

# Magneto-reheating constraints from curvature perturbations

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**Abstract.** As additional perturbative degrees of freedom, it is known that magnetic fields of inflationary origin can source curvature perturbations on super-Hubble scales. By requiring the magnetic generated curvature to remain smaller than its inflationary adiabatic counterpart during inflation and reheating, we derive new constraints on the maximal field value today, the reheating energy scale and its equation of state parameter. These bounds end up being stronger by a few order of magnitude than those associated with a possible backreaction of the magnetic field onto the background. Our results are readily applicable to any slow-roll single field inflationary models and any magnetic field having its energy density scaling as  $a^\gamma$  during inflation. As an illustrative example, massive inflation is found to remain compatible with a magnetic field today  $B_0 = 5 \times 10^{-15}$  G for some values of  $\gamma$  only if a matter dominated reheating takes place at energies larger than  $10^5$  GeV. Conversely, assuming  $\gamma = -1$ , massive inflation followed by a matter dominated reheating cannot explain large scale magnetic fields larger than  $10^{-20}$  G today.

**Keywords:** Cosmic Inflation, Magnetic Fields, Reheating, Cosmic Microwave Background

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## 1 Introduction

Current measurements of magnetic fields filling the intergalactic medium show that they are not vanishing on the largest length scales. Using the spectra of blazars, and under the safest assumptions, Ref. [1] finds the two-sigma limit  $B_0 > 10^{-17}$  G whereas other data yield that  $B_0 > 5 \times 10^{-15}$  G [2–4] and  $B_0 > 10^{-20}$  G [5]. A non-vanishing magnetic field today, and on the largest length scales, suggests that it has a primordial origin. Indeed, as any other perturbation modes, large scale magnetic fields are necessarily of super-Hubble wavelengths at higher redshifts, and this triggers the problem of an astrophysical generation mechanism [6–8]. Since inflation solves exactly the same problem for the generation of the primordial curvature perturbations, it sounds the best candidate to generate magnetic field [9]. However, in flat space [10], the conformal invariance of electromagnetism prevents an amplification mechanism to take place during inflation. Even worse, as the magnetic energy density redshifts as  $\rho_B \propto 1/a^4$ , its contribution during inflation can become dominant and leads to a severe backreaction problem [11–15]. In fact, backreaction can already appear during the reheating era as soon as the mean equation of state parameter  $w_{\text{reh}} < 1/3$ . As shown in Ref. [16], this yields some non-trivial constraints between the energy density  $\rho_{\text{reh}}$  at the end of the reheating era, the equation of state parameter  $w_{\text{reh}}$  and the present Hubble scale value of the magnetic field  $B_0$ . As a result, if magnetic fields have an inflationary origin, conformal invariance has to be broken, at least during inflation [17–27].

Assuming the background evolution is under control, one still has to consider the gravitational effects of the magnetic degrees of freedom onto the evolution of the cosmological perturbations. Among others, one expects the generation of primordial non-Gaussianities [28–32] and a non-conservation of the curvature perturbation on super-Hubble scales, in a way similar to the presence of entropy modes [28, 33–35]. In this context, magnetic fields should not induce too large curvature perturbations on super-Hubble scales which would otherwise spoil the standard adiabatic contribution of inflationary origin. In this paper, we look into this issue, both during inflation and reheating, and use a phenomenological model to describe the evolution of the magnetic field during inflation as  $B \propto a^\gamma$ . Although our approach does not allow to derive the backreaction of the perturbations onto the magnetic field itself, our results should give the correct order of magnitude. In particular, and in addition to  $\gamma$ , we show that the maximal allowed value of  $B_0$  on Hubble scales today depends on the way the reheating

proceeded and on the inflation model itself. By assuming only slow-roll during inflation and a constant equation of state during reheating, we derive a generic formulae giving the upper bound of  $B_0$  in terms of the energy scale of reheating  $\rho_{\text{reh}}^{1/4}$  and the corresponding equation of state parameter  $w_{\text{reh}}$ . Such a result therefore generalizes the background constraints derived in Ref. [16] and extend the results of Ref. [34] to any reheating history. Our findings are then applied to large field massive inflation and the small field inflationary models. For instance, we find that massive inflation can only be compatible with a matter dominated reheating and a magnetic field value today of  $B_0 = 5 \times 10^{-15}$  G if the energy scale of reheating is higher than  $E_{\text{reh}} \gtrsim 10^5$  GeV, independently of  $\gamma$ . If some assumptions are made on  $\gamma$ , the bounds can be much stronger and are represented in Fig. 1. Our main formulae are Eq. (2.7) and Eqs. (2.11) to (2.13) which explicitly give the amplitude of the curvature perturbation given an inflationary potential, the magnetic field today and the reheating parameter.

## 2 Super-Hubble curvature perturbations from magnetic fields

The super-Hubble scale curvature perturbation  $\zeta$  on the constant energy density hypersurface sourced by an inhomogeneous primordial magnetic field is given by [34],

$$\zeta(t) = - \int_{t_*}^t dt_1 \frac{H(t_1)}{\rho(t_1) + P(t_1)} \delta P_{\text{rel}}(t_1) + \frac{8\pi G}{3} \int_{t_*}^t \frac{dt_1}{a^3(t_1)} \int_{t_*}^{t_1} dt_2 a^3(t_2) \Pi(t_2), \quad (2.1)$$

where we have imposed an initial condition that  $\zeta(t_*) = 0$ . In the actual situation,  $t_*$  may be taken to be a Hubble crossing time. The first term, which we denote by  $\zeta_1$ , represents the contribution due to the relative entropy perturbation,  $\delta P_{\text{rel}} \equiv \delta P_B - \frac{P}{\rho} \delta \rho_B$  where  $\delta P_B$  and  $\delta \rho_B$  denote pressure and energy density perturbation of the electromagnetic field. The quantities  $P$  and  $\rho$  are the total pressure and the total energy density, respectively. Assuming that the electromagnetic field obeys the Maxwell equations after the end of inflation,  $\zeta_1$  becomes independent of time after reheating. On the other hand, the second term, which we denote by  $\zeta_2$ , is sourced by the anisotropic stress of the magnetic field. Since the anisotropic stress persists even after reheating,  $\zeta_2$  still continues to evolve during and after reheating.

### 2.1 Phenomenological model

In order to evaluate Eq. (2.1), we need to specify the time evolution of  $\delta P_{\text{rel}}$  and  $\Pi$  during inflation, which requires specification of the generation model of the magnetic field. Since we do not want to concentrate on the particular model of the magnetogenesis, in this paper, we will take a phenomenological approach and make the following ansatz:

$$\delta P_{\text{rel}}(t) = \alpha \rho_{B_{\text{end}}} \left[ \frac{a(t)}{a_{\text{end}}} \right]^\gamma, \quad \Pi(t) = \beta \rho_{B_{\text{end}}} \left[ \frac{a(t)}{a_{\text{end}}} \right]^\gamma, \quad (2.2)$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are constant parameters and  $\rho_{B_{\text{end}}}$  is the energy density of the magnetic field at the end of inflation, which is treated as first order perturbation.

Then, adopting the slow-roll approximation and assuming a constant equation of state during reheating, Eq. (2.1) can be explicitly integrated in terms of the number of e-folds  $N$ :

$$\zeta_1 = -\alpha \rho_{B_{\text{end}}} \int_{N_*}^{N_{\text{end}}} \frac{e^{\gamma(N-N_{\text{end}})}}{\dot{\phi}^2} dN - \frac{1}{3} \frac{\rho_{B_{\text{end}}}}{\rho_{\text{end}}} \int_{N_{\text{end}}}^{N_{\text{reh}}} \frac{(1-3w_{\text{reh}})}{1+w_{\text{reh}}} e^{(-1+3w_{\text{reh}})(N-N_{\text{end}})} dN, \quad (2.3)$$

where a dot stands for differentiation with respect to  $N$ . The first term can be further simplified by assuming slow-roll while the second can be integrated explicitly. In reduced Planck mass units ( $M_{\text{Pl}}^2 = 8\pi G = 1$ ), and at first order in the Hubble flow functions, one gets

$$\zeta_1 = 3\alpha\rho_{B_{\text{end}}} \int_{\phi_*}^{\phi_{\text{end}}} \frac{V^2(\phi)}{V_{,\phi}^3(\phi)} \left[1 - \frac{\epsilon_1(\phi)}{3}\right] \exp\left[\gamma \int_{\phi}^{\phi_{\text{end}}} \frac{V(\varphi)}{V_{,\varphi}(\varphi)} d\varphi\right] d\phi + \frac{\rho_{B_{\text{end}}}}{\rho_{\text{end}}} \frac{R_{\text{rad}}^4 - 1}{3(1 + w_{\text{reh}})}. \quad (2.4)$$

In this expression

$$R_{\text{rad}} \equiv \frac{a_{\text{end}}}{a_{\text{reh}}} \left(\frac{\rho_{\text{end}}}{\rho_{\text{reh}}}\right)^{1/4} = \left(\frac{\rho_{\text{reh}}}{\rho_{\text{end}}}\right)^{\frac{1-3w_{\text{reh}}}{12(1+w_{\text{reh}})}}, \quad (2.5)$$

is the reheating parameter [36–39]. The labels “end” and “reh” denote respectively the end of inflation and the end of reheating, *i.e.* the beginning of the radiation era. Since after inflation we assume the magnetic field to obey Maxwell equations, one can express the ratio  $\rho_{B_{\text{end}}}/\rho_{\text{end}}$  in terms of energy densities today [16]. In particular, one has  $\rho_{B_{\text{end}}} = \rho_{B_0}(1+z_{\text{end}})^4$  where  $z_{\text{end}}$  is the redshift at the end of inflation. Assuming instantaneous transitions [40] and making use of the reheating parameter, one has

$$1 + z_{\text{end}} = \frac{1}{R_{\text{rad}}} \left(\frac{\rho_{\text{end}}}{\tilde{\rho}_{\gamma_0}}\right)^{1/4}, \quad \tilde{\rho}_{\gamma_0} \equiv \mathcal{Q}_{\text{reh}}\rho_{\gamma_0}. \quad (2.6)$$

Here  $\rho_{\gamma_0} = 3H_0^2\Omega_\gamma^0$  is the total radiation density today, and  $\mathcal{Q}_{\text{reh}} \equiv q_0^{4/3}g_{\text{reh}}/(q_{\text{reh}}^{4/3}g_0)$  is the measure of the change of relativistic degrees of freedom between the reheating epoch and today, where  $q$  and  $g$  respectively denotes the number of entropy and energetic relativistic degrees of freedom at the epoch of interest. Plugging Eq. (2.6) into Eq. (2.4), and making use of the Friedmann–Lemaître equations to express  $\rho_{\text{end}}$  in terms of the field potential  $V_{\text{end}} \equiv V(\phi_{\text{end}})$ , one finally gets

$$\zeta_1 = \frac{1}{R_{\text{rad}}^4} \frac{\rho_{B_0}}{\tilde{\rho}_{\gamma_0}} \left[ \frac{9\alpha}{3 - \epsilon_{1\text{end}}} \int_{\phi_*}^{\phi_{\text{end}}} \frac{V_{\text{end}}V^2}{V_{,\phi}^3} \left(1 - \frac{\epsilon_1}{3}\right) e^{\gamma\Delta N(\phi)} d\phi + \frac{R_{\text{rad}}^4 - 1}{3(1 + w_{\text{reh}})} \right]. \quad (2.7)$$

Here  $\Delta N(\phi) < 0$  is the number of e-folds before the end of inflation and can be obtained from the slow-roll trajectory

$$\Delta N(\phi) \equiv N(\phi) - N_{\text{end}} \simeq - \int_{\phi_{\text{end}}}^{\phi} \frac{V}{V_{,\varphi}} d\varphi. \quad (2.8)$$

The first Hubble flow function  $\epsilon_1(\phi)$  is evaluated along the field trajectory and is also uniquely determined by the potential in the slow-roll approximation [41]

$$\epsilon_1(\phi) = \frac{\dot{\phi}^2}{2H^2} \simeq \frac{1}{2} \left(\frac{V_{,\phi}}{V}\right)^2. \quad (2.9)$$

By definition, the quantity  $\epsilon_{1\text{end}} \equiv \epsilon_1(\phi_{\text{end}})$  is unity for inflationary models ending by slow-roll violation but can be much smaller than unity for inflationary models ending by tachyonic instabilities. This formula allows us to evaluate  $\zeta_1$  once the inflation model and the reheating are specified.

In a similar manner, straightforward calculations yield  $\zeta_2$  at any e-fold  $N$  during the radiation era. For this, it is convenient to split the integral over the three domains, inflation for  $N_* < N < N_{\text{end}}$ , reheating for  $N_{\text{end}} < N < N_{\text{reh}}$  and radiation era with  $N > N_{\text{reh}}$ :

$$\zeta_2(N) = \zeta_2^{(\text{inf})} + \zeta_2^{(\text{reh})} + \zeta_2^{(\text{rad})}(N). \quad (2.10)$$

Again assuming slow-roll during inflation, and a constant equation of state during reheating, one gets, at first order in the Hubble flow functions:

$$\begin{aligned} \zeta_2^{(\text{inf})} = & -\frac{1}{R_{\text{rad}}^4} \frac{\rho_{B_0}}{\tilde{\rho}_{\gamma_0}} \frac{3\beta}{(3+\gamma)(3-\epsilon_{1\text{end}})} \int_{\phi_*}^{\phi_{\text{end}}} \frac{V_{\text{end}}}{V_{,\phi}} \left[ \left( 1 - \frac{6+\gamma}{9+3\gamma} \epsilon_1 \right) e^{\gamma\Delta N(\phi)} \right. \\ & \left. - \sqrt{\frac{V}{V_*}} \left( 1 - \frac{1}{6} \epsilon_1 \right) e^{-3\Delta N(\phi)} e^{(3+\gamma)\Delta N_*} \right] d\phi, \end{aligned} \quad (2.11)$$

$$\begin{aligned} \zeta_2^{(\text{reh})} = & \frac{2\beta}{R_{\text{rad}}^4} \frac{\rho_{B_0}}{\tilde{\rho}_{\gamma_0}} \left\{ -\frac{R_{\text{rad}}^4 - 1}{1 - 9w_{\text{reh}}^2} + \frac{(R_{\text{rad}}^2)^{\frac{3w_{\text{reh}}-3}{3w_{\text{reh}}-1}} - 1}{(3+\gamma)(3w_{\text{reh}}-3)} \right. \\ & \left. \times \left[ 1 - \frac{\epsilon_{1\text{end}}}{3+\gamma} - \frac{6+2\gamma}{3w_{\text{reh}}+1} - \sqrt{\frac{3V_{\text{end}}}{(3-\epsilon_{1\text{end}})V_*}} e^{(3+\gamma)\Delta N_*} \right] \right\}, \end{aligned} \quad (2.12)$$

$$\begin{aligned} \zeta_2^{(\text{rad})}(N) = & \frac{\beta}{R_{\text{rad}}^4} \frac{\rho_{B_0}}{\tilde{\rho}_{\gamma_0}} \left\{ R_{\text{rad}}^4 (N - N_{\text{reh}}) + \frac{1 - 3w_{\text{reh}}}{1 + 3w_{\text{reh}}} R_{\text{rad}}^4 \right. \\ & \left. + \frac{(R_{\text{rad}}^2)^{\frac{3w_{\text{reh}}-3}{3w_{\text{reh}}-1}}}{3+\gamma} \left[ 1 - \frac{\epsilon_{1\text{end}}}{3+\gamma} - \frac{6+2\gamma}{3w_{\text{reh}}+1} - \sqrt{\frac{3V_{\text{end}}}{(3-\epsilon_{1\text{end}})V_*}} e^{(3+\gamma)\Delta N_*} \right] \right\}. \end{aligned} \quad (2.13)$$

As before,  $\epsilon_1(\phi)$  stands for the first Hubble flow function evaluated along the field trajectory and  $\epsilon_{1\text{end}} = \epsilon_1(\phi_{\text{end}})$ . The quantity  $\Delta N_* \equiv N_* - N_{\text{end}} < 0$  is the number of e-folds before the end of inflation at which the pivot scale  $k_*$  crossed the Hubble radius during inflation. It does not depend on  $\phi$  in the previous equations but, as discussed below, it is an inflationary model-dependent function of the reheating parameter  $R_{\text{rad}}$  (see Ref. [39]). The same remark holds for  $V_* \equiv V(\phi_*)$ . Finally, when relevant, we have dropped all terms involving  $\epsilon_{1*}$  as we always have  $\epsilon_{1*} \ll 1$  for all inflationary models.

Let us stress that the quantity  $\zeta_2^{(\text{rad})}(N)$  grows during the radiation era, as  $N - N_{\text{reh}}$ . However, Eq. (2.13) only makes sense if the perturbation mode under consideration is super-Hubble. As a result, an upper bound for  $N - N_{\text{reh}}$  is given by the number of e-fold *after* the end of reheating at which the pivot scale  $k_*$  re-enters the Hubble radius.

Finally, let us notice that the expressions for  $\zeta_1$ ,  $\zeta_2^{(\text{inf})}$ ,  $\zeta_2^{(\text{reh})}$  and  $\zeta_2^{(\text{rad})}$  are all proportional to the factor  $\rho_{B_{\text{end}}}/\rho_{\text{end}} = R_{\text{rad}}^{-4} \rho_{B_0}/\tilde{\rho}_{\gamma_0}$ . As shown in Ref. [16], imposing this factor to be smaller than unity is equivalent to avoid magnetic field backreaction over the background energy density during reheating. As here we are requiring that  $|\zeta| < 10^{-5}$ , our constraints are expected to be typically, and at least, 2.5 order of magnitude stronger than those derived from the background evolution (see Fig. 1).

## 2.2 Reheating consistent slow-roll

As already mentioned, both  $\zeta_1$  and  $\zeta_2$  depends on parameters of the inflationary model as well as the reheating energy scale. In particular, one needs to determine the value of all “\*”

quantities, *i.e.* evaluated  $\Delta N_*$  e-folds before the end of inflation. These parameters cannot be freely chosen as one wants them to be compatible with the observed amplitude of the CMB fluctuations. In fact, as shown in Ref. [36],  $\Delta N_*$  is itself a function of  $R_{\text{rad}}$ . In order to ensure consistency with reheating, we here follow the slow-roll approach of Ref. [39] that we briefly recap.

The e-fold  $N_*$  is by definition solution of  $k_*/a(N_*) = H(N_*)$  during inflation. This equation can be recast in terms of “observable” quantities, namely

$$\frac{k_*}{a_0}(1+z_{\text{end}})e^{-\Delta N_*} = H_* . \quad (2.14)$$

The right hand side of this equation can be fixed by the amplitude of the adiabatic primordial power spectrum, which is a well measured quantity. At leading order in slow-roll

$$P_* = \frac{H_*^2}{8\pi^2\epsilon_{1*}M_{Pl}^2} . \quad (2.15)$$

From Eqs. (2.5) and (2.6),  $z_{\text{end}}$  can be expressed in terms of  $R_{\text{rad}}$  and  $\rho_{\text{end}}$ , the energy density at the end of inflation. The latter can, in turn, be further simplified using the Friedmann–Lemaître equations for a scalar field (in Planck units)

$$\rho_{\text{end}} = \frac{3V_{\text{end}}}{3-\epsilon_{1\text{end}}} = \frac{V_{\text{end}}}{V_*} \frac{3V_*}{3-\epsilon_{1\text{end}}} = 3H_*^2 \frac{V_{\text{end}}}{V_*} \frac{3-\epsilon_{1*}}{3-\epsilon_{1\text{end}}} . \quad (2.16)$$

As before, one can drop the term in  $\epsilon_{1*} \ll 3$ . The advantage of this last expression is that it does no longer depend on the potential normalization but involves only  $H_*$ . Plugging everything back into Eq. (2.14),  $\Delta N_*$  is a solution of the algebraic equation [36]

$$\Delta N_* = -\ln R_{\text{rad}} + N_0 + \frac{1}{4} \ln \left[ \frac{9}{\epsilon_{1*}(3-\epsilon_{1\text{end}})} \frac{V_{\text{end}}}{V_*} \right] - \frac{1}{4} \ln(8\pi^2 P_*) . \quad (2.17)$$

Here the quantity  $|N_0|$  roughly measures the number of e-folds of deceleration and is defined in Planck units by

$$N_0 \equiv \ln \left[ \frac{k_*/a_0}{(3\mathcal{Q}_{\text{reh}}\Omega_\gamma^0 H_0^2)^{1/4}} \right] . \quad (2.18)$$

Notice that the trajectory is needed to evaluate the right hand side of Eq. (2.17) as  $V_*$  and  $\epsilon_{1*}$  are functions of  $\phi(N_*)$ . One can nevertheless render this equation more explicit in terms of  $\phi_*$  and  $\rho_{\text{reh}}$  by assuming slow-roll and expanding  $R_{\text{rad}}$  from Eqs. (2.5) and (2.16), *i.e.*

$$\ln R_{\text{rad}} = \frac{1-3w_{\text{reh}}}{3+3w_{\text{reh}}} \ln(\rho_{\text{reh}}^{1/4}) - \frac{1-3w_{\text{reh}}}{12(1+w_{\text{reh}})} \ln \left[ \frac{9\epsilon_{1*}}{3-\epsilon_{1\text{end}}} \frac{V_{\text{end}}}{V_*} \right] - \frac{1-3w_{\text{reh}}}{12(1+w_{\text{reh}})} \ln(8\pi^2 P_*) . \quad (2.19)$$

One finally gets

$$\begin{aligned} \Delta N_* = \int_{\phi_*}^{\phi_{\text{end}}} \frac{V}{V_\varphi} d\varphi &= -\frac{1-3w_{\text{reh}}}{3+3w_{\text{reh}}} \ln(\rho_{\text{reh}}^{1/4}) + N_0 - \frac{1+3w_{\text{reh}}}{2(3+3w_{\text{reh}})} \ln(8\pi^2 P_*) \\ &+ \frac{1}{3+3w_{\text{reh}}} \ln \left[ \frac{9}{(\epsilon_{1*})^{\frac{3w_{\text{reh}}+1}{2}}} \frac{V_{\text{end}}}{(3-\epsilon_{1\text{end}})V_*} \right] . \end{aligned} \quad (2.20)$$

Once the slow-roll trajectory is integrated, this expression is explicit in  $\phi_*$  and can be solved by specifying only  $E_{\text{reh}} \equiv \rho_{\text{reh}}^{1/4}$  and  $w_{\text{reh}}$ . Plugging the result into Eqs. (2.11), (2.12), (2.13), (2.19), (2.20) and finally Eq. (2.7) allows to determine  $\zeta(N_*)/\rho_{B_0}$  uniquely from the input of  $E_{\text{reh}}$  and  $w_{\text{reh}}$ . Imposing that  $|\zeta(N_*)|^2 \ll P_*$  actually yields the reheating-dependent upper bound on  $\rho_{B_0}$ .

### 3 Application to some representative models

#### 3.1 Large field models

The potential energy for the large field models is given by

$$V(\phi) = M^4 \phi^p, \quad (3.1)$$

where  $M$  is a constant of mass dimension (in Planck unit) and  $p$  is a positive number. In order to be definite, we assume that this potential is correct not only for large  $\phi$  responsible for inflation but also for small  $\phi$  relevant for oscillating period and reheating. For this potential, the field value at which inflation terminates is given by

$$\phi_{\text{end}} = \frac{p}{\sqrt{2}}, \quad (3.2)$$

the solution of  $\epsilon_1(\phi_{\text{end}}) = 1$  where

$$\epsilon_1(\phi) \simeq \frac{p^2}{2\phi^2}. \quad (3.3)$$

The slow-roll evolution of  $\phi$  during inflation is given by integrating Eq. (2.8). In Planck units, one gets

$$\Delta N = \frac{1}{2p} (\phi_{\text{end}}^2 - \phi^2), \quad (3.4)$$

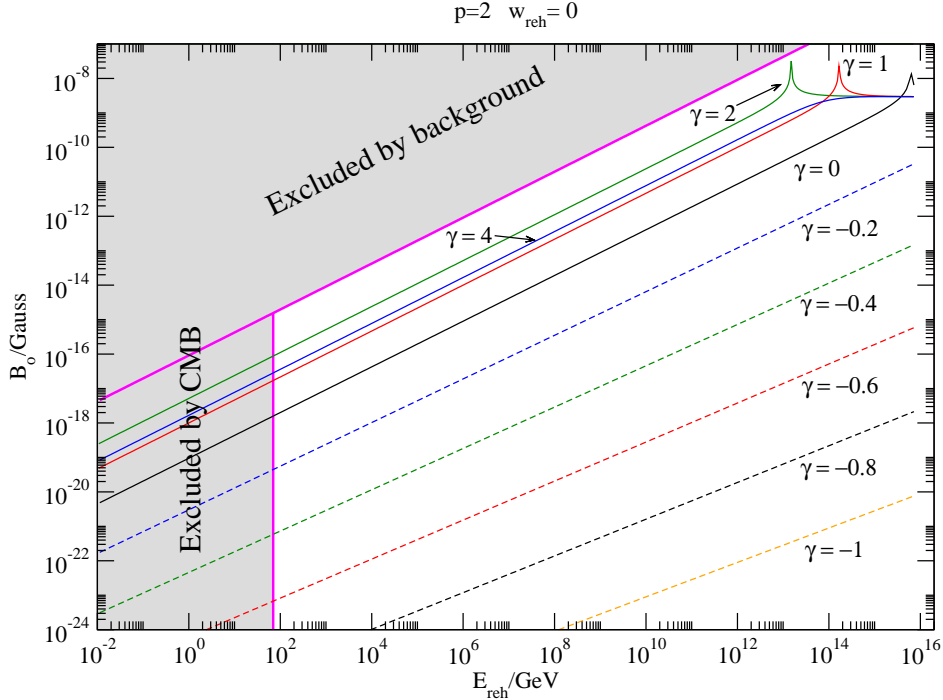
where, as before  $\Delta N = N - N_{\text{end}}$ , is the number of e-fold measured from the end of inflation. After inflation,  $\phi$  oscillates around the minimum such that the natural equation of state parameter is given by  $w_{\text{reh}} = (p - 2)/(p + 2)$  [42].

Plugging Eqs. (3.1), (3.2), (3.3) and (3.4) into Eq. (2.7) and Eqs. (2.11) to (2.13) gives the expression of  $\zeta(N_*)/\rho_{B_0}$  in terms of the potential parameter  $p$ ,  $R_{\text{rad}}$  and  $\phi_*$  only (plus the magnetic parameters). What remains to do is to solve Eq. (2.20) to get  $\phi_*$  in terms of  $p$  and  $\rho_{\text{reh}}$ , from which  $R_{\text{rad}}$  is determined by Eq. (2.19).

For the large field model, an analytic solution of Eq. (2.20) can be found in terms of the Lambert function [39], but as the various integrations entering into the expression of  $\zeta(N_*)$  that can only be performed numerically, we have here preferred to solve this equation numerically. The various cosmological parameters have been set to their preferred values from the WMAP data [43–45], *i.e.*  $P_* \simeq 2.16 \times 10^{-9}$ ,  $h \simeq 0.72$ ,  $\Omega_\gamma^0 = 4.6 \times 10^{-5}$ , and we have set all  $\mathcal{Q}$  to unity for simplicity (they have only a small effect). With a pivot scale chosen at  $k_* = 0.05 \text{ Mpc}^{-1}$ , one has  $N_0 \simeq -62$ . The final result is a parametric curve  $B_0 = \mathcal{B}_{\alpha,\beta,\gamma,p}(E_{\text{reh}}, w_{\text{reh}})$ , solution of

$$\zeta(N_*) = \sqrt{P_*}, \quad (3.5)$$

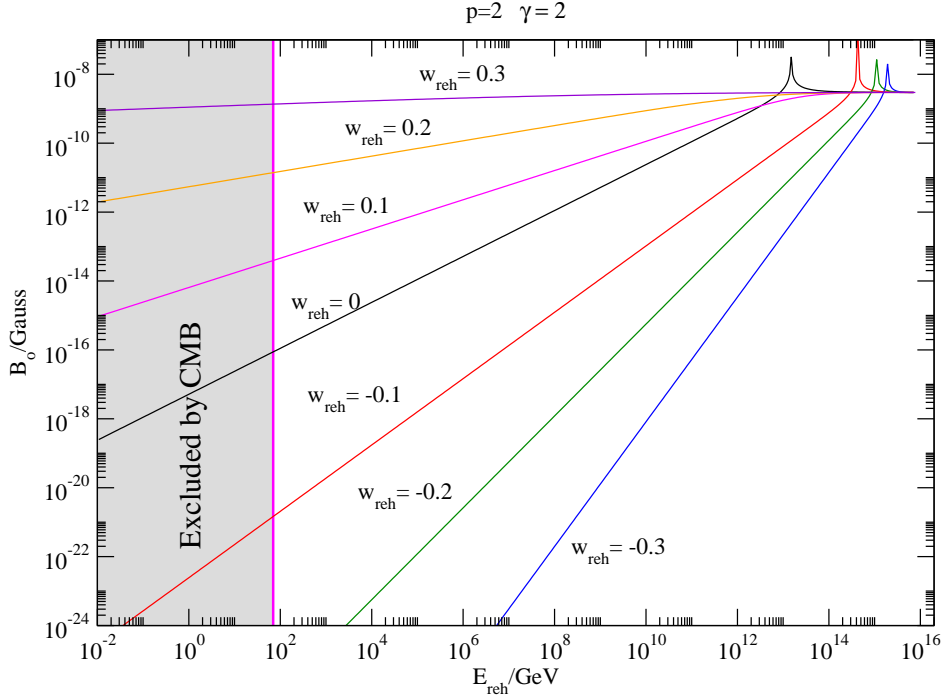
which separates the plane  $(E_{\text{reh}}, B_0)$  in two regions. Above this curve, the combination of the magnetic field value today and the energy scale of reheating would be such that the



**Figure 1.** Limits on the amplitude of the present magnetic field of inflationary origin with respect to the reheating energy scale. We have assumed a large field model with  $p = 2$  (and generic values for  $\alpha = 4/3$ ,  $\beta = 1$ ). Each line is the solution of Eq. (3.5) for various values of  $\gamma$ . The region above the corresponding line is excluded. The reheating energy scale  $E_{\text{reh}} \equiv \rho_{\text{reh}}^{1/4}$  varies from 100 MeV (BBN energy scale) to  $\rho_{\text{end}}^{1/4}$ , the energy at which inflation ends, see Eq. (2.16). For convenience, the CMB lower bound for the large field with  $p = 2$  reheating energy has been reported  $E_{\text{reh}} > 70$  GeV (95%), see Ref. [38]. We have also represented the region excluded by magnetic field backreaction over the background energy density during reheating, see Ref. [16].

magnetically generated super-Hubble curvature perturbation would be equal or larger than the adiabatic modes generated during inflation. The allowed region therefore lies under this curve. Let us notice that this is a very conservative upper limit as current constraints on isocurvature modes show that they cannot exceed 10% of the adiabatic counterparts [46, 47].

In Fig. 1, we have plotted these limits for the large field model with  $p = 2$  and for various values of  $\gamma$  (taking  $\alpha = 4/3$  and  $\beta = 1$  as reference values of  $\mathcal{O}(1)$  parameters). Negative values of  $\gamma$  provides the tightest bounds on the magnetic field. This is expected since in that situation the magnetic contribution increases more and more for  $a \rightarrow 0$ , *i.e.* deep during inflation. As it is already known, standard values for  $\gamma = -4$  during inflation would even generate a strong backreaction problem [11, 12, 15]. Here, even for  $\gamma = -1$ , curvature perturbations generated by the magnetic fields become important during inflation and  $\zeta(N_*) \simeq \zeta_1 + \zeta_2^{(\text{inf})}$ . We see on the figure that, for  $\gamma = -1$ , one cannot actually generate a magnetic field today larger than  $B_0 = 10^{-20}$  G. In the opposite situation,  $\gamma > 0$ , deep during inflation the magnetic effects are very small and  $\zeta(N_*) \simeq \zeta_2^{(\text{reh})} + \zeta_2^{(\text{rad})}(N_*)$  is mostly generated after inflation. Remembering that after inflation  $\rho_B \propto a^{-4}$ , at fixed  $B_0$ , a longer period of reheating, *i.e.* a lower reheating temperature, results in higher magnetic field values at the end of inflation; and hence a larger  $\zeta$ . As for the background case discussed in



**Figure 2.** Limits on the amplitude of the present magnetic field of inflationary origin with respect to the reheating energy scale for a large field model  $p = 2$ ,  $\gamma = 2$  and for various values of  $w_{\text{reh}}$ .

Ref. [16], this effect is enhanced if the energy density of the universe decreases more slowly than radiation, as this is the case for  $w_{\text{reh}} = 0$  here. As can be checked in Eqs. (2.12) and (2.13), the  $\zeta$ -dependency of these two terms with respect to  $\gamma$  is very weak (as opposed to  $\zeta_2^{(\text{inf})}$ ). In Fig. 1, one indeed sees that the bounds become almost insensitive to  $\gamma$  as soon as  $\gamma \gtrsim 2$  such that this limit is actually very conservative and somehow model-independent. At around  $E_{\text{reh}} = 10^{14}$  GeV, a spiky bump is observed for  $\gamma = 1$  curve (and similar one for  $\gamma = 0$  curve at higher value of  $E_{\text{reh}}$ ). This is due to the occasional vanishing of  $\zeta$ , which is possible since  $\zeta$  is a function of  $R_{\text{rad}}$  and hence of  $E_{\text{reh}}$ . At that point,  $\zeta$  vanishes irrespective of the amplitude of the magnetic field and, as a result, we have no constraint on the magnetic field, which shows up as the bump.

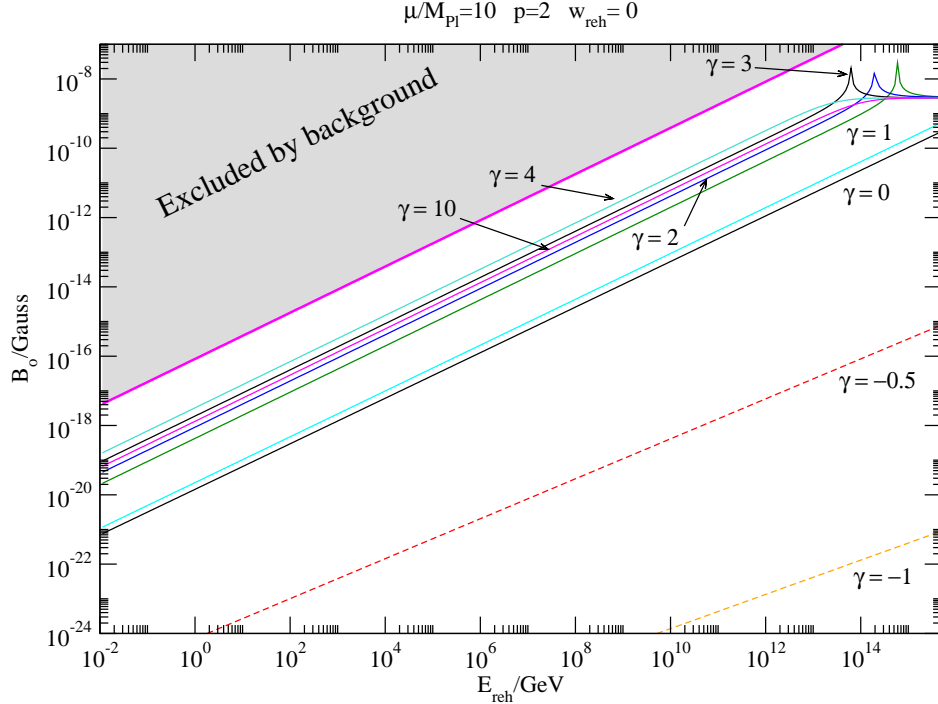
In Fig. 2, we have plotted the magneto-reheating constraints, still for a large field model with  $p = 2$ , but assuming another equation of state parameter  $w_{\text{reh}}$ . As expected, the more  $w_{\text{reh}}$  becomes negative, the tighter the limits are whereas there is no constraints for a radiation-like reheating era.

### 3.2 Small field models

Let us next consider the small field models for which the potential is given by

$$V(\phi) = M^4 \left[ 1 - \left( \frac{\phi}{\mu} \right)^p \right], \quad (3.6)$$

where  $M$  and  $\mu$  are constants of mass dimension in Planck unit and  $p$  is a positive number. Notice that this potential is valid only for the inflationary regime  $\phi/\mu \ll 1$  and has to be



**Figure 3.** Limits on the amplitude of the present magnetic field of inflationary origin assuming the small field models with  $p = 2$  and  $\mu = 10 M_{\text{Pl}}$ . For each labelled value of  $\gamma$ , the region above the corresponding line is excluded. These bounds are almost unchanged for larger values of  $\mu$  (see text).

replaced by another function after the end of inflation. If this is a quadratic function, one can assume  $w_{\text{reh}} \simeq 0$ . For this potential, normalizing the field value by  $\mu$  as  $\chi = \phi/\mu$ , the slow-roll trajectory reads (in Planck units)

$$\Delta N = -\frac{\mu^2}{2p} \left[ \left( \chi^2 + \frac{2}{2-p} \chi^{2-p} \right) - \left( \chi_{\text{end}}^2 + \frac{2}{2-p} \chi_{\text{end}}^{2-p} \right) \right], \quad (3.7)$$

where  $\chi_{\text{end}}$  is determined by  $\epsilon_1(\chi_{\text{end}}) = 1$ . From

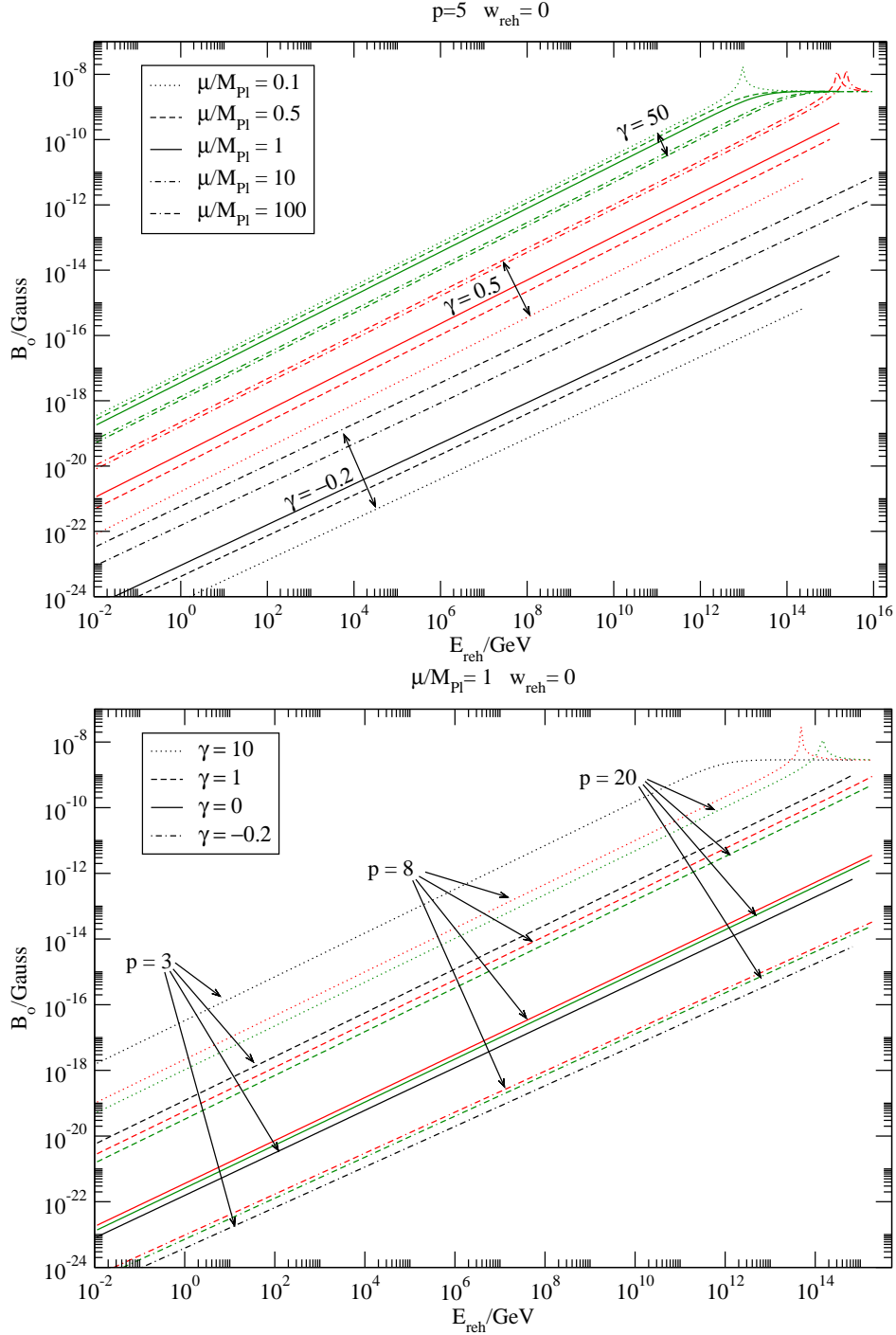
$$\epsilon_1(\chi) = \frac{1}{2} \left( \frac{p}{\mu} \frac{\chi^{p-1}}{1 - \chi^p} \right)^2, \quad (3.8)$$

one finds a transcendental equation for  $\chi_{\text{end}}$ :

$$\chi_{\text{end}}^{p-1} = \sqrt{2} \frac{\mu}{p} (1 - \chi_{\text{end}}^p). \quad (3.9)$$

Equation (3.7) can also be applied to the case  $p = 2$  by taking the limit  $p \rightarrow 2$ , which yields terms in  $\ln(\chi/\chi_{\text{end}})$ . Along the lines detailed for the large field models, these expressions are enough to completely determine the upper bound  $B_0 = \mathcal{B}(E_{\text{reh}}, w_{\text{reh}})$  parametrized by  $\mu$  and  $p$  (see Sect. 3.1).

In Fig. 3, we have plotted the limits on the amplitude of the present magnetic field of inflationary origin as a function of the reheating temperature assuming small field models with  $p = 2$  and  $\mu = 10 M_{\text{Pl}}$ . Notice that for  $w_{\text{reh}} = 0$ , there is currently no bounds on the



**Figure 4.** This panel shows the limit on the amplitude of the present magnetic field of inflationary origin assuming the small field models with various values of  $p$  and at fixed  $\mu = M_{\text{Pl}}$ . As before, the region above the lines is excluded and  $\alpha = \frac{4}{3}$ ,  $\beta = 1$ .

reheating energy scale coming from CMB alone [38]. Let us also recap that the models having  $p = 2$  are compatible with the current spectral index and tensor-to-scalar ratio only for large

values of  $\mu$ , typically  $\mu \gg M_{\text{Pl}}$  [36]. The bounds of Fig. 3 exhibits the same behaviour as the large field models, *i.e.* tighter bounds on the magnetic field for lower reheating temperatures and for lower values of  $\gamma$ . We also find that those results remain insensitive to larger values of  $\mu$ . This is not surprising since, for the small field models, the first two-Hubble flow functions become independent of  $\mu$  in the large  $\mu$ -limit [38]. Moreover, the above equations imply that  $1 - \chi_{\text{end}} = \mathcal{O}(M_{\text{Pl}}/\mu)$  and  $\chi_{\text{end}} - \chi_* = \mathcal{O}(M_{\text{Pl}}/\mu)$ . Therefore the integral in Eq. (2.7) scales as  $M_{\text{Pl}}^3/\mu^3$  and hence  $\zeta_1$  becomes also independent of  $\mu$  in the large  $\mu$ -limit.

In the top panel of Fig. 4, the limits on the amplitude of the magnetic field have been derived for the case  $p = 5$ , and we have made both  $\gamma$  and  $\mu$  vary. Contrary to the case  $p = 2$ , sub-Planckian values for  $\mu$  can be made compatible with CMB data [38]. The magneto-reheating bounds become generically stronger at small  $\mu$ ; but such a behaviour can be transiently inverted for some very large values of  $\gamma \gg 1$ . For instance, this is the case for  $\gamma = 50$ , but for values of  $\mu < 10^{-2}$  the upper bound moves again downwards in Fig. 4 (not represented). As before, for  $\gamma < 0$  the bounds are driven by the behaviour of  $\zeta_1$  because the magnetic effects are dominant deep in inflation. For  $\gamma > 0$ , magnetic effects during reheating are the most important, and  $\zeta_2^{(\text{reh})}$  is the dominant term such that the  $\mu$ -dependence ends up being related to the values of  $\gamma$ . In the limit  $\mu/M_{\text{Pl}} \ll 1$ , one can nevertheless use some crude approximations to guess the dependency in  $\mu$ . One has  $\chi_* \simeq [p(p-2)\mu^2/|\Delta N_*|]^{1/(p-2)}$  such that the dominant terms in Eq. (2.7) scale as (in Planck units)

$$\int_{\chi_*}^{\chi_{\text{end}}} \frac{\mu^4}{p^3} \chi^{3(1-p)} \exp\left[-\gamma \frac{\mu^2}{p(p-2)} \chi^{2-p}\right] d\chi \propto \mu^{-2p/(p-2)}. \quad (3.10)$$

Similarly, one can use Eq. (2.19) to get  $R_{\text{rad}}^4 \propto \epsilon_{1*}^{(3w_{\text{reh}}-1)/(3+3w_{\text{reh}})}$ . In the limit  $\mu \ll 1$ , Eq. (3.8) shows that  $\epsilon_{1*} \propto \mu^{2p/(p-2)}$  and, for  $w_{\text{reh}} = 0$ , one finally gets that  $\zeta_1 \propto \epsilon_{1*}^{-2/3}$ . As expected, this quantity increases as  $\mu$  decreases. For  $\gamma > 0$ , the behaviour of  $\zeta_2$  is now driven by powers of  $R_{\text{rad}}$ , which again increases when  $\mu$  decreases, albeit in a different way.

In the bottom panel of Fig. 4, we have fixed  $\mu = M_{\text{Pl}}$  and plotted the upper limits for  $p = 3$ ,  $p = 8$  and  $p = 20$ . Since  $p$  only appears in the equation as an order unity factor, the final dependency in  $p$  remains weak. Notice again the change of behaviour between large and low values of  $\gamma$  that can, as for  $\mu$ , be traced back to which part of  $\zeta$  contributes the most.

## 4 Conclusion

Recent observations of the cosmic rays reveal that the magnetic fields are ubiquitous in the universe. Lack of the convincing astrophysical explanation for the origin of such magnetic fields has led some theorists to seriously consider the possibility that those magnetic fields are produced during the primordial inflation. In addition to the obvious condition on the model of the inflationary magnetogenesis that it must produce the observed amplitude of the magnetic field at the relevant scales, there is another condition that must be taken into account for whatever the model of the magnetogenesis is. Since the inhomogeneous magnetic fields enter the right hand side of the Einstein equation as perturbations of the energy-momentum tensor, they induce the metric perturbation in addition to the standard adiabatic one produced by the inflaton fluctuation. The condition must be satisfied that this additional metric perturbation should not exceed the observed one in order for any inflationary magnetogenesis model to be embedded consistently in the standard model of the early universe.

In this paper, we first derived general expression of the resulting super-horizon scale curvature perturbation sourced by the primordial magnetic field, without resorting to the specific model of inflationary magnetogenesis but with a phenomenological assumption that the magnetic field energy density and the associated anisotropic stress scale as  $a^\gamma$ , where  $\gamma$  is a free parameter, so that our result provides a wide coverage for any magnetogenesis model satisfying the above scaling behavior. Our result can be also applied to any canonical single field inflation model followed by the oscillations of the inflaton with any equation of state and by reheating with any reheating energy scale. Given the inflation model and the measurement of the curvature power spectrum at the pivot scale, our formula allows us to put bound on the combination of the amplitude of the today's magnetic field, reheating energy scale and its equation of state parameter. In practice, our formula requires numerical solution of the algebraic equation to find the inflaton field value corresponding to the Hubble crossing of the pivot scale and one dimensional numerical integration for evaluating the curvature perturbation, both of which are quite feasible to implement. Our perturbation bound is tighter by a few orders of magnitude than the one given in the literature, which is derived from the requirement that the magnetic field energy density be smaller than the background energy density so that it does not destroy the homogeneous and isotropic Friedmann–Lemaître metric.

We then applied our formula to the large field inflation models and the small field inflation models, which are representative models of inflation. In either case, the upper bound on the value of the magnetic field today becomes tighter for negative value of  $\gamma$ , where the magnetic curvature perturbation is dominantly produced deep inside inflation. In particular, the magnetic field strength of  $10^{-15}$  G, which has the observational relevance, with  $\gamma < -1$  is completely incompatible with the perturbation bound. On the other hand, the upper bound is insensitive for positive  $\gamma$  and saturates at  $\gamma = \mathcal{O}(1)$ . Thus we can interpret the upper bound for  $\gamma = \mathcal{O}(1)$  as the fairly conservative bound. With  $\gamma$  being fixed, the upper bound on the magnetic field becomes severer as the reheating energy scale becomes lower if the reheating equation of state parameter is less than  $1/3$ . This is expected since the magnetic field energy density contributes more to the total energy density as we go back into the past. For example, for the chaotic inflation model with a quadratic potential, the magnetic field today must be less than  $10^{17}$  G if the reheating energy scale is as low as 70 GeV that is the possible minimal energy scale allowed by WMAP 7yr data. If the cosmic magnetic fields actually turn out to be of the inflationary origin in the future and search for the realistic model of the inflationary magnetogenesis becomes an indispensable pillar in constructing a realistic evolutionary scenario of the early universe, our perturbation bound derived in this paper provides one of the necessary conditions that must be considered for constraining the magnetogenesis models.

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