{aligned}
\dot{S}_1 &= u(S_{1f} - S_1) - k_1\mu_1(S_1)X_1 \\
\dot{X}_1 &= \mu_1(S_1)X_1 - auX_1 \\
\dot{S}_2 &= u(S_{2f} - S_2) + k_2\mu_1(S_1)X_1 - k_3\mu_2(S_2)X_2 \\
\dot{X}_2 &= \mu_2(S_2)X_2 - auX_2
\end{aligned} \tag{18}$$

For system (18) the state vector is defined as $x = \in \mathbb{R}^4$ with $x_1 = S_1$, $x_2 = X_1$, $x_3 = S_2$ and $x_4 = X_2$. The inlet concentration of the primary organic substrate and VFA are denotes be S_{1f} and S_{2f}, respectively. The control input u is the dilution rate of the bioreactor, while k_1, k_2, k_3 are yield coefficients. The parameter $a \in [0, 1]$ represents the proportion of the bacterial population that are affected by the

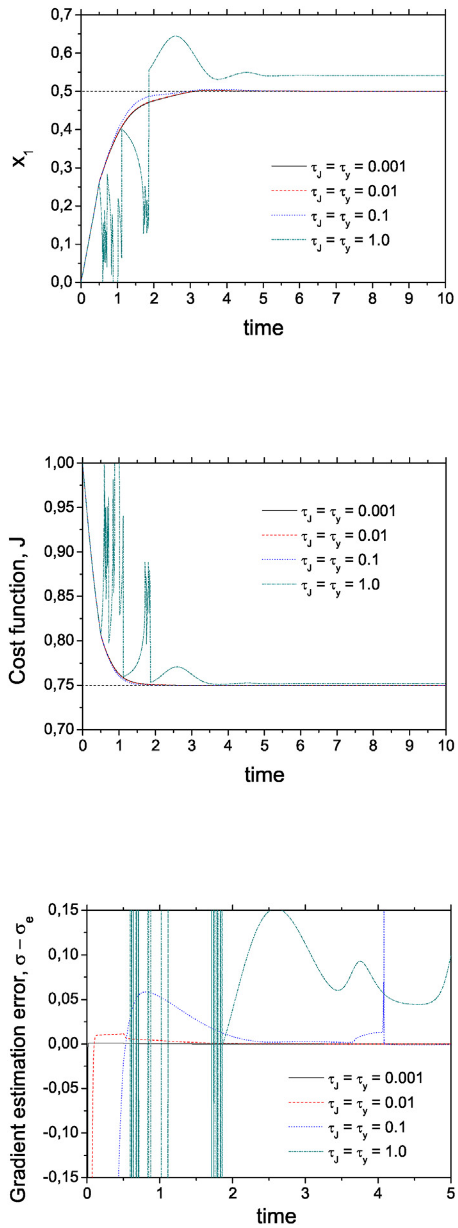

Fig. 4. Effect of the gradient estimator parameters τ_y and τ_J on the ESC performance for the system (14).

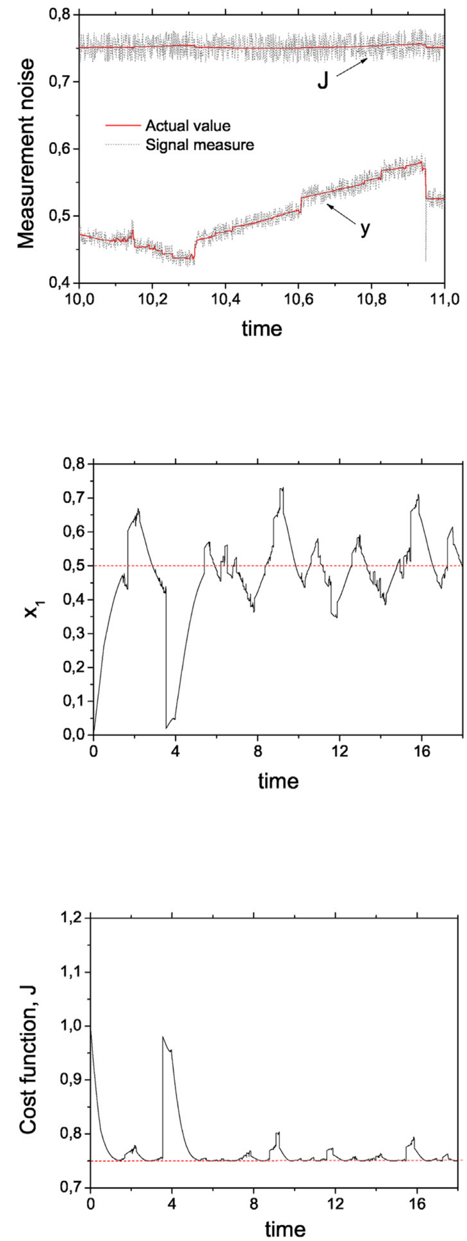

Fig. 5. Effect of measure-noise on the signal J and y for the system (14).

digester dilution; so, $a = 0$ and $a = 1$ correspond to an ideal fixed bed reactor and to an ideal CSTR, respectively [25]. The methane outflow rate can be modeled as [26]:

$$Q = k_2\mu(S_2)X_2 \tag{19}$$

where Q is the methane outflow rate and $k_2 > 0$ is a yield coefficient. The specific growth rates for the acidogenic and methanogenic populations, in [25] are assumed to be described by the Monod and Haldane expressions, respectively, i.e.,

$$\mu_1(S_1) = \frac{\mu_{1,\max}S_1}{K_{S1} + S_1} \tag{20}$$

$$\mu_2(S_2) = \frac{\mu_{2,\max}S_2}{K_{S2} + S_2 + S_2^2/K_{I2}} \tag{21}$$

where $\mu_{1,max}$, K_{S1}, $\mu_{2,max}$, K_{S2}, and K_{I2} are kinetic parameters for the bacterial growth rates.

We assume that only the methane outflow rate and the VFA concentration are available for online monitoring, *i.e.*, the specific growth rates μ_1, μ_2 and the bacterial concentrations X_1 and X_2 are not available for on-line measurement. Hence the control objective is to design an ESC control scheme with u as the control action such that the methane outflow rate Q achieves its maximum value.

In order to apply the design methodology described in Section 3, the AD model is rewritten in the form (1), with

$$f(x) = \begin{bmatrix} -k_1\mu_1(x_1)x_2 \\ \mu_1(x_1)x_2 \\ k_2\mu_1(x_1)x_2 - k_3\mu_2(x_3)x_4 \\ \mu_2(x_3)x_4 \end{bmatrix}$$

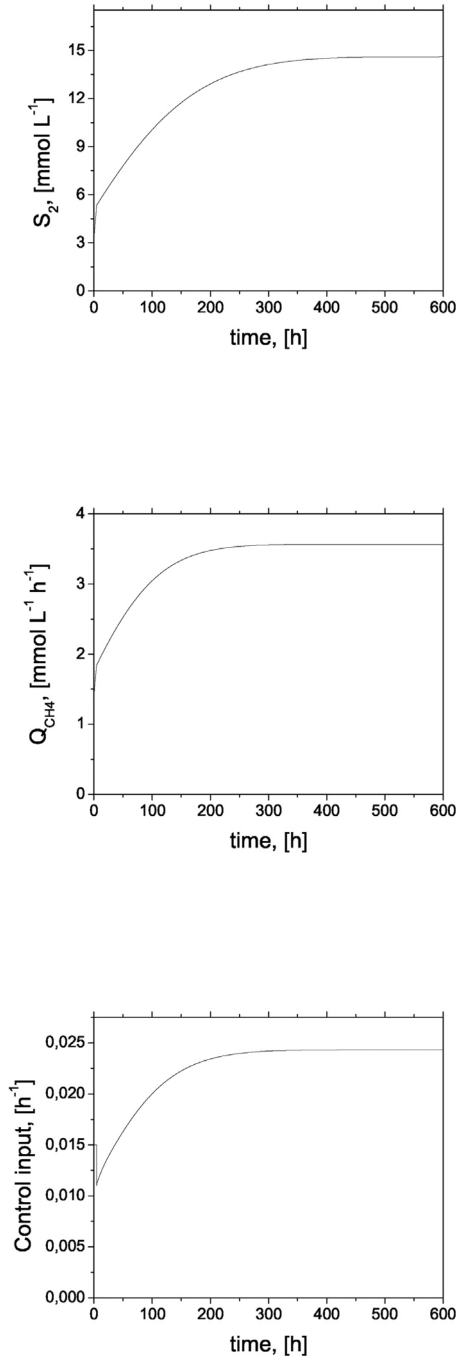

Fig. 6. Closed-loop performance for the AD process without fluctuation in the inlet composition, i.e. $\Delta S_f = 0$.

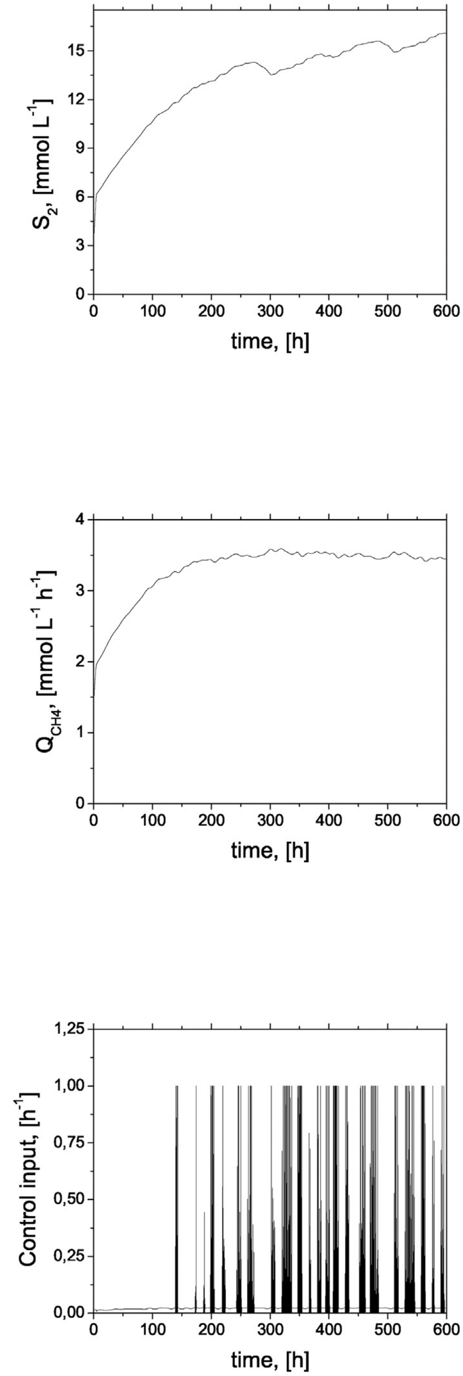

Fig. 7. Closed-loop performance for the AD process with fluctuation in the inlet composition $\Delta S_f \neq 0$.

$$g(x) = \begin{bmatrix} S_{1f} - x_1 \\ -ax_2 \\ S_{2f} - x_3 \\ -ax_4 \end{bmatrix}$$

and $y = h(x) = x_3$. The performance measurable function for this case is related with the methane outflow rate, i.e., $J = Q$ given by (19). Now we can define a function for lumping the uncertainties of the bioreaction system as follows

$$\eta(t) = k_2\mu_1(x_1)x_2 - k_3\mu_2(x_3)x_4 + \Delta S_f$$

we assume that the inlet composition S_{2f} is an uncertain function varying around the nominal value, i.e., $S_{2f} = \bar{S_f} + \Delta S_f$, with $\bar{S_f}$ is a constant such that it provides a nominal value of the inlet concentration ($\bar{S_f} \neq 0 \in \mathbb{R}$).

In this way, the ESC scheme for the AD model is

$$\begin{aligned} u &= \frac{1}{\bar{S_f} - x_3}\left(-\hat{\eta} + k_c\hat{\sigma}\right) \\ \dot{\hat{x_3}} &= u(\bar{S_f} - x_3) + \hat{\eta} + \Gamma\kappa_1(x_3 - \hat{x_3}) \\ \dot{\hat{\eta}} &= \Gamma^2\kappa_2(x_3 - \hat{x_3}) \end{aligned} \tag{22}$$

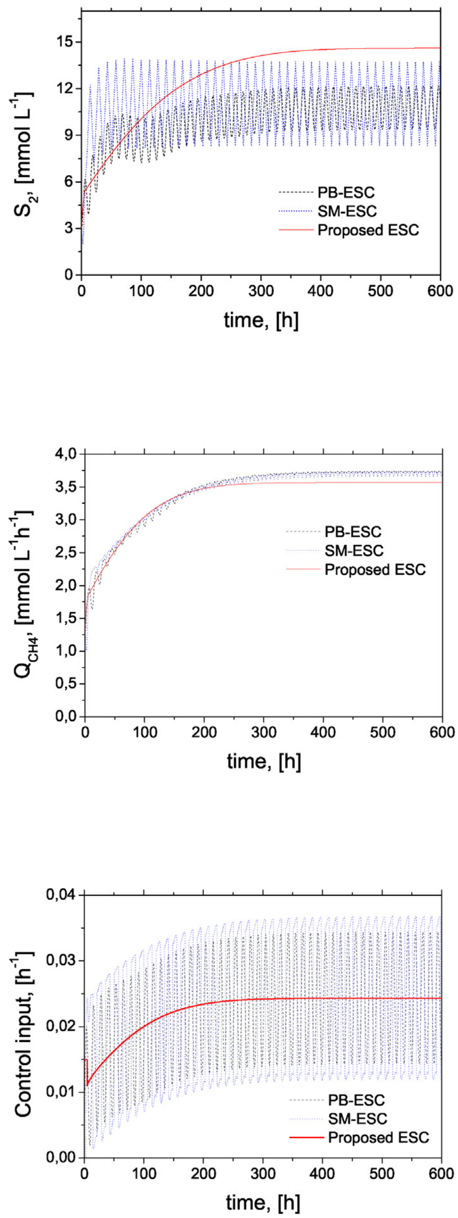

Fig. 8. Comparison of the ESC performance with the perturbation-based and sliding mode based ESC approaches; Perturbation-based ESC scheme (PB-ESC), sliding mode-based ESC (SM-ESC) and proposed Model-based approach (Proposed ESC).

where the estimation value for the gradient $\hat{\sigma}$ is given by ((11)-(12)).

The simulation results are shown in Figs. 6 and 7 . The parameters of the AD model are the same reported in [25]. The controller parameters are set to the following values: $k_c = 0.2$, $\kappa_1 = 1$, $\kappa_2 = 2$, $\Gamma = 2.5$, $\tau_y = 0.1$ and $\tau_J = 0.1$. As we can see in Fig. 6 the ESC scheme (22) achieves the robust convergence for the closed-loop system at the optimal operating conditions. It is interesting to note that the proposed ESC scheme is able to stabilizes more complex unknown systems near the optimal unknown operating conditions. As we can see in Fig. 3(b), the control scheme is able to reach the on-line optimization of the process despite unmodeled dynamics and fluctuations in the inlet composition.

In order to compare the performance of the proposed ESC scheme with respect to the traditional perturbation-based ESC and sliding mode-based approaches [11,7–10,16,27], numerical simulation have been done for the AD model (18). For the case of the free-model perturbation-based ESC we use the basic structure with a high-gain filter [12], for the case of the AD process we have the following ESC controller

$$
\begin{aligned}
u &= \hat{u} + A_m \sin(\omega t) \\
\dot{\hat{u}} &= k\xi \\
\xi &= (Q_{CH4} - \eta_{AD}) \sin(\omega t) \\
\dot{\eta}_{AD} &= -\omega_h \eta_{AD} + \omega_h Q_{CH4}
\end{aligned}
\tag{23}
$$

The control parameters were tuned as $k = 0.0003$, $A_m = 0.01$, $\omega = 0.5$ and $\omega_h = 0.4$. In the case of the sliding mode-based scheme we consider the ESC scheme developed in [27] with the same control parameters. The convergence rate does not has significant difference to reach the optimal operating conditions for the AD process (Fig. 8). However, the response of the control signal presents a clear advantage in terms of the smoothness for the proposed ESC scheme with respect to the other approaches (see Fig. 8).

The simulations results show that the ESC approach proposed here is a good alternative to avoid the drawback of excessive oscillatory behavior in the control signal due to the continuous external perturbation (dither signal) in traditional ESC schemes. On the way to the implementation of the proposed ESC, it is necessary to develop a tuning guidelines for choosing the appropriate control parameters. Also is desirable a more exhaustive convergence analysis taking into account the stability properties of the gradient estimation algorithm. These issues will be studied in a future work.

5. Conclusion

In this paper an extremum-seeking control scheme for a class of nonlinear uncertain systems was proposed. The feedback control configuration was based on the modeling-error compensation approach using the input–output linearizing method coupled with a first-order gradient estimator. Unlike to the conventional ESC schemes, the proposed controller is a model-based algorithm without using exogenous perturbation signals, which avoid the drawback of excessive oscillatory behavior in the controlled signal. Numerical simulations showed that the ESC scheme is able to stabilize uncertain systems at about unknown optimal operating conditions despite unmodeled dynamics and external disturbances.

Conflict of interest

None.

Acknowledgements

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