Delft University of Technology
Lecture for the course CT5312
23 May 2003

Some aspects of turbulence closure schemes for marine models

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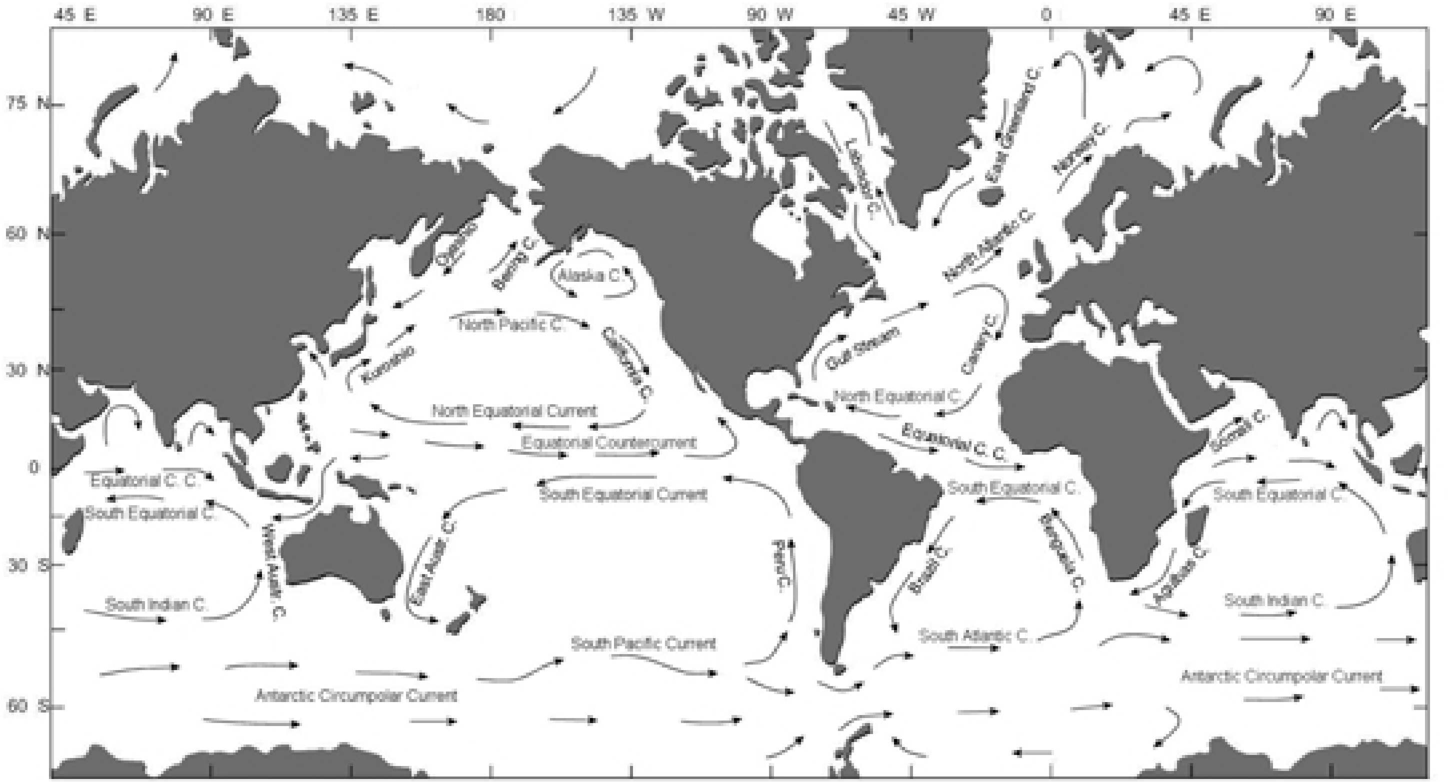
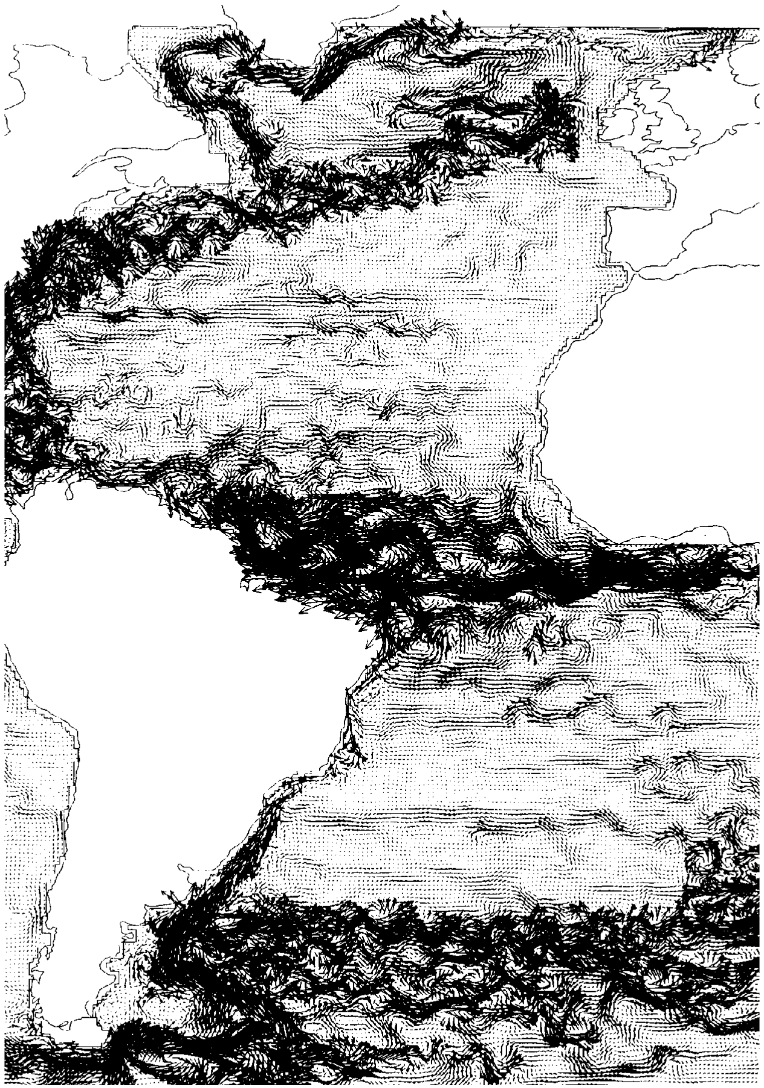
Marine/oceanic variability (I)



A classic sketch of the surface ocean general circulation...

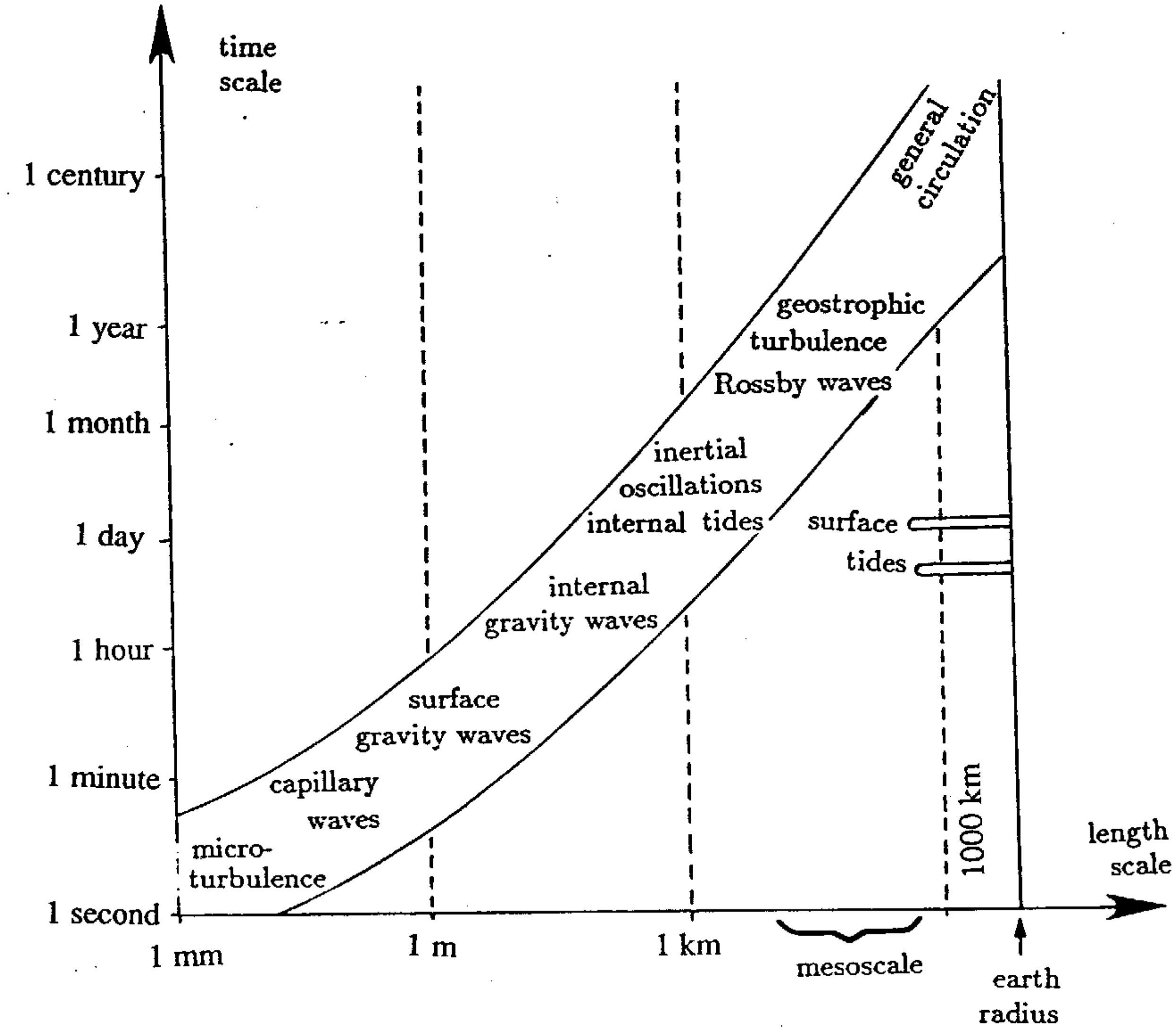
Marine/oceanic variability (II)



But, if we have a closer look at the ocean surface velocity field...

Marine/oceanic variability (III)

The range of time and space scales of marine/oceanic processes is enormous!



Need for closure

- It is impossible to resolve all time-scales of motion. Therefore, it is necessary to delineate the border between the phenomena which will be represented explicitly, i.e. resolved, and those which will not be resolved.

- Subgrid-scale, or unresolved, motions lead to an additional flux:

$$\frac{\partial\psi}{\partial t} \;=\; s \;-\; \nabla\bullet\left[\mathbf{u}\psi - \kappa\nabla\psi + \langle\mathbf{u}'\psi'\rangle\right]$$

where $\kappa \lesssim 10^{-6}\,\mathrm{m}^2\mathrm{s}^{-1}$ is the molecular diffusivity, and $\langle\ \rangle$ is an appropriate low-pass filter, filtering out subgrid-scale motions.

- The additional flux $\langle\mathbf{u}'\psi'\rangle$ has to be expressed/parameterised in terms of resolved variables, which is not that easy...

Fourier-Fick parameterisations

• Subgrid-scale motions are generally assumed to lead to mixing. Hence, Fourier-Fick parameterisations are often used:

$$\langle \mathbf{u}'\psi' \rangle = - \mathbf{K} \bullet \nabla\psi ,$$

where **K** is the diffusivity tensor, which needs to be symmetric and positive-definite.

• Turbulent fluxes are generally much larger than molecular diffusion fluxes: $|\mathbf{K}| >> \kappa$.

• 3D turbulence ($T \approx 10^0 - 10^2$ s, $L \approx 10^{-3} - 10^0$ m) is associated with motions that are — very — roughly speaking isotropic. Therefore, the relevant parameterisations simplifies to $\langle \mathbf{u}'\psi' \rangle = -\lambda\nabla\psi$, where the eddy diffusivity is $\lambda \approx 10^{-4} - 10^{-1}\,\mathrm{m}^2\mathrm{s}^{-1}$.

FAQs (I)

1. Why must tensor **K** be symmetric?

Answer: Formulate **K** as the sum of its symmetric part, $\mathbf{K}_s$, and anti-symmetric part, $\mathbf{K}_a$, so that $\mathbf{K} = \mathbf{K}_s + \mathbf{K}_a$. The divergence of the subgrid-scale flux reads:

$$\nabla \bullet (-\mathbf{K} \bullet \nabla\psi) = \nabla \bullet (-\mathbf{K}_s \bullet \nabla\psi) + \nabla \bullet (-\mathbf{K}_a \bullet \nabla\psi) \ .$$

As one always has $-\mathbf{K}_a \bullet \nabla\psi = \mathbf{v} \times \nabla\psi$, where **v** is an appropriate vector, the anti-symmetric part of the divergence above is, in fact, an advection-like term:

$$\nabla \bullet (-\mathbf{K}_a \bullet \nabla\psi) = \nabla \bullet (\mathbf{v} \times \nabla\psi) = (\nabla \times \mathbf{v}) \bullet \nabla\psi \ .$$

This has nothing to do with diffusion. QED.

(See e.g. Griffies 1998, and Deleersnijder et al. 2001)

FAQs (II)

2. Why must tensor **K** be positive definite?

Answer: Consider domain Ω limited by surface $\delta\Omega$, which is impermeable. If $\mathbf{n}$ is the outward unit normal to $\delta\Omega$, the impermeability conditions read:

$$\text{on } \delta\Omega : \quad \mathbf{u} \bullet \mathbf{n} = 0 \;\; \text{and} \;\; (\mathbf{K} \bullet \nabla\psi) \bullet \mathbf{n} = 0 \; .$$

If ψ is neither produced not destroyed (s=0), the evolution of ψ is governed by equation

$$\frac{\partial\psi}{\partial t} \; = \; -\nabla \bullet (\mathbf{u}\psi - \mathbf{K} \bullet \nabla\psi) \; .$$

As there is no production/destruction and incoming/outgoing fluxes, the domain-averaged value of ψ must remain constant:

FAQs (III)

$$\frac{d\overline{\psi}}{dt} = \frac{d}{dt}\left(\frac{1}{\Omega}\int_{\Omega}\psi d\mathbf{x}\right) = -\frac{1}{\Omega}\int_{\delta\Omega}(\mathbf{u}\psi - \mathbf{K}\bullet\nabla\psi)\bullet\mathbf{n}\ \ d\mathbf{x}^{\delta\Omega} = 0\ .$$

Then, the variance of ψ obeys

$$\frac{d}{dt}\left[\frac{1}{\Omega}\int_{\Omega}(\psi - \overline{\psi})^2\,d\mathbf{x}\right] = -\frac{2}{\Omega}\int_{\Omega}\nabla\psi\bullet\mathbf{K}\bullet\nabla\psi\ \ d\mathbf{x}\ .$$

If the subgrid-scale flux has a mixing/diffusive effect, the variance of ψ should decrease monotonically. This is guaranteed if and only if $\nabla\psi\bullet\mathbf{K}\bullet\nabla\psi < 0$ for any $\nabla\psi \neq 0$. Thus, **K** must be positive definite. QED.

FAQs (IV)

3. <u>Why is horizontal diffusion related to 3D turbulence negligeable</u>?

Answer: Most marine/oceanic processes have a small aspect ratio:

$$\frac{\text{vertical length scale}}{\text{horizontal length scale}} = \frac{L_v}{L_h} \ll 1 .$$

Vertical and horizontal diffusion terms are such that

$$\frac{\dfrac{\partial}{\partial z}\left(\lambda \dfrac{\partial \psi}{\partial z}\right)}{\nabla_h \bullet (\lambda \nabla_h \psi)} \approx \left(\frac{L_v}{L_h}\right)^{-2} \gg 1 .$$

Therefore, for small-aspect ratio flows, i.e. for most flows addressed in marine/oceanic studies, horizontal diffusion parameterising the impact of 3D turbulence may be neglected. QED.

FAQs (V)

4. Why is a horizontal diffusion term needed?

Answer: If 3D turbulence is the only subgrid-scale phenomenon, then the grid of the numerical model is ≈ isotropic, with grid sizes of the order $\Delta x \approx \Delta y \approx \Delta z \approx 10^0 - 10^1\,\mathrm{m}$. In most models, the grid is not isotropic: $\Delta x \approx \Delta y >> \Delta z$.

So, there is a range of motions which are essentially “horizontal”, have a small aspect ratio and are unresolved. Their impact on the resolved flow must be parameterised: horizontal diffusion is often resorted to, with diffusivity $\lambda_h >> \lambda$, leading to generic equation:

$$\frac{\partial \psi}{\partial t} = s - \nabla \bullet (\mathbf{u}\psi) + \nabla_h \bullet (\lambda_h \nabla_h \psi) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial \psi}{\partial z}\right).$$

QED.

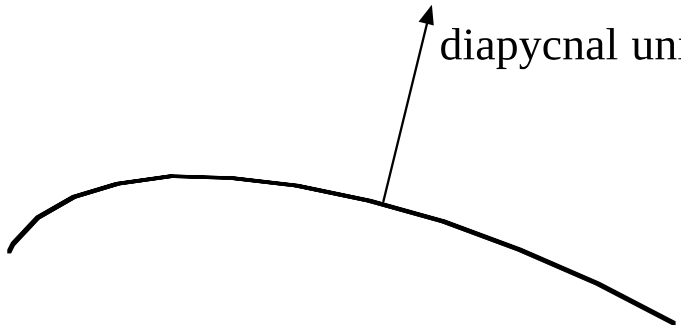
FAQs (VI)

5. <u>What is isopycnal diffusion</u>?

Answer: For horizontal-vertical diffusion, the diffusivity tensor is

$$\mathbf{K} = \begin{pmatrix} \lambda_h & 0 & 0 \\ 0 & \lambda_h & 0 \\ 0 & 0 & \lambda \end{pmatrix}.$$

This model is not appropriate for representing the impact of meso-scale eddies. For a model which does not resolve these eddies, there should be isopycnal-diapycnal diffusion.



(See e.g. Redi, 1982)

FAQs (VII)

Two diffusivities are needed: K_I, the isopycnal diffusivity, and K_D, the diapycnal diffusivity, with $\lambda_h \approx K_I >> K_D \approx \lambda$. Then, the difusivity tensor is

$$\mathbf{K} = K_I(\mathbf{I} - \mathbf{ee}) + K_D\mathbf{ee} ,$$

or

$$\mathbf{K} = \frac{K_I}{\rho_x{}^2 + \rho_y{}^2 + \rho_z{}^2} \begin{pmatrix} \rho_z{}^2 + \rho_y{}^2 + \frac{K_D}{K_I}\rho_x{}^2 & \left(\frac{K_D}{K_I} - 1\right)\rho_x\,\rho_y & \left(\frac{K_D}{K_I} - 1\right)\rho_x\rho_z \\ \left(\frac{K_D}{K_I} - 1\right)\rho_x\rho_y & \rho_z{}^2 + \rho_x{}^2 + \frac{K_D}{K_I}\rho_y{}^2 & \left(\frac{K_D}{K_I} - 1\right)\rho_y\rho_z \\ \left(\frac{K_D}{K_I} - 1\right)\rho_x\rho_z & \left(\frac{K_D}{K_I} - 1\right)\rho_y\rho_z & \rho_x{}^2 + \rho_y{}^2 + \frac{K_D}{K_I}\rho_z{}^2 \end{pmatrix},$$

with $\rho_x = \dfrac{\partial\rho}{\partial x}$, $\rho_y = \dfrac{\partial\rho}{\partial y}$, and $\rho_z = \dfrac{\partial\rho}{\partial z}$.

FAQs (VIII)

Let the diapycnal and isopycnal gradient operators be defined as

$$\nabla_D = \mathbf{e}(\mathbf{e} \bullet \nabla \quad \text{and} \quad \nabla_I = \nabla - \nabla_D \;.$$

Hence, the diffusive flux may be split easily into its isopycnal and diapycnal components:

$$- \; \mathbf{K} \bullet \nabla\psi \;\; = \; - \;\; K_I \bullet \nabla_I \psi \; - \;\; K_D \bullet \nabla_D \psi \;.$$

The isopycnal-diapycnal diffusivity tensor is symmetric: this is obvious — no need for a demonstration. It is also positive-definite as, for any vector $\mathbf{y} \neq 0$, one has

$$\mathbf{y} \bullet \mathbf{K} \bullet \mathbf{y} \;\; = \;\; K_I \left|\mathbf{y} \times \mathbf{e}\right|^2 \; + \; K_D (\mathbf{y} \bullet \mathbf{e})^2 \;\; > \;\; 0 \;,$$

if $K_I, K_D > 0$.

Note: the isopycnal diffusion operator generally is applied to scalar variables only.

How to estimate the eddy coefficients (I)

• The diffusivity λ depends of the flow. There are many recipes for estimating eddy coefficients. Usually, there is a difference between the eddy viscosity (momentum), K_M, and the eddy diffusivity (scalar variables), K_H.

• The simplest approach: *constant eddy coefficients*. Too simple: no way to account for the mechanisms producing and destroying turbulence. However, it is used for idealised studies in order to obtain an analytical solution.

• Taking eddy coefficients as *prescribed functions of time and position*, which usually reduce to functions of the vertical coordinate only (e.g. Bryan and Lewis, 1979). Too simple, but still used in large-scale ocean models with coarse time and space resolution.

How to estimate the eddy coefficients (II)

• To take into account in a simple way the impact of stratification, which tends to inhibit turbulence, the eddy coefficients can be *decreasing functions of the Richardson number*,

$$Ri = \frac{\dfrac{\partial b}{\partial z}}{\left|\dfrac{\partial \mathbf{u}_h}{\partial z}\right|^2} = \frac{\dfrac{\partial b}{\partial z}}{\left(\dfrac{\partial u}{\partial z}\right)^2 + \left(\dfrac{\partial v}{\partial z}\right)^2} ,$$

where b is the buoyancy

$$b = -g(\rho - \rho_0)/\rho_0 ,$$

with g the gravitational acceleration, and ρ_0 a constant, reference value of the density.

How to estimate the eddy coefficients (III)

The Munk-Anderson (1949) closure is as follows:

$$K_M = \frac{K_M^0}{(1+\beta_M Ri)^{\alpha_M}} \quad \text{and} \quad K_H = \frac{K_H^0}{(1+\beta_H Ri)^{\alpha_H}} .$$

Commonly used values are: $\alpha_M = 1/2$, $\beta_M = 10$, $\alpha_H = 3/2$, and $\beta_H = 10/3$. The neutral values, K_M^0 and K_H^0, are obtained for zero stratification and must be specified arbitrarily.

The Pacanowski-Philander (1981) closure is as follows:

$$K_M = \frac{K_M^0}{(1+\beta_M Ri)^{\alpha_M}} + K_M^b \quad \text{and} \quad K_H = \frac{K_M}{1+\alpha Ri} + K_H^b .$$

Commonly used values are: $\alpha_M = 2$ and $\beta_M = 5$. Positive constants K_M^0, K_M^b and K_H^b must be specified arbitrarily.

How to estimate the eddy coefficients (IV)

• *One-equation turbulence closure scheme* aka “k-model”: every eddy coefficient is of the form

$$K_X = lqS_X \ ,$$

where:

l is the turbulence macro-scale $\propto$ size of the energy-containing eddies,

$q^2/2$ is the turbulent kinetic energy,

S_X is an O(1) dimensionless function, termed “stability function”, in the Mellor-Yamada (1982) approach.

The product lS_X is derived from an empirical formula, which is often a function of position and a measure of stratification. Many expressions exist. All of them are consistent with the log. layer, i.e. lS_X is a linear function of the distance to the wall near a wall.

How to estimate the eddy coefficients (V)

The turbulent kinetic energy (TKE) is governed by an evolution equation taking into account key-phenomena in the TKE budget:

$$\frac{\partial}{\partial t}\frac{q^2}{2} = K_M \left|\frac{\partial \mathbf{u}_h}{\partial z}\right|^2 - K_H \frac{\partial b}{\partial z} - \varepsilon + (\mathit{transport\ \ terms}) \,,$$

where $K_M \left|\frac{\partial \mathbf{u}_h}{\partial z}\right|^2$: energy transfer from the mean flow,

$K_H \frac{\partial b}{\partial z}$: TKE conversion into potential energy,

ε : viscous dissipation of TKE, parameterised as $\propto q^3 /(lS_q)$,

the *transport terms* often simplify to $\frac{\partial}{\partial z}\left(K_q \frac{\partial q^2}{\partial z} \right)$.

How to estimate the eddy coefficients (VI)

• *Two-equation turbulence closure scheme*: same as the one-equation, except that there is a second evolution equation from which l is derived.

There is no consensus as to what the second equation should be:

- In the k-ε model, there is an equation for ε (see Burchard, 2002, for state-of-art review and references);
- In the k-kl or Mellor-Yamada level 2.5 model, the unknown of the second equation is $q^2 l$;
- Other options are possible, such as the k-ω model, in which the second equation is for a frequency ω (e.g. Umlauf et al., 2003).

All these models may be seen as members of the same family (Launder and Spalding, 1974).

The Mellor-Yamada level 2.5 scheme (I)

- The viscous dissipation of turbulence kinetic energy is parameterised as $\varepsilon = \dfrac{q^2}{16.6l}$.

- The second equation of the Mellor-Yamada level 2.5 turbulence closure scheme is as follows (Yamada, 1977):

$$\frac{\partial}{\partial t}(q^2 l) = 1.8 l K_M \left|\frac{\partial \mathbf{u}_h}{\partial z}\right|^2 - 1.8 l K_H \frac{\partial b}{\partial z} - \frac{W q^3}{16.6} + (\textit{transport terms})$$

where the *transport terms* often simplify to $\dfrac{\partial}{\partial z}\left(K_q \dfrac{\partial q^2}{\partial z} \right)$,

$K_q = lqS_q$, with $S_q = 0.2$,

W is the so-called “wall proximity function”.

The Mellor-Yamada level 2.5 scheme (II)

• The wall proximity function is a dimensionless *non-local* term. A similar term is needed in all the models belonging to the family of the two-equation turbulence closure schemes, except the k-ε model, which can represent a log. layer without the introduction of a non-local term.

• The original stability functions for momentum and scalar variables are (Mellor and Yamada, 1982):

$$S_M = \frac{0.698 - 9.288 G_H}{1 - 36.71 G_H + 5.078 G_M + 187.387 G_H^2 - 88.814 G_H G_M},$$

$$S_H = \frac{0.730 - 4.533 G_H + 0.907 G_M}{1 - 36.71 G_H + 5.078 G_M + 187.387 G_H^2 - 88.814 G_H G_M},$$

with $G_M = l^2 u_z^2 / q^2$ and $G_H = -l^2 b_z / q^2$.

The Mellor-Yamada level 2.5 scheme (III)

• Alternative stability functions have been proposed, such as Galperin's et al (1988) quasi-equilibrium ones:

$$S_M = \frac{0.393 - 3.09 G_H}{1 - 40.8 G_H + 212 G_H^2} \quad \text{and} \quad S_H = \frac{0.494}{1 - 34.7 G_H} .$$

There are also those of Kantha and Clayson (1994), or Canuto et al. (2001).

• Some stability functions seem to lead to unstable solutions, in which spurious oscillations arise. See, for instance, Deleersnijder and Luyten (1994), Burchard and Deleersnijder (2001), Mellor (2003), or Deleersnijder and Burchard (2003). In this respect, a tentative stability analysis is outlined below.

The Kato-Phillips test case (I)

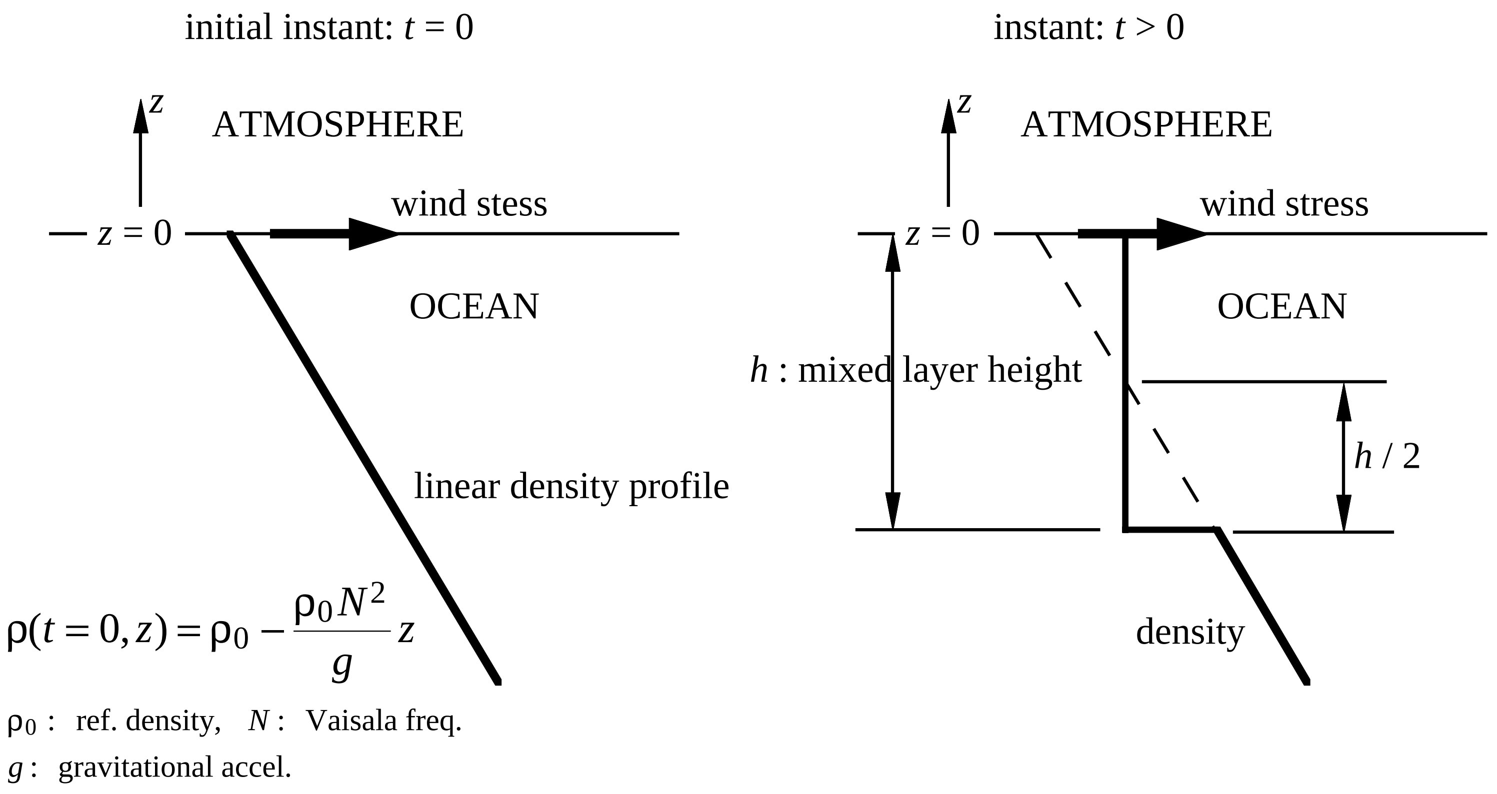
• Above, several turbulence closure schemes of various levels of complexity were presented. How to select the *appropriate level of complexity*? There is no universally-accepted answer; the choice depends on the type of flow under consideration and the purposes of the study.

• Keep in mind that the skills of the model also depends on its numerics. There is now a largely-accepted approach to the discretisation of two-equations turbulence closure schemes (see Burchard, 2002, and http://www.gotm.net).

• A widely-used test case for stratified flow models is that of Kato and Phillips (1969).

The Kato-Phillips test case (II)

• Kato and Phillips conducted laboratory experiments on the stress-driven penetration of a turbulent layer into stratified fluid initially motionless, which may be re-scaled to become relevant to the dynamics of the upper layers of the ocean.



The Kato-Phillips test case (III)

- Model equations:

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial z}\left(K_M \frac{\partial u}{\partial z}\right) \quad \text{and} \quad \frac{\partial b}{\partial t} = \frac{\partial}{\partial z}\left(K_H \frac{\partial b}{\partial z}\right).$$

- Initial values: $u(t=0,z)=0$, $b(t=0,z)=N^2 z$.

- Boundary conditions:

$$K_M \partial u / \partial z]_{z=0} = u_*^2 \ , \quad [K_H \partial b / \partial z]_{z=0} = 0 \ .$$

- Price's (1979) mixed layer growth : $h(t) = (6/5)^{1/4} u_* N^{-1/2} t^{1/2}$.

$\Rightarrow \dfrac{1}{h}\dfrac{dh}{dt} = \dfrac{1}{2t}$: growth rates dimishes as the density contrast at the bottom of the mixed layer increases, since the work to be done to entrain unperturbed water into the mixed layer increases.

The Kato-Phillips test case (IV)

- Model results:

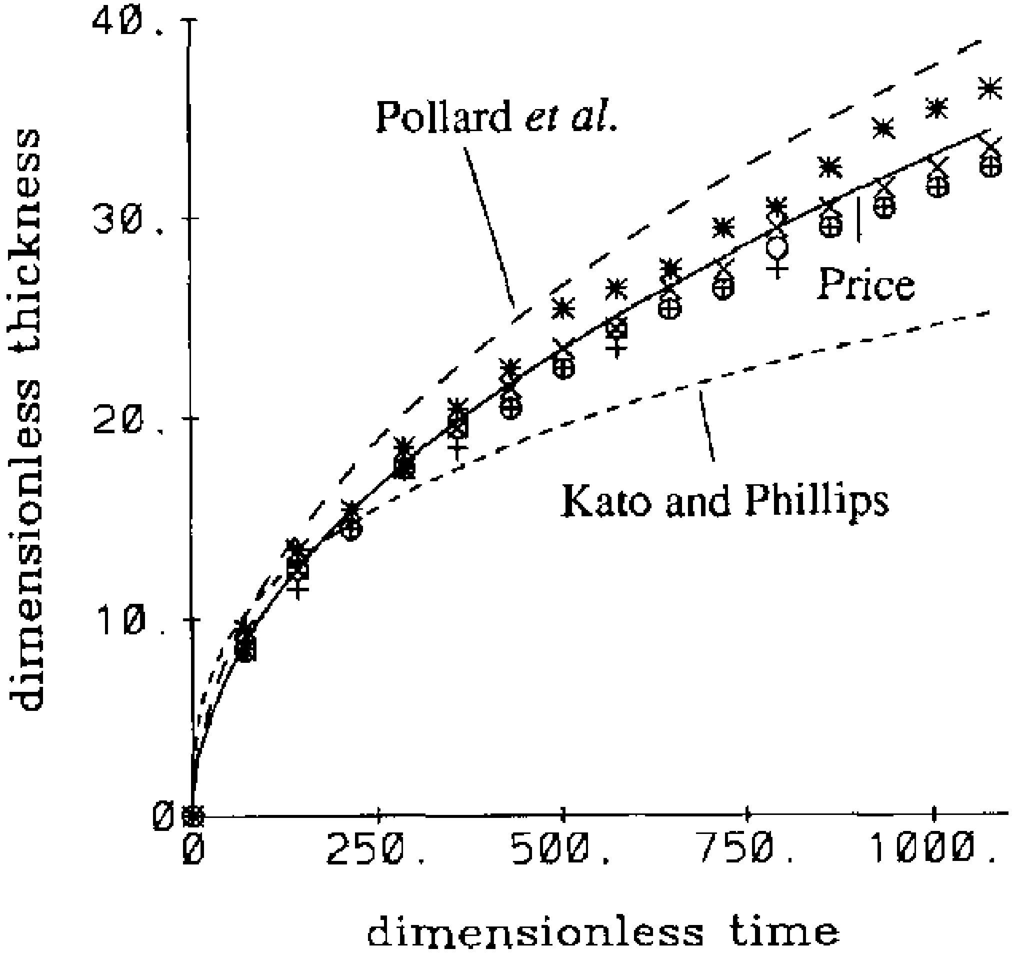
  * Constant or prescribed-profile eddy coefficients fail.

  * Richardson-dependent eddy coefficients can be calibrated so as to produced a mixed layer depth deepening similar to Price's. However, a version calibrated for a given wind stress fails for another wind stress. Validation is thus impossible.

  * One-equation models are about right, but the formulation of the length scale *l* remains tricky.

  * Two-equation models perform well, without any tuning.

The Kato-Phillips test case (V)



(from Deleersnijder and Luyten, 1994)

On the stability of the solutions (I)

• A salient feature of turbulent parameterisations is that the eddy coefficients may depend on the gradient of some of the unknowns — which would not occur with molecular diffusivities only.

• In the domain $t \geq 0$ and $z_{\min} \leq z \leq z_{\max}$, this suggests examining first a highly idealised water column model:

$$\frac{\partial\psi}{\partial t} = \frac{\partial}{\partial z}\left(\lambda \frac{\partial\psi}{\partial z}\right)$$

with $\lambda(\psi_z) > 0$, $\left|\psi(t=0,z)\right| < \infty$, $\psi_z(t, z_{\min}) = 0 = \psi_z(t, z_{\max})$.

The eddy coefficient is assumed to be a function the vertical gradient of the unknown, i.e. $\psi_z = \partial\psi / \partial z$.

On the stability of the solutions (II)

• The solution is bounded as

$$\frac{d}{dt}\int_{z_{\min}}^{z_{\max}} \psi^2 \; dz = -\; 2 \int_{z_{\min}}^{z_{\max}} \lambda \psi_z^2 \; dz \;\leq\; 0 \,.$$

However, a global measure of the gradient of the solution,

$$\frac{d}{dt}\int_{z_{\min}}^{z_{\max}} \psi_z^2 \; dz = -\; 2 \int_{z_{\min}}^{z_{\max}} \tilde{\lambda} \psi_{zz}^2 \; dz \,,$$

may well increase, if the effective diffusivity, $\tilde{\lambda} = \dfrac{\partial \lambda}{\partial \psi_z}\psi_z + \lambda$, is not positive definite.

• This suggests the possible existence — at least for some time — of **bounded solutions with small-scale oscillations**.

On the stability of the solutions (III)

• Now, consider a generic water column, i.e. horizontally homogeneous, model:

$$\frac{\partial \psi_m}{\partial t} = s_m + \frac{\partial}{\partial z}\left(\lambda_m \frac{\partial \psi_m}{\partial z}\right), \qquad m = 1, 2, \ldots M ,$$

with: s_m is source/sink term;

for instance: $\psi_1 = u$ (velocity), $\psi_2 = b$ (buoyancy), etc.

• If $\hat{\psi}(t,z)$ denotes a small-amplitude perturbation of solution $\psi(t,z)$, then linearised equations may be derived:

$$\frac{\partial \hat{\psi}_m}{\partial t} = \sum_{n=1}^{M} \left(A_{m,n} \hat{\psi}_n + B_{m,n} \frac{\partial \hat{\psi}_n}{\partial z} + C_{m,n} \frac{\partial^2 \hat{\psi}_n}{\partial z^2} \right),$$

where matrices A, B and C only depend on $\psi(t,z)$ and its derivatives — and are independent of $\hat{\psi}(t,z)$.

On the stability of the solutions (IV)

• Only matrix C is important: $C_{m,n} \; = \; \dfrac{\partial \lambda_m}{\partial \psi_{n,z}} \psi_{m,z} \; + \; \lambda_m \delta_{m,n}$.

• If the time- and space-scales of the perturbations $\hat{\psi}(t, z)$ are much smaller than those associated with $\psi(t, z)$, the equations governing the evolution of the perturbation may be simplified to

$$\frac{\partial \hat{\psi}_m}{\partial t} \; = \; \sum_{n=1}^{M} C_{m,n} \frac{\partial^2 \hat{\psi}_n}{\partial z^2} \, ,$$

where C may be regarded as locally constant. Then, for the perturbation to be stable, it is necessary that **the real part of the eigenvalues of *C* be positive**. The eigenvalues of C may be regarded as effective diffusivities.

• Consider the Mellor-Yamada level 2.5 model.

On the stability of the solutions (V)

- There is an evolution equation for $\psi_3 = q^2$ and $\psi_4 = q^2 l$. There are different versions of the closure, depending only on the expression of S_M and S_H. In all versions, $\lambda_3 = \lambda_4 = K_q = lqS_q$, with $S_q = 0.2$.

- Matrix C is:

$$C = lq \begin{pmatrix} 2\dfrac{\partial S_M}{\partial G_M} G_M + S_M & \dfrac{\partial S_M}{\partial G_H}\dfrac{G_H u_z}{b_z} & 0 & 0 \\ 2\dfrac{\partial S_H}{\partial G_M}\dfrac{G_M b_z}{u_z} & \dfrac{\partial S_H}{\partial G_H} G_H + S_H & 0 & 0 \\ 0 & 0 & S_q & 0 \\ 0 & 0 & 0 & S_q \end{pmatrix}.$$

On the stability of the solutions (VI)

• There are two identical, positive eigenvalues, lqS_q, which lead to no stability problem. Of interest is the reduced stability matrix:

$$C_r \;=\; lq \begin{pmatrix} 2\,\dfrac{\partial S_M}{\partial G_M}\,G_M + S_M & \dfrac{\partial S_M}{\partial G_H}\,\dfrac{G_H u_z}{b_z} \\ 2\,\dfrac{\partial S_H}{\partial G_M}\,\dfrac{G_M b_z}{u_z} & \dfrac{\partial S_H}{\partial G_H}\,G_H + S_H \end{pmatrix}.$$

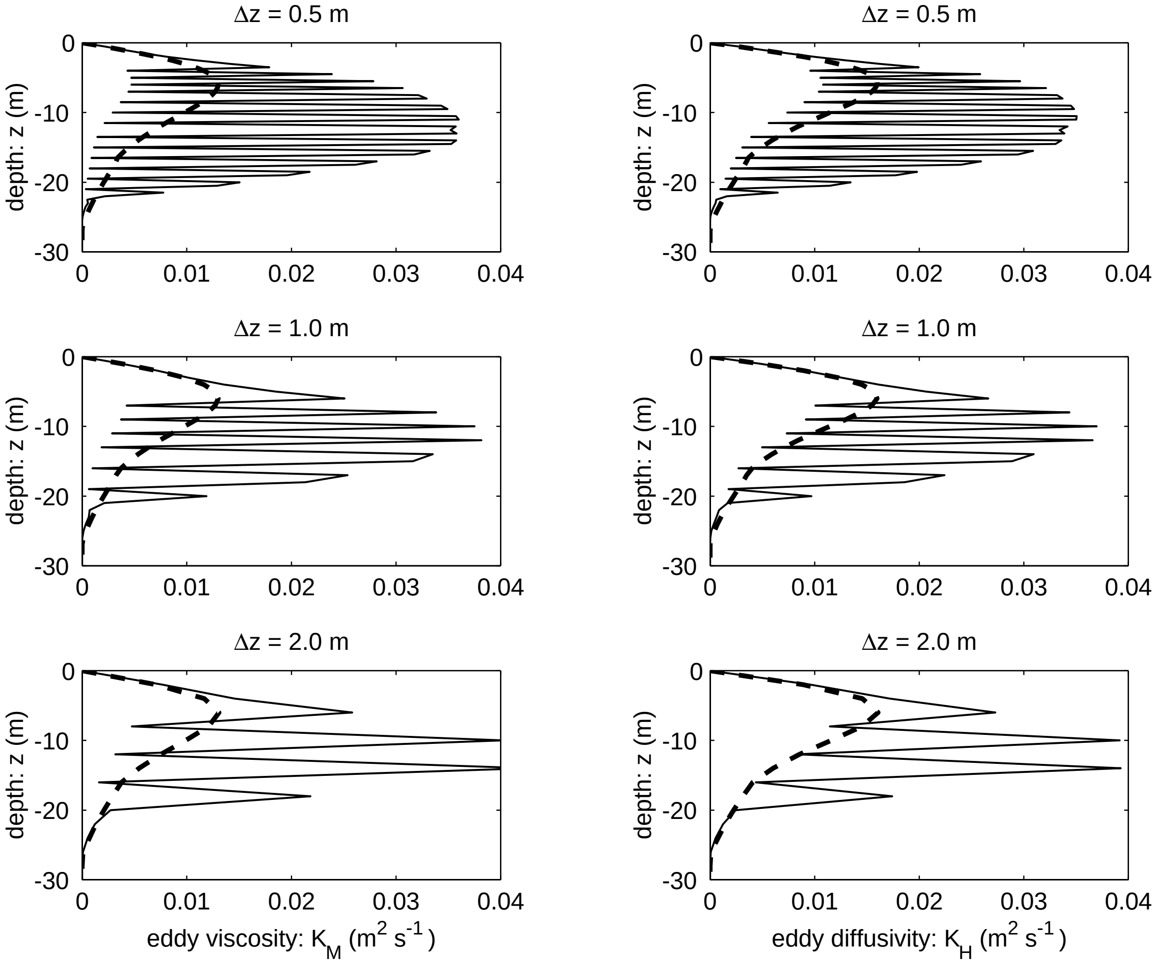
For the original version of the closure, the eigenvalues of the reduced stability matrix may be negative, implying that instabilities may occur.

For Galperin's quasi-equilibrium version, the eigenvalues are always positive, so that the solution should be stable.

On the stability of the solutions (VII)

• Numerical results after 20 hours: dotted curve (quasi-equilibrium closure) and solid curve (original closure).

Spurious oscillations are **grid-dependent**, as it is the smallest-scale perturbations that are the most unstable.



On the stability of the solutions (VIII)

- The spurious oscillations are not a numerical artefact, though they are largely influenced by the details of the discretisation.

- The stability criterion suggested above seems to be relevant, though it rests on many simplifying hypotheses. It is further supported by an earlier numerical sensitivity study (Deleersnijder and Luyten, 1994), indicating that the spurious oscillations are sensitive mostly to modifications of the stability functions in the diffusive terms, i.e. modifying elsewhere the stability functions does not cause spurious oscillations.

- It is suggested that any new turbulence closure development take into account the stability criterion developed above.

A 3D model sensitivity analysis in the Rhine plume region (I)

(From Ruddick et al., 1995)

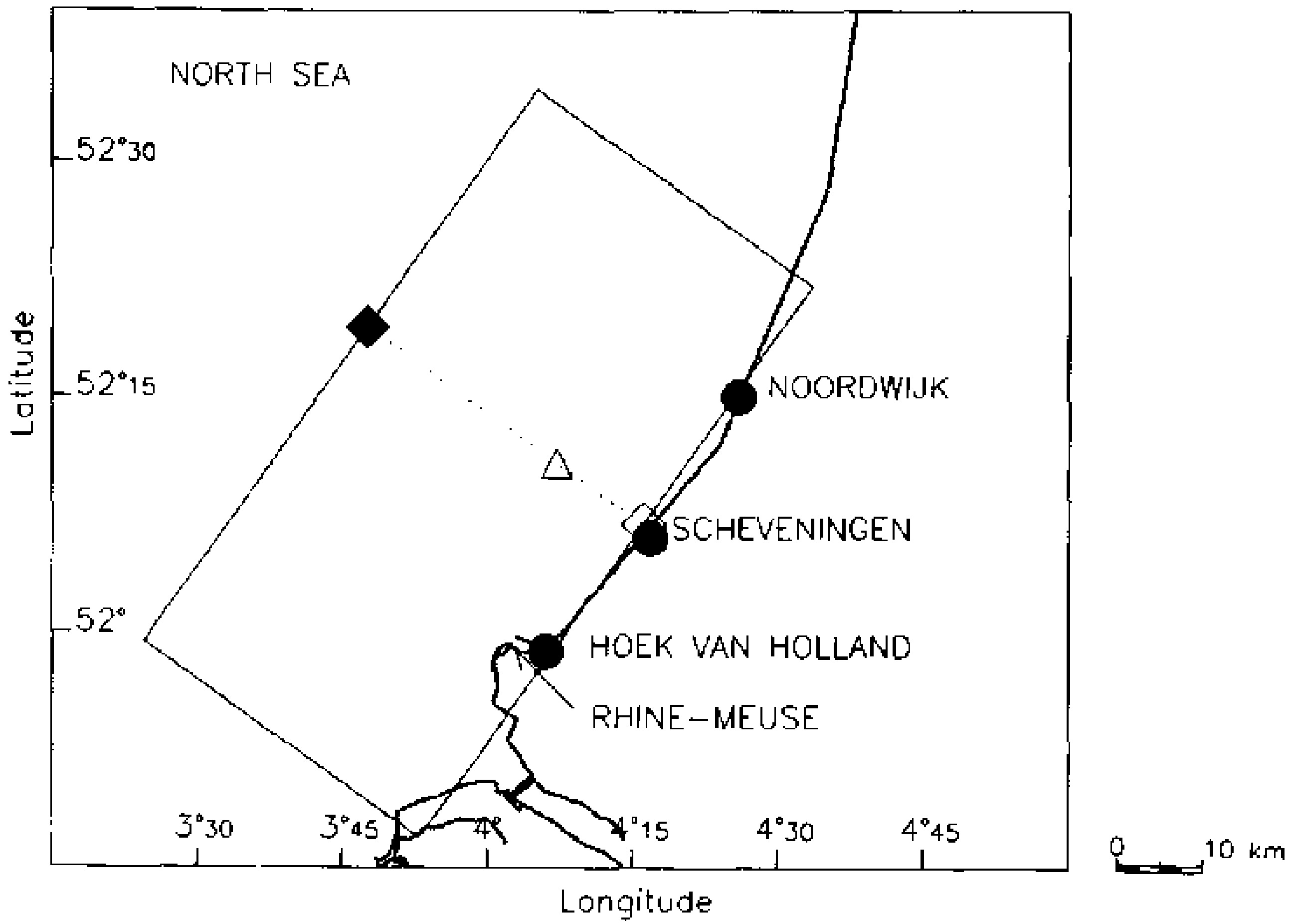
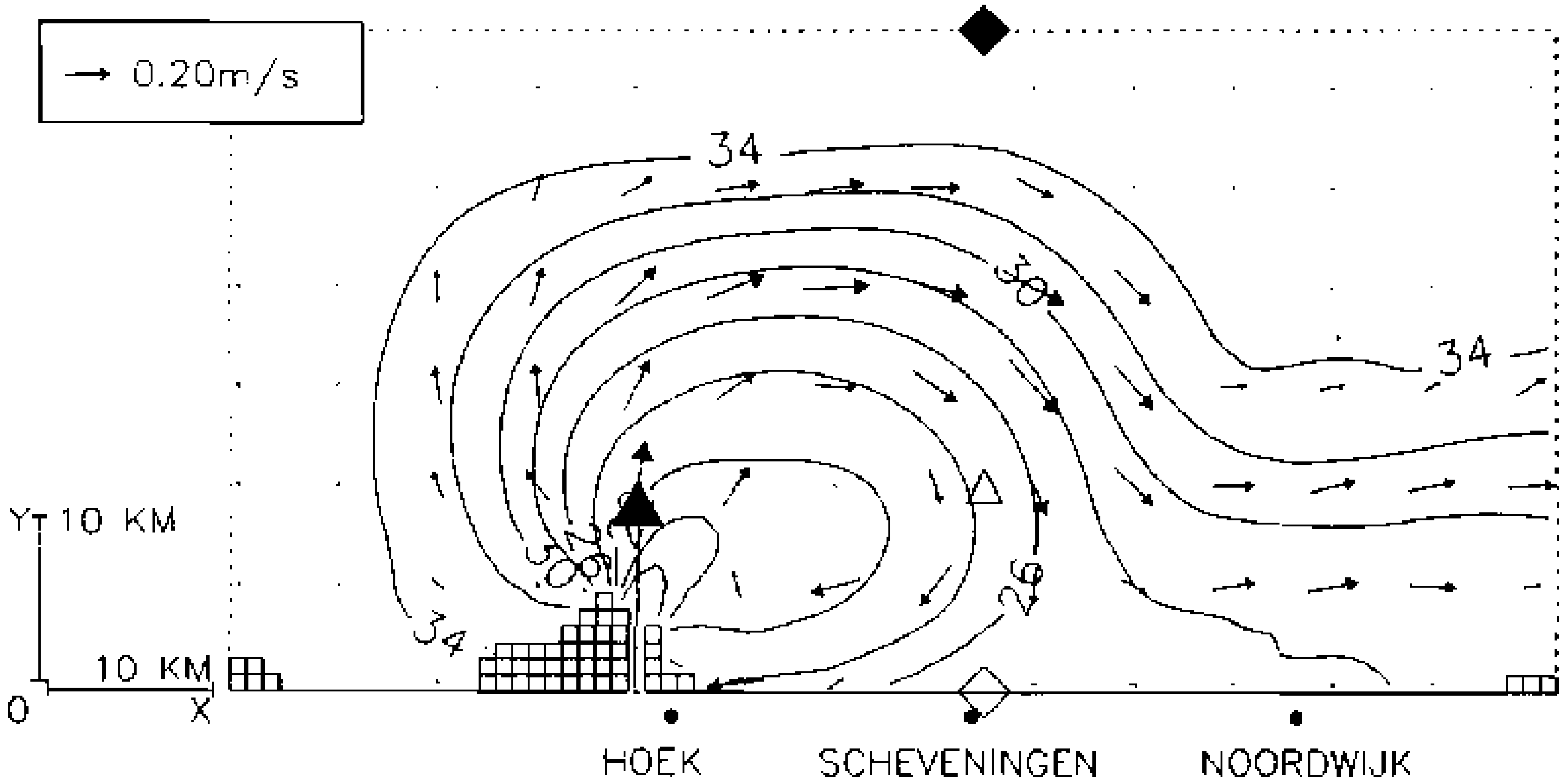


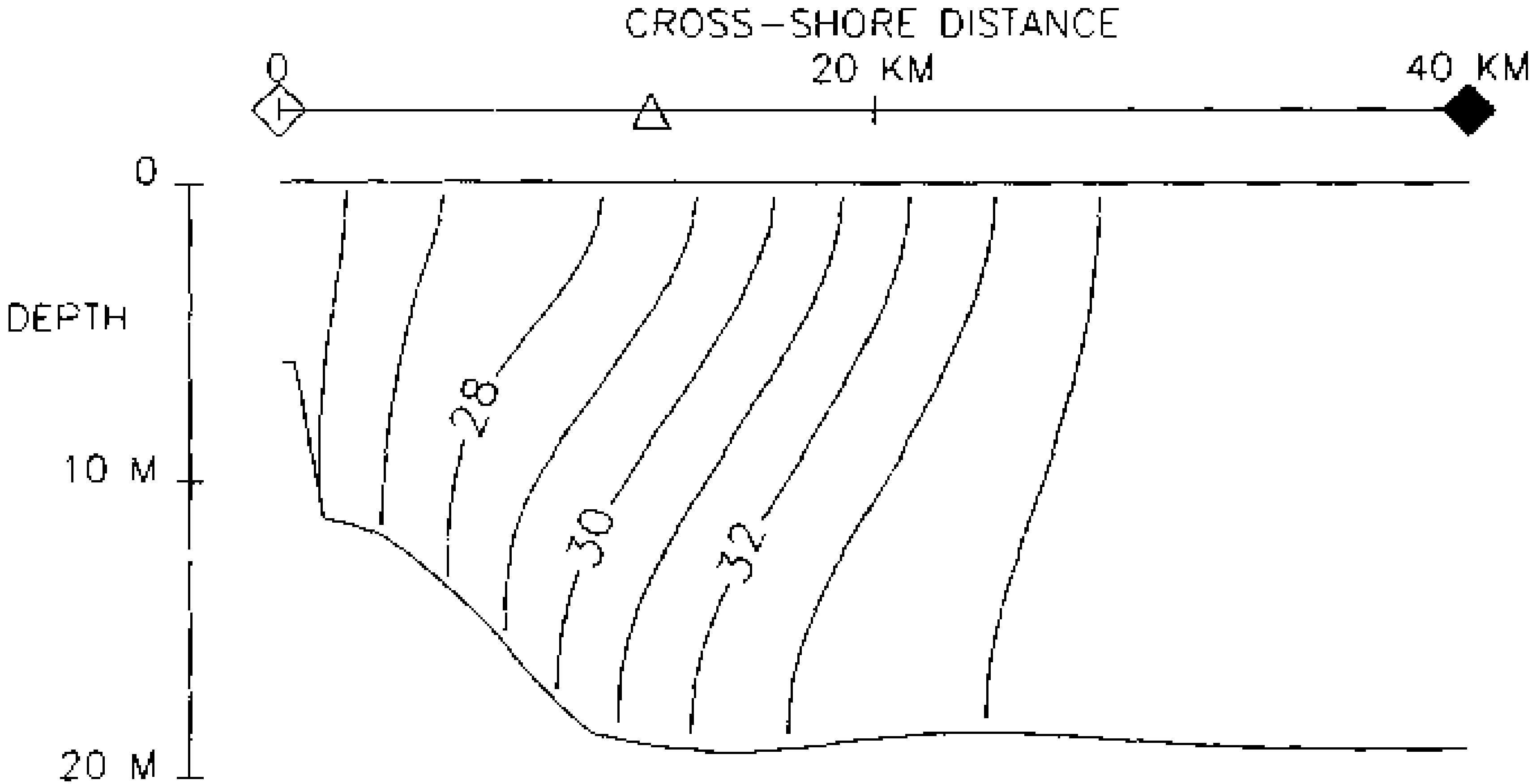
Fig. 1. The domain of interest showing the location of the grid, the cross-shore transect ◇ ⟷ ◆ and the "mooring" △ used for the presentation of results.

A 3D model sensitivity analysis in the Rhine plume region (II)



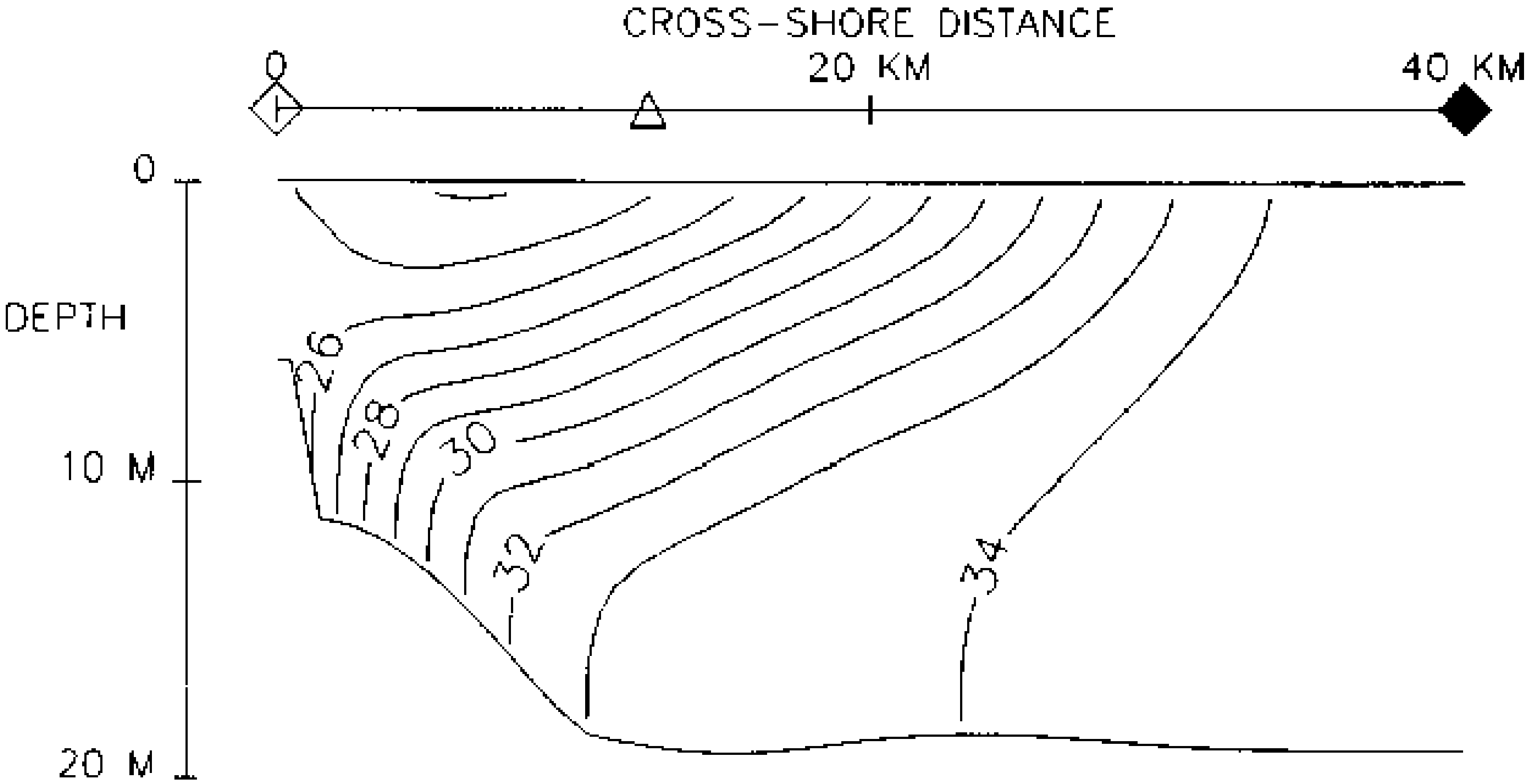
Tidally-averaged surface current and surface salinity (PSU), obtained from a k-model.

A 3D model sensitivity analysis in the Rhine plume region (III)



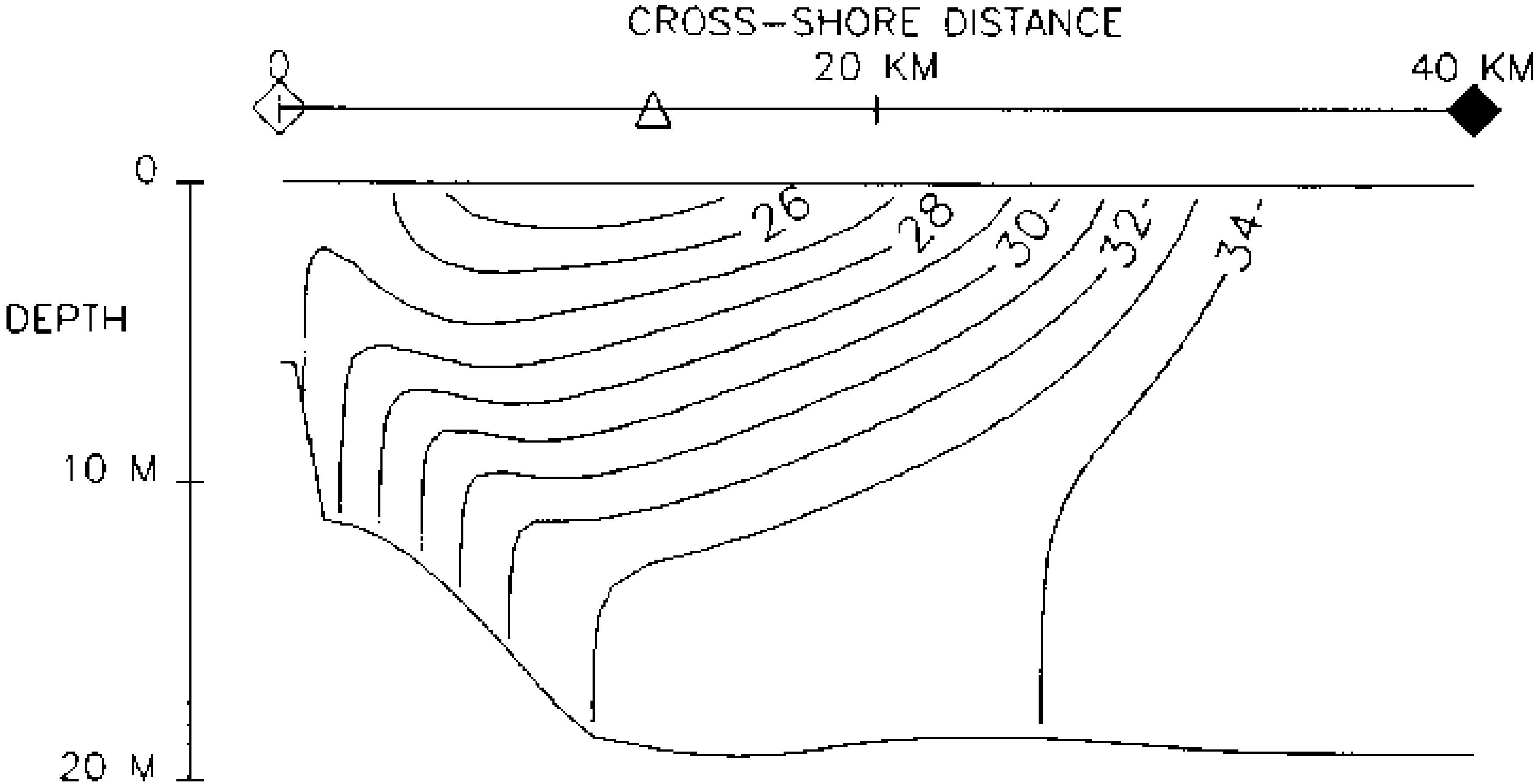
Tidally-averaged salinity (PSU), obtained from a constant eddy coefficient model. Stratification almost absent.

A 3D model sensitivity analysis in the Rhine plume region (IV)



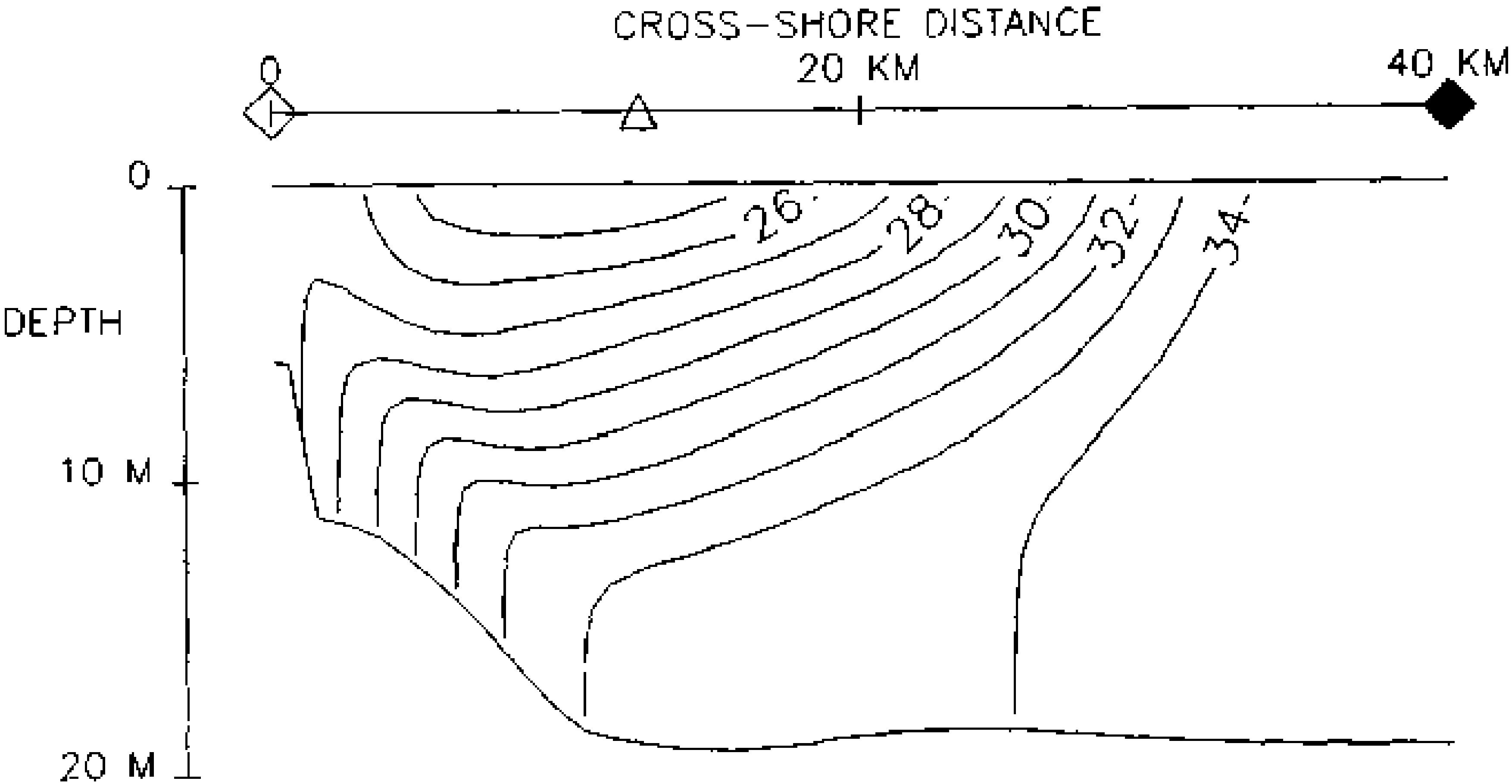
Tidally-averaged salinity (PSU), obtained from a Pacanowski-Philander-like model. Stratification insufficient.

A 3D model sensitivity analysis in the Rhine plume region (V)



Tidally-averaged salinity (PSU), obtained from a k model.
Stratification qualitatively acceptable.

A 3D model sensitivity analysis in the Rhine plume region (VI)



Tidally-averaged salinity (PSU), obtained from a k-kl (Mellor-Yamada 2.5) model. Stratification qualitatively acceptable.

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