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Exploration of externally post-tensioned spatial structures using 3D graphic statics

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Abstract
Bending-free structures are of enormous interest due to their high material efficiency. However, structural concerns are commonly conditioned by other design criteria, such as function or aesthetics. In order to overcome these limitations, Todisco et al. developed a methodology that allowed achieving bending-free behavior in two-dimensional structures whose geometry departed from the ideal funicular shape. An external system of post-tensioned cables was defined using graphic statics to introduce new loads into the structure, which permitted minimizing bending. The aim of this paper is to present the basis of a new procedure to extend this methodology to three-dimensional structures using the new developments of 3D graphic statics. Simple spatial structures under different loading conditions have been studied, exploring the wide range of solutions allowed by the indeterminacies of the problem. In addition, reduced-scale models have been constructed for several configurations, as first conceptual proof of the method. This procedure allows exploiting the potentialities of 3D graphic statics for the design of bending-free spatial structures. This is applicable to explore a new and wide range of high-efficient geometries whose shape is imposed *a priori*.

Keywords: equilibrium, 3D graphic statics, structural design, post-tensioning systems, funicular structures.

1. Introduction
Material efficiency in spatial structures is intimately related to their shape. The geometries that allow minimizing bending in the elements of the structure and obtaining an axial-only behavior are referred as funicular. However, this ideal shape is not always compatible with non-structural requirements such as function or aesthetics.

Todisco et al. [1, 2] developed a methodology using graphic statics that permits attaining bending-free behavior for curved structures in two dimensions whose geometry departs from the ideal funicular shape. The desired state of equilibrium is obtained through the introduction of an additional set of loads into the structure that is deduced using graphic statics from the structure's reciprocal polygon of forces. In order to generate these new loads, an external system of post-tensioned cables is designed based on the same graphical construction.

The procedure exposed by Todisco is applicable for any two-dimensional curved geometry. In addition, two parameters, or indeterminacies, governing the internal forces in the elements of the structure and the cables, enable the optimization of the solutions and the exploration of wide and varied range of bending-free structures.

This paper explores the possibilities of expanding the methodology proposed by Todisco for its application to spatial structures, exploiting the recent developments of three-dimensional graphic statics.

2. Theoretical framework

2.1. Introduction

The approach presented in this paper aims to extend the two-step design procedure employed with planar structures to 3D geometries. In this study, structures composed of discrete elements have been analyzed; therefore, the loads have been supposed to be applied at the nodes of the structure.

The pinned structures addressed by the method will behave, in general, as mechanisms under an arbitrary set of loads. They will exclusively attain equilibrium when loaded with certain specific distributions of external loads. The goal of the method is to find these distributions of loads and to introduce them into the structure by means of an external system of post-tensioned cables.

2.2. Distribution of loads provoking funicular behavior

The first step of the proposed procedure consists in obtaining the additional loads that have to be introduced into the structure to achieve a funicular behavior. The polyhedron-based approach to 3D graphic statics, as developed by Akbarzadeh et al. [3, 4], has proven to be well adapted for this purpose.

For a given spatial structure composed by discrete bar elements and subjected to external loads applied on its nodes, it is possible to obtain an open polyhedron of forces, whose faces represent the internal axial forces in the bars and the loads applied at the nodes. For each of the nodes of the structure, the faces of the polyhedron are perpendicular to the bars and forces converging at the node. Only the direction, and not the area of the faces corresponding to the bars of the structure, is known *a priori*. Combining the polyhedrons representing all the nodes of the structure, it is possible to obtain a global polyhedron of forces for the structure. This polyhedron will not be closed at this point. The forces that need to be introduced into the structure to achieve funicular equilibrium can be deduced from the area and direction of the faces that are needed to close the polyhedron.

As proved by Maxwell [5], the reciprocal relationship between a structure and its force polyhedron requires the structure to be composed of planar faces. If the structure and the external loads comply with this condition, it will be possible to obtain a closed polyhedron of forces modifying only the area of the faces corresponding to the external loads. Therefore, the loads to be introduced into the structure will act in the same direction as the external loads as shown in Figure 1.

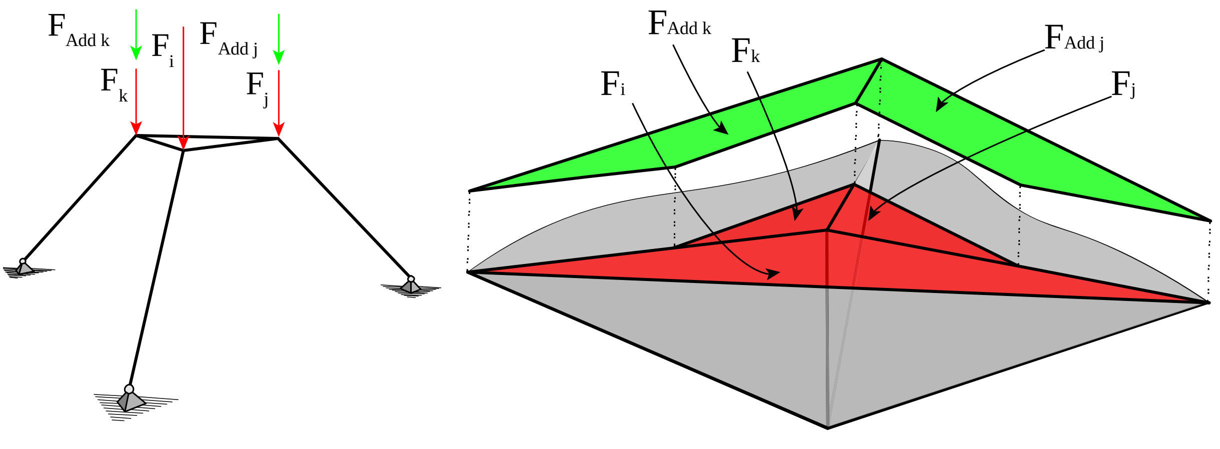

Figure 1: Structure with planar faces under external loads F_i, F_j, and F_k, in red, and additional loads to obtain funicular equilibrium F_{Addj} and F_{Addk} in green (left); reciprocal polyhedron of forces with faces corresponding to external loads F_i, F_j, and F_k in red, and closing faces corresponding to F_{Addj} and F_{Addk} in green (right).

If, on the contrary, the elements of the structure and the external loads do not comply with the planarity condition, the polyhedral cells corresponding to each of the nodes of the structure will not be compatible, and the global polyhedron will present open spaces. It will be possible to add new faces to the polyhedron of forces to close these spaces, and thus obtain a polyhedron compatible with funicular equilibrium. The new faces will represent new forces to be introduced into the structure, whose directions will not coincide in general with those of the external loads. They will depend on the disposition chosen for the faces that close the polyhedron (Figure 2).

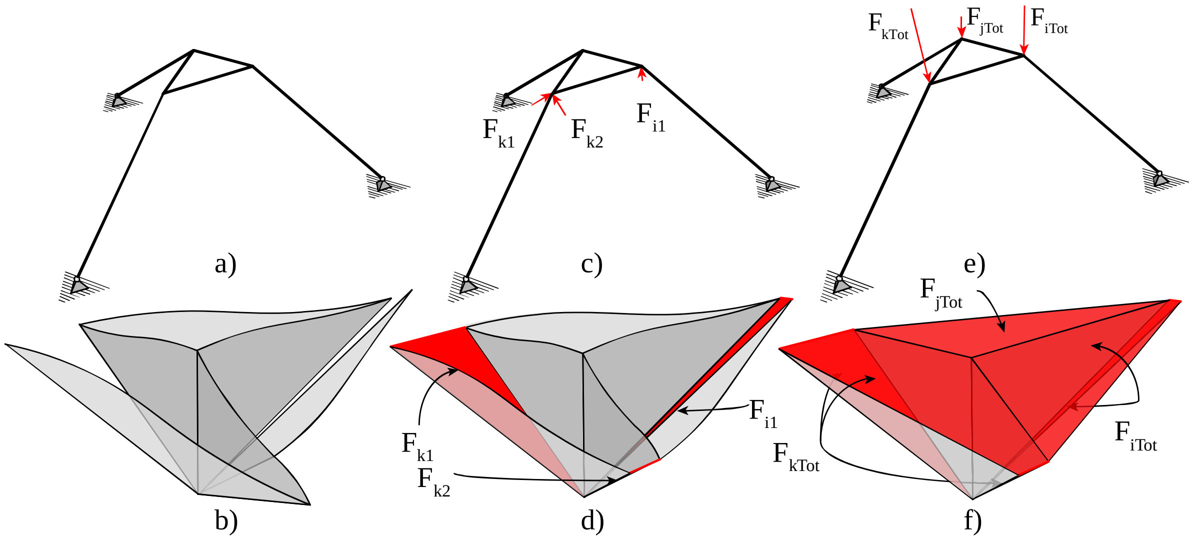

Figure 2: Computation of the loads to obtain funicular behavior for a structure with non-planar faces: a) original structure without any loads; b) open polyhedron with faces perpendicular to the elements of the structure; c) forces applied on the structure to obtain compatible cells F_{i1}, F_{k1} and F_{k2}; d) polyhedron of forces with faces closing the open spaces between cells; e) forces that need to be applied to obtain funicular equilibrium F_{iTot}, F_{jTot} and F_{kTot}; f) corresponding closed polyhedron of forces representing funicular equilibrium.

2.2. Design of the system of post-tensioned cables to introduce additional loads into the structure

Once the loads that provoke the funicular behavior of the structure are known, the second step of the procedure requires finding a way to introduce these forces into the structure. Following the proposal by Todisco [2], an external system of post-tensioned cables is designed (Figure 3). However, contrary to the two-dimensional case, the higher complexity of spatial structures prohibits, in general, the use of a unique graphical construction for the two steps of the procedure.

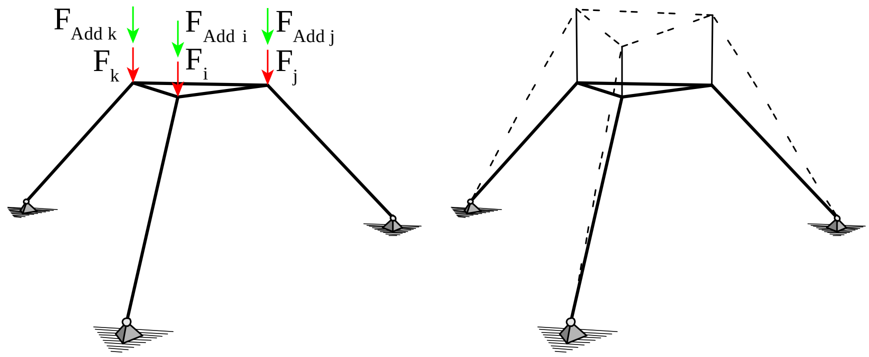

Figure 3: Structure under external loads and additional forces (left) and system of post-tensioned cables (right).

In two dimensions, the cable system constituted a funicular structure itself when loaded with the forces to be introduced into the structure. Finding a spatial funicular configuration for a given set of loads is a complex problem. Polyhedron-based 3D graphic statics will only be applicable to design the cable system in certain very specific cases, when the resulting structure respects the condition of having planar faces. If the structure does not respect this condition, other procedures can be employed for the design of the cable system. If all the loads to be introduced into the structure are parallel, the Thrust Network Analysis (TNA) developed by Block and Ochsendorf [6, 7] can be used. A simpler approach consists in designing a statically determinate structure. In this last case, in order to maintain the graphic nature of the method, the vector-based approach to 3D graphic statics [8] can be employed. Figure 4 shows an example for the cable system for each of the three design alternatives proposed.

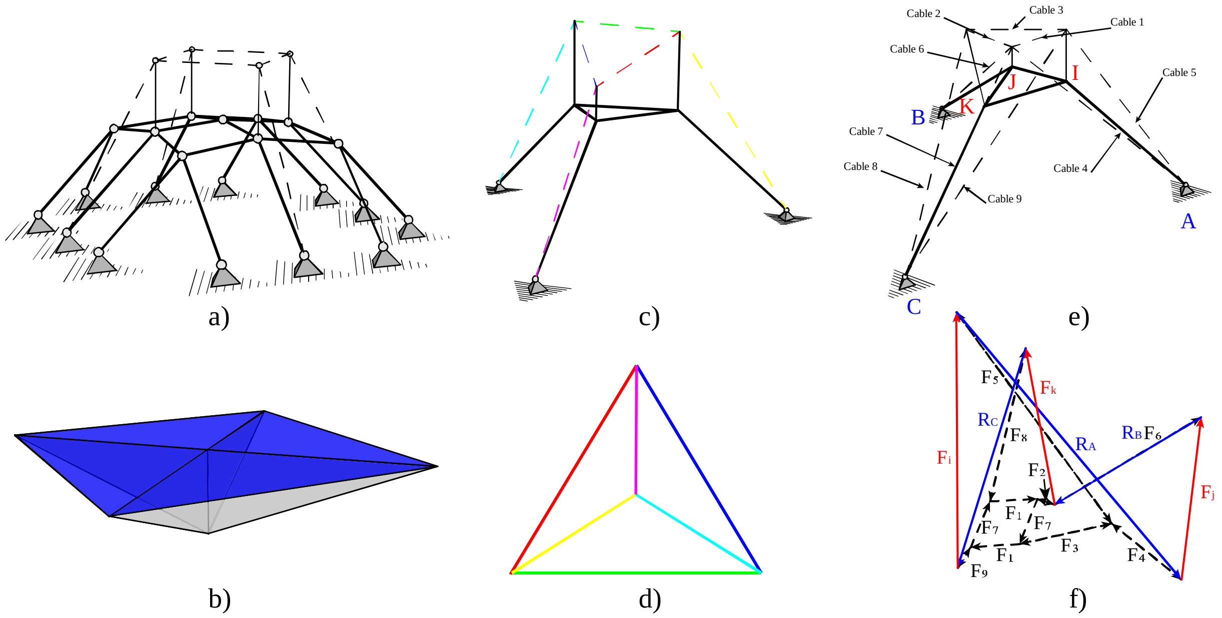

Figure 4: Design of the external system of post-tensioned cables: a) funicular structure for the cable system using the polyhedron approach to 3D graphic statics and b) polyhedron of forces corresponding to the cable system; c) funicular structure for the cable system using TNA and d) corresponding dual diagram of forces; e) statically determinate structure for the cable system and f) diagram of forces using vector-based 3D graphic statics.

If polyhedron-based 3D graphic statics can be employed, it should be possible to obtain an aggregated polyhedron of forces for both the original structure and the external cable system.

2.3. Indeterminacies of the method

Todisco identified two indeterminacies that allowed exploring the solution space of externally post-tensioned funicular structures in two dimensions. A similar reasoning can be adopted to explore the possibilities of the three-dimensional version of the method, although, some differences arise due to the higher complexity of spatial structures.

2.3.1. First indeterminacy

For spatial structures, analogously to the two-dimensional case, the first indeterminacy is related to the infinite possible solutions that exist changing the magnitude of the compressive internal forces that are induced in the original structure. This indeterminacy is governed by the shape and size of the polyhedron of forces representing the state of internal forces in the structure.

Depending on the geometric degrees of freedom available to reshape the polyhedron, the distribution of internal forces compatible with a certain set of external loads will be unique or not. The first case is equivalent to the two-dimensional problem, where it is only possible to change the intensity of the internal forces and the loads applied on the structure but not their distribution. This is achieved modifying the size (i.e. scaling) of the polyhedron of forces, which is visually explained in Figure 5.

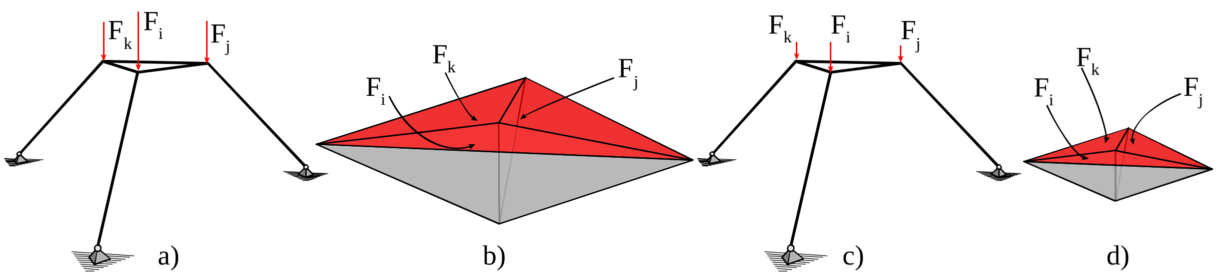

Figure 5: Effect of the 1st indeterminacy: a) structure under a set of loads of higher magnitude and b) reciprocal polyhedron of forces; c) structure under a set of loads of lower magnitude and d) reciprocal polyhedron of forces.

In the second case, the geometric degrees of freedom of the polyhedron allow exploring different possible distributions of internal forces compatible with a certain set of loads applied on the structure as shown in Figure 6. In this situation it is not possible to control the internal forces in the structure modifying the loads that are applied on it.

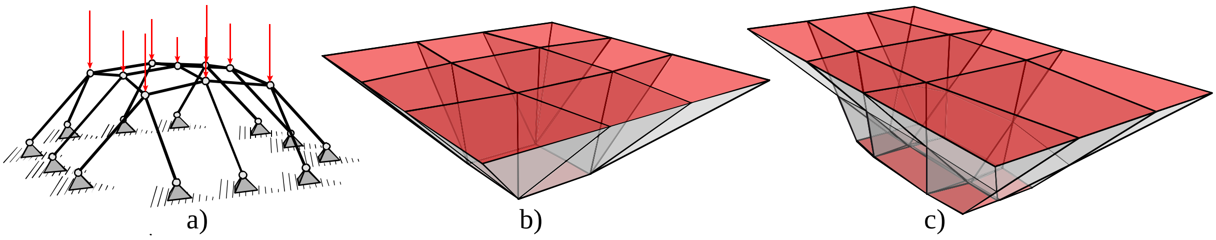

Figure 6: Effect of the 1st indeterminacy: a) structure under a symmetric set of loads; b) 1st polyhedron of forces compatible with the external loads; c) 2nd polyhedron of forces compatible with the external loads.

2.3.2. Second indeterminacy

The second indeterminacy relates to the position of the post-tensioned cables and the prestressing forces in them. The interpretation of this indeterminacy will depend on the approach adopted to design the cable system. If the cable system is designed using polyhedron-based 3D graphic statics, the different configurations for the post-tensioned cables can be explored changing the shape of the polyhedron of forces reciprocal to the cable system. If TNA is used, the second indeterminacy will be controlled through the dual diagram of forces. Finally, if the structure chosen for the cable system is statically determinate there will be as many parameters corresponding to the second indeterminacy as there are nodes in the cable structure. The effect of the variation of the parameters of the second indeterminacy in the cables and the corresponding diagrams of forces for each approach are illustrated in Figure 7.

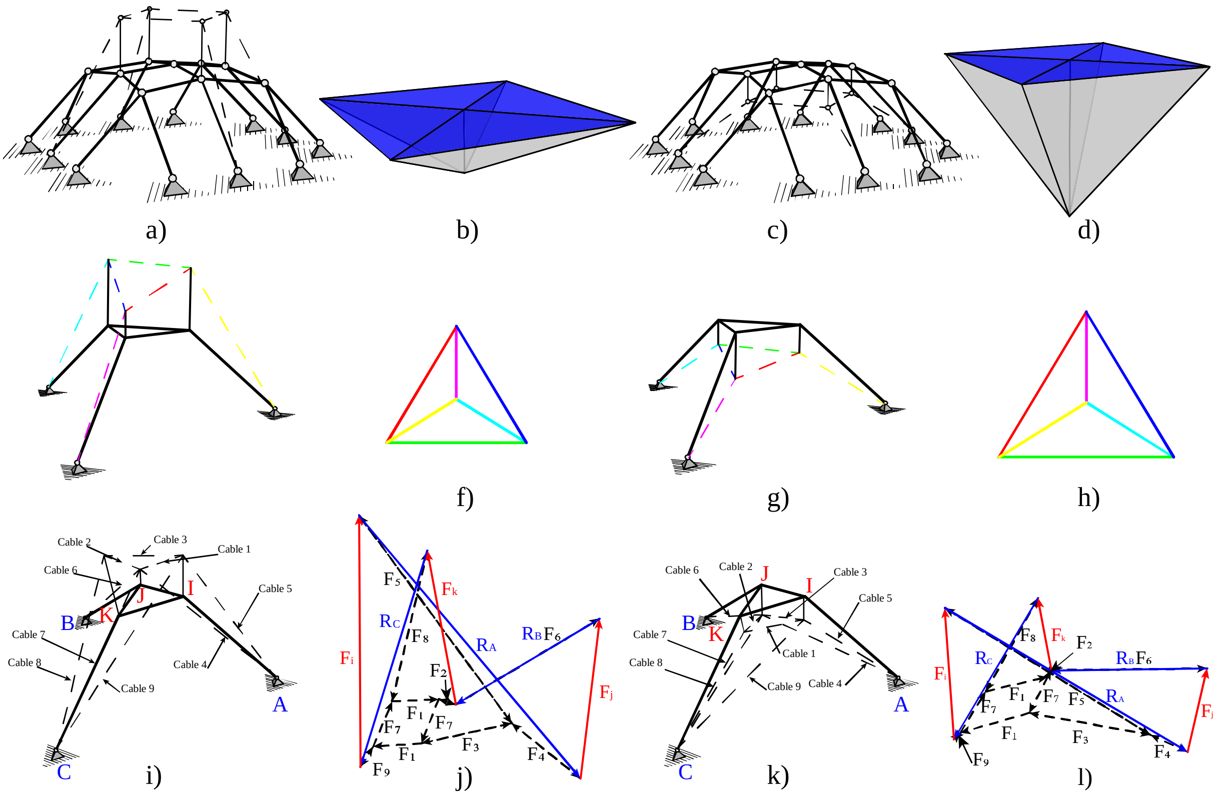

Figure 7: Effect of the second indeterminacy: a), c) possible funicular configurations for the cable system with planar faces and b), d) corresponding polyhedrons of forces; e), g) possible funicular configurations for the cable system with non-planar faces and f), h) corresponding dual diagrams of forces (TNA); i), k) statically determinate structures for the cable system and j), l) corresponding spatial diagrams of forces.

3. Case studies

The methodology described above has been implemented to achieve funicular behavior in two structures of progressive complexity. Both structures have been built respecting the planarity restriction, which limits the possible configurations attainable. The structures are constructed ensuring a minimum number of degrees of freedom. If the structure was statically determinate or indeterminate, axial-only behavior would always be possible, given that axial stiffness was assumed to be much higher than flexural stiffness. For the present study, all external loads act on the vertical direction.

The first structure studied consists of 8 simply supported nodes, and 4 interior nodes, on which loads are applied. The structure has a total of 12 nodes, 12 bars and 9 faces. Several configurations, all of them respecting the planarity restriction, with different conditions of symmetry were studied (Figure 8).

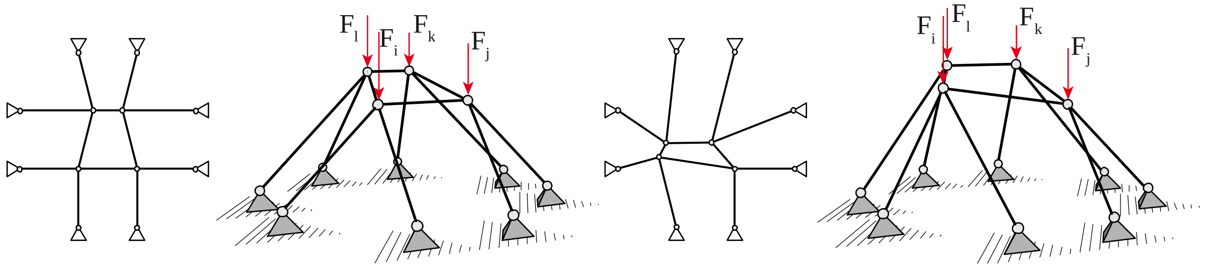

Figure 8: View in plan and perspective of the first structure studied for a symmetric (left) and a non-symmetric (right) configuration.

Developing the translational equilibrium equations, it was found that the symmetry conditions had an impact on the redistribution of the external loads. While the symmetric configurations of the structure can only attain funicular equilibrium under certain sets of external loads, non-symmetric configurations are always capable of developing axial-only internal forces in response to external loads.

In order to allow considering all the mechanic degrees of freedom of the structure through the geometric degrees of freedom of the corresponding polyhedron of forces, the principle of transmissibility of forces was taken into account to represent external loads with one or two parallel faces as shown in Figure 9.

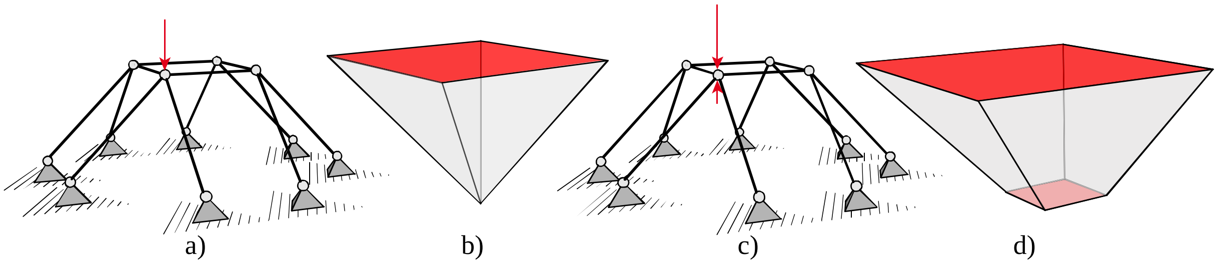

Figure 9: a) External load applied on a node of the structure and b) reciprocal polyhedral cell; c) decomposition of the external load applied on the structure and d) reciprocal polyhedral cell.

Exploiting the geometric degrees of freedom of the polyhedron of forces, it is possible to adapt the shape of the polyhedron in such a way, that some of the faces representing the external loads are proportional to the corresponding loads. As a result, for the 12-node structure studied, it is possible to match the faces of the polyhedron with three external loads when the structure is symmetric. When the configuration is non-symmetric, all the faces representing external loads can be made proportional to the corresponding loads. Hence, the polyhedrons graphically depict the conditions to obtain funicular behavior that were deduced from the equilibrium equations. Moreover, the polyhedron of forces shows that for symmetric configurations, the distribution of internal forces in the structure is not determined for a given set of external loads, while for non-symmetric configurations, the state of internal forces in the structure is unique for each load case, which allows controlling it through the first indeterminacy.

Consequently, for symmetric configurations, funicular behavior can be obtained introducing only one additional load into the structure. For non-symmetric configurations, axial-only behavior is always possible. However, a fully compressed state in the structure can be achieved adequately modifying one of the loads that acts on it. In both cases, only one load needs to be added to the structure to obtain the desired behavior. Thus, the cable system to introduce this load could theoretically be as simple as a pair of post-tensioned cables coplanar with the additional load or a system of three cables.

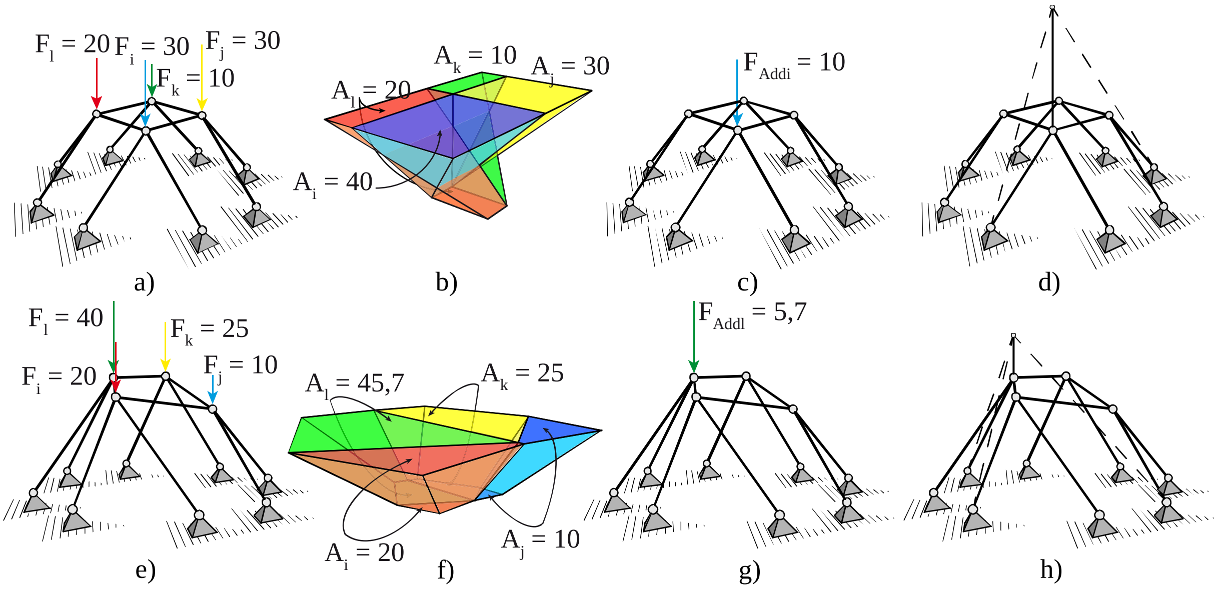

Figure 10: 12-node structure: a) symmetric configuration under external loads; b) polyhedron of forces matching three faces with external loads; c) load needed to achieve funicular behavior and d) cable system introducing the load into the structure; e) non-symmetric configuration under external loads; f) polyhedron of forces matching three faces; g) load needed to achieve full compression and h) cable system introducing the load into the structure.

A second, more complex, structure is studied; it is composed of 12 exterior nodes, corresponding to hinged supports, 9 interior nodes, where loads are applied, and a total of 24 bars. Diverse configurations have been studied with different conditions of symmetry. In Figure 11, a case is presented where the distribution of loads that make the structure funicular, allows designing the cable system using the polyhedron-based approach to 3D graphic statics. It should be possible to obtain an aggregated polyhedron of forces for both the original structure and the cable system. A first proposal is presented in Figure 11.f, but further study of this type of polyhedron is required.

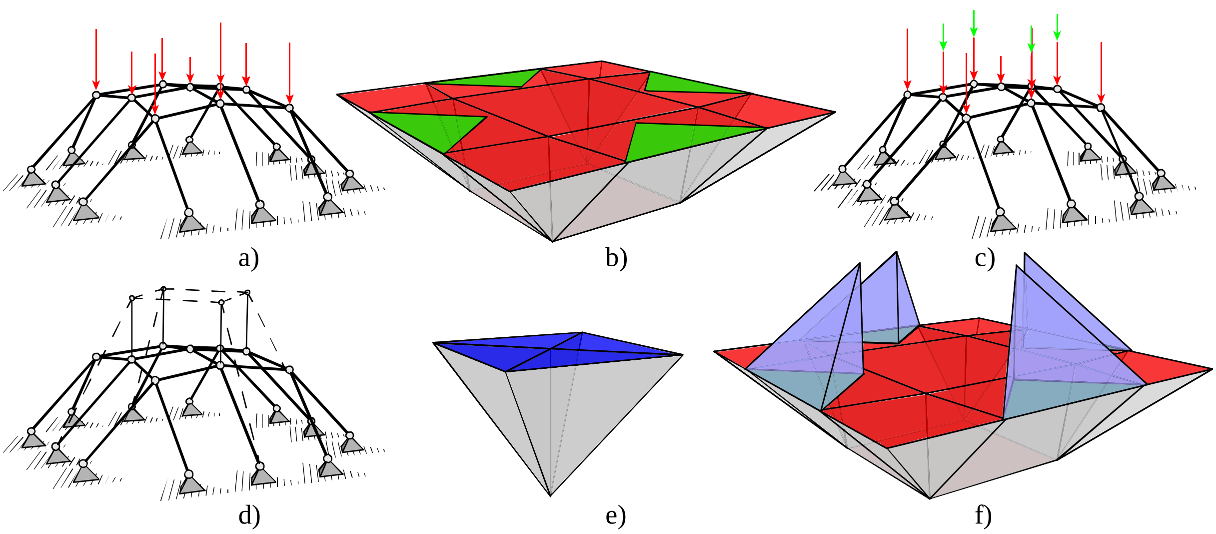

Figure 11: a) Structure under self-weight; b) polyhedron of forces with faces that need to be modified in green; c) loads to be added to the structure in green; d) cable system to introduce additional loads; e) polyhedron of forces for the cable system; and f) aggregated polyhedron for the compressed structure and the cable system.

4. Reduced-scale models

Reduced-scale structures were fabricated using additive manufacturing for the base geometry and the struts, and two types of thin elastic strands to reproduce the post-tensioning system. Despite several approximations, the models provide a useful visual evidence of the concepts presented before.

Figure 12: Reduced-scale model of an externally post-tensioned spatial funicular structure.

5. Conclusion

Designing spatial structures constitutes a complex challenge in which many factors must be taken into account. Choosing a bending-free geometry would be the optimal solution under a material-efficient point of view, but this is often not possible due to non-structural requirements. This project, following the design philosophy developed by Todisco, offers an alternative for the design of spatial bending-free structures. The methodology is founded on the transformation of the internal force distribution in the structure through the introduction of a new set of loads using an external post-tensioning system. Furthermore, the applicability of the method based on 3D graphic statics has been tested on two case studies. Finally, reduced-scale physical models have provided further validation of the method.

The research presented in this paper is just a first step for understanding the structural behavior of externally post-tensioned 3D structures. Therefore, further studies would be needed to apply the method to continuous curved structures, develop systems to control the internal forces distribution and to study the conditions that ensure tension in the external cable system, among other possible lines of research.

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