Structure of certain periodic rings and near rings

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Using commutativity of rings satisfying $(xy)^{n(x,y)} = xy$ proved by Searcoid and MacHale [3], Ligh and Luh [2] have given a direct sum decomposition for rings with the mentioned condition. Further Bell and Ligh [1] sharpened the result and obtained a decomposition theorem for rings with the property $xy = (xy)^2 f(x,y)$ where $f(X,Y) \in Z < X,Y >$, the ring of polynomials in two noncommuting indeterminates. In this talk we discuss the structure of certain rings and near rings satisfying the following condition which is more general than the mentioned conditions : $xy = p(x,y)$, where $p(x,y)$ is an admissible polynomial in $Z < X,Y >$. Moreover we deduce the commutativity of such rings.

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Commuting Maps

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Let R be ring with center $Z(R)$. In the sequel, for $x, y \in R$ we write $[x, y]$ for the commutator of x and y. Let X be a subset of R. A map $f : X \to R$ is said to be commuting (resp. centralizing) on X if

$$[f(x), x] = 0 \text{ (resp. } [f(x), x] \in Z(R)) \text{ for all } x \in X.$$

The study of commuting and centralizing mappings was initiated by Divinsky in 1955 and Posner in 1957. In [1] Divinsky proved tha a simple artian ring is commutative if it has a commuting non-identity automorphism. An additive map $d : R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. In [1] Posner proved that a prime ring must be commutative if it possesses a non zero centralizing derivation. After Posner's theorem, a number of results concerning commuting or centralizing maps on certain subsets have been obtained in the literature. This theorem has been extremely influential and at least indirectly it initiated many issues which will be discussed in this talk.

Much more recently commuting additive maps, or even more generally, the traces of a multiadditive map has been examined. The initial results on such maps were obtained in 1996 by M. Brešar in [3]. Since then there has been a lot of activity on this subject. The main reason for describing commuting traces of multiadditive maps is a wide variety of applications. One of them is the solution of a long-standing open problem by Herstein on Lie isomorphisms of associative rings. Commuting maps also naturally appear in some linear preserver problems. This is another relevant area of applications. In this talk we shall also briefly deal with various extensions of the notion of a commuting map. For more detail, see [4].

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A NOTE ON GENERALIZED DERIVATION IN PRIME RINGS

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Let R be a prime ring of characteristic different from 2 and $f(x_1, \ldots, x_n)$ be a non-central multilinear polynomial over C. Suppose that F, G and H are three generalized derivations on R such that

$$F\Big(G(f(r))f(r)\Big) = H(f(r)^2)$$

for all $r = (r_1, \ldots, r_n) \in R^n$, then one of the following holds:

(1) $H = 0$ and either $F = 0$ or $G = 0$;
(2) there exist $a, b \in U$ such that $F(x) = ax$, $G(x) = bx$ and $H(x) = abx$ for all $x \in R$;
(3) there exist $\mu \in C$, $a, b \in U$ such that $F(x) = ax + xb$, $G(x) = \mu x$ and $H(x) = \mu(ax + xb)$ for all $x \in R$;
(4) $f(r_1, \ldots, r_n)^2$ is central valued on R and there exist $a, b, p, c, u \in U$ such that $F(x) = ax + xb$, $G(x) = px$ and $H(x) = cx + xu$ for all $x \in R$ with $F(p) = c + u$;
(5) there exist $a, c \in U$, $\alpha \in C$ and a derivation d of R such that $F(x) = ax + d(x)$, $G(x) = \alpha x$ and $H(x) = cx + \alpha d(x)$ for all $x \in R$ with $\alpha a - c \in C$ and $\alpha a - c + d(\alpha) = 0$;
(6) there exist $a, c, k \in U$, $\alpha \in C$ and a derivation d of R such that $F(x) = ax + d(x)$, $G(x) = \alpha x$ and $H(x) = cx + \alpha d(x) + [k, x]$ for all $x \in R$ with $f(r_1, \ldots, r_n)^2$ is central valued and $\alpha a - c + d(\alpha) = 0$.

Some identities related to multiplicative (generalized)-derivations in prime and semiprime rings

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Let R be a semiprime ring with center $Z(R)$ and λ a nonzero left ideal of R. A mapping $F : R \to R$ (not necessarily additive) is said to be a multiplicative (generalized)-derivation on R, if there exists a map d (not necessarily an additive map or derivation) on R such that $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in R$. Suppose that F and G are two multiplicative (generalized)-derivations of R associated with the maps d and g respectively on R. Main aim is to study the following situations:

(1) $F([x, y]) + G(yx) + d(x)F(y) + xy \in Z(R)$,
(2) $F(x \circ y) + G(yx) + d(x)F(y) + xy \in Z(R)$,
(3) $F(xy) + G(yx) + d(x)F(y) \pm [x, y] \in Z(R)$,
(4) $F([x, y]) + G(xy) + d(x)F(y) + yx \in Z(R)$,
(5) $F(x \circ y) + G(xy) + d(x)F(y) + yx \in Z(R)$,
(6) $F([x, y]) + G(yx) + d(y)F(x) - xy \in Z(R)$,
(7) $F(x)F(y) - G(yx) - xy + yx \in Z(R)$; for all $x, y \in \lambda$.

Functional Identities of Degree 2 Vanishing on Zero Products of XY and YX

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[1]

Let R be a unital prime ring with characteristic $\mathrm{char}(R) \neq 2$ and $Q_{ml}(R)$ be the maximal left ring of quotients. Suppose that R contains a nontrivial idempotent. We completely characterize the form of additive maps $F_1, F_2, G_1, G_2 : R \to Q_{ml}(R)$ such that

$$F_1(x)y + F_2(y)x + xG_1(y) + yG_2(x) = 0$$

whenever $x, y \in R$ satisfy $xy = yx = 0$. Our result partially generelizes a result of T. K. Lee in [Bi-additive maps of ζ-Lie product type vanishing on zero products of XY and YX. Comm. Algebra. 2017;45(8):3449-3467].

[1] Joint work with with Çagri Demir.

Clean Semirings and Exchange Semirings

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In this paper, we introduce the concept of clean semiring as a generalization of clean ring. A semiring is said to be clean if its every nonzero element can be written as the sum of a unit and an idempotent. We study the notion of clean semiring and obtain some important characterization of clean semiring. We also introduce the concept of exchange semiring as a generalization of exchange ring. Finally, we characterize the relation between clean semiring and exchange semiring.

A Certain Functional Identity Involving Inverses in Division Rings

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Let D be a division ring and $u \in D$ a fixed element of D. Let further $f, g : D \to D$ be two additive maps satisfying the functional identity

$$f(x)x^{-1} + xg(x^{-1}) = u \tag{1}$$

for every nonzero $x \in D$. In the talk, we will completely determine all possible forms of the additive maps f and g that satisfy (1). Our result generalizes some earlier results obtained in [1] and [2]. If time permits, we will also consider (1) in the context of unital local rings.

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Multiplicative lattices

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A multiplicative lattice is a complete lattice $(L, \vee, \wedge)$ equipped with a further binary operation $\cdot$ (multiplication) satisfying $x \cdot y \leq x \wedge y$ for all $x, y \in L$. For instance, the lattice of all two-sided ideals of a ring R with the usual product of ideals is a multiplicative lattice. Another example is given by the lattice of all normal subgroups of a group G with the commutator $[-, -]$ (Recall that the commutator $[H, L]$ of two normal subgroups H, N of G is a normal subgroup of G contained in $H \cap L$.) Multiplicative lattices allow to explain notions like those of Zariski spectrum, solvability of a group, hyperabelian groups. The talk is based on two papers: "A. Facchini, C. A. Finocchiaro and G. Janelidze, Abstractly constructed prime spectra, arXiv:2104.09840" and "A. Facchini, F. de Giovanni and M. Trombetti, Spectra of groups". Both papers were submitted for publication this year.

Some elementary generalizations of Boolean Rings

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A remakable application to the ring theory in its infancy J.H.M.Wedderburn in Trans. Amer. Math. Soc. 6 (1905), 349-352 discovered that every finite division ringis necessarily commutative. Besides its own intrinsic beauty and striking applications, the Wedderburn Theorem has served as a jumping-off point for investigating commutativity of rings under many polynomial constraints. The aim of the present talk is to discuss some results on commutativity of a certain class of rings including Boolean-like rings of Foster mentioned in Trans. Amer. Math. 59 (1946), 146-167.

Generalized Derivations Commuting on Lie Ideals in Prime Rings

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Let R be a non-commutative prime ring of characteristic different from 2 with Utumi quotient ring U and extended centroid C, L a non-central Lie ideal of R, F, G and H are three non-zero generalized derivations of R. If

$$[F(u)G(u) - uH(u), u] = 0$$

for all $u \in L$, then one of the following holds:

(1) there exist $a, c, q, p, p' \in U$ such that $F(x) = ax$, $G(x) = cx + xq$ and $H(x) = px + xp'$ with $a, ac - p, p' - aq \in C$;
(2) there exist $a, c, p \in U$ such that $F(x) = xa$, $G(x) = cx$ and $H(x) = px$ with $ac - p \in C$;
(3) there exist $p, c \in U$, $\lambda, \mu \in C$ and a derivation h such that $F(x) = \mu x$, $G(x) = cx + \lambda h(x)$ and $H(x) = px + h(x)$ for all $x \in R$ with $\mu c - p \in C$ and $\lambda\mu = 1$;
(4) R satisfies s_4.

Two-step nilpotent Leibniz algebras

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Leibniz algebras were first introduced by J.-L. Loday in [1] as a non-antisymmetric version of Lie algebras, and many results of Lie algebras have been extended to Leibniz algebras. Earlier, such algebraic structures had been considered by A. Blokh, who called them D-algebras [3]. Leibniz algebras play a significant role in different areas of mathematics and physics.
In [2] Lie algebras with small dimensional commutator ideals have been considered. The purpose of this talk is to classify the two-step nilpotent Leibniz algebras over a field of characteristic different from 2 in terms of Kronecker modules associated with pairs of bilinear forms. We show that there are only three classes of nilpotent Leibniz algebras with one-dimensional commutator ideal. Up to isomorphisms, the first class is determined by the companion matrix of the power of a monic irreducible polynomial, and the other two are unique. In this way, we give the definition of *Heisenberg Leibniz algebra*, *Kronecker Leibniz algebra* and *Dieudonné Leibniz algebra*. Then we describe the complex and the real case of the indecomposable *Heisenberg Leibniz algebras* as a generalization of the classical $(2n+1)-$dimensional Heisenberg Lie algebra $\mathfrak{h}_{2n+1}$. Moreover, when the dimension is 3, we determine all the isomorphism classes of these algebras.

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On strongly m-clean rings and m-semiperfect rings

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The notion of clean ring was first initiated by W. K. Nicholson in his study of "Lifting idempotents and exchange rings" in 1977. A ring is clean if each of its elements can be written as a sum of a unit and an idempotent element. A ring is strongly clean if each of its elements can be written as a sum of an idempotent and a unit which are commutative. Various generalizations of clean rings and strongly clean rings have been explored such as weakly clean ring, strongly J-clean ring, $*$-clean ring, nil clean ring, Zhou nil clean ring, neat rings etc. It is quite natural to ask what happens if we replace the idempotent element in the definition of a clean ring by an m-potent element. An element 'a' of a ring R is said to be m-potent if $a^m = a$. Letting m to be a positive integer greater than equal to 2, a ring R is said to be m-clean if each element of R can be written as a sum of a unit and an m-potent element. A ring R is said to be strongly m-clean if each element a of R can be written as a sum of a unit and an m-potent element which are commutative. In 1994, Camillo and Yu proved that a ring R is a semiperfect ring if and only if R is clean and orthogonally finite. Inspired by this work, we prove a similar characterization theorem for strongly m-clean ring in terms of m-semiperfect ring.

Generalized skew derivations preserving Jordan product on Lie ideals in prime rings

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Let R be an associative ring. A map $F : R \to R$ is said to be *strong commutativity preserving* on $S \subseteq R$ if $[F(x), F(y)] = [x, y]$, for all $x, y \in S$. Several results concerning strong commutativity preservers have been obtained applying the technique of Functional Identities Theory in rings (see for instance [1, 2, 3]). Starting from the definition of commutativity preserver, one may naturally introduce the concept of maps preserving the Jordan product ($x \circ y = xy + yx$) in case of rings having characteristic different from 2. More precisley a map $F : R \to R$ is said to be *strong Jordan product preserving* on $S \subseteq R$ if $F(x) \circ F(y) = x \circ y$, for all $x, y \in S$.

Here we will describe the form of a non-zero generalized skew derivation F of a non-commutative prime ring R having characteristic different from 2, in the case

$$\Big(F(x) \circ F(y)\Big)^n = \Big(x \circ y\Big)^n$$

for all $x, y \in L$, where $n \geq 1$ is a fixed integer and L is a non-central Lie ideal of R.

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Generalized skew derivations having a Lie-type behavior

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Let R be an associative ring, α an automorphism of R. An additive map $d : R \longrightarrow R$ is called a *skew derivation* of R if

$$d(xy) = d(x)y + \alpha(x)d(y)$$

for all $x, y \in R$ (α is called an *associated automorphism* of d).
A *generalized skew derivation* of R is an additive map $F : R \longrightarrow R$ such that

$$F(xy) = F(x)y + \alpha(x)d(y)$$

for all $x, y \in R$, where d is a skew derivation of R with associated automorphism α.
An additive map $\Delta : R \mapsto R$ acts as a *Lie derivation* on R, if

$$\Delta([x, y]) = [\Delta(x), y] + [x, \Delta(y)]$$

for all $x, y \in R$.
Here we discuss the structure of a prime ring R in the case there exists a generalized skew derivation $F : R \mapsto R$ satisfying the following Lie-type condition:

$$F([x, y]_k) = [F(x), y]_k + [x, F(y)]_k$$

for all $x, y \in R$, with $k \geq 1$ fixed integer.

Generalized g-derivations on prime rings

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We introduce the definitions of g-derivations and generalized g-derivations on a ring R. The main objective of the paper is to describe the structure of a prime ring R in which g-derivations and generalized g-derivations satisfy certain algebraic identities. Some well-known results concerning derivations, generalized derivations, skew derivations and generalized skew derivations in prime rings, have been generalized to the case of g-derivations and generalized g-derivations.

Commuting product of generalized derivations on multilinear polynomials in prime rings

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In recent years additive maps with automorphisms are in demand amongst the authors treating maps in prime rings. The usual goal when treating an additive map is to describe its form if possible or to state some structural results of the ring.

Let R be a prime ring. The first result on commuting maps is Posner's Theorem in [1] which states that there exists no nonzero commuting derivation on a prime ring R unless R is commutative.

The characterisation of commuting maps of prime rings raised by Brešar [2]. He proved that if an additive map F of a prime ring R is commuting on R, then there exists some $\lambda \in C$ (C the extended centroid of R) and an additive map $\mu : R \to C$ such that $F(x) = \lambda x + \mu(x)$ for all $x \in R$.

Related to this line of research commuting derivations, generalized derivations, anti-homomorphisms and multi-additive mappings have been worked by a number of authors [3],[4],[5],[6].

Following this line of investigation, this talk will focus on a joint work vith Vincenzo De Filippis, considering the two generalized derivations F and G of a prime ring R, in the case when FG is commuting on elements of the subset S,where S is the set of all evaluations in R of a multilinear polynomial $f(x_1, \ldots, x_n)$ over C in n non-commuting variables.

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