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Behaviour of the Absorptive Part of the $W^\pm$ Electromagnetic Vertex

M. Beuthe[1], R. Gonzalez Felipe, G. López Castro[2] and J. Pestieau

Institut de Physique Théorique, Université catholique de Louvain,

B-1348 Louvain-la-Neuve, Belgium

Abstract

The absorptive part of the $WW\gamma$ vertex induced by massive fermion loops is considered for different kinematical configurations. We show that the axial part of this vertex is different from zero not only when massive fermions are involved but also for massless fermion loops, if one of the W bosons is space-like and the other is time-like. We also discuss in what sense Low's soft photon theorem is satisfied.

[1]IISN researcher under contract 4.4509.86

[2]Permanent address: Departamento de Física, CINVESTAV del IPN, Apartado Postal 14-740, 07000 México D.F., México

1 Introduction

The unstable $W^{\pm}$ and Z^0 gauge bosons cannot be prepared as asymptotic states. They are first produced in collisions of stable particles and subsequently we observe their decay products. The intermediate states associated to resonances in these processes are described by propagators in the physical amplitude. The latter blows up in the resonance kinematical region, unless an imaginary part, which softens this singularity, is added to the propagators. The solution to the above problem in field theory requires to include an infinite summation of vacuum polarization diagrams in the gauge boson propagator. At a given order in perturbation theory, this procedure can destroy the gauge invariance of the resulting amplitudes and therefore, can induce important changes in observable effects [1]-[3].

Different prescriptions to obtain gauge-invariant amplitudes have been presented in the literature [1]-[7]. In the presence of unstable particles, this can be achieved either, (a) by making a Laurent expansion of the full amplitude around the complex pole associated to the resonance in the second Riemann sheet [1, 2, 5, 6] or, (b) by using full propagators and vertex functions in the amplitudes which include only resummation of (gauge-invariant) fermion loops [3]-[5]. (For a different approach, see also [7]).

In this paper we shall discuss the electromagnetic gauge invariance in processes where a real photon is emitted from an internal $W^{\pm}$ gauge boson. Since gauge invariance in a given process is guaranteed by the fulfillment of the Ward identities among different Green functions, we will focus our analysis on the latter requirement. This issue is relevant for instance in processes involving the production and decay of a W boson such as $e^+e^- \rightarrow e^-\bar{\nu}_e ud$ [3, 6], $q\bar{q}' \rightarrow \ell\nu_\ell\gamma$ [4], the top quark decay $t \rightarrow b\ell\nu_\ell\gamma$, or $\gamma e^- \rightarrow \nu_e\tau^-\bar{\nu}_\tau$ [7].

In previous papers [3]-[5] the problem of gauge invariance has been considered including the imaginary parts induced by massless fermion loops in the $W^{\pm}$ propagator and the $WW\gamma$ vertex. Here we shall address this problem in the case when the fermions in the loops are massive. Generally speaking, mass effects of light fermions in the loops turn out to be negligible for invariant masses of the W bosons large enough. However, we would like to stress two new relevant features that appear in both the massless and massive cases, namely, the appearance of axial-vector contributions to the $WW\gamma$ vertex and the violation of Low's photon energy expansion [8] (see section 3) for specific kinematical configurations of W's four-momenta.

2 Electromagnetic gauge invariance for unstable $W^\pm$ bosons

In this section we will consider the electromagnetic Ward identity relating the W propagator and the $WW\gamma$ vertex in the one-loop approximation. Since we are interested in gauge invariance in presence of a finite width for the W boson, we will study only the absorptive parts of the one-loop corrections. Following Refs. [3, 4], we will consider the kinematical situations where the absorptive parts of two- and three-point functions are generated by fermion loops.

For definiteness let us consider the W propagator in the unitary gauge. Dyson resummation of vacuum polarization graphs leads to the following form of the W propagator:

$$D^{\mu\nu}(q) = \frac{-i}{q^2 - M_W^2 + i\mathrm{Im}\,\Pi^T(q^2)}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + \frac{i}{M_W^2 - i\mathrm{Im}\,\Pi^L(q^2)}\frac{q^\mu q^\nu}{q^2}, \tag{1}$$

and its inverse

$$(iD^{\mu\nu}(q))^{-1} = \left(q^2 - M_W^2 + i\mathrm{Im}\,\Pi^T(q^2)\right)\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) - \left(M_W^2 - i\mathrm{Im}\,\Pi^L(q^2)\right)\frac{q^\mu q^\nu}{q^2}, \tag{2}$$

where the transverse (T) and longitudinal (L) pieces of the W vacuum polarization are defined as:

$$\mathrm{Im}\,\Pi^{\alpha\beta}(q) = \mathrm{Im}\,\Pi^T(q^2)\left(g^{\alpha\beta} - \frac{q^\alpha q^\beta}{q^2}\right) + \mathrm{Im}\,\Pi^L(q^2)\frac{q^\alpha q^\beta}{q^2}. \tag{3}$$

To define the $WW\gamma$ vertex we use the convention $W^-_\alpha(q_1) \to W^-_\beta(q_2) + \gamma_\mu(k)$, where α, β, μ denote the four-polarization indices and q_1, q_2, k the corresponding four-momenta. In what follows we will assume that the W bosons are virtual and the photon is real. After including the one-loop corrections (cf. Fig. 1) we obtain the following form of the full $WW\gamma$ vertex:

$$-ie\Gamma^{\alpha\beta\mu} = -ie\left(\Gamma_0^{\alpha\beta\mu} + \Gamma_1^{\alpha\beta\mu}\right), \tag{4}$$

where

$$\Gamma_0^{\alpha\beta\mu} = g^{\alpha\beta}(q_1+q_2)^\mu - g^{\alpha\mu}(q_1+k)^\beta - g^{\beta\mu}(q_2-k)^\alpha \tag{5}$$

corresponds to the tree-level expression and $\Gamma_1^{\alpha\beta\mu}$ denotes the absorptive part of the one-loop correction.

The QED Ward identity which relates the two- and three-point functions involving the W boson is given by

$$k_\mu\Gamma^{\alpha\beta\mu} = (iD^{\alpha\beta}(q_1))^{-1} - (iD^{\alpha\beta}(q_2))^{-1}. \tag{6}$$

This identity provides (at any order of perturbation theory) a linear relation between the vertex correction and the vacuum polarization, which reads for the present case

$$k_\mu\Gamma_1^{\alpha\beta\mu} = i\mathrm{Im}\,\Pi^{\alpha\beta}(q_1) - i\mathrm{Im}\,\Pi^{\alpha\beta}(q_2). \tag{7}$$

Using the explicit expressions for these corrections we can verify the validity of Eq.(7). Indeed, for simplicity let us consider only the contribution of the (ν_τ, τ^-) doublet to the fermion loops. Using the standard cutting rules, the vacuum polarization is given by

$$\mathrm{Im}\,\Pi_\tau^{\alpha\beta}(q) = -\frac{g^2}{8}\int\frac{d\Omega}{32\pi^2}\mathrm{Tr}\left\{\not{p}_\tau\gamma^\alpha(\not{q}-\not{p}_\tau)\gamma^\beta(1-\gamma^5)\right\}\frac{q^2-m_\tau^2}{q^2}\theta(q^2-m_\tau^2), \tag{8}$$

where $\theta(x)$ denotes the step function, g is the $SU(2)$ weak coupling constant, m_τ and p_τ^μ are the τ-lepton mass and four-momentum respectively.

On the other hand, the vertex correction contains two terms corresponding to two different cut diagrams depicted in Figs. (1a) and (1b), namely,

$$\Gamma_1^{\alpha\beta\mu} = i\left(I_\tau^{\alpha\beta\mu}(q_1) + \widetilde{I}_\tau^{\alpha\beta\mu}(q_2)\right), \tag{9}$$

where

$$\begin{aligned} I_\tau^{\alpha\beta\mu}(q_1) &= \frac{Q_\tau g^2}{8}\int \frac{d\Omega}{32\pi^2}\frac{q_1^2 - m_\tau^2}{2(k\cdot p_\tau)q_1^2}\,\theta(q_1^2 - m_\tau^2) \\ &\quad \times \mathrm{Tr}\left\{\not{p}_\tau\gamma^\alpha(\not{q}_1 - \not{p}_\tau)\gamma^\beta(2p_\tau^\mu - \not{k}\gamma^\mu)(1-\gamma^5)\right\}, \end{aligned} \tag{10}$$

$$\begin{aligned} \widetilde{I}_\tau^{\alpha\beta\mu}(q_2) &= -\frac{Q_\tau g^2}{8}\int \frac{d\Omega}{32\pi^2}\frac{q_2^2 - m_\tau^2}{2(k\cdot p_\tau)q_2^2}\,\theta(q_2^2 - m_\tau^2) \\ &\quad \times \mathrm{Tr}\left\{\not{p}_\tau(2p_\tau^\mu + \gamma^\mu\not{k})\gamma^\alpha(\not{q}_2 - \not{p}_\tau)\gamma^\beta(1-\gamma^5)\right\}; \end{aligned} \tag{11}$$

$Q_\tau = -1$ is the τ - lepton electric charge in units of the positron charge e.

It is straightforward to check that the QED Ward identity is satisfied when we insert Eqs.(8)-(11) into Eq.(7). In other words, the electromagnetic gauge invariance is guaranteed in a process involving the production and decay of W bosons, if Eqs.(1) and (4) are used for the W propagator and $WW\gamma$ vertex function, respectively.

Let us now consider the relationship between the above results and those of Ref.[6]. If the fermions in the loops are massless then $\mathrm{Im}\,\Pi^L(q^2) = 0$, $\mathrm{Im}\,\Pi^T(q^2) = q^2\Gamma_W/M_W = \sum g^2q^2/48\pi$ (the sum runs over doublets and colours) and we obtain [4] when $q^2 > 0$:

$$D^{\mu\nu}(q) = \frac{-i}{q^2 - M_W^2 + iq^2\gamma_W}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{M_W^2}(1 + i\gamma_W)\right), \tag{12}$$

where $\gamma_W \equiv \Gamma_W/M_W$ and Γ_W denotes the total width of the W boson for massless fermions. In this case the $WW\gamma$ vertex receives only vector contributions and we have [4] (see also Eqs.(15),(17) and (24) below)

$$-ie\Gamma^{\alpha\beta\mu} = -ie\,(1 + i\gamma_W)\,\Gamma_0^{\alpha\beta\mu}. \tag{13}$$

Note that the propagator (12) can be rewritten as follows:

$$D^{\mu\nu}(q) = \frac{-i(1+i\overline{\gamma}_W)^{-1}}{q^2 - \overline{M}_W^2 + i\overline{M}_W\overline{\Gamma}_W}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{\overline{M}_W^2 - i\overline{M}_W\overline{\Gamma}_W}\right), \quad (14)$$

with $\overline{M}_W^2 = M_W^2 + \Gamma_W^2$ and $\overline{\gamma}_W \equiv \overline{\Gamma}_W/\overline{M}_W = \Gamma_W/M_W$.

The form of the propagator given in Eq.(14) was obtained in [6] using a Laurent expansion of the full propagator around the complex pole position $\overline{M}_W^2 - i\overline{M}_W\overline{\Gamma}_W$, where $(1+i\overline{\gamma}_W)^{-1}$ plays the role of a (complex) renormalization wave function for the W boson. Thus, the form of the propagator derived in Ref.[6], which is valid near the resonance, is identical to the full W propagator in the limit of massless fermions in loop corrections. Since the full $WW\gamma$ vertex in the massless case has the same tensor structure as the tree-level expression[3], the Ward identity is also satisfied. We note that $|1+i\gamma_W|^2 \simeq 1.0007$ is a modest correction.

3 Absorptive corrections to $WW\gamma$ vertex: massless fermions

In this section we consider explicitly the absorptive part of the one-loop corrections to the $WW\gamma$ vertex (Fig. 1) in the case of massless fermions (except for the top quark). We will focus our attention on the structure of these vertex corrections for two different kinematical configurations involved in processes of current interest, namely, (a) $q_1^2 < 0, q_2^2 > 0$ and (b) $q_1^2 > 0, q_2^2 > 0$. In particular, we will discuss the behaviour of the corrections *in the soft photon limit* and the non-zero contribution to the axial-vector part induced by the decoupling of the top quark from the absorptive vertex corrections.

The momenta in the $WW\gamma$ vertex flow according to $W_\alpha^-(q_1) \to W_\beta^-(q_2) + \gamma_\mu(k)$, with $q_1 = q_2 + k$ and the photon being on-shell. Since the top quark ($m_t = 175 \pm 6$ GeV) [9] is far from being massless, the doublet of fermions (t, b) containing the top quark does not contribute to absorptive corrections when $q_{1,2}^2 < m_t^2$. In this case, the vertex corrections can be written as

[3]Indeed, if we keep the non-leading renormalization wave function factor in Eq.(14), the Ward identity forces the $WW\gamma$ vertex to have the form given in Eq.(13).

$$\Gamma_1^{\alpha\beta\mu} = ic\left\{(3Q_L + 6Q_D - 6Q_U)\,\overline{V}^{\alpha\beta\mu} + i\,(3Q_L + 6Q_D + 6Q_U)\,\overline{A}^{\alpha\beta\mu}\right\}, \tag{15}$$

where Q_i $(i = L, U, D)$ denote the electric charges of leptons and quarks in units of the positron charge e and

$$c \equiv -\frac{g^2}{64\pi(k\cdot q_1)}. \tag{16}$$

The vector and axial-vector contributions in Eq.(15) are given, respectively, by

$$\overline{V}^{\alpha\beta\mu} = V_f^{\alpha\beta\mu}(q_1)\,\theta(q_1^2) + \widetilde{V}_f^{\alpha\beta\mu}(q_2)\,\theta(q_2^2), \tag{17}$$

$$\overline{A}^{\alpha\beta\mu} = A_f^{\alpha\beta\mu}(q_1)\,\theta(q_1^2) + \widetilde{A}_f^{\alpha\beta\mu}(q_2)\,\theta(q_2^2), \tag{18}$$

where f refers to the fermion that emits the photon.

Note that if the top contributed to the absorptive corrections ($q_1^2, q_2^2 \gg m_t^2$), we would have to add the term $3ic(Q_D - Q_U)V^{\alpha\beta\mu} - 3c(Q_D + Q_U)A^{\alpha\beta\mu}$ to the r.h.s. of Eq.(15). The coefficient of the axial-vector term would become $-3c(Q_L + 3Q_D + 3Q_U) = 0$, which assures the vanishing of this contribution as noticed in [3].

Using the results given in the Appendix we can write the explicit expressions for the terms in Eqs.(17),(18) in the limit where all fermions except the t-quark have zero mass:

$$\begin{aligned} V_f^{\alpha\beta\mu}(q_1) \;&=\; \frac{4}{3}\left(q_1^2 g^{\alpha\beta} - q_1^\alpha q_1^\beta\right) q_1^\mu - 2q_1^2\left(k^\beta g^{\alpha\mu} - k^\alpha g^{\beta\mu}\right) \\ &\quad + \frac{1}{3}\left(q_1^2 q_1^\alpha - \frac{q_1^4}{k\cdot q_1}k^\alpha - 6q_1^\alpha (k\cdot q_1)\right)\left(g^{\beta\mu} - \frac{k^\beta q_1^\mu}{k\cdot q_1}\right) \\ &\quad + \frac{1}{3}\left(q_1^2 q_1^\beta - \frac{q_1^4}{k\cdot q_1}k^\beta + 3k^\beta q_1^2\right)\left(g^{\alpha\mu} - \frac{k^\alpha q_1^\mu}{k\cdot q_1}\right), \end{aligned} \tag{19}$$

$$\widetilde{V}_f^{\alpha\beta\mu}(q_2) = V_f^{\beta\alpha\mu}(-q_2), \tag{20}$$

$$A_f^{\alpha\beta\mu}(q_1) = q_1^2\varepsilon^{\alpha\beta\mu\sigma}(q_1 - k)_\sigma + 2\varepsilon^{\beta\mu\rho\sigma}q_{1\rho}k_\sigma q_1^\alpha + \frac{q_1^2}{k\cdot q_1}\left(\varepsilon^{\alpha\beta\rho\sigma}q_1^\mu - \varepsilon^{\beta\mu\rho\sigma}k^\alpha\right) q_{1\rho}k_\sigma, \tag{21}$$

$$\widetilde{A}_f^{\alpha\beta\mu}(q_2) = -A_f^{\beta\alpha\mu}(-q_2). \tag{22}$$

The above corrections satisfy the following identities:

$$\begin{aligned}
k_\mu V_f^{\alpha\beta\mu}(q_1) &= \frac{4}{3}(k\cdot q_1)\left(q_1^2 g^{\alpha\beta} - q_1^\alpha q_1^\beta\right), \\
k_\mu \widetilde{V}_f^{\alpha\beta\mu}(q_2) &= -\frac{4}{3}(k\cdot q_2)\left(q_2^2 g^{\alpha\beta} - q_2^\alpha q_2^\beta\right), \\
k_\mu A_f^{\alpha\beta\mu}(q_1) &= k_\mu \widetilde{A}_f^{\alpha\beta\mu}(q_2) = 0.
\end{aligned} \tag{23}$$

Let us first consider the case $q_1^2 > 0, q_2^2 > 0$. This kinematical situation is present, for instance, in the process $q\bar{q}' \to \ell\nu_\ell\gamma$ [4, 5]. In this case both cuts in Eqs.(17),(18) contribute and we get

$$\begin{aligned}
\overline{V}^{\alpha\beta\mu} &= \frac{4}{3}(k\cdot q_1)\Gamma_0^{\alpha\beta\mu}, \\
\overline{A}^{\alpha\beta\mu} &= 0,
\end{aligned} \tag{24}$$

where $\Gamma_0^{\alpha\beta\mu}$ is the tree-level $WW\gamma$ vertex defined in Eq.(5).

If we insert the above expressions into Eq.(15) we recover Eq.(13) *i.e.* the result of [4], namely, one-loop corrections to the $WW\gamma$ vertex accounts for the rescaling of the tree-level expression by a factor $1 + i\Gamma_W/M_W$. Notice however that this factorization of the $WW\gamma$ vertex is valid only when $q_{1,2}^2 > 0$.

Observe that the axial contribution vanishes when we add the two cuts associated to the photon emission from a given fermionic line (cf. Eqs. (18),(21),(22)). In other words, when $q_1^2 > 0, q_2^2 > 0$, the axial-vector contribution vanishes, regardless of the condition for the cancellation of anomalous terms, $Q_L + 3Q_D + 3Q_U = 0$.

A second comment concerns the behaviour of vertex corrections in the soft photon limit. According to the Low's soft-photon theorem [8], the leading terms (of order ω^{-1} and ω^0 , ω is the photon energy) in the amplitude for the emission of a soft photon in a radiative process are determined by the electromagnetic gauge invariance and the corresponding non-radiative process. Terms of the form $O(\omega^{-1})$ arise only from photon emission off external charged particles, while photon emission off internal lines starts at order $O(\omega^0)$ and cannot be of the form $k^\alpha/k\cdot q$ (k is the photon momentum and q is the momentum of an internal line). Now, if we insert Eq.(24) into Eq.(15) we can easily check that terms proportional to $k\cdot q_1$ drop and the vector contribution satisfies in this case Low's theorem. As we shall see in the next section, this property remains valid in the massive case for kinematical configurations when all possible cuts (for the photon emission from a given fermionic line) are allowed.

Let us now consider the case $q_1^2 < 0, q_2^2 > 0$. Such kinematical configuration is present *e.g.* in the process $\gamma e^- \rightarrow \nu_e \tau^- \bar{\nu}_\tau$ [7] with the appropriate change of the photon momentum $k \rightarrow -k$. According to Eqs.(17),(18) only one cut diagram for each loop (Figs. 1b, 1d, 1f) is allowed in this case and we obtain from Eq.(15):

$$\Gamma_1^{\alpha\beta\mu} = ic\left\{(3Q_L + 6Q_D - 6Q_U)\,\widetilde{V}_f^{\alpha\beta\mu}(q_2) + iQ_\tau \widetilde{A}_f^{\alpha\beta\mu}(q_2)\right\}, \tag{25}$$

where we have used the fact that $Q_L + 3Q_U + 3Q_D = 0$. Therefore, the axial contribution is not zero in contradiction with what is claimed in Ref.[3]. This term does not vanish because the heavy top quark decouples from the absorptive vertex corrections and leaves a net contribution coming from leptons of the third generation.

A second relevant feature in the case $q_1^2 < 0, q_2^2 > 0$ concerns the validity of Low's expansion with respect to k. As can be easily realized by inserting Eqs.(20),(22) into Eq.(25), the vector and axial contributions that appear in the latter equation contain terms of order $(k\cdot q_2)^{-1}$ and $k^\alpha/(k\cdot q_2)$. This is at variance with Low's expansion with respect to k. Of course, Low's soft photon theorem [8] is not violated because it is impossible to have $k \rightarrow 0$ when $q_1^2 < 0$ and $q_2^2 > 0$.

Finally we note that, when $q_1^2 < 0$, the factorization appearing in Eq.(13) does not remain valid.

4 Absorptive corrections to $WW\gamma$ vertex: massive fermions

In this section we derive the expressions for the vector and axial-vector contributions to the absorptive part of the $WW\gamma$ vertex when the masses of the fermions are taken into account in the loops. To illustrate our work, we will assume that $m_t^2 > q_{1,2}^2 > m_\tau^2$ and that only the t, b and c quarks and the τ lepton are massive fermions. All the possible cut diagrams giving contributions to the absorptive part of the $WW\gamma$ vertex are shown in Fig. 1 and the general expressions for such contributions are provided in the Appendix for the case of arbitrary fermion masses in the loops.

The correction to the vertex can be split into vector and axial-vector terms,

$$\Gamma_1^{\alpha\beta\mu} = ic\left(V^{\alpha\beta\mu} + iA^{\alpha\beta\mu}\right), \tag{26}$$

with

$$\begin{aligned} V^{\alpha\beta\mu} &= \sum_{L=e,\mu,\tau} Q_L\left(V_L^{\alpha\beta\mu}(q_1) + \widetilde{V}_L^{\alpha\beta\mu}(q_2)\right) + 3\sum_{D=s,d} Q_D\left(V_D^{\alpha\beta\mu}(q_1) + \widetilde{V}_D^{\alpha\beta\mu}(q_2)\right) \\ &\quad -3\sum_{U=u,c} Q_U\left(V_U^{\alpha\beta\mu}(q_1) + \widetilde{V}_U^{\alpha\beta\mu}(q_2)\right) \end{aligned} \tag{27}$$

and

$$\begin{aligned} A^{\alpha\beta\mu} &= \sum_{L=e,\mu,\tau} Q_L\left(A_L^{\alpha\beta\mu}(q_1) + \widetilde{A}_L^{\alpha\beta\mu}(q_2)\right) + 3\sum_{D=s,d} Q_D\left(A_D^{\alpha\beta\mu}(q_1) + \widetilde{A}_D^{\alpha\beta\mu}(q_2)\right) \\ &\quad +3\sum_{U=u,c} Q_U\left(A_U^{\alpha\beta\mu}(q_1) + \widetilde{A}_U^{\alpha\beta\mu}(q_2)\right), \end{aligned} \tag{28}$$

For simplicity we have neglected the Kobayashi-Maskawa mixing. Note also that the two different cuts for the emission of the photon from a given fermionic line are determined by q_1^2 and q_2^2, such that $q_1^2 - q_2^2 = 2(k \cdot q_1) = 2(k \cdot q_2)$.

Let us first consider the vector contributions. The expressions for the coefficients $V_f^{\alpha\beta\mu}(q_1)$ and $\widetilde{V}_f^{\alpha\beta\mu}(q_2)$ ($f = L, U, D$) are given in the Appendix for the general case when all the fermion masses are different from zero. Here we will explicitly assume $m_e = m_\mu = m_u = m_d = m_s = 0$, but m_c and m_τ are kept finite. Using the Eqs.(A1) and (A2) given in the Appendix we obtain

$$V^{\alpha\beta\mu} = (3Q_L + 6Q_D - 6Q_U)\,\frac{4}{3}(k\cdot q_1)\Gamma_0^{\alpha\beta\mu} + Q_\tau \widehat{V}_\tau^{\alpha\beta\mu} + 3Q_s\widehat{V}_s^{\alpha\beta\mu} - 3Q_c\widehat{V}_c^{\alpha\beta\mu} \tag{29}$$

where $\widehat{V}_\tau^{\alpha\beta\mu}$, $\widehat{V}_s^{\alpha\beta\mu}$ and $\widehat{V}_c^{\alpha\beta\mu}$ correspond to the photon emission from the τ lepton and the s and c quarks, respectively, and they vanish in the limit of massless fermions. The explicit expressions for these terms are

$$\begin{aligned}
\widehat{V}_\tau^{\alpha\beta\mu} \;=\;& \Bigg\{-\frac{2m_\tau^6(k\cdot q_1)(q_1^2+q_2^2)}{3q_1^4q_2^4}g^{\alpha\beta}q_1^\mu - 2\left[-\frac{m_\tau^2}{k\cdot q_1}\ln\left(\frac{q_1^2}{q_2^2}\right) + \frac{2m_\tau^4(k\cdot q_1)(2q_2^2-q_1^2)}{q_1^4q_2^4}\right.\\
&\left. -\frac{4m_\tau^6(k\cdot q_1)(q_1^2+2q_2^2)}{3q_1^6q_2^4} + \frac{2m_\tau^2}{q_2^2}\right]q_1^\alpha q_1^\beta q_1^\mu - \left[4m_\tau^2 + \frac{3m_\tau^4}{q_1^2} + \frac{m_\tau^4}{q_2^2} + \frac{4m_\tau^6(k\cdot q_1)}{3q_1^4q_2^2}\right.\\
&\left. -\frac{2m_\tau^2(q_2^2+m_\tau^2)}{k\cdot q_1}\ln\left(\frac{q_1^2}{q_2^2}\right) + \frac{2m_\tau^2(k\cdot q_1)(m_\tau^2-2q_1^2)}{q_1^4}\right]k^\alpha\left(g^{\beta\mu} - \frac{k^\beta q_1^\mu}{k\cdot q_1}\right)\\
&+2(k\cdot q_1)\left[\frac{2m_\tau^2}{q_2^2} - \frac{m_\tau^2}{k\cdot q_1}\ln\left(\frac{q_1^2}{q_2^2}\right) + \frac{m_\tau^4(3q_2^2-q_1^2)}{q_1^2q_2^4} - \frac{2m_\tau^6(q_1^2+q_2^2)}{3q_1^4q_2^4}\right]g^{\alpha\mu}q_1^\beta\\
&+\frac{2m_\tau^2(k\cdot q_1)}{q_1^4q_2^4}\left[2q_1^2q_2^2 - m_\tau^2(q_1^2+q_2^2)\right]q_2^\alpha q_1^\beta q_1^\mu\Bigg\} + \Big\{\alpha\leftrightarrow\beta, q_1\leftrightarrow -q_2\Big\}
\end{aligned} \tag{30}$$

and

$$\begin{aligned}
\widehat{V}_s^{\alpha\beta\mu} \;=\;& \frac{4m_c^4(k\cdot q_1)}{3q_1^4q_2^4}\Bigg\{-m_c^2(q_1^2+q_2^2)g^{\alpha\beta}q_1^\mu - \frac{(q_1^2+2q_2^2)(3q_1^2-2m_c^2)}{q_1^2}q_1^\alpha q_1^\beta q_1^\mu\\
&-\frac{(q_2^2+2q_1^2)(3q_2^2-2m_c^2)}{q_2^2}q_2^\alpha q_2^\beta q_2^\mu + \frac{3}{2}\left(q_2^4q_2^\alpha g^{\beta\mu} + q_1^4q_1^\beta g^{\alpha\mu}\right)\\
&+\frac{1}{2}\left(3q_1^2q_2^2 - 2m_c^2(q_1^2+q_2^2)\right)\left(q_2^\alpha g^{\beta\mu} + q_1^\beta g^{\alpha\mu}\right) + 3(q_1^2+q_2^2)q_2^\alpha q_1^\beta q_1^\mu\\
&+\frac{1}{2}\left(2m_c^2 - 3q_1^2 - 3q_2^2\right)\left(q_1^2k^\beta g^{\alpha\mu} - q_2^2k^\alpha g^{\beta\mu} - 2k^\alpha k^\beta q_1^\mu\right)\Bigg\}
\end{aligned} \tag{31}$$

The expression for $\widehat{V}_c^{\alpha\beta\mu}$ is obtained from $\widehat{V}_\tau^{\alpha\beta\mu}$ by replacing $m_\tau \to m_c$.

Next we discuss the vertex corrections arising from axial-vector contributions. In our case of interest, where $m_e = m_\mu = m_u = m_d = m_s = 0$ but m_c , m_τ different from zero, the explicit expression for the total axial-vector contribution becomes (cf. Appendix)

$$
\begin{aligned}
A^{\alpha\beta\mu} &= 2\left\{\varepsilon^{\alpha\beta\mu\sigma}k_\sigma\left(Q_\tau B_\tau + 3Q_c B_c\right) - \varepsilon^{\beta\mu\rho\sigma}q_{1\rho}k_\sigma q_1^\alpha\left(Q_\tau A_{1\tau} + 3Q_s D_{1c} + 3Q_c A_{1c}\right)\right. \\
&\quad \left. + \varepsilon^{\alpha\mu\rho\sigma}q_{2\rho}k_\sigma q_2^\beta\left(Q_\tau A_{2\tau} + 3Q_s D_{2c} + 3Q_c A_{2c}\right)\right\}, \qquad (32)
\end{aligned}
$$

where (f denotes τ or c)

$$
\begin{aligned}
A_{1f} &= -\frac{m_f^2}{k\cdot q_1}\ln\left(\frac{q_1^2}{q_2^2}\right) + \frac{2m_f^2}{q_1^2} + D_{1f}, \\
A_{2f} &= A_{1f}\ (q_1 \leftrightarrow -q_2), \\
D_{1f} &= -\frac{2m_f^4(k\cdot q_1)}{q_1^4 q_2^2}, \qquad (33) \\
D_{2f} &= D_{1f}\ (q_1 \leftrightarrow -q_2), \\
B_f &= q_1^2 A_{1f} + q_2^2 A_{2f}\ .
\end{aligned}
$$

Finally, let us comment that our expressions for the vector contributions to the $WW\gamma$ vertex corrections (cf. Eqs.(29)-(31)) satisfy the QED Ward identity given in Eq.(7). Note also that the axial-vector piece of the vertex corrections (cf. Eqs.(32)-(33)) is gauge-invariant by itself and therefore does not contribute to this Ward identity. Our results are in agreement with Low's soft photon theorem [8]. Indeed, the expansion of Eqs.(30),(31) and (33) around $(k\cdot q_1) = 0$ yields an overall factor $(k\cdot q_1)$, which cancels the $(k\cdot q_1)^{-1}$ factor present in the coefficient c (Eq.(16)) of Eq.(26).

5 Conclusions

The amplitude for a radiative process involving production and decay of an unstable $W^\pm$ boson (*e.g.* $q\bar{q}' \to \ell\nu_\ell\gamma, e^+e^- \to e^-\bar{\nu}_e ud$, $t \to b\ell\nu_\ell\gamma$ or $\gamma e^- \to \nu_e\tau^-\bar{\nu}_\tau$) can be made gauge-invariant by resuming the vacuum polarization of the W and of the vertex $WW\gamma$ corrections induced by loops of fermions [3]-[7]. In this paper we have explicitly shown that the absorptive corrections indeed satisfy the corresponding electromagnetic Ward identity in the general case when loops with massive fermions are considered.

We have also computed the absorptive parts of the $WW\gamma$ vertex generated by massive fermions in the one-loop corrections. Contrary to the case of massless fermions, where vertex corrections account for a simple rescaling of the tree-level $WW\gamma$ vertex [3, 4] when $q_1^2 > 0, q_2^2 > 0$ (see Eq.(13)), we find that the Lorentz tensor structure of this vertex is sensibly different. In particular, additional corrections to the magnetic dipole and electric quadrupole moments of W are induced, as clearly seen from Eqs.(29)-(31).

The axial-vector structure in the $WW\gamma$ vertex is a novel feature in the case of loops with massive fermions as well as in the case of massless fermions for specific kinematical configurations of the W's four-momenta in the vertex. Such a structure induces non-zero electric dipole and magnetic quadrupole moments of the W gauge boson (see Eqs.(32)-(33)).

Our results for the vector and axial-vector pieces of the $WW\gamma$ vertex have the correct threshold behaviour (they vanish when $q_{1,2}^2 \rightarrow (m_f + m_{f'})^2$) and are in agreement with Low's soft photon theorem [8], when the two cuts are allowed, *i.e.* $q_{1,2}^2 > (m_f + m_{f'})^2$, for the emission of the photon from a given fermionic line. When $q_1^2 < 0$ and $q_2^2 > 0$, Low's photon energy expansion is not satisfied and the axial-vector structure in the $WW\gamma$ vertex is not zero even in the massless fermion case.

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Appendix

In this Appendix we provide the general expressions for the vector and axial-vector contributions to the absorptive part of the $WW\gamma$ vertex in the case of massive fermions. As is well known, we have to include all cut diagrams allowed by kinematics with on-shell fermions in the loops.

(i) *Vector contributions*

If we consider the cut diagrams with q_1 (Figs. 1a, 1c and 1e) we obtain the following expressions for the coefficients $V_f^{\alpha\beta\mu}(q_1)$ in Eq.(27) $(f = L, U, D)$:

$$
\begin{aligned}
V_f^{\alpha\beta\mu}(q_1) \;=\;& \frac{8E_1\upsilon_1}{\sqrt{q_1^2}}\left[-\frac{2E_1^2\upsilon_1^2}{3} + E_1\sqrt{q_1^2} - m_f^2\right] g^{\alpha\beta}q_1^{\mu} \\
&+\frac{16E_1^2\upsilon_1}{q_1^2\sqrt{q_1^2}}\left[E_1\left(1+\frac{\upsilon_1^2}{3}\right) - \sqrt{q_1^2}\right] q_1^{\alpha}q_1^{\beta}q_1^{\mu} + \frac{8E_1\upsilon_1}{q_1^2}\left(E_1 - \sqrt{q_1^2}\right) q_1^{\alpha}T^{\beta\mu} \\
&+\frac{2}{k\cdot q_1}\left(-\frac{8E_1^3\upsilon_1^3}{3\sqrt{q_1^2}} - m_f^2\ln\left(\frac{1+\upsilon_1}{1-\upsilon_1}\right) + 2\upsilon_1E_1^2\right)\left(q_1^{\alpha}T^{\beta\mu} + q_1^{\beta}T^{\alpha\mu}\right) \\
&-\frac{4E_1\sqrt{q_1^2}}{(k\cdot q_1)^2}\left(-\frac{4E_1^2\upsilon_1^3}{3} - m_f^2\ln\left(\frac{1+\upsilon_1}{1-\upsilon_1}\right) + 2\upsilon_1E_1^2\right)\left(k^{\alpha}T^{\beta\mu} + k^{\beta}T^{\alpha\mu}\right) \\
&-\frac{2}{k\cdot q_1}\left(m_f^2\ln\left(\frac{1+\upsilon_1}{1-\upsilon_1}\right) - 2\upsilon_1E_1\sqrt{q_1^2}\right) k^{\alpha}T^{\beta\mu} + \frac{4E_1\upsilon_1}{k\cdot q_1}\left(E_1 - \sqrt{q_1^2}\right) k^{\beta}T^{\alpha\mu}.
\end{aligned}
\tag{A1}
$$

Similarly, the cut diagrams characterized by q_2 (Figs. 1b, 1d, 1f) give rise to the following expression for the coefficients $\widetilde{V}_f^{\alpha\beta\mu}(q_2)$ in Eq.(27):

$$
\widetilde{V}_f^{\alpha\beta\mu}(q_2) = V_f^{\beta\alpha\mu}(-q_2). \tag{A2}
$$

In the above equations the following definitions have been introduced:

$$
\begin{aligned}
E_{1,2} \;&=\; \frac{q_{1,2}^2 + m_f^2 - m_{f'}^2}{2\sqrt{q_{1,2}^2}}, \qquad \upsilon_{1,2} = \sqrt{1 - \frac{m_f^2}{E_{1,2}^2}}, \\
T^{\rho\mu} \;&=\; (k\cdot q_1)g^{\rho\mu} - k^{\rho}q_1^{\mu} = (k\cdot q_2)g^{\rho\mu} - k^{\rho}q_2^{\mu},
\end{aligned}
\tag{A3}
$$

where m_f denotes the mass of the fermion that emits the photon and $m_{f'}$, its isospin partner in the loop. Note that the last equality in (A3) supposes the transversality condition $\varepsilon \cdot k = 0$.

The following identities are easily verified:

$$\begin{aligned} ck_\mu \sum_{f,\text{colours}} Q_f V_f^{\alpha\beta\mu}(q_1) &= \operatorname{Im}\Pi^{\alpha\beta}(q_1), \\ ck_\mu \sum_{f,\text{colours}} Q_f \widetilde{V}_f^{\alpha\beta\mu}(q_2) &= -\operatorname{Im}\Pi^{\alpha\beta}(q_2), \end{aligned} \tag{A4}$$

with c defined in Eq.(16).

(ii) *Axial-vector contributions*

To obtain the general expressions for the axial-vector coefficients in Eq.(28), we consider again the cut diagrams characterized by the momenta q_1 (Figs. 1a, 1c and 1e) and q_2 (Figs. 1b, 1d and 1f). Finally we find ($f = L, U, D$):

$$\begin{aligned} A_f^{\alpha\beta\mu}(q_1) &= 2\left(-m_f^2 \ln\left(\frac{1+v_1}{1-v_1}\right) + 2v_1 E_1^2\right)\left\{\varepsilon^{\alpha\beta\mu\sigma} q_{1\sigma} + \left(\varepsilon^{\alpha\beta\rho\sigma} q_1^\mu - \varepsilon^{\beta\mu\rho\sigma} k^\alpha\right)\frac{q_{1\rho} k_\sigma}{k \cdot q_1}\right\} \\ &\quad + \frac{4E_1 v_1}{q_1^2}\left(E_1 - \sqrt{q_1^2}\right)\left\{q_1^2 \varepsilon^{\alpha\beta\mu\sigma} k_\sigma - 2\varepsilon^{\beta\mu\rho\sigma} q_{1\rho} k_\sigma q_1^\alpha\right\} \end{aligned} \tag{A5}$$

$$\widetilde{A}_f^{\alpha\beta\mu}(q_2) = -A_f^{\beta\alpha\mu}(-q_2). \tag{A6}$$

If $q_1^2, q_2^2 > (m_f + m_{f'})^2$ (*i.e.* if both cuts contribute to the absorptive part) then we obtain a simplified expression for the total axial-vector contribution:

$$\begin{aligned} A_f^{\alpha\beta\mu}(q_1) + \widetilde{A}_f^{\alpha\beta\mu}(q_2) &= \left(q_1^2 \varepsilon^{\alpha\beta\mu\sigma} k_\sigma - 2\varepsilon^{\beta\mu\rho\sigma} q_{1\rho} k_\sigma q_1^\alpha\right)\frac{a_{12}}{k \cdot q_1} \\ &\quad - \left(q_2^2 \varepsilon^{\alpha\beta\mu\sigma} k_\sigma + 2\varepsilon^{\alpha\mu\rho\sigma} q_{2\rho} k_\sigma q_2^\beta\right)\frac{a_{21}}{k \cdot q_2}, \end{aligned} \tag{A7}$$

where

$$\begin{aligned} a_{12} &= m_f^2 \ln\left(\frac{(1+v_2)(1-v_1)}{(1-v_2)(1+v_1)}\right) + 2\left(v_1 E_1^2 - v_2 E_2^2\right) \\ &\quad + \frac{4E_1 v_1 (k \cdot q_1)}{q_1^2}\left(E_1 - \sqrt{q_1^2}\right), \\ a_{21} &= a_{12}\,(q_1 \leftrightarrow -q_2); \end{aligned} \tag{A8}$$

$E_{1,2}$ and $\upsilon_{1,2}$ are defined in Eqs.(A3). In deriving Eq.(A7) we have used the following cyclic identity for an arbitrary four-vector a^μ:

$$\varepsilon^{\alpha\beta\rho\sigma}a^\mu + \varepsilon^{\beta\rho\sigma\mu}a^\alpha + \varepsilon^{\rho\sigma\mu\alpha}a^\beta + \varepsilon^{\sigma\mu\alpha\beta}a^\rho + \varepsilon^{\mu\alpha\beta\rho}a^\sigma = 0. \tag{A9}$$

Notice also that in the limit of massless fermions (and with $q_{1,2}^2 > 0$) $\upsilon_{1,2} = 1, E_{1,2} = \frac{1}{2}\sqrt{q_{1,2}^2}$ and we obtain $a_{12} = a_{21} = 0$. Thus, the axial-vector contribution vanishes for any doublet of massless fermions.

References

[1] R.G. Stuart, Phys. Lett. **B262** (1991) 113; Phys. Lett. **B272** (1991) 353; Phys. Rev. Lett. **70** (1993) 3193 and references therein.

[2] S. Willenbrock and G. Valencia, Phys. Lett. **B259** (1991) 373.

[3] E.N. Argyres, W. Beenakker, G.J. van Oldenborgh, A. Denner, S. Dittmaier, J. Hoogland, R. Kleiss, C.G. Papadopoulos, G. Passarino, Phys. Lett. **B358** (1995) 339.

[4] U. Baur and D. Zeppenfeld, Phys. Rev. Lett. **75** (1995) 1002.

[5] D. Wackeroth and W. Hollik, preprint hep-th/9606398.

[6] G. López Castro, J.L. Lucio and J. Pestieau, Mod. Phys. Lett. **A 6** (1991) 3679; Int. J. Mod. Phys. **A 11** (1996) 563.

[7] J. Papavassiliou and A. Pilaftsis, Phys. Rev. Lett. **75** (1995) 3060; Phys. Rev. **D 53** (1996) 2128.

[8] F.E. Low, Phys. Rev. **110** (1958) 974.

[9] A. Blondel, *Experimental Status of Electroweak Interactions*, talk given at ICHEP'96, Warsaw, Poland.

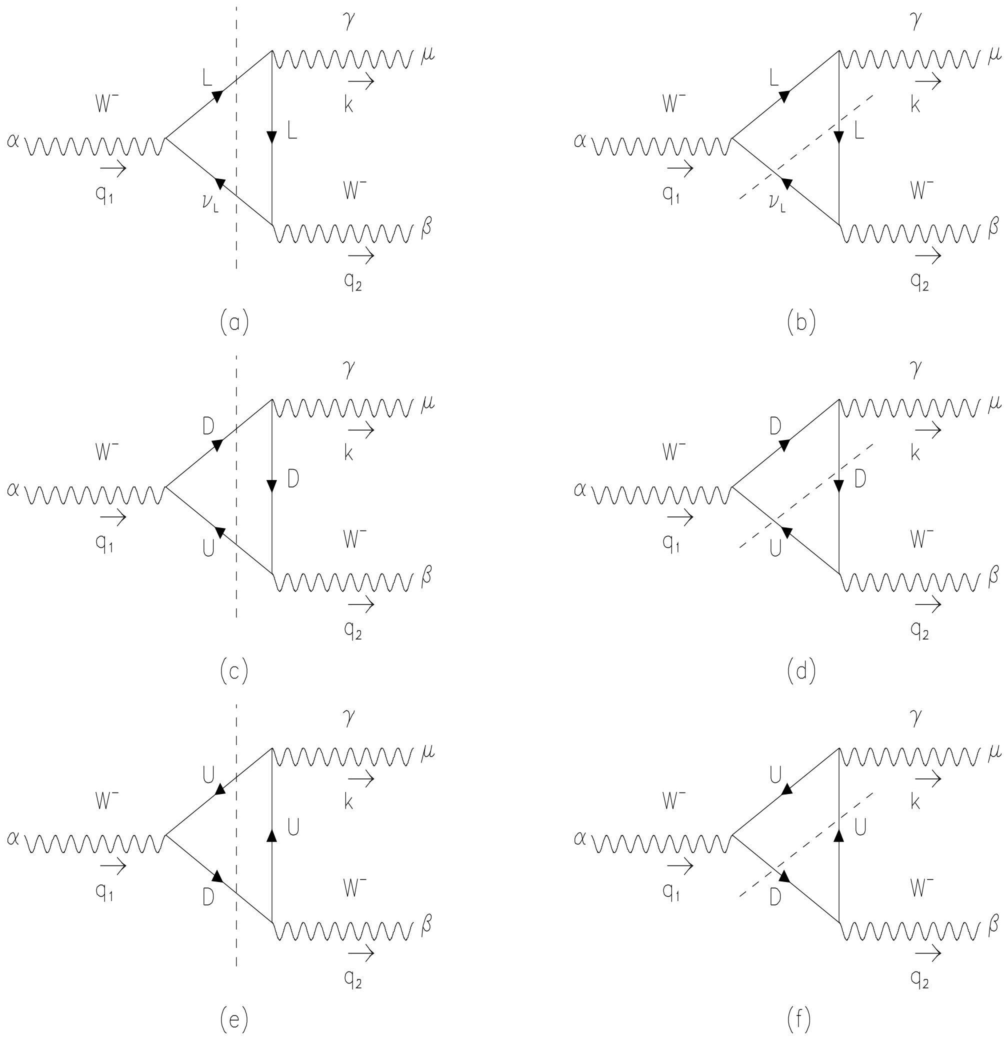

Figure 1: Cut diagrams for the one-loop fermionic corrections to the $WW\gamma$ vertex; L, U, D denote the charged leptons, up and down quarks, respectively.