Optimal Policy With Tradable And Bankable Pollution Permits : Taking The Market Microstructure Into Account[1]

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Abstract

This paper analyses how the way emission permits are traded –their market microstructure– impacts the optimal policy to be adopted by the environmental agency. The microstructure used is one of a *quote driven market* type, which characterizes many financial markets : market makers act as intermediaries for trading the permits by setting a ask price and a bid price. The possibility of permit banking is also introduced in our dynamic two-period model.

We show that when the market makers and the agency do not know the technology of the producers with certainty, a positive spread may be set by market makers and that, under some conditions, banking increases the expected welfare given this market microstructure.

If such a microstructure takes place or is organised on markets for pollution permits, we recommend to allow for banking (a) if the marginal willingness to pay for the environment increases much over time, (b) if the pollutant is rather a stock than a flow one, and/or (c) if the incomplete information faced by the intermediaries and by the agency is severe. In the first period, the environmental agency will then have to define and allocate a larger amount of permits than if banking is not allowed, but a lower or a same amount of permits in the second period.

1 Introduction

Following Dales (1968) idea, Montgomery (1972) shows that the implementation of a market for pollution permits is an efficient instrument to reach an environmental target, provided that the market is competitive. Several authors have then relaxed this latter assumption. For instance, Hahn (1984) looks at market power and Stavins (1995) analyses the effect of different kinds of transaction costs. They both emphasize the impact of the initial permit allocation among firms on the efficiency properties of the instrument.

Another interesting issue to look at is how trades in permits do actually take place. Taking the allocation of permits among firms as given, Germain, Lovo and van Steenberghe (2000) analyse the impact of a market microstructure on the optimal environmental policy, and more particularly on the total number of permits to be allocated. The microstructure used is one of a *quote driven market* type: market makers act as intermediaries for trading the permits by setting an ask price (i.e. a price at which they are ready to sell permits) and a bid price (i.e., a price at which they are ready to buy permits), the former being larger or equal to the latter. Market makers may then diffuse their prices either through informal channels, for instance by phone or via Internet, or in organized exchanges. This kind of microstructure is particularly relevant for large markets such as the one under the US *Acid Rain Program* or the forthcoming world markets in greenhouse gases emission permits.

Their results, obtained in a static context, are the following ones. When the market maker is a monopolist, the optimal environmental policy consists in defining fewer permits than if the market is walrasian, that is when there are no intermediaries. Taking the microstructure into account has, however, no effect on the optimal environmental policy whenever market makers compete 'à la Bertrand'. Nevertheless, these results do not hold anymore if the environmental agency and the market maker(s) are uncertain about the polluters production functions. In this case, the market makers set a positive spread and the optimal policy consists in allocating an amount of permits that is either higher or lower than if no microstructure is taken into account. The expected welfare may then be higher in the presence of competing or even monopolist market makers than under walrasian markets. These results under uncertainty are largely driven by the fact that market markers are left with unsold permits in some states of the world.

The aim of the present paper is to extend this analysis into a dynamic setting. The reason is twofold. Firstly, we introduce a system of banking, which is an important feature of existing and envisioned pollution permits markets. This would allow for a less wasteful use of permits which have not been sold by the market makers by allowing to use them in subsequent periods. Secondly, the dynamic setting allows to resorb (part of) the uncertainty (incomplete information) faced by the environmental agency and by market makers.

Dynamic models with tradable pollution permits have recently received much attention. Cronshaw and Kruse (1996) and Rubin (1996) extend the work of Montgomery (1972) and show that banking can decrease the total costs of achieving a fixed emissions target over all periods. But, when it is used, banking (and, symetrically, borrowing) of permits leads in some periods to levels of ambient pollution which do not necessarily correspond to those specified by the total amount of permits distributed. While Kling and Rubin (1997) have shown that this leads the economy to a state which is not always socially efficient, Requate (1998) points out that allowing banking under uncertainty can improve welfare in some situations. Finally, Phaneuf and Requate (1998) argue that banking may distort firms incentives to invest in cleaner technology ([1]).

We work in a discrete-time two-period model. As intermediary results, we reach similar conclusions to those of Requate (1998), but in a slightly different context. Indeed, rather than investigating the effetct of demand uncertainty on polluters' output, we focus on the incomplete information about the polluters production functions that has to be faced by the environnmental agency and by market makers if any. Although both settings lead to uncertain aggregate demand and supply of permits, our setting implies that agents learn the production functions by observing trades in permits in the first period ([2]) and do not face any uncertainty in the second period.

The structure of the paper is the following one. In the next section, we introduce the model and solve the problem of the social planner.

In section 3, we analyse the optimal policy to be taken by the environmental agency if the tradable emission permits instrument is used. At this stage, we do not introduce any microstructure. We rather assume that the price of permits is formed on a walrasian market. This section is divided in two parts. In the first part, we assume that information is complete and show that allowing banking may forbid the social planner's optimum to be reached. In the second part, however, when the agency does not know the producers technology with certainty, banking may increase welfare.

A realistic permit price formation mechanism, namely a quote driven market microstrure, is introduced in section 4. Under complete information, Bertrand-competition between the market markers leads to the same outcome as under a walrasian market. Things are, however, very different under incomplete information: banking is more often recommended than when the market is walrasian.

[1] See also the interesting paper of Hagem and Westlog (1998), dealing with market power, banking and borrowing in a world with certainty, as well as Schennach (2000) who looks at the effects of technological innovations, growth in electricity demand or changes in environmental regulations under uncertainty.

[2] Aggregate demand and supply of permits might however also change through changes in productivity or in the demand for outputs, which would only lead to partial learning by the agents.

Finally, section 5 concludes.

2 The model

As mentionned above, the aim of the analysis is to introduce a microstructure for a pollution permits market in a dynamic context. For this purpose, time is divided into two periods ($t = \{1, 2\}$). In each period, the environmental agency defines a certain amount of permits to be distributed to the firms according to an exogenous sharing rule. The firms are thus not allowed to emit more pollutants than the amount of permits in their possession in each period. However, banking of permits is allowed, i.e., permits which have been defined and distributed in the first period but which have not been used in this period may be used by the firms during the second period.

Each firm is characterized by an exogenous couple of parameters (ρ_1, ρ_2) with $0 \leq \rho_t \leq \overline{\rho}_t$, $t = 1, 2$, on which the allocation of emission permits will be based. Denoting by $n(\rho_1, \rho_2)$ the number of firms which enjoy the characteristics (ρ_1, ρ_2), the total amount of firms is $N = \int_0^{\overline{\rho}_1} \int_0^{\overline{\rho}_2} n(\rho_1, \rho_2) \, d\rho_2 d\rho_1$. The number of firms enjoying characteristic ρ_t at period t is then given by $N_t(\rho_t) = \int_0^{\overline{\rho}_{3-t}} n(\rho_1, \rho_2) \, d\rho_{3-t}$. Note also that we need to have $\int_0^{\overline{\rho}_1} \rho_1 N_1(\rho_1) d\rho_1 = 1 = \int_0^{\overline{\rho}_2} \rho_2 N_2(\rho_2) d\rho_2$.

We assume that all firms have the same production function :

$$y_t = mg(x_t) \tag{1}$$

where y_t is the output of a firm in period t, x_t is its amount of emissions in the same period and m is a positive parameter. $g(.)$ is strictly concave and differentiable, with $\gamma(x_t) = \frac{dg}{dx_t} > 0$. Assuming that production functions differ would not modify our results ; this is briefly discussed in section 3.1.1.

The total amount of pollutants emitted at period t is then

$$z_t = \int_0^{\overline{\rho}_1} \int_0^{\overline{\rho}_2} x_t(\rho_1, \rho_2) \, n(\rho_1, \rho_2) \, d\rho_2 d\rho_1 \tag{2}$$

and contributes to a stock of pollution M_t at period t in the following way:

$$M_t = \delta M_{t-1} + z_t \tag{3}$$

where $[1 - \delta]$ $(0 \leq \delta \leq 1)$ is the depreciation rate of the stock ([3]). Without loss of generality, we set $M_0 = 0$. Focusing on market microstructure rather than on intertemporal aspects, we suppose that the discount rate is null. This stock of pollutants then causes damages $\Omega_t(M_t)$ at period t. We assume linear damages :

$$\Omega_t(M_t) = \pi_t M_t \tag{4}$$

[3] The pollutant is called a *flow pollutant* if $\delta = 0$, while it is called a *stock pollutant* if $0 < \delta \leq 1$.

where π_t may be interpreted as the social marginal willingness to pay for a reduction in the stock of pollutants.

Finally, the total production in period t is writen as

$$Y_t = \int_0^{\overline{\rho}_1} \int_0^{\overline{\rho}_2} mg\left(x_t\left(\rho_1, \rho_2\right)\right) n\left(\rho_1, \rho_2\right) d\rho_2 d\rho_1. \tag{5}$$

The problem of the social planner is thus

$$\max_{\{x_1, x_2\}} U \equiv \sum_{t=1}^{2} \left[Y_t - \pi_t M_t\right] \tag{6}$$

subject to (3) and (5). The solution of this problem leads to

$$m\gamma\left(x_1\left(\rho_1, \rho_2\right)\right) = \pi_1 + \delta\pi_2 \tag{7}$$

$$m\gamma\left(x_2\left(\rho_1, \rho_2\right)\right) = \pi_2 \tag{8}$$

i.e., at each period, each firm should emit an amount of pollutants such that the marginal cost of pollution abatement in that period equals the social marginal damage caused by these pollutants in the same period and in the subsequent periods if any. This is nothing else than the Samuelson condition for public goods. This may also be written as

$$x_1 = \eta\left(\tfrac{\pi_1 + \delta\pi_2}{m}\right) \tag{9}$$

$$x_2 = \eta\left(\tfrac{\pi_2}{m}\right) \tag{10}$$

with $\eta(.) \equiv \gamma^{-1}(.)$. η is decreasing as g is concave.

We now introduce a market for pollution permits and analyse whether this instrument is susceptible to lead the economy to the social planner's optimum or not.

3 Introducing a walrasian market for pollution permits

The sequence of decisions is as follows. In each period t, the environmental agency defines a certain amount of permits $\overline{z}_t$ and allocates them to the firms according to their characteristic ρ_t, $t = 1, 2$. Each firm thus receives $\overline{x}_t = \rho_t \overline{z}_t$ permits at period t. These maximise their profits and trade permits among each other. Each agent takes into account the impact of his decision on the subsequent decisions. In particular, the agency takes the firms behaviour into account when defining the amount of permits.

This two-period model is thus solved recursively : (i) firms maximise their profits in period $t = 2$; (ii) the agency computes the amount of permits to be distributed in period $t = 2$; (iii) firms maximise their profits in period $t = 1$; (iv) the agency computes the amount of permits to be distributed in period $t = 1$.

Two sitations are analysed. In the next subsection, we assume complete information for all agents. This assumption is relaxed in the second subsection where we consider that the environmental agency does not know the producers technology with certainty.

3.1 Complete information (W)[4]

3.1.1 The last period

The firms

At the second period, each firm solves

$$\max_{\{x_2 \geq 0\}} mg(x_2) - \sigma_2 \left[x_2 - \rho_2 \overline{z}_2 - B^f(\rho_1, \rho_2) \right] \tag{11}$$

where σ_2 is the price of the permits at the second period and $B^f(\rho_1, \rho_2)$ $(0 \leq B^f \leq \rho_1 \overline{z}_1)$ is the amount of permits that have been banked by the firm from the first period. This gives a level of emissions

$$x_2 = \eta \left(\tfrac{\sigma_2}{m} \right) \tag{12}$$

and a net demand of permits $\widehat{x}_2 = \eta \left(\frac{\sigma_2}{m} \right) - \rho_2 \overline{z}_2 - B^f(\rho_1, \rho_2)$.

As not all firms have the same characteristics (ρ_1, ρ_2), their net demand differs. A similar consequence would be induced by assuming different production functions: (ρ_1, ρ_2) given, the lower a firm's abatement costs, the lower its net demand ([5]).

The market

The aggregate excess demand of permits $ED_2 \left(\frac{\sigma_2}{m} \right)$ then writes as

$$ED_2 \left(\tfrac{\sigma_2}{m} \right) = \int_0^{\overline{\rho}_2} \left[\eta \left(\tfrac{\sigma_2}{m} \right) - \rho_2 \overline{z}_2 \right] N_2(\rho_2) d\rho_2 - B^F \tag{13}$$

where $B^F \equiv \int_0^{\overline{\rho}_1} \int_0^{\overline{\rho}_2} B^f(\rho_1, \rho_2) n(\rho_1, \rho_2) d\rho_2 d\rho_1$. At equilibrium, $ED_2 \left(\frac{\sigma_2}{m} \right) = 0$, that is $N\eta \left(\frac{\sigma_2}{m} \right) = \overline{z}_2 + B^F$, or

$$\sigma_2 = m\gamma \left(\frac{\overline{z}_2 + B^F}{N} \right). \tag{14}$$

The agency

Knowing firms' behaviour[6] and the market equilibrium condition, the environmental agency defines the amount of permits $\overline{z}_2$ to be distributed at the second period according to the exogenousely given proportion ρ_2. It does so by solving

$$\max_{\{\overline{z}_2\}} Y_2 - \pi_2 M_2 \tag{15}$$

[4]'W' will be used to denote the equilibrium values of the decision variables when the market is walrasian.

[5]Note also that adding to the production function (1) a fixed cost component –which would differ accross firms– would somehow relax the assumption of identical production functions while not modifying firms' beheviour.

[6]And having observed in their account the permits that they have banked.

subject to the market clearing condition (14) and where the agregate production is given by

$$Y_2 = \int_0^{\overline{\rho}_1} \int_0^{\overline{\rho}_2} g\left(\eta\left(\tfrac{\sigma_2}{m}\right)\right) n(\rho_1,\rho_2) d\rho_2 d\rho_1 = Ng\left(\eta\left(\tfrac{\sigma_2}{m}\right)\right), \tag{16}$$

the stock of pollutants is

$$M_2 = \delta M_1 + z_2 = \delta M_1 + N\eta\left(\tfrac{\sigma_2}{m}\right) \tag{17}$$

and the agregate emissions are

$$z_2 = \int_0^{\overline{\rho}_1} \int_0^{\overline{\rho}_2} \eta\left(\tfrac{\sigma_2}{m}\right) n(\rho_1,\rho_2) d\rho_2 d\rho_1 = N\eta\left(\tfrac{\sigma_2}{m}\right). \tag{18}$$

The first order conditions of this problem lead to

$$\sigma_2 = \pi_2 \tag{19}$$

and thus the environmental agency chooses

$$\overline{z}_2^W = N\eta\left(\tfrac{\pi_2}{m}\right) - B^F, \tag{20}$$

which leads the economy to the social planner's optimum at the second period.

Indeed, condition (19) states that the price of the permits should be equal to the social marginal damages caused by pollution. Therefore (19) and (12) together give conditions (8) or (10) which characterize the social planner's optimum. By (18), condition (20) may be rewritten as $z_2 = \overline{z}_2 + B^F$. The total amount of emissions allowed and realized at the second period corresponds to the amount of permits defined at that period and the permits banked from the first period.

3.1.2 The first period

In the first period, each agent has rational expectations on the behaviour of all the other agents.

The firms

Each firm solves

$$\begin{aligned} &\max_{\{x_1, Bf\geq 0\}} mg(x_1) - \sigma_1 \left[x_1 - \rho_1\overline{z}_1 + B^f(\rho_1,\rho_2)\right] + mg(\eta\left(\tfrac{\pi_2}{m}\right)) \\ &-\pi_2 \left[\eta\left(\tfrac{\pi_2}{m}\right) - \rho_2\left[N\eta\left(\tfrac{\pi_2}{m}\right) - B^F\right] - B^f(\rho_1,\rho_2)\right] \end{aligned} \tag{21}$$

where σ_1 is the price of permits in the first period. Note that, by arbitrage, we cannot have $\sigma_1 < \sigma_2$. Then, the firms problem leads to emissions $x_1 = \eta\left(\frac{\sigma_1}{m}\right)$, and to $B^f(\rho_1,\rho_2) = 0$ if $\sigma_1 > \sigma_2$ and $B^f(\rho_1,\rho_2) = \rho_1\overline{z}_1 - \eta\left(\frac{\sigma_1}{m}\right) > 0$ if $\sigma_1 = \sigma_2$. The net demand of permits by a firm is thus $\widehat{x}_1 = \eta\left(\frac{\sigma_1}{m}\right) - \rho_1\overline{z}_1 + B^f(\rho_1,\rho_2)$.

The market

The market clears when the agregate excess is zero, that is

$$ED_1\left(\tfrac{\sigma_1}{m}\right) = \int_0^{\overline{\rho}_1} \left[\eta\left(\tfrac{\sigma_1}{m}\right) - \rho_1 \overline{z}_1\right] N_1(\rho_1) d\rho_1 + B^F = 0 \tag{22}$$

Thus when $\overline{z}_1 < \overline{z}_2 = N\eta\left(\frac{\pi_2}{m}\right) - B^F$, we have

$$\sigma_1 = m\gamma\left(\frac{\overline{z}_1}{N}\right) \tag{23}$$

as well as $\sigma_1 > \sigma_2$ and $B^F = 0$, while when $\overline{z}_1 \geq \overline{z}_2$ we have

$$\sigma_1 = m\gamma\left(\frac{\overline{z}_1 + \overline{z}_2}{2N}\right) \tag{24}$$

with $\sigma_1 = \sigma_2$ and $B^F = \frac{\overline{z}_1 - \overline{z}_2}{2} \geq 0$.

The agency

The environmental agency's problem is then

$$\max_{\{\overline{z}_1\}} U = Nmg\left(\eta\left(\tfrac{\sigma_1}{m}\right)\right) - \pi_1 N\eta\left(\tfrac{\sigma_1}{m}\right) + Nmg\left(\eta\left(\tfrac{\pi_2}{m}\right)\right) - \pi_2 N\left[\delta\eta\left(\tfrac{\sigma_1}{m}\right) + \eta\left(\tfrac{\pi_2}{m}\right)\right] \tag{25}$$

where σ_1 and σ_2 are given by (23) or by (24) according to whether $\overline{z}_1 < \overline{z}_2$ or $\overline{z}_1 \geq \overline{z}_2$. U is a non-analytical function of $\overline{z}_1$, i.e., its analytical expression changes on its interval of definition. Two regimes emerge ; they are illustrated in figures 1.a and 1.b.

Figure 1.a : Regime 1

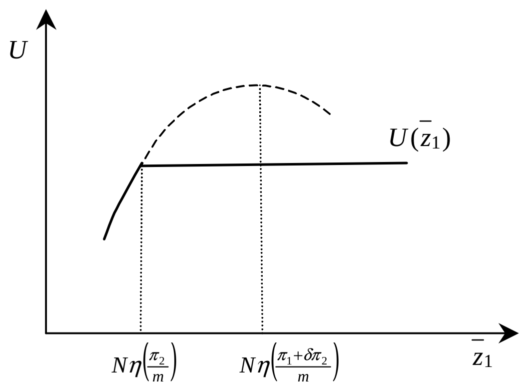
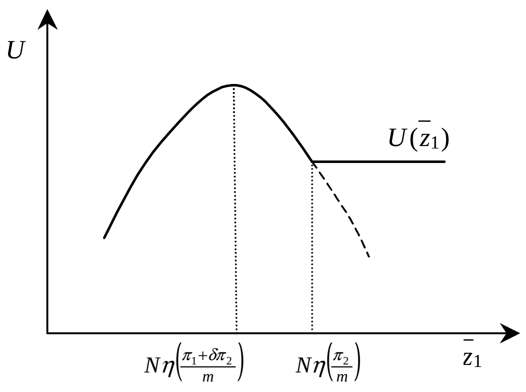


Figure 1.b : Regime 2



Proposition 1 *If* $\pi_1 < [1-\delta]\,\pi_2$ *(Regime 1), the environmental agency chooses*

$$N\eta\left(\tfrac{\pi_2}{m}\right) \leq \overline{z}_1^W \leq 2N\eta\left(\tfrac{\pi_2}{m}\right) \quad and \quad \overline{z}_2^W = 2N\eta\left(\tfrac{\pi_2}{m}\right) - \overline{z}_1^W,$$

banking takes place ($B^F = \frac{\overline{z}_1 - \overline{z}_2}{2}$) *and the social planner's optimum is reached in the second period but not in the first one.*
If $\pi_1 \geq [1-\delta]\,\pi_2$ *(Regime 2), then the environmental agency chooses*

$$\overline{z}_1^W = N\eta\left(\frac{\pi_1 + \delta\pi_2}{m}\right) \quad \textit{and} \quad \overline{z}_2^W = N\eta\left(\frac{\pi_2}{m}\right)),$$

banking does not take place and the social planner's optimum is reached at both periods.
■

These results are driven by the relative size of the marginal willingness to pay in each period. In Regime 2, that is when the marginal willingness to pay for the environment –corrected for the decay of the pollutants emitted in the first period ($\delta\pi_2$)– is smaller in the second period than in the first one ($\pi_1 \geq [1-\delta]\,\pi_2$), the environmental agency issues fewer permits in the first period than in the second one ($\overline{z}_1^W \leq \overline{z}_2^W$), which results in higher permits prices at the first period ($\sigma_1 \geq \sigma_2$) and no banking by firms ($B^f(\rho_1,\rho_2) = 0$). In these circumstances, the economy reaches the social planner's optimum. Indeed, $\overline{z}_1^W = N\eta\left(\frac{\pi_1+\delta\pi_2}{m}\right)$ and $\overline{z}_2^W = N\eta\left(\frac{\pi_2}{m}\right)$, together with (23), (14) and with $B^f(\rho_1,\rho_2) = 0$, lead to conditions (9) and (10), i.e. $x_1 = \eta\left(\frac{\pi_1+\delta\pi_2}{m}\right)$ and $x_2 = \eta\left(\frac{\pi_2}{m}\right)$.

This will, however, not be the case in Regime 1 ($\pi_1 < [1-\delta]\,\pi_2$). In the first period, the environmental agency should define and allocate an amount of permits at least as high as the optimal amount of emissions to be realized in the second period. If the agency defines more permits than this amount, then firms are banking permits in such a way ($B^F = \frac{\overline{z}_1 - \overline{z}_2}{2} > 0$) that their price equalizes accross periods ($\sigma_1 = \sigma_2$). Note that $\overline{z}_1 \leq 2N\eta\left(\frac{\pi_2}{m}\right)$ ensures that $\overline{z}_2 \geq 0$. Then, $x_2 = \eta\left(\frac{\pi_2}{m}\right) = x_1 < \eta\left(\frac{\pi_1+\delta\pi_2}{m}\right)$ and the economy does not reach the social planner's optimum in the first period as condition (9) is not satisfied. Indeed, banking of permits does not allow the total amount of emissions –and thus of production– to be larger in the first period than in the second one, although it would be desirable to do so as the marginal willingness to pay is relatively small in the first period ($\pi_1 < [1-\delta]\,\pi_2$).

This is in line with the results of Kling and Rubin (1997) who consider simultaneously the possibilities of banking and borrowing (with a discout rate) but set $\delta = 0$.

3.2 Uncertainty on the production functions (WU)[7]

We now assume that, in the first period, the technology is known perfectly by the firms but not by the agency. In the second period, the environmental agency has learned the production function by observing either trades in permits or the level of production in the first period.

[7]'WU' will be used to denote the equilibrium values of the decision variables under uncertainty when the market is walrasian.

The uncertainty faced by the environemental agency in the first period takes the form of a distribution of probabilities $p^I, p^{II}, ..., p^R$ on the technology parameter $m \in \{m^I, m^{II}, ..., m^R\}$, with $m^I < m^{II} < ... < m^R$, $\sum_{r=1}^{R} p^r = 1$ and $p^r \geq 0\ \forall r$.

3.2.1 The last period

Because the technology parameter is known by the environmental agency in the second period, the resolution of the agents problems is similar to the one under certainty (section 3.1). The marginal willingness to pay being constant, the price of the permits does not depend on the technology parameter in the second period and $\sigma_2^r = \pi_2\ \forall r$. The optimal amount of permits to be defined by the environmental agency is then

$$\overline{z}_2^{r^{WU}} = N\eta\left(\frac{\pi_2}{m^r}\right) - B^{F,r}, \tag{26}$$

which is similar to the result without uncertainty on the technology.

3.2.2 The first period

The firms

In the first period, each firm knows m^r and emits $x_1^r = \eta\left(\frac{\sigma_1^r}{m^r}\right)$. It sets its net demand of permits as follows:

$$\widehat{x}_1^r = \left\{ \begin{array}{ll} \eta\left(\frac{\sigma_1^r}{m^r}\right) - \rho_1\overline{z}_1 & \text{if } \pi_2 < \sigma_1^r \\ \eta\left(\frac{\sigma_1^r}{m^r}\right) - \rho_1\overline{z}_1 + B^{f,r} & \text{if } \pi_2 = \sigma_1^r \\ \infty & \text{if } \pi_2 > \sigma_1^r \end{array} \right\}$$

The market

The market equilibrium is then characterized by

$$\sigma_1^r = m^r\gamma\left(\frac{\overline{z}_1}{N}\right) \quad \text{and} \quad B^{F,r} = 0$$

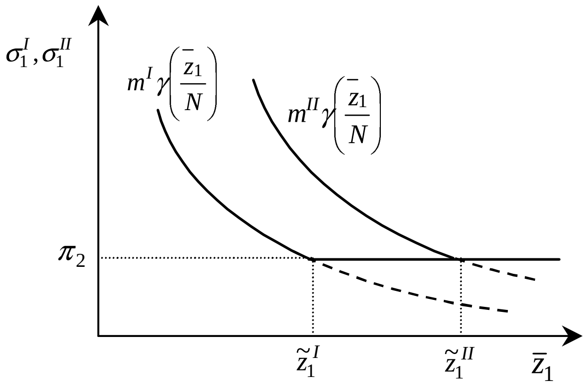
if $\exists\sigma_1^r > \pi_2$ such that $ED_1\left(\frac{\sigma_1^r}{m^r}\right) = \int_0^{\overline{\rho}_1}\left[\eta\left(\frac{\sigma_1^r}{m^r}\right) - \rho_1\overline{z}_1\right]N_1(\rho_1)d\rho_1 = 0$ (i.e., the market clears), and by

$$\sigma_1^r = \pi_2 \quad \text{and} \quad B^{F,r} = \overline{z}_1 - N\eta\left(\frac{\pi_2}{m^r}\right)$$

otherwise. Let $\widetilde{z}_1^r$ be the amount of permits for which the market just clears at the price $\sigma_1^r = \pi_2$. In other words, $\widetilde{z}_1^r$ is such that $ED_1\left(\frac{\pi_2}{m^r}\right) = \int_0^{\overline{\rho}_1}\left[\eta\left(\frac{\pi_2}{m^r}\right) - \rho_1\widetilde{z}_1^r\right]N_1(\rho_1)d\rho_1 = 0$, that is $\widetilde{z}_1^r = N\eta\left(\frac{\pi_2}{m^r}\right)$.

Figure 2 illustrates the price of the permits in the first period for $r = I, II$. If the total amount of permits distributed by the agency, $\overline{z}_1$, is above $\widetilde{z}_1^{II}$, banking takes place and the price will not go below π_2. If $\widetilde{z}_1^I < \overline{z}_1 < \widetilde{z}_1^{II}$, banking takes place only for $r = I$ and thus $\sigma_1^I = \pi_2$ and $\sigma_1^{II} > \pi_2$. Finally, banking does not take place and $\sigma_1^I, \sigma_1^{II} > \pi_2$ if $\overline{z}_1 < \widetilde{z}_1^I$.

Figure 2



The agency

We solve the problem of the environmental agency for the case where $m \in \left\{m^I, m^{II}\right\}$ with $\widetilde{m} \equiv p^I m^I + p^{II} m^{II}$. It writes

$$\max_{\{\overline{z}_1\}} E\left[U\left(\overline{z}_1\right)\right] = p^I U^I\left(\overline{z}_1\right) + p^{II} U^{II}\left(\overline{z}_1\right)$$

where

$$U^r = \left\{ \begin{array}{ll} U^{r,a} = N m^r g\left(\frac{\overline{z}_1}{N}\right) - \pi_1 \overline{z}_1 + N m^r g\left(\eta\left(\frac{\pi_2}{m^r}\right)\right) - \pi_2\left[\delta \overline{z}_1 + N\eta\left(\frac{\pi_2}{m^r}\right)\right] & \text{if} \quad \overline{z}_1 \leq \widetilde{z}_1^r, \\ U^{r,b} = N m^r g\left(\eta\left(\frac{\pi_2}{m^r}\right)\right) - \pi_1 N\eta\left(\frac{\pi_2}{m^r}\right) + N m^I g\left(\eta\left(\frac{\pi_2}{m^r}\right)\right) - \pi_2\left[1+\delta\right] N\eta\left(\frac{\pi_2}{m^r}\right) & \text{if} \quad \overline{z}_1 \geq \widetilde{z}_1^r \end{array} \right\} \tag{27}$$

for $r = I, II$.

Then :

Proposition 2 *At the first period, the environmental agency chooses*

$$\overline{z}_1^{WU} \left\{ \begin{array}{llll} \geq N\eta\left(\frac{\pi_2}{m^{II}}\right) & \textit{if} & \pi_1 \leq \left[1-\delta\right]\pi_2 & \textit{(Regime 1)} \\ = N\eta\left(\frac{\pi_1 + \delta\pi_2}{m^{II}}\right) & \textit{if} & \left[1-\delta\right]\pi_2 < \pi_1 \leq \overline{\pi} & \textit{(Regime 2.a)} \\ = N\eta\left(\frac{\pi_1 + \delta\pi_2}{\widetilde{m}}\right) & \textit{if} & \overline{\pi} \leq \pi_1 & \textit{(Regime 2.b)} \end{array} \right\}$$

where $\overline{\pi} \in \left]\left(\frac{\widetilde{m}}{m^I} - \delta\right)\pi_2; \left(\frac{m^{II}}{m^I} - \delta\right)\pi_2\right[$ *is the value of* π_1 *such that*

$$p^I U^{I,a}\left(N\eta\left(\frac{\pi_1+\delta\pi_2}{\widetilde{m}}\right)\right) + p^{II} U^{II,a}\left(N\eta\left(\frac{\pi_1+\delta\pi_2}{\widetilde{m}}\right)\right) = p^I U^{I,b}\left(N\eta\left(\frac{\pi_1+\delta\pi_2}{m^{II}}\right)\right) + p^{II} U^{II,a}\left(N\eta\left(\frac{\pi_1+\delta\pi_2}{m^{II}}\right)\right) \tag{28}$$

Proof. See appendix 1. ■

Note that $\overline{\pi}$ is necessarily increasing with $\frac{m^{II}}{m^I}$ and decreasing with $\frac{p^I}{p^{II}}$ and with δ as $\overline{\pi} \in \left] \left[\frac{\widetilde{m}}{m^I} - \delta\right] \pi_2; \left[\frac{m^{II}}{m^I} - \delta\right] \pi_2 \right[$.

Figure 3

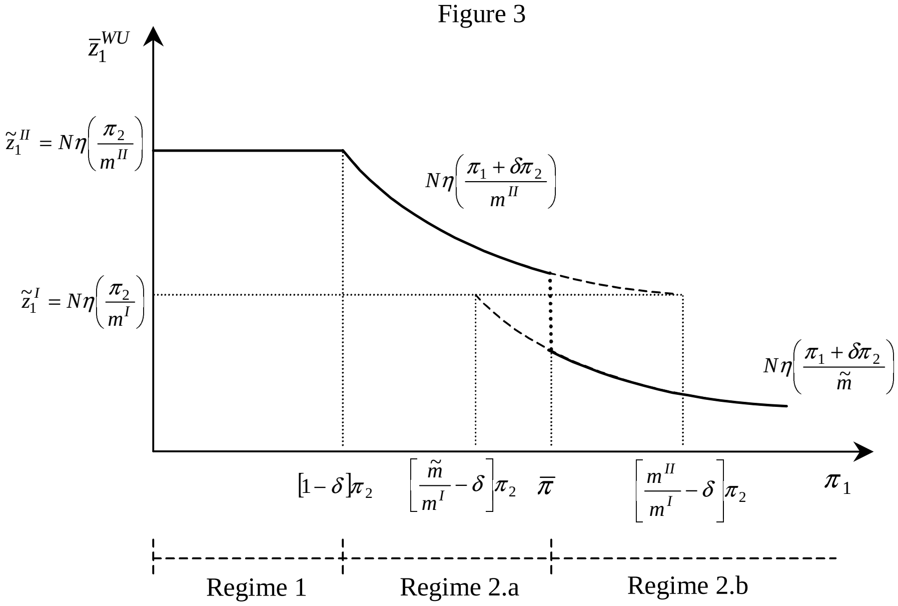
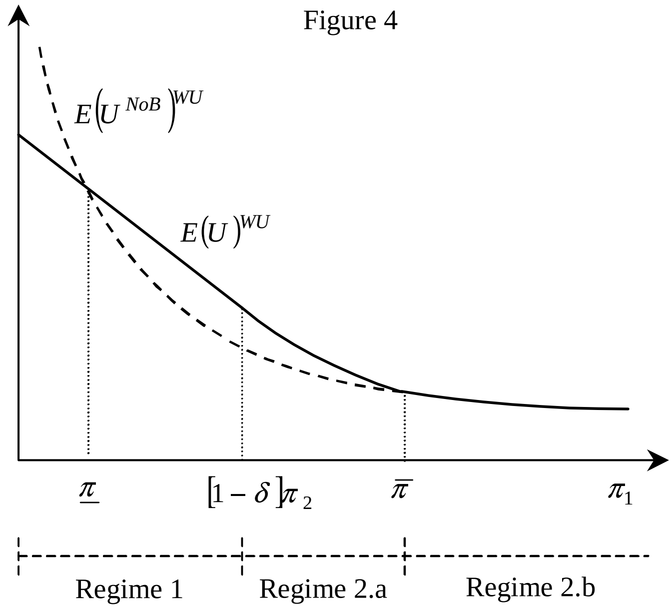


Figure 3 illustrates the solution. In Regime 1, the marginal willingness to pay for the environment in the first period, π_1, is relatively small. We would therefore expect the environmental agency to allow for a relatively high level of pollution –and thus also of production– in that period. However, the level of the ambient pollution cannot be increased above $z_1^I = N\eta\left(\frac{\pi_2}{m^I}\right)$ and $z_1^{II} = N\eta\left(\frac{\pi_2}{m^{II}}\right)$ because the prices in the first period cannot fall below π_2, i.e., $\sigma_1^I = \sigma_1^{II} = \pi_2$, due to the possibility of banking, which is used for $m = m^I$ and for $m = m^{II}$ if $\overline{z}_1 > N\eta\left(\frac{\pi_2}{m^{II}}\right)$; it is used only for $m = m^I$ if $\overline{z}_1 = N\eta\left(\frac{\pi_2}{m^{II}}\right)$. In this Regime 1, the levels of production and of pollution are thus independant of π_1 and the expected utility is linear in π_1, as suggested in figure 4.

Figure 4



In Regime 2.a, $\sigma_1^I = \pi_2$ and $\sigma_1^{II} = m^{II}\frac{\pi_1+\delta\pi_2}{m^{II}}$. The price of the permits in the first period is no longer fixed at π_2 when $m = m^{II}$ and banking takes place only when $m = m^I$.

In Regime 2.b, $\sigma_1^I = m^I\frac{\pi_1+\delta\pi_2}{\widetilde{m}}$ and $\sigma_1^{II} = m^{II}\frac{\pi_1+\delta\pi_2}{\widetilde{m}}$, and there is no banking. The level of pollution is thus the same under both possible values of the parameter m: $z_1^I = N\eta\left(\frac{\pi_1+\delta\pi_2}{\widetilde{m}}\right) = z_1^{II}$.

On the role of banking

What happens if banking is not allowed ? In the second period, the analysis of section 3.2.1 applies but without any permits being banked from the first period. The environmental agency then chooses $\overline{z}_2^r = N\eta\left(\frac{\pi_2}{m^r}\right)$, which leads the economy to exactly the same level of expected utility.

In the first period, the permit market price forms independently from the second period's permits price as banking is not allowed : $\sigma_1^r = m^r\gamma\left(\frac{\overline{z}_1}{N}\right)$. The problem of the environmental agency is thus

$$\begin{aligned} & \max_{\{\overline{z}_1\}} E\left(U^{NoB}\right) \\ = \; & p^I\left[Nm^I g\left(\tfrac{\overline{z}_1}{N}\right) - \pi_1\overline{z}_1 + Nm^I g\left(\eta\left(\tfrac{\pi_2}{m^I}\right)\right) - \pi_2\left[\delta\overline{z}_1 + N\eta\left(\tfrac{\pi_2}{m^I}\right)\right]\right] \\ & + p^{II}\left[Nm^{II} g\left(\tfrac{\overline{z}_1}{N}\right) - \pi_1\overline{z}_1 + Nm^{II} g\left(\eta\left(\tfrac{\pi_2}{m^{II}}\right)\right) - \pi_2\left[\delta\overline{z}_1 + N\eta\left(\tfrac{\pi_2}{m^{II}}\right)\right]\right], \end{aligned}$$

which leads to $\overline{z}_1 = N\eta\left(\frac{\pi_1+\delta\pi_2}{\widetilde{m}}\right)$, that is the same solution as in Regime 2.b when banking is allowed.

Therefore, and as expected, allowing banking does not modify the expected utility in Regime 2.b. It might however do so in Regimes 1 and 2.a.

Indeed, consider figure 4 which illustrates simultaneously the expected utility with and without the possiblity of banking as a function of π_1 ($E(U)^{WU}$ and $E\left(U^{NoB}\right)^{WU}$ respectively). Firstly note that $E\left(U^{NoB}\right)^{WU} = E(U)^{WU}$ for $\overline{\pi} \leq \pi_1$. Secondly, it can be shown that $E(U)^{WU} > E\left(U^{NoB}\right)^{WU}$ for $\pi_1 = [1-\delta]\,\pi_2$ ([8]). Then, by monotonicity of $E(U)^{WU}$ and $E\left(U^{NoB}\right)^{WU}$ on $\pi_1 \in [(1-\delta)\,\pi_2; \overline{\pi}[$, we necessarily have that $E(U)^{WU} > E\left(U^{NoB}\right)^{WU}$ for $[1-\delta]\,\pi_2 \leq \pi_1 < \overline{\pi}$ (i.e., in Regime 2.a).

Hence, as suggested in figure 4, allowing banking decreases the expected utility for relatively low values of π_1 ($\pi_1 < \underline{\pi}$) but increases this utility for higher values of π_1 ($\underline{\pi} < \pi_1$), where

$$\underline{\pi} \left\{ \begin{array}{c} \text{is the value of } \pi_1 \in [0; [1-\delta]\,\pi_2[\text{ which solves } E\left(U^{NoB}\right)^{WU} - E(U)^{WU} = 0 \\ \text{if such a solution exists} \\ = 0 \quad \text{otherwise.} \end{array} \right\} \tag{29}$$

Thus $\underline{\pi}$ may be null, in which case allowing banking always increases the expected utility in Regimes 1 and 2.a.

Proposition 3 *Allowing banking :*

$$\begin{array}{lrcl} \textit{decreases the expected utility} & \textit{if } \pi_1 & < & \underline{\pi} \\ \textit{increases the expected utility} & \textit{if } \underline{\pi} & < & \pi_1 < \overline{\pi} \\ \textit{does not affect the expected utility} & \textit{if } \overline{\pi} & < & \pi_1 \end{array}$$

where $\underline{\pi} \in [0; [1-\delta]\,\pi_2[$ *is defined by (29) and* $\overline{\pi} \in \left]\left(\frac{\widetilde{m}}{m^I} - \delta\right)\pi_2; \left(\frac{m^{II}}{m^I} - \delta\right)\pi_2\right[$ *is defined by (28).* ■

The reason why the possibility of banking does not modify the expected utility when $\pi_1 > \overline{\pi}$ (Regime 2.b) is clear : the amount of permits defined at the first period by the agency is so low that banking is never used.

However, when $\pi_1 < \overline{\pi}$ (Regimes 1 and 2.a), banking takes place and is responsible for two effects. The first one is due to the fact that the amount of permits banked is never the same for both possible values of the productivity parameter m : this amount is always larger for $m = m^I$; it is even nul for $m = m^{II}$ when $\pi_1 > [1-\delta]\,\pi_2$. The level of pollution will therefore be different according to the value of m, which is not the case

[8] This amounts to showing that

$$p^I \left[m^I g\left(\eta\left(\tfrac{\pi_2}{m^I}\right)\right) - \pi_2 \eta\left(\tfrac{\pi_2}{m^I}\right)\right] + p^{II}\left[m^{II} g\left(\eta\left(\tfrac{\pi_2}{m^{II}}\right)\right) - \pi_2\eta\left(\tfrac{\pi_2}{m^{II}}\right)\right] > \widetilde{m} g\left(\eta\left(\tfrac{\pi_2}{\widetilde{m}}\right)\right) - \pi_2\eta\left(\tfrac{\pi_2}{\widetilde{m}}\right).$$

This can be expressed as $p^I F(m^I) + p^{II} F(m^{II}) > F(\widetilde{m})$, which is always true as $F'(m) > 0$ and $F''(m) > 0$.

when banking is not allowed. This tends to increase the expected utility as both levels of pollution and production vary in the same way according to the productivity of the emissions: the levels of pollution and production are lower when the productivity is low than when it is high. This is not the case when banking is not allowed. This first effect therefore plays in favor of a bankable permit system.

The second effect concerns the comparison of the levels of pollution and production between periods rather than between the possible values of the productivity parameter. As in the case without uncertainty (section 3.1), banking forbids high levels of pollution and production to take place in the first period although this would increase welfare for relativeley low values of the first period marginal willingness to pay for the environment, π_1. This second effect plays against allowing banking, exactly as in section 3.1.

When $[1-\delta]\,\pi_2 < \pi_1 < \overline{\pi}$ (Regime 2.a), banking never takes place for $m = m^{II}$ while it does take place to some extend for $m = m^I$. Therefore, even if the agency allocates the same amount of permits as it would do if banking was not allowed, the expected utility may only increase: the levels of pollution and of production will be the same for $m = m^{II}$, but lower for $m = m^I$, which tends to increase the expected utility with respect to the situation where banking is not allowed. Thus only the first effect is present and the expected utility can only increase.

When $\pi_1 < [1-\delta]\,\pi_2$ (Regime 1), both effects are present, and $\underline{\pi}$ ($\underline{\pi} < [1-\delta]\,\pi_2$) is the level of π_1 such that these two effects just compensate. Banking is thus recommended when $\pi_1 \in [\underline{\pi};\overline{\pi}]$.

How does this range of values $[\underline{\pi};\overline{\pi}]$ vary with the different parameters characterizing the economy ? To answer this question, we run numerical simulations for a particular case. We consider a uniform distribution of the firms, that is

$$n\left(\rho_1,\rho_2\right) = n \tag{30}$$

and

$$\overline{\rho}_1 = \overline{\rho}_2 = 1 \tag{31}$$

which implies $N = N_t\left(\rho_t\right) = n = 2$ ([9]). Another simplification is to consider iso-elastic production functions, i.e.

$$mg(x_t) = m\frac{x^{1-\alpha}}{1-\alpha} \tag{32}$$

with $0 < \alpha \leq 1$.

Simulations show that (i) $\underline{\pi}$ is always decreasing with $\frac{m^{II}}{m^I}$, so that $[\overline{\pi}-\underline{\pi}]$ is always increasing with $\frac{m^{II}}{m^I}$, (ii) $\underline{\pi}$ is also decreasing with $\frac{p^I}{p^{II}}$ when choosing $E\left(m\right) = 1$, so that $[\overline{\pi}-\underline{\pi}]$ is is increasing with $\frac{p^I}{p^{II}}$ when $E\left(m\right) = 1$ and (iii) $\underline{\pi}$ and $\overline{\pi}$ are always deceasing

[9] Indeed, $N = \int_0^1\int_0^1 n d\rho_1 d\rho_2 = n$ and $N_t\left(\rho_t\right) = \int_0^1 n d\rho_{3-t} = n$. Then we need to have $\int_0^1 \rho_t N_t\left(\rho_t\right) d\rho_t = n\left[\frac{\rho_t^2}{2}\right]_0^1 = \frac{n}{2} = 1$, that is $n = 2$.

with δ ([10]). In other words, and as expected, allowing banking is recommended for a larger range of values when the uncertainty is high or when a low productivity (m^I) is likely to occur with a high probability. It is also recommended for low values of π_1 when the depreciation rate of the stock of pollution, δ, is low ([11]).

4 Introducing a microstructure: competing market makers

Exchanges are actually not taking place on walrasian markets. In this section, we introduce a microstructure characterized by the presence of market makers who compete 'à la Bertrand' by setting bid and ask prices in each period. Without loss of generality, we assume that only two market makers –indexed by j– compete. In each period, each market maker j sets its bid price b_t^j –the price at which he is ready to buy permits– and its ask price a_t^j –the price at which he is ready to sell permits.

The sequence of actions is thus the following: at the first period, (i) the environmental agency defines a certain amount of permits and allocates them to the firms, (ii) market makers set their bid and ask prices, (iii) firms demand and supply permits at those prices and trades as well as production take place. Then, the same sequence of actions takes place at the second period.

Exactly as in section 3, we analyse the problem under complete information and in situation in which the environmental agency and the market makers do not know the producers technology with certainty.

4.1 Complete information (D)[12]

4.1.1 The last period

The firms

At the second –and last– period, each firm solves

$$\max_{\{x_2 \geq 0\}} mg(x_2) - a_2 \max\left\{\eta\left(\tfrac{a_2}{m}\right) - \rho_2\overline{z}_2 - B^f, 0\right\} + b_2 \max\left\{\rho_2\overline{z}_2 + B^f - \eta\left(\tfrac{b_2}{m}\right), 0\right\}, \tag{33}$$

[10]Note that neither $\underline{\pi}$ nor $\overline{\pi}$ do vary with α.

[11]These results are in line with those of Requate (1998), although our model is slightly different than his one. Indeed, the author works with non linear damage functions, illustrates the optimal policy for the case of linear marginal abatement costs and quadratic damage functions and concentrates on the two polar cases of pure flow pollution ($\delta = 0$) and pure stock pollution ($\delta = 1$). Furthermore, Requate assumes that polluters face some uncertainty while, in our framework, each firms knows its own production function and the environmental agency faces incomplete information which is resolved in the second period.

[12]'D' will be used to denote the equilibrium values of the decision variables in the presence of duopolist market makers.

where the ask (bid) price considered is the lowest (highest) of each market maker's ask (bid) price, that is

$$a_2 = \min\left\{a_2^1, a_2^2\right\} \qquad \text{and} \qquad b_2 = \max\left\{b_2^1, b_2^2\right\} \tag{34}$$

and where B^f $(0 \leq B^f \leq \rho_1 \overline{z}_1)$ is the amount of permits that have been banked by the firm from the first period. Then its level of emissions is given by

$$x_2 = \left\{ \begin{array}{ccc} \eta\left(\frac{a_2}{m}\right) & \text{if} & \eta\left(\frac{a_2}{m}\right) - \rho_2\overline{z}_2 - B^f > 0 \\ \rho_2\overline{z}_2 + B^f & \text{if} & \eta\left(\frac{a_2}{m}\right) \leq \rho_2\overline{z}_2 - B^f \leq \eta\left(\frac{b_2}{m}\right) \\ \eta\left(\frac{b_2}{m}\right) & \text{if} & \eta\left(\frac{b_2}{m}\right) - \rho_2\overline{z}_2 - B^f < 0 \end{array} \right\}. \tag{35}$$

The market

Consider for the moment that firms either have no incentives to bank permits or are indifferent between banking and no banking. This statement will be proved below in section 4.1.2. Assuming that firms do not bank any permits when they are indifferent between doing so or not, we may set $B^f = 0$ for all firms.

In this case, the agregate demand and supply of permits by firms are respectively

$$D_2 = \int_0^{\frac{1}{\overline{z}_2}\eta\left(\frac{a_2}{m}\right)} \left[\eta\left(\frac{a_2}{m}\right) - \rho_2\overline{z}_2\right] N_2(\rho_2) d\rho_2 \tag{36}$$

and

$$S_2 = \int_{\frac{1}{\overline{z}_2}\eta\left(\frac{b_2}{m}\right)}^{\overline{\rho}_2} \left[\rho_2\overline{z}_2 - \eta\left(\frac{b_2}{m}\right)\right] N_2(\rho_2) d\rho_2. \tag{37}$$

Each market maker then maximizes its profit under the constraint that he cannot sell more permits than the sum of those he buys and those he has banked (B^j) from the first period. Therefore, each of them chooses $\left\{a_2^j, b_2^j\right\}$ in order to solve

$$\max_{\left\{a_2^j, b_2^j\right\}} \Pi_2^j = a_2^j D_2^j\left(a_2^j, a_2^{3-j}\right) - b_2^j S_2^j\left(b_2^j, b_2^{3-j}\right) \tag{38}$$

subject to

$$D_2^j\left(a_2^j, a_2^{3-j}\right) = S_2^j\left(b_2^j, b_2^{3-j}\right) + B^j \tag{39}$$

and

$$a_2^j \geq b_2^j \tag{40}$$

with

$$D_2^j\left(a_2^j, a_2^{3-j}\right) = D_2\left(a_2^j\right) 1_{\left\{a_2^j < a_2^{3-j}\right\}} \tag{41}$$

$$S_2^j\left(b_2^j, b_2^{3-j}\right) = S_2\left(b_2^j\right) 1_{\left\{b_2^j > b_2^{3-j}\right\}} \tag{42}$$

where $1_{\{.\}}$ is the indicator function : $1_{\{x<y\}} = \left\{ \begin{array}{ll} 1 & \text{if } x<y \\ 0 & \text{otherwise} \end{array} \right\}$, and $\left\{a_2^{3-j}, b_2^{3-j}\right\}$ given. Note that constraint (39) can be interpreted as the production function of market maker j.

Proposition 4 *In the second period, an equilibrium is reached when the market makers choose* $a_2^1 = a_2^2 = b_2^1 = b_2^2 = \sigma_2$ *; then, they enjoy a positive profit* $\Pi_2^j = \sigma_2 B^j \geq 0$ *for* $j = 1, 2$.

Proof. See appendix 2. ■

As there is no spread, the market clearing condition takes the following form :

$$ED_2\left(\tfrac{\sigma_2}{m}\right) = B^J \tag{43}$$

where $ED_2\left(\frac{\sigma_2}{m}\right)$, the agregate excess demand of firms, is

$$ED_2\left(\tfrac{\sigma_2}{m}\right) = \int_0^{\overline{\rho}_1}\int_0^{\overline{\rho}_2}\left[\eta\left(\tfrac{\sigma_2}{m}\right) - \rho_2\overline{z}_2\right] n(\rho_1,\rho_2)d\rho_2 d\rho_1 \tag{44}$$

and $B^J =_{def} \sum_{j=1}^{2} B^j$. (43) may then be written as $N\eta\left(\frac{\sigma_2}{m}\right) - \overline{z}_2 = B^J$, or

$$\sigma_2 = m\gamma\left(\tfrac{\overline{z}_2 + B^J}{N}\right) \tag{45}$$

The agency

The environmental agency's problem is thus similar to the one it faces with a walrasian market and leads to $\sigma_2 = \pi_2$ or

$$\overline{z}_2^D = N\eta\left(\tfrac{\pi_2}{m}\right) - B^J. \tag{46}$$

4.1.2 The first period

The firms

In the first period, each firm solves

$$\begin{aligned} &\max_{\{x_1\}} mg(x_1) - a_1\max\left\{\eta\left(\tfrac{a_1}{m}\right) - \rho_1\overline{z}_1, 0\right\} + b_1\max\left\{\rho_1\overline{z}_1 - \eta\left(\tfrac{b_1}{m}\right), 0\right\} \\ &+mg(x_2) - \pi_2\left[d_2(\pi_2) - s_2(\pi_2)\right] \end{aligned} \tag{47}$$

subject to $x_2 = \eta\left(\frac{\pi_2}{m}\right)$ and where

$$a_1 = \min\left\{a_1^1, a_1^2\right\} \quad \text{and} \quad b_1 = \max\left\{b_1^1, b_1^2\right\}. \tag{48}$$

This leads to

$$x_1 = \left\{ \begin{array}{ccc} \eta\left(\frac{a_1}{m}\right) & \text{if} & \eta\left(\frac{a_1}{m}\right) - \rho_1 \overline{z}_1 > 0 \\ \rho_1 \overline{z}_1 & \text{if} & \eta\left(\frac{a_1}{m}\right) \leq \rho_1 \overline{z}_1 \leq \eta\left(\frac{b_1}{m}\right) \\ \eta\left(\frac{b_1}{m}\right) & \text{if} & \eta\left(\frac{b_1}{m}\right) - \rho_1 \overline{z}_1 < 0 \end{array} \right\}. \tag{49}$$

The market

Agregate demand and supply by firms are then

$$D_1 = \int_0^{\frac{1}{\overline{z}_1}\eta\left(\frac{a_1}{m}\right)} \left[\eta\left(\frac{a_1}{m}\right) - \rho_1 \overline{z}_1\right] N_1(\rho_1) d\rho_1 \tag{50}$$

and

$$S_1 = \int_{\frac{1}{\overline{z}_1}\eta\left(\frac{b_1}{m}\right)}^{\overline{\rho}_1} \left[\rho_1 \overline{z}_1 - \eta\left(\frac{b_1}{m}\right)\right] N_1(\rho_1) d\rho_1. \tag{51}$$

Each market maker chooses $\left\{a_1^j, b_1^j\right\}$ so as to solve

$$\max_{\left\{a_1^j, b_1^j, B^j \geq 0\right\}} \Pi^j = a_1^j D_1^j\left(a_1^j, a_1^{3-j}\right) - b_1^j S_1^j\left(b_1^j, b_1^{3-j}\right) + \pi_2 B^j \tag{52}$$

subject to (46), $a_1^j \geq b_1^j \geq \pi_2$ and

$$D_1^j\left(a_1^j, a_1^{3-j}\right) + B^j = S_1^j\left(b_1^j, b_1^{3-j}\right), \tag{53}$$

with

$$D_1^j\left(a_1^j, a_1^{3-j}\right) = D_1\left(a_1^j\right) 1_{\left\{a_1^j < a_1^{3-j}\right\}} \tag{54}$$

$$S_1^j\left(b_1^j, b_1^{3-j}\right) = S_1\left(b_1^j\right) 1_{\left\{b_1^j > b_1^{3-j}\right\}} \tag{55}$$

We then have :

Proposition 5 *In the first period, an equilibrium is reached when the market makers set their prices* $\left\{a_1^1, a_1^2, b_1^1, b_1^2\right\}$ *such that* $a_1^1 = a_1^2 = b_1^1 = b_1^2 = \sigma_1$ *; their profits over both periods are then null, i.e.,* $\Pi^j = 0$ *for* $j = 1, 2$.

Proof. See appendix 3. ■

However, up to now we have considered that firms do no bank permits either because they have no incentives to do so or because they are indifferent between banking and not banking. We now prove that this is indeed the case.

Banking of permits by firms depends on the level of $\{a_2, b_2\}$ with respect to $\{a_1, b_1\}$, knowing that $a_1 \geq b_1$ and that $a_2 = b_2 = \pi_2$. We cannot have $a_1 < \pi_2$ because this would lead to intertemporal arbitrage: firms would buy and bank and infinite amount of permits in order to sell them at the second period.

Furthermore, we cannot have $b_1 < \pi_2$ because then firms would not sell any permits at the first period but would rather save and sell them at a higher price at the second period. Then, there would be no trade of permits at the first period as there is no supply, and market makers would need to set a_1 relatively high in order to satisfy their first period constraint (53). They would then make no profits at the first period. But this cannot be an equilibrium for the market makers: given π_2, there is a profitable deviation consisting in proposing $b_1^j = \pi_2$ or $b_1^j = \pi_2 + \varepsilon$ where ε is a very small number (that is $S_1^j(.) > 0$ and no banking by firms) while decreasing a_1^j and keeping (53) satisfied. Indeed, by deviating and choosing $\left\{a_1^j, b_1^j\right\}$ such that $b_1^j = \pi_2(+\varepsilon)$ and $0 < D_1^j\left(a_1^j\right) \leq S_1^j(\pi_2(+\varepsilon))$, market maker j enjoys a profit $\Pi^j = \left[a_1^j - \pi_2\right] D_1^j - [\pi_2(+\varepsilon) - \pi_2] S_1^j$, which is positive as $[\pi_2(+\varepsilon) - \pi_2]$ is nul or very small.

Thus market makers will set prices such that firms either have no incentives to bank permits ($b_1 > \pi_2$) or are indifferent between banking and not banking ($b_1 = \pi_2$).

The agency

The environmental agency thus faces a problem similar to the one under a walrasian market. Consequently, $\overline{z}_1^D = \overline{z}_1^W$ and $B^J = \max\left\{\frac{\overline{z}_1 - \overline{z}_2}{2}, 0\right\}$.

As expected, this microstructure characterized by competing market makers mimiks the walrasian market, but banking –if any– may be realized by market makers rather than by firms. This is however not the case when market makers and the environmental agency face some uncertainty on the polluters' production functions.

4.2 Uncertainty on the production functions (DU)[13]

4.2.1 The last period

The technology parameter is known by all agents at the second period because, as for the environmental agency, market makers learn firms' production functions by observing their demand and supply of permits at the first period.

By the same argument as the one developed in section 4.1, we may consider that firms do not bank permits. The resolution of the agents problems is thus similar to the one under certainty (section 4.1) and is also characterized by a zero spread.

The results are identical to those presented above in the case of a walrasian market, but where banking –if any– is made by market makers rather than by firms. The environmental agency therefore defines

$$\overline{z}_2^{r^{DU}} = N\eta\left(\frac{\pi_2}{m^r}\right) - B^{J,r}, \tag{56}$$

permits, which leads the economy to the social planner's optimum in the second period.

[13]'DU' will be used to denote the equilibrium values of the decision variables under uncertainty in the presence of duopolist market makers.

4.2.2 The first period

The firms and the market

In the first period, each firm knows m^r and solves a similar problem to the one described by (47) (complete information) and where the emissions in the second period are $x_2^r = \eta\left(\frac{\pi_2}{m^r}\right)$. Firms' agregate demand and supply at the first period are described by (50)-(51) where $m = m^r$. Hence, market makers face a similar problem to (52)-(54) but where (52) is replaced by the expected profit.

Proposition 6 *An equilibrium is reached when market makers set their prices in such a way that*

$$\begin{aligned} a_1 &= b_1 = \pi_2 \quad \text{if } \overline{z}_1 \geq N\eta\left(\tfrac{\pi_2}{m^R}\right) \\ a_1 &> b_1 > \pi_2 \quad \text{otherwise.} \end{aligned}$$

Then, (i) their expected profits are null, i.e.,

$$E\left(\Pi\right) = 0, \tag{57}$$

(ii) there is no banking only when $m = m^R$ and $\overline{z}_1 < N\eta\left(\frac{\pi_2}{m^R}\right)$, i.e.

$$B^{J,r} > 0 \;\; \forall r \neq R \quad \text{and} \quad B^{J,R} \left\{ \begin{array}{l} > 0 \;\; \text{if } \overline{z}_1 > N\eta\left(\frac{\pi_2}{m^R}\right) \\ = 0 \;\; \text{otherwise} \end{array} \right\} \tag{58}$$

and (iii), for $\overline{z}_1 < N\eta\left(\frac{\pi_2}{m^R}\right)$, there is no profitable deviation, i.e.,

$$\frac{\partial E\left(\Pi^j\right)}{\partial b_1^j} < 0 \;\; \text{and} \;\; \frac{\partial E\left(\Pi^j\right)}{\partial a_1^j} + \frac{\partial E\left(\Pi^j\right)}{\partial b_1^j}\frac{db_1^j}{da_1^j} > 0 \;\; \text{with } \left\{a_1^{3-j}, b_1^{3-j}\right\} \text{ given} \tag{59}$$

where $\frac{db_1^j}{da_1^j} = \frac{\partial D_1\left(\frac{a_1}{m^R}\right)/\partial a_1^j}{\partial S_1\left(\frac{b_1}{m^R}\right)/\partial b_1^j}$.

Proof. See appendix 4. ■

When $b_1 > \pi_2$, we necessarily have a positive spread ($a_1 > b_1$) eventhough market makers expected profits are null. Indeed, the permits which are banked (necessarily for at least one of the two values of parameter m because ask and bid prices do not depend on the realization of m) are sold at the price π_2 although they have been bought at the price $b_1 > \pi_2$. Market makers would thus enjoy negative expected profits if they do not set $a_1 > b_1$.

The agency

Agregate production is then

$$Y_2^r = Nm^r g\left(\eta\left(\tfrac{\pi_2}{m^r}\right)\right) \tag{60}$$

in the second period and

$$Y_1^r = m^r g\left(\eta\left(\tfrac{a_1}{m^r}\right)\right)\int_0^{\frac{1}{\overline{z}_1}\eta\left(\frac{a_1}{m^r}\right)} N_1(\rho_1)d\rho_1 + m^r \int_{\frac{1}{\overline{z}_1}\eta\left(\frac{a_1}{m^r}\right)}^{\frac{1}{\overline{z}_1}\eta\left(\frac{b_1}{m^r}\right)} g\left(\rho_1\overline{z}_1\right) N_1(\rho_1)d\rho_1$$
$$+m^r g\left(\eta\left(\frac{b_1}{m^r}\right)\right)\int_{\frac{1}{\overline{z}_1}\eta\left(\frac{b_1}{m^r}\right)}^{\overline{\rho}_1} N_1(\rho_1)d\rho_1 \qquad (61)$$

in the first period. The total amount of emissions is respectively

$$z_2^r = N\eta\left(\tfrac{\pi_2}{m^r}\right) \qquad (62)$$

and

$$z_1^r = \eta\left(\tfrac{a_1}{m^r}\right)\int_0^{\frac{1}{\overline{z}_1}\eta\left(\frac{a_1}{m^r}\right)} N_1(\rho_1)d\rho_1 + \int_{\frac{1}{\overline{z}_1}\eta\left(\frac{a_1}{m^r}\right)}^{\frac{1}{\overline{z}_1}\eta\left(\frac{b_1}{m^r}\right)} \rho_1\overline{z}_1 N_1(\rho_1)d\rho_1 + \eta\left(\frac{b_1}{m^r}\right)\int_{\frac{1}{\overline{z}_1}\eta\left(\frac{b_1}{m^r}\right)}^{\overline{\rho}_1} N_1(\rho_1)d\rho_1$$

which leads to the stocks of pollution

$$M_2^r = \delta M_1^r + N\eta\left(\tfrac{\pi_2}{m^r}\right) \qquad (63)$$

and

$$M_1^r = z_1^r. \qquad (64)$$

The environmental agency's problem is thus to solve

$$\max_{\{\overline{z}_1,\overline{z}_2\}} \sum_{r=1}^{R} p_r\left[Y_1^r - \pi_1 M_1^r + Y_2^r - \pi_2 M_2^r\right] \qquad (65)$$

subject to conditions (57),(58) and (59), and where Y_1^r, Y_2^r, M_1^r and M_2^r are respectively defined by (61), (60), (64) and (63). Because the conditions associated to this problem are difficult to interpret, we solve it for the particular case presented in section 3.2.2.

4.2.3 Solving for a particular case

Recall that the particular case presented in section 3.2.2 is characterized by a uniform distribution of the firms ($n(\rho_1, \rho_2) = 2$, $\overline{\rho}_1 = \overline{\rho}_2 = 1$ and $N = 2$) and by an isoelastic production function ($g(x_t) = \frac{x^{1-\alpha}}{1-\alpha}$ with $0 < \alpha \leq 1$) which is influenced in a multiplicative way by the technology parameter m. We furthermore assume that this parameter may take two values $\left\{m^I, m^{II}\right\}$ with $m^I < m^{II}$ with probabilities $\left\{p^I, p^{II}\right\}$. Problem (65) under these specifications is presented in appendix 5.

This problem is solved numerically([14]).

Claim 1 *We observe that :*

When $\pi_1 < [1-\delta]\,\pi_2$ (Regime 1):

[14] By use of the MATLAB 5.2 Optimization toolbox.

(i) the agency chooses the same amount of permits as under walrasian markets

(ii) there is no spread in the first period and $a_1 = b_1 = \pi_2$*,*

(iii) there is always banking and

(iv) the expected utility is the same as under walrasian markets.

When $\pi_1 > [1-\delta]\,\pi_2$ *(Regimes 2.a and 2.b):*

(i) the agency chooses

$$\begin{array}{rcl} \overline{z}_1^{DU} & > & \overline{z}_1^{WU} \\ \overline{z}_2^{I^{DU}} & < & \overline{z}_2^{I^{WU}} \\ \overline{z}_2^{II^{DU}} & = & \overline{z}_2^{II^{WU}} \end{array}$$

that is more permits than under walrasian markets in the first period, fewer permits in the second period when $m = m^I$ *and the same amount of permits in the second period when* $m = m^{II}$*,*

(ii) there is a positive spread in the first period and $a_1 > b_1 > \pi_2$*,*

(iii) there is banking for $m = m^I$ *but not for* $m = m^{II}$*, and*

(iv) the expected utility may be either higher or lower than the one obtained under walrasian markets. ■

Thus, in Regime 1, there is banking for both values of m and the competition between market makers drives their prices to the price of the second period, π_2.

In Regime 2, two effects resulting from the positive spread ($a_1 > b_1$ whenever $\pi_1 > [1-\delta]\,\pi_2$) play in opposite ways. On the one hand, this spread is responsible for the non equalization of the polluters' marginal abatement costs, which means that, for a given amount of emissions (pollution), the production is lower than if the market is walrasian.

On the other hand, because market makers must set their prices under uncertainty (their prices are 'rigid'), the amount of permits that are banked is always different from the walrasian equilibrium depending on the actual value of the productivity parameter m. In Regime 2.b, conversely to the situation prevailing under walrasian markets (where no banking takes place), the levels of pollution and production at the first period therefore always differ according to the value of m ; they are closer to the social planner's optimum –that is lower for $m = m^I$ than for $m = m^{II}$ – than they would be under walrasian markets. In Regime 2.a, the banking which takes place for $m = m^I$ (and not for $m = m^{II}$) has a similar role in both settings –duopolist market makers and walrasian markets– ; however, the rigidity of the prices offered by the market makers emphasizes this effect.

When the second effect dominates the first one, the expected utility is higher in the presence of market makers than under walrasian markets.

In the first period in Regime 2, the environmental agency defines more permits than under walrasian markets because banking is used to a larger extend in the presence of the duopolist market makers. Indeed, although more permits are defined, the actual level of emissions is lower when $m = m^I$ ($z_1^{I^{DU}} < z_1^{I^{WU}}$), but higher when $m = m^{II}$ ($z_1^{II^{DU}} > z_1^{II^{WU}}$) because there is then no banking in either setting. In the second period, there is no spread and the agency enjoying complete information may define an amount of permits which varies according to the value of m and which results in a level of emissions such that the social planner's optimum is always reached. This level of emissions is the same under both the microstructure and the walrasian markets ($z_2^{I^{DU}} = z_2^{I^{WU}}$ and $z_2^{II^{DU}} = z_2^{II^{WU}}$), but the agency should define fewer permits in the presence of market makers when $m = m^I$ ($\overline{z}_2^{I^{DU}} < \overline{z}_2^{I^{WU}}$) because more banking has taken place in the first period due to the spread. This is not the case when $m = m^{II}$ ($\overline{z}_2^{II^{DU}} = \overline{z}_2^{II^{WU}}$) because then no banking has taken place.

4.2.4 The role of banking

It is now interesting to compare the situations prevailing with and without the possibiliy of banking, given the presence of the market makers.

Proposition 7 *If banking is not allowed and if an equilibrium with trades exists, market makers set $a_2 = b_2 = \pi_2$ and $a_1 > b_1$ with $D_1(a_1, ., m^r) < S_1(b_1, ., m^r)$ $\forall m^r \neq m^R$ and $D_1(a_1, ., m^R) = S_1(b_1, ., m^R)$.*

Proof. See appendix 6. ■

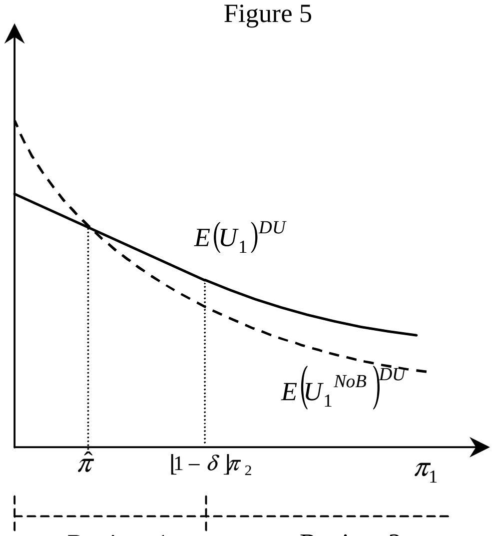
The situation at each period is therefore the same as the one presented in the static model of Germain et al. (2000). The market makers and the environmental agency face some uncertainty at the first period, but not at the second one. In this case, market makers are left with unsold permits at the first period when $m \neq m^R$. These unsold permits are not used as banking is not allowed. In order to enjoy a non negative expected profit, market makers must set a positive spread.

However, numerical simulations on the basis of the particular case (presented above in section 4.2.3) show that there exists no equilibrium with trades for relatively high values of $\frac{m^{II}}{m^I}$ or $\frac{p^I}{p^{II}}$, or for α relatively small. Indeed, in these circumstances the uncertainty becomes so high that it is impossible for market makers to set prices such that their budgetary constraints are binding ($S_t^{j,r} - D_t^{j,r} \geq 0$, $\forall j, r, t$) and their expected profits are not negative (see Germain et al., 2000). Hence, as mentionned above, without the possibility of banking, market makers are necessarily left, at the first period, with permits that cannot be sold when $m = m^I$. When banking is allowed, these permits, which have been bought at price b_1 by market markers, are then banked and sold by them at the second period at the price π_2. The loss that they incur due to these unsold permits in the first period is thus necessarily lower when banking may take place. It is then easier for the market makers to satisfy both their budgetary and their non negative expected profits constraints. This emphasizes a first role of banking in the presence of such a microstructure: allowing for the existence of equilibria with trades which would not have been reached otherwise.

The same resonning may be applied in order to explain the following observation: for $\pi_1 > [1-\delta]\,\pi_2$ (Regime 2), the expected utility is always higher when banking is allowed[15]. The lower loss under banking allows the market makers to set a lower spread, which leads to less inefficiency in production as the marginal abatement costs are closer to equalization across firms. In this regime, the agency then always defines, in the first period, more permits than if banking is not allowed ($\overline{z}_1^{DU} > \left(\overline{z}_1^{DU}\right)^{NoB}$). In the second period, it defines the same amount of permits when $m = m^{II}$ ($\overline{z}_2^{II^{DU}} = \left(\overline{z}_2^{II^{DU}}\right)^{NoB}$), but fewer permits when $m = m^I$ ($\overline{z}_2^{I^{DU}} < \left(\overline{z}_2^{I^{DU}}\right)^{NoB}$) because then some permits have been banked. The levels of emissions in the second period are then the same with and without banking.

However, when $\pi_1 < [1-\delta]\,\pi_2$ (Regime 1), another effect plays in the other direction : there is banking for both $m = m^I$ and $m = m^{II}$, and the level of pollution is determined (fixed) by the second period permits price π_2. Therefore, banking does not allow to increase the level of pollution (and of production) at the first period although this would increase the expected utility. The lower π_1, the stronger this second effect.

Figure 5



[15] Provided that an equilibrium with trades exists when banking is not allowed.

As it is illustrated in figure 5, this second effect might dominate the first one for some low values of π_1. Call $\widehat{\pi}$ the value of π_1 such that both effects just compensate, i.e.

$$\widehat{\pi}\left\{\begin{array}{c}\text{is the value of } \pi_1 \in [0;[1-\delta]\,\pi_2[\text{ which solves } E\left(U^{NoB}\right)^D - E\left(U\right)^D = 0 \\ \text{if such a solution exists} \\ = 0 \quad \text{otherwise.}\end{array}\right\} \tag{66}$$

Thus, to summarize :

Claim 2 *We observe that :*

(i)(a) when $\pi_1 < [1-\delta]\,\pi_2$ (Regime 1) the optimal amount of permits when banking is allowed is not unique, so that it may be higher or lower than when banking is not allowed, while

(b) when $\pi_1 > [1-\delta]\,\pi_2$ (Regime 2), the agency chooses

$$\begin{array}{rcl}\overline{z}_1^{DU} & > & \left(\overline{z}_1^{DU}\right)^{NoB} \\ \overline{z}_2^{I\,DU} & < & \left(\overline{z}_2^{I\,DU}\right)^{NoB} \\ \overline{z}_2^{II\,DU} & = & \left(\overline{z}_2^{II\,DU}\right)^{NoB}\end{array}$$

that is more permits when banking is allowed in the first period, fewer permits in the second period when $m = m^I$ and the same amount of permits in the second period when $m = m^{II}$,

(ii) allowing banking guarantees the existence of equilibria with trades which do not exist otherwise for some values of the parameters (high uncertainty) and

(iii) if an equilibrium with trades exists when banking is not allowed, allowing banking

$$\begin{array}{lrcl}\textit{decreases the expected utility} & \textit{if } \pi_1 & < & \widehat{\pi} \\ \textit{increases the expected utility} & \textit{if } \widehat{\pi} & < & \pi_1\end{array}$$

where $\widehat{\pi} \in [0;[1-\delta]\,\pi_2[$ is defined by (66). ■

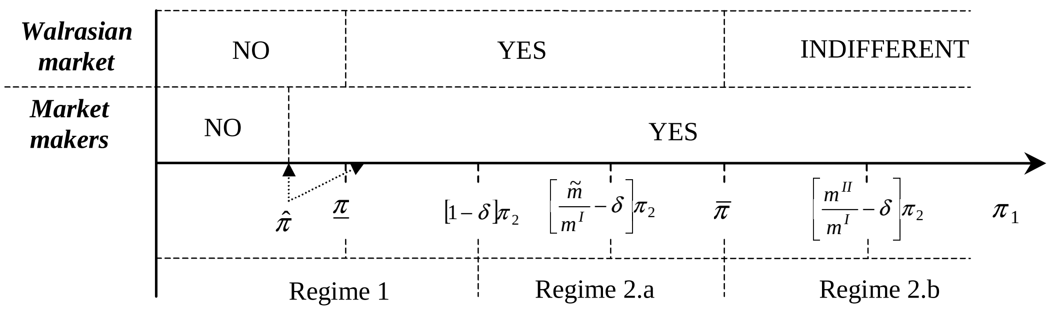
To allow or not to allow for banking : the importance of considering the market microstructure

We observe that $\widehat{\pi} \neq \underline{\pi}$, with usually $\widehat{\pi} < \underline{\pi}$, except for relatively high values of $\frac{m^{II}}{m^I}$ or $\frac{p^I}{p^{II}}$ ([16]). Hence recommendations on whether or not to allow for banking

[16] $\pi < \underline{\pi}$ if and only if for $\pi_1 \in [0;[1-\delta]\,\pi_2[$ the expected utility under walrasian markets without banking is higher than the expected utility under duopolist market makers without banking. This is usually the case, except for relatively high values of $\frac{m^{II}}{m^I}$ or $\frac{p^I}{p^{II}}$. However such high values of $\frac{m^{II}}{m^I}$ or $\frac{p^I}{p^{II}}$ may undermine the existence of equilibria with trades when banking is not allowed, as suggested above.

are different when a microstructure is considered. Figure 6 helps to summarize the comparison. Allowing banking is usually more often recommended in the presence of the microstructure than under the walrasian market.

Figure 6 : Should the agency allow for banking under incomplete information ?



Recall that $\widehat{\pi}$ and $\underline{\pi}$ might be nul, in which case banking is always recommended in the presence of competing market makers and that we might have $\widehat{\pi} > \underline{\pi}$ for relatively high levels of uncertainty.

5 Conclusions and recommendations

The purpose of this paper has been to analyse, in a two periods model, how the market structure of emission permits –their microstructure– influences the optimal policy to be adopted by the environmental agency. The microstructure used is one of a *quote driven market* type, which characterizes many financial markets: market makers serve as intermediaries for trading permits by setting an ask price (i.e. a price at which they are ready to sell permits) and a bid price (i.e. a price at which they are ready to buy permits). The possibility of banking permits from one period to the other has also been introduced.

As intermediary results, we show that, under complete information, the competition between the market makers leads to no spread, that is to the situation prevailing under 'walrasian' markets, i.e., when no market microstructure is modeled. Allowing for banking is then not recommended.

More interesting is the case where the market makers and the agency do not know the producers/polluters technology with certainty. Then, considering two possible levels of firms' productivity, we show that (i) a positive spread may be set by market makers, (ii) banking may increase the expected welfare and (iii) banking is usually more often recommended when a market microstructure (of a quote driven type) is introduced.

If such a microstructure takes place or is organised on markets for pollution permits, we recommend to allow for banking (a) if the marginal willingness to pay for the environment increases much over time, (b) if the pollutant is rather a stock than a flow one, and/or (c) if the incomplete information faced by the intermediaries and by the agency is severe.

In the first period, the environmental agency will then have to define and allocate a larger amount of permits than if banking is not allowed, but a lower or a same amount of permits in the second period.

The main limitations of our analysis are twofold. On the one hand, we only consider linear damage functions, so that the marginal willingness to pay for the environment is constant in each period ; we, however, allow it to be different in each period. On the other hand, the results under uncertainty (incomplete information) are obtained with only two possible values of the firms' productivity and a generalisation to a continuum of possible values is not straightforward. These limitations could be tackled in a first extension of this paper.

Another interesting extension would be to look at the optimal policy if the intermediaries are not competing. As a starting point, one could model a microstructure characterized by the presence of a monopolistic market marker, as it is introduced in the static model of Germain et al. (2000).

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7 Appendix

7.1 Proof of proposition 2

Define first $\overset{\circ}{z}{}_1^{I}$ as the solution of $\max_{\{\overline{z}_1\}} U^{Ia}$ and $\overset{\circ}{z}{}_1^{II}$ as the solution of $\max_{\{\overline{z}_1\}} U^{IIa}$, where U^{Ia} and U^{IIa} are defined by (27) and correspond to the social utility (respectively for each of the two possible values of the productivity parameter m) when banking is not used. This directly gives

$$\begin{aligned} \overset{\circ}{z}{}_1^{I} &= N\eta\left(\tfrac{\pi_1+\delta\pi_2}{m^I}\right) \\ \overset{\circ}{z}{}_1^{II} &= N\eta\left(\tfrac{\pi_1+\delta\pi_2}{m^{II}}\right), \end{aligned}$$

and therefore $\overset{\circ}{z}{}_1^{I} < \overset{\circ}{z}{}_1^{II}$.

Recall that

$$\begin{aligned} \widetilde{z}_1^{I} &= N\eta\left(\tfrac{\pi_2}{m^I}\right) \\ \widetilde{z}_1^{II} &= N\eta\left(\tfrac{\pi_2}{m^{II}}\right), \end{aligned}$$

and thus $\widetilde{z}_1^{I} < \widetilde{z}_1^{II}$.

Regime 1: $\pi_1 + \delta\pi_2 \leq \pi_2 \;\Leftrightarrow\; \pi_1 \leq [1-\delta]\,\pi_2$ (see figure A 1)

In this case $\widetilde{z}_1^{I} \leq \overset{\circ}{z}{}_1^{I}$ and $\widetilde{z}_1^{II} \leq \overset{\circ}{z}{}_1^{II}$, and both U^I and U^{II} are non decreasing functions of $\overline{z}_1$. The expected utility thus reaches its maximum and is constant on $[\widetilde{z}_1^{II};\rightarrow$. Therefore, the agency chooses

$$\overline{z}_1^{WU} \geq N\eta\left(\frac{\pi_2}{m^{II}}\right) = \widetilde{z}_1^{II} \quad \text{if} \quad \pi_1 \leq [1-\delta]\,\pi_2. \tag{67}$$

Regime 2: $\pi_1 + \delta\pi_2 > \pi_2 \;\Leftrightarrow\; \pi_1 > [1-\delta]\,\pi_2$

We have $\widetilde{z}_1^{I} > \overset{\circ}{z}{}_1^{I}$ and $\widetilde{z}_1^{II} > \overset{\circ}{z}{}_1^{II}$. In this case, U^r is strictly increasing in $\overline{z}_1$ on $[0;\overset{\circ}{z}{}_1^{r}[$, strictly decreasing on $]\overset{\circ}{z}{}_1^{r};\widetilde{z}_1^{r}[$ and constant on $[\widetilde{z}_1^{r};\rightarrow$, $r = I, II$. We then need to consider two cases.

Regime 2.1: $\pi_2 < \pi_1 + \delta\pi_2 < \left[\frac{m^{II}}{m^I} - \delta\right]\pi_2 \;\Leftrightarrow\; [1-\delta]\,\pi_2 < \pi_1 < \left[\frac{m^{II}}{m^I} - \delta\right]\pi_2$ (see figures A 2.1.1 and A 2.1.2)

This leads to $\widetilde{z}_1^{I} < \overset{\circ}{z}{}_1^{II}$ and thus $\overset{\circ}{z}{}_1^{I} < \widetilde{z}_1^{I} < \overset{\circ}{z}{}_1^{II} < \widetilde{z}_1^{II}$. This situation may lead to two local maxima for $E(U_1)$, namely $\overline{z}_1^{2.1.1}$ on the interval $]\widetilde{z}_1^{I};\widetilde{z}_1^{II}[$ and $\overline{z}_1^{2.1.2}$ on the interval $\left[0;\widetilde{z}_1^{I}\right]$. $\overline{z}_1^{2.1.1}$ is the solution of $\max_{\{\overline{z}_1\}} p^I U^{Ib} + p^{II} U^{IIa}$ and $\overline{z}_1^{2.1.2}$ is the solution of $\max_{\{\overline{z}_1\}} p^I U^{Ia} + p^{II} U^{IIa}$, which gives

$$\begin{aligned} \overline{z}_1^{2.1.1} &= N\eta\left(\frac{\pi_1+\delta\pi_2}{m^{II}}\right) = \overset{\circ}{z}{}_1^{II} \\ \overline{z}_1^{2.1.2} &= N\eta\left(\frac{\pi_1+\delta\pi_2}{\widetilde{m}}\right). \end{aligned}$$

The global maximum is thus

$$\overline{z}_1^{WU} = \arg\max\left\{E\left(U\left(\overline{z}_1^{2.1.1}\right)\right); E\left(U\left(\overline{z}_1^{2.1.2}\right)\right)\right\} \quad \text{if} \quad [1-\delta]\,\pi_2 < \pi_1 < \left[\tfrac{m^{II}}{m^I} - \delta\right]\pi_2.$$

Figure A 1

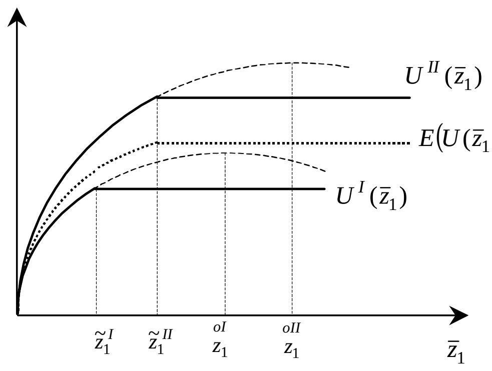

Figure A 2.1.1

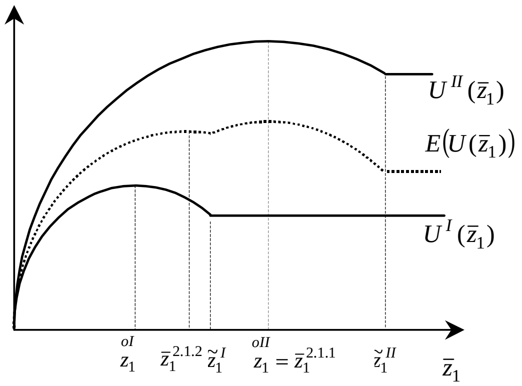

Figure A 2.1.2

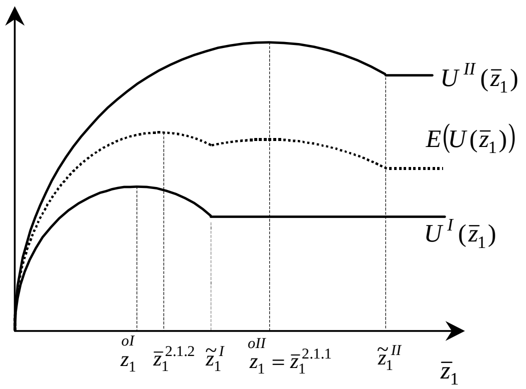

Figure A 2.2

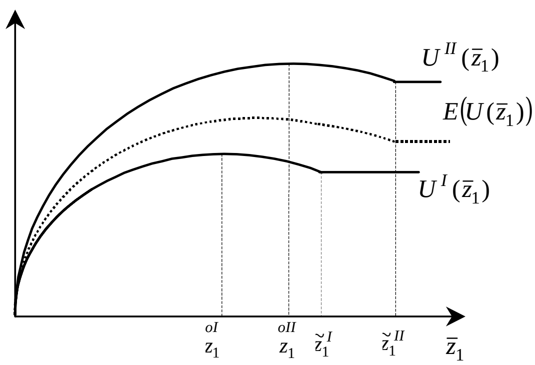

Figure 1:

Call then $\overline{\pi}$ the value of π_1 such that $E\left(U\left(\overline{z}_1^{2.1.1}\right)\right) = E\left(U\left(\overline{z}_1^{2.1.2}\right)\right)$. This defines the two following cases :

Regime 2.1.1: $[1-\delta]\,\pi_2 < \pi_1 \leq \overline{\pi}$

Then the agency chooses

$$\overline{z}_1^{WU} = \overline{z}_1^{2.1.1} \quad \text{if} \quad [1-\delta]\,\pi_2 < \pi_1 \leq \overline{\pi}.$$

Regime 2.1.2: $\overline{\pi} \leq \pi_1 < \left[\frac{m^{II}}{m^I} - \delta\right]\pi_2$

Then the agency chooses

$$\overline{z}_1^{WU} = \overline{z}_1^{2.1.2} \quad \text{if} \quad \overline{\pi} \leq \pi_1 < \left[\frac{m^{II}}{m^I} - \delta\right]\pi_2.$$

Regime 2.2: $\pi_1 + \delta\pi_2 \geq \left[\frac{m^{II}}{m^I} - \delta\right]\pi_2 \;\Leftrightarrow\; \pi_1 \geq \left[\frac{m^{II}}{m^I} - \delta\right]\pi_2$ (see figure A 2.2)

This leads to $\overset{\circ}{z}_1^{II} < \tilde{z}_1^I$ and thus $\overset{\circ}{z}_1^{I} < \overset{\circ}{z}_1^{II} < \tilde{z}_1^I < \tilde{z}_1^{II}$ (see figure A 2.2). The problem of the agency is then $\max_{\{\overline{z}_1\}} p^I U^{Ia} + p^{II} U^{IIb}$, which gives

$$\overline{z}_1^{WU} = N\eta\left(\frac{\pi_1 + \delta\pi_2}{\widetilde{m}}\right) \quad \text{if} \quad \pi_1 \geq \left[\frac{m^{II}}{m^I} - \delta\right]\pi_2. \tag{68}$$

Thus, to summarize,

$$\overline{z}_1^{WU} \left\{ \begin{array}{lll} \geq N\eta\left(\frac{\pi_2}{m^{II}}\right) & \text{if } \pi_1 \leq [1-\delta]\,\pi_2 & \text{(Regime 1)} \\ = N\eta\left(\frac{\pi_1+\delta\pi_2}{m^{II}}\right) & \text{if } [1-\delta]\,\pi_2 < \pi_1 \leq \overline{\pi} & \text{(Regime 2.1.1)} \\ = N\eta\left(\frac{\pi_1+\delta\pi_2}{\widetilde{m}}\right) & \text{if } \overline{\pi} \leq \pi_1 < \left[\frac{m^{II}}{m^I} - \delta\right]\pi_2 & \text{(Regime 2.1.2)} \\ = N\eta\left(\frac{\pi_1+\delta\pi_2}{\widetilde{m}}\right) & \text{if } \left[\frac{m^{II}}{m^I} - \delta\right]\pi_2 \leq \pi_1 & \text{(Regime 2.2)} \end{array} \right\}.$$

Note that $\overline{\pi} < \left[\frac{m^{II}}{m^I} - \delta\right]\pi_2$ (otherwise we are in Regime 2.2 instead of Regime 2.1) and $\overline{\pi} > \left[\frac{\widetilde{m}}{m^I} - \delta\right]\pi_2$ because we necessarily have $\overline{z}_1 > \overline{z}_1^{2.1.1}$ (otherwise we are in Regime 1 instead of Regime 2.1).

Finally, rename the Regimes as follows:

$$\begin{array}{rrcl} \text{Regime 2.1.1 becomes Regime 2.a} & ([1-\delta]\,\pi_2 & < & \pi_1 \leq \overline{\pi}) \\ \text{Regimes 2.1.2 and 2.2 become Regime 2.b} & (\overline{\pi} & \leq & \pi_1) \end{array}$$

while Regime 1 ($\pi_1 \leq [1-\delta]\,\pi_2$) keeps the same name. ■

7.2 Proof of proposition 4

This follows from a standard Bertrand competition analysis where, however, demand and supply of permits depend on ask and bid prices.

Assuming that market maker j (MM^j) captures all demands and supplies of permits (i.e. $a_2^{3-j} >> a_2^j$ and $b_2^j >> b_2^{3-j}$), he will set a_2^j and b_2^j such that $D^j(a_2^j) = S^j\left(b_2^j\right) + B^j$. Indeed, by condition (39), we cannot have $D^j(a_2^j) > S^j\left(b_2^j\right) + B^j$. Furthermore, MM^j will not choose its prices such that $D^j(a_2^j) < S^j\left(b_2^j\right) + B^j$ because, given a_2^j, s/he always could increase its profits by decreasing b_2^j until $D(a_2) = S\,(b_2) + B^j$ as $\frac{\partial S^j(b_2^j,.)}{\partial b_2^j} > 0$.

But market markers are actually competing as $a_2 = \min\left\{a_2^1, a_2^2\right\}$ and $b_2 = \max\left\{b_2^1, b_2^2\right\}$. Therefore, from any $a_2^{3-j} > a_2^j > b_2^j > b_2^{3-j}$ with $D(a_2) = S\,(b_2) + B^j$, MM^{3-j} has an incentive to propose $\left\{a_2^{3-j}, b_2^{3-j}\right\} = \left\{a_2^j - \varepsilon, b_2^j + \varepsilon\right\}$ –where ε is a very small number– in order to capture all demands and supplies of permits, which increases its own profit. But we now have $\Pi^j < \Pi^{3-j}$ and MM^j would then increase its profit by undercutting MM^{3-j} 's prices. And so on untill $a_2^j = a_2^{3-j} = b_2^j = b_2^{3-j} =_{def} \sigma_2$ with $D(\sigma_2) = S\,(\sigma_2) + B^J$. At this stage, no MM has an incentive to undercut because this would lead to arbitrage, i.e. to an infinite loss for the MM.

By (38), this 'no spread' equilibrium is thus characterized by $\Pi^j = \sigma_2 B^j$ for $j = 1, 2$.■

7.3 Proof of proposition 5

Recall first that the prices with walrasian markets (under certainty) are $\sigma_1 = m\gamma\left(\frac{\overline{z}_1}{N}\right) > \sigma_2 = \pi_2$ when $\overline{z}_1 < \overline{z}_2$ and $\sigma_1 = m\gamma\left(\frac{\overline{z}_1+\overline{z}_2}{2N}\right) = \sigma_2 = \pi_2$ when $\overline{z}_1 \geq \overline{z}_2$. By combining (52) and (53), market makers profits are

$$\Pi^j = \left[a_1^j - \pi_2\right] D_1\left(a_1^j, a_1^{3-j}\right) - \left[b_1^j - \pi_2\right] S_1\left(b_1^j, b_1^{3-j}\right).$$

For $a_1^1 = a_1^2 = b_1^1 = b_1^2 = \sigma_1 = \sigma_2 = \pi_2$ (i.e. $\overline{z}_1 \geq \overline{z}_2$), we directly have $\Pi^j = 0$. There are thus no deviations which would increase the profit of a market maker. Furthermore, the market clearing condition at the first period is $ED_1(\sigma_1) + B^J = 0$ with $ED_1(\sigma_1) = \int_0^{\overline{\rho}_1}\left[\eta(\sigma_1) - \rho_1\overline{z}_1\right] N_1(\rho_1)\, d\rho_1$. Thus $B^J = \frac{\overline{z}_1 - \overline{z}_2}{2} \geq 0$.

For $a_1^1 = a_1^2 = b_1^1 = b_1^2 = \sigma_1 > \sigma_2 = \pi_2$ (i.e. $\overline{z}_1 < \overline{z}_2$), we have

$$\Pi^j = \left[\sigma_1 - \pi_2\right]\left[D_1(\sigma_1) - S_1(\sigma_1)\right].$$

As condition (53) requires $D_1(\sigma_1) - S_1(\sigma_1) \leq 0$, market makers need to set σ_1 such that $D_1(\sigma_1) - S_1(\sigma_1) = 0$, i.e. $B^J = 0$, in order to enjoy non negative profits. Thus $\Pi^j = 0$ and there are then no deviations which would increase the profit of a market maker.■

7.4 Proof of propostion 6

The expected profits of market markers writes as

$$E(\Pi) = E\left(a_1 D_1\left(\frac{a_1}{m}\right) - b_1 S_1\left(\frac{b_1}{m}\right) + \pi_2 B^J(m)\right)$$

with $B^J(.) = S_1\left(\frac{b_1}{m}\right) - D_1\left(\frac{a_1}{m}\right)$. We thus have

$$E(\Pi) = E\left(\left[a_1 - \pi_2\right] D_1\left(\frac{a_1}{m}\right) - \left[b_1 - \pi_2\right] S_1\left(\frac{b_1}{m}\right)\right). \tag{69}$$

Then three cases may be distinguished:

Case 1:	$a_1 \geq b_1 > \pi_2$
Case 2:	$a_1 \geq b_1 = \pi_2$
Case 3:	$a_1 \geq \pi_2 > b_1$

By the same argument as the one developed in section 4.1.2 showing that we don't need to consider banking by firms, *Case 3* does not need to be considered.

In *Case 2* ($a_1 \geq b_1 = \pi_2$), we necessarily have $a_1 = b_1 = \pi_2$ and $E(\Pi) = 0$. Indeed, $b_1 = \pi_2$ implies that

$$E(\Pi) = E\left(\left[a_1 - \pi_2\right] D_1\left(\frac{a_1}{m}\right)\right)$$

which always leads to $E(\Pi) > 0$ for $a_1 > \pi_2$. The Bertrand competition then drives the expected profits to zero, that is up to $a_1 = \pi_2 = b_1$. At this point, no deviation may increase market markers'expected profits.

In *Case 1* ($a_1 \geq b_1 > \pi_2$), we necessarily have $a_1 > b_1 > \pi_2$ and $E(\Pi) = 0$. Indeed, $b_1 > \pi_2$ implies that (69) is maximized for $S_1\left(\frac{b_1}{m}\right)$ as small as possible (market makers lose on the bid side). But market makers must satisfy their production or feasibility constraints :

$$S_1\left(\frac{b_1}{m^r}\right) - D_1\left(\frac{a_1}{m^r}\right) = B^{J,r} \geq 0, \quad \forall r. \tag{70}$$

For any $a_1 \geq b_1$, they therefore choose b_1 such that $S_1\left(\frac{b_1}{m^R}\right) - D_1\left(\frac{a_1}{m^R}\right) = B^{J,R} = 0$ because this minimizes $S_1\left(\frac{b_1}{m^r}\right)$ $\forall r$ while necessarily satisfying constraints (70). This implies

$$S_1\left(\frac{b_1}{m^r}\right) - D_1\left(\frac{a_1}{m^r}\right) = B^{J,r} > 0, \quad \forall r \neq R,$$

or $S_1\left(\frac{b_1}{m^r}\right) > D_1\left(\frac{a_1}{m^r}\right)$ $\forall r \neq R$. This latter relation and $b_1 > \pi_2$ then means that market makers need to set $a_1 > b_1$ for their expected profits to be non negative. The Bertrand competiton will then drive the expected profits to zero and any couple of prices $\{a_1, b_1\}$ is an equilibrium if there are no profitable deviations, i.e. if (59) is satisfied.

We now show that $\overline{z}_1 = N\eta\left(\frac{\pi_2}{m^R}\right)$ is the limit case between *Cases 1* and *2*. Indeed, when $a_1 = \pi_2 = b_1$ (*Case 2*), the market clearing condition is

$$N\eta\left(\frac{\pi_2}{m^r}\right) - \overline{z}_1 + B^{J,r} = 0, \quad \forall r$$

where $N\eta\left(\frac{\pi_2}{m^r}\right) - \overline{z}_1 = D_1\left(\frac{a_1}{m^r}\right) - S_1\left(\frac{b_1}{m^r}\right)$. Hence, because $B^{J,r} \geq 0\ \forall r$, this condition implies that $\overline{z}_1 \geq N\eta\left(\frac{\pi_2}{m^r}\right)$ $\forall r$, and thus also $\overline{z}_1 \geq N\eta\left(\frac{\pi_2}{m^R}\right)$. Then, $\overline{z}_1 > N\eta\left(\frac{\pi_2}{m^R}\right) \iff B^{J,R} > 0$, which necessarily implies $B^{J,r} > 0\ \forall r \neq R$. ■

7.5 Calculations for the particular case

Agregate demand and supply of permits at each period are then

$$D_2\left(\frac{\pi_2}{m^r}\right) = \frac{1}{\overline{z}_2}\eta\left(\frac{\pi_2}{m^r}\right)^2 \tag{71}$$

$$S_2\left(\frac{\pi_2}{m^r}\right) = \frac{1}{\overline{z}_2}\left[\overline{z}_2 - \eta\left(\frac{\pi_2}{m^r}\right)\right]^2 \tag{72}$$

$$D_1\left(\frac{a_1}{m^r}\right) = \frac{1}{\overline{z}_1}\eta\left(\frac{a_1}{m^r}\right)^2 \tag{73}$$

$$S_1\left(\frac{b_1}{m^r}\right) = \frac{1}{\overline{z}_1}\left[\overline{z}_1 - \eta\left(\frac{b_1}{m^r}\right)\right]^2 \tag{74}$$

The total amount of emissions at each period is

$$z_2 = 2\eta\left(\frac{\pi_2}{m^r}\right) \tag{75}$$

and

$$z_1 = \frac{1}{\overline{z}_1}\eta\left(\frac{a_1}{m^r}\right)^2 - \frac{1}{\overline{z}_1}\eta\left(\frac{b_1}{m^r}\right)^2 + 2\eta\left(\frac{b_1}{m^r}\right) \tag{76}$$

while the stocks of pollutants are

$$M_2 = \delta M_1 + z_2 \tag{77}$$

and

$$M_1 = \delta M_0 + z_1. \tag{78}$$

Agregate production is

$$Y_2 = \frac{2m^r}{1-\alpha}\eta\left(\frac{\pi_2}{m^r}\right)^{1-\alpha} \tag{79}$$

at the second period and

$$Y_1 = \frac{2m^r}{[2-\alpha]\,\overline{z}_1}\left[\eta\left(\frac{a_1}{m^r}\right)^{2-\alpha} - \eta\left(\frac{b_1}{m^r}\right)^{2-\alpha}\right] + \frac{2m^r}{1-\alpha}\eta\left(\frac{b_1}{m^r}\right)^{1-\alpha} \tag{80}$$

at the first one.

The environmental agency then solves

$$\begin{aligned}&\max_{\{\overline{z}_1\}} p^I\left[Y_1^I - \pi_1\left[\delta M_0 + z_1^I\right] + Y_2^I - \pi_2\left[\delta^2 M_0 + \delta z_1^I + z_2^I\right]\right]\\ &+p^{II}\left[Y_1^{II} - \pi_1\left[\delta M_0 + z_1^{II}\right] + Y_2^{II} - \pi_2\left[\delta^2 M_0 + \delta z_1^{II} + z_2^{II}\right]\right]\end{aligned} \tag{81}$$

subject to market equilibrium conditions

$$\frac{1}{\overline{z}_2^I}\eta\left(\frac{\pi_2}{m^I}\right)^2 = \frac{1}{\overline{z}_2^I}\left[\overline{z}_2^I - \eta\left(\frac{\pi_2}{m^I}\right)\right]^2 + B^{J,I} \tag{82}$$

$$\frac{1}{\overline{z}_2^{II}}\eta\left(\frac{\pi_2}{m^{II}}\right)^2 = \frac{1}{\overline{z}_2^{II}}\left[\overline{z}_2^{II} - \eta\left(\frac{\pi_2}{m^{II}}\right)\right]^2 + B^{J,II} \tag{83}$$

$$\frac{1}{\overline{z}_1}\left[\overline{z}_1 - \eta\left(\frac{b_1}{m^I}\right)\right]^2 = \frac{1}{\overline{z}_1}\eta\left(\frac{a_1}{m^I}\right)^2 + B^{J,I} \tag{84}$$

$$\frac{1}{\overline{z}_1}\left[\overline{z}_1 - \eta\left(\frac{b_1}{m^{II}}\right)\right]^2 = \frac{1}{\overline{z}_1}\eta\left(\frac{a_1}{m^{II}}\right)^2 + B^{J,II} \tag{85}$$

$$\begin{aligned}E(\Pi) &= 0 = p^I\left[[a_1-\pi_2]\,\eta\left(\frac{a_1}{m^I}\right)^2 - [b_1-\pi_2]\left[\overline{z}_1 - \eta\left(\frac{b_1}{m^I}\right)\right]^2\right]\\ &\quad +p^{II}\left[[a_1-\pi_2]\,\eta\left(\frac{a_1}{m^{II}}\right)^2 - [b_1-\pi_2]\left[\overline{z}_1 - \eta\left(\frac{b_1}{m^{II}}\right)\right]^2\right]\end{aligned} \tag{86}$$

$$\left\{\begin{array}{c} B^{J,II} = 0 \text{ if } \overline{z}_1 \geq N\eta\left(\frac{\pi_2}{m^{II}}\right) \\ b_1 = \pi_2 \text{ if } \overline{z}_1 < N\eta\left(\frac{\pi_2}{m^{II}}\right)\end{array}\right\} \tag{87}$$

If $\overline{z}_1 \geq N\eta\left(\frac{\pi_2}{m^{II}}\right)$, $B^{J,II} = 0$ gives

$$\overline{z}_1 = \eta\left(\frac{a_1}{m^{II}}\right) + \eta\left(\frac{b_1}{m^{II}}\right). \tag{88}$$

If $\overline{z}_1 < N\eta\left(\frac{\pi_2}{m^{II}}\right)$, $b_1 = \pi_2$ implies

$$[a_1 - \pi_2]\left[p^I\eta\left(\frac{a_1}{m^I}\right)^2 + p^{II}\eta\left(\frac{a_1}{m^{II}}\right)^2\right] = 0. \tag{89}$$

Consequently, $a_1 = \pi_2$ and

$$\overline{z}_1 = 2\eta\left(\frac{\pi_2}{m^I}\right) + B^{J,I} = 2\eta\left(\frac{\pi_2}{m^{II}}\right) + B^{J,II}. \tag{90}$$

■

7.6 Proof of proposition 7

a) $a_2 = b_2 = \pi_2$. This directly follows from proof of proposition 4 where $B^{j,r} = 0$.
b) $a_1 > b_1$ *with* $D_1(a_1, ., m^r) < S_1(b_1, ., m^r)\ \forall m^r \neq m^R$ and $D_1(a_1, ., m^R) = S_1(b_1, ., m^R)$. See Germain et al. (2000). ■