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### ABSTRACT

Measurements of the double-differential  $\pi^\pm$  production cross-section in the range of momentum  $0.5 \text{ GeV} \leq p \leq 8.0 \text{ GeV}$  and angle  $0.025 \text{ rad} \leq \theta \leq 0.25 \text{ rad}$  in interactions of charged pions on beryllium, carbon, aluminium, copper, tin, tantalum and lead are presented. These data represent the first experimental campaign to systematically measure forward pion hadroproduction. The data were taken with the large acceptance HARP detector in the T9 beam line of the CERN PS. Incident particles, impinging on a 5% nuclear interaction length target, were identified by an elaborate system of beam detectors. The tracking and identification of the produced particles was performed using the forward spectrometer of the HARP detector. Results are obtained for the double-differential cross-sections  $\frac{d^2\sigma}{dp d\Omega}$  mainly at four incident pion beam momenta (3 GeV, 5 GeV, 8 GeV and 12 GeV). The measurements are compared with the GEANT4 and MARS Monte Carlo simulation

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# Forward production of charged pions with incident $\pi^\pm$ on nuclear targets measured at the CERN PS.

HARP Collaboration

April 7, 2010

## Abstract

Measurements of the double-differential  $\pi^\pm$  production cross-section in the range of momentum  $0.5 \text{ GeV}/c \leq p \leq 8.0 \text{ GeV}/c$  and angle  $0.025 \text{ rad} \leq \theta \leq 0.25 \text{ rad}$  in interactions of charged pions on beryllium, carbon, aluminium, copper, tin, tantalum and lead are presented. These data represent the first experimental campaign to systematically measure forward pion hadroproduction.

The data were taken with the large acceptance HARP detector in the T9 beam line of the CERN PS. Incident particles, impinging on a 5% nuclear interaction length target, were identified by an elaborate system of beam detectors. The tracking and identification of the produced particles was performed using the forward spectrometer of the HARP detector. Results are obtained for the double-differential cross-sections  $d^2\sigma/dpd\Omega$  mainly at four incident pion beam momenta (3 GeV/ $c$ , 5 GeV/ $c$ , 8 GeV/ $c$  and 12 GeV/ $c$ ). The measurements are compared with the GEANT4 and MARS Monte Carlo simulation.

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# 1 Introduction

The HARP experiment [1] at the CERN PS was designed to measure hadron yields from a large range of nuclear targets and for incident particle momenta from 1.5 GeV/ $c$  to 15 GeV/ $c$ . With incident protons, this corresponds to a momentum region of great interest for neutrino beams and far from the coverage by earlier dedicated hadroproduction experiments [2, 3, 4]. Measurements with incident charged pions are relevant for the calculation of cosmic-ray muon and neutrino fluxes using extended air shower simulations. These data are also important to simulate re-interactions in particle detectors and neutrino beam production targets. At present such calculations rely on models with large uncertainties given the lack of experimental data in this energy region.

Covering an extended range of solid targets in the same experiment, it is also possible to perform systematic comparison of hadron production models with measurements at different incoming beam momenta over a large range of target atomic number  $A$ .

This paper presents our final measurements of the double-differential cross-section,  $d^2\sigma^\pi/dp d\Omega$  for  $\pi^\pm$  forward production by incident charged pions of 3 GeV/ $c$ , 5 GeV/ $c$ , 8 GeV/ $c$  and 12 GeV/ $c$  momentum impinging on a thin beryllium, carbon, aluminium, copper, tin, tantalum or lead target of 5% nuclear interaction length ( $\lambda_I$ ). In addition, high statistics data at 8.9 GeV/ $c$  (12.9 GeV/ $c$ ) on a thin beryllium (aluminium) target are presented.

Some HARP results on pion production in the forward region with incident protons have already been published in papers [5, 6, 7, 8].

Pion production data at low momenta and in the forward region from incident pions are extremely scarce [9]. HARP is the first experiment to provide a high statistics data set, taken with many different targets, full particle identification and large-acceptance detector. The collected statistics, for the different nuclear targets, are reported in Table 1.

Other data with an incident pion beam, up to higher momenta, has been collected by the Fermilab E970-MIPP experiment [10] and more data at other momenta would be available with its proposed upgrade [11].

The paper is organized as follows. In subsection 1.1 we briefly describe the HARP detector, while section 2 is devoted to the main features of the analysis procedure. Section 3 presents results. It is followed by a summary presented in section 4.

## 1.1 Experimental apparatus

The HARP experiment makes use of a large-acceptance spectrometer consisting of a forward and large-angle detection system. The HARP detector is shown in Fig. 1. A detailed description of the experimental apparatus can be found in Ref. [12]. The forward spectrometer – based on five modules of large area drift chambers (NDC1-5) [13] and a dipole magnet complemented by a set of detectors for particle identification (PID): a time-of-flight wall (TOFW) [14], a large Cherenkov detector (CHE) and an electromagnetic calorimeter (ECAL) – covers polar angles up to 250 mrad. The muon contamination of the beam is measured with a muon identifier consisting of thick iron absorbers and scintillation counters. The large-angle spectrometer – based on a Time Projection Chamber (TPC) and Resistive Plate Chambers (RPCs) located inside a solenoidal magnet – has a large acceptance in momentum and angular range for the pions relevant to the production of the muons in a neutrino factory. For the analysis described here only the forward spectrometer and the beam instrumentation are used.

The HARP experiment, located in the T9 beam of the CERN PS, took data in 2001 and 2002. The momentum definition of the T9 beam is known with a precision of the order of 1% [15].

The target is placed inside the inner field cage (IFC) of the TPC, in an assembly that can be moved in and out of the solenoid magnet. The targets used for the measurements reported here have a cylindrical shape with a nominal diameter of about 30 mm. Their thickness is equivalent to about 5%  $\lambda_I$ .

A set of four multi-wire proportional chambers (MWPCs) measures the position and direction of the incoming beam particles with an accuracy of  $\approx 1$  mm in position and  $\approx 0.2$  mrad in angle per projection. A beam time-of-flight system (BTOF) measures the time difference of particles over a 21.4 m path-length. It is made of two identical scintillation hodoscopes, TOFA and TOFB (originally built for the NA52 experiment [16]), which, together with a small target-defining trigger counter, TDS (also used for the trigger), provide particle identification at low momenta. This provides separation of pions, kaons and protons up to 5 GeV/ $c$  and determines the initial time at the interaction vertex ( $t_0$ ). The timing resolution of the combined BTOF system is about 70 ps. A system of two N<sub>2</sub>-filled Cherenkov detectors (BCA and BCB) is used to tag electrons at low energies and pions at higher energies. The electron and pion tagging efficiency is found to be close to 100%. At the beam energy used for this analysis the Cherenkov counters select all particles lighter than protons, while the BTOF is used to reject ions. A set of trigger detectors completes the beam instrumentation.

The selection of beam pions is performed as described in [12]. At 3 GeV/ $c$  BCB gas pressure is set to tag  $e^\pm$  (BCA is evacuated to reduce multiple scattering of the beam) while the TOFs are capable of resolving pions from protons. At 5 GeV/ $c$  pions are separated from protons by using joint information from the TOFs and one of the Cherenkovs (usually BCB), while the gas pressure in the other Cherenkov (BCA) is set to tag  $e^\pm$ . At higher momenta the  $e^\pm$  contamination drops below 1% and the TOF system becomes unable to separate pions from protons efficiently: the beam Cherenkov detectors are used for  $\pi$ -p separation. At the highest beam energy the gas pressure in one of the Cherenkov detectors has been set to distinguish protons from kaons. This allows us to estimate the kaon component to be negligible in the pion beam selection at all momenta.

Table 1: Total number of events and tracks used in the various nuclear 5%  $\lambda_I$  target data sets and the number of incident pions on target as calculated from the pre-scaled incident beam triggers. Numbers are for incident  $\pi^+$  (in parenthesis for incident  $\pi^-$ ) in units of  $10^3$  events.

| Data set (GeV/ $c$ )                            |      | 3           | 5           | 8           | 8.9  | 12          | 12.9 |
|-------------------------------------------------|------|-------------|-------------|-------------|------|-------------|------|
| Total DAQ events                                | (Be) | 1113 (2233) | 1296 (1798) | 1935 (1585) | 5868 | 1207 (1227) |      |
|                                                 | (C)  | 1345 (1831) | 2628 (1279) | 1846 (1399) |      | 1062 (646)  |      |
|                                                 | (Al) | 1159 (1523) | 1789 (920)  | 1707 (1059) |      | 619 (741)   | 4713 |
|                                                 | (Cu) | 624 (3325)  | 2079 (1805) | 2089 (1615) |      | 745 (591)   |      |
|                                                 | (Sn) | 1637 (1972) | 2828 (1625) | 2404(1408)  |      | 1803(937)   |      |
|                                                 | (Ta) | 1783 (994)  | 2084(1435)  | 1965(1505)  |      | 866(961)    |      |
|                                                 | (Pb) | 1911 (1282) | 2111(2074)  | 2266 (1496) |      | 487(1706)   |      |
| Accepted beam pions with forward interaction    | (Be) | 246 (731)   | 384 (914)   | 341 (826)   | 1278 | 76 (693)    |      |
|                                                 | (C)  | 257 (299)   | 754 (530)   | 358 (772)   |      | 41 (352)    |      |
|                                                 | (Al) | 213(486)    | 523 (308)   | 335 (611)   |      | 27 (435)    | 332  |
|                                                 | (Cu) | 168 (1185)  | 611 (850)   | 397 (966)   |      | 33 (347)    |      |
|                                                 | (Sn) | 467 (778)   | 819 (732)   | 481 (804)   |      | 79 (584)    |      |
|                                                 | (Ta) | 561 (426)   | 600 (671)   | 388 (893)   |      | 37 (536)    |      |
|                                                 | (Pb) | 611 (473)   | 594 (997)   | 444 (896)   |      | 20 (839)    |      |
| <b>Final state</b> $\pi^-$<br>selected with PID | (Be) | .9 (24.9)   | 3.5 (32.1)  | 6.3 (31.0)  | 26.7 | 2.0 (27.5)  |      |
|                                                 | (C)  | .7 (9.1)    | 5.9 (15.0)  | 5.9 (24.2)  |      | 1.0 (13.7)  |      |
|                                                 | (Al) | .5 (10.3)   | 4.0 (6.8)   | 5.6 (13.4)  |      | .7 (13.4)   | 9.8  |
|                                                 | (Cu) | .3 (18.9)   | 4.3 (19.0)  | 6.0 (25.1)  |      | .9 (12.9)   |      |
|                                                 | (Sn) | .7 (9.1)    | 4.8 (14.4)  | 6.9 (16.3)  |      | 1.9k (19.4) |      |
|                                                 | (Ta) | .6 (3.7)    | 3.1 (11.5)  | 4.9 (16.2)  |      | .9k (16.6)  |      |
|                                                 | (Pb) | .6 (3.4)    | 3.0 (16.0)  | 5.2 (16.3)  |      | .5k (25.5)  |      |
| <b>Final state</b> $\pi^+$<br>selected with PID | (Be) | 8.9 (1.3)   | 14 (4.8)    | 13.8 (10.6) | 50.0 | 3.4 (13.1)  |      |
|                                                 | (C)  | 8.0 (.5)    | 23.8 (2.5)  | 13.3 (8.0)  |      | 1.7 (6.7)   |      |
|                                                 | (Al) | 5.2 (.5)    | 15 (1.3)    | 12.1 (5.6)  |      | 1.2 (7.0)   | 15.1 |
|                                                 | (Cu) | 2.8 (1.6)   | 14.8 (3.5)  | 12.2 (9.6)  |      | 1.4 (6.6)   |      |
|                                                 | (Sn) | 5.9 (.6)    | 16.8 (2.8)  | 13.3 (6.1)  |      | 3.1 (10.2)  |      |
|                                                 | (Ta) | 5.3 (.2)    | 10.8 (2.2)  | 9.4 (6.0)   |      | 1.4 (8.7)   |      |
|                                                 | (Pb) | 5.0 (.2)    | 9.8 (2.9)   | 10.3 (6.1)  |      | .7 (13.1)   |      |

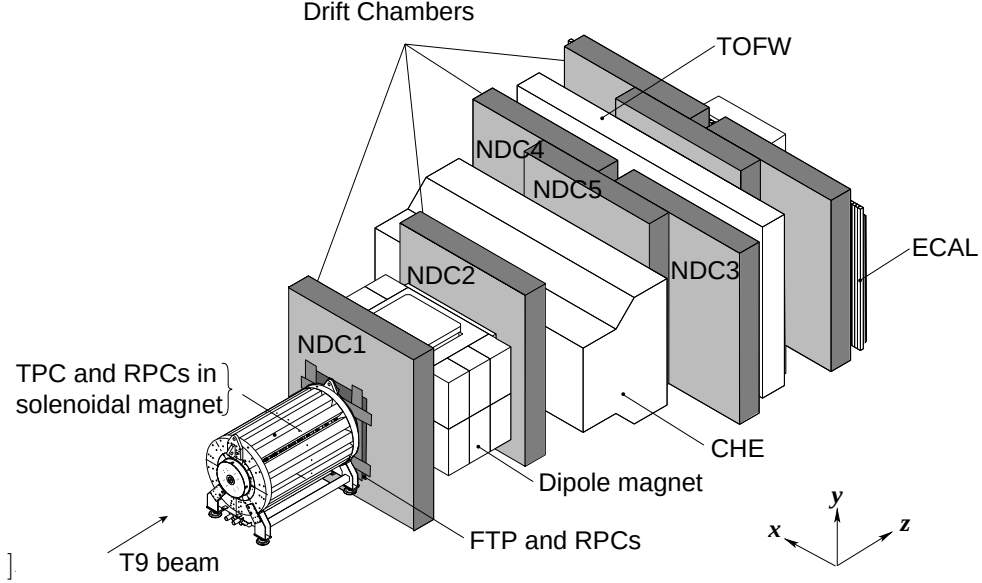


Figure 1: Schematic layout of the HARP detector. The convention for the coordinate system is shown in the lower-right corner.

A downstream trigger in the forward scintillator trigger plane (FTP) was required to record the event, accepting only tracks with a trajectory outside the central hole (60 mm) which allows beam particles to pass. The trigger counter covers the spectrometer acceptance fully and has a high efficiency ( $>99.8\%$ ).

The length of the accelerator spill was 400 ms with a typical intensity of 15 000 beam particles per spill. The average number of events recorded by the data acquisition ranged from 300 to 350 per spill.

The absolute normalization of the number of incident pions was performed using ‘incident-particle’ triggers. These are triggers where the same selection on the beam particle was applied but no selection on the interaction was performed. The rate of this trigger was down-scaled by a factor 64.

## 2 Data Analysis

### 2.1 Event and particle selection

A detailed description of the experimental techniques used for data analysis in the HARP forward spectrometer can be found in Ref. [5, 17]. Further details of the improved analysis techniques can be found in [6, 7]. For the current analysis we have used identical reconstruction and PID algorithms, while at the final stage of the analysis the unfolding technique introduced as UFO in [5] has been applied. This technique has already been described in details in Ref. [19].

At the first stage of the analysis a beam pion is selected using the beam time of flight system (TOF-A, TOF-B) and the Cherenkov counters (BCA, BCB) as described in section 1.1. We always require time measurements in TOF-A, TOF-B and/or TDS to be present which are needed for calculating the arrival time of the beam pion at the target.

Secondary track selection criteria, described in [7], are optimized to ensure the quality of momentum reconstruction and a clean time-of-flight measurement while maintaining a high reconstruction efficiency.

The background induced by interactions of beam particles in the materials outside the target is measured by taking data without a target in the target holder (“empty target data”). These data are subject to the same event and track selection criteria as the standard data sets.

To take into account this background the number of particles of the observed type ( $\pi^+$ ,  $\pi^-$ ) in the “empty target data” are subtracted bin-by-bin (momentum and angular bins) from the number of particles of the same type. The uncertainty induced by this method is discussed in section 2.3 and labeled “empty target subtraction”. The event statistics is summarized in Table 1.

The negative beam consists of  $e^-$  and  $\pi^-$  (with a dominant fraction of  $\pi^-$ ), while the positive beam is dominated by protons at high momenta and by  $\pi^+$  at low momenta <sup>1</sup>. The kaon background is estimated to be <0.5% and is neglected in the analysis, while the muon contamination (tagged by the muon identifier) is around 3% and is subtracted.

The small fraction of pions in the positively charged beam explains the significantly different statistics of the data-sets with incident  $\pi^-$  and  $\pi^+$ . The relatively small fraction of accepted final state pions compared to the accepted beam pions is due to the stringent quality cuts applied in the analysis. The major loss is due to the small acceptance of the dipole magnet in the vertical plane, and the fact that only focused particles are accepted for analyses. The latter requirement reduces the statistics by a factor two but improves the quality of reconstruction and reduces the background [6].

## 2.2 Cross-section calculation

The cross-section is calculated as follows

$$\frac{d^2\sigma^\alpha}{dpd\Omega}(p_i, \theta_j) = \frac{A}{N_A \rho t} \cdot \frac{1}{N_{\text{pot}}} \cdot \frac{1}{\Delta p_i \Delta \Omega_j} \cdot \sum_{p'_i, \theta'_j, \alpha'} \mathcal{M}_{p_i \theta_j \alpha p'_i \theta'_j \alpha'}^{\text{cor}} \cdot N^{\alpha'}(p'_i, \theta'_j), \quad (1)$$

where

- $\frac{d^2\sigma^\alpha}{dpd\Omega}(p_i, \theta_j)$  is the cross-section in mb/(GeV/c sr) for the particle type  $\alpha$  (p,  $\pi^+$  or  $\pi^-$ ) for each true momentum and angle bin  $(p_i, \theta_j)$  covered in this analysis;
- $N^{\alpha'}(p'_i, \theta'_j)$  is the number of particles of type  $\alpha'$  in bins of reconstructed momentum  $p'_i$  and angle  $\theta'_j$  in the raw data;
- $\mathcal{M}_{p_i \theta_j \alpha p'_i \theta'_j \alpha'}^{\text{cor}}$  is the correction matrix which accounts for efficiency and resolution of the detector;
- $\frac{A}{N_A \rho t}$ ,  $\frac{1}{N_{\text{pot}}}$  and  $\frac{1}{\Delta p_i \Delta \Omega_j}$  are normalization factors, namely:
  - $\frac{N_A \rho t}{A}$  is the number of target nuclei per unit area <sup>2</sup>;
  - $N_{\text{pot}}$  is the number of incident beam particles on target (particles on target);
  - $\Delta p_i$  and  $\Delta \Omega_j$  are the bin sizes in momentum and solid angle, respectively <sup>3</sup>.

We do not make a correction for the attenuation of the incoming beam in the target, so that strictly speaking the cross-sections are valid for  $\lambda_I = 5\%$  targets.

The calculation of the correction matrix  $\mathcal{M}_{p_i \theta_j \alpha p'_i \theta'_j \alpha'}^{\text{cor}}$  is a rather difficult task. Various techniques are described in the literature to obtain this matrix. As discussed in Ref. [5] for the p-Al analysis of HARP data at 12.9 GeV/c, two complementary analyses have been performed to cross-check internal consistency and possible biases in the respective procedures. A comparison of both analyses shows that the results are consistent within the overall systematic error [5].

In the first method – called “Atlantic” in [5] – the correction matrix  $\mathcal{M}_{p_i \theta_j \alpha p'_i \theta'_j \alpha'}^{\text{cor}}$  is decomposed into distinct independent contributions, which are computed mostly using the data themselves. The second

<sup>1</sup> the proton fraction in the incoming beam goes from 35% at 3 GeV/c to about 92% at 12 GeV/c

<sup>2</sup>  $A$  - atomic mass,  $N_A$  - Avogadro number,  $\rho$  - target density and  $t$  - target thickness

<sup>3</sup>  $\Delta p_i = p_i^{\text{max}} - p_i^{\text{min}}$ ,  $\Delta \Omega_j = 2\pi(\cos(\theta_j^{\text{min}}) - \cos(\theta_j^{\text{max}}))$



method – called “UFO” in [5] – is the unfolding method introduced by D’Agostini [18]<sup>4</sup>. This method has been used in the recent HARP publications [8, 19, 20, 21, 22] and it is also applied in the analysis described here (see [7] for additional information).

The Monte Carlo simulation of the HARP setup is based on the GEANT4 package [23, 24]. The detector materials are accurately described in this simulation as well as the relevant features of the detector response and the digitization process. All relevant physics processes are considered, including multiple scattering, energy loss, absorption and re-interactions. The simulation is independent of the beam momentum and particle type because it only generates for each event exactly one secondary particle of a specific particle type inside the target material and propagates it through the complete detector. A small difference (at the few percent level) is observed between the efficiency calculated for events simulated with the single-particle Monte Carlo and with a simulation using a multi-particle hadron-production model. A similar difference is seen between the single-particle Monte Carlo and the efficiencies measured directly from the data. The single-particle Monte Carlo predicts an efficiency more in agreement with the efficiency measured with the data. A momentum-dependent correction factor determined using the efficiency measured with the data is applied to take this into account. The track reconstruction algorithms used in this analysis and the simulation are identical to the ones used for the  $\pi^+$  production in p-Be collisions [6]. A detailed description of the corrections and their magnitude can be found there.

The reconstruction efficiency (inside the geometrical acceptance) is larger than 95% above 1.5 GeV/c and drops to 80% at 0.5 GeV/c. The requirement of a match with a TOFW hit has an efficiency between 90% and 95% above 1.0 GeV/c and is responsible for the drop in efficiency towards lower momenta. The electron veto rejects about 1% of the pions and protons below 3 GeV/c with a remaining electron background of less than 0.5%. Below Cherenkov threshold the TOFW separates pions and protons with negligible background and an efficiency of  $\approx 98\%$  for pions. Above Cherenkov threshold the efficiency for pions is greater than 99% with only 1.5% of the protons mis-identified as a pion. The kaon background in the pion spectra is smaller than 1%.

The absorption and decay of particles is simulated by the Monte Carlo. The generated single particle can re-interact and produce background particles by hadronic or electromagnetic processes, thus giving rise to tracks in the forward spectrometer (“tertiaries”). In such cases also the additional measurements are entered into the migration matrix thereby taking into account the combined effect of the generated particle and any secondaries it creates. The absorption correction is on average 20%, approximately independent of momentum. Uncertainties in the absorption of secondaries in the dipole spectrometer material are taken into account by a variation of 10% of this effect in the simulation. The effect of pion decay is treated in the same way as the absorption and is 20% at 500 MeV/c and negligible at 3 GeV/c.

The uncertainty in the production of background due to tertiary particles is larger. The average correction is  $\approx 10\%$  and up to 20% at 1 GeV/c. The correction includes re-interactions in the detector material as well as a small component coming from re-interactions in the target. The validity of the generators used in the simulation was checked by an analysis of HARP data with incoming protons and charged pions on aluminium and carbon targets at lower momenta (3 GeV/c and 5 GeV/c). A 30% variation of the secondary production was applied. The average empty-target subtraction amounts to  $\approx 20\%$ .

Owing to the redundancy of the tracking system downstream of the target the reconstruction efficiency is very robust under the usual variations of the detector performance during the long data taking periods. Since the momentum is reconstructed without making use of the upstream drift chamber module (which is more sensitive in its performance to the beam intensity) the reconstruction efficiency is uniquely determined by the downstream system. No variation of the overall efficiency has been observed. The performance of the TOFW and CHE system have been monitored to be constant for the data taking periods used in this analysis. The calibration of the detectors was performed on a day-by-day basis.

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<sup>4</sup> The unfolding method tries to put in correspondence the vector of measured observables (such as particle momentum, polar angle and particle type)  $x_{\text{meas}}$  with the vector of true values  $x_{\text{true}}$  using a migration matrix:  $x_{\text{meas}} = M_{\text{migr}} \times x_{\text{true}}$ . The goal of the method is to compute a transformation (correction matrix) to obtain the expected values for  $x_{\text{true}}$  from the measured ones. The most simple and obvious solution, based on simple matrix inversion  $M_{\text{migr}}^{-1}$ , is usually unstable and is dominated by large variances and strong negative correlations between neighbouring bins. In the method of D’Agostini, the correction matrix  $M^{\text{UFO}}$  tries to connect the measurement space (effects) with the space of the true values (causes) using an iterative Bayesian approach, based on Monte Carlo simulations to estimate the probability for a given effect to be produced by a certain cause.

### 2.3 Error estimation

Different types of sources induce systematic errors for the analysis described here: track yield corrections ( $\sim 5\%$ ), particle identification ( $\sim 0.1\%$ ), momentum and angular reconstruction ( $\sim 0.5\%$ )<sup>5</sup>. The dominant error is due to track yield correction, in particular due to the subtraction of tertiary particles, with smaller contributions from particle absorption, empty target subtraction and reconstruction efficiency. The strategy to calculate these systematic errors and the different methods used for their evaluation are described in [7] where the numerical values are tabulated. An additional source of error is due to misidentified secondary kaons, which are not considered in the particle identification method used for this analysis and are subtracted on the basis of a Monte Carlo simulation, as in [7]. No explicit correction is made for pions coming from decays of other particles created in the target, as they give a very small contribution according to the selection criteria applied in the analysis.

Examples of experimental uncertainties are shown in Figure 2 for Be target with incident  $\pi^-$  for  $\pi^\pm$  production at two incident beam momenta (5 and 8 GeV/c). They are very similar for the other beam momenta and targets. Going from lighter (Be, C) to heavier targets (Ta, Pb) the corrections for  $\pi^0$  conversion and absorption/tertiaries increase slightly.

The overall normalization has an uncertainty of  $\sim 2\%$  and is mainly due to the uncertainty in the efficiency that beam pions counted in the normalization actually hit the target, with smaller components from the target density and the beam particle counting procedure. On average the total integrated systematic error is around 5 – 6%, with a differential bin-to-bin systematic error of the order of 10 – 11%, to be compared with a statistical integrated (bin-to-bin differential) error of  $\sim 2-3\%$  ( $\sim 10-13\%$ ). Systematic and statistical errors are roughly of the same order.

## 3 Results

The measured double-differential cross-sections for the production of  $\pi^+$  and  $\pi^-$  in the laboratory system as a function of the momentum and the polar angle for each incident beam momentum are shown in Figures 3 to 9 for targets from Be to Pb. The error bars shown are the square-roots of the diagonal elements in the covariance matrix, where statistical and systematic uncertainties are combined in quadrature. The correlation of the statistical errors (introduced by the unfolding procedure) are typically smaller than 20% for adjacent momentum bins and even smaller for adjacent angular bins. The correlations of the systematic errors are larger, typically 80% for adjacent bins. The overall scale error ( $< 2\%$ ) is not shown. The results of this analysis are also tabulated in Appendix A. We have not included the scale errors in the tables to make it possible to calculate e.g. integrated particle ratios taking the scale errors into account only when applicable, i.e. when different beams are compared.

The dependence of the averaged pion yields on the atomic number  $A$  is shown from Fig. 10 to Fig. 13. The pion yields, averaged over two angular regions ( $0.05 \text{ rad} \leq \theta < 0.15 \text{ rad}$  and  $0.15 \text{ rad} \leq \theta < 0.25 \text{ rad}$ ) and four momentum regions ( $0.5 \text{ GeV}/c \leq p < 1.5 \text{ GeV}/c$ ,  $1.5 \text{ GeV}/c \leq p < 2.5 \text{ GeV}/c$ ,  $2.5 \text{ GeV}/c \leq p < 3.5 \text{ GeV}/c$  and  $3.5 \text{ GeV}/c \leq p < 4.5 \text{ GeV}/c$ ), are shown for four different beam momenta. One observes a smooth behaviour of the averaged yields. The most striking feature is the difference between same-sign and opposite-sign pion production compared to the beam particle. Here one observes a stronger beam energy dependence for opposite-sign production. The  $A$ -dependence is similar for  $\pi^-$  and  $\pi^+$  production if one compares opposite-sign and same-sign production, although one observes that especially at lower beam momenta the opposite-sign pion production saturates earlier towards higher  $A$ .

In the following we will show only some comparisons of the HARP data with publicly available Monte Carlo simulations: GEANT4 [23] (version 9.1p02) and MARS [25], not previously tuned to our data-sets, by using different models. The comparison will be shown for a limited set of plots and only for the Be (Figures 14 to 18) and Ta (Figures 19 to 23) targets, as examples of a light and a heavy target.

At intermediate energies (up to 5–10 GeV), GEANT4 uses two types of intra-nuclear cascade models: the

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<sup>5</sup> The quoted error in parenthesis refers to fractional error of the integrated cross-section in the kinematic range covered by the HARP experiment

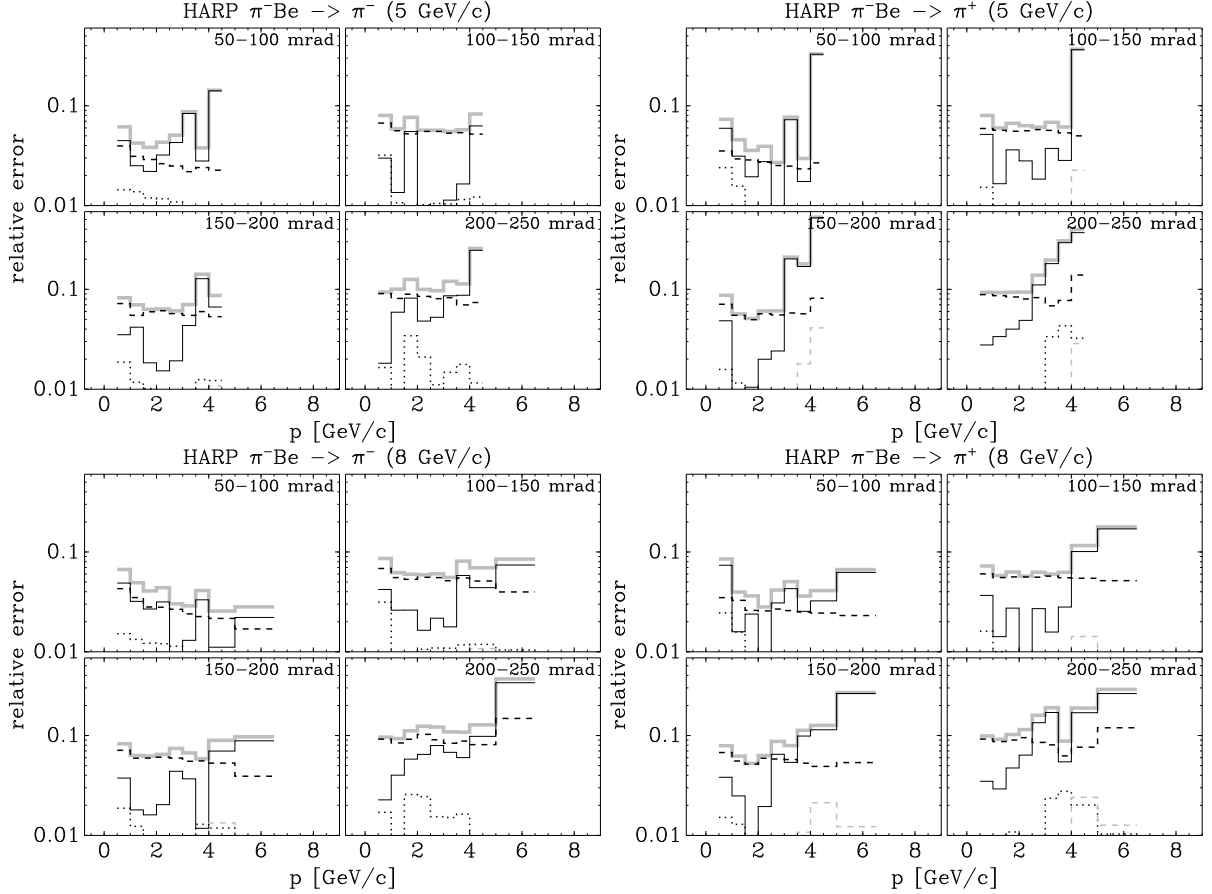


Figure 2: Systematic errors as a function of momentum of the outgoing pions for the particular case of 5 and 8 GeV/c  $\pi^-$  interacting on a beryllium target. Plots in the upper left panel are for incident 5 GeV/c  $\pi^-$  producing  $\pi^-$ , plots in the upper right panel are for 5 GeV/c  $\pi^-$  producing  $\pi^+$ , plots in the lower left panels are for 8 GeV/c  $\pi^-$  producing  $\pi^-$  and plots in the lower right panel are for 8 GeV/c  $\pi^-$  producing  $\pi^+$ . Total systematic error (grey solid line) are displayed together with the main components: black short-dashed line for absorption+tertiaries interactions, black dotted line for track efficiency and target pointing efficiency, black dot-dashed line for  $\pi^0$  subtraction, black solid line for momentum scale+resolution and angle scale, grey short-dashed line for PID.

Bertini model [27, 28] (valid up to  $\sim 10$  GeV) and the binary model [29] (with a validity range up to  $\sim 3$  GeV). Both models treat the target nucleus in detail, taking into account density variations and tracking in the nuclear field. The binary model is based on hadron collisions with nucleons, giving resonances that decay according to their quantum numbers. The Bertini model is based on the cascade code reported in [30] and hadron collisions are assumed to proceed according to free-space partial cross-sections and final-state distributions measured for the incident particle types.

At higher energies, instead, two parton string models, the quark-gluon string (QGS) model [27, 31] and the Fritiof (FTP) model [31] are used, in addition to a High Energy Parametrized model (HEP) derived from the high energy part of the GHEISHA code used inside GEANT3 [32].

The parametrized models of GEANT4 (HEP and LEP) are intended to be fast, but conserve energy and momentum on average and not event by event.

A realistic GEANT4 simulation is built by combining models and physics processes into what is called a “physics list”. In high energy calorimetry the two most commonly used are the QGSP physics list, based on the QGS model, the pre-compound nucleus model and some of the Low Energy Parametrized (LEP)

model <sup>6</sup> and the LHEP physics list [26] based on the parametrized LEP model and HEP models.

Currently is also popular the QGSP-BERT physics list in which the Bertini model is used at 3 and 5 GeV/c and the QGSP model at 8 and 12 GeV/c.

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<sup>6</sup> Also this model, at low energy, has its root in the Gheisha code inside GEANT3.

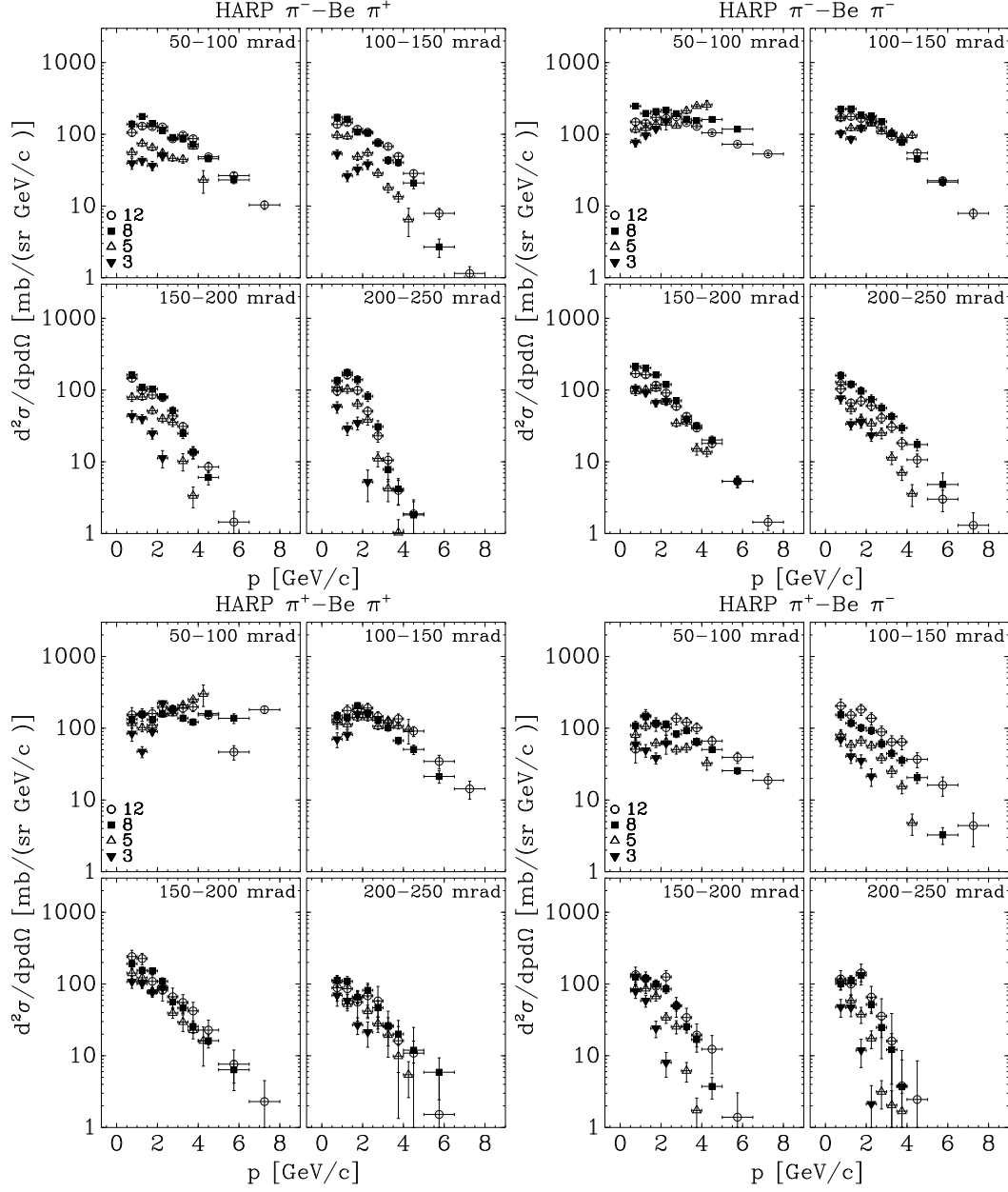


Figure 3:  $\pi^-$ -Be (top) and  $\pi^+$ -Be (bottom) differential cross-sections for different incoming beam momenta (from 3 to 12 GeV/c). Left panels show the  $\pi^+$  production, while right panels show the  $\pi^-$  production. In the top right corner of each plot the covered angular range is shown in mrad.

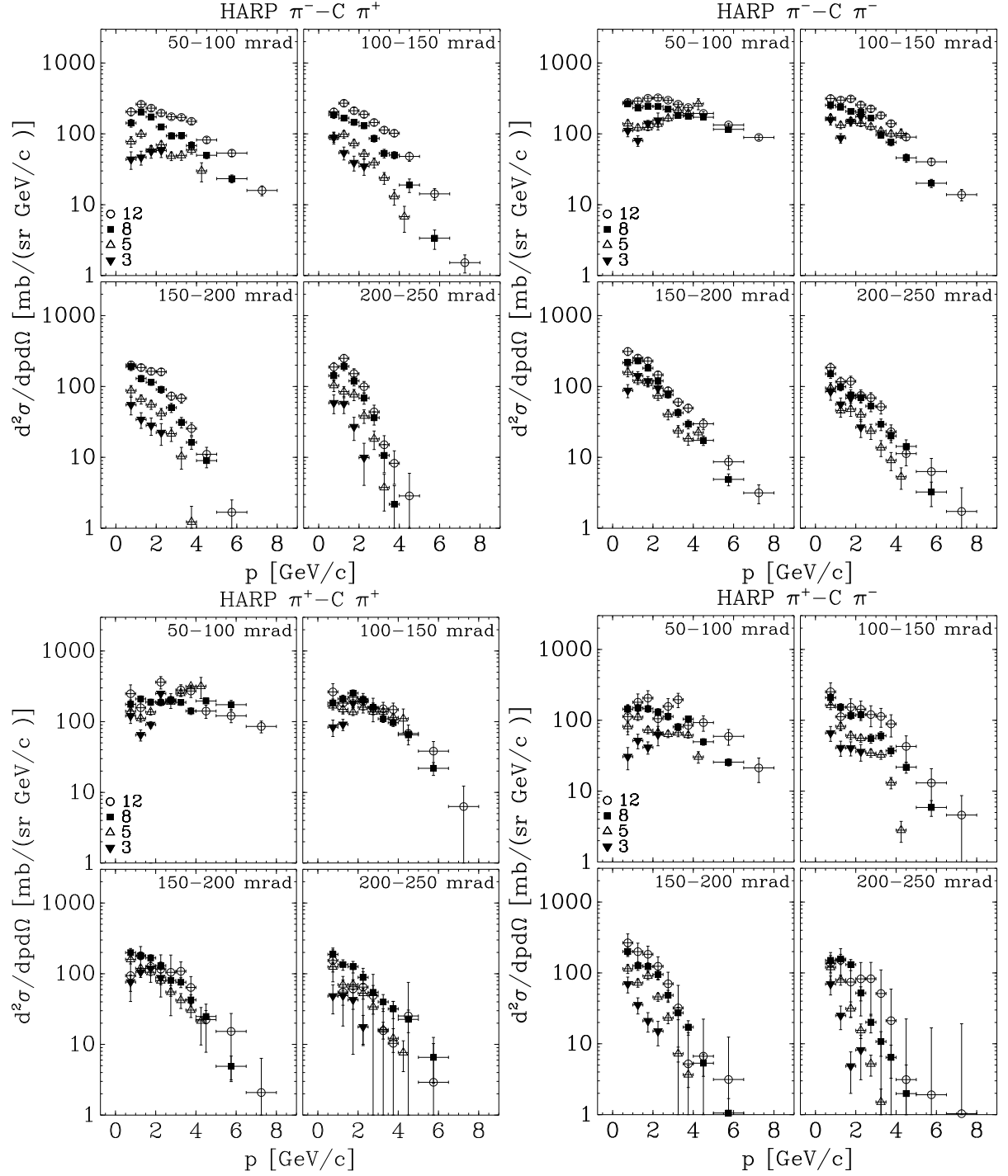


Figure 4:  $\pi^-$ -C (top) and  $\pi^+$ -C (bottom) differential cross-sections for different incoming beam momenta (from 3 to 12  $\text{GeV}/c$ ). Left panels show the  $\pi^+$  production, while right panels show the  $\pi^-$  production. In the top right corner of each plot the covered angular range is shown in mrad.

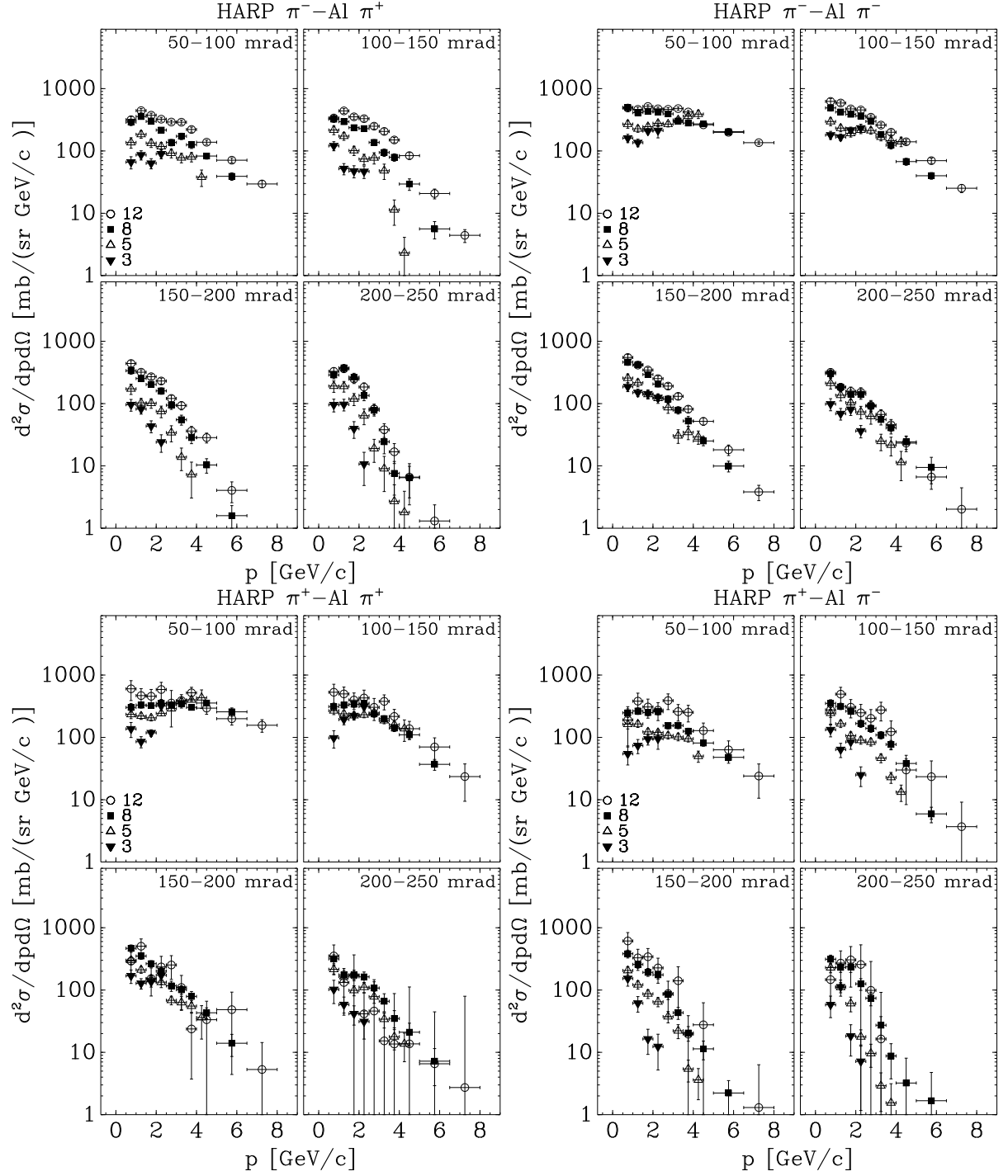


Figure 5:  $\pi^-$ -Al (top) and  $\pi^+$ -Al (bottom) differential cross-sections for different incoming beam momenta (from 3 to 12 GeV/c). Left panels show the  $\pi^+$  production, while right panels show the  $\pi^-$  production. In the top right corner of each plot the covered angular range is shown in mrad.

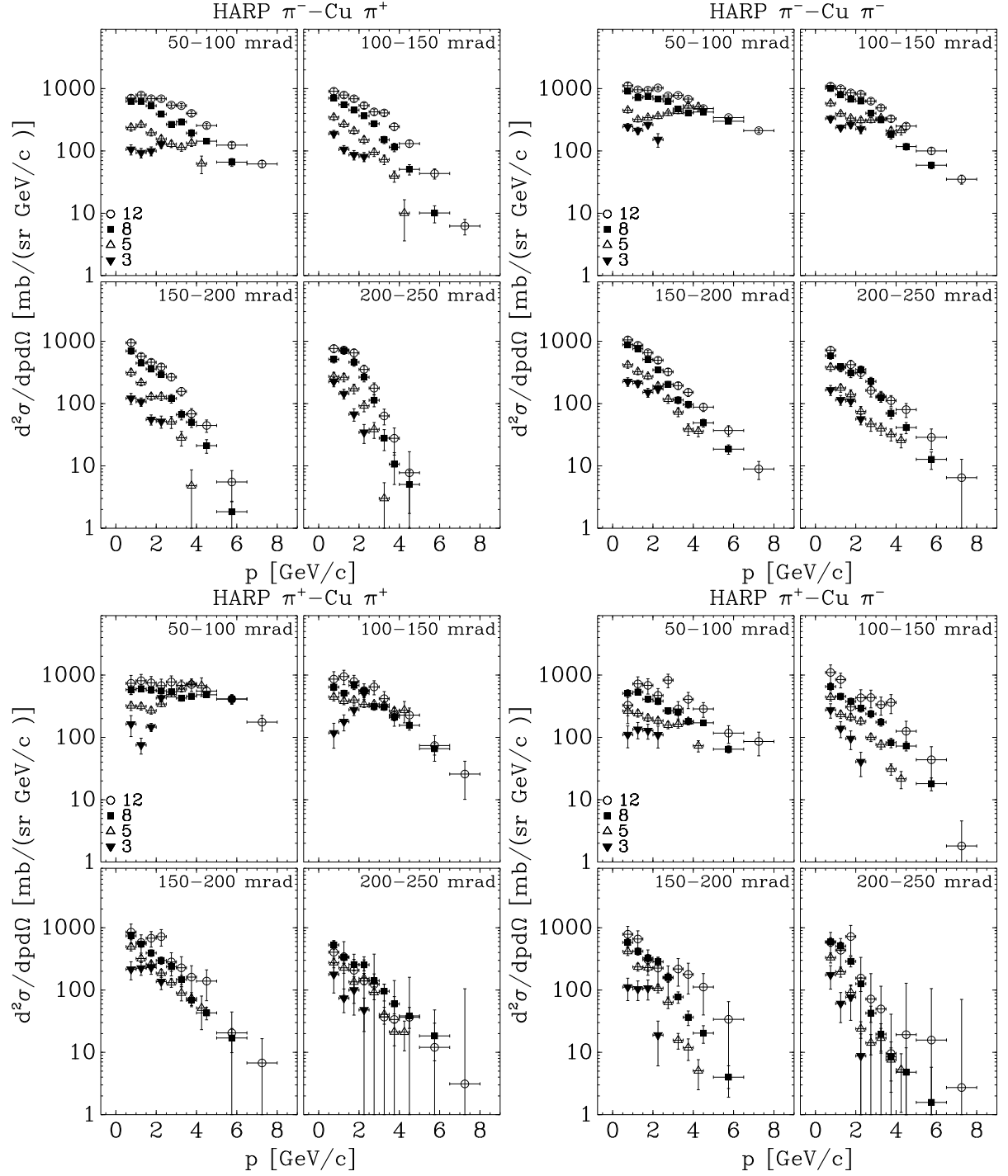


Figure 6:  $\pi^-$ -Cu (top) and  $\pi^+$ -Cu (bottom) differential cross-sections for different incoming beam momenta (from 3 to 12 GeV/c). Left panels show the  $\pi^+$  production, while right panels show the  $\pi^-$  production. In the top right corner of each plot the covered angular range is shown in mrad.



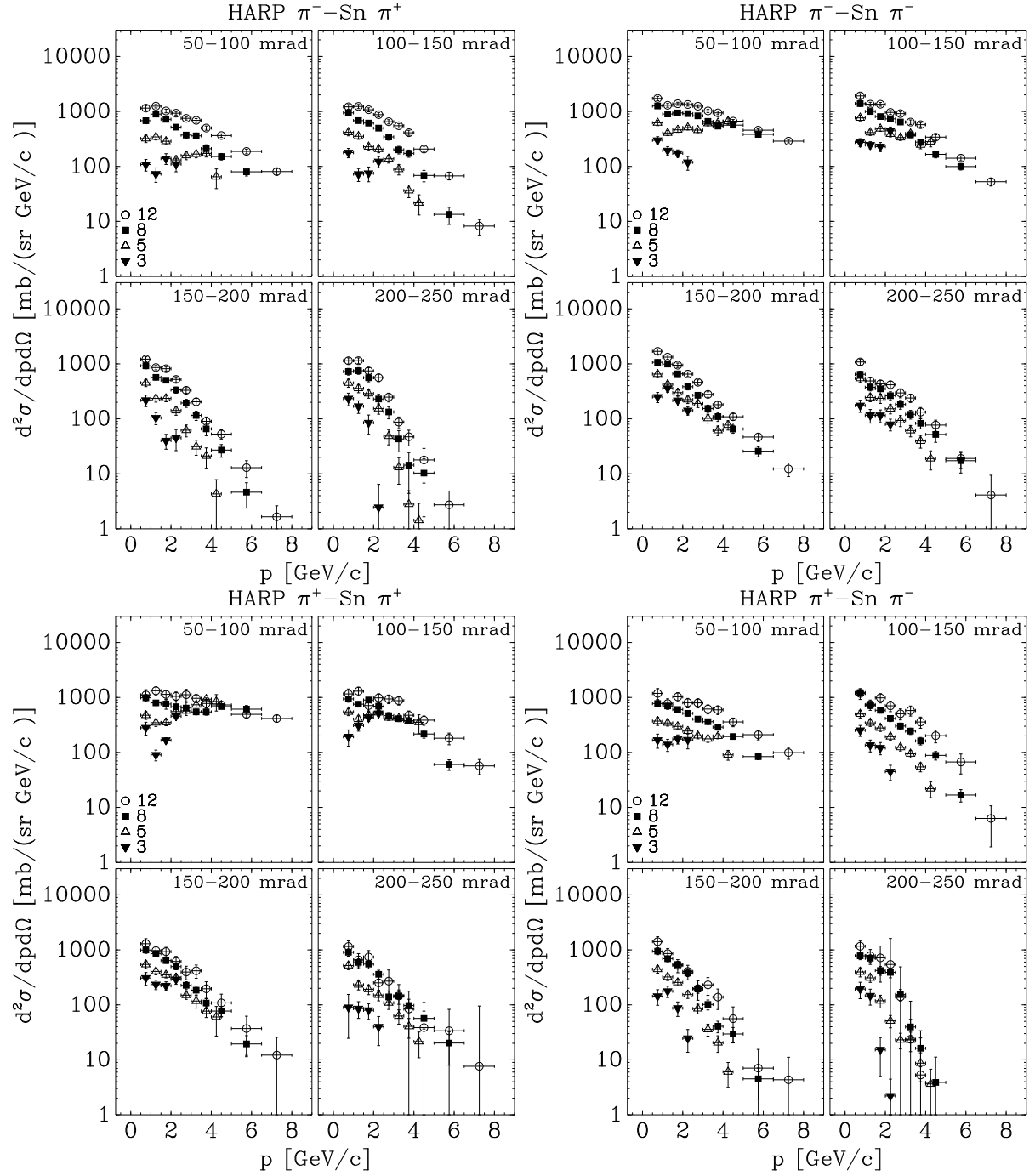


Figure 7:  $\pi^-$ -Sn (top) and  $\pi^+$ -Sn (bottom) differential cross-sections for different incoming beam momenta (from 3 to 12  $\text{GeV}/c$ ). Left panels show the  $\pi^+$  production, while right panels show the  $\pi^-$  production. In the top right corner of each plot the covered angular range is shown in mrad.

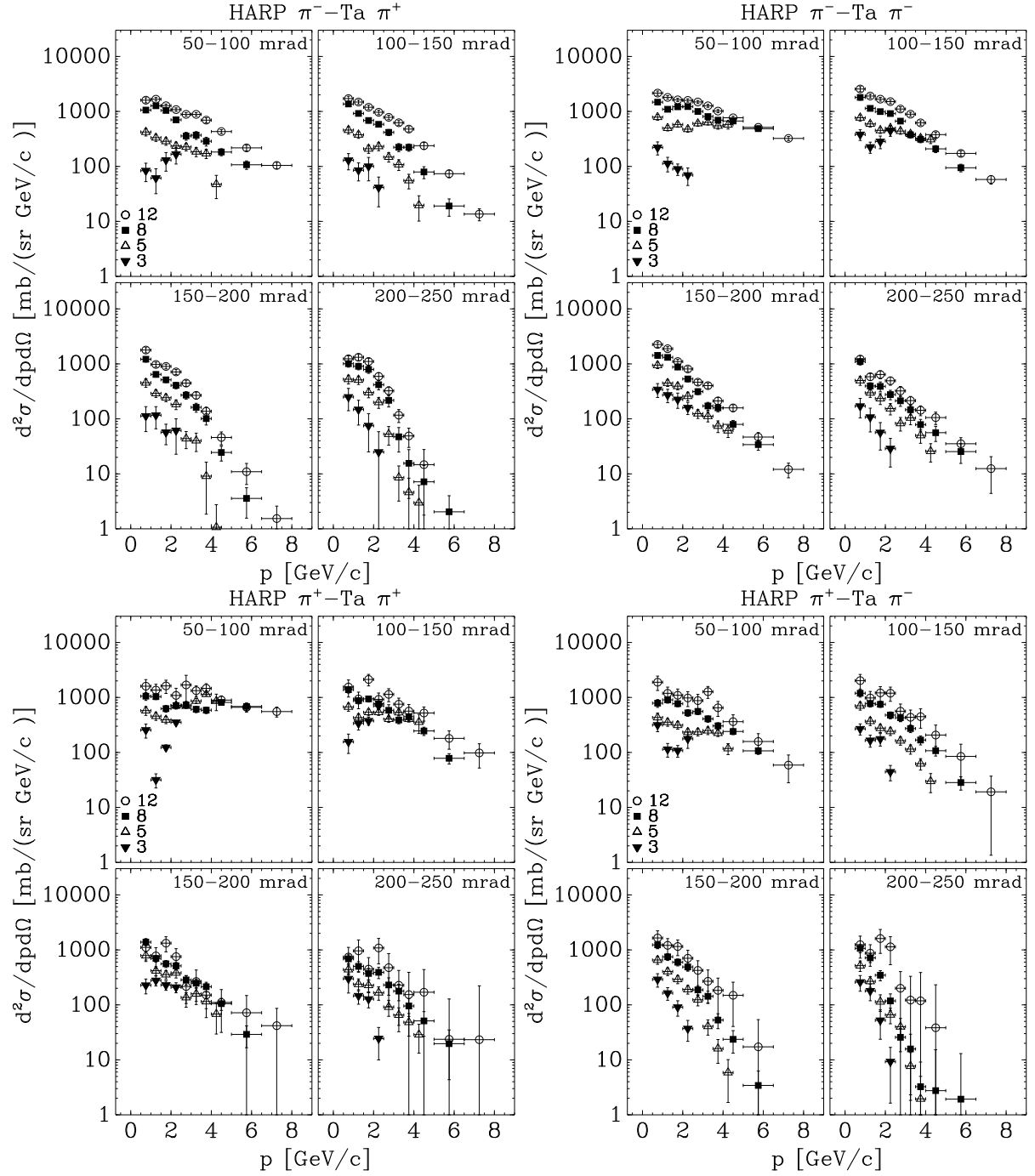


Figure 8:  $\pi^-$ -Ta (top) and  $\pi^+$ -Ta (bottom) differential cross-sections for different incoming beam momenta (from 3 to 12  $\text{GeV}/c$ ). Left panels show the  $\pi^+$  production, while right panels show the  $\pi^-$  production. In the top right corner of each plot the covered angular range is shown in mrad.

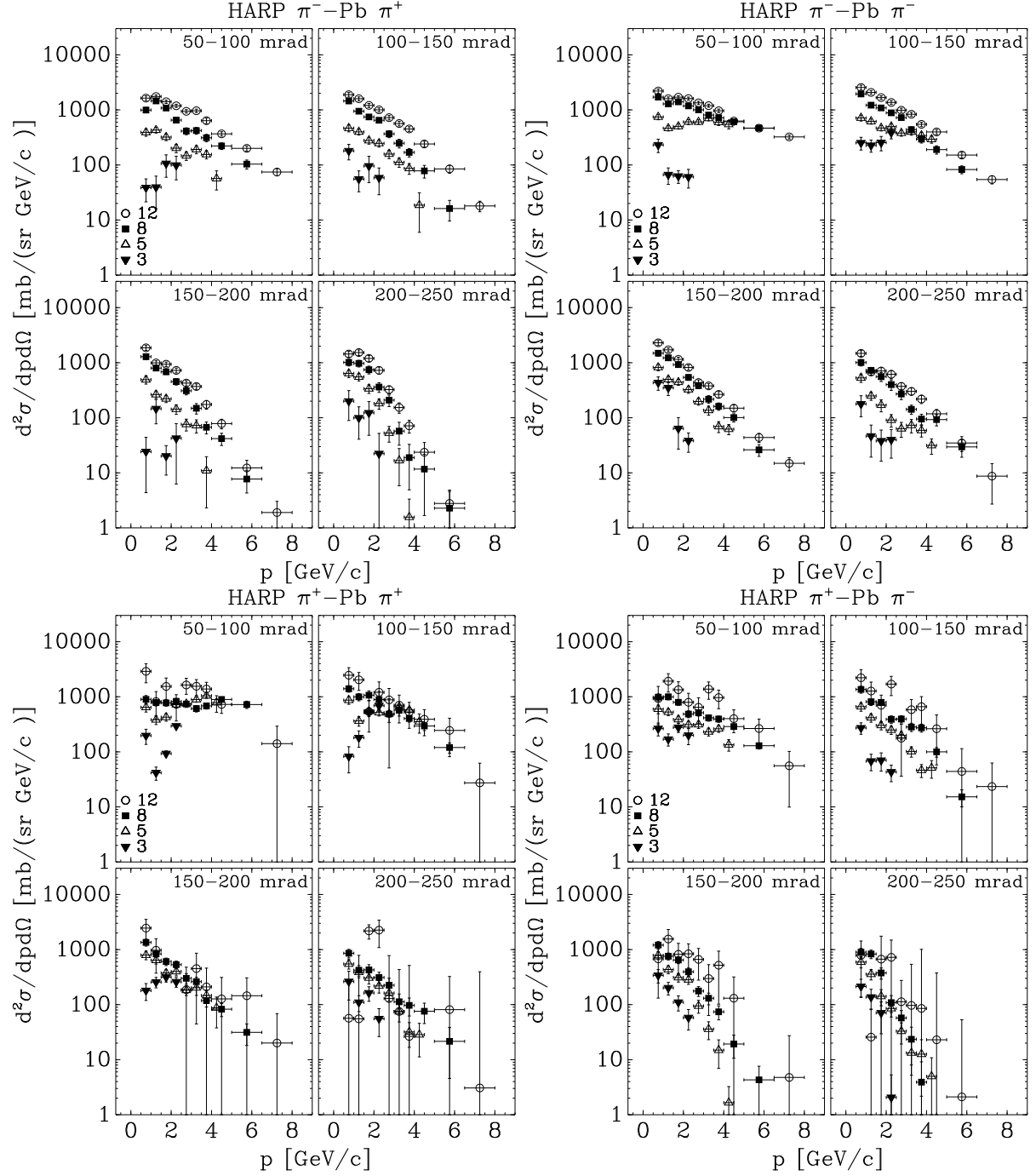


Figure 9:  $\pi^-$ -Pb (top) and  $\pi^+$ -Pb (bottom) differential cross-sections for different incoming beam momenta (from 3 to 12 GeV/c). Left panels show the  $\pi^+$  production, while right panels show the  $\pi^-$  production. In the top right corner of each plot the covered angular range is shown in mrad.

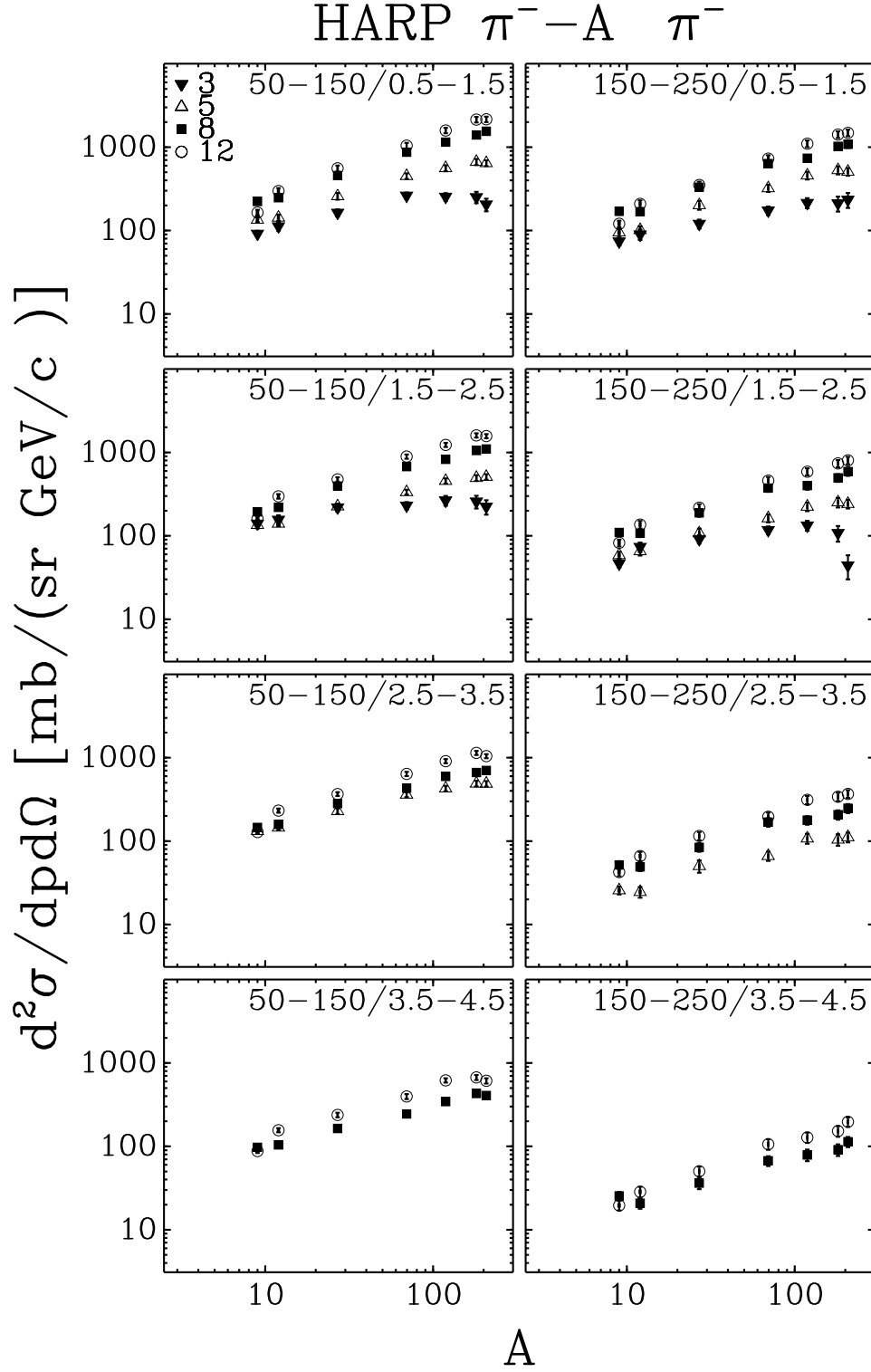


Figure 10: The dependence on the atomic number  $A$  of the  $\pi^-$  production yields in  $\pi^-$ -Be,  $\pi^-$ -C,  $\pi^-$ -Al,  $\pi^-$ -Cu,  $\pi^-$ -Sn,  $\pi^-$ -Ta,  $\pi^-$ -Pb interactions averaged over two forward angular region ( $0.05 \text{ rad} \leq \theta < 0.15 \text{ rad}$  and  $0.15 \text{ rad} \leq \theta < 0.25 \text{ rad}$ ) and four momentum regions ( $0.5 \text{ GeV}/c \leq p < 1.5 \text{ GeV}/c$ ,  $1.5 \text{ GeV}/c \leq p < 2.5 \text{ GeV}/c$ ,  $2.5 \text{ GeV}/c \leq p < 3.5 \text{ GeV}/c$  and  $3.5 \text{ GeV}/c \leq p < 4.5 \text{ GeV}/c$ ), for the four different incoming beam momenta (from  $3 \text{ GeV}/c$  to  $12 \text{ GeV}/c$ ).

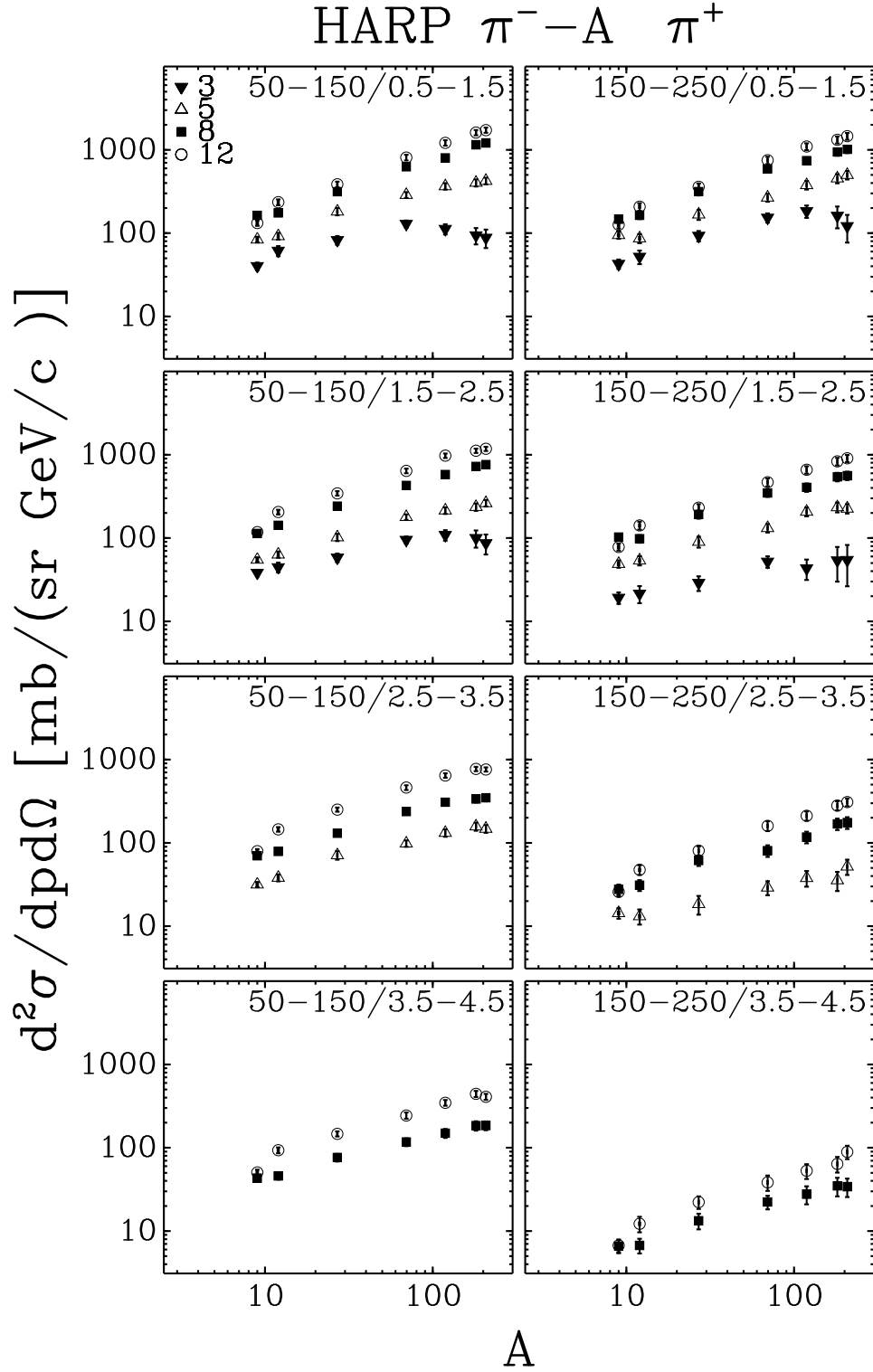


Figure 11: The dependence on the atomic number  $A$  of the  $\pi^+$  production yields in  $\pi^-$ -Be,  $\pi^-$ -C,  $\pi^-$ -Al,  $\pi^-$ -Cu,  $\pi^-$ -Sn,  $\pi^-$ -Ta,  $\pi^-$ -Pb interactions averaged over two forward angular region ( $0.05 \text{ rad} \leq \theta < 0.15 \text{ rad}$  and  $0.15 \text{ rad} \leq \theta < 0.25 \text{ rad}$ ) and four momentum regions ( $0.5 \text{ GeV}/c \leq p < 1.5 \text{ GeV}/c$ ,  $1.5 \text{ GeV}/c \leq p < 2.5 \text{ GeV}/c$ ,  $2.5 \text{ GeV}/c \leq p < 3.5 \text{ GeV}/c$  and  $3.5 \text{ GeV}/c \leq p < 4.5 \text{ GeV}/c$ ), for the four different incoming beam momenta (from  $3 \text{ GeV}/c$  to  $12 \text{ GeV}/c$ ).

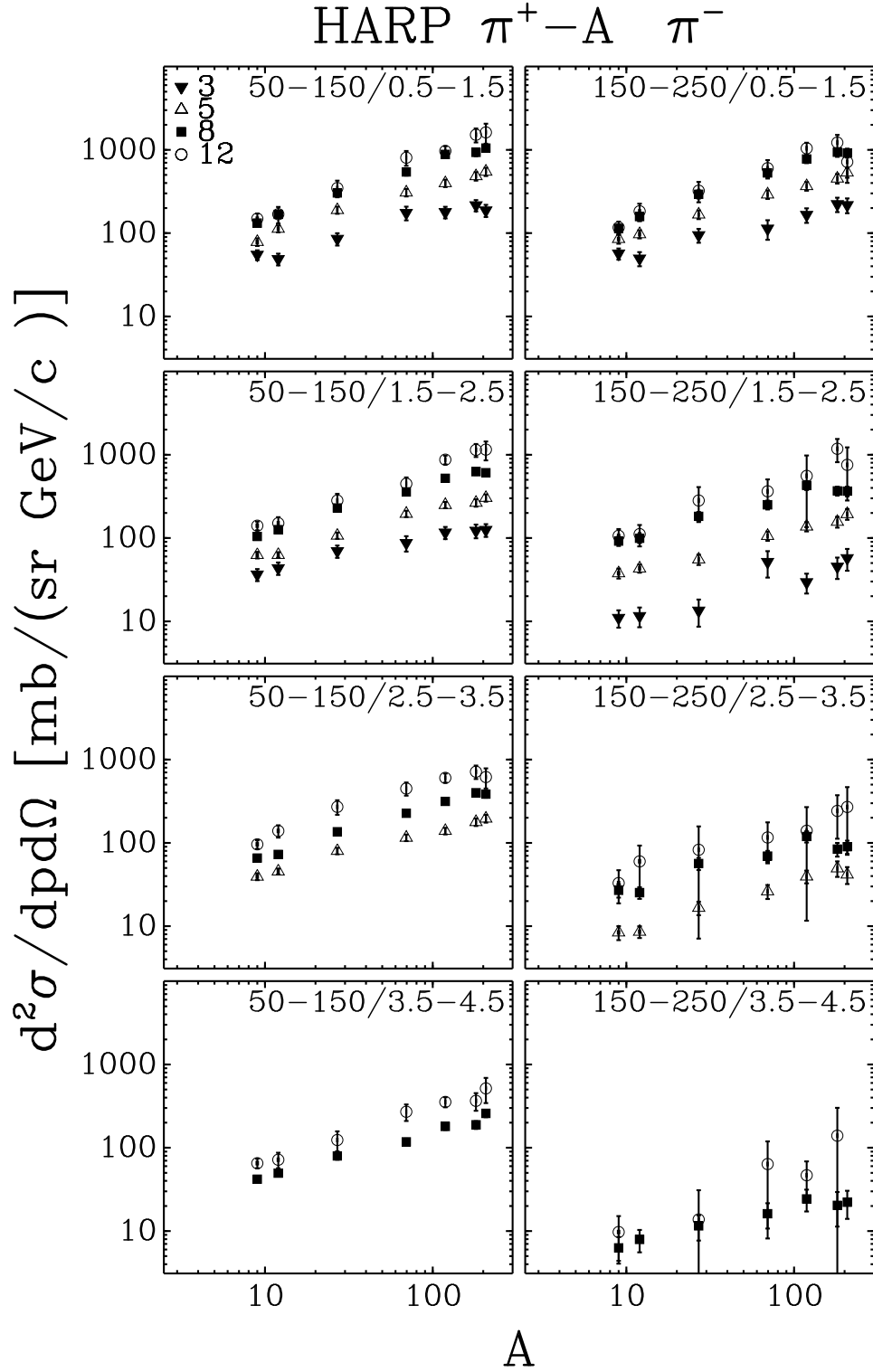


Figure 12: The dependence on the atomic number  $A$  of the  $\pi^-$  production yields in  $\pi^+$ -Be,  $\pi^+$ -C,  $\pi^+$ -Al,  $\pi^+$ -Cu,  $\pi^+$ -Sn,  $\pi^+$ -Ta,  $\pi^+$ -Pb interactions averaged over two forward angular region ( $0.05 \text{ rad} \leq \theta < 0.15 \text{ rad}$  and  $0.15 \text{ rad} \leq \theta < 0.25 \text{ rad}$ ) and four momentum regions ( $0.5 \text{ GeV}/c \leq p < 1.5 \text{ GeV}/c$ ,  $1.5 \text{ GeV}/c \leq p < 2.5 \text{ GeV}/c$ ,  $2.5 \text{ GeV}/c \leq p < 3.5 \text{ GeV}/c$  and  $3.5 \text{ GeV}/c \leq p < 4.5 \text{ GeV}/c$ ), for the four different incoming beam momenta (from  $3 \text{ GeV}/c$  to  $12 \text{ GeV}/c$ ).

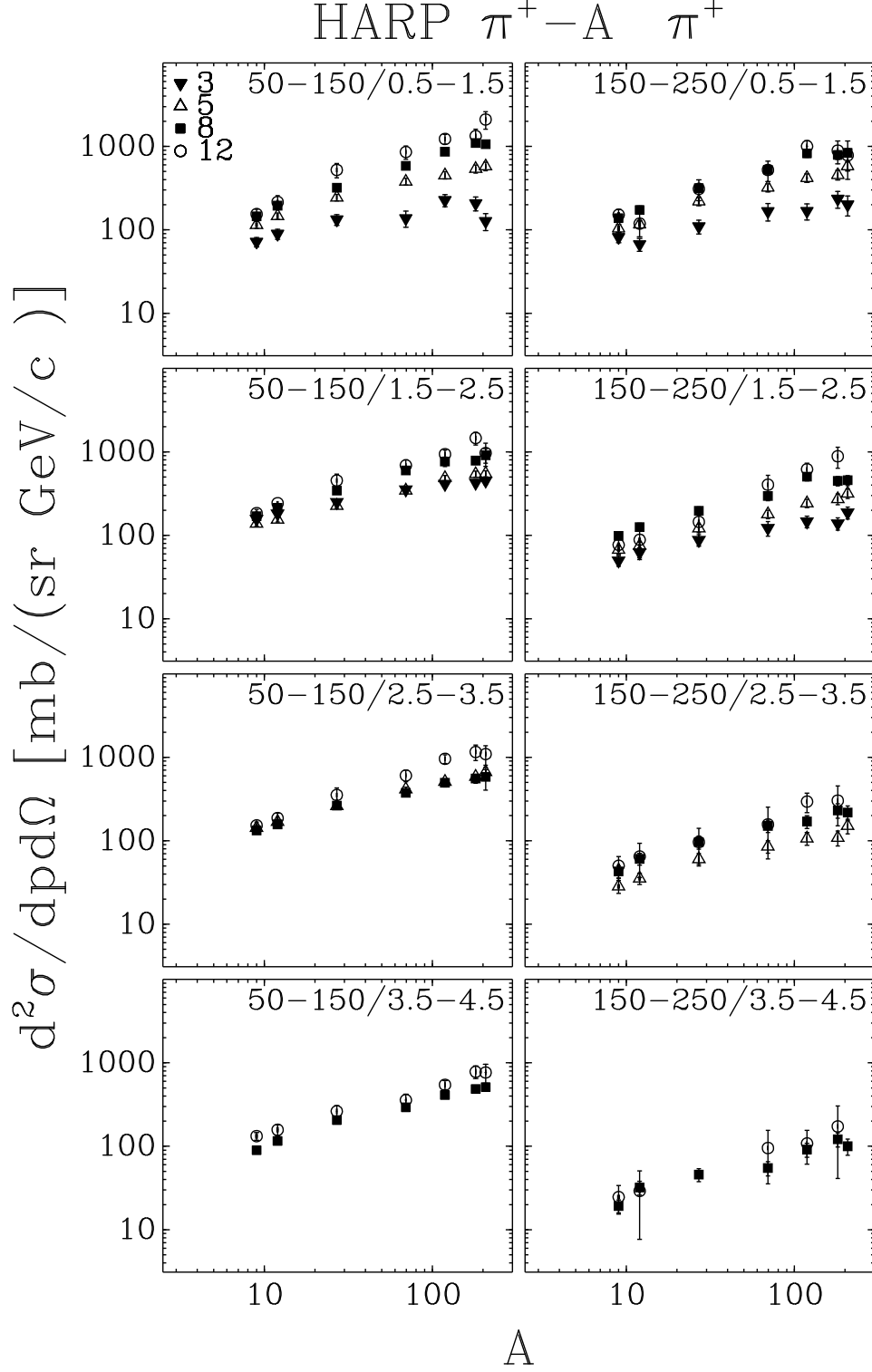


Figure 13: The dependence on the atomic number  $A$  of the  $\pi^+$  production yields in  $\pi^+$ -Be,  $\pi^+$ -C,  $\pi^+$ -Al,  $\pi^+$ -Cu,  $\pi^+$ -Sn,  $\pi^+$ -Ta,  $\pi^+$ -Pb interactions averaged over two forward angular region ( $0.05 \text{ rad} \leq \theta < 0.15 \text{ rad}$  and  $0.15 \text{ rad} \leq \theta < 0.25 \text{ rad}$ ) and four momentum regions ( $0.5 \text{ GeV}/c \leq p < 1.5 \text{ GeV}/c$ ,  $1.5 \text{ GeV}/c \leq p < 2.5 \text{ GeV}/c$ ,  $2.5 \text{ GeV}/c \leq p < 3.5 \text{ GeV}/c$  and  $3.5 \text{ GeV}/c \leq p < 4.5 \text{ GeV}/c$ ), for the four different incoming beam momenta (from  $3 \text{ GeV}/c$  to  $12 \text{ GeV}/c$ ).

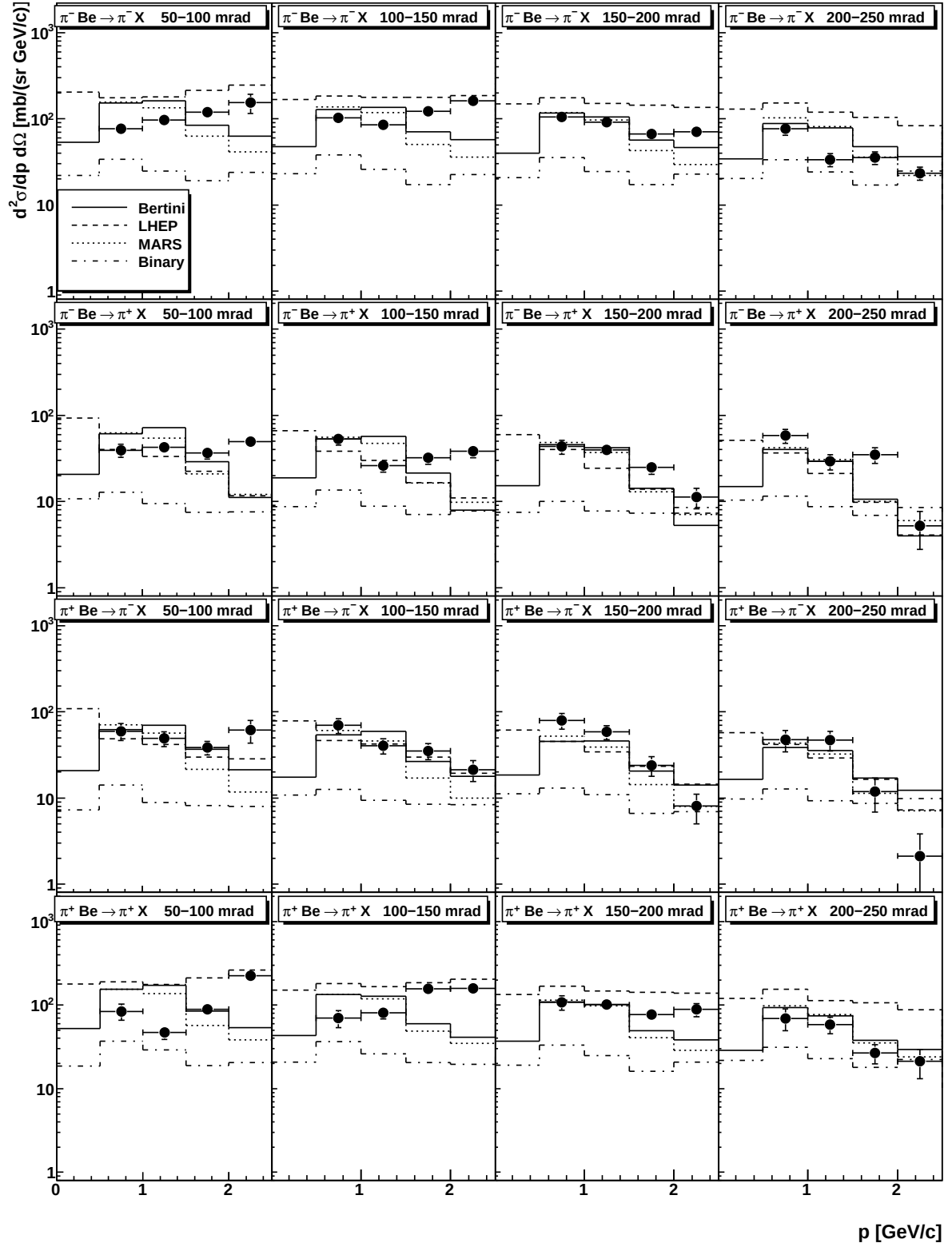


Figure 14: Comparison of HARP double-differential  $\pi^\pm$  cross-sections for  $\pi$ -Be at 3 GeV/c with GEANT4 and MARS MC predictions, using several generator models.



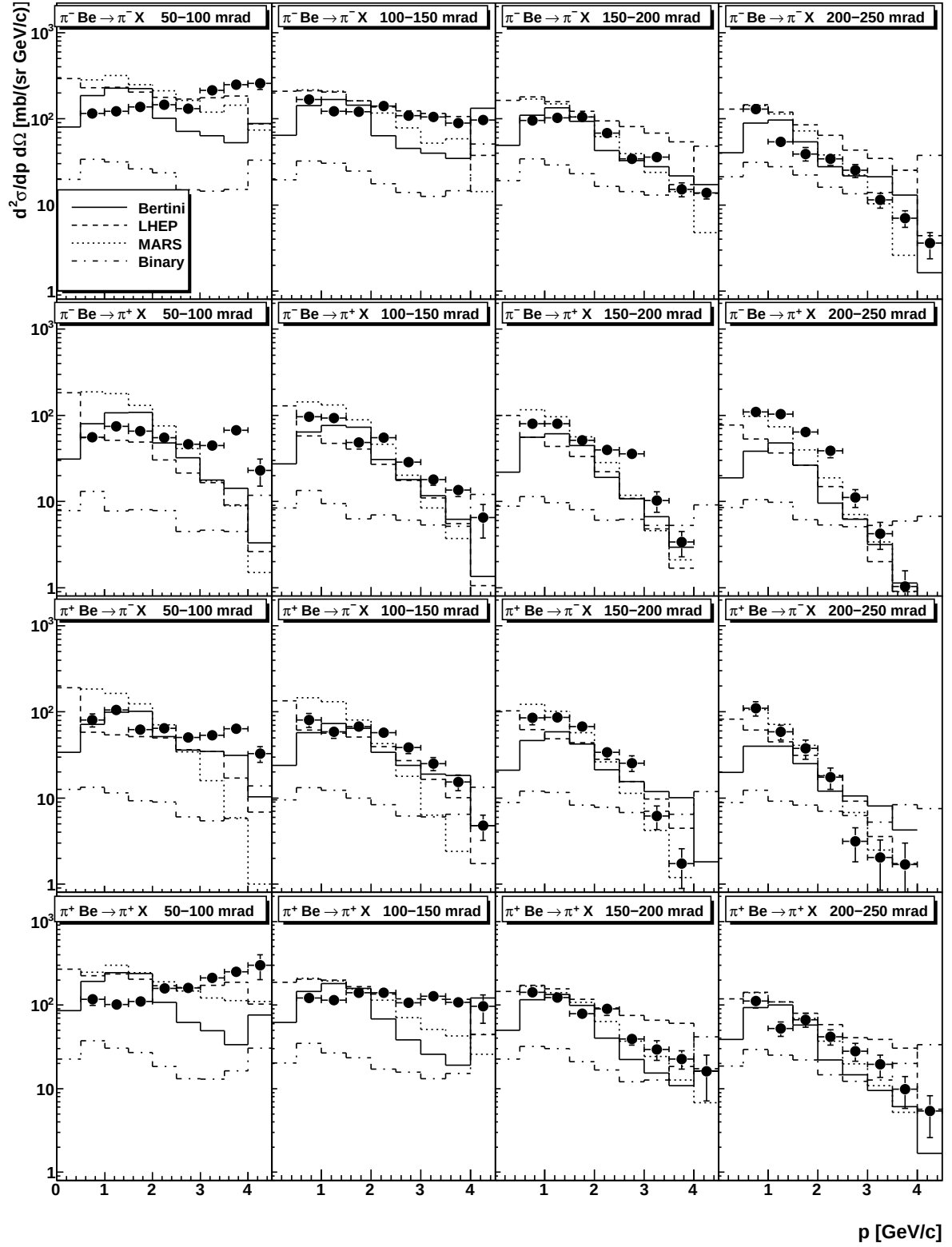


Figure 15: Comparison of HARP double-differential  $\pi^\pm$  cross-sections for  $\pi$ -Be at 5 GeV/c with GEANT4 and MARS MC predictions, using several generator models.

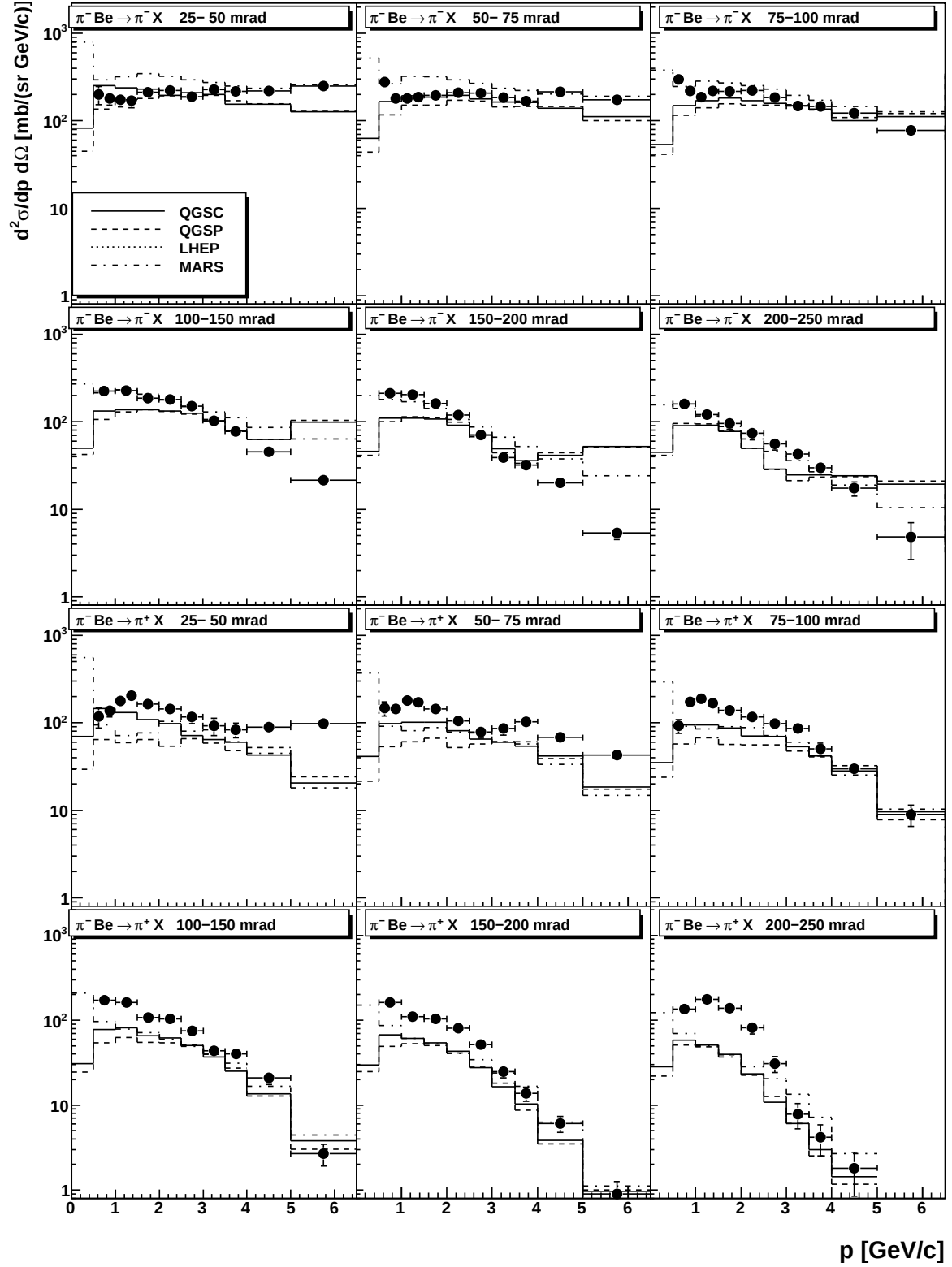


Figure 16: Comparison of HARP double-differential  $\pi^\pm$  cross-sections for  $\pi^-$ -Be at 8 GeV/c with GEANT4 and MARS MC predictions, using several generator models.

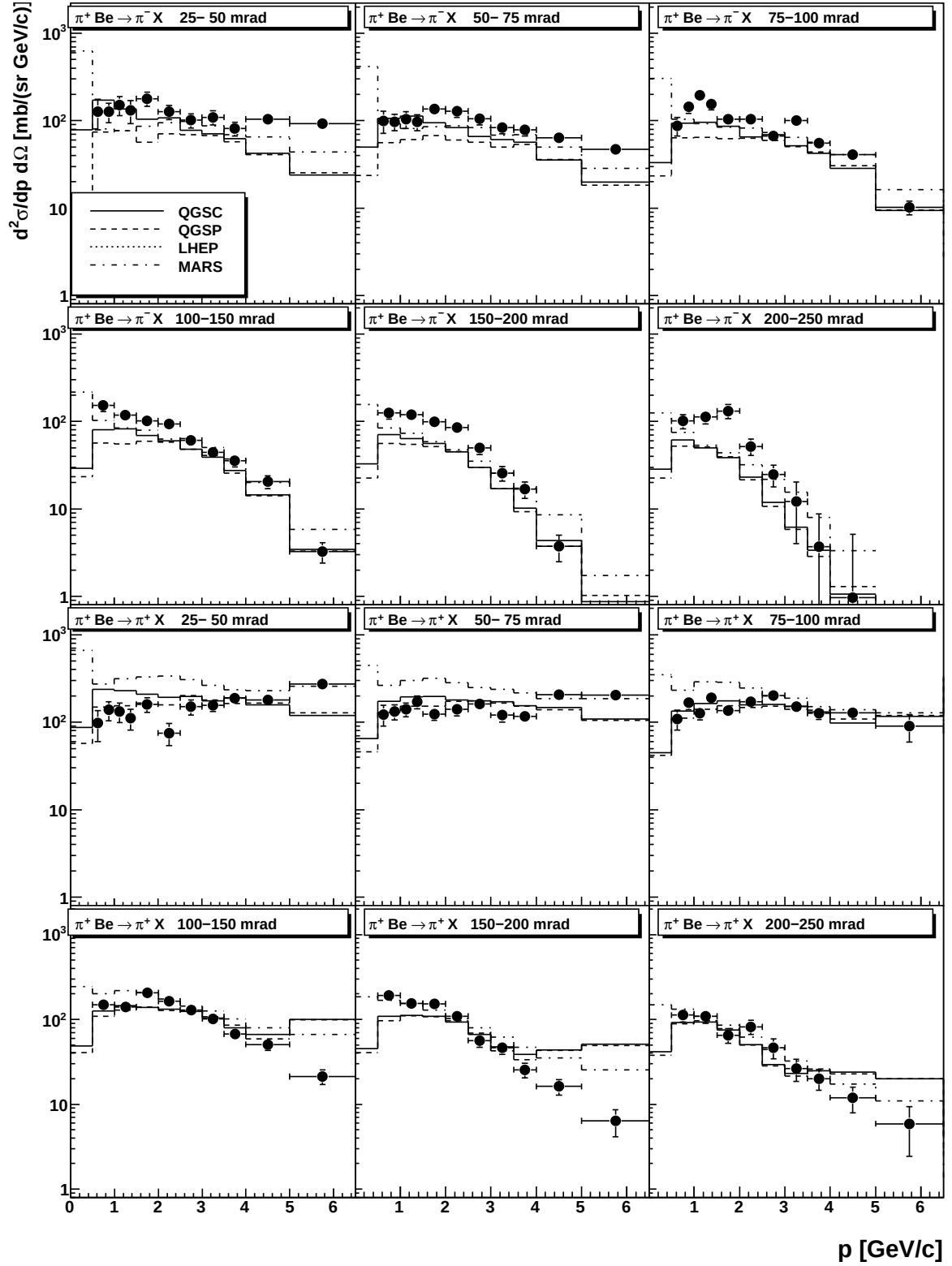


Figure 17: Comparison of HARP double-differential  $\pi^\pm$  cross-sections for  $\pi^\pm$ -Be at 8 GeV/c with GEANT4 and MARS MC predictions, using several generator models.

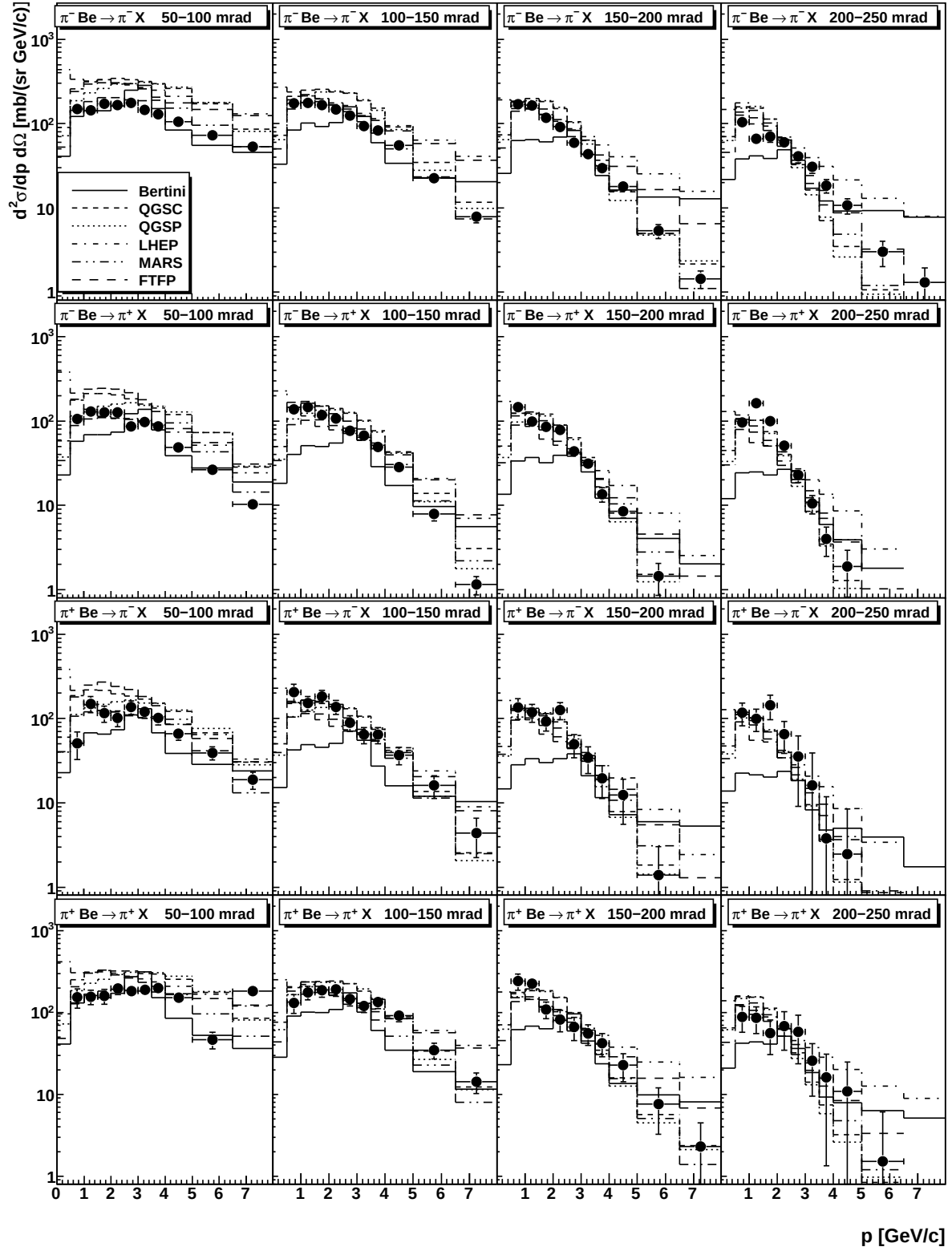


Figure 18: Comparison of HARP double-differential  $\pi^\pm$  cross-sections for  $\pi$ -Be at 12 GeV/c with GEANT4 and MARS MC predictions, using several generator models.

The MARS code system [25] uses as basic model an inclusive approach to multiparticle production originated by R. Feynman in 1974. At each interaction vertex, a particle cascade tree is constructed by using a fixed number of particles with weights that in the simplest case are equal to the partial mean multiplicity of that event. Above 3 GeV phenomenological particle production models are used, while below 5 GeV a cascade-exciton model [33] combined with the Fermi break-up model, the coalescence model, an evaporation model and a multifragmentation extension are used instead.

HARP measurements have been compared to the available models, first renormalizing MC predictions to data and then computing the  $\chi^2$  between models and data themselves. Normalization factors are shown in Tables 2 and 3. An absolute comparison yields  $\chi^2$  values that are too large to be meaningful. Over the full energy range covered by the HARP experiment, the best comparison is obtained with the MARS Monte Carlo which is on average 10% below the data, with a maximum shift of 30%. This may be owing to the fact that MARS is using different models in different energy regions, equivalent to use a collection of models as implemented in the “physics lists” of GEANT4. Unfortunately, all models have very bad  $\chi^2$  compared with the data, showing significant inconsistencies in the shape distribution. This is worse than the situation with incident protons and may be explained by the fact that in the latter case some experimental data were available for tuning of the Monte Carlo simulations. This was not the case with incident pions in the momentum range studied here (below 15 GeV/c).

At higher energies the FTF model (from GEANT4) and the MARS model describe the data better, while at the lowest energies the Bertini and binary cascade models (from GEANT4) seem more appropriate. Parametrized models, such as LHEP from GEANT4, show important discrepancies.

Table 2: Normalization factors data over MC for some models, incident  $\pi^+$ .

| Generator | Be      |         | Ta      |         | Be      |         | Ta      |         | Be      |         | Ta      |         | Be       |         | Ta       |         |
|-----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|----------|---------|----------|---------|
|           | 3 GeV/c |         | 3 GeV/c |         | 5 GeV/c |         | 5 GeV/c |         | 8 GeV/c |         | 8 GeV/c |         | 12 GeV/c |         | 12 GeV/c |         |
|           | $\pi^+$ | $\pi^-$ | $\pi^+$ | $\pi^-$ | $\pi^+$ | $\pi^-$ | $\pi^+$ | $\pi^-$ | $\pi^+$ | $\pi^-$ | $\pi^+$ | $\pi^-$ | $\pi^+$  | $\pi^-$ | $\pi^+$  | $\pi^-$ |
| Bertini   | 1.1     | 1.1     | 0.9     | 0.6     | 1.3     | 1.3     | 1.1     | 1.1     | 1.7     | 1.4     | 1.5     | 1.4     | 2.0      | 1.4     | 1.9      | 1.6     |
| LHEP      | 1.3     | 0.6     | 2.8     | 1.0     | 1.5     | 0.8     | 2.5     | 1.1     | 1.3     | 0.8     | 1.0     | 0.5     | 1.0      | 0.8     | 0.7      | 0.5     |
| QGSC      | -       | -       | -       | -       | -       | -       | -       | -       | 1.6     | 1.0     | 1.2     | 0.7     | 0.8      | 0.7     | 1.0      | 0.8     |
| QGSP      | -       | -       | -       | -       | -       | -       | -       | -       | 1.9     | 1.0     | 1.2     | 0.6     | 0.9      | 0.8     | 0.8      | 0.6     |
| FTFP      | -       | -       | -       | -       | -       | -       | -       | -       | -       | -       | 0.8     | 0.7     | 1.1      | 1.0     | 0.9      | 0.8     |
| MARS      | 1.3     | 1.2     | 1.2     | 0.9     | 0.9     | 1.0     | 0.7     | 0.6     | 1.0     | 1.1     | 0.8     | 0.6     | 0.8      | 1.0     | 0.8      | 0.7     |

Table 3: Normalization factors data over MC for some models, incident  $\pi^-$ .

| Generator | Be      |         | Ta      |         | Be      |         | Ta      |         | Be      |         | Ta      |         | Be       |         | Ta       |         |
|-----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|----------|---------|----------|---------|
|           | 3 GeV/c |         | 3 GeV/c |         | 5 GeV/c |         | 5 GeV/c |         | 8 GeV/c |         | 8 GeV/c |         | 12 GeV/c |         | 12 GeV/c |         |
|           | $\pi^+$ | $\pi^-$ | $\pi^+$ | $\pi^-$ | $\pi^+$ | $\pi^-$ | $\pi^+$ | $\pi^-$ | $\pi^+$ | $\pi^-$ | $\pi^+$ | $\pi^-$ | $\pi^+$  | $\pi^-$ | $\pi^+$  | $\pi^-$ |
| Bertini   | 1.0     | 1.1     | 0.5     | 0.6     | 1.2     | 1.4     | 0.9     | 1.2     | 1.6     | 2.1     | 1.4     | 1.9     | 1.2      | 1.6     | 1.6      | 2.1     |
| LHEP      | 0.5     | 1.6     | 0.7     | 2.9     | 0.8     | 1.9     | 1.0     | 2.6     | 0.9     | 1.7     | 0.6     | 1.2     | 0.6      | 1.0     | 0.5      | 0.7     |
| QGSC      | -       | -       | -       | -       | -       | -       | -       | -       | 1.1     | 1.8     | 0.7     | 1.3     | 0.6      | 0.7     | 0.8      | 1.1     |
| QGSP      | -       | -       | -       | -       | -       | -       | -       | -       | 1.2     | 2.2     | 0.7     | 1.4     | 0.6      | 0.8     | 0.6      | 0.9     |
| FTFP      | -       | -       | -       | -       | -       | -       | -       | -       | -       | -       | 0.7     | 1.1     | 0.7      | 1.0     | 0.8      | 1.1     |
| MARS      | 1.1     | 1.2     | 0.7     | 1.0     | 0.9     | 0.9     | 0.6     | 0.6     | 1.2     | 1.2     | 0.7     | 0.9     | 0.8      | 0.7     | 0.7      | 0.9     |

The full set of HARP data, taken with targets spanning the full periodic table of elements, with small total errors and full coverage of the solid angle in a single detector will definitely help the validation of models used in hadronic simulations in the difficult energy range between 3 GeV/c and 15 GeV/c of incident momentum with incident pions, where data were previously missing.

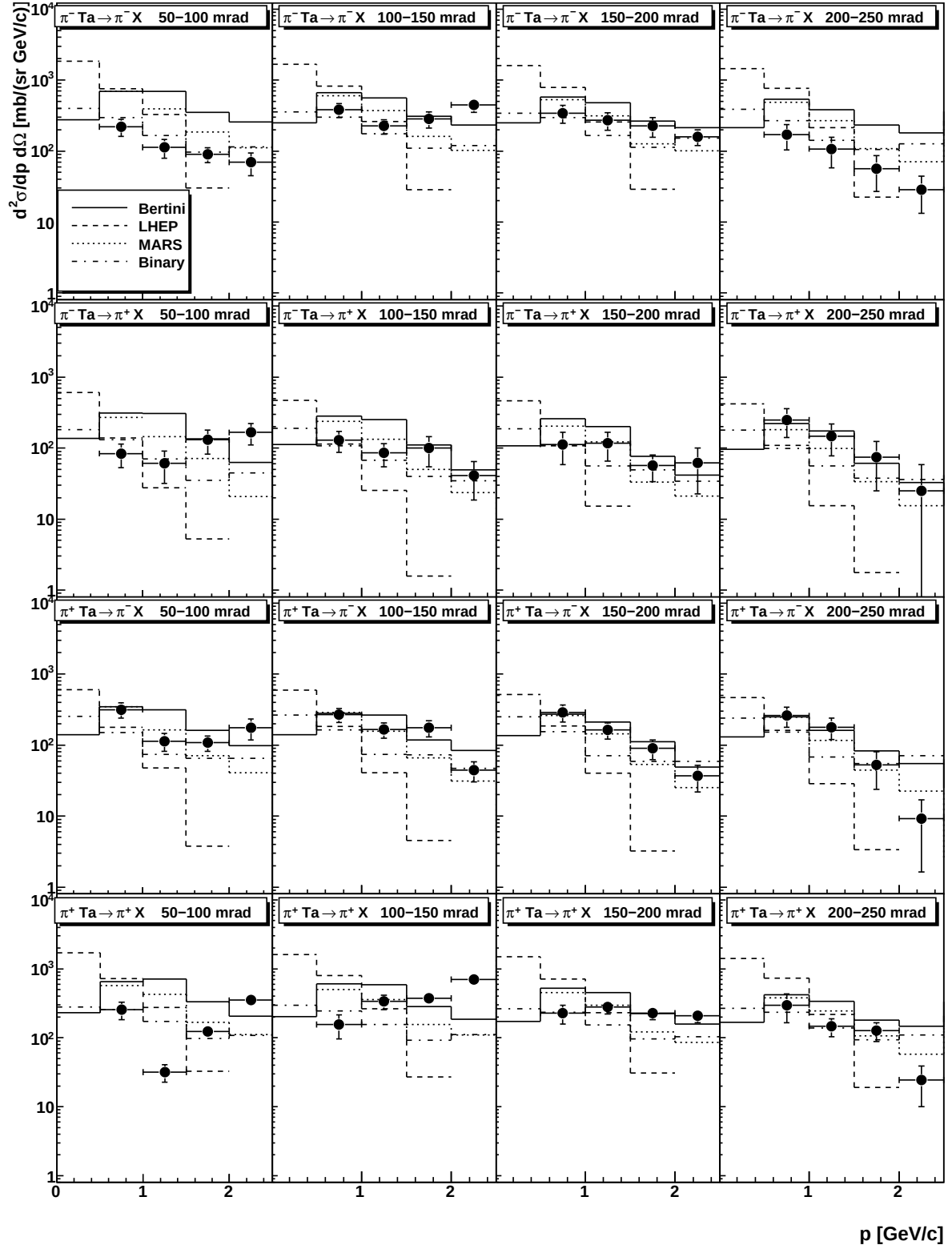


Figure 19: Comparison of HARP double-differential  $\pi^\pm$  cross-sections for  $\pi$ -Ta at 3 GeV/c with GEANT4 and MARS MC predictions, using several generator models.

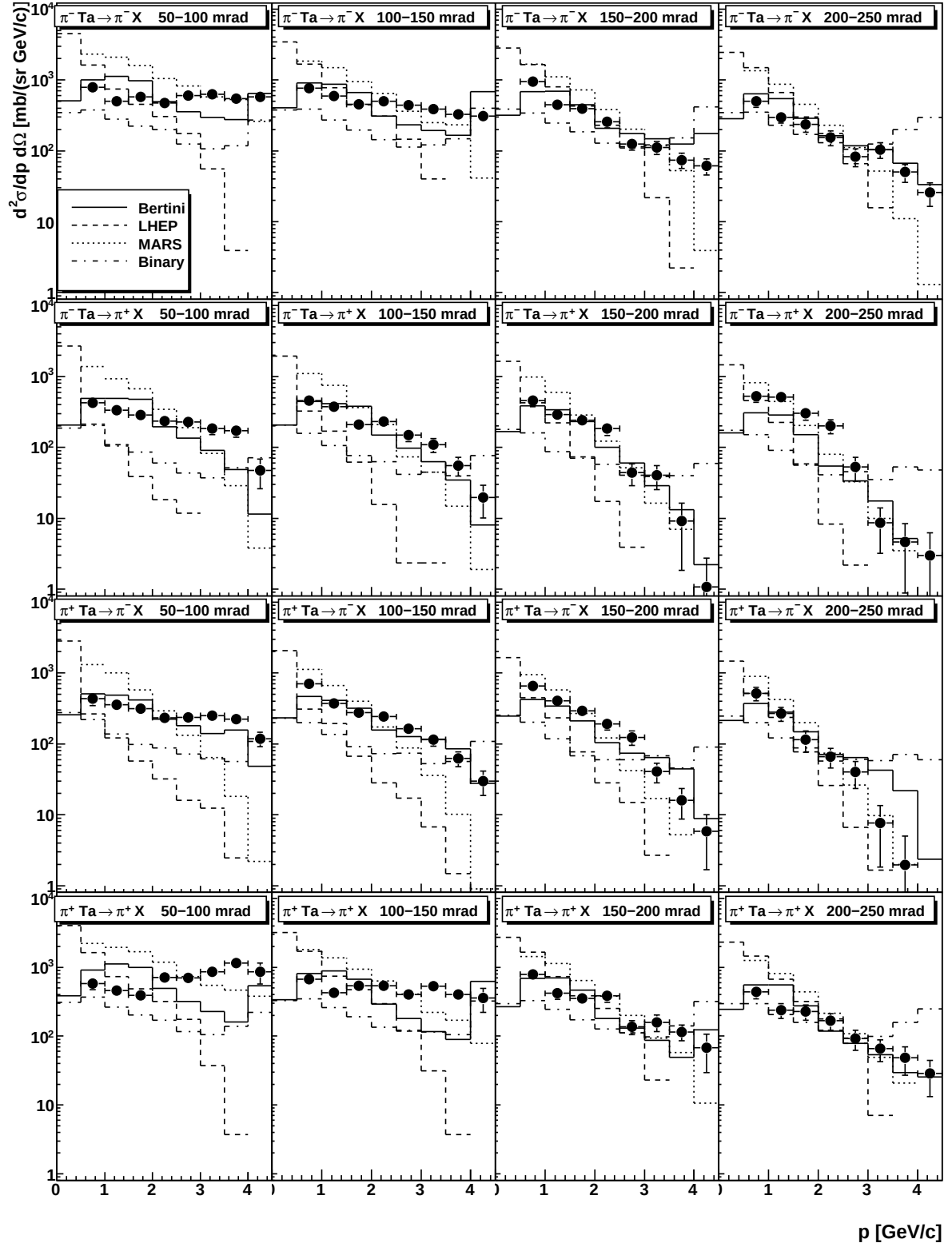


Figure 20: Comparison of HARP double-differential  $\pi^\pm$  cross-sections for  $\pi$ -Ta at 5 GeV/c with GEANT4 and MARS MC predictions, using several generator models.

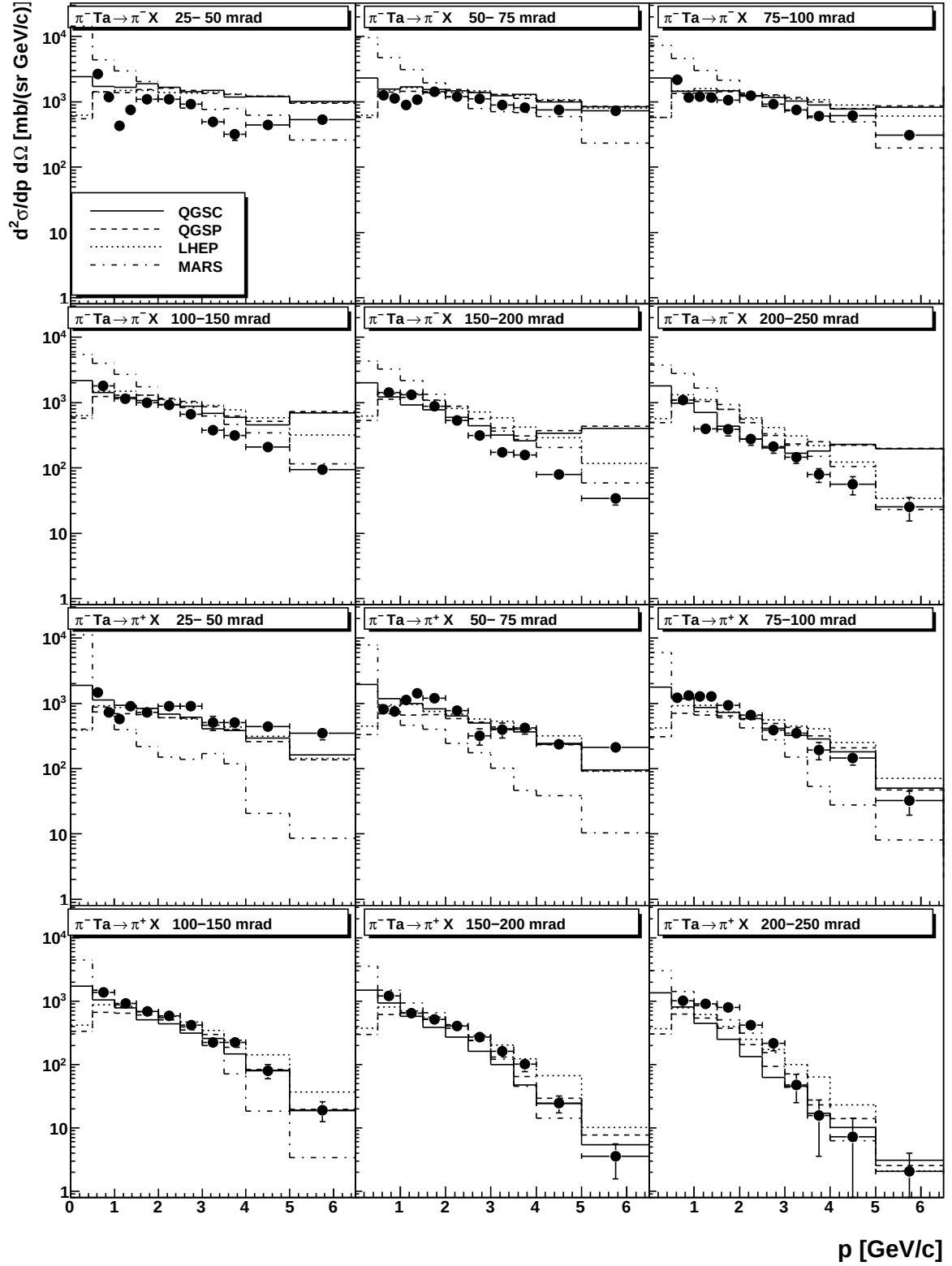


Figure 21: Comparison of HARP double-differential  $\pi^\pm$  cross-sections for  $\pi^-$ -Ta at 8 GeV/c with GEANT4 and MARS MC predictions, using several generator models.



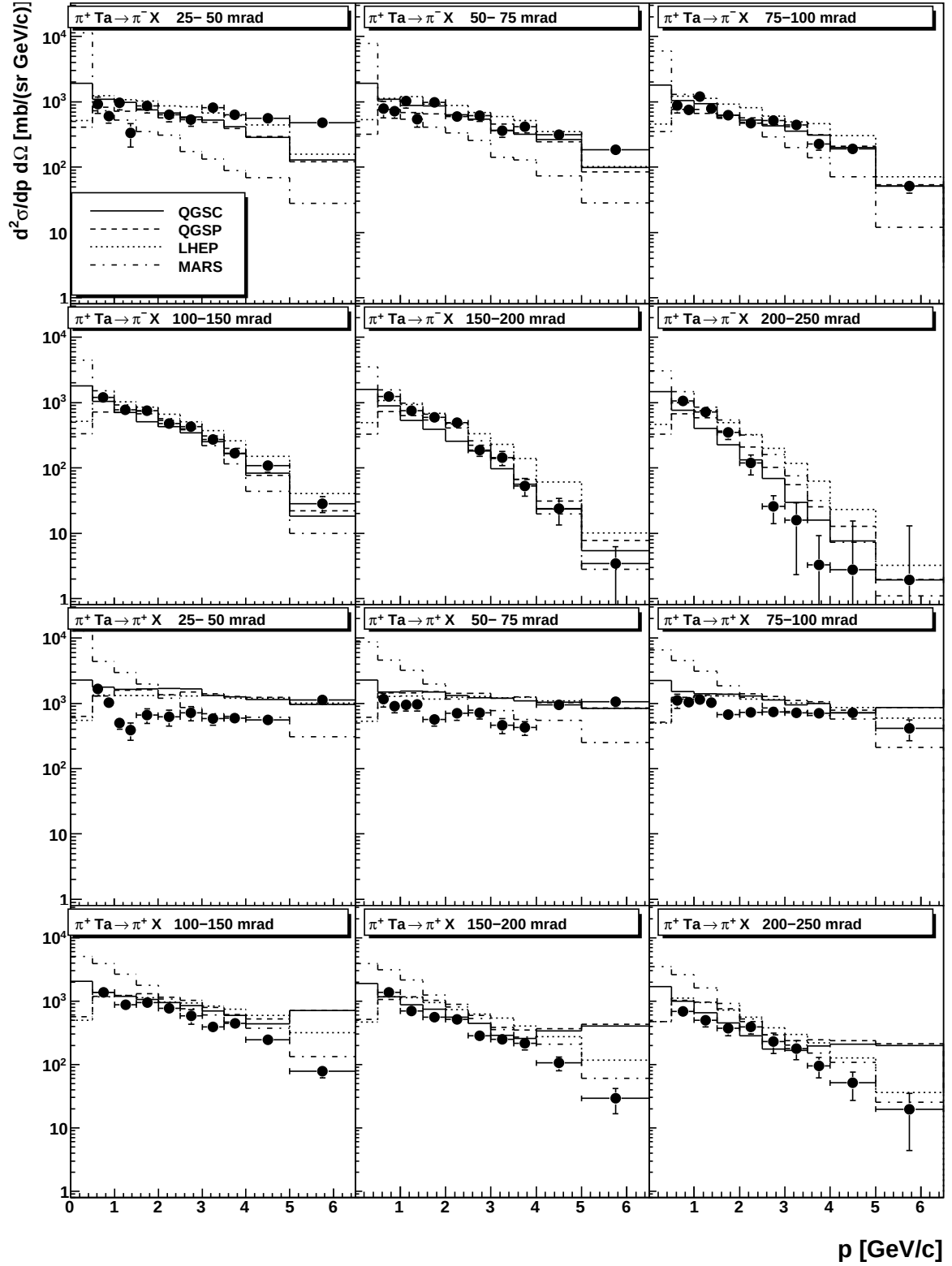


Figure 22: Comparison of HARP double-differential  $\pi^\pm$  cross-sections for  $\pi^+$ -Ta at 8 GeV/c with GEANT4 and MARS MC predictions, using several generator models.

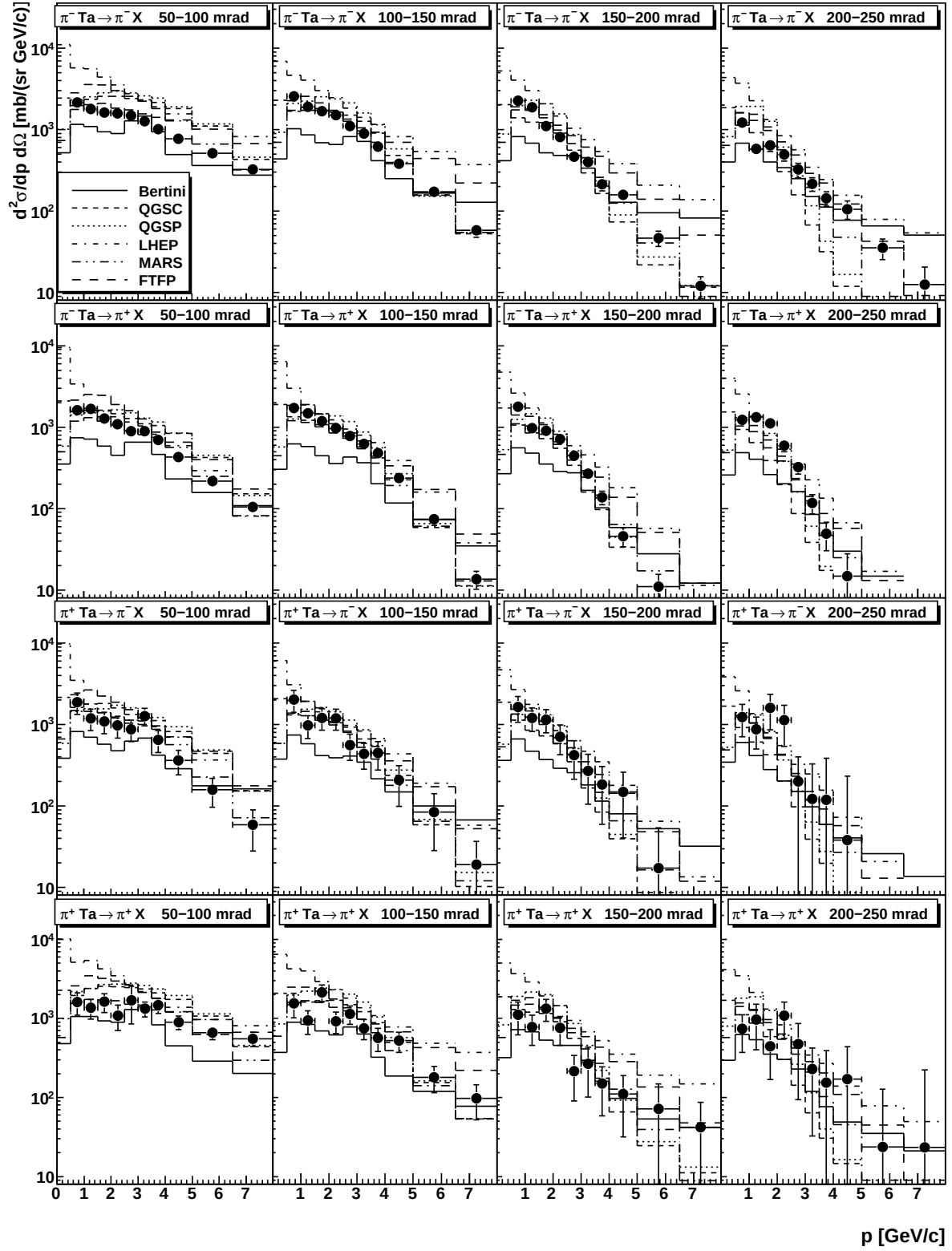


Figure 23: Comparison of HARP double-differential  $\pi^\pm$  cross-sections for  $\pi$ -Ta at 12 GeV/c with GEANT4 and MARS MC predictions, using several generator models.

## 4 Summary and conclusions

Double-differential cross-sections for the production of positive and negative pions in the kinematic range  $0.5 \text{ GeV}/c \leq p_\pi \leq 8 \text{ GeV}/c$  and  $0.025 \text{ rad} \leq \theta_\pi \leq 0.25 \text{ rad}$  from the interactions of 3, 5, 8 and 12  $\text{GeV}/c$   $\pi^\pm$  on beryllium, carbon, aluminium, copper, tin, tantalum and lead targets of 5% interaction length thickness have been reported.

We should stress that the HARP data are the first precision measurements with incoming charged pion beams in this kinematic region and may have a major impact on the tuning of Monte Carlo generators at low energies. Quantitative comparisons with a variety of generators currently in use have been presented.

The pion yield averaged over different momentum and angular ranges increase smoothly with the atomic number  $A$  of the target and with the energy of the incoming pion beam.

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## A Cross-section data

The following tables report the measured differential cross-section for positive and negative pion production in interactions of 3, 5, 8 and 12 GeV/ $c$  momentum charged pions on different types of nuclear targets. The data are presented in the kinematic range of  $0.5 \text{ GeV}/c \leq p_\pi \leq 8 \text{ GeV}/c$  and  $0.05 \text{ rad} \leq \theta_\pi \leq 0.25 \text{ rad}$ . The overall normalization uncertainty ( $\leq 2\%$ ) is not included in the reported errors.

Results at higher incoming beam momenta, from 8 GeV/ $c$  to 12.9 GeV/ $c$ , are also presented extending to a lower value of the polar angle  $\theta$  (0.025 rad), using a finer binning. This second set of tables contains additional data for collisions of 12.9 GeV/ $c$   $\pi^\pm$  on Al and 8.9 GeV/ $c$   $\pi^\pm$  on Be.

The cross-section values in some bins of the following tables are omitted and replaced by the symbol  $* \pm *$ , due to instabilities in the unfolding procedure coming from the low statistics available.

Table 4: HARP results for the double-differential  $\pi^+$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Be interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.05                     | 0.10                     | 0.50                  | 1.00                  | 0.038±0.007                                          | 0.054±0.007 | 0.14±0.02   | 0.106±0.012 |
|                          |                          | 1.00                  | 1.50                  | 0.042±0.006                                          | 0.073±0.007 | 0.177±0.011 | 0.130±0.010 |
|                          |                          | 1.50                  | 2.00                  | 0.035±0.005                                          | 0.064±0.006 | 0.141±0.009 | 0.127±0.010 |
|                          |                          | 2.00                  | 2.50                  | 0.048±0.007                                          | 0.053±0.005 | 0.112±0.008 | 0.127±0.008 |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.045±0.004 | 0.090±0.008 | 0.086±0.006 |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.044±0.005 | 0.086±0.008 | 0.097±0.008 |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.066±0.005 | 0.072±0.007 | 0.087±0.007 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.022±0.008 | 0.046±0.004 | 0.049±0.004 |
|                          | 0.15                     | 5.00                  | 6.50                  | ±                                                    | ±           | 0.023±0.003 | 0.027±0.002 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.010±0.001 |
|                          |                          | 0.50                  | 1.00                  | 0.052±0.008                                          | 0.094±0.011 | 0.17±0.02   | 0.137±0.015 |
|                          |                          | 1.00                  | 1.50                  | 0.026±0.004                                          | 0.091±0.009 | 0.162±0.013 | 0.145±0.012 |
|                          |                          | 1.50                  | 2.00                  | 0.032±0.005                                          | 0.047±0.005 | 0.107±0.010 | 0.117±0.010 |
|                          |                          | 2.00                  | 2.50                  | 0.037±0.006                                          | 0.054±0.006 | 0.104±0.009 | 0.107±0.010 |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.028±0.004 | 0.075±0.007 | 0.077±0.007 |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.018±0.003 | 0.044±0.005 | 0.068±0.006 |
| 0.15                     | 0.20                     | 3.50                  | 4.00                  | ±                                                    | 0.013±0.002 | 0.040±0.004 | 0.049±0.005 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.006±0.003 | 0.021±0.004 | 0.028±0.003 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.003±0.001 | 0.008±0.001 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.001±0.000 |
|                          |                          | 0.50                  | 1.00                  | 0.042±0.008                                          | 0.078±0.010 | 0.16±0.02   | 0.15±0.02   |
|                          |                          | 1.00                  | 1.50                  | 0.039±0.005                                          | 0.078±0.008 | 0.110±0.010 | 0.099±0.009 |
|                          |                          | 1.50                  | 2.00                  | 0.024±0.004                                          | 0.050±0.005 | 0.104±0.009 | 0.085±0.008 |
|                          |                          | 2.00                  | 2.50                  | 0.011±0.003                                          | 0.039±0.005 | 0.081±0.008 | 0.078±0.008 |
|                          | 0.25                     | 2.50                  | 3.00                  | ±                                                    | 0.035±0.005 | 0.052±0.007 | 0.044±0.006 |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.010±0.003 | 0.025±0.004 | 0.031±0.004 |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.003±0.001 | 0.014±0.003 | 0.014±0.003 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.001±0.000 | 0.006±0.001 | 0.008±0.001 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.001±0.000 | 0.001±0.001 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | ±           |
|                          |                          | 0.50                  | 1.00                  | 0.057±0.010                                          | 0.107±0.014 | 0.13±0.02   | 0.097±0.013 |
|                          |                          | 1.00                  | 1.50                  | 0.028±0.006                                          | 0.101±0.013 | 0.18±0.02   | 0.16±0.02   |
| 0.20                     | 0.25                     | 1.50                  | 2.00                  | 0.034±0.007                                          | 0.063±0.009 | 0.14±0.02   | 0.099±0.014 |
|                          |                          | 2.00                  | 2.50                  | 0.005±0.002                                          | 0.038±0.006 | 0.082±0.013 | 0.051±0.008 |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.011±0.003 | 0.031±0.007 | 0.023±0.004 |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.004±0.001 | 0.008±0.003 | 0.011±0.003 |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.001±0.001 | 0.004±0.002 | 0.004±0.002 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | ±           | 0.002±0.001 | 0.002±0.001 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | ±           | ±           |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.000±0.001 |

Table 5: HARP results for the double-differential  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Be interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.05                     | 0.10                     | 0.50                  | 1.00                  | 0.075±0.010                                          | 0.113±0.012 | 0.25±0.02   | 0.15±0.02   |
|                          |                          | 1.00                  | 1.50                  | 0.094±0.010                                          | 0.119±0.009 | 0.195±0.014 | 0.144±0.012 |
|                          |                          | 1.50                  | 2.00                  | 0.116±0.009                                          | 0.134±0.009 | 0.208±0.013 | 0.172±0.011 |
|                          |                          | 2.00                  | 2.50                  | 0.15±0.04                                            | 0.142±0.010 | 0.217±0.014 | 0.167±0.011 |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.128±0.009 | 0.194±0.011 | 0.176±0.011 |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.21±0.02   | 0.163±0.011 | 0.145±0.008 |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.241±0.013 | 0.155±0.011 | 0.129±0.007 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.25±0.04   | 0.161±0.009 | 0.105±0.006 |
|                          | 0.15                     | 5.00                  | 6.50                  | ±                                                    | ±           | 0.118±0.006 | 0.073±0.004 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.053±0.003 |
|                          |                          | 0.50                  | 1.00                  | 0.100±0.012                                          | 0.16±0.02   | 0.22±0.02   | 0.17±0.02   |
|                          |                          | 1.00                  | 1.50                  | 0.083±0.009                                          | 0.119±0.010 | 0.23±0.02   | 0.177±0.014 |
|                          |                          | 1.50                  | 2.00                  | 0.119±0.012                                          | 0.118±0.011 | 0.186±0.014 | 0.165±0.013 |
|                          |                          | 2.00                  | 2.50                  | 0.16±0.02                                            | 0.137±0.011 | 0.180±0.014 | 0.147±0.012 |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.107±0.009 | 0.150±0.012 | 0.124±0.010 |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.103±0.008 | 0.103±0.009 | 0.093±0.007 |
| 0.15                     | 0.20                     | 3.50                  | 4.00                  | ±                                                    | 0.087±0.007 | 0.078±0.008 | 0.083±0.006 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.094±0.009 | 0.045±0.005 | 0.055±0.005 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.022±0.003 | 0.023±0.002 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.008±0.001 |
|                          |                          | 0.50                  | 1.00                  | 0.103±0.014                                          | 0.093±0.012 | 0.21±0.02   | 0.17±0.02   |
|                          |                          | 1.00                  | 1.50                  | 0.089±0.010                                          | 0.099±0.010 | 0.20±0.02   | 0.163±0.014 |
|                          |                          | 1.50                  | 2.00                  | 0.065±0.008                                          | 0.103±0.010 | 0.163±0.014 | 0.118±0.011 |
|                          |                          | 2.00                  | 2.50                  | 0.069±0.009                                          | 0.067±0.007 | 0.120±0.011 | 0.091±0.008 |
|                          | 0.25                     | 2.50                  | 3.00                  | ±                                                    | 0.034±0.004 | 0.071±0.007 | 0.059±0.006 |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.035±0.004 | 0.039±0.005 | 0.043±0.004 |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.015±0.003 | 0.032±0.003 | 0.030±0.003 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.013±0.002 | 0.020±0.003 | 0.018±0.002 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.005±0.001 | 0.005±0.001 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.001±0.000 |
|                          |                          | 0.50                  | 1.00                  | 0.075±0.012                                          | 0.13±0.02   | 0.16±0.02   | 0.104±0.014 |
|                          |                          | 1.00                  | 1.50                  | 0.033±0.006                                          | 0.053±0.008 | 0.120±0.014 | 0.066±0.009 |
| 0.20                     | 0.25                     | 1.50                  | 2.00                  | 0.035±0.006                                          | 0.038±0.007 | 0.096±0.014 | 0.070±0.011 |
|                          |                          | 2.00                  | 2.50                  | 0.023±0.004                                          | 0.033±0.005 | 0.074±0.012 | 0.060±0.009 |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.025±0.004 | 0.056±0.009 | 0.041±0.006 |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.011±0.002 | 0.043±0.006 | 0.031±0.005 |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.007±0.002 | 0.030±0.005 | 0.018±0.003 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.003±0.001 | 0.017±0.003 | 0.011±0.002 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.005±0.002 | 0.003±0.001 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.001±0.001 |



Table 6: HARP results for the double-differential  $\pi^+$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Be interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.050                    | 0.100                    | 0.50                  | 1.00                  | 0.08±0.02                                            | 0.12±0.02   | 0.13±0.02   | 0.15±0.04   |
|                          |                          | 1.00                  | 1.50                  | 0.047±0.008                                          | 0.102±0.013 | 0.16±0.02   | 0.16±0.03   |
|                          |                          | 1.50                  | 2.00                  | 0.089±0.009                                          | 0.110±0.012 | 0.131±0.013 | 0.16±0.03   |
|                          |                          | 2.00                  | 2.50                  | 0.22±0.02                                            | 0.158±0.014 | 0.158±0.015 | 0.20±0.03   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.161±0.012 | 0.18±0.02   | 0.18±0.03   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.21±0.02   | 0.138±0.014 | 0.19±0.03   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.251±0.014 | 0.122±0.012 | 0.20±0.03   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.30±0.10   | 0.161±0.010 | 0.15±0.02   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.14±0.02   | 0.047±0.011 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.18±0.02   |
|                          | 0.150                    | 0.50                  | 1.00                  | 0.07±0.02                                            | 0.12±0.02   | 0.15±0.02   | 0.13±0.03   |
|                          |                          | 1.00                  | 1.50                  | 0.081±0.013                                          | 0.114±0.014 | 0.14±0.02   | 0.18±0.03   |
|                          |                          | 1.50                  | 2.00                  | 0.16±0.02                                            | 0.14±0.02   | 0.21±0.02   | 0.19±0.03   |
|                          |                          | 2.00                  | 2.50                  | 0.16±0.02                                            | 0.14±0.02   | 0.16±0.02   | 0.19±0.03   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.107±0.012 | 0.130±0.014 | 0.15±0.02   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.128±0.013 | 0.102±0.011 | 0.12±0.02   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.108±0.011 | 0.067±0.008 | 0.13±0.02   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.10±0.04   | 0.051±0.008 | 0.091±0.014 |
| 0.150                    | 0.200                    | 0.50                  | 1.00                  | 0.11±0.02                                            | 0.14±0.02   | 0.19±0.03   | 0.24±0.05   |
|                          |                          | 1.00                  | 1.50                  | 0.101±0.015                                          | 0.122±0.015 | 0.16±0.02   | 0.23±0.04   |
|                          |                          | 1.50                  | 2.00                  | 0.077±0.011                                          | 0.079±0.010 | 0.15±0.02   | 0.11±0.02   |
|                          |                          | 2.00                  | 2.50                  | 0.09±0.02                                            | 0.090±0.015 | 0.109±0.013 | 0.08±0.02   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.039±0.006 | 0.056±0.009 | 0.07±0.02   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.030±0.008 | 0.046±0.008 | 0.06±0.02   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.023±0.006 | 0.026±0.005 | 0.042±0.013 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.016±0.009 | 0.016±0.003 | 0.023±0.009 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.006±0.002 | 0.008±0.004 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.002±0.002 |
|                          | 0.250                    | 0.50                  | 1.00                  | 0.07±0.02                                            | 0.11±0.02   | 0.11±0.02   | 0.09±0.03   |
|                          |                          | 1.00                  | 1.50                  | 0.059±0.013                                          | 0.053±0.010 | 0.11±0.02   | 0.09±0.03   |
|                          |                          | 1.50                  | 2.00                  | 0.027±0.007                                          | 0.067±0.013 | 0.065±0.013 | 0.06±0.03   |
|                          |                          | 2.00                  | 2.50                  | 0.021±0.008                                          | 0.042±0.009 | 0.08±0.02   | 0.07±0.03   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.028±0.007 | 0.047±0.012 | 0.06±0.03   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.019±0.006 | 0.026±0.008 | 0.03±0.02   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.010±0.004 | 0.020±0.005 | 0.016±0.015 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.005±0.003 | 0.012±0.004 | 0.011±0.014 |
| 0.200                    | 0.250                    | 5.00                  | 6.50                  | ±                                                    | ±           | 0.006±0.003 | 0.002±0.005 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.000±0.011 |

Table 7: HARP results for the double-differential  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Be interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^\pi/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|--------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                          | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.050                    | 0.100                    | 0.50                  | 1.00                  | 0.060±0.013                                      | 0.081±0.014 | 0.11±0.02   | 0.05±0.02   |
|                          |                          | 1.00                  | 1.50                  | 0.049±0.010                                      | 0.106±0.013 | 0.14±0.02   | 0.15±0.03   |
|                          |                          | 1.50                  | 2.00                  | 0.038±0.007                                      | 0.062±0.008 | 0.118±0.013 | 0.12±0.02   |
|                          |                          | 2.00                  | 2.50                  | 0.06±0.02                                        | 0.064±0.008 | 0.114±0.012 | 0.10±0.02   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.050±0.007 | 0.083±0.009 | 0.14±0.02   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.054±0.008 | 0.093±0.009 | 0.12±0.02   |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.063±0.007 | 0.065±0.007 | 0.10±0.02   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.033±0.007 | 0.050±0.005 | 0.066±0.011 |
|                          | 0.150                    | 5.00                  | 6.50                  | ±                                                | ±           | 0.026±0.003 | 0.039±0.007 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.019±0.004 |
|                          |                          | 0.50                  | 1.00                  | 0.070±0.014                                      | 0.081±0.015 | 0.15±0.02   | 0.20±0.05   |
|                          |                          | 1.00                  | 1.50                  | 0.041±0.008                                      | 0.059±0.009 | 0.118±0.014 | 0.15±0.03   |
|                          |                          | 1.50                  | 2.00                  | 0.035±0.007                                      | 0.067±0.010 | 0.101±0.012 | 0.18±0.03   |
|                          |                          | 2.00                  | 2.50                  | 0.021±0.006                                      | 0.057±0.008 | 0.093±0.011 | 0.14±0.03   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.038±0.006 | 0.061±0.008 | 0.09±0.02   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.025±0.004 | 0.044±0.006 | 0.064±0.014 |
| 0.150                    | 0.200                    | 3.50                  | 4.00                  | ±                                                | 0.015±0.003 | 0.036±0.005 | 0.064±0.014 |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.005±0.002 | 0.020±0.004 | 0.037±0.009 |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.003±0.001 | 0.016±0.005 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.004±0.002 |
|                          |                          | 0.50                  | 1.00                  | 0.08±0.02                                        | 0.086±0.014 | 0.13±0.02   | 0.14±0.04   |
|                          |                          | 1.00                  | 1.50                  | 0.059±0.011                                      | 0.086±0.012 | 0.12±0.02   | 0.12±0.03   |
|                          |                          | 1.50                  | 2.00                  | 0.024±0.006                                      | 0.068±0.010 | 0.099±0.013 | 0.09±0.02   |
|                          |                          | 2.00                  | 2.50                  | 0.008±0.003                                      | 0.034±0.006 | 0.085±0.012 | 0.13±0.03   |
|                          | 0.250                    | 2.50                  | 3.00                  | ±                                                | 0.025±0.005 | 0.050±0.008 | 0.05±0.02   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.006±0.002 | 0.026±0.005 | 0.034±0.012 |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.002±0.001 | 0.017±0.004 | 0.019±0.008 |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.001±0.001 | 0.004±0.001 | 0.012±0.007 |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | *±*         | 0.001±0.002 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.000±0.001 |
|                          |                          | 0.50                  | 1.00                  | 0.048±0.013                                      | 0.11±0.02   | 0.10±0.02   | 0.12±0.04   |
|                          |                          | 1.00                  | 1.50                  | 0.047±0.012                                      | 0.059±0.011 | 0.11±0.02   | 0.10±0.03   |
| 0.200                    | 0.250                    | 1.50                  | 2.00                  | 0.012±0.005                                      | 0.038±0.010 | 0.13±0.02   | 0.14±0.05   |
|                          |                          | 2.00                  | 2.50                  | 0.002±0.002                                      | 0.017±0.005 | 0.052±0.011 | 0.07±0.03   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.003±0.001 | 0.025±0.007 | 0.04±0.03   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.002±0.001 | 0.012±0.008 | 0.016±0.023 |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.002±0.001 | 0.004±0.005 | 0.004±0.008 |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.001±0.001 | 0.001±0.004 | 0.002±0.006 |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.000±0.004 | 0.000±0.001 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.000±0.006 |

Table 8: HARP results for the double-differential  $\pi^+$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -C interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.05                     | 0.10                     | 0.50                  | 1.00                  | 0.044±0.012                                          | 0.078±0.013 | 0.14±0.02   | 0.21±0.03   |
|                          |                          |                       | 1.00                  | 0.046±0.010                                          | 0.099±0.012 | 0.204±0.014 | 0.26±0.02   |
|                          |                          | 1.50                  | 2.00                  | 0.057±0.011                                          | 0.060±0.009 | 0.174±0.011 | 0.23±0.02   |
|                          |                          |                       | 2.50                  | 0.058±0.012                                          | 0.070±0.009 | 0.126±0.011 | 0.20±0.02   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.049±0.007 | 0.094±0.010 | 0.175±0.014 |
|                          |                          |                       | 3.50                  | ±                                                    | 0.050±0.007 | 0.094±0.010 | 0.171±0.014 |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.059±0.008 | 0.068±0.009 | 0.150±0.013 |
|                          |                          |                       | 5.00                  | ±                                                    | 0.030±0.009 | 0.050±0.005 | 0.082±0.008 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.023±0.003 | 0.053±0.005 |
|                          |                          |                       | 8.00                  | ±                                                    | ±           | ±           | 0.016±0.003 |
|                          |                          | 0.50                  | 1.00                  | 0.09±0.02                                            | 0.090±0.014 | 0.19±0.02   | 0.20±0.03   |
|                          |                          |                       | 1.50                  | 0.054±0.011                                          | 0.098±0.012 | 0.167±0.015 | 0.27±0.03   |
|                          |                          |                       | 2.00                  | 0.039±0.009                                          | 0.073±0.010 | 0.146±0.013 | 0.21±0.02   |
|                          |                          |                       | 2.50                  | 0.035±0.009                                          | 0.052±0.008 | 0.130±0.012 | 0.19±0.02   |
|                          |                          |                       | 3.00                  | ±                                                    | 0.039±0.006 | 0.086±0.010 | 0.145±0.015 |
|                          |                          |                       | 3.50                  | ±                                                    | 0.024±0.005 | 0.053±0.007 | 0.113±0.012 |
|                          |                          |                       | 4.00                  | ±                                                    | 0.013±0.003 | 0.050±0.006 | 0.102±0.011 |
|                          |                          |                       | 5.00                  | ±                                                    | 0.007±0.003 | 0.019±0.004 | 0.048±0.007 |
| 0.10                     | 0.15                     | 0.50                  | 1.00                  | 0.06±0.02                                            | 0.09±0.02   | 0.19±0.02   | 0.20±0.03   |
|                          |                          |                       | 1.50                  | 0.034±0.008                                          | 0.066±0.010 | 0.130±0.013 | 0.19±0.02   |
|                          |                          | 1.50                  | 2.00                  | 0.028±0.008                                          | 0.056±0.008 | 0.115±0.011 | 0.16±0.02   |
|                          |                          |                       | 2.50                  | 0.022±0.008                                          | 0.042±0.007 | 0.091±0.010 | 0.16±0.02   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.022±0.005 | 0.050±0.007 | 0.073±0.010 |
|                          |                          |                       | 3.50                  | ±                                                    | 0.010±0.004 | 0.031±0.005 | 0.068±0.010 |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.001±0.001 | 0.016±0.003 | 0.026±0.005 |
|                          |                          |                       | 5.00                  | ±                                                    | 0.001±0.001 | 0.009±0.002 | 0.011±0.003 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.001±0.000 | 0.002±0.001 |
|                          |                          |                       | 8.00                  | ±                                                    | ±           | ±           | ±           |
|                          |                          | 0.50                  | 1.00                  | 0.06±0.02                                            | 0.10±0.02   | 0.14±0.02   | 0.19±0.03   |
|                          |                          |                       | 1.50                  | 0.06±0.02                                            | 0.08±0.02   | 0.19±0.02   | 0.25±0.03   |
|                          |                          |                       | 2.00                  | 0.027±0.010                                          | 0.077±0.014 | 0.12±0.02   | 0.15±0.03   |
|                          |                          |                       | 2.50                  | 0.010±0.006                                          | 0.039±0.008 | 0.069±0.012 | 0.10±0.02   |
|                          |                          |                       | 3.00                  | ±                                                    | 0.018±0.005 | 0.036±0.008 | 0.044±0.011 |
|                          |                          |                       | 3.50                  | ±                                                    | 0.004±0.002 | 0.011±0.005 | 0.015±0.005 |
|                          |                          |                       | 4.00                  | ±                                                    | 0.001±0.001 | 0.002±0.002 | 0.008±0.004 |
|                          |                          |                       | 5.00                  | ±                                                    | ±           | 0.001±0.001 | 0.003±0.003 |
| 0.15                     | 0.20                     | 0.50                  | 1.00                  | 0.06±0.02                                            | 0.10±0.02   | 0.14±0.02   | 0.19±0.03   |
|                          |                          |                       | 1.50                  | 0.06±0.02                                            | 0.08±0.02   | 0.19±0.02   | 0.25±0.03   |
|                          |                          | 1.50                  | 2.00                  | 0.027±0.010                                          | 0.077±0.014 | 0.12±0.02   | 0.15±0.03   |
|                          |                          |                       | 2.50                  | 0.010±0.006                                          | 0.039±0.008 | 0.069±0.012 | 0.10±0.02   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.018±0.005 | 0.036±0.008 | 0.044±0.011 |
|                          |                          |                       | 3.50                  | ±                                                    | 0.004±0.002 | 0.011±0.005 | 0.015±0.005 |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.001±0.001 | 0.002±0.002 | 0.008±0.004 |
|                          |                          |                       | 5.00                  | ±                                                    | ±           | 0.001±0.001 | 0.003±0.003 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | ±           | ±           |
|                          |                          |                       | 8.00                  | ±                                                    | ±           | ±           | 0.000±0.003 |
|                          |                          | 0.50                  | 1.00                  | 0.06±0.02                                            | 0.10±0.02   | 0.14±0.02   | 0.19±0.03   |
|                          |                          |                       | 1.50                  | 0.06±0.02                                            | 0.08±0.02   | 0.19±0.02   | 0.25±0.03   |
|                          |                          |                       | 2.00                  | 0.027±0.010                                          | 0.077±0.014 | 0.12±0.02   | 0.15±0.03   |
|                          |                          |                       | 2.50                  | 0.010±0.006                                          | 0.039±0.008 | 0.069±0.012 | 0.10±0.02   |
|                          |                          |                       | 3.00                  | ±                                                    | 0.018±0.005 | 0.036±0.008 | 0.044±0.011 |
|                          |                          |                       | 3.50                  | ±                                                    | 0.004±0.002 | 0.011±0.005 | 0.015±0.005 |
|                          |                          |                       | 4.00                  | ±                                                    | 0.001±0.001 | 0.002±0.002 | 0.008±0.004 |
|                          |                          |                       | 5.00                  | ±                                                    | ±           | 0.001±0.001 | 0.003±0.003 |

Table 9: HARP results for the double-differential  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -C interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.05                     | 0.10                     | 0.50                  | 1.00                  | 0.11±0.02                                            | 0.14±0.02   | 0.27±0.03   | 0.28±0.03   |
|                          |                          | 1.00                  | 1.50                  | 0.080±0.014                                          | 0.122±0.013 | 0.23±0.02   | 0.29±0.03   |
|                          |                          | 1.50                  | 2.00                  | 0.14±0.02                                            | 0.123±0.013 | 0.25±0.02   | 0.32±0.02   |
|                          |                          | 2.00                  | 2.50                  | 0.15±0.04                                            | 0.138±0.014 | 0.24±0.02   | 0.32±0.02   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.17±0.02   | 0.222±0.013 | 0.30±0.02   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.22±0.02   | 0.183±0.014 | 0.26±0.02   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.19±0.02   | 0.177±0.014 | 0.23±0.02   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.27±0.05   | 0.172±0.011 | 0.194±0.013 |
|                          | 0.15                     | 5.00                  | 6.50                  | ±                                                    | ±           | 0.116±0.008 | 0.134±0.007 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.089±0.007 |
|                          |                          | 0.50                  | 1.00                  | 0.16±0.03                                            | 0.17±0.02   | 0.25±0.03   | 0.31±0.04   |
|                          |                          | 1.00                  | 1.50                  | 0.087±0.013                                          | 0.133±0.015 | 0.24±0.02   | 0.30±0.03   |
|                          |                          | 1.50                  | 2.00                  | 0.15±0.02                                            | 0.15±0.02   | 0.21±0.02   | 0.31±0.03   |
|                          |                          | 2.00                  | 2.50                  | 0.17±0.03                                            | 0.140±0.014 | 0.20±0.02   | 0.26±0.02   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.126±0.013 | 0.167±0.014 | 0.23±0.02   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.110±0.012 | 0.097±0.010 | 0.18±0.02   |
| 0.15                     | 0.20                     | 3.50                  | 4.00                  | ±                                                    | 0.100±0.011 | 0.076±0.008 | 0.140±0.014 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.10±0.02   | 0.046±0.006 | 0.090±0.009 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.020±0.003 | 0.040±0.005 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.014±0.003 |
|                          |                          | 0.50                  | 1.00                  | 0.09±0.02                                            | 0.16±0.02   | 0.22±0.03   | 0.31±0.04   |
|                          |                          | 1.00                  | 1.50                  | 0.14±0.02                                            | 0.12±0.02   | 0.23±0.02   | 0.25±0.03   |
|                          |                          | 1.50                  | 2.00                  | 0.12±0.02                                            | 0.112±0.013 | 0.18±0.02   | 0.23±0.02   |
|                          |                          | 2.00                  | 2.50                  | 0.095±0.014                                          | 0.074±0.010 | 0.120±0.013 | 0.15±0.02   |
|                          | 0.25                     | 2.50                  | 3.00                  | ±                                                    | 0.041±0.007 | 0.077±0.008 | 0.087±0.011 |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.024±0.004 | 0.042±0.006 | 0.060±0.008 |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.018±0.003 | 0.029±0.004 | 0.050±0.007 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.022±0.004 | 0.017±0.003 | 0.030±0.005 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.005±0.001 | 0.009±0.002 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.003±0.001 |
|                          |                          | 0.50                  | 1.00                  | 0.08±0.02                                            | 0.10±0.02   | 0.15±0.02   | 0.19±0.03   |
|                          |                          | 1.00                  | 1.50                  | 0.056±0.013                                          | 0.046±0.009 | 0.099±0.013 | 0.12±0.02   |
| 0.20                     | 0.25                     | 1.50                  | 2.00                  | 0.07±0.02                                            | 0.048±0.010 | 0.076±0.014 | 0.12±0.02   |
|                          |                          | 2.00                  | 2.50                  | 0.027±0.007                                          | 0.040±0.008 | 0.070±0.012 | 0.075±0.015 |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.024±0.006 | 0.053±0.009 | 0.069±0.013 |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.014±0.003 | 0.029±0.005 | 0.052±0.011 |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.009±0.003 | 0.020±0.004 | 0.023±0.006 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.005±0.002 | 0.014±0.003 | 0.011±0.004 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.003±0.001 | 0.006±0.003 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.002±0.002 |

Table 10: HARP results for the double-differential  $\pi^+$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -C interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |                |                |                 |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|----------------|----------------|-----------------|
|                          |                          |                       |                       | <b>3 GeV/c</b>                                       | <b>5 GeV/c</b> | <b>8 GeV/c</b> | <b>12 GeV/c</b> |
| 0.050                    | 0.100                    | 0.50                  | 1.00                  | 0.12±0.03                                            | 0.14±0.02      | 0.18±0.03      | 0.25±0.08       |
|                          |                          |                       | 1.00                  | 0.065±0.012                                          | 0.112±0.013    | 0.21±0.02      | 0.16±0.05       |
|                          |                          | 1.50                  | 2.00                  | 0.091±0.010                                          | 0.137±0.013    | 0.19±0.02      | *±*             |
|                          |                          | 2.00                  | 2.50                  | 0.25±0.02                                            | 0.191±0.014    | 0.19±0.02      | 0.36±0.07       |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.191±0.012    | 0.20±0.02      | 0.20±0.05       |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.25±0.02      | 0.19±0.02      | 0.28±0.05       |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.315±0.015    | 0.14±0.02      | 0.27±0.05       |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.32±0.10      | 0.196±0.014    | 0.14±0.03       |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±              | 0.17±0.03      | 0.12±0.02       |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±              | ±              | 0.09±0.02       |
|                          | 0.150                    | 0.50                  | 1.00                  | 0.08±0.02                                            | 0.17±0.02      | 0.18±0.03      | 0.26±0.08       |
|                          |                          |                       | 1.00                  | 0.09±0.02                                            | 0.15±0.02      | 0.21±0.02      | 0.18±0.05       |
|                          |                          | 1.50                  | 2.00                  | 0.18±0.02                                            | 0.136±0.014    | 0.25±0.03      | 0.22±0.06       |
|                          |                          | 2.00                  | 2.50                  | 0.20±0.02                                            | 0.160±0.015    | 0.21±0.03      | 0.19±0.06       |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.138±0.013    | 0.16±0.02      | 0.16±0.06       |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.138±0.013    | 0.110±0.012    | 0.15±0.04       |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.115±0.010    | 0.097±0.011    | 0.15±0.04       |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.11±0.04      | 0.065±0.011    | 0.07±0.02       |
| 0.150                    | 0.200                    | 5.00                  | 6.50                  | ±                                                    | ±              | 0.022±0.005    | 0.038±0.014     |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±              | ±              | 0.006±0.006     |
|                          |                          | 0.50                  | 1.00                  | 0.08±0.02                                            | 0.16±0.02      | 0.20±0.03      | 0.09±0.05       |
|                          |                          |                       | 1.00                  | 0.10±0.02                                            | 0.119±0.014    | 0.18±0.02      | 0.18±0.06       |
|                          |                          | 1.50                  | 2.00                  | 0.12±0.02                                            | 0.105±0.011    | 0.17±0.02      | 0.13±0.05       |
|                          |                          | 2.00                  | 2.50                  | 0.09±0.02                                            | 0.080±0.010    | 0.13±0.02      | 0.12±0.07       |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.055±0.007    | 0.080±0.013    | 0.10±0.08       |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.042±0.010    | 0.076±0.012    | 0.11±0.04       |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.031±0.007    | 0.043±0.008    | 0.06±0.03       |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.022±0.012    | 0.025±0.005    | 0.022±0.015     |
| 0.200                    | 0.250                    | 5.00                  | 6.50                  | ±                                                    | ±              | 0.005±0.002    | 0.015±0.012     |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±              | ±              | 0.002±0.004     |
|                          |                          | 0.50                  | 1.00                  | 0.05±0.02                                            | 0.12±0.02      | 0.19±0.03      | 0.15±0.08       |
|                          |                          |                       | 1.00                  | 0.049±0.013                                          | 0.070±0.012    | 0.13±0.02      | 0.05±0.04       |
|                          |                          | 1.50                  | 2.00                  | 0.043±0.011                                          | 0.071±0.012    | 0.13±0.02      | 0.06±0.05       |
|                          |                          | 2.00                  | 2.50                  | 0.018±0.008                                          | 0.053±0.009    | 0.09±0.02      | 0.06±0.05       |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.034±0.007    | 0.055±0.014    | 0.05±0.05       |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.016±0.004    | 0.040±0.011    | 0.016±0.016     |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.012±0.005    | 0.032±0.009    | 0.010±0.030     |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.008±0.004    | 0.023±0.008    | 0.025±0.050     |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±              | 0.007±0.004    | 0.003±0.010     |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±              | ±              | 0.000±0.027     |

Table 11: HARP results for the double-differential  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -C interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^\pi/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|--------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                          | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.050                    | 0.100                    | 0.50                  | 1.00                  | 0.031±0.011                                      | 0.082±0.013 | 0.14±0.02   | 0.11±0.05   |
|                          |                          | 1.00                  | 1.50                  | 0.052±0.012                                      | 0.111±0.012 | 0.15±0.02   | 0.18±0.06   |
|                          |                          | 1.50                  | 2.00                  | 0.041±0.008                                      | 0.073±0.008 | 0.15±0.02   | 0.21±0.06   |
|                          |                          | 2.00                  | 2.50                  | 0.06±0.02                                        | 0.067±0.007 | 0.131±0.015 | 0.11±0.03   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.064±0.007 | 0.113±0.013 | 0.16±0.05   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.067±0.008 | 0.081±0.009 | 0.20±0.04   |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.062±0.006 | 0.104±0.011 | 0.08±0.02   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.030±0.006 | 0.050±0.005 | 0.09±0.02   |
|                          | 0.150                    | 5.00                  | 6.50                  | ±                                                | ±           | 0.026±0.003 | 0.059±0.015 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.021±0.008 |
|                          |                          | 0.50                  | 1.00                  | 0.07±0.02                                        | 0.16±0.02   | 0.21±0.03   | 0.25±0.08   |
|                          |                          | 1.00                  | 1.50                  | 0.041±0.010                                      | 0.082±0.011 | 0.15±0.02   | 0.11±0.04   |
|                          |                          | 1.50                  | 2.00                  | 0.040±0.009                                      | 0.061±0.008 | 0.117±0.014 | 0.15±0.05   |
|                          |                          | 2.00                  | 2.50                  | 0.036±0.009                                      | 0.056±0.007 | 0.120±0.015 | 0.14±0.04   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.034±0.004 | 0.056±0.008 | 0.12±0.04   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.033±0.004 | 0.060±0.009 | 0.11±0.03   |
| 0.150                    | 0.200                    | 3.50                  | 4.00                  | ±                                                | 0.013±0.002 | 0.037±0.006 | 0.09±0.03   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.003±0.001 | 0.022±0.004 | 0.04±0.02   |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.006±0.001 | 0.013±0.008 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.005±0.004 |
|                          |                          | 0.50                  | 1.00                  | 0.07±0.02                                        | 0.11±0.02   | 0.20±0.03   | 0.27±0.09   |
|                          |                          | 1.00                  | 1.50                  | 0.035±0.009                                      | 0.073±0.010 | 0.13±0.02   | 0.20±0.06   |
|                          |                          | 1.50                  | 2.00                  | 0.021±0.006                                      | 0.091±0.011 | 0.12±0.02   | 0.18±0.06   |
|                          |                          | 2.00                  | 2.50                  | 0.015±0.006                                      | 0.046±0.006 | 0.095±0.013 | 0.12±0.04   |
|                          | 0.250                    | 2.50                  | 3.00                  | ±                                                | 0.023±0.004 | 0.048±0.008 | 0.07±0.03   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.007±0.002 | 0.027±0.005 | 0.032±0.035 |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.004±0.001 | 0.017±0.004 | 0.005±0.009 |
|                          |                          | 4.00                  | 5.00                  | ±                                                | ±           | 0.005±0.002 | 0.007±0.016 |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.001±0.001 | 0.003±0.009 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.000±0.001 |
|                          |                          | 0.50                  | 1.00                  | 0.07±0.02                                        | 0.12±0.02   | 0.15±0.03   | 0.14±0.06   |
|                          |                          | 1.00                  | 1.50                  | 0.025±0.009                                      | 0.080±0.013 | 0.16±0.03   | 0.15±0.07   |
| 0.200                    | 0.250                    | 1.50                  | 2.00                  | 0.005±0.003                                      | 0.031±0.008 | 0.13±0.02   | 0.07±0.05   |
|                          |                          | 2.00                  | 2.50                  | 0.008±0.005                                      | 0.015±0.004 | 0.052±0.012 | 0.08±0.06   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.005±0.002 | 0.020±0.006 | 0.08±0.06   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.002±0.001 | 0.011±0.004 | 0.051±0.059 |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.000±0.000 | 0.006±0.003 | 0.021±0.038 |
|                          |                          | 4.00                  | 5.00                  | ±                                                | ±           | 0.002±0.003 | 0.003±0.019 |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.000±0.001 | 0.002±0.015 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.001±0.018 |

Table 12: HARP results for the double-differential  $\pi^+$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Al interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.05                     | 0.10                     | 0.50                  | 1.00                  | 0.066±0.015                                          | 0.14±0.03   | 0.29±0.03   | 0.32±0.04   |
|                          |                          |                       | 1.00                  | 0.086±0.015                                          | 0.18±0.02   | 0.36±0.03   | 0.44±0.03   |
|                          |                          | 1.50                  | 2.00                  | 0.063±0.012                                          | 0.13±0.02   | 0.30±0.02   | 0.37±0.03   |
|                          |                          |                       | 2.50                  | 0.090±0.015                                          | 0.12±0.02   | 0.22±0.02   | 0.32±0.02   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.092±0.015 | 0.14±0.02   | 0.29±0.02   |
|                          |                          |                       | 3.50                  | ±                                                    | 0.078±0.014 | 0.17±0.02   | 0.29±0.02   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.080±0.014 | 0.13±0.02   | 0.22±0.02   |
|                          |                          |                       | 5.00                  | ±                                                    | 0.038±0.011 | 0.083±0.009 | 0.138±0.011 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.039±0.005 | 0.071±0.006 |
|                          |                          |                       | 8.00                  | ±                                                    | ±           | ±           | 0.030±0.004 |
|                          |                          | 0.50                  | 1.00                  | 0.12±0.02                                            | 0.22±0.03   | 0.32±0.04   | 0.34±0.04   |
|                          |                          |                       | 1.50                  | 0.052±0.011                                          | 0.17±0.02   | 0.29±0.03   | 0.44±0.04   |
|                          |                          |                       | 2.00                  | 0.047±0.010                                          | 0.10±0.02   | 0.24±0.02   | 0.35±0.03   |
|                          |                          |                       | 2.50                  | 0.047±0.011                                          | 0.074±0.014 | 0.23±0.02   | 0.33±0.03   |
|                          |                          |                       | 3.00                  | ±                                                    | 0.08±0.02   | 0.14±0.02   | 0.25±0.02   |
|                          |                          |                       | 3.50                  | ±                                                    | 0.048±0.013 | 0.094±0.013 | 0.21±0.02   |
|                          |                          |                       | 4.00                  | ±                                                    | 0.011±0.005 | 0.079±0.011 | 0.150±0.015 |
|                          |                          |                       | 5.00                  | ±                                                    | 0.002±0.002 | 0.029±0.006 | 0.084±0.011 |
|                          |                          |                       | 6.50                  | ±                                                    | ±           | 0.006±0.002 | 0.021±0.004 |
|                          |                          |                       | 8.00                  | ±                                                    | ±           | ±           | 0.004±0.001 |
| 0.15                     | 0.20                     | 0.50                  | 1.00                  | 0.10±0.02                                            | 0.17±0.03   | 0.34±0.04   | 0.44±0.05   |
|                          |                          |                       | 1.50                  | 0.082±0.014                                          | 0.10±0.02   | 0.25±0.03   | 0.32±0.03   |
|                          |                          | 1.50                  | 2.00                  | 0.043±0.009                                          | 0.10±0.02   | 0.20±0.02   | 0.27±0.03   |
|                          |                          |                       | 2.50                  | 0.024±0.008                                          | 0.08±0.02   | 0.16±0.02   | 0.23±0.03   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.034±0.010 | 0.094±0.013 | 0.121±0.015 |
|                          |                          |                       | 3.50                  | ±                                                    | 0.014±0.005 | 0.055±0.010 | 0.093±0.014 |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.007±0.004 | 0.028±0.006 | 0.036±0.007 |
|                          |                          |                       | 5.00                  | ±                                                    | 0.001±0.001 | 0.010±0.003 | 0.028±0.005 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.002±0.001 | 0.004±0.001 |
|                          |                          |                       | 8.00                  | ±                                                    | ±           | ±           | 0.001±0.000 |
|                          |                          | 0.50                  | 1.00                  | 0.10±0.02                                            | 0.19±0.04   | 0.29±0.04   | 0.33±0.05   |
|                          |                          |                       | 1.50                  | 0.10±0.02                                            | 0.19±0.04   | 0.38±0.05   | 0.36±0.05   |
|                          |                          |                       | 2.00                  | 0.039±0.012                                          | 0.12±0.03   | 0.27±0.04   | 0.25±0.04   |
|                          |                          |                       | 2.50                  | 0.011±0.006                                          | 0.06±0.02   | 0.13±0.02   | 0.18±0.03   |
|                          |                          |                       | 3.00                  | ±                                                    | 0.019±0.008 | 0.078±0.015 | 0.08±0.02   |
|                          |                          |                       | 3.50                  | ±                                                    | 0.009±0.005 | 0.024±0.009 | 0.038±0.009 |
|                          |                          |                       | 4.00                  | ±                                                    | 0.003±0.002 | 0.008±0.004 | 0.017±0.006 |
|                          |                          |                       | 5.00                  | ±                                                    | 0.002±0.002 | 0.006±0.003 | 0.007±0.004 |
|                          |                          |                       | 6.50                  | ±                                                    | ±           | 0.001±0.001 | 0.001±0.001 |
|                          |                          |                       | 8.00                  | ±                                                    | ±           | ±           | 0.000±0.004 |

Table 13: HARP results for the double-differential  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Al interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^\pi/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|--------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                          | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.05                     | 0.10                     | 0.50                  | 1.00                  | 0.16±0.02                                        | 0.27±0.04   | 0.50±0.05   | 0.49±0.05   |
|                          |                          | 1.00                  | 1.50                  | 0.14±0.02                                        | 0.23±0.03   | 0.41±0.03   | 0.46±0.04   |
|                          |                          | 1.50                  | 2.00                  | 0.21±0.02                                        | 0.25±0.03   | 0.43±0.03   | 0.52±0.03   |
|                          |                          | 2.00                  | 2.50                  | 0.21±0.05                                        | 0.28±0.03   | 0.42±0.03   | 0.47±0.03   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.27±0.03   | 0.39±0.02   | 0.46±0.03   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.31±0.03   | 0.30±0.02   | 0.48±0.03   |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.36±0.03   | 0.28±0.02   | 0.42±0.02   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.39±0.06   | 0.27±0.02   | 0.26±0.02   |
|                          | 0.15                     | 5.00                  | 6.50                  | ±                                                | ±           | 0.204±0.013 | 0.199±0.010 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.136±0.009 |
|                          |                          | 0.50                  | 1.00                  | 0.18±0.03                                        | 0.30±0.04   | 0.49±0.05   | 0.62±0.07   |
|                          |                          | 1.00                  | 1.50                  | 0.17±0.02                                        | 0.23±0.03   | 0.42±0.04   | 0.59±0.05   |
|                          |                          | 1.50                  | 2.00                  | 0.22±0.03                                        | 0.19±0.02   | 0.39±0.03   | 0.47±0.04   |
|                          |                          | 2.00                  | 2.50                  | 0.23±0.04                                        | 0.22±0.03   | 0.36±0.03   | 0.46±0.04   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.21±0.03   | 0.30±0.03   | 0.35±0.03   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.17±0.02   | 0.18±0.02   | 0.26±0.02   |
| 0.15                     | 0.20                     | 3.50                  | 4.00                  | ±                                                | 0.15±0.02   | 0.12±0.02   | 0.20±0.02   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.14±0.02   | 0.068±0.009 | 0.140±0.013 |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.040±0.005 | 0.070±0.007 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.025±0.004 |
|                          |                          | 0.50                  | 1.00                  | 0.19±0.03                                        | 0.26±0.04   | 0.46±0.05   | 0.55±0.06   |
|                          |                          | 1.00                  | 1.50                  | 0.15±0.02                                        | 0.22±0.03   | 0.42±0.04   | 0.42±0.04   |
|                          |                          | 1.50                  | 2.00                  | 0.14±0.02                                        | 0.14±0.02   | 0.29±0.03   | 0.35±0.03   |
|                          |                          | 2.00                  | 2.50                  | 0.12±0.02                                        | 0.12±0.02   | 0.21±0.02   | 0.25±0.03   |
|                          | 0.25                     | 2.50                  | 3.00                  | ±                                                | 0.09±0.02   | 0.12±0.02   | 0.19±0.02   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.031±0.008 | 0.079±0.011 | 0.131±0.015 |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.035±0.008 | 0.053±0.010 | 0.081±0.010 |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.029±0.007 | 0.025±0.004 | 0.052±0.007 |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.010±0.002 | 0.018±0.003 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.004±0.001 |
|                          |                          | 0.50                  | 1.00                  | 0.10±0.02                                        | 0.21±0.04   | 0.30±0.04   | 0.32±0.04   |
|                          |                          | 1.00                  | 1.50                  | 0.069±0.015                                      | 0.14±0.03   | 0.18±0.03   | 0.18±0.03   |
| 0.20                     | 0.25                     | 1.50                  | 2.00                  | 0.08±0.02                                        | 0.10±0.02   | 0.14±0.03   | 0.16±0.03   |
|                          |                          | 2.00                  | 2.50                  | 0.036±0.008                                      | 0.07±0.02   | 0.14±0.02   | 0.15±0.03   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.06±0.02   | 0.09±0.02   | 0.09±0.02   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.025±0.007 | 0.056±0.010 | 0.067±0.013 |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.022±0.007 | 0.041±0.008 | 0.046±0.010 |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.011±0.006 | 0.024±0.006 | 0.023±0.006 |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.009±0.004 | 0.007±0.002 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.002±0.002 |



Table 14: HARP results for the double-differential  $\pi^+$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Al interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.050                    | 0.100                    | 0.50                  | 1.00                  | 0.14±0.04                                            | 0.23±0.03   | 0.30±0.05   | 0.60±0.21   |
|                          |                          | 1.00                  | 1.50                  | 0.09±0.02                                            | 0.22±0.03   | 0.33±0.04   | 0.47±0.14   |
|                          |                          | 1.50                  | 2.00                  | 0.118±0.014                                          | 0.21±0.02   | 0.32±0.03   | 0.46±0.13   |
|                          |                          | 2.00                  | 2.50                  | 0.32±0.03                                            | 0.24±0.02   | 0.35±0.03   | 0.59±0.18   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.29±0.02   | 0.33±0.03   | 0.36±0.21   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.38±0.04   | 0.35±0.03   | 0.39±0.09   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.40±0.02   | 0.30±0.03   | 0.52±0.12   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.43±0.14   | 0.36±0.02   | 0.29±0.06   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.26±0.04   | 0.20±0.04   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.16±0.04   |
|                          | 0.100                    | 0.50                  | 1.00                  | 0.10±0.03                                            | 0.27±0.04   | 0.31±0.04   | 0.53±0.18   |
|                          |                          | 1.00                  | 1.50                  | 0.19±0.03                                            | 0.24±0.03   | 0.33±0.04   | 0.50±0.15   |
|                          |                          | 1.50                  | 2.00                  | 0.22±0.03                                            | 0.22±0.03   | 0.34±0.04   | 0.40±0.12   |
|                          |                          | 2.00                  | 2.50                  | 0.31±0.04                                            | 0.23±0.02   | 0.35±0.04   | 0.43±0.14   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.25±0.03   | 0.24±0.03   | 0.30±0.12   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.19±0.02   | 0.20±0.02   | 0.38±0.10   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.16±0.02   | 0.14±0.02   | 0.22±0.06   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.14±0.05   | 0.11±0.02   | 0.14±0.04   |
| 0.150                    | 0.200                    | 5.00                  | 6.50                  | ±                                                    | ±           | 0.037±0.007 | 0.07±0.03   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.023±0.014 |
|                          |                          | 0.50                  | 1.00                  | 0.17±0.04                                            | 0.30±0.04   | 0.47±0.06   | 0.29±0.13   |
|                          |                          | 1.00                  | 1.50                  | 0.13±0.02                                            | 0.21±0.03   | 0.35±0.04   | 0.50±0.16   |
|                          |                          | 1.50                  | 2.00                  | 0.14±0.02                                            | 0.15±0.02   | 0.26±0.03   | 0.15±0.07   |
|                          |                          | 2.00                  | 2.50                  | 0.17±0.03                                            | 0.13±0.02   | 0.20±0.03   | 0.24±0.11   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.069±0.011 | 0.12±0.02   | 0.25±0.10   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.06±0.02   | 0.10±0.02   | 0.11±0.06   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.055±0.013 | 0.08±0.02   | 0.02±0.02   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.04±0.02   | 0.043±0.009 | 0.03±0.03   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.014±0.005 | 0.05±0.04   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.005±0.009 |
|                          | 0.200                    | 0.50                  | 1.00                  | 0.10±0.04                                            | 0.22±0.03   | 0.32±0.05   | 0.35±0.18   |
|                          |                          | 1.00                  | 1.50                  | 0.06±0.02                                            | 0.16±0.03   | 0.17±0.03   | 0.13±0.09   |
|                          |                          | 1.50                  | 2.00                  | 0.042±0.014                                          | 0.10±0.02   | 0.17±0.03   | 0.17±0.19   |
|                          |                          | 2.00                  | 2.50                  | 0.031±0.015                                          | 0.11±0.02   | 0.16±0.03   | 0.042±0.048 |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.08±0.02   | 0.11±0.03   | 0.046±0.101 |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.034±0.010 | 0.07±0.02   | 0.015±0.027 |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.018±0.007 | 0.035±0.012 | 0.014±0.074 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.014±0.007 | 0.021±0.008 | 0.014±0.098 |
| 0.200                    | 0.250                    | 5.00                  | 6.50                  | ±                                                    | ±           | 0.007±0.004 | 0.007±0.038 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.003±0.077 |

Table 15: HARP results for the double-differential  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Al interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^\pi/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|--------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                          | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.050                    | 0.100                    | 0.50                  | 1.00                  | 0.05±0.02                                        | 0.16±0.03   | 0.25±0.04   | 0.19±0.12   |
|                          |                          | 1.00                  | 1.50                  | 0.07±0.02                                        | 0.16±0.02   | 0.27±0.03   | 0.38±0.13   |
|                          |                          | 1.50                  | 2.00                  | 0.09±0.02                                        | 0.12±0.02   | 0.25±0.03   | 0.31±0.10   |
|                          |                          | 2.00                  | 2.50                  | 0.09±0.03                                        | 0.120±0.014 | 0.26±0.03   | 0.27±0.09   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.107±0.013 | 0.15±0.02   | 0.39±0.11   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.102±0.014 | 0.16±0.02   | 0.26±0.08   |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.095±0.011 | 0.13±0.02   | 0.25±0.08   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.050±0.010 | 0.081±0.009 | 0.13±0.04   |
|                          | 0.150                    | 5.00                  | 6.50                  | ±                                                | ±           | 0.048±0.006 | 0.06±0.02   |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.024±0.013 |
|                          |                          | 0.50                  | 1.00                  | 0.13±0.03                                        | 0.24±0.04   | 0.35±0.05   | 0.28±0.12   |
|                          |                          | 1.00                  | 1.50                  | 0.06±0.02                                        | 0.17±0.02   | 0.31±0.04   | 0.49±0.14   |
|                          |                          | 1.50                  | 2.00                  | 0.08±0.02                                        | 0.11±0.02   | 0.26±0.03   | 0.31±0.10   |
|                          |                          | 2.00                  | 2.50                  | 0.025±0.009                                      | 0.090±0.012 | 0.17±0.02   | 0.25±0.09   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.084±0.012 | 0.14±0.02   | 0.20±0.08   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.047±0.007 | 0.109±0.015 | 0.27±0.09   |
| 0.150                    | 0.200                    | 3.50                  | 4.00                  | ±                                                | 0.023±0.005 | 0.077±0.012 | 0.12±0.06   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.013±0.004 | 0.038±0.007 | 0.03±0.02   |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.006±0.002 | 0.02±0.02   |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.004±0.005 |
|                          |                          | 0.50                  | 1.00                  | 0.15±0.04                                        | 0.21±0.03   | 0.38±0.06   | 0.61±0.22   |
|                          |                          | 1.00                  | 1.50                  | 0.06±0.02                                        | 0.12±0.02   | 0.26±0.03   | 0.33±0.12   |
|                          |                          | 1.50                  | 2.00                  | 0.016±0.007                                      | 0.087±0.014 | 0.19±0.03   | 0.34±0.12   |
|                          |                          | 2.00                  | 2.50                  | 0.012±0.007                                      | 0.065±0.011 | 0.18±0.02   | 0.23±0.10   |
|                          | 0.250                    | 2.50                  | 3.00                  | ±                                                | 0.037±0.008 | 0.085±0.015 | 0.09±0.05   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.022±0.006 | 0.043±0.009 | 0.14±0.10   |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.005±0.002 | 0.020±0.005 | 0.02±0.02   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.004±0.002 | 0.011±0.004 | 0.03±0.03   |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.002±0.001 | *±*         |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.001±0.005 |
|                          |                          | 0.50                  | 1.00                  | 0.06±0.02                                        | 0.23±0.04   | 0.31±0.05   | 0.15±0.09   |
|                          |                          | 1.00                  | 1.50                  | 0.11±0.03                                        | 0.11±0.02   | 0.23±0.04   | 0.27±0.15   |
| 0.200                    | 0.250                    | 1.50                  | 2.00                  | 0.018±0.010                                      | 0.06±0.02   | 0.24±0.04   | 0.30±0.19   |
|                          |                          | 2.00                  | 2.50                  | 0.007±0.006                                      | 0.018±0.005 | 0.13±0.03   | 0.257±0.275 |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.010±0.004 | 0.07±0.02   | 0.10±0.19   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.003±0.002 | 0.027±0.010 | 0.016±0.075 |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.002±0.002 | 0.009±0.005 | 0.000±0.014 |
|                          |                          | 4.00                  | 5.00                  | ±                                                | ±           | 0.003±0.005 | 0.000±0.016 |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.002±0.003 | 0.000±0.003 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.000±0.037 |

Table 16: HARP results for the double-differential  $\pi^+$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Cu interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.05                     | 0.10                     | 0.50                  | 1.00                  | 0.11±0.02                                            | 0.24±0.03   | 0.63±0.06   | 0.70±0.08   |
|                          |                          | 1.00                  | 1.50                  | 0.09±0.02                                            | 0.26±0.03   | 0.62±0.04   | 0.79±0.07   |
|                          |                          | 1.50                  | 2.00                  | 0.10±0.02                                            | 0.20±0.02   | 0.53±0.03   | 0.69±0.06   |
|                          |                          | 2.00                  | 2.50                  | 0.13±0.02                                            | 0.16±0.02   | 0.39±0.03   | 0.69±0.05   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.13±0.02   | 0.27±0.03   | 0.54±0.04   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.11±0.02   | 0.29±0.03   | 0.53±0.04   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.13±0.02   | 0.19±0.03   | 0.40±0.04   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.06±0.02   | 0.143±0.014 | 0.26±0.02   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.066±0.010 | 0.124±0.014 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.062±0.008 |
|                          |                          | 0.50                  | 1.00                  | 0.19±0.03                                            | 0.35±0.04   | 0.70±0.07   | 0.91±0.10   |
|                          |                          | 1.00                  | 1.50                  | 0.11±0.02                                            | 0.27±0.03   | 0.55±0.05   | 0.79±0.08   |
|                          |                          | 1.50                  | 2.00                  | 0.09±0.02                                            | 0.21±0.02   | 0.45±0.04   | 0.69±0.07   |
|                          |                          | 2.00                  | 2.50                  | 0.081±0.014                                          | 0.15±0.02   | 0.37±0.03   | 0.54±0.06   |
| 0.10                     | 0.15                     | 2.50                  | 3.00                  | ±                                                    | 0.095±0.014 | 0.27±0.03   | 0.42±0.05   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.073±0.013 | 0.15±0.02   | 0.41±0.04   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.040±0.008 | 0.11±0.02   | 0.25±0.03   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.010±0.006 | 0.051±0.010 | 0.13±0.02   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.010±0.003 | 0.043±0.008 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.006±0.002 |
|                          |                          | 0.50                  | 1.00                  | 0.12±0.02                                            | 0.32±0.04   | 0.70±0.07   | 0.95±0.11   |
|                          |                          | 1.00                  | 1.50                  | 0.11±0.02                                            | 0.22±0.03   | 0.45±0.04   | 0.57±0.06   |
|                          |                          | 1.50                  | 2.00                  | 0.055±0.010                                          | 0.13±0.02   | 0.36±0.03   | 0.46±0.05   |
|                          |                          | 2.00                  | 2.50                  | 0.052±0.011                                          | 0.13±0.02   | 0.29±0.03   | 0.39±0.05   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.051±0.011 | 0.12±0.02   | 0.27±0.04   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.028±0.007 | 0.067±0.012 | 0.16±0.02   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.005±0.004 | 0.050±0.009 | 0.068±0.015 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | ±           | 0.021±0.005 | 0.045±0.010 |
| 0.15                     | 0.20                     | 5.00                  | 6.50                  | ±                                                    | ±           | 0.002±0.001 | 0.006±0.003 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.000±0.001 |
|                          |                          | 0.50                  | 1.00                  | 0.22±0.04                                            | 0.27±0.04   | 0.51±0.07   | 0.76±0.11   |
|                          |                          | 1.00                  | 1.50                  | 0.14±0.03                                            | 0.26±0.04   | 0.71±0.08   | 0.72±0.10   |
|                          |                          | 1.50                  | 2.00                  | 0.07±0.02                                            | 0.17±0.03   | 0.46±0.06   | 0.65±0.10   |
|                          |                          | 2.00                  | 2.50                  | 0.035±0.012                                          | 0.09±0.02   | 0.27±0.04   | 0.35±0.06   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.039±0.011 | 0.11±0.02   | 0.18±0.04   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.003±0.002 | 0.028±0.010 | 0.06±0.02   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | ±           | 0.011±0.006 | 0.028±0.013 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | ±           | 0.005±0.003 | 0.008±0.009 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | ±           | 0.000±0.001 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.000±0.009 |
|                          |                          | 0.50                  | 1.00                  | 0.22±0.04                                            | 0.27±0.04   | 0.51±0.07   | 0.76±0.11   |
|                          |                          | 1.00                  | 1.50                  | 0.14±0.03                                            | 0.26±0.04   | 0.71±0.08   | 0.72±0.10   |
|                          |                          | 1.50                  | 2.00                  | 0.07±0.02                                            | 0.17±0.03   | 0.46±0.06   | 0.65±0.10   |
|                          |                          | 2.00                  | 2.50                  | 0.035±0.012                                          | 0.09±0.02   | 0.27±0.04   | 0.35±0.06   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.039±0.011 | 0.11±0.02   | 0.18±0.04   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.003±0.002 | 0.028±0.010 | 0.06±0.02   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | ±           | 0.011±0.006 | 0.028±0.013 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | ±           | 0.005±0.003 | 0.008±0.009 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | ±           | 0.000±0.001 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.000±0.009 |

Table 17: HARP results for the double-differential  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Cu interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.05                     | 0.10                     | 0.50                  | 1.00                  | 0.24±0.03                                            | 0.45±0.05   | 0.91±0.08   | 1.12±0.12   |
|                          |                          | 1.00                  | 1.50                  | 0.21±0.03                                            | 0.32±0.03   | 0.71±0.05   | 0.96±0.08   |
|                          |                          | 1.50                  | 2.00                  | 0.26±0.03                                            | 0.34±0.03   | 0.75±0.04   | 0.94±0.07   |
|                          |                          | 2.00                  | 2.50                  | 0.15±0.04                                            | 0.36±0.03   | 0.68±0.04   | 1.03±0.07   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.41±0.03   | 0.63±0.04   | 0.76±0.06   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.43±0.04   | 0.47±0.04   | 0.78±0.06   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.53±0.04   | 0.41±0.03   | 0.68±0.05   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.51±0.08   | 0.42±0.03   | 0.48±0.03   |
|                          | 0.15                     | 5.00                  | 6.50                  | ±                                                    | ±           | 0.30±0.02   | 0.34±0.02   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.21±0.02   |
|                          |                          | 0.50                  | 1.00                  | 0.33±0.04                                            | 0.58±0.06   | 1.01±0.11   | 1.10±0.12   |
|                          |                          | 1.00                  | 1.50                  | 0.24±0.03                                            | 0.40±0.04   | 0.80±0.06   | 1.00±0.09   |
|                          |                          | 1.50                  | 2.00                  | 0.26±0.03                                            | 0.34±0.03   | 0.67±0.05   | 0.85±0.08   |
|                          |                          | 2.00                  | 2.50                  | 0.22±0.04                                            | 0.31±0.03   | 0.63±0.05   | 0.82±0.07   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.31±0.03   | 0.40±0.03   | 0.63±0.06   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.34±0.03   | 0.31±0.03   | 0.49±0.05   |
| 0.15                     | 0.20                     | 3.50                  | 4.00                  | ±                                                    | 0.21±0.03   | 0.18±0.03   | 0.33±0.03   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.21±0.04   | 0.118±0.014 | 0.25±0.03   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.059±0.007 | 0.100±0.013 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.035±0.006 |
|                          |                          | 0.50                  | 1.00                  | 0.22±0.04                                            | 0.42±0.05   | 0.88±0.09   | 1.05±0.13   |
|                          |                          | 1.00                  | 1.50                  | 0.21±0.03                                            | 0.33±0.04   | 0.76±0.07   | 0.86±0.09   |
|                          |                          | 1.50                  | 2.00                  | 0.15±0.02                                            | 0.27±0.03   | 0.51±0.05   | 0.65±0.07   |
|                          |                          | 2.00                  | 2.50                  | 0.17±0.02                                            | 0.19±0.02   | 0.35±0.03   | 0.50±0.05   |
|                          | 0.25                     | 2.50                  | 3.00                  | ±                                                    | 0.12±0.02   | 0.20±0.02   | 0.32±0.04   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.073±0.012 | 0.114±0.015 | 0.19±0.03   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.039±0.008 | 0.097±0.012 | 0.15±0.02   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.036±0.007 | 0.049±0.007 | 0.088±0.013 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.019±0.003 | 0.037±0.007 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.009±0.003 |
|                          |                          | 0.50                  | 1.00                  | 0.17±0.03                                            | 0.39±0.06   | 0.59±0.07   | 0.72±0.10   |
|                          |                          | 1.00                  | 1.50                  | 0.11±0.02                                            | 0.18±0.03   | 0.39±0.05   | 0.38±0.06   |
| 0.20                     | 0.25                     | 1.50                  | 2.00                  | 0.11±0.02                                            | 0.14±0.03   | 0.31±0.05   | 0.42±0.07   |
|                          |                          | 2.00                  | 2.50                  | 0.056±0.011                                          | 0.07±0.02   | 0.35±0.05   | 0.31±0.06   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.047±0.011 | 0.23±0.04   | 0.16±0.04   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.040±0.009 | 0.13±0.02   | 0.13±0.03   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.032±0.007 | 0.070±0.013 | 0.11±0.02   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.026±0.006 | 0.041±0.008 | 0.08±0.02   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.013±0.004 | 0.029±0.010 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.006±0.006 |

Table 18: HARP results for the double-differential  $\pi^+$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Cu interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.050                    | 0.100                    | 0.50                  | 1.00                  | 0.16±0.06                                            | 0.32±0.05   | 0.58±0.09   | 0.74±0.25   |
|                          |                          | 1.00                  | 1.50                  | 0.08±0.02                                            | 0.32±0.04   | 0.60±0.06   | 0.81±0.21   |
|                          |                          | 1.50                  | 2.00                  | 0.15±0.02                                            | 0.27±0.03   | 0.57±0.06   | 0.75±0.20   |
|                          |                          | 2.00                  | 2.50                  | 0.43±0.05                                            | 0.34±0.03   | 0.56±0.06   | 0.68±0.18   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.49±0.04   | 0.54±0.05   | 0.77±0.19   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.60±0.06   | 0.43±0.05   | 0.68±0.15   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.73±0.04   | 0.46±0.05   | 0.71±0.16   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.68±0.22   | 0.48±0.03   | 0.55±0.10   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.41±0.06   | 0.41±0.07   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.18±0.05   |
| 0.100                    | 0.150                    | 0.50                  | 1.00                  | 0.12±0.05                                            | 0.45±0.06   | 0.64±0.08   | 0.86±0.27   |
|                          |                          | 1.00                  | 1.50                  | 0.18±0.05                                            | 0.38±0.05   | 0.51±0.06   | 0.95±0.25   |
|                          |                          | 1.50                  | 2.00                  | 0.28±0.05                                            | 0.40±0.05   | 0.68±0.07   | 0.79±0.20   |
|                          |                          | 2.00                  | 2.50                  | 0.51±0.07                                            | 0.34±0.04   | 0.56±0.06   | 0.56±0.16   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.34±0.04   | 0.31±0.04   | 0.65±0.16   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.33±0.03   | 0.31±0.04   | 0.42±0.13   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.27±0.03   | 0.21±0.03   | 0.23±0.07   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.27±0.10   | 0.16±0.03   | 0.23±0.07   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.066±0.014 | 0.07±0.03   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.03±0.02   |
| 0.150                    | 0.200                    | 0.50                  | 1.00                  | 0.21±0.07                                            | 0.50±0.07   | 0.74±0.10   | 0.85±0.30   |
|                          |                          | 1.00                  | 1.50                  | 0.22±0.05                                            | 0.32±0.04   | 0.55±0.06   | 0.58±0.19   |
|                          |                          | 1.50                  | 2.00                  | 0.23±0.05                                            | 0.26±0.03   | 0.39±0.05   | 0.68±0.19   |
|                          |                          | 2.00                  | 2.50                  | 0.14±0.04                                            | 0.19±0.03   | 0.30±0.04   | 0.72±0.21   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.13±0.02   | 0.24±0.04   | 0.28±0.11   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.09±0.02   | 0.15±0.03   | 0.23±0.11   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.07±0.02   | 0.069±0.015 | 0.16±0.08   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.05±0.03   | 0.043±0.010 | 0.14±0.07   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.017±0.007 | 0.021±0.024 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.007±0.010 |
| 0.200                    | 0.250                    | 0.50                  | 1.00                  | 0.18±0.09                                            | 0.28±0.05   | 0.53±0.08   | 0.41±0.22   |
|                          |                          | 1.00                  | 1.50                  | 0.07±0.03                                            | 0.23±0.04   | 0.33±0.06   | 0.34±0.26   |
|                          |                          | 1.50                  | 2.00                  | 0.10±0.04                                            | 0.14±0.03   | 0.26±0.05   | 0.21±0.17   |
|                          |                          | 2.00                  | 2.50                  | 0.05±0.03                                            | 0.15±0.03   | 0.25±0.05   | 0.14±0.20   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.09±0.02   | 0.14±0.04   | 0.124±0.253 |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.041±0.012 | 0.10±0.03   | 0.037±0.087 |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.021±0.009 | 0.06±0.02   | 0.034±0.108 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.021±0.011 | 0.038±0.014 | 0.037±0.124 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.018±0.011 | 0.012±0.036 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.003±0.102 |

Table 19: HARP results for the double-differential  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Cu interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.050                    | 0.100                    | 0.50                  | 1.00                  | 0.11±0.04                                            | 0.27±0.04   | 0.51±0.07   | 0.33±0.16   |
|                          |                          | 1.00                  | 1.50                  | 0.13±0.04                                            | 0.24±0.03   | 0.54±0.06   | 0.72±0.21   |
|                          |                          | 1.50                  | 2.00                  | 0.13±0.03                                            | 0.20±0.03   | 0.41±0.05   | 0.68±0.19   |
|                          |                          | 2.00                  | 2.50                  | 0.11±0.04                                            | 0.19±0.02   | 0.38±0.04   | 0.47±0.14   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.16±0.02   | 0.27±0.03   | 0.83±0.20   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.16±0.02   | 0.25±0.03   | 0.29±0.09   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.18±0.02   | 0.18±0.02   | 0.41±0.12   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.07±0.02   | 0.17±0.02   | 0.29±0.08   |
|                          | 0.150                    | 5.00                  | 6.50                  | ±                                                    | ±           | 0.065±0.008 | 0.12±0.04   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.09±0.04   |
|                          |                          | 0.50                  | 1.00                  | 0.27±0.07                                            | 0.45±0.06   | 0.65±0.09   | 1.10±0.35   |
|                          |                          | 1.00                  | 1.50                  | 0.14±0.04                                            | 0.23±0.03   | 0.46±0.05   | 0.85±0.22   |
|                          |                          | 1.50                  | 2.00                  | 0.10±0.03                                            | 0.21±0.03   | 0.38±0.04   | 0.31±0.12   |
|                          |                          | 2.00                  | 2.50                  | 0.04±0.02                                            | 0.18±0.02   | 0.29±0.04   | 0.43±0.15   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.100±0.015 | 0.24±0.03   | 0.44±0.14   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.077±0.013 | 0.18±0.02   | 0.34±0.11   |
| 0.150                    | 0.200                    | 3.50                  | 4.00                  | ±                                                    | 0.031±0.006 | 0.082±0.014 | 0.36±0.12   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.022±0.007 | 0.073±0.012 | 0.13±0.06   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.018±0.004 | 0.04±0.03   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.002±0.003 |
|                          |                          | 0.50                  | 1.00                  | 0.11±0.04                                            | 0.42±0.06   | 0.58±0.08   | 0.79±0.27   |
|                          |                          | 1.00                  | 1.50                  | 0.10±0.04                                            | 0.23±0.03   | 0.42±0.06   | 0.66±0.23   |
|                          |                          | 1.50                  | 2.00                  | 0.11±0.04                                            | 0.23±0.03   | 0.33±0.04   | 0.31±0.13   |
|                          |                          | 2.00                  | 2.50                  | 0.019±0.013                                          | 0.11±0.02   | 0.28±0.04   | 0.23±0.11   |
|                          | 0.250                    | 2.50                  | 3.00                  | ±                                                    | 0.064±0.013 | 0.16±0.03   | 0.16±0.09   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.016±0.004 | 0.08±0.02   | 0.22±0.10   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.012±0.005 | 0.037±0.010 | 0.18±0.09   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.005±0.003 | 0.020±0.006 | 0.11±0.07   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.004±0.002 | 0.03±0.03   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | ±           |
|                          |                          | 0.50                  | 1.00                  | 0.17±0.07                                            | 0.33±0.06   | 0.60±0.09   | 0.58±0.26   |
|                          |                          | 1.00                  | 1.50                  | 0.06±0.03                                            | 0.20±0.04   | 0.51±0.08   | 0.44±0.23   |
| 0.200                    | 0.250                    | 1.50                  | 2.00                  | 0.08±0.04                                            | 0.09±0.02   | 0.29±0.05   | 0.72±0.37   |
|                          |                          | 2.00                  | 2.50                  | 0.009±0.011                                          | 0.024±0.007 | 0.13±0.03   | 0.16±0.18   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.014±0.005 | 0.043±0.012 | 0.07±0.11   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.017±0.007 | 0.020±0.009 | 0.050±0.065 |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.008±0.004 | 0.008±0.006 | 0.010±0.031 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.005±0.004 | 0.005±0.007 | 0.019±0.109 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.002±0.004 | 0.016±0.090 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.003±0.068 |

Table 20: HARP results for the double-differential  $\pi^+$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Sn interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.05                     | 0.10                     | 0.50                  | 1.00                  | 0.11±0.03                                            | 0.32±0.05   | 0.68±0.08   | 1.15±0.12   |
|                          |                          | 1.00                  | 1.50                  | 0.07±0.02                                            | 0.34±0.04   | 0.89±0.07   | 1.25±0.09   |
|                          |                          | 1.50                  | 2.00                  | 0.14±0.03                                            | 0.29±0.04   | 0.72±0.05   | 1.02±0.08   |
|                          |                          | 2.00                  | 2.50                  | 0.11±0.03                                            | 0.13±0.02   | 0.52±0.05   | 0.93±0.07   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.16±0.03   | 0.37±0.04   | 0.75±0.05   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.17±0.02   | 0.36±0.05   | 0.69±0.06   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.17±0.02   | 0.21±0.04   | 0.50±0.04   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.06±0.03   | 0.15±0.02   | 0.36±0.03   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.080±0.013 | 0.19±0.02   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.081±0.011 |
|                          |                          | 0.50                  | 1.00                  | 0.18±0.03                                            | 0.42±0.06   | 0.94±0.10   | 1.21±0.12   |
|                          |                          | 1.00                  | 1.50                  | 0.07±0.02                                            | 0.35±0.04   | 0.68±0.07   | 1.23±0.11   |
|                          |                          | 1.50                  | 2.00                  | 0.08±0.02                                            | 0.23±0.03   | 0.61±0.06   | 1.07±0.09   |
|                          |                          | 2.00                  | 2.50                  | 0.12±0.03                                            | 0.21±0.03   | 0.50±0.05   | 0.87±0.08   |
| 0.10                     | 0.15                     | 2.50                  | 3.00                  | ±                                                    | 0.14±0.02   | 0.34±0.04   | 0.65±0.06   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.09±0.02   | 0.20±0.03   | 0.55±0.05   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.036±0.009 | 0.17±0.03   | 0.41±0.04   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.022±0.009 | 0.07±0.02   | 0.21±0.02   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.013±0.005 | 0.067±0.011 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.008±0.003 |
|                          |                          | 0.50                  | 1.00                  | 0.22±0.05                                            | 0.45±0.07   | 0.92±0.11   | 1.22±0.13   |
|                          |                          | 1.00                  | 1.50                  | 0.10±0.03                                            | 0.24±0.03   | 0.57±0.06   | 0.86±0.08   |
|                          |                          | 1.50                  | 2.00                  | 0.040±0.012                                          | 0.24±0.03   | 0.50±0.05   | 0.82±0.07   |
|                          |                          | 2.00                  | 2.50                  | 0.05±0.02                                            | 0.14±0.03   | 0.34±0.04   | 0.52±0.06   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.06±0.02   | 0.19±0.03   | 0.33±0.04   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.031±0.010 | 0.12±0.02   | 0.20±0.03   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.021±0.008 | 0.07±0.02   | 0.09±0.02   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.004±0.004 | 0.027±0.007 | 0.053±0.010 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.005±0.002 | 0.013±0.004 |
| 0.15                     | 0.20                     | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.002±0.001 |
|                          |                          | 0.50                  | 1.00                  | 0.23±0.06                                            | 0.45±0.08   | 0.73±0.10   | 1.14±0.15   |
|                          |                          | 1.00                  | 1.50                  | 0.17±0.04                                            | 0.36±0.06   | 0.75±0.10   | 1.14±0.14   |
|                          |                          | 1.50                  | 2.00                  | 0.08±0.03                                            | 0.29±0.05   | 0.56±0.09   | 0.75±0.11   |
|                          |                          | 2.00                  | 2.50                  | 0.002±0.005                                          | 0.15±0.03   | 0.23±0.05   | 0.56±0.09   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.05±0.02   | 0.13±0.03   | 0.25±0.05   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.013±0.007 | 0.04±0.02   | 0.09±0.02   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.003±0.002 | 0.014±0.010 | 0.047±0.015 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.001±0.001 | 0.010±0.009 | 0.018±0.011 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.001±0.001 | 0.003±0.002 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.001±0.009 |
|                          |                          | 0.50                  | 1.00                  | 0.23±0.06                                            | 0.45±0.08   | 0.73±0.10   | 1.14±0.15   |
|                          |                          | 1.00                  | 1.50                  | 0.17±0.04                                            | 0.36±0.06   | 0.75±0.10   | 1.14±0.14   |
|                          |                          | 1.50                  | 2.00                  | 0.08±0.03                                            | 0.29±0.05   | 0.56±0.09   | 0.75±0.11   |
|                          |                          | 2.00                  | 2.50                  | 0.002±0.005                                          | 0.15±0.03   | 0.23±0.05   | 0.56±0.09   |
| 0.20                     | 0.25                     | 2.50                  | 3.00                  | ±                                                    | 0.05±0.02   | 0.13±0.03   | 0.25±0.05   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.013±0.007 | 0.04±0.02   | 0.09±0.02   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.003±0.002 | 0.014±0.010 | 0.047±0.015 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.001±0.001 | 0.010±0.009 | 0.018±0.011 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.001±0.001 | 0.003±0.002 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.001±0.009 |

Table 21: HARP results for the double-differential  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Sn interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.05                     | 0.10                     | 0.50                  | 1.00                  | 0.30±0.05                                            | 0.62±0.07   | 1.26±0.11   | 1.72±0.16   |
|                          |                          | 1.00                  | 1.50                  | 0.19±0.04                                            | 0.41±0.04   | 0.90±0.07   | 1.29±0.10   |
|                          |                          | 1.50                  | 2.00                  | 0.17±0.03                                            | 0.46±0.05   | 0.94±0.06   | 1.38±0.09   |
|                          |                          | 2.00                  | 2.50                  | 0.12±0.03                                            | 0.51±0.05   | 0.91±0.07   | 1.34±0.09   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.46±0.04   | 0.83±0.06   | 1.24±0.08   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.61±0.06   | 0.66±0.06   | 1.02±0.07   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.62±0.06   | 0.54±0.05   | 0.94±0.07   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.66±0.10   | 0.56±0.05   | 0.67±0.05   |
|                          | 0.15                     | 5.00                  | 6.50                  | ±                                                    | ±           | 0.38±0.03   | 0.46±0.03   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.29±0.02   |
|                          |                          | 0.50                  | 1.00                  | 0.27±0.05                                            | 0.76±0.09   | 1.39±0.15   | 1.91±0.20   |
|                          |                          | 1.00                  | 1.50                  | 0.25±0.04                                            | 0.41±0.05   | 1.00±0.09   | 1.35±0.11   |
|                          |                          | 1.50                  | 2.00                  | 0.23±0.04                                            | 0.49±0.05   | 0.81±0.07   | 1.35±0.11   |
|                          |                          | 2.00                  | 2.50                  | 0.44±0.08                                            | 0.39±0.04   | 0.73±0.06   | 0.96±0.08   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.33±0.04   | 0.64±0.06   | 0.91±0.08   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.40±0.05   | 0.37±0.04   | 0.64±0.06   |
| 0.15                     | 0.20                     | 3.50                  | 4.00                  | ±                                                    | 0.25±0.04   | 0.28±0.04   | 0.57±0.05   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.28±0.06   | 0.17±0.02   | 0.34±0.03   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.100±0.013 | 0.141±0.015 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.053±0.009 |
|                          |                          | 0.50                  | 1.00                  | 0.25±0.05                                            | 0.65±0.09   | 1.07±0.13   | 1.69±0.18   |
|                          |                          | 1.00                  | 1.50                  | 0.36±0.06                                            | 0.43±0.05   | 0.99±0.09   | 1.33±0.12   |
|                          |                          | 1.50                  | 2.00                  | 0.22±0.04                                            | 0.30±0.04   | 0.66±0.07   | 0.95±0.09   |
|                          |                          | 2.00                  | 2.50                  | 0.14±0.02                                            | 0.22±0.03   | 0.38±0.05   | 0.65±0.06   |
|                          | 0.25                     | 2.50                  | 3.00                  | ±                                                    | 0.19±0.03   | 0.27±0.03   | 0.46±0.05   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.10±0.02   | 0.15±0.02   | 0.28±0.03   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.062±0.012 | 0.11±0.02   | 0.18±0.03   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.077±0.015 | 0.065±0.011 | 0.11±0.02   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.026±0.005 | 0.047±0.008 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.012±0.003 |
|                          |                          | 0.50                  | 1.00                  | 0.17±0.04                                            | 0.55±0.09   | 0.65±0.09   | 1.08±0.14   |
|                          |                          | 1.00                  | 1.50                  | 0.12±0.03                                            | 0.24±0.04   | 0.37±0.05   | 0.49±0.07   |
| 0.20                     | 0.25                     | 1.50                  | 2.00                  | 0.12±0.03                                            | 0.24±0.05   | 0.35±0.07   | 0.43±0.07   |
|                          |                          | 2.00                  | 2.50                  | 0.08±0.02                                            | 0.15±0.03   | 0.26±0.05   | 0.42±0.07   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.09±0.02   | 0.18±0.03   | 0.30±0.06   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.061±0.014 | 0.12±0.02   | 0.24±0.04   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.040±0.010 | 0.08±0.02   | 0.13±0.03   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.019±0.007 | 0.052±0.015 | 0.08±0.02   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.017±0.007 | 0.019±0.006 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.004±0.005 |



Table 22: HARP results for the double-differential  $\pi^+$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Sn interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.050                    | 0.100                    | 0.50                  | 1.00                  | 0.28±0.07                                            | 0.48±0.08   | 0.99±0.14   | 1.11±0.27   |
|                          |                          |                       | 1.00                  | 0.09±0.02                                            | 0.35±0.05   | 0.80±0.08   | 1.32±0.23   |
|                          |                          | 1.50                  | 2.00                  | 0.17±0.02                                            | 0.35±0.05   | 0.76±0.15   | 1.14±0.21   |
|                          |                          | 2.00                  | 2.50                  | 0.46±0.04                                            | 0.55±0.05   | 0.68±0.21   | 1.05±0.21   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.57±0.04   | 0.64±0.17   | 1.12±0.28   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.74±0.07   | 0.55±0.06   | 0.96±0.15   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.93±0.05   | 0.55±0.07   | 0.78±0.13   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.85±0.28   | 0.68±0.06   | 0.73±0.10   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.61±0.10   | 0.49±0.07   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.41±0.06   |
| 0.100                    | 0.150                    | 0.50                  | 1.00                  | 0.19±0.06                                            | 0.55±0.08   | 0.94±0.12   | 1.16±0.26   |
|                          |                          |                       | 1.00                  | 0.31±0.07                                            | 0.40±0.06   | 0.75±0.08   | 1.30±0.24   |
|                          |                          | 1.50                  | 2.00                  | 0.43±0.06                                            | 0.48±0.06   | 0.90±0.09   | 0.71±0.15   |
|                          |                          | 2.00                  | 2.50                  | 0.51±0.06                                            | 0.53±0.05   | 0.69±0.09   | 0.98±0.18   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.43±0.05   | 0.47±0.06   | 0.94±0.17   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.42±0.04   | 0.41±0.05   | 0.87±0.15   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.37±0.04   | 0.38±0.04   | 0.47±0.09   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.35±0.13   | 0.22±0.03   | 0.39±0.07   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.060±0.013 | 0.18±0.04   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.06±0.02   |
| 0.150                    | 0.200                    | 0.50                  | 1.00                  | 0.31±0.08                                            | 0.54±0.09   | 0.99±0.13   | 1.29±0.31   |
|                          |                          |                       | 1.00                  | 0.23±0.05                                            | 0.41±0.05   | 0.85±0.09   | 0.97±0.20   |
|                          |                          | 1.50                  | 2.00                  | 0.22±0.04                                            | 0.35±0.04   | 0.64±0.07   | 0.93±0.18   |
|                          |                          | 2.00                  | 2.50                  | 0.29±0.05                                            | 0.31±0.04   | 0.50±0.06   | 0.62±0.15   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.15±0.02   | 0.23±0.04   | 0.40±0.11   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.12±0.03   | 0.18±0.03   | 0.42±0.11   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.08±0.02   | 0.11±0.02   | 0.20±0.07   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.06±0.03   | 0.08±0.02   | 0.11±0.05   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.019±0.008 | 0.04±0.03   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.012±0.014 |
| 0.200                    | 0.250                    | 0.50                  | 1.00                  | 0.09±0.07                                            | 0.52±0.08   | 0.91±0.14   | 1.15±0.31   |
|                          |                          |                       | 1.00                  | 0.09±0.03                                            | 0.23±0.05   | 0.59±0.10   | 0.66±0.20   |
|                          |                          | 1.50                  | 2.00                  | 0.08±0.03                                            | 0.19±0.04   | 0.55±0.10   | 0.74±0.23   |
|                          |                          | 2.00                  | 2.50                  | 0.04±0.02                                            | 0.15±0.03   | 0.36±0.07   | 0.25±0.11   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.11±0.03   | 0.14±0.04   | 0.27±0.16   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.06±0.02   | 0.14±0.04   | 0.15±0.09   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.04±0.02   | 0.10±0.03   | 0.084±0.093 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.022±0.011 | 0.06±0.02   | 0.039±0.072 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.020±0.012 | 0.034±0.049 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.008±0.087 |

Table 23: HARP results for the double-differential  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Sn interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^\pi/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|--------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                          | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.050                    | 0.100                    | 0.50                  | 1.00                  | 0.17±0.05                                        | 0.37±0.06   | 0.78±0.10   | 1.18±0.27   |
|                          |                          | 1.00                  | 1.50                  | 0.14±0.03                                        | 0.34±0.04   | 0.70±0.08   | 0.78±0.17   |
|                          |                          | 1.50                  | 2.00                  | 0.18±0.03                                        | 0.30±0.04   | 0.60±0.06   | 1.03±0.19   |
|                          |                          | 2.00                  | 2.50                  | 0.17±0.05                                        | 0.25±0.03   | 0.51±0.06   | 0.80±0.16   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.20±0.03   | 0.40±0.04   | 0.80±0.15   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.18±0.02   | 0.36±0.04   | 0.62±0.11   |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.20±0.02   | 0.29±0.03   | 0.60±0.11   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.09±0.02   | 0.19±0.02   | 0.36±0.06   |
|                          | 0.150                    | 5.00                  | 6.50                  | ±                                                | ±           | 0.084±0.010 | 0.21±0.04   |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.10±0.02   |
|                          |                          | 0.50                  | 1.00                  | 0.25±0.06                                        | 0.50±0.08   | 1.22±0.16   | 1.18±0.26   |
|                          |                          | 1.00                  | 1.50                  | 0.14±0.03                                        | 0.34±0.05   | 0.71±0.08   | 0.74±0.16   |
|                          |                          | 1.50                  | 2.00                  | 0.12±0.03                                        | 0.28±0.04   | 0.58±0.07   | 0.98±0.19   |
|                          |                          | 2.00                  | 2.50                  | 0.045±0.014                                      | 0.19±0.03   | 0.42±0.05   | 0.71±0.15   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.12±0.02   | 0.30±0.04   | 0.51±0.11   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.094±0.015 | 0.24±0.03   | 0.58±0.12   |
| 0.150                    | 0.200                    | 3.50                  | 4.00                  | ±                                                | 0.055±0.010 | 0.16±0.03   | 0.36±0.08   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.022±0.007 | 0.09±0.02   | 0.20±0.05   |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.017±0.004 | 0.07±0.03   |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.006±0.004 |
|                          |                          | 0.50                  | 1.00                  | 0.14±0.05                                        | 0.45±0.07   | 0.95±0.13   | 1.41±0.32   |
|                          |                          | 1.00                  | 1.50                  | 0.18±0.04                                        | 0.32±0.04   | 0.69±0.08   | 0.86±0.20   |
|                          |                          | 1.50                  | 2.00                  | 0.09±0.03                                        | 0.26±0.04   | 0.54±0.07   | 0.52±0.14   |
|                          |                          | 2.00                  | 2.50                  | 0.025±0.011                                      | 0.15±0.02   | 0.38±0.05   | 0.40±0.12   |
|                          | 0.250                    | 2.50                  | 3.00                  | ±                                                | 0.09±0.02   | 0.20±0.03   | 0.20±0.08   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.036±0.009 | 0.10±0.02   | 0.23±0.08   |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.021±0.007 | 0.041±0.010 | 0.14±0.06   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.006±0.003 | 0.030±0.009 | 0.06±0.04   |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.005±0.003 | 0.007±0.008 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.004±0.007 |
|                          |                          | 0.50                  | 1.00                  | 0.19±0.06                                        | 0.40±0.08   | 0.78±0.12   | 1.17±0.30   |
|                          |                          | 1.00                  | 1.50                  | 0.15±0.05                                        | 0.31±0.05   | 0.70±0.11   | 0.78±0.24   |
| 0.200                    | 0.250                    | 1.50                  | 2.00                  | 0.015±0.010                                      | 0.12±0.03   | 0.42±0.08   | 0.72±0.46   |
|                          |                          | 2.00                  | 2.50                  | 0.002±0.002                                      | 0.051±0.013 | 0.39±0.08   | 0.54±1.07   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.023±0.007 | 0.15±0.04   | 0.14±0.35   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.023±0.009 | 0.04±0.02   | 0.02±0.09   |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.009±0.005 | 0.016±0.009 | 0.01±0.03   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.004±0.003 | 0.004±0.007 | 0.000±0.003 |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.000±0.002 | 0.000±0.002 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.000±0.036 |

Table 24: HARP results for the double-differential  $\pi^+$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Ta interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.05                     | 0.10                     | 0.50                  | 1.00                  | 0.08±0.03                                            | 0.42±0.06   | 1.06±0.11   | 1.60±0.17   |
|                          |                          | 1.00                  | 1.50                  | 0.06±0.03                                            | 0.33±0.05   | 1.27±0.09   | 1.67±0.13   |
|                          |                          | 1.50                  | 2.00                  | 0.13±0.05                                            | 0.29±0.04   | 1.04±0.07   | 1.27±0.09   |
|                          |                          | 2.00                  | 2.50                  | 0.17±0.06                                            | 0.24±0.04   | 0.70±0.07   | 1.08±0.08   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.23±0.04   | 0.36±0.06   | 0.89±0.07   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.19±0.03   | 0.37±0.06   | 0.89±0.07   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.17±0.03   | 0.29±0.05   | 0.69±0.07   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.05±0.02   | 0.18±0.03   | 0.43±0.04   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.11±0.02   | 0.22±0.03   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.104±0.014 |
| 0.10                     | 0.15                     | 0.50                  | 1.00                  | 0.13±0.04                                            | 0.46±0.07   | 1.37±0.14   | 1.72±0.17   |
|                          |                          | 1.00                  | 1.50                  | 0.09±0.03                                            | 0.38±0.05   | 0.92±0.09   | 1.48±0.13   |
|                          |                          | 1.50                  | 2.00                  | 0.10±0.05                                            | 0.21±0.04   | 0.68±0.07   | 1.19±0.10   |
|                          |                          | 2.00                  | 2.50                  | 0.04±0.02                                            | 0.23±0.04   | 0.58±0.06   | 0.97±0.09   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.15±0.03   | 0.42±0.05   | 0.78±0.07   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.11±0.02   | 0.22±0.04   | 0.62±0.06   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.06±0.02   | 0.22±0.03   | 0.48±0.05   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.020±0.010 | 0.08±0.02   | 0.24±0.03   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.019±0.007 | 0.074±0.012 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.014±0.003 |
| 0.15                     | 0.20                     | 0.50                  | 1.00                  | 0.11±0.05                                            | 0.45±0.08   | 1.22±0.14   | 1.78±0.19   |
|                          |                          | 1.00                  | 1.50                  | 0.12±0.05                                            | 0.29±0.05   | 0.64±0.07   | 0.98±0.10   |
|                          |                          | 1.50                  | 2.00                  | 0.06±0.02                                            | 0.24±0.04   | 0.51±0.06   | 0.90±0.09   |
|                          |                          | 2.00                  | 2.50                  | 0.06±0.04                                            | 0.18±0.04   | 0.41±0.05   | 0.71±0.08   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.04±0.02   | 0.27±0.04   | 0.45±0.06   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.040±0.015 | 0.16±0.03   | 0.27±0.04   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.009±0.007 | 0.10±0.02   | 0.14±0.03   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.001±0.002 | 0.024±0.007 | 0.046±0.012 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.004±0.002 | 0.011±0.005 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.002±0.001 |
| 0.20                     | 0.25                     | 0.50                  | 1.00                  | 0.25±0.11                                            | 0.53±0.10   | 1.01±0.14   | 1.24±0.17   |
|                          |                          | 1.00                  | 1.50                  | 0.15±0.07                                            | 0.51±0.09   | 0.90±0.12   | 1.32±0.17   |
|                          |                          | 1.50                  | 2.00                  | 0.07±0.05                                            | 0.30±0.06   | 0.79±0.12   | 1.11±0.16   |
|                          |                          | 2.00                  | 2.50                  | 0.02±0.03                                            | 0.20±0.04   | 0.42±0.08   | 0.59±0.10   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.05±0.02   | 0.22±0.05   | 0.32±0.06   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.009±0.005 | 0.05±0.02   | 0.12±0.03   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.005±0.004 | 0.016±0.012 | 0.05±0.02   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.003±0.003 | 0.007±0.007 | 0.015±0.013 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.002±0.002 | 0.001±0.001 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.000±0.013 |

Table 25: HARP results for the double-differential  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Ta interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.05                     | 0.10                     | 0.50                  | 1.00                  | 0.22±0.06                                            | 0.79±0.09   | 1.48±0.13   | 2.16±0.20   |
|                          |                          | 1.00                  | 1.50                  | 0.11±0.03                                            | 0.50±0.06   | 1.10±0.09   | 1.80±0.14   |
|                          |                          | 1.50                  | 2.00                  | 0.09±0.02                                            | 0.58±0.06   | 1.22±0.08   | 1.63±0.11   |
|                          |                          | 2.00                  | 2.50                  | 0.07±0.02                                            | 0.48±0.06   | 1.23±0.10   | 1.59±0.11   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.61±0.07   | 1.00±0.07   | 1.49±0.10   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.63±0.07   | 0.81±0.08   | 1.27±0.08   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.55±0.07   | 0.69±0.07   | 1.02±0.07   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.57±0.10   | 0.67±0.06   | 0.77±0.05   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.49±0.05   | 0.51±0.03   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.32±0.03   |
|                          |                          | 0.50                  | 1.00                  | 0.38±0.08                                            | 0.77±0.10   | 1.80±0.20   | 2.56±0.27   |
|                          |                          | 1.00                  | 1.50                  | 0.22±0.05                                            | 0.59±0.07   | 1.14±0.10   | 1.91±0.16   |
|                          |                          | 1.50                  | 2.00                  | 0.28±0.07                                            | 0.45±0.06   | 0.99±0.09   | 1.68±0.13   |
|                          |                          | 2.00                  | 2.50                  | 0.45±0.09                                            | 0.50±0.06   | 0.91±0.08   | 1.51±0.13   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.44±0.05   | 0.67±0.06   | 1.11±0.10   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.39±0.05   | 0.38±0.05   | 0.89±0.08   |
| 0.10                     | 0.15                     | 3.50                  | 4.00                  | ±                                                    | 0.33±0.04   | 0.31±0.04   | 0.62±0.06   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.31±0.05   | 0.21±0.03   | 0.38±0.04   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.095±0.015 | 0.17±0.02   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.058±0.010 |
|                          |                          | 0.50                  | 1.00                  | 0.34±0.10                                            | 0.95±0.13   | 1.43±0.17   | 2.25±0.25   |
|                          |                          | 1.00                  | 1.50                  | 0.27±0.07                                            | 0.45±0.06   | 1.31±0.12   | 1.89±0.17   |
|                          |                          | 1.50                  | 2.00                  | 0.22±0.07                                            | 0.39±0.06   | 0.88±0.09   | 1.10±0.11   |
|                          |                          | 2.00                  | 2.50                  | 0.16±0.04                                            | 0.26±0.05   | 0.53±0.06   | 0.81±0.08   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.12±0.02   | 0.31±0.04   | 0.46±0.05   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.11±0.02   | 0.17±0.03   | 0.40±0.05   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.07±0.02   | 0.16±0.02   | 0.21±0.03   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.06±0.02   | 0.079±0.014 | 0.16±0.02   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.034±0.007 | 0.047±0.010 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.012±0.004 |
|                          |                          | 0.50                  | 1.00                  | 0.17±0.07                                            | 0.50±0.09   | 1.10±0.14   | 1.22±0.16   |
|                          |                          | 1.00                  | 1.50                  | 0.11±0.05                                            | 0.30±0.05   | 0.40±0.06   | 0.58±0.08   |
| 0.15                     | 0.20                     | 1.50                  | 2.00                  | 0.06±0.03                                            | 0.23±0.05   | 0.39±0.08   | 0.64±0.10   |
|                          |                          | 2.00                  | 2.50                  | 0.03±0.02                                            | 0.15±0.04   | 0.28±0.05   | 0.49±0.08   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.08±0.02   | 0.21±0.04   | 0.32±0.07   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.10±0.03   | 0.15±0.03   | 0.21±0.04   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.050±0.014 | 0.08±0.02   | 0.14±0.03   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.026±0.009 | 0.06±0.02   | 0.11±0.03   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.025±0.010 | 0.035±0.010 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.012±0.008 |
|                          |                          | 0.50                  | 1.00                  | 0.17±0.07                                            | 0.50±0.09   | 1.10±0.14   | 1.22±0.16   |
|                          |                          | 1.00                  | 1.50                  | 0.11±0.05                                            | 0.30±0.05   | 0.40±0.06   | 0.58±0.08   |
|                          |                          | 1.50                  | 2.00                  | 0.06±0.03                                            | 0.23±0.05   | 0.39±0.08   | 0.64±0.10   |
|                          |                          | 2.00                  | 2.50                  | 0.03±0.02                                            | 0.15±0.04   | 0.28±0.05   | 0.49±0.08   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.08±0.02   | 0.21±0.04   | 0.32±0.07   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.10±0.03   | 0.15±0.03   | 0.21±0.04   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.050±0.014 | 0.08±0.02   | 0.14±0.03   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.026±0.009 | 0.06±0.02   | 0.11±0.03   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.025±0.010 | 0.035±0.010 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.012±0.008 |

Table 26: HARP results for the double-differential  $\pi^+$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Ta interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |           |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-----------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c   | 8 GeV/c     | 12 GeV/c    |
| 0.050                    | 0.100                    | 0.50                  | 1.00                  | 0.26±0.07                                            | 0.58±0.10 | 1.06±0.17   | 1.60±0.51   |
|                          |                          | 1.00                  | 1.50                  | 0.032±0.009                                          | 0.46±0.07 | 1.03±0.12   | 1.36±0.38   |
|                          |                          | 1.50                  | 2.00                  | 0.123±0.015                                          | 0.39±0.06 | 0.63±0.09   | 1.62±0.44   |
|                          |                          | 2.00                  | 2.50                  | 0.35±0.03                                            | 0.71±0.07 | 0.71±0.10   | 1.09±0.38   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.70±0.06 | 0.73±0.09   | 1.69±0.84   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.87±0.09 | 0.61±0.08   | 1.33±0.29   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 1.15±0.07 | 0.59±0.08   | 1.46±0.31   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.86±0.29 | 0.81±0.06   | 0.90±0.17   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±         | 0.68±0.10   | 0.66±0.12   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±         | ±           | 0.55±0.11   |
| 0.100                    | 0.150                    | 0.50                  | 1.00                  | 0.16±0.06                                            | 0.67±0.11 | 1.38±0.18   | 1.55±0.53   |
|                          |                          | 1.00                  | 1.50                  | 0.34±0.08                                            | 0.43±0.08 | 0.88±0.11   | 0.94±0.31   |
|                          |                          | 1.50                  | 2.00                  | 0.37±0.06                                            | 0.54±0.07 | 0.94±0.12   | 2.13±0.51   |
|                          |                          | 2.00                  | 2.50                  | 0.70±0.09                                            | 0.54±0.07 | 0.77±0.14   | 0.92±0.28   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.40±0.05 | 0.58±0.15   | 1.14±0.30   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.53±0.06 | 0.39±0.06   | 0.75±0.22   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.40±0.05 | 0.45±0.06   | 0.56±0.18   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.36±0.14 | 0.25±0.04   | 0.52±0.15   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±         | 0.08±0.02   | 0.18±0.07   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±         | ±           | 0.10±0.05   |
| 0.150                    | 0.200                    | 0.50                  | 1.00                  | 0.23±0.07                                            | 0.79±0.13 | 1.38±0.20   | 1.11±0.49   |
|                          |                          | 1.00                  | 1.50                  | 0.28±0.06                                            | 0.42±0.08 | 0.70±0.10   | 0.78±0.32   |
|                          |                          | 1.50                  | 2.00                  | 0.23±0.04                                            | 0.36±0.06 | 0.56±0.08   | 1.33±0.41   |
|                          |                          | 2.00                  | 2.50                  | 0.21±0.04                                            | 0.38±0.07 | 0.51±0.08   | 0.75±0.30   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.14±0.03 | 0.28±0.06   | 0.22±0.13   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.16±0.04 | 0.25±0.04   | 0.27±0.16   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.12±0.03 | 0.21±0.04   | 0.15±0.09   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.07±0.04 | 0.11±0.03   | 0.11±0.08   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±         | 0.029±0.012 | 0.072±0.076 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±         | ±           | 0.042±0.045 |
| 0.200                    | 0.250                    | 0.50                  | 1.00                  | 0.30±0.13                                            | 0.44±0.09 | 0.69±0.13   | 0.74±0.37   |
|                          |                          | 1.00                  | 1.50                  | 0.15±0.04                                            | 0.24±0.06 | 0.50±0.11   | 0.96±0.56   |
|                          |                          | 1.50                  | 2.00                  | 0.13±0.04                                            | 0.23±0.06 | 0.37±0.09   | 0.45±0.28   |
|                          |                          | 2.00                  | 2.50                  | 0.024±0.014                                          | 0.17±0.05 | 0.39±0.09   | 1.09±0.53   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.09±0.03 | 0.23±0.08   | 0.48±0.38   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.07±0.02 | 0.18±0.06   | 0.23±0.20   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.05±0.02 | 0.10±0.03   | 0.154±0.237 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.03±0.02 | 0.05±0.02   | 0.17±0.27   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±         | 0.02±0.02   | 0.02±0.10   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±         | ±           | 0.02±0.20   |

Table 27: HARP results for the double-differential  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Ta interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^\pi/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|--------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                          | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.050                    | 0.100                    | 0.50                  | 1.00                  | 0.32±0.08                                        | 0.43±0.09   | 0.79±0.13   | 1.89±0.55   |
|                          |                          | 1.00                  | 1.50                  | 0.11±0.03                                        | 0.36±0.06   | 0.90±0.11   | 1.19±0.35   |
|                          |                          | 1.50                  | 2.00                  | 0.11±0.03                                        | 0.31±0.05   | 0.77±0.09   | 1.10±0.33   |
|                          |                          | 2.00                  | 2.50                  | 0.18±0.06                                        | 0.23±0.03   | 0.53±0.07   | 0.98±0.30   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.24±0.04   | 0.56±0.07   | 0.88±0.26   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.25±0.04   | 0.41±0.05   | 1.27±0.31   |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.22±0.03   | 0.30±0.04   | 0.65±0.20   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.12±0.03   | 0.24±0.03   | 0.36±0.12   |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.107±0.015 | 0.16±0.06   |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.06±0.03   |
|                          | 0.150                    | 0.50                  | 1.00                  | 0.27±0.06                                        | 0.70±0.12   | 1.19±0.18   | 2.02±0.60   |
|                          |                          | 1.00                  | 1.50                  | 0.17±0.04                                        | 0.37±0.06   | 0.78±0.10   | 0.98±0.31   |
|                          |                          | 1.50                  | 2.00                  | 0.18±0.04                                        | 0.28±0.05   | 0.75±0.09   | 1.21±0.36   |
|                          |                          | 2.00                  | 2.50                  | 0.044±0.014                                      | 0.24±0.04   | 0.48±0.07   | 1.20±0.35   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.16±0.03   | 0.43±0.06   | 0.56±0.20   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.11±0.02   | 0.27±0.04   | 0.44±0.15   |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.062±0.015 | 0.17±0.03   | 0.45±0.17   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.030±0.011 | 0.11±0.02   | 0.21±0.11   |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.028±0.008 | 0.08±0.06   |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.02±0.02   |
| 0.150                    | 0.200                    | 0.50                  | 1.00                  | 0.29±0.08                                        | 0.65±0.11   | 1.23±0.18   | 1.65±0.57   |
|                          |                          | 1.00                  | 1.50                  | 0.16±0.04                                        | 0.41±0.06   | 0.75±0.11   | 1.21±0.39   |
|                          |                          | 1.50                  | 2.00                  | 0.09±0.03                                        | 0.29±0.05   | 0.60±0.09   | 1.15±0.37   |
|                          |                          | 2.00                  | 2.50                  | 0.04±0.02                                        | 0.19±0.03   | 0.49±0.08   | 0.70±0.28   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.12±0.03   | 0.19±0.04   | 0.42±0.21   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.041±0.013 | 0.14±0.03   | 0.27±0.16   |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.016±0.007 | 0.05±0.02   | 0.18±0.12   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.006±0.004 | 0.024±0.010 | 0.15±0.11   |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.003±0.003 | 0.017±0.037 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.000±0.001 |
| 0.200                    | 0.250                    | 0.50                  | 1.00                  | 0.26±0.08                                        | 0.52±0.11   | 1.07±0.18   | 1.24±0.54   |
|                          |                          | 1.00                  | 1.50                  | 0.18±0.06                                        | 0.27±0.06   | 0.72±0.13   | 0.87±0.37   |
|                          |                          | 1.50                  | 2.00                  | 0.05±0.03                                        | 0.11±0.04   | 0.35±0.07   | 1.61±0.74   |
|                          |                          | 2.00                  | 2.50                  | 0.009±0.008                                      | 0.07±0.02   | 0.12±0.04   | 1.14±0.60   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.04±0.02   | 0.026±0.012 | 0.20±0.20   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.008±0.006 | 0.016±0.013 | 0.12±0.21   |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.00±0.00   | 0.00±0.01   | 0.12±0.27   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.00±0.00   | 0.00±0.01   | 0.04±0.19   |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.00±0.01   | 0.00±0.04   |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.00±0.10   |

Table 28: HARP results for the double-differential  $\pi^+$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Pb interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.05                     | 0.10                     | 0.50                  | 1.00                  | 0.04±0.02                                            | 0.39±0.06   | 0.99±0.11   | 1.64±0.16   |
|                          |                          | 1.00                  | 1.50                  | 0.04±0.02                                            | 0.42±0.05   | 1.46±0.10   | 1.75±0.12   |
|                          |                          | 1.50                  | 2.00                  | 0.10±0.05                                            | 0.32±0.04   | 1.08±0.07   | 1.41±0.10   |
|                          |                          | 2.00                  | 2.50                  | 0.10±0.04                                            | 0.20±0.03   | 0.65±0.07   | 1.19±0.08   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.14±0.03   | 0.41±0.06   | 0.94±0.07   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.19±0.03   | 0.42±0.06   | 0.96±0.07   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.15±0.03   | 0.31±0.05   | 0.64±0.07   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.06±0.02   | 0.22±0.03   | 0.37±0.03   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.10±0.02   | 0.20±0.02   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.074±0.011 |
| 0.10                     | 0.15                     | 0.50                  | 1.00                  | 0.18±0.06                                            | 0.46±0.07   | 1.45±0.15   | 1.89±0.18   |
|                          |                          | 1.00                  | 1.50                  | 0.05±0.02                                            | 0.39±0.05   | 0.95±0.09   | 1.59±0.13   |
|                          |                          | 1.50                  | 2.00                  | 0.09±0.05                                            | 0.27±0.04   | 0.75±0.08   | 1.21±0.10   |
|                          |                          | 2.00                  | 2.50                  | 0.06±0.03                                            | 0.24±0.04   | 0.65±0.07   | 1.00±0.09   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.16±0.03   | 0.37±0.05   | 0.72±0.07   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.11±0.02   | 0.25±0.04   | 0.57±0.06   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.09±0.02   | 0.17±0.03   | 0.45±0.04   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.018±0.012 | 0.08±0.02   | 0.24±0.03   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.016±0.007 | 0.085±0.014 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.018±0.004 |
| 0.15                     | 0.20                     | 0.50                  | 1.00                  | 0.02±0.02                                            | 0.48±0.08   | 1.29±0.15   | 1.86±0.19   |
|                          |                          | 1.00                  | 1.50                  | 0.14±0.07                                            | 0.26±0.04   | 0.80±0.08   | 0.99±0.09   |
|                          |                          | 1.50                  | 2.00                  | 0.020±0.011                                          | 0.22±0.03   | 0.68±0.07   | 0.94±0.09   |
|                          |                          | 2.00                  | 2.50                  | 0.04±0.04                                            | 0.14±0.03   | 0.45±0.06   | 0.73±0.07   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.07±0.02   | 0.31±0.05   | 0.43±0.05   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.07±0.02   | 0.15±0.03   | 0.37±0.04   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.011±0.009 | 0.07±0.02   | 0.17±0.03   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.001±0.001 | 0.042±0.010 | 0.08±0.02   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.008±0.003 | 0.012±0.005 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.002±0.001 |
| 0.20                     | 0.25                     | 0.50                  | 1.00                  | 0.20±0.11                                            | 0.63±0.10   | 1.01±0.14   | 1.44±0.18   |
|                          |                          | 1.00                  | 1.50                  | 0.10±0.06                                            | 0.54±0.08   | 0.97±0.13   | 1.53±0.18   |
|                          |                          | 1.50                  | 2.00                  | 0.12±0.07                                            | 0.33±0.06   | 0.75±0.12   | 1.20±0.16   |
|                          |                          | 2.00                  | 2.50                  | 0.02±0.03                                            | 0.18±0.04   | 0.36±0.07   | 0.73±0.11   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.05±0.02   | 0.21±0.05   | 0.32±0.05   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.016±0.011 | 0.06±0.03   | 0.15±0.03   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.002±0.002 | 0.019±0.014 | 0.07±0.02   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.000±0.001 | 0.012±0.010 | 0.024±0.012 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±           | 0.002±0.002 | 0.003±0.002 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±           | ±           | 0.001±0.009 |

Table 29: HARP results for the double-differential  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Pb interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^\pi/(dpd\Omega)$<br>(barn/(GeV/c sr)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|--------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                          | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.05                     | 0.10                     | 0.50                  | 1.00                  | 0.23±0.06                                        | 0.73±0.08   | 1.71±0.15   | 2.20±0.19   |
|                          |                          | 1.00                  | 1.50                  | 0.07±0.02                                        | 0.46±0.05   | 1.29±0.10   | 1.60±0.13   |
|                          |                          | 1.50                  | 2.00                  | 0.06±0.02                                        | 0.49±0.05   | 1.40±0.10   | 1.70±0.11   |
|                          |                          | 2.00                  | 2.50                  | 0.06±0.02                                        | 0.60±0.06   | 1.18±0.08   | 1.61±0.11   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.59±0.06   | 1.00±0.08   | 1.34±0.09   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.69±0.07   | 0.81±0.08   | 1.19±0.08   |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.59±0.08   | 0.72±0.08   | 0.96±0.07   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.54±0.09   | 0.61±0.06   | 0.63±0.05   |
|                          | 0.15                     | 5.00                  | 6.50                  | ±                                                | ±           | 0.47±0.04   | 0.47±0.03   |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.32±0.03   |
|                          |                          | 0.50                  | 1.00                  | 0.25±0.06                                        | 0.70±0.09   | 1.95±0.21   | 2.55±0.26   |
|                          |                          | 1.00                  | 1.50                  | 0.22±0.06                                        | 0.61±0.07   | 1.22±0.10   | 2.08±0.16   |
|                          |                          | 1.50                  | 2.00                  | 0.25±0.07                                        | 0.46±0.05   | 1.09±0.10   | 1.67±0.13   |
|                          |                          | 2.00                  | 2.50                  | 0.38±0.09                                        | 0.49±0.06   | 0.89±0.08   | 1.37±0.11   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.37±0.05   | 0.72±0.07   | 0.99±0.09   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.39±0.05   | 0.44±0.05   | 0.83±0.07   |
| 0.15                     | 0.20                     | 3.50                  | 4.00                  | ±                                                | 0.34±0.05   | 0.29±0.04   | 0.55±0.05   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.29±0.04   | 0.19±0.03   | 0.40±0.04   |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.083±0.013 | 0.15±0.02   |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.054±0.009 |
|                          |                          | 0.50                  | 1.00                  | 0.43±0.12                                        | 0.81±0.11   | 1.49±0.17   | 2.30±0.24   |
|                          |                          | 1.00                  | 1.50                  | 0.34±0.10                                        | 0.49±0.06   | 1.23±0.12   | 1.71±0.15   |
|                          |                          | 1.50                  | 2.00                  | 0.06±0.04                                        | 0.44±0.05   | 0.92±0.09   | 1.15±0.11   |
|                          |                          | 2.00                  | 2.50                  | 0.038±0.014                                      | 0.32±0.05   | 0.54±0.06   | 0.82±0.09   |
|                          | 0.25                     | 2.50                  | 3.00                  | ±                                                | 0.19±0.03   | 0.38±0.04   | 0.44±0.05   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.13±0.03   | 0.22±0.03   | 0.38±0.04   |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.07±0.02   | 0.16±0.02   | 0.27±0.03   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.061±0.013 | 0.10±0.02   | 0.15±0.02   |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.026±0.006 | 0.044±0.008 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.015±0.004 |
|                          |                          | 0.50                  | 1.00                  | 0.18±0.07                                        | 0.52±0.09   | 1.01±0.14   | 1.48±0.18   |
|                          |                          | 1.00                  | 1.50                  | 0.05±0.03                                        | 0.25±0.04   | 0.73±0.10   | 0.68±0.09   |
| 0.20                     | 0.25                     | 1.50                  | 2.00                  | 0.04±0.02                                        | 0.17±0.04   | 0.56±0.10   | 0.71±0.10   |
|                          |                          | 2.00                  | 2.50                  | 0.04±0.02                                        | 0.09±0.02   | 0.40±0.09   | 0.62±0.10   |
|                          |                          | 2.50                  | 3.00                  | ±                                                | 0.06±0.02   | 0.27±0.06   | 0.37±0.06   |
|                          |                          | 3.00                  | 3.50                  | ±                                                | 0.07±0.02   | 0.14±0.03   | 0.30±0.05   |
|                          |                          | 3.50                  | 4.00                  | ±                                                | 0.058±0.014 | 0.09±0.02   | 0.22±0.04   |
|                          |                          | 4.00                  | 5.00                  | ±                                                | 0.031±0.010 | 0.09±0.02   | 0.12±0.02   |
|                          |                          | 5.00                  | 6.50                  | ±                                                | ±           | 0.030±0.010 | 0.035±0.011 |
|                          |                          | 6.50                  | 8.00                  | ±                                                | ±           | ±           | 0.009±0.006 |



Table 30: HARP results for the double-differential  $\pi^+$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Pb interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c sr)) |           |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------|-----------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                              | 5 GeV/c   | 8 GeV/c     | 12 GeV/c    |
| 0.050                    | 0.100                    | 0.50                  | 1.00                  | 0.20±0.06                                            | 0.64±0.12 | 0.90±0.16   | 2.89±1.10   |
|                          |                          |                       | 1.50                  | 0.042±0.011                                          | 0.38±0.07 | 0.78±0.10   | 0.82±0.42   |
|                          |                          | 1.50                  | 2.00                  | 0.093±0.011                                          | 0.43±0.07 | 0.77±0.10   | 1.54±0.65   |
|                          |                          | 2.00                  | 2.50                  | 0.29±0.03                                            | 0.72±0.08 | 0.82±0.11   | 0.73±0.36   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.75±0.07 | 0.74±0.10   | 1.64±0.53   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.89±0.09 | 0.61±0.08   | 1.55±0.50   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 1.04±0.07 | 0.68±0.09   | 1.38±0.45   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.78±0.26 | 0.89±0.07   | 0.73±0.23   |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±         | 0.72±0.11   | *±*         |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±         | ±           | 0.14±0.15   |
|                          | 0.150                    | 0.50                  | 1.00                  | 0.08±0.04                                            | 0.87±0.14 | 1.40±0.19   | 2.46±0.91   |
|                          |                          |                       | 1.50                  | 0.18±0.06                                            | 0.36±0.07 | 1.00±0.15   | 2.03±0.73   |
|                          |                          | 1.50                  | 2.00                  | 0.52±0.08                                            | 0.55±0.08 | 1.07±0.21   | 0.54±0.31   |
|                          |                          | 2.00                  | 2.50                  | 0.69±0.09                                            | 0.52±0.07 | 0.89±0.36   | 1.20±0.65   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.49±0.06 | 0.49±0.43   | 0.88±0.53   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.66±0.08 | 0.57±0.12   | 0.70±0.37   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.57±0.06 | 0.40±0.06   | 0.53±0.26   |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.35±0.13 | 0.29±0.05   | 0.39±0.19   |
| 0.150                    | 0.200                    | 5.00                  | 6.50                  | ±                                                    | ±         | 0.12±0.03   | 0.24±0.16   |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±         | ±           | 0.027±0.035 |
|                          |                          | 0.50                  | 1.00                  | 0.18±0.06                                            | 0.79±0.14 | 1.35±0.19   | 2.46±1.08   |
|                          |                          |                       | 1.50                  | 0.26±0.06                                            | 0.64±0.10 | 0.83±0.11   | 0.97±0.60   |
|                          |                          | 1.50                  | 2.00                  | 0.31±0.06                                            | 0.37±0.06 | 0.61±0.09   | *±*         |
|                          |                          | 2.00                  | 2.50                  | 0.26±0.05                                            | 0.40±0.07 | 0.53±0.09   | *±*         |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.18±0.04 | 0.30±0.07   | 0.19±0.29   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.20±0.06 | 0.26±0.05   | 0.45±0.41   |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.15±0.04 | 0.12±0.03   | 0.210±0.250 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.09±0.05 | 0.08±0.02   | 0.127±0.185 |
|                          |                          | 5.00                  | 6.50                  | ±                                                    | ±         | 0.031±0.013 | 0.145±0.162 |
|                          |                          | 6.50                  | 8.00                  | ±                                                    | ±         | ±           | 0.020±0.049 |
|                          | 0.250                    | 0.50                  | 1.00                  | 0.26±0.14                                            | 0.55±0.11 | 0.86±0.15   | 0.057±0.212 |
|                          |                          |                       | 1.50                  | 0.11±0.04                                            | 0.39±0.08 | 0.42±0.09   | 0.06±0.73   |
|                          |                          | 1.50                  | 2.00                  | 0.16±0.05                                            | 0.31±0.07 | 0.43±0.10   | 2.18±0.64   |
|                          |                          | 2.00                  | 2.50                  | 0.06±0.03                                            | 0.22±0.06 | 0.31±0.08   | 2.25±1.17   |
|                          |                          | 2.50                  | 3.00                  | ±                                                    | 0.16±0.05 | 0.23±0.08   | 0.13±0.65   |
|                          |                          | 3.00                  | 3.50                  | ±                                                    | 0.07±0.03 | 0.11±0.04   | 0.075±0.359 |
|                          |                          | 3.50                  | 4.00                  | ±                                                    | 0.03±0.02 | 0.10±0.03   | 0.026±0.488 |
|                          |                          | 4.00                  | 5.00                  | ±                                                    | 0.03±0.02 | 0.08±0.03   | *±*         |
| 0.200                    | 0.250                    | 5.00                  | 6.50                  | ±                                                    | ±         | 0.02±0.02   | 0.08±0.24   |

Table 31: HARP results for the double-differential  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Pb interactions at 3,5,8,12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}$ ,  $\theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^-}/(dpd\theta)$<br>(barn/(GeV/c rad)) |             |             |             |
|--------------------------|--------------------------|-----------------------|-----------------------|-------------------------------------------------------|-------------|-------------|-------------|
|                          |                          |                       |                       | 3 GeV/c                                               | 5 GeV/c     | 8 GeV/c     | 12 GeV/c    |
| 0.050                    | 0.100                    | 0.50                  | 1.00                  | 0.26±0.07                                             | 0.60±0.11   | 0.98±0.14   | 0.93±0.62   |
|                          |                          | 1.00                  | 1.50                  | 0.17±0.04                                             | 0.53±0.07   | 1.00±0.12   | 1.92±0.70   |
|                          |                          | 1.50                  | 2.00                  | 0.28±0.05                                             | 0.39±0.06   | 0.79±0.10   | 1.34±0.53   |
|                          |                          | 2.00                  | 2.50                  | 0.20±0.06                                             | 0.31±0.04   | 0.48±0.07   | 0.79±0.36   |
|                          |                          | 2.50                  | 3.00                  | ±                                                     | 0.31±0.05   | 0.51±0.07   | 0.64±0.31   |
|                          |                          | 3.00                  | 3.50                  | ±                                                     | 0.23±0.04   | 0.41±0.05   | 1.38±0.48   |
|                          |                          | 3.50                  | 4.00                  | ±                                                     | 0.26±0.04   | 0.39±0.05   | 0.96±0.36   |
|                          |                          | 4.00                  | 5.00                  | ±                                                     | 0.13±0.03   | 0.29±0.03   | 0.40±0.18   |
|                          | 0.150                    | 5.00                  | 6.50                  | ±                                                     | ±           | 0.13±0.02   | 0.27±0.13   |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | ±           | ±           | 0.06±0.05   |
|                          |                          | 0.50                  | 1.00                  | 0.27±0.06                                             | 0.66±0.12   | 1.36±0.20   | 2.21±0.90   |
|                          |                          | 1.00                  | 1.50                  | 0.07±0.02                                             | 0.42±0.07   | 0.81±0.10   | 1.27±0.58   |
|                          |                          | 1.50                  | 2.00                  | 0.07±0.03                                             | 0.30±0.05   | 0.79±0.10   | 0.70±0.37   |
|                          |                          | 2.00                  | 2.50                  | 0.043±0.014                                           | 0.25±0.04   | 0.39±0.06   | 1.70±0.65   |
|                          |                          | 2.50                  | 3.00                  | ±                                                     | 0.20±0.03   | 0.39±0.06   | 0.18±0.14   |
|                          |                          | 3.00                  | 3.50                  | ±                                                     | 0.10±0.02   | 0.28±0.05   | 0.58±0.31   |
| 0.150                    | 0.200                    | 3.50                  | 4.00                  | ±                                                     | 0.046±0.012 | 0.27±0.05   | 0.66±0.35   |
|                          |                          | 4.00                  | 5.00                  | ±                                                     | 0.05±0.02   | 0.10±0.02   | 0.26±0.20   |
|                          |                          | 5.00                  | 6.50                  | ±                                                     | ±           | 0.015±0.005 | 0.04±0.07   |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | ±           | ±           | 0.023±0.039 |
|                          |                          | 0.50                  | 1.00                  | 0.34±0.09                                             | 0.79±0.13   | 1.21±0.19   | 0.68±0.55   |
|                          |                          | 1.00                  | 1.50                  | 0.20±0.05                                             | 0.43±0.07   | 0.76±0.11   | 1.56±0.76   |
|                          |                          | 1.50                  | 2.00                  | 0.11±0.03                                             | 0.31±0.05   | 0.65±0.09   | 0.82±0.48   |
|                          |                          | 2.00                  | 2.50                  | 0.06±0.02                                             | 0.28±0.05   | 0.40±0.07   | 0.84±0.43   |
|                          | 0.250                    | 2.50                  | 3.00                  | ±                                                     | 0.09±0.02   | 0.18±0.04   | 0.66±0.38   |
|                          |                          | 3.00                  | 3.50                  | ±                                                     | 0.036±0.013 | 0.13±0.03   | 0.30±0.23   |
|                          |                          | 3.50                  | 4.00                  | ±                                                     | 0.015±0.008 | 0.07±0.02   | 0.52±0.42   |
|                          |                          | 4.00                  | 5.00                  | ±                                                     | 0.002±0.002 | 0.019±0.009 | 0.13±0.19   |
|                          |                          | 5.00                  | 6.50                  | ±                                                     | ±           | 0.004±0.003 | 0.001±0.006 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | ±           | ±           | 0.005±0.023 |
|                          |                          | 0.50                  | 1.00                  | 0.21±0.08                                             | 0.59±0.13   | 0.90±0.16   | 0.78±0.65   |
|                          |                          | 1.00                  | 1.50                  | 0.14±0.05                                             | 0.36±0.08   | 0.83±0.15   | 0.03±0.06   |
| 0.200                    | 0.250                    | 1.50                  | 2.00                  | 0.07±0.04                                             | 0.14±0.05   | 0.38±0.09   | 0.67±1.08   |
|                          |                          | 2.00                  | 2.50                  | ±                                                     | 0.09±0.03   | 0.11±0.04   | 0.72±0.77   |
|                          |                          | 2.50                  | 3.00                  | ±                                                     | 0.033±0.014 | 0.06±0.02   | 0.11±0.16   |
|                          |                          | 3.00                  | 3.50                  | ±                                                     | 0.013±0.008 | 0.02±0.02   | 0.10±0.44   |
|                          |                          | 3.50                  | 4.00                  | ±                                                     | 0.013±0.010 | 0.00±0.01   | 0.09±0.93   |
|                          |                          | 4.00                  | 5.00                  | ±                                                     | 0.00±0.01   | 0.00±0.01   | 0.02±0.35   |
|                          |                          | 5.00                  | 6.50                  | ±                                                     | ±           | ±           | 0.00±0.05   |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | ±           | ±           | 0.00±0.20   |

chop chop

Table 32: HARP results for the double-differential  $\pi^+$  and  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Be interactions at 8 and 12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. A finer angular binning than in the previous set of tables is used. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |             | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |             |
|--------------------------|--------------------------|-----------------------|-----------------------|-------------------------------------------------------|-------------|-------------------------------------------------------|-------------|
|                          |                          |                       |                       | 8 GeV/c                                               | 12 GeV/c    | 8 GeV/c                                               | 12 GeV/c    |
| 0.025                    | 0.050                    | 0.50                  | 0.75                  | 0.12±0.03                                             | 0.07±0.02   | 0.20±0.05                                             | 0.13±0.03   |
|                          |                          | 0.75                  | 1.00                  | 0.14±0.02                                             | 0.11±0.02   | 0.18±0.03                                             | 0.15±0.02   |
|                          |                          | 1.00                  | 1.25                  | 0.18±0.02                                             | 0.09±0.02   | 0.17±0.03                                             | 0.12±0.02   |
|                          |                          | 1.25                  | 1.50                  | 0.20±0.02                                             | 0.13±0.03   | 0.17±0.03                                             | 0.14±0.02   |
|                          |                          | 1.50                  | 2.00                  | 0.16±0.02                                             | 0.13±0.02   | 0.21±0.02                                             | 0.17±0.02   |
|                          |                          | 2.00                  | 2.50                  | 0.14±0.02                                             | 0.13±0.02   | 0.22±0.02                                             | 0.17±0.02   |
|                          |                          | 2.50                  | 3.00                  | 0.12±0.02                                             | 0.15±0.02   | 0.19±0.02                                             | 0.14±0.02   |
|                          |                          | 3.00                  | 3.50                  | 0.09±0.02                                             | 0.12±0.02   | 0.23±0.02                                             | 0.16±0.02   |
|                          |                          | 3.50                  | 4.00                  | 0.08±0.02                                             | 0.094±0.014 | 0.22±0.02                                             | 0.132±0.014 |
|                          |                          | 4.00                  | 5.00                  | 0.089±0.013                                           | 0.064±0.009 | 0.22±0.02                                             | 0.129±0.012 |
|                          |                          | 5.00                  | 6.50                  | 0.098±0.011                                           | 0.065±0.007 | 0.25±0.02                                             | 0.117±0.010 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.053±0.006 | ±                                                     | 0.142±0.010 |
|                          | 0.075                    | 0.50                  | 0.75                  | 0.15±0.03                                             | 0.08±0.02   | 0.28±0.04                                             | 0.17±0.03   |
|                          |                          | 0.75                  | 1.00                  | 0.14±0.02                                             | 0.10±0.02   | 0.18±0.02                                             | 0.11±0.02   |
|                          |                          | 1.00                  | 1.25                  | 0.18±0.02                                             | 0.12±0.02   | 0.18±0.02                                             | 0.13±0.02   |
|                          |                          | 1.25                  | 1.50                  | 0.17±0.02                                             | 0.13±0.02   | 0.19±0.02                                             | 0.16±0.02   |
|                          |                          | 1.50                  | 2.00                  | 0.145±0.012                                           | 0.119±0.012 | 0.20±0.02                                             | 0.15±0.02   |
|                          |                          | 2.00                  | 2.50                  | 0.105±0.012                                           | 0.133±0.012 | 0.21±0.02                                             | 0.159±0.014 |
|                          |                          | 2.50                  | 3.00                  | 0.079±0.012                                           | 0.088±0.009 | 0.21±0.02                                             | 0.158±0.014 |
|                          |                          | 3.00                  | 3.50                  | 0.086±0.014                                           | 0.106±0.011 | 0.18±0.02                                             | 0.163±0.014 |
|                          |                          | 3.50                  | 4.00                  | 0.103±0.012                                           | 0.096±0.010 | 0.17±0.02                                             | 0.135±0.011 |
|                          |                          | 4.00                  | 5.00                  | 0.068±0.007                                           | 0.056±0.006 | 0.21±0.02                                             | 0.131±0.009 |
| 0.075                    | 0.100                    | 5.00                  | 6.50                  | 0.043±0.005                                           | 0.037±0.004 | 0.174±0.012                                           | 0.100±0.006 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.018±0.003 | ±                                                     | 0.086±0.005 |
|                          |                          | 0.50                  | 0.75                  | 0.09±0.02                                             | 0.10±0.02   | 0.30±0.04                                             | 0.20±0.03   |
|                          |                          | 0.75                  | 1.00                  | 0.17±0.02                                             | 0.128±0.015 | 0.22±0.02                                             | 0.109±0.013 |
|                          |                          | 1.00                  | 1.25                  | 0.19±0.02                                             | 0.124±0.013 | 0.19±0.02                                             | 0.15±0.02   |
|                          |                          | 1.25                  | 1.50                  | 0.167±0.014                                           | 0.141±0.013 | 0.22±0.02                                             | 0.14±0.02   |
|                          |                          | 1.50                  | 2.00                  | 0.139±0.010                                           | 0.133±0.012 | 0.22±0.02                                             | 0.185±0.013 |
|                          |                          | 2.00                  | 2.50                  | 0.117±0.010                                           | 0.122±0.010 | 0.222±0.015                                           | 0.172±0.013 |
|                          |                          | 2.50                  | 3.00                  | 0.098±0.009                                           | 0.085±0.007 | 0.184±0.012                                           | 0.189±0.013 |
|                          |                          | 3.00                  | 3.50                  | 0.086±0.008                                           | 0.091±0.009 | 0.147±0.012                                           | 0.133±0.009 |
|                          |                          | 3.50                  | 4.00                  | 0.050±0.009                                           | 0.080±0.009 | 0.146±0.013                                           | 0.124±0.008 |
|                          |                          | 4.00                  | 5.00                  | 0.030±0.004                                           | 0.043±0.004 | 0.123±0.009                                           | 0.086±0.006 |
|                          |                          | 5.00                  | 6.50                  | 0.009±0.002                                           | 0.019±0.002 | 0.078±0.005                                           | 0.053±0.004 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.005±0.001 | ±                                                     | 0.030±0.002 |

Table 33: HARP results for the double-differential  $\pi^+$  and  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Be interactions at 8 and 8.9 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. A finer angular binning than in the previous set of tables is used. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |             | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |             |
|--------------------------|--------------------------|-----------------------|-----------------------|-------------------------------------------------------|-------------|-------------------------------------------------------|-------------|
|                          |                          |                       |                       | 8 GeV/c                                               | 8.9 GeV/c   | 8 GeV/c                                               | 8.9 GeV/c   |
| 0.025                    | 0.050                    | 0.50                  | 0.75                  | 0.10±0.04                                             | 0.13±0.04   | 0.13±0.05                                             | 0.10±0.03   |
|                          |                          |                       | 1.00                  | 0.14±0.03                                             | 0.09±0.03   | 0.13±0.03                                             | 0.12±0.03   |
|                          |                          | 1.00                  | 1.25                  | 0.13±0.03                                             | 0.14±0.03   | 0.15±0.04                                             | 0.11±0.03   |
|                          |                          |                       | 1.25                  | 0.11±0.03                                             | 0.13±0.03   | 0.13±0.04                                             | 0.16±0.03   |
|                          |                          | 1.50                  | 2.00                  | 0.16±0.03                                             | 0.15±0.03   | 0.18±0.03                                             | 0.13±0.02   |
|                          |                          |                       | 2.50                  | 0.08±0.02                                             | 0.15±0.02   | 0.13±0.02                                             | 0.12±0.02   |
|                          |                          | 2.50                  | 3.00                  | 0.15±0.03                                             | 0.19±0.02   | 0.10±0.02                                             | 0.07±0.02   |
|                          |                          |                       | 3.50                  | 0.16±0.02                                             | 0.15±0.02   | 0.11±0.02                                             | 0.10±0.02   |
|                          |                          | 3.50                  | 4.00                  | 0.19±0.03                                             | 0.21±0.02   | 0.081±0.014                                           | 0.081±0.015 |
|                          |                          |                       | 5.00                  | 0.18±0.02                                             | 0.21±0.02   | 0.104±0.014                                           | 0.085±0.012 |
|                          |                          | 5.00                  | 6.50                  | 0.27±0.02                                             | 0.22±0.02   | 0.092±0.012                                           | 0.079±0.009 |
|                          |                          |                       |                       |                                                       |             |                                                       |             |
|                          |                          | 0.50                  | 0.75                  | 0.12±0.03                                             | 0.12±0.03   | 0.10±0.03                                             | 0.09±0.02   |
|                          |                          |                       | 1.00                  | 0.13±0.03                                             | 0.10±0.02   | 0.10±0.02                                             | 0.11±0.02   |
|                          |                          | 1.00                  | 1.25                  | 0.14±0.03                                             | 0.11±0.02   | 0.10±0.02                                             | 0.12±0.02   |
|                          |                          |                       | 1.25                  | 0.17±0.03                                             | 0.16±0.02   | 0.10±0.02                                             | 0.12±0.02   |
| 0.050                    | 0.075                    | 1.50                  | 2.00                  | 0.12±0.02                                             | 0.15±0.02   | 0.14±0.02                                             | 0.14±0.02   |
|                          |                          |                       | 2.50                  | 0.14±0.02                                             | 0.14±0.02   | 0.13±0.02                                             | 0.129±0.014 |
|                          |                          | 2.50                  | 3.00                  | 0.16±0.02                                             | 0.14±0.02   | 0.11±0.02                                             | 0.096±0.013 |
|                          |                          |                       | 3.50                  | 0.12±0.02                                             | 0.17±0.02   | 0.083±0.013                                           | 0.106±0.011 |
|                          |                          | 3.50                  | 4.00                  | 0.12±0.02                                             | 0.17±0.02   | 0.079±0.012                                           | 0.084±0.009 |
|                          |                          |                       | 5.00                  | 0.21±0.02                                             | 0.173±0.012 | 0.064±0.009                                           | 0.062±0.007 |
|                          |                          | 5.00                  | 6.50                  | 0.203±0.013                                           | 0.136±0.010 | 0.047±0.006                                           | 0.035±0.004 |
|                          |                          |                       |                       |                                                       |             |                                                       |             |
|                          |                          | 0.50                  | 0.75                  | 0.11±0.03                                             | 0.15±0.03   | 0.09±0.02                                             | 0.12±0.02   |
|                          |                          |                       | 1.00                  | 0.17±0.03                                             | 0.15±0.02   | 0.14±0.02                                             | 0.08±0.02   |
|                          |                          | 1.00                  | 1.25                  | 0.13±0.02                                             | 0.18±0.02   | 0.20±0.03                                             | 0.14±0.02   |
|                          |                          |                       | 1.25                  | 0.19±0.03                                             | 0.18±0.02   | 0.16±0.02                                             | 0.16±0.02   |
|                          |                          | 1.50                  | 2.00                  | 0.14±0.02                                             | 0.19±0.02   | 0.104±0.014                                           | 0.132±0.012 |
|                          |                          |                       | 2.50                  | 0.17±0.02                                             | 0.188±0.014 | 0.105±0.014                                           | 0.118±0.010 |
|                          |                          | 2.50                  | 3.00                  | 0.20±0.02                                             | 0.150±0.011 | 0.067±0.009                                           | 0.088±0.008 |
|                          |                          |                       | 3.50                  | 0.15±0.02                                             | 0.168±0.013 | 0.100±0.012                                           | 0.078±0.008 |
| 0.075                    | 0.100                    | 3.50                  | 4.00                  | 0.13±0.02                                             | 0.143±0.011 | 0.055±0.008                                           | 0.061±0.006 |
|                          |                          |                       | 5.00                  | 0.128±0.010                                           | 0.109±0.009 | 0.041±0.005                                           | 0.036±0.004 |
|                          |                          | 5.00                  | 6.50                  | 0.09±0.03                                             | 0.074±0.005 | 0.010±0.002                                           | 0.011±0.002 |
|                          |                          |                       |                       |                                                       |             |                                                       |             |

Table 34: HARP results for the double-differential  $\pi^+$  and  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -C interactions at 8 and 12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. A finer angular binning than in the previous set of tables is used. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |             | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |             |
|--------------------------|--------------------------|-----------------------|-----------------------|-------------------------------------------------------|-------------|-------------------------------------------------------|-------------|
|                          |                          |                       |                       | 8 GeV/c                                               | 12 GeV/c    | 8 GeV/c                                               | 12 GeV/c    |
| 0.025                    | 0.050                    | 0.50                  | 0.75                  | 0.06±0.02                                             | 0.13±0.04   | 0.22±0.06                                             | 0.21±0.07   |
|                          |                          | 0.75                  | 1.00                  | 0.15±0.03                                             | 0.25±0.05   | 0.24±0.04                                             | 0.30±0.05   |
|                          |                          | 1.00                  | 1.25                  | 0.20±0.03                                             | 0.12±0.03   | 0.18±0.03                                             | 0.24±0.05   |
|                          |                          | 1.25                  | 1.50                  | 0.24±0.03                                             | 0.22±0.05   | 0.23±0.04                                             | 0.22±0.04   |
|                          |                          | 1.50                  | 2.00                  | 0.17±0.03                                             | 0.25±0.04   | 0.29±0.03                                             | 0.31±0.04   |
|                          |                          | 2.00                  | 2.50                  | 0.16±0.02                                             | 0.26±0.04   | 0.24±0.02                                             | 0.34±0.05   |
|                          |                          | 2.50                  | 3.00                  | 0.15±0.03                                             | 0.24±0.03   | 0.21±0.03                                             | 0.25±0.03   |
|                          |                          | 3.00                  | 3.50                  | 0.05±0.02                                             | 0.22±0.03   | 0.23±0.03                                             | 0.33±0.04   |
|                          |                          | 3.50                  | 4.00                  | 0.11±0.02                                             | 0.17±0.03   | 0.17±0.03                                             | 0.24±0.03   |
|                          |                          | 4.00                  | 5.00                  | 0.08±0.02                                             | 0.13±0.02   | 0.23±0.03                                             | 0.24±0.03   |
|                          |                          | 5.00                  | 6.50                  | 0.098±0.015                                           | 0.119±0.014 | 0.24±0.03                                             | 0.27±0.03   |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.108±0.015 | ±                                                     | 0.30±0.02   |
|                          | 0.075                    | 0.50                  | 0.75                  | 0.15±0.03                                             | 0.24±0.05   | 0.30±0.05                                             | 0.29±0.06   |
|                          |                          | 0.75                  | 1.00                  | 0.13±0.02                                             | 0.22±0.04   | 0.25±0.03                                             | 0.21±0.03   |
|                          |                          | 1.00                  | 1.25                  | 0.23±0.03                                             | 0.26±0.04   | 0.24±0.03                                             | 0.29±0.04   |
|                          |                          | 1.25                  | 1.50                  | 0.19±0.02                                             | 0.20±0.03   | 0.18±0.02                                             | 0.23±0.03   |
|                          |                          | 1.50                  | 2.00                  | 0.18±0.02                                             | 0.20±0.03   | 0.25±0.02                                             | 0.34±0.04   |
|                          |                          | 2.00                  | 2.50                  | 0.14±0.02                                             | 0.17±0.02   | 0.25±0.02                                             | 0.29±0.03   |
|                          |                          | 2.50                  | 3.00                  | 0.10±0.02                                             | 0.17±0.02   | 0.22±0.02                                             | 0.26±0.03   |
|                          |                          | 3.00                  | 3.50                  | 0.09±0.02                                             | 0.18±0.02   | 0.22±0.02                                             | 0.28±0.03   |
|                          |                          | 3.50                  | 4.00                  | 0.090±0.013                                           | 0.17±0.02   | 0.20±0.02                                             | 0.24±0.03   |
|                          |                          | 4.00                  | 5.00                  | 0.083±0.010                                           | 0.096±0.012 | 0.21±0.02                                             | 0.23±0.02   |
| 0.075                    | 0.100                    | 5.00                  | 6.50                  | 0.045±0.006                                           | 0.073±0.009 | 0.17±0.02                                             | 0.173±0.013 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.026±0.005 | ±                                                     | 0.140±0.012 |
|                          |                          | 0.50                  | 0.75                  | 0.12±0.02                                             | 0.14±0.03   | 0.30±0.04                                             | 0.37±0.06   |
|                          |                          | 0.75                  | 1.00                  | 0.16±0.02                                             | 0.23±0.03   | 0.22±0.02                                             | 0.22±0.03   |
|                          |                          | 1.00                  | 1.25                  | 0.21±0.02                                             | 0.27±0.03   | 0.25±0.02                                             | 0.29±0.03   |
|                          |                          | 1.25                  | 1.50                  | 0.18±0.02                                             | 0.30±0.03   | 0.25±0.03                                             | 0.33±0.04   |
|                          |                          | 1.50                  | 2.00                  | 0.166±0.013                                           | 0.25±0.02   | 0.25±0.02                                             | 0.30±0.03   |
|                          |                          | 2.00                  | 2.50                  | 0.114±0.012                                           | 0.21±0.02   | 0.24±0.02                                             | 0.34±0.03   |
|                          |                          | 2.50                  | 3.00                  | 0.092±0.010                                           | 0.18±0.02   | 0.23±0.02                                             | 0.32±0.03   |
|                          |                          | 3.00                  | 3.50                  | 0.096±0.010                                           | 0.17±0.02   | 0.16±0.02                                             | 0.25±0.02   |
|                          |                          | 3.50                  | 4.00                  | 0.053±0.012                                           | 0.136±0.014 | 0.16±0.02                                             | 0.23±0.02   |
|                          |                          | 4.00                  | 5.00                  | 0.026±0.005                                           | 0.072±0.008 | 0.144±0.013                                           | 0.171±0.014 |
|                          |                          | 5.00                  | 6.50                  | 0.008±0.003                                           | 0.039±0.006 | 0.078±0.006                                           | 0.105±0.008 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.008±0.002 | ±                                                     | 0.052±0.006 |

Table 35: HARP results for the double-differential  $\pi^+$  and  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+-C$  interactions at 8 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. A finer angular binning than in the previous set of tables is used. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c rad))<br>8 GeV/c | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c rad))<br>8 GeV/c |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------------------|------------------------------------------------------------------|
| 0.025                    | 0.050                    | 0.50                  | 0.75                  | 0.15±0.06                                                        | 0.27±0.08                                                        |
|                          |                          | 0.75                  | 1.00                  | 0.19±0.05                                                        | 0.19±0.05                                                        |
|                          |                          | 1.00                  | 1.25                  | 0.05±0.02                                                        | 0.12±0.03                                                        |
|                          |                          | 1.25                  | 1.50                  | 0.07±0.03                                                        | 0.18±0.06                                                        |
|                          |                          | 1.50                  | 2.00                  | 0.11±0.03                                                        | 0.17±0.04                                                        |
|                          |                          | 2.00                  | 2.50                  | 0.15±0.04                                                        | 0.12±0.03                                                        |
|                          |                          | 2.50                  | 3.00                  | 0.18±0.04                                                        | 0.09±0.02                                                        |
|                          |                          | 3.00                  | 3.50                  | 0.23±0.03                                                        | 0.15±0.05                                                        |
|                          |                          | 3.50                  | 4.00                  | 0.17±0.02                                                        | 0.10±0.02                                                        |
|                          |                          | 4.00                  | 5.00                  | 0.21±0.02                                                        | 0.11±0.02                                                        |
|                          |                          | 5.00                  | 6.50                  | 0.36±0.02                                                        | 0.097±0.014                                                      |
|                          |                          | 0.50                  | 0.75                  | 0.18±0.05                                                        | 0.16±0.04                                                        |
|                          |                          | 0.75                  | 1.00                  | 0.17±0.03                                                        | 0.12±0.03                                                        |
|                          |                          | 1.00                  | 1.25                  | 0.14±0.03                                                        | 0.17±0.04                                                        |
| 0.050                    | 0.075                    | 1.25                  | 1.50                  | 0.25±0.04                                                        | 0.15±0.03                                                        |
|                          |                          | 1.50                  | 2.00                  | 0.17±0.03                                                        | 0.15±0.02                                                        |
|                          |                          | 2.00                  | 2.50                  | 0.16±0.03                                                        | 0.16±0.02                                                        |
|                          |                          | 2.50                  | 3.00                  | 0.22±0.03                                                        | 0.11±0.02                                                        |
|                          |                          | 3.00                  | 3.50                  | 0.17±0.03                                                        | 0.081±0.014                                                      |
|                          |                          | 3.50                  | 4.00                  | 0.14±0.02                                                        | 0.13±0.02                                                        |
|                          |                          | 4.00                  | 5.00                  | 0.26±0.02                                                        | 0.065±0.010                                                      |
|                          |                          | 5.00                  | 6.50                  | 0.25±0.02                                                        | 0.046±0.006                                                      |
|                          |                          | 0.50                  | 0.75                  | 0.13±0.03                                                        | 0.17±0.04                                                        |
|                          |                          | 0.75                  | 1.00                  | 0.22±0.04                                                        | 0.13±0.02                                                        |
|                          |                          | 1.00                  | 1.25                  | 0.23±0.03                                                        | 0.14±0.03                                                        |
|                          |                          | 1.25                  | 1.50                  | 0.21±0.03                                                        | 0.15±0.02                                                        |
|                          |                          | 1.50                  | 2.00                  | 0.20±0.02                                                        | 0.15±0.02                                                        |
|                          |                          | 2.00                  | 2.50                  | 0.20±0.03                                                        | 0.11±0.02                                                        |
| 0.075                    | 0.100                    | 2.50                  | 3.00                  | 0.19±0.02                                                        | 0.12±0.02                                                        |
|                          |                          | 3.00                  | 3.50                  | 0.20±0.02                                                        | 0.081±0.011                                                      |
|                          |                          | 3.50                  | 4.00                  | 0.14±0.02                                                        | 0.088±0.013                                                      |
|                          |                          | 4.00                  | 5.00                  | 0.153±0.014                                                      | 0.039±0.005                                                      |
|                          |                          | 5.00                  | 6.50                  | 0.12±0.04                                                        | 0.011±0.002                                                      |

Table 36: HARP results for the double-differential  $\pi^+$  and  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Al interactions at 8 and 12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. A finer angular binning than in the previous set of tables is used. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |             | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |             |
|--------------------------|--------------------------|-----------------------|-----------------------|-------------------------------------------------------|-------------|-------------------------------------------------------|-------------|
|                          |                          |                       |                       | 8 GeV/c                                               | 12 GeV/c    | 8 GeV/c                                               | 12 GeV/c    |
| 0.025                    | 0.050                    | 0.50                  | 0.75                  | 0.12±0.04                                             | 0.32±0.09   | 0.49±0.11                                             | 0.50±0.12   |
|                          |                          | 0.75                  | 1.00                  | 0.27±0.05                                             | 0.37±0.06   | 0.41±0.06                                             | 0.42±0.07   |
|                          |                          | 1.00                  | 1.25                  | 0.22±0.04                                             | 0.33±0.06   | 0.36±0.06                                             | 0.38±0.06   |
|                          |                          | 1.25                  | 1.50                  | 0.42±0.07                                             | 0.38±0.06   | 0.34±0.06                                             | 0.50±0.09   |
|                          |                          | 1.50                  | 2.00                  | 0.33±0.05                                             | 0.36±0.06   | 0.45±0.05                                             | 0.51±0.06   |
|                          |                          | 2.00                  | 2.50                  | 0.32±0.05                                             | 0.48±0.06   | 0.42±0.05                                             | 0.49±0.07   |
|                          |                          | 2.50                  | 3.00                  | 0.19±0.04                                             | 0.45±0.06   | 0.40±0.05                                             | 0.37±0.05   |
|                          |                          | 3.00                  | 3.50                  | 0.15±0.04                                             | 0.30±0.04   | 0.38±0.04                                             | 0.43±0.05   |
|                          |                          | 3.50                  | 4.00                  | 0.17±0.04                                             | 0.31±0.04   | 0.31±0.04                                             | 0.41±0.04   |
|                          |                          | 4.00                  | 5.00                  | 0.15±0.03                                             | 0.26±0.03   | 0.32±0.04                                             | 0.41±0.04   |
|                          |                          | 5.00                  | 6.50                  | 0.14±0.02                                             | 0.19±0.02   | 0.35±0.04                                             | 0.39±0.03   |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.16±0.02   | ±                                                     | 0.40±0.03   |
|                          |                          | 0.50                  | 0.75                  | 0.28±0.06                                             | 0.23±0.05   | 0.55±0.09                                             | 0.48±0.08   |
|                          |                          | 0.75                  | 1.00                  | 0.25±0.04                                             | 0.38±0.05   | 0.45±0.05                                             | 0.38±0.05   |
|                          |                          | 1.00                  | 1.25                  | 0.34±0.04                                             | 0.39±0.05   | 0.32±0.04                                             | 0.38±0.05   |
| 0.050                    | 0.075                    | 1.25                  | 1.50                  | 0.30±0.04                                             | 0.43±0.06   | 0.33±0.05                                             | 0.43±0.06   |
|                          |                          | 1.50                  | 2.00                  | 0.33±0.03                                             | 0.35±0.04   | 0.46±0.04                                             | 0.48±0.05   |
|                          |                          | 2.00                  | 2.50                  | 0.20±0.03                                             | 0.34±0.04   | 0.40±0.04                                             | 0.47±0.05   |
|                          |                          | 2.50                  | 3.00                  | 0.13±0.03                                             | 0.28±0.03   | 0.39±0.04                                             | 0.45±0.04   |
|                          |                          | 3.00                  | 3.50                  | 0.18±0.03                                             | 0.35±0.04   | 0.33±0.04                                             | 0.48±0.05   |
|                          |                          | 3.50                  | 4.00                  | 0.17±0.02                                             | 0.24±0.03   | 0.33±0.04                                             | 0.48±0.04   |
|                          |                          | 4.00                  | 5.00                  | 0.12±0.02                                             | 0.16±0.02   | 0.33±0.03                                             | 0.31±0.03   |
|                          |                          | 5.00                  | 6.50                  | 0.073±0.010                                           | 0.098±0.011 | 0.29±0.02                                             | 0.27±0.02   |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.049±0.007 | ±                                                     | 0.23±0.02   |
|                          |                          | 0.50                  | 0.75                  | 0.26±0.05                                             | 0.25±0.04   | 0.57±0.08                                             | 0.63±0.09   |
|                          |                          | 0.75                  | 1.00                  | 0.35±0.04                                             | 0.40±0.05   | 0.43±0.04                                             | 0.43±0.05   |
|                          |                          | 1.00                  | 1.25                  | 0.36±0.04                                             | 0.52±0.05   | 0.44±0.04                                             | 0.49±0.05   |
|                          |                          | 1.25                  | 1.50                  | 0.40±0.04                                             | 0.42±0.04   | 0.50±0.05                                             | 0.52±0.06   |
|                          |                          | 1.50                  | 2.00                  | 0.28±0.02                                             | 0.39±0.03   | 0.41±0.03                                             | 0.54±0.04   |
|                          |                          | 2.00                  | 2.50                  | 0.23±0.02                                             | 0.31±0.03   | 0.44±0.04                                             | 0.48±0.04   |
| 0.075                    | 0.100                    | 2.50                  | 3.00                  | 0.14±0.02                                             | 0.30±0.02   | 0.40±0.03                                             | 0.48±0.04   |
|                          |                          | 3.00                  | 3.50                  | 0.17±0.02                                             | 0.25±0.02   | 0.28±0.03                                             | 0.47±0.03   |
|                          |                          | 3.50                  | 4.00                  | 0.10±0.02                                             | 0.21±0.02   | 0.24±0.03                                             | 0.38±0.03   |
|                          |                          | 4.00                  | 5.00                  | 0.060±0.009                                           | 0.124±0.012 | 0.23±0.02                                             | 0.23±0.02   |
|                          |                          | 5.00                  | 6.50                  | 0.015±0.005                                           | 0.052±0.006 | 0.140±0.011                                           | 0.146±0.011 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.016±0.003 | ±                                                     | 0.071±0.007 |

Table 37: HARP results for the double-differential  $\pi^+$  and  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Al interactions at 8 and 12.9 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. A finer angular binning than in the previous set of tables is used. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |             | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |             |
|--------------------------|--------------------------|-----------------------|-----------------------|-------------------------------------------------------|-------------|-------------------------------------------------------|-------------|
|                          |                          |                       |                       | 8 GeV/c                                               | 12.9 GeV/c  | 8 GeV/c                                               | 12.9 GeV/c  |
|                          |                          |                       |                       |                                                       |             |                                                       |             |
| 0.025                    | 0.050                    | 0.50                  | 0.75                  | 0.37±0.11                                             | 0.51±0.16   | 0.15±0.06                                             | 0.35±0.16   |
|                          |                          | 0.75                  | 1.00                  | 0.38±0.08                                             | 0.43±0.13   | 0.37±0.09                                             | 0.38±0.12   |
|                          |                          | 1.00                  | 1.25                  | 0.24±0.07                                             | 0.36±0.12   | 0.27±0.07                                             | 0.36±0.14   |
|                          |                          | 1.25                  | 1.50                  | 0.11±0.04                                             | 0.27±0.10   | 0.48±0.12                                             | 0.13±0.08   |
|                          |                          | 1.50                  | 2.00                  | 0.23±0.05                                             | 0.54±0.10   | 0.26±0.06                                             | 0.30±0.09   |
|                          |                          | 2.00                  | 2.50                  | 0.27±0.06                                             | 0.42±0.09   | 0.24±0.05                                             | 0.34±0.11   |
|                          |                          | 2.50                  | 3.00                  | 0.35±0.06                                             | 0.63±0.10   | 0.21±0.04                                             | 0.27±0.07   |
|                          |                          | 3.00                  | 3.50                  | 0.31±0.05                                             | 0.47±0.08   | 0.25±0.04                                             | 0.35±0.08   |
|                          |                          | 3.50                  | 4.00                  | 0.38±0.05                                             | 0.49±0.09   | 0.21±0.04                                             | 0.40±0.08   |
|                          |                          | 4.00                  | 5.00                  | 0.37±0.04                                             | 0.39±0.06   | 0.19±0.03                                             | 0.21±0.04   |
|                          |                          | 5.00                  | 6.50                  | 0.50±0.03                                             | 0.41±0.05   | 0.19±0.03                                             | 0.14±0.03   |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.42±0.05   | ±                                                     | 0.16±0.03   |
| 0.050                    | 0.075                    | 0.50                  | 0.75                  | 0.38±0.09                                             | 0.48±0.11   | 0.22±0.06                                             | 0.21±0.08   |
|                          |                          | 0.75                  | 1.00                  | 0.33±0.06                                             | 0.64±0.10   | 0.15±0.04                                             | 0.27±0.07   |
|                          |                          | 1.00                  | 1.25                  | 0.33±0.06                                             | 0.36±0.08   | 0.24±0.05                                             | 0.40±0.09   |
|                          |                          | 1.25                  | 1.50                  | 0.21±0.05                                             | 0.36±0.08   | 0.18±0.04                                             | 0.39±0.09   |
|                          |                          | 1.50                  | 2.00                  | 0.28±0.04                                             | 0.43±0.07   | 0.23±0.04                                             | 0.43±0.06   |
|                          |                          | 2.00                  | 2.50                  | 0.37±0.05                                             | 0.52±0.07   | 0.24±0.04                                             | 0.33±0.06   |
|                          |                          | 2.50                  | 3.00                  | 0.34±0.05                                             | 0.45±0.06   | 0.16±0.03                                             | 0.24±0.05   |
|                          |                          | 3.00                  | 3.50                  | 0.33±0.05                                             | 0.46±0.06   | 0.21±0.03                                             | 0.32±0.06   |
|                          |                          | 3.50                  | 4.00                  | 0.29±0.04                                             | 0.42±0.06   | 0.18±0.03                                             | 0.29±0.05   |
|                          |                          | 4.00                  | 5.00                  | 0.42±0.04                                             | 0.41±0.04   | 0.13±0.02                                             | 0.17±0.03   |
|                          |                          | 5.00                  | 6.50                  | 0.40±0.03                                             | 0.29±0.03   | 0.080±0.011                                           | 0.14±0.02   |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.21±0.03   | ±                                                     | 0.052±0.012 |
| 0.075                    | 0.100                    | 0.50                  | 0.75                  | 0.26±0.06                                             | 0.35±0.09   | 0.32±0.07                                             | 0.24±0.08   |
|                          |                          | 0.75                  | 1.00                  | 0.26±0.05                                             | 0.59±0.08   | 0.26±0.04                                             | 0.30±0.07   |
|                          |                          | 1.00                  | 1.25                  | 0.34±0.05                                             | 0.47±0.07   | 0.33±0.05                                             | 0.41±0.07   |
|                          |                          | 1.25                  | 1.50                  | 0.40±0.06                                             | 0.45±0.07   | 0.28±0.04                                             | 0.41±0.07   |
|                          |                          | 1.50                  | 2.00                  | 0.35±0.04                                             | 0.43±0.06   | 0.26±0.03                                             | 0.36±0.05   |
|                          |                          | 2.00                  | 2.50                  | 0.34±0.04                                             | 0.54±0.06   | 0.27±0.03                                             | 0.36±0.04   |
|                          |                          | 2.50                  | 3.00                  | 0.33±0.04                                             | 0.34±0.04   | 0.15±0.02                                             | 0.29±0.04   |
|                          |                          | 3.00                  | 3.50                  | 0.36±0.04                                             | 0.48±0.05   | 0.12±0.02                                             | 0.27±0.03   |
|                          |                          | 3.50                  | 4.00                  | 0.31±0.05                                             | 0.36±0.04   | 0.09±0.02                                             | 0.29±0.03   |
|                          |                          | 4.00                  | 5.00                  | 0.31±0.02                                             | 0.25±0.03   | 0.043±0.007                                           | 0.12±0.02   |
|                          |                          | 5.00                  | 6.50                  | 0.16±0.05                                             | 0.14±0.02   | 0.025±0.005                                           | 0.051±0.010 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.080±0.012 | ±                                                     | 0.015±0.004 |



Table 38: HARP results for the double-differential  $\pi^+$  and  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Cu interactions at 8 and 12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. A finer angular binning than in the previous set of tables is used. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |             | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |             |
|--------------------------|--------------------------|-----------------------|-----------------------|-------------------------------------------------------|-------------|-------------------------------------------------------|-------------|
|                          |                          |                       |                       | 8 GeV/c                                               | 12 GeV/c    | 8 GeV/c                                               | 12 GeV/c    |
|                          |                          |                       |                       |                                                       |             |                                                       |             |
| 0.025                    | 0.050                    | 0.50                  | 0.75                  | 0.44±0.10                                             | 0.73±0.19   | 1.03±0.16                                             | 1.23±0.27   |
|                          |                          | 0.75                  | 1.00                  | 0.47±0.06                                             | 0.68±0.12   | 0.55±0.07                                             | 1.08±0.16   |
|                          |                          | 1.00                  | 1.25                  | 0.42±0.05                                             | 0.64±0.11   | 0.46±0.06                                             | 0.54±0.10   |
|                          |                          | 1.25                  | 1.50                  | 0.70±0.09                                             | 0.80±0.14   | 0.55±0.09                                             | 0.89±0.17   |
|                          |                          | 1.50                  | 2.00                  | 0.55±0.07                                             | 0.71±0.11   | 0.78±0.07                                             | 0.93±0.12   |
|                          |                          | 2.00                  | 2.50                  | 0.50±0.07                                             | 0.54±0.09   | 0.68±0.06                                             | 0.80±0.11   |
|                          |                          | 2.50                  | 3.00                  | 0.46±0.07                                             | 0.73±0.11   | 0.57±0.06                                             | 0.73±0.10   |
|                          |                          | 3.00                  | 3.50                  | 0.24±0.06                                             | 0.76±0.11   | 0.42±0.05                                             | 0.79±0.10   |
|                          |                          | 3.50                  | 4.00                  | 0.24±0.05                                             | 0.61±0.09   | 0.35±0.05                                             | 0.63±0.08   |
|                          |                          | 4.00                  | 5.00                  | 0.29±0.05                                             | 0.33±0.05   | 0.37±0.05                                             | 0.61±0.07   |
|                          |                          | 5.00                  | 6.50                  | 0.24±0.04                                             | 0.34±0.04   | 0.41±0.05                                             | 0.57±0.06   |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.27±0.04   | ±                                                     | 0.61±0.05   |
| 0.050                    | 0.075                    | 0.50                  | 0.75                  | 0.55±0.09                                             | 0.81±0.15   | 1.12±0.15                                             | 1.25±0.22   |
|                          |                          | 0.75                  | 1.00                  | 0.63±0.07                                             | 0.72±0.11   | 0.77±0.07                                             | 1.09±0.14   |
|                          |                          | 1.00                  | 1.25                  | 0.59±0.06                                             | 0.85±0.12   | 0.61±0.06                                             | 0.99±0.13   |
|                          |                          | 1.25                  | 1.50                  | 0.67±0.07                                             | 0.83±0.12   | 0.62±0.07                                             | 0.81±0.11   |
|                          |                          | 1.50                  | 2.00                  | 0.61±0.05                                             | 0.63±0.08   | 0.76±0.06                                             | 0.92±0.10   |
|                          |                          | 2.00                  | 2.50                  | 0.37±0.05                                             | 0.60±0.07   | 0.70±0.06                                             | 0.93±0.10   |
|                          |                          | 2.50                  | 3.00                  | 0.24±0.04                                             | 0.49±0.06   | 0.62±0.05                                             | 0.67±0.08   |
|                          |                          | 3.00                  | 3.50                  | 0.29±0.05                                             | 0.56±0.07   | 0.50±0.06                                             | 0.70±0.08   |
|                          |                          | 3.50                  | 4.00                  | 0.25±0.04                                             | 0.47±0.07   | 0.43±0.06                                             | 0.63±0.09   |
|                          |                          | 4.00                  | 5.00                  | 0.23±0.03                                             | 0.28±0.04   | 0.55±0.06                                             | 0.59±0.05   |
|                          |                          | 5.00                  | 6.50                  | 0.12±0.02                                             | 0.17±0.02   | 0.43±0.04                                             | 0.48±0.04   |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.12±0.02   | ±                                                     | 0.34±0.03   |
| 0.075                    | 0.100                    | 0.50                  | 0.75                  | 0.55±0.09                                             | 0.72±0.12   | 1.08±0.13                                             | 1.42±0.19   |
|                          |                          | 0.75                  | 1.00                  | 0.75±0.07                                             | 0.59±0.08   | 0.70±0.06                                             | 0.74±0.09   |
|                          |                          | 1.00                  | 1.25                  | 0.67±0.06                                             | 0.87±0.10   | 0.82±0.07                                             | 1.10±0.12   |
|                          |                          | 1.25                  | 1.50                  | 0.58±0.05                                             | 0.65±0.07   | 0.75±0.07                                             | 0.89±0.10   |
|                          |                          | 1.50                  | 2.00                  | 0.48±0.04                                             | 0.73±0.08   | 0.74±0.05                                             | 0.96±0.08   |
|                          |                          | 2.00                  | 2.50                  | 0.40±0.04                                             | 0.75±0.07   | 0.67±0.05                                             | 1.10±0.09   |
|                          |                          | 2.50                  | 3.00                  | 0.29±0.03                                             | 0.58±0.05   | 0.63±0.05                                             | 0.83±0.07   |
|                          |                          | 3.00                  | 3.50                  | 0.30±0.03                                             | 0.51±0.05   | 0.45±0.05                                             | 0.84±0.08   |
|                          |                          | 3.50                  | 4.00                  | 0.16±0.04                                             | 0.35±0.04   | 0.39±0.04                                             | 0.71±0.08   |
|                          |                          | 4.00                  | 5.00                  | 0.082±0.014                                           | 0.24±0.03   | 0.33±0.03                                             | 0.40±0.03   |
|                          |                          | 5.00                  | 6.50                  | 0.024±0.007                                           | 0.089±0.014 | 0.21±0.02                                             | 0.24±0.02   |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.018±0.005 | ±                                                     | 0.122±0.013 |

Table 39: HARP results for the double-differential  $\pi^+$  and  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ - Cu interactions at 8 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. A finer angular binning than in the previous set of tables is used. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c rad))<br>8 GeV/c | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c rad))<br>8 GeV/c |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------------------|------------------------------------------------------------------|
| 0.025                    | 0.050                    | 0.50                  | 0.75                  | 0.22±0.08                                                        | 0.45±0.15                                                        |
|                          |                          | 0.75                  | 1.00                  | 0.32±0.08                                                        | 0.45±0.11                                                        |
|                          |                          | 1.00                  | 1.25                  | 0.26±0.07                                                        | 0.37±0.10                                                        |
|                          |                          | 1.25                  | 1.50                  | 0.49±0.12                                                        | 0.31±0.11                                                        |
|                          |                          | 1.50                  | 2.00                  | 0.32±0.08                                                        | 0.64±0.12                                                        |
|                          |                          | 2.00                  | 2.50                  | 0.34±0.09                                                        | 0.36±0.08                                                        |
|                          |                          | 2.50                  | 3.00                  | 0.61±0.12                                                        | 0.27±0.06                                                        |
|                          |                          | 3.00                  | 3.50                  | 0.32±0.06                                                        | 0.40±0.07                                                        |
|                          |                          | 3.50                  | 4.00                  | 0.47±0.07                                                        | 0.31±0.05                                                        |
|                          |                          | 4.00                  | 5.00                  | 0.40±0.05                                                        | 0.32±0.04                                                        |
|                          |                          | 5.00                  | 6.50                  | 0.76±0.05                                                        | 0.30±0.04                                                        |
|                          |                          | 0.50                  | 0.75                  | 0.57±0.14                                                        | 0.44±0.11                                                        |
|                          |                          | 0.75                  | 1.00                  | 0.59±0.11                                                        | 0.49±0.09                                                        |
|                          |                          | 1.00                  | 1.25                  | 0.49±0.09                                                        | 0.48±0.10                                                        |
|                          |                          | 1.25                  | 1.50                  | 0.66±0.11                                                        | 0.56±0.10                                                        |
|                          |                          | 1.50                  | 2.00                  | 0.60±0.08                                                        | 0.44±0.07                                                        |
| 0.050                    | 0.075                    | 2.00                  | 2.50                  | 0.59±0.09                                                        | 0.36±0.06                                                        |
|                          |                          | 2.50                  | 3.00                  | 0.51±0.08                                                        | 0.31±0.05                                                        |
|                          |                          | 3.00                  | 3.50                  | 0.45±0.08                                                        | 0.30±0.05                                                        |
|                          |                          | 3.50                  | 4.00                  | 0.44±0.07                                                        | 0.22±0.04                                                        |
|                          |                          | 4.00                  | 5.00                  | 0.58±0.06                                                        | 0.23±0.03                                                        |
|                          |                          | 5.00                  | 6.50                  | 0.62±0.04                                                        | 0.12±0.02                                                        |
|                          |                          | 0.50                  | 0.75                  | 0.64±0.15                                                        | 0.61±0.12                                                        |
|                          |                          | 0.75                  | 1.00                  | 0.53±0.09                                                        | 0.47±0.08                                                        |
|                          |                          | 1.00                  | 1.25                  | 0.66±0.10                                                        | 0.46±0.08                                                        |
|                          |                          | 1.25                  | 1.50                  | 0.59±0.08                                                        | 0.64±0.09                                                        |
|                          |                          | 1.50                  | 2.00                  | 0.55±0.07                                                        | 0.39±0.05                                                        |
|                          |                          | 2.00                  | 2.50                  | 0.54±0.07                                                        | 0.40±0.05                                                        |
|                          |                          | 2.50                  | 3.00                  | 0.56±0.07                                                        | 0.24±0.03                                                        |
|                          |                          | 3.00                  | 3.50                  | 0.41±0.05                                                        | 0.22±0.03                                                        |
|                          |                          | 3.50                  | 4.00                  | 0.47±0.07                                                        | 0.15±0.03                                                        |
|                          |                          | 4.00                  | 5.00                  | 0.41±0.04                                                        | 0.13±0.02                                                        |
| 0.075                    | 0.100                    | 5.00                  | 6.50                  | 0.27±0.09                                                        | 0.024±0.005                                                      |

Table 40: HARP results for the double-differential  $\pi^+$  and  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Sn interactions at 8 and 12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. A finer angular binning than in the previous set of tables is used. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |             | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |           |
|--------------------------|--------------------------|-----------------------|-----------------------|-------------------------------------------------------|-------------|-------------------------------------------------------|-----------|
|                          |                          |                       |                       | 8 GeV/c                                               | 12 GeV/c    | 8 GeV/c                                               | 12 GeV/c  |
|                          |                          |                       |                       |                                                       |             |                                                       |           |
| 0.025                    | 0.050                    | 0.50                  | 0.75                  | 0.64±0.14                                             | 1.56±0.30   | 1.39±0.21                                             | 1.86±0.34 |
|                          |                          | 0.75                  | 1.00                  | 0.53±0.07                                             | 1.16±0.19   | 0.70±0.08                                             | 1.53±0.17 |
|                          |                          | 1.00                  | 1.25                  | 0.36±0.05                                             | 0.94±0.13   | 0.29±0.04                                             | 0.93±0.12 |
|                          |                          | 1.25                  | 1.50                  | 0.55±0.09                                             | 1.15±0.17   | 0.55±0.10                                             | 1.34±0.20 |
|                          |                          | 1.50                  | 2.00                  | 0.63±0.10                                             | 1.01±0.76   | 0.80±0.08                                             | 1.38±0.15 |
|                          |                          | 2.00                  | 2.50                  | 0.84±0.12                                             | 1.07±0.36   | 0.75±0.09                                             | 1.19±0.16 |
|                          |                          | 2.50                  | 3.00                  | 0.53±0.11                                             | 0.88±0.15   | 0.70±0.09                                             | 1.08±0.12 |
|                          |                          | 3.00                  | 3.50                  | 0.24±0.08                                             | 0.92±0.16   | 0.45±0.07                                             | 1.04±0.11 |
|                          |                          | 3.50                  | 4.00                  | 0.37±0.09                                             | 0.81±0.10   | 0.27±0.05                                             | 0.95±0.13 |
|                          |                          | 4.00                  | 5.00                  | 0.36±0.07                                             | 0.58±0.07   | 0.43±0.07                                             | 0.91±0.09 |
|                          |                          | 5.00                  | 6.50                  | 0.25±0.05                                             | 0.43±0.05   | 0.63±0.09                                             | 0.73±0.07 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.39±0.05   | ±                                                     | 0.84±0.07 |
| 0.050                    | 0.075                    | 0.50                  | 0.75                  | 0.55±0.11                                             | 0.84±0.16   | 1.19±0.17                                             | 1.98±0.28 |
|                          |                          | 0.75                  | 1.00                  | 0.57±0.09                                             | 0.99±0.13   | 1.01±0.11                                             | 1.13±0.14 |
|                          |                          | 1.00                  | 1.25                  | 0.79±0.10                                             | 1.01±0.13   | 0.82±0.10                                             | 1.20±0.14 |
|                          |                          | 1.25                  | 1.50                  | 0.96±0.12                                             | 1.24±0.17   | 0.88±0.12                                             | 1.17±0.14 |
|                          |                          | 1.50                  | 2.00                  | 0.77±0.07                                             | 1.01±0.11   | 0.93±0.09                                             | 1.50±0.14 |
|                          |                          | 2.00                  | 2.50                  | 0.58±0.08                                             | 0.95±0.11   | 0.84±0.09                                             | 1.20±0.11 |
|                          |                          | 2.50                  | 3.00                  | 0.32±0.07                                             | 0.68±0.07   | 0.69±0.08                                             | 1.19±0.11 |
|                          |                          | 3.00                  | 3.50                  | 0.39±0.08                                             | 0.87±0.10   | 0.82±0.11                                             | 1.10±0.11 |
|                          |                          | 3.50                  | 4.00                  | 0.31±0.06                                             | 0.60±0.07   | 0.58±0.09                                             | 1.03±0.11 |
|                          |                          | 4.00                  | 5.00                  | 0.23±0.04                                             | 0.41±0.05   | 0.70±0.09                                             | 0.79±0.07 |
|                          |                          | 5.00                  | 6.50                  | 0.16±0.03                                             | 0.26±0.03   | 0.56±0.06                                             | 0.67±0.05 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.14±0.02   | ±                                                     | 0.45±0.04 |
| 0.075                    | 0.100                    | 0.50                  | 0.75                  | 0.73±0.12                                             | 1.09±0.16   | 1.61±0.20                                             | 2.13±0.26 |
|                          |                          | 0.75                  | 1.00                  | 0.80±0.10                                             | 1.54±0.16   | 1.13±0.11                                             | 1.56±0.15 |
|                          |                          | 1.00                  | 1.25                  | 1.08±0.11                                             | 1.37±0.13   | 0.98±0.09                                             | 1.31±0.12 |
|                          |                          | 1.25                  | 1.50                  | 0.73±0.08                                             | 1.30±0.11   | 0.88±0.10                                             | 1.43±0.15 |
|                          |                          | 1.50                  | 2.00                  | 0.68±0.06                                             | 1.03±0.09   | 0.94±0.08                                             | 1.29±0.10 |
|                          |                          | 2.00                  | 2.50                  | 0.48±0.05                                             | 0.92±0.07   | 0.96±0.09                                             | 1.43±0.11 |
|                          |                          | 2.50                  | 3.00                  | 0.41±0.05                                             | 0.80±0.07   | 0.93±0.08                                             | 1.28±0.10 |
|                          |                          | 3.00                  | 3.50                  | 0.34±0.04                                             | 0.56±0.05   | 0.54±0.08                                             | 0.97±0.07 |
|                          |                          | 3.50                  | 4.00                  | 0.14±0.05                                             | 0.43±0.05   | 0.51±0.07                                             | 0.88±0.07 |
|                          |                          | 4.00                  | 5.00                  | 0.09±0.02                                             | 0.33±0.03   | 0.47±0.05                                             | 0.57±0.06 |
|                          |                          | 5.00                  | 6.50                  | 0.021±0.008                                           | 0.14±0.02   | 0.26±0.03                                             | 0.31±0.02 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.037±0.008 | ±                                                     | 0.17±0.02 |

Table 41: HARP results for the double-differential  $\pi^+$  and  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Sn interactions at 8 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. A finer angular binning than in the previous set of tables is used. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c rad))<br>8 GeV/c | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c rad))<br>8 GeV/c |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------------------|------------------------------------------------------------------|
| 0.025                    | 0.050                    | 0.50                  | 0.75                  | 0.78±0.18                                                        | 0.58±0.14                                                        |
|                          |                          | 0.75                  | 1.00                  | 0.63±0.10                                                        | 0.57±0.11                                                        |
|                          |                          | 1.00                  | 1.25                  | 0.34±0.07                                                        | 0.68±0.14                                                        |
|                          |                          | 1.25                  | 1.50                  | 0.27±0.08                                                        | 0.34±0.11                                                        |
|                          |                          | 1.50                  | 2.00                  | 0.63±0.13                                                        | 0.82±0.16                                                        |
|                          |                          | 2.00                  | 2.50                  | 0.55±0.13                                                        | 0.59±0.11                                                        |
|                          |                          | 2.50                  | 3.00                  | 0.59±0.13                                                        | 0.67±0.11                                                        |
|                          |                          | 3.00                  | 3.50                  | 0.52±0.09                                                        | 0.48±0.08                                                        |
|                          |                          | 3.50                  | 4.00                  | 0.54±0.07                                                        | 0.45±0.07                                                        |
|                          |                          | 4.00                  | 5.00                  | 0.61±0.06                                                        | 0.52±0.07                                                        |
|                          |                          | 5.00                  | 6.50                  | 0.98±0.06                                                        | 0.39±0.06                                                        |
|                          |                          | 0.50                  | 0.75                  | 0.85±0.19                                                        | 0.88±0.19                                                        |
|                          |                          | 0.75                  | 1.00                  | 0.61±0.12                                                        | 0.54±0.11                                                        |
|                          |                          | 1.00                  | 1.25                  | 0.60±0.11                                                        | 0.66±0.13                                                        |
|                          |                          | 1.25                  | 1.50                  | 0.97±0.16                                                        | 0.81±0.15                                                        |
|                          |                          | 1.50                  | 2.00                  | 0.85±0.11                                                        | 0.49±0.08                                                        |
| 0.050                    | 0.075                    | 2.00                  | 2.50                  | 0.67±0.11                                                        | 0.61±0.10                                                        |
|                          |                          | 2.50                  | 3.00                  | 0.54±0.09                                                        | 0.41±0.07                                                        |
|                          |                          | 3.00                  | 3.50                  | 0.44±0.09                                                        | 0.42±0.07                                                        |
|                          |                          | 3.50                  | 4.00                  | 0.39±0.08                                                        | 0.39±0.06                                                        |
|                          |                          | 4.00                  | 5.00                  | 0.77±0.08                                                        | 0.28±0.04                                                        |
|                          |                          | 5.00                  | 6.50                  | 0.88±0.06                                                        | 0.14±0.02                                                        |
|                          |                          | 0.50                  | 0.75                  | 1.33±0.26                                                        | 0.91±0.17                                                        |
|                          |                          | 0.75                  | 1.00                  | 1.01±0.15                                                        | 0.74±0.11                                                        |
|                          |                          | 1.00                  | 1.25                  | 0.82±0.12                                                        | 0.67±0.11                                                        |
|                          |                          | 1.25                  | 1.50                  | 0.79±0.12                                                        | 0.68±0.10                                                        |
|                          |                          | 1.50                  | 2.00                  | 0.70±0.23                                                        | 0.69±0.08                                                        |
|                          |                          | 2.00                  | 2.50                  | 0.68±0.35                                                        | 0.44±0.06                                                        |
|                          |                          | 2.50                  | 3.00                  | 0.72±0.28                                                        | 0.40±0.05                                                        |
|                          |                          | 3.00                  | 3.50                  | 0.62±0.08                                                        | 0.32±0.05                                                        |
|                          |                          | 3.50                  | 4.00                  | 0.65±0.11                                                        | 0.22±0.03                                                        |
|                          |                          | 4.00                  | 5.00                  | 0.62±0.08                                                        | 0.13±0.02                                                        |
| 0.075                    | 0.100                    | 5.00                  | 6.50                  | 0.42±0.15                                                        | 0.044±0.009                                                      |

Table 42: HARP results for the double-differential  $\pi^+$  and  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Ta interactions at 8 and 12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. A finer angular binning than in the previous set of tables is used. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |             | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |           |
|--------------------------|--------------------------|-----------------------|-----------------------|-------------------------------------------------------|-------------|-------------------------------------------------------|-----------|
|                          |                          |                       |                       | 8 GeV/c                                               | 12 GeV/c    | 8 GeV/c                                               | 12 GeV/c  |
|                          |                          |                       |                       |                                                       |             |                                                       |           |
| 0.025                    | 0.050                    | 0.50                  | 0.75                  | 1.47±0.25                                             | 1.65±0.32   | 2.65±0.39                                             | 2.44±0.39 |
|                          |                          | 0.75                  | 1.00                  | 0.72±0.08                                             | 1.11±0.15   | 1.18±0.12                                             | 1.34±0.16 |
|                          |                          | 1.00                  | 1.25                  | 0.57±0.07                                             | 1.02±0.15   | 0.43±0.06                                             | 0.89±0.13 |
|                          |                          | 1.25                  | 1.50                  | 0.90±0.12                                             | 1.19±0.18   | 0.76±0.13                                             | 1.61±0.24 |
|                          |                          | 1.50                  | 2.00                  | 0.72±0.11                                             | 1.21±0.17   | 1.09±0.11                                             | 1.51±0.19 |
|                          |                          | 2.00                  | 2.50                  | 0.90±0.15                                             | 1.26±0.17   | 1.10±0.12                                             | 1.80±0.23 |
|                          |                          | 2.50                  | 3.00                  | 0.90±0.15                                             | 1.26±0.16   | 0.92±0.13                                             | 1.34±0.18 |
|                          |                          | 3.00                  | 3.50                  | 0.51±0.12                                             | 1.49±0.18   | 0.50±0.08                                             | 1.50±0.16 |
|                          |                          | 3.50                  | 4.00                  | 0.51±0.11                                             | 0.77±0.14   | 0.32±0.06                                             | 1.08±0.13 |
|                          |                          | 4.00                  | 5.00                  | 0.44±0.08                                             | 0.55±0.08   | 0.44±0.08                                             | 0.93±0.10 |
|                          |                          | 5.00                  | 6.50                  | 0.35±0.07                                             | 0.55±0.07   | 0.53±0.09                                             | 0.92±0.11 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.36±0.05   | ±                                                     | 0.96±0.09 |
|                          |                          | 0.50                  | 0.75                  | 0.82±0.14                                             | 1.39±0.25   | 1.27±0.18                                             | 2.13±0.31 |
|                          |                          | 0.75                  | 1.00                  | 0.75±0.11                                             | 1.38±0.18   | 1.12±0.12                                             | 1.35±0.18 |
|                          |                          | 1.00                  | 1.25                  | 1.12±0.14                                             | 1.27±0.17   | 0.89±0.11                                             | 1.62±0.20 |
|                          |                          | 1.25                  | 1.50                  | 1.41±0.16                                             | 1.87±0.25   | 1.08±0.15                                             | 1.92±0.22 |
| 0.050                    | 0.075                    | 1.50                  | 2.00                  | 1.21±0.11                                             | 1.23±0.13   | 1.43±0.12                                             | 1.64±0.16 |
|                          |                          | 2.00                  | 2.50                  | 0.77±0.11                                             | 1.16±0.12   | 1.21±0.13                                             | 1.70±0.15 |
|                          |                          | 2.50                  | 3.00                  | 0.32±0.09                                             | 1.02±0.11   | 1.11±0.11                                             | 1.43±0.14 |
|                          |                          | 3.00                  | 3.50                  | 0.40±0.10                                             | 0.95±0.11   | 0.89±0.13                                             | 1.41±0.13 |
|                          |                          | 3.50                  | 4.00                  | 0.42±0.08                                             | 0.87±0.12   | 0.82±0.13                                             | 1.06±0.11 |
|                          |                          | 4.00                  | 5.00                  | 0.24±0.04                                             | 0.54±0.06   | 0.75±0.10                                             | 0.88±0.08 |
|                          |                          | 5.00                  | 6.50                  | 0.21±0.04                                             | 0.31±0.04   | 0.73±0.09                                             | 0.73±0.06 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.19±0.03   | ±                                                     | 0.53±0.05 |
|                          |                          | 0.50                  | 0.75                  | 1.21±0.19                                             | 1.42±0.22   | 2.19±0.26                                             | 2.67±0.34 |
|                          |                          | 0.75                  | 1.00                  | 1.31±0.14                                             | 2.10±0.22   | 1.17±0.12                                             | 2.23±0.21 |
|                          |                          | 1.00                  | 1.25                  | 1.27±0.12                                             | 1.77±0.17   | 1.19±0.12                                             | 1.78±0.17 |
|                          |                          | 1.25                  | 1.50                  | 1.27±0.12                                             | 1.71±0.16   | 1.16±0.13                                             | 1.86±0.20 |
|                          |                          | 1.50                  | 2.00                  | 0.93±0.08                                             | 1.30±0.11   | 1.06±0.09                                             | 1.62±0.13 |
|                          |                          | 2.00                  | 2.50                  | 0.66±0.07                                             | 1.03±0.10   | 1.24±0.11                                             | 1.51±0.13 |
|                          |                          | 2.50                  | 3.00                  | 0.39±0.06                                             | 0.79±0.08   | 0.92±0.08                                             | 1.53±0.12 |
|                          |                          | 3.00                  | 3.50                  | 0.35±0.05                                             | 0.84±0.08   | 0.75±0.09                                             | 1.17±0.09 |
| 0.075                    | 0.100                    | 3.50                  | 4.00                  | 0.19±0.06                                             | 0.57±0.06   | 0.61±0.07                                             | 0.98±0.08 |
|                          |                          | 4.00                  | 5.00                  | 0.14±0.03                                             | 0.35±0.04   | 0.61±0.07                                             | 0.69±0.06 |
|                          |                          | 5.00                  | 6.50                  | 0.032±0.013                                           | 0.15±0.02   | 0.31±0.04                                             | 0.36±0.03 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.040±0.009 | ±                                                     | 0.17±0.02 |

Table 43: HARP results for the double-differential  $\pi^+$  and  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Ta interactions at 8 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. A finer angular binning than in the previous set of tables is used. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c rad))<br>8 GeV/c | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c rad))<br>8 GeV/c |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------------------|------------------------------------------------------------------|
| 0.025                    | 0.050                    | 0.50                  | 0.75                  | 1.66±0.37                                                        | 0.92±0.25                                                        |
|                          |                          | 0.75                  | 1.00                  | 1.02±0.16                                                        | 0.60±0.13                                                        |
|                          |                          | 1.00                  | 1.25                  | 0.50±0.10                                                        | 0.96±0.21                                                        |
|                          |                          | 1.25                  | 1.50                  | 0.39±0.12                                                        | 0.33±0.13                                                        |
|                          |                          | 1.50                  | 2.00                  | 0.66±0.16                                                        | 0.87±0.19                                                        |
|                          |                          | 2.00                  | 2.50                  | 0.62±0.17                                                        | 0.63±0.14                                                        |
|                          |                          | 2.50                  | 3.00                  | 0.72±0.18                                                        | 0.54±0.11                                                        |
|                          |                          | 3.00                  | 3.50                  | 0.58±0.12                                                        | 0.82±0.15                                                        |
|                          |                          | 3.50                  | 4.00                  | 0.59±0.11                                                        | 0.63±0.11                                                        |
|                          |                          | 4.00                  | 5.00                  | 0.56±0.08                                                        | 0.56±0.09                                                        |
|                          |                          | 5.00                  | 6.50                  | 1.12±0.08                                                        | 0.48±0.07                                                        |
|                          |                          | 0.50                  | 0.75                  | 1.15±0.27                                                        | 0.79±0.22                                                        |
|                          |                          | 0.75                  | 1.00                  | 0.90±0.19                                                        | 0.72±0.16                                                        |
|                          |                          | 1.00                  | 1.25                  | 0.95±0.19                                                        | 1.02±0.22                                                        |
|                          |                          | 1.25                  | 1.50                  | 0.96±0.20                                                        | 0.54±0.13                                                        |
|                          |                          | 1.50                  | 2.00                  | 0.57±0.12                                                        | 0.98±0.16                                                        |
| 0.050                    | 0.075                    | 2.00                  | 2.50                  | 0.70±0.15                                                        | 0.60±0.11                                                        |
|                          |                          | 2.50                  | 3.00                  | 0.72±0.14                                                        | 0.62±0.12                                                        |
|                          |                          | 3.00                  | 3.50                  | 0.46±0.12                                                        | 0.36±0.07                                                        |
|                          |                          | 3.50                  | 4.00                  | 0.42±0.10                                                        | 0.41±0.08                                                        |
|                          |                          | 4.00                  | 5.00                  | 0.95±0.11                                                        | 0.31±0.05                                                        |
|                          |                          | 5.00                  | 6.50                  | 1.06±0.08                                                        | 0.18±0.03                                                        |
|                          |                          | 0.50                  | 0.75                  | 1.11±0.27                                                        | 0.88±0.21                                                        |
|                          |                          | 0.75                  | 1.00                  | 1.04±0.18                                                        | 0.75±0.14                                                        |
|                          |                          | 1.00                  | 1.25                  | 1.15±0.19                                                        | 1.19±0.20                                                        |
|                          |                          | 1.25                  | 1.50                  | 1.03±0.17                                                        | 0.79±0.14                                                        |
|                          |                          | 1.50                  | 2.00                  | 0.67±0.11                                                        | 0.62±0.10                                                        |
|                          |                          | 2.00                  | 2.50                  | 0.72±0.12                                                        | 0.47±0.08                                                        |
|                          |                          | 2.50                  | 3.00                  | 0.74±0.12                                                        | 0.51±0.08                                                        |
|                          |                          | 3.00                  | 3.50                  | 0.72±0.10                                                        | 0.44±0.07                                                        |
|                          |                          | 3.50                  | 4.00                  | 0.70±0.11                                                        | 0.22±0.04                                                        |
|                          |                          | 4.00                  | 5.00                  | 0.71±0.07                                                        | 0.19±0.03                                                        |
| 0.075                    | 0.100                    | 5.00                  | 6.50                  | 0.41±0.15                                                        | 0.051±0.011                                                      |

Table 44: HARP results for the double-differential  $\pi^+$  and  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^-$ -Pb interactions at 8 and 12 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. A finer angular binning than in the previous set of tables is used. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |             | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c rad)) |           |
|--------------------------|--------------------------|-----------------------|-----------------------|-------------------------------------------------------|-------------|-------------------------------------------------------|-----------|
|                          |                          |                       |                       | 8 GeV/c                                               | 12 GeV/c    | 8 GeV/c                                               | 12 GeV/c  |
| 0.025                    | 0.050                    | 0.50                  | 0.75                  | 1.60±0.27                                             | 1.58±0.29   | 3.24±0.43                                             | 2.52±0.41 |
|                          |                          | 0.75                  | 1.00                  | 0.87±0.09                                             | 1.37±0.16   | 1.26±0.14                                             | 1.74±0.18 |
|                          |                          | 1.00                  | 1.25                  | 0.56±0.07                                             | 1.06±0.14   | 0.54±0.07                                             | 0.88±0.13 |
|                          |                          | 1.25                  | 1.50                  | 0.76±0.11                                             | 1.25±0.18   | 0.54±0.09                                             | 0.84±0.14 |
|                          |                          | 1.50                  | 2.00                  | 0.82±0.13                                             | 1.40±0.18   | 1.27±0.14                                             | 1.53±0.20 |
|                          |                          | 2.00                  | 2.50                  | 1.12±0.16                                             | 1.24±0.17   | 1.24±0.14                                             | 1.66±0.20 |
|                          |                          | 2.50                  | 3.00                  | 0.76±0.15                                             | 1.28±0.16   | 1.02±0.13                                             | 0.87±0.13 |
|                          |                          | 3.00                  | 3.50                  | 0.41±0.12                                             | 0.92±0.14   | 0.63±0.10                                             | 1.27±0.16 |
|                          |                          | 3.50                  | 4.00                  | 0.53±0.12                                             | 0.85±0.13   | 0.42±0.09                                             | 1.05±0.13 |
|                          |                          | 4.00                  | 5.00                  | 0.47±0.09                                             | 0.56±0.09   | 0.54±0.10                                             | 0.73±0.10 |
|                          |                          | 5.00                  | 6.50                  | 0.40±0.08                                             | 0.44±0.07   | 0.65±0.11                                             | 0.62±0.09 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.33±0.06   | ±                                                     | 0.77±0.10 |
|                          | 0.075                    | 0.50                  | 0.75                  | 0.92±0.15                                             | 1.45±0.22   | 1.91±0.24                                             | 2.46±0.33 |
|                          |                          | 0.75                  | 1.00                  | 0.75±0.11                                             | 1.33±0.17   | 1.39±0.15                                             | 1.34±0.18 |
|                          |                          | 1.00                  | 1.25                  | 1.00±0.13                                             | 1.82±0.20   | 1.12±0.14                                             | 1.56±0.19 |
|                          |                          | 1.25                  | 1.50                  | 1.72±0.20                                             | 1.70±0.20   | 1.15±0.16                                             | 1.49±0.18 |
|                          |                          | 1.50                  | 2.00                  | 1.19±0.10                                             | 1.38±0.14   | 1.43±0.14                                             | 1.66±0.16 |
|                          |                          | 2.00                  | 2.50                  | 0.71±0.10                                             | 1.21±0.12   | 1.24±0.13                                             | 1.52±0.15 |
|                          |                          | 2.50                  | 3.00                  | 0.36±0.10                                             | 1.00±0.10   | 0.95±0.11                                             | 1.29±0.13 |
|                          |                          | 3.00                  | 3.50                  | 0.41±0.11                                             | 1.04±0.11   | 0.90±0.14                                             | 1.16±0.12 |
|                          |                          | 3.50                  | 4.00                  | 0.42±0.08                                             | 0.76±0.10   | 0.84±0.13                                             | 0.93±0.11 |
|                          |                          | 4.00                  | 5.00                  | 0.36±0.06                                             | 0.43±0.06   | 0.82±0.12                                             | 0.76±0.08 |
| 0.075                    | 0.100                    | 5.00                  | 6.50                  | 0.20±0.04                                             | 0.29±0.04   | 0.67±0.08                                             | 0.65±0.06 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.13±0.02   | ±                                                     | 0.51±0.05 |
|                          |                          | 0.50                  | 0.75                  | 0.96±0.16                                             | 1.81±0.23   | 2.21±0.26                                             | 2.93±0.34 |
|                          |                          | 0.75                  | 1.00                  | 1.25±0.15                                             | 1.83±0.18   | 1.28±0.13                                             | 1.92±0.19 |
|                          |                          | 1.00                  | 1.25                  | 1.75±0.16                                             | 1.77±0.16   | 1.27±0.12                                             | 1.59±0.15 |
|                          |                          | 1.25                  | 1.50                  | 1.30±0.13                                             | 1.71±0.14   | 1.53±0.18                                             | 1.71±0.19 |
|                          |                          | 1.50                  | 2.00                  | 1.00±0.09                                             | 1.42±0.12   | 1.38±0.11                                             | 1.73±0.13 |
|                          |                          | 2.00                  | 2.50                  | 0.61±0.07                                             | 1.17±0.10   | 1.14±0.10                                             | 1.67±0.12 |
|                          |                          | 2.50                  | 3.00                  | 0.45±0.06                                             | 0.90±0.08   | 1.04±0.10                                             | 1.38±0.11 |
|                          |                          | 3.00                  | 3.50                  | 0.43±0.06                                             | 0.91±0.08   | 0.74±0.11                                             | 1.22±0.09 |
|                          |                          | 3.50                  | 4.00                  | 0.24±0.06                                             | 0.55±0.07   | 0.63±0.09                                             | 0.99±0.10 |
|                          |                          | 4.00                  | 5.00                  | 0.12±0.03                                             | 0.32±0.04   | 0.46±0.07                                             | 0.54±0.06 |
|                          |                          | 5.00                  | 6.50                  | 0.033±0.014                                           | 0.14±0.02   | 0.32±0.04                                             | 0.33±0.03 |
|                          |                          | 6.50                  | 8.00                  | ±                                                     | 0.032±0.008 | ±                                                     | 0.19±0.02 |

Table 45: HARP results for the double-differential  $\pi^+$  and  $\pi^-$  production cross-section in the laboratory system,  $d^2\sigma^\pi/(dpd\Omega)$ , for  $\pi^+$ -Pb interactions at 8 GeV/c. Each row refers to a different ( $p_{\min} \leq p < p_{\max}, \theta_{\min} \leq \theta < \theta_{\max}$ ) bin, where  $p$  and  $\theta$  are the pion momentum and polar angle, respectively. A finer angular binning than in the previous set of tables is used. The central value as well as the square-root of the diagonal elements of the covariance matrix are given.

| $\theta_{\min}$<br>(rad) | $\theta_{\max}$<br>(rad) | $p_{\min}$<br>(GeV/c) | $p_{\max}$<br>(GeV/c) | $d^2\sigma^{\pi^+}/(dpd\Omega)$<br>(barn/(GeV/c rad))<br>8 GeV/c | $d^2\sigma^{\pi^-}/(dpd\Omega)$<br>(barn/(GeV/c rad))<br>8 GeV/c |
|--------------------------|--------------------------|-----------------------|-----------------------|------------------------------------------------------------------|------------------------------------------------------------------|
| 0.025                    | 0.050                    | 0.50                  | 0.75                  | 1.72±0.38                                                        | 0.75±0.21                                                        |
|                          |                          | 0.75                  | 1.00                  | 1.04±0.16                                                        | 0.85±0.17                                                        |
|                          |                          | 1.00                  | 1.25                  | 0.42±0.09                                                        | 0.73±0.17                                                        |
|                          |                          | 1.25                  | 1.50                  | 0.58±0.15                                                        | 0.79±0.25                                                        |
|                          |                          | 1.50                  | 2.00                  | 0.73±0.17                                                        | 0.66±0.17                                                        |
|                          |                          | 2.00                  | 2.50                  | 0.63±0.20                                                        | 0.72±0.16                                                        |
|                          |                          | 2.50                  | 3.00                  | 0.49±0.15                                                        | 0.60±0.13                                                        |
|                          |                          | 3.00                  | 3.50                  | 0.65±0.13                                                        | 0.51±0.10                                                        |
|                          |                          | 3.50                  | 4.00                  | 0.59±0.10                                                        | 0.67±0.12                                                        |
|                          |                          | 4.00                  | 5.00                  | 0.63±0.08                                                        | 0.58±0.09                                                        |
|                          |                          | 5.00                  | 6.50                  | 1.18±0.08                                                        | 0.49±0.08                                                        |
|                          |                          | 0.50                  | 0.75                  | 0.66±0.18                                                        | 0.75±0.20                                                        |
|                          |                          | 0.75                  | 1.00                  | 0.77±0.18                                                        | 0.73±0.16                                                        |
|                          |                          | 1.00                  | 1.25                  | 0.43±0.12                                                        | 0.69±0.17                                                        |
|                          |                          | 1.25                  | 1.50                  | 0.75±0.18                                                        | 1.00±0.20                                                        |
|                          |                          | 1.50                  | 2.00                  | 0.69±0.13                                                        | 0.77±0.13                                                        |
| 0.050                    | 0.075                    | 2.00                  | 2.50                  | 0.74±0.15                                                        | 0.69±0.13                                                        |
|                          |                          | 2.50                  | 3.00                  | 0.85±0.16                                                        | 0.49±0.10                                                        |
|                          |                          | 3.00                  | 3.50                  | 0.55±0.13                                                        | 0.51±0.10                                                        |
|                          |                          | 3.50                  | 4.00                  | 0.50±0.11                                                        | 0.52±0.09                                                        |
|                          |                          | 4.00                  | 5.00                  | 1.09±0.12                                                        | 0.40±0.06                                                        |
|                          |                          | 5.00                  | 6.50                  | 1.11±0.09                                                        | 0.22±0.03                                                        |
|                          |                          | 0.50                  | 0.75                  | 1.04±0.27                                                        | 1.05±0.23                                                        |
|                          |                          | 0.75                  | 1.00                  | 1.02±0.19                                                        | 1.27±0.20                                                        |
|                          |                          | 1.00                  | 1.25                  | 0.86±0.16                                                        | 1.30±0.21                                                        |
|                          |                          | 1.25                  | 1.50                  | 0.99±0.17                                                        | 0.91±0.15                                                        |
|                          |                          | 1.50                  | 2.00                  | 0.84±0.13                                                        | 0.81±0.12                                                        |
|                          |                          | 2.00                  | 2.50                  | 0.88±0.15                                                        | 0.33±0.06                                                        |
|                          |                          | 2.50                  | 3.00                  | 0.66±0.11                                                        | 0.53±0.08                                                        |
|                          |                          | 3.00                  | 3.50                  | 0.66±0.09                                                        | 0.34±0.06                                                        |
|                          |                          | 3.50                  | 4.00                  | 0.82±0.13                                                        | 0.30±0.06                                                        |
|                          |                          | 4.00                  | 5.00                  | 0.75±0.07                                                        | 0.21±0.03                                                        |
| 0.075                    | 0.100                    | 5.00                  | 6.50                  | 0.44±0.15                                                        | 0.065±0.013                                                      |



| Normalization factors, incident $\pi^+$ |          |         |          |         |          |         |          |         |          |         |          |         |           |         |
|-----------------------------------------|----------|---------|----------|---------|----------|---------|----------|---------|----------|---------|----------|---------|-----------|---------|
| Generator                               | Be 3 GeV |         | Ta 3 GeV |         | Be 5 GeV |         | Ta 5 GeV |         | Be 8 GeV |         | Ta 8 GeV |         | Be 12 GeV |         |
|                                         | $\pi^+$  | $\pi^-$ | $\pi^+$  | $\pi^-$ | $\pi^+$  | $\pi^-$ | $\pi^+$  | $\pi^-$ | $\pi^+$  | $\pi^-$ | $\pi^+$  | $\pi^-$ | $\pi^+$   | $\pi^-$ |
| bert                                    | 1.1      | 1.1     | 0.9      | 0.6     | 1.3      | 1.3     | 1.1      | 1.1     | 1.7      | 1.4     | 1.5      | 1.4     | 2.0       | 1.1     |
| lhep                                    | 1.3      | 0.6     | 2.8      | 1.0     | 1.5      | 0.8     | 2.5      | 1.1     | 1.3      | 0.8     | 1.0      | 0.5     | 1.0       | 0.6     |
| qgsb                                    | 4.8      | 1.8     | 2.1      | 1.5     | 3.8      | 1.3     | 2.2      | 1.1     | 1.9      | 1.0     | 1.4      | 0.8     | 0.9       | 0.6     |
| qgsc                                    | -        | -       | -        | -       | -        | -       | -        | -       | 1.6      | 1.0     | 1.2      | 0.7     | 0.8       | 0.6     |
| qgsp                                    | -        | -       | -        | -       | -        | -       | -        | -       | 1.9      | 1.0     | 1.2      | 0.6     | 0.9       | 0.6     |
| ftfp                                    | -        | -       | -        | -       | -        | -       | -        | -       | -        | -       | 0.8      | 0.7     | 1.1       | 1.1     |
| mars                                    | 1.3      | 1.2     | 1.2      | 0.9     | 0.9      | 1.0     | 0.7      | 0.6     | 1.0      | 1.1     | 0.8      | 0.6     | 0.8       | 1.1     |
| rpg                                     | 1.4      | 0.6     | 2.7      | 0.8     | 1.2      | 0.7     | 1.8      | 0.9     | 1.1      | 0.6     | 0.7      | 0.4     | 0.9       | 0.6     |

| Normalization factors, incident $\pi^+$ |          |         |          |         |          |         |          |         |          |         |          |         |           |         |
|-----------------------------------------|----------|---------|----------|---------|----------|---------|----------|---------|----------|---------|----------|---------|-----------|---------|
| Generator                               | Be 3 GeV |         | Ta 3 GeV |         | Be 5 GeV |         | Ta 5 GeV |         | Be 8 GeV |         | Ta 8 GeV |         | Be 12 GeV |         |
|                                         | $\pi^+$  | $\pi^-$ | $\pi^+$  | $\pi^-$ | $\pi^+$  | $\pi^-$ | $\pi^+$  | $\pi^-$ | $\pi^+$  | $\pi^-$ | $\pi^+$  | $\pi^-$ | $\pi^+$   | $\pi^-$ |
| bert                                    | 1.0      | 1.1     | 0.5      | 0.6     | 1.2      | 1.4     | 0.9      | 1.2     | 1.6      | 2.1     | 1.4      | 1.9     | 1.2       | 1.1     |
| lhep                                    | 0.5      | 1.6     | 0.7      | 2.9     | 0.8      | 1.9     | 1.0      | 2.6     | 0.9      | 1.7     | 0.6      | 1.2     | 0.6       | 1.1     |
| qgsb                                    | 1.7      | 4.8     | 1.1      | 1.7     | 1.2      | 4.0     | 0.9      | 2.3     | 1.2      | 2.2     | 0.8      | 1.7     | 0.6       | 0.6     |
| qgsc                                    | -        | -       | -        | -       | -        | -       | -        | -       | 1.1      | 1.8     | 0.7      | 1.3     | 0.6       | 0.6     |
| qgsp                                    | -        | -       | -        | -       | -        | -       | -        | -       | 1.2      | 2.2     | 0.7      | 1.4     | 0.6       | 0.6     |
| ftfp                                    | -        | -       | -        | -       | -        | -       | -        | -       | -        | -       | 0.7      | 1.1     | 0.7       | 1.1     |
| mars                                    | 1.1      | 1.2     | 0.7      | 1.0     | 0.9      | 0.9     | 0.6      | 0.6     | 1.2      | 1.2     | 0.7      | 0.9     | 0.8       | 0.6     |
| rpg                                     | 0.6      | 1.7     | 0.7      | 2.8     | 0.7      | 1.7     | 0.8      | 1.9     | 0.8      | 1.5     | 0.5      | 0.9     | 0.5       | 0.6     |