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The Only Dance in Town: Unique Equilibrium in a Generalized Model of Price Competition*

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Abstract

We study a canonical model of simultaneous price competition between firms that sell a homogeneous good to consumers who are characterized by the number of prices they are exogenously aware of. Our setting subsumes many employed in the literature over the last several decades. We show there is a unique equilibrium if and only if there exist *some* consumers who are aware of *exactly two* prices. The equilibrium we derive is in symmetric mixed strategies. Furthermore, when there are no consumers aware of exactly two prices, we show there is an uncountable-infinity of asymmetric equilibria in addition to the symmetric equilibrium. Our results show the paradigm generically produces a unique equilibrium. We also show that the commonly-sought symmetric equilibrium (which also nests the textbook Bertrand pure strategy equilibrium as a special case) is robust to perturbations in consumer behavior, while the asymmetric equilibria are not. *(JEL: D43, L11)*

Keywords: price competition; price dispersion; unique equilibrium

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1 Introduction

Prices for seemingly homogeneous goods are typically dispersed (see e.g., De los Santos, Hortaçsu, and Wildenbeest, 2012; Gorodnichenko, Sheremirov, and Talavera, 2018; Kaplan and Menzio, 2015; Lach and Moraga-González, 2017). The theoretical industrial organization literature offers an elegant rationalization of this phenomenon via games in which firms simultaneously compete in prices for consumers who differ in the number of prices they compare.

We study the elementary and oft-studied such setting in which $n \geq 2$ firms each sell a homogeneous good at a common marginal cost, $c \geq 0$, and simultaneously set prices in a one-shot game. A mass of consumers are each willing to pay up to $r > c$ for one unit of the good. Consumers exogenously differ in the number of prices they know: $I_m \geq 0$ are aware of $m \in \{1, \ldots, n\}$ random prices, where we assume $I_1 > 0$ and $I_m > 0$ for at least some $m > 1$.[1] In this setting, it is known that equilibrium price dispersion is produced via mixed-strategies: opposing forces lead firms to "tango" (to use the term of Baye, Kovenock, and De Vries, 1992).

There is exactly one symmetric equilibrium. Researchers have almost exclusively relied upon this equilibrium in their analyses. However, a potentially-uncomfortable fact about these popular models is that they can produce very many equilibria. For example, under the assumptions that $I_1, I_n > 0$ and $I_2, \ldots, I_{n-1} = 0$, Baye, Kovenock, and De Vries (1992) show there is an uncountable infinity of asymmetric equilibria in addition to the symmetric equilibrium.

We contribute by pin-pointing the source of this multiplicity and characterizing when the symmetric equilibrium is in fact the only equilibrium. Specifically, we show that there is a unique equilibrium if and only if $I_2 > 0$. The result is stark: if $I_2 > 0$ the symmetric equilibrium is the unique equilibrium, but if $I_2 = 0$ we show there is a continuum of asymmetric equilibria.

In the absence of consumers who make the minimal number of comparisons ($I_2 = 0$) each of the infinitely-many asymmetric equilibria feature at least one firm that charges the monopoly price, r, with positive probability. When these firms charge r, they sell only to their share of "captive" consumers (I_1/n) instead of competing for "contested" (non-captive) consumers by setting lower prices. In contrast, if $I_2 > 0$ each firm competes head-to-head with each other firm for some consumers. This gives firms an incentive to compete over all undominated prices, preventing mass points an any one price (including r), dismantling the asymmetric equilibria.

We highlight four main implications of our result. First, the key determinant of equilibrium uniqueness is not the number of firms, as implied by some studies, but the configuration of consumers' consideration sets. Second, by nesting the vast majority of settings found in the literature, we reconcile existing findings and pinpoint conditions for uniqueness of the commonly-

[1] It is well-known that if $I_1 = 0$ then at least two firms price at marginal cost and all earn zero profit (we discuss the classic Bertrand equilibrium later), and if $I_1 > 0$ but $I_m = 0$ for all $m > 1$ then all firms set the monopoly price r.

studied symmetric equilibrium. Third, and more generally, the framework may become more attractive to researchers because multiplicity only surfaces in the special case of $I_2 = 0$. Fourth, even if a researcher adopts $I_2 = 0$ and hence faces multiplicity, we provide a novel "stability" rationale in terms of consumer behavior for selecting the symmetric equilibrium: the symmetric equilibrium strategy when $I_2 = 0$ is equal to the unique equilibrium strategy as $I_2 \downarrow 0$.

We next discuss related literature, then provide the model and derive our results. We then detail the equilibrium in several applied settings and discuss the wider implications of our findings.

2 Literature

Models of price competition with heterogeneously-informed consumers offered an early rationalization of price dispersion in homogeneous-goods markets (foundational studies include Rosenthal, 1980; Narasimhan, 1988; Shilony, 1977; Varian, 1980). Since then, the framework has been applied to, or featured in, a wide range of settings including: consumer search (both theoretical, e.g., Atayev, 2019; Burdett and Judd, 1983; Janssen and Moraga-González, 2004; Stahl, 1989, and empirical studies, e.g., De los Santos, Hortaçsu, and Wildenbeest, 2012; De los Santos, 2018; Honka, 2014; Honka and Chintagunta, 2016; Pires, 2016); price discrimination (Armstrong and Vickers, 2018; Fabra and Reguant, 2018); product substitutability (Inderst, 2002); strategic clearing-houses such as comparison websites (Baye and Morgan, 2001; Moraga-González and Wildenbeest, 2012; Ronayne, 2019; Ronayne and Taylor, 2019; Shelegia and Wilson, 2017); competition with behavioral or boundedly-rational consumers (e.g., Carlin, 2009; Chioveanu and Zhou, 2013; Gu and Wenzel, 2014; Heidhues, Johnen, and Koszegi, 2018; Inderst and Obradovits, 2018; Johnen, 2018; Piccione and Spiegler, 2012; Spiegler, 2006, 2016); and switching-cost models (for a review, see Farrell and Klemperer, 2007).

The almost-ubiquitous assumption made is that consumers are aware of symmetrically- and randomly-drawn prices (without replacement), which is the setting we study. As such, each consumer's information is completely characterized by the number of prices they know. Perhaps the most well-known is the "Model of Sales" of Varian (1980, 1981), which assumes $I_1, I_n > 0$ and $I_{m\neq 1,n} = 0$. There, Baye, Kovenock, and De Vries (1992) show there is an uncountable infinity of equilibria in addition to the symmetric equilibrium when $n > 2$. Most of the literature deal with this multiplicity by focusing on the symmetric equilibrium (for example, and in addition to those cited above: Armstrong, 2015; Armstrong, Vickers, and Zhou, 2009; Lach and Moraga-González, 2017; Moraga-González, Sándor, and Wildenbeest, 2017; Nermuth, Pasini, Pin, and Weidenholzer, 2013). Uniqueness has only been found in some special cases of our setting e.g., when $n = 2$ (Baye, Kovenock, and De Vries, 1992), or when consumer's awareness of any two prices is independent (Spiegler, 2006).[2] As we prove, those results follow because some consumers know precisely two prices ($I_2 > 0$), the exact condition for uniqueness.

[2]Szech (2011) extends Spiegler's result to show uniqueness extends to the asymmetric independent case.

In the multiple equilibria identified by Baye, Kovenock, and De Vries (1992), firms earn the same profits, but may set price via very different distributions. Equilibrium price distributions are of key interest in many models of consumer search. They drive consumers' incentive to become informed (e.g., Armstrong, Vickers, and Zhou, 2009; Baye and Morgan, 2001; Burdett and Judd, 1983; Fershtman and Fishman, 1994; Moraga-González, Sándor, and Wildenbeest, 2017), and their comparative static properties are the central focus of many studies (e.g. Janssen and Moraga-González, 2004; Moraga-González, Sándor, and Wildenbeest, 2017; Nermuth, Pasini, Pin, and Weidenholzer, 2013).[3] The robustness of results in these papers depends on the existence of asymmetric equilibria. Our finding that asymmetric equilibria are knife-edge phenomena implies that predictions derived with symmetric equilibria are the relevant ones.

Arguments in favor of the symmetric equilibrium have been made in some settings. In an extension of their main analysis, Baye, Kovenock, and De Vries (1992) allow those consumers willing to check exactly one firm to choose which firms to buy from. They show that game has a unique solution where firms adopt symmetric pricing strategies. In our more general setting, we provide a distinct argument for the symmetric equilibrium based on continuity, and without extending or otherwise changing the game's structure.

A related literature is that on all-pay auctions. Perhaps the most relevant paper there is Baye, Kovenock, and De Vries (1996), which documents the equilibria in an all-pay auction with complete information and so does not explore the role of different information sets.[4]

A few have made progress examining equilibria in some particular asymmetric settings, including, e.g., Baye, Kovenock, and De Vries (1992, Section V), Inderst (2002); Ireland (1993); McAfee (1994); Narasimhan (1988); Szech (2011).

Allowing for more general asymmetries in consumers' information sets is challenging and little is known about equilibria there. Armstrong and Vickers (2019) is an exception, offering a rich characterization when $n = 3$. In this paper, we analyze the standard (symmetric and random) configuration of consumers' information.

[3]Extending the framework by adding a preliminary stage in which firms set list prices (the upper bound of final retail prices), Myatt and Ronayne (2019) produce an equilibrium in pure strategies and compare the comparative statics to those of the mixed strategies of several of the single-stage models cited here.

[4]Baye, Kovenock, and De Vries (1996, Theorem 3) indicates that asymmetries (e.g., in marginal costs) between firms (bidders) in settings where consumers see either one price or all (à la Varian, 1980), would lead to asymmetric equilibria in which some players mix continuously while others demonstrate complete rigidity at the monopoly price via a common pure strategy. In the context of pricing with otherwise similar firms, the coexistence of such polar strategies seems empirically unattractive. These equilibria are also not robust to $I_2 > 0$. In addition, although symmetry is a restrictive assumption, firms having the same marginal cost seems reasonable in some important settings featuring price dispersion, e.g., online platforms who charge the same fees to multiple sellers.

3 Model and Equilibrium

Model. There are $n \geq 2$ firms indexed $i = 1, \ldots, n$ that produce a homogeneous product to sell to consumers who wish to buy one unit of the good and have a common and finite willingness to pay, $r > 0$.[5] Firms face a constant marginal cost, $c \in [0, r)$, and simultaneously choose price, where the price of firm i is denoted p_i. Consumers differ by the number of prices they are exogenously aware of i.e., the size of their "information" or "consideration" sets.[6] Consumers buy from the firm with the lowest price in their consideration set. Where there is a tie in the lowest price, any interior tie-breaking rule may be assumed. The mass of consumers informed of $m \in \{1, \ldots, n\}$ prices is denoted $I_m \geq 0$, where $I_1 > 0$ and $I_m > 0$ for some $m > 1$.[7] For each type of consumer, consideration sets are symmetrically and randomly distributed across firms. This means, for example, that I_2 comprises of the same share, equal to $I_2/\binom{n}{2}$, of consumers with each consideration set $\{1, 2\}, \{1, 3\}, \{1, 4\}, \ldots$, etc. We refer to the I_1 consumers as captive and all others as contested. Before deriving our result we illustrate it for the case of $n = 3$.

Example. Consider a triopoly in which no consumer sees exactly two prices ($I_2 = 0$), but some consumers observe one ($I_1 > 0$) and others all three ($I_3 > 0$).

Baye, Kovenock, and De Vries (1992) show that the following equilibria exist. Two firms, say 1 and 2, randomize continuously over an interval $\left[\underline{p}, r\right]$. The remaining firm, 3, randomizes continuously over some $\left[\underline{p}, x\right) \cup r$ with $x \in \left(\underline{p}, r\right]$, placing a mass point at r whenever $x < r$.[8] This is an equilibrium for all $x \in \left[\underline{p}, r\right]$, hence there is an uncountable infinity of equilibria.[9] When $x = r$, equilibrium strategies are symmetric. The asymmetric equilibria require firm 3 to have a mass point on r. In all equilibria, each firm's profit is determined by what they can earn from charging the monopoly price to their captive consumers, their minmax payoff, $(r - c)I_1/3$.

In each equilibrium, 1 and 2 trade-off exploiting captive consumers and competing for the I_3 contested consumers. But the asymmetric equilibria "sideline" 3: 1 and 2 compete head-to-head for I_3 by mixing over a common interval such that 3 does not gain by joining the competition, and can therefore focus more on exploiting its captive consumers by placing a mass point on r.

[5] Under an alternative (unrealistic) assumption that demand is unbounded, mixed-strategy equilibria can exist, as shown by Kaplan and Wettstein (2000) in the case that $I_n > 0$ and $I_m = 0$ for $m < n$.

[6] Some evidence indicates that such "fixed-sample" or "simultaneous" search often describes consumer behavior well. De los Santos, Hortaçsu, and Wildenbeest (2012) and Honka and Chintagunta (2016) examine data from markets for books and auto insurance, respectively. Both studies fail to find a relationship between the prices consumers have seen and their decision to search on, consistent with the premise of simultaneous search.

[7] As we also detail later, information on how many offers consumers consider is available to firms in at least some markets. For example, the Consumer Financial Protection Bureau (2015) reported that approximately 45% of mortgage borrowers only seriously consider one lender, 40% two, and 10% three.

[8] $\underline{p}$ is the lowest undominated price; see (2). Equilibrium strategies for this example are in the proof of Lemma 16.

[9] More generally for $n \geq 2$ (and $I_1, I_n > 0$ and $I_2, \ldots, I_{n-1} = 0$), at least two firms continuously mix over $\left[\underline{p}, r\right)$ in any equilibrium, while all other firms may have mass points on r.

In contrast, when $I_2 > 0$ each firm competes head-to-head with each of its rivals for some consumers. This incentivizes each firm to compete for contested consumers: no firm can sit on the sideline in equilibrium so the asymmetric equilibria no longer exist. To see this more precisely, take our example, but add some consumers who compare two prices so that $I_2 > 0$, and suppose asymmetric strategies are played in which firm 3 places a mass point at r. Now when charging r, firm 3 loses all contested consumers and earns $(r-c)I_1/3$. In contrast, because $I_2 > 0$ and firm 3 has a mass point on r, firms 1 and 2 each sell to $I_2/3$ contested consumers with positive probability, even at prices arbitrarily close to r. Thus, firms 1 and 2 earn strictly more than $(r-c)I_1/3$. But then firm 3 can increase its profit by competing for the $2 \cdot I_2/3$ contested consumers who see firm 3 and exactly one other firm by charging prices below r (it has a strict incentive to shift the probability mass from r to lower prices).

Our result generalizes this intuition for any $n \geq 2$. If $I_2 > 0$, each firm competes head-to-head with each rival for at least some contested consumers. This rules out the possibility of sidelined firms in equilibrium and leads the uniqueness of the symmetric equilibrium.

Analysis. Firms are guaranteed a profit of at least $\pi_i = (r-c)I_1/n > 0$ by setting a price of r which sells to their I_1/n captive consumers regardless of other prices. Profit is zero for $p_i > r$, and so such prices are strictly dominated by r. The highest profit a price p_i below r can generate is found when the firm sells with certainty to all consumers who are aware of its price:

$$(p_i - c)\sum_{m=1}^{n} I_m \frac{\binom{n-1}{m-1}}{\binom{n}{m}} = (p_i - c)\sum_{m=1}^{n} I_m m/n, \tag{1}$$

hence only prices in $\left[\underline{p}, r\right]$ arise in equilibrium, where $\underline{p}$ is the lowest undominated price:[10]

$$(p_i - c)\sum_{m=1}^{n} I_m m/n = (r-c)I_1/n \Leftrightarrow \underline{p} = \frac{rI_1 + c\sum_{m=2}^{n} I_m m}{\sum_{m=1}^{n} I_m m} > c. \tag{2}$$

Proposition 1. *There is a unique equilibrium if and only if $I_2 > 0$. The equilibrium is symmetric: firms continuously mix over the support $\left[\underline{p}, r\right]$ via a common CDF that solves* (12)*. When $I_2 = 0$, there are uncountably-many equilibria.*

We prove Proposition 1 via two lemmas. Lemma 15 shows there is a unique equilibrium when $I_2 > 0$, and Lemma 16 shows there are uncountably-many equilibria when $I_2 = 0$. We construct our proof of Lemma 15 through a sequence of intermediate Lemmas.

Denote firm i's price distribution by G_i, and $\underline{s}_i$ and $\bar{s}_i$ as the minimum and maximum of the support of firm i's prices. Lemmas 1 to 5 do not require $I_2 > 0$. They say that at least two firms have r as the maximum of their support in equilibrium.

[10] We follow convention to ignore the possibility that firms choose suboptimal prices with probability zero.

Lemma 1. *If some firm i has a mass point at $\bar{\bar{s}} \equiv \max_j\{\bar{s}_j\}$, i sells only to its I_1/n captive consumers when it sets $p_i = \bar{\bar{s}}$.*

Proof. Suppose instead there was a positive probability that i sells to some contested consumers when setting $p_i = \bar{\bar{s}}$. Then some other firm, $j \neq i$, also has a mass point at $\bar{\bar{s}}$, implying $\lim_{p\uparrow\bar{\bar{s}}} \pi_i(p) > \pi_i(\bar{\bar{s}})$, a contradiction.[11] □

Lemma 2. $\exists\, i\colon \bar{s}_i = r$.

Proof. Denote $i\colon \bar{s}_i = \max_j\{\bar{s}_j\}$ and suppose $\bar{s}_i < r$. Suppose some firm, j, has a mass point at $\bar{s}_i$. By Lemma 1, $\pi_j(\bar{s}_i) = (\bar{s}_i - c)I_1/n < (r-c)I_1/n = \pi_j(r)$. If no firm has a mass point at $\bar{s}_i$, $\lim_{p\uparrow\bar{s}_i} \pi_i(p) < \pi_i(r)$. □

Lemma 3. $\exists\, i, j\colon i \neq j \;\&\; \bar{s}_i = \bar{s}_j = r$.

Proof. From Lemma 2 we know one firm, say i, has $\bar{s}_i = r$. Denote $j \neq i$ as a firm with the second-highest support-maximum and suppose $\bar{s}_j < r$. Note that as a result, firm i places no mass on prices in $(\bar{s}_i, r)$. Suppose some firm, k, has a mass point at $\bar{s}_j$. By the same argument as in Lemma 1, firm k only ever sells to two types of consumers when it sets $p_k = \bar{s}_j$: its captive consumers, and contested consumers who only see the price of i and k. But then $\pi_k(\bar{s}_j) < \lim_{p\uparrow r} \pi_k(p)$. If no firm has a mass point at $\bar{s}_j$, $\lim_{p\uparrow\bar{s}_j} \pi_j(p) < \lim_{p\uparrow r} \pi_j(p)$. □

Lemma 4. $\pi_i = \pi_j \;\forall\, i, j$.

Proof. Suppose $\pi_i < \pi_j$ for some i and j. Then $\lim_{p\uparrow\bar{s}_j} \pi_i(p) = \pi_j > \pi_i$. □

Lemma 5. *No firm places a mass point at any $p \in \left[\underline{p}, r\right)$.*

Proof. Suppose that firm i has a mass point at $p_i \in \left[\underline{p}, r\right)$. There exists some interval $(p_i, p_i + \epsilon)$ in which no other firm puts probability mass (suppose some firms did, and let $j \neq i$ be a firm with $p_j > p_i$ in its support such that no other firm $k \neq i, j$ has (p_i, p_j) in its support: because $I_m > 0$ for some $m > 1$, there are consumers informed of i's and j's price, hence $\lim_{p\uparrow p_i} \pi_j(p) > \pi_j(p_i + \delta)$ for $\delta \in (0, p_j - p_i)$). But then $\pi_i(p_i + \delta) > \pi_i(p_i)$ for $\delta \in (0, \epsilon)$. □

Lemma 6. *If $I_2 > 0$, at most one firm places a mass point at r.*

Proof. Suppose i and j place mass points at r. Because $I_2 > 0$, there are consumers who are informed of i's and j's price and no other price, so $\pi_i(r) < \lim_{p\uparrow r} \pi_i(p)$. □

Lemma 7. *If $I_2 > 0$, no firm has a mass point at r.*

Proof. By Lemma 6, at most one firm has a mass point at r. Suppose exactly one firm, i, has a mass point at r. By Lemma 1, $\pi_i = (r-c)I_1/n$. By Lemma 3 there is some $j \neq i$ with $\bar{s}_j = r$.

[11] The limit argument means firm i can increase profits by shifting probability mass from the mass point at $\bar{\bar{s}}$ to just below $\bar{\bar{s}}$. We use similar limit arguments throughout.

Because $I_2 > 0$ there are consumers who are informed of the prices from firm i and j and no other price, hence $\lim_{p\uparrow r} \pi_j(p) > \pi_i$, contradicting Lemma 4. □

Lemma 8. *If $I_2 > 0$, $\pi_i = (r-c)I_1/n \; \forall \; i$.*

Proof. From Lemma 3, $\exists\, i, j\colon i \neq j$ and $\bar{s}_i = \bar{s}_j = r$. By Lemma 7, no firm has a mass point at r, hence $\pi_i = \pi_j = (r-c)I_1/n$. By Lemma 4, all firms make this profit. □

Lemma 9. *If $I_2 > 0$, $\exists\, i, j\colon \underline{s}_i = \underline{s}_j = \underline{p}$.*

Proof. Index firms such that $\underline{s}_1 \leq \underline{s}_2 \leq \cdots \leq \underline{s}_n$ and suppose $\underline{s}_1 < \underline{s}_2$. Firm 1 strictly increases profit by shifting the mass it places on prices in $[\underline{s}_1, \underline{s}_2)$, to prices slightly below $\underline{s}_2$, hence $\underline{s}_2 = \underline{s}_1$. Next, suppose $\underline{s}_1 = \underline{s}_2 > \underline{p}$. By Lemma 5 no firm places a mass point on $\underline{s}_1$. But then $\lim_{p\uparrow \underline{s}_1} \pi_i(p) > (r-c)I_1/n$ for any i, contradicting Lemma 8. □

We now introduce notation to characterize i's expected profits. First, D_i^m, which is the expected share of consumers i sells to among those who see m prices by setting a price $p_i < r$. To write D_i^m we use $S_{m-1}(i)$, the set of all vectors of length $m-1$ of distinct firms that do not include i:

$$S_{m-1}(i) \equiv \{a | a = (j_1, ..., j_{m-1}) \not\ni i, \text{ and } j_k \neq j_l \; \forall j_k, j_l \in a\}, \tag{3}$$

$$D_i^m \equiv \begin{cases} n^{-1} & m = 1, \\ \binom{n}{m}^{-1} \sum\limits_{a \in S_{m-1}(i)} \prod\limits_{k \in a} (1 - G_k) & m = 2, \ldots, n. \end{cases} \tag{4}$$

Term D_i^1 is the share of consumers who see one price and buy from i. More generally, let a be a consideration set of a consumer of length m that includes firm i and $m-1$ other firms. Consumers with this consideration set buy from i if $p_i \leq p_k$ for all $k \in a$.[12] The probability of this is $\prod_{k\in a}(1-G_k)$. Summing over all sets of firms $a \in S_{m-1}(i)$ that are in a consideration set with i, and dividing by the number of consumers who see m prices, $\binom{n}{m}$, gives the expected share of consumers that firm i attracts among consumers who see m prices.

Similarly, we define D_{ij}^m using the set of vectors of non-equal firms excluding i and $j \neq i$:

$$S_{m-1}(ij) \equiv \{a | a = (j_1, ..., j_{m-1}) \not\ni i, j\colon i \neq j, \text{ and } j_k \neq j_l, \forall j_k, j_l \in a\}, \tag{5}$$

$$D_{ij}^m \equiv \begin{cases} n^{-1} & m = 1 \\ \binom{n}{m}^{-1} \sum\limits_{a \in S_{m-1}(ij)} \prod\limits_{k \in a} (1 - G_k) & m = 2, \ldots, n-1 \\ 0 & m = n. \end{cases} \tag{6}$$

[12] By Lemma 5, ties are zero-probability events so we do not need to specify a tie-breaking rule.

Relating D_i^m and D_{ij}^m, we find

$$D_i^m = D_{ij}^m + \frac{\binom{n}{m-1}}{\binom{n}{m}}(1 - G_j)D_{ij}^{m-1}, \qquad m = 2, \ldots, n \text{ and } i \neq j. \tag{7}$$

This decomposition will be useful for the proceeding results. Note that $D_i^n = \prod_{j=1;j\neq i}^n (1 - G_j)$, $D_{ij}^{n-1} = \frac{1}{n}\prod_{k=1;k\neq i,j}^{n-1}(1-G_k)$, and $D_i^n = (1-G_j)nD_{ij}^{n-1}$. The total profits at a price p from consumers who see m prices is $K^m(p) \equiv (p - c)I_m$ for $m = 1, \ldots, n$, so that the expected profit of firm i is

$$B_i(p_i) \equiv \sum_{m=1}^{n} K^m(p_i)D_i^m(p_i), \qquad i = 1, \ldots, n. \tag{8}$$

$K^m(p_i)$ is the total profit from consumers who see m prices when they pay p_i. $D_i^m(p_i)$ is the expected share of consumers of firm i when charging p_i among consumers who see m prices. Thus, $K^m(p_i)D_i^m(p_i)$ is the expected profit of firm i from consumers who see m prices, and $B_i(p_i)$ is the expected profit of firm i given that each rival j plays G_j.

Lemma 10. *If $I_2 > 0$, $B_i(p_i)$ is constant and equal to $(r - c)I_1/n$ at the points of increase of G_i on $\left[\underline{p}, r\right)$ for all i.*

Proof. If p_i is a point of increase of G_i, then firm i must earn the equilibrium profit at p_i, which by Lemma 8 is $(r - c)I_1/n$. □

Lemma 11. *If p is a point of increase of G_i and G_j, then $G_i = G_j$ at p almost surely. $B_i(p)$ is continuous at every point of increase of G_i.*[13]

Proof. First, we can use (7) to rewrite

$$B_i(p) = \tfrac{1}{n}K^1(p) + \sum_{m=2}^{n-1} D_{ij}^m(p)K^m(p) + (1 - G_j(p)) \sum_{m=2}^{n} \frac{\binom{n}{m-1}}{\binom{n}{m}} D_{ij}^{m-1}(p)K^m(p). \tag{9}$$

Because p is a point of increase of G_i and G_j, Lemma 4 implies that $B_i(p) = B_j(p)$. Using $D_{ij}^m = D_{ji}^m$ and $D_{ij}^{m-1} = D_{ji}^{m-1}$, it follows that $B_i(p) = B_j(p)$:

$$\begin{aligned} &\Rightarrow (1 - G_j(p)) \sum_{m=2}^{n} \frac{\binom{n}{m-1}}{\binom{n}{m}} D_{ij}^{m-1}(p)K^m(p) = (1 - G_i(p)) \sum_{m=2}^{n} \frac{\binom{n}{m-1}}{\binom{n}{m}} D_{ij}^{m-1}(p)K^m(p) \\ &\Rightarrow G_i(p) = G_j(p). \end{aligned} \tag{10}$$

Because $D_{ij}^1 = n^{-1}$, the summations are strictly positive, allowing the last implication. To show that $B_i(p)$ is continuous at every point of increase p of G_i, we use that all firms j who also have a point of increase at p set $G_j(p) = G_i(p)$. For all other firms k, $G_k(p)$ is constant around p.

[13] A 'point of increase of G_i' is p such that i plays prices in $(p - \epsilon, p + \epsilon)$ with strictly positive probability $\forall\ \epsilon > 0$.

Additionally, observe that $K^m(p)$ is continuous for all m and p, and $(r-c)I_1/n$ is continuous. Thus, $B_i(p) = (r-c)I_1/n$ pins down a continuous function $G_i(p)$ around p. We conclude that $B_i(p)$ is continuous at p. □

Lemma 12. *If $I_2 > 0$, for every i and every point of increase p of G_i in $\left[\underline{p}, r\right)$, there is at least one G_j with $j \neq i$ such that G_j increases at p.*

Proof. Suppose instead there is no such firm $j \neq i$. Because p is a point of increase of G_i, $\frac{dB_i(p)}{dp} = 0$ by Lemma 10. By Lemma 11, we can differentiate $B_i(p)$ with respect to p, leading to

$$\sum_{m=1}^{n} D_i^m(p)\frac{dK^m}{dp} = 0. \tag{11}$$

Note that because no firm $j \neq i$ has a point of increase at p, $\frac{dD_i^m}{dp} = 0$ for all $m > 1$; hence $\frac{dD_i^m}{dp}$ terms are absent from (11). Because $D_i^m(p)\frac{dK^m}{dp} \geq 0$ for all m and $D_i^1\frac{dK^1}{dp} = \frac{1}{n}I_1 > 0$, the left-hand-side of (11) is strictly positive, but the right-hand-side is zero, a contradiction. We conclude that $\frac{dD_i^m}{dp} < 0$ for some m, implying that at least one G_j has to increase at p. □

Lemma 13. *If $I_2 > 0$ and G_i is strictly increasing on some open interval (x, y), $\underline{p} < x < y < r$, then G_i is strictly increasing on the whole interval $\left[\underline{p}, y\right)$.*

Proof. Suppose instead that G_i is, without loss of generality, constant on (z, x) for some $z \in \left[\underline{p}, x\right)$. By Lemma 5, there are no mass points at z or x, hence $G_i(z) = G_i(x)$. At least two firms, k and l, must charge prices with positive probability in $(x-\epsilon, x)$ for some $\epsilon > 0$. (If no firm charges a price in $(x-\epsilon, x)$, take $\bar{\epsilon}$ to be the supremum of the set of $\epsilon > 0$ such that this is true. Firms charging prices in some neighborhood around $x - \bar{\epsilon}$ strictly increase profits by moving probability mass into $(x-\bar{\epsilon}, x)$. Thus, at least one firm charges prices with positive probability in $(x-\epsilon, x)\ \forall\ \epsilon > 0$. By Lemma 12, this extends to two firms.)

By Lemma 10, for every $p \in (x-\epsilon, x)$, $B_k(p) = B_l(p) = (r-c)I_1/n$. And because there are no mass points on $\left[\underline{p}, r\right]$ (Lemmas 5 and 7), $B_i(x) = B_k(x) = B_l(x) = (r-c)I_1/n$. Because $x < r$ and i charges prices in (x, y) with positive probability, Lemma 11 implies that

$$G_k(x) = G_l(x) = G_i(x) < 1.$$

But with $B_i(x) = B_l(x) = B_l(p)$ for all $p \in (x-\epsilon, x)$, it must be that $B_i(p) \leq B_l(p)$ for all $p \in (x-\epsilon, x)$, because these p are not in i's support. But then by the same arguments used in Lemma 11, $B_i(p) \leq B_l(p)$ implies $G_i(p) \leq G_l(p)$. But this contradicts $G_i(x) = G_l(x)$, because $G_l(p)$ is increasing on $(x-\epsilon, x)$, but $G_i(p)$ is constant. □

Lemma 14. *If $I_2 > 0$, all firms mix continuously via some symmetric $G(p)$ over the support $\left[\underline{p}, r\right]$, in any equilibrium.*

Proof. We first show that $\bar{s}_i = r \ \forall i$. Suppose instead there exists a firm i with $\bar{s}_i < r$. Then by Lemmas 5 and 13, this firm charges prices on $\left[\underline{p}, \bar{s}_i\right)$. By Lemma 11, all firms that charge prices $p \in \left[\underline{p}, \bar{s}_i\right)$ play symmetric CDFs $G(\cdot)$ for these prices. Because some firms have r in their support (Lemma 3) and charge prices in $\left[\underline{p}, r\right)$ (Lemmas 7 and 13), we have $G\left(\bar{s}_i\right) < 1$, a contradiction. We conclude that $\bar{s}_i = r \ \forall i$.

Because $\bar{s}_i = r \ \forall i$ and no firm has a mass point at r (Lemma 7), Lemma 13 implies that all firms charge prices on every open subinterval of $\left[\underline{p}, r\right]$ with strictly positive probability. Lemma 11 then implies that all firms play a symmetric price distribution over the whole support. □

Lemma 15. *If $I_2 > 0$, there is a unique equilibrium which is symmetric. The equilibrium CDF, G, solves*

$$(p-c)\sum_{m=1}^{n}\frac{m}{n}I_m(1-G(p))^{m-1} = (r-c)\frac{I_1}{n}, \quad \forall p \in \left[\underline{p}, r\right]. \tag{12}$$

Proof. By Lemma 14 all firms play symmetric price distributions, G over the support $\left[\underline{p}, r\right]$, in any equilibrium. By Lemma 10, G must solve $B(p) = (r-c)I_1/n$ (12) for all $p \in \left[\underline{p}, r\right]$. Equation (12) is satisfied at $G\left(\underline{p}\right) = 0$ and $G(r) = 1$. Because $(p-c)$ is strictly increasing and continuous in p, the equation pins down a unique strictly increasing and continuous $G(p)$ for all $p \in \left[\underline{p}, r\right]$. To complete the specification, let $G(p) = 0$ for $p < \underline{p}$, and $G(p) = 1$ for $p \geq r$. □

Lemma 16. *If $I_2 = 0$, infinitely many equilibria exist.*

Proof. We show there are uncountably-many asymmetric equilibria in which one firm has a mass point at r when $I_2 = 0$. Specifically, the following strategies constitute Nash Equilibria, parameterized by $x \in \left(\underline{p}, r\right)$. Firms $i = 1, \ldots, n-1$ have support $\left[\underline{p}, r\right]$, while one firm, n, has support $\left[\underline{p}, x\right] \cup \{r\}$. Over $\left[\underline{p}, x\right)$, all firms play the symmetric mixed strategy $G(\cdot)$ that solves (12). Firm n places its remaining probability mass $1-G(x)$ at r. Over $[x, r)$, firms $i = 1, \ldots, n-1$ play the symmetric mixed strategy $\hat{G}(\cdot)$. Adjusting (9) appropriately, we see that $\hat{G}(\cdot)$ solves

$$\frac{1}{n}K^1(p)+\sum_{m=2}^{n-1}\frac{\binom{n-2}{m-1}}{\binom{n}{m}}(1-\hat{G}(p))^{m-1}K^m(p)+(1-G(x))\sum_{m=2}^{n}\frac{\binom{n-2}{m-2}}{\binom{n}{m}}(1-\hat{G}(p))^{m-2}K^m(p) = (r-c)\frac{I_1}{n}. \tag{13}$$

As $p \uparrow r$, the left-hand-side converges to $K^1(p)/n = (p-c)I_1/n$. Because $I_2 = 0$, firms that charge prices arbitrarily close to r cannot attract any contested consumers from firm n when n charges r. Thus, even arbitrarily close to r, firms $i \neq n$ still only earn $(p-c)I_1/n$ (in contrast to the proof of Lemma 7 when $I_2 > 0$). These equilibria exist for $I_2 = 0$ and any $I_3, I_4, \ldots, I_m \geq 0$. If $I_2 = 0$ and $I_3 = 0$, there are equilibria where two firms have a mass point at r, and so on. □

Lemmas 15 and 16 yield Proposition 1, which generalizes the intuition from the triopoly example. In each equilibrium at least two firms put no mass at r and compete fully for contested consumers. If $I_2 = 0$, remaining firms may be sidelined: they do not benefit by competing for contested consumers, instead placing mass at r to focus on exploiting their captive base. If $I_2 > 0$, however, no firm sits on the sideline in equilibrium. Each firm goes head-to-head with each rival for at least some consumers, giving each an incentive to compete for contested consumers. Hence no firm puts mass at r and only the symmetric equilibrium survives.

4 Discussion

Proposition 1 tells us there is a unique equilibrium when $I_2 > 0$. The unique equilibrium CDF (the solution to the polynomial (12)) does not generally have an analytic solution except in some cases. We now report the unique equilibrium for some such settings that feature $I_2 > 0$. To do so, we normalize (without loss of generality) $\sum_{m=1}^{n} I_m = 1$, and express (12) more parsimoniously as a probability generating function associated with the number of rivals faced by each firm:[14]

$$\phi(x) = \sum_{m=1}^{n} a_m x^{m-1}, \tag{14}$$

where $a_m \equiv I_m m/n$ and the equilibrium CDF is the solution to

$$\frac{\phi(1 - G(p))}{\phi(0)} = \frac{r - c}{p - c}, \quad \text{where} \quad \underline{p} \equiv \frac{r\phi(0) + c(\phi(1) - \phi(0))}{\phi(1)}. \tag{15}$$

Example 1. Suppose consumers only check one or two prices i.e., $I_1, I_2 > 0$ and $I_{m>2} = 0$. Then $\phi(x) = a_1 + a_2 x$. This setting is particularly relevant. First, it is the model of Varian (1980), i.e., $I_1, I_n > 0$ and $I_1 + I_n = 1$, with $n = 2$. The unique equilibrium is reported by Baye, Kovenock, and De Vries in their full characterization of equilibria in Varian's model. Our Proposition 1 shows uniqueness is obtained not because $n = 2$, but because $I_2 > 0$. Second, our analysis treated consumers' information as exogenous. In their canonical consumer search model, Burdett and Judd (1983) show ex-ante symmetric consumers endogenously choose to search either once or twice (i.e., $I_1, I_2 > 0$ and $I_{m>2} = 0$) and derive a symmetric equilibrium.[15] Our Proposition 1 shows this is in fact the unique equilibrium in such a setting.

Example 2. Extending the first example, suppose that consumers are again aware of few prices, but either one, two, or three (i.e., $I_1, I_2, I_3 > 0$ and $I_{m>3} = 0$). Then $\phi(x) = a_1 + a_2 x + a_3 x^2$, for which one of the two solutions to (15) is valid, which by our result gives the unique equilibrium. More generally, the solution to the setting with $I_1, \ldots, I_k > 0$ and $I_{m>k} = 0$ quickly becomes intractable as k grows. However, some approaches simplify (14), as the next example shows.

[14] We are grateful to Mark Armstrong for this suggestion and adopt the notation of Armstrong and Vickers (2019).
[15] Burdett and Judd (1983) assume a continuum of firms, but the equilibrium strategy is the same for any $n \geq 2$.

Example 3. Consider a setting in which each consumer is independently aware of each firm's price with probability $\alpha \in (0,1)$. Then $I_m = \binom{n}{m}\alpha^m(1-\alpha)^{n-m} > 0$ for $m = 1, \ldots, n$. Application of the binomial theorem to (14) yields $\phi(x) = \alpha(1-\alpha(1-x))^{n-1}$. This is the symmetric version of the models of awareness and advertising by Ireland (1993) and McAfee (1994), the equilibrium of which is shown to be unique by Spiegler (2006). Our more general result shows that uniqueness follows in this special setting because $I_2 > 0$, which is implied by $\alpha \in (0,1)$.

Textbook Bertrand. Consider perhaps the simplest version of Bertrand competition where all consumers compare prices ($I_1 = 0$). With $n > 2$ it is well-known that there are infinitely-many equilibria (two charge marginal cost while others charge any prices). Our results offer an intuitive rationale to focus on the symmetric (price-equals-marginal-cost) equilibrium: When $I_1, I_2 > 0$ (and any $I_m \geq 0$ for $m > 2$) Proposition 1 shows the only equilibrium is symmetric; and as $I_1 \downarrow 0$ ceteris paribus, profits go to zero and the equilibrium pricing distribution converges to the degenerate distribution with all its probability mass on marginal cost.

Other Implications. More generally, our result may make this classic price competition framework more attractive to researchers. For those adopting it, we argue that our result makes the symmetric equilibrium the practically- and theoretically-relevant one.

Firstly, the assumption that some consumers check two prices ($I_2 > 0$) appears quite reasonable. It seems intuitive that a wide range of search-cost distributions would lead some consumers to compare two prices. Indeed, Burdett and Judd (1983) find that even with homogeneous (and linear) search costs, a positive mass of consumers gather two price quotations in equilibrium.[16] Empirical evidence also broadly supports this. The Consumer Financial Protection Bureau (2015) find that around 90% of home-buyers consider either one or two mortgage lenders/brokers, consistent with the estimate of Woodward and Hall (2012). De los Santos, Hortaçsu, and Wildenbeest (2012) find that around 70% of consumers visited one or two online book stores before purchasing, while De los Santos (2018) finds the average number to be 1.27. In the US auto-insurance market, Honka (2014) finds 35% obtain two quotes.

From a theoretical perspective, the framework as a whole may be more attractive to researchers because we now know that multiple equilibria only arise in the special case of $I_2 = 0$. Furthermore, even in that special case, the symmetric equilibrium is robust in the sense that the strategy is right-continuous at $I_2 = 0$: as $I_2 \downarrow 0$, the equilibrium strategy is equal to the symmetric equilibrium strategy when $I_2 = 0$. Conversely, suppose one predicts an asymmetric equilibrium when $I_2 = 0$ and consider a perturbation in consumer behavior $I_2 = \epsilon > 0$, ceteris paribus. Following the shock, the unique and symmetric equilibrium strategies of each firm jumps, and can be qualitatively very different from firms' supposed asymmetric strategies before the change. This offers a stability justification for the restriction to symmetric equilibria made in prior work.

[16] Such search behavior is also found in the "high-intensity" equilibria of Janssen and Moraga-González (2004).

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