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Determining an optimal multiplier in dynamic core-satellite strategies

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ABSTRACT This article investigates the performance of various multiplier techniques in reducing downside risk for dynamic core-satellite portfolios. Using Monte Carlo simulations calibrated on monthly data for three different portfolios over a 10-year period, we show that the dynamic IR/TE multiplier offers the best level of capital protection, as the specified floor is violated in less than 1 per cent of the cases. Even though other multipliers might offer higher average excess returns, the IR/TE multiplier still captures a significant fraction of the satellite excess return. In addition, it delivers an almost constant average floor violation rate.
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INTRODUCTION

Although diversification is a cornerstone of modern finance, the literature has showed that diversification does not help much in stressful market conditions. As a result, well-diversified investors may still suffer dramatic losses in extreme circumstances, precisely when diversification is needed the most. To limit the maximum loss on their portfolios, investors have progressively been looking beyond the standard static asset allocation methodology and increasingly employed dynamic asset allocation techniques to stabilize the risk-return tradeoff of their portfolios.

One of the most popular dynamic approaches is the so-called 'Constant Proportion Portfolio Insurance' (CPPI) technique. It consists in providing investors with a percentage guarantee of the initial investment at maturity (the floor) as well as a positive risk exposure when the market performs well. On the one hand, the downside protection is obtained by investing in a relatively safe asset. On the other hand, positive risk exposure is determined by a buffer, also called the cushion. The cushion is equal to the difference between the net asset value and the discounted floor, that is, the net present value of the floor. This cushion is then leveraged by a 'multiplier' to determine the desired level of risk exposure. The CPPI technique has recently been extended to a relative framework known as the 'Dynamic Core Satellite' (DCS) methodology. The goal of DCS is to limit the underperformance of the portfolio relative to a given benchmark, that is, the 'core', as well as to offer potential outperformance delivered by a risky asset, that is, the 'satellite'.

The purpose of this article is to address one of the main challenges of the DCS strategy, that is, the identification of the best method for computing the multiplier. In this article, we test six different DCS methods and compare them with the static strategy. Various levels of risk exposures are taken into account. The Monte Carlo simulations are carried out for three portfolios characterized by different levels of correlation between the core and the satellite. Particular attention is paid to the number of floor violations and the negative excess returns. We also measure outperformance in terms of 'upside gains', that is, positive excess returns. In short, we find that the dynamic IR/TE multiplier offers the best tradeoff between downside risk control and positive excess return participation. It also delivers an almost constant average floor violation rate, while the other multipliers are more sensitive to market trends.

This article is structured as follows. The next section includes a brief introduction to the DCS approach. In the subsequent section, we describe the data and explain the methodology. The empirical results are given in the latter section. In the penultimate section, we test the sensitivity of our results with respect to the data sample used to calibrate the simulations. We conclude in the last section.

THE DCS APPROACH

The DCS approach can be considered as an extension of CPPI, which is also called the 'cushion method'. Although the CPPI technique was originally designed to ensure the respect of absolute performance (Black and Jones, 1987), the DCS methodology is applied to a relative return framework. Like the standard CS technique, the DCS approach divides the portfolio into a core component, which fully replicates the investor's designated benchmark, and a performance-seeking component, made up of one or more satellites, which allows for higher tracking error. Although weights are static in the standard approach, they are not in the dynamic version: the proportion invested in the satellite can also fluctuate as a function of the current cumulative outperformance of the overall portfolio.

Amenc *et al* (2004) show that the standard CPPI can be extended to offer a relative performance guarantee, implying that underperformance relative to the benchmark is capped. Conventional CPPI techniques still apply as long as the risky asset is reinterpreted as the satellite portfolio (which contains risk relative to the benchmark) and the risk-free asset is reinterpreted as the core portfolio (which contains no risk relative to the benchmark). The key difference with standard CPPI is that the core (or benchmark) portfolio can itself be risky.

The DCS technique makes it possible to reduce a satellite's negative impact on performance during periods of relative underperformance while maximizing its positive impact during periods of outperformance. In other words, the DCS technique permits to reduce the loss of upside potential while maintaining good downside protection.

Compared with the standard CPPI, the DCS technique increases the fraction allocated to the satellite when it outperforms the benchmark and reduces this fraction when the satellite underperforms the benchmark. This dual objective is met by extending the CPPI to a relative risk management framework (Amenc *et al*, 2010). The portfolio P_t is still broken into a floor F_t and a cushion C_t while B_t is the benchmark. The floor is given by:

$$F_t = k^\star B_t$$

where k is a constant lower than 1. The investment in the satellite (S) is:

$$e_t = w^\star S_t = m^\star C_t = m^\star (P_t - F_t)$$

where m is a constant multiplier greater than 1 and w is the fraction invested in the satellite. The remainder of the portfolio invested in the benchmark is:

$$P_t - e_t = (1-w)^\star B_t$$

In a relative return environment, the core will contain trackers of a given benchmark and the satellite will be composed of assets that can potentially outperform the benchmark. An accumulation of past outperformance results in an increase in the cushion and therefore increases the potential for a more aggressive strategy in the future. However, if the satellite has underperformed the benchmark, the fraction invested in the satellite will decrease in order to make sure that the relative performance objective will be met.

Setting the multiplier

The attractiveness of the CPPI and DCS methods lies in the leverage effect of the multiplier. In practice, the multiplier is often set in function of the investor's level of risk tolerance based upon the following principle: the higher the multiplier, the higher the exposure to the active portfolio, and the higher the gains if the satellite outperforms the benchmark. Conversely, the poorer is the performance of the satellite, the more significant is the loss. The portfolio return therefore increases with the multiplier, but so does the risk. Assuming the cushion must be preserved, a lower multiplier is therefore recommended if the tracking error between the satellite and the benchmark is prone to large and sudden price jumps.

The main hurdle lies in the determination of the target multiplier. If the risky asset drops, the value of the cushion must remain superior or equal to zero for the portfolio value to stay above the floor. Without requiring any portfolio rebalancing, the cushion will absorb a shock inferior or equal to the inverse of the multiplier. This is particularly crucial for banks that directly bear the risk of the guaranteed portfolios they sell. At maturity, if the guaranteed floor is not reached, they have to compensate the loss with their own capital.

In the traditional CPPI framework, an unconditional multiplier is defined once and for all and might not be adapted anymore to changing market conditions.[1] Such unconditional setting methods do not take into account that the risk of the underlying asset changes. Conditional multipliers have therefore been developed to modify the risky exposure depending on the asset type and the market conditions.

In the literature, the target multiplier is computed in different ways. Qualitative methods are based on the investor's forecast profiles in terms of risk and return. Quantitative approaches incorporate market volatility measures: historical, current and/or implicit volatilities. In such a case, the multiplier is calculated as the inverse of the risky asset volatility. Otherwise, the multiplier is very often set as the inverse of the maximum historical drawdown (that is, drop) in the risky asset price. Some of the other factors of determination include the expected return of the risky asset, the effective interest rate levels or the Value-at-Risk (VaR). For example, Hamidi *et al* (2010) model different multipliers based on different estimation methods of the VaR. The shortfall constraint is then defined such that the probability of the portfolio value falling below a specified disaster level is limited to a specified disaster probability. The risky asset exposition is thus driven by a conditional multiplier defined as the inverse of a shortfall constraint quantified by the VaR. For the CPPI-type portfolio to remain protected, the multiplier has to stay smaller than the inverse of the risky asset maximum drawdown until the manager can rebalance the allocation. In addition, to obtain a convex payoff function, the multiplier is required to be higher than one. For a guaranteed floor level at a significance level α per cent, the multiplier must be lower than the inverse of the αth conditional quantile of the risky asset return distribution. The target multiplier can thus be interpreted as the inverse of the maximum loss that can bear the cushion at a given confidence level before any rebalancing.

METHODOLOGY AND DATA

The goal of this article is to identify the best performing optimal multiplier in the DCS framework. For this purpose, we compare different computational methods of the dynamic multiplier. Particular attention is given on whether the tested multiplier leads to a consistent adjustment of the risky allocation with respect to market conditions. A multiplier will outperform another when the dynamic control of downside risk is improved. This will be the case when the number of floor violations and the negative excess returns are reduced. We also measure outperformance in terms of 'upside gains', that is, positive excess returns obtained by using the multiplier.

In the empirical analysis, rebalancing is not expressed in absolute returns (like in the standard CPPI approach) but in relative returns (like in the DCS approach). If we assume that the floor is set at 90 per cent, it implies that the portfolio should then never deliver a return lower than 90 per cent of the benchmark return.

In practice, the risky allocation is often constrained. We therefore set a minimum of 0 per cent and a maximum of 60 per cent to the level of risk exposure. A 'no trade area' of 10 per cent is also included: It implies that no rebalancing occurs if the difference between the 'observed' risky exposure (due to asset price variations) and the theoretical exposure (resulting from the CPPI formula) is less than 10 per cent. As result, the theoretical exposure is computed as follows:

$$\mathrm{Max}(\mathrm{Min}((m\times C),\ \mathrm{MaxExposure}),\ \mathrm{MinExposure})$$
$$\text{or } \mathrm{Max}(\mathrm{Min}((m\times C),\ 60\text{ per cent}),\ 0\text{ per cent})$$

The decision to rebalance or not is taken every month, with transaction costs equal to 10 basis points. The investment horizon is 60 months, which is the typical horizon used by DCS investors. We use Monte Carlo simulations that are calibrated on historical monthly assets prices, volatilities and correlations, from the 1st of January 2000 to the 1st of November 2010. A total of 500 iterations are run for each computational method of the multiplier.

To assess the added value provided by the DCS strategy, we also simulate a 'static alternative' that consists in investing equally with no rebalancing during the whole investment period.

We run simulations on three portfolios characterized by different levels of correlation between the core and the satellite, as shown in Table 1.

Data are extracted from Bloomberg. The governmental and corporate bond indices are the BOfA/Merrill Lynch total return bond indices. The S&P GSCI Precious Metals Index is a sub-index of the S&P GSCI class of indices and provides a benchmark for investment performance in the precious metals commodity market.

As mentioned at the end of the previous section, various methods to compute the multiplier have been developed over the past few years. Some of them are more complex to implement and more difficult to understand for the customers. In this article, we focus on three methods that are believed to offer both suitable and practical solutions. These three methods are defined in both absolute and relative terms in Table 2.

Absolute multipliers are computed based on the historical returns of the risky asset (that is, the satellite) whereas the relative versions are computed on the basis of its excess return compared with the benchmark (that is, the core).

Each dynamic multiplier described in Table 2 is a time-weighted sum of two components: an unconditional multiplier and a 1-year rolling period conditional multiplier that varies according to the evolution of market conditions. The unconditional multiplier is computed by including all the historical data used to calibrate the simulations. The conditional multiplier is computed over the trailing period, that is, the last 12 months. The first 12 months are the last months of the historical data used to calibrate the Monte Carlo simulations, that is, from the 1st of December 2009 to the 1st of November 2010. The weighting (w) is a decreasing function of time and is computed as follows:

$$w = (60 - t)/60$$

where t is the t^{th} month of the investment period and 60 representing the total number of periods, that is, 60 months. The dynamic multiplier is then computed as

$$m_{dyn} = w^{\star} m_{unc} + (1 - w)^{\star} m_{cond}$$

where m_{dyn} is the dynamic multiplier, w is the weight, m_{unc} is the unconditional multiplier based on the historical data, and m_{con} is the conditional multiplier computed over a 1-year rolling period.

The average floor violation rate will be computed for each type of multiplier. This criterion is of particular interest for portfolio managers who want to make sure their portfolio does not underperform the benchmark by a predetermined percentage margin (10 per cent in this case). In the empirical analysis, a floor violation occurs if the cumulated portfolio return is below 90 per cent of the cumulated benchmark return at any time during the investment horizon. We also analyze the evolution of the risky

Table 1: Composition of the three simulated portfolios

Portfolio	*Core*	*Satellite*	*Correlation (%)*
(1) Short-term + Long-term Euro govies	AAA 3–5 years European governmental bonds	AAA +15 years European governmental bonds	86.88
(2) Medium-term Euro govies and corporate bonds + equity	60% AAA 5–10 years European governmental bonds + 40% A 5–7 years European corporate bonds	Euro Stoxx 50	10.83
(3) Short-term Euro govies + equity	AAA 3–5 years European governmental bonds	S&P GSCI Precious Metals	−10.59

Table 2: Multipliers

Absolute version	*Calculation*	*Relative version*	*Calculation*
Volatility multiplier	1/volatility	TE multiplier	1/tracking error
Max drawdown multiplier	1/(minimum negative return)	Relative max drawdown multiplier	1/(minimum negative excess return)
Sharpe/volatility multiplier	Sharpe ratio/volatility	IR/TE multiplier	Information ratio/tracking error

allocation during a typical bearish scenario in order to determine whether the multiplier leads to a consistent adjustment of the risk exposure to the relative risky asset performance.

EMPIRICAL ANALYSIS

The results in Table 3 for Portfolio 1 indicate that the average excess returns (after transaction costs) and the information ratio are lower for the first two multipliers. The Sharpe/volatility multiplier and its relative version, the IR/TE multiplier, deliver the lowest average excess returns, that is, 1.16 and 1.07 per cent respectively versus 1.52 and 1.58 per cent for the four other multipliers. The lower levels of excess returns might be explained by higher average transaction costs: 0.74 and 0.97 per cent respectively, against 0.14–0.20 per cent for the other four multipliers. The IR is also lower for the first two multipliers: 0.67 and 0.64 respectively, against 0.80–0.84 for the other four multipliers.

Except for these first two multipliers, all strategies deliver higher excess returns than the static strategy, which points to rebalancing benefits. However, those higher excess returns are obtained at the cost of a higher tracking error and a lower information ratio than in the case of the static strategy.

The first two multipliers perform in terms of the maximum drawdown, the violation rate, and the tracking error. For example, the IR/TE multiplier offers the lowest max drawdown on a 1-year rolling period. It is also the only strategy to beat the static strategy in this respect. The most significant benefit offered by the dynamic Sharpe/volatility and IR/TE multipliers is a significantly lower average floor violation rate of only 0.67 and 0.66 per cent, respectively.

Interestingly, absolute multipliers always offer a slightly lower floor violation rate than their relative counterparts, except for the dynamic Sharpe/volatility and IR/TE multipliers that have similar outcomes. Finally, lower floor violation rates seem to be associated with higher transaction costs.

Figure 1 shows that no DCS strategy brings much added value in terms of VaR reduction compared with the static strategy.

Figure 2 displays the evolution of the allocation to the risky satellite during a typical bearish scenario of the core component. With the last four multipliers given in Table 3, exposure remains almost constant, at its maximum (60 per cent) or at a very high level. For instance, the TE and volatility multipliers take too much time to adapt the risky allocation to bad market conditions. Only the Sharpe/volatility and IR/TE multipliers lead to rapidly changing exposures.

Table 3: Statistical results – Portfolio 1 (short-term + long-term govies)

DCS strategies	*Max 1-year drawdown (%)*	*Average floor violation rate (%)*	*Average transaction costs (%)*	*Average excess return (%)*	*TE (%)*	*IR*[a]
IR/TE multiplier	–35.74	0.66	0.97	1.07	1.67	0.64
Sharpe/volatility multiplier	–36.72	0.67	0.74	1.16	1.72	0.67
Relative max drawdown multiplier	–36.93	1.94	0.14	1.58	1.89	0.84
Max drawdown multiplier	–37.76	1.33	0.17	1.56	1.88	0.83
TE multiplier	–37.08	1.73	0.14	1.58	1.89	0.83
Volatility multiplier	–36.67	1.15	0.19	1.52	1.88	0.81
EUGOVIESAAA3–5	–35.41	—	—	0.00	0.00	—
EUGOVIESAAA15+	–40.18	—	—	2.61	2.97	0.88
Static strategy	–36.14	1.01	0.10	1.40	1.58	0.89

[a]IR is the modified information ratio introduced by Israelsen (2005).

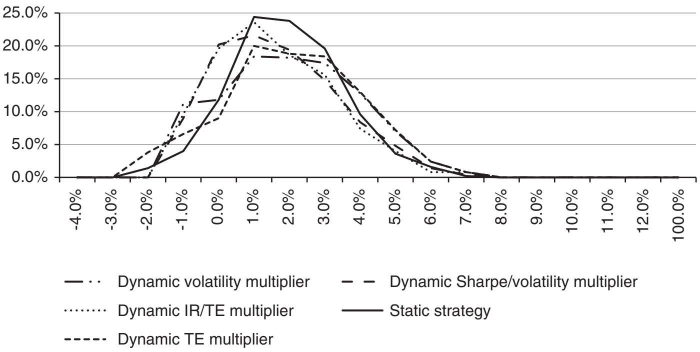

Figure 1: Distribution of portfolio excess return – Portfolio 1.

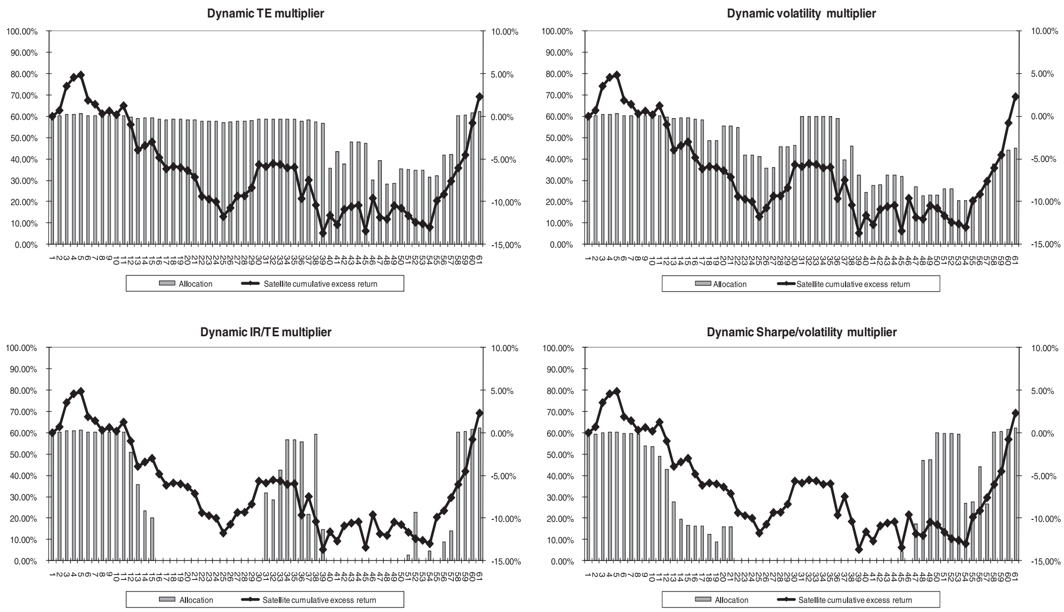

Figure 2: Risky allocation and cumulative excess return of the satellite – Bearish scenario.

Although the IR/TE and Sharpe/volatility multipliers do not follow the risky asset performance perfectly, Figure 3 confirms that these two multipliers offer a better match between the multiplier value and the risky allocation.

Figure 3 also demonstrates that all multipliers take incoherent values. In general, a 'static' multiplier between 0 and 5 is often suggested in the literature. When the multiplier varies according to the asset features and the evolution of market conditions, consistent values for the multiplier are typically assumed to vary between 0 and 8. With a floor equal to 90 per cent, multiplier values greater than 8 do not make much sense. It would lead to a risky exposure superior to 80 per cent, which would be unrealistic in practice.

For portfolio 2, Table 4 indicates that all the DCS strategies perform better than the static strategy in terms of the floor violation rate and the max 1-year drawdown. While the static strategy violates the floor in almost 60 per cent of the cases, the worst result obtained by a DCS strategy is a floor violation rate equal to 29.56 per cent. Compared with Portfolio 1, the floor violation rate is

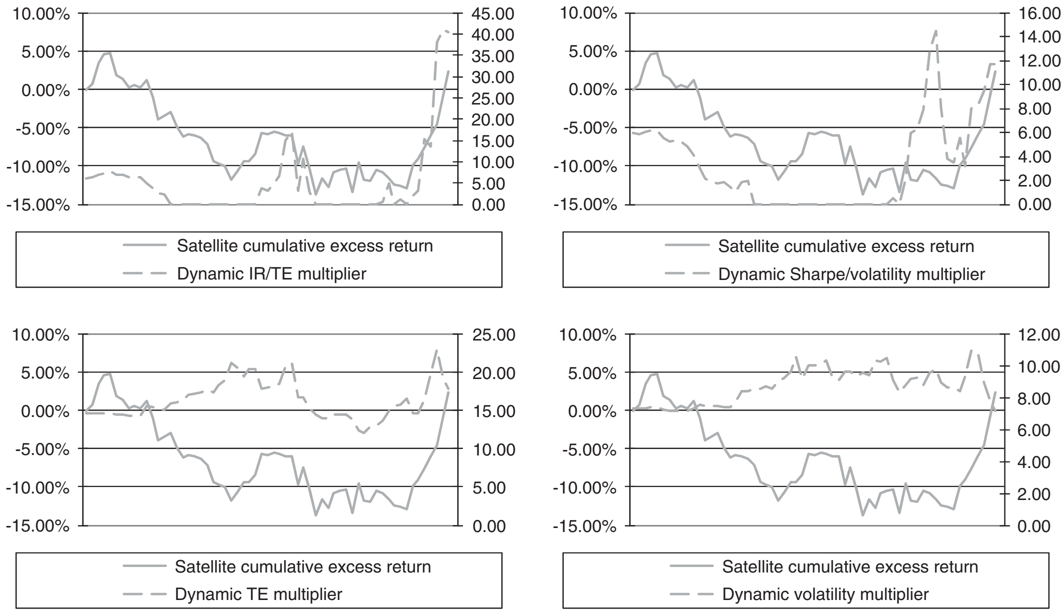

Figure 3: Multipliers and cumulative excess return of the satellite in a bearish scenario.

Table 4: Statistical results – Portfolio 2 (medium-term govies and corporate bonds + Euro Stoxx 50)

	Max 1-year drawdown (%)	*Average floor violation rate (%)*	*Average transaction costs (%)*	*Average excess return (%)*	*TE (%)*	*IR*
IR/TE multiplier	−37.89	0.59	0.34	−0.48	0.72	−3.45E-05
Sharpe/volatility multiplier	−38.00	1.84	0.49	−0.82	0.90	−7.42E-05
Relative max drawdown multiplier	−42.64	28.41	0.34	−2.00	0.88	−1.76E-04
Max drawdown multiplier	−43.52	29.56	0.33	−2.00	0.88	−1.77E-04
TE multiplier	−40.67	25.00	0.31	−1.96	0.79	−1.56E-04
Volatility multiplier	−40.88	26.75	0.31	−1.96	0.82	−1.62E-04
MTGovCorp(60/40)	−38.29	—	—	0.00	0.00	—
Euro Stoxx 50	−84.43	—	—	−15.94	9.90	−1.58E-02
Static strategy	−44.74	58.37	0.10	−5.91	3.22	−1.90E-03

nevertheless much higher for all DCS strategies, except for the IR/TE and Sharpe/volatility multipliers that deliver stable rates in both portfolios. This is explained by the higher volatility of the satellite and the constant overexposure to the risky asset when it performs badly, except for the IR/TE and Sharpe/volatility multipliers.

Each of the six DCS strategies exhibits higher information ratios than the static strategy. The tracking error is also systematically lower for any of the DCS strategies than in the static case. Combined with higher excess returns, it shows that DCS strategies allows for participating in 'positive' tracking errors (that is, when the satellite performs well) while avoiding taking part in 'negative' tracking errors (that is, when the satellite performs poorly). As shown in Portfolio 2, the performance of the satellite (that is, Euro Stoxx 50) does not perform well and each DCS strategy is nevertheless not particularly affected because no significant deviation from the benchmark occurs. In Portfolio 1, the satellite was able to deliver positive excess returns and it impacted the

DCS strategies positively through higher tracking errors relatively to the static strategy.

The IR/TE multiplier is clearly the outperformer for Portfolio 2. Closely followed by the Sharpe/volatility multiplier, it leads to the lowest tracking error, the highest average excess return, the lowest floor violation rate and the lowest maximum 1-year drawdown. This outperformance is obtained at the cost of marginally higher transaction costs.

Figure 4 indicates that compared with the static strategy, DCS strategies truncate the excess return probability distribution so as to allocate the probability weights away from severe underperformance relative to the benchmark. The IR/TE multiplier is again the best alternative to the static case. Nevertheless, no DCS strategy delivers positive average excess returns due to the poor performance of the satellite.

By reducing to zero its allocation to the satellite, the Sharpe/volatility and IR/TE multipliers protect against poor performance, as indicated in Figure 5 by the sharp fall in the satellite cumulative excess return. The other multipliers (not shown to save space) still maintain some positive exposure to the risky asset even when it generates negative excess returns for a prolonged period of time.

Figure 6 indicates that the dynamic IR/TE and Sharpe/volatility multipliers are close to zero at any time during the bearish trend. As such, they protect the portfolio value against the continuous drop in the performance of the satellite and keep the invested capital in the core component

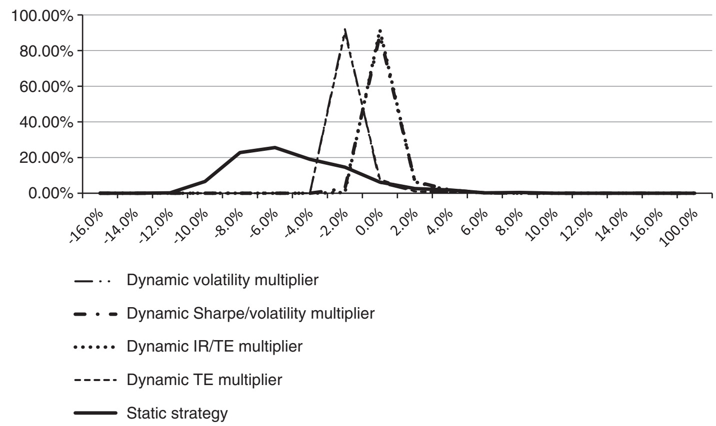

Figure 4: Distribution of portfolio excess return – Portfolio 2.

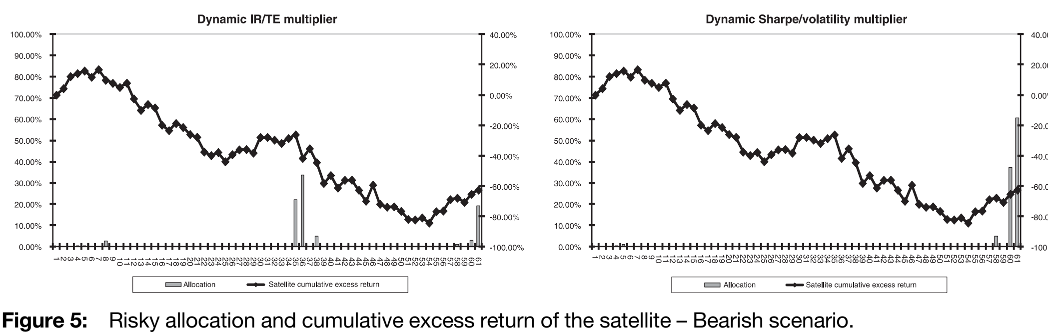

Figure 5: Risky allocation and cumulative excess return of the satellite – Bearish scenario.

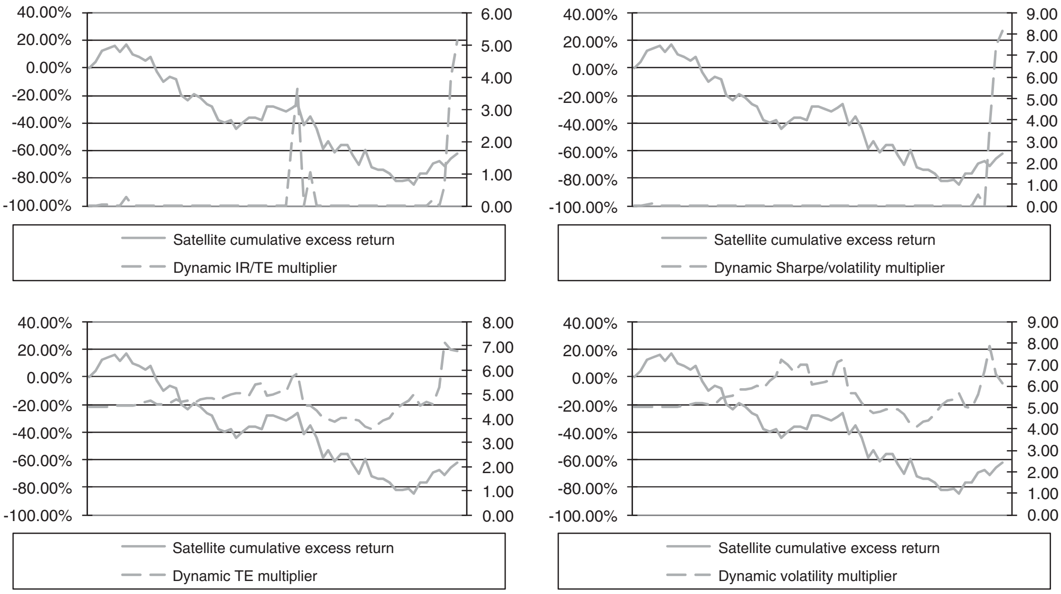

Figure 6: Multipliers and cumulative excess return of the satellite in a bearish scenario.

Table 5: Statistical results – Portfolio 3 (short-term govies + S&P GSCI Precious Metals)

	Max 1-year drawdown (%)	*Average floor violation rate (%)*	*Average transaction costs (%)*	*Average excess return (%)*	*TE (%)*	*IR*
IR/TE multiplier	–30.99	0.88	0.81	1.31	3.34	0.39
Sharpe/volatility multiplier	–32.23	8.00	0.70	2.36	5.00	0.47
Relative max drawdown multiplier	–32.35	12.21	0.37	2.59	5.54	0.47
Max drawdown multiplier	–32.33	12.65	0.37	2.67	5.51	0.48
TE multiplier	–31.92	7.08	0.41	2.61	5.25	0.50
Volatility multiplier	–32.34	9.82	0.38	2.60	5.43	0.48
EUGOVIESAAA3–5	–35.41	—	—	0.00	0.00	—
S&P GSCI Metals	–52.11	—	—	5.58	9.75	0.57
Static strategy	–26.47	12.35	0.10	3.55	5.55	0.64

of the portfolio. By contrast, the risky allocation remains too high when the TE and volatility multipliers are used, both based on volatility.

Table 5 indicates that the maximum drawdown on a 1-year rolling period is somewhat lower for Portfolio 3 than for the first two portfolios. This can be explained by diversification benefits obtained by investing in two negatively correlated assets such as short-term governmental bonds and precious metals.

The IR/TE multiplier scores best when it comes to minimizing the maximum drawdown and the floor violation rate. Interestingly, the IR/TE multiplier delivers the lowest floor violation rate in all three portfolios. Nevertheless, better floor protection leads to higher transaction costs and lower excess return, as shown by the IR/TE multiplier. Compared with the first two portfolios, the performance of the Sharpe/volatility multiplier is much lower and the relative poor performance of the two maximum drawdown multipliers is confirmed, with floor violation rates above 12 per cent.

In portfolio 3, the highest excess returns are obtained by applying the static strategy. This can be explained by the exposure of the

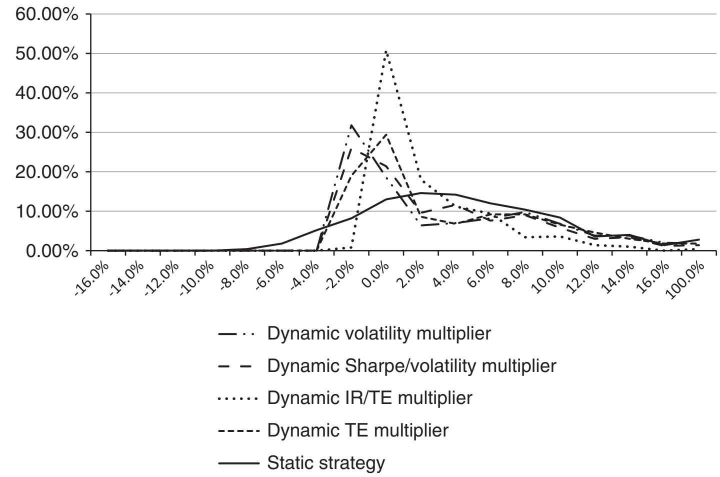

Figure 7: Distribution of portfolio excess return – Portfolio 3.

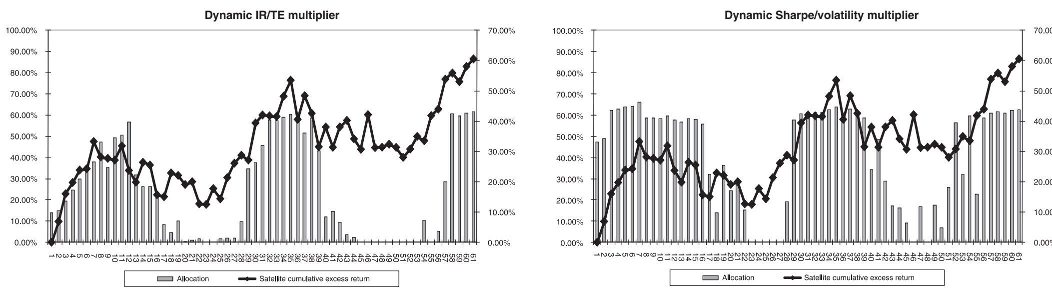

Figure 8: Risky allocation and cumulative excess return of the satellite – Bearish scenario.

static strategy to the satellite that performs quite well compared with the core. As expected, those higher excess returns are obtained through higher positive tracking error than those related to the IR/TE multiplier. Nevertheless, the gain in excess returns outweighs the 'cost' in tracking error when the static strategy is used instead of the IR/TE multiplier.

One of the key benefits of the DCS strategies is the potential truncation of the relative return distribution so as to allocate the probability weights away from severe underperformance to greater potential outperformance. The introduction of risk-controlled strategies makes it possible to move the distribution of the excess returns to the right. As shown on Figure 7, the dynamic IR/TE multiplier shifts the distribution of returns away from extreme negative outcomes although at the (reasonable) cost of a lower probability of extreme positive excess returns.

When the same type of bearish scenario is simulated for the core component of Portfolio 3 (as it was done for the first two portfolios), the satellite (that is, the so-called 'risky' asset) performs well because of the negative correlation between the core and the satellite. Whereas the volatility and TE multipliers keep an exposure very close to 60 per cent almost constantly (not shown to save space), Figure 8 shows how the IR/TE and Sharpe/volatility multipliers adjust the allocation to the satellite over time.

Contrary to the TE and volatility multipliers, the IR/TE and Sharpe/volatility multipliers implement a more consistent and

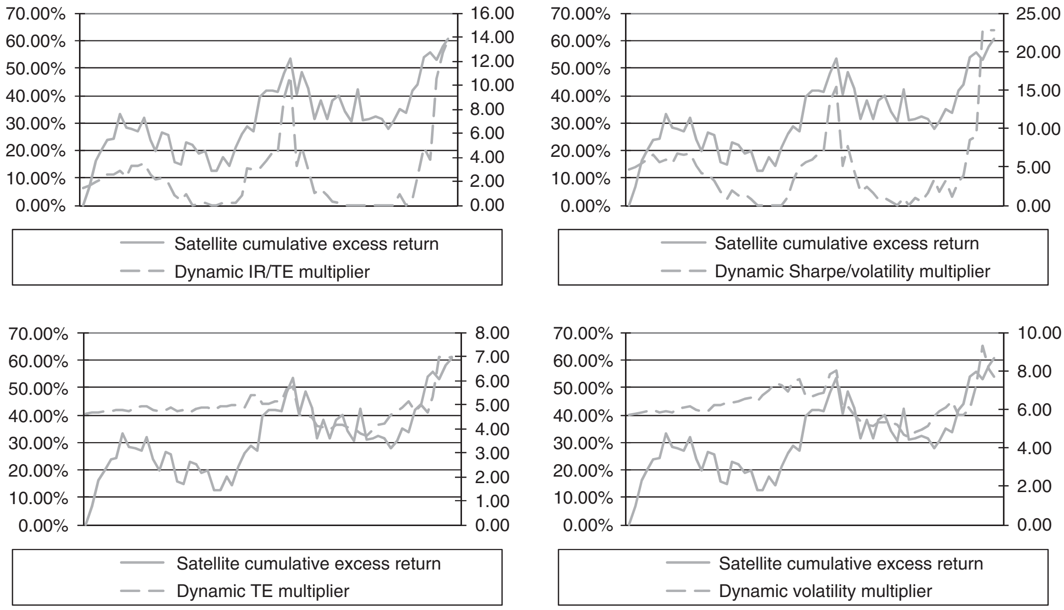

Figure 9: Multipliers and cumulative excess return of the satellite in a bearish scenario.

dynamic risky allocation. If the risky allocation is at some time a little bit superior to 60 per cent (despite the maximum limit set at that level), this is due to the 'no trade area' rule implemented in order to avoid high transaction costs.

Figure 9 confirms that the IR/TE and Sharpe/volatility multipliers better adapt to market conditions. They increase in value as the cumulative excess return of the satellite goes up while the other multipliers do not show much dynamism. Of course, the values taken by any of these multipliers are sometimes inconsistent as they would result in a very high risk exposure.

Overall, the IR/TE and Sharpe/volatility multipliers perform best when the satellite underperforms relatively to the core, limiting the impact of the negative excess returns. When market conditions are expected to be more favorable to the satellite, the use of the other multipliers would be more appropriate.

Regarding the floor violation rate, the worst performance is achieved by the two (absolute and relative) maximum drawdown multipliers: They fail to reduce the floor violation rate and lead to transaction costs as high as if the other multipliers were used. The best performance is achieved by the IR/TE multiplier. While the Sharpe/volatility and IR/TE multipliers exhibit similar floor violation rates for Portfolios 1 and 2, the first underperforms the second for Portfolio 3, that is, when the core and the satellite are negatively correlated.

The IR/TE multiplier clearly enhances downside risk protection but it does not capture the outperformance of the risky asset as well as other multipliers do. Although downside risk protection is the main objective of these strategies, it expectedly comes at the price of lower expected returns. Nevertheless, when transaction costs are compared with floor violation rates, the IR/TE multiplier is never dominated by another strategy, as shown in Figure 10. This outcome is confirmed when excess returns are compared with floor violation rates (not shown to save space).

ROBUSTNESS CHECKS

The sensitivity of the results is first tested by changing the data sample used to calibrate the

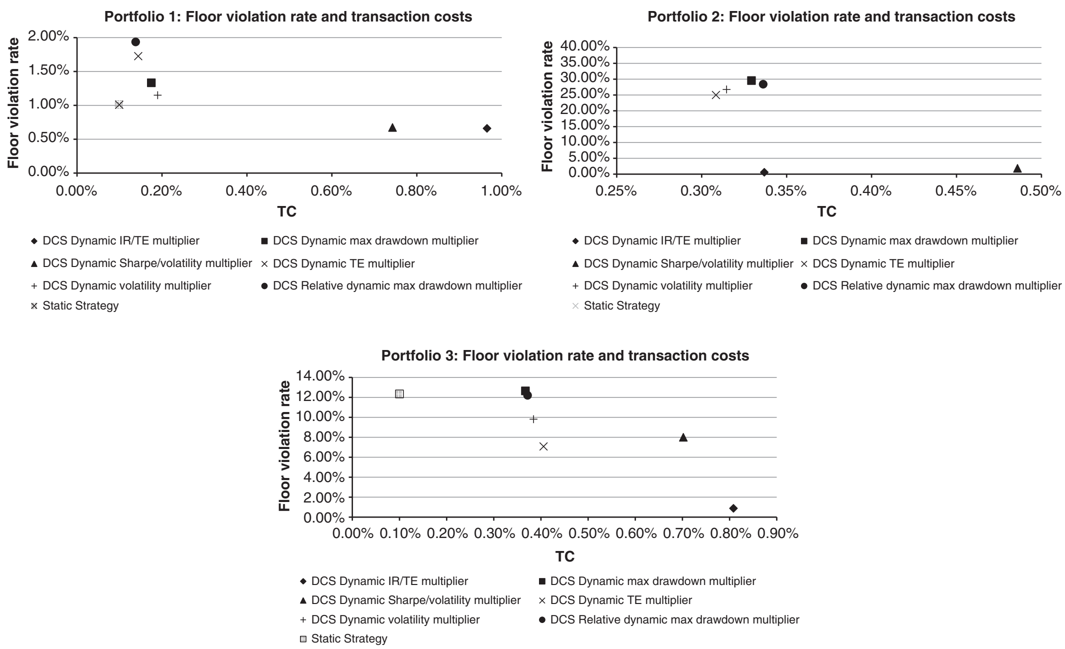

Figure 10: Average floor violation rates and transaction costs.

Table 6: Statistical results after 30 000 iterations – Portfolio 2 (medium-term govies and corporate bonds+Euro Stoxx 50)

	Max 1-year drawdown (%)	*Average floor violation rate (%)*	*Average transaction costs (%)*	*Average excess return (%)*	*TE (%)*	*IR*
IR/TE multiplier	−30.01	0.46	0.49	−0.36	1.09	−3.91E-05
Sharpe/volatility multiplier	−30.42	3.40	0.61	−0.67	1.75	−1.18E-04
TE multiplier	−33.73	18.33	0.34	−1.33	2.06	−2.75E-04
Volatility multiplier	−34.17	19.80	0.34	−1.34	2.09	−2.79E-04
MTGovCorp(60/40)	−7.93	—	—	0.00	—	—
Euro Stoxx 50	−31.22	—	—	−11.06	12.65	−1.40E-02
Static strategy	−35.20	43.54	0.10	−3.90	4.46	−1.74E-03

Monte Carlo simulations. Calibration is done by using the first 5 years of data and a 1-month rolling window. The starting date in the simulations therefore varies between 01/01/2000 and 01/01/2005. Sixty different time windows are used to generate the simulations, leading to a total of 30 000 iterations (that is, 60 simulations times 500 iterations).

Given the significant amount of time and computing resources required, only Portfolio 2 is considered as it is the most likely combination of assets used in DCS-type strategies. We also drop the two maximum drawdown multipliers due to their obvious underperformance.

The results in Table 6 are very close to those previously obtained in Table 4. All strategies perform somewhat better on average given the better performance of the satellite on some time periods used for calibration. The ranking remains the same nevertheless. The IR/TE and Sharpe/volatility multipliers still outperform the TE and volatility multipliers. Compared with Table 4, all multipliers achieve a lower average floor violation rate, except the Sharpe/volatility for which the rate almost

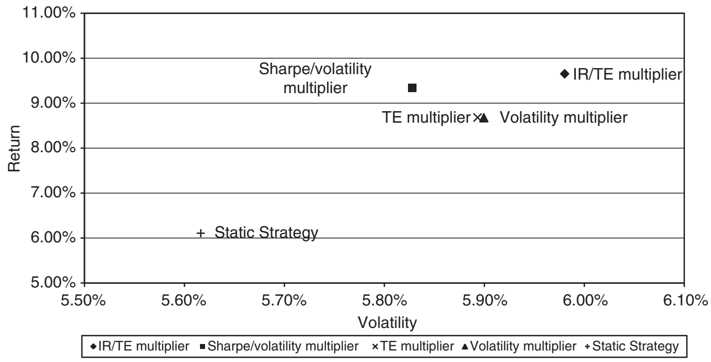

Figure 11: Risk-return tradeoff offered by the different strategies.

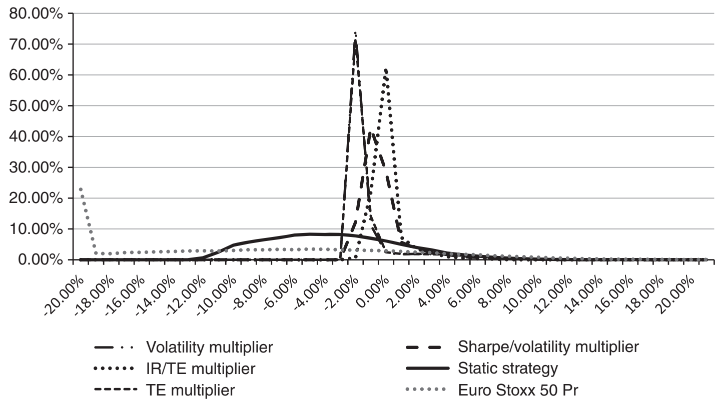

Figure 12: Risk-return tradeoff offered by the different strategies.

doubles. Transaction costs are almost unchanged for the TE and volatility multipliers (+0.03 per cent points) while they have increased for the Sharpe/volatility (+0.12 per cent points) and IR/TE multipliers (+0.15 per cent points).

In Figure 11, the risk-return tradeoff indicates that the TE and volatility multipliers are clearly dominated by the Sharpe/volatility multiplier.

The distribution of excess returns obtained from the 30 000 iterations (Figure 12) confirms the results displayed on Figure 4. The dynamic Sharpe/volatility and IR/TE multipliers shift the distribution of relative returns further to the right, away from severe underperformance, but the IR/TE multiplier is clearly the best method when it comes to truncating the distribution of excess return. The TE and volatility multipliers display the same performance and are almost undistinguishable.

The sensitivity of the results is now tested with respect to the trends observed during the time period used for calibration. Figure 13 shows the historical annualized return, volatility and correlation of Portfolio 2's core and satellite across each of the 60 five-year rolling windows used for calibration. Window 31 corresponds to the 01/01/2002–07/01/2007 time period. From that point, the subprime bubble bursts. Returns of both the core and the satellite decline sharply and correlation between the core and the satellite becomes positive. At window 47 (that is, 01/11/2003–1/11/2008 time period), the collapse of stock indices around the world leads to negative 5-year returns for the

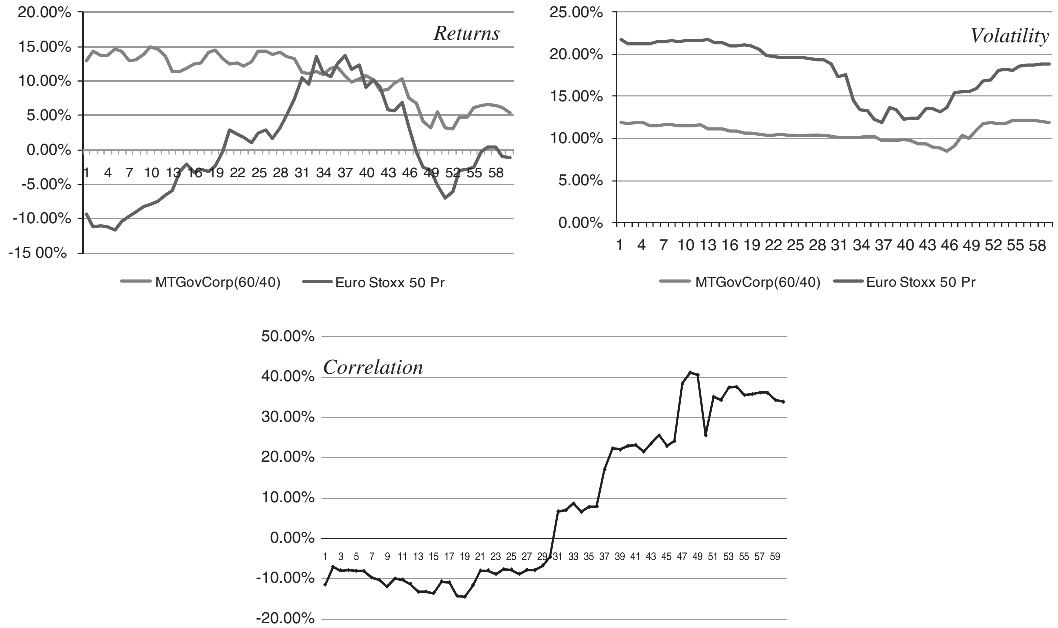

Figure 13: Annualized returns, volatility and correlation of Portfolio 2's core and satellite over the 60 five-year rolling windows.

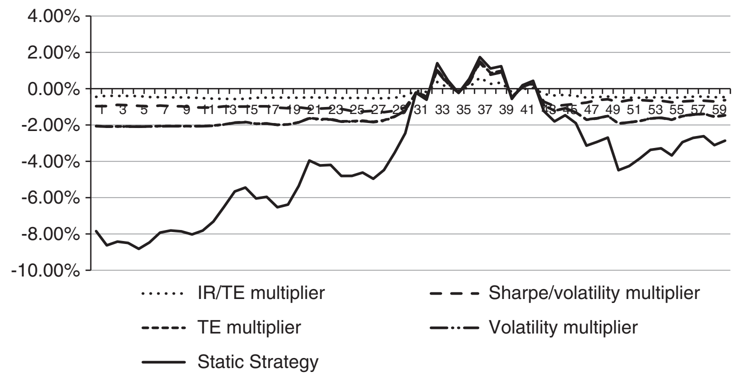

Figure 14: Average excess return across the 60 five-year rolling windows.

satellite (that is, Euro Stoxx 50) and volatility keeps rising. Correlation is around 40 per cent.

Figure 14 displays the average excess return delivered by each multiplier method over the 60 windows used for calibration. The empirical results of the previous section are confirmed. The TE and volatility multipliers are dominated by the IR/TE and Sharpe/volatility multipliers. Even when markets returns for the satellite are high (between windows 29 and 43, more or less), they fail to deliver superior returns than the Sharpe/volatility multiplier. Most interestingly, the static strategy is clearly outperformed by any of the four DCS strategies.

The IR/TE multiplier is the best method for limiting the impact of negative excess returns when market conditions are not favorable. In particular, it best limits the negative impact of the increase in correlation observed on Figure 13. This feature is important because a DCS strategy does not bring any added value if it fails to move the

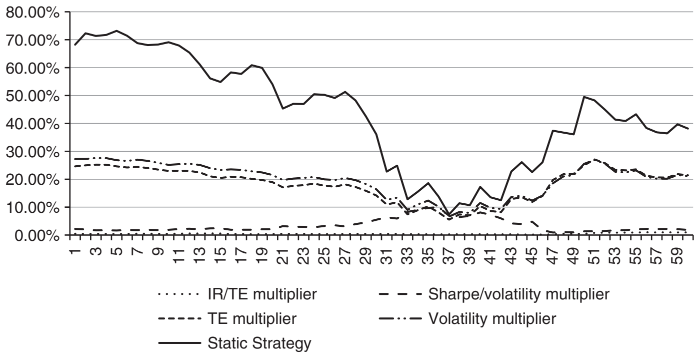

Figure 15: Average floor violation rate across the 60 five-year rolling windows.

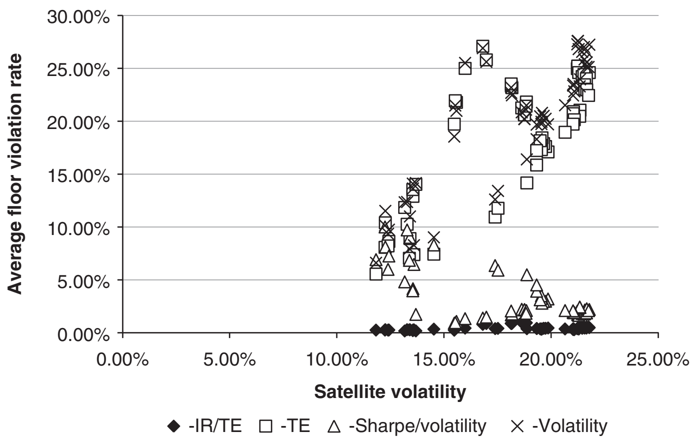

Figure 16: Average floor violation rate and volatility of the satellite.

invested capital back to the core when the satellite does not provide any return and diversification benefit. However, it comes at the price since the IR/TE multiplier does not capture all the positive excess returns when the satellite performs well.

Figure 15 indicates that the IR/TE multiplier offers the lowest and most constant floor violation rate. Interestingly, the Sharpe/volatility multiplier is the only strategy which does not lead to a lower number of floor violations during the ‘bullish period’ characterized by higher returns and lower volatility for the satellite (that is, Euro Stoxx). Since the Sharpe/volatility multiplier is equal to the Sharpe ratio divided by the volatility (or the average return divided by the volatility square), it takes higher values for two reasons: the higher average returns *and* the lower volatility. This in turn leads to an overexposure to the satellite, which might trigger floor violations more often (given the high level of the floor especially). Its relative counterpart (that is, the IR/TE multiplier) avoids this pitfall as its computation is based on the excess returns relative to the core. Although the Sharpe/volatility might be suitable for absolute return strategies, its relative version makes more sense when the goal is to track and outperform a benchmark.

The above robustness checks confirm the empirical findings of the previous section: The IR/TE multiplier clearly outperforms the other computational methods when it comes to optimizing the dynamic control of the downside risk. It offers a high level of floor protection and truncates the distribution of relative returns appropriately. Even though other multipliers might offer higher average excess returns, the

IR/TE multiplier still captures a significant fraction of the excess return delivered by the satellite. In addition, the sensitivity analysis shows that the IR/TE multiplier delivers an almost constant average floor violation rate, while the other multipliers are more sensitive to market trends. The dynamic IR/TE multiplier represents the best tradeoff between positive excess return participation and capital protection, whatever the market conditions.

In Figure 16, the average floor violation rate is measured at different volatility levels of the satellite for each of the four multipliers. Once again, it clearly points to the sharp reduction in the violation rate variability brought by the IR/TE multiplier.

CONCLUSION

DCS strategies are designed to control for downside risk that is particularly acute in stressful market conditions. These strategies provide investors with a percentage guarantee of the initial investment at maturity. Capital protection may however be endangered because of significant gap risk or incorrect implementation. In order to optimize the preservation of the floor, several methods for computing the multiplier have been developed.

In this article, we test six DCS strategies and compare them with the static strategy. We show that the dynamic IR/TE multiplier offers the best level of capital protection as the specified floor is violated in less than 1 per cent of the cases. This clearly is an enviable feature for practitioners who may not underperform a given benchmark by more than a certain percentage. Even though other multipliers might offer higher average excess returns, the IR/TE multiplier still captures a significant fraction of the excess return delivered by the satellite. In short, the dynamic IR/TE multiplier offers the best tradeoff between downside risk control and positive excess return participation. In addition, it delivers an almost constant average floor violation rate, while the other multipliers are more sensitive to market trends.

However, further work may still be done to improve the proportion of the satellite's positive excess returns that the IR/TE multiplier can capture when market conditions are favorable. In this article, the simulations have been realized with a constant floor equal to 90 per cent of the benchmark performance. Obviously, the floor can take other values and be computed in many other ways. For example, the floor can be equal to a certain percentage of the *current* net asset value. A link could also be established between the floor level and the multiplier. A multiplier of 15 might be associated to a floor of 99 per cent while it might not be acceptable with a floor of 90 per cent as it would incur an exposure of 150 per cent.

Another parameter of interest to be further investigated is rebalancing frequency. A monthly rebalancing might not be precise enough to limit the risky exposure when the satellite does not perform well. A weekly rebalancing might be more efficient although the impact of higher transaction costs must also be taken into account. Rebalancing frequency may also be increased during volatile market conditions and reduced in tranquil markets. Dynamic rebalancing frequency can also be introduced by triggering rebalancing when market returns move by a certain percentage or when the deviation from the desired level of risk exposure becomes too high. Dynamic rebalancing frequency could improve the performance of the dynamic IR/TE multiplier since it might imply lower transaction costs and enable the multiplier to capture a higher fraction of the positive excess returns.

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NOTE

1. For example, an unconditional multiplier is often simply based on the investor's risk aversion.

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