Interference of lateralized distractors on arithmetic problem solving: A functional role for attention shifts in mental calculation

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Abstract

Solving arithmetic problems has been shown to induce shifts of spatial attention in simple probe-detection tasks, subtractions orienting attention to the left side and additions to the right side of space. Whether these attentional shifts constitute epiphenomena or are critically linked to the calculation process is still unknown. In the present study we investigate participants' performance on addition and subtraction solving while they have to detect central or lateralized targets. The results show that lateralized distractors presented in the hemifield congruent to the operation to be solved interfere with arithmetical solving: participants are slower to solve subtractions or additions when distractors are located on the left or on the right, respectively. These results converge with previous data to show that attentional shifts underlie not only number processing but also mental arithmetic. They extend them as they reveal the reverse effect of the one previously reported by showing that inducing attention shifts interferes with the solving of arithmetic problems. They also demonstrate that spatial attentional shifts are part of the calculation procedure of solving mentally arithmetic problems. Their functional role is to access, from the first operand, the representation of the result in a direction congruent to the operation.

Keywords: mental arithmetic; visuo-spatial attention; attentional shifts

Introduction

The association between numbers and space has been conceptualized by the idea of the mental number line (MNL), a mental representation of the magnitude of numbers in which small numbers are spatially represented at the left and large numbers at the right of a line (Dehaene, 1992). Accordingly, processing small-magnitude numbers has been shown to orient attention and/or gaze towards the left part of space and processing large-magnitude numbers towards the right. For example, participants detect targets displayed on the left part of a screen more rapidly after they have processed small-magnitude numbers, and on the right part of screen after they have processed large-magnitude numbers (Fischer, Castel, Dodd, & Pratt, 2003; Fischer, Warlop, Hill, & Fias, 2004). Conversely, lateralized targets affect the speed of number processing, showing that the link between number magnitude and space is bidirectional. Indeed, the presentation of leftward/rightward visual signals accelerates the participants' response to small/large digits respectively in comparison to a standard or parity judgment tasks (Stoianov, Kramer, Umiltà, & Zorzi, 2008). Rightward optokinetic stimulation, a technique that moderates attention orientation by means of reflexive eye pursuits of meaningless visual moving stimuli, could also facilitate the processing of large numbers in numerical comparison tasks (Ranzini, et al., 2014). Studies that investigated spatial-numerical interaction in left unilateral neglect, a neuropsychological disease impairing patients' ability to orient their visuo-spatial attention to the contra-lesional part of space, showed that patients with right damage, thus neglecting the left part of space, were biased towards large numerical magnitudes in numerical interval bisection tasks (e.g., Cappelletti, Freeman, & Cipolotti, 2007; Hoeckner et al., 2008; Zorzi, Priftis, & Umiltà, 2002), or were slower to process numbers larger than a reference in numerical comparison tasks (Masson, Pesenti, & Dormal, 2013; van Dijck, Gevers, Lafosse, & Fias, 2012; Vuilleumier, Ortigue, & Brugger, 2004; Zorzi et al., 2012). Using leftward optokinetic stimulation (Priftis, Pitteri, Meneghello, Umiltà, & Zorzi, 2012), random dot kinematograms (Salillas, Granà, Juncadella, Rico, & Semenza, 2009) or prismatic adaptation (Rossetti et al., 2004) on left neglect patients not only improved attention orientation deficits towards left space, but also restored numerical processing by suppressing the deviation towards larger numbers in numerical interval bisection or the abnormal slowing to process larger numbers in a numerical comparison task to a standard. Altogether, these findings suggest that processing number magnitude may rely on spatio-attentional mechanisms.

This spatial–numerical orientation bias has recently been extended to mental arithmetic problem solving with the aim of testing whether mental calculation would also involve cognitive mechanisms akin to those supporting spatial attention (for a review, see Fischer & Shaki, 2014). A few studies reported that participants over- or underestimate the result of addition or subtraction problems respectively when asked to give an approximate response (McCrink, Dehaene, & Dehaene-Lambertz, 2007). This effect, called *operational momentum* (OM), has been replicated and is mainly observed with non-symbolic material (Knops, Dehaene, Berteletti, & Zorzi, 2014; Knops, Viarouge, & Dehaene, 2009a; Knops, Zitzmann, & McCrink, 2013; Lindemann & Tira, 2011; McCrink & Wynn, 2009; Pinhas & Fischer, 2008). It seems to arise for non-carry problems that are supposed to be solved by attentional strategies, but not for carry problems that might rely more on verbal strategies (Lindemann & Tira, 2011). OM has mostly been interpreted as the result of an attentional move along the MNL that goes too far in the direction related to the operation (Knops et al., 2009b; 2013; McCrink et al., 2007), although this interpretation is still debated because the mere observation of over- and underestimation biases in addition and subtraction solving is not sufficient to infer that space processing is implicated in mental arithmetic. Indeed, this bias could have a non-spatial cause. In most circumstances, the simple heuristic that adding means more and subtracting means less is accurate. Applying this principle would make a participant most likely to accept a larger/smaller proposed result for addition/subtraction solving. Another possible explanation of OM that does not require attentional shifts is that it might be the consequence of the compression of the representation of numbers and an uncompressed operation (Mc Crink et al., 2007; for a computational model, see Chen & Verguts, 2012). For instance, adding 20 and 8 that are internally represented as *log 20=1.301* and *log 8=0.903* would result in the *log 160* (i.e., *log 20*8*) which is larger than *log 28*. Thus, adding two compressed numbers will result in overestimating the result of the operation. The bias described here is however much larger than what is actually found in OM experiments. It is suggested that when performing an addition or a subtraction, the representation of numbers is less compressed which produce less over or underestimation biases. Moreover, underestimation bias for tie additions (Charras, Brod, & Lupianez, 2012; Charras, Molina, & Lupianez, 2013), or a reverse estimation bias in children (Knops et al., 2013), have recently been reported, which questions the reliability of OM. Strong evidence of the spatial account of OM is thus still needed.

At the neurofunctional level, it has been shown that the brain activations elicited by the resolution of additions with large operands (e.g., 50 ± 26) in the posterior part of the superior parietal lobule (PSPL) resemble activations induced by rightward eye saccades (Knops, Thirion, Hubbard, Michel, & Dehaene, 2009a). Indeed, a classifier built to predict whether participants were solving additions or subtractions on the basis of the activations of the PSPL when shifting eyes to the right or to the left could accurately sort addition trials although no such precision could be obtained with subtractions.

Until recently, very few studies had reported actual shifts of spatial attention as a result of arithmetic problem solving. In a target detection task with arithmetic problems as prime, the detection of targets located in the left hemifield was accelerated by the solving of subtractions of small magnitude whereas the detection of right targets was facilitated by the solving of additions of large magnitude (Masson & Pesenti, 2014). This study suggested that the spatial association between arithmetic and space might rely on a semantic link rather than on actual mental movements on a spatial representation of numbers because problems with zero as second operand, which require no mental movements on a MNL, induced similar attentional shifts. Such spatial associations with zero problems were also reported when participants were asked to provide their estimated response by pointing on a line flanked by numbers presented on a touchscreen (Pinhas & Fischer, 2008). Indeed, participants deviated the location of their answer to the right or to the left when the problem was, respectively, an addition or a subtraction. This effect was reversed when the flanked line was oriented from right to left (Pinhas, Shaki & Fischer, in press). In another recent study requiring participants to select the correct answer of arithmetic problems by moving a mouse cursor, hand movements were found to be deflected to the right/left respectively when answering an addition/subtraction problem (Marghetis, Nunez, & Bergen, 2014). It has also been shown that when asked to approximate the results of addition or subtraction problems and afterwards to select among several proposed results the one that was the closest to their own estimation, participants selected the proposals located at the upper right of the display for additions and at the upper left for subtractions (Knops et al., 2009a). These findings thus supply some evidence that solving arithmetic problems goes along with orientation of attention. However, it is still an open question whether these shifts of spatial attention underlie arithmetic processes or are just epiphenomena not critical for mental calculation. They might indeed simply go along with or result from the calculation processes while not being involved in any critical step of the solving procedures.

To assess whether orienting attention to the right or to the left is an underlying process rather than a by-product of solving addition and subtraction problems, we submitted healthy participants to a calculation task while lateralized stimuli were flickering on the left or on the right of the screen. This manipulation differs from our previous study (Masson & Pesenti, 2014) as lateralized stimuli were presented while participants were solving the problem rather than after the participants had given their answer. Therefore, while the previous study investigated the effect of solving arithmetic problems on attention shifts, here we investigated the reverse effect, namely how shifting attention may or may not affect calculation. If attention orienting is somehow linked to a critical step in the calculation process, then participants' performance should be affected when a lateralized flickering stimuli is presented. This influence would be greater if participants orient their attention to this portion of space when solving the problem. Participants’ performance is expected to be selectively affected when the flickering target is located in the hemifield congruent with the operation (i.e., right or left for additions or subtraction, respectively). Experiment 1 aimed at testing whether centrally presented flickering signals would interfere positively or negatively with arithmetic problem solving. Experiment 2 tested whether the lateralization of the target to be detected would affect selectively additions or subtractions. If orienting attention is not part of a critical calculation process but just a by-product of it, then flickering stimuli should have little or no specific impact on participants' performance.

Method

Participants

Twenty-two French-speaking students (14 females; mean age = 21 years; SD = 2; 19 right-handed) gave their informed consent to participate in this experiment, for which they received course credits. None was aware of the objectives of the study. The experiments were non-invasive, were performed in accordance with the ethical standards established by the Declaration of Helsinki, and were approved by the Ethics Committee of the Institut de Recherche en Sciences Psychologiques of the Université catholique de Louvain.

Material, tasks and stimuli

The experiments were conducted on a Dell PC equipped with a 17-inch screen. Stimulus presentation and data collection were programmed using E-Prime 1 (Schneider, Eschman, & Zuccolotto, 2002). Arithmetical problems were presented auditorily through headphones. The latencies of the verbal responses to arithmetic problems were recorded with an interfaced microphone while accuracy was assessed on-line by the experimenter.

In each experiment, participants were asked to perform two tasks successively at each trial: (i) solve an arithmetical problem, then (ii) report whether or on which side of the screen (i.e., left or right) a flickering target appeared while they were solving the problem. In the arithmetical task, the participants were asked to respond aloud to a list of 2-digit ± 1-digit problems generated on the basis of the following considerations. The first operand (O1) was between 22 and 89. Three ranges of second operand (O2) were used: Small (2, 3), Medium (4, 5, 6) and Large (7, 8). We controlled that the same amount of carry and non-carry problems were presented for additions and subtractions, which resulted in a total of 144 different problems, with 12 problems per condition (e.g., Addition/Carry/LargeO2; see Appendix). The means of results of the problems of each condition were equilibrated (range: 23-89; mean for additions 58 ± 16; subtractions: 57 ± 18) such that, when conducting an ANOVA on the magnitude of the results with OPERATION (Addition; Subtraction), O2 RANGE (Large; Medium; Small), and TYPE (Carry; Non-Carry) as within-subject variables, no main effect or interaction reached significance (all p-values > 0.1), thus excluding that the magnitude of the results alone could induce shifts of spatial attention. In the detection task, a flickering target was presented while the participants were solving the problem, either at the center of the screen (Experiment 1) or at 12 cm from the center (i.e., 14° of visual angle) on the congruent side or on the incongruent side of the screen (Experiment 2). Congruent targets would be located on the right/left for addition/subtraction and incongruent targets on the left/right for addition/subtraction, respectively. The target to detect was a 3-cm height (i.e., 3.5°) white star flickering three times. The participants had to report the presence of the target (Experiment 1) or the side of the screen where it appeared (Experiment 2).

For Experiment 1, a sample of 18 additions and 18 subtractions was selected from the 144 problems such that half of them for each operation required a carrying or borrowing process. These problems were presented randomly twice (i.e., once with and once without the distractor). For Experiment 2, the whole set of 144 problems were repeated only twice (i.e., once with a spatially congruent and once with a spatially incongruent target) to ensure that participants would not memorize the results; the problems were presented in six successive blocks in a pseudo-random order.

Procedure

Participants were seated 50 cm from the computer screen such that the midline of their face was aligned with the center of the screen, with their head positioned in a chin-rest to limit inappropriate movements as much as possible.

All participants started with the arithmetical task coupled with the central target detection task (Experiment 1). The sequence of events was as follows (see Figure 1). A fixation dot was presented for 500 ms followed by a blank screen during the auditory presentation of the arithmetic problem, which lasted for 3000 ms. For half of the trials, 300 milliseconds after the second operand had been presented, a centered target flickered three times (i.e., 3 cycles of 200 ms on screen interleaved with 150-ms blanks, for a total duration of 900 ms), then a blank screen remained until the answer; for the other half of the trials, the screen remained blank until the answer. The verbal answer to the arithmetic problem launched the appearance of a question mark at the center of the screen to which the participants were instructed to press the space bar as fast as possible with their right hand if a flickering target had been presented. The next trial began after a 2000-ms pause.

After a short break, the participants performed the arithmetical task coupled with the lateralized target detection task (Experiment 2). The sequence of events was identical to previously, except that the target to detect was displayed on the left or on the right of the screen for half of the trials respectively. At the appearance of the question mark, the participants had to specify on which side of the screen the target had been presented by pressing with their left or right hand the left or right response key (i.e., “f” and “j” of an AZERTY keyboard respectively).

The whole experiment lasted about 50 minutes.

Results

Experiment 1: Arithmetical solving with central target detection

No error was made, by any of the participants, in the target detection task. For the arithmetical task, trials where the answer to the problem was incorrect (4.68%; 74/1584 trials) or where the microphone failed to trigger (4.23%; 67/1584 trials) were removed from the response latency (RL) analysis. A repeated measures analysis of variance (ANOVA) was carried out on the median RLs using OPERATION (Addition; Subtraction) and TARGET (With; Without) as within-subject variables. There was a main effect of OPERATION ($F(1,21)=19.602$, MSE=55,262, $p<.001$; $\eta^2_p=.483$) indicating that participants were slower to solve subtractions (2634±520 ms) than to solve additions (2412±428 ms). There was also a main effect of TARGET ($F(1,21)=954.612$, MSE=64,468, $p<.001$; $\eta^2_p=.978$) showing that the presence of a target drastically slowed the solving of the problems (With: 3360±547 ms; Without: 1687±398 ms). The interaction between OPERATION and TARGET was not significant ($F(1,21)<1$, MSE=61,218, *ns*; $\eta^2_p=.006$).

A similar ANOVA was performed on error rates. It showed only a significant main effect of TARGET ($F(1,21)=5.332$, MSE=12,891, $p<.04$, $\eta^2_p=.202$) indicating that participants made more errors when the flickering target was present (5.56±5.42%) than when it was not (3.79±4.66). No other main effect or interaction reached significance (all *p*-values >.3).

Experiment 2: Arithmetical solving with lateralized target detection

Trials where the answer to the arithmetic problem was incorrect (5.81%; 368/6336 trials), where the microphone failed to trigger (3.48%; 231/6336 trials), or where participants failed to report correctly the localization of the target (1.21%; 77/6336 trials) were removed from the response latency analysis.

We conducted a repeated measures ANOVA on the median RLs of correctly solved problems with OPERATION (Addition; Subtraction), TARGET (Congruent; Incongruent), O2 RANGE (Small; Medium; Large) and TYPE (Carry; Non-Carry) as factors. There was a main effect of OPERATION ($F(1,21)=21.837$, MSE=365,303, $p<.001$, $\eta^2_p=.51$) indicating that additions (3359±512 ms) were solved faster than subtractions (3604±704 ms). A main effect of O2 RANGE ($F(2,42)=54.229$, MSE=175, 435, $p<.001$, $\eta^2_p=.721$) showed that small problems (3242±508 ms) were solved faster than medium problems (3497±626 ms; $t(21)=$, 6.317, $p<.001$), which were solved faster than large ones (3706±696ms; $t(21)=6.361$, $p<.001$). There was also a significant main effect of TYPE ($F(1,21)=119.939$, MSE=743,107, $p<.001$, $\eta^2_p=.851$), participants being slower to respond to carry problems (3892±747 ms) than to non-carry problems (3071±481 ms). There was a significant interaction between OPERATION and TYPE ($F(1,21)=16.37$, MSE=154,924, $p<.001$, $\eta^2_p=.438$). Paired sample *t*-tests revealed that there was a larger difference between operations for carry (additions: 3700±602 ms; subtractions: 4085±905 ms; $t(21)=4.893$, $p<.001$) than for non-carry problems (additions: 3017±446 ms; subtractions: 3124±531 ms; $t(21)=2.593$, $p<.02$). A significant interaction between TYPE and O2 RANGE ($F(2,42)=83.377$, MSE=149,826, $p<.001$, $\eta^2_p=.799$) revealed that carry problems with large o2 (4372±930 ms) were solved more slowly than those with medium o2 (3930±807 ms; $t(21)=7.388$, $p<.001$), and that medium problems were solved more slowly than small problems (3376±540 ms; $t(21)=7.793$, $p<.001$). For non-carry problems, participants responded equally fast to all ranges of problems (all p-values >.1). Most importantly, a main effect of TARGET ($F(1,21)=6.363$, MSE=25,292, $p<.02$, $\eta^2_p=.233$) was found. When the target was presented in the hemifield congruent with the operation, participants were slower to respond to the arithmetic problem (Congruent: 3499±602 ms; Incongruent: 3464±605). This indicates that participants were slower to solve an addition when the target was presented in the right side (3377±529 ms) than when it was presented in the left side of the screen (3340±502 ms). Conversely, subtractions were slowed down by the presentation of a left-sided target (3621±696 ms) in comparison to a right-sided target (3588±717 ms; see Figure 2). This congruency effect was identical in both operations as no interaction between operation and target was observed ($F(1,21)=0.011$, MSE:76,457, *ns;* $\eta^2_p=.001$). Finally, the effect of congruency was not moderated by o2 range as TARGET and O2 RANGE did not interact ($F(2,42)=0.48$, MSE=78,785 *ns*, $\eta^2_p=.002$). No other interaction was observed (all p-values >.05)

A similar ANOVA on the error rates revealed a main effect of OPERATION ($F(1,21)=19.206$, MSE=31.666, $p<.001$; $\eta^2_p=.478$) indicating that participants made more errors for subtractions (6.88±2.81 %) than for additions (4.73±2.84 %). There was also a main effect of O2 RANGE ($F(2,42)=22.224$, MSE=54.156, $p<.001$; $\eta^2_p=.729$) showing that participants were better at solving small o2 problems (2.98±1.62 %) than medium o2 problems (6.3±4 %; $t(21)=3.967$, $p<.001$), which were themselves better solved than large o2 problems (8.14±3.87 %; $t(21)2.254$, $p<.04$). A main effect of TYPE was also found ($F(1,21)=36.328$, MSE=119.924, $p<.001$; $\eta^2_p=.634$), participants being better at solving non-carry (2.94±2.17 %) than carry (8.68±4.31 %) problems. Finally, there was a significant interaction between O2 RANGE and TYPE

(F(2,42)=36.450, MSE=42.156, p<.001; η^2_p=.77). Participants made more errors when a carry or a borrow was needed for large o2 problems (Carry: 14.3±7.14 %; Non-Carry: 1.99±2.84 %; t(21)=7.566, p<.001) and for medium o2 problems (Carry: 8.33±5.42 %; Non-Carry: 4.26±3.83 %; t(21)=3.885, p<.005), but not for small o2 problems (Carry: 3.41±2.84 %; Non-Carry: 2.56±2.65; t(21)=.901, *ns*). No other main effect or interaction reached significance (all p-value >.1).

Discussion

It has been suggested that arithmetic problem solving involves mechanisms akin to those underlying spatial attention orientation. Solving subtractions and additions has been associated to leftward or rightward shifts of attention in several behavioral (Masson & Pesenti, 2014; Marghetis et al., 2014; Pinhas & Fischer 2008; Werner & Raab, 2014; Wiemers, Bekkering, & Lindemann, 2014) and brain imaging studies (Knops et al., 2009a). However, it was still a matter of debate whether orienting spatial attention functionally underlies arithmetic processes or if these compatibility effects were epiphenomena not required for mental arithmetic. This study aimed at investigating the impact of spatial attention manipulation on calculation by means of flickering targets that were presented while solving additions or subtractions. We analyzed participants' performances on addition and subtraction problem solving.

Because larger magnitudes are themselves associated to the right part of space and smaller magnitudes to the left part of space (e.g., Fischer et al., 2003), it was critical to control for the magnitude of the results of additions and subtractions. Indeed, using larger results for additions than for subtractions would make it difficult to disentangle whether lateralized signals interfere with the type of operation or with the magnitude of the numbers that are manipulated. Given that the present set of problems was matched for the mean magnitude of the results of additions and subtractions, the magnitude of the results of the problems cannot be confounded with an effect of the operation. It is also worth noting that the more difficult the problems are, the longer the participants take to solve them and the higher the error rate. Indeed, subtraction problems are usually more difficult than additions, and the magnitude of the operands as well as the need to manage carrying and borrowing are known to make the solving more difficult (Ashcraft, 1992; Campbell, 2005). The present findings fit with these classical effects, which is important as they show that the participants did perform the arithmetic task as expected.

Considering now the influence of the targets that were presented during arithmetic solving, we first showed that the mere presentation of a centered flickering target interferes with addition and subtraction solving (Experiment 1), probably because it captures part of the attentional resources needed to solve the problem. Importantly, this interfering effect was similar for the two operations. Thus, in the context of this global slowing down, one might expect this interference to be greater if attention was oriented by the resolution of the arithmetic problem towards the location of the flickering target. Since addition solving orients attention towards the right side of space and subtraction towards the left side of space (Masson & Pesenti, 2014), left-sided flickering targets would interfere more with subtraction solving while right-sided targets would interfere more with addition solving if attention orientation is part of the solving process and not a by-product of it. This was exactly what we observed (Experiment 2): participants were indeed slower to solve an addition while a right-sided distractor was presented and slower to solve a subtraction with a left-sided distractor. This space congruency effect was equally strong for both additions and subtractions as no interaction between congruency and operation was observed. Previous findings have already revealed that solving additions and subtractions produce shifts of spatial attention to the right or to the left respectively. The present study extends these existing findings by documenting the reverse effect, namely that shifting attention affects the solving procedure, thus showing that attention orientation mechanisms are actually functionally involved during arithmetic problem solving and are not just irrelevant by-products of it. These results are in line with previous observation that the direction of hand movements (Wiemers et al., 2014) or of the entire body in the horizontal (Anelli, Lugli, Baroni, Borghi, & Nicoletti, 2014) or vertical planes (Lugli, Baroni, Anelli, Borghi, & Nicoletti, 2013) impacts arithmetic solving mechanisms.

However, how spatial attention shifts play a role in arithmetic solving remains a matter of debate. A recent computational model suggested that spatial attentional mechanisms help to determine the solution of an arithmetic problem by producing a mental move after remapping the first operand onto a left-to-right oriented continuum (Chen & Verguts, 2012). Thus, additions or subtractions would be accompanied by spatial attentional shifts to the right or to the left, respectively, of a scale determined by the magnitude of the second operand. Interestingly, the congruency effect found irrespective of the magnitude of the second operand suggests that if attention orientation is actually part of the solving process, it does not simply correspond to an actual move or "walk" on the MNL. Indeed, in case of an actual move, the larger the second operand, the larger the move should be, hence the larger the congruency effect. This was not observed here and this absence of enhancing congruency effect for problems with a larger second operand does not fit with the idea that arithmetic solving can be related to a simple move on a MNL with spatial characteristics isomorphic to real space. We will come back to this point below. Alternatively, Pinhas & Fischer (2008) suggested that spatial biases related to arithmetic may be the consequence of a competition between localized activation of the operands, the result, and a semantic link between the type of operation and space (i.e., right-addition; left-subtraction). This idea stems from the observation of spatial bias in various behavioral tasks for problems with zero as second operand, which excludes any movement along a mental continuum (Lindemann & Tira, 2011; Marghetis et al., 2014; Masson & Pesenti, 2014; Pinhas & Fischer, 2008). It has also been reported that the magnitude of the operands and of the results of the problem may modulate attentional shifts (Marghetis, et al., 2014; Masson & Pesenti, 2014; Pinhas et al., 2014; Pinhas & Fischer, 2008) or switch the direction of OM (Charras et al., 2014; Klein, Huber, Nuerk, & Moeller, 2014).

Interestingly, operation signs themselves can evoke spatial associations. A recent study recorded eye movements of participants while solving additions or subtractions and showed early upward gaze shifts when hearing the *plus* sign and downward shifts when hearing the *minus* sign, thus before starting the calculation process itself since the second operand is not yet known (Hartmann, Mast, & Fischer, 2015). Moreover, when participants were asked to classify *plus* and *minus* signs by pressing a right or a left response key, responses were indeed faster when the *plus* sign was associated with the right response key and the *minus* sign with the left response key (Pinhas, Shaki, & Fischer, 2014). However, this "operation sign spatial association" (OSSA) was very weak for the *minus* sign and was only observed when preceded by other numerical tasks that highlighted a mathematical context to the operation sign. We argue that, alone, such a small OSSA effect is unlikely to cause an impairment to solving subtractions in left neglect patients (Dormal, Schuller, Nihoul, Pesenti, & Andres, 2014). Rather, we suggest that when solving a problem, part of the solving procedure implies representing first a starting location (i.e., O1) and then, relative to this location, the approximate location of the result that will be on the left for a subtraction. In healthy individuals, the same process takes place also to the right for additions.

More generally, we propose that an attentional shift whose direction is semantically driven by the operation is needed to access the representation of the result of the problem. Indeed, a mere attentional account of the observed spatial association that would simply reduce it to a procedural calculation strategy consisting in moving along a MNL is not enough to account for the data. As supported by the above-mentioned effects (i.e., OSSA, plus or minus zero, absence of second operand magnitude), at least part of the attentional shift requires and stems from a semantic association between a given arithmetical operation and a given side of space. It is this semantic association that drives the direction of attention shifts. In this perspective, our participants might have been disturbed in this visualization by the presentation of a flickering target located in the locus of their attentional shift. This assumption fits with the recent report of altered performance for subtraction solving of left neglect patients (Dormal, et al., 2014). Indeed, these patients, who experience some difficulty orienting their attention to the left side of space, were also impaired when solving subtractions. The authors suggested that left neglect could hamper the ability to access the representation of the correct answer of a subtraction that is located on the left relative to the first operand whatever the absolute position of the solution on a spatial continuum. These mechanisms may also be recruited in numerical comparison. Indeed, the impaired ability of left neglect patients to mentally orient their attention towards the representation of numbers located on the left side of a given reference, whatever its magnitude (Masson et al., 2013; Vuilleumier et al., 2004), corroborates the impairment that has been described for left neglect patients in subtractions solving. Attention shifts may thus be functionally involved whenever a number has to be represented to the left or to the right relative to another on a mental continuum. Further work is needed in order to investigate this assumption by submitting left but also right neglect patients to numerical comparison tasks and arithmetic problems.

Shifts of spatial attention related to number magnitude or problem solving fit and are often interpreted as supporting the MNL as commonly conceptualized (i.e., a left-to-right horizontally spread and compressed mental space; Dehaene, 1992). However, it is worth noting that the evidence so far in fact also fits with any type of mental representations, as long as they somehow preserve or represent spatial and/or ordinal relationships between numbers. In any case, a key aspect of the present and some of the previous findings is that a first-order isomorphism of this representation and of its usage is not supported, and thus not mandatory. Whether a second-order isomorphism offers a viable framework to interpret the observed attentional shifts is still under debate. Here also further work is thus needed to properly assess the possible spatial properties and metrics of the representation giving rise to the observed attentional shifts.

In conclusion, converging evidence suggests that solving arithmetic problems involves shifting attention on a mental continuum with addition and subtraction causing, respectively, rightward or leftward attentional shifts. The present study aimed at testing whether attentional shifts are critical in arithmetic solving or constitute epiphenomena by disturbing attentional locus with flickering targets. Our data suggest that attentional shifts are functionally linked to mental arithmetic through semantic associations, showing that attentional displacement can affect operation solving by accessing the representation of the result of the problem from the representation of a first operand.

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Figure captions

Figure 1: Sequence of events and temporal attributes for one trial.

Figure 2: Mean response latencies (±S.E.) to respond to the arithmetic problems as a function of operation (addition vs. subtraction) and target (congruent vs. incongruent) for Experiment 2.

Appendix – List of problems used in Experiment 2.

Operation	Carry	Second Operand Range	First Operand	Second Operand	Result	Average Magnitude *
Addition	Carry	Large	29	8	37	24,7
			38	8	46	30,7
			29	7	36	24,0
			65	8	73	48,7
			56	7	63	42,0
			65	7	72	48,0
			46	8	54	36,0
			44	8	52	34,7
			38	7	45	30,0
			47	7	54	36,0
			67	8	75	50,0
			74	7	81	54,0
		Medium	26	5	31	20,7
			66	6	72	48,0
			29	4	33	22,0
			47	4	51	34,0
			48	6	54	36,0
			68	5	73	48,7
			77	5	82	54,7
			38	4	42	28,0
			57	6	63	42,0
			75	6	81	54,0
			39	6	45	30,0
			59	5	64	42,7
		Small	38	3	41	27,3
			69	3	72	48,0
			29	2	31	20,7
			58	3	61	40,7
			59	2	61	40,7
			79	3	82	54,7
			39	2	41	27,3
			29	3	32	21,3
			49	3	52	34,7
			78	3	81	54,0
			69	2	71	47,3
			68	3	71	47,3
	Non-Carry	Large	41	7	48	32,0
			81	8	89	59,3
			22	7	29	19,3
			71	8	79	52,7
			61	7	68	45,3
			62	7	69	46,0
			31	7	38	25,3
			32	7	39	26,0
			31	8	39	26,0
			51	8	59	39,3
			71	7	78	52,0

			52	7	59	39,3
		Medium	35	4	39	26,0
			74	5	79	52,7
			33	5	38	25,3
			52	4	56	37,3
			63	6	69	46,0
			61	4	65	43,3
			62	6	68	45,3
			44	4	48	32,0
			41	5	46	30,7
			32	5	37	24,7
			71	6	77	51,3
			53	4	57	38,0
		Small	62	2	64	42,7
			54	3	57	38,0
			81	3	84	56,0
			45	2	47	31,3
			63	3	66	44,0
			45	3	48	32,0
			62	3	65	43,3
			71	2	73	48,7
			63	2	65	43,3
			36	3	39	26,0
			37	2	39	26,0
			54	2	56	37,3
Subtraction	Carry	Large	62	8	54	41,3
			53	8	45	35,3
			31	8	23	20,7
			52	7	45	34,7
			63	7	56	42,0
			45	8	37	30,0
			64	8	56	42,7
			61	7	54	40,7
			85	7	78	56,7
			74	7	67	49,3
			86	8	78	57,3
			86	7	79	57,3
		Medium	51	4	47	34,0
			85	6	79	56,7
			71	5	66	47,3
			72	5	67	48,0
			52	6	46	34,7
			34	5	29	22,7
			42	4	38	28,0
			74	6	68	49,3
			83	5	78	55,3
			51	6	45	34,0
			73	6	67	48,7
			53	4	49	35,3
		Small	51	3	48	34,0
			52	3	49	34,7
			82	3	79	54,7
			62	3	59	41,3

		51	2	49	34,0
		41	2	39	27,3
		81	3	78	54,0
		31	2	29	20,7
		71	3	68	47,3
		72	3	69	48,0
		61	2	59	40,7
		41	3	38	27,3
Non-Carry	Large	49	7	42	32,7
		79	7	72	52,7
		38	7	31	25,3
		79	8	71	52,7
		59	8	51	39,3
		48	7	41	32,0
		68	7	61	45,3
		58	7	51	38,7
		39	7	32	26,0
		89	7	82	59,3
		89	8	81	59,3
		69	8	61	46,0
	Medium	78	6	72	52,0
		89	6	83	59,3
		57	4	53	38,0
		86	5	81	57,3
		59	5	54	39,3
		67	6	61	44,7
		46	4	42	30,7
		37	5	32	24,7
		79	4	75	52,7
		68	4	64	45,3
		35	4	31	23,3
		48	5	43	32,0
	Small	56	3	53	37,3
		89	2	87	59,3
		45	3	42	30,0
		89	3	86	59,3
		45	2	43	30,0
		67	2	65	44,7
		34	3	31	22,7
		67	3	64	44,7
		48	3	45	32,0
		56	2	54	37,3
		33	2	31	22,0
		78	2	76	52,0
	Addition Mean	52,65	5	57,65	38,4
	sd	16,06	2,08	16,07	10,71
	Subtraction Mean	61,375	5	56,375	40,9
	sd	17,19	2,08	16,93	11,46

* = average magnitude of First Operand, Second Operand and Result