PUBLIC PENSIONS AND LTC INSURANCE WITH FAMILY SOLIDARITY

Y. Nishimura, Pierre Pestieau

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Public pensions and LTC insurance with family solidarity.*

Y. Nishimura†and P. Pestieau‡

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Abstract

As income rises, the risk of disability in old age declines, while life expectancy increases. These correlations strengthen the case for public long-term care (LTC) insurance over public pension systems. However, this perspective shifts when considering family solidarity—specifically, the informal care provided by spouses and children to elderly relatives. When viewed through the lens of altruistic caregiving motives, the argument for social LTC insurance becomes more nuanced. The interplay between formal and informal care is a key factor in shaping optimal policy. In this paper, we demonstrate that when family members reliably provide informal care, the design of a comprehensive public LTC system depends on the existence of a private insurance and on the degree of substitutability between informal and formal care.

Keywords: long-term care, mortality risk, disability risk, informal care

JEL classification: H2, H5.

1 Introduction

The provision of long-term care (LTC) has emerged as a critical challenge for modern welfare states. While the need for LTC has always existed, it was not traditionally recognized as a distinct social risk requiring dedicated policy attention. This paradigm has shifted dramatically in recent years, spawning a rich body of literature on social insurance for LTC. Two primary factors drive this increased focus: the rapid aging of populations, particularly among the very elderly who require intensive care, and the declining capacity of families to serve as primary caregivers for disabled elderly relatives. The theoretical foundation for LTC social insurance builds upon Rochet's seminal work[1], which

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†University of Osaka

‡CORE, U. de Louvain and HEC, U. de Liège

[1]Rochet (1991), Cremer and Pestieau, (2014)

demonstrates that social insurance becomes optimal when the risk being insured correlates negatively with income.

Empirical evidence has established that higher income and wealth correlate with lower disability risk in old age, while simultaneously predicting increased longevity. This inverse relationship actually presents a stronger argument for public LTC insurance compared to traditional pension systems. However, the calculus becomes more complex when considering family solidarity, particularly the informal care provided by spouses and children to elderly relatives. When viewed through the lens of altruistic caregiving motivations, the optimal design of social LTC insurance requires careful consideration of the relationship between formal and informal care provision. Our research yields three significant findings. First, the complementarity between formal and informal care strengthens the case for public LTC expenditure. Second, in situations where formal and informal care act as substitutes, family solidarity toward disabled elderly individuals positively influences public pension systems. Third, uncertainty regarding the reliability of informal care arrangements reinforces the necessity for social LTC coverage when substitutability exists between care types. These three findings are shown to hold as long as there is no actuarially fair private insurance. If such insurance exists for all, the role of the public LTC program is merely redistributive. In other words LTC private insurance has the effect of neutralizing the insurance role of public LTC program.

These conclusions rest on two key stylized facts. The first, documented by Lefebvre et al.(2018, 2025)[2], establishes the relationship between income, longevity, and disability risk, demonstrating positive correlation between income and longevity while confirming negative correlation between income and disability risk in old age. The second fact addresses the interaction between formal and informal care in managing age-related disability. Following Bonsang (2010) and Bolin et al. (2008), we employ a utility function for dependent individuals that incorporates both care types. However, the precise nature of their relationship—whether substitutive or complementary—remains an open empirical question, with preliminary evidence suggesting that complementarity may increase with disability severity.

This paper is structured as follows. Section 2 explores how informal care provided by altruistic children influences the optimal level of public LTC insurance. Section 3 examines the relative social desirability of public pensions versus public LTC insurance in the presence of informal care. Section 4 analyzes the impact of uncertainty surrounding informal care on public LTC insurance. Finally, Section 5 concludes.

2 No uncertain lifetime.

The basic model used involves parent-child families, where children's contributions to their elderly parents' care depend on their individual market productivity and their degree of altruism. Families differ in the levels of wages w_i and

[2]see also Connolly et al. (2025)

ω_i of the parent and the child and the disability risk π_i. Parent's lifetime utility can be written as:

$$U_i^p = u(w_i(1-\tau)l_i - s_i - I_i - v(l_i)) + (1-\pi_i)\, u(s_i) + \pi_i H\left(a_i, I_i/\pi_i + s_i + g\right)$$

where $H(a,m)$ is the LTC utility that depends on informal care provided by children, a_i, and formal cares m_i, financed by the insurance scheme I_i, saving s_i and public LTC benefit, g. LTC public benefits are financed by a payroll tax paid by the parents when young. We assume zero interest rate and no time preference. A word on the LTC utility function $H(a,m)$ is in order. In the theoretical literature, this utility has most often just an argument, which means that formal and informal care are perfect substitutes. Another specification consists in distinguishing between standard consumption c and health status h, in which case we have $H(c,h)$. This specification is often used in health economics. In this paper, we deliberately distinguish between formal and formal care and focus on whether these two types of care are substitutes or complements.

The timing of the game is the following:

1. The government chooses the tax rate and the benefit level that maximize the parents' welfare.

2. Parents choose their labor supply, their saving and the LTC insurance premium, if any .

3. Children help their parents in case of disability.

2.1 Child's problem

As standard, we proceed backward and thus look first at the child problem. The child's utility depends on whether the parent is dependent or not. It can be written as:

- in case of healthy parent: $U_i^c = u(d_i) = u(\omega_i)$
- in case of disabled parent: $U_i^c = u(d_i) + \beta H\left(a_i, m_i\right) = u\left(\omega_i\left(1-a_i\right)\right) + \beta H\left(a_i, m_i\right)$,

where β denotes the degree of altruism and $m_i = I_i/\pi_i + s_i + g$ denotes formal care.

When the parent is healthy, the child consumes his/her earning ω_i. In case of loss of autonomy, he/she devotes a fraction of time a_i to caring. The FOC that defines the optimal a_i is simply

$$\Delta = -u'(d_i)\omega_i + \beta H_1\left(a_i, m_i\right) = 0.$$

From this condition, we obtain a supply function for caring time: $a_i = a\left(m_i; \beta, \omega_i\right)$. Differentiating the above FOC we get:

$$\frac{\partial a_i}{\partial m_i} = \frac{\beta H_{12}}{-\Delta_{a_i}} \lesseqgtr 0 \Leftrightarrow H_{12} \lesseqgtr 0.$$

Note that in the separable case where $H_{12} = 0$, elder care would only depend on ω_i and β. In case of complementarity, elder care increases with m_i and in case of substitutability, we have some crowding out. We might be interested by the effect of ω_i on a_i. This can be obtained by differentiating the above FOC:

$$\frac{da_i}{d\omega_i} = \frac{u'(d_i)[R_R(d_i) - 1]}{-\Delta_{a_i}}$$

In other words, the effect of ω_i on a_i is positive or negative depending on whether the coefficient of relative risk aversion is higher or lower than 1. In this paper, we posit that the relative risk aversion coefficient as well as the distribution of children's wages are such that they have a small impact on informal care.

Throughout this paper, we assume that children care only for their disabled parents out of altruism, meaning they do not provide assistance when their parents are healthy. Additionally, we intentionally selected altruism as the primary motive for caregiving. An alternative explanation could be the influence of social norms, in which case informal care would remain independent of the variable m_i.[3]

2.2 Parent's problem

Before knowing his/her health status, the parent chooses l_i, s_i and I_i so as to maximize lifetime expected utility U_i^p subject to the caring function $a\left(m_i;\beta,\omega_i\right)$.

Parent's lifetime utility can be written as:

$$U_i^p = u(w_i(1-\tau)l_i - s_i - I_i - v(l_i)) + (1-\pi_i)\,u(s_i) + \pi_i H\left(a_i, I_i/\pi_i + s_i + g\right)$$

The FOCs are:

$$w_i(1-\tau) - v'(l_i) = 0$$

$$-u'(c_i) + (1-\pi_i)\,u'(s_i) + \pi_i H_2\left(a_i, m_i\right) + \pi_i H_1\left(a_i, m_i\right)\frac{\partial a_i}{\partial s_i} = 0$$

$$-u'(c_i) + H_2\left(a_i, m_i\right) + \pi_i H_1\left(a_i, m_i\right)\frac{\partial a_i}{\partial I_i} \leq 0.$$

where $\frac{\partial a_i}{\partial I_i} = \frac{\partial a_i}{\partial m_i}\frac{1}{\pi_i}$ and $\frac{\partial a_i}{\partial s_i} = \frac{\partial a_i}{\partial I_i} = \frac{\partial a_i}{\partial g} = \frac{\partial a_i}{\partial m_i}$. Note that granted that the Inada conditions apply to $u(d)$, the FOC with respect to s is aways interior.

In case of interior solution with $I_i > 0$,we have:

$$-u'(c_i) + H_2\left(a_i, m_i\right) + \pi_i H_1\left(a_i, m_i\right)\frac{\partial a_i}{\partial I_i} = 0; u'(c_i) = u'(s_i).$$

[3]Klimavciute et al. (2017)

Otherwise, when $I_i = 0$, we have

$$-u'(c_i) + H_2(a_i, m_i) + \pi_i H_1(a_i, m_i)\frac{\partial a_i}{\partial I_i} < 0.$$

We will consider those two cases.

2.3 Government's problem

2.3.1 Interior solution.

We now turn to the social welfare maximization when $I_i > 0$. We posit that the government is only concerned by the welfare of the elderly's generation.We write the following Lagrangian to be maximized:

$$\mathcal{L} = E\{u(w(1-\tau)l - s - I - v(l)) + (1-\pi)u(s) + \pi H(a(.), I/\pi + s + g)$$

$$+\mu[\tau wl - \pi g]\},$$

where $Ez = \sum n_i z_i$ and μ denotes the multiplier associated with the revenue constraint.

We obtain the FOCs:

$$\frac{\partial \mathcal{L}}{\partial g} = E\left[(-u'(c) + (1-\pi)u'(d) + \pi H_2)\frac{\partial s}{\partial g} + (-u'(c_i) + H_2)\frac{\partial I}{\partial g} + \pi H_2 + \pi H_1\frac{da}{dg} - \mu\pi\right] = 0$$

$$\frac{\partial \mathcal{L}}{\partial \tau} = -E\left[u'(c)wl - \mu\left(wl + \tau w\frac{\partial l}{\partial \tau}\right)\right] = 0.$$

Using the FOCs of the parent:

$$\frac{\partial \mathcal{L}}{\partial g} = E\left[-\pi H_1[\frac{\partial a}{\partial s}\frac{\partial s}{\partial g} + \frac{\partial a}{\partial I}\frac{\partial I}{\partial g}] + \pi H_2 + \pi H_1\frac{da}{dg} - \mu\pi\right] = 0$$

or

$$\frac{\partial \mathcal{L}}{\partial g} = E\left[\pi H_2 + \pi H_1\frac{\partial a}{\partial g} - \mu\pi\right] = 0$$

where we have used the equality: $\frac{da_i}{dg} = \frac{\partial a_i}{\partial m_i}\left[1 + \frac{\partial m_i}{\partial s_i}\frac{\partial s_i}{\partial g} + \frac{\partial m_i}{\partial I_i}\frac{\partial I_i}{\partial g}\frac{1}{\pi}\right]$.

Combining those two FOCs, we obtain the compensated effect of the tax denoted $\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial \tau} = \frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial \tau} + \frac{dg}{d\tau}\frac{\partial \mathcal{L}}{\partial g}$:

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial \tau} = -Eu'(c)wl + E\pi H_2\frac{Ewl}{E\pi} + E\pi H_1\frac{\partial a}{\partial m}\frac{Ewl}{E\pi} + \mu\tau Ew\frac{\partial l}{\partial \tau} = 0$$

This can be rewritten as:

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial \tau} = -Eu'(c)wl + E\pi H_2\frac{Ewl}{E\pi} + E\pi(u'(c_i) - H_2)\frac{Ewl}{E\pi} + \mu\tau Ew\frac{\partial l}{\partial \tau} = 0,$$

or

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial \tau} = -cov\left(u'(c), wl\right) + cov\left(u'(c), \pi\right)\frac{Ewl}{E\pi} + \mu\tau Ew\frac{\partial l}{\partial \tau} = 0.$$

And thus :

$$\tau = \frac{-cov\left(u'(c), wl\right) + cov\left(u'(c), \pi\right)\frac{Ewl}{E\pi}}{-\mu Ew\frac{\partial l}{\partial \tau}}$$

The first covariance is clearly negative. Given that it is generally assumed that the risk of disability is negatively correlated with income, the second covariance is positive. We thus have that in case of interior solutions, family solidarity does not affect public LTC insurance. In this case, private insurance neutralizes the insurance role of the public scheme.

2.3.2 LTC insurance unavailable

Let us now assume that for the various reasons that are discussed within the issue of the LTC insurance puzzle (see Pestieau and Ponthière (2012)[4], such insurance does not exist. The choice of the parent is thus reduced to l_i and s_i that are used to maximize lifetime expected utility U_i^p subject to the caring function $a\left(m_i; \beta, \omega_i\right)$.

The problem of the government is expressed by the following Lagrangian to be maximized:

$$\mathcal{L} = E\{u(w(1-\tau)l - s - v(l)) + (1-\pi)\,u(s) + \pi H\left(a\left(.\right), s+g\right)$$

$$+\mu\left[\tau wl - \pi g\right]$$

Interior solutions are obvious here and thus, we obtain the FOCs:

$$\frac{\partial \mathcal{L}}{\partial g} = E\left[\pi H_1\frac{\partial a}{\partial m} + \pi H_2 - \mu\pi\right] = 0$$

$$\frac{\partial \mathcal{L}}{\partial \tau} = -E\left[u'(c)wl - \mu\left(wl + \tau w\frac{\partial l}{\partial \tau}\right)\right] = 0.$$

Combining those two FOCs, we obtain the compensated effect of the tax

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial \tau} = -E\left[u'(c)wl - \pi H_2\frac{Ewl}{E\pi} - \pi H_1\frac{\partial a}{\partial m}\frac{Ewl}{E\pi} - \mu\tau w\frac{\partial l}{\partial \tau}\right] =$$

$$-cov\left(u'(c), wl\right) + cov\left(\pi, H_2\right)\frac{Ewl}{E\pi} + EwlE\left[H_2 - u'(c)\right] + E\pi H_1\frac{\partial a}{\partial m}\frac{Ewl}{E\pi} - \mu\tau Ew\frac{\partial l}{\partial \tau} = 0.$$

[4] See also Pestieau (2026) .

And thus :

$$\tau = \frac{-cov\left(u'(c), wl\right) + cov\left(\pi, H_2\right)\frac{Ewl}{E\pi} + EwlE\left[H_2 - u'(c)\right] + E\pi H_1 \frac{\partial a}{\partial m}\frac{Ewl}{E\pi}}{-\mu E w \frac{\partial l}{\partial \tau}}$$

The first covariance is clearly negative and the second is positive. The third term is expected to be positive. Indeed in the First Best, we have that $H_2 - u'(c) = 0$, which can be implemented with an actuarial fair private insurance. In the absence of such insurance, the level of m is suboptimal.As to the last term of the denominator, it is positive as long as $H_{12} > 0$. We thus have that family solidarity fosters public spending in case of complementarity between a and m.

Proposition 1

Complementarity (substitutability) between formal and informal care fosters (reduces) public LTC spending when private LTC insurance is not available.

3 Uncertain lifetime.

We now introduce another risk, namely the mortality risk that makes the length of life uncertain and calls for an annuitization of pensions and of savings.. We want to see the role of family solidarity in the choice of public pensions and LTC insurance.

3.1 Simple model.

The basic model used involves parent-child families, where children's contributions to their elderly parents' care depend on their individual market productivity and their degree of altruism. Families differ in the levels of wages w_i and ω_i of the parent and the child, the survival probability ϕ_i and the disability risk π_i. Parent's lifetime utility can be written as:

$$U_i^p = u(w_i(1-\tau)l_i - s_i - I_i - v(l_i)) + \phi_i\left(1-\pi_i\right)u(s_i/\phi_i + P)$$

$$+\phi_i \pi_i H\left(a_i, I_i/\pi_i + s_i/\phi_i + P + g\right),$$

where $H(a, m)$ is the LTC utility that depends on informal care provided by children, a_i, and formal cares m_i, financed by the insurance scheme I_i, annuitized saving s_i , public pensions P, and public LTC benefit, g. LTC public benefits and pensions are financed by a payroll tax paid by the parents when young.

The timing of the game is the following:

1. The government chooses the tax rate and the benefit levels that maximize the parents' welfare.

2. The parent chooses labor supply, annuitized saving and the LTC insurance premium

3. The child chooses to help his/her parent in case of disability.

3.1.1 Child's problem

As standard, we proceed backward and thus look first at the child problem. The child's utility depends on whether the parent is dependent or not. In case of disability, the dependent parent receives informal care and formal care financed by $m_i = I_i/\pi_i\phi_i + s_i/\phi_i + P + g$. As in the previous section we have that:

$$\frac{\partial a_i}{\partial m_i} \lesseqgtr 0 \Leftrightarrow H_{12} \lesseqgtr 0.$$

3.1.2 Parent's problem

Before knowing whether they survive the first period and if so, their health status, the parent chooses l_i, s_i and I_i so as to maximize their lifetime expected utility U_i^p subject to the caring function $a\left(m_i; \beta, \omega_i\right)$.

Parent's lifetime utility is maximized with respect to l_i, s_i, and I_i. We obtain the following FOCs :

$$w_i(1-\tau) - v'(l_i) = 0$$

$$-u'(c_i) + (1-\pi_i)\, u'(s_i/\phi_i) + \pi_i H_2\left(a_i, m_i\right) + \pi H_1\left(a_i, m_i\right) \frac{\partial a_i}{\partial m_i} = 0$$

$$-u'(c_i) + H_2\left(a_i, m_i\right) + H_1\left(a_i, m_i\right) \frac{\partial a_i}{\partial m_i} \leqslant 0.$$

In case $I_i > 0$. we have that $c_i = d_i$.

3.1.3 Government's problem

We now turn to the social welfare maximization. We write the following Lagrangian:

$$\mathcal{L} = E\{u(w(1-\tau)l - s - I - v(l)) + \phi\,(1-\pi)\, u(s/\phi + P) + \phi\pi H\left(a\left(.\right), m\right)$$

$$+\mu\left[\tau wl - \phi\pi g - \phi P\right]$$

The FOCs are:

$$\frac{\partial \mathcal{L}}{\partial g} = E[(-u'(c) + (1-\pi)\, u'(d) + \pi H_2)\frac{\partial s}{\partial g} + (-u'(c_i) + H_2)\frac{\partial I}{\partial g} + \phi\pi H_2 + \phi\pi H_1 \frac{da}{dg} - \mu\phi\pi] = 0$$

$$\frac{\partial \mathcal{L}}{\partial P} = E[(-u'(c) + (1-\pi)\, u'(d) + \pi H_2)\, \frac{\partial s}{\partial P} + (-u'(c_i) + H_2)\, \frac{\partial I}{\partial P}$$

$$+\phi\,(1-\pi)\, u'(d) + \phi\pi H_2 + \phi\pi H_1 \frac{da}{dP} - \mu\phi = 0].$$

$$\frac{\partial \mathcal{L}}{\partial \tau} = -E\left[u'(c)wl - \mu\left(wl + \tau w \frac{\partial l}{\partial \tau}\right)\right] = 0.$$

Using the FOCs of the parents, we can rewrite the first two conditions:

$$\frac{\partial \mathcal{L}}{\partial g} = E[-\phi\pi H_1 \frac{\partial a}{\partial s}\frac{\partial s}{\partial g} - \phi\pi H_1 \frac{\partial a}{\delta I}\frac{\partial I}{\partial g} + \phi\pi H_2 + \phi\pi H_1 \frac{da}{dg} - \mu\phi\pi] = 0$$

$$\frac{\partial \mathcal{L}}{\partial P} = E[-\phi\pi H_1 \frac{\partial a}{\partial s}\frac{\partial s}{\partial P} - \phi\pi H_1 \frac{\partial a}{\partial I}\frac{\partial I}{\partial P} + \phi\,(1-\pi)\, u'(d)$$

$$+\phi\pi H_2 + \phi\pi H_1 \frac{da}{dP} - \mu\phi] = 0$$

Using the fact that $\frac{\partial a}{\partial g} = \frac{\partial a}{\partial m}\left[1 + \frac{\partial s}{\partial g}\frac{1}{\phi} + \frac{\partial I}{\partial g}\frac{1}{\pi\phi}\right]$, this can be rewritten as:

$$\frac{\partial \mathcal{L}}{\partial g} = E\left[\phi\pi H_2 + \phi\pi H_1 \frac{\partial a}{\partial g} - \mu\phi\pi\right] = 0$$

$$\frac{\partial \mathcal{L}}{\partial P} = E\left[\phi\,(1-\pi)\, u'(d) + \phi\pi H_2 + \phi\pi H_1 \frac{\partial a}{\partial P} - \mu\phi\right] = 0$$

We want to know whether for given revenue, increasing P at the expense of g is socially desirable when family solidarity intervenes. We thus have to sign the following expression:

$$\frac{\partial \mathcal{L}^C}{\partial P} = \frac{\partial \mathcal{L}}{\partial P} + \frac{\partial \mathcal{L}}{\partial g}\frac{\partial dg}{dP} = E\left[\phi\,(1-\pi)\, u'(d) + \phi\pi H_2 - \phi\pi H_2 \frac{E\phi}{E\phi\pi} + \phi\pi H_1 \frac{\partial a^C}{\partial P}\right] \tag{1}$$

where $\frac{\partial a^C}{\partial P} = \frac{\partial a}{\partial P} - \frac{E\phi}{E\phi\pi}\frac{\partial a}{\partial g} = \frac{\partial a}{\partial m}\frac{E\phi(\pi-1)}{E\phi\pi} < 0$.

Expression (1) can be rewritten as:

$$\frac{\partial \mathcal{L}^C}{\partial P} = cov\,(\phi\,(1-\pi)\,, u'(d)) + E(\phi(1-\pi))E\left[u'(d) - \frac{\phi\pi}{E\phi\pi}\left(H_2 + H_1 \frac{\partial a}{\partial m}\right)\right] \tag{2}$$

As in the previous section, we distinguish between the case where LTC insurance is available or not. In the case $I > 0$, the Euler equation yields:

$$\left(H_2 + H_1 \frac{\partial a}{\partial m}\right)\frac{\phi\pi}{E\phi\pi} = u'(d)\frac{\phi\pi}{E\phi\pi}.$$

Thus, (1) can be rewritten as

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial P} = cov\left(\phi\left(1-\pi\right), u'(d)\right) + E(\phi(1-\pi))cov\left(u'(d), \frac{E\phi\pi - \phi\pi}{E\phi\pi}\right)$$

or

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial P} = cov\left(\phi\left(1-\pi\right), u'(d)\right) - \frac{E(\phi(1-\pi))}{E\phi\pi}cov\left(u'(d), \phi\pi\right)$$

The first covariance is positive and the second negative (Nishimura and Pestieau, 2022), which implies that priority should be given to LTC over pensions.

If, however, we have $I = 0$, these simplifications are not possible and we are back to equation (2). Then we have:

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial P} = cov\left(\phi\left(1-\pi\right), u'(d)\right) - cov\left(\phi\pi, u'(d)\right)\frac{E\phi\left(1-\pi\right)}{E\phi\pi}$$

$$+E\phi\left(1-\pi\right)E\left[\frac{\phi\pi}{E\phi\pi}\left(u'(d) - H_2\right)\right] - E\phi\left(1-\pi\right)E\frac{\phi\pi}{E\phi\pi}H\frac{\partial a}{\partial m}$$

The first covariance is negative and the second positive. Given that $H_2 - u'(c) > 0$, the third term is also negative. The sign of the fourth one depends on that of H_{12}.

Proposition 2

The negative effect on social welfare of a budget-balancing increase in public pension offset by a decrease in public LTC is not influenced by family solidarity when private LTC insurance is available. When it is not available, family solidarity strengthens (weakens) this effect in case of complementarity (substitutability).

4 Uncertain child altruism.

When disabled parents rely on family members for care and assistance, various life events can disrupt these support systems without warning. Let us cite the most notable of these disruptions. Death of caregivers creates an immediate and often permanent gap in support. When an adult child who provides daily assistance passes away, the disabled parent may be left without their primary caregiver and emotional support. Illness can affect caregivers limiting their ability to provide care. Migration for employment, education, or personal reasons can physically separate caregivers from those who need care. Economic hardship can reduce the caregiver's availability. Finally, family disputes and relationship breakdowns can sever caregiving arrangements.

These disruptions modify the design of public policy as we now see. The child's problem is unchanged. We thus turn to the parent's problem.

4.1 Parent's problem.

Let us denote the uniform probability of informal care by ζ. This means that with a probability $1-\zeta$, the disabled parent does not receive any informal care $(a = 0)$.

The lifetime utility of the parent facing a probability π of becoming disabled and a probability of getting aid from the child ζ is the following

$$U_i^p = u(w_i(1-\tau)l_i - s_i - I_i - v(l_i)) + (1-\pi)\,u(s_i)$$

$$+\pi_i\,(1-\zeta)\,H\,(0, I_i/\pi_i + s_i + g) + \pi_i\zeta H\,(a_i, I_i/\pi_i + s_i + g)$$

The FOCs are:

$$w_i(1-\tau) - v'(l_i) = 0$$

$$-u'(c_i) + (1-\pi_i)\,u'(s_i) + \pi_i\zeta\left[H_2\,(a_i, m_i) + H_1\,(a_i, m_i)\,\frac{\partial a_i}{\partial m_i}\right]$$

$$+\pi_i\,(1-\zeta)\,H_2\,(0, m_i) = 0$$

$$-u'(c_i) + \zeta\left[H_2\,(a_i, m_i) + H_1\,(a_i, m_i)\,\frac{\partial a_i}{\partial m_i}\right] + (1-\zeta)\,[H_2\,(0, m_i)] \leq 0.$$

4.2 Government's problem

The problem of the government is to maximize the following Lagrangian:

$$\mathcal{L} = E\{u(w(1-\tau)l - s - I - v(l)) + (1-\pi)\,u(s) + \pi\,(1-\zeta)\,H\,(0, m)$$

$$+\pi\zeta H\,(a, m) + \mu\,[\tau wl - \pi g]\}$$

Given $a = a\,(m)$, we have the following FOCs:

$$\frac{\partial \mathcal{L}}{\partial g} = E\{(-u'(c) + (1-\pi)\,u'(d) + \pi\Xi H_2)\,\frac{\partial s}{\partial g} + (-u'(c_i) + \Xi H_2)\,\frac{\partial I}{\partial g}$$

$$+\zeta\left[\pi H_2\,(a, m) + \pi H_1\,(a, m)\,\frac{da}{dg}\right] + (1-\zeta)\,\pi H_2\,(0, m) - \mu\pi\} = 0,$$

where $\Xi H_2 = \zeta H_2\,(a, m) + (1-\zeta)\,H_2\,(0, m)\,.$

Using the FOCs of the parent, it can be rewritten as:

$$\frac{\partial \mathcal{L}}{\partial g} = E\left[\zeta\pi H_1(a, m)\frac{\partial a}{\partial g} + \pi\Xi H_2 - \mu\pi\right] = 0 \qquad (3)$$

$$\frac{\partial \mathcal{L}}{\partial \tau} = -E\left[u'(c)wl - \mu\left(wl + \tau w \frac{\partial l}{\partial \tau}\right)\right] = 0.$$

We now combine those two equations to obtain:

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial \tau} = -E\left[u'(c)wl - \pi \Xi H_2 \frac{Ewl}{E\pi} - \pi\zeta H_1(a,m)\frac{\partial a}{\partial m}\frac{Ewl}{E\pi} - \mu\tau w\frac{\partial l}{\partial \tau}\right] \quad (4)$$

$$= -cov\,(u'(c), wl) + \Xi cov(\pi, H_2)\frac{Ewl}{E\pi}$$

$$-Ewl\left[Eu'(c) - \Xi H_2\right] + \zeta E\pi H_1(a,m)\frac{\partial a}{\partial m}\frac{Ewl}{E\pi} - \mu\tau Ew\frac{\partial l}{\partial \tau} = 0,$$

where $\Xi cov(\pi, H_2) = \zeta cov\,(\pi, H_2(a,m)) + (1-\zeta)\,cov\,(\pi, H_2(0,m))$

We distinguish two cases. If $I > 0$, from the FOC of the parent , we can rewrite this equation as :

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial \tau} = -cov\,(u'(c), wl) + cov\,(u'(c), \pi)\frac{Ewl}{E\pi} + \mu\tau Ew\frac{\partial l}{\partial \tau} = 0$$

.

And thus :

$$\tau = \frac{-cov\,(u'(c), wl) + cov\,(u'(c), \pi)\frac{Ewl}{E\pi}}{-\mu Ew\frac{\partial l}{\partial \tau}}$$

If instead, $I = 0$, we have

$$\tau = \frac{-cov\,(u'(c), wl) + \Xi cov(\pi, H_2)\frac{Ewl}{E\pi} + Ewl\left[Eu'(c) - \Xi H_2\right] + \zeta E\pi H_1(a,m)\frac{\partial a}{\partial m}\frac{Ewl}{E\pi}}{-\mu Ew\frac{\partial l}{\partial \tau}}$$

To see the impact of ζ on the level taxation $[\tau(\zeta)]$,we look at the two extreme cases:

$$\tau(0) = \frac{-cov\,(u'(c), wl) + cov(\pi, H_2(0,m))\frac{Ewl}{E\pi}}{-\mu Ew\frac{\partial l}{\partial \tau}}$$

$$\tau(1) = \frac{-cov\,(u'(c), wl) + cov(\pi, H_2(a,m))\frac{Ewl}{E\pi} + E\pi H_1(a,m)\frac{\partial a}{\partial m}\frac{Ewl}{E\pi}}{-\mu Ew\frac{\partial l}{\partial \tau}}$$

It appears that when $H_{12} = 0$, the two formulas, $\tau(0)$ and $\tau(1)$, are very similar. When $H_{12} > 0$, the level of public spending seems to benefit from an increase in ζ, whereas when $H_{12} < 0$, we observe the opposite effect. This is pretty intuitive. In case of separability, there is no interaction between g and a. Family solidarity thus has a negligible impact on the desired level of public spending. In case of strong complementarity between g and a, there is a clear link between formal and informal care. An increase in a makes public spending more attractive.

Finally, in case of strong substitutability between a and g, an increase in family solidarity crowds out formal care and thus discourages public spending. We can summarize these findings in a proposition.

Proposition 3

The risk of default of informal care has no impact on the level of social LTC spending when every individual purchases private insurance. In case private insurance is not available, the incidence depends on the sign of $H_{12.}$ In case of complementarity, family solidarity fosters public spending. In case of substitutability, it discourages it.

4.3 Public pensions versus public LTC insurance.

Let us now see whether or not the risk of solidarity default impacts the choice of public LTC insurance versus public pensions when there is uncertain lifetime. The government problem is thus:

$$\mathcal{L} = E\{u(w(1-\tau)l - s - I - v(l)) + \phi\,(1-\pi)\,u(s/\phi + P) + \phi\pi\,(1-\zeta)\,H\,(0,m)$$

$$+\phi\pi\zeta H\,(a,m) + \mu\,[\tau wl - \phi\pi g - \phi P]\}$$

We obtain the following FOCs with respect to P and τ:

$$\frac{\partial\mathcal{L}}{\partial P} = E\left[\phi\,(1-\pi)\,u'(d) + \zeta\phi\pi H_1(a,m)\frac{\partial a}{\partial P} + \phi\pi\Xi H_2 - \mu\phi\right]$$

.

$$\frac{\partial\mathcal{L}}{\partial g} = E\left[\zeta\phi\pi H_1(a,m)\frac{\partial a}{\partial g} + \phi\pi\Xi H_2 - \mu\pi\phi\right]$$

.We now look at the compensated effect of public pensions:

$$\frac{\partial\mathcal{L}^{\mathcal{C}}}{\partial P} = E\left[\phi\,(1-\pi)\,u'(d) + \left[1 - \frac{E\phi}{E\phi\pi}\right]\phi\pi\Xi H_2 + \zeta E\phi\pi H_1(a,m)\frac{\partial a^C}{\partial P}\right]$$

In case $I > 0$, we have from the parent's FOC:

$$-u'(c_i) + \Xi H_2 + \zeta H_1\,(a_i, m_i)\,\frac{\partial a_i}{\partial m_i} = 0; c_i = d_i.$$

Thus, after some simplifications, we obtain:

$$\frac{\partial\mathcal{L}^{\mathcal{C}}}{\partial P} = E\left[\phi\,(1-\pi)\,u'(d) - \frac{E\phi\,(1-\pi)}{E\phi\pi}u'(c)\right]$$

$$= cov\,(\phi\,(1-\pi)\,, u'(c)) - \frac{E(\phi(1-\pi))}{E\phi\pi}cov\,(\phi\pi, u'(c))$$

Naturally, when $I = 0$, those simplifications are not possible and we have that the desirability of public LTC increases in the case of complementarity.

Proposition 4

When family solidarity is uncertain, it does not impact the social desirability of public LTC insurance if there is private insurance. In case private insurance is unavailable, whatever the reasons, then the complementarity between informal and formal care strengthens the desirability of public LTC insurance versus public pensions.

5 Conclusions

Before concluding, two remarks are in order. Throughout this paper, we have examined two polar cases regarding private insurance: either it was non-existent or it was actuarially fair. We could have introduced intermediate scenarios with loading factors that make insurance less attractive. In such cases, private insurance would not fully neutralize the impact of informal care on the desirability of public long-term care spending. In other words, family assistance would either positively or negatively influence the level of public spending.

Regarding the concept of uncertain altruism, there is a substantial body of literature on the topic[5]. However, these studies differ from our approach in three key ways. First, they explore two methods by which public spending can be associated with private spending: topping up and opting out. In contrast, our paper adopts the former, assuming from the outset that public spending can supplement other sources of financing for formal care. Second, these studies focus solely on formal care, whereas our paper examines the substitutability between formal and informal care. Finally, our paper addresses the issue of triple heterogeneity (wages, disability risk, and longevity), while most of the other studies consider identical individuals.

It is well established that the risk of disability in old age decreases with higher income and wealth, while longevity tends to increase with these factors. One key implication of these correlations, as shown by Nishimura and Pestieau (2020), is that the case for a public long-term care (LTC) scheme is stronger than that for public pensions. In this paper, we demonstrate that this dynamic may shift when family solidarity—specifically informal care provided by a spouse or children—is taken into account.

Assuming altruism as the primary motivator for caregiving, we demonstrate that the generosity of a social long-term care insurance is highly sensitive to both the degree of substitutability between formal and informal care, as well as the availability of private insurance. When everyone purchases actuarially fair insurance, family solidarity does not significantly impact public LTC spending. However, if private insurance is not universally adopted, the presence of informal care can either support or reduce public LTC spending, depending on whether formal and informal care are seen as complementary or substitutive. Given the limited availability of LTC insurance in many countries, it is likely that public LTC expenditure will be shaped by the strength of family solidarity.

[5] Cremer et al. (2013), Canta et al. (2020), Canta and Cremer (2021)

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