CHAPTER 1:

THE SEMANTIC NUMERICAL REPRESENTATION

Introduction

This thesis is about the study of numerical cognition in a particular population: congenitally or early blind people. By the examination of how blind people represent and process numbers and numerosities, we will have the opportunity to see whether the visual modality is necessary in the elaboration of human adult numerical representations and skills. However in the domain of numerical cognition, the study of this population is only at his premises, meaning that our thesis presents a certain exploratory character. Consequently, the most reasonable way to start the investigation of our issue is to base our work and research on the literature on numerical cognition that presents robustness and encounters a large consensus.

Therefore, the first chapter of our thesis will be firstly devoted to the presentation of the usual behavioural effects repeatedly observed when normal sighted adult participants are submitted to numerical tasks. These effects are: the distance, the size and the SNARC effects[1] that led to the well-known metaphor of the mental number line on which numbers are suggested to be represented by mental magnitudes. Secondly, the different classical theoretical models made on the mental number line will be reviewed, followed by more recent proposals, which emphasised the role of the experience with numbers and numerosities on the development of numerical processing abilities.

1. Classical behavioural effects

Throughout the study of numerical cognition, several effects have been repeatedly observed in the literature: the distance effect, the size effect and the SNARC effect. These

[1] We would like to point out that these three behavioural effects are not the only effects that can be observed when participants are submitted to numerical tasks. Nevertheless, we will concentrate on them as they are robust effects expected in the classical numerical tasks to which our experimental population was submitted in the studies conducted throughout our thesis. In addition, these three effects are the ones that have been repeatedly taken into consideration in order to conceptualize how numbers are mentally represented.

effects have allowed the elaboration of different theoretical approaches trying to conceptualize how numbers and numerical quantities are represented in the human brain. In the first section of this chapter, these three classical effects are defined and a large section of the literature devoted to their observation is reviewed.

1.1. The distance and the size effects

The distance effect corresponds to a decline in performance, both in response time and accuracy, as the distance between the numbers to discriminate decreases. It reflects the fact, for example, that it is more difficult to compare numbers 4 and 5 (i.e., slower response time and less accuracy), which are only distant from each other by 1 unit, than numbers 1 and 5, which are more distant from each other (i.e., 4 units). The size effect indicates a decrease of performance (both in reaction times and accuracy) as a function of the number size, for a given distance: the larger the numbers and numerical quantities, the more approximate and imprecise their processing. For example, reaction times are faster and subjects are more accurate in comparing 1 and 3, than comparing 7 and 9. Both the distance and the size effects reflect that numerical processing obeys Weber's law in the same way as in psychophysics the perception of physical stimuli such as loudness or brightness (Dehaene, 2001): the larger the stimuli and the smaller the difference between them, the worse their processing. The following formulas for Weber's law were defined to account for two observations:

- physical sensations, as well as more abstract parameters such as numerical representations, vary according to the logarithm of the initial stimulation or stimulus: $S = k \times \log(I)$ (S = perceived sensation; I = stimulation intensity, k = constant; log = logarithmic function);
- reaction times are a logarithmic function of the distance (physical or numerical) between the items to process: $RT = k \times \log(D)$ (RT = reaction time; D = distance between the stimuli to process; k = constant; log = logarithmic function).

Moyer and Landauer (1967) were the first to report the distance and the size effects in adults submitted to a speeded comparison task of single Arabic digits, in which they had to press a key on the side of the largest number, or on the side of the smallest. They observed that numerical judgements were similar, analogous to judgements of inequality of stimuli varying along physical continua, such as loudness, pitch or colour: the larger the numbers to process and the smaller the difference between them, the longer the time required to perform the task. Therefore, according to the authors, numbers might be converted into analogue mental magnitudes in order to be compared in a same way as properties of physical stimuli.

From then on, the distance and the size effects have been extensively replicated in the literature on numerical cognition. They have been repeatedly observed in numerical comparison tasks with different numerical material: Arabic single-digit numbers (Aiken & Williams, 1968; Buckley & Gillman, 1974; Dehaene, 1996; Foltz, Poltrock, & Potts, 1984; Henik & Tzelgov, 1982), Arabic two-digit numbers (Dehaene, 1989; Dehaene, Dupoux, & Mehler, 1990; Fias, Lammertyn, Reynvoet, Dupont, & Orban, 2003; Hinrichs, Yurko, & Hu, 1981; Lavidor, Brinskman, & Göbel, 2004; Restle, 1970), written verbal numerals (Dehaene, 1996; Foltz et al., 1984) and non-symbolic quantifiable dimensions such as dot patterns (Buckley & Gillman, 1974), line length (Fias et al., 2003; Johnson, 1939), angle size (Fias et al., 2003) and object size (Moyer, 1973).

The distance effect has also been found in experiments in which the numerical magnitude of the stimuli was irrelevant to solve the task. Henik and Tzelgov (1982) observed a distance effect in a comparison task of the physical size of single Arabic digits. In another study, Dehaene and Akhavein (1995) submitted participants to a same-different judgement task of pairs of numerals with a numerical matching condition (response "same" to pairs of stimuli such as 2 - TWO and response "different" to pairs such as 2 - 8) and a physical matching condition (response "different" to pairs of stimuli such as 2 - TWO and response "same" to pairs such as 2 - 8). Their results revealed a distance effect in both conditions: the closer the two numbers constituting the pairs to compare, the slower the "different" responses, even when subjects had only to process the physical cue of the stimuli.

Moyer and Landauer (1967) and Restle (1970) suggested that the distance and the size effects indicate that numbers are represented by mental magnitudes along a linear analogue representation. Both effects were assumed to constitute salient markers of an automatic access to the semantic numerical representation, as they have been systematically observed in the literature, even in tasks in which numerical information was not relevant (Dehaene & Akhavein, 1995). Furthermore, as both effects have been found with different numerical formats such as symbolic (verbal and Arabic numerals) and non-symbolic (e.g., dot patterns) inputs, it was postulated that the semantic numerical representation is abstract (i.e., accessed whatever format has the numerical input) (Dehaene & Akhavein, 1995; Dehaene, Dehaene-Lambertz, & Cohen, 1998).

1.2. The SNARC effect

The SNARC effect is a third robust effect extensively replicated in the literature on numerical cognition since its first description and definition in the seminal paper of Dehaene, Bossini, and Giraux (1993). This effect corresponds, within the numerical range tested, to faster reaction times to smaller numbers when they are responded on the left side and to larger numbers when they are responded on the right side. Following its observation, Dehaene et al. (1993) postulated the spatial orientation of the semantic numerical representation on which numbers are represented according to their numerical size, with small numbers on the left side and large numbers on the right side (but see Gevers, Verguts, Reynvoet, Caessens, & Fias, 2006[2]). Since then, the SNARC effect has been widely investigated in the literature on numerical cognition and many behavioural studies have given further support to this postulate of the existence of a close link between

[2] Recently, Gevers et al. (2006) proposed an alternative model to account for the origin of the SNARC effect. According to this model, the link between magnitude information and response codes reflected by the SNARC effect is not the result of a spatial orientation of the semantic numerical representation itself. But it is supposed to be the result of a mapping, taking place after the numbers have been processed semantically, between the automatic activation of their magnitude information and their associated spatial code on different representational levels (see Figure 21 in the general conclusion for an illustration and further discussion of this model).

numbers and space. Throughout all these studies, it appears that the SNARC presents several characteristics:

A) The SNARC effect is task-independent:

The first observation of what was called later the SNARC effect was made in Dehaene et al. (1990). In this study, participants were submitted to a comparison task to 65 in which target numbers ranged from 31 to 99 were visually presented. Half of the participants had to press the right-hand response key if the target number was larger than 65, or the left-hand response key if the target number was smaller than 65. The other half of the participants performed the task with the reverse response assignment. The results indicated a response-side effect: the reaction times were slower in the larger-left condition than in the larger-right condition. This finding was interpreted as the evidence of the existence of a close link between numerical and spatial concepts, with the association between large magnitudes and right response side.

From then on, the SNARC effect has been defined and observed in many studies using different experimental paradigms. By submitting human adults to a series of parity judgements tasks, Dehaene and colleagues (1993) further explored the occurrence of the association between numbers and space. In all their experiences, they found a significant interaction between the response side and the magnitude of the target numbers. Following their findings, the authors named this numerical-spatial interaction the Spatial-Numerical Association of Response Codes and made the strong postulates that the SNARC effect reflects the fact that (1) the semantic numerical representation has a spatial left-to-right orientation, and that (2) magnitude information is automatically accessed.

Fias, Brysbaert, Geypens and d'Ydewalle (1996) replicated the SNARC effect in a parity judgement task but with the introduction of another statistical analysis than Dehaene et al. (1993), which better captures the nature of this effect. Their analysis consists in applying a regression analysis on the difference in reaction times between the right- and the left-hand response times with number magnitude as predictor variable. The observation of a significant negative relation between number magnitude and the difference in reaction times between the right and the left hand indicates the occurrence of the SNARC effect (see Figure 1). In a second and a third experiment, Fias et al. (1996) also found a SNARC effect in a phoneme monitoring task, in which participants had to decide whether the /e/ phoneme was present in the target Arabic numbers ranging from 1 to 9. The demonstration of a SNARC effect in these tasks showed that magnitude information was also accessed during Arabic-to-verbal transcoding.

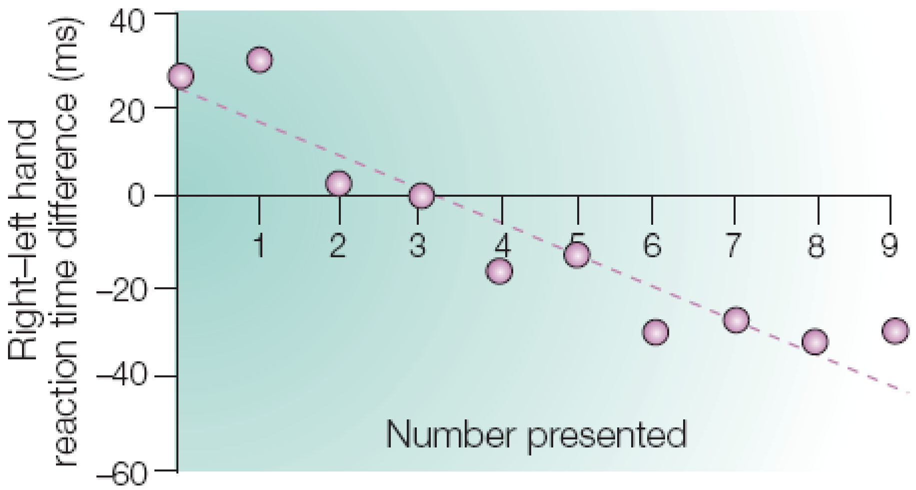

Figure 1. The SNARC effect in a parity judgement task: when participants have to judge the parity of target numbers, their responses are faster to larger numbers on the right side of space and faster to smaller numbers on the left side. The graph illustrates a SNARC effect observed with the statistical analysis proposed by Fias et al. (1996). It represents the response-time difference between right-hand and left-hand key presses. For smaller numbers, values greater than 0 are found, indicating a left-hand advantage. For larger numbers, values less than 0 are found, indicating a right-hand advantage *(reproduced from Hubbard, Piazza, Pinel, & Dehaene, 2005)*.

The SNARC effect was also observed in tasks in which numbers were irrelevant cues to solve the task. In Fias, Lauwereyns and Lammertyn's (2001) study, participants had to respond to an orientation-discrimination task on a triangle or a line superimposed on an Arabic digit ranging from 0 to 9. The results indicated the influence of the irrelevant digit's magnitude: small numbers elicited faster left responses, whereas large numbers elicited faster right responses.[3] These findings give further support to the postulate that the SNARC effect reflects an automatic access to numerical magnitude.

B) The SNARC effect is effector-independent:

In their study on the SNARC effect, Dehaene et al. (1993) submitted participants to a parity judgement task with their hands crossed. If the SNARC effect is effector-dependent (i.e., determined by the responding hand), then with hand crossed it should be reversed. However, the results showed that the SNARC effect was not reversed, suggesting that it is not due to a strict dependency on the hand performing the response. Therefore, the authors concluded that the SNARC effect is effector-independent and operates at an abstract level of representation. However, recently, Wood, Nuerk, & Willmes (2006) did not succeed to replicate the observation of a SNARC effect with crossed hands (i.e., they found a non significant SNARC effect). These results indicate that when the saliency of hand-based coordinates is high (i.e., hand crossed), than the representational (i.e., small numbers-left/large numbers-right on the mental number line) and the hand-based (i.e., small numbers-left hand positioned on the right side/large numbers-right hand positioned on the left side) spatial-numerical associations are both activated and interfere with each other, leading to a null SNARC effect[4]. In consequence, it seems that the SNARC effect is not completely effector-independent and that the spatial context can have an influence on

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[3] In this study, participants also undertook colour- or shape-discrimination task. These two other tasks were introduced as the authors tried to investigate whether the degree of neural overlap of structures dedicated to process relevant (orientation, colour or shape) and irrelevant (Arabic digit) information could have an impact on the observation or not of the SNARC effect. It is well known that the processing of numerical information relies on parietal cortex activation (Chochon, Cohen, Van De Moortele, & Dehaene, 1999; Pesenti, Thioux, Seron, & De Volder, 2000). The orientation processing also depends on parietal cortex, whereas colour or shape processing relies only minimally on parietal activation. The authors found an influence of the irrelevant digit's magnitude (SNARC effect) only for the processing of the relevant orientation, but not for the processing of colour or shape. Therefore according to them, a contamination in reaction times of the target processing by the distracter processing is possible essentially when there is neural overlap in parietal cortex between them.

[4] If a hand-based association had been the only determinant of the SNARC effect, then with crossed hand the SNARC should have been inverted.

its observation. Di Luca, Grana, Semenza, Seron & Pesenti (2006) found results in the same vein with Wood and al.'s (2006) results. In their study, participants had to identify Arabic digits by pressing one of ten keys with all the ten fingers. The direction of the finger-digit mapping was varied by manipulating the global direction of the hand-digit mapping as well as the direction of the finger-digit mapping within each hand. In this particular experimental design, a mapping congruent with the participants' prototypical finger-counting strategy (i.e., small-right superiority) led to better performance than did a mapping congruent with a SNARC effect (i.e., small-left superiority). Consequently, taken together, Wood et al.'s (2006) and Di Luca et al.'s (2006) results indicate that the semantic numerical representation as it is generally conceptualized (i.e., a mental number line oriented from left to right) may not be the only spatial frame of reference leading to spatial congruency effects in numerical tasks. Indeed, according to the experimental context, other spatial-numerical associations (i.e., hand-based in Wood et al. (2006) - finger-counting in Di Luca et al. (2006)) can interfere with this representational association or even overcome it.

However, even if the SNARC effect is not completely effector-independent, several studies give support to the postulate that the SNARC effect reflects a space-related representation of numerical magnitudes, rather than an over-learned motor association between numbers and manual responses. Fischer (2003) conducted a study in which small or large Arabic numbers (1, 2, 8 or 9) were presented at central fixation and participants had to point to a left or right target according to the parity of the Arabic numbers presented. The participants showed a SNARC effect with manual pointing: their movement execution was faster when left-responses were made to small digits and right-responses to large digits. Schwarz and Keus (2004) demonstrated the occurrence of a similar SNARC effect in saccadic and manual parity judgement tasks. Furthermore, Lavidor, Brinksman and Göbel (2004) found that the SNARC effect can arise from spatial congruity of the stimulus, rather than the response. They conducted a comparison task to 55 and a comparison task to 65, in which they manipulated visual field presentation, magnitude and distance from target number. Their results revealed an interaction between these three factors: for large distance, numbers of small magnitude were responded faster when they were presented in the left visual field, whereas numbers of large magnitude were responded faster when they were presented in the right visual field. This result constitutes a SNARC-like effect as it reflects the spatial orientation from left-to-right of the semantic numerical representation

C) The SNARC effect is input format- and modality-independent:

The SNARC effect has been found with different numerical input stimuli and in different modalities. Dehaene et al. (1993) and Fias (2001) were the first to demonstrate that the SNARC effect does not occur only with Arabic numbers, but also with number written words. In a further attempt to examine whether the SNARC effect could be observed whatever the notation or the modality of the numerical input, Nuerk, Wood and Willmes (2005) submitted human adults to parity judgement tasks on auditory number words, visual Arabic numbers, visual number words and visual dice patterns. In each condition, a SNARC effect was observed. Moreover the size of the SNARC effect did not differ across modality. Therefore, the authors concluded that the SNARC effect corresponds to a general index of the access to the semantic numerical representation, which is central and modality-independent. Moreover, Fias and Fischer (2005) reported the existence of non-published SNARC studies with particular number formats, such as Roman or Chinese numerals, dot patterns, counting fingers, auditory or tactile magnitude information, supporting the idea of a supramodal number representation.

D) The SNARC effect is culturally determined:

The SNARC effect appears to be culturally determined. More precisely, it seems to be determined by the orientation of reading habits. Western people who read from left to right demonstrated a left-to-right SNARC effect. On the other hand, Iranian people who read from right-to-left showed a weaker or even reversed SNARC effect (Dehaene et al., 1993). Therefore it can be assumed that, in Western cultures, the left-to-right orientation of the cognitive strategies for the processing of information, such as letters, words, sentences, but also numbers, equations, etc. might be responsible for the orientation of the semantic numerical representation and in consequence for the observation of the SNARC effect following its access. This postulate was reinforced by the observation that monolingual Arabic speaker from Beirut processed two Arabic numbers more easily when the larger number was presented to the left of the smaller number, compared to a display with the larger number on the right (Zebian, 2001, cited in Fias & Fischer, 2005). Nevertheless, only a few studies have been conducted on the postulate of the cultural origin of the SNARC effect and more research are needed to strengthen and further examine this postulate.

E) The SNARC effect is number-specific:

In their paper, Dehaene et al. (1993) wanted to study whether the SNARC effect is governed by the ordered sequential structure of the numerical stimuli or whether it is specific to the semantic representation of numbers. They submitted their participants to two tasks, similar to a parity judgement task on numbers, but this time on letters of the alphabet. The first task consisted in a letter classification task, in which participants had to press one key when presented with the letters A, C or E and another key when presented with the letters B, D, or F. The second task was a consonant-vowel classification task. No SNARC effect was found in both tasks on letters, leading the authors to conclude that the SNARC effect has a genuine numerical origin. In his study on manual pointing, Fischer (2003) gave further support to the idea that the SNARC effect is number-specific. Participants were submitted to a pointing task with letters of the alphabet, in which they had to point to target letters according to their consonant/vowels status. The results of this experiment also demonstrated the absence of SNARC-like effect on letters of the alphabet.

Zorzi, Priftis, Menegello et al.'s (2006) tackled from a new perspective the issues of the spatial nature of the semantic numerical representation and of the number-specific nature of the SNARC effect: they studied the representation of numbers in patients with neglect[5]. In their study, patients with left neglect undertook a visual line bisection task and three mental bisection tasks on intervals of numbers, letters of the alphabet and months of the year. The mental bisection tasks consisted for the patients in telling which number,

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[5] Spatial neglect generally occurs following a parietal lesion, often in the right hemisphere. Patients suffering from spatial neglect have difficulty to process and represent the contralesional space (i.e., left space), involving the impaired ability to direct their attention to this side of space and to respond to stimuli located in it (Bisiach and Vallar, 1988). Moreover, neglect can also affect spatial mental representation, for example in imagery (e.g., Guariglia, Padovani, Pantano, & Pizzamiglio, 1991).

letter of the alphabet or month of the year occupies the middle position in a given interval (e.g., 6 is the middle number of the numerical interval (3-9)). In the visual line bisection task, Zorzi et al. (2006) replicated the classical clinical demonstration of neglect: the patients missed the true midpoint of the visual lines and showed a right bias in their bisection, which was modulated by line length. In the numerical interval bisection tasks, neglect patients demonstrated error patterns that mirrored those observed in the visual task: a rightward shift of the subjective bisection point that was modulated by the interval length[6]. In contrast, no such effects were observed in the bisection tasks of non-numerical intervals. According to the authors, these findings reinforced the conclusion that the spatial layout of the semantic numerical representation is number-specific, instead of being a general characteristic of ordered sequences.

However, discrepant results have been observed on this last statement on the SNARC effect. Indeed, Gevers, Reynvoet, and Fias (2003) found a SNARC effect with non-numerical ordered sequences, such as letters of the alphabet and months of the year in both order relevant (before/after a given standard) and order irrelevant (consonant/vowel classification for letters, letter monitoring task for months) tasks. Therefore, according to the authors, the association between numbers and space observed by means of the SNARC effect might not be number-specific and necessarily attributable to the quantitative meaning of numbers, but perhaps to their ordinal property. Or it can be postulated that a shared representation handle numerical and ordinal information depending on the task context. Turconi, Jemel, Rossion & Seron (2004) also found a SNARC effect with letters of the alphabet while investigating whether the processing of number magnitude and order rely on common or different functional processes and neural substrates using event-related potentials (ERPs). They submitted participants to a quantity task (classification of numbers

[6] These findings have been recently replicated with healthy adults submitted to bisection tasks on physical lines and numerical intervals (Longo & Lourenco, 2007). This study showed that the slight leftward bias (pseudoneglect) generally observed when bisecting physical lines, is also found when bisecting numerical intervals. Moreover, this bias increased with the average of the numbers to bisect, reflecting Weber's law. Therefore, these findings further demonstrate that the hemispheric asymmetries generally observed in spatial attention operate similarly in physical and numerical space.

as smaller or larger than 15), an order task on the same materiel (classification of numbers as coming before or after 15), and an order task on letters of the alphabet (classification of letters as coming before or after M). In addition of a classical distance effect, the behavioural results showed for all tasks a SNARC effect: left-hand responses were significantly longer for larger/after items, than for smaller/before items. However, the electrophysiological data indicated that even if numerical quantity and order processing led to similar behavioural effects, they were associated with different spatio-temporal courses in parietal and prefrontal cortices. So, it seems that further research need to be conducted on this last specific issue on the SNARC effect, as currently no straightforward conclusion can be drawn in view of the different results found.

Taken together, all the data presented here reveal how, in the study of numerical cognition, the SNARC effect has gathered a lot of attention. This attention is due to the fact that the SNARC effect is often considered as a striking evidence of the existence of a strong connection between numbers and space (Hubbard, Piazza, Pinel, & Dehaene, 2005). Moreover, many behavioural, neuropsychological and neuro-imaging data further support the idea that numbers and space are linked together, as it will be discussed later in our thesis (see, for reviews, Fias & Fischer, 2005; Hubbard et al., 2005).

Following the observation of the three classical behavioural effects described above, the popular metaphor of a continuous, analogue mental number line (distance and size effects) oriented from left to right (SNARC effect) on which numbers are represented by mental magnitudes emerges in the literature on numerical cognition (Brysbaert, 1995; Dehaene, 1992, 1997; Gallistel & Gelman, 1992, 2000). This metaphor constitutes nowadays a view on which there is a large consensus in numerical cognition research (Dehaene, 2003; Zorzi et al., 2006).

In the next section of this chapter, the two prominent classical theoretical proposals made on the mental number line are presented. These two theories conceptualized in a different way how numerical processing obeys Weber's law. According to the first view, the mental number line is logarithmically compressed (Brysbaert, 1995; Dehaene, 1992, 1997): the relation between numbers and their analogue magnitudes is logarithmic.

Alternatively, according to the accumulator model, the mental number line is linear with scalar variability (Gallistel & Gelman, 1992, 2000): numerical magnitudes are represented equally spaced, but the mapping from numbers to their magnitude representations displays scalar variability.

2. Classical theoretical models

2.1. The model of the mental number line

The metaphor of a mental number line on which numerical quantities are represented was firstly proposed by Restle (1970), following the observation of the distance effect. According to Restle (1970), human adults represent numerical symbols on an analogue system having distinctive markers: the closer two numbers, the more likely their representations on the number line lie within a common region. Dehaene (1992, 1997, 2001, 2003) took over this metaphor of a mental number line on which numbers are represented by equal distributions of activation (i.e., the mental number line is a place-coding system). But he also assumed that this mental number line presents a spatial orientation from left-to-right (SNARC effect) and a compressive scale (size effect): numerical magnitudes are farther apart on the low end of the number line than on its high end (see Figure 2). Therefore according to this model: 1) the more important the difference between the numbers or numerosities to compare, the more distant their distributions of activation on the mental number line, and the more salient their numerical difference (distance effect); 2) with the logarithmic compression, for a given distance, the larger the numerosities or numbers to compare, the closer their distributions of activation, and the more difficult their discrimination (size effect).

The mental number line takes place in what Dehaene (1992, 1997) called the triple-code model suggesting that human adults possess three different mental representations of numbers: two symbolic representations (verbal numerals and Arabic numerals) and the mental number line. Transcoding routes enable numerical information to be translated from one code to the other and each numerical procedure is tied to a specific input and output code (e.g., numerical comparison depends on the access to the analogue magnitude code).

According to the triple code model, the analogue magnitude representations on the mental number line constitute the only "semantic" numerical representations, as they provide the understanding of the quantity that the numerals represent.

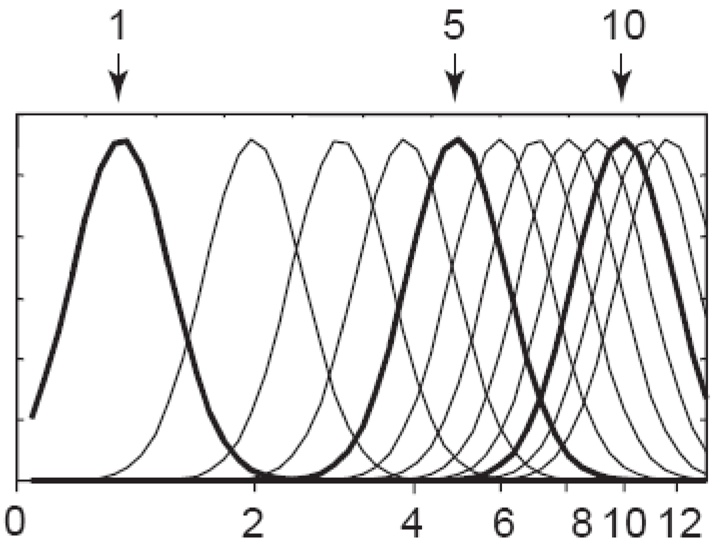


Figure 2. The model of the mental number line with a logarithmic compression: mental activations are depicted on the Y axis as a function of numerosity on the X axis *(reproduced from Feigenson, Dehaene & Spelke, 2004)*.

2.2. The accumulator model

According to Gallistel and Gelman (1992, 2000), numbers are represented by magnitudes on an analogue linear representation with scalar variability (see Figure 3 – right panel). Their theoretical proposal is generally called the accumulator model, as it is based on Meck and Church's (1983) accumulator model to account for numerical competences observed in rats (see Figure 3a). According to this model, when a number or a quantity has to be processed, mental magnitudes are generated by a nonverbal counting process: each enumerated unit is represented in a mental accumulator by an additional fixed increment of magnitude. When the preverbal counting process is over, the final state of the accumulator corresponds to the cardinal value of the counted target, which is then read into noisy memory: the trial-to-trial variability in remembered magnitudes is proportional to these magnitudes. This variability has been called scalar variability and conceptualizes why numerical processing obeys Weber's law. Gallistel and Gelman (1992,

2000) assumed that humans share with animals this nonverbal counting process that allows the representation of numerosities by the generation of mental magnitudes on a linear mental number line with scalar variability. Their model is a magnitude-coding system: the mental representation of a number is analogue to the magnitude it represents (i.e., a given number activates a portion of the mental number line, and this portion constitutes a subset of the portion activated for a larger number; see Figure 3b). In consequence, numbers are presented by unequal distributions of activation with scalar variability on the mental number line, accounting like Dehaene (1992, 1997) for the distance and size effects: the more distant and the smaller the numerosities or numbers to process, the less their activation distributions overlap.

a)

b)

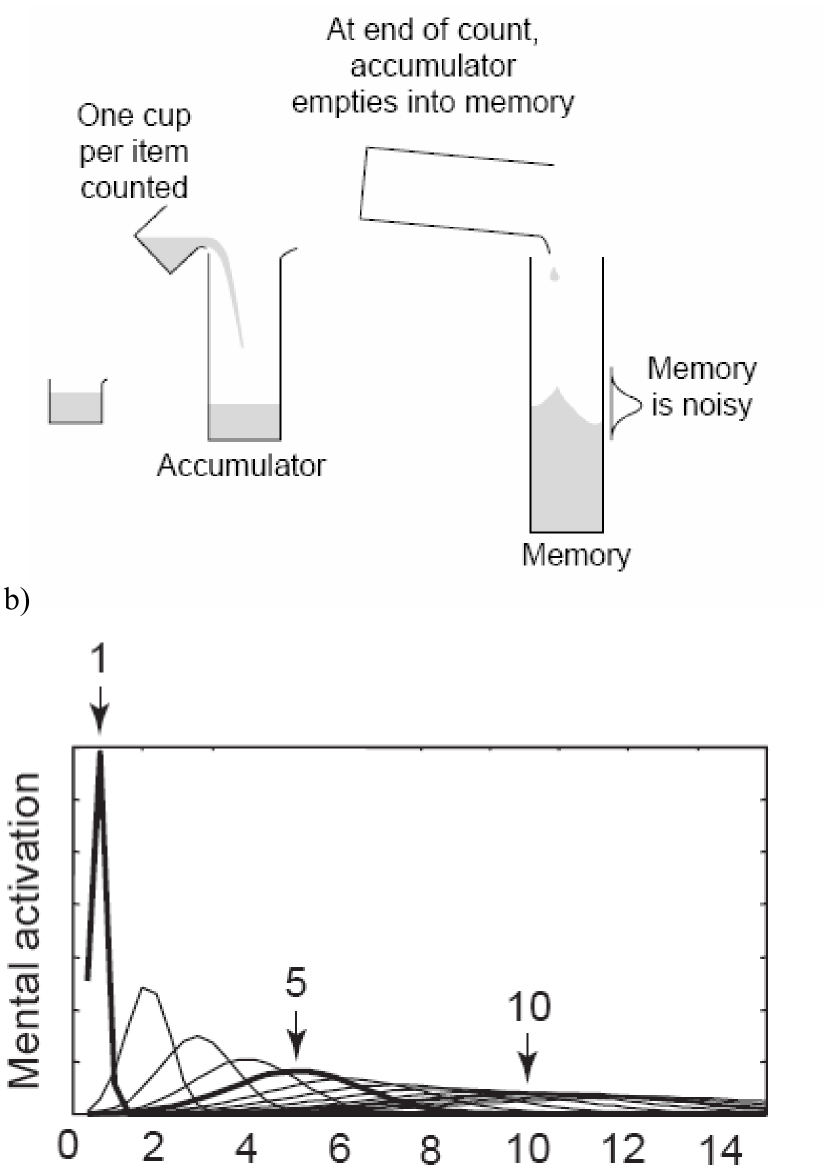


Figure 3. The accumulator model (a) *(reproduced from Gallistel & Gelman, 2000)* and the model of the mental linear number line with scalar variability (b) *(reproduced from Feigenson, Dehaene, & Spelke, 2004)*: mental activations are depicted on the Y axis as a function of numerosity on the X axis.

This model has been supported by Whalen, Gallistel and Gelman's (1999) study, in which human adults undertook two numerical estimation tasks. In the first one, participants had to produce approximately the magnitude of target numbers, by pressing a response key at rates that made vocal or subvocal counting difficult or impossible. In the second one, participants had to estimate the number of flashes in a rapid and randomly timed sequence. In both tasks, participants demonstrated patterns of performance supporting the hypothesis according to which numerosities are represented by mental magnitudes with scalar variability: the means of estimates and the variability in the numerical estimates were proportional to target magnitude, resulting in constant coefficient of variation across target size. These results have been replicated in another key-press experiment by Cordes, Gelman, Gallistel and Whalen (2001).

The two classical theoretical proposals presented here on the mental number line conceptualize in a different way why the processing of numerosities obeys Weber's law. However, they are similar at a functional level. The psychological predictions of the linear (Gallistel & Gelman, 1992; 2000) and logarithmic (Dehaene, 1992, 1997) models are essentially equivalent (Dehaene, 1992, 2001, 2003). Therefore, the use of behavioural studies, with the psychophysical paradigms known currently, does not allow disentangling the two models. Moreover, in recent theoretical proposals on numerical cognition, they are no longer considered as diverging, but rather as coexisting (see, for example, Huntley-Fenner, 2001; Siegler & Opfer, 2003; Verguts, Fias, & Stevens, 2005).

However, new theoretical proposals have been made in the literature on numerical cognition. These new perspectives raise two central points: 1) they question the assumption that numerical processing necessarily obeys Weber's law; 2) they suggest that the experience humans have with numbers and numerosities could have an impact on the way they represent or access their corresponding magnitudes, and consequently could have an impact on their numerical abilities.

In the following section, these new theoretical approaches of the semantic numerical representation are reviewed.

3. New theoretical proposals

Both hypotheses of the existence of a mental number line with either a compressed scaling (Dehaene, 1992, 1997) or an increasing variability (Gallistel & Gelman, 1992, 2000) imply an asymmetry between small and large numbers with the observation of a size effect not only in number comparison task, but in every task involving the access to the semantic numerical representation. In other words, both theoretical proposals assume a necessary approximate numerical processing of large numerosities. However, they encountered difficulties to account for behavioural data observed in several numerical tasks in which this obligatory asymmetry between small and large numbers reflecting Weber's law was not found: 1) the observation of symmetric priming in number naming (Reynvoet & Brysbaert, 1999; 2004; Reynvoet, Brysbaert, & Fias, 2002) and in parity judgement (Reynvoet & Brysbaert, 1999, 2004; Reynvoet, Caessens, & Brysbaert, 2002) tasks (see Figure 4); 2) the absence of a size effect in tasks that show involvement of the mental number line (distance-related priming effect and/or SNARC effect), like number naming (Butterworth, Zorzi, Girelli, & Jonckheere, 2001; Reynvoet, Brysbaert, & Fias, 2002) and parity judgement tasks (Dehaene et al., 1993; Fias et al., 1996; Reynvoet, Caessens, & Brysbaert, 2002); 3) the observation of different patterns of performance (logarithmic-approximate vs. linear-accurate) in numerical estimation tasks, according to age, numerical context and mathematical achievement (Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003). In consequence, three different theoretical proposals on the mental number line have been made questioning the systematic numerical processing's obedience to Weber's law and putting stress on the role of experience with numbers and numerosities in the way numerical magnitudes are represented or accessed. These models are: 1) the model of multiple semantic numerical representations; 2) the model of exact small-number representation; 3) the model of better mapping abilities.

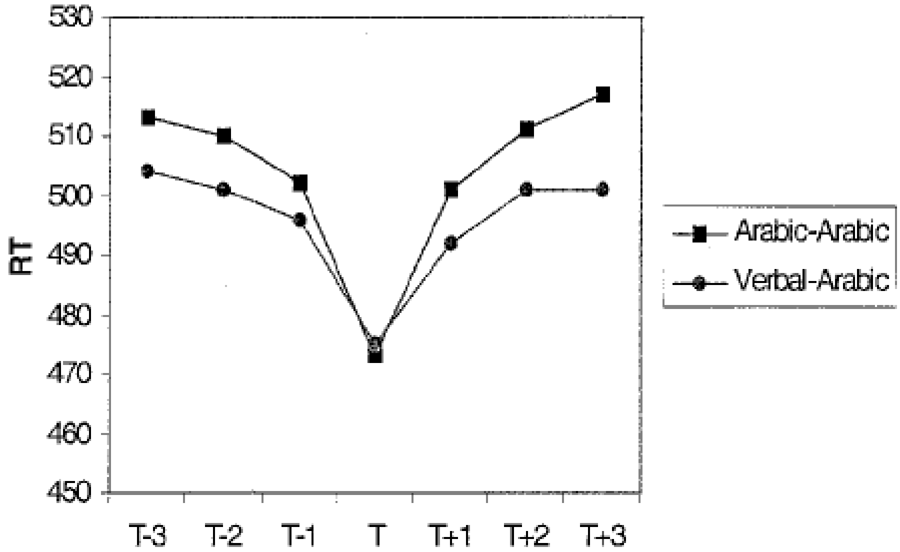


Figure 4. The symmetric priming effect in an Arabic number naming task with Arabic and verbal primes: equal prime-target distances lead to equal priming effects (i.e., same RTs to name target number 5 or judging it to be odd, whether it has been primed by 3 or 7). The classical views (Dehaene, 1992, 1997; Gallistel & Gelman, 1992, 2000) would imply an asymmetric priming effect between small and large primes with equal prime-target distances: RTs to name target number 5 should be faster when it has been primed by 7 compared to when it has been primed by 3 *(reproduced from Reynvoet et al., 2002).*

3.1. The model of multiple semantic numerical representations

Following Campbell & Clark's (1988) encoding complex model suggesting an integrated network of multiple numerical representations to account for numerical processing and Case and Okamoto's (1996) proposal that children of different ages use different numerical representations[7], some authors assume that individuals know and use multiple representations of numerical quantities: the representations obeying Weber's law, implied by the logarithmic ruler (Dehaene, 1992, 1997) and the accumulator (Gallistel & Gelman, 1992, 2000) models, but also a linear-ruler representation of quantitative

[7] Case and Okamoto (1996) suggested that at a given age, children use a single representation for numbers: from 4- to 5-year-old, children possess only a qualitative central conceptual structure for representing numbers ("a little" vs. "a lot"); at 6-year-old, they possess and consistently rely on a linear structure to represent numbers.

magnitudes that does not entail scalar variability (Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003). According to this model, numerical quantity representations obeying Weber's law develop early in life, constitute a default option for quantitative computations and are mainly used for unfamiliar numerical ranges. The linear numerical representation without scalar variability is on the contrary acquired later, during middle childhood, and used for familiar numerical ranges. Its acquisition and extension is the result of numerical experience and mathematics instruction, as it correlates with age and mathematical achievement. Therefore, as individuals gain experience with numbers and numerical quantities, their linear semantic numerical representation may be extended to wider ranges. The multiple numerical representations coexist and compete with each other, the use of one representation instead of the other depending on the expertise of the subject with the range of numbers tested, its age, but also the numerical context involved in the task.

The underlying rationale of this model is that the existence of several numerical quantity representations is very useful for individuals as different representations are optimal in different situations. A representation obeying Weber's law is ideal in situations in which a better distinction across small numbers and numerosities is important. Indeed, this representation involves a better discriminability of the magnitude representations of small numbers and numerosities compared to those of large numbers and numerosities. In the beginning of mathematics instruction, the use of a semantic numerical representation allowing great discrimination of small numerical magnitudes is useful for children, as small numbers are often encountered with the learning of counting, simple additions, subtractions, etc. On the contrary, a linear representation of numerical magnitudes without scalar variability is ideal when all parts of the range are equally important, as it allows a great discrimination in the range of large numbers and numerosities as in the range of small numbers and numerosities. With the development of numerical and arithmetic abilities, the acquisition of a linear numerical representation without scalar variability is helpful as children are submitted to situations in which they have to be able to discriminate among large numbers and numerosities (estimation of the answers of arithmetical problems, understanding of the results of multi-digit additions, subtractions, etc.).

This new perspective emerged through a set of three different studies conducted by Siegler and colleagues. In their first study, Siegler and Opfer (2003) submitted 3 groups of children from 8- to 12-year-old (second, fourth and sixth graders) and a group of adults (undergraduate students) to two numerical estimation tasks. In the first task, called the number-to-position task, target numbers were shown to participants, who had to estimate their position on a number line. In the second task, called the position-to-number task, positions were shown on a number line and the participants had to estimate the numbers corresponding to these positions. Two numerical contexts were introduced in both tasks: number lines with a scale from 0 to 100; number lines with a scale from 0 to 1000. Firstly, the authors observed that with development, children rely increasingly on a formally appropriate linear representation rather than the intuitive logarithmic one. On the 0-1000 number lines, between second graders and six graders, patterns of estimates progressed from consistently logarithmic, to mixture of logarithmic and linear, to a primarily linear pattern (see Figure 5). Secondly, it appears that children know and use multiple representations of numerical quantity, as the same target numbers could elicit either a logarithmic or a linear pattern of estimates depending on the numerical context. For the target numbers presented in both tasks: 1) in the number-to-position task with the 0-100 number lines, the linear function fitted the second graders' median estimates, whereas in the same task but with the 0-1000 number lines, the logarithmic function better fitted their pattern of performance; 2) the linear function fitted generally median estimates much better in the number-to-position task 0-100 context than in the same task 0-1000 context. Thirdly, given that in other numerical studies, adults demonstrated numerical processing obeying Weber's law (e.g., Cordes et al., 2001; Whalen et al., 1999), whereas in this study the linear function rather than the logarithmic function fitted their patterns of performance, human adults also appear to know and use multiple numerical magnitude representations. Taken together these findings indicate that with age, participants demonstrate improvement in their numerical estimation abilities. According to the authors, this improvement might be due to the fact that with age, individuals get more experience with numbers. Therefore, they develop a linear representation without variability of these experienced numbers; and they are able to rely on this representation and use it adequately according to the numerical context.

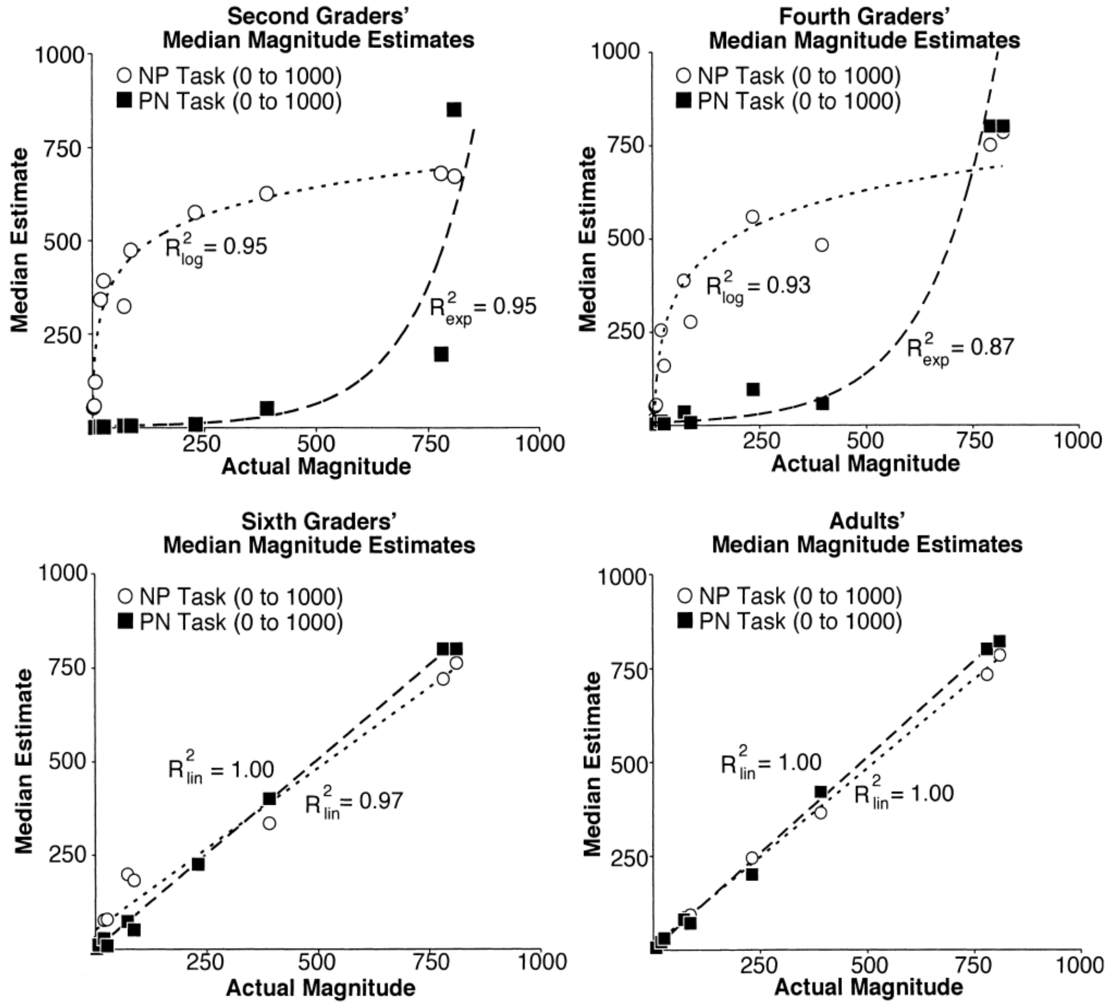


Figure 5: Developmental course in numerical processing observed in Siegler and Opfer (2003): from a predominant reliance on a logarithmic representation (second graders), to a mixture of reliance on logarithmic and linear representations (fourth graders), to a predominant reliance on a linear representation (sixth graders and adults). The graphs represent by group of subjects the median estimates on the number-to-position (NP) and position-to-number (PN) tasks for the 0-1000 number lines. The dotted and dashed lines indicate the best fitting functions, and for each function, the R^2 indicates the percentage of variance accounted for.

In a second study, children aged from 6- to 8-year-old (kindergartners, first and second graders) undertook a number-to-position task on 0-100 number lines and a standardized achievement test for mathematics (Siegler & Booth, 2004). The same developmental pattern as in Siegler & Opfer (2003) was observed: from predominant reliance on a logarithmic representation to a mixture of reliance on logarithmic and linear representations to predominant reliance on a linear representation. Moreover, it appeared that mathematical achievement correlated with the estimation accuracy. Therefore, the authors demonstrated for the second time that by gaining experience with numbers, children's reliance on a linear numerical magnitude representation increases, inducing better numerical estimation abilities.

Booth and Siegler (2006) conducted a third study in which children from kindergarten to fourth grade had to solve four types of estimation tasks: a computational (approximation of the result of addition problems), a numerosity (estimation of the number of objects in a set), a measurement (line length estimation) and a number line (number-to-position) estimation task. Two numerical contexts were examined. In a first experiment, children from kindergarten to third grade were submitted to the tasks with numbers ranged from 0 to 100; whereas in a second experiment, children from second to fourth grades were submitted with problems involving numbers ranged from 0 to 1000. For the third time, on several types of numerical estimation task and during the same period of development, the authors observed the developmental shift from reliance on a logarithmic representation of numerical magnitudes to reliance on a linear representation of them. Moreover, estimation skills appeared again to be positively correlated to mathematical achievement.

Taken together these findings led the authors to conclude that various numerical magnitude representations persist over time. When adults and older children have to process numbers in unfamiliar ranges (i.e., numbers/numerosities for which they have only a few or no experience), they may rely on the intuitive and widely applicable logarithmic representation. When they have to process numbers in familiar ranges (i.e., numbers/numerosities for which they have experience), they may rely on the acquired linear representation. However, it is worth pointing out that the linear patterns of performance observed in Siegler and colleagues' studies might be related to a particular recurrent characteristic of the numerical context involved in their experimental tasks. Indeed in each task for which the authors found patterns of performance reflecting the use of a linear numerical representation without scalar variability, participants could base their numerical processing on numerical landmarks given in the experimental procedure. In the number-line tasks, number 0 was always printed below the left end of the number lines presented and number 100 or 1000 below their right end (Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003). In the same way, numerical references were given in the measurement and the numerosity estimation tasks in Booth and Siegler (2006). In the measurement task, participants had to produce desired line lengths of x "zips"[8] on sheets of paper. On the top of each sheet of paper given for each line length to produce, there were two pre-printed lines of reference: the first line was very short and corresponded to the reference one "zip", the second line was much longer and corresponded to the reference one thousand "zips". In the numerosity estimation task, children had to fill a box presented on a computer screen with an "increase" and a "decrease" button to make dots appear in or disappear. But two other boxes, an empty box with zero dots in it and a full box with 1000 dots in it, always remained on the screen as references for the children (Booth & Siegler, 2006). It was only in the fourth computational task in Booth and Siegler's (2006) study that no references of the low and the high ends of the numerical estimation were specified to the subjects. However, in this task, the calculation of linearity scores was impossible. So the authors could not investigate the developmental progression from logarithmic to linear representations in this single task without numerical indications. In consequence, the observation of patterns of performance reflecting the use of a linear numerical representation without variability might be specific to particular numerical contexts involving numerical landmarks on which the participants can base their processing.

3.2. The model of exact small-number representation

Following Feigenson, Dehaene, and Spelke's (2004) suggestion to distinguish two core systems of number, one for representing precisely small numbers and one for representing approximately large numbers, Verguts et al., (2005) postulate the existence of a "hybrid" mental number line: linear with fixed variability in the range of small numbers, but compressed with increasing variability in the range of large numbers. This model was suggested to account for the symmetric priming effect and the absence of size effect in several numerical tasks. According to this model, called the model of exact small-number

[8] Unit of measurement created by the authors in order to avoid the requirement of knowledge of conventional measurements units to solve the task (one "zip" = 0.034 cm).

representation, the magnitude representations for small numbers are precise and exact, whereas the magnitude representations for large numbers become fuzzier and approximate. This difference between the magnitude representations of small and large numbers is interpreted in terms of frequency. The boundary between small exact and large fuzzy numerical magnitude representations depends on the frequency of exposure to numbers. As large numbers have low frequency compared to small numbers, fewer number line units might be dedicated to them. For example children's exact magnitude representations should be restricted to small numbers, whereas adults' exact magnitude representations should at least go beyond single digit numbers, as they have been more exposed to a wider range of numbers.

According to this model, the number line corresponds to a set of units, one for each number. The magnitude representation of small numbers is characterized by three principles:

- *place-coding system*: each number activates an equal amount of units on the number line.
- *linear scaling*: the distance between numbers' magnitude representations on the number line is equal for all numbers with a fixed distance.
- *constant variability*: the amount of variability in representing numerical magnitudes is unrelated to numerical magnitude. The activation curves induced on the number line have the same width for each number.

A crucial factor in this model is the input format of the stimuli. According to the type of input, this semantic numerical magnitude system is assumed to behave differently, especially in the range of small-frequent numbers. On the one hand, the representation of non-symbolic quantities is characterized by place-coding and increasing variability, so the system behaves in approximate way with this kind of stimuli. On the other hand, the representation of symbolic numerical input is characterized by place-coding and constant variability, so the system behaves in an exact way with this kind of stimuli (Fias & Verguts, 2004; Verguts et al., 2005). In addition, the authors assumed that the size effect is inherent to the comparison process. This effect is considered as a consequence of nonlinear mapping from the number line to the comparison output system. Therefore, in comparison tasks, a size effect should be observed with symbolic (nonlinear mapping) and non-symbolic (increasing variability and nonlinear mapping) input stimuli; while in other numerical tasks, the size effect is only expected with non-symbolic stimuli (increasing variability). The authors trained this connectionist model on naming, parity judgement and comparison. They reported that it accounted for the distance and size effects in number comparison, but also in number naming and parity judgement for the presence of a symmetric priming effect and the absence of a size effect.

3.3. The model of better mapping abilities

A third theoretical proposal developed from studies conducted with children, that we will call the model of better mapping abilities, emphasizes the role of experience in the way children access numerical magnitude representations. According to their counting abilities, children appear to present better numerical estimation performance (Huntley-Fenner, 2001; Lipton & Spelke, 2005), suggesting that by gaining experience with numbers, children and adults improve the mapping between the symbolic representation of numbers, which is suggested to be linear, and their corresponding magnitude representations on the mental number line obeying Weber's law with logarithmic compression (Feigenson, Dehaene, & Spelke., 2004) or with scalar variability (Huntley-Fenner, 2001; Lipton & Spelke, 2005). In other words, by gaining experience with numbers, children learn to compensate for Weber's law on the mental number line.

Huntley-Fenner (2001) conducted a study in which he investigated the hypothesis that with age, children improve their estimation skills (decrease in their trial-to-trial variability). He submitted a group of children aged from 5- to 7-year-old to an estimation task of the number of items (ranged from 5 to 11) presented in visual displays on a computer screen. Children had to give their answer by pointing to a numeral on a number line. Children, like adults, showed the pattern of scalar variability in their numerical estimates: mean and standard deviation of estimates increased in direct proportion with target magnitude, resulting in the observation of constant coefficients of variation across target size (Whalen et al., 1999). Moreover, with age, the size of children's coefficient of variation scores decreased towards the range of adults' coefficient of variation scores.

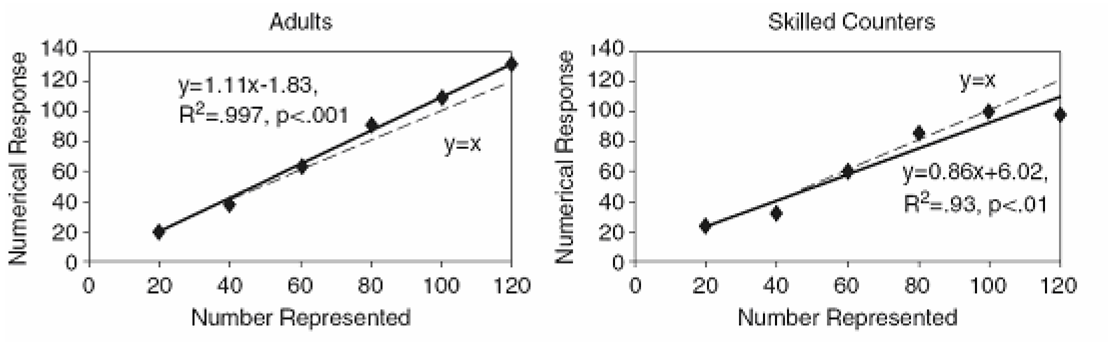
These findings support the idea that with age and experience with numbers, children improve their estimation abilities that become less variable. These results led the author to conclude that with age, children develop an intuitive sense of quantity and their experience with numbers by counting. Therefore they might also improve the quality of the mapping process between the symbolic representations of numbers (numerals) and their corresponding magnitude representations, resulting in better numerical skills.

Throughout our development, our numerical abilities improve. Already during infancy, infants' numerical abilities get better: at 6-month-old, babies are able to discriminate two numerosities at a ratio 1/2 (4 vs. 8 and 8 vs. 16, but not 4 vs. 6 or 8 vs. 12), while at 9-month-old their numerosity discrimination ratio is 2/3 (4 vs. 6 and 8 vs. 12, but not 4 vs. 5 or 8 vs. 10) (Lipton & Spelke, 2003, 2004). During childhood, this ratio still enhances (Siegler and Opfer, 2003; Huntley-Fenner, 2001) until adulthood, when it reaches 7/8 (Temple & Posner, 1998).

Taken together these results led Lipton and Spelke (2005) to investigate whether the enhancement observed in children in their numerical estimation skills correlates with the acquisition of counting abilities on large numbers; in other words, whether these better numerical abilities can be attributed to the development of mapping abilities between number words and their non-symbolic numerical magnitude representations. They assessed the counting abilities up to 100 of a group of 5-year-old children and a group of adults and they submitted them to four numerical tasks. Of these tasks, three involved assignment of number words to large sets of objects: 1) the estimation task: participants had to estimate the number of elements presented on visual displays (ranged from 4 to 120); 2) the number word comprehension task: a target number word was given to the particpants, who had to choose the corresponding array of elements among two arrays (one with the target numerosity, one half or twice as numerous as the target numerosity); 3) the number word ordering task: two sets differing in numerosity by a 1/2 ratio were presented, the verbal cardinal value of one array was given to the participants, who had to estimate the number of elements in the second array. The fourth task was a simple non-symbolic numerosity comparison task, in which participants had to indicate between two sets of elements the one with more items. This last task was conducted to make sure that the errors occurring in the three first tasks could be attributed to failure in the mapping between number words and their corresponding numerical magnitude representations, instead of being attributed to failure to discriminate non-symbolic numerosities. On the one hand, in the three first tasks, children who were skilled counter (able to count to 100) showed like adults very good patterns of performance. Moreover on the estimation and number word ordering tasks, skilled counters' and adults' estimates followed a linear function[9] (see Figure 6). On the other hand, unskilled counters (unable to count reliably to 100) succeeded the tasks only in their counting range but failed for larger number words. In the non-symbolic numerical discrimination task, all the participants showed high performance, indicating that errors observed previously, especially with the unskilled counter children, stemmed from either their lack of knowledge of the number words or errors in the mapping from number words to non-symbolic numerosities, but not from the discrimination or comparison of the non-symbolic numerosities. In consequence, these findings support the hypothesis that with the development of counting abilities, children learn to map symbolic numerical representations (number words) to non-symbolic numerical magnitude representations, resulting in accurate estimation skills. Moreover, as skilled counters and adults demonstrated patterns of performance fitted by a linear function in numerical tasks in which no specific numerical landmarks were given, it seems that Siegler and colleagues' results can be replicated in numerical tasks without this particular experimental characteristic.

[9] Whether the linear function fitted the patterns of performance in the number word comprehension task was not examined.

a)



b)

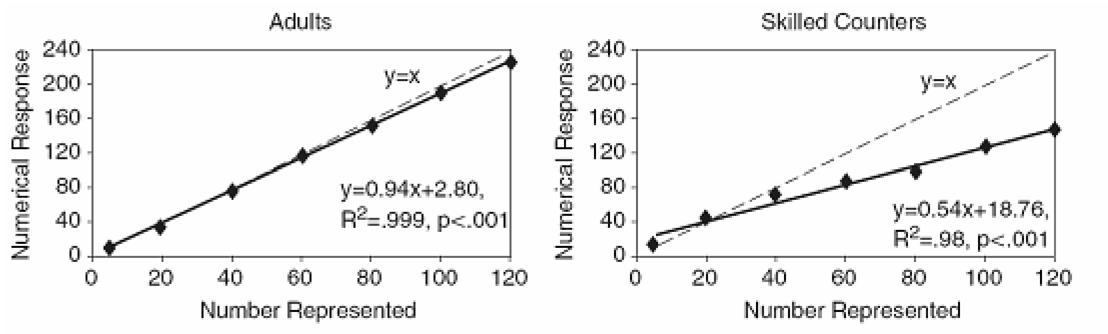


Figure 6: Illustration of Lipton and Spelke's (2005) results: the performance (diamonds), the best-fitting linear regressions (solid lines) and ideal performance (dotted lines) for adults and children categorized as skilled counters in the estimation task (a), and for adults and skilled counter children in the number word ordering task (b).

Taken together, this set of data indicates that, with the development of their counting abilities and their experience with numbers, children improve their numerical abilities. The fact that children (and adults) showed accurate and precise numerical estimation skills cannot be accounted by the classical view assuming that numerical processing systematically obeys Weber's law. Therefore, recent evidence (Booth & Siegler, 2006; Huntley-Fenner, 2001; Lipton & Spelke, 2005; Siegler & Booth, 2004; Siegler & Opfer, 2003) demonstrate that children quickly learn to map symbolic numerical input onto their pre-existing numerical magnitude representations obeying Weber's law. This mapping is initially logarithmic, resulting in approximate numerical estimation. But with age, the development of counting abilities and experience with numbers, this mapping becomes linear. In other words, children learn to compensate for Weber's law, resulting in accurate and precise numerical estimation (Feigenson et al., 2004).

The new theoretical assumptions reviewed here propose different conceptualizations of the semantic numerical representations to account for recent behavioural data. Siegler and Opfer's (2003) and Verguts et al.'s (2005) models are functionally equivalent. Firstly, both assumed that experience and exposure to numbers and numerosities give rise to the elaboration of a linear, exact representation of them, either on an acquired linear mental number line without scalar variability (Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003), or on a linear portion with no scalar variability of the same mental number line as the one initially postulated by Dehaene (1992, 1997) and Gallistel and Gelman (1992, 2000)(Fias & Verguts, 2004; Verguts et al., 2005). Secondly, the classical theoretical proposals are combined rather than clearly differentiated. Siegler and Opfer (2003) assumed the co-existence of a logarithmic and a linear semantic numerical representation with scalar variability. Verguts et al. (2005) suggested that the processing of low-frequent and large numbers obey Weber's law following the compression and the scalar variability of their mental magnitude representations. The last new theoretical proposal (Feigenson et al., 2004; Huntley-Fenner, 2001; Lipton & Spelke, 2005) is more in accordance with the two classical models, as it suggests that the semantic numerical representation obeys Weber's law. Nevertheless, it adds the assumption that with experience, the mapping process between the symbolic representations of numbers and their related magnitude representations on the mental number line improves in a way that subjects are able to compensate for Weber's law.

Conclusion

To sum up, three principal behavioural effects have been largely observed in research on numerical cognition: the distance, the size and the SNARC effects. And consequently, these effects have been used to conceptualize how numbers and numerosities are mentally represented in the human brain. Following the observation of the distance and the SNARC effects, it was postulated that numbers are represented by magnitudes on an analogue mental continuum oriented from left to right, usually called the mental number line. Following the observation of the size effect (along with the distance effect), it appears that numerical processing obeys Weber's law. Two predominant theoretical models have been made to account for this third property of the semantic numerical representation: the mental number line and the accumulator models. Although these two theoretical proposals conceptualize in a different way how numerical processing obeys Weber's law, they are equivalent at a functional level. However, both of them encounter difficulties to account for several recent behavioural data, which do not show systematic asymmetry between small and large numbers as expected by the mental number line and the accumulator models. Consequently, new theoretical proposals have been made lately, questioning the numerical processing necessary obedience to Weber's law and crucially suggesting that experience with numbers has an important impact on the way people access or represent numerosities.

At the end of this first chapter, we have now a clear idea on what are the classical behavioural effects that should be observed when participants are submitted to a numerical task, and what are the properties of the postulated underlying semantic numerical representation accessed to solve this task. At this point, one may wonder: where does this semantic numerical representation come from? Are we born with a mental number line and an innate concept of numbers? Or do we have to acquire and develop our numerical representations and skills with age? A large part of the literature on numerical cognition has been devoted to this question. Principally two main trends, presented in the following chapters, have been made on this issue: the nativist and the empiricist hypotheses.