

AN ENTROPY MEASURE OF DIVERSIFICATION

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While there exist various descriptive measures of horizontal and vertical concentration, and an endless discussion of their respective merits, there has been, till recently, a lack of comparable reflections and proposals in the measurement of conglomerate concentration or diversification.

Two aspects are usually mentionned as being important determinants of the degree of diversification : on the one hand, the number of industries in which a firm is active, and on the other hand, the proportion of its activity which is not exercised in its primary industry.

Some authors have tried to combine these two complementary elements as in the following expression :

$$D = \left(\frac{B}{A + B} \right) n,$$

where A is the firm's sales (or another size indicator) in its primary industry, B is the corresponding sales in the secondary industries and n is the number of industries represented within the firm's total sales. (1)

(1) See L. R. AMEY, Diversified Manufacturing Businesses, Journal of the Royal Statistical Society, Series A, vol. 27, Part II, 1964 ; M. GORT, Diversification and Integration in American Industry, National Bureau of Economic Research, Princeton, 1962.

In a suggestive article, Ch. BERRY has proposed a more refined index of diversification (2) :

$$D_H = 1 - \sum_{i=1}^n P_i^2$$

where P_i is the ratio of the firm's sales in the i th industry to the firm's total sales in its n industries. This application of the Herfindhal index of industrial concentration has all the properties of the index itself : for example, when a firm is equally active in each of several industries, $D_H = 1 - \frac{1}{n}$, where n is the number of industries in which the firm is active.

Yet, this index suffers from a difficulty related to a limitation of the Herfindhal index. The computed degree of diversification will greatly differ, according to the level of disaggregation of the Industriel Classification, i.e., 2-, 3- or 4- digit, and this for two reasons : the degree of disaggregation affects the proportion of a firm's activity in each industry or sector, as well as the number of industries figuring in the summation. For the 460 largest (Fortune) corporations in 1965, Ch. BERRY finds that at the 2- digit level, the diversification index equals .379 while at the 4- digit level it is .645. It then becomes necessary to compute the index at different levels of diversification but without the possibility of making weighted comparisons of the respective role of these different levels in the firm's diversification process. More specifically, the Herfindhal index cannot be decomposed into additive elements (3) and the contribution of each level of diversi-

(2) Ch. BERRY, Corporate Growth and Diversification, The Journal of Law and Economics, October 1971, p. 373.

(3) See H. THEIL, Economics and Information Theory, North-Holland, Amsterdam, 1967, Appendix to chapter 8.

fication to total diversification cannot be defined. "More appropriate would be measures distinguishing for each firm that 4- digit diversification which is within 2- digit industry groups from that which arises as a consequence of diversification accross new 2- digit categories".

(4)

This is the reason why we suggest an alternative measure, namely a direct (5) entropy measure of diversification :

$$D_E = \sum_{i=1}^n P_i \log \frac{1}{P_i}.$$

Clearly, this index takes the value 0 when a firm is active in a single industry and becomes $D_E = \log n$ when it is equally active in each of the n relevant industries. But the main interest of our proposed measure is that it can be decomposed into an intra - group component and an inter - group component. Consider a given firm which is active in n 4- digit industries. These industries could be aggregated in S 2- digit sectors. For a given sector s , the combined shares of the firm among the corresponding industries is given by :

$$P_s = \sum_{i \in s} P_i.$$

The diversification inside this sector becomes :

$$D_s = \sum_{i \in s} \frac{P_i}{P_s} \cdot \log \frac{P_s}{P_i},$$

(4) Ch. BERRY, op. cit., p. 381

(5) By contrast, entropy as an indicator of concentration is an inverse measure. To have a direct measure, it is necessary to take the reciprocal of the antilogarithm of entropy = $C_E = 1/\text{antilog } E = \prod_{i=1}^n P_i$ where the shares of individual firms are the weights in a weighted geometric mean. See Ch. MARFELS, Absolute and Relative Measures of Concentration Reconsidered, *Kyklos*, 1971, fasc.4.

where P_s = the ratio of the firm's sales in the s sector to the firm's total sales in S sectors.

The total diversification inside the S sectors weighted by the relative importance of these sectors is defined as :

$$\sum_{s=1}^S P_s \cdot D_s$$

In accord with the general formula of the disaggregated entropy measure (6), the measure of total diversification is written :

$$D_E = \sum_{s=1}^S P_s \log \frac{1}{P_s} + \sum_{s=1}^S P_s \cdot D_s$$

The first term of the expression is inter-sectorial diversification whereas the second term is intra-sectorial diversification.

Illustrative values of the entropy-type measure as well as of the Herfindhal-type are provided in the following table. (7) We assume that there are $n = 5$ industries and $S = 2$ two-digit sectors including respectively industries (1) and (2), and industries (3), (4) and (5).

(6) See for example A. JACQUEMIN and A. M. KUMPS, Changes in the Size Structure of the Largest European Firms : an Entropy Measure, The Journal of Industrial Economics, Nov. 1971, n° 1.

(7) We use the numbers chosen by BERRY, op.cit., in order to make the comparison.

Illustrated Values of the Herfindhal-Type (D_H) and the Entropy-Type (D_E) Indexes of Diversification								
Percentage of Firm's sales In Sector and Industry					D_H	D_E		
I (1)	(2)	II (3)	(4)	(5)		Total	Between Sectors	Within Sectors
100	-	-	-	-	0	0	0	0
95	5	-	-	-	.10	.1545	0	.1545
90	10	-	-	-	.18	.3249	0	.3249
80	10	10	-	-	.34	.8791	.3249	.5542
60	40	-	-	-	.48	.6727	0	.6727
60	10	10	10	10	.60	1.2257	.6093	.6164
50	20	20	10	-	.66	1.2189	.6093	.6096
40	20	20	10	10	.74	1.4695	.6727	.7968
30	20	20	20	10	.78	1.5569	.6931	.8638
20	20	20	20	20	.80	1.6091	.6727	.9364

A first aspect of the comparison between the two measures is that the use of the logarithm tempers the impact of the largest shares on the results, hence the effect of shifts in market shares within industries without a real increase in diversification.

A second aspect is that by taking into account the intensity of diversification, the D_E index may correct questionable results given by D_H . Two examples appear in the table, one with the "inter-sectorial" aspect and the other with the "intra-sectorial" dimension.

Comparing lines 4 and 5, the Herfindhal-Type index suggests that the first situation is less diversified than the second ; but the reality is

clearly the inverse as indicated by the entropy-type index. The different results stem from the fact that "inter-sectorial" diversification is taken into account by D_E , and not by D_H . (8) Correspondingly, if we go from line 6 to 7, D_H suggests an increase of diversification while in fact the firm has retreated from one industry ; D_E indicates a decrease of diversification, given the decrease in "intra-sectorial" diversification for a given level of "inter-sectorial" diversification. (9)

Hence, this index allows one to distinguish between the diversification corresponding to activities in unrelated areas of productive endeavor and the diversification among closely related products. It is therefore possible to determine, from a structural standpoint, the intensity of diversification : the relative corporate specialization within a group of industries may be contrasted with diversification to new industry groups where the entering firm is far a way from its initial area. A classification of firms according to the respective level of their-sectorial and intra-sectorial diversification is also possible. Over-time, it could be determined to what extent corporate growth is within or among broad sectors like 2-digit industry groups. Regression analysis could relate various performance indicators (growth, profitability, variances of growth and profitability, and so on) to the two additive

(8) However, if the grouping of industries was such that sector I includes industries (1), (2), and (3), the situation expressed by line 4 becomes less diversified than the following one, in terms of D_E as well as of D_H . Indeed D_E would be reduced to .6390 because of the absence of "intra-sectorial" diversification.

(9) This decrease is the net effect of two conflicting phenomena : first, the firm drops out of one industry, and secondly the repartition of shares in sector I is more balanced. The measure, correctly we think, gives more weight to the first effect. If we assume a situation such that in sector I, we have 60 and 10 at line 7 as well as at line 6, together with the changes indicated in the table in sector II, the "within sector" number would fall even more : from .6164 to .4777.

components of the total diversification measure and clearly state the explanatory power of each of them.

To summarize, the entropy measure of diversification may help the observer to disentangle the kinds of diversification that are in process and to determine their significance in terms of competition and entry.

x x
x