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Nature versus Nurture in Social Mobility Under Private and Public Education Systems

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Abstract

This paper analyzes the roles of innate talent versus family background in shaping intergenerational mobility and social welfare under different education systems. We establish an overlapping-generations model in which the allocation of workforce between a high-paying skilled labor sector and a low-paying unskilled labor sector depends on talent, parental human capital, and educational resources, and the wage rate of skilled workers is governed by their average talent. Our model suggests that under the private education system, income inequality is inversely associated with social mobility, and the steady-state average talent of skilled labor declines as parents increase educational spending. The introduction of public education, which makes the allocation of workforce depend more on talent and less on family background, tends to increase both inequality and mobility and improve welfare under reasonable conditions. Our simulations show that

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if the government diverts public school funding to redistribution, the economy has lower inequality and mobility in the steady state. Moving from elitist to meritocratic systems yields lower inequality and greater mobility.

Keywords
innate ability, private education, public education, intergenerational mobility

JEL Classifications: H20, H31, H50, O11

Introduction

To achieve future career success, is it better to be born rich or to be born intelligent? The answer may depend on the existing education system. The private school system gives affluent families an edge. A wealthy parent can offer generous funding for her child's schooling to place him to an elite university and then a high-paying job, even if he is not very smart. The further down the economic ladder a parent is, the less likely is it that her child will attain adequate educational resources. Poor children have a slim chance of escaping poverty not because they lack talent but because material disadvantage holds them back. In contrast, the public school system provides all children with an equal learning opportunity, which helps to mitigate the effects of adverse family backgrounds and to highlight the role of innate ability in the determination of educational attainment. Equally gifted children with low-income backgrounds thus stand on the same ground to compete with rich peers, and the gates of upward mobility open for them.

This paper contributes to the literature by analyzing the roles of individual innate ability ("nature") versus family and school inputs ("nurture") in determining social mobility and social welfare under different education systems. Intergenerational mobility has received growing attention in the economics literature. Many empirical studies demonstrate its negative relationship with economic inequality, which is known as the "Great Gatsby curve" (Corak, 2013; Jerrim and Macmillan, 2015). While inequality indicates the income gap between the rich and the poor of a generation, mobility reflects the extent to which children's socioeconomic outcomes differ from those of their parents. Mitigating inequality and promoting mobility are both salient from the welfare perspective, and the poor tend to resent inequality less if their children are more assured of equal opportunity.[1]

Innate ability and educational expenditure are two main determinants of child outcome (Becker and Tomes, 1986; Galor and Tsiddon, 1997; Abbott et al., 2019). Endow- ed with different levels of ability, individuals may differ in productivity and hence income (Spence, 1973).[2] In a growth model of ability-biased technological transition in the evolution of technology, human capital, and wage gap, Galor and Moav (2000) suggest that technological progress leads to an increased return to ability and the average wage of skilled workers; human capital intensive industries exhibit a productivity level closely linked to the *average talent* of labor force. For instance, a software developer's creativity depends more on his innate ability than school performance; Bill Gates and Steve Jobs are gifted innovators who made pioneering contributions to the development of IT industry and generated positive externalities for other developers. We follow Lucas's (1988) idea of human capital spillovers and the role of average human capital in economic development but attend to one aspect of human capital, namely innate ability.

While innate endowments are fairly evenly distributed across the whole population (Papageorge and Thom, 2020), this may not be the case for educational resources provided by family and school (see our detailed discussion in section 2). Such differences lead educational attainment and career success to be weighted in favor of the haves over the have nots. For example, a kindergartner who grows up in a low-income family and achieves high test scores for academic talent has 30% chance of getting a college degree and a desirable entry-level job in adulthood, compared to a 70% chance for his affluent peers who get low scores (Carnevale et al., 2019). About 24% of high-potential individuals born to low-income fathers graduate from college, compared to 63% of those born to high-income fathers, while 27% of low-potential people with high-income fathers graduate from college, which is a greater proportion than that of smartest people from poorest families (Papageorge and Thom, 2020). These marked contrasts imply that family background, and relatedly educational expenditure, play a prominent role in shaping a child's outcome.

In section 3, we build an overlapping generations model with a high-paying skilled labor sector and a low-paying unskilled labor sector. A child receives education, and her prospects of becoming a skilled worker depend on her innate ability, educational resources, and parental human capital. Innate ability is exogenous and random among the population. An adult works for a wage, which can be spent on her consumption and her child's education. To capture human capital spillovers, we assume that the productivity of skilled workers is governed by their average ability. Given educational resources, skilled workers have an advantage in educating children than unskilled workers.

Section 4 analyzes how private and public school systems allocate children with different innate abilities and family backgrounds between the skilled and unskilled labor sectors.[3] Under the private school system, rich children receive more educational investments than poor children not only because their parents have a higher income but also because their parents perform more efficiently in using educational resources. For children with equal ability, those from disadvantaged families are less likely to become high-paid skilled workers, which hinders social mobility. In the *laissez-faire* equilibrium, low-ability children with skilled parents have a good prospect of becoming skilled workers, hurting the productivity of skilled workers and keeping their wage rate low. Our analysis also suggests that income inequality is inversely related to social mobility in a given period, which implies the existence of the Gatsby curve. A related paper by Bouchard St-Amant, Garon, and Marceau (2020) builds a model that analyzes the conditions for Gatsby curves to arise, showing that a Gatsbian economy may go turn into a non-Gatsbian economy. Our paper extends their model in emphasizing the role of average innate ability in the skilled sector and distinguishing private and public education in an augmented normative analysis.

Under the public school system, the government imposes a proportional income tax on workers and uses the revenues to provide every child with equal educational expenditure. The allocation of workforce thus relies more on innate ability and less on family background, which promotes intergenerational mobility. Compared with private education, public education gives talented people an advantage of being employed in the skilled labor sector, leading to an increase in the average ability of skilled workers and hence their productivity and wage rate. The wider pay gap between skilled and unskilled workers makes income inequalities worse. We also derive the condition under which a "top-up system" (i.e., parents make private investments in their children's education in addition to public schooling) is operative in equilibrium.

In section 5, we perform a simulation comparing inequality, mobility, and welfare under various fiscal and educational policies. We first demonstrate a Great Gatsby curve along the *dynamic* path under *laissez-faire*, which implicates that more equal societies tend to be more mobile over time. The introduction of public school system increases both inequality and mobility, which violates the Great Gatsby curve. Nevertheless, if the government diverts public school funding toward income redistribution (i.e., policy shift from "transfer in kind" to "transfer in cash"), both inequality and mobility will be lower. In addition, moving from elitist to meritocratic educational systems leads to a lower income inequality and an improvement in intergenerational mobility. Our welfare analysis suggests that transfer in cash tends to yield a higher social welfare than transfer in kind. Yet if some poor parents are myopic and spend little on their children's education,[4] transfer in kind enhances intergenerational mobility and achieves the highest welfare.

Empirical Motivation

This paper is motivated by some striking cases of rich parents spending a tremendous amount of money on their children's education to send them to prestigious universities. The following is a 2012 CNN report.[5]

> *How much would you pay to get your child into an Ivy League university? For Gerard and Lily Chow, it seems the sky was the limit. In 2007, the Hong Kong couple enlisted Harvard-lecturer-turned-admissions-consultant Mark Zimny to steer their two sons through elite U.S. boarding schools into a top-ranked university preferably Harvard. For a monthly $4000 fee per child, their total education management package included extensive admissions counseling, arranging homestays, private tutoring, and extra-curricular activities, whereby Zimny and his team functioned as parents away from parents for their sons. The Chows later switched to a retainer of $1 million per child.*

The urge to secure children's success has even led to an array of college admissions scandals. In May 2019, the family of a Stanford student was reported to have paid $6.5 million to help her get admitted. The student confessed in a webcast that her IQ is not quite high and her early academic performance was mediocre.[6]

While rich parents are willing to make enormous outlays to secure an academic certificate for their children, poor parents lack the financial resources to compete and find themselves helpless. The wide gap in educational resources between rich and poor manifests itself in at least four ways. First, rich parents tend to buy expensive houses in nice neighborhoods with the top-performing school districts. For example, as rising income inequality has translated into a residential sorting effect for American households with children since 1990, rich and poor have become increasingly unlikely to share the same neighborhoods (Owens, 2016). Income inequality has also been identified as a major predictor of income segregation between school districts (Owens, Reardon, and Jencks, 2016). Neighborhoods influence children's long-term outcome through childhood exposure effects. Chetty, Hendren, and Katz (2016) demonstrate that moving to a richer neighborhood before the teenage years increases the likelihood of college attendance and earnings. In a study of more than 7 million families that moved across US counties, Chetty and Hendren (2018) show that the neighborhood in which a child grows up considerably shapes his social mobility.

Second, in societies where free public education plays a limited role, parents have to consider sending their children to private schools, which usually charge very high tuition and accommodation fees. In that case, only wealthy children can go to school, or at least, to a high-quality school. Poor children often have to attend low-quality schools, drop out after a few years of schooling, or even do not attend school at all. Skiba et al. (2008) suggest that poor children are more likely to attend schools with a high rate of teacher turnover, fewer experienced teachers, and larger class sizes. Chetty, Friedman, and Rockoff (2014) find that a better school with high-quality teachers substantially increases students' educational attainment.

Third, even in developed countries where all children attend primary and middle schools under a free public school system, parents need to pay for children's pre-school education. Rich children have access to better early-age educational resources in a high-quality kindergarten where teachers often receive professional training. It is well noted in the literature that early childhood intervention lays the foundation for subsequent academic performance (Shonkoff and Phillips, 2000) and plays a powerful role in shaping human capital formation (Heckman, Pinto, and Savelyev, 2013).

Fourth, rich parents spend more on their children's after-school training. The global private tutoring market is booming, with East Asia leading the way. South Korea's household expenses on private tutoring were about 2.8% of GDP in 2006, equivalent to 80% of government spending on primary and secondary school education (Kim and Lee, 2010). In Hong Kong, the average tutoring spending for middle school students was 8.7% of household income in 2010 (Bray et al., 2014). Children from some ultra-rich British families take lessons at home with a rotation of tutors who charge £60–100 per hour.[7] Poor families, however, do not have money to hire private tutors.

Model Setup

Consider an overlapping generations economy where a mass of individuals lives for two periods – childhood and adulthood. Each generation consists of high abilities (a^H) and low abilities ($a^L := 0$). Such attribution is random, and the probability of being high ability is denoted as $\lambda \in (0, 1)$. An individual's ability is unrevealed in childhood and remains publicly unobservable in adulthood (see Bernasconi and Profeta 2012). Each adult bears a single child such that the population size of each generation is stable and, for simplicity, is assumed to be one.

Adults in period t are labelled as "generation t". A fraction θ_t of them work in the skilled labor sector (s), while the others work in the unskilled labor sector (u). Both sectors are composed of high ability and low ability workers and produce the unique final goods competitively using their own type of labor with a constant-returns-to-scale technology. With the price of final goods in each period t being normalized to unity, the productivity in sector $i \in \{s, u\}$ equals the equilibrium wage w_t^i. Let ϕ and $\delta\bar{a}_t$ be the productivity (wage) of the unskilled and skilled workers in period t, respectively, where the positive parameter δ measures the marginal return to skilled workers' average ability, $\bar{a}_t$. Assume that a skilled worker is more productive and is paid a higher wage than an unskilled worker, namely

$$w_t^s = \delta\bar{a}_t > w_t^u = \phi > 0. \tag{1}$$

Under this assumption, an individual's ability causes no direct impact on her wage. Moreover, $\bar{a}_t \in (0, a^H)$ can be expressed by

$$\bar{a}_t = \mu_t a^H, \tag{2}$$

where μ_t denotes the fraction of high abilities in the skilled workers of generation t. Equation (2) suggests that only the high ability workers contribute to the sectoral productivity, which reflects the observation that talented employees, as opposed to the "trivial many", are making a big difference and are the main driver of a company's success (e.g., Chamorro-Premuzic, 2016).

As in, for instance, Fan and Zhang (2013) and Fan, Pang, and Pestieau (2020), we consider that for each i-type worker of generation t, the quality of her child is measured by the *ex ante* probability that the child becomes a skilled worker in the future, namely $q_t^i \in [0, 1]$. This probability, which increases with the child's ability, a_{t+1}, and her received educational spending, $e_t^i \geqslant 1$, is specified in the following Cobb-Douglas functional form:[8]

$$q_t^i = \ln(1 + a_{t+1}) + \beta^i \ln e_t^i = \lambda \ln(1 + a^H) + \beta^i \ln e_t^i. \tag{3}$$

The first term of the right-hand-side captures the role of "nature" in shaping child outcome, while the second term reflects the effect of "nurture" (including family environment and school input). An increase in the received educational spending makes an individual have a higher chance to enter the skilled market, thereby raising her expected productivity and wage. The parameter $\beta^i > 0$ measures the effectiveness of converting education into a successful entry to the skilled labor sector. Following Becker et al. (2018), we assume $\beta^s \geqslant \beta^u$, which means that skilled parents perform no worse at using educational resources than unskilled parents.[9]

Furthermore, we measure the intergenerational social mobility by the odds ratio – the probability that children of unskilled workers become skilled relative to the probability that children of skilled workers become skilled:

$$M_t : = \frac{q_t^u}{q_t^s} = \frac{\lambda \ln(1 + a^H) + \beta^u \ln e_t^u}{\lambda \ln(1 + a^H) + \beta^s \ln e_t^s}. \tag{4}$$

Given $\beta^u \leqslant \beta^s$ by assumption and $e^u \leqslant e^s$ in equilibrium, we have $M_t \in (0, 1)$.

Each i-type worker of generation t spends her wage, w_t^i, on her consumption, c_t^i, and her child's education, e_t^i, without leaving bequests. Her budget constraint amounts to

$$w_t^i = c_t^i + e_t^i. \tag{5}$$

Her utility takes the Cobb-Douglas form, which depends positively on private consumption and the quality of her child, as below:

$$v_t^i = \ln c_t^i + \alpha q_t^i, \tag{6}$$

where the parameter $\alpha > 0$ represents the preference over the quality of the child. In the beginning of period t, an individual expects to obtain v_t^s with a probability θ_t and v_t^u with a probability $1 - \theta_t$. Social welfare in period t is defined as the expected individual utility:

$$V_t = \theta_t v_t^s + (1 - \theta_t) v_t^u. \tag{7}$$

We proceed to discuss more demographic characteristics from the macroeconomic perspective. In period t, skilled and unskilled workers expect to have a total of $q_t^s \theta_t$ and $q_t^u (1 - \theta_t)$, respectively, of children who will become skilled workers in the future. In generation $t + 1$, the proportion of skilled workers therefore amounts to $\theta_{t+1} = q_t^s \theta_t + q_t^u (1 - \theta_t)$, which can be rewritten as

$$\theta_{t+1} = \lambda \ln(1 + a^H) + \beta^s \theta_t \ln e_t^s + \beta^u (1 - \theta_t) \ln e_t^u, \tag{8}$$

which governs the transition dynamics of θ_t. Moreover, the proportion of the high ability in skilled workers of generation t can be expressed by

$$\mu_{t+1} = \frac{\lambda\left[\ln(1+a^H) + \beta^s \ln e_t^s\right]\theta_t + \lambda\left[\ln(1+a^H) + \beta^u \ln e_t^u\right](1-\theta_t)}{\theta_{t+1}} \\ = \lambda\left[1 + \frac{(1-\lambda)\ln(1+a^H)}{\theta_{t+1}}\right] > \lambda, \tag{9}$$

which implies that the majority of high ability individuals work in the skilled labor sector. Combining (2) and (9) yields the average innate ability of skilled workers of generation $t+1$, which is higher than the average innate ability of the whole population:

$$\bar{a}_{t+1} = \lambda a^H\left[1 + \frac{(1-\lambda)\ln(1+a^H)}{\theta_{t+1}}\right] > \lambda a^H. \tag{10}$$

Finally, we rely on the Gini coefficient (G_t) to measure the economic inequality in period t. If the government imposes an income tax at the rate of $\tau_t \in [0, 1)$ in period t and makes no transfer payment, then G_t is calculated as the fraction of high-paid workers' after-tax income minus their population share, which can be further written as a function of (ϕ, δ, $\bar{a}_t$, θ_t) as below:

$$G_t = \frac{(1-\tau_t)\delta\bar{a}_t\theta_t}{(1-\tau_t)\delta\bar{a}_t\theta_t + (1-\tau_t)\phi(1-\theta_t)} - \theta_t \\ = \theta_t\left[\frac{1}{\theta_t + (1-\theta_t)\phi/(\delta\bar{a}_t)} - 1\right]. \tag{11}$$

Equilibrium under Different Education Systems

In this section, we characterize the educational spending per child (e_t^i), the proportion of skilled workers (θ_t), the average ability of skilled workers ($\bar{a}_t$), and intergenerational mobility (M_t) under private and public education systems. We first consider that parents send their children to private schools and optimally choose their educational investments. Next, we discuss the case in which the government funds public schools by imposing an (optimal) income tax on all workers, who then decide on private after-school education for their children.

*Private Education (*Laissez-faire*)*

We start with the case where people make decentralized decisions on their educational spending without any government intervention. Taking into account the budget constraint (5) and the quality of child (3), every i-type member of generation t, where $i \in \{s, u\}$, obtains a utility at the level of

$$v_t^i = \alpha\lambda \ln(1 + a^H) + \ln\left(w_t^i - e_t^i\right) + \alpha\beta^i \ln e_t^i, \tag{12}$$

To maximize her utility, she chooses her educational investment in accordance with

$$e_t^i = \gamma^i w_t^i \quad \text{where} \quad \gamma^{\mathrm{i}} \equiv \frac{\alpha\beta^{\mathrm{i}}}{\alpha\beta^{\mathrm{i}} + 1} \in (0, 1), \tag{13}$$

which shows that an i-type worker allocates a constant share γ^i of her wage to her child's education in equilibrium given the Cobb-Douglas utility function. It follows that a skilled worker pays a premium for her child's education ($e_t^s > e_t^u$) not only because she receives a higher income ($\delta\bar{a} > \phi$) but also because she is more efficient in using educational resources ($\gamma^s \geqslant \gamma^u$ under the assumption $\beta^s \geqslant \beta^u$). The next lemma follows directly from (3) and (13):

> **Lemma 1**. Under the private school system, a skilled parent is more likely to have a skilled child than an unskilled parent.

Lemma 1 suggests that a child is more likely to become skilled if her parent is a skilled worker. As equation (3) demonstrates, a rich child's advantage comes from two sources. First, she receives more educational resources from her parent ($e^s > e^u$). Second, her parent performs at least as well as a poor parent in using these resources ($\beta^s > \beta^u$). The intergenerational transmission of human capital and labor earnings is advocated in earlier theoretical work (Becker and Tomes, 1986) and receives some support from empirical examinations (Schneider, Hastings, and LaBriola, 2018).

Note that parents' educational investments have a positive effect on the probability of their children becoming skilled workers but do not affect the wage rate of skilled workers, which depends only on the average innate ability of skilled workers. Moreover, as larger educational investments (especially by rich parents) translate into a lower average ability among skilled workers of the next generation, educational investments generate an indirect adverse effect on the wage of the skilled labor sector. This unfortunate viewpoint of the role of education investments has not been explored in the prior literature.

We now relate intergenerational mobility to income inequality. Inserting (13) into (4) rewrites intergenerational mobility in period t as

$$M_t = \frac{\lambda \ln(1 + a^H) + \beta^u \ln(\gamma^u \phi)}{\lambda \ln(1 + a^H) + \beta^s \ln(\gamma^s \delta \bar{a}_t)}. \tag{14}$$

Because $\tau_t = 0$ under laissez-faire, G_t can be expressed as in (11). Based on (11) and (14), we develop the following proposition given θ_t in period t:

Proposition 1

(i) An increase in δ leads to a lower M_t and a higher G_t;
(ii) An increase in $\bar{a}$ leads to a lower M_t and a higher G_t;
(iii) An increase in ϕ leads to a higher M_t and a lower G_t.

Proof. See Appendix. ■

All else being equal, narrowing the wage gap (small $\delta\bar{a}_t$ or large ϕ) helps to lower intragenerational earnings inequality. The income effect suggests that the rich will then tend to spend less on education, with the result that their children are less likely to become rich in the future, while the poor will do the opposite, resulting in more opportunities for their children. This is consistent with the empirical finding that educational attainment mediates the link between social origin and destination (Jerrim and Macmillan, 2015). Proposition 1 suggests that a change in $(\delta, \bar{a}, \phi)$ drives M_t and G_t to move in opposite directions.

We proceed to characterize the evolution of $(\theta_t, \bar{a}_t)$. Plugging (13) into (8) and (10) obtains

$$\theta_{t+1}(\bar{a}_t, \theta_t) = \lambda \ln(1 + a^H) + \beta^s \theta_t \ln(\gamma^s \delta \bar{a}_t) + \beta^u (1 - \theta_t) \ln(\gamma^u \phi) \tag{15}$$

$$\bar{a}_{t+1}(\bar{a}_t, \theta_t) = \left[1 + \frac{(1 - \lambda) \ln(1 + a^H)}{\ln(1 + a^H) + \beta^s \theta_t \ln(\gamma^s \delta \bar{a}_t) + \beta^u (1 - \theta_t) \ln(\gamma^u \phi)}\right] \lambda a^H. \tag{16}$$

These two equations show that θ_t and $\bar{a}_t$ are both path-dependent. Define the steady state of the economy as that both θ_t and $\bar{a}_t$ achieve their time-invariant levels, θ and $\bar{a}$.

Definition 1. The economy reaches the steady state when $\theta_t = \theta$ and $\bar{a}_t = \bar{a}$.

We characterize the steady-state solutions in the following proposition:

Proposition 2. Under the private school system, the steady state of the economy is determined by

$$\left[1+\frac{\beta^u \ln(\gamma^u\phi)}{\lambda\ln(1+a^H)}\right]\frac{\bar{a}}{a^H}+\beta^s(1-\lambda)\ln(\gamma^s\delta\bar{a}) = 1+\left[1-\lambda+\frac{1}{\ln(1+a^H)}\right]\beta^u\ln(\gamma^u\phi), \tag{17}$$

$$\theta=\frac{\lambda(1-\lambda)a^H\ln(1+a^H)}{\bar{a}-\lambda a^H}. \tag{18}$$

Proof. See Appendix. ■

Equation (17) demonstrates that the average innate ability of skilled workers in the steady state, $\bar{a}$, is governed by six parameters, namely $\bar{a}(a^H, \lambda, \alpha, \beta^s, \beta^u, \delta, \phi)$. Inserting $\bar{a}$ solved in equation (17) into equation (18) derives the solution to θ. As equation (18) shows, holding (a^H, λ) constant, θ and $\bar{a}$ are inversely correlated in the steady state. Since a skilled worker is more likely to be high ability than an unskilled worker on average (equation (9)), the average talent of skilled workers tends to decline when workers are increasingly moved from the unskilled to the skilled labor sector.

We proceed to examine the comparative statics of $\bar{a}$ in the following proposition:

Proposition 3. Under the private school system, the steady-state average ability of skilled workers $\bar{a}$: (i) increases with a^H and λ, (ii) decreases with α, (iii) decreases with β^i where $i \in \{s, u\}$, and (iv) decreases with δ and ϕ.

Proof. See Appendix. ■

Proposition 3(*i*) is straightforward: all else being equal, if talented people have a higher level of talent and account for a larger population size, then the average talent of skilled workers will be higher in the steady state. The other parts of Proposition 3 can be interpreted from the perspective of parents' educational investments. A parent tends to invest more in her child's schooling if she is more concerned about the educational attainment of her child (larger α), more efficient in using educational resources (larger β^s or β^u), or earns a higher wage (larger δ or ϕ). By (3), education and innate ability are substitutes in governing an individual's skill. As a rise in educational spending mitigates the relative weight of ability in occupational choice, more low-ability individuals born into rich families enter the skilled labor market, which harms the productivity of skilled occupation.

In our model, since natural talent is usually exogenously given, children from poor and rich families tend to have the same probability of having high innate ability. Therefore, an economy becomes more efficient if intergenerational mobility is higher, which will lead to a higher level of innate ability among skilled workers. The next question that arises is how to enhance the efficiency and equity of society by reallocating educational resources across families. This goal can be achieved by taxing the rich and subsidizing the poor. The income effect means that after income redistribution, the rich will spend less on their children's education whereas the poor will spend more. Another solution would be the provision of public education, which aims to provide children with equal educational spending. In the next subsections, we investigate the equilibrium outcome when the government finances public education by levying an income tax.

Public Education

In this section, we consider the public education system, under which the government imposes a proportional income tax on all workers and spends tax revenues to fund public schools. Public education system generates two benefits. First, rich families give pecuniary supports to poor families through this in-kind redistribution. Second, public education narrows the gap in the educational resources between rich and poor children, which in turn promotes social mobility. It is also noted that parents may provide their children, who receive formal education at public schools, with supplementary remedial classes. After-school private tutoring (often referred to as shadow education) is an example of programs existing alongside the formal education system with an increasing scale (Bray et al., 2014). Our analysis accommodates private education in addition to public education.

The interactions between the government and workers in period t proceed in a two-stage game. The government moves first to fund public education by imposing the welfare-maximizing income tax on all workers. We consider a scheme of linear taxation. With the tax rate $\tau_t \in (0, 1)$ and the public educational expenditure per child e_t^P in period t, the balanced government budget implies:

$$e_t^P = \tau_t[\theta_t \delta \bar{a}_t + (1 - \theta_t)\phi], \tag{19}$$

Next, workers choose the optimal private educational investments for their children. Because tutoring is often less effective than mainstream schooling, we consider the case where the government choose sufficiently large public educational expenditure in equilibrium so that a poor parent needs not to spend on tutoring.[10] In other words, a poor child receives public education only, while a rich child may receive both private and public education, namely,

$$e_t^u = e_t^P, \quad e_t^s = e_t^P + \kappa e_t^A, \tag{20}$$

where $\kappa \in (0, 1)$ measures the discount in private tutoring effectiveness (relative to public education), and e^A is the additional tuition fee paid by a rich parent for her child's private education.

We first examine the choice of a skilled worker. By using (3) and (6), we write her utility as

$$v_t^s = \alpha\lambda \ln(1 + a^H) + \ln\left[(1 - \tau_t)\delta\bar{a}_t - e_t^A\right] + \alpha\beta^s \ln\left(e_t^P + \kappa e_t^A\right).$$

Given the tax rate τ_t and the public educational expenditure per child e_t^P, a skilled worker optimally chooses private educational expenditure to maximize her utility. Taking the first order condition of the above equation with respect to e_t^A and then rearranging obtains

$$e_t^A = \gamma^s(1 - \tau_t)\delta\bar{a}_t - (1 - \gamma^s)\frac{e_t^P}{\kappa}. \tag{21}$$

The negative relationship between e_t^A and e_t^P implies a substitution effect between public schooling and private tutoring. The next proposition compares the private and public school systems:

> **Proposition 4**. Compared with the private school system, the public school system leads rich children to receive less education and poor children more. Hence, intergenerational mobility improves.
>
> *Proof.* See Appendix. ■

If the private education system is replaced by a public one, educational resources tend to gravitate from rich to poor children. This result follows directly from the income effect: the public school system involves a redistributive process that effectively transfers wealth from rich to poor families. In equilibrium, more talented poor children attaining a college degree, crowding out some mediocre wealthy children. Consequently, a poor child faces a higher probability of becoming rich in the future while a rich child has a lower probability, which mitigates the transmission of earnings inequality across generations. The public school system creates a more equal society in which an individual's career success hinges more on innate ability and less on family background.[11] Proposition 4 not only provides a rationale for the wide adoption of public education around the world but also finds some empirical support. For example, Neidhöfer (2019) shows that public education is positively associated with intergenerational mobility in Latin American countries.

In period t, the government chooses the tax rate to maximize social welfare as expressed by

$$\begin{aligned} V_t = {} & \theta_t \ln\left\{\kappa(1-\tau_t)\delta\bar{a}_t + \tau_t[\theta_t\delta\bar{a}_t + (1-\theta_t)\phi]\right\} + (1-\theta_t)\ln[\phi(1-\tau_t)] \\ & + \alpha\beta^s\theta_t \ln\left\{\kappa(1-\tau_t)\delta\bar{a}_t - \tau_t\big[\theta_t\delta\bar{a}_t + (1-\theta_t)\phi\big]\right\} + \alpha\beta^u(1-\theta_t)\ln\tau_t \\ & + \alpha\beta^u(1-\theta_t)\ln[\theta_t\delta\bar{a}_t + (1-\theta_t)\phi] + \alpha\lambda\ln(1+a^H) + \theta_t\ln\frac{1-\gamma^s}{\kappa} \\ & + \alpha\beta^s\theta_t\ln\gamma^s. \end{aligned} \tag{22}$$

We denote $\tau_t^* \in (0, 1)$ as the optimal income tax rate in period t and propose the next proposition:

Proposition 5. Under the public school system, the optimal income tax rate satisfies

$$\begin{aligned} & \frac{(\theta_t-\kappa)\delta\bar{a}_t + (1-\theta_t)\phi}{\kappa(1-\tau_t^*)\delta\bar{a}_t + \tau_t^*[\theta_t\delta\bar{a}_t + (1-\theta)\phi]} \\ & \quad - \frac{\alpha\beta^s[(\theta_t+\kappa)\delta\bar{a}_t + (1-\theta_t)\phi]}{\kappa(1-\tau_t^*)\delta\bar{a}_t - \tau_t^*[\theta_t\delta\bar{a}_t + (1-\theta)\phi]} \\ & \quad = \frac{1-\theta_t}{\theta_t}\left(\frac{1}{1-\tau_t^*} - \frac{\alpha\beta^u}{\tau_t^*}\right). \end{aligned} \tag{23}$$

The rich will provide their children with extra private education in period t if and only if

$$\alpha\beta^s\kappa\left(\frac{1}{\tau_t^*}-1\right) > \theta_t + (1-\theta_t)\frac{\phi}{\delta\bar{a}_t}. \tag{24}$$

Proof. See Appendix. ■

Proposition 5 presents a sufficient and necessary condition for tutoring classes to operate (i.e., $e^A > 0$). Condition (24) is more likely to hold if the rich are greatly concerned by the quality of their children (large α) and are efficient in using educational resources (large β^s) and if private tutoring is efficient (large κ). Moreover, this condition is more likely to hold in period t if the government provides a small educational fund (small e_t^P and thus small τ_t^*), the wage gap is wide (small $\frac{\phi}{\delta\bar{a}_t}$), and skilled workers make up a small fraction in the working-age population (small θ_t).

Analogous to Proposition 2, the next proposition examines the steady state of the economy:

Proposition 6. Suppose that condition (24) is violated under the public school system.

(i) The steady state is determined by the following two equations:

$$\theta - [\beta^s\theta + \beta^u(1-\theta)]\ln\frac{\lambda\delta a^H[(1-\lambda)\ln(1+a^H)+\theta] + (1-\theta)\phi}{\{\alpha[\beta^s\theta+\beta^u(1-\theta)]\}^{-1}+1} = \lambda\ln(1+a^H), \tag{25}$$

$$\bar{a} = \lambda a^H\left[1+\frac{(1-\lambda)\ln(1+a^H)}{\theta}\right]. \tag{26}$$

(ii) If $\beta^i = \beta$, then the steady-state average ability of skilled workers, $\bar{a}$, decreases with (α, δ, ϕ) if and only if

$$\beta < \frac{(\delta\bar{a}-\phi)\theta+\phi}{\lambda a^H-\phi}. \tag{27}$$

Proof. See Appendix. ■

Equation (25) shows that under the public school system, the steady-state proportion of skilled workers, θ, is governed by (a^H, λ, α, β^s, β^u, δ, ϕ). It can hence be inferred from (26) that the average innate ability of skilled workers in the steady state is also influenced by the seven parameters. Furthermore, τ and e^P are a constant in the steady state when $(\theta, \bar{a})$ remain constant.

Proposition 6(ii) investigates the comparative statics of $\bar{a}$ with a focus on the special case of perfect meritocracy ($\beta^s = \beta^u$). We present a result similar to that in Proposition 3, with the only difference being that condition (27) is imposed. This condition emphasizes that parents' efficiency in using educational expenditure should be sufficiently low (small β). Parents who care little about the quality of their children (small α) or earn a low income (small δ and ϕ) tend to spend a small amount of money on their children's education. Limited educational resources, coupled with low efficiency in using educational resources, will make innate ability a predominant determinant of one's placement. Intelligent children are thus very likely to become skilled workers, which in turn enhances the average ability of the skilled labor sector.

A Quantitative Analysis

To compare the equilibrium outcomes under private and public educational systems introduced in section 4, we rely on a quantitative example in this section. We conduct three sets of simulations: (i) the dynamic relationship between income inequality and intergenerational social mobility under *laissez-faire* and government intervention; (ii) the impact of a transition from elitist to meritocratic educational systems on inequality and mobility; and (iii) the welfare consequence under different policies and educational systems. Our numerical simulations are performed based on the parameter values given in Table 1.

Figure 1 illustrates the dynamic trajectory of income inequality versus social mobility with and without government intervention. Starting from the initial state where the Gini coefficient is 0.1873 and mobility equals 0.5149, the *laissez-faire* economy moves up and left along the gray path until it achieves the steady state at point A, where $G = 0.1551$ and $M = 0.5473$. Interpreted literally, the transitional dynamics of the unregulated economy exhibits a simultaneous improvement in equality and mobility.[12] This pattern suggests the validity of the Great Gatsby curve along the *dynamic* path, which complements the implication of Proposition 1 that there exists a *static* negative relationship between economic inequality and social mobility in the absence of government intervention. As recent research based on cross-sectional data find, higher inequality in childhood is associated with lower mobility in adulthood in Latin America (Neidhöfer, 2019).

We then consider three different fiscal policies at point A. The first one is that the government imposes an income tax at the optimal rate of τ_t^* and devotes all the revenues to supporting public education, as analyzed in section 4.2. Our parameter configuration suggests that skilled parents provide their children with extra private education (i.e., condition (24) holds). The introduction of public education leads to new transitional dynamics along the dark curve toward the steady state at point B, where the optimal tax rate equals 0.354. As the government intervenes, income inequality remains unaffected, whereas social mobility increases from 0.5473 to 0.6682. Intuitively, a proportional income tax combined with in-kind transfers has no impact on the dispersion of disposable incomes among the population, but it effectively narrows the educational resources gap between rich and poor children. Despite this immediate effect, the economy will reach point B, where $G = 0.1563$ and $M = 0.6673$, in the long run. The dynamic transition displays a slightly higher Gini coefficient and a lower social mobility. We will explain this long-run pattern later in the discussion of Table 2. Overall, a change from private to public education system tends not to generate Gatsby curves.

The second policy differs from the first one only in that the government returns the same tax revenue equally to workers in a lump-sum manner. The adoption of the redistributive policy causes the economy to move along the dashed curve to the new steady state at point C, where $G = 0.1001$ and $M = 0.5983$. Note that to be consistent with the construction of the Gini in the model setup, G is computed on the after-tax income. Redistributing

Table 1. Benchmark Values for Key Parameters.

Parameter	Description	Value
a^H	Level of high ability	3
λ	Probability that an individual is endowed with high ability	0.15
α	Preference over the quality of the child	5
β^s	Educational efficiency of rich families	0.2
β^u	Educational efficiency of poor families	0.12
δ	Impact of average ability on skilled labor productivity	5
ϕ	Unskilled labor productivity	5
θ_0	The initial fraction of skilled workers	0.3
κ	Discount of after-school private education	0.8

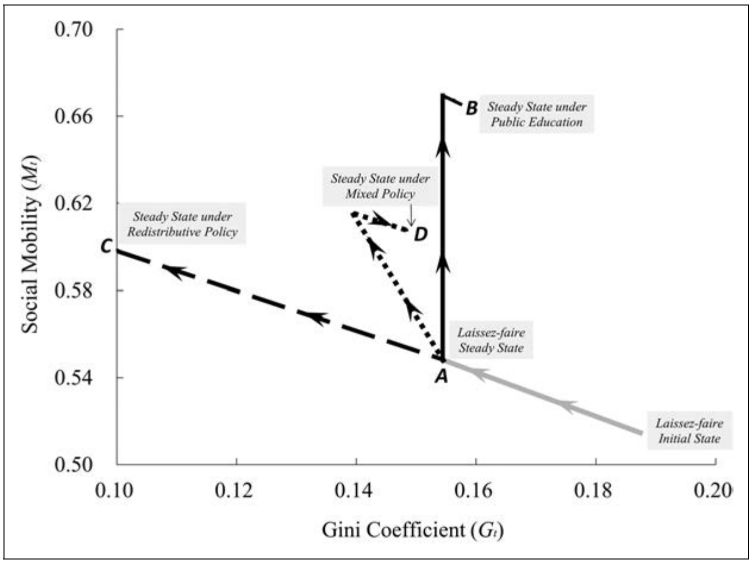

Figure 1. Inequality versus Mobility Under Different Fiscal and Educational Policies.
Notes. The arrows show the economy's direction of motion. A government policy is introduced at point A (i.e., the laissez-faire steady state). Points B, C, and D indicate the steady states under the public education system, income redistribution, and mixed policy, respectively.

wages from the rich to the poor via cash transfers reduce income dispersion, thereby driving the Gini coefficient down. Meanwhile, intergenerational mobility tends to improve because the poor, who become less poor after the redistribution, provide their children with better private education to boost the prospects of upward mobility, while the rich have to do the opposite. The key message is that equality and mobility move together under the redistributive policy, which means that the Great Gatsby curve holds true.

Finally, we consider a mix of public education and social protection, under which the government receives the same income taxes as in section 4.2 and then allocates 80% of tax revenues to public education and 20% to the program of cash transfer, while the poor do not invest in their children's private education. The dotted curve traces the transitional dynamics under this mixed policy. The Gini coefficient falls immediately after the

Table 2. The Steady-state Properties Under Different Fiscal and Educational Policies.

	θ	$\bar{a}$	M	G	G_c
Private School System					
[1] Laissez-faire	0.3701	1.8827	0.5473	0.1551	0.0994
[2] Income redistribution	0.3703	1.8820	0.5983	0.1001	0.0451
Public School System					
[3] No transfer-in-cash	0.3674	1.8933	0.6673	0.1563	0.0813
[4] 20% transfer-in-cash	0.3488	1.9702	0.6075	0.1485	0.0510

policy change, thanks to the income redistribution. Also, mobility improves as educational resources become more equally distributed among children with different family backgrounds. The economy ultimately reaches the steady state at point D. In the long run, the path AD follows an increasing trend in income inequality and a downtrend in social mobility, which is similar to the path AB. As Hassler, Rodríguez Mora, and Zeira (2007) suggest, public subsidies to education play a key role in explaining the negative correlation between inequality and mobility. From the laissez-faire steady state at point A to the new steady state at point D, we observe a simultaneous increase in mobility and equality in general. In other words, a Great Gatsby curve emerges under this mixed policy.

Table 2 summarizes the simulated outcomes of (θ, $\bar{a}$, M, G) in each of four scenarios of Figure 1, namely [1] *laissez-faire*, [2] income redistribution, [3] public education, and [4] mixed policy. We see from the first column that under the private school system, income redistribution enables a greater proportion of population to work in the skilled labor sector (larger θ). Transfer in cash is a useful instrument to help more poor children obtain a college degree and then a skilled occupation, though it inevitably undermines the quality of rich children through the income effect. Given the parameter configuration, the poor's benefits outweigh the rich's costs, thereby leading to the overall improvement in children's quality. When public education is available, there is a smaller share of skilled labor in the steady state. This is largely because the introduction of the public educational system results in a lower level of educational spending on rich children, worsening their prospects of becoming skilled workers. In particular, comparing [3] with [4] indicates that a reallocation of government spending away from public education toward income redistribution will further reduce educational resources received by children, making them less likely to be skilled in the future.

Note that equation (18) characterizes a tradeoff between $\bar{a}$ and θ in the steady state. Compared with a private education system, a public one is more effective in placing smart individuals into the skilled labor sector. In contrast, the introduction of redistributive policy may result in a lower level of average ability of skilled workers in the steady state.

The third column of Table 2 shows that both transfers in cash and in kind help to make future generations more mobile, as we have presented in Figure 1. Social mobility is at its highest under the public school system without cash transfers, which echoes Proposition 4 that public schooling offers a relatively equal opportunity of achieving skilled professions. Yet the fourth column shows that the public school system gives rise to a greater Gini coefficient. This result in in accord with Glomm and Kaganovich's (2003) viewpoint that increased spending on public education may lead to a higher degree of inequality. We provide an explanation as follows. The public school system facilitates high-ability people from disadvantaged families to crowd out silver-spooned low-ability people when competing for jobs in skilled occupations. As the increased average talent of skilled workers raises their wage rate, the between-sector pay gap widens given that the wage of unskilled workers remains constant. A downsizing of high-paid skilled workers (small θ) also helps to fuel the increase in the Gini coefficient. By comparing [2], [3], and [4], we see that if the government is more concerned about economic equality than social mobility, its policy target moves from public education to income redistribution. It is noteworthy that the Great Gatsby curve is violated when going from pure income redistribution to public education or vice versa. This result holds even if we substitute the indicator of after-tax income inequality with an alternative indicator, such as the inequality measured by consumption (i.e., disposable income net of educational investments, as represented by G_c in the last column).

We turn next to examine how the changing degrees of meritocracy of educational systems (as measured by the gap between β^s and β^u) affects economic inequality and social mobility. Figure 2 graphs the steady-state mobility (M) versus the steady-state Gini coefficient (G) with and without government intervention based on the parameter values in Table 1 except that β^u changes between 0.1 and 0.2. We draw four plots, each representing a different regime, namely (a) *laissez-faire*, (b) income redistribution, (c) public education, and (d) mixed policy. It is shown that all four curves slope downward: changing the values of β^u traces out an inverse relationship between G and M, regardless of the policy. In general, as the economy moves from an elitist educational system to a meritocratic one, mobility will increase as well as equality.

We proceed to examine the steady-state levels of social welfare under *laissez-faire*, redistributive policy, and public education regime. In Figure 3, we first chart the welfare consequence using parameter values in Table 1 (the benchmark case). It shows that the private school system with income redistribution leads to a higher social welfare than other systems. Through the income effect, educational investments increase in poor families and fall in rich families, which enhances social welfare due to the law of diminishing marginal utility (e.g., Boadway and Keen, 2000; Boadway and Sato, 2015). This result sheds light on the practice in many countries of focusing government interference on redistributing wealth from the rich to the poor. We also demonstrate that the public school system results in the lowest welfare level (even lower than the *laissez-faire* outcome). This is largely because poor parents are very inefficient in using the educational resources although their children attain more resources after the implementation of public education.

The middle block investigates how the introduction of myopic parents changes the simulation outcome. Consider that a fraction η of poor parents and none of rich parents are myopic. When deciding on their

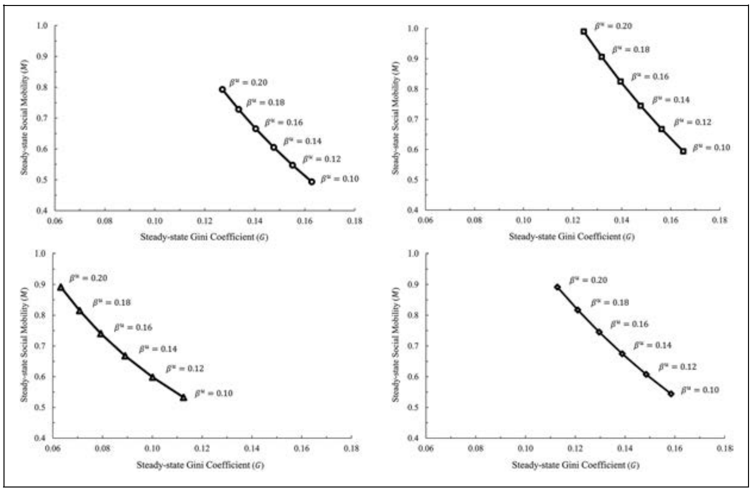

Figure 2. The Great Gatsby Curve: From Elitist to Meritocratic Educational Systems.

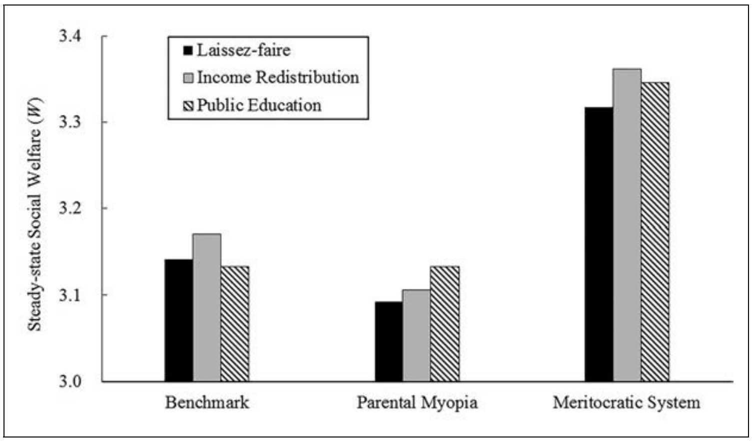

Figure 3. Welfare Consequences under Different Policies.

educational investments, myopic parents will optimally choose the minimum level ($e = 1$). In that case, intergenerational social mobility amounts to

$$M_t = \frac{\ln(1 + a^H) + (1 - \eta)\beta^u \ln(\gamma\phi)}{\ln(1 + a^H) + \beta^s \ln(\gamma\delta\bar{a}_t)}.$$

Comparing this expression with (14) indicates that the economy becomes less mobile when myopia matters. Under the public school system, the benevolent government ignores the myopic acts so that policies (and therefore the equilibrium outcomes) remain the same as in section 4.2. To run the simulation, we set the proportion of myopic people in poor parents to 60% (i.e., $\eta = 0.6$). The presence of myopia makes public education more appealing: the public school system now yields a higher social welfare than any case under the private school system.

The block on the right examines the meritocratic educational system, under which poor parents perform as efficient as rich parents in using educational resources ($\beta^u = 0.2$). With the improved efficiency of poor parents, public education is preferable to the laissez-faire outcome. Moreover, transfer in cash is more desirable than transfer in kind when poor parents have no disadvantage in using educational resources. The reason why welfare is lower with public education than with pure redistribution can be explained by our static utilitarian welfare function. If mobility were added as an additional argument to social welfare, the result would be different. Because social welfare remains lowest under *laissez-faire*, government intervention can always make people better off.

Conclusion

Market-based inequality has been growing in many countries of the world, which poses serious challenges to the governments (e.g., Boadway and Cuff, 2013). Also, one reason behind the populist vote in traditional welfare states is that policies have aimed at alleviating economic inequality while neglecting the issue of social mobility. What is observed in these states is a quite stable income distribution, in great part due to proactive redistributive policies, but little concern for what has been called a broken social elevator. To illustrate, 60 years ago over 90% of parents expected their children to do better than them; today this fraction has fallen to one half.

This paper develops an overlapping-generations model analyzing the influences of educational policies on the intergenerational mobility of individuals with different innate abilities and family backgrounds. We consider that an economy's productivity and income depend on both the fraction of skilled individuals and their average talent. Our model suggests that enhancing intergenerational mobility may promote efficiency and equity simultaneously, which in turn increases social welfare.

In the *laissez-faire* economy where parents invest in their own children's education, the income effect suggests that the rich tend to spend more on education, leading to rich children being more likely to become rich in the future, while the poor do the opposite, leaving their children with fewer opportunities. There is long-run intergenerational persistence in human capital under the private education system. Our comparative static analysis shows that, under some reasonable conditions, the proportion of skilled workers in the steady state increases with parental concern about children's educational attainment and parental income; however, the average ability of skilled workers decreases with these factors.

We then suggest that the introduction of public education helps to improve social mobility by reallocating educational resources toward poor families. Our model derives the condition in which rich parents top up their children's education will be operative in equilibrium. We also show that income inequality may go worse under the public school system. As public education prompt smart individuals from poor families to replace mediocre peers from rich families in skilled occupations, the average worker productivity and wage rate in the skilled labor increases, which in turn widens the wage gap between the rich and the poor.

Our simulation demonstrate that the Great Gatsby curve is robust in many cases: (1) the transitional dynamics towards the steady state under *laissez-faire*, (2) government intervention through income redistribution, and (3) the evolution from elitist to meritocratic educational systems. Two exceptions are government intervention through public education and a policy change from income redistribution to public education or vice versa. We further suggest that when some poor parents are myopic, it may be welfare-improving to implement public education. As the education system becomes increasingly meritocratic in the economy, both social mobility and income equality will improve in the steady state. In that case, government intervention will be socially preferable.

One limitation of our stylized model is a restrictive assumption that an individual's own talent is not rewarded in the labor market. Under this assumption, we derive the steady state. An alternative assumption that a skilled worker's wage depends on both her own talent and the average talent would complicate analysis without a qualitative change in our results under transitional dynamics. The steady states under such an assumption would be technically difficult to derive, since it would imply that individual heterogeneity would increase over time. Also, future research may extend our analysis to a setting in which an economy is concerned with not only the wellbeing of the current generation but also the equity of future generations. In that case, the provision of public education ("transfer in kind") is presumably preferable to income redistribution ("transfer in cash"). It is conceivable that a greater concern about social mobility leads to a lower level of public educational expenditure per child. This is because skilled workers are often more efficient in utilizing educational resources that their children receive from public schools, and overly generous educational expenses may put poor children at a disadvantage. This consideration would imply a potential tradeoff between efficiency and *dynamic* equity.

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Notes

1. Roemer and Trannoy (2016: p. 1289) address that "*equality of opportunity can be described as seeking to offset differences in outcomes attributable to luck, but not those differences in outcomes for which individuals are responsible.*" Also see Fleurbaey (1995) and Leroux and Ponthière (2013).
2. The literature on the "signaling" role of education in the labor market posits that education can tell the employer about a job candidate's ability, although it contributes little in itself to worker productivity.
3. Earlier literature that compares private with public education includes Glomm and Ravikumar (1992) and de la Croix and Doepke (2004, 2009) among others.
4. Myopia is a widely recognized topic in the public economics literature. See Cremer et al. (2007) and Cremer and Pestieau (2011) among others.
5. See "Hong Kong in hot pursuit of Ivy League education." by Alexis Lai, CNN.com, 3 December 2012.
6. See "Hard work got me into Stanford University, says Chinese student in viral video after parents paid US$6.5 million to get her accepted." by Laurie Chen, *South China Morning Post*, 3 May 2019.
7. See "School's out forever: Why super-rich parents are opting to educate their children using private tutors?" by Joshi Herrmann, *Evening Standard*, 13 February 2015.
8. To make the model tractable, we assume specific functional forms. For example, we assume log functions for both ability (a) and educational expenditure (e) to capture that both factors are subject to the law of diminishing returns. Since a only takes two values, the choice of functional forms may not affect much the analytical result. In Section 5, log functional forms appear to lead more reasonable simulation results than some other forms (e.g., linear functional forms). In other words, log functions serve best for our simulation.
9. Educated parents tend to be better at encouraging and motivating their children, have a more positive attitude toward learning, own more books and read regularly, and provide healthier diets (e.g., Papageorge and Thom 2020).
10. For example, after-school tutoring often takes place off campus; to participate in private tutoring therefore incurs a fixed adjustment cost, such as the time and expense of commuting between school and the tutoring center (Fashola, 2001). Besides, the curriculum emphasized by tutoring institutions may not match well with that in school. The outcome that family socioeconomic status

is positively related with student participation in private tutoring has a clear empirical counterpart in, for example, South Korea (Kim and Lee, 2010) and Hong Kong (Bray et al., 2014).

11. In a different model setup, Hassler and Rodríguez Mora (2000) make a similar point: when the world changes slowly, children of skilled workers have an informational advantage for skilled occupations, but if the world changes a great deal over time, parents' information becomes less valuable and innate ability becomes more important in social selection. Rather than discuss economic growth, our paper addresses the role of education systems.
12. In our example as shown by Figure 1 the transition to the steady state at point A, we observe higher equality and mobility. Note that the initial conditions are important. Another set of initial conditions could imply more inequality and immobility.

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Appendix

Proof of Proposition 1

Equation (14) shows that M_t decreases with δ and $\bar{a}_t$ while increases with ϕ. Equation (11) shows that given the proportion of skilled workers, θ_t, in a certain period t, G_t decreases with ϕ and increases with δ and $\bar{a}_t$.

Proof of Proposition 2

In the steady state, equations (15) and (10) can be rewritten as

$$\begin{aligned} \theta &= \frac{\lambda \ln(1+a^H) + \beta^u \ln(\gamma^u \phi)}{1 - \beta^s \ln(\gamma^s \delta \bar{a}) + \beta^u \ln(\gamma^u \phi)}, \\ \bar{a} &= \lambda a^H \left[1 + \frac{(1-\lambda)\ln(1+a^H)}{\theta} \right]. \end{aligned} \tag{A.1}$$

Combining the above two expressions obtains

$$\begin{aligned}\frac{\bar{a}}{\lambda a^H}-1&=\frac{(1-\lambda)\ln(1+a^H)[1-\beta^s\ln(\gamma^s\delta\bar{a})+\beta^u\ln(\gamma^u\phi)]}{\lambda\ln(1+a^H)+\beta^u\ln(\gamma^u\phi)}\\ &\Leftrightarrow\frac{\bar{a}}{a^H}+\frac{\beta^u\bar{a}\ln(\gamma^u\phi)}{\lambda a^H\ln(1+a^H)}+(1-\lambda)\beta^s\ln(\gamma^s\delta\bar{a})\\ &=1+(1-\lambda)\beta^u\ln(\gamma^u\phi)+\frac{\beta^u\ln(\gamma^u\phi)}{\ln(1+a^H)},\end{aligned}\tag{A.2}$$

which can be rewritten as (17). Rearranging the expression of $\bar{a}$ in (A.1) obtains (18).

Proof of Proposition 3

(i) Totally differentiating (17) with respect to $\bar{a}$ and a^H and then using (10) obtains

$$\begin{aligned}&\left[1+\frac{\beta^u\ln(\gamma^u\phi)}{\lambda\ln(1+a^H)}\right]\frac{d\bar{a}}{a^H}+\frac{\beta^s(1-\lambda)d\bar{a}}{\bar{a}}\\ &-\left\{\frac{\bar{a}}{(a^H)^2}+\frac{\beta^u\bar{a}\ln(\gamma^u\phi)\left[\frac{a^H}{1+a^H}+\ln(1+a^H)\right]}{\lambda[\ln(1+a^H)a^H]^2}\right\}da^H\\ &=-\frac{\beta^u\ln(\gamma^u\phi)}{[\ln(1+a^H)]^2(1+a^H)}da^H\\ &\Leftrightarrow a^h\Omega d\bar{a}=\left\{\bar{a}+\frac{\beta^u\ln(\gamma^u\phi)[\bar{a}a^h+\bar{a}(1+a^h)\ln(1+a^h)-\lambda a^{H2}]}{\lambda[\ln(1+a^h)]^2(1+a^h)}\right\}da^h\\ &\Leftrightarrow\frac{d\bar{a}}{da^h}=\frac{1}{a^h\Omega}\left\{\bar{a}+\frac{[\lambda(1-\lambda)a^{H2}/\theta+\bar{a}(1+a^h)]\beta^u\ln(\gamma^u\phi)}{\lambda(1+a^h)\ln(1+a^h)}\right\}>0.\end{aligned}\tag{A.3}$$

where, for notational simplification, we denote $\Omega:=1+\frac{\beta^u\ln(\gamma^u\phi)}{\lambda\ln(1+a^H)}+\frac{\beta^s(1-\lambda)a^H}{\bar{a}}>0$. Totally differentiating (17) with respect to $\bar{a}$ and λ and using

(13) obtains

$$\begin{aligned}
&\left[1+\frac{\beta^u \ln(\gamma^u\phi)}{\lambda \ln(1+a^H)}\right]\frac{d\bar{a}}{a^H}+\frac{\beta^s(1-\lambda)d\bar{a}}{\bar{a}}-\frac{\beta^u \ln(\gamma^u\phi)\bar{a}d\lambda}{\lambda^2 a^H \ln(1+a^H)}\\
&\quad=[\beta^s \ln(\gamma^s\delta\bar{a})-\beta^u \ln(\gamma^u\phi)]d\lambda\\
&\quad\Leftrightarrow \frac{d\bar{a}}{d\lambda}=\frac{1}{\Omega}\left\{\frac{\beta^u\bar{a}\ln(\gamma^u\phi)}{\lambda^2\ln(1+a^H)}+a^H\Big[\beta^s \ln(\gamma^s\delta\bar{a})-\beta^u \ln(\gamma^u\phi)\Big]\right\}>0.
\end{aligned} \tag{A.4}$$

(*ii*) Totally differentiating (17) with respect to $\bar{a}$ and α and then using (13) obtains

$$\begin{aligned}
&\left[1+\frac{\beta^u \ln(\gamma^u\phi)}{\lambda \ln(1+a^H)}\right]\frac{d\bar{a}}{a^H}+\frac{\beta^s(1-\lambda)d\bar{a}}{\bar{a}}+\frac{\bar{a}}{a^H}\frac{\beta^u}{\lambda\ln(1+a^H)}\frac{\beta^u d\alpha}{\gamma^u(\alpha\beta^u+1)^2}\\
&\quad+\frac{\beta^s(1-\lambda)}{\gamma^s}\frac{\beta^s d\alpha}{(\alpha\beta^s+1)^2}=\beta^u\left[1-\lambda+\frac{1}{\ln(1+a^H)}\right]\frac{\beta^u d\alpha}{\gamma^u(\alpha\beta^u+1)^2}\\
&\quad\Leftrightarrow \frac{\Omega d\bar{a}}{a^H}=\left[1-\lambda+\frac{1}{\ln(1+a^H)}\right]\frac{\gamma^u d\alpha}{\alpha^2}-\frac{\bar{a}}{a^H}\frac{\gamma^u d\alpha}{\alpha^2\lambda a^H}-\frac{(1-\lambda)\gamma^s d\alpha}{\alpha^2}\\
&\quad\Leftrightarrow \frac{d\bar{a}}{d\alpha}=-\frac{a^H}{\alpha^2\Omega}\left[(1-\lambda)(\gamma^s-\gamma^u)+\frac{\gamma^u(\bar{a}-\lambda a^H)}{\lambda\ln(1+a^H)}\right]<0.
\end{aligned} \tag{A.5}$$

(*iii*) Totally differentiating (17) with respect to $\bar{a}$ and β^s obtains

$$\begin{aligned}
&\left[1+\frac{\beta^u \ln(\gamma^u\phi)}{\lambda \ln(1+a^H)}\right]\frac{d\bar{a}}{a^H}+\frac{\beta^s(1-\lambda)d\bar{a}}{\bar{a}}+(1-\lambda)\ln(\gamma^s\delta\bar{a})d\beta^s+\frac{\beta^s(1-\lambda)\alpha}{\gamma^s(\alpha\beta^s+1)^2}d\beta^s=0\\
&\quad\Leftrightarrow \frac{\Omega d\bar{a}}{a^H}=-(1-\lambda)\left[\ln(\gamma^s\delta\bar{a})+\frac{1}{\alpha\beta^s+1}\right]d\beta^s\\
&\quad\Leftrightarrow \frac{d\bar{a}}{d\beta^s}=-\frac{(1-\lambda)a^H}{\Omega}\left[\ln(\gamma^s\delta\bar{a})+1-\gamma^s\right]<0.
\end{aligned} \tag{A.6}$$

Totally differentiating (17) with respect to $\bar{a}$ and β^u and using (10) obtains

$$\left[1+\frac{\beta^u \ln(\gamma^u \phi)}{\lambda \ln(1+a^H)}\right]\frac{d\bar{a}}{a^H}+\frac{\beta^s(1-\lambda)d\bar{a}}{\bar{a}}+\frac{\bar{a}}{\lambda a^H \ln(1+a^H)}$$
$$\left[\ln(\gamma^u\phi)+\frac{\alpha\beta^u}{\gamma^u(\alpha\beta^u+1)^2}\right]d\beta^u=\left(1-\lambda+\frac{1}{a^H}\right)\left[\ln(\gamma^u\phi)+\frac{\alpha\beta^u}{\gamma^u(\alpha\beta^u+1)^2}\right]d\beta^u$$
$$\Leftrightarrow \frac{\Omega d\bar{a}}{a^H}=\left[\ln(\gamma^u\phi)+\frac{1}{\alpha\beta^u+1}\right]\left[1-\lambda+\frac{\lambda a^H-\bar{a}}{\lambda a^H\ln(1+a^H)}\right]d\beta^u$$
$$\Leftrightarrow \frac{d\bar{a}}{d\beta^u}=-\frac{a^H(1-\lambda)(1-\theta)[\ln(\gamma^u\phi)+1-\gamma^u]}{\theta\Omega}<0. \tag{A.7}$$

(iv) Totally differentiating (17) with respect to $\bar{a}$ and δ and rearranging obtains

$$\left[1+\frac{\beta^u \ln(\gamma^u \phi)}{\lambda \ln(1+a^H)}\right]\frac{d\bar{a}}{a^H}+\beta^s(1-\lambda)\left(\frac{d\bar{a}}{\bar{a}}+\frac{d\delta}{\delta}\right)=0 \quad \Leftrightarrow$$
$$\frac{d\bar{a}}{d\delta}=-\frac{\beta^s(1-\lambda)a^H}{\delta\Omega}<0. \tag{A.8}$$

Totally differentiating (17) with respect to $\bar{a}$ and ϕ and using (10) obtains

$$\left[1+\frac{\beta^u \ln(\gamma^u \phi)}{\lambda \ln(1+a^H)}\right]\frac{d\bar{a}}{a^H}+\frac{\beta^s(1-\lambda)d\bar{a}}{\bar{a}}+\frac{\bar{a}}{a^H}\frac{\beta^u d\phi}{\lambda\ln(1+a^H)\phi}$$
$$=\left[1-\lambda+\frac{1}{\ln(1+a^H)}\right]\frac{\beta^u d\phi}{\phi}$$
$$\Leftrightarrow \Omega d\bar{a}=\frac{\beta^u}{\phi}\left[(1-\lambda)a^H+\frac{\lambda a^H-\bar{a}}{\lambda\ln(1+a^H)}\right]d\phi$$
$$\Leftrightarrow \frac{d\bar{a}}{d\phi}=-\frac{(1-\theta)(1-\lambda)a^H\beta^u}{\theta\phi\Omega}<0. \tag{A.9}$$

Proof of Proposition 4

Given $\kappa, \tau_t \in (0,1)$, it is easy to infer from (20) with (21) that $e_t^s=\gamma^s[\kappa(1-\tau_t)\delta\bar{a}_t-e_t^P]$, which is smaller than $e_t^s=\gamma^s\delta\bar{a}_t$ in (13). Under the top-up education system, the proportional income tax leads to an effective rise in an unskilled worker's disposable income, which means that $e_t^u>\gamma^u\phi$. It follows from equations (4) and (14) that $M_t^{private}<M_t^{public}$.

Proof of Proposition 5

Taking the first order condition of V_t in (22) with respect to τ_t yields

$$\frac{\partial V_t}{\partial \tau_t} = \theta_t \frac{-\kappa\delta\bar{a}_t + \theta_t\delta\bar{a}_t + (1-\theta_t)\phi}{\kappa(1-\tau_t)\delta\bar{a}_t + \tau_t[\theta_t\delta\bar{a}_t + (1-\theta)\phi]} + (1-\theta_t)\frac{-1}{1-\tau_t}$$
$$+ \alpha\beta^s\theta_t \frac{-\kappa\delta\bar{a}_t - \theta_t\delta\bar{a}_t - (1-\theta_t)\phi}{\kappa(1-\tau_t)\delta\bar{a}_t - \tau_t[\theta_t\delta\bar{a}_t + (1-\theta)\phi]} + \alpha\beta^u(1-\theta_t)\frac{1}{\tau_t} = 0. \tag{A.10}$$

which can be rearranged as (23). By (19) and (21), rich children will receive private tutoring in equilibrium if and only if $e_t^A > 0$, namely

$$\gamma^s(1-\tau^*)\delta\bar{a}_t > \frac{\tau_t^*[\theta_t\delta\bar{a}_t + (1-\theta_t)\phi]}{(1+\alpha\beta^s)\kappa} \quad \Leftrightarrow$$
$$\frac{\alpha\beta^s\kappa}{1+\alpha\beta^s}\cdot\frac{1-\tau^*}{\tau^*} > \frac{\theta_t\delta\bar{a}_t + (1-\theta_t)\phi}{(1+\alpha\beta^s)\delta\bar{a}_t}.$$

which can be rearranged as (24).

Proof of Proposition 6

(i) If condition (24) fails, we have $e^A = 0$ and $e^u = e^s = e^P$. It follows that (7) can be rewritten as

$$V_t = \ln(1-\tau_t) + \alpha[\beta^s\theta_t + \beta^u(1-\theta_t)]\{\ln\tau_t + \ln[\theta_t\delta\bar{a}_t + (1-\theta_t)\phi]\}$$
$$+ \theta_t\ln(\delta\bar{a}_t) + (1-\theta_t)\ln\phi + \alpha\lambda\ln(1+a^H).$$

Given $(\theta_t, \bar{a}_t)$, differentiating V_t in with respect to τ_t and setting it to zero yields

$$\frac{dV_t}{d\tau_t} = \frac{\alpha[\beta^s\theta_t + \beta^u(1-\theta_t)]}{\tau_t} - \frac{1}{1-\tau_t} = 0 \quad \Leftrightarrow$$
$$\tau_t = 1 - \frac{1}{\alpha[\beta^s\theta_t + \beta^u(1-\theta_t)] + 1}. \tag{A.11}$$

Substituting (A.11) into (19) and then using (10) yields e_t^P as a function of θ_t:

$$e_t^P = \frac{\lambda\delta a^H[(1-\lambda)a^H + \theta_t] + (1-\theta_t)\phi}{\{\alpha[\beta^s\theta_t + \beta^u(1-\theta_t)]\}^{-1} + 1}. \tag{A.12}$$

Inserting (A.12) into (8) and then focusing on the steady state obtains

$$\theta = \ln(1 + a^H) + [\beta^s\theta + \beta^u(1 - \theta)] \ln\left\{\frac{\lambda\delta a^H[(1-\lambda)\ln(1+a^H) + \theta] + (1-\theta)\phi}{\{\alpha[\beta^s\theta + \beta^u(1-\theta)]\}^{-1} + 1}\right\}, \tag{A.13}$$

which can be rearranged as (25). Inserting (25) into (10) and rearranging yields (26).

(ii) Given $\beta^s = \beta^u = \beta$, we simplify (A.11) as $\tau = \frac{\alpha\beta}{\alpha\beta+1}$ and thus rewrite (25) as

$$K = \theta - \lambda a^H - \beta \ln\frac{\alpha\beta}{\alpha\beta + 1} - \beta \ln\left\{\delta\lambda a^H[(1-\lambda)a^H + \theta] + (1-\theta)\phi\right\} = 0,$$

where $K = K(\theta; \alpha, \beta, \delta, \phi, \lambda, a^H)$. We have the following four partial derivatives:

$$\frac{\partial K}{\partial \theta} = 1 - \frac{\beta(\delta\lambda a^H - \phi)}{\delta\lambda a^H[(1-\lambda)\ln(1+a^H) + \theta] + (1-\theta)\phi} = 1 - \beta\frac{\delta\lambda a^H - \phi}{(\delta\bar{a} - \phi)\theta + \phi}, \tag{A.14}$$

$$\frac{\partial K}{\partial \alpha} = -\beta \cdot \frac{\alpha\beta + 1}{\alpha\beta} \cdot \frac{\beta(\alpha\beta+1) - \alpha\beta^2}{(\alpha\beta+1)^2} = -\frac{\beta}{\alpha(\alpha\beta+1)} < 0, \tag{A.15}$$

$$\frac{\partial K}{\partial \delta} = -\frac{\beta\lambda a^H[(1-\lambda)\ln(1+a^H) + \theta]}{\delta\lambda a^H[(1-\lambda)\ln(1+a^H) + \theta] + (1-\theta)\phi} < 0, \tag{A.16}$$

$$\frac{\partial K}{\partial \phi} = -\frac{\beta(1-\theta)}{\delta\lambda a^H[(1-\lambda)\ln(1+a^H) + \theta] + (1-\theta)\phi} < 0, \tag{A.17}$$

where $\frac{\partial K}{\partial \theta} > 0$ if and only if (27) is satisfied. In that case, taking the total differentiation of K with respect to θ, α, δ, ϕ and rearranging implies

$$\frac{\partial K}{\partial \theta}d\theta + \frac{\partial K}{\partial \alpha}d\alpha = 0 \quad \Leftrightarrow \quad \frac{d\theta}{d\alpha} = -\frac{\partial K}{\partial \alpha}\Big/\frac{\partial K}{\partial \theta} > 0. \tag{A.18}$$

$$\frac{\partial K}{\partial \theta}d\theta + \frac{\partial K}{\partial \delta}d\delta = 0 \quad \Leftrightarrow \quad \frac{d\theta}{d\delta} = -\frac{\partial K}{\partial \delta}\Big/\frac{\partial K}{\partial \theta} > 0. \tag{A.19}$$

$$\frac{\partial K}{\partial \theta} d\theta + \frac{\partial K}{\partial \phi} d\phi = 0 \quad \Leftrightarrow \quad \frac{d\theta}{d\phi} = -\frac{\partial K}{\partial \phi} \Big/ \frac{\partial K}{\partial \theta} > 0. \tag{A.20}$$

Because $\bar{a}$ and θ are inversely related by equation (18), we can infer that $\bar{a}$ decreases with (α, δ, ϕ) if and only if condition (27) holds.