

Einstein-Gauss-Bonnet gravity in 4-dimensional space-time

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In this *Letter* we present a general covariant modified theory of gravity in $D = 4$ space-time dimensions which propagates only the massless graviton *and* bypasses the Lovelock's theorem. The theory we present is formulated in $D > 4$ dimensions and its action consists of the Einstein-Hilbert term with a cosmological constant, and the Gauss-Bonnet term multiplied by a factor $1/(D-4)$. The four-dimensional theory is defined as the limit $D \rightarrow 4$. In this singular limit the Gauss-Bonnet invariant gives rise to non-trivial contributions to gravitational dynamics, while preserving the number of graviton degrees of freedom and being free from Ostrogradsky instability. We report several appealing new predictions of this theory, including the corrections to the dispersion relation of cosmological tensor and scalar modes, singularity resolution for spherically symmetric solutions, and others.

Introduction. According to the Lovelock's theorem [1–3], Einstein's general relativity with the cosmological constant is the unique theory of gravity if we assume: (i) the space time is 3+1 dimensional, (ii) diffeomorphism invariance, (iii) metricity, and (iv) second order equations of motion. In this *Letter* we demonstrate a way to bypass the conclusions of Lovelock's theorem, and present a model respecting all the assumptions (i–iv), but nevertheless exhibiting modified dynamics.

It is believed that the most general theory in four dimensional space-time consists of the Einstein-Hilbert action and a cosmological constant,

$$S_{\text{EH}}[g_{\mu\nu}] = \int d^D x \sqrt{-g} \left[\frac{M_{\text{P}}^2}{2} R - \Lambda_0 \right], \quad (1)$$

where $D = 4$. This theory contains two parameters – the reduced Planck mass M_{P} characterizing the gravitational coupling strength, and the (bare) cosmological constant Λ_0 playing the role of vacuum energy.

In higher dimensions, however, there are more terms – higher order Lovelock invariants – satisfying conditions (i–iv). First such term appears in five dimensions,

$$S_{\text{GB}}[g_{\mu\nu}] = \int d^D x \sqrt{-g} \alpha \mathcal{G}, \quad (2)$$

where α is a dimensionless coupling constant and $\mathcal{G}$ is the Gauss-Bonnet invariant, $\mathcal{G} = R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\mu\nu} - 4R^{\mu}{}_{\nu} R^{\nu}{}_{\mu} + R^2 = 6R^{\mu\nu}{}_{[\mu\nu} R^{\rho\sigma}{}_{\rho\sigma]}$. In $D = 4$ the Gauss-Bonnet invariant is a total derivative, and hence does not contribute to the gravitational dynamics. This is exhibited by its contribution to Einstein's equation,

$$\frac{g_{\nu\rho}}{\sqrt{-g}} \frac{\delta S_{\text{GB}}}{\delta g_{\mu\rho}} = 15\alpha \delta^{\mu}{}_{[\nu} R^{\rho\sigma}{}_{\rho\sigma} R^{\alpha\beta}{}_{\alpha\beta]} = -2R^{\mu\alpha}{}_{\rho\sigma} R^{\rho\sigma}{}_{\nu\alpha} + 4R^{\mu\alpha}{}_{\nu\beta} R^{\beta}{}_{\alpha} + 4R^{\mu}{}_{\alpha} R^{\alpha}{}_{\nu} - 2R R^{\mu}{}_{\nu} + \frac{1}{2} \mathcal{G} \delta^{\mu}{}_{\nu}. \quad (3)$$

being anti-symmetrized over five indices, and vanishing identically in $D=4$, but not in $D \geq 5$. An explicit manifestation of this can be seen by taking the trace of (3),

$$\frac{g_{\mu\nu}}{\sqrt{-g}} \frac{\delta S_{\text{GB}}}{\delta g_{\mu\nu}} = (D-4) \times \frac{\alpha}{2} \mathcal{G}, \quad (4)$$

which is proportional to a vanishing factor $(D-4)$ in four space-time dimensions. One might wonder whether this feature is specific just to the trace equation or whether it is a general feature of Einstein's equation. This question was addressed previously in the literature in [4, 5] with the conclusion that the Gauss-Bonnet term contribution to all the components of Einstein's equation are in fact proportional to $(D-4)$, regardless of the space-time symmetries. For instance, for an even dimensional space-time with $D > 4$, we have the Einstein-Lovelock equation written in terms of differential form [4]

$$\sum_{p=0}^{\frac{D}{2}-1} \alpha_p (D-2p) \epsilon_{a_1 \dots a_D} R^{a_1, a_2} \wedge \dots \wedge R^{a_{2p-1}, a_{2p}} \wedge e^{a_{2p+1}} \wedge \dots \wedge e^{a_{D-1}} = 0. \quad (5)$$

We e^a is the *vielbein*, and we obtain the factor of $(D-4)$ for the Gauss-Bonnet term where $p = 2$. Noted that the space-time indices are suppressed in the above equation, and there is one less e^a for the odd dimensional space-time. This proportionality to $(D-4)$ has also been observed in the dynamical equation of motion for graviton in the ADM $D=d+1$ decomposition analysis [5].

The idea we investigate in this *Letter* is the following. What if we rescale the coupling constant,

$$\alpha \rightarrow \alpha/(D-4), \quad (6)$$

of the Gauss-Bonnet term, and then consider the limit $D \rightarrow 4$? This idea is reminiscent of the way in which finite terms are generated by dimensional regularization in quantum field theory, after the divergences are absorbed by counterterms. It is particularly similar to

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the way in which the conformal (trace) anomaly arises in quantum field theory in curved space-times [6]. However, contrary to dimensional regularization, here there are no divergent contributions that need to be subtracted, but rather the singular coefficient is introduced to extract a finite contribution from the Gauss-Bonnet term. Therefore, we consider this prescription to define a *classical theory of gravity*.

Furthermore, what distinguishes this theory from the conformal anomaly is an attractive feature that the number of degrees of freedom does not change as $\alpha \rightarrow 0$ in any number of dimensions, thus it smoothly connects to general relativity, and is free from the Ostrogradsky instability [7]. The same cannot be said of conformal anomaly which introduces additional degrees of freedom due to the introduction of higher derivative terms (but if treated in the same spirit in which they arise – perturbatively – this issue can be circumvented [8, 9]).

Therefore, there is no obstacle to consider the Gauss-Bonnet contribution on the same level as the Einstein-Hilbert term. Nevertheless, because of Lovelock’s theorem, we are prompted to ask whether this theory is actually equivalent to Einstein’s gravity? As will be demonstrated in the remainder of this *Letter*, the answer is no.

Maximally Symmetric Space-time. Let us consider a pure gravity theory given by the action $S = S_{\text{EH}} + S_{\text{GB}}$, *i.e.* by

$$S[g_{\mu\nu}] = \int d^D x \sqrt{-g} \left[\frac{M_{\text{P}}^2}{2} R - \Lambda_0 + \frac{\alpha}{D-4} \mathcal{G} \right], \quad (7)$$

where α is a finite non-vanishing dimensionless constant in $D=4$. Assuming a maximally symmetric solution of the theory, the Riemann tensor is given by $M_{\text{P}}^2 R^{\mu\nu}{}_{\rho\sigma} = (\delta_\rho^\mu \delta_\sigma^\nu - \delta_\sigma^\mu \delta_\rho^\nu) \Lambda / (D-1)$, with Λ being an effective cosmological constant. The Gauss-Bonnet contribution in this case evaluates to

$$\frac{g_{\nu\rho}}{\sqrt{-g}} \frac{\delta S_{\text{GB}}}{\delta g_{\mu\rho}} = \frac{\alpha}{D-4} \times \frac{(D-2)(D-3)(D-4)}{2(D-1)M_{\text{P}}^4} \times \Lambda^2 \delta_\nu^\mu, \quad (8)$$

Note that the divergent factor $1/(D-4)$ coming from the rescaling (6) cancels out the vanishing factor $(D-4)$ from the variation of the Gauss-Bonnet action. The same

feature is exhibited by all the equations of motion given in the remainder of the *Letter*. In the limit $D \rightarrow 4$, the contribution above evaluates to $\alpha \Lambda^2 \delta_\nu^\mu / (3M_{\text{P}}^4)$.

There are two branches of solutions for the effective cosmological constant,

$$\Lambda_\pm \equiv M_{\text{P}}^2 R / D = \frac{3M_{\text{P}}^4}{4\alpha} \left[-1 \pm \sqrt{1 + \frac{8\alpha\Lambda_0}{3M_{\text{P}}^4}} \right]. \quad (9)$$

In case of a hierarchy $|\alpha\Lambda_0| \ll M_{\text{P}}^4$, the Einstein-Hilbert term balances out the bare cosmological constant term in the first branch, with the Gauss-Bonnet term providing a small correction,

$$\Lambda_+ \simeq \Lambda_0 \left(1 - \frac{2\alpha\Lambda_0}{3M_{\text{P}}^4} \right), \quad (10)$$

while in the second branch, reversely, the Einstein-Hilbert term balances out the Gauss-Bonnet term, while the bare cosmological constant only provides a small correction,

$$\Lambda_- \simeq -\frac{3M_{\text{P}}^4}{2\alpha} - \Lambda_0. \quad (11)$$

The existence of two branches of de Sitter solutions (or AdS solutions depending on the signs of α and Λ_0) in higher dimensional ($D \geq 5$) Einstein-Gauss-Bonnet gravity is well known in the literature. For instance, see Ref. [10] for an early work. In the four dimensional limit of that solution the second branch in (11) is removed, and only the first branch in (10) remains as a solution. However, in our setup, both of these branches remain in four dimensional space-time as legitimate solutions due to the rescaling in Eq. (6).

The question from the end of the Introduction section can be posed here in a precise way: being in one of branches of the maximally symmetric solutions, can we discriminate our theory from general relativity, at least at the level of perturbation theory? To this end, we perturb the metric

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}. \quad (12)$$

where $\bar{g}_{\mu\nu}$ is the background (anti-)de Sitter metric. A straightforward computation gives us the full equation of motion for linearized graviton evaluated in $D=4$,

$$\left(1 + \frac{4\alpha}{3} \frac{\Lambda}{M_{\text{P}}^4} \right) \left[\nabla^\rho \nabla^\mu h_{\nu\rho} + \nabla_\nu \nabla_\rho h^{\mu\rho} - \square h^\mu{}_\nu - \nabla^\mu \nabla_\nu h^\rho{}_\rho + \delta_\nu^\mu \left(\square h^\rho{}_\rho - \nabla_\rho \nabla_\sigma h^{\rho\sigma} \right) + \frac{\Lambda}{M_{\text{P}}^2} \left(\delta_\nu^\mu h^\rho{}_\rho - 2h^\mu{}_\nu \right) \right] = 0. \quad (13)$$

The correction arising from Gauss-Bonnet term only appears in the overall factor of the equation of motion, while all terms in the brackets coincide with the ones from Einstein gravity. This result warrants two remarks. Firstly,

the equation of motion being identical to the one of Einstein gravity implies that a graviton has only two degrees of freedom, which is consistent to what we expected from the beginning. Secondly, it implies that the effect of the

Gauss-Bonnet action is only to shift the Planck mass by a constant and thus its contribution to the linearized dynamics is trivial. However, the sign of the overall factor in (13) would imply that the second branch (11) is unstable regardless of the sign of α (as noted in [10] for $\alpha > 0$), due to the overall “wrong” sign in front of the linearized graviton action. This instability however cannot indicate a spatially homogeneous decay since the only FLRW solutions of (7) are the maximally symmetric de Sitter solutions. This is in contrast to as the conformal anomaly (*e.g.* [12]), where the richer dynamics of the scale factor is attributable to the extra degrees of freedom.

From (13) we are unable, to discriminate our theory from general relativity, at the level of perturbation theory in a maximally symmetric space-time. It is still possible though that this degeneracy is specific to the maximal symmetry of space-time, rather than of a more fundamental origin. Next we shall consider two less symmetric space-times: cosmological FLRW space-time and static spherically symmetric space-time.

FLRW Cosmology. In order to study cosmology we consider the theory in (7) together with a scalar field, namely $S = S_{\text{EH}} + S_{\text{GB}} + S_\phi$, where the scalar is canonical and minimally coupled to gravity,

$$S_\phi[g_{\mu\nu}, \phi] = \int d^D x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right], \quad (14)$$

Assuming the FLRW ansatz $ds^2 = -dt^2 + a^2 d\mathbf{x}^2$, the Friedmann equations in $D \rightarrow 4$ limit read,

$$3M_{\text{P}}^2 H^2 + 6\alpha H^4 = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad (15)$$

$$-M_{\text{P}}^2 \Gamma \dot{H} = \frac{1}{2} \dot{\phi}^2, \quad (16)$$

where we have defined a dimensionless parameter $\Gamma \equiv 1 + 4\alpha H^2/M_{\text{P}}^2$. The two Friedmann equations above are consistent with each other, provided that the scalar field satisfies the equation of motion $\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$. Therefore, the Bianchi identity holds.

One of the key observables in a FLRW universe is the transverse and traceless part of the metric fluctuation – gravitational waves or tensor modes – which we define as

$$g_{ij} = a^2 (\delta_{ij} + \gamma_{ij}), \quad (17)$$

where γ_{ij} satisfies $\partial_i \gamma_{ij} = 0$ and $\gamma_{ii} = 0$. At the linear level these tensor modes are gauge invariant and decouple from the vector and scalar modes due to the spatial $SO(3)$ rotational symmetry. Their equation of motion reads

$$\ddot{\gamma}_{ij} + 3H \left(1 - \frac{8\alpha\epsilon H^2}{3M_{\text{P}}^2 \Gamma} \right) \dot{\gamma}_{ij} - c_s^2 \frac{\partial^2 \gamma_{ij}}{a^2} = 0, \quad (18)$$

where $\epsilon \equiv -\dot{H}/H^2$, and $c_s^2 \equiv 1 - 8\alpha\epsilon H^2/(M_{\text{P}}^2 \Gamma)$, and an overall factor of Γ has been omitted. Here again the $D \rightarrow 4$ limit is well defined since the divergent factor in (6) is cancelled by the vanishing one in (3). The

Gauss-Bonnet term modifies both the sound speed and the Hubble friction term compared to the general relativity limit $\alpha = 0$. During the early universe, the inflationary epoch for instance, when H^2/M_{P}^2 is not as small as nowadays, we would expect some non-trivial observational effects, given a reasonably sized α . At late times, however, H^2/M_{P}^2 is tiny and we thus expect the predictions from gravitational waves sector are consistent with all current astrophysical and cosmological observations, including the multi-messenger gravitational waves detection of binary neutron star merger [11].

Another important observable in the FLRW universe is the scalar cosmological perturbation, which is essentially due to the single scalar field ϕ in the matter sector and the scalar polarization of metric fluctuation it induces. We define the scalar perturbation on the metric as follows,

$$g_{00} = -(1 + 2\chi), \quad g_{0i} = \partial_i \beta, \quad g_{ij} = a^2 e^{2\zeta} (\delta_{ij} + \partial_i \partial_j E). \quad (19)$$

We have to perturb the scalar field as well,

$$\phi(t, \mathbf{x}) = \phi(t) + \delta\phi(t, \mathbf{x}). \quad (20)$$

Noted that the theory possesses full space-time diffeomorphisms, and therefore we can safely remove $\delta\phi$ and $\partial_i \partial_j E$ by performing the following coordinate transformation,

$$t \rightarrow t + \xi^0, \quad x^i \rightarrow x^i + \partial_i \xi, \quad (21)$$

given proper function of ξ^0 and ξ . Among the rest of three scalar variables, χ , β and ζ , we find χ and β are non-dynamical. We can eliminate these two non-dynamical modes by solving the (00) and (0i) components of Einstein equations, *i.e.* solving the Hamiltonian constraint and momentum constraint equations. Doing so results in the equation of motion for the scalar mode,

$$\ddot{\zeta} + 3H \left(1 + \frac{\eta}{3} - \frac{8\alpha\epsilon H^2}{3M_{\text{P}}^2 \Gamma} \right) \dot{\zeta} - \frac{\partial^2 \zeta}{a^2} = 0, \quad (22)$$

where $\eta \equiv \dot{\epsilon}/H\epsilon$, and again the overall factor $\epsilon\Gamma$ has been omitted. We see that the Hubble friction term of the scalar mode is modified, while its sound speed is unity. The sound speed of scalar mode is generally different from the one of gravitational waves. However, this deviation is tiny in the late universe, as it is proportional to H^2/M_{P}^2 .

The tensor and the scalar perturbations are all the physical degrees of freedom in the theory given by $S_{\text{EH}} + S_{\text{GB}} + S_\phi$, as is expected since the Gauss-Bonnet action does not give rise to any additional degrees of freedom when added to the Einstein-Hilbert one in any number of space-time dimensions. Therefore, no vector modes are expected, which we have confirmed by checking that they are all eliminated by solving for the momentum constraint equations.

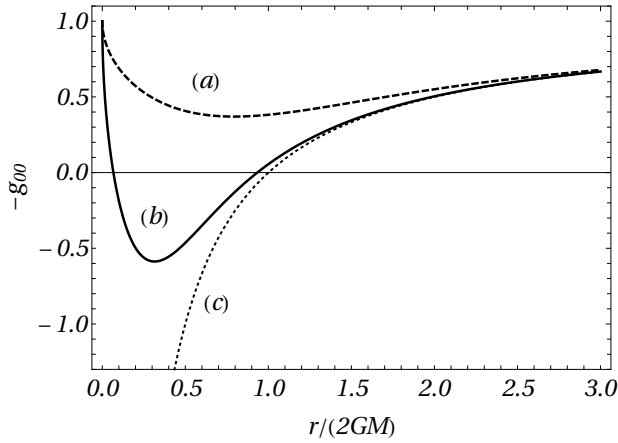


FIG. 1. Radial dependence of gravitational potential g_{00} in the four-dimensional Einstein-Gauss-Bonnet gravity in cases (a) $\alpha = M^2 G / (4\pi)$ ($M = M_*/2$, dashed line) and (b) $\alpha = M^2 G / (64\pi)$ ($M = 2M_*$, full line), and in (c) general relativity ($\alpha = 0$, dotted line).

Static Spherically Symmetric Solution. We now derive the static spherically symmetric solution for the theory given by $S_{\text{EH}} + S_{\text{GB}}$, with the vanishing bare cosmological constant. It is clear from the onset that the Schwarzschild metric does not solve the vacuum Einstein's equations, on the account that the Riemann tensor, which appears explicitly in (3), does not vanish. As we shall see shortly, vacuum equations with the Gauss-Bonnet term allow for solutions free from the singularity issue of general relativity. The solutions for a static and spherically symmetric ansatz in an arbitrary number of dimensions $D \geq 5$,

$$ds^2 = -e^{2\omega} dt^2 + e^{2\lambda} dr^2 + r^2 d\Omega_{D-2}^2 \quad (23)$$

were already found in Ref. [10]. These are extended to $D = 4$ solutions of our theory in (7) by making the rescaling (6), and then taking the limit $D \rightarrow 4$,

$$\begin{aligned} -g_{00} &= e^{2\omega} = e^{-2\lambda} \\ &= 1 + \frac{r^2}{32\pi\alpha G} \left[1 \pm \left(1 + \frac{128\pi\alpha G^2 M}{r^3} \right)^{1/2} \right]. \end{aligned} \quad (24)$$

Here instead of the reduced Planck mass we give results in terms of more customary Newton's constant, $G = 1/(8\pi M_{\text{P}}^2)$, and M is a test point mass. There are two branches of solutions if $\alpha > 0$. However, if $\alpha < 0$, there is no real solution at short radial distances for which $r^3 < -128\pi\alpha G^2 M$. The absence of real solutions at short distances implies the static spherically symmetric ansatz in Eq. (23) is not a good assumption, and probably we need a more general ansatz to find real solutions. This problem is beyond the scope of the *Letter* at hand and we will leave it for future investigations. In this section we focus on the case $\alpha > 0$.

At large distances the two branches behave asymptotically as

$$-g_{00} \xrightarrow{r \rightarrow \infty} 1 - \frac{2GM}{r} \quad \text{or} \quad 1 + \frac{r^2}{16\pi\alpha G} + \frac{2GM}{r}, \quad (25)$$

i.e. they reduce to a Schwarzschild solution with positive gravitational mass, or to a Schwarzschild-de Sitter solution with negative gravitational mass, respectively. We are more interested in the first branch, the one with minus sign inside of brackets in Eq. (24), where we have asymptotic Schwarzschild metric at large distance. Note that even though this branch is a vacuum solution, the Ricci scalar does not vanish due to the contributions from the Gauss-Bonnet term to the vacuum Einstein's equations. The physical properties of this branch differ depending whether the mass M is larger or smaller than the critical mass given by

$$M_* = \sqrt{\frac{16\pi\alpha}{G}}, \quad (26)$$

and in Fig. 1 we plot the radial dependence of g_{00} to illustrate it for (a) $M < M_*$, and (b) $M > M_*$. In both cases the gravitational potential has a minimum, and gravity is therefore attractive to the right of the minimum, and repulsive to the left of it. What distinguishes the two cases is that in the first case the gravitational potential is always positive, and there are no horizons that form, and hence no black hole solutions, while in the second case the gravitational potential crosses zero at two points defining two horizons,

$$r_{\pm}^H = GM \left[1 \pm \sqrt{1 - \frac{16\pi\alpha}{GM^2}} \right]. \quad (27)$$

The horizon at r_+^H is the event horizon of a black hole, which envelops a white hole with the event horizon at r_-^H . We expect the gravitational collapse comes to a halt when the size of system reaches the one corresponding to the bottom of the gravitational potential for a collapsing dust model. In a realistic stellar collapse, the gravitational collapse ceases at somewhere between the bottom of the potential and the event horizon of a black hole due to the stellar internal pressure.

Another important property is the resolution for the singularity problem. At short distances $r \rightarrow 0$, the gravitational potential approaches a finite value $-g_{00} \rightarrow 1$, while the curvature invariant $R \propto r^{-3/2}$ diverges at short distance limit (so does the gravitational force). Nevertheless, the gravitational force is repulsive at short distance and thus an infalling particle never reaches $r = 0$ point. In this sense, our theory is practically free from singularity problem. This is in contrast to Einstein's general relativity, where an infalling particle will eventually hit the singularity and effective theory breaks down.

Conclusion and Discussion. The Gauss-Bonnet action does not contribute to the dynamics of the four

dimensional space-time, as its contribution to Einstein's equation vanishes identically in $D=4$ space-time dimensions. We multiply the Gauss-Bonnet action by a factor of $1/(D-4)$ to compensate for this and to produce a finite non-vanishing contribution to Einstein's equations in $D=4$. Thus the Gauss-Bonnet action becomes a non-trivial ghost-free extension of the Einstein-Hilbert action. It should be noted that the limit $D \rightarrow 4$ has to be taken in the continuous sense, at the level of the equations of motion, rather than at the level of the action. It is in general possible to take this limit [4, 5], however, in practice it should be taken with due care.

In the several examples we presented we were able to take the continuous $D \rightarrow 4$ limit in a natural and straightforward way due to the assumed symmetry between a number coordinates of the space-time solving Einstein's equations. It is not obvious though, that this works in less symmetric space-times. In our prescription the additional dimensions have no physical meaning, and only serve to define the limit. Therefore, in practice one can extend the dimensionality of space-time in a way that the symmetries between coordinates are restricted to the fiducial dimensions and one physical spatial dimension. Thus the limit is finite and well defined. However, additional important insight can be obtained from different formulations of the theory, other than the tensor formalism we utilize here.

In [5] the Gauss-Bonnet term was examined in the $D=d+1$ ADM decomposition. Upon rescaling (6) of the coupling constant we employed, one can read off from the canonical equations that the dynamical (ij) Einstein's equations are manifestly finite in $D=4$ as the singular term explicitly cancels (Eq. (76) from [5]). This is however not manifest in the constraint sector, where the limit has to be taken carefully. Nevertheless, the precise prescription of the limit in the constraint sector cannot be of physical concern, as the constraint sector has to be chosen by hand anyway, *i.e.* we have to specify gauge conditions.

Even more insight is provided by formulating the Gauss-Bonnet action in terms of differential forms. In $D=4$ it is just an Euler density, $S_{\text{GB}} \sim \int \epsilon_{a_1 \dots a_4} R^{a_1 a_2} \wedge R^{a_3 a_4}$, which is just a total derivative. In $D > 4$ the Gauss-Bonnet term reads $S_{\text{GB}} \sim \int \epsilon_{a_1 \dots a_D} R^{a_1 a_2} \wedge R^{a_3 a_4} (\wedge e^a)^{D-4}$, which is an exterior product of a total derivative and a $(D-4)$ -form. Taking the variation with respect to the *vielbein* gives rise to the vanishing factor $(D-4)$ which is precisely cancelled out by the singular factor in the coupling

constant rescaling eq. (6), and we thus expect that all components of the Einstein equation are regular.

Similar idea to the one presented here has been considered before motivated by the study of quantum corrections arising from integrating out matter fields [13, 14]. The perspective that we take is that the Gauss-Bonnet action should be considered a classical modified gravity theory, defined by a modified action principle, rather than a one-loop perturbative correction. In that sense it is on an equal footing with general relativity.

The Gauss-Bonnet extension to Einstein's gravity presented here satisfies the criteria of Lovelock's theorem. In general it leads to very different phenomenologies. For the spherically symmetric static solution it predicts singularity resolution. Generally there are two event horizons for a spherical static solution in vacuum. The interior horizon is an event horizon of a white hole, enveloped by the event horizon of a black hole, so a gravitational collapse ceases with a typical length scale somewhere in between. Cosmological applications of our theory imply a modified dispersion relation for the tensor modes. This has potential observational relevance as it provides a possibility of the parametric resonance, and the production of gravitational waves during the reheating epoch.

We expect a similar prescription presented here to apply to higher order Lovelock invariants. These are of sub-sub-leading effects in Einstein equation in a weak field limit, compared to the Einstein-Hilbert term and the finite Gauss-Bonnet term. Therefore, this class of theories bypasses the conclusions of Lovelock's theorem on the account of modifying the action principle, and challenges the distinctive role of general relativity as the unique non-linear theory describing gravitational interactions in the four dimensional space-time.

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