Title: Hypersensitivity-to-interference in memory as a possible cause of difficulty in arithmetic facts storing

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Abstract

Difficulty in memorizing arithmetic facts is a common problem encountered in people suffering from dyscalculia. We suggest that hypersensitivity-to-interference in memory could account for this profile of difficulty.

We first report the description of a patient showing a deficit in storing multiplication facts together with a hypersensitivity-to-interference. We hypothesized that similarity between arithmetic facts would provoke interference and that learners who are hypersensitive-to-interference would encounter difficulties in storing arithmetic facts in memory. This hypothesis was first tested in 4^{th} grade children: Children with weak arithmetic facts learning showed higher sensitivity-to-interference in memory compared to the control group. We then showed in adults, that this hypothesis specifically explained difficulties in arithmetic facts solving but not in a global math test. Finally, we created a measure of the proactive interference weight for each multiplication problem and showed that it accounted for both difference of performance between the problems and between individuals.

Keywords: interference, inhibition, memory, dyscalculia, arithmetic fact, children, patient

1. Introduction

Children spend most of their first four grades in primary school learning to solve simple arithmetical operations. The expectation is that, by the end of the fourth grade approximately, children are able to use mature strategies to solve most of the calculations and that they have stored in memory some of the answers; in particular, the products. Yet, for some children, this learning is problematic. More specifically, this is often so in the case of math learning disabilities or of dyscalculia (Geary, Hoard & Hamson, 1999, Garnett & Fleischner, 1983).

In this chapter, we focus on multiplications because the memorization of the answers of the problem is particularly stressed in school compared to other operations. Much previous research assumed that additions were first solved through counting strategies and that, at some point, children would store the answers of at least the small additions (this idea is however debated, see Barrouillet & Thevenot, 2013). In contrast, children learn the meaning of multiplication by performing repeated additions but very soon they are encouraged to learn the answers by heart as other strategies such as counting or repeated additions are too time-consuming and error-prone.

Despite a number of differences between the existing theoretical models, there is a general consensus that arithmetic facts are organized in an interrelated network in long-term memory (e.g., Campbell, 1995; McCloskey & Lindeman, 1992; Verguts & Fias, 2005) and that when confronted with a multiplication problem, not only the correct answer but also other related incorrect answers are activated. Consequently, the set of associated but incorrect answers constitutes competing answers that interfere with the retrieval of the correct answer (e.g., Campbell, 1987a, 1987b, 1995; Campbell & Tarling, 1996). Based on this idea, Barrouillet, Fayol, and Lathulière (1997, see also Barrouillet's chapter in this volume) proposed that difficulties in recalling multiplication answers might be due to difficulties in inhibiting the competitors. To support this view, they presented multiple-choice multiplications to adolescents with learning disabilities. The adolescents erred more often when the false answers were multiples of one of the operands (e.g., 4 × 6 = 18, 24, 28, or 30?) than when they were not (e.g., 4 × 6 = 21, 24, 25, or 27?) or when the false answers did not come from multiplication tables (e.g., 4 × 6 = 22, 23, 24, or 26?). However, one should be cautious when drawing conclusions from this study. First, these adolescents did not present specific math learning disability but presented global learning disability with abnormally low IQ. Second, there was no group control to ensure that their profile was specific. Following this study, some authors proposed that inhibition weakness might account for difficulties in retrieving multiplication answers from memory.

2. Are difficulties in arithmetic due to inhibition problems?

A series of research tested whether general math performance in math correlated with inhibition capacities and many of them found significant correlations (Bull & Scerif , 2001; Espy, McDiarmid, Cwik, Stalets, Hamby, Senn, 2004; Gilmore, Attridge, Clayton, Cragg, Johnson, Marlow, et al., 2013 ; St Clair-Thompson & Gathercole, 2006 and see Cragg & Gilmore, 2014, for a recent review). Others tested the hypothesis that children with math learning disabilities would present weaker inhibition skills than typically developing children but failed to find an inhibition deficit in these math disabled children (e.g., van der Sluis, de Jong, and van der Leij, 2004; Censabella & Noël, 2005). But only a few studies specifically examined whether inhibition capacities are specifically related to performance in simple arithmetic. In particular, Censabella and Noël (2008) addressed this question by testing three inhibitory functions in children with or without math learning difficulties, and also in children with a specific deficit in arithmetic facts. These three functions were the suppression of irrelevant information from working memory, the inhibition of proponent response and the interference control of distractors. The first of these was assessed using the listening span task: the child listens to a series of sentences, judges whether they are true or false and then has to recall the last word of each sentence. The authors examined the number of intrusions of not-ending words in the child's recall. The inhibition of proponent response was assessed by a Stroop task. An Erikson task, in which the participant has to judge a central target surrounded by the same or different distractors, assessed resistance to the interference caused by external distractors. The authors found no significant differences of performance in any of these inhibition tasks between control children and either all the children presenting arithmetic disabilities or those with difficulties in arithmetic facts retrieval. In the same vein, Bellon, Fias and De Smedt (2016) tested typically developing third graders and found no correlation between performance in an addition or multiplication verification task and inhibition abilities tested in numerical and colour Stroop tasks.

Censabella and Noël (2008) then reasoned that most of these studies reported here above tested inhibition capacities; that is, the active suppression of irrelevant or competing stimuli or responses. However, interference can also arise in long-term memory when the retrieval of information is based on a process of spreading activation (Anderson, 1983). In this view, the ease of retrieving a fact is a function of the level of activation of this fact relative to the activation of other associated facts. If other associated facts are also activated, then the retrieval of the target fact will suffer interference and the retrieval will be less efficient. Following this idea, Censabella and Noël (2004) tested in adult participants whether arithmetic fact performance was more closely related to performance in a typical inhibition task (the color Stroop task) or in an activation-based interference paradigm. The latter was assessed using the fan effect paradigm (Anderson, 1974): participants had to learn sets of sentences such as "this character is in this location". Yet one character could be associated with either a single or several locations. The number of associations between the character and the location(s) is called the fan. In this experiment, the fan was either one or three. After the learning phase, participants were asked to verify propositions such as “this character is in this location”. Typically, it is found that participants are slower and less accurate when the fan is large. The assumption is that the character’s name acts as a memory probe and spreads activation to associated location node(s). Yet, as the capacity of spreading activation is supposed to be limited, it leads to full activation of associated location nodes in the case of a fan of one, but to reduced activation if the fan is larger. Indeed, in the case of a larger fan, the activation is divided into the different connected pathways and this leads to a reduced accessibility of the associated location concepts.

Censabella and Noël (2004) found no correlation between arithmetical abilities and performance in the Stroop task but found a significant correlation with the measure of activation-based interference: the larger the fan effect in a participant, the longer his/her response time in multiplications. The authors concluded that, in arithmetic facts models, the key determinant of performance and retrieval efficiency is the associative strength of the correct answer relative to the associative strengths of all answers. This relative associative strength entails retrieval interference: the correct answer’s relative associative strength is weaker as the number of associated but incorrect answers increases. This activation-based interference does not necessarily require active inhibitory mechanisms, as the reduced associative strength of the correct answer is sufficient to account for weaker performances. This interference is a passive one due to the “over-facilitation” of competitors (Anderson & Bjork, 1994; see also Logan, 1994).

All these previous studies consider that interference could occur when the person is attempting to retrieve the correct answer of a problem and suppose that arithmetic facts have been stored in memory, where the problems are associated with the correct and incorrect related answers. However, the study of one single case showing a dramatic impairment in storing multiplication facts in memory led us to consider that some other types of interference could already play a role at the learning stage.

3. A case study with arithmetic fact impairment and hypersensitivity-to-interference in memory

DB was a 42-year-old woman previously diagnosed with developmental dyscalculia and high intellectual potential (De Visscher & Noël, 2013). When we assessed her numerical skills, we found a very specific profile of dyscalculia characterized by extremely slow answers in single-digit multiplications, and to a lesser extent in additions. This extreme slowness was explained by her use of computational and finger-counting strategies, while the control subjects used retrieval. Further investigations revealed that she actually stored only very few products. Indeed, when we forced her to produce the first answer activated by the problem, by using a time-limited multiplication production task, she performed as well as control participants only for 2 out of 8 small problems (2 x 3 and 3 x 3) and gave no correct answers for the 8 large problems except for 8 x 8. This suggests that she only stored the answers of a very few items: some of the small products and some of the ties. Furthermore, in a table membership judgment task where participants had to decide whether the number displayed was a possible result of a multiplication of two single-digits, control participants were 95% correct while she reached only 66% and based her answers mainly on the parity status of the item: she accepted all the even numbers as table facts and dismissed too many odd ones.

We therefore explored what might explain such a severe and persistent difficulty in memorizing arithmetic facts, in the context of high intelligence, good memory and high motivation in practicing the times tables. Deep cognitive investigation showed that none of the hypotheses currently found in the literature to account for dyscalculia or difficulties in learning multiplication facts were supported. Indeed, DB had perfectly normal performance in tasks tapping the approximate number system, access to number magnitude from Arabic symbols or rote verbal memory. Eventually, we considered a new hypothesis based on hypersensitivity-to-interference in memory.

The rationale was that single-digit multiplication facts are made up of three elements: the two operands, selected from ten possibilities (0-9), and the product, corresponding to a combination of two elements from these ten possibilities. If we consider the 64 problems from 2 x 2 to 9 x 9, any given operand appears in 15 problems and several problems share the same product (for instance, 24 is the product of 8 x 3, 3 x 8, 4 x 6, 6 x 4 and 18 is the product of 6 x 3, 3 x 6 and 9 x 2 and 2 x 9). In the memory literature, it is well known that during the storage stage, memorization is hindered when the items are similar to one another. This detrimental effect of similarity has been observed in many different paradigms both in long-term (e.g., paired-associated learning such as in Hall, 1971) and short-term memory (e.g., immediate recall paradigms such as Conrad and Hull, 1964; or Conlin et al., 2005). According to Oberauer and Kliegl (2006), any two items have a certain degree of overlap regarding their features, with more similar items sharing a larger proportion of feature units. When items have to be stored in memory, the features of item representations interact with one another, which leads to mutual interference. This interference partially degrades the memory traces, which in turn leads to slower processing and to retrieval errors. When we learn multiplication facts by rote, we have to learn distinct associations of three elements with many features in common. For instance, we have to learn that 9 x 4 = 36 but also that 6 x 4 = 24, 8 x 4 = 32, and so on. Considering the high overlap of features, arithmetic facts can be qualified as prone to interference. Accordingly, we proposed that a heightened sensitivity to interference could prevent the storage of arithmetic facts in long-term memory.

Using different kinds of paradigms assessing interference in working memory and in learning in long-term memory, we tested this hypothesis on the patient DB (De Visscher & Noël, 2013). In association learning tasks, DB performed perfectly when the verbal associates were composed of dissimilar items, but she had impaired performance when the associates were composed of similar items or when the associations included shared items. In this type of experiment, the sensitivity to item similarity is typically due to greater proactive interference (Underwood, 1983). The hypothesis of high susceptibility to proactive interference in DB was directly tested using the recent-probes task (originally from Monsell, 1978). In a visual version of the task, two unknown visual signs were presented for 3000 ms followed by one visual sign and participants had to decide whether this sign was one of the two just previously seen. DB had no difficulty in accepting the sign when it was a part of the stimulus trial (positive trials) or in rejecting a sign that had not been seen before (negative trials). However, she had considerable difficulty in rejecting lures that were seen in a previous trial (negative lures, see Figure 1). Thus, her difficulties were not due to memory problems per se, but to a very high sensitivity to proactive interference between the previous trial's target set and the current trial's negative response (see Jonides and Nee, 2006).

Insert Figure 1 around here

This single-case is particularly interesting as it described the case of an adult with very good general cognitive abilities but with a circumscribed deficit in arithmetic fact storage. We found that she suffered from hypersensitivity-to-interference in memory and we argued that this prevented her from storing very similar associations in memory, such as arithmetic facts. From this new observation, we decided to investigate whether DB was one isolated case of dyscalculia or whether this new theoretical explanation could also account for arithmetic facts deficit in other people.

4. Hypersensitivity to interference in children with low arithmetical skills

The hypersensitivity-to-interference account of difficulties in storing arithmetic facts in memory was first tested in children at the age when they are supposed to build this memory network (De Visscher & Noël, 2014a), namely in fourth grade. Two groups of fourth-graders were compared: one characterized by weak arithmetical fluency (i.e., ability to solve many addition or multiplication facts within a given time limit) and the other showing normal arithmetical fluency. Children from the two groups were matched for school, gender and age. We designed a simple and child-friendly task to assess their sensitivity-to-proactive interference in the context of associative memory. Children were presented with pairs of pictures associating one famous cartoon character with a location (e.g., beach, shop, garage,etc.) and were asked to remember where each character was. In each task block, children successively saw three different associations of characters and locations. Directly after the three presentations, a verification phase started, offering associations of character-location that the child had to verify (pressing the 'true' or 'false' button). After three verifications, children were instructed that the characters were travelling and that they had to learn new associations of character-location. The next block then started with the presentation of three new character-location associations. Twenty of these blocks were presented, corresponding to a total of 60 verifications. Half of them were true and half were false. Regarding the true character-location verifications, half of the trials were low-interfering because they were new items that were never seen before. The other half were high-interfering because the same pictures were associated differently in the previous block, causing proactive interference. For example, in the first block Asterix was in the mountain and Marsupilami was in the shop, and in the second block Asterix was in the shop in both the learning and the verification phase. Regarding the false character-location verifications, some parts of the trials were low-interfering because they were composed of one picture from the learning phase and one totally new picture. The other parts of the false verifications were high-interfering, either because they associated differently one character and one location that had both been encountered during the current block, or because they associated differently a character and a location used in the previous block. The two groups of children performed equally well in the verification of low-interfering character-location associations. However, and in accordance with the hypersensitivity-to-interference hypothesis, children with low fluency in arithmetic performed worse than the other group in the high-interfering condition (see figure 2). Their impairment in the high-interference condition is a clear sign of an abnormal proactive interference, while their normal performance in the low-interference condition indicates that they had typical associative memory in general. These results corroborate the findings from the DB case study and support the hypothesis that hypersensitivity-to-interference in memory can prevent storage of arithmetic facts in memory in children.

Insert figure 2 around here

5. Hypersensitivity-to-interference as an explanation for specific impairment in arithmetic facts

Our suggestion is that people who are very sensitive to interference in memory have great difficulties in learning multiplication facts because arithmetic facts are associations made of very similar features. This hypothesis is thus one possible explanation for one very specific difficulty that is frequently encountered in dyscalculia. Yet dyscalculia is heterogeneous and other explanations should be considered for other types of numerical disabilities. Accordingly, we conducted a study that aimed first to examine whether we could replicate the pattern of results of DB in a group of adults presenting dyscalculia and, second, to test the specificity of our hypothesis, i.e., testing whether hypersensitivity-to-interference accounts for dyscalculia characterized by a specific difficulty in learning arithmetic facts (De Visscher, Szmalec, Van der Linden, & Noël, 2015). With these aims in mind, we compared adults with no learning difficulties to two groups of adults with dyscalculia: one group who presented global math impairment and one group selectively impaired in arithmetic facts. We predicted that only the latter would show hypersensitivity-to-interference. Regarding the former group, we tested another hypothesis that has emerged in the scientific literature, namely, the hypothesis of a serial-order learning deficit.

Recently, a number of researchers have questioned the quality of ordinality processing in dyscalculia. Rubinsten and Sury (2011) found that adults with dyscalculia experienced difficulties in judging whether three stimuli (dots or digits) were correctly ordered. Kaufmann, Vogel, Starke, Kremser and Schocke (2009) showed different brain activations between dyscalculic and control children during ordinal judgment tasks of numerical (digits) and non-numerical (physical size of symbols) stimuli. Furthermore, Lyons and Beilock (2011) found that the ability to judge whether three Arabic digits are presented in increasing order is highly correlated with arithmetical ability. However, in all these studies, ordinality processing was confounded with magnitude processing as researchers used digits, amounts of dots or physical size of items, all of which have a magnitude aspect. A deficit in magnitude processing could in itself explain these results. Accordingly, we (De Visscher, Szmalec, Van der Linden, & Noël, 2015) selected a task of serial-order learning that would not involve any processing of magnitude. We hypothesized that a deficit at that level would already impair early numerical learning - in particular, learning the counting sequence - and would lead to global math disability.

To test these hypotheses, we selected the Hebb repetition-learning paradigm. In this paradigm, participants have to recall the order of presentation of nine syllables presented sequentially in the center of the screen. After the presentation of the sequence, all syllables are randomly positioned on a virtual circle around a question mark and the participants have to click on the nine syllables in the same order as the presentation. In the Hebb learning paradigm, the sequences' order is random and always different for all the fillers but one sequence's order is repeated throughout the experiment, without the participants being aware of it. What is typically found is that participants benefit from the repetition of the order and show an improvement in remembering that specific repeated sequence as the number of repetitions increases. In our experiment, two sets of meaningless consonant–vowel syllables were used to build three types of sequence: non-repeated sequences (fillers), interfering repeated sequence and non-interfering repeated sequence. So, in our paradigm, two sequences were repeated throughout the experiment. The interfering repeated sequence was composed of the same syllables as the fillers, hence increasing proactive interference. Conversely, the non-interfering repeated sequence was constructed from different syllables than those involved in the fillers (and therefore in the interfering sequence), hence reducing proactive interference. The experiment ended when the participant correctly reproduced two successive non-interfering repeated trials, with a maximum of 20 repetitions.

In line with our hypotheses, not only did the performance of participants with dyscalculia differ from the controls but the profile of performance between the two groups with dyscalculia also differed. In agreement with serial-order learning deficit, the group with global dyscalculia was poorer than the controls in retrieving the order of the fillers, thereby already revealing problems in processing order in short-term memory. Regarding long-term memory ability, as measured by the Hebb repetition effect, they needed more repetitions than the control group to learn the non-interfering repeated sequence. Moreover, after learning the non-interfering repeated sequence, we asked participants to travel within the sequence by giving the syllable that came immediately after or two places after a given syllable. They produced many more errors in this task than did the controls. Thus, although they reached the same stopping criterion indicating that they had learned the sequence, long-term consolidation of the acquired sequential information was not as efficient as that of the controls. Importantly, these difficulties were not due to a general memory problem. Indeed, they did not differ from the controls in a word list memory task that did not involve any order constraint. These results support the view of a serial-order learning deficit in this group with global math deficit.

By contrast, the profile of adults with a specific problem with arithmetic facts showed a typical Hebb effect and reached the stopping criterion with the same number of repetitions as the control participants. In addition, the quality of their memory trace for the non-interfering repeated sequence was similar to that of the controls, as evidenced by their performance in answering which syllable came immediately after or two places after a given syllable. These results support the assumption of good serial-order learning skills in this group. Contrariwise, in the interfering situation, their performance dropped drastically and they showed an interference effect while the two other groups did not (see Figure 3).

Insert figure 3 around here

In sum, these findings supplied new evidence for a link between an arithmetic facts deficit and hypersensitivity-to-interference in adults with arithmetic fact dyscalculia, and showed its specific character as it is found in certain profiles of dyscalculia, not in all of them.

6. An index of interference for each multiplication problem

So far, we have shown that hypersensitivity-to-interference in memory could account for difficulties in learning arithmetic facts, mostly multiplication facts. This has been shown in a case study as well as in groups of children or adults performing poorly in arithmetic fact tasks. All these studies assumed that arithmetic facts interfere because of features overlap but they did not test this assumption directly.

Such a similarity-based interference in arithmetic facts was already proposed in the network interference model by Campbell (1995). According to this model, when a problem is presented, it activates a physical code and a magnitude code. The former corresponds to the physical representation of the two operands and the operation sign and it activates other problems according to the feature overlap of these operands and sign. A weight of interference is given according to the type of feature overlap (a weight for each common operand and sign). The second corresponds to the magnitude representation of the answers. Globally, the problem node activation depends on the total similarity corresponding to the sum of the feature matching of operands and sign (i.e., physical code) and magnitude-similarity of the answers (magnitude code).

Campbell mainly tried to account for differences in errors and response times between the different multiplication problems in typical adults who are supposed to have built a multiplication fact network. He was mainly interested in accounting for differences between problems in the retrieval stage of the answers. Here, we (De Visscher and Noël, (2014b) were interested in understanding how children develop their arithmetic fact network in order to explain why some never succeed in learning these facts. Taking a learning perspective, one could consider that the first problem-answer associations that a child learns probably do not lead to great difficulties. However, learning would become more problematic if children are hypersensitive-to-interference and learn other problem-answer associations that share many features with the already stored facts. Based on this view, the order of learning arithmetic facts and the degree of overlap between each problem and the already learned problems should matter in the process of learning. Of course, the order in which problems are learned varies from one education system to another but in general, the small tables are learned before the large ones. Accordingly, we assumed that table 2 is learned before table 3, which itself is learned before table 4 and so on. Second, we assumed that learning a new multiplication fact is subject to proactive interference. More precisely, the more features of this new multiplication fact that overlap with the previously learned ones, the weaker its memory trace will be. To calculate the degree of features overlap we calculated the frequency of co-occurrences of digits associations between the target problem and the previously learned ones (for both the operands and the answers).

The problems learned in the first place are 2 x 2 = 4 and 2 x 3 = 6. These do not have any associations of digits in common and therefore have a level of interference of 0. The third problem learned is 2 x 4 = 8, which shares the association of 2 and 4 with the problem 2 x 2 = 4. The interference level is thus 1. Let us consider a larger problem, 3 x 9 = 27. Its level of interference is 9. Indeed, this problem shares the association of the 2 and 3 with four previously learned problems (3 x 2 = 6, 4 x 3 = 12, 3 x 7 = 21, 8 x 3 = 24), of 2 and 7 with two previously learned problems (2 x 7 = 14, 3 x 7 = 21), of 2 and 9 with one other problem (9 x 2 = 18), of 3 and 7 with one other problem (3 x 7 = 21) and of 3 and 9 (3 x 3 = 9), thus leading to a total of 9. It is important to note that the problems encountered later are not necessarily the most interfering. The problem 7 x 7 = 49, for instance, is learned 31st but ranked 16th in interference weight. Conversely, the problem 4 x 8 = 32 is learned 20th but is the most interfering problem. The measure of the features overlap between the problem and the previously learned ones is therefore a proxy aiming to measure the degree of proactive interference that a problem has undergone during the learning stage. The position of the digits in the associative structure of problem-answer is not considered since the positional value of the features has been shown to not play a role in the feature-overlap model of Oberauer & Lange (2008). We intended to use the most parsimonious proxy of proactive interference, but this index could be refined on different levels. With this interference parameter, we wanted to test whether the features overlap in arithmetic fact learning impacts on performance across problems.

We first tested whether this interference parameter could explain differences of performance across multiplication problems by analyzing data published by Campbell (1997). These data were error and response times for solving each multiplication problem collected on 44 undergraduates. We considered jointly the interference parameter and an index of the problem size. Indeed, the problem size is one of the more powerful factors accounting for performance variations across problems and indicates that small problems (i.e. made of small operands) are usually solved more accurately and more rapidly than large problems (Campbell & Graham, 1985, Zbrodoff & Logan, 2005). Both the problem size and the interference parameter played a significant and unique role in accounting for differences in reaction time and accuracy across multiplication problems.

In addition, we collected data from third-grade children, fifth-grade children and undergraduates and analyzed the error rates and response times according to the problem size and interference parameter. Regarding accuracy, the problem size explained a substantial part of the variance in performance in each of the three age groups, while the interference parameter explained variance in the third-grade group only. Regarding response time, the interference parameter was significant in all age groups and the problem size accounted for independent and significant explanatory power only in undergraduates (and was marginally significant in 3rd and 5th graders). Concretely, the time needed to solve a multiplication increases as the level of interference increases and this, throughout the child's development.

We were also interested in accounting for inter-individual differences. Accordingly, we measured sensitivity to the interference parameter and/or sensitivity to the problem size for each individual (based on the regression slopes of the person's response times). The two factors significantly accounted for inter-individual differences in multiplication response time in each of the three groups. More precisely, children or adults who were more sensitive to the interference parameter were globally slower to solve multiplications.

Coming back to our patient DB, we examined her sensitivity to the interference parameter and to the problem size. It appeared that the interference parameter but not the problem size accounted for her response time in multiplication. Moreover, DB's sensitivity to the interference parameter was larger than that of the controls. These current results corroborate the previous findings and clearly show that the abnormally slow response time of DB in multiplication was due to her hypersensitivity to the interference weight of the problems.

Finally, we considered the data obtained in fourth-grade children who had poor *versus* normal arithmetic facts fluency (see De Visscher & Noël, 2014a) and examined whether the interference parameter and/or the problem size effect predicted the individual differences in multiplication in fourth-grade as well as one year later, when they were in fifth grade. Again, the slope of the problem size and the interference parameter were calculated for each child and then used to account for inter-individual differences in accuracy and speed in multiplication. Sensitivity to the interference parameter proved to be a significant explanation of differences in multiplication accuracy, with higher sensitivity being associated with lower accuracy. Being sensitive to the problem size was less influential. Finally, only the interference parameter significantly accounted for differences in time needed for solving multiplications. When considering multiplication performance one year later when the children were in grade 5, sensitivity to the interference parameter remained strongly associated with lower accuracy and longer response times in multiplications.

We then verified the assumption according to which hypersensitivity-to-interference disturbs the learning stage and consequently hampers retrieval from memory, by using a time-limited multiplication task and a multiplication table membership judgment task. The multiplication production task without time limit allowed children to use different strategies, such as computing strategies or retrieval strategies. We therefore used a multiplication production task in which children had only 2 seconds to answer, forcing them to use a retrieval strategy. Results showed that children who were more sensitive to the interference parameter of multiplication were also those who performed worse in the time-limited multiplication task. This finding supports the idea that hypersensitivity-to-interference prevents storage of arithmetic facts in memory, hampering the retrieval of the answer to arithmetic problems.

Then, we used a multiplication table membership judgment task in order to distinguish between a storage *versus* an access deficit. Indeed, when a child can recognize whether a number belongs to a multiplication table's answers or not, it means that he/she has to a certain extent created an arithmetic facts network. Conversely, difficulties in this task reveal a storage deficit. Results showed that children who were the more sensitive to the interference parameter of multiplication were also slower in the table membership judgment task.

In summary, this interference parameter accounted for variation of performance across multiplication problems in adults (Campbell's data in 1997 and our own data) as well as in children (third-grade and fifth-grade). Sensitivity to this interference parameter also accounted for inter-individual differences with a higher sensitivity being observed in children and adults who are slow in multiplication solving, in children with low arithmetical fluency and in the patient DB. Finally, children who are more sensitive to this interference parameter are performing weaker in a time-limited multiplication task and in a table-membership judgment task, suggesting that they failed to build an appropriate arithmetical fact network.

7. Index of interference and problem size

As already mentioned, Campbell (1995) proposed that both a physical and a magnitude code are activated when a problem is presented. The index of interference just described is a way of assessing the interference that can be associated with the physical code. The magnitude code would be activated for the answer and would explain the problem size effect: i.e. the fact that smaller single-digit multiplications are solved more quickly and accurately than larger single-digit multiplications (e.g. 2 x 3 compared to 7 x 8). Indeed, problems would activate the magnitude representation of the answers and since the magnitude representation is increasingly compressed with larger numbers (Dehaene, 1992), the magnitude representations of large answers would be more difficult to discriminate than those of small answers. Due to their higher similarity in the magnitude code, larger problems would create more interference, which would increase the retrieval time. Together with De Smedt (De Visscher, Noël and De Smedt, 2016) we have measured the unique influence of the physical representation and the numerical magnitude representation on the multiplication performance of 79 fourth-graders. To measure the influence of the magnitude code, we presented a magnitude comparison task to the participants in which they had to compare two numbers, which actually corresponded to two close answers of multiplication problems (e.g., 42 and 45). The underlying idea was that if they have a very precise number magnitude representation, they would perform better in this task and show less overlap of magnitude code for problems close in magnitude; according to Campbell's model, this should lead to better performance in multiplication.

The time taken to compare two close answers was indeed related to children's performance in multiplication but it was also related to their performance in addition, subtraction, division, in a mixture of all these as well as to performance in a general math test that included word problems, geometry, and other mathematical knowledge. The ease of processing number magnitude is therefore a factor related to all kinds of mathematical dimensions since all numerical tasks require the processing of number magnitude.

By contrast, sensitivity to the interference parameter was specifically related to performance in operations that are mainly tapping facts stored in memory (i.e., additions and multiplications) but not to subtractions, mixed operation calculations, or to a general mathematics test. Moreover, sensitivity to this physical similarity interference uniquely explained the use of retrieval strategies. Indeed, more interfering problems were less often solved via retrieval and individuals with higher sensitivity to physical interference also had lower frequencies of retrieval use. Physical representation therefore seems to have an important and specific role in storing and retrieving arithmetic facts in long-term memory.

8. Hypersensitivity to interference in memory and close concepts

Finally, in order to better understand this "hypersensitivity to interference in memory", we tried to disentangle it from other closely related concepts. Firstly, we explored whether sensitivity to the interference parameter in multiplication reflected a general sensitivity to interference in memory or if it was specific to numbers. We measured the correlation between two measures of sensitivity to interference, the one measured in the multiplication task (sensitivity to the interference parameter) and one measured in a non-numerical task, the associative memory task of character-location explained here above. A positive correlation was obtained between sensitivity to the interference parameter in multiplication and sensitivity to interference in this non-numerical associative memory task (Spearman's rho = .506).

Secondly, we wanted to determine whether this sensitivity to interference was a specific concept or whether it was confused with closely related concepts such as inhibition capacities, verbal memory or associative memory capacities. To that end, we tested inhibition capacities with a Stroop colour task, verbal memory capacities with a word list memory task and associative memory capacities with a (non-interfering) paired-associates memory task.

Sensitivity to interference in the non-numerical associative memory task did not correlate with any of these measures, showing the specificity of that concept.

In summary, the sensitivity to the interference parameter in multiplication reflects a general sensitivity to interference in memory. This sensitivity to interference in memory is a specific concept that should not be confounded with close concepts such as inhibition, memory or associative memory.

Discussion

This chapter aimed at understanding what could account for the difficulty in storing arithmetic facts in memory that is frequently encountered in children with math learning disability, and considered hypersensitivity-to-interference in memory as a possible cause.

After the learning stage, arithmetic facts are stored in an interconnected network in memory in which not only the correct response but also other associated incorrect responses are activated (e.g., Campbell, 1987a, 1987b). Accordingly, some researchers have proposed that difficulty in retrieving arithmetic facts from memory might be due to weak inhibition capacities preventing the individual from suppressing the interference of competitors (Barrouillet & al., 1997). However, this hypothesis has not received much empirical support (Censabella & Noël, 2008; Bellon & al, 2016).

More recently, we (De Visscher and Noël, 2013) considered the learning stage of arithmetic facts and proposed that arithmetic fact deficit might be caused by hypersensitivity-to-interference in memory. We argued that memorizing arithmetic facts amounts to storing associations of operands with answers and that these associations share many common features, which creates interference. Individuals who show heightened sensitivity to this kind of interference would be hampered in creating distinct memory traces for each of the arithmetic facts.

In a way, this hypothesis is congruent with previous finding of Censabella and Noël (2004) who found that the time taken to solve multiplications was correlated with adults' sensitivity to the fan effect. Indeed, several authors consider that the fan effect is due to proactive interference in memory. For instance, Hasher, Tonev, Lustig and Zacks (2001) wrote, "These fan effects are widely seen as the basis for proactive interference" (p. 293, see also, Bunting, Conway & Heitz, 2004).

This hypothesis of a hypersensitivity-to-interference was proposed to explain the profile of a patient showing a specific impairment in arithmetic facts (De Visscher & Noël, 2013) and was later supported in a group study of children (De Visscher & Noël, 2014a) and of adults (De Visscher & al., 2015) with difficulties in arithmetic facts. Furthermore, it was shown to be a hypothesis accounting for specific disorder in learning arithmetic facts rather than global math difficulties (De Visscher & al., 2015).

Subsequently, we (De Visscher and Noël, 2014b) went further in understanding how this proactive interference took place when learning multiplication facts. We created an index for each multiplication problem, aiming at assessing the amount of proactive interference each problem received from the previous learned ones. This index of interference accounted for a significant part of the variance across the different multiplication problems, beyond the well-known problem size effect. The more a problem had a high index of interference, the slower and the more error-prone was the answer, and the less frequent the retrieval strategy was used (De Visscher & al., 2016). Sensitivity to this index of interference also accounted for differences across individuals. Individuals with high sensitivity to this index of interference were slower to solve multiplications and used fewer retrieval strategies (De Visscher et al., 2016). This has been shown in populations of unselected children as well as in adults (De Visscher & Noël, 2014b). Regarding atypical arithmetic development, high sensitivity to this index of interference was characteristic of the patient DB as well as of children with weak arithmetic fact fluency (De Visscher & Noël, 2014b).

Sensitivity to the parameter of interference specifically accounted for inter-individual differences in arithmetical tasks that mostly required recalling facts from memory and not in arithmetical operations that are usually solved through procedural strategies or performance in a global math test (De Visscher & al., 2016). In addition, in the case of atypical development, this sensitivity to the interference parameter in multiplication correlated with a measure of non-numerical sensitivity to interference in memory. That is, when testing participants with arithmetic fact deficit, both adults and children, a correlation between sensitivity to interference in arithmetic and in a non-numerical domain was found. Finally, sensitivity to interference has been distinguished from the capacities to inhibit dominant responses or global memory capacities (De Visscher & Noël, 2014b). This concept thus proved to be very specific and to account for very specific difficulties in mathematics.

Several of these studies have compared individual(s) with impairment in arithmetic facts to control individuals. We do not yet know whether one needs to have a certain degree of resistance to interference in memory to be able to constitute a normal arithmetic fact network, or whether there is a linear relationship between arithmetic fact storing and sensitivity to interference. Further research might also consider the consequences of becoming more sensitive to interference, due to aging or brain damage: to what extent would this affect the person's ability to retrieve answers from an already constituted arithmetic fact network or the ability to learn new arithmetic facts?

Further work should also examine what other areas of learning could suffer from hypersensitivity-to-interference and could thus be expected to be impaired along with arithmetic facts. One possible avenue to consider is dyslexia. Indeed, people with dyslexia often report problems with fact retrieval (see Göbel, 2015 for a review). Bogaerts, Szmalec, Hachmann, Page, Woumans, and Duyck (2015) found a greater difficulty in resisting proactive interference in adults with dyslexia than in typical adults. This sensitivity to interference may perhaps account for some of the reading difficulties of people with dyslexia and for associated deficits in arithmetic. However, one needs to clarify the process by which this sensitivity to interference might account for reading problems. One possibility is to consider learning the grapheme-phoneme conversions, which may include several shared features. For instance, in French, we have proper sounds for the combinations of the letters in, on, an, ain, un, en, ou etc.

It is interesting to note that at the brain level, the left angular gyrus is involved in both phonological as well as in calculation tasks (Göbel, 2015). Dysfunction of brain areas centered in and around the left angular gyrus is found in dyslexia and functional dysconnectivity is mainly found in tasks that require phonological assembly (Pugh, Mencl, Shaywitz, B.A., Shaywitz, S.E., Fulbright, & al. 2000). This region also proved to be sensitive to the level of interference of multiplication problems. De Visscher, Berens, Keidel, Noël, and Bird (2015) compared the brain activation of adults while they were verifying multiplication problems. They contrasted small and large problems and, in each of these categories, problems characterized by a low or high index of interference. They found specific modulation of brain activity according to the level of interference of the problem in the left angular gyrus with greater activation for low interfering problems than for high interfering problems. Further research might try to understand if there is a common underlying process that accounts for this activation in the left angular gyrus for both phonological tasks and problems according to their level of interference.

According to Ansari (2008), the angular gyrus is engaged in general mapping processes. For instance, it is considered to be involved in associating symbols with non-symbolic events, such as written letters with sounds (Booth, Burman, Meyer, Gitelman, Parrish, & Mesulam, 2004). In arithmetic, Grabner, Ansari, Koschutnig, Reishofer and Ebner (2013) suggested that it could mediate the automatic mapping of arithmetic problems onto answers in long-term memory. As the level of activation of the angular gyrus was seen to be affected by the degree of interference of multiplication problems (De Visscher & al., 2015), one possibility would be to consider that this mapping to information stored in memory would vary according to the level of proactive interference in establishing the memory trace. In the particular case of multiplication, the low-interfering problems would lead to a highly automated mapping with their corresponding answers in memory compared to high-interfering problems.

All this work supports the view that similarity between arithmetic facts provokes interference already during the learning stage, and that individuals who are hypersensitive-to-interference struggle in learning those facts. The next and most important question, then, is what can be done to help those individuals with hypersensitivity-to-interference learning arithmetic facts? Clinical research should find ways to improve their ability to resist interference or ways to increase the distinctiveness of the different arithmetic facts so as to decrease similarity-based interference (perhaps by adding differentiating -but irrelevant-features). Further research is needed to answer these questions.

Bibliography

Anderson, J.R. (1974). Retrieval of propositional information from long-term memory. *Cognitive Psychology, 6*, 451-474.

Ansari, D. (2008). Effects of development and enculturation on number representation in the brain. *Nature Reviews Neuroscience*, *9*, 278–291.

Barrouillet P., Thevenot C., (2013). On the problem-size effect in small additions: can we really discard any counting-based account? *Cognition* 128(1), pp. 35-44.

Barrouillet, P., Fayol, M., & Lathulière, E. (1997). Selecting between competitors in multiplication tasks: An explanation of the errors produced by adolescents with learning difficulties. *International Journal of Behavioral Development, 21*, 253-275.

Bellon, E.; Fias, W. & De Smedt, B. (2016). Are individual differences in arithmetic fact retrieval in children related to inhibition? *Frontiers in Psychology, June*, vol 7, Article 825.

Bogaerts, L., Szmalec, A., Hachmann, W.M., Page, M.P.A., Woumans, E. & Duyck, W. (2015). Increased susceptibility to proactive interference in adults with dyslexia ? *Memory* 23, 268–277. doi: 10.1080/09658211.2014. 882957

Booth, J.R., Burman, D.D., Meyer, J.R., Gitelman, D.R., Parrish, T.B., & Mesulam, M.M. (2004). Development of brain mechanisms for processing orthographic and phonologic representations. *Journal of Cognitive Neuroscience, 16*, 1234–1249.

Bull R, Scerif G. (2001). Executive functioning as a predictor of children's mathematics ability: inhibition, switching and working memory. *Developmental Neuropsychology;19(3):*273–93

Bunting, M.F., Conway, A.R.A. & Heitz, R.P. (2004). Individual differences in the fan effect and working memory capacity . *Journal of Memory and Language, 51*, 604-622.

Campbell, J. I. D. (1987a). Network interference and mental multiplication. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *13*(1), 109–123.

Campbell, J. I. D. (1987b). Production, verification, and priming of multiplication facts. *Memory & Cognition, 15*, 349-364.

Campbell, J. I. D. (1995). Mechanisms of simple addition and multiplication: A modified network-interference theory and simulation. *Mathematical Cognition*, *1*(2), 121–164.

Campbell, J. I. D. (1997). On the relation between skilled performance of simple division and multiplication. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *23*(5), 1140–1159.

Campbell, J. I. D., & Graham, D. J. (1985). Mental multiplication skill: Structure, process, and acquisition. *Canadian Journal of Psychology*, *39*, 338–366.

Campbell, J. I. D., & Tarling, D. P. M. (1996). Retrieval processes in arithmetic production and verification. *Memory & Cognition, 24*, 156-172.

Censabella, S. & Noël, M-P. (2005). The inhibition of exogenous information in children with learning disabilities. *Journal of Learning Disabilities*, 38, 400-410.

Censabella, S. & Noël, M-P. (2008). The inhibition capacities of children with mathematical disabilities. *Child Neuropsychology,* 14(1), 1-20.

Censabella, S., & Noël, M. P. (2004). Interference in arithmetic facts: Are active suppression processes involved when performing simple mental arithmetic? *Cahiers de Psychologie Cognitive/ Current Psychology of Cognition, 22*, 635–671.

Conlin JA, Gathercole SE, and Adams JW. (2005) Stimulus similarity decrements in children's working memory span. *Quarterly Journal of Experimental Psychology Section A - Human Experimental Psychology, 58(8):* 1434-1446.

Conrad R and Hull AJ. (1964). Information, acoustic confusion and memory span. *British Journal of Psychology, 55*: 429-432.

Cragg, L. & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education, 3,* 63-68.

De Visscher, A. ; Berens, S.C. ; Keidel, J.L.; Noël, M-P. ; Bird, C.M. (2015). The interference effect in arithmetic fact solving: an fMRI study. *NeuroImage Aug 1;116:92-101. doi: 10.1016/j.neuroimage.2015.04.063*

De Visscher, A., & Noël, M. P. (2013). A case study of arithmetic facts dyscalculia caused by a hypersensitivity-to-interference in memory. *Cortex*, *49*(1), 50–70.

De Visscher, A., & Noël, M.-P. (2014a). Arithmetic facts storage deficit: the hypersensitivity-to-interference in memory hypothesis. *Developmental Science*, 1–9. doi:10.1111/desc.12135

De Visscher, A. & Noël, M-P. (2014b). The detrimental effect of interference in multiplication facts storing: Typical development and individual differences *Journal of Experimental Psychology: General; 143(6), 2380-2400.*

De Visscher, A., Noël, M-P. & De Smedt, B. (2016). The role of physical digit representation and numerical magnitude representation in children's multiplication fact retrieval. *Journal of Experimental Child Psychology, 152*, 41-53.

De Visscher, A., Szmalec, A., Van der Linden, L. & Noël, M-P. (2015). Serial-order learning impairment and hypersensitivity-to-interference. *Cognition*, 144, 38-48.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition, 44*, 1–42.

Espy KA, McDiarmid MM, Cwik MF, Stalets MM, Hamby A, Senn TE (2004). The contribution of executive functions to emergent mathematic skills in pre- school children. *Developmental Neuropsychology; 26(1):*465–86

Garnett, K., & Fleischner, J.E. (1983). Automatization and basic fact performance of normal and learning disabled children. *Learning Disability Quarterly, 6*, 223–230.

Geary, D.C., Hoard, M.K., & Hamson, C.O. (1999). Numerical and arithmetical cognition: patterns of functions and deficits in children at risk for a mathematical disability. *Journal of Experimental Child Psychology, 74(3),* 213–239.

Gilmore C, Attridge N, Clayton S, Cragg L, Johnson S, Marlow N, et al. (2013). Individual differences in inhibitory control, not non-verbal number acuity, correlate with mathematics achievement. *PLoS One;8(6):*e67374

Göbel, S. (2015). Number Processing and Arithmetic in Children and Adults with Reading Difficulties, In: Roi Cohen Kadosh and Ann Dowker, *The Oxford Handbook of Numerical Cognition.*, Oxford Library of Psychology, 2015, p. 680-715.

Hall JF. (1971) Formal intralist response similarity: Its role in paired- associate learning. *American Journal of Psychology, 84(4):* 521-528.

Hasher, L., Tonev, S.T., Lustig, C. & Zacks, R.T. (2001). Inhibitory control, environmental support, and selft-initiated processing in ageing. Chapter 23, p. 286-297, in M. Naveh-Benjamin, M. Moscovictch & H.L. Roediger, (eds.). *Perspectives on human memory and cognitive aging.* Psychology Press, New york.

Jonides J and Nee DE. (2006). Brain mechanisms of proactive interference in working memory. *Neuroscience, 139(1):* 181-193.

Kaufmann, L., Vogel, S. E., Starke, M., Kremser, C., & Schocke, M. (2009). Numerical and non-numerical ordinality processing in children with and without developmental dyscalculia: Evidence from fMRI. *Cognitive Development*, *24*(4), 486–494. doi:10.1016/j.cogdev.2009.09.001

Logan, G.D. (1994). On the ability to inhibit thought and action: A users' guide to the stop signal paradigm. In D. Dagenbach & T. H. Carr (Eds.), *Inhibitory processes in attention, memory, and language* (pp. 189-239). San Diego, CA: Academic Press.

Lyons, I. M., & Beilock, S. L. (2011). Numerical ordering ability mediates the relation between number-sense and arithmetic competence. *Cognition*, *121*(2), 256–61. doi:10.1016/j.cognition.2011.07.009

McCloskey, M., & Lindemann, A.M. (1992). MATHNET: preliminary results from a distributed model of arithmetic fact retrieval. In J. I. D. Campbell (Ed.), *The nature and origins of mathematical skills* (pp. 365-409). Amsterdam: Elsevier.

Monsell S. (1978). Recency, immediate recognition, and reaction time. *Cognitive Psychology, 10*: 465-501.

Oberauer K and Kliegl R. (2006). A formal model of capacity limits in working memory. *Journal of Memory and Language, 55(4):* 601-626.

Oberauer, K., & Lange, E. B. (2008). Interference in verbal working memory: Distinguishing similarity-based confusion, feature overwriting, and feature migration. *Journal of Memory and Language*, *58*(3), 730–745. doi:http://dx.doi.org/10.1016/j.jml.2007.09.006

Pugh, K.R., Mencl, W.E., Shaywitz, B.A., Shaywitz, S.E., Fulbright, R.K., Constable, R. T., ... Gore, J.C. (2000). The angular gyrus in developmental dyslexia: task-specific differences in functional connectivity within posterior cortex. *Psychological Science, 11,* 51–56.

Rubinsten, O., & Sury, D. (2011). Processing ordinality and quantity: the case of developmental dyscalculia. *PloS One*, *6*(9), e24079. doi:10.1371/journal.pone.0024079

St Clair-Thompson HL, Gathercole SE (2006). Executive functions and achievements in school: shifting, updating, inhibition, and working memory. *Quarterly Journal of Experimental Psychology; 59(4)*:745–59

Underwood BJ. (1983). Attributes of Memory. Glenview, IL: Scott, Foresman.

van der Sluis, S., de Jong, P.F., & van der Leij, A. (2004). Inhibition and shifting in children with learning deficits in arithmetic and reading. *Journal of Experimental Child Psychology, 87*, 239- 266.

Verguts, T., & Fias, W. (2005). Interacting neighbours: A connectionist model of retrieval in single-digit multiplication. *Memory & Cognition, 36*, 1-16.

Zbrodoff, N.J. & Logan, G.D. (2005). What everyone finds: The problem-size effect. In J. I. D. Campbell, ed. *Handbook of Mathematical Cognition*. Psychology Press, New York, p. 331-346.

Figure caption

Figure 1

Percentage of correct responses for DB and the control participants in the recent probe task for the positive, negative and negative lure trials.

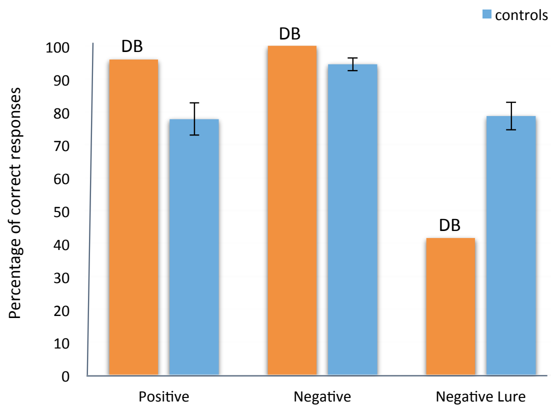
Figure 2

Percentage of accuracy for associations with low and high interference in both typical children and children with low arithmetic fluency.

Figure 3

Mean correct binding (or correct succession of two syllables) in the Hebb paradigm for the interfering and non-interfering repeated sequences in the three groups of participants: controls, with global dyscalculia and with specific problems in arithmetic facts.

Figure 1



controls
DB
DB
DB
Percentage of correct responses
100
90
80
70
60
50
40
30
20
10
0
Positive
Negative
Negative Lure

Figure 2

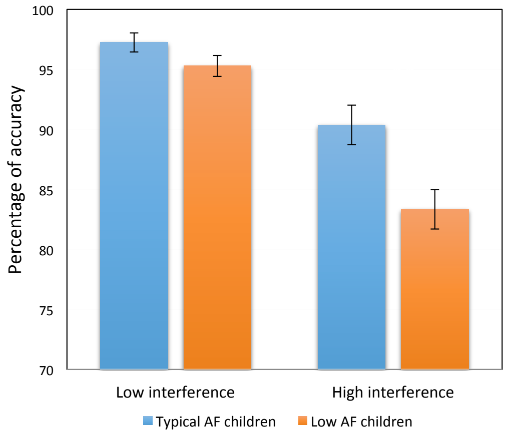
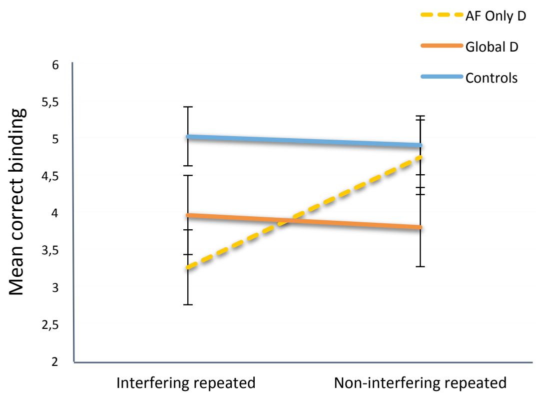

100
95
90
85
80
75
70
Percentage of accuracy
Low interference
High interference
Typical AF children
Low AF children

Figure 3


AF Only D
Global D
Controls
Mean correct binding
6
5,5
5
4,5
4
3,5
3
2,5
2
Interfering repeated
Non-interfering repeated