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Resource Allocation Techniques in Relay-Aided Multicell OFDMA-based Wireless Communication Systems

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Abstract

This thesis studies resource allocation (RA) problems in multi-cell orthogonal frequency division multiple access (OFDMA) downlink systems with co-channel interference (CCI). This thesis contains four parts.

In the first part, four signal models used for formulating RA problems in multi-cell OFDMA downlink systems are overviewed. For comparison, the classic sum rate maximized RA problem with per-station individual power constraints is considered. Applying the four signal models, exact formulations of the considered RA problem are proposed. Properties of the different formulations are unveiled and compared. Based on existing RA methods, suggestions about choosing a suitable signal model and problem formulation are given. Finally, an interesting property of the recommended formulations is unveiled.

In the second part, the sum (over all cells and all destinations) rate maximized problem is considered in multi-cell decode-and-forward (DF) relay-aided OFDMA downlink systems. An iterative RA algorithm is proposed to optimize mode selection (decision whether the relay should help or not), subcarrier assignment and pairing (MSSAP) and power allocation (PA) in an alternate way. The proposed algorithm is coordinate ascent (CA) based and can reach a local optimum in polynomial time.

In the third part, user fairness is further considered in multi-cell DF relay-aided OFDMA downlink systems. The problem of maximizing the weighted sum of per cell min-rate (WSMR) with per-cell total power constraints is formulated and its per-cell maximum fairness property is proven. An iterative RA algorithm is proposed to obtain a local optimum of the problem. The convergence and the per-cell user fairness of the proposed RA algorithm are proven.

In the fourth part, multiple DF relay stations (RSs) are considered to assist data transmission in each cell. An iterative semi-distributed RA algorithm is firstly proposed to reach a local optimum with guaranteed convergence. Then a modified iterative waterfilling (IWF) algorithm is further proposed to solve the problem autonomously. An optimum algorithm is proposed to solve the local RA problem in each cell.

Contents

Notations and Acronyms

Notations

x	a real number
$\|x\|$	the absolute value of x
$\frac{\partial f(x)}{\partial x}$	the derivative of function $f(x)$ with respect to x
$\|\|\mathbf{X}\|\|$	the second order vector norm of $\mathbf{X}$, i.e., $\|\|\mathbf{X}\|\| = \left(\sum_i \|x_i\|^2\right)^{1/2}$
$\mathbf{X}$	a vector
$\arg\min_{x\in\mathbf{X}} f(x)$	the set of $x \in \mathbf{X}$ minimizing $f(x)$
$\arg\max_{x\in\mathbf{X}} f(x)$	the set of $x \in \mathbf{X}$ maximizing $f(x)$
$[x]_+$	$\max(x, 0)$
$(\cdot)^T$	the transposition of a vector or a matrix
N	number of cells
U	number of users
K	number of subcarriers
J	number of relay stations
$\mathbf{A}$	binary variables for the direct transmission mode
$\mathbf{B}$	binary variables for the relay aided transmission mode
$\mathbf{P}$	power variables
G	channel gain
s_n	the base station in cell n
r_{jn}	the jth relay station in cell n
d_{un}	the uth user in cell n

Acronyms

AWGN	additive white Gaussian noise
AF	amplify and forward (AF)
BPA	best power allocation
BSs	base stations
CA	centralized algorithm
CC	central controller
CCI	cochannel interference
CFR	channel frequency response
CGP	complementary geometric program
CIR	channel impulse response
CSI	channel state information
DF	decode and forward (DF)
DSL	digital subscriber lines
DR	direct rounding
DR-MSSA	direct rounding based MSSA
GP	geometric program
HSE	high spectrum efficiency
IP	integer program
IWF	iterative waterfilling
KKT	Karush-Kuhn-Tucker
LAP	linear assignment problem
LSE	low spectrum efficiency
MILPs	mixed integer linear programs
MSSA	mode selection and subcarrier assignment
MSSAP	mode selection, subcarrier assignment and pairing
MSs	mobile stations
OFDM	orthogonal frequency division modulation
OFDMA	orthogonal frequency division multiple access
OR	opportunistic relaying
PA	power allocation
RA	resource allocation
RR	randomized rounding
RR-MSSA	randomized rounding based MSSA
RSs	relay stations

Acronyms

SA	subcarrier allocation
SC	single condensation
SCA	successive convex approximation
SCGP	single condensation and geometric programming
SCLD	single condensation and Lagrange duality
SGP	standard geometric programming
SINR	signal-to-interference-plus-noise ratio
SP	subcarrier pairing
TSs	time slots
WSMR	weighted sum of per cell min-rate
WSR	weighted sum rate

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Introduction

1

This chapter first explains motivations of this thesis, then states the outline and contributions, and finally lists related publications.

1.1 Motivation

In the last three decades, the communication industry has experienced a phenomenal growth. Not surprisingly, wireless communication is the fastest growing segment, which plays an important role in everyday life of billions of people. To provide better services for more users, high data rate and ubiquitous coverage are strongly required in the next-generation wireless communication networks. To achieve this goal, orthogonal frequency division multiple access (OFDMA) technology is receiving a lot of interest due to its inherent ability to combat frequency-selective multi-path fading and its flexibility in applying dynamic radio resource allocation (RA) for performance improvement. Here the resources to be allocated are time slots, subcarriers, power, etc. When channel state information (CSI) is available at the OFDMA transmitters, dynamic RA plays a key role for performance improvement and is quite challenging with all resources jointly optimized.

On the other hand, the emerging relaying technology is highly favored by both academia and industry due to its attractive feature of low cost, coverage extension and data rate improvement [1]. Therefore, it is promising to incorporate OFDMA with relaying technologies in next generation cellular systems.

Concerning relay aided OFDM(A) transmissions, two low-complexity yet efficient types of relaying have been proposed in [2] and [3], namely amplify and forward (AF) and decode and forward (DF). With AF/DF relaying, symbols are transmitted in two time slots (TSs). During the first TS, the source broadcasts symbols on all subcarriers with the relay keeping quiet. During the second TS, except for the relay, the source might also broadcast symbols on subcarriers not used by the relay, as will be elaborated later. Each subcarrier used by the relay during the second TS is paired with a subcarrier during the first TS. In particular, these two subcarriers are termed as paired subcarriers, referred to as the relay aided transmission mode hereafter. Adopting subcarrier-by-subcarrier pairing based DF relaying, RA problems for downlink OFDMA have been investigated intensively [4–23].

In particular, the works in [4–7] have considered RA for the DF relay-aided OFDM systems when the destination can not or hardly hear from the source, meaning the source-to-destination link is unavailable. Every subcarrier during the first TS is paired with a subcarrier during the second TS using the relay aided transmission mode. Under a total power constraint, it has been proven that the optimum sum rate can be reached by ordered subcarrier pairing, i.e. the best source-to-relay subcarrier should be paired with the strongest relay-to-destination subcarrier, and so on.

Considering the case when the S-D link is available, [8–23] have studied RA in systems with opportunistic relaying (sometimes termed as selection relaying). To start with, a low spectrum efficiency (LSE) protocol was studied in [8–13], when only the relay broadcasts symbols during the second TS. Specifically, every symbol can be transmitted in either the relay aided mode or the direct mode. In the relay aided mode, a subcarrier during the first TS is paired with a subcarrier during the second TS for the transmission. While in the direct mode, the symbol is transmitted on a subcarrier during the first TS directly to the targeted destination without the help of the relay. Note that, using this LSE protocol, unpaired subcarriers during the second TS are actually unused, which causes a waste of the limited spectrum resource. To address this issue, [14–22] have proposed and studied improved high spectrum efficiency (HSE) protocols, which allow new symbols to be transmitted on the unpaired subcarriers during the second TS. Note that the HSE protocols do not really modify the implementation of DF relaying during the second TS, but simply let the source use unpaired subcarriers during the second TS for direct-mode transmission. To further improve the system performance, a novel subcarrier-pair based opportunistic DF relaying protocol is proposed in [23]. Note that this protocol truly improves the implementation of DF relaying by using transmit beamforming at a subcarrier during the second TS. Specifically, this novel protocol is the same as the HSE protocol, except that the source and relay implement transmit beamforming to transmit the symbol at a paired subcarrier during the second TS.

Note that all these papers consider RA in single cell situations with the cochannel interference (CCI) modeled as additive background noise, which is reasonable only when the frequency reuse factor is low. The frequency reuse factor is 1/W, where W is the number of cell clusters which cannot use the same frequencies for transmission. However in next generation cellular systems, aggressive frequency reuse is recommended due to its ability to achieve higher system capacity [24]. When the frequency reuse factor is high or even up to 1, the CCI becomes a key factor affecting the system performance and thus can not be ignored [25,26].

Considering the CCI, RA problems in multi-cell DF relay aided OFDMA systems become more interesting and challenging. Although important, these problems have been investigated only by few papers. In this thesis, RA problems (with/without fairness consideration) will be investigated in multi-cell DF relay aided OFDMA systems.

1.2 Outline and contributions

The structure of this thesis will be as follows. We first overview different signal models used for formulating RA problems in multi-cell OFDMA downlink systems with co-channel interference (CCI). After comparison, a suitable signal model is then adopted to facilitate the algorithm design for the considered problems. Finally, based on the signal model, RA problems (with/without fairness consideration) are formulated and solved by designed algorithms. Specifically, the outline and contributions of this thesis are listed as follows:

In Chapter 2, some basic preliminaries and mathematical backgrounds are provided.

In Chapter 3, we overview different signal models used for formulating RA problems in multi-cell OFDMA downlink systems with CCI. Four signal models are summarized, which bring the same RA problem with formulations of different complexities. For comparison, we consider the classic sum rate maximized RA problem in multi-cell OFDMA downlink systems with per-station individual power constraints. Applying the four signal models, exact formulations of the considered RA problem are proposed. Properties of the different formulations are uncovered and compared. Based on existing RA methods, suggestions about choosing a suitable signal model and problem formulation are given. Finally, an interesting property of the recommended formulations is unveiled.

In Chapter 4, we consider multi-cell DF relay-aided OFDMA downlink systems, in which all sources and relays are coordinated by a central controller for RA. The improved subcarrier-pair based opportunistic DF relaying protocol proposed and studied in [8] and [23] is applied. This protocol has a HSE in the sense that all unpaired subcarriers are utilized for data transmission during the second TS. In particular, the sum (over all cells and all destinations) rate maximized problem with a total power constraint in each cell is formulated. To solve this problem, an iterative RA algorithm is proposed to optimize mode selection (decision whether the relay should help or not), subcarrier assignment and pairing (MSSAP) and power allocation (PA) in an alternate way. As for the MSSAP stage of each iteration, the formulated problem is decoupled into subproblems with the tentative PA results. Each subproblem can be easily solved by using the optimal results of a Linear Assignment Problem (LAP), which is then solved by the Hungarian Algorithm in polynomial time. As for the PA stage of each iteration, an algorithm based on single condensation and geometric programming (SCGP) is proposed to optimize PA in polynomial time with the tentative MSSAP results. The proposed algorithm is coordinate ascent (COA) based and therefore can reach a local optimum in polynomial time. Finally, the convergence and effectiveness of the proposed algorithm, the impact of relay position and total power on the system performance as well as the benefits of using subcarrier pairing (SP) and the HSE protocol are illustrated through numerical experiments.

In Chapter 5, we consider the same system as that of Chapter 4, except that SP is not considered here. Compared to Chapter 4, per-cell user fairness is further studied here. In particular, the problem of maximizing the weighted sum of per cell min-rate (WSMR) with per-cell total power constraints is formulated and its per-cell maximum fairness property is proven. An iterative RA algorithm is proposed to optimize mode selection (decision whether the relay should help or not), subcarrier assignment (MSSA) and PA alternatively. Each iteration is composed of the MSSA stage and the PA stage. During the MSSA stage, the original problem is decoupled into mixed integer linear programs (MILPs) with the tentative PA results, which can be solved by typical MILP solvers. To solve the MILPs more efficiently in polynomial time, a randomized rounding based MSSA (RR-MSSA) algorithm and a direct rounding based MSSA (DR-MSSA) algorithm are further proposed. During the PA stage, an algorithm based on SCGP is designed to optimize PA with the tentative MSSA results. The convergence and the per-cell user fairness of the proposed RA algorithm are proven. Finally, the performance of the RA algorithm and the benefits of using opportunistic relaying (OR) and the HSE protocol are illustrated through numerical experiments.

In Chapter 6, we consider the same system as that of Chapter 5, except that multiple DF relay stations (RSs) are placed in each cell. The problem considered is the maximization of the system sum rate with a total power constraint in each cell. An iterative semi-distributed resource allocation (RA) algorithm is first proposed to optimize MSSA and PA alternatively. During the MSSA stage, the problem is decoupled into subproblems which can be solved distributively in linear time. During the PA stage, an algorithm based on single condensation and Lagrange duality (SCLD) is designed to optimize PA with the tentative MSSA results. The convergence of the SCLD based RA algorithm is theoretically guaranteed and an local optimum is reached after convergence. To solve the formulated problem autonomously, a modified iterative waterfilling (IWF) algorithm is further proposed. Specifically, each cell autonomously optimizes its own sum rate with the estimated power values of the received interferences from the other cells. An optimum algorithm is proposed to solve the local RA problem in each cell. Through numerical experiments, the convergence of the two proposed algorithms as well as their benefits compared with the centralized algorithm (CA) proposed in Chapter 4 are illustrated.

In Chapter 7, a summary and perspectives for future work are presented.

1.3 Related publications

The content of Chapter 4 has been published or accepted in

- Jin Zhiwen, Vandendorpe Luc, '*Resource Allocation in Multi-Cellular DF Relayed OFDMA Systems*,' IEEE Global Communications Conference (IEEE GLOBECOM 2011), Houston, Texas, USA, Dec. 2011.

- Jin Zhiwen, Wang Tao, Wei Jibo, Vandendorpe Luc, '*Sum Rate Maximization for Multi-Cell Downlink OFDMA with Subcarrier-pair based Opportunistic DF Relaying*,' submitted to EURASIP Journal on Wireless Communications and Networking (EURASIP JWCN), 2014.

The content of Chapter 5 has been published or submitted in

- Jin Zhiwen, Wang Tao, Wei Jibo, Vandendorpe Luc, '*Resource Allocation for Maximizing Weighted Sum of Per Cell Min-Rate in Multi-Cell DF Relay Aided Downlink OFDMA Systems*,' IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (IEEE PIMRC 2012), Sydney, Australia, Sept. 2012.

- Jin Zhiwen, Wang Tao, Wei Jibo, Vandendorpe Luc, '*Weighted Sum of Per Cell Min-Rate Maximization for Multi-Cell Downlink OFDMA with Opportunistic DF Relaying*,' submitted to EURASIP Journal on Wireless Communications and Networking (EURASIP JWCN), 2014.

Part of the content of Chapter 6 has been published in

- Jin Zhiwen, Wang Tao, Wei Jibo, Vandendorpe Luc, '*A Low-Complexity Resource Allocation Algorithm in Multi-Cell DF Relay Aided OFDMA Systems*,' IEEE Wireless Communications and Networking Conference (IEEE WCNC 2013), Shanghai, China, April 2013.

Preliminaries and Mathematical Backgrounds

2

In this chapter, some preliminaries and mathematical backgrounds about RA in OFDMA downlink systems will be introduced.

2.1 Preliminaries about RA in OFDMA systems

As a realization of multicarrier modulation, OFDM provides an elegant solution to the problem of intersymbol interference induced by multipath channels. The principle of multicarrier modulation is to split the transmission bandwidth into several narrowband subcarriers for parallel data transmission. Each subcarrier experiences flat fading. Equalization is performed on a subcarrier basis and thus is much simpler than that of the single carrier transmission systems. A single fade only affects a limited number of subcarriers. By coding across the symbols in different subcarriers, the signal transmitted by those affected subcarriers can be recovered at the receiver [27].

Modern multicarrier systems propose a very challenging multiuser communication problem. Multiple users in the same geographic area require high data rates in a finite bandwidth. Multiple access techniques allow different users to share the available bandwidth by allocating each user some fraction of the total system resources. Combining the frequency division multiple access technology with OFDM, OFDMA allocates the available subcarriers among multiple users. Each user receives a unique carrier frequency and bandwidth. There are a number of ways to perform this allocation. The simplest method is a static allocation of subcarriers to each user. In this way, it is possible that high data rate users are allocated with more subcarriers than lower rate users, which is uneven. To improve it, dynamic subcarrier allocation based upon channel state informations is recommended.

Similarly to OFDM, OFDMA has the advantage of robust multipath suppression. Thanks to its multiple access feature, the OFDMA can accommodate multiple users with widely varying applications, data rates and quality of service requirements through dynamic and efficient resource (subcarrier, power, etc.) allocation. The main motivation for dynamic RA in OFDMA is to exploit multiuser diversity, which describes the gains available by selecting a user from a subset of users that have good conditions.

In the past decade, RA problems for OFDMA networks have attracted a lot of interest [28], [29], [30], [31], [32], [33]. In general, those problems can be classified into two categories. One is margin adaptive which minimizes the total power consumption subject to prescribed rate requirements for users [28]. The other is rate adaptive which maximizes the sum rate subject to a total power constraint [34]. Specifically, [28], [30], [29], [32], [33] propose suboptimal algorithms which tradeoff between complexity and performance. Recently, there has also been interest in extending the scope of RA to ensure fairness among users, provide quality of service guarantees, and cope with mobility and network dynamics, both of which brings uncertainty to the wireless channel. Fairness and quality of service guarantees can be effected by maximizing a suitable utility function of average user rates and introducing minimum rate constraints per user [35], [36], [37], [38], [39], [40], [41]. Channel uncertainty on the other hand, can be accommodated through on-line channel-adaptive scheduling schemes that essentially learn the underlying channel distribution [38], [37].

2.2 Mathematical backgrounds

In this section, some mathematical backgrounds regarding RA in OFDMA downlink systems will be introduced.

2.2.1 Lagrange dual method

Let us consider a constrained minimization problem as follows:

$$\begin{aligned} & f_0^* = \min_{\mathbf{x}} f_0(\mathbf{x}) \\ & s.t. \\ & g_i(\mathbf{x}) = 0, \ \forall i, \\ & h_j(\mathbf{x}) \leq 0, \ \forall j. \end{aligned} \tag{2.1}$$

where, $\mathbf{x} \in R^n$ is a vector of optimization variables, $f_0 : R^n \to R$, $g_i : R^n \to R, \forall i$ and $h_j : R^n \to R, \forall j$ are continuously differentiable functions. By applying the Lagrange dual approach, problem (2.1) can be solved by forming its Lagrangian dual [42], [43] as

$$L_p(\mathbf{x}, \lambda, \mu) = f_0(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i g_i(\mathbf{x}) + \sum_{j=1}^{r} \mu_j h_j(\mathbf{x}), \tag{2.2}$$

where λ_i is the Lagrangian multiplier for the equality constraint $g_i(\mathbf{x}) = 0$, μ_j is the Lagrangian multiplier for the inequality constraint $h_j(\mathbf{x}) \leq 0$, $\lambda = \{\lambda_i\}$ and $\mu = \{\mu_j\}$.

We define the dual objective function of problem (2.1) as follows

$$q(\lambda, \mu) = \min_{\mathbf{x}} L_p(\mathbf{x}, \lambda, \mu). \tag{2.3}$$

Then the dual optimization problem of (2.1) is denoted as follows

$$q^* = \max_{\lambda, \mu \geq 0} q(\lambda, \mu), \tag{2.4}$$

Note problem (2.4) is now concave [42], [43].

Let us denote by $\mathbf{x}^*$ the local minimum of the primal problem (2.1). Then there exist unique Lagrange multipliers vectors $\lambda^* = \{\lambda_i^*\}$ and $\mu^* = \{\mu_j^*\}$ such that

$$\begin{aligned} & g_i(\mathbf{x}^*) = 0; \ \forall i, \\ & h_j(\mathbf{x}^*) \leq 0; \ \forall j, \\ & \mu_j^* \geq 0; \ \forall j, \\ & \frac{\partial L_p(\mathbf{x}^*, \lambda^*, \mu^*)}{\partial \mathbf{x}} = 0, \\ & \mu_j^* h_j(\mathbf{x}^*) = 0 \ \forall j. \end{aligned} \tag{2.5}$$

Here the equations in (2.5) form the Karush-Kuhn-Tucker(KKT) conditions, which are necessary and sufficient for optimality. The last equation in (2.5) is the complementary slackness constraint, which guarantees that the inequality constraints of (2.1) are satisfied with equality if and only if the corresponding Lagrange multipliers are strictly greater than zero. Note we always have $q^* \leq f_0^*$ where the difference between these two optimal values is defined as the optimal duality gap [42], [43]

$$f_0^* - q^*. \tag{2.6}$$

When the objective function and the constraint functions are convex, problem (2.1) is a standard convex optimization problem. Then according to [44], the optimal duality gap is zero, meaning that the primal problem (2.1) and the dual problem (2.4) have the same optimal value. If problem (2.1) is nonconvex, the duality gap can be non-zero. In this case, the optimal dual value provides a lower bound for the optimal primal value. More details about the Lagrangian dual methods can be found in [44], [45], [42], [46] and [47].

Let us further consider the following constrained maximization problem

$$\begin{aligned}
&f^* = \max_{\mathbf{x}} f(\mathbf{x}) \\
&s.t. \\
&g_i(\mathbf{x}) = 0, \ \forall i, \\
&h_j(\mathbf{x}) \leq 0, \ \forall j.
\end{aligned} \tag{2.7}$$

Note that the maximization of the function f is equivalent to the minimization of its negative. Thus the maximization problem (2.7) can be solved by using the above Lagrange dual method.

2.2.2 Successive convex approximation (SCA) method

Let us consider a nonconvex RA problem in OFDMA downlink systems denoted as follows:

$$\begin{aligned}
&\min \ f_0(\mathbf{X}) \\
&s.t. \ f_i(\mathbf{X}) \leq 1, \forall i,
\end{aligned} \tag{2.8}$$

where $f_0(\mathbf{X})$ is convex without loss of generality and $f_i(\mathbf{X}), \forall i$ is nonconvex. Note if $f_0(\mathbf{X})$ is nonconvex, we can introduce a slack variable t and rewrite the problem as:

$$
\begin{aligned}
&\min \ t \\
&s.t. \ f_i(\mathbf{X}) \leq 1, \forall i, \\
&f_0(\mathbf{X}) \leq t.
\end{aligned}
\tag{2.9}
$$

As it is very difficult to solve the nonconvex problem directly, we may solve it by using the successive convex approximation method. Specifically, the nonconvex problem can be approximated into a series of convex problems, which can be solved by standard convex optimization methods easily. Each approximated convex problem has the same expression as (2.8) except that $f_i(\mathbf{X})$ is replaced by its approximation $\tilde{f}_i^m(\mathbf{X})$ at the m-th approximation iteration. According to the corollary 1 of [48], if all the approximations satisfy the following three conditions, then this successive approximation method is a general inner approximation algorithm [48] and will converge to a local optimum satisfying the KKT conditions of problem (2.8):

1. Bounding condition: $\forall \mathbf{X}$,
$$f_i(\mathbf{X}) \leq \tilde{f}_i^m(\mathbf{X}), \ \forall i.$$
2. Tightness condition: At the beginning of iteration m,
$$f_i(\mathbf{X}^{m-1}) = \tilde{f}_i^m(\mathbf{X}^{m-1}), \ \forall i.$$
3. Differential condition: At the beginning of iteration m, $\forall x \in \mathbf{X}$,
$$\frac{\partial f_i(\mathbf{X}^{m-1})}{\partial x} = \frac{\partial \tilde{f}_i^m(\mathbf{X}^{m-1})}{\partial x}, \ \forall i.$$

Here $\mathbf{X}^m$ is the solution obtained at the m-th approximation iteration of the nonconvex problem (2.8). The bounding condition guarantees that the approximation $\tilde{f}_i^m(\mathbf{X})$ is tightening the constraints in (2.8), and any solution of the approximated problem will be a feasible point of the original problem (2.8). The tightness condition guarantees that $\mathbf{X}^{m-1}$ is a feasible solution of the approximated convex problem at iteration m. Thus the solution of each approximated problem will decrease the cost function, meaning $f_0(\mathbf{X}^m) \leq f_0(\mathbf{X}^{m-1})$. The differential condition guarantees that the KKT conditions of problem (2.8) will be satisfied after the convergence of approximations [48], [49].

2.2.3 Complementary geometric program

According to [49] and [50], a complementary GP (CGP) is a constrained optimization problem formulated in terms of a rational function of posynomials defined as sums of monomials. A monomial is a function $g(\mathbf{x})$ of positive variables $\mathbf{x} = \{x_n, n = 1, 2, ..., N\}$ having the form $b \prod_{n=1}^{N} x_n^{c_n}$, where b is a positive real constant and $c_n, \forall n$, are real constants. Therefore a posynomial $f(\mathbf{x})$ has the following form:

$$\sum_{k=1}^{K} b_k \prod_{n=1}^{N} x_n^{c_{nk}}. \tag{2.10}$$

The general form of a CGP is:

$$\begin{aligned} & \min \ f_0(\mathbf{x}) \\ & s.t. \\ & f_i(\mathbf{x}) \leq 1, \ i = 1, 2, ..., I, \\ & h_j(\mathbf{x}) = 1, \ j = 1, 2, ..., J, \\ & x_n > 0, \ n = 1, 2, ..., N, \end{aligned} \tag{2.11}$$

where $f_i(\mathbf{x})$ and $h_j(\mathbf{x})$ are all rational functions with the following structure:

$$\frac{A(\mathbf{x}) - B(\mathbf{x})}{C(\mathbf{x}) - D(\mathbf{x})}, \tag{2.12}$$

where the A, B, C and D are all posynomials and some of them may be absent.

Actually, a CGP is an extension of a standard GP. As a matter of fact, when $f_i(\mathbf{x}), i = 0, 1, ..., I$, are posynomials and $h_j(\mathbf{x}), j = 1, 2, ..., J$, are monomials, problem (43) becomes a standard GP. For GPs, the global optimum can always be found in an efficient manner. For CGPs, only a local optimum can be obtained efficiently. But prohibitive computations are required to find the global optimum [50].

2.2.4 Standard interference function

In paper [51], the author introduces a special function named the standard interference function $\mathbf{F}(\mathbf{X})$. This kind of function has some special properties which can be used to solve the set of equations $\mathbf{X} = \mathbf{F}(\mathbf{X})$ iteratively. The solution of the equation set is named as a fixed point.

Specifically, $\mathbf{F}(\mathbf{X})$ is a standard interference function if for all $\mathbf{X} \geq 0$ the following properties are satisfied:

1. Positivity: $\mathbf{F}(\mathbf{X}) > 0$,
2. Monotonicity: If $\mathbf{X} \geq \mathbf{X}'$, then $\mathbf{F}(\mathbf{X}) \geq \mathbf{F}(\mathbf{X}')$,
3. Scalability: $\forall c > 1, c\mathbf{F}(\mathbf{X}) > \mathbf{F}(c\mathbf{X})$.

Here the inequalities should be understood as element-wise.

According to the Theorem 1 and Theorem 2 in [51], the equation set $\mathbf{X} = \mathbf{F}(\mathbf{X})$ has a unique fixed point solution, which can be found iteratively as follows:

$$\mathbf{X}^{m+1} = \mathbf{F}(\mathbf{X}^{m}). \tag{2.13}$$

3 Overview of Signal Models and Problem Formulations for RA in Multi-Cell OFDMA Systems

In this Chapter, we overview different signal models used for formulating RA problems in multi-cell OFDMA downlink systems with co-channel interference (CCI). Considering the CCI, resource allocation problems in multi-cell OFDMA systems have been investigated by lots of papers [25, 26, 52–67]. Specifically, [52–62] have studied RA problems in multi-cell OFDMA systems and [25,26,63–67] have investigated RA problems in multi-cell OFDMA downlink systems with relaying of either the amplify-and-forward (AF) type or the decode-and-forward (DF) type. In these papers, although the considered systems and RA problems are different, the same issue of modeling the transmitted signals to formulate rate expressions and RA problems has to be considered. Considering this issue, different signal models are introduced, which bring the same problem with formulations of different complexities. Yet, no paper has clearly indicated its motivation for choosing its signal model, which causes ambiguity for understanding. Thus in this Chapter, we overview all the signal models used for formulating RA problems in multicell OFDMA systems to help readers understand better the hidden insights of them. Specifically, the contributions of this Chapter are listed below:

- We classify the existing signal models used for RA in multi-cell OFDMA systems into four categories and unveil the hidden insights of them.

- We consider the classic sum rate maximized joint RA problem in multi-cell OFDMA downlink systems with per-station individual power constraints. Using the four signal models, exact system models and problem formulations are proposed. Considering the existing RA methods, a suitable signal model and problem formulation is suggested.

- We unveil and prove an interesting property of the recommended formulations.

The rest of this Chapter is organized as follows. First, four signal models used for formulating RA problems in multi-cell orthogonal frequency division multiple access (OFDMA) downlink systems are overviewed. Then, applying the four signal models, exact formulations for the classic sum rate maximized RA problem are derived and compared. Finally, an interesting property of the recommended formulations are unveiled.

3.1 Summary of four signal models

In the papers [25,26,52–67], although the considered systems and RA problems are a little bit different, the same issue of modeling the transmitted signals to formulate rate expressions and RA problems in Multi-Cell OFDMA has to be considered. Specifically, there exist four models for the transmitted signals, as will be elaborated in the following classic system.

Let us consider a multi-cell OFDMA downlink system with N cells. In each cell, a signal is broadcast from the base station (BS) to U mobile stations (MSs). The broadcasting channel is assumed to be frequency selective and transformed into K parallel subchannels by using OFDM with sufficiently long cyclic prefix. With OFDMA, each subcarrier can be allocated to at most one MS in each cell. This means, at this subcarrier, either the BS keeps silent or the broadcast signal can be decoded by at most one MS.

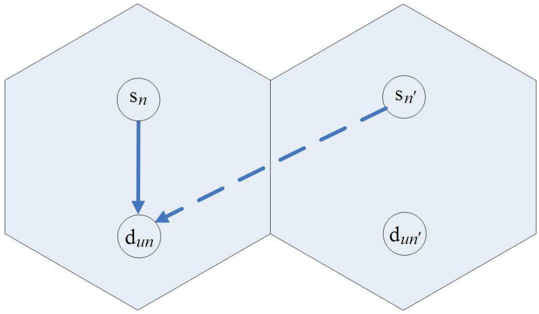


Figure 3.1 Illustration of a transmission in cell n with cochannel interference from cell n'. The solid line and the dashed line represent communication link and interfering link respectively.

With this system, the sum rate maximization problem is considered here. To formulate the rate expression on subcarrier k for MS d_{un} in cell n, we now denote the transmitted signal on subcarrier k for MS d_{un} at BS s_n and the total transmitted signal on subcarrier k at BS s_n as $x^{k,Tx}_{\mathrm{s}_n,\mathrm{d}_{un}}$ and $x^{k,Tx}_{\mathrm{s}_n}$ respectively, which will later be formulated using four signal models. As shown in Figure 3.1, signal $x^{k,Tx}_{\mathrm{s}_n,\mathrm{d}_{un}}$ is first broadcast from BS s_n to the MS d_{un}. Simultaneously in an interfering cell n', a signal $x^{k,Tx}_{\mathrm{s}_{n'}}$ is also broadcast from the interfering BS $\mathrm{s}_{n'}$ on the same subcarrier. The targeted destination d_{un} receives signals from all BSs at subcarrier k, which can be expressed as

$$\begin{aligned} y^k_{\mathrm{d}_{un}} =& h^k_{\mathrm{s}_n,\mathrm{d}_{un}} x^{k,Tx}_{\mathrm{s}_n,\mathrm{d}_{un}} + v^k_{\mathrm{d}_{un}} + \\ & \sum_{n'=1,n'\neq n}^{N} h^k_{\mathrm{s}_{n'},\mathrm{d}_{un}} x^{k,Tx}_{\mathrm{s}_{n'}} \end{aligned} \tag{3.1}$$

where $h^k_{\mathrm{s}_{n'},\mathrm{d}_{un}}$ denotes the CFR of subcarrier k from $\mathrm{s}_{n'}$ to d_{un} and $v^k_{\mathrm{d}_{un}}$ denotes the AWGN corrupting d_{un} at subcarrier k during the first TS.

Let us denote by $x^k_{\mathrm{s}_n,\mathrm{d}_{un}}$ the normalized symbol (meaning $E\{|x^k_{\mathrm{s}_n,\mathrm{d}_{un}}|^2\} = 1$) transmitted from BS s_n to MS d_{un} at subcarrier k and $P^k_{\mathrm{s}_n,\mathrm{d}_{un}}$ the corresponding signal power for d_{un}. Considering OFDMA, it is natural to introduce a binary variable a^k_{un}, where $a^k_{un} = 1$ indicates that subcarrier k is allocated to d_{un}. Thus $x^{k,Tx}_{\mathrm{s}_n,\mathrm{d}_{un}}$ and $x^{k,Tx}_{\mathrm{s}_n}$ can be modeled as $a^k_{un}\sqrt{P^k_{\mathrm{s}_n,\mathrm{d}_{un}}}x^k_{\mathrm{s}_n,\mathrm{d}_{un}}$ and $\sum_u a^k_{un}\sqrt{P^k_{\mathrm{s}_n,\mathrm{d}_{un}}}x^k_{\mathrm{s}_n,\mathrm{d}_{un}}$ respectively, which are used by [52–54, 64] and named as signal model 1 here. Applying signal model 1, the achievable transmission rate on subcarrier k for MS d_{un} is calculated as

$$R_{un}^{k,1} = \ln\left(1 + \Gamma_{\mathrm{d}_{un}}^{k,1}\right) \tag{3.2}$$

in nats per symbol time, where

$$\Gamma_{\mathrm{d}_{un}}^{k,1} = \frac{a_{un}^{k} P_{\mathrm{s}_n,\mathrm{d}_{un}}^{k} G_{\mathrm{s}_n,\mathrm{d}_{un}}^{k}}{f_{\mathrm{d}_{un}}^{k,1}} \tag{3.3}$$

denotes the SINR associated with the decoding of $x_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,Tx}$ from $y_{\mathrm{d}_{un}}^{k}$ at d_{un}. $G_{\mathrm{s}_n,\mathrm{d}_{un}}^{k}$ denotes the channel gain of subcarrier k from s_n to d_{un}.

$$f_{\mathrm{d}_{un}}^{k,1} = \sigma^2 + \sum_{n'=1,n'\neq n}^{N} \sum_{u'} a_{u'n'}^{k} P_{\mathrm{s}_{n'},\mathrm{d}_{u'n'}}^{k} G_{\mathrm{s}_{n'},\mathrm{d}_{un}}^{k} \tag{3.4}$$

denotes the sum power of the AWGN and the interference received by d_{un} at subcarrier k. Here we have in total NKU power variables and NKU binary variables to be optimized.

When using the system model 1, it is important to note that the achievable rate $R_{un}^{k,1}$ is a nonlinear function of both the binary variables $\{a_{un}^{k}\}$ and the power variables $\{P_{\mathrm{s}_n,\mathrm{d}_{un}}^{k}\}$, which makes the objective function of the sum rate maximization problem complex. In order to simplify the rate expression, [55–57, 65] have proposed another signal model (named model 2 here), where $x_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,Tx} = \sqrt{P_{\mathrm{s}_n,\mathrm{d}_{un}}^{k}}\, x_{\mathrm{s}_n,\mathrm{d}_{un}}^{k}$ and $x_{\mathrm{s}_n}^{k,Tx} = \sum_u \sqrt{P_{\mathrm{s}_n,\mathrm{d}_{un}}^{k}}\, x_{\mathrm{s}_n,\mathrm{d}_{un}}^{k}$. This means that, instead of introducing a_{un}^{k} to indicate whether BS s_n transmits data to MS d_{un} on subcarrier k or not, we use the corresponding power value $P_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,Tx}$ to do so. Specifically, $P_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,Tx} > 0$ means that s_n transmits data to MS d_{un} on subcarrier k and $P_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,Tx} = 0$ means that s_n does not transmit. Applying signal model 2, the achievable transmission rate on subcarrier k for MS d_{un} is calculated as

$$R_{un}^{k,2} = \ln\left(1 + \Gamma_{\mathrm{d}_{un}}^{k,2}\right) \tag{3.5}$$

in nats per symbol time, where

$$\Gamma_{\mathrm{d}_{un}}^{k,2} = \frac{P_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,Tx} G_{\mathrm{s}_n,\mathrm{d}_{un}}^{k}}{f_{\mathrm{d}_{un}}^{k,2}}, \tag{3.6}$$

$$f_{\mathrm{d}_{un}}^{k,2} = \sigma^2 + \sum_{n'=1,n'\neq n}^{N} P_{\mathrm{s}_{n'},\mathrm{d}_{u'n'}}^{k,Tx} G_{\mathrm{s}_{n'},\mathrm{d}_{un}}^{k}. \tag{3.7}$$

In this case, the achievable rate $R_{un}^{k,2}$ is only a function of the power variables $\{P_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,Tx}\}$, which is simplified compared to that of system model 1. As will be discussed later, to facilitate the description of the multiple access feature in OFDMA, a_{un}^k will still be introduced in the constraints to confine the value of $P_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,Tx}$. Also, considering the multiple access feature of OFDMA, $R_{un}^{k,2}$ is denoted as:

$$a_{u'n'}^k \ln\left(1+\Gamma_{\mathrm{d}_{un}}^{k,2}\right) \tag{3.8}$$

in paper [55–57, 65]. Thus we still have in total NKU power variables and NKU binary variables to be optimized.

In order to lower down the number of optimization variables, [26] and [63] have proposed another signal model, which is named model 3 here. Specifically in this model, $x_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,Tx}$ and $x_{\mathrm{s}_n}^{k,Tx}$ are modeled as $a_{un}^k\sqrt{P_{\mathrm{s}_n}^k}x_{\mathrm{s}_n,\mathrm{d}_{un}}^k$ and $\sqrt{P_{\mathrm{s}_n}^k}\sum_u a_{un}^k x_{\mathrm{s}_n,\mathrm{d}_{un}}^k$ respectively. Here $P_{\mathrm{s}_n}^k$ denotes the power allocated to subcarrier k at BS s_n, which can be used by at most one MS in cell n. Note in signal model 3, the number of power variables is NK, much less than those of the signal model 1. Applying signal model 3, the achievable transmission rate on subcarrier k for MS d_{un} is calculated as

$$R_{un}^{k,3} = \ln\left(1+\Gamma_{\mathrm{d}_{un}}^{k,3}\right) \tag{3.9}$$

in nats per symbol time, where

$$\Gamma_{\mathrm{d}_{un}}^{k,3} = \frac{a_{un}^k P_{\mathrm{s}_n}^k G_{\mathrm{s}_n,\mathrm{d}_{un}}^k}{f_{\mathrm{d}_{un}}^{k,3}}, \tag{3.10}$$

$$f_{\mathrm{d}_{un}}^{k,3} = \sigma^2 + \sum_{n'=1,n'\neq n}^{N} P_{\mathrm{s}_{n'}}^k\left(\sum_{u'} a_{u'n'}^k\right)G_{\mathrm{s}_{n'},\mathrm{d}_{un}}^k. \tag{3.11}$$

Note $R_{un}^{k,3}$ is still a nonlinear function of both the binary variables $\{a_{un}^k\}$ and the power variables $\{P_{\mathrm{s}_n,\mathrm{d}_{un}}^k\}$. In order to formulate $R_{un}^{k,3}$ as a linear function of the binary variables $\{a_{un}^k\}$, [25, 58–62, 66, 67] have further proposed signal model 4. Here $x_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,Tx}$ and $x_{\mathrm{s}_n}^{k,Tx}$ are modeled in the same way as model 3. However when modeling the inter-cell CCI power of a subcarrier k in a selected cell n, instead of using integer variables $\sum_{u'} a_{u'n'}^k$ to indicate whether a BS in an interfering cell n' interferes on this subcarrier or not, we use the corresponding power value $P_{\mathrm{s}_{n'}}^{k,Tx}$ to do it. Specifically, $P_{\mathrm{s}_{n'}}^{k,Tx} > 0$ means that BS $\mathrm{s}_{n'}$ interferes

Table 3.1 List of power variables

$P^{k}_{\mathrm{s}_n,\mathrm{d}_{un}}$	allocated power for user u at subcarrier k in cell n
$P^{k,Tx}_{\mathrm{s}_n,\mathrm{d}_{un}}$	transmitted power for user u at subcarrier k in cell n
$P^{k}_{\mathrm{s}_n}$	allocated power for subcarrier k in cell n
$P^{k,Tx}_{\mathrm{s}_n}$	transmitted power for subcarrier k in cell n

at subcarrier k and $P^{k,Tx}_{\mathrm{s}_{n'}} = 0$ means it does not. Applying signal model 4, the achievable transmission rate on subcarrier k for MS d_{un} is calculated as

$$R^{k,4}_{un} = \ln\left(1 + \Gamma^{k,4}_{\mathrm{d}_{un}}\right) \tag{3.12}$$

in nats per symbol time, where

$$\Gamma^{k,4}_{\mathrm{d}_{un}} = \frac{a^{k}_{un} P^{k,Tx}_{\mathrm{s}_n} G^{k}_{\mathrm{s}_n,\mathrm{d}_{un}}}{f^{k,4}_{\mathrm{d}_{un}}}, \tag{3.13}$$

$$f^{k,4}_{\mathrm{d}_{un}} = \sigma^2 + \sum_{n'=1,n'\neq n}^{N} P^{k,Tx}_{\mathrm{s}_{n'}} G^{k}_{\mathrm{s}_{n'},\mathrm{d}_{un}}. \tag{3.14}$$

So far, we have summarized the four signal models used in present papers considering RA in multicell OFDMA systems. Note the four signal models are different mainly due to their different definitions of power variables, as summarized in Table 3.1. In the following section, the models will be used and compared for the same RA problem .

3.2 Comparison of formulations using four signal models

In this section, the classic sum rate maximization problem is considered in the multicell downlink OFDMA system. Problem formulations using the four signal models will be presented and compared. After comparison, some advices will be given according to different RA methods.

Let us first denote the sum rate expression of all cells using signal model $i \in \{1, 2, 3, 4\}$ as

$$R^{i} = \sum_{n}\sum_{k}\sum_{u} R^{k,i}_{un}. \tag{3.15}$$

With signal model 1, the RA problem can be formulated as:

$$\begin{aligned}
&\max_{\mathbf{P},\mathbf{A}} R^1\left(\mathbf{P},\mathbf{A}\right) \\
&s.t. \\
&C3.1: \ a_{un}^k \in \{0,1\}, \ \forall u,n,k, \\
&C3.2: \ \sum_{u=1}^{U} a_{un}^k \leq 1, \ \forall n,k, \\
&C3.3: \ P_{\mathrm{s}_n,\mathrm{d}_{un}}^k \geq 0, \ \forall u,n,k, \\
&C3.4: \ \sum_{k=1}^{K}\sum_{u=1}^{U} a_{un}^k P_{\mathrm{s}_n,\mathrm{d}_{un}}^k \leq P_{t,n}, \ \forall n,
\end{aligned} \tag{3.16}$$

where $\mathbf{P} = \{P_{\mathrm{s}_n,\mathrm{d}_{un}}^k\}$ and $\mathbf{A} = \{a_{un}^k\}$. *Note $P_{\mathrm{s}_n,\mathrm{d}_{un}}^k$ and $a_{un}^k P_{\mathrm{s}_n,\mathrm{d}_{un}}^k$ denote the allocated power and the transmitted power on subcarrier k at BS s_n for MS d_{un} respectively. Also $\sum_u a_{un}^k P_{\mathrm{s}_n,\mathrm{d}_{un}}^k$ denotes the transmitted power on subcarrier k at BS s_n, which causes interference on this subcarrier for the other cells.* Here, $C3.1$ and $C3.2$ ensure that each subcarrier k can be allocated for data transmission to at most one MS d_{un}. If subcarrier k is not used, the transmitted power at BS s_n should be zero, causing no interference for the other cells on this subcarrier. Moreover, $C3.3$ and $C3.4$ ensure that the consumed sum power of each BS is less than its available sum power.

Using signal model 2, the RA problem is formulated as:

$$\begin{aligned}
&\max_{\mathbf{P}^{Tx},\mathbf{A}} R^2\left(\mathbf{P}^{Tx}\right) \\
&s.t. \ C3.1, C3.2, C3.3 \\
&C3.4': \ \sum_{k=1}^{K}\sum_{u=1}^{U} P_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,Tx} \leq P_{t,n}, \ \forall n, \\
&C3.5: \ P_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,Tx}\left(1 - a_{un}^k\right) = 0, \ \forall n,k,u,
\end{aligned} \tag{3.17}$$

where $\mathbf{P}^{Tx} = \{P_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,Tx}\}$. *Note $P_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,Tx}$ denotes both the allocated power and the transmitted power on subcarrier k at BS s_n for MS d_{un}. Also $\sum_u P_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,Tx}$ denotes the transmitted power on subcarrier k at BS s_n.* Here we need $C3.1$, $C3.2$ and $C3.5$ together to ensure that, for each subcarrier in each cell, at most one user's transmitted power can be non-zero. This means each subcarrier k can be allocated for data transmission to at most one MS d_{un}. If subcarrier k is not used, the transmitted power at BS s_n is zero. Note $C3.5$ is necessary to describe the multiple access feature of the OFDMA system.

Using signal model 3, the RA problem is formulated as:

$$\begin{aligned}
&\max_{\mathbf{P}_1,\mathbf{A}} R^3\left(\mathbf{P}_1,\mathbf{A}\right)\\
&s.t.\ C3.1, C3.2,\\
&C3.3':\ P_{\mathrm{s}_n}^k \geq 0,\ \forall n,k,\\
&C3.4'':\ \sum_{k=1}^{K}(\sum_{u=1}^{U} a_{un}^k)P_{\mathrm{s}_n}^k \leq P_{t,n},\ \forall n,
\end{aligned} \tag{3.18}$$

where $\mathbf{P}_1 = \{P_{\mathrm{s}_n}^k\}$. *Note that* $P_{\mathrm{s}_n}^k$ *should be understood as the allocated power on subcarrier* k *at BS* s_n*, which is further allocated to each MS* d_{un} *with* $a_{un}^k P_{\mathrm{s}_n}^k$. $\sum_u a_{un}^k$ *indicates whether subcarrier* k *is used or not. The transmitted power on this subcarrier at BS* s_n *is denoted as* $P_{\mathrm{s}_n}^k(\sum_u a_{un}^k)$. Here $C3.1$ and $C3.2$ ensure that each subcarrier k can be allocated for data transmission towards at most one MS d_{un}. If a subcarrier is not used, then the transmitted power on this subcarrier at BS s_n is zero.

Using signal model 4, the RA problem is formulated as:

$$\begin{aligned}
&\max_{\mathbf{P}_1^{Tx},\mathbf{A}} R^4\left(\mathbf{P}_1^{Tx},\mathbf{A}\right)\\
&s.t.\ C3.1, C3.2, C3.3'\\
&C3.4''':\ \sum_{k=1}^{K} P_{\mathrm{s}_n}^{k,Tx} \leq P_{t,n},\ \forall n,\\
&C3.5':\ P_{\mathrm{s}_n}^{k,Tx}\left(1-(\sum_{u=1}^{U} a_{un}^k)\right) = 0,\ \forall n,k,
\end{aligned} \tag{3.19}$$

where $\mathbf{P}_1^{Tx} = \{P_{\mathrm{s}_n}^{k,Tx}\}$. *Here* $P_{\mathrm{s}_n}^{k,Tx}$ *should be understood as both the allocated power and the transmitted power on subcarrier* k *at BS* s_n*, which is further allocated to each user* d_{un} *with* $a_{un}^k P_{\mathrm{s}_n}^{k,Tx}$. $C3.1, C3.2$ ensure that each subcarrier k can be allocated for data transmission towards at most one MS d_{un}. $C3.5'$ ensures that if a subcarrier is not used in cell n, then the transmitted power on this subcarrier at BS s_n is zero.

Regarding the formulated problems, there are basically three types of RA methods: the game theory based method, the duality-based joint optimization method and the variable decoupling based method, where the integer variables and the continuous variables are decoupled and solved separately [25, 26, 52–55, 58–67]. Specifically, with the game theory based method, each cell autonomously maximizes its own utility function with the estimated power values of the received interferences from the other cells. Actually, this method do not really solve the formulated sum rate maximization problem. However, through numerical experiments, it is illustrated that the system sum rate calculated with the RA results of this method is close to that obtained by using centralized RA methods. Due to its autonomous property, the game theory based method provides a good tradeoff between complexity and performance [58]. Also, [64] proposes the duality-based joint optimization method, where the original mixed integer non-convex program is transformed into a convex program with continuous variables by introducing additional CCI constraints and variable relaxations. Finally, several variable decoupling based methods are proposed in [25, 26, 52–55, 59–63, 65–67]. These methods consist of two steps: the subcarrier allocation (SA) step and the power allocation (PA) step. In particular, [55, 66, 67] assume fixed PA (e.g. uniform PA) for each subcarrier at each BS and only design algorithm for SA. [52–54, 59–61, 65] implement SA and PA separately in two steps. [25, 26, 62, 63] adopt the coordinate ascent method and carry out SA and PA iteratively.

Considering three types of methods, let us now discuss the properties of the four RA formulations and give advices on choosing a suitable formulation to facilitate the algorithm design. As discussed before, signal model 1 is a straightforward model, while the related RA formulation (named as formulation 1) is more complex than formulations with the other signal models. There are in total NKU binary variables and NKU non-negative power variables to be optimized. The objective function of formulation 1 is a non-linear function of all variables. There are NK constraints for the binary variables and N total power constraints for the continuous variables coupled with the corresponding binary variables. Compared with formulation 1, the formulation 2 with signal model 2 has the same number of optimization variables. While the objective function of formulation 2 is simplified, as it is now a linear function of the binary variables and a non-linear function of the continuous variables. Also, the N constraints for the continuous variables are decoupled from the corresponding binary variables. As a cost of these simplification, there are now NKU additional constraints for the continuous variables, which is denoted as C3.5.

Compared with formulation 1, formulation 3 with signal model 3 has only

NK continuous variables, much less than that of formulation 1. The objective function is still a non-linear function of all variables. The number of constraints is still the same as that of formulation 1. Compared with formulation 3, formulation 4 with signal model 4 has the same number of variables, while the objective function is now a linear function of the binary variables. The N total power constraints for the continuous variables are decoupled from the corresponding binary variables. As a cost of these simplification, there are now NK additional constraints for the continuous variables, which are denoted in C3.5′.

For all methods, we recommend signal model 3 and signal model 4, as the corresponding formulations have less optimization variables. Considering the game theory based methods, each local utility function is optimized with the CCI fixed and calculated by RA results at the initial stage or at the previous iteration. Thus the objective function is a linear function of the binary variables, which makes the advantage of using signal model 4 negligible. Compared to signal model 4, model 3 has less constraints and thus more recommended. Also considering the duality based joint optimization methods, additional CCI constraints are introduced to turn the non-convex RA problem into a convex one, where the objective function is also a linear function of the binary variables. Thus similar as the game theory based methods, we recommend model 3 and formulation 3.

Considering the variable decoupling based methods, the binary variables and the continuous variables are decoupled and optimized separately in two steps (the SA step and the PA step). For each step, one type of variables are optimized with the other variables fixed. Using formulation 3 and 4, the optimization sub-problems at the PA step are the same. While at the SA step, the sub-problem of formulation 4 is less complex than that of formulation 3. This is due to the fact that, the objective function with formulation 4 is a linear function of the binary variables, while the objective function with formulation 3 is a non-linear one. The sub-problem of formulation 4 is now a linear integer program (IP) while the sub-problem of formulation 3 is a non-linear IP. Thus signal model 4 and formulation 4 are recommended for this type of RA methods.

3.3 An interesting property of formulation 3 and formulation 4

In this section, an interesting property will be unveiled for the recommended formulation 3 and 4. Let us now consider Theorem 1.

Theorem 3.1. *Finding the optimum (the optimal variables and the optimal objective function values) of formulation 3 (F3) and formulation 4 (F4) is equivalent to finding the optimum of F4$'$. Here formulation F4$'$ is the same as F4 except that the constraint C3.5$'$ is not included.*

Proof. The intuitive idea of this proof is that: at the optimum, the amount of power that is allocated to a subcarrier with the corresponding indicators zero valued is wasted. In other words, we can still improve the system sum rate by reallocating this amount of power to other subcarriers with non-zero indicators. Let us introduce variable $c_n^k = \sum_u a_{un}^k$ and vector $\mathbf{C} = \{c_n^k\}$. Then $F3$ can be rewritten as the following problem $F3'$.

$$\begin{aligned} &\max_{\mathbf{P}_1,\mathbf{A},\mathbf{C}} R^5\left(\mathbf{P}_1,\mathbf{A},\mathbf{C}\right) \\ &s.t.\ C3.1, C3.2, C3.3', C3.4'', \\ &C3.6:\ c_n^k = \sum_u a_{un}^k,\ \forall n,k, \end{aligned} \tag{3.20}$$

where $R^5\left(\mathbf{P}_1,\mathbf{A},\mathbf{C}\right)$ is denoted as

$$\sum_n\sum_k\sum_u \ln\left(1+\frac{a_{un}^k P_{\mathrm{s}_n}^k G_{\mathrm{s}_n,\mathrm{d}_{un}}^k}{\sigma^2+\sum\limits_{n'=1,n'\neq n}^{N} P_{\mathrm{s}_{n'}}^k c_{n'}^k G_{\mathrm{s}_{n'},\mathrm{d}_{un}}^k}\right) \tag{3.21}$$

Also, $F4'$ is rewritten as

$$\begin{aligned} &\max_{\mathbf{P}_1,\mathbf{A},\mathbf{C}} R^6\left(\mathbf{P}_1,\mathbf{A},\mathbf{C}\right) \\ &s.t.\ C3.1, C3.2, C3.3', C3.4''', C3.6, \end{aligned} \tag{3.22}$$

where $R^6\left(\mathbf{P}_1,\mathbf{A},\mathbf{C}\right)$ is a constant function of $\mathbf{C}$ and has the same expression as $R^4\left(\mathbf{P}_1,\mathbf{A}\right)$. According to $C3.1$, $C3.2$ and $C3.6$, we introduce two feasible sets $\mathcal{A}_f$ and $\mathcal{C}_f$ for binary variables $\mathbf{A}$ and $\mathbf{C}$ respectively in the original problem

$F3'$. Note that the equivalent problem $F4'$ has the same constraints (C3.1, C3.2) on binary variables as problem $F3'$. Thus $\mathcal{A}_f$ and $\mathcal{C}_f$ are also the feasible sets of $\mathbf{A}$ and $\mathbf{C}$ respectively in problem $F4'$. Let us randomly choose one feasible solution for the binary variables from $\{\mathcal{A}_f, \mathcal{C}_f\}$ and denote it as $\{\mathbf{A}_0, \mathbf{C}_0\}$. Suppose $\mathbf{P}_{opt,f}(\mathbf{A}_0, \mathbf{C}_0)$ and $\mathbf{P}'_{opt,f}(\mathbf{A}_0, \mathbf{C}_0)$ are the optimal solutions for $\mathbf{P}$ in problem $F3'$ and problem $F4'$ respectively when $\mathbf{A} = \mathbf{A}_0$ and $\mathbf{C} = \mathbf{C}_0$. Then the optimal binary variables in problem $F3'$ and problem $F4'$ can be expressed as

$$(\mathbf{A}_{opt}, \mathbf{C}_{opt}) = \underset{\mathbf{A}_0, \mathbf{C}_0}{\mathrm{argmax}} R^5(\mathbf{P}_{opt,f}, \mathbf{A}_0, \mathbf{C}_0) \tag{3.23}$$

and

$$(\mathbf{A}'_{opt}, \mathbf{C}'_{opt}) = \underset{\mathbf{A}_0, \mathbf{C}_0}{\mathrm{argmax}} R^6(\mathbf{P}'_{opt,f}, \mathbf{A}_0, \mathbf{C}_0). \tag{3.24}$$

respectively. The optimal solutions for $\mathbf{P}$ in problem $F3'$ and problem $F4'$ are given by $\mathbf{P}_{opt} = \mathbf{P}_{opt,f}(\mathbf{A}_{opt}, \mathbf{C}_{opt})$ and $\mathbf{P}'_{opt} = \mathbf{P}'_{opt,f}(\mathbf{A}'_{opt}, \mathbf{C}'_{opt})$ respectively. As will be proven, $\mathbf{P}_{opt,f}(\mathbf{A}_0, \mathbf{C}_0) = \mathbf{P}'_{opt,f}(\mathbf{A}_0, \mathbf{C}_0)$ and $R^5(\mathbf{P}_{opt,f}, \mathbf{A}_0, \mathbf{C}_0) = R^6(\mathbf{P}'_{opt,f}, \mathbf{A}_0, \mathbf{C}_0)$ for each feasible $\{\mathbf{A}_0, \mathbf{C}_0\}$. Thus we have $\mathbf{A}_{opt} = \mathbf{A}'_{opt}$, $\mathbf{C}_{opt} = \mathbf{C}'_{opt}$ and $\mathbf{P}_{opt} = \mathbf{P}'_{opt}$. In other words, the optimal solutions of problem $F3'$ and problem $F4'$ are equal and the optimal objective function values of problem $F3'$ and problem $F4'$ are also equal. Thus by definition, problem $F3'$ and problem $F4'$ are equivalent.

Let us now prove the previous claim in two cases when $\mathbf{A} = \mathbf{A}_0$ and $\mathbf{C} = \mathbf{C}_0$. Case 1 corresponds to $\forall u, n, k$, where $a^k_{un} = 0, c^k_n = 0$ (meaning that the subcarrier is deactivated for all subcarriers in all cells). It is obvious that problem $F3'$ has the same expression as problem $F4'$. Then we certainly have $\mathbf{P}_{opt,f}(\mathbf{A}_0, \mathbf{C}_0) = \mathbf{P}'_{opt,f}(\mathbf{A}_0, \mathbf{C}_0)$ and $R^5(\mathbf{P}_{opt,f}, \mathbf{A}_0, \mathbf{B}_0, \mathbf{C}_0) = R^6(\mathbf{P}'_{opt,f}, \mathbf{A}_0, \mathbf{C}_0)$.

Case 2 corresponds to $\exists u, n, k$, where $a^k_{un} = 1, c^k_n = 1$ (meaning that there exist at least one subcarrier activated in at least one cell). We first introduce two power vectors $\mathbf{P}_2 = \{P^k_{s_n} | \forall n, k, c^k_n = 0\}$ and $\mathbf{P}_3$, which contains all power variables not in $\mathbf{P}_2$. Note that $\forall u, n, k, R^{k,5}_{un}$ is independent on $\mathbf{P}_2$. Thus we further introduce the following rate functions as:

$$\tilde{R}^{k,5}_{un}(\mathbf{P}_3) = R^{k,5}_{un}(\{\mathbf{P}_2, \mathbf{P}_3\}),\ \forall u, n, k, \tag{3.25}$$

Then problem $F3'$ and problem $F4'$ can be rewritten as

$$\begin{aligned}
&\max_{\mathbf{P}_2,\mathbf{P}_3} R^5\left(\{\mathbf{P}_2,\mathbf{P}_3\},\mathbf{A}_0,\mathbf{C}_0\right),\\
&s.t.\ C3.3',\\
&C3.4'':\ \sum_{k,c_n^k=1} P_{\mathrm{s}_n}^k \le P_{t,n},\ \forall n,
\end{aligned} \tag{3.26}$$

and

$$\begin{aligned}
&\max_{\mathbf{P}_2,\mathbf{P}_3} R^6\left(\{\mathbf{P}_2,\mathbf{P}_3\},\mathbf{A}_0,\mathbf{C}_0\right),\\
&s.t.\ C3.5,\\
&C3.4''':\ \sum_{k} P_{\mathrm{s}_n}^k \le P_{t,n},\ \forall n,
\end{aligned} \tag{3.27}$$

respectively. Here

$$R^5 = \sum_{nuk,a_{un}^k=1} \tilde{R}_{un}^{k,5}(\mathbf{P}_3), \tag{3.28}$$

$$R^6 = \sum_{nuk,a_{un}^k=1} \tilde{R}_{un}^{k,6}(\mathbf{P}_2,\mathbf{P}_3), \tag{3.29}$$

where

$$\tilde{R}_{un}^{k,5} = \sum_n\sum_k\sum_u \ln\left(1+\frac{a_{un}^k P_{\mathrm{s}_n}^k G_{\mathrm{s}_n,\mathrm{d}_{un}}^k}{\sigma^2+f_{\mathrm{d}_{un}}^{k,5}}\right), \tag{3.30}$$

$$\tilde{R}_{un}^{k,6} = \sum_n\sum_k\sum_u \ln\left(1+\frac{a_{un}^k P_{\mathrm{s}_n}^k G_{\mathrm{s}_n,\mathrm{d}_{un}}^k}{\sigma^2+f_{\mathrm{d}_{un}}^{k,6}}\right) \tag{3.31}$$

$$f_{\mathrm{d}_{un}}^{k,5} = \sum_{n'\neq n,c_{n'}^k=1} P_{\mathrm{s}_{n'}}^k G_{\mathrm{s}_{n'},\mathrm{d}_{un}}^k, \tag{3.32}$$

$$f_{\mathrm{d}_{un}}^{k,6} = f_{\mathrm{d}_{un}}^{k,5} + \sum_{n'\neq n,c_{n'}^k=0} P_{\mathrm{s}_{n'}}^k G_{\mathrm{s}_{n'},\mathrm{d}_{un}}^k. \tag{3.33}$$

As for problem (3.26), we note that $R^5(\mathbf{P}_2,\mathbf{P}_3)$ does actually not depend on $\mathbf{P}_2$ when case 2 is considered. Thus $\forall n,k$, where $c_n^k = 0$, the optimal $P_{\mathrm{s}_n}^k$ of problem (3.26), which is denoted by $P^k_{\mathrm{s}_n,opt,f}$, can take any non-negative value. Specifically, we may choose $P^k_{\mathrm{s}_n,opt,f} = 0, \forall n,k$, where $c^k_n = 0$. Then problem (3.26) is equivalent to the following problem

$$\begin{aligned} &\max_{\mathbf{P}_3} R^5 \left(\{\mathbf{P}_{2,opt,f}, \mathbf{P}_3\}, \mathbf{A}_0, \mathbf{C}_0 \right), \\ &s.t.\ C3.3',\ C3.4'', \end{aligned} \tag{3.34}$$

where $\mathbf{P}_{2,opt,f} = \{P^k_{\mathrm{s}_n,opt,f} | \forall n,k, c^k_n = 0\}$.

As for problem (3.34), it is interesting to note that $\forall n,u,k,n'$, where $n' \neq n$, $a^k_{un} = 1$ and $c^k_{n'} = 0$, $\tilde{R}^{k,6}_{un}$ decreases when $P^k_{\mathrm{s}_{n'}}$ increases. Also $c^{k'}_{n'} = 0$ definitely leads to $a^{k'}_{u'n'} = 0, \forall u',k'$, which makes $\tilde{R}^{k',6}_{u'n'}$ zero-weighted in R^6. Therefore R^6 decreases when $P^k_{\mathrm{s}_{n'}}$ increases, meaning that $P^k_{\mathrm{s}_{n'}}$ has to be zero valued at the optimum to maximize R^6. Thus $\forall n,k$, such that $c^k_n = 0$, the optimal $P^k_{\mathrm{s}_n}$ for problem (3.34), which is denoted by $P'^k_{\mathrm{s}_n,opt,f}$, is to select $P'^k_{\mathrm{s}_n,opt,f} = 0$. Therefore problem (3.34) is equivalent to the following problem:

$$\begin{aligned} &\max_{\mathbf{P}_3} R^6 \left(\{\mathbf{P}_{2,opt,f}, \mathbf{P}_3\}, \mathbf{A}_0, \mathbf{C}_0 \right), \\ &s.t.\ C3.5,\ C3.4'' : \sum_k P^k_{\mathrm{s}_n} \leq P_{t,n},\ \forall n, \end{aligned} \tag{3.35}$$

where $\mathbf{P}'_{2,opt,f} = \{P'^k_{\mathrm{s}_n,opt,f} | \forall n,k, c^k_n = 0\}$. This clearly shows that problem (3.34) and problem (3.35) have exactly the same formulation, which proves problem (3.26) and problem (3.27) are equivalent. Consequently $\mathbf{P}_{opt,f}(\mathbf{A}_0, \mathbf{C}_0) = \mathbf{P}'_{opt,f}(\mathbf{A}_0, \mathbf{C}_0)$ and $R^5(\mathbf{P}_{opt,f}, \mathbf{A}_0, \mathbf{C}_0) = R^6(\mathbf{P}'_{opt,f}, \mathbf{A}_0, \mathbf{C}_0)$ at case 2.

Considering the conclusion reach for case 1 and case 2, we have that $\mathbf{P}_{opt,f}(\mathbf{A}_0, \mathbf{C}_0) = \mathbf{P}'_{opt,f}(\mathbf{A}_0, \mathbf{C}_0)$ and $R^5(\mathbf{P}_{opt,f}, \mathbf{A}_0, \mathbf{C}_0) = R^6(\mathbf{P}'_{opt,f}, \mathbf{A}_0, \mathbf{C}_0)$ for each feasible $\{\mathbf{A}_0, \mathbf{C}_0\}$. According to the previous discussion, problem $F3$ and problem $F4'$ have the same optimum. Note that problem $F3$ is equivalent to problem $F4$. Thus problem $F4$ and problem $F4'$ have the same optimum. This concludes the proof of theorem 3.1. $\square$

4 Sum Rate Maximization for Multi-Cell Downlink OFDMA with Subcarrier-pair based Opportunistic DF Relaying

In this Chapter, we consider multi-cell DF relay-aided OFDMA downlink systems, in which all sources and relays are coordinated by a central controller for RA. The improved subcarrier-pair based opportunistic DF relaying protocol proposed and studied in [8] and [23] is applied. The sum (over all cells and all destinations) rate maximized problem with a total power constraint in each cell is formulated.

For RA problems in such multi-cell systems, some algorithms have been proposed in [68] and [69] when powers are uniformly allocated to all stations. Several RA algorithms have been proposed in [52, 61, 62, 70] for multi-cell OFDMA systems without DF relaying. However, these methods can not be extended directly to solve RA problems jointly optimizing transmission mode selection, subcarrier assignment and pairing (MSSAP) as well as power allocation (PA) in multi-cell OFDMA systems with subcarrier-pair based opportunistic DF relaying. Paper [64] has recently considered and formulated the joint RA and scheduling problem in multi-cell DF relayed OFDMA systems taking CCI as well as user data rate requirements into account. However, both opportunistic relaying and subcarrier pairing (SP) were not considered there. CCI from the interfering sources to the considered destination and relay is ignored for practical and simplification consideration. Moreover, to solve this problem, an additional interference power constraint has been imposed to transform the original problem into a standard convex one.

Compared with the above existing works, the contributions of this Chapter are listed below:

- We formulate the sum rate maximized joint RA problem in multi-cell OFDMA downlink systems aided by a DF relay station (RS) in each cell. In particular, the subcarrier-pair based opportunistic relaying is adopted. To start with, the HSE protocol with SP is considered here without signal combining and transmit beamforming. It will be shown that the system performance can be improved by using SP and the HSE protocol. Note that, when modeling the inter-cell CCI of a subcarrier in a selected cell, instead of using an additional integer variable to indicate whether a node in an interfering cell transmits data on this subcarrier or not, we use the corresponding power value to do it. This choice is motivated to simplify the system sum rate expression and facilitate the algorithm design. Specifically, the rate expression would be a non-linear function of the integer variables when using an integer variable to indicate whether a subcarrier k in a cell n is used or not, while it is a linear function of the integer variables when using the corresponding power value to do it.

- We propose an iterative RA algorithm to solve the RA problem, which optimizes MSSAP and the power allocation (PA) in an alternate way with the sum rate keeping increasing until convergence. As for the MSSAP stage of each iteration, the formulated problem is decoupled into subproblems with the tentative PA results. Each subproblem can be easily solved by using the optimal results of a Linear Assignment Problem (LAP), which is then solved by the Hungarian Algorithm in polynomial time. As for the PA stage of each iteration, an algorithm based on single condensation and geometric programming (SCGP) is proposed to optimize PA in polynomial time with the tentative MSSAP results. The proposed algorithm is coordinate ascent based and therefore can reach a local optimum in polynomial time.

The rest of this Chapter is organized as follows. First, the system model and the problem formulation for the considered system are presented. Then, the proposed algorithms are described. Finally, the effectiveness and convergence of the proposed RA algorithm as well as the benefits of using SP and the HSE protocol are illustrated by numerical experiments.

4.1 System model and problem formulation

4.1.1 System model

We consider a cellular OFDMA system with N cells coordinated by a central controller for RA. In each cell, downlink transmission is carried out from a source to U destinations with the help of a relay, which is assumed to be of the DF type. For each link, the channel is assumed to be frequency selective and transformed into K parallel subchannels by using OFDM with sufficiently long cyclic prefix. The data transmission is carried out in two TSs. During the first TS, a symbol is first broadcast by the source at a subcarrier k, which is in either relay aided mode or direct mode. Both the relay and the targeted destination receive this symbol. If the relay aided mode is used, the relay decodes the received symbol and forwards it to the targeted destination over a subcarrier l with the source keeping quiet on this subcarrier during the second TS. Here subcarrier l is paired with subcarrier k for transmitting the same symbol. The destination only decodes the symbol received during the second TS. If the direct mode is used, the targeted destination decodes the symbol received during the first TS. Also, another symbol is broadcast by the source at an unpaired subcarrier l during the second TS, which is received and only decoded by the destination selected for the second TS.

With OFDMA, each subcarrier is allocated to only one destination in each cell. For the relay-aided transmission, subcarrier k during the first TS can be paired with only one subcarrier l during the second TS and vice versa. Throughout this Chapter, perfect timeing and carrier synchronization is assumed. The RA is carried out by a central controller which has perfect knowledge of the system channel state information. Moreover, the coherence time of each link is assumed to be sufficiently long for the RA to be implemented. Lastly we assume the optimized RA can be correctly distributed to all nodes.

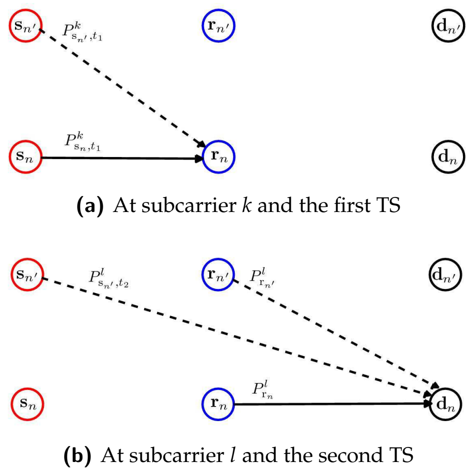


Figure 4.1 Illustration of a relay aided transmission in cell n with cochannel interference from cell n'. The solid lines and the dashed lines represent communication links and interfering links respectively.

Note that an upper bound on the system performances is obtained by assuming the above idealities.

The data transmission procedure in every cell is identical. Therefore we only analyze the downlink data transmission inside one selected cell n, which is impaired by cochannel interference from the other cells. Specifically in cell n, symbols are transmitted in either relay-aided mode or direct mode, as will be elaborated in the following.

Let us first see more closely the relay aided data transmission. As illustrated in Fig. 1(a), the source s_n first produces a symbol $\sqrt{P^k_{\mathrm{s}_n,t_1}}x^k_{\mathrm{s}_n,t_1}$ at subcarrier k during the first TS, while the transmitter of the relay r_n remains idle. Here $x^k_{\mathrm{s}_n,t_1}$ denotes the normalized symbol (meaning $E\{|x^k_{\mathrm{s}_n,t_1}|^2\} = 1$) transmitted by s_n at subcarrier k during the first TS and $P^k_{\mathrm{s}_n,t_1}$ denotes the corresponding transmit power. Simultaneously in an interfering cell n', a symbol $\sqrt{P^k_{\mathrm{s}_{n'},t_1}}x^k_{\mathrm{s}_{n'},t_1}$ is also produced from the interfering source $\mathrm{s}_{n'}$ at the same subcarrier. Here again $E\{|x^k_{\mathrm{s}_{n'},t_1}|^2\} = 1$. Note that, instead of using an additional integer variable to indicate whether $\mathrm{s}_{n'}$ transmits data on subcarrier k or not, we use $\sqrt{P^k_{\mathrm{s}_{n'},t_1}}$ to do it. This corresponds to the formulation 4 in Chapter 3. Specifically, $P^k_{\mathrm{s}_{n'},t_1} > 0$ means that $\mathrm{s}_{n'}$ uses subcarrier k for data transmission during the first TS and $P^k_{\mathrm{s}_{n'},t_1} = 0$ means that $\mathrm{s}_{n'}$ transmits nothing at the subcarrier k during the first TS. This choice is motivated to simplify the system sum rate expression and facilitate the algorithm design. Specifically, when using an integer variable to indicate whether a subcarrier k in a cell n is used or not, the rate expression would contain integer variables in both the denominators and the numerators, which makes it a non-linear function of the integer variables. When using the corresponding power value to do it, the rate expression contains integer variables only in the numerators, which makes it a linear function of the integer variables. At the end of the first TS, the signal received by r_n for subcarrier k can be expressed as

$$\begin{aligned} y^k_{\mathrm{r}_n} =& \sqrt{P^k_{\mathrm{s}_n,t_1}} h^k_{\mathrm{s}_n,\mathrm{r}_n} x^k_{\mathrm{s}_n,t_1} + v^k_{\mathrm{r}_n} + \\ & \sum_{n'=1,n'\neq n}^{N} \sqrt{P^k_{\mathrm{s}_{n'},t_1}} h^k_{\mathrm{s}_{n'},\mathrm{r}_n} x^k_{\mathrm{s}_{n'},t_1} \end{aligned} \tag{4.1}$$

where $v^k_{\mathrm{r}_n}$ denotes the additive white Gaussian noise (AWGN) at subcarrier k and r_n during the first TS. $h^k_{\mathrm{s}_{n'},\mathrm{r}_n}$ denotes the channel frequency response (CFR) for subcarrier k from $\mathrm{s}_{n'}$ to r_n.

During the second TS, as illustrated in Fig. 1(b), r_n reencodes the decoded symbol and forwards $\sqrt{P^l_{\mathrm{r}_n}} x^l_{\mathrm{r}_n}$ at a subcarrier l. Here $x^l_{\mathrm{r}_n} = x^k_{\mathrm{s}_n,t_1}$, meaning that subcarrier l during the second TS is paired with subcarrier k during the first TS for transmitting the same symbol in cell n. The source s_n transmits nothing on this paired subcarrier l, meaning that $P^l_{\mathrm{s}_n,t_2} = 0$. Here $P^l_{\mathrm{r}_n}$ and $P^l_{\mathrm{s}_n,t_2}$ denote the transmit power allocated to r_n and s_n respectively at subcarrier l during the second TS. At the same time, in an interfering cell n', $\mathrm{r}_{n'}$ and $\mathrm{s}_{n'}$ also transmit $\sqrt{P^l_{\mathrm{r}_{n'}}} x^l_{\mathrm{r}_{n'}}$ and $\sqrt{P^l_{\mathrm{s}_{n'},t_2}} x^l_{\mathrm{s}_{n'},t_2}$ at subcarrier l, at most only one power value out of $\sqrt{P^l_{\mathrm{r}_{n'}}} x^l_{\mathrm{r}_{n'}}$ and $\sqrt{P^l_{\mathrm{s}_{n'},t_2}} x^l_{\mathrm{s}_{n'},t_2}$ can be non-zero. More specifically when subcarrier l is paired with a subcarrier used during the first TS in cell n', $P^l_{\mathrm{s}_{n'},t_2} = 0$. Otherwise, $P^l_{\mathrm{r}_{n'}} = 0$. Here again $E\{|x^l_{\mathrm{r}_{n'}}|^2\} = 1$ and $E\{|x^l_{\mathrm{s}_{n'},t_2}|^2\} = 1$. At the end of the second TS, the signal received by the targeted destination

d_{un} at subcarrier l can be expressed as

$$\begin{aligned} y^l_{\mathrm{d}_{un},t_2} = & \sqrt{P^l_{\mathrm{r}_n}} h^l_{\mathrm{r}_n,\mathrm{d}_{un}} x^l_{\mathrm{r}_n} + v^l_{\mathrm{d}_{un},t_2} + \\ & \sum_{n'=1,n'\neq n}^{N} \sqrt{P^l_{\mathrm{s}_{n'},t_2}} h^l_{\mathrm{s}_{n'},\mathrm{d}_{un}} x^l_{\mathrm{s}_{n'},t_2} + \\ & \sum_{n'=1,n'\neq n}^{N} \sqrt{P^l_{\mathrm{r}_{n'}}} h^l_{\mathrm{r}_{n'},\mathrm{d}_{un}} x^l_{\mathrm{r}_{n'}} \end{aligned} \tag{4.2}$$

where $v^l_{\mathrm{d}_{un},t_2}$ denotes the AWGN corrupting d_{un} at subcarrier l during the second TS , $h^l_{\mathrm{r}_{n'},\mathrm{d}_{un}}$ denotes the CFR for subcarrier l from $\mathrm{r}_{n'}$ to d_{un}. $h^l_{\mathrm{s}_{n'},\mathrm{d}_{un}}$ denotes the CFR of subcarrier l from $\mathrm{s}_{n'}$ to d_{un}.

We assume $\{v^k_{\mathrm{r}_n}, v^l_{\mathrm{d}_{un},t_2}\}$ are independent zero-mean circular Gaussian random variables with the same variance σ^2. After some mathematical calculations, the signal-to-interference-plus-noise ratio (SINR) associated with decoding $x^k_{\mathrm{s}_n,t_1}$ from $y^k_{\mathrm{r}_n}$ at r_n during the first TS is expressed by

$$\Gamma^k_{\mathrm{r}_n} = \frac{P^k_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{r}_n}}{f^k_{\mathrm{r}_n}} \tag{4.3}$$

where $f^k_{\mathrm{r}_n} = \sigma^2 + \sum_{n',n'\neq n} P^k_{\mathrm{s}_{n'},t_1} G^k_{\mathrm{s}_{n'},\mathrm{r}_n}$ denotes the sum power of the AWGN and the interference received by r_n at subcarrier k during the first TS. $G^k_{\mathrm{s}_n,\mathrm{r}_n} = |h^k_{\mathrm{s}_n,\mathrm{r}_n}|^2$ denotes the channel gain of subcarrier k from s_n to r_n.

Also the SINR associated with decoding $x^l_{\mathrm{r}_n}$, which equals $x^k_{\mathrm{s}_n,t_1}$, from $y^l_{\mathrm{d}_{un},\mathrm{r}_n}$ at d_{un} during the second TS is expressed by

$$\Gamma^l_{\mathrm{d}_{un},t_2} = \frac{P^l_{\mathrm{r}_n} G^l_{\mathrm{r}_n,\mathrm{d}_{un}}}{f^l_{\mathrm{d}_{un},t_2}} \tag{4.4}$$

where $f^l_{\mathrm{d}_{un},t_2} = \sigma^2 + \sum_{n'=1,n'\neq n}^{N} P^l_{\mathrm{s}_{n'},t_2} G^l_{\mathrm{s}_{n'},\mathrm{d}_{un}} + \sum_{n'=1,n'\neq n}^{N} P^l_{\mathrm{r}_{n'}} G^l_{\mathrm{r}_{n'},\mathrm{d}_{un}}$ denotes the sum power of the AWGN and the interference received by d_{un} at subcarrier l during the second TS. $G^l_{\mathrm{r}_n,\mathrm{d}_{un}} = |h^l_{\mathrm{r}_n,\mathrm{d}_{un}}|^2$ denotes the channel gain of subcarrier l from r_n to d_{un}. $G^l_{\mathrm{s}_{n'},\mathrm{d}_{un}} = |h^l_{\mathrm{s}_{n'},\mathrm{d}_{un}}|^2$ denotes the channel gain of subcarrier l from $\mathrm{s}_{n'}$ to d_{un}.

The maximum achievable rate for the subcarrier pair (k,l) when allocated to d_{un} is given by [3]

$$R^{kl}_{un,2} = \min\left(\ln\left(1+\Gamma^k_{\mathrm{r}_n}\right), \ln\left(1+\Gamma^l_{\mathrm{d}_{un},t_2}\right)\right) \tag{4.5}$$

in nats/two-TSs.

Let us describe further the direct data transmission in cell n. During the first TS, s_n broadcasts $\sqrt{P^k_{\mathrm{s}_n,t_1}}x^k_{\mathrm{s}_n,t_1}$ at subcarrier k. The targeted destination d_{un} receives signals from all sources. Finally, the signal received by d_{un} at subcarrier k can be expressed as

$$\begin{aligned} y^k_{\mathrm{d}_{un},t_1} =& \sqrt{P^k_{\mathrm{s}_n,t_1}}h^k_{\mathrm{s}_n,\mathrm{d}_{un}}x^k_{\mathrm{s}_n,t_1} + v^k_{\mathrm{d}_{un},t_1} + \\ & \sum_{n'=1,n'\neq n}^{N} \sqrt{P^k_{\mathrm{s}_{n'},t_1}}h^k_{\mathrm{s}_{n'},\mathrm{d}_{un}}x^k_{\mathrm{s}_{n'},t_1} \end{aligned} \tag{4.6}$$

where $v^k_{\mathrm{d}_{un},t_1}$ denotes the AWGN corrupting d_{un} at subcarrier k during the first TS.

During the second time slot, another symbol $\sqrt{P^l_{\mathrm{s}_n,t_2}}x^l_{\mathrm{s}_n,t_2}$ is broadcast by s_n at an unpaired subcarrier l and received by a destination d_{vn}. The received signal can be expressed as:

$$\begin{aligned} y^l_{\mathrm{d}_{vn},t_2} =& \sqrt{P^l_{\mathrm{s}_n,t_2}}h^l_{\mathrm{s}_n,\mathrm{d}_{vn}}x^l_{\mathrm{s}_n,t_2} + v^l_{\mathrm{d}_{vn},t_2} + \\ & \sum_{n'=1,n'\neq n}^{N} \sqrt{P^l_{\mathrm{s}_{n'},t_2}}h^l_{\mathrm{s}_{n'},\mathrm{d}_{vn}}x^l_{\mathrm{s}_{n'},t_2} + \\ & \sum_{n'=1,n'\neq n}^{N} \sqrt{P^l_{\mathrm{r}_{n'}}}h^l_{\mathrm{r}_{n'},\mathrm{d}_{vn}}x^l_{\mathrm{r}_{n'}} \end{aligned} \tag{4.7}$$

where $v^l_{\mathrm{d}_{vn},t_2}$ denotes the AWGN corrupting d_{vn} at subcarrier l during the second TS.

Thus the achievable sum rate for subcarrier k during the first TS and subcarrier l during the second TS is given by

$$R^{kl}_{uvn,1} = \ln\left(1 + \Gamma^k_{\mathrm{d}_{un},t_1}\right) + \ln\left(1 + \Gamma^l_{\mathrm{d}_{vn},t_2}\right) \tag{4.8}$$

in nats/two-TSs, where

$$\Gamma^k_{\mathrm{d}_{un},t_1} = \frac{P^k_{\mathrm{s}_n,t_1}G^k_{\mathrm{s}_n,\mathrm{d}_{un}}}{f^k_{\mathrm{d}_{un},t_1}} \tag{4.9}$$

denotes the SINR associated with the decoding of $x^k_{\mathrm{s}_n,t_1}$ from $y^k_{\mathrm{d}_{un},t_1}$ at d_{un} during the first TS. $f^k_{\mathrm{d}_{un},t_1} = \sigma^2 + \sum\limits_{n'=1,n'\neq n}^{N} P^k_{\mathrm{s}_{n'},t_1}G^k_{\mathrm{s}_{n'},\mathrm{d}_{un}}$ denotes the sum power of the AWGN and the interference received by d_{un} at subcarrier k during the first TS.

$$\Gamma^l_{\mathrm{d}_{vn},t_2} = \frac{P^l_{\mathrm{s}_n,t_2} G^l_{\mathrm{s}_n,\mathrm{d}_{vn}}}{f^l_{\mathrm{d}_{vn},t_2}} \tag{4.10}$$

denotes the SINR associated with the decoding of $x^l_{\mathrm{s}_n,t_2}$ from $y^l_{\mathrm{d}_{vn},t_2}$ at d_{vn} during the second TS. $f^l_{\mathrm{d}_{vn},t_2} = \sigma^2 + \sum_{n'=1,n'\neq n}^{N} P^l_{\mathrm{s}_{n'},t_2} G^l_{\mathrm{s}_{n'},\mathrm{d}_{vn}} + \sum_{n'=1,n'\neq n}^{N} P^l_{\mathrm{r}_{n'}} G^l_{\mathrm{r}_{n'},\mathrm{d}_{vn}}$ denotes the sum power of the AWGN and the interference received by d_{vn} at subcarrier l during the second TS.

4.1.2 Problem formulation

In order to formulate the RA problem, we now introduce binary variables a^{kl}_{uvn} and b^{kl}_{un} to describe the mode selection, subcarrier assignment and pairing in both TSs. To be more specific, $a^{kl}_{uvn} = 1$ indicates that subcarrier k is allocated for data transmission to d_{un} in direct mode during the first TS, and so is subcarrier l to d_{vn} during the second TS. Note in direct mode, although used for transmitting two different symbols, subcarrier k and subcarrier l are virtually seen as a subcarrier pair for data transmission during two TSs. $b^{kl}_{un} = 1$ indicates that subcarrier k used during the first TS is paired with subcarrier l used during the second TS and they are both allocated for data transmission to d_{un} aided by r_n.

We consider maximizing the sum rate of all destinations in all cells under per cell total power constraints. The optimization variables are the transmission mode for each subcarrier, the subcarrier assignments and pairings as well as the subcarrier power allocations at the sources and the relays. According to the system model, the considered RA problem can be formulated as:

$$
\begin{aligned}
&\max_{\mathbf{P},\mathbf{A},\mathbf{B}} R\left(\mathbf{P},\mathbf{A},\mathbf{B}\right)\\
&s.t.\\
&C4.1: \sum_{l=1}^{K}\Big(\sum_{u=1}^{U}\sum_{v=1}^{U} a_{uvn}^{kl}+\sum_{u=1}^{U} b_{un}^{kl}\Big)\leq 1,\ \forall n,k,\\
&C4.2: \sum_{k=1}^{K}\Big(\sum_{u=1}^{U}\sum_{v=1}^{U} a_{uvn}^{kl}+\sum_{u=1}^{U} b_{un}^{kl}\Big)\leq 1,\ \forall n,l,\\
&C4.3: a_{uvn}^{kl}\in\{0,1\}, b_{un}^{kl}\in\{0,1\},\ \forall u,v,n,k,l,\\
&C4.4: \sum_{k=1}^{K} P_{\mathrm{s}_n,t_1}^{k}+\sum_{l=1}^{K} P_{\mathrm{s}_n,t_2}^{l}+\sum_{l=1}^{K} P_{\mathrm{r}_n}^{l}\leq P_{t,n},\ \forall n,\\
&C4.5: P_{\mathrm{s}_n,t_1}^{k}\geq 0, P_{\mathrm{s}_n,t_2}^{l}\geq 0, P_{\mathrm{r}_n}^{l}\geq 0,\ \forall n,k,l,\\
&C4.6: P_{\mathrm{s}_n,t_1}^{k}\left(1-\sum_{l=1}^{K}\Big(\sum_{u=1}^{U}(\sum_{v=1}^{U} a_{uvn}^{kl}+b_{un}^{kl})\Big)\right)=0,\ \forall n,k,\\
&C4.7: P_{\mathrm{s}_n,t_2}^{l}\Big(1-\sum_{k=1}^{K}\sum_{u=1}^{U}\sum_{v=1}^{U} a_{uvn}^{kl}\Big)=0,\ \forall n,l,\\
&C4.8: P_{\mathrm{r}_n}^{l}\Big(1-\sum_{k=1}^{K}\sum_{u=1}^{U} b_{un}^{kl}\Big)=0,\ \forall n,l.
\end{aligned}
\tag{4.11}
$$

where $\mathbf{P}=\{P_{\mathrm{s}_n,t_1}^{k}, P_{\mathrm{s}_n,t_2}^{l}, P_{\mathrm{r}_n}^{l}\}$, $\mathbf{A}=\{a_{uvn}^{kl}\}$, $\mathbf{B}=\{b_{un}^{kl}\}$ and

$$
\begin{aligned}
R\left(\mathbf{P},\mathbf{A},\mathbf{B}\right)=&\sum_{n=1}^{N}\sum_{u=1}^{U}\sum_{v=1}^{U}\sum_{k=1}^{K}\sum_{l=1}^{K} a_{uvn}^{kl}R_{un,1}^{kl}\left(\mathbf{P}\right)+\\
&\sum_{n=1}^{N}\sum_{u=1}^{U}\sum_{k=1}^{K}\sum_{l=1}^{K} b_{un}^{kl}R_{un,2}^{kl}\left(\mathbf{P}\right).
\end{aligned}
\tag{4.12}
$$

Here, $C4.1$, $C4.2$ and $C4.3$ ensure that each subcarrier k during the first TS can be paired or virtually paired with only one subcarrier l during the second TS in each cell n. Also subcarrier l can be paired or virtually paired with only one subcarrier k. For each subcarrier pair (k,l), only one mode (direct/relay-aided) can be selected for data transmission. If the direct mode is used, unique destinations (d_{un} and d_{vn}) should be targeted for subcarrier k and subcarrier l respectively. If the relay aided mode is selected, the subcarrier pair (k,l) can be allocated to only one destination. Moreover, $C4.4$ and $C4.5$ ensure that the consumed sum power for each cell is less than its available sum power. This type of power constraints gives an upper bound of the system performance. In practice, each node (source, relay) in each cell will have an individual power constraint. Finally, $C4.6$, $C4.7$ and $C4.8$ guarantee that no data is transmitted on an unused subcarrier and subcarrier l is used by only one node (either the source or the relay) in each cell during the second TS.

4.2 Algorithm development

4.2.1 Algorithm overview

Note that problem (4.11), which contains both integer and continuous variables, is a mixed combinatorial and non-convex optimization. Here the non-convexity is due to the existence of the inter-cell co-channel interference terms in (4.12). In general, an exhaustive search is needed to find the global optimum. Let us consider a system with N cells, U users and K subcarriers in each cell. There are K^{NK} possible subcarrier pairs, each of which can be assign to one of the U possible users with 2 possible transmission modes. Thus there are $(2U)^{K^{NK}}$ possible MSSAPs, which limits the scalability.

In order to make the problem tractable, we opt for a coordinate ascent approach made of two stages: the MSSAP stage and the PA stage. We propose an iterative algorithm which optimizes the two stages in an alternate way, as depicted by Algorithm 4.1. Integer m indicates the iteration number and a variable with the superscript m denotes the value obtained at the end of iteration m. Specifically in the proposed RA algorithm, we first set $m = 0$ and

Algorithm 4.1 Overall RA Optimization Algorithm

1: **Initialize:** set $m = 0$ and initialize $\mathbf{P}^0$ by the UPA algorithm;
2: **repeat**
3: $\quad m = m + 1$;
4: $\quad$ **The MSSAP stage:** Compute the optimal MSSAP result $\{\mathbf{A}^m, \mathbf{B}^m\}$ for (4.11) with $\mathbf{P} = \mathbf{P}^{m-1}$ using the method proposed in section 4.2.2;
5: $\quad$ **The PA stage:** Compute the optimized PA result $\mathbf{P}^m$ for (4.11) using the SCGP algorithm proposed in section 4.2.3, when $\mathbf{A} = \mathbf{A}^m$ and $\mathbf{B} = \mathbf{B}^m$;
6: **until** $|R^m - R^{m-1}| < \epsilon_1$ or m=M
7: **Output:** $\mathbf{A}^m$, $\mathbf{B}^m$ and $\mathbf{P}^m$.

initialize the power $\mathbf{P}^0$ by uniform power allocation (UPA). Each iteration is made of the MSSAP stage followed by the PA stage.

During the MSSAP stage of iteration m, problem (4.11) is solved with $\mathbf{P} = \mathbf{P}^{m-1}$ and the output delivered is denoted as $\{\mathbf{A}^m, \mathbf{B}^m\}$. As will be shown in section 4.2.2, when $\mathbf{P}$ is fixed, problem (4.11) is decoupled into n subproblems which can be easily solved using the optimal results of n Linear Assignment Problems (LAP). Note as each LAP can be solved by the Hungarian Algorithm in polynomial time [71], $\{\mathbf{A}^m, \mathbf{B}^m\}$ will be obtained in polynomial time. Finally we have $R\left(\mathbf{P}^{m-1}, \mathbf{A}^{m-1}, \mathbf{B}^{m-1}\right) \leq R\left(\mathbf{P}^{m-1}, \mathbf{A}^{m}, \mathbf{B}^{m}\right)$

During the PA stage of iteration m, problem (4.11) is solved with $\mathbf{A} = \mathbf{A}^{m-1}$ and $\mathbf{B} = \mathbf{B}^{m-1}$. The output delivered is denoted as $\mathbf{P}^m$. However, when $\{\mathbf{A}, \mathbf{B}\}$ is fixed, problem (4.11) is still non-convex and hard to solve. Fortunately by using the algorithm based on single condensation and geometric programming (SCGP) proposed in section 4.2.3, this non-convex problem can be approximated into a series of standard geometric programs (GP), which can be solved by common GP solvers. As will be shown in section 4.2.3, the solutions of the approximated GP problems converge to a local optimum satisfying the Karush-Kuhn-Tucker (KKT) conditions of the non-convex problem. Note as each standard GP can be solved in polynomial time [72], $\mathbf{P}^m$ will be obtained in polynomial time after the convergence of the SCGP algorithm. Finally we have $R\left(\mathbf{P}^{m-1}, \mathbf{A}^{m}, \mathbf{B}^{m}\right) \leq R\left(\mathbf{P}^{m}, \mathbf{A}^{m}, \mathbf{B}^{m}\right)$.

Considering the proposed RA algorithm, we now have:

$$\begin{aligned} & R\left(\mathbf{P}^{m-1}, \mathbf{A}^{m-1}, \mathbf{B}^{m-1}\right) \\ E1: \ & \leq R\left(\mathbf{P}^{m-1}, \mathbf{A}^{m}, \mathbf{B}^{m}\right) \\ E2: \ & \leq R\left(\mathbf{P}^{m}, \mathbf{A}^{m}, \mathbf{B}^{m}\right), \end{aligned}$$

where E1 and E2 are due to effects of the MSSAP stage and the PA stage respectively. This means that Algorithm 4.1 yields non-decreasing sum rates along with iterations. Moreover, due to the total power constraint in each cell, the optimum sum rate is upper bounded and the sum rate values will not increase indefinitely along with iterations. This means that the iterations will eventually converge [62]. Algorithm 4.1 will stop when the sum rate increase is below a prescribed value ϵ_1 or when m reaches a prescribed value M.

Note that both the MSSAP stage and the SCGP stage can be solved in polynomial time. As will be illustrated by experiments, the proposed RA algorithm converges in limited number of iterations. Thus problem (4.11) can be solved in polynomial time. At the end of iteration m, a local optimum of (4.11) is obtained at the SCGP stage, which is then improved at the MSSAP stage of the next iteration. After that, a better local optimum of (4.11) can be calculated at the SCGP stage of iteration $m+1$. Finally, a good local optimum can be obtained after convergence.

4.2.2 MSSAP optimization

In this subsection, the MSSAP stage for iteration m is considered. After setting the power variable $\mathbf{P}$ to $\mathbf{P}^{m-1}$, R becomes a linear function of all indicators. Note that each cell has independent constraints for its local indicators. Therefore problem (4.11) can be decoupled into N subproblems. As for subproblem n_0, the sum rate of the corresponding cell is maximized subject to its local constraints. Specifically, subproblem n_0 is formulated as:

$$
\begin{aligned}
&\max_{\mathbf{A}_{n_0},\mathbf{B}_{n_0},\mathbf{C}_{n_0}} \quad R_{n_0}\left(\mathbf{A}_{n_0},\mathbf{B}_{n_0}\right) \\
&s.t.\ C4.1': \ \sum_{l=1}^{K} c_{n_0}^{kl} \leq 1,\ \forall k, \\
&C4.2': \ \sum_{k=1}^{K} c_{n_0}^{kl} \leq 1,\ \forall l, \\
&C4.3': \ a_{uvn_0}^{kl} \in \{0,1\}, b_{un_0}^{kl} \in \{0,1\},\ \forall u,v,k,l, \\
&C4.4': \ c_{n_0}^{kl} = \sum_{u=1}^{U}\sum_{v=1}^{U} a_{uvn_0}^{kl} + \sum_{u=1}^{U} b_{un_0}^{kl},\ \forall k,l, \\
&C4.6': \ P_{\mathrm{s}_{n_0},t_1}^{k,m-1}\left(1-\sum_{l=1}^{K}\Big(\sum_{u=1}^{U}(\sum_{v=1}^{U} a_{uvn_0}^{kl} + b_{un_0}^{kl})\Big)\right) = 0,\ \forall k, \\
&C4.7': \ P_{\mathrm{s}_{n_0},t_2}^{l,m-1}\Big(1-\sum_{k=1}^{K}\sum_{u=1}^{U}\sum_{v=1}^{U} a_{uvn_0}^{kl}\Big) = 0,\ \forall l, \\
&C4.8': \ P_{\mathrm{r}_{n_0}}^{l,m-1}\Big(1-\sum_{k=1}^{K}\sum_{u=1}^{U} b_{un_0}^{kl}\Big) = 0,\ \forall l.
\end{aligned}
\tag{4.13}
$$

where $\mathbf{A}_{n_0} = \{a_{uvn}^{kl} | n = n_0\}$, $\mathbf{B}_{n_0} = \{b_{un}^{kl} | n = n_0\}$, $\mathbf{C}_{n_0} = \{c_{n_0}^{kl}\}$ and

$$R_{n_0} = \sum_{k=1}^{K}\sum_{l=1}^{K}\sum_{u=1}^{U}\sum_{v=1}^{U} a_{uvn_0}^{kl} R_{uvn_0,1}^{kl}\left(\mathbf{P}^{m-1}\right) + \sum_{k=1}^{K}\sum_{l=1}^{K}\sum_{u=1}^{U} b_{un_0}^{kl} R_{un_0,2}^{kl}\left(\mathbf{P}^{m-1}\right).$$

As will be explained later, problem (4.13) has the same optimum solution as the following problem, where the constraints $C4.6' - C4.8'$ are removed.

$$\begin{aligned}&\max_{\mathbf{A}_{n_0},\mathbf{B}_{n_0},\mathbf{C}_{n_0}} \quad R_{n_0}\left(\mathbf{A}_{n_0}, \mathbf{B}_{n_0}\right)\\ &s.t.\ C4.1' - C4.4'.\end{aligned} \tag{4.14}$$

Specifically for each subcarrier k_0, when $P_{s_{n_0},t_1}^{k_0,m-1} > 0$, we certainly have $R_{uvn_0,1}^{k_0 l} > 0, \forall l, u, v$. Therefore at the optimum of problem (4.14), we have $\sum_{l=1}^{K}\left(\sum_{u=1}^{U}(\sum_{v=1}^{U} a_{uvn_0}^{k_0 l} + b_{un_0}^{k_0 l})\right) = 1$, meaning that this subcarrier k_0 should be used for data transmission. Otherwise, we can still improve R_{n_0} by randomly selecting one variable of the set $\{a_{uvn}^{kl} | k = k_0, n = n_0\}$ and setting it to 1. Thus in this case, we have $P_{s_{n_0},t_1}^{k_0,m-1}\left(1 - \sum_{l=1}^{K}\left(\sum_{u=1}^{U}(\sum_{v=1}^{U} a_{uvn_0}^{k_0 l} + b_{un_0}^{k_0 l})\right)\right) = 0$. Also, when $P_{s_{n_0},t_1}^{k_0,m-1} = 0$, we certainly have $P_{s_{n_0},t_1}^{k_0}\left(1 - \sum_{l=1}^{K}\left(\sum_{u=1}^{U}(\sum_{v=1}^{U} a_{uvn_0}^{k_0 l} + b_{un_0}^{k_0 l})\right)\right) = 0$. Thus at the optimum of problem (4.14), the constraint $C4.6'$ is fulfilled. Similarly, at the optimum of problem (4.14), the constraints $C4.7'$ and $C4.8'$ are fulfilled. This means that the optimum solution of problem (4.14) is also feasible for problem (4.13). Note that problem (4.13) has the same expression as problem (4.14) except that its feasible set is a subset of that of problem (4.14). Thus the optimum solution of problem (4.14) is also the optimum solution of problem (4.13).

Due to constraints $C4.1'$ and $C4.2'$, we find that the inequality

$$\sum_{u=1}^{U}\sum_{v=1}^{U} a_{uvn_0}^{kl} R_{uvn_0,1}^{kl} + \sum_{u=1}^{U} b_{un_0}^{kl} R_{un_0,2}^{kl} \le c_{n_0}^{kl} Q_{n_0}^{kl} \tag{4.15}$$

holds, where $Q_{n_0}^{kl} = \max\{\max_{uv} R_{uvn_0,1}^{kl}, \max_u R_{un_0,2}^{kl}\}$. The objective function of problem (4.14) is upper bounded by that of the following problem.

$$\begin{aligned} &\max_{\mathbf{A}_{n_0},\mathbf{B}_{n_0},\mathbf{C}_{n_0}} \quad R_{n_0,UB}\left(\mathbf{C}_{n_0}\right) \\ &s.t.\ C4.1' - C4.4', \end{aligned} \tag{4.16}$$

where $R_{n_0,UB} = \sum_{k=1}^{K} \sum_{l=1}^{K} c_{n_0}^{kl} Q_{n_0}^{kl}$.

According to $C4.1', C4.2', C4.3'$ and $C4.4'$, we certainly have $c_{n_0}^{kl} \in \{0,1\},\ \forall k,l$. Considering variables $\mathbf{A}_{n_0}$ and $\mathbf{B}_{n_0}$ of (4.16) take values only based on $\mathbf{C}_{n_0}$, problem (4.16) can be reformulated as:

$$\begin{aligned} &\max_{\mathbf{C}_{n_0}} R_{n_0,UB}\left(\mathbf{C}_{n_0}\right) \\ &s.t.\ C4.1',\ C4.2', \\ &C4.5':\ c_{n_0}^{kl} \in \{0,1\},\ \forall k,l. \end{aligned} \tag{4.17}$$

Let us now consider theorem 4.1.

Theorem 4.1. *An optimal solution of (4.14) ($\mathbf{A}_{n_0}^*$ and $\mathbf{B}_{n_0}^*$) can be easily obtained using the optimal solution of (4.17) ($\mathbf{c}_{n_0,UB}^{*,kl}$). Specifically, $\mathbf{A}_{n_0}^*$ and $\mathbf{B}_{n_0}^*$ are obtained by assigning for every combination of k and l, all entries in $\{a_{uvn_0}^{kl}|\forall u,v\}$ and $\{b_{un_0}^{kl}|\forall u\}$ to zero, except for the one with the metric equal to $Q_{n_0}^{kl}$ to $\mathbf{c}_{n_0,UB}^{*,kl}$.*

Proof. Let us view $R_{uvn_0,1}^{kl}$ and $R_{un_0,2}^{kl}$ as metric for $a_{uvn_0}^{kl}$ and $b_{un_0}^{kl}$ respectively. Note that, $\forall c_{n_0}^{kl} = 0$, inequality (4.15) is tightened when all entries of $\{a_{uvn_0}^{kl}|\forall u,v\}$ and $\{b_{un_0}^{kl}|\forall u\}$ are set to zero. While $\forall c_{n_0}^{kl} > 0$, inequality (4.15) is tightened when all entries of $\{a_{uvn_0}^{kl}|\forall u,v\}$ and $\{b_{un_0}^{kl}|\forall u\}$ are set to zero, except that the one with the metric equal to $Q_{n_0}^{kl}$ is assigned to $c_{n_0}^{kl}$.

We now denote by $\{\mathbf{C}_{n_0}^*\}$ the optimal variables of (4.17). Thus, $\exists \mathbf{A}_{n_0}^*, \mathbf{B}_{n_0}^*$, so that $R_{n_0}\left(\mathbf{A}_{n_0}^*, \mathbf{B}_{n_0}^*\right) = R_{n_0}^{UB}\left(\mathbf{C}_{n_0}^*\right)$. Here $\mathbf{A}_{n_0}^*$ and $\mathbf{B}_{n_0}^*$ are obtained by assigning for every combination of k and l, all entries in $\{a_{uvn_0}^{kl}|\forall u,v\}$ and $\{b_{un_0}^{kl}|\forall u\}$ to zero, except for the one with the metric equal to $Q_{n_0}^{kl}$ to $\mathbf{c}_{n_0,UB}^{*,kl}$. It is important to know that $\{\mathbf{A}_{n_0}^*, \mathbf{B}_{n_0}^*, \mathbf{C}_{n_0}^*\}$ belongs to the feasible set of (4.14).

We further note that, $\forall$ feasible $\{\mathbf{A}_{n_0}, \mathbf{B}_{n_0}, \mathbf{C}_{n_0}\}$, $R_{n_0}\left(\mathbf{A}_{n_0}, \mathbf{B}_{n_0}\right) \leq R_{n_0}^{UB}\left(\mathbf{C}_{n_0}\right) \leq R_{n_0}^{UB}\left(\mathbf{C}_{n_0}^*\right)$. Recalling $R_{n_0}\left(\mathbf{A}_{n_0}^*, \mathbf{B}_{n_0}^*\right) = R_{n_0}^{UB}\left(\mathbf{C}_{n_0}^*\right)$, we obtain the optimal variables of problem (4.14) as $\{\mathbf{A}_{n_0}^*, \mathbf{B}_{n_0}^*, \mathbf{C}_{n_0}^*\}$. This concludes the proof of theorem 4.1. $\square$

Let us introduce two sets, $\mathbf{F}$ and $\mathbf{F1}$ ($\mathbf{F1} \subset \mathbf{F}$). Here $\mathbf{F}$ is the feasible sets of (4.17). $\mathbf{F1}$ is the same as $\mathbf{F}$ except that both $C4.1'$ and $C4.2'$ are saturated. Note that $\forall \mathbf{C}_{n_0} \in \mathbf{F} \backslash \mathbf{F1}$, there must $\exists k_0, \sum_{l=1}^{K} c_{n_0}^{k_0 l} = 0$ or $\exists l_0, \sum_{k=1}^{K} c_{n_0}^{k l_0} = 0$. If $\exists k_0, \sum_{l=1}^{K} c_{n_0}^{k_0 l} = 0$, according to the constraints, there must $\exists l_0$ such that $c_{n_0}^{k_0 l_0} = 0$ and $\sum_{k=1}^{K} c_{n_0}^{k l_0} = 0$. Let us denote the sets of these k_0 and l_0 as $\mathbf{K}_0$ and $\mathbf{L}_0$ respectively. Thus we can find $\mathbf{C}_{n_0}^{'} \in \mathbf{F1}$, where $c_{n_0}^{',kl} = 1, \forall k \in \mathbf{K}_0, l \in \mathbf{L}_0$ and $c_{n_0}^{',kl} = c_{n_0}^{kl}, \forall k \notin \mathbf{K}_0 \; or \; l \notin \mathbf{L}_0$, such that $R_{n_0,UB}\left(\mathbf{C}_{n_0}^{'}\right) \geq R_{n_0,UB}\left(\mathbf{C}_{n_0}\right)$. The optimal variables of the following problem is also optimal for problem (4.17).

$$
\begin{aligned}
&\max_{\mathbf{C}_{n_0}} R_{n_0,UB}\left(\mathbf{C}_{n_0}\right) \\
&s.t. \\
&C4.1'': \sum_{l=1}^{K} c_{n_0}^{kl} = 1, \; \forall k, \\
&C4.2'': \sum_{k=1}^{K} c_{n_0}^{kl} = 1, \; \forall l, \\
&C4.5': \; c_{n_0}^{kl} \in \{0,1\}, \; \forall k, l.
\end{aligned}
\tag{4.18}
$$

Note that (4.18) is the same as (4.17) except that the feasible set of (4.18) $\mathbf{F1}$ is a subset of that of (4.17) $\mathbf{F}$. (4.18) is actually a LAP. Hence it can be solved by means of typical LAP algorithms, like the Hungarian Algorithm.

4.2.3 PA optimization

In this subsection, the PA stage for iteration m is considered. After setting the indicators to $\{\mathbf{A}^m, \mathbf{B}^m\}$, the transmission modes are fixed in all cells. Let us denote by $\mathcal{S}_n(d)$ ($\mathcal{S}_n(r)$) the set of subcarriers in direct (relay aided) mode during the first TS in cell n. $\forall k$, the associated destination u during the first TS and the subcarrier l with which it is paired or virtually paired during the second TS are decided. $\forall k \in \mathcal{S}_n(d)$, the associated destination during the second TS is also decided. Thus the objective function of problem (4.11) can consequently be rewritten as:

$$\begin{aligned} R\left(\mathbf{P}\right) = & \sum_{n} \sum_{k \in \mathcal{S}_n(d)} \ln\left(1+\Gamma^{k}_{\mathrm{d}_{un},t_1}\right) + \\ & \sum_{n} \sum_{k \in \mathcal{S}_n(d)} \ln\left(1+\Gamma^{l}_{\mathrm{d}_{vn},t_2}\right) + \\ & \sum_{n} \sum_{k \in \mathcal{S}_n(r)} \ln\left(\min\left(1+\Gamma^{k}_{\mathrm{r}_n}, 1+\Gamma^{l}_{\mathrm{d}_{un,t_2}}\right)\right), \end{aligned} \tag{4.19}$$

where variables l, u and v are all constants. Note that R is a non-convex function due to the presence of interfering power terms in the denominators of $\Gamma^{k}_{\mathrm{d}_{un},t_1}$, $\Gamma^{l}_{\mathrm{d}_{vn},t_2}$, $\Gamma^{k}_{\mathrm{r}_n}$ and $\Gamma^{l}_{\mathrm{d}_{un,t_2}}$. To solve problem (4.11), we first replace it with an equivalent complementary geometric program (CGP) that is then addressed by means of the SCGP algorithm.

The equivalent CGP is obtained by formulating problem (4.11) as a minimization problem. We first formulate problem (4.11) as follows:

$$\begin{aligned} & \min_{\mathbf{P}} e^{-R(\mathbf{P})} \\ & s.t.\ C4.4 - C4.8, \end{aligned} \tag{4.20}$$

where

$$\begin{aligned} e^{-R(\mathbf{P})} = & \prod_{n} \prod_{k \in \mathcal{S}_n(d)} \frac{1}{1+\Gamma^{k}_{\mathrm{d}_{un},t_1}} \frac{1}{1+\Gamma^{l}_{\mathrm{d}_{vn},t_2}} \\ & \prod_{n} \prod_{k \in \mathcal{S}_n(r)} \max\left(\frac{1}{1+\Gamma^{k}_{\mathrm{r}_n}}, \frac{1}{1+\Gamma^{l}_{\mathrm{d}_{un,t_2}}}\right), \end{aligned}$$

$$\frac{1}{1+\Gamma^{k}_{\mathrm{d}_{un},t_1}} = \frac{f^{k}_{\mathrm{d}_{un},t_1}}{f^{k}_{\mathrm{d}_{un},t_1} + P^{k}_{\mathrm{s}_n,t_1} G^{k}_{\mathrm{s}_n,\mathrm{d}_{un}}},$$

$$\frac{1}{1+\Gamma^{l}_{\mathrm{d}_{vn},t_2}} = \frac{f^{l}_{\mathrm{d}_{vn},t_2}}{f^{l}_{\mathrm{d}_{vn},t_2} + P^{l}_{\mathrm{s}_n,t_2} G^{l}_{\mathrm{s}_n,\mathrm{d}_{vn}}},$$

$$\frac{1}{1+\Gamma^{l}_{\mathrm{d}_{un,t_2}}} = \frac{f^{l}_{\mathrm{d}_{un},t_2}}{f^{l}_{\mathrm{d}_{un},t_2} + P^{l}_{\mathrm{r}_n} G^{l}_{\mathrm{r}_n,\mathrm{d}_{un}}},$$

$$\frac{1}{1+\Gamma^k_{\mathrm{r}_n}} = \frac{f^k_{\mathrm{r}_n}}{f^k_{\mathrm{r}_n} + P^k_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{r}_n}}.$$

Next, an equivalent formulation is obtained by introducing slack variables $\mathbf{\Psi}_r = \{\Psi^{n,k}_r, \forall n, k \in \mathcal{S}_n(r)\}$. Then problem (4.20) can be formulated as:

$$\begin{aligned}
&\min_{\mathbf{P},\mathbf{\Psi}_r} e^{-R'(\mathbf{P},\mathbf{\Psi}_r)} \\
&s.t.\ C4.4 - C4.8, \\
&C4.9:\ \frac{f^k_{\mathrm{r}_n}}{g^k_{\mathrm{r}_n}\Psi^{n,k}_r} \leq 1,\ \forall n,k \in \mathcal{S}_n(r), \\
&C4.10:\ \frac{f^l_{\mathrm{d}_{un},t_2}}{g^l_{\mathrm{d}_{un},t_2}\Psi^{n,k}_r} \leq 1,\ \forall n,k \in \mathcal{S}_n(r),
\end{aligned} \tag{4.21}$$

where

$$\begin{aligned}
e^{-R'(\mathbf{P},\mathbf{\Psi}_r)} =& \prod_n \prod_{k\in\mathcal{S}_n(d)} \frac{f^k_{\mathrm{d}_{un},t_1}}{g^k_{\mathrm{d}_{un},t_1}} \\
&\prod_n \prod_{k\in\mathcal{S}_n(d)} \frac{f^l_{\mathrm{d}_{vn},t_2}}{g^l_{\mathrm{d}_{vn},t_2}} \prod_n \prod_{k\in\mathcal{S}_n(r)} \Psi^{n,k}_r,
\end{aligned}$$

$$\begin{aligned}
g^k_{\mathrm{d}_{un},t_1} &= f^k_{\mathrm{d}_{un},t_1} + P^k_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{d}_{un}}, \\
g^l_{\mathrm{d}_{vn},t_2} &= f^l_{\mathrm{d}_{vn},t_2} + P^l_{\mathrm{s}_n,t_2} G^l_{\mathrm{s}_n,\mathrm{d}_{vn}}, \\
g^l_{\mathrm{d}_{un},t_2} &= f^l_{\mathrm{d}_{un},t_2} + P^l_{\mathrm{r}_n} G^l_{\mathrm{r}_n,\mathrm{d}_{un}}, \\
g^k_{\mathrm{r}_n} &= f^k_{\mathrm{r}_n} + P^k_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{r}_n}.
\end{aligned}$$

Problem (4.21) is made of an objective function and bounding constraints which all are ratios of two posynomials, making the problem belong to the class of CGP [49]. More details about CGP can be found in section 2.2.3. Problem (4.21) can not be made convex and is NP-hard [72]. In order to solve it, the SCGP algorithm is now proposed. The proposed SCGP algorithm approximates the non-convex problem (4.21) into a series of standard GPs. Therefore, it belongs to the class of successive convex approximation methods [48]. The SCGP is described in Algorithm 4.2. Integer t indicates the current iteration number and a variable with superscript m' denotes the value obtained at the end of iteration m'.

Algorithm 4.2 SCGP Algorithm

1: **Input:** feasible power variables $\mathbf{P}_{ini}$ of problem (4.21)
2: **Initialize:** set $m' = 0$ and initialize $\mathbf{P}^0$ with $\mathbf{P}_{ini}$;
3: **repeat**
4: $m' = m' + 1$;
5: **The SC stage:** Apply a condensation rule and condense all denominator posynomials in problem (4.21) $\{g^k_{\mathrm{d}_{un},t_1}, g^l_{\mathrm{d}_{vn},t_2}, g^l_{\mathrm{d}_{un},t_2}, g^k_{\mathrm{r}_n}\}$ into monomials $\{\tilde{g}^k_{\mathrm{d}_{un},t_1}, \tilde{g}^l_{\mathrm{d}_{vn},t_2}, \tilde{g}^l_{\mathrm{d}_{un},t_2}, \tilde{g}^k_{\mathrm{r}_n}\}$ using $\mathbf{P}^{m'-1}$ as the initial power values;
6: **The SGP stage:** Solve the condensed problem using a standard GP solver and assign the results to $\mathbf{P}^{m'}$;
7: **until** $||\mathbf{P}^{m'} - \mathbf{P}^{m'-1}|| \leq \epsilon_2$, m′=M′
8: **Output:** $\mathbf{P}^t$.

Specifically in the proposed SCGP algorithm, we first set $m' = 0$ and initialize the PA vector $\mathbf{P}^0$ with $\mathbf{P}_{ini}$. Each iteration contains an SC stage followed by a standard geometric programming (SGP) stage.

During the SC stage of iteration m', a condensation rule is first introduced and depicted in Algorithm 4.3. The goal of the condensation rule is to construct a GP approximation of problem (4.21) by condensing all denominator posynomials $\{g^k_{\mathrm{d}_{un},t_1}, g^l_{\mathrm{d}_{vn},t_2}, g^l_{\mathrm{d}_{un},t_2}, g^k_{\mathrm{r}_n}\}$ into monomials $\{\tilde{g}^k_{\mathrm{d}_{un},t_1}, \tilde{g}^l_{\mathrm{d}_{vn},t_2}, \tilde{g}^l_{\mathrm{d}_{un},t_2}, \tilde{g}^k_{\mathrm{r}_n}\}$. Functions $h^{n,k}_1 = \frac{f^k_{\mathrm{r}_n}}{g^k_{\mathrm{r}_n}\Psi^{n,k}_r}$, $h^{n,k}_2 = \frac{f^l_{\mathrm{d}_{un},t_2}}{g^l_{\mathrm{d}_{un},t_2}\Psi^{n,k}_r}$ and $h^{n,k}_3 = \prod_n \prod_{k \in \mathcal{S}_n(d)} \frac{f^k_{\mathrm{d}_{un},t_1}}{g^k_{\mathrm{d}_{un},t_1}} \frac{f^l_{\mathrm{d}_{vn},t_2}}{g^l_{\mathrm{d}_{vn},t_2}} \prod_n \prod_{k \in \mathcal{S}_n(r)} \Psi^{n,k}_r$ are approximated by respectively $\tilde{h}^{n,k}_1 = \frac{f^k_{\mathrm{r}_n}}{\tilde{g}^k_{\mathrm{r}_n}\Psi^{n,k}_r}$, $\tilde{h}^{n,k}_2 = \frac{f^l_{\mathrm{d}_{un},t_2}}{\tilde{g}^l_{\mathrm{d}_{un},t_2}\Psi^{n,k}_r}$ and $\tilde{h}^{n,k}_3 = \prod_n \prod_{k \in \mathcal{S}_n(d)} \frac{f^k_{\mathrm{d}_{un},t_1}}{\tilde{g}^k_{\mathrm{d}_{un},t_1}} \frac{f^l_{\mathrm{d}_{vn},t_2}}{\tilde{g}^l_{\mathrm{d}_{vn},t_2}} \prod_n \prod_{k \in \mathcal{S}_n(r)} \Psi^{n,k}_r$. According to the proposition 3 in [49], all the approximations satisfy the three conditions proposed in [48] for the convergence of the successive approximation method. By denoting $\mathbf{\Theta} = \{\mathbf{P}, \mathbf{\Psi}_r\}$, the three conditions are listed as follows:

1. Bounding condition: $\forall \mathbf{\Theta}$,

$$h^{n,k}_i(\mathbf{\Theta}) \leq \tilde{h}^{n,k}_i(\mathbf{\Theta}),\ \forall i = 1, 2, 3.$$

2. Tightness condition: At the beginning of iteration m',

$$h_i^{n,k}(\mathbf{\Theta}^{m,m'-1}) = \tilde{h}_i^{n,k}(\mathbf{\Theta}^{m,m'-1}), \ \forall i = 1, 2, 3.$$

3. Differential condition: At the beginning of iteration m', $\forall \theta \in \mathbf{\Theta}$,

$$\frac{\partial h_i^{n,k}(\mathbf{\Theta}^{m,m'-1})}{\partial \theta} = \frac{\partial \tilde{h}_i^{n,k}(\mathbf{\Theta}^{m,m'-1})}{\partial \theta}, \ \forall i = 1, 2, 3.$$

Algorithm 4.3 Condensation Rule

1: **Input:** a posynomial $g(x) = \sum_i u_i(x)$ to be approximated and an initial input value x_0;
2: (1) Evaluate $g(x_0)$;
3: (2) Compute the weight for each term of the posynomial,
$$\alpha_i = \frac{u_i(x_0)}{g(x_0)};$$
4: (3) Condense $g(x)$ into a monomial $\tilde{g}(x)$:
$$\tilde{g}(x) = \prod_i \left(\frac{u_i(x)}{\alpha_i} \right)^{\alpha_i}.$$
5: **Output:** the condensed monomial $\tilde{g}(x)$.

After the SC stage, problem (4.21) is formulated by a standard GP. The SGP stage amounts to solving the GP by means of a standard GP solver, e.g. the software provided at [73]. The output provided by this stage corresponds to $\mathbf{P}^{m'}$.

Thanks to the three conditions fulfilled during the SC stage, our proposed SCGP algorithm is a general inner approximation algorithm [48], which will converge to a local optimum satisfying the KKT conditions of problem (4.21) according to the corollary 1 of [48]. In practice, the iterations of the SCGP algorithm will be stopped when $||\mathbf{P}^{m'} - \mathbf{P}^{m'-1}|| \leq \epsilon_2$ or when m' exceeds a prescribed value M'.

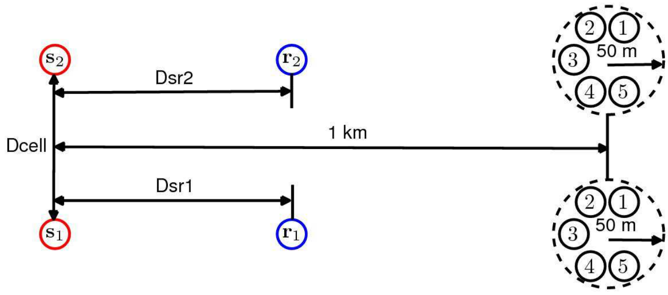


Figure 4.2 A random system setup.

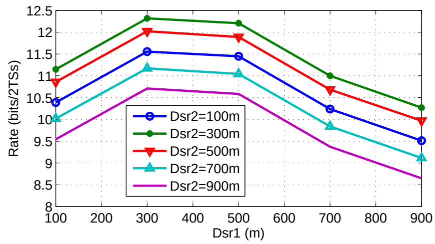
4.3 Numerical experiments

In order to illustrate the benefits of using SP and the high efficiency protocol respectively, we now compare the proposed protocol, named P1, with two other protocols, named P2 and P3 respectively. Specifically, P2 is the same as P1 except that subcarrier k in TS 1 is always paired with the same subcarrier k in TS 2, meaning that no subcarrier pairing is considered. P3 is also the same as P1 except that all sources keep silent during TS 2. For both P2 and P3, Algorithm 4.1 can easily be applied by adding additional constraints. Both UPA and best power allocation (BPA) are used to initialize the proposed algorithm. Here BPA is the PA method used in [23] where the CCI is set to 0. Specifically with BPA, the CCI is treated as if it does not exist during optimization but is considered during the sum rate calculation.

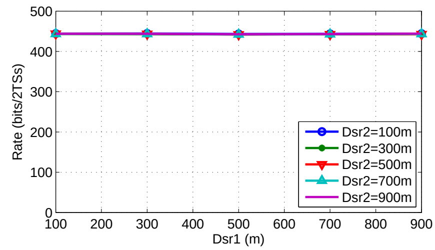
Impact of relay deployment on average sum rate is first studied and proper relay positions are selected for further experiments. Then RA and sum rate results are presented for one particular channel realization. The convergence of the proposed RA algorithm is illustrated for all three protocols. Finally, results are provided and discussed for the performance of those protocols averaged over many channel realizations.

4.3.1 System setup

As shown in Figure 4.1, a multi-cell OFDMA system with $N = 2$ coordinated cells and $K = 32$ available subcarriers is considered for illustration. Each cell contains one source, one relay and $U = 5$ destinations, which are uniformly distributed in a circular region of radius 50 m. In each cell, the



(a) 10 dBm

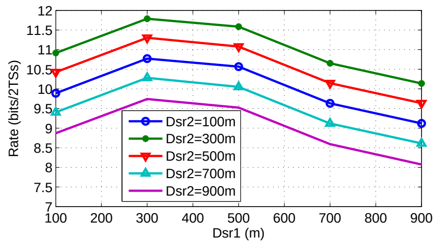


(b) 40 dBm

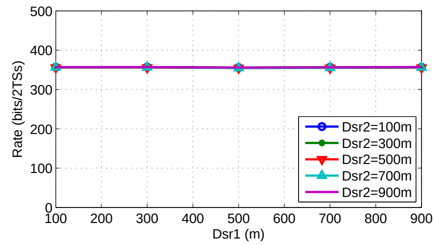
Figure 4.3 Impact of relay positions on average sum rate

relay is located along the line between the source and the center of the user region. The source-to-relay distances in cell 1 and cell 2 are denoted as D_{sr1} and D_{sr2} respectively. We set $P_T^n = P_T, \forall n$, $\sigma^2 = -65dBm$, $\epsilon_1 = 0.1$ and $\epsilon_2 = R_{ini}/100$. Here R_{ini} denotes the sum rate calculated with the initial RA results at the beginning of each iteration.

The channel impulse response (CIR) of each link is drawn randomly from an 8-tap delay line model, where each tap i has a circular complex gaussian distribution with zero mean and variance σ_i^2. We further assume $\frac{\sigma_i^2}{\sigma_{i+1}^2} = e^3$, meaning that the tap power decreases exponentially with a coefficient 3. Moreover $\sum_i \sigma_i^2 = d^{-2.5}$, meaning that the received power decreases exponentially with distance d and the propagation exponent equals 2.5. Finally, the CFR of each link is computed from its CIR using the K-point FFT.



(a) 10 dBm



(b) 50 dBm

Figure 4.4 Impact of relay positions on average sum rate with uncoordinated CCI

4.3.2 Impact of relay deployment on average sum rate

We have chosen a set of values for $Dsr1$ and $Dsr2$, ranging from 100 m to 900 m. For each possible combination of $Dsr1$ and $Dsr2$, 100 sets of random realizations of channels are generated and the average optimized sum rate is computed by using the proposed algorithm with the UPA initialization method. We then compare the obtained average sum rates of all combinations to illustrate how the deployment of relays influences the system performance.

As shown in Figure 4.3, in low SNR scenario when $P_T = 10dBm$, the average sum rate increases as the relay moves towards the middle between the source and the destination in each cell. This is because the achievable rate for a relay aided transmission is the minimum between the rate of the source-to-relay link and the rate of the relay-to-destination link. As the relays move towards the middle, both the source-to-relay channels and the relay-to-destination channels are in good conditions. Thus none of the good link is detrimental and the minimum rate of both links is very likely to be high. However, in high SNR scenario when $P_T = 40dBm$, the average sum rates for each possible combination of $Dsr1$ and $Dsr2$ are almost the same. As interpreted in [23], this is because the direct transmission is more desirable to maximize the sum rate than the relay aided transmission when the system sum power is very high.

We further study the impact of relay deployment considering uncoordinated out-of-cluster CCI. Specifically, we place one uncoordinated cell on the top of the 2 coordinated cells and another one at the bottom of the 2 coordinated cells. In each uncoordinated cell, the source-to-relay distance is set to 300 m and the power is uniformly allocated. Considering the interferences generated from two uncoordinated cells, the proposed resource allocation algorithm is used for the two coordinated cells. Simulation results are given in Figure 4.4 both when $P_T = 10dBm$ and when $P_T = 50dBm$. As in Figure 4.3, the same insights can be found in Figure 4.4 except that the system sum rate is decreased due to the existence of the out-of-cluster CCI, especially in high SNR scenario when the CCI is quite strong.

In the following simulations, we set $Dsr1 = 300m$ and $Dsr2 = 300m$.

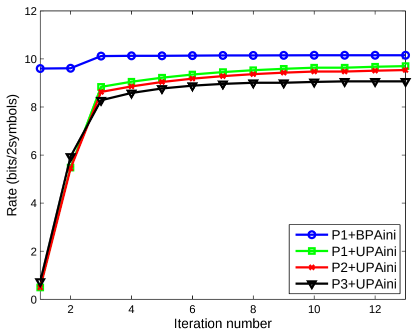
4.3.3 Results for a random realization of channels

4.3.3.1 Convergence of the proposed RA algorithm

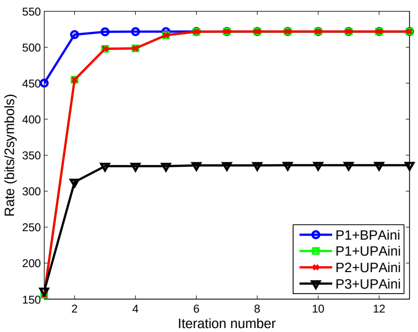
In order to illustrate the convergence of the proposed RA algorithm, we plot the sum rates calculated during iterations with the three protocols and two initialization methods when $P_T = 10dBm$ and $P_T = 40dBm$.

As shown in Figure 4.5, the sum rates for all three protocols using all initialization methods converge smoothly to the final rate without much fluctuation. After convergence, the sum rates are improved significantly except when $P_T = 10dBm$ and the BPA initialization method is used. When using protocol P1 and setting $P_T = 10dBm$, the converged sum rate with BPA initialization method is higher than that obtained with UPA initialization method. This is because the BPA is near-optimal in low SNR scenario when the CCI is small. Interestingly, in high SNR scenario when $P_T = 40dBm$, both BPA and UPA lead to the same sum rate after convergence.

Initialized by the UPA method, the optimized sum rate of P1 is much higher then that of P3 both when $P_T = 10dBm$ and when $P_T = 40dBm$. Thus the benefit of using the high spectrum efficiency (HSE) protocol is very high. When $P_T = 10dBm$, the optimized sum rate of P1 is slightly higher than that



(a) 10 dBm



(b) 40 dBm

Figure 4.5 Convergence for three protocols with UPA and BPA

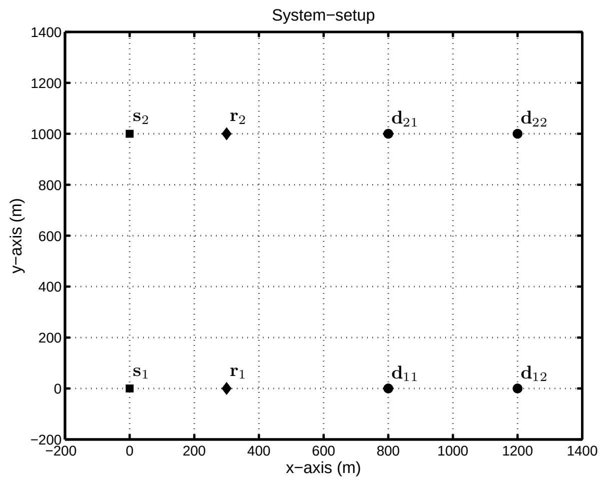


Figure 4.6 A random system setup

of P2. However, when $P_T = 40dBm$, P1 and P2 leads to the same optimized sum rate. Thus the benefit of using subcarrier pairing is limited. This can be understood as most of the subcarriers are in direct mode, for which pairing does not have any impact.

4.3.3.2 Benefits illustration of the HSE protocol

In order to further illustrate the benefit of using the HSE protocol, we now compare the RA results and sum rates of protocol P2 with another protocol P4. Here P4 is the same as P2 except that all sources keep silent during the second TS. A multi-cell OFDMA system with $N = 2$ coordinated cells and $K = 16$ available subcarriers is considered in this subsection. Each cell contains $U = 2$ destinations. Figure 4.6 shows the positions of the BSs, the relays and the destinations expressed in meters. Specifically, the coordinates of source 1 and source 2 are $(0, 0)$ and $(0, 1000)$ respectively. The coordinates of relay 1 and relay 2 are $(300, 0)$ and $(300, 1000)$ respectively. The coordinates of d_{11} and d_{12} are $(800, 0)$ and $(1200, 0)$ respectively. The coordinates of d_{21} and d_{22} are $(800, 1000)$ and $(1200, 1000)$ respectively. Finally, we set $P_T = 30dBm$.

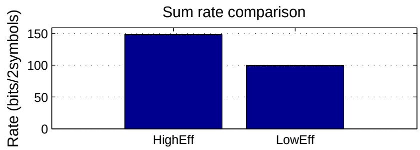
As shown in Figure 4.7 (e), (f) and (g), each cell appears to allocate the dominant part of its power to only a few subcarriers during each TS for both protocols. Once a cell assigns a significant part of its power to a subcarrier during a TS, the other cells tend to allocate only a small part of their power to this subcarrier during this TS, so as to reduce CCI. As shown in Figure 4.7 (b), (c) and (d), the optimized indicators for both P2 and P4 are the same. *However, it is very interesting to note that, compared to P4, P2 exploits the freedom of allocating power to BSs during the second TS, as shown in Figure 4.7 (f), and eventually leads to a much higher sum rate after optimization, as shown in Figure 4.7 (a).* Thus, the effectiveness of using the HSE protocol is illustrated.

Finally according to Figure 4.7 (c) and (d), no resources are allocated to the remote destinations (d_{11} in cell 1 and d_{21} in cell 2) for both protocols. Actually, this confirms the unfair nature of sum rate maximization. Therefore, fairness will be considered in our future work.

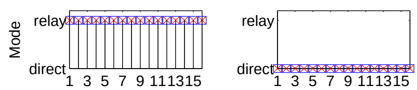
4.3.4 Results averaged over channel distribution

In order to illustrate the average performance of our proposed RA algorithm, 100 random realizations of channels are generated. Three benchmark algorithms (BA1, BA2 and BA3) are introduced. Here BA1 applies the RA algorithm proposed in [23] where CCI is always set to 0. Specifically with BA1, the CCI is treated as if it does not exist during optimization and is considered during the sum rate calculation. BA2 uses UPA together with our algorithm proposed for MSSAP. BA3 also uses UPA but with the transmission modes and destinations randomly selected for each subcarrier. We denote the distance between two parallel cells as *Dcell* and increase it from 200m to 5200m.

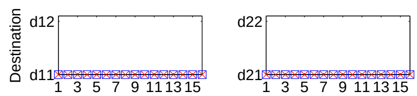
Let us fisrt compare the average performances for three protocols using the proposed algorithm. As shown in Figure 4.8, when $P_T = 10dbm$, the proposed protocol P1 performs slightly better than P2 and P3. When $P_T = 40dbm$, P1 performs similarly as P2 and much better than P3. This illustrates that the proposed protocol is more effective than other ones and moreover that the benefits of using the high efficiency protocol is much larger than that of using SP. Note that, although the benefit of using SP is acceptable, the complexity of the system with SP is $K!$ times higher than that of the system without SP. As the effect of using SP is well understood for the setup of this chapter, a similar benefit can be expected for related setups. In order not to enlarge the calculation time under consideration, we decide not to incorporate SP in the next two chapters. In the future, it would be interesting to check whether the benefit of using SP is similar in the other system setups.



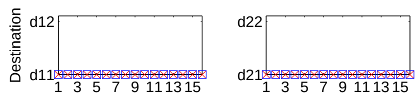
(a) Sum Rate Comparison



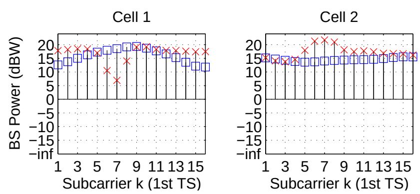
(b) Mode Selection



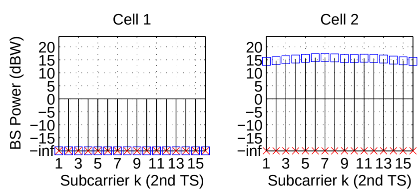
(c) Subcarrier Allocation at TS1



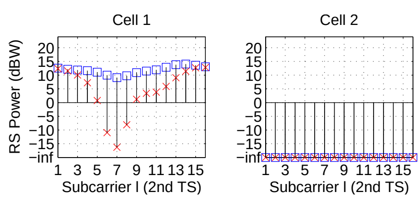
(d) Subcarrier Allocation at TS2



(e) Power Allocation for the source at TS1



(f) Power Allocation for the source at TS2

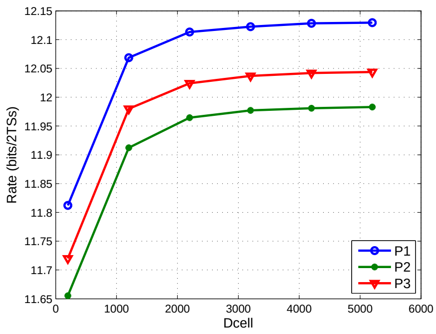


(g) Power Allocation for the relay

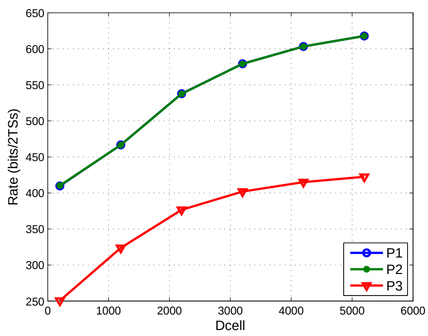
Figure 4.7 RA results for P2 and P4 when $P_t = 30dBmW$, where the blue squares and the red crosses indicate P2 and P4 respectively.

We then compare our proposed algorithm with the benchmark algorithms. Figure 4.9 reports the average sum rates for BA1, BA2, BA3 and the proposed algorithm, which is initialized by either UPA or BPA. As shown in Figure 4.9, for both initialization methods, the proposed algorithm (named as CA in the figure) results in similar sum rates after optimization. Regardless of the value of P_T, the proposed algorithm outperforms all three benchmark algorithms. The improvements are especially significant when $P_T = 40dbm$.

Note that, in low SNR scenario when $P_T = 10dbm$, the sum rates are similar regardless of the distance between two cells. This is because the CCI is already low even when two cells are close to each other. However in high SNR scenario when $P_T = 40dbm$, the sum rates of all algorithms increase as $Dcell$ increases. This is because the CCI is quite high when two cells are close to each other and it decreases as they move away from each other. Note when $Dcell = 5200m$, meaning two cells are quite far from each other, BA 1 performs similarly to our proposed algorithms. This is because BA 1 is near-optimal when the CCI is small.

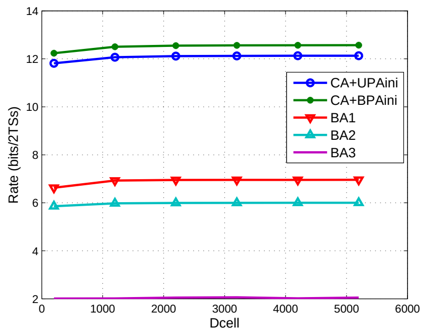


(a) 10 dBm

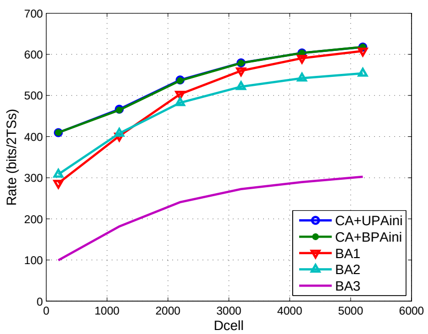


(b) 40 dBm

Figure 4.8 Comparison of average sum rates for 3 protocols



(a) 10 dBm



(b) 40 dBm

Figure 4.9 Comparison of average sum rates for 4 algorithms

5 Weighted Sum of Per Cell Min-Rate Maximization for Multi-Cell Downlink OFDMA with Opportunistic DF Relaying

In this Chapter, we further consider per-cell user fairness in the same system as that of Chapter 4 except that subcarrier pairing is not considered here. Recently in [62], the maximization of the weighted sum of per cell min-rate (WSMR) was considered for multi-cell OFDMA systems without relaying. The corresponding optimum RA leads to per-cell maximum fairness and different priorities in different cells. The authors only illustrate the per-cell fairness by experiments without mathematical proof. Taking WSMR into consideration, the contributions of this Chapter are listed below:

- We formulate the WSMR maximized joint RA problem in multi-cell OFDMA downlink systems aided by a RS in each cell. To start with, the DF protocol with OR and HSE is considered without signal combining. It will be shown that the system performance can be enhanced largely by using OR and the HSE protocol. Note that, when modeling the inter-cell CCI of a subcarrier in a selected cell, instead of using an additional integer variable to indicate whether a node in an interfering cell transmits data on this subcarrier or not, we use the corresponding power value to do it. This choice is motivated to simplify the system sum rate expression and facilitate the algorithm design.

- We unveil and prove the per-cell maximum fairness feature of the formulated RA problem. Specifically, with any feasible mode selection and subcarrier assignment (MSSA) result, the optimum power allocation results in the same transmission rates for all users in the same cell.

- We propose an iterative RA algorithm to optimize the MSSA and the power allocation (PA) alternatively with the WSMR keeping increasing. Each iteration is composed of the MSSA stage and the PA stage. During the MSSA stage, the original problem is decoupled into mixed integer linear programs (MILPs), which can be solved by typical MILP solvers. To solve the MILPs more efficiently in polynomial time, we further propose the randomized rounding based MSSA (RR-MSSA) algorithm and the direct rounding based MSSA (DR-MSSA) algorithm at this stage. During the PA stage, an algorithm based on single condensation and geometric programming PA (SCGP) is used to optimize PA with the tentative MSSA results. The convergence and the per-cell user fairness of the proposed RA algorithm are proven.

The rest of this Chapter is organized as follows. First, the considered system model is presented. Then, the RA problem is formulated and its fairness property is discussed. After that, the proposed algorithms are described. Finally, the effectiveness and convergence of the proposed RA algorithms as well as the benefits of using OR and the HSE protocol are illustrated by numerical experiments.

5.1 Weighted sum of per cell minimum rate (WSMR)

In this chapter, the considered system is a special case of that in chapter 4. Here subcarrier k in TS 1 is always paired with the same subcarrier k in TS

2. Considering the transmission procedure described in section 4.1.1, the sum rate achievable for the direct mode subcarrier k is given by

$$R^k_{un,1} = \ln\left(1 + \Upsilon^k_{\mathrm{d}_{un},t_1}\right) + \ln\left(1 + \Upsilon^k_{\mathrm{d}_{un},t_2}\right) \tag{5.1}$$

in nats/two-TSs, where

$$\Upsilon^k_{\mathrm{d}_{un},t_1} = \frac{P^k_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{d}_{un}}}{f^k_{\mathrm{d}_{un},t_1}} \tag{5.2}$$

denotes the SINR associated with the decoding of $x^k_{\mathrm{s}_n,t_1}$ from $y^k_{\mathrm{d}_{un},t_1}$ at d_{un} during the first TS. $f^k_{\mathrm{d}_{un},t_1} = \sigma^2 + \sum_{n'=1,n'\neq n}^{N} P^k_{\mathrm{s}_{n'},t_1} G^k_{\mathrm{s}_{n'},\mathrm{d}_{un}}$ denotes the sum power of the AWGN and the interference received by d_{un} at subcarrier k during the first TS. $G^k_{\mathrm{s}_{n'},\mathrm{d}_{un}} = |h^k_{\mathrm{s}_{n'},\mathrm{d}_{un}}|^2$ denotes the channel gain of subcarrier k from $\mathrm{s}_{n'}$ to d_{un}.

$$\Upsilon^k_{\mathrm{d}_{un},t_2} = \frac{P^k_{\mathrm{s}_n,t_2} G^k_{\mathrm{s}_n,\mathrm{d}_{un}}}{f^k_{\mathrm{d}_{un},t_2}} \tag{5.3}$$

denotes the SINR associated with the decoding of $x^k_{\mathrm{s}_n,t_2}$ from $y^k_{\mathrm{d}_{un},t_2}$ at d_{un} during the second TS. $f^k_{\mathrm{d}_{un},t_2} = \sigma^2 + \sum_{n'=1,n'\neq n}^{N} P^k_{\mathrm{s}_{n'},t_2} G^k_{\mathrm{s}_{n'},\mathrm{d}_{un}} + \sum_{n'=1,n'\neq n}^{N} P^k_{\mathrm{r}_{n'}} G^k_{\mathrm{r}_{n'},\mathrm{d}_{un}}$ denotes the sum power of the AWGN and the interference received by d_{un} at subcarrier k during the second TS. $G^k_{\mathrm{r}_n,\mathrm{d}_{un}} = |h^k_{\mathrm{r}_n,\mathrm{d}_{un}}|^2$ denotes the channel gain of subcarrier k from r_n to d_{un}.

Also the maximum achievable rate for subcarrier k when allocated to d_{un} in relay aided mode is given by [3]

$$R^k_{un,2} = \min\left(\ln\left(1 + \Gamma^k_{\mathrm{r}_n}\right), \ln\left(1 + \Gamma^k_{\mathrm{d}_{un},t_2}\right)\right) \tag{5.4}$$

in nats/two-TSs, where

$$\Gamma^k_{\mathrm{r}_n} = \frac{P^k_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{r}_n}}{f^k_{\mathrm{r}_n}} \tag{5.5}$$

denotes the signal-to-interference-plus-noise ratio (SINR) associated with decoding $x^k_{\mathrm{s}_n,t_1}$ from $y^k_{\mathrm{r}_n}$ at r_n during the first TS, $f^k_{\mathrm{r}_n} = \sigma^2 + \sum_{n',n'\neq n} P^k_{\mathrm{s}_{n'},t_1} G^k_{\mathrm{s}_{n'},\mathrm{r}_n}$ denotes the sum power of the AWGN and the interference received by r_n at subcarrier k during the first TS. $G^k_{\mathrm{s}_n,\mathrm{r}_n} = |h^k_{\mathrm{s}_n,\mathrm{r}_n}|^2$ denotes the channel gain of subcarrier k from s_n to r_n.

$$\Gamma^k_{\mathrm{d}_{un},t_2} = \frac{P^k_{\mathrm{r}_n} G^k_{\mathrm{r}_n,\mathrm{d}_{un}}}{f^k_{\mathrm{d}_{un},t_2}} \tag{5.6}$$

denotes the SINR associated with decoding $x^k_{\mathrm{r}_n}$, which equals $x^k_{\mathrm{s}_n,t_1}$, from $y^k_{\mathrm{d}_{un},\mathrm{r}_n}$ at d_{un} during the second TS.

Thus the WSMR of the considered system is denoted as:

$$\begin{aligned} R\left(\mathbf{P},\mathbf{A},\mathbf{B}\right) = \sum_{n=1}^{N} \omega_n \min_{u\in\mathcal{U}_n} \Big(\sum_{k=1}^{K} a^k_{un} R^k_{un,1}\left(\mathbf{P}^k\right) + \\ \sum_{k=1}^{K} b^k_{un} R^k_{un,2}\left(\mathbf{P}^k\right) \Big), \end{aligned} \tag{5.7}$$

where $\mathbf{P}^k = [[\mathbf{P}^k_1]^{\mathrm{T}}, ..., [\mathbf{P}^k_N]^{\mathrm{T}}]^{\mathrm{T}}$, $\mathbf{P}^k_n = [P^k_{\mathrm{s}_n,t_1}, P^k_{\mathrm{s}_n,t_2}, P^k_{\mathrm{r}_n}]^{\mathrm{T}}$, $\mathbf{A} = \{a^k_{un}\}$, $\mathbf{B} = \{b^k_{un}\}$. ω_n denotes the weight of the minimal rate in cell n, $\mathcal{U}_n$ denotes the MS set in cell n.

5.2 Problem formulation and its property

5.2.1 Problem formulation

We consider maximizing the WSMR under per cell total power constraints. The optimization variables are the transmission mode for each subcarrier, the subcarrier assignments and the power allocations at the sources and the relays. According to the system model, the considered RA problem can be formulated as:

$$
\begin{aligned}
&\max_{\mathbf{P},\mathbf{A},\mathbf{B}} R\left(\mathbf{P},\mathbf{A},\mathbf{B}\right)\\
&s.t.\\
&C5.1:\ \left(\sum_{u=1}^{U} a_{un}^{k}+\sum_{u=1}^{U} b_{un}^{k}\right)\leq 1,\ \forall n,k,\\
&C5.2:\ a_{un}^{k}\in\{0,1\}, b_{un}^{k}\in\{0,1\},\ \forall u,n,k,\\
&C5.3:\ \sum_{k=1}^{K} P_{\mathrm{s}_n,t_1}^{k}+\sum_{k=1}^{K} P_{\mathrm{s}_n,t_2}^{k}+\sum_{k=1}^{K} P_{\mathrm{r}_n}^{k}\leq P_{t,n},\ \forall n,\\
&C5.4:\ P_{\mathrm{s}_n,t_1}^{k}\geq 0, P_{\mathrm{s}_n,t_2}^{k}\geq 0, P_{\mathrm{r}_n}^{k}\geq 0,\ \forall n,k,\\
&C5.5:\ P_{\mathrm{s}_n,t_1}^{k}\left(1-\sum_{u=1}^{U} a_{un}^{k}-\sum_{u=1}^{U} b_{un}^{k}\right)=0,\ \forall n,k,\\
&C5.6:\ P_{\mathrm{s}_n,t_2}^{k}\left(1-\sum_{u=1}^{U} a_{un}^{k}\right)=0,\ \forall n,k,\\
&C5.7:\ P_{\mathrm{r}_n}^{k}\left(1-\sum_{u=1}^{U} b_{un}^{k}\right)=0,\ \forall n,k.
\end{aligned}
\tag{5.8}
$$

Here, $C5.1$ and $C5.2$ ensure that each subcarrier k during the first TS can select only one mode (direct/relay-aided) to transmit data towards only one destination d_{un}. Moreover, $C5.3$ and $C5.4$ ensure that the consumed sum power for each cell is less than its available sum power. This type of power constraints gives an upper bound of the system performance. In practice, each node (source, relay) in each cell will have an individual power constraint. Finally, $C5.5$, $C5.6$ and $C5.7$ guarantee that no data is transmitted on an unused subcarrier and subcarrier k is used by only one node (either the source or the relay) in each cell during the second TS.

5.2.2 Fairness property

To investigate the fairness property of the formulated problem, let us denote by R_{un} the sum rate of each user u in each cell n, $\mathcal{U}_n$ the set of users in cell n, $\tilde{\mathcal{U}}_n \subset \mathcal{U}_n$ the set of users with the minimum rate in cell n, $\tilde{\mathcal{V}}_n$ the set of other users (the complimentary set of $\tilde{\mathcal{U}}_n$) and $\mathcal{S}_{\tilde{u}n}$ ($\mathcal{S}_{\tilde{v}n}$) the set of subcarriers allocated to MS $d_{\tilde{u}n}$ ($d_{\tilde{v}n}$), where $\tilde{u} \in \tilde{\mathcal{U}}_n$ ($\tilde{v} \in \tilde{\mathcal{V}}_n$). Note that $\tilde{\mathcal{V}}_n \neq \emptyset$ means that

$$\min_{\tilde{v}\in\tilde{\mathcal{V}}_n} R_{\tilde{v}n} > \min_{\tilde{u}\in\tilde{\mathcal{U}}_n} R_{\tilde{u}n} = \min_{u\in\mathcal{U}_n} R_{un}. \tag{5.9}$$

Then for each subcarrier k, there exists three states considering which one of the two user sets does the allocated user belong to in each cell. Specifically, state 1 corresponds to $k \in \mathcal{S}_{\tilde{u}n}, \forall n$, meaning that in each cell, subcarrier k is allocated to a user with the minimum rate of that cell. State 2 corresponds to $k \in \mathcal{S}_{\tilde{v}n}, \forall n$, meaning that in each cell, subcarrier k is allocated to a user with a rate larger than the minimum rate of that cell. State 3 corresponds to $\exists\ n_1, n_2$ such that $k \in \mathcal{S}_{\tilde{u}n_1}$ and $k \in \mathcal{S}_{\tilde{v}n_2}$, meaning other possible allocation results for subcarrier k in all cells. Note when all subcarriers are in state 1, per-cell user fairness is reached, meaning all users inside the same cell obtain the same rate.

We first consider the following lemma, which will be used later in this section.

Lemma 5.1. *Given any feasible values of variables* $\mathbf{A}$, $\mathbf{B}$ *and* $\mathbf{P}$, *if there exists n and k where* $\tilde{\mathcal{V}}_n \neq \emptyset$ *and* $k \in \mathcal{S}_{\tilde{v}n}$ *(meaning that there exists subcarrier k in state 2 or state 3), then part of the feasible powers at this subcarrier k in each cell (*$\mathbf{P}_n^k = \{P^k_{\mathrm{s}_n,t_1}, P^k_{\mathrm{s}_n,t_2}, P^k_{\mathrm{r}_n}\}$ *and* $\mathbf{P}^k_{n'}, \forall\ n' \neq n$*) can be saved while keeping the WSMR of the system non-decreased.*

Proof. Considering the subcarrier k in cell n where $\tilde{\mathcal{V}}_n \neq \emptyset$ and $k \in \mathcal{S}_{\tilde{v}n}$, we now introduce two sets of cells $\mathcal{N}_1^k = \{n|k \in \mathcal{S}_{\tilde{u}n}\}$ and $\mathcal{N}_2^k = \{n|\tilde{\mathcal{V}}_n \neq \emptyset, k \in \mathcal{S}_{\tilde{v}n}\}$. Note when k is in state 2, $\mathcal{N}_1^k = \emptyset$ and $\mathcal{N}_2^k \neq \emptyset$. Also when k is in state 3, $\mathcal{N}_1^k \neq \emptyset$ and $\mathcal{N}_2^k \neq \emptyset$.

Let us consider the case when k is in state 3. We define

$$\tilde{\mathbf{P}}_n^k(s^k) = \begin{cases} s^k * \mathbf{P}_n^k & n \in \mathcal{N}_2^k \\ \mathbf{P}_n^k & n \in \mathcal{N}_1^k \end{cases} \tag{5.10}$$

where $s^k \in (0,1]$, $\tilde{R}^k_{un}\left(\tilde{\mathbf{P}}^k(s^k)\right) = a^k_{un}R^k_{un,1}\left(\tilde{\mathbf{P}}^k(s^k)\right) + b^k_{un}R^k_{un,2}\left(\tilde{\mathbf{P}}^k(s^k)\right)$ and $\tilde{\mathbf{P}}^k(s^k) = [[\tilde{\mathbf{P}}^k_1(s^k)]^{\mathrm{T}}, ..., [\tilde{\mathbf{P}}^k_N(s^k)]^{\mathrm{T}}]^{\mathrm{T}}$. Please note that s^k is the same for all cells.

$\forall\ n \in \mathcal{N}_2^k$, $\Upsilon^k_{\mathrm{d}_{\tilde{v}n},t_1}$ can be denoted as

$$\frac{s^k P^k_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{d}_{\tilde{v}n}}}{\sigma^2 + \sum\limits_{n'\in\mathcal{N}_1^k} P^k_{\mathrm{s}_{n'},t_1} G^k_{\mathrm{s}_{n'},\mathrm{d}_{\tilde{v}n}} + \sum\limits_{n'\neq n, n'\in\mathcal{N}_2^k} s^k P^k_{\mathrm{s}_{n'},t_1} G^k_{\mathrm{s}_{n'},\mathrm{d}_{\tilde{v}n}}}, \tag{5.11}$$

which can be further written as

$$\frac{P^k_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{d}_{\tilde{v}n}}}{\frac{\sigma^2}{s^k} + \sum\limits_{n' \in \mathcal{N}_1^k} \frac{P^k_{\mathrm{s}_{n'},t_1} G^k_{\mathrm{s}_{n'},\mathrm{d}_{\tilde{v}n}}}{s^k} + \sum\limits_{n' \neq n, n' \in \mathcal{N}_2^k} P^k_{\mathrm{s}_{n'},t_1} G^k_{\mathrm{s}_{n'},\mathrm{d}_{\tilde{v}n}}}. \tag{5.12}$$

It is obvious that $\Upsilon^k_{\mathrm{d}_{\tilde{v}n},t_1}$ is an increasing function of s^k. Similarly, $\Upsilon^k_{\mathrm{d}_{\tilde{v}n},t_2}$, $\Gamma^k_{\mathrm{r}_n}$ and $\Gamma^k_{\mathrm{d}_{\tilde{v}n},t_2}$ are also increasing functions of s^k. Thus $\tilde{R}^k_{\tilde{v}n}\left(\tilde{\mathbf{P}}^k(s^k)\right)$ is an increasing function of s^k.

$\forall\ n \in \mathcal{N}_1^k$, $\Upsilon^k_{\mathrm{d}_{\tilde{u}n},t_1}$ can be denoted as

$$\frac{P^k_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{d}_{\tilde{u}n}}}{\sigma^2 + \sum\limits_{n' \neq n, n' \in \mathcal{N}_1^k} P^k_{\mathrm{s}_{n'},t_1} G^k_{\mathrm{s}_{n'},\mathrm{d}_{\tilde{u}n}} + \sum\limits_{n' \in \mathcal{N}_2^k} s^k P^k_{\mathrm{s}_{n'},t_1} G^k_{\mathrm{s}_{n'},\mathrm{d}_{\tilde{u}n}}}. \tag{5.13}$$

It is obvious that $\Upsilon^k_{\mathrm{d}_{\tilde{u}n},t_1}$ is an decreasing function of s^k. Similarly, $\Upsilon^k_{\mathrm{d}_{\tilde{u}n},t_2}$, $\Gamma^k_{\mathrm{r}_n}$ and $\Gamma^k_{\mathrm{d}_{\tilde{u}n},t_2}$ are also decreasing functions of s^k. Thus $\tilde{R}^k_{\tilde{u}n}\left(\tilde{\mathbf{P}}^k(s^k)\right)$ is a decreasing function of s^k.

Note that $\tilde{\mathbf{P}}^k(s^k) \leq \tilde{\mathbf{P}}^k(1) = \mathbf{P}^k$, where the inequality should be understood as element wise. Thus $\exists\ s_0^k \in (0,1), \epsilon_n^k > 0, \eta^k_{n'} > 0$, such that $\forall\ n \in \mathcal{N}_2^k, \tilde{R}^k_{\tilde{v}n}\left(\tilde{\mathbf{P}}^k(s_0^k)\right) = \tilde{R}^k_{\tilde{v}n}\left(\tilde{\mathbf{P}}^k(1)\right) - \epsilon_n^k = R^k_{\tilde{v}n}\left(\mathbf{P}^k\right) - \epsilon_n^k, \tilde{R}_{\tilde{v}n} = R_{\tilde{v}n} - \epsilon_n^k > \min\limits_{u \in \mathcal{U}_n} R_{un}$ and $\forall\ n' \in \mathcal{N}_1^k, \tilde{R}^k_{\tilde{u}n'}\left(\tilde{\mathbf{P}}^k(s_0^k)\right) = R^k_{\tilde{u}n'}\left(\tilde{\mathbf{P}}^k(1)\right) + \eta^k_{n'} = R^k_{\tilde{u}n'}\left(\mathbf{P}^k\right) + \eta^k_{n'}, \tilde{R}_{\tilde{u}n'} = R_{\tilde{u}n'} + \eta^k_{n'} > \min\limits_{u \in \mathcal{U}_{n'}} R_{un'}$. Note the WSMR of the system is non-decreased, while the power $(1 - s_0^k)\mathbf{P}_n^k$ is saved from subcarrier k in cell $n \in \mathcal{N}_2^k$.

Let us introduce $R_n^{min} = \min\limits_{u \in \mathcal{U}_n} R_{un}, \forall n$. After substituting $\mathbf{P}_n^{\prime k}$ with $\tilde{\mathbf{P}}_n^k(s_0^k)$, we now have $\forall n \in \mathcal{N}_2^k, k \in \mathcal{S}_{\tilde{v}n}, \tilde{R}_{\tilde{v}n} > R_n^{min}$ and $\forall n \in \mathcal{N}_1^k, k \in \mathcal{S}_{\tilde{u}n}, \tilde{R}_{\tilde{u}n} > R_n^{min}$. Let us define $\check{\mathbf{P}}_n^k(c^k) = c^k * \mathbf{P}_n^{\prime k}$ where $c^k \in (0,1)$. Thus $\Upsilon^k_{\mathrm{d}_{un},t_1}$ can be denoted as

$$\frac{P^{\prime k}_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{d}_{un}}}{\frac{\sigma^2}{c^k} + \sum\limits_{n' \neq n} P^{\prime k}_{\mathrm{s}_{n'},t_1} G^k_{\mathrm{s}_{n'},\mathrm{d}_{un}}}. \tag{5.14}$$

It is obvious that $\Upsilon^k_{\mathrm{d}_{un},t_1}$ is an increasing function of c^k. Similarly, $\Upsilon^k_{\mathrm{d}_{un},t_2}$, $\Gamma^k_{\mathrm{r}_n}$ and $\Gamma^k_{\mathrm{d}_{un},t_2}$ are also increasing functions of c^k. Thus $\check{R}^k_{un}\left(\check{\mathbf{P}}^k(c^k)\right)$ is an increasing function of c^k. There exists a $c_0^k \in (0,1), \alpha_n^k > 0$ such that $\forall\ n, \check{R}_{un}^k\left(\check{\mathbf{P}}^k(c_0^k)\right) = \check{R}_{un}^k\left(\check{\mathbf{P}}^k(1)\right) - \alpha_n^k = R_{un}^k\left(\mathbf{P}'^k\right) - \alpha_n^k = \tilde{R}_{un}^k\left(\tilde{\mathbf{P}}^k(s_0^k)\right) - \alpha_n^k, \check{R}_{un} = \tilde{R}_{un} - \alpha_n^k > R_n^{min}$. The power $(1-c_0^k)\tilde{\mathbf{P}}_n^k(s_0^k)$ can be saved from subcarrier k in each cell n while keeping the WSMR of the system non-decreased.

We now consider the case when k is in state 2, meaning $n \in \mathcal{N}_2^k, \forall n$. By using the previous derivations, we can still find $s_1^k \in (0,1)$, such that the power $(1-s_1^k)\mathbf{P}_n^k$ can be saved from subcarrier k in each cell n while keeping the WSMR of the system non-decreased. This concludes the proof of Lemma 1. □

We now state the main result of this section. Based on Lemma 1, the following theorem is proposed, which sheds light on the fairness property of the formulated problem.

Theorem 5.1. *With any feasible values $\mathbf{A} = \mathbf{A}_0$ and $\mathbf{B} = \mathbf{B}_0$, the optimum power allocation $\mathbf{P}_{opt}$ of the formulated problem results in the same transmission rates for all users in the same cell.*

Proof. When $\mathbf{A} = \mathbf{A}_0$ and $\mathbf{B} = \mathbf{B}_0$, we assume at the optimum, $\exists\ n_0$ where $\tilde{\mathcal{V}}_{n_0} \neq \emptyset$ and a subcarrier $k_0 \in \mathcal{S}_{\tilde{v}n}$ in state 2 or state 3. Then, $\forall\ n, k \in \mathcal{S}_{\tilde{u}n}$ where $\tilde{u} \in \tilde{\mathcal{U}}_n$, subcarrier k can be in either state 3 or state 1.

When subcarrier k is in state 3, as discussed in the proof of Lemma 1, we can still improve $R_{\tilde{u}n}^k$ by decreasing the interfering powers of subcarrier k in cell n', while keeping the minimum user rate of cell n' non-decreased.

When subcarrier k is in state 1, as at the optimum $\exists\ k_0 \neq k$ that is in state 2 or state 3, we can save power from $\mathbf{P}_n^{k_0}$ in each cell n while keeping the WSMR of the system non-decreased. Let us denote the saved power in each cell n as ΔP_n and define $\hat{\mathbf{P}}_n^k(t^k) = t^k * \mathbf{P}_n^k$ where $t^k > 1$. Thus $\Upsilon_{\mathrm{d}_{\tilde{u}n},t_1}^k$ can be denoted as

$$\frac{P_{\mathrm{s}_n,t_1}^k G_{\mathrm{s}_n,\mathrm{d}_{\tilde{u}n}}^k}{\frac{\sigma^2}{t^k} + \sum\limits_{n' \neq n, n' \in \mathcal{N}_1^k} P_{\mathrm{s}_{n'},t_1}^k G_{\mathrm{s}_{n'},\mathrm{d}_{\tilde{u}n}}^k}. \tag{5.15}$$

It is obvious that $\Upsilon_{\mathrm{d}_{\tilde{u}n},t_1}^k$ is an increasing function of t^k. Similarly, $\Upsilon_{\mathrm{d}_{\tilde{u}n},t_2}^k$, $\Gamma_{\mathrm{r}_n}^k$ and $\Gamma_{\mathrm{d}_{\tilde{u}n},t_2}^k$ are also increasing functions of t^k. Thus for the subcarrier k in state 1, $\tilde{R}_{\tilde{u}n}^k\left(\hat{\mathbf{P}}^k(t^k)\right)$ is an increasing function of t^k. Thus, there exists a $t_0^k > 1$ such that $\tilde{R}_{\tilde{u}n}^k\left(\hat{\mathbf{P}}^k(t_0^k)\right) > R_{\tilde{u}n}^k$ and $\sum_{k \in \hat{\mathcal{K}}}(t_0^k - 1)\|\mathbf{P}_n^k\| \leq \Delta P_n$. Here $\hat{\mathcal{K}}$ denotes the set of subcarriers in state 1.

Therefore, $\forall\ k \in \mathcal{S}_{\tilde{u}n}$ in cell n, $R^k_{\tilde{u}n}$ can still be improved. Thus the minimum user rate of each cell can still be improved by adjusting power allocation. Specifically we can find matrices $\mathbf{S}, \mathbf{C}, \mathbf{T}, \mathbf{P}_1 = \mathbf{SCTP}_{opt}$, according to the previous discussions, such that $R(\mathbf{P}_1, \mathbf{A}_0, \mathbf{B}_0) > R(\mathbf{P}_{opt}, \mathbf{A}_0, \mathbf{B}_0)$. Here each element of $\mathbf{S}, \mathbf{C}$ and $\mathbf{T}$ takes value from $(0, 1]$, $(0, 1]$ and $[1, \infty)$ respectively. This obviously contradicts the optimum assumption and concludes the proof of Theorem 5.1. $\square$

5.3 Algorithm development

In this section, we will propose an iterative RA algorithm, which obtains a local optimum of the formulated problem after convergence. The proposed algorithm is based on the coordinate ascent approach, which divides the whole set of optimization variables into several sets of variables and iteratively optimizes each set of variables with other sets of variables fixed.

Specifically, variables $\{\mathbf{P}, \mathbf{A}, \mathbf{B}\}$ are divided into two sets: the integral variable set $\{\mathbf{A}, \mathbf{B}\}$ and the continuous variable set $\{\mathbf{P}\}$. During each iteration, the MSSA stage is first carried out followed by the PA stage. Let us introduce integer m to indicate the iteration number. At the MSSA stage of iteration m, the integral variables $\{\mathbf{A}, \mathbf{B}\}$ are optimized with $\mathbf{P} = \mathbf{P}^{m-1}$. While at the PA stage of iteration m, the continuous variables $\{\mathbf{P}\}$ are optimized with $\mathbf{A} = \mathbf{A}^m$ and $\mathbf{B} = \mathbf{B}^m$. Here a variable with the superscript m denotes the value obtained at the end of iteration m. In the following, we first design algorithms for both stages, based on which the iterative RA algorithm will then be stated. Finally, several characteristics of the proposed RA algorithm will be discussed.

5.3.1 MSSA optimization

In this subsection, the MSSA stage for iteration m is considered. After setting $\mathbf{P}$ to $\mathbf{P}^{m-1}$,the formulated problem can be rewritten as:

$$\begin{aligned} &\max_{\mathbf{A},\mathbf{B}} R\left(\mathbf{P}^{m-1}, \mathbf{A}, \mathbf{B}\right) \\ &s.t.\ C5.1, C5.2, C5.5, C5.6, C5.7. \end{aligned} \tag{5.16}$$

Note that each cell has independent constraints for its local integral variables. Therefore problem (5.16) can be decoupled into N subproblems. As for subproblem n_0, the minimum rate of all users in the cell n_0 is maximized subject to the local constraints. Specifically, subproblem n_0 is formulated as:

$$\begin{aligned}
&\max_{\mathbf{A}_{n_0},\mathbf{B}_{n_0}} \ R_{n_0}\left(\mathbf{P}^{m-1},\mathbf{A}_{n_0},\mathbf{B}_{n_0}\right)\\
&s.t.\\
&\mathrm{C5.1}':\ \sum_{u=1}^{U} a_{un_0}^k + \sum_{u=1}^{U} b_{un_0}^k \leq 1,\ \forall k,\\
&\mathrm{C5.2}':\ a_{un_0}^k \in \{0,1\}, b_{un_0}^k \in \{0,1\},\ \forall u,k,\\
&\mathrm{C5.5}':\ P_{\mathrm{s}_{n_0},t_1}^{k,m-1}\left(1-\sum_{u=1}^{U} a_{un_0}^k - \sum_{u=1}^{U} b_{un_0}^k\right) = 0,\ \forall k,\\
&\mathrm{C5.6}':\ P_{\mathrm{s}_{n_0},t_2}^{k,m-1}\left(1-\sum_{u=1}^{U} a_{un_0}^k\right) = 0,\ \forall k,\\
&\mathrm{C5.7}':\ P_{\mathrm{r}_{n_0}}^{k,m-1}\left(1-\sum_{u=1}^{U} b_{un_0}^k\right) = 0,\ \forall k.
\end{aligned} \tag{5.17}$$

where $\mathbf{A}_{n_0} = \{a_{un}^k | n = n_0\}$, $\mathbf{B}_{n_0} = \{b_{un}^k | n = n_0\}$ and

$$\begin{aligned}
R_{n_0} = \min_{u\in\mathcal{U}_{n_0}} \Big(& \sum_{k=1}^{K} a_{un_0}^k R_{un_0,1}^k \left(\mathbf{P}^{m-1}\right) + \\
& \sum_{k=1}^{K} b_{un_0}^k R_{un_0,2}^k \left(\mathbf{P}^{m-1}\right) \Big).
\end{aligned}$$

By introducing a slack variable ξ_{n_0}, problem (5.17) can be reformulated as:

$$\begin{aligned}
&\max_{\mathbf{A}_{n_0},\mathbf{B}_{n_0},\xi_{n_0}} \ \xi_{n_0}\\
&s.t.\ C5.1', C5.2', C5.5', C5.6', C5.7' \text{and}\\
&C5.3':\ \sum_{k=1}^{K}\left(a_{un_0}^k R_{un_0,1}^k + b_{un_0}^k R_{un_0,2}^k\right) \geq \xi_{n_0},\ \forall u \in \mathcal{U}_n.
\end{aligned} \tag{5.18}$$

Note that (5.18) is actually a mixed integer linear program (MILP). Hence it can be solved by means of typical MILP solvers, like TOMLAB [74]. In order to solve it much faster in polynomial time, we now propose a randomized rounding based MSSA (RR-MSSA) algorithm and a direct rounding based

MSSA (DR-MSSA) algorithm. Specifically, both algorithms consist of three steps: the relaxation step, the rounding step and the decision step.

During the relaxation step, we first relax constraint $C5.2'$ as $C5.2'' : a^k_{un_0} \in [0,1], b^k_{un_0} \in [0,1], \ \forall u, k$. Then the MILP is relaxed as a linear program (LP), which can easily be solved by any standard LP algorithm in polynomial time (time that is bounded by a fixed polynomial of the length of the input). Note as the feasible set of the LP is larger than that of the MILP, the optimal objective function value of the LP is an upper bound of that of the MILP.

During the rounding step, we round the fractional optimal solution $(\{\mathbf{A}^*_{n_0}, \mathbf{B}^*_{n_0}\})$ of the LP into an integer solution $(\{\mathbf{A}'_{n_0}, \mathbf{B}'_{n_0}\})$. Specifically, each value of the fractional solution is viewed as the probability that this value should be rounded to 1. Then a direct rounding (DR) algorithm is $\forall k$,

$$a^{k,\prime}_{un_0} = \begin{cases} 1 & a^{k,*}_{un_0} = \max\limits_u \{a^{k,*}_{un_0}, b^{k,*}_{un_0}\} \\ 0 & a^{k,*}_{un_0} \neq \max\limits_u \{a^{k,*}_{un_0}, b^{k,*}_{un_0}\}, \end{cases}$$

$$b^{k,\prime}_{un_0} = \begin{cases} 1 & b^{k,*}_{un_0} = \max\limits_u \{a^{k,*}_{un_0}, b^{k,*}_{un_0}\} \\ 0 & b^{k,*}_{un_0} \neq \max\limits_u \{a^{k,*}_{un_0}, b^{k,*}_{un_0}\}. \end{cases}$$

After this rounding, the R_{n_0} calculated with $\{\mathbf{A}'_{n_0}, \mathbf{B}'_{n_0}\}$ is decreased, compared to that calculated with $\{\mathbf{A}^*_{n_0}, \mathbf{B}^*_{n_0}\}$.

In order to make $R_{n_0}\left(\mathbf{P}^{m-1}, \mathbf{A}'_{n_0}, \mathbf{B}'_{n_0}\right)$ closer to $R_{n_0}\left(\mathbf{P}^{m-1}, \mathbf{A}^*_{n_0}, \mathbf{B}^*_{n_0}\right)$, we now use the randomized rounding (RR) algorithm, where N_s samples of binary values are generated for variables $\{\mathbf{A}_{n_0}, \mathbf{B}_{n_0}\}$ by a randomized process with probability $(\{\mathbf{A}^*_{n_0}, \mathbf{B}^*_{n_0}\})$. Each sample is denoted as $\{\mathbf{A}^{(s)}_{n_0}, \mathbf{B}^{(s)}_{n_0}\}$. Finally the feasible sample with the maximum R_{n_0} is assigned to $\{\mathbf{A}'_{n_0}, \mathbf{B}'_{n_0}\}$.

During the decision step, we assign $\{\mathbf{A}^m_{n_0}, \mathbf{B}^m_{n_0}\}$ with

$$\begin{aligned} \underset{\mathbf{A}_{n_0}, \mathbf{B}_{n_0}}{\operatorname{argmax}} (& R_{n_0}\left(\mathbf{P}^{m-1}, \mathbf{A}'_{n_0}, \mathbf{B}'_{n_0}\right), \\ & R_{n_0}\left(\mathbf{P}^{m-1}, \mathbf{A}^{m-1}_{n_0}, \mathbf{B}^{m-1}_{n_0}\right)). \end{aligned}$$

5.3.2 PA optimization

In this subsection, the PA stage for iteration m is considered. After setting the indicators to $\{\mathbf{A}^m, \mathbf{B}^m\}$, the transmission modes are fixed in all cells. Let us denote by $\mathcal{S}_{un}(d)$ ($\mathcal{S}_{un}(r)$) the set of subcarriers allocated to MS d_{un} in direct (relay aided) mode. Then the objective function of problem (5.8) can be rewritten as $R\left(\mathbf{P}, \mathbf{A}^m, \mathbf{B}^m\right)$, given by:

$$\begin{aligned}\sum_n \omega_n \min_{u\in\mathcal{U}_n}\Bigg(&\sum_{k\in\mathcal{S}_{un}(d)} \left(\ln\left(1+\Upsilon^k_{\mathrm{d}_{un},t_1}\right)+\ln\left(1+\Upsilon^k_{\mathrm{d}_{un},t_2}\right)\right)\\ &+\sum_{k\in\mathcal{S}_{un}(r)} \ln\min\left(1+\Gamma^k_{\mathrm{r}_n}, 1+\Gamma^k_{\mathrm{d}_{un},t_2}\right)\Bigg).\end{aligned} \tag{5.19}$$

Note that R is a non-convex function due to the presence of interfering power terms in the denominators of $\Upsilon^k_{\mathrm{d}_{un},t_1}, \Upsilon^k_{\mathrm{d}_{un},t_2}, \Gamma^k_{\mathrm{r}_n}$ and $\Gamma^k_{\mathrm{d}_{un},t_2}$. To solve problem (5.8), we first replace it with an equivalent complementary geometric program (CGP) that is then addressed by means of the algorithm based on single condensation and geometric programming (SCGP). Note this methodology has also been used in [25] to tackle another optimization problem.

The equivalent CGP is obtained by formulating problem (5.8) as a minimization problem. Problem (5.8) is first converted into an equivalent one given by:

$$\begin{aligned}&\min_{\mathbf{P}}\ e^{-R(\mathbf{P},\mathbf{A}^m,\mathbf{B}^m)}\\ &s.t.\ C5.3 - C5.7,\end{aligned} \tag{5.20}$$

where $e^{-R(\mathbf{P},\mathbf{A}^m,\mathbf{B}^m)}$ is given by

$$\begin{aligned}\prod_n \max_{u\in\mathcal{U}_n}\Bigg(&\prod_{k\in\mathcal{S}_{un}(d)} \frac{1}{1+\Upsilon^k_{\mathrm{d}_{un},t_1}} \frac{1}{1+\Upsilon^k_{\mathrm{d}_{un},t_2}}\\ &\prod_{k\in\mathcal{S}_{un}(r)} \max\left(\frac{1}{1+\Gamma^k_{\mathrm{r}_n}}, \frac{1}{1+\Gamma^k_{\mathrm{d}_{un},t_2}}\right)\Bigg)^{\omega_n},\end{aligned} \tag{5.21}$$

and

$$\frac{1}{1+\Upsilon^k_{\mathrm{d}_{un},t_1}} = \frac{f^k_{\mathrm{d}_{un},t_1}}{f^k_{\mathrm{d}_{un},t_1} + P^k_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{d}_{un}}}, \tag{5.22}$$

$$\frac{1}{1+\Upsilon^k_{\mathrm{d}_{un},t_2}} = \frac{f^k_{\mathrm{d}_{un},t_2}}{f^k_{\mathrm{d}_{un},t_2} + P^k_{\mathrm{s}_n,t_2} G^k_{\mathrm{s}_n,\mathrm{d}_{un}}}, \tag{5.23}$$

$$\frac{1}{1+\Gamma^k_{\mathrm{d}_{un},t_2}} = \frac{f^k_{\mathrm{d}_{un},t_2}}{f^k_{\mathrm{d}_{un},t_2} + P^k_{\mathrm{r}_n} G^k_{\mathrm{r}_n,\mathrm{d}_{un}}}, \tag{5.24}$$

$$\frac{1}{1+\Gamma^k_{\mathrm{r}_n}} = \frac{f^k_{\mathrm{r}_n}}{f^k_{\mathrm{r}_n} + P^k_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{r}_n}}. \tag{5.25}$$

Second, another equivalent formulation is obtained by introducing slack variables $\{R'_n\}$ and $\mathbf{\Psi}_r = \{\Psi^{unk}_r, \forall u, n, k \in \mathcal{S}_{un}(r)\}$. Then problem (5.20) can be formulated as:

$$\begin{aligned}
&\min_{\mathbf{P},\{R'_n\},\mathbf{\Psi}_r} \prod_n R'_n \\
&s.t.\ C5.3 - C5.7, \\
&C5.8:\ \frac{1}{R'_n}\left(\prod_{k\in\mathcal{S}_{un}(d)} \frac{f^k_{\mathrm{d}_{un},t_1}}{g^k_{\mathrm{d}_{un},t_1}} \frac{f^k_{\mathrm{d}_{un},t_1}}{g'^k_{\mathrm{d}_{un},t_2}} \prod_{k\in\mathcal{S}_{un}(r)} \Psi^{unk}_r\right)^{\omega_n} \le 1,\ \forall u,n, \\
&C5.9:\ \frac{f^k_{\mathrm{r}_n}}{g^k_{\mathrm{r}_n}\Psi^{unk}_r} \le 1,\ \forall u,n,k \in \mathcal{S}_{un}(r), \\
&C5.10:\ \frac{f^k_{\mathrm{d}_{un},t_2}}{g^k_{\mathrm{d}_{un},t_2}\Psi^{unk}_r} \le 1,\ \forall u,n,k \in \mathcal{S}_{un}(r),
\end{aligned} \tag{5.26}$$

where

$$g^k_{\mathrm{d}_{un},t_1} = f^k_{\mathrm{d}_{un},t_1} + P^k_{\mathrm{s}_n,t1} G^k_{\mathrm{s}_n,\mathrm{d}_{un}}, \tag{5.27}$$

$$g'^k_{\mathrm{d}_{un},t_2} == f^k_{\mathrm{d}_{un},t_2} + P^k_{\mathrm{s}_n,t2} G^k_{\mathrm{s}_n,\mathrm{d}_{un}}, \tag{5.28}$$

$$g^k_{\mathrm{d}_{un},t_2} = f^k_{\mathrm{d}_{un},t_2} + P^k_{\mathrm{r}_n} G^k_{\mathrm{r}_n,\mathrm{d}_{un}}, \tag{5.29}$$

$$g^k_{\mathrm{r}_n} = f^k_{\mathrm{r}_n} + P^k_{\mathrm{s}_n,t1} G^k_{\mathrm{s}_n,\mathrm{r}_n}. \tag{5.30}$$

Problem (5.26) consists of an objective function and bounding constraints which all are ratios of two posynomials, making the problem belong to the class of CGP [49]. Problem (5.26) can not be made convex and is NP-hard [72]. In order to solve it, the SCGP algorithm is now proposed. The proposed SCGP algorithm approximates the non-convex problem (5.26) into a series of standard GPs. Therefore, it belongs to the class of successive convex approximation methods [48]. The SCGP is described in algorithm 5.1. Integer m' indicates the current iteration number and a variable with superscripts m and m' denotes the value obtained at the end of inner iteration m' in outer iteration m.

Specifically in the proposed SCGP algorithm, we first set $m' = 0$ and initialize the PA vector $\mathbf{P}^{m,0}$ with $\mathbf{P}_{ini}$. Each iteration m' contains an SC stage followed by a standard geometric programming (SGP) stage.

Algorithm 5.1 SCGP Algorithm

1: **Input:** feasible power variables $\mathbf{P}_{ini}$ of Problem (5.26)
2: **Initialize:** set $m' = 0$ and initialize $\mathbf{P}^{m,0}$ with $\mathbf{P}_{ini}$;
3: **repeat**
4: $\quad m' = m' + 1$;
5: $\quad$ **The SC stage:** Apply a condensation rule and condense all denominator posynomials in Problem (5.26) $\{g^k_{\mathrm{d}_{un},t_1}, g'^k_{\mathrm{d}_{un},t_2}, g^k_{\mathrm{r}_n}, g^k_{\mathrm{d}_{un},t_2}\}$ into monomials $\{\tilde{g}^k_{\mathrm{d}_{un},t_1}, \tilde{g}'^k_{\mathrm{d}_{un},t_2}, \tilde{g}^k_{\mathrm{r}_n}, \tilde{g}^k_{\mathrm{d}_{un},t_2}\}$ using $\mathbf{P}^{m,m'-1}$ as the initial power values;
6: $\quad$ **The SGP stage:** Solve the condensed problem using a standard GP solver and assign the results to $\mathbf{P}^{m,m'}$;
7: **until** $||\mathbf{P}^{m,m'} - \mathbf{P}^{m,m'-1}|| \leq \epsilon_1$, $m' = M'$
8: **Output:** $\mathbf{P}^{m,m'}$.

During the SC stage, a GP approximation of Problem (5.26) is constructed by using the method of Lemma 1 in paper [49] to condense all denominator posynomials $\{g^k_{\mathrm{d}_{un},t_1}, g'^k_{\mathrm{d}_{un},t_2}, g^k_{\mathrm{r}_n}, g^k_{\mathrm{d}_{un},t_2}\}$ into monomials $\{\tilde{g}^k_{\mathrm{d}_{un},t_1}, \tilde{g}'^k_{\mathrm{d}_{un},t_2}, \tilde{g}^k_{\mathrm{r}_n}, \tilde{g}^k_{\mathrm{d}_{un},t_2}\}$. Functions $h^{n,k}_1 = \frac{f^k_{\mathrm{r}_n}}{g^k_{\mathrm{r}_n}\Psi^{unk}_r}$, $h^{n,k}_2 = \frac{f^k_{\mathrm{d}_{un},t_2}}{g^k_{\mathrm{d}_{un},t_2}\Psi^{unk}_r}$ and $h^{n,k}_3 = \frac{1}{R'_n}\left(\prod_{k\in\mathcal{S}_{un}(d)} \frac{f^k_{\mathrm{d}_{un},t_1}}{g^k_{\mathrm{d}_{un},t_1}} \frac{f^k_{\mathrm{d}_{un},t_1}}{g'^k_{\mathrm{d}_{un},t_2}} \prod_{k\in\mathcal{S}_{un}(r)} \Psi^{unk}_r\right)^{\omega_n}$ are approximated by respectively $\tilde{h}^{n,k}_1 = \frac{f^k_{\mathrm{r}_n}}{\tilde{g}^k_{\mathrm{r}_n}\Psi^{unk}_r}$, $\tilde{h}^{n,k}_2 = \frac{f^k_{\mathrm{d}_{un},t_2}}{\tilde{g}^k_{\mathrm{d}_{un},t_2}\Psi^{unk}_r}$ and $\tilde{h}^{n,k}_3 = \frac{1}{R'_n}\left(\prod_{k\in\mathcal{S}_{un}(d)} \frac{f^k_{\mathrm{d}_{un},t_1}}{\tilde{g}^k_{\mathrm{d}_{un},t_1}} \frac{f^k_{\mathrm{d}_{un},t_1}}{\tilde{g}'^k_{\mathrm{d}_{un},t_2}} \prod_{k\in\mathcal{S}_{un}(r)} \Psi^{unk}_r\right)^{\omega_n}$. According to proposition 3 in [49], all the approximations satisfy the three conditions proposed in [48] for the convergence of the successive approximation method. By denoting $\mathbf{\Theta} = \{\mathbf{P}, \{R'_n\}, \mathbf{\Psi}_r\}$, the three conditions are listed as follows:

1. Bounding condition: $\forall \mathbf{\Theta}$,

$$h^{n,k}_i(\mathbf{\Theta}) \leq \tilde{h}^{n,k}_i(\mathbf{\Theta}),\ \forall i = 1, 2, 3.$$

2. Tightness condition: At the beginning of iteration m',

$$h^{n,k}_i(\mathbf{\Theta}^{m,m'-1}) = \tilde{h}^{n,k}_i(\mathbf{\Theta}^{m,m'-1}),\ \forall i = 1, 2, 3.$$

3. Differential condition: At the beginning of iteration m', $\forall \theta \in \mathbf{\Theta}$,

$$\frac{\partial h_i^{n,k}(\mathbf{\Theta}^{m,m'-1})}{\partial \theta} = \frac{\partial \tilde{h}_i^{n,k}(\mathbf{\Theta}^{m,m'-1})}{\partial \theta}, \ \forall i = 1, 2, 3.$$

After the SC stage, problem (5.26) is formulated by a standard GP. The SGP stage amounts to solving the GP by means of a standard GP solver, e.g. the software provided at [73]. The output provided by this stage corresponds to $\mathbf{P}^{m,m'}$.

Thanks to the three conditions fulfilled during the SC stage, our proposed SCGP algorithm is a general inner approximation algorithm [48], which will converge to a local optimum satisfying the KKT conditions of problem (5.26) according to the corollary 1 of [48]. In practice, the iterations of the SCGP algorithm will be stopped when $||\mathbf{P}^{m,m'} - \mathbf{P}^{m,m'-1}|| \le \epsilon_1$ or when m' exceeds a prescribed value M'.

5.3.3 The proposed RA algorithm and its characteristics

Based on the algorithms proposed at both the MSSA stage and the PA stage, we now propose the overall RA algorithm as follows. Specifically in

Algorithm 5.2 Overall RA Optimization Algorithm

1: **Initialize:** set $m = 0$ and initialize $\mathbf{P}^0$ by the UPA algorithm;
2: **repeat**
3: $m = m + 1$;
4: **The MSSA stage:** Compute the optimal MSSA result $\{\mathbf{A}^m, \mathbf{B}^m\}$ for (5.11) with $\mathbf{P} = \mathbf{P}^{m-1}$ using the method proposed in section 5.3.1;
5: **The PA stage:** Compute the optimized PA result $\mathbf{P}^m$ for (5.11) using the SCGP algorithm proposed in section 5.3.2, when $\mathbf{A} = \mathbf{A}^m$ and $\mathbf{B} = \mathbf{B}^m$;
6: **until** $R^m - R^{m-1} < \epsilon_2$ or $m = M$
7: **Output:** $\mathbf{A}^m$, $\mathbf{B}^m$ and $\mathbf{P}^m$.

the proposed RA algorithm, we first set $m = 0$ and initialize the power $\mathbf{P}^0$ by uniform power allocation (UPA). Each iteration consists of the MSSA stage and the PA stage. During the MSSA stage of iteration m, we set $\mathbf{P} = \mathbf{P}^{m-1}$ and decouple problem (5.8) into n MILPs, which can be solved by either typical MILP solvers (named the MSSA algorithm) or the RR-MSSA algorithm and the DR-MSSA algorithm proposed in section 5.3.1. Note that both the RR-MSSA algorithm and the DR-MSSA algorithm can be solved in polynomial time. The output delivered is denoted as $\mathbf{A}^m$ and $\mathbf{B}^m$. Finally we have $R\left(\mathbf{P}^{m-1}, \mathbf{A}^{m-1}, \mathbf{B}^{m-1}\right) \leq R\left(\mathbf{P}^{m-1}, \mathbf{A}^{m}, \mathbf{B}^{m}\right)$

During the PA stage of iteration m, we set $\mathbf{A} = \mathbf{A}^m$, $\mathbf{B} = \mathbf{B}^m$ and solve problem (5.8) using the SCGP algorithm proposed in section 5.3.2. The output delivered is denoted as $\mathbf{P}^m$. As shown in section 5.3.2, the solutions of the approximated GP problems converge to a local optimum satisfying the Karush-Kuhn-Tucker (KKT) conditions of the non-convex problem. Note as each standard GP can be solved in polynomial time [72], $\mathbf{P}^m$ will be obtained in polynomial time after the convergence of the SCGP algorithm. Finally we have $R\left(\mathbf{P}^{m-1}, \mathbf{A}^{m}, \mathbf{B}^{m}\right) \leq R\left(\mathbf{P}^{m}, \mathbf{A}^{m}, \mathbf{B}^{m}\right)$

Considering the proposed RA algorithm, we now have:

$$\begin{aligned} & R\left(\mathbf{P}^{m-1}, \mathbf{A}^{m-1}, \mathbf{B}^{m-1}\right) \\ E1: \ & \leq R\left(\mathbf{P}^{m-1}, \mathbf{A}^{m}, \mathbf{B}^{m}\right) \\ E2: \ & \leq R\left(\mathbf{P}^{m}, \mathbf{A}^{m}, \mathbf{B}^{m}\right), \end{aligned}$$

where E1 and E2 are due to effects of the MSSA stage and the PA stage respectively. This means that algorithm 5.2 yields non-decreasing sum rates along with iterations. Moreover, due to the total power constraint in each cell, the optimum sum rate is upper bounded and the sum rate values will not increase indefinitely along with iterations. This means that the iterations will eventually converge [62]. Algorithm 5.2 will stop when the sum rate increase is below a prescribed value ϵ_2 or when m reaches a prescribed value M.

At the end of iteration m, a local optimum of the formulated problem is obtained at the SCGP stage, which is then improved at the MSSA stage of the next iteration. After that, a better local optimum can be calculated at the SCGP stage of iteration $m+1$. Finally, a good local optimum can be obtained after convergence. Note that both problems at the two stages can be solved by the proposed algorithms in polynomial time. As will be illustrated by experiments, the proposed algorithm 5.2 converges in a limited number of iterations. Thus the formulated problem can be solved by algorithm 5.2 in polynomial time.

We now propose theorem 5.2 as follows.

Theorem 5.2. *After the convergence of algorithm 5.2, per-cell user fairness is reached, meaning that every user in the same cell obtains the same transmission rate.*

Proof. As mentioned before, after the convergence of algorithm 5.2, a local optimum of problem (5.8) is obtained. Let us denote the optimal solutions as $\mathbf{A}^*$, $\mathbf{B}^*$ and $\mathbf{P}^*$. Thus for the local optimum, there exists $r > 0$, such that $\forall \mathbf{P} \in \mathbf{B}_r(\mathbf{P}^*), R(\mathbf{P}, \mathbf{A}^*, \mathbf{B}^*) \leq R(\mathbf{P}^*, \mathbf{A}^*, \mathbf{B}^*)$. Here $\mathbf{B}_r(\mathbf{P}^*) = \{\mathbf{P} \mid d(\mathbf{P}, \mathbf{P}^*) < r\}$ is the open ball with center $\mathbf{P}^*$ and radius r, where $d(\mathbf{P}, \mathbf{P}^*)$ denotes the Euclidean distance between $\mathbf{P}$ and $\mathbf{P}^*$. Let us assume $\exists n$ where $\tilde{\mathcal{V}}_n \neq \emptyset$. Following the methodology similar to the proof of Theorem 5.1, there exists matrices $\mathbf{S}, \mathbf{C}, \mathbf{T}$ and $\mathbf{P}_1 = \mathbf{SCTP}^* \in \mathbf{B}_r(\mathbf{P}^*)$, such that $R(\mathbf{P}_1, \mathbf{A}^*, \mathbf{B}^*) > R(\mathbf{P}^*, \mathbf{A}^*, \mathbf{B}^*)$. Here each element of $\mathbf{S}, \mathbf{C}$ and $\mathbf{T}$ takes value from $(0, 1], (0, 1]$ and $[1, \infty)$ respectively. This obviously contradicts the previous optimality statement, and $\tilde{\mathcal{V}}_n = \emptyset$, which concludes the proof of Theorem 5.2. $\square$

5.4 Numerical experiments

In this section, the performance of our proposed RA algorithm as well as the benefits of using OR and the high spectrum efficiency (HSE) protocol will be discussed. To illustrate the convergence and effectiveness of the algorithm, rates of all users and the WSMRs are first presented for one particular channel realization. Then results are provided and discussed for the performance of the algorithms averaged over many channel realizations. Note that, we introduce the MSSA based algorithm, the RR-MSSA based algorithm and the DR-MSSA based algorithm to denote the proposed iterative RA algorithms which adopt the MSSA algorithm, the RR-MSSA algorithm and the DR-MSSA algorithm respectively at the MSSA stage.

5.4.1 System setup

For the purpose of illustration, a multi-cell OFDMA system with $N = 3$ coordinated cells and $K = 32$ available subcarriers is considered. Each cell contains $U = 4$ MSs. Figure 5.1 shows the positions of the BSs, the RSs and the MSs expressed in meters. Specifically, the coordinates of BS 1, 2 and

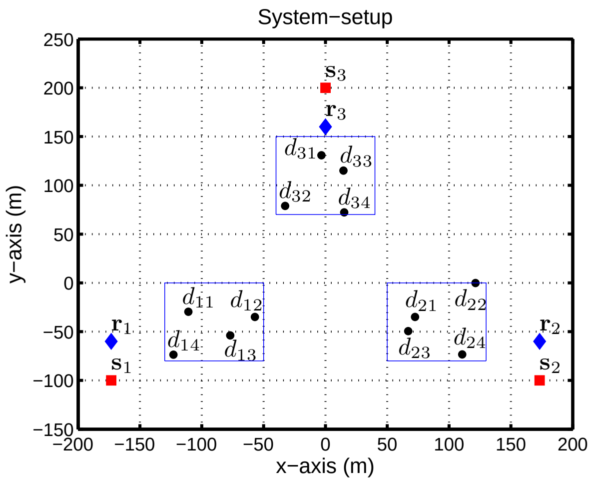


Figure 5.1 A random system setup

3 are $(-100\sqrt{3}, -100)$, $(100\sqrt{3}, -100)$ and $(0, 200)$ respectively. Also, the coordinates of RS 1, 2 and 3 are $(-100\sqrt{3}, -60)$, $(100\sqrt{3}, -60)$ and $(0, 160)$ respectively. Each member of the MS group $\mathcal{U}_n$ in cell n is located randomly and uniformly inside a confined region of that cell, which is depicted as a blue box in Figure 5.1.

The channel impulse response (CIR) of each link is drawn randomly from an 8-tap delay line model, where each tap has a circular complex gaussian distribution with zero mean and variance as σ_i^2. We further assume $\frac{\sigma_i^2}{\sigma_{i+1}^2} = e^3$, meaning that the tap power decreases exponentially with a coefficient 3. Moreover $\sum_i \sigma_i^2 = d^{-3}$, meaning that the received power decreases exponentially with distance d and the propagation exponent equals 3. Finally, the CFR of each link is computed from its CIR using the K-point FFT. Each cell is assumed to have the same total power constraint and we set $\omega_n = 1, P_{t,n} = P_t, \forall n$, $\sigma^2 = -70dBm$, $\epsilon_1 = R_{ini}/100$ and $\epsilon_2 = R_{ini}/100$. Here R_{ini} denotes the sum rate calculated with the initial RA results at the beginning of each iteration.

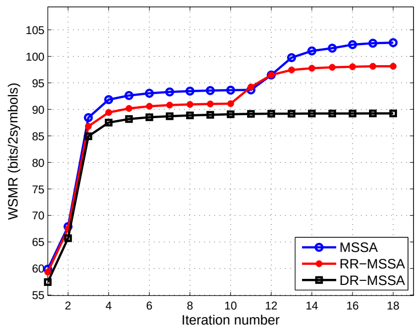


Figure 5.2 Convergence of the WSMR

5.4.2 Results for a random realization of channels

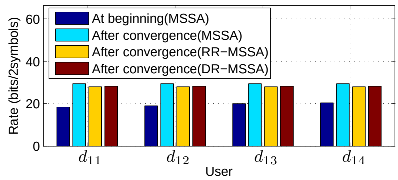
In order to illustrate the convergence and effectiveness of three RA algorithms, we set $P_t = 20dBm$ and $U = 4$. The positions of MSs and a set of channels are randomly generated. Specifically, the coordinates of d_{11}, d_{12}, d_{13}, d_{14}, d_{21}, d_{22}, d_{23}, d_{24}, d_{31}, d_{32}, d_{33} and d_{34} are $(-110.9, -29.6)$, $(-57, -34.9)$, $(-76.9, -53.9)$, $(-122.9, -73.6)$, $(72.5, -35)$, $(121.4, -0.1)$, $(66.9, -49.5)$, $(110.6, -73.4)$, $(-3.2, 130.8)$, $(-32.6, 78.9)$, $(14.6, 115.1)$ and $(15.2, 72.2)$ respectively.

As shown in Figure 5.2, using three proposed algorithms, the WSMRs keep increasing continuously and converge smoothly to the final rate. After convergence, the WSMRs are increased by around 74%, 73% and 55% using the MSSA based algorithm, the RR-MSSA based algorithm and the DR-MSSA based algorithm respectively. Compared to the MSSA based algorithm, the RR-MSSA based algorithm has similar performance and less complexity. Thus it is preferred for practical implementation.

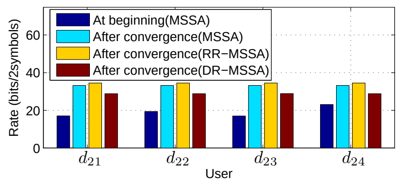
Figure 5.3 shows the optimized user rates in three cells using three RA algorithms. After the convergence of each RA algorithm, the minimal user rate in each cell is enhanced and all users in the same cell obtain the same rates. Therefore, the fairness of our algorithms is illustrated.

5.4.3 Results averaged over channel distribution

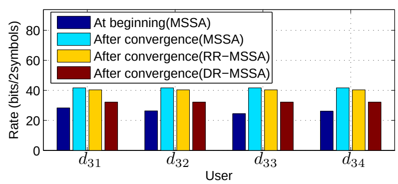
In order to illustrate the average performance of our proposed RA algorithms, 100 random realizations of channels are generated with $U = 8$. We set



(a) Rates for users in cell 1



(b) Rates for users in cell 2



(c) Rates for users in cell 3

Figure 5.3 Rate results for the random realization of channels.

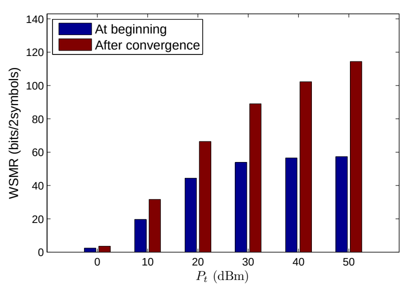


Figure 5.4 Average WSMRs over 100 random realizations of channels

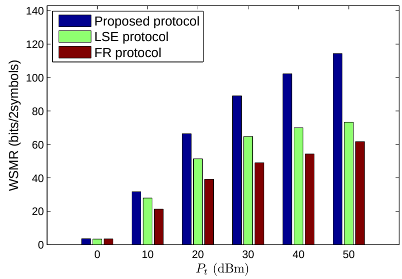


Figure 5.5 Comparison of average WSMRs for 3 protocols

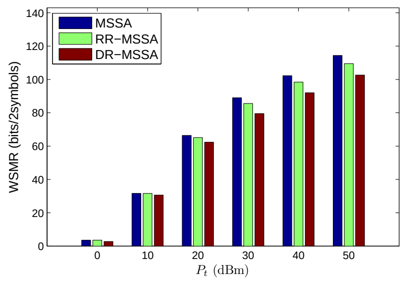


Figure 5.6 Comparison of average WSMRs with 3 RA algorithms

up different total power constraints ranging from 0 dBm to 50 dBm. To show the benefit of using OR and the HSE protocol, we now compare the proposed protocol with a fixed relaying (FR) protocol and a low spectrum efficiency (LSE) protocol. Specifically, the FR protocol is the same as the proposed protocol except that all subcarriers are forced to be in relay aided mode. The LSE protocol is also the same as the proposed protocol except that all direct mode subcarriers are activated only during the first TS. We also compare the performances of the MSSA based algorithm, the RR-MSSA based algorithm and the DR-MSSA based algorithm.

Figure 5.4 adopts the MSSA based algorithm. It appears that the average WSMR is increased significantly after optimization, especially when P_t takes high values. This confirms the effectiveness of our proposed algorithm. Figure 5.5 provides the average WSMRs for the FR protocol, the LSE protocol and the proposed protocol. We can see that the proposed protocol performs much better than the other two protocols. Thus the effectiveness of using OR and the HSE protocol is illustrated. Figure 5.6 provides the average WSMRs of using the MSSA based algorithm, the RR-MSSA based algorithm and the DR-MSSA based algorithm. We can see that the RR-MSSA based algorithm performs similarly as the MSSA based algorithm and better than the DR-MSSA based algorithm. Note that the RR-MSSA based algorithm can be solved in polynomial time. Thus it offers a good trade-off between the performance and the complexity.

6 Distributed Algorithms for Sum Rate Maximization in Multi-Cell Downlink OFDMA with Opportunistic DF Relaying

In this chapter, we consider the same system as that of Chapter 5, except that multiple DF relay stations (RSs) are placed in each cell. When a subcarrier is in relay aided mode, one helping RS has to be selected. The problem considered is the maximization of the system sum rate with a total power constraint in each cell. After some modification, the centralized RA algorithm proposed in Chapter 4 can be applied here. However, it is quite heavy to implement in practice. Specifically, the contributions of this Chapter are as follows:

- We formulate the sum rate maximized joint RA problem in multi-cell OFDMA downlink systems aided by multiple RSs in each cell.
- We propose an iterative semi-distributed RA algorithm to optimize the mode selection, subcarrier assignment (MSSA) and the power allocation (PA) alternatively with the sum rate keeping increasing. For the MSSA stage, the optimization problem is decoupled into subproblems which can be solved distributively in linear time. For the PA stage, the

algorithm based on single condensation and Lagrange duality (SCLD) is designed to optimize PA iteratively with an analytical solution at each iteration. As will be illustrated through numerical results, the proposed algorithm converges quite fast and is more practical compared to the centralized algorithm (CA) of [25].

- We further propose a modified iterative waterfilling (IWF) algorithm to solve the formulated RA problem autonomously. Specifically, each cell autonomously optimizes its own sum rate with the estimated power values of the received interferences from the other cells. An optimum algorithm is proposed to solve the local RA problem in each cell. As will be illustrated, the convergence of the modified IWF algorithm is always observed in simulations, although it seems intractable to derive the conditions of convergence theoretically. Through numerical results, the modified IWF algorithm provides a good tradeoff between complexity and performance. Thus it is recommended for practical implementation.

The rest of this Chapter is organized as follows. First, the system model and the problem formulation for the considered system are presented. Then, the proposed SCLD based RA algorithm and the modified IWF algorithm are described. Finally, the convergence of the proposed algorithms and their benefits compared with a CA are illustrated by numerical experiments.

6.1 System model and problem formulation

6.1.1 System model

We consider a cellular OFDMA system with N cells coordinated by a central controller for message passing among cells. In each cell, downlink transmission is carried out from a base station (BS) to U mobile stations (MSs) with the help of J relay stations (RSs), which are assumed to be of the DF type. For each link, the channel is assumed to be frequency selective and transformed into K parallel subchannels by using OFDM with sufficiently long cyclic prefix. The data transmission is carried out in two TSs. During the first TS, a symbol is first broadcast by the BS at a subcarrier k, which is in either relay aided mode or direct mode. Both a selected RS and a targeted MS receive this symbol. If the relay aided mode is used, the RS decodes the received symbol and forwards it to the targeted MS over the subcarrier k with the BS keeping quiet on this subcarrier during the second TS. The MS only decodes the symbol received during the second TS. If the direct mode is used, the targeted MS decodes the symbol received during the first TS. Also, another symbol is broadcast by the BS at the same subcarrier during the second TS, which is received and only decoded by the targeted MS.

With OFDMA, each subcarrier is allocated to only one MS in each cell. Throughout this Chapter, we assume the timing and carrier synchronization is perfect. We further assume that the central controller can obtain/distribute messages from/to all nodes. Moreover, we assume that the coherence time of each link is sufficiently long for implementing the RA. Note that by assuming the above idealities, an upper bound on the system performances is obtained.

The data transmission procedure in every cell is identical. Thus we only analyze the downlink data transmission inside one selected cell n, which is impaired by cochannel interference from the other cells. Specifically in cell n, data transmissions are carried out in either relay-aided mode or direct mode, as will be elaborated in the following.

Let us first see more closely the relay aided data transmission. The BS s_n first produces a symbol $\sqrt{P^k_{\mathrm{s}_n,t_1}}x^k_{\mathrm{s}_n,t_1}$ at subcarrier k during the first TS, while the transmitter of the selected RS r_{jn} remains idle. Here $x^k_{\mathrm{s}_n,t_1}$ denotes the normalized symbol (meaning $E\{|x^k_{\mathrm{s}_n,t_1}|^2\} = 1$) transmitted by s_n at subcarrier k during the first TS and $P^k_{\mathrm{s}_n,t_1}$ denotes the corresponding transmit power. Simultaneously in an interfering cell n', a symbol $\sqrt{P^k_{\mathrm{s}_{n'},t_1}}x^k_{\mathrm{s}_{n'},t_1}$ is also produced from the interfering BS $\mathrm{s}_{n'}$ at the same subcarrier. Here again $E\{|x^k_{\mathrm{s}_{n'},t_1}|^2\} = 1$. Note that, instead of using an additional integer variable to indicate whether $\mathrm{s}_{n'}$ transmits data on subcarrier k or not, we use $\sqrt{P^k_{\mathrm{s}_{n'},t_1}}$ to do it. Specifically, $P^k_{\mathrm{s}_{n'},t_1} > 0$ means that $\mathrm{s}_{n'}$ uses subcarrier k for data transmission during the first TS and $P^k_{\mathrm{s}_{n'},t_1} = 0$ means that $\mathrm{s}_{n'}$ transmits nothing at the subcarrier k during the first TS. This choice is motivated to simplify the system sum rate expression and facilitate the algorithm design, as discussed in Chapter 4. At the end of the first TS, the signal received by r_{jn} for subcarrier k can be expressed as

$$
\begin{aligned}
y^k_{\mathrm{r}_{jn}} = & \sqrt{P^k_{\mathrm{s}_n,t_1}} h^k_{\mathrm{s}_n,\mathrm{r}_{jn}} x^k_{\mathrm{s}_n,t_1} + v^k_{\mathrm{r}_{jn}} + \\
& \sum_{n'=1,n'\neq n}^{N} \sqrt{P^k_{\mathrm{s}_{n'},t_1}} h^k_{\mathrm{s}_{n'},\mathrm{r}_{jn}} x^k_{\mathrm{s}_{n'},t_1}
\end{aligned} \tag{6.1}
$$

where $v^k_{\mathrm{r}_{jn}}$ denotes the additive white Gaussian noise (AWGN) at subcarrier k and r_{jn} during the first TS. $h^k_{\mathrm{s}_{n'},\mathrm{r}_{jn}}$ denotes the channel frequency response (CFR) for subcarrier k from $\mathrm{s}_{n'}$ to r_{jn}.

During the second TS, the selected RS r_{jn} reencodes the decoded symbol and forwards $\sqrt{P^k_{\mathrm{r}_{jn}}} x^k_{\mathrm{r}_{jn}}$ at the same subcarrier k. Here $x^k_{\mathrm{r}_{jn}} = x^k_{\mathrm{s}_n,t_1}$. The BS s_n transmits nothing on this subcarrier, meaning that $P^k_{\mathrm{s}_n,t_2} = 0$. Here $P^k_{\mathrm{r}_{jn}}$ and $P^k_{\mathrm{s}_n,t_2}$ denote the transmit power allocated to r_{jn} and s_n respectively at subcarrier k during the second TS. At the same time, in an interfering cell n', $\mathrm{r}_{j'n'}$ and $\mathrm{s}_{n'}$ also transmit $\sqrt{P^k_{\mathrm{r}_{j'n'}}} x^k_{\mathrm{r}_{j'n'}}$ and $\sqrt{P^k_{\mathrm{s}_{n'},t_2}} x^k_{\mathrm{s}_{n'},t_2}$ at subcarrier k, where at most only one power value out of $\sqrt{P^k_{\mathrm{r}_{j'n'}}} x^k_{\mathrm{r}_{j'n'}}$ and $\sqrt{P^k_{\mathrm{s}_{n'},t_2}} x^k_{\mathrm{s}_{n'},t_2}$ can be non-zero. More specifically when subcarrier k of cell n' chooses the direct mode for data transmission, $P^k_{\mathrm{r}_{j'n'}} = 0$. Otherwise, $P^k_{\mathrm{s}_{n'},t_2} = 0$. Here again $E\{|x^k_{\mathrm{r}_{j'n'}}|^2\} = 1$ and $E\{|x^k_{\mathrm{s}_{n'},t_2}|^2\} = 1$. At the end of the second TS, the signal received by the targeted MS d_{un} at subcarrier k can be expressed as

$$
\begin{aligned}
y^k_{\mathrm{d}_{un},t_2} = & \sqrt{P^k_{\mathrm{r}_{jn}}} h^k_{\mathrm{r}_{jn},\mathrm{d}_{un}} x^k_{\mathrm{r}_{jn}} + v^k_{\mathrm{d}_{un},t_2} + \\
& \sum_{n'=1,n'\neq n}^{N} \sqrt{P^k_{\mathrm{s}_{n'},t_2}} h^k_{\mathrm{s}_{n'},\mathrm{d}_{un}} x^k_{\mathrm{s}_{n'},t_2} + \\
& \sum_{n'=1,n'\neq n}^{N} \sum_{j'=1}^{J} \sqrt{P^k_{\mathrm{r}_{j'n'}}} h^k_{\mathrm{r}_{j'n'},\mathrm{d}_{un}} x^k_{\mathrm{r}_{j'n'}}
\end{aligned} \tag{6.2}
$$

where $v^k_{\mathrm{d}_{un},t_2}$ denotes the AWGN corrupting d_{un} at subcarrier k during the second TS , $h^k_{\mathrm{r}_{j'n'},\mathrm{d}_{un}}$ denotes the CFR for subcarrier k from $\mathrm{r}_{j'n'}$ to d_{un}. $h^k_{\mathrm{s}_{n'},\mathrm{d}_{un}}$ denotes the CFR of subcarrier k from $\mathrm{s}_{n'}$ to d_{un}.

We assume $\{v^k_{\mathrm{r}_{jn}}, v^k_{\mathrm{d}_{un},t_2}\}$ are independent zero-mean circular Gaussian random variables with the same variance σ^2. After some mathematical calculations, the signal-to-interference-plus-noise ratio (SINR) associated with decoding $x^k_{\mathrm{s}_n,t_1}$ from $y^k_{\mathrm{r}_{jn}}$ at r_{jn} during the first TS is expressed by

$$\Gamma^k_{\mathrm{r}_{jn}} = \frac{P^k_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{r}_{jn}}}{f^k_{\mathrm{r}_{jn}}} \tag{6.3}$$

where $f^k_{\mathrm{r}_{jn}} = \sigma^2 + \sum\limits_{n',n'\neq n} P^k_{\mathrm{s}_{n'},t_1} G^k_{\mathrm{s}_{n'},\mathrm{r}_{jn}}$ denotes the sum power of the AWGN and the interference received by r_{jn} at subcarrier k during the first TS. $G^k_{\mathrm{s}_n,\mathrm{r}_{jn}} = |h^k_{\mathrm{s}_n,\mathrm{r}_{jn}}|^2$ denotes the channel gain of subcarrier k from s_n to r_{jn}.

Also the SINR associated with decoding $x^k_{\mathrm{r}_{jn}}$, which equals $x^k_{\mathrm{s}_n,t_1}$, from $y^k_{\mathrm{d}_{un},\mathrm{r}_{jn}}$ at d_{un} during the second TS is expressed by

$$\Gamma^k_{\mathrm{d}_{un},t_2} = \frac{P^k_{\mathrm{r}_{jn}} G^k_{\mathrm{r}_{jn},\mathrm{d}_{un}}}{f^k_{\mathrm{d}_{un},t_2}} \tag{6.4}$$

where $f^k_{\mathrm{d}_{un},t_2} = \sigma^2 + \sum\limits_{n'=1,n'\neq n}^{N} P^k_{\mathrm{s}_{n'},t_2} G^k_{\mathrm{s}_{n'},\mathrm{d}_{un}} + \sum\limits_{n'=1,n'\neq n}^{N} \sum\limits_{j'=1}^{J} P^k_{\mathrm{r}_{j'n'}} G^k_{\mathrm{r}_{j'n'},\mathrm{d}_{un}}$ denotes the sum power of the AWGN and the interference received by d_{un} at subcarrier k during the second TS. $G^k_{\mathrm{r}_{jn},\mathrm{d}_{un}} = |h^k_{\mathrm{r}_{jn},\mathrm{d}_{un}}|^2$ denotes the channel gain of subcarrier k from r_{jn} to d_{un}. $G^k_{\mathrm{s}_{n'},\mathrm{d}_{un}} = |h^k_{\mathrm{s}_{n'},\mathrm{d}_{un}}|^2$ denotes the channel gain of subcarrier k from $\mathrm{s}_{n'}$ to d_{un}.

The maximum achievable rate for the subcarrier k when allocated to d_{un} is given by [3]

$$R^k_{ujn,2} = \min\left(\ln\left(1+\Gamma^k_{\mathrm{r}_{jn}}\right), \ln\left(1+\Gamma^k_{\mathrm{d}_{un},t_2}\right)\right) \tag{6.5}$$

in nats/two-TSs.

Let us describe further the direct data transmission in cell n. During the first TS, s_n broadcasts $\sqrt{P^k_{\mathrm{s}_n,t_1}} x^k_{\mathrm{s}_n,t_1}$ at subcarrier k. The targeted MS d_{un} receives signals from all BSs. Finally, the signal received by d_{un} at subcarrier k can be expressed as

$$\begin{aligned} z^k_{\mathrm{d}_{un},t_1} =& \sqrt{P^k_{\mathrm{s}_n,t_1}} h^k_{\mathrm{s}_n,\mathrm{d}_{un}} x^k_{\mathrm{s}_n,t_1} + v^k_{\mathrm{d}_{un},t_1} + \\ & \sum_{n'=1,n'\neq n}^{N} \sqrt{P^k_{\mathrm{s}_{n'},t_1}} h^k_{\mathrm{s}_{n'},\mathrm{d}_{un}} x^k_{\mathrm{s}_{n'},t_1} \end{aligned} \tag{6.6}$$

where $v^k_{\mathrm{d}_{un},t_1}$ denotes the AWGN corrupting d_{un} at subcarrier k during the first TS.

During the second time slot, another symbol $\sqrt{P^k_{\mathrm{s}_n,t_2}}x^k_{\mathrm{s}_n,t_2}$ is broadcast by s_n at subcarrier k and received by the same MS d_{un}. The received signal can be expressed as:

$$
\begin{aligned}
z^k_{\mathrm{d}_{un},t_2} = &\sqrt{P^k_{\mathrm{s}_n,t_2}}h^k_{\mathrm{s}_n,\mathrm{d}_{un}}x^k_{\mathrm{s}_n,t_2} + v^k_{\mathrm{d}_{un},t_2} + \\
&\sum_{n'=1,n'\neq n}^{N} \sqrt{P^k_{\mathrm{s}_{n'},t_2}}h^k_{\mathrm{s}_{n'},\mathrm{d}_{un}}x^k_{\mathrm{s}_{n'},t_2} + \\
&\sum_{n'=1,n'\neq n}^{N}\sum_{j'=1}^{J} \sqrt{P^k_{\mathrm{r}_{j'n'}}}h^k_{\mathrm{r}_{j'n'},\mathrm{d}_{un}}x^k_{\mathrm{r}_{j'n'}}.
\end{aligned} \tag{6.7}
$$

Thus the achievable sum rate for subcarrier k during two TSs is given by

$$R^k_{un,1} = \ln\left(1 + \Upsilon^k_{\mathrm{d}_{un},t_1}\right) + \ln\left(1 + \Upsilon^k_{\mathrm{d}_{un},t_2}\right) \tag{6.8}$$

in nats/two-TSs, where

$$\Upsilon^k_{\mathrm{d}_{un},t_1} = \frac{P^k_{\mathrm{s}_n,t_1}G^k_{\mathrm{s}_n,\mathrm{d}_{un}}}{f^k_{\mathrm{d}_{un},t_1}} \tag{6.9}$$

denotes the SINR associated with the decoding of $x^k_{\mathrm{s}_n,t_1}$ from $y^k_{\mathrm{d}_{un},t_1}$ at d_{un} during the first TS. $f^k_{\mathrm{d}_{un},t_1} = \sigma^2 + \sum_{n'=1,n'\neq n}^{N} P^k_{\mathrm{s}_{n'},t_1}G^k_{\mathrm{s}_{n'},\mathrm{d}_{un}}$ denotes the sum power of the AWGN and the interference received by d_{un} at subcarrier k during the first TS.

$$\Upsilon^k_{\mathrm{d}_{un},t_2} = \frac{P^k_{\mathrm{s}_n,t_2}G^k_{\mathrm{s}_n,\mathrm{d}_{un}}}{f^k_{\mathrm{d}_{un},t_2}} \tag{6.10}$$

denotes the SINR associated with the decoding of $x^k_{\mathrm{s}_n,t_2}$ from $y^k_{\mathrm{d}_{un},t_2}$ at d_{un} during the second TS.

6.1.2 RA problem formulation

In order to formulate the RA problem, we now introduce binary variables a^k_{un} and b^k_{ujn} to describe the mode selection and subcarrier assignment in both TSs. To be more specific, $a^k_{un} = 1$ indicates that subcarrier k is allocated for data transmission to d_{un} in direct mode during two TSs. $b^k_{ujn} = 1$ indicates that subcarrier k is allocated for data transmission to d_{un} aided by r_{jn}.

We consider maximizing the sum rate of all MSs in all cells under per cell total power constraints. The optimization variables are the transmission mode for each subcarrier, the subcarrier assignments and the power allocations at the BSs and the RSs. According to the system model, the considered RA problem can be formulated as:

$$\begin{aligned}
&\max_{\mathbf{P},\mathbf{A},\mathbf{B}} R\left(\mathbf{P},\mathbf{A},\mathbf{B}\right)\\
&s.t.\\
&C6.1:\ \left(\sum_{u=1}^{U} a_{un}^{k}+\sum_{u=1}^{U}\sum_{j=1}^{J} b_{ujn}^{k}\right)\leq 1,\ \forall n,k,\\
&C6.2:\ a_{un}^{k}\in\{0,1\}, b_{ujn}^{k}\in\{0,1\},\ \forall u,j,n,k,\\
&C6.3:\ \sum_{k=1}^{K} P_{\mathrm{s}_n,t_1}^{k}+\sum_{k=1}^{K} P_{\mathrm{s}_n,t_2}^{k}+\sum_{j=1}^{J}\sum_{k=1}^{K} P_{\mathrm{r}_{jn}}^{k}\leq P_{t,n},\ \forall n,\\
&C6.4:\ P_{\mathrm{s}_n,t_1}^{k}\geq 0, P_{\mathrm{s}_n,t_2}^{k}\geq 0, P_{\mathrm{r}_{jn}}^{k}\geq 0,\ \forall j,n,k,\\
&C6.5:\ P_{\mathrm{s}_n,t_1}^{k}\left(1-\left(\sum_{u=1}^{U} a_{un}^{k}+\sum_{u=1}^{U}\sum_{j=1}^{J} b_{ujn}^{k}\right)\right)=0,\ \forall n,k,\\
&C6.6:\ P_{\mathrm{s}_n,t_2}^{k}\left(1-\sum_{u=1}^{U} a_{un}^{k}\right)=0,\ \forall n,k,\\
&C6.7:\ P_{\mathrm{r}_{jn}}^{k}\left(1-\sum_{u=1}^{U} b_{ujn}^{k}\right)=0,\ \forall j,n,k,
\end{aligned}\tag{6.11}$$

where

$$\begin{aligned}
R\left(\mathbf{P},\mathbf{A},\mathbf{B}\right)=&\sum_{n=1}^{N}\sum_{u=1}^{U}\sum_{k=1}^{K} a_{un}^{k} R_{un,1}^{k}\left(\mathbf{P}_n^k\right)+\\
&\sum_{n=1}^{N}\sum_{u=1}^{U}\sum_{j=1}^{J}\sum_{k=1}^{K} b_{ujn}^{k} R_{ujn,2}^{k}\left(\mathbf{P}_n^k\right),
\end{aligned}\tag{6.12}$$

$\mathbf{P}=[[\mathbf{P}_1]^{\mathrm{T}},...,[\mathbf{P}_N]^{\mathrm{T}}]^{\mathrm{T}}$, $\mathbf{P}_n=[[\mathbf{P}_n^1]^{\mathrm{T}},...,[\mathbf{P}_n^K]^{\mathrm{T}}]^{\mathrm{T}}$, $\mathbf{P}_n^k=[P_{\mathrm{s}_n,t_1}^k, P_{\mathrm{s}_n,t_2}^k, P_{\mathrm{r}_{1n}}^k,...,P_{\mathrm{r}_{Jn}}^k]^{\mathrm{T}}$, $\mathbf{A}=\{a_{un}^k\}$, $\mathbf{B}=\{b_{ujn}^k\}$. Here, $C6.1$ and $C6.2$ ensure that each subcarrier k during the first TS can select only one mode (direct/relay-aided) to transmit data towards only one MS d_{un}. If the relay aided mode is selected, the data transmission can be helped by only one RS r_{jn}. Moreover, $C6.3$ and $C6.4$ ensure that the consumed sum power for each cell is less than its available sum power. This type of power constraints gives an upper bound of the system performance. In practice, each node (BS, RS) in each cell will have an individual power constraint. Finally, $C6.5$, $C6.6$ and $C6.7$ guarantee that no data is transmitted on an unused subcarrier and subcarrier k is used by only one node (either the BS or the RS) in each cell during the second TS.

6.2 SCLD based coordinate ascent RA algorithm

6.2.1 The overall RA algorithm

In order to solve problem (6.11), an iterative coordinate ascent approach is adopted. Each iteration is carried out in two stages: the MSSA stage and the PA stage. We introduce integer m to indicate the iteration number and add superscript m to variables obtained at the end of iteration m. We now propose the overall RA algorithm, as depicted in Algorithm 1.

Specifically in the proposed RA algorithm, we first set $m = 0$ and initial-

Algorithm 6.1 Overall RA Optimization Algorithm

1: **Initialize:** set $m = 0$ and initialize $\mathbf{P}^0$ by the UPA algorithm;
2: **repeat**
3: $m = m + 1$;
4: **The MSSA stage:** Compute the optimal MSSA result $\{\mathbf{A}^m, \mathbf{B}^m\}$ for (6.11) with $\mathbf{P} = \mathbf{P}^{m-1}$ using the method proposed in section 6.2.2;
5: **The PA stage:** Compute the optimized PA result $\mathbf{P}^m$ for (6.11) using the SCLD PA algorithm proposed in section 6.2.3, when $\mathbf{A} = \mathbf{A}^m$ and $\mathbf{B} = \mathbf{B}^m$;
6: **until** $R^m - R^{m-1} < \epsilon_1$ or $m = M$
7: **Output:** $\mathbf{A}^m$, $\mathbf{B}^m$ and $\mathbf{P}^m$.

ize the power $\mathbf{P}^0$ either by the uniform power allocation (UPA) or by using the optimal algorithm, proposed in [16] when CCI is ignored. Each iteration consists of the MSSA stage and the PA stage. During the MSSA stage of iteration m, we set $\mathbf{P} = \mathbf{P}^{m-1}$ and decouple problem (6.11) into $n * k$ ILPs, which can easily be solved in linear time, as will be shown in section 6.2.2. After optimization, $\{\mathbf{A}^m, \mathbf{B}^m\}$ will be obtained. Finally we have $R\left(\mathbf{P}^{m-1}, \mathbf{A}^{m-1}, \mathbf{B}^{m-1}\right) \leq R\left(\mathbf{P}^{m-1}, \mathbf{A}^m, \mathbf{B}^m\right)$.

During the PA stage of iteration m, we set $\mathbf{A} = \mathbf{A}^m$, $\mathbf{B} = \mathbf{B}^m$ and solve problem (6.11) using the SCLD PA algorithm proposed in section 6.2.3. The output delivered is denoted as $\mathbf{P}^m$. As will be shown in section 6.2.3, the SCLD PA algorithm approximates the original nonconvex problem into a series of convex problems. The solutions of the approximated convex problems converge to a local optimum satisfying the Karush-Kuhn-Tucker (KKT) conditions of the non-convex problem. After convergence, the power vector $\mathbf{P}^m$ will be output. Finally we have $R\left(\mathbf{P}^{m-1}, \mathbf{A}^m, \mathbf{B}^m\right) \leq R\left(\mathbf{P}^m, \mathbf{A}^m, \mathbf{B}^m\right)$.

Considering the proposed RA algorithm, we now have:

$$
\begin{aligned}
& R\left(\mathbf{P}^{m-1}, \mathbf{A}^{m-1}, \mathbf{B}^{m-1}\right) \\
E1: \ & \leq R\left(\mathbf{P}^{m-1}, \mathbf{A}^{m}, \mathbf{B}^{m}\right) \\
E2: \ & \leq R\left(\mathbf{P}^{m}, \mathbf{A}^{m}, \mathbf{B}^{m}\right),
\end{aligned}
$$

where E1 and E2 are due to the MSSA stage and the PA stage respectively. This means that Algorithm 6.1 yields non-decreasing sum rates along with iterations. Moreover, due to the total power constraint in each cell, the optimum sum rate is upper bounded and the sum rate values will not increase indefinitely along with iterations. This means that the iterations will eventually converge [62]. Algorithm 6.1 will stop when the sum rate increase is below a prescribed value ϵ_1 or when m reaches a prescribed value M.

At the end of iteration m, a local optimum of the formulated problem is obtained at the PA stage, which is then improved at the MSSA stage of the next iteration. After that, a better local optimum can be calculated at the PA stage of iteration $m+1$. Finally, a good local optimum can be obtained after convergence. Note that each stage is carried out with analytical equations. Thus the proposed RA algorithm has a lower-complexity compared to the CA of [25]. As will be elaborated in section 6.2.2 and section 6.2.3, the proposed MSSA algorithm can be implemented distributively in each cell and the SCLDPA algorithm can be implemented semi-distributively in each cell. Thus the proposed iterative RA algorithm can be implemented semi-distributively in each cell with the help of a central controller (CC) exchanging messages among cells.

6.2.2 MSSA optimization

In this subsection, the MSSA stage for iteration m is considered. After setting $\mathbf{P}$ to $\mathbf{P}^{m-1}$, problem (6.11) can be rewritten as:

$$\begin{aligned}&\max_{\mathbf{A},\mathbf{B}} R\left(\mathbf{P}^{m-1}, \mathbf{A}, \mathbf{B}\right)\\ &s.t.\ C6.1, C6.2, C6.5, C6.6, C6.7.\end{aligned} \tag{6.13}$$

Note that each subcarrier in each cell has independent constraints. Therefore problem (6.13) can be decoupled into $N * K$ subproblems. Specifically, subproblem i, which corresponds to subcarrier k_0 in cell n_0 is formulated as:

$$\begin{aligned}&\max_{\mathbf{A}_i,\mathbf{B}_i} R_i\left(\mathbf{P}^{m-1}, \mathbf{A}_i, \mathbf{B}_i\right)\\ &s.t.\\ &C6.1' :\ \sum_u a_{un_0}^{k_0} + \sum_u \sum_j b_{ujn_0}^{k_0} \leq 1,\\ &C6.2' :\ a_{un_0}^{k_0} \in \{0,1\}, b_{ujn_0}^{k_0} \in \{0,1\},\ \forall u, j,\\ &C6.5' :\ P_{\mathrm{s}_{n_0},t_1}^{k_0,m-1}\left(1 - \left(\sum_{u=1}^{U} a_{un_0}^{k_0} + \sum_{u=1}^{U}\sum_{j=1}^{J} b_{ujn_0}^{k_0}\right)\right) = 0,\\ &C6.6' :\ P_{\mathrm{s}_{n_0},t_2}^{k_0,m-1}\left(1 - \sum_{u=1}^{U} a_{un_0}^{k_0}\right) = 0,\\ &C6.7' :\ P_{\mathrm{r}_{jn_0}}^{k_0,m-1}\left(1 - \sum_{u=1}^{U} b_{ujn_0}^{k_0}\right) = 0,\ \forall j,\end{aligned} \tag{6.14}$$

where $\mathbf{A}_i = \{a_{un}^k | n = n_0, k = k_0\}$, $\mathbf{B}_i = \{b_{ujn}^k | n = n_0, k = k_0\}$ and R_i is given by

$$R_i = \sum_u a_{un_0}^{k_0} R_{un_0,1}^{k_0}\left(\mathbf{P}^{m-1}\right) + \sum_u \sum_j b_{ujn_0}^{k_0} R_{ujn_0,2}^{k_0}\left(\mathbf{P}^{m-1}\right).$$

As will be explained later, problem (6.14) has the same optimum solution as the following problem, where the constraints $C6.5' - C6.7'$ are removed.

$$\begin{aligned}&\max_{\mathbf{A}_i,\mathbf{B}_i} R_i\left(\mathbf{P}^{m-1}, \mathbf{A}_i, \mathbf{B}_i\right)\\ &s.t.\ C6.1', C6.2'.\end{aligned} \tag{6.15}$$

Specifically, $\forall n_0, k_0$, when $P_{\mathrm{s}_{n_0},t_1}^{k_0,m-1} > 0$, we certainly have $R_{un_0,1}^{k_0} > 0, \forall u$. Therefore at the optimum of problem (6.15), we have $\sum_{u=1}^{U} a_{un_0}^{k_0} +$

$\sum_{u=1}^{U}\sum_{j=1}^{J} b_{ujn_0}^{k_0} = 1$, meaning that this subcarrier k_0 should be used for data transmission. Otherwise, we can still improve R_i by randomly selecting one variable of the set $\{a_{un}^{k}|k = k_0, n = n_0\}$ and setting it to 1. Thus in this case, we have $P_{\mathrm{s}_{n_0},t_1}^{k_0,m-1}\left(1-\left(\sum_{u=1}^{U} a_{un_0}^{k_0}+\sum_{u=1}^{U}\sum_{j=1}^{J} b_{ujn_0}^{k_0}\right)\right) = 0$. Also, when $P_{\mathrm{s}_{n_0},t_1}^{k_0,m-1} = 0$, we certainly have $P_{\mathrm{s}_{n_0},t_1}^{k_0,m-1}\left(1-\left(\sum_{u=1}^{U} a_{un_0}^{k_0}+\sum_{u=1}^{U}\sum_{j=1}^{J} b_{ujn_0}^{k_0}\right)\right) = 0$. Thus at the optimum of problem (6.15), the constraint $C6.5'$ is fulfilled. Similarly, at the optimum of problem (6.15), the constraints $C6.6'$ and $C6.7'$ are fulfilled. This means that the optimum solution of problem (6.15) is also feasible for problem (6.14). Note that problem (6.14) is the same as problem (6.15) except that its feasible set is a subset of that of problem (6.15). Thus the optimum solution of problem (6.15) is also the optimum solution of problem (6.14).

According to the expression of R_i and the constraints $(C6.1', C6.2')$, this subproblem (6.15) can easily be solved by searching the optimal mode, MS and RS which provide the maximum rate for subcarrier k_0 in cell n_0 with $\mathbf{P} = \mathbf{P}^{m-1}$.

6.2.3 PA optimization

In this subsection, the PA stage for iteration m is considered. After setting the indicators to $\{\mathbf{A}^m, \mathbf{B}^m\}$, the transmission modes are fixed in all cells. We denote by $\mathcal{S}_{un}(d)$ the set of subcarriers allocated to MS d_{un} in direct mode and $\mathcal{S}_{ujn}(r)$ the set of subcarriers allocated to MS d_{un} and RS r_{jn} in relay aided mode. Then the objective function of problem (6.11) can be rewritten as $R\left(\mathbf{P}, \mathbf{A}^m, \mathbf{B}^m\right)$, given by:

$$\sum_n\Big(\sum_{k\in\mathcal{S}_{un}(d)}\log\left(1+\Upsilon_{\mathrm{d}_{un},t_1}^{k}\right)+\log\left(1+\Upsilon_{\mathrm{d}_{un},t_2}^{k}\right)$$
$$\sum_{k\in\mathcal{S}_{ujn}(r)}\log\min\left(1+\Gamma_{\mathrm{r}_{jn}}^{k},1+\Gamma_{\mathrm{d}_{un,t_2}}^{k}\right)\Big).$$

Note that R is a non-convex function due to the presence of interfering power terms in the denominators of $\Upsilon_{\mathrm{d}_{un},t_1}^{k}, \Upsilon_{\mathrm{d}_{un},t_2}^{k}, \Gamma_{\mathrm{r}_{jn}}^{k}$ and $\Gamma_{\mathrm{d}_{un,t_2}}^{k}$. To solve problem (6.11), we first replace it with a minimization problem that is then solved by using the SC-LDPA algorithm.

The equivalent minimization problem is obtained by two steps. Problem (6.11) is first converted into an equivalent one given by:

$$\begin{aligned}&\min_{\mathbf{P}} \; -R\left(\mathbf{P}, \mathbf{A}^m, \mathbf{B}^m\right)\\ &s.t.\ C6.3 - C6.7,\end{aligned} \tag{6.16}$$

where $-R\left(\mathbf{P}, \mathbf{A}^m, \mathbf{B}^m\right)$ is given by

$$\begin{aligned}\sum_n \Bigg(&\sum_{k\in\mathcal{S}_{un}(d)} \left(\log \frac{1}{1+\Upsilon^k_{\mathrm{d}_{un},t_1}} + \log \frac{1}{1+\Upsilon^k_{\mathrm{d}_{un},t_2}}\right)\\ &\sum_{k\in\mathcal{S}_{ujn}(r)} \log\max\left(\frac{1}{1+\Gamma^k_{\mathrm{r}_{jn}}}, \frac{1}{1+\Gamma^k_{\mathrm{d}_{un,t_2}}}\right)\Bigg),\end{aligned} \tag{6.17}$$

$$\frac{1}{1+\Upsilon^k_{\mathrm{d}_{un},t_1}} = \frac{f^k_{\mathrm{d}_{un},t_1}}{f^k_{\mathrm{d}_{un},t_1} + P^k_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{d}_{un}}}, \tag{6.18}$$

$$\frac{1}{1+\Upsilon^k_{\mathrm{d}_{un},t_2}} = \frac{f^k_{\mathrm{d}_{un},t_2}}{f^k_{\mathrm{d}_{un},t_2} + P^k_{\mathrm{s}_n,t_2} G^k_{\mathrm{s}_n,\mathrm{d}_{un}}}, \tag{6.19}$$

$$\frac{1}{1+\Gamma^k_{\mathrm{d}_{un,t_2}}} = \frac{f^k_{\mathrm{d}_{un},t_2}}{f^k_{\mathrm{d}_{un},t_2} + P^k_{\mathrm{r}_{jn}} G^k_{\mathrm{r}_{jn},\mathrm{d}_{un}}}, \tag{6.20}$$

$$\frac{1}{1+\Gamma^k_{\mathrm{r}_{jn}}} = \frac{f^k_{\mathrm{r}_{jn}}}{f^k_{\mathrm{r}_{jn}} + P^k_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{r}_{jn}}}. \tag{6.21}$$

After that, another equivalent formulation is obtained by introducing slack variables $\mathbf{\Psi}_r = \{\Psi^{nk}_r, \forall n, k \in \mathcal{S}_{ujn}(r)\}$. Then problem (6.16) can be formulated as:

$$\begin{aligned}&\min_{\mathbf{P},\mathbf{\Psi}_r} \sum_n \Bigg(\sum_{k\in\mathcal{S}_{un}(d)} (\log \frac{f^k_{\mathrm{d}_{un},t_1}}{g^k_{\mathrm{d}_{un},t_1}} \frac{f^k_{\mathrm{d}_{un},t_2}}{g^k_{\mathrm{s}_n\mathrm{d}_{un},t_2}} + \sum_{k\in\mathcal{S}_{ujn}(r)} \Psi^{nk}_r \Bigg)\\ &s.t.\ C6.3 - C6.7,\\ &C6.8:\ \log \frac{f^k_{\mathrm{r}_{jn}}}{g^k_{\mathrm{r}_{jn}}} - \Psi^{nk}_r \le 0,\ \forall n, k \in \mathcal{S}_{ujn}(r),\\ &C6.9:\ \log \frac{f^k_{\mathrm{d}_{un},t_2}}{g^k_{\mathrm{r}_{jn}\mathrm{d}_{un},t_2}} - \Psi^{nk}_r \le 0,\ \forall n, k \in \mathcal{S}_{ujn}(r),\end{aligned} \tag{6.22}$$

where $g^k_{\mathrm{d}_{un},t_1} = f^k_{\mathrm{d}_{un},t_1} + P^k_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{d}_{un}}$, $g^k_{\mathrm{s}_n\mathrm{d}_{un},t_2} = f^k_{\mathrm{d}_{un},t_2} + P^k_{\mathrm{s}_n,t_2} G^k_{\mathrm{s}_n,\mathrm{d}_{un}}$, $g^k_{\mathrm{r}_{jn}\mathrm{d}_{un},t_2} = f^k_{\mathrm{d}_{un},t_2} + P^k_{\mathrm{r}_{jn}} G^k_{\mathrm{r}_{jn},\mathrm{d}_{un}}$ and $g^k_{\mathrm{r}_{jn}} = f^k_{\mathrm{r}_{jn}} + P^k_{\mathrm{s}_n,t_1} G^k_{\mathrm{s}_n,\mathrm{r}_{jn}}$.

Problem (6.22) is still non-convex. In order to solve it, we now propose the SCLD PA algorithm. To derive it, two steps are involved. During the first step, a convex approximation of problem (6.22) is constructed. We first use the method of Lemma 1 in paper [49] to condense all denominator posynomials $\{g^k_{\mathrm{d}_{un},t_1}, g^k_{\mathrm{s}_n\mathrm{d}_{un},t_2}, g^k_{\mathrm{r}_{jn}\mathrm{d}_{un},t_2}, g^k_{\mathrm{r}_{jn}}\}$ into monomials $\{\hat{g}^k_{\mathrm{d}_{un},t_1}, \hat{g}^k_{\mathrm{s}_n\mathrm{d}_{un},t_2}, \hat{g}^k_{\mathrm{r}_{jn}\mathrm{d}_{un},t_2}, \hat{g}^k_{\mathrm{r}_{jn}}\}$ using tentative PA results $\mathbf{P}_0 = \{P^k_{\mathrm{s}_n,t_1,0}, P^k_{\mathrm{s}_n,t_2,0}, P^k_{\mathrm{r}_{jn},0}\}$. Functions $h^{nk}_1 = \log \frac{f^k_{\mathrm{r}_{jn}}}{g^k_{\mathrm{r}_{jn}}} - \Psi^{nk}_r$, $h^{nk}_2 = \log \frac{f^k_{\mathrm{d}_{un},t_2}}{g^k_{\mathrm{r}_{jn}\mathrm{d}_{un},t_2}} - \Psi^{nk}_r$ and $h^{nk}_3 = \sum_n \Big(\sum_{k\in\mathcal{S}_{un}(d)} \log \frac{f^k_{\mathrm{d}_{un},t_1}}{g^k_{\mathrm{d}_{un},t_1}} \frac{f^k_{\mathrm{d}_{un},t_2}}{g^k_{\mathrm{s}_n\mathrm{d}_{un},t_2}} + \sum_{k\in\mathcal{S}_{ujn}(r)} \Psi^{nk}_r \Big)$ are approximated by respectively $\hat{h}^{nk}_1 = \log \frac{f^k_{\mathrm{r}_{jn}}}{\hat{g}^k_{\mathrm{r}_{jn}}} - \Psi^{nk}_r$, $\hat{h}^{nk}_2 = \log \frac{f^k_{\mathrm{d}_{un},t_2}}{\hat{g}^k_{\mathrm{r}_{jn}\mathrm{d}_{un},t_2}} - \Psi^{nk}_r$ and $\hat{h}^{nk}_3 = \sum_n \Big(\sum_{k\in\mathcal{S}_{un}(d)} \log \frac{f^k_{\mathrm{d}_{un},t_1}}{\hat{g}^k_{\mathrm{d}_{un},t_1}} \frac{f^k_{\mathrm{d}_{un},t_2}}{\hat{g}^k_{\mathrm{s}_n\mathrm{d}_{un},t_2}} + \sum_{k\in\mathcal{S}_{ujn}(r)} \Psi^{nk}_r \Big)$. Then, by introducing the logarithmic change of the variables (e.g. $\tilde{x} = \log x$), the approximated problem is formulated as:

$$
\begin{aligned}
&\underset{\tilde{\mathbf{P}},\tilde{\mathbf{\Psi}}_r}{\text{minimize}} \sum_n R^n(\tilde{\mathbf{P}}, \tilde{\mathbf{\Psi}}_r) \\
&s.t.\ C6.3,\ C6.5 - C6.7, \\
&C6.4: \quad \frac{1}{P^n_T} \sum_k \big(e^{\tilde{P}^k_{\mathrm{r}_{jn}}} + e^{\tilde{P}^k_{\mathrm{s}_n,t_1}} + e^{\tilde{P}^k_{\mathrm{s}_n,t_2}}\big) \leq 1, \forall n, \\
&C6.8: \qquad \log \frac{f^k_{\mathrm{r}_{jn}}(e^{\tilde{\mathbf{P}}})}{\hat{g}^k_{\mathrm{r}_{jn}}(e^{\tilde{\mathbf{P}}}) e^{\tilde{\Psi}^{nk}_r}} \leq 0, \forall n, k \in \mathcal{S}_{ujn}(r), \\
&C6.9: \qquad \log \frac{f^k_{\mathrm{d}_{un},t_2}(e^{\tilde{\mathbf{P}}})}{\hat{g}^k_{\mathrm{r}_{jn}\mathrm{d}_{un},t_2}(e^{\tilde{\mathbf{P}}}) e^{\tilde{\Psi}^{nk}_r}} \leq 0, \forall n, k \in \mathcal{S}_{ujn}(r),
\end{aligned}
\tag{6.23}
$$

where $R^n(\tilde{\mathbf{P}}, \tilde{\mathbf{\Psi}}_r)$ is given by:

$$
\sum_{k\in\mathcal{S}_{un}(d)} \log \frac{f^k_{\mathrm{d}_{un},t_1}(e^{\tilde{\mathbf{P}}})}{\hat{g}^k_{\mathrm{d}_{un},t_1}(e^{\tilde{\mathbf{P}}})} \frac{f^k_{\mathrm{d}_{un},t_2}(e^{\tilde{\mathbf{P}}})}{\hat{g}^k_{\mathrm{s}_n\mathrm{d}_{un},t_2}(e^{\tilde{\mathbf{P}}})} + \sum_{k\in\mathcal{S}_{ujn}(r)} \log e^{\tilde{\Psi}^{nk}_r}.
$$

$\tilde{\mathbf{P}}$ denotes the vector of logarithmic power variables, $\tilde{\mathbf{\Psi}}_r$ denotes the vector of logarithmic slack variables $\tilde{\Psi}^{nk}_r$.

Note that the method we used in this step belongs to the class of successive convex approximation methods [48]. After some mathematical calculations, it is easy to verify that all the approximations of problem (6.22) satisfy the three conditions proposed in [48] for the convergence of the successive approximation method. By denoting $\mathbf{\Theta} = \{\mathbf{P}, \mathbf{\Psi}_r\}$, the three conditions are listed as follows:

1. Bounding condition: $\forall \mathbf{\Theta}$,

$$h_i^{n,k}(\mathbf{\Theta}) \leq \tilde{h}_i^{n,k}(\mathbf{\Theta}), \ \forall i = 1,2,3.$$

2. Tightness condition: At the beginning of iteration t,

$$h_i^{n,k}(\mathbf{\Theta}^{m,t-1}) = \tilde{h}_i^{n,k}(\mathbf{\Theta}^{m,t-1}), \ \forall i = 1,2,3.$$

3. Differential condition: At the beginning of iteration t, $\forall \theta \in \mathbf{\Theta}$,

$$\frac{\partial h_i^{n,k}(\mathbf{\Theta}^{m,t-1})}{\partial \theta} = \frac{\partial \tilde{h}_i^{n,k}(\mathbf{\Theta}^{m,t-1})}{\partial \theta}, \ \forall i = 1,2,3.$$

During the second step, the Lagrange dual method is applied to solve the approximated convex problem. Let's denote the Lagrange multipliers of $C6.4$, $C6.5$ and $C6.6$ by $\lambda = \{\lambda_n\}$, $\mu = \{\mu_{nk}\}_{k \in \mathcal{S}_{ujn}(r)}$ and $\nu = \{\nu_{nk}\}_{k \in \mathcal{S}_{ujn}(r)}$ respectively. Then the dual problem of (6.23) is written as $\max\limits_{\lambda,\mu,\nu}(\min\limits_{\tilde{\mathbf{P}},\tilde{\mathbf{\Psi}}_r} L(\lambda,\mu,\nu,\tilde{\mathbf{P}},\tilde{\mathbf{\Psi}}_r))$. Here $L(\lambda,\mu,\nu,\tilde{\mathbf{P}},\tilde{\mathbf{\Psi}}_r)$ denotes the Lagrange of problem (6.16).

After taking the derivative of L w.r.t. $\tilde{\Psi}_r^{nk}$ and setting it to zero, we obtain $\mu_{nk} + \nu_{nk} = 1, \ \forall n,k \in \mathcal{S}_{ujn}(r)$. By applying it, the dual variables are updated with the subgradient method as:

$$\lambda_n^{(i+1)} = \lambda_n^{(i)} + \delta_{\lambda,n}^{(i)} \left(\frac{1}{P_T^n} \sum_k (P_{\mathrm{r}_{jn}}^{k,(i)} + P_{\mathrm{s}_n}^{k,(i)} + P_{\mathrm{s}_n,t_2}^{k,(i)}) - 1 \right), \tag{6.24}$$

$$\mu_{nk}^{(i+1)} = \mu_{nk}^{(i)} + \delta_{\mu,nk}^{(i)} \log \frac{f_{\mathrm{r}_{jn}}^k(\mathbf{P}^{(i)}) \hat{g}_{\mathrm{r}_{jn}\mathrm{d}_{un},t_2}^k(\mathbf{P}^{(i)})}{\hat{g}_{\mathrm{r}_{jn}}^k(\mathbf{P}^{(i)}) f_{\mathrm{d}_{un},t_2}^k(\mathbf{P}^{(i)})}, \tag{6.25}$$

$$\nu_{nk}^{(i+1)} = 1 - \mu_{nk}^{(i+1)}. \tag{6.26}$$

Here, $\delta_{\lambda,n}^{(i)} = \alpha_{\lambda,n}/\sqrt{i}$ and $\delta_{\mu,nk}^{(i)} = \alpha_{\mu,nk}/\sqrt{i}$ are the stepsizes choosen for $\lambda_n^{(i)}$ and $\mu_{nk}^{(i)}$ respectively. We assume that each receiver in a cell n can estimate the received power from each of the interfering stations (either the BS or a RS of an interfering cell). Thus $f_{\mathrm{r}_{jn}}^k(\mathbf{P}^{(i)}), f_{\mathrm{d}_{un},t_2}^k(\mathbf{P}^{(i)}), \hat{g}_{\mathrm{r}_{jn}\mathrm{d}_{un},t_2}^k(\mathbf{P}^{(i)})$ and $\hat{g}_{\mathrm{r}_{jn}}^k(\mathbf{P}^{(i)})$ can be calculated locally in the cell n.

We further take the derivative of L w.r.t. $\tilde{P}_{\mathrm{s}_n,t_1}^k, \tilde{P}_{\mathrm{r}_{jn}}^k$ and $\tilde{P}_{\mathrm{s}_n,t_2}^k$, and force them to zero. With the logarithmic power variables transformed back to the original P-space, fixed-point equations for $P_{\mathrm{s}_n,t_1}^{k,(i)}, \forall n,k$, $P_{\mathrm{r}_{jn}}^{k,(i)}, \forall n,k \in \mathcal{S}_{ujn}(r)$ and $P_{\mathrm{s}_n,t_2}^{k,(i)}, \forall n,k \in \mathcal{S}_{un}(d)$ can be formulated as

$$h_4^{nk}(\lambda_n^{(i)}, \mu^{(i)}, \mathbf{P}^{(i)}) = \frac{X_1^{nk}(\mathbf{P}_0) + X_2^{nk}(\mu^{(i)}, \mathbf{P}_0)}{\frac{\lambda_n^{(i)}}{P_T^n} + Y_1^{nk}(\mathbf{P}^{(i)}) + Y_2^{nk}(\mu^{(i)}, \mathbf{P}^{(i)})}, \tag{6.27}$$

$$h_5^{nk}(\lambda_n^{(i)}, \nu^{(i)}, \mathbf{P}^{(i)}) = \frac{X_3^{nk}(\mathbf{P}_0) + X_4^{nk}(\nu^{(i)}, \mathbf{P}_0)}{\frac{\lambda_n^{(i)}}{P_T^n} + Y_3^{nk}(\mathbf{P}^{(i)}) + Y_4^{nk}(\nu^{(i)}, \mathbf{P}^{(i)})}, \tag{6.28}$$

and

$$h_6^{nk}(\lambda_n^{(i)}, \nu^{(i)}, \mathbf{P}^{(i)}) = \frac{X_5^{nk}(\mathbf{P}_0) + X_6^{nk}(\nu^{(i)}, \mathbf{P}_0)}{\frac{\lambda_n^{(i)}}{P_T^n} + Y_5^{nk}(\mathbf{P}^{(i)}) + Y_6^{nk}(\nu^{(i)}, \mathbf{P}^{(i)})}, \tag{6.29}$$

respectively. Here, $\mu^{(i)} = \{\mu_{nk}^{(i)}\}, \nu^{(i)} = \{\nu_{nk}^{(i)}\}$,

$$X_1^{nk}(\mathbf{P}_0) = \sum_{n',k \in \mathcal{S}_{u'n'}(d)} \frac{P_{\mathrm{s}_n,0}^k G_{\mathrm{s}_n,\mathrm{d}_{u'n'}}^k}{g_{\mathrm{d}_{u'n'},t_1}^k(\mathbf{P}_0)},$$

$$X_2^{nk}(\mu^{(i)}, \mathbf{P}_0) = \sum_{n',k \in \mathcal{S}_{u'j'n'}(r)} \mu_{n'k}^{(i)} \frac{P_{\mathrm{s}_n,0}^k G_{\mathrm{s}_n,\mathrm{r}_{j'n'}}^k}{g_{\mathrm{r}_{j'n'}}^k(\mathbf{P}_0)},$$

$$X_3^{nk}(\mathbf{P}_0) = \sum_{n',k \in \mathcal{S}_{u'n'}(d)} \frac{P_{\mathrm{r}_{jn},0}^k G_{\mathrm{r}_{jn},\mathrm{d}_{u'n'}}^k}{g_{\mathrm{d}_{u'n'},t_2}^k(\mathbf{P}_0)},$$

$$X_4^{nk}(\nu^{(i)}, \mathbf{P}_0) = \sum_{n',k \in \mathcal{S}_{u'j'n'}(r)} \nu_{n'k}^{(i)} \frac{P_{\mathrm{r}_{jn},0}^k G_{\mathrm{r}_{jn},\mathrm{d}_{u'n'}}^k}{g_{\mathrm{d}_{u'n'},t_2}^k(\mathbf{P}_0)},$$

$$X_5^{nk}(\mathbf{P}_0) = \sum_{n',k \in \mathcal{S}_{u'n'}(d)} \frac{P_{\mathrm{s}1_n,0}^k G_{\mathrm{s}_n,\mathrm{d}_{u'n'}}^k}{g_{\mathrm{d}_{u'n'},t_2}^k(\mathbf{P}_0)},$$

$$X_6^{nk}(\nu^{(i)}, \mathbf{P}_0) = \sum_{n',k \in \mathcal{S}_{u'j'n'}(r)} \nu_{n'k}^{(i)} \frac{P_{\mathrm{s}1_n,0}^k G_{\mathrm{s}_n,\mathrm{d}_{u'n'}}^k}{g_{\mathrm{d}_{u'n'},t_2}^k(\mathbf{P}_0)},$$

$$Y_1^{nk}(\mathbf{P}^{(i)}) = \sum_{n' \neq n,k \in \mathcal{S}_{u'n'}(d)} \frac{G_{\mathrm{s}_n,\mathrm{d}_{u'n'}}^k}{f_{\mathrm{d}_{u'n'},t_1}^k(\mathbf{P}^{(i)})},$$

$$Y_2^{nk}(\mu^{(i)}, \mathbf{P}^{(i)}) = \sum_{n' \neq n,k \in \mathcal{S}_{u'j'n'}(r)} \mu_{n'k}^{(i)} \frac{G_{\mathrm{s}_n,\mathrm{r}_{j'n'}}^k}{f_{\mathrm{r}_{j'n'}}^k(\mathbf{P}^{(i)})},$$

$$Y_3^{nk}(\mathbf{P}^{(i)}) = \sum_{n' \neq n,k \in \mathcal{S}_{u'n'}(d)} \frac{G_{\mathrm{r}_{jn},\mathrm{d}_{u'n'}}^k}{f_{\mathrm{d}_{u'n'},t_2}^k(\mathbf{P}^{(i)})},$$

$$Y_4^{nk}(\nu^{(i)}, \mathbf{P}^{(i)}) = \sum_{n' \neq n,k \in \mathcal{S}_{u'j'n'}(r)} \nu_{n'k}^{(i)} \frac{G_{\mathrm{r}_{jn},\mathrm{d}_{u'n'}}^k}{f_{\mathrm{d}_{u'n'},t_2}^k(\mathbf{P}^{(i)})},$$

$$Y_5^{nk}(\mathbf{P}^{(i)}) = \sum_{n' \neq n,k \in \mathcal{S}_{u'n'}(d)} \frac{G_{\mathrm{s}_n,\mathrm{d}_{u'n'}}^k}{f_{\mathrm{d}_{u'n'},t_2}^k(\mathbf{P}^{(i)})},$$

$$Y_6^{nk}(\nu^{(i)}, \mathbf{P}^{(i)}) = \sum_{n' \neq n,k \in \mathcal{S}_{u'j'n'}(r)} \nu_{n'k}^{(i)} \frac{G_{\mathrm{s}_n,\mathrm{d}_{u'n'}}^k}{f_{\mathrm{d}_{u'n'},t_2}^k(\mathbf{P}^{(i)})}.$$

We now consider the following theorem.

Theorem 6.1. $\mathbf{H}(\mathbf{P}) = \{h_4^{nk}(\mathbf{P}), h_5^{nk}(\mathbf{P}), h_6^{nk}(\mathbf{P})\}$ *satisfies the three properties of the standard interference function described in section 2.2.4.*

Proof. Taking equations h_4^{nk} for example, we denote the right-hand-side functions as

$$h_4^{nk}(\mathbf{P}) = \frac{X_1^{nk}(\mathbf{P}_0) + X_2^{nk}(\mu, \mathbf{P}_0)}{\frac{\lambda_n}{P_T^n} + Y_1^{nk}(\mathbf{P}) + Y_2^{nk}(\mu, \mathbf{P})},$$

$$Y_1^{nk}(\mathbf{P}) = \sum_{n' \neq n,k \in \mathcal{S}_{u'n'}(d)} \frac{G_{\mathrm{s}_n,\mathrm{d}_{u'n'}}^k}{\sigma^2 + I_{\mathrm{d}_{u'n'},t_1}^k(\mathbf{P})},$$

$$Y_2^{nk}(\mu, \mathbf{P}) = \sum_{n' \neq n,k \in \mathcal{S}_{u'j'n'}(r)} \mu_{n'k} \frac{G_{\mathrm{s}_n,\mathrm{r}_{j'n'}}^k}{\sigma^2 + I_{\mathrm{r}_{j'n'}}^k(\mathbf{P})},$$

Here the values of $\lambda_n^{(i)}$, $\mu_{n'k}^{(i)}$, $X_1^{nk}(\mathbf{P}_0)$ and $X_2^{nk}(\mu^{(i)}, \mathbf{P}_0)$ are all constant and positive. Therefore it is obvious that $h_4^{nk}(\mathbf{P}) > 0$. Also, both $I_{\mathrm{d}_{u'n'},t_1}^k(\mathbf{P})$ and $I_{\mathrm{r}_{j'n'}}^k(\mathbf{P})$

are linear functions of power variables. Thus it is obvious that $h_4^{nk}(\mathbf{P})$ is a monotonic increasing function of $\mathbf{P}$, meaning that $h_4^{nk}(\mathbf{P}) > h_4^{nk}(\mathbf{P}'), \ \forall \mathbf{P} > \mathbf{P}'$. Here the inequality should be understood as element-wise. Note for all $\alpha > 1$, we have

$$\alpha h_4^{nk}(\mathbf{P}) = \frac{X_1^{nk}(\mathbf{P}_0) + X_2^{nk}(\mu, \mathbf{P}_0)}{\frac{\lambda_n}{\alpha P_T^n} + \frac{Y_1^{nk}(\mathbf{P})}{\alpha} + \frac{Y_2^{nk}(\mu, \mathbf{P})}{\alpha}}$$
$$> \frac{X_1^{nk}(\mathbf{P}_0) + X_2^{nk}(\mu, \mathbf{P}_0)}{\frac{\lambda_n}{P_T^n} + \check{Y}_1^{nk}(\alpha, \mathbf{P}) + \check{Y}_2^{nk}(\alpha, \mu, \mathbf{P})}$$

where,

$$\check{Y}_1^{nk}(\alpha, \mathbf{P}) = \sum_{n' \neq n, k \in \mathcal{S}_{u'n'}(d)} \frac{G^k_{\mathrm{s}_n, \mathrm{d}_{u'n'}}}{\sigma^2 + \alpha I^k_{\mathrm{d}_{u'n'}, t_1}(\mathbf{P})},$$
$$\check{Y}_2^{nk}(\alpha, \mu, \mathbf{P}) = \sum_{n' \neq n, k \in \mathcal{S}_{u'j'n'}(r)} \mu_{n'k} \frac{G^k_{\mathrm{s}_n, \mathrm{r}_{j'n'}}}{\sigma^2 + \alpha I^k_{\mathrm{r}_{j'n'}}(\mathbf{P})}.$$

According to the expression of $I^k_{\mathrm{d}_{un}, t_1}(\mathbf{P})$ and $I^k_{\mathrm{r}_{jn}}(\mathbf{P})$, it is obvious that $\alpha I^k_{\mathrm{d}_{un}, t_1}(\mathbf{P}) = I^k_{\mathrm{d}_{un}, t_1}(\alpha \mathbf{P})$ and $\alpha I^k_{\mathrm{r}_{jn}}(\mathbf{P}) = I^k_{\mathrm{r}_{jn}}(\alpha \mathbf{P})$. Thus we have

$$\alpha h_4^{nk}(\mathbf{P}) > h_4^{nk}(\alpha \mathbf{P}), \forall \alpha > 1, n, k.$$

Similarly, we have $h_5^{nk}(\mathbf{P}) > 0, \alpha h_5^{nk}(\mathbf{P}) > h_5^{nk}(\alpha \mathbf{P}), \forall \alpha > 1, n, k \in \mathcal{S}_{ujn}(r)$ and $h_6^{nk}(\mathbf{P}) > 0, \alpha h_6^{nk}(\mathbf{P}) > h_6^{nk}(\alpha \mathbf{P}), \forall \alpha > 1, n, k \in \mathcal{S}_{un}(d)$. Both $h_5^{nk}(\mathbf{P})$ and $h_6^{nk}(\mathbf{P})$ are monotonic increasing functions of $\mathbf{P}$.

Note that each element of $\mathbf{H}(\mathbf{P})$ satisfies the three properties (positivity, monotonicity, scalability) of the standard interference function defined in [51]. Thus $\mathbf{H}(\mathbf{P})$ is a standard interference function.

□

Thanks to the Theorem 6.1 and Theorem 6.2 in [51], the standard interference function $\mathbf{H}(\mathbf{P})$ has a unique fixed point solution, which can be found iteratively as follows:

$$P^{k,(i,i'+1)}_{\mathrm{s}_n, t_1} = h_4^{nk}(\lambda_n^{(i)}, \mu^{(i)}, \mathbf{P}^{(i,i')}), \ \forall n, k, i, \tag{6.30}$$
$$P^{k,(i,i'+1)}_{\mathrm{r}_{jn}} = h_5^{nk}(\lambda_n^{(i)}, \nu^{(i)}, \mathbf{P}^{(i,i')}), \ \forall n, k \in \mathcal{S}_{ujn}(r), i, \tag{6.31}$$
$$P^{k,(i,i'+1)}_{\mathrm{s}_n, t_2} = h_6^{nk}(\lambda_n^{(i)}, \nu^{(i)}, \mathbf{P}^{(i,i')}), \ \forall n, k \in \mathcal{S}_{un}(d), i. \tag{6.32}$$

Let us denote $\mathbf{X}^{nk,n'} = \{\frac{P^k_{\mathrm{s}n,0}G^k_{\mathrm{s}n,\mathrm{d}_{u'n'}}}{g^k_{\mathrm{d}_{u'n'},t_1}(\mathbf{P}_0)}, \mu^{(i)}_{n'k}\frac{P^k_{\mathrm{s}n,0}G^k_{\mathrm{s}n,\mathrm{r}_{j'n'}}}{g^k_{\mathrm{r}_{j'n'}}(\mathbf{P}_0)}, \nu^{(i)}_{n'k}\frac{P^k_{\mathrm{r}jn,0}G^k_{\mathrm{r}jn,\mathrm{d}_{u'n'}}}{g^k_{\mathrm{d}_{u'n'},t_2}(\mathbf{P}_0)}, \frac{P^k_{\mathrm{r}jn,0}G^k_{\mathrm{r}jn,\mathrm{d}_{u'n'}}}{g^k_{\mathrm{d}_{u'n'},t_2}(\mathbf{P}_0)}, \nu^{(i)}_{n'k}\frac{P^k_{\mathrm{s}1n,0}G^k_{\mathrm{s}n,\mathrm{d}_{u'n'}}}{g^k_{\mathrm{d}_{u'n'},t_2}(\mathbf{P}_0)}, \frac{P^k_{\mathrm{s}1n,0}G^k_{\mathrm{s}n,\mathrm{d}_{u'n'}}}{g^k_{\mathrm{d}_{u'n'},t_2}(\mathbf{P}_0)}\}$, $\mathbf{Y}^{nk,n'} = \{\frac{1}{f^k_{\mathrm{d}_{u'n'},t_1}(\mathbf{P}^{(i)})}, \frac{\mu^{(i)}_{n'k}}{f^k_{\mathrm{r}_{j'n'}}(\mathbf{P}^{(i)})}, \frac{\nu^{(i)}_{n'k}}{f^k_{\mathrm{d}_{u'n'},t_2}(\mathbf{P}^{(i)})}, \frac{G^k_{\mathrm{r}jn,\mathrm{d}_{u'n'}}}{f^k_{\mathrm{d}_{u'n'},t_2}(\mathbf{P}^{(i)})}, \nu^{(i)}_{n'k}\frac{G^k_{\mathrm{s}n,\mathrm{d}_{u'n'}}}{f^k_{\mathrm{d}_{u'n'},t_2}(\mathbf{P}^{(i)})}, \frac{G^k_{\mathrm{s}n,\mathrm{d}_{u'n'}}}{f^k_{\mathrm{d}_{u'n'},t_2}(\mathbf{P}^{(i)})}\}$. As we assume that each receiver in each cell can estimate the received power from each of the interfering stations (either the BS or a RS of an interfering cell), both $\mathbf{X}^{nk,n'}$ and $\mathbf{Y}^{nk,n'}$ can be calculated locally in each interfering cell n' for a considered cell n. We further assume that the central controller (CC) has access to estimates of all the cross gains ($G^k_{\mathrm{s}n,\mathrm{d}_{u'n'}}, G^k_{\mathrm{s}n,\mathrm{r}_{j'n'}}, G^k_{\mathrm{r}jn,\mathrm{d}_{u'n'}}$) between two different cells. Thus $\mathbf{X}^n = \{X^{nk}_1, X^{nk}_2, X^{nk}_3, X^{nk}_4, X^{nk}_5, X^{nk}_6\}$ and $\mathbf{Y}^n = \{Y^{nk}_1, Y^{nk}_2, Y^{nk}_3, Y^{nk}_4, Y^{nk}_5, Y^{nk}_6\}$ can be calculated in the CC with the uploaded values of $\mathbf{X}^{nk,n'}$ and $\mathbf{Y}^{nk,n'}$ from each cell. Finally, the SCLD PA algorithm is described in Algorithm 6.2. Note that, through estimation of the received power and message passing, the SCLD PA algorithm is semi-distributed with the help of a CC.

6.3 Modified IWF algorithm

In order to solve the formulated problem autonomously in each cell, we now propose the modified IWF algorithm, which is inspired by the non-cooperative game-theoretic approach (named as the IWF algorithm) proposed by [75] in digital subscriber lines (DSL) systems. In there, with the interfering users' power values fixed, each user measures the interference plus noise levels locally and allocates its power greedily to maximize its own rate. Although its convergence is guaranteed under limited cases where the crosstalk interferences are weak, the IWF algorithm has become a popular candidate for multicell power allocation due to its interesting trade-off between the complexity and the performance.

In this section, we mimic the idea of the IWF algorithm and formulate a competitive game defining each cell as a player, which optimizes its own utility function (meaning the sum rate of this cell) given the RA results of the other cells. In each cell, both the indicators and the power variables are optimized which makes the local problem more interesting than that of [75]. The

Algorithm 6.2 SCLD PA Algorithm

Input: RA results of outer iteration $m-1$, i.e. $\mathbf{P}^{m-1}$, $\mathbf{A}^m$ and $\mathbf{B}^m$;
Initialize: set $t = 0$, $\mathbf{P}_0 = \mathbf{P}^{m-1}$, initialize $\mathcal{S}_{ujn}(r)$ and $\mathcal{S}_{un}(d)$ using $\mathbf{A}^m$ and $\mathbf{B}^m$;
repeat
  Initialize: set $i = 0$, $\mu_{nk}^{(i)} = \mu_0, \forall n, k \in \mathcal{S}_{ujn}(r)$, $\lambda_n^{(i)} = \lambda_0, \forall n, k$. In each cell n', estimate the received power from each of the interfering stations, calculate $\mathbf{X}^{nk,n'}, \mathbf{Y}^{nk,n'}, \forall k, n \neq n'$ and send them to the CC. In the CC, calculate $\mathbf{X}^n, \mathbf{Y}^n$, $\forall n$, distribute them to each corresponding cell n;
  repeat
    $i = i + 1$;
    Update power:
    Initialize: set $i' = 0$ and $\mathbf{P}^{(i,i')} = \mathbf{P}_0$;
    repeat
      $i' = i' + 1$;
      In each cell n, update power variables $\mathbf{P}_n^{(i,i')}$ using equations (6.30), (6.31) and (6.32) with $\mathbf{X}^n, \mathbf{Y}^n$; estimate the received power from each of the interfering stations, calculate $\mathbf{X}^{n'k,n}, \mathbf{Y}^{n'k,n}, \forall k, n' \neq n$ and send them to the CC;
      In the CC, update $\mathbf{X}^n, \mathbf{Y}^n$ with $\mathbf{X}^{nk,n'}, \mathbf{Y}^{nk,n'}$ and distribute them to each corresponding cell n;
    until $i' = I'$
    Update Lagrange factors: Update Lagrange factors $\{\lambda^{(i)}, \mu^{(i)}, \nu^{(i)}\}$ distributively in each cell using equations (6.24), (6.25) and (6.26);
  until $||\lambda^{(i)} - \lambda^{(i-1)}|| \leq \epsilon_2$
  $t = t + 1$;
  Assign the results $\mathbf{P}^{(i)}$ to $\mathbf{P}_0$;
until $||\mathbf{R}^{m,t} - \mathbf{R}^{m,t-1}|| \leq \epsilon_3$
Output: $\mathbf{P}^m = \mathbf{P}_0$.

CCI received by the cell under consideration is fixed and treated as an additive noise. The received power of the interference and noise at each subcarrier k of each receiver (either a RS or a MS) in cell n is a constant. Specifically, $f^k_{\mathrm{r}_{jn}} = \sigma^2 + \sum_{n', n' \neq n} P^k_{\mathrm{s}_{n'}, t_1} G^k_{\mathrm{s}_{n'}, \mathrm{r}_{jn}}, \forall j, k,\ f^k_{\mathrm{d}_{un}, t_2} = \sigma^2 + \sum_{n'=1, n' \neq n}^{N} P^k_{\mathrm{s}_{n'}, t_2} G^k_{\mathrm{s}_{n'}, \mathrm{d}_{un}} + \sum_{n'=1, n' \neq n}^{N} \sum_{j'=1}^{J} P^k_{\mathrm{r}_{j'n'}} G^k_{\mathrm{r}_{j'n'}, \mathrm{d}_{un}}, \forall u, k$ and $f^k_{\mathrm{d}_{un}, t_1} = \sigma^2 + \sum_{n'=1, n' \neq n}^{N} P^k_{\mathrm{s}_{n'}, t_1} G^k_{\mathrm{s}_{n'}, \mathrm{d}_{un}}, \forall u, k.$ Thus, the achievable sum rate of cell n is denoted as:

$$R_n \left(\mathbf{P}_n, \mathbf{A}_n, \mathbf{B}_n \right) = \sum_{u=1}^{U} \sum_{k=1}^{K} a^k_{un} R^k_{un,1} \left(\mathbf{P}^k_n \right) + \sum_{u=1}^{U} \sum_{j=1}^{J} \sum_{k=1}^{K} b^k_{ujn} R^k_{ujn,2} \left(\mathbf{P}^k_n \right). \tag{6.33}$$

where $\mathbf{A}_n = \{a^k_{un'} | n' = n\}$, $\mathbf{B}_n = \{b^k_{ujn'} | n' = n\}$, $R^k_{un,1} = \ln \left(1 + \Upsilon^k_{\mathrm{d}_{un}, t_1} \right) + \ln \left(1 + \Upsilon^k_{\mathrm{d}_{un}, t_2} \right)$, $R^k_{ujn,2} = \min \left(\ln \left(1 + \Gamma^k_{\mathrm{r}_{jn}} \right), \ln \left(1 + \Gamma^k_{\mathrm{d}_{un}, t_2} \right) \right)$, $\Upsilon^k_{\mathrm{d}_{un}, t_1} = P^k_{\mathrm{s}_n, t_1} \Lambda^{k, t_1}_{\mathrm{s}_n, \mathrm{d}_{un}}$, $\Upsilon^k_{\mathrm{d}_{un}, t_2} = P^k_{\mathrm{s}_n, t_2} \Lambda^{k, t_2}_{\mathrm{s}_n, \mathrm{d}_{un}}$, $\Gamma^k_{\mathrm{r}_{jn}} = P^k_{\mathrm{s}_n, t_1} \Lambda^k_{\mathrm{s}_n, \mathrm{r}_{jn}}$, $\Gamma^k_{\mathrm{d}_{un}, t_2} = P^k_{\mathrm{r}_{jn}} \Lambda^k_{\mathrm{r}_{jn}, \mathrm{d}_{un}}$,

$$\Lambda^{k, t_1}_{\mathrm{s}_n, \mathrm{d}_{un}} = \frac{G^k_{\mathrm{s}_n, \mathrm{d}_{un}}}{f^k_{\mathrm{d}_{un}, t_1}}, \tag{6.34}$$

$$\Lambda^{k, t_2}_{\mathrm{s}_n, \mathrm{d}_{un}} = \frac{G^k_{\mathrm{s}_n, \mathrm{d}_{un}}}{f^k_{\mathrm{d}_{un}, t_2}}, \tag{6.35}$$

$$\Lambda^k_{\mathrm{s}_n, \mathrm{r}_{jn}} = \frac{G^k_{\mathrm{s}_n, \mathrm{r}_{jn}}}{f^k_{\mathrm{r}_{jn}}}, \tag{6.36}$$

$$\Lambda^k_{\mathrm{r}_{jn}, \mathrm{d}_{un}} = \frac{G^k_{\mathrm{r}_{jn}, \mathrm{d}_{un}}}{f^k_{\mathrm{d}_{un}, t_2}}. \tag{6.37}$$

The local problem of cell n can be formulated as:

$$
\begin{aligned}
&\max_{\mathbf{P}_n,\mathbf{A}_n,\mathbf{B}_n} \; R_n\left(\mathbf{P}_n,\mathbf{A}_n,\mathbf{B}_n\right)\\
&s.t.\\
&C6.1':\;\left(\sum_{u=1}^{U} a_{un}^{k}+\sum_{u=1}^{U}\sum_{j=1}^{J} b_{ujn}^{k}\right)\leq 1,\;\forall k,\\
&C6.2':\;a_{un}^{k}\in\{0,1\}, b_{ujn}^{k}\in\{0,1\},\;\forall u,j,k,\\
&C6.3':\;\sum_{k=1}^{K} P_{\mathrm{s}_n,t_1}^{k}+\sum_{k=1}^{K} P_{\mathrm{s}_n,t_2}^{k}+\sum_{j=1}^{J}\sum_{k=1}^{K} P_{\mathrm{r}_{jn}}^{k}\leq P_{t,n},\\
&C6.4':\;P_{\mathrm{s}_n,t_1}^{k}\geq 0, P_{\mathrm{s}_n,t_2}^{k}\geq 0, P_{\mathrm{r}_{jn}}^{k}\geq 0,\;\forall j,k\\
&C6.5':\;P_{\mathrm{s}_n,t_1}^{k}=F\left(P_{\mathrm{s}_n,t_1}^{k},\left(\sum_{u=1}^{U} a_{un}^{k}+\sum_{u=1}^{U}\sum_{j=1}^{J} b_{ujn}^{k}\right)\right),\;\forall k,\\
&C6.6':\;P_{\mathrm{s}_n,t_2}^{k}=F\left(P_{\mathrm{s}_n,t_2}^{k},\sum_{u=1}^{U} a_{un}^{k}\right),\;\forall k,\\
&C6.7':\;P_{\mathrm{r}_{jn}}^{k}=F\left(P_{\mathrm{r}_{jn}}^{k},\sum_{u=1}^{U} b_{ujn}^{k}\right),\;\forall j,k.
\end{aligned}
\tag{6.38}
$$

Note that, in the case when $f_{\mathrm{d}_{un},t_1}^{k}=f_{\mathrm{d}_{un},t_2}^{k}$, the optimal RA algorithm proposed in section 3 of [16] can be applied here with simple extension. However, when $f_{\mathrm{d}_{un},t_1}^{k}\neq f_{\mathrm{d}_{un},t_2}^{k}$, the optimization problem becomes more interesting and the algorithm of [16] can not be applied here easily. To find the optimal RA result for the formulated problem, we will first propose the optimal PA algorithm followed by the optimal MSSA algorithm. To obtain the optimal PA result, we now propose two theorems as follows.

Theorem 6.2. *For a relayed subcarrier k in cell n, the DF constraint is active at the optimum point of problem (6.38), meaning that* $\ln\left(1+\Gamma_{\mathrm{r}_{jn}}^{k}\right)=\ln\left(1+\Gamma_{\mathrm{d}_{un},t_2}^{k}\right)$.

Proof. We assume that the optimal RA scheme is obtained, leading to the optimal sum rate. For a relayed subcarrier k in cell n, the optimal MS u and RS j are fixed. The optimal BS power and the optimal RS power are denoted as $P_{\mathrm{s}_n,t_1}^{k,opt}$ and $P_{\mathrm{r}_{jn}}^{k,opt}$ respectively. Then we have

$$\Gamma_{\mathrm{r}_{jn}}^{k}=P_{\mathrm{s}_n,t_1}^{k,opt}\Lambda_{\mathrm{s}_n,\mathrm{r}_{jn}}^{k}, \tag{6.39}$$

$$\Gamma^k_{\mathrm{d}_{un},t_2} = P^{k,opt}_{\mathrm{r}_{jn}} \Lambda^k_{\mathrm{r}_{jn},\mathrm{d}_{un}}. \tag{6.40}$$

If $\Gamma^k_{\mathrm{r}_{jn}} > \Gamma^k_{\mathrm{d}_{un},t_2}$, then $R^k_{ujn,2}$ is determined only by $\Gamma^k_{\mathrm{d}_{un},t_2}$. Note that both $\Lambda^k_{\mathrm{s}_n,\mathrm{r}_{jn}}$ and $\Lambda^k_{\mathrm{r}_{jn},\mathrm{d}_{un}}$ are constants. Thus $R^k_{ujn,2}$ is determined only by $P^{k,opt}_{\mathrm{r}_{jn}}$. In this case, we can still improve the sum rate R_n by borrowing some power from $P^{k,opt}_{\mathrm{s}_n,t_1}$ to $P^{k,opt}_{\mathrm{r}_{jn}}$ while keeping $\Gamma^k_{\mathrm{r}_{jn}} > \Gamma^k_{\mathrm{d}_{un},t_2}$. This obviously contradicts the previous optimal sum rate assumption.

Similarly, if $\Gamma^k_{\mathrm{r}_{jn}} < \Gamma^k_{\mathrm{d}_{un},t_2}$, the optimal sum rate R_n can also be improved by borrowing some power from $P^{k,opt}_{\mathrm{r}_{jn}}$ to $P^{k,opt}_{\mathrm{s}_n,t_1}$ while keeping the inequality. This again contradicts the previous optimal sum rate assumption. Therefore, at the optimal point, $\Gamma^k_{\mathrm{r}_{jn}} = \Gamma^k_{\mathrm{d}_{un},t_2}$ and $\ln\left(1+\Gamma^k_{\mathrm{r}_{jn}}\right) = \ln\left(1+\Gamma^k_{\mathrm{d}_{un},t_2}\right)$. This concludes the proof of Theorem 6.2. □

According to Theorem 6.2, we now have $\Gamma^k_{\mathrm{r}_{jn}} = \Gamma^k_{\mathrm{d}_{un},t_2}$, which means:

$$P^k_{\mathrm{s}_n,t_1} = P^k_{\mathrm{r}_{jn}} \alpha^k_{un} = P^k_n \frac{\alpha^k_{un}}{1+\alpha^k_{un}}, \quad \forall k \in S_n(r) \tag{6.41}$$

$$\alpha^k_{un} = \frac{\Lambda^k_{\mathrm{r}_{jn},\mathrm{d}_{un}}}{\Lambda^k_{\mathrm{s}_n,\mathrm{r}_{jn}}}, \quad \forall k \in S_n(r) \tag{6.42}$$

$$P^k_n = \begin{cases} P^k_{\mathrm{s}_n,t_1} + P^k_{\mathrm{s}_n,t_2} & k \in S_n(d) \\ P^k_{\mathrm{s}_n,t_1} + P^k_{\mathrm{r}_{jn}} & k \in S_n(r) \end{cases} \tag{6.43}$$

where $S_n(d)$ denotes the set of subcarriers with direct mode in cell n, $S_n(r)$ denotes the set of subcarriers with relay aided mode in cell n.

Theorem 6.3. *For a direct mode subcarrier k in cell n, at the optimum point of problem (6.38), we have*

$$P^{k,opt}_{\mathrm{s}_n,t_1} = [\frac{P^{k,opt}_n}{2} + \frac{\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}} - \Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}}{2\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}} \Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}}]_+, \tag{6.44}$$

$$P^{k,opt}_{\mathrm{s}_n,t_2} = [\frac{P^{k,opt}_n}{2} + \frac{\Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}} - \Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}}}{2\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}} \Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}}]_+. \tag{6.45}$$

Here $[.]_+$ stands for max $[0,.]$ and $P^{k,opt}_n$ denotes the optimal value of P^k_n for problem (6.38).

Proof. Let us introduce the following problem:

$$\begin{aligned} &\max_{P^k_{\mathrm{s}_n,t_1}, P^k_{\mathrm{s}_n,t_2}} \quad R^k_{un,1}\left(P^k_{\mathrm{s}_n,t_1}, P^k_{\mathrm{s}_n,t_2}\right) \\ &s.t.\ P^k_{\mathrm{s}_n,t_1} + P^k_{\mathrm{s}_n,t_2} = P^{k,opt}_n, \end{aligned} \tag{6.46}$$

where $R^k_{un,1} = \ln\left(1 + P^k_{\mathrm{s}_n,t_1}\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}}\right) + \ln\left(1 + P^k_{\mathrm{s}_n,t_2}\Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}\right)$. By introducing a Lagrange multiplier ζ, the Lagrange function of problem (6.46) is denoted as

$$L = R^k_{un,1} - \zeta(P^k_{\mathrm{s}_n,t_1} + P^k_{\mathrm{s}_n,t_2} - P^{k,opt}_n). \tag{6.47}$$

After taking the derivative of L w.r.t $P^k_{\mathrm{s}_n,t_1}$ and setting it to zero, we certainly have $\frac{\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}}}{1+\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}}P^k_{\mathrm{s}_n,t_1}} = \zeta$. Similarly, we have $\frac{\Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}}{1+\Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}P^k_{\mathrm{s}_n,t_2}} = \zeta$. Considering the sum power constraint, we can easily calculate the optimal values for $P^k_{\mathrm{s}_n,t_1}$ and $P^k_{\mathrm{s}_n,t_2}$ of problem (6.46) as $[\frac{P^{k,opt}_n}{2} + \frac{\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}} - \Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}}{2\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}}\Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}}]_+$ and $[\frac{P^{k,opt}_n}{2} + \frac{\Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}} - \Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}}}{2\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}}\Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}}]_+$ respectively.

Thus for a direct mode subcarrier k in cell n, at the optimum point of problem (6.38), we have $P^{k,opt}_{\mathrm{s}_n,t_1} = [\frac{P^{k,opt}_n}{2} + \frac{\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}} - \Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}}{2\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}}\Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}}]_+$ and $P^{k,opt}_{\mathrm{s}_n,t_2} = [\frac{P^{k,opt}_n}{2} + \frac{\Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}} - \Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}}}{2\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}}\Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}}]_+$. Otherwise, we can still improve $R^k_{un,1}$ and R_n. This concludes the proof of Theorem 6.3. $\square$

According to Theorem 6.2, the rate of a relayed subcarrier ($R^k_{ujn,2}$) when allocated to a MS d_{un} with the help of a RS r_{jn} in cell n is denoted as

$$\ln\left(1 + P^k_n \frac{\Lambda^k_{\mathrm{s}_n,\mathrm{r}_{jn}}\alpha^k_{un}}{1+\alpha^k_{un}}\right). \tag{6.48}$$

Also according to Theorem 6.3, the rate of a direct mode subcarrier ($R^k_{un,1}$) when allocated to a MS d_{un} in cell n is denoted as

$$\begin{aligned} &\ln\left(1 + \frac{P^{k,opt}_n\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}}}{2} + \frac{\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}} - \Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}}{2\Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}}\right) \\ &+ \ln\left(1 + \frac{P^{k,opt}_n\Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}}{2} + \frac{\Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}} - \Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}}}{2\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}}}\right), \end{aligned} \tag{6.49}$$

which can be further denoted as

$$\ln\left(1 + P_n^k \frac{\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1} + \Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}}{2} + (P_n^k)^2 \frac{\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1}\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}}{4} + \frac{(\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1} - \Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2})^2}{4\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1}\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}}\right). \tag{6.50}$$

Let us first propose the PA algorithm with the MSSA results fixed. By introducing a Lagrange multiplier γ_n, the Lagrange function is denoted as

$$L_n = R_n - \gamma_n \left(\sum_{k=1}^{N_t} P_n^k - P_t^n\right). \tag{6.51}$$

For a direct mode subcarrier k, after taking the derivative of L_n w.r.t. $P_n^k, k \in S_n(d)$ and setting $-4\frac{\partial L_n}{\partial P_n^k} = 0$, we have $F_2(P_n^k) = 0$, where $F_2(P_n^k)$ is denoted as follow.

$$\begin{aligned}
&(P_n^k)^2\gamma_n\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1}\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}\\
&+ P_n^k[2\gamma_n(\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1} + \Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}) - 2\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1}\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}]\\
&+ \frac{\gamma_n(\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1} + \Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2})^2}{\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1}\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}}\\
&- 2(\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1} + \Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}).
\end{aligned}$$

After some mathematical calculations, the discriminant of $F_2(P_n^k)$ is denoted as $4(\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1}\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2})^2$. The roots are denoted as $\frac{2}{\gamma_n} - \frac{\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1} + \Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}}{\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1}\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}}$ and $-\frac{\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1} + \Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}}{\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1}\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}}$. Note that $-\frac{\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1} + \Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}}{\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1}\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}} < 0$. Thus by setting $F_2(P_n^k)$ to zero, the allocated total power for the direct mode subcarrier k in cell n is given by:

$$P_n^k = \left[\frac{2}{\gamma_n} - \frac{\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1} + \Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}}{\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1}\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}}\right]_+ \tag{6.52}$$

where $[.]_+$ stands for max $[0, .]$.

For a relayed subcarrier k, after taking the derivative of L_n w.r.t. $P_n^k, k \in S_n(r)$ and setting it to zero, we obtain

$$P_n^k = \left[\frac{1}{\gamma_n} - \frac{1 + \alpha_{un}^k}{\Lambda_{\mathrm{s}_n,\mathrm{r}_{jn}}^k \alpha_{un}^k}\right]_+. \tag{6.53}$$

The derivations show that the constrained optimization problem can be solved using the waterfilling algorithm, applied to a water container built either from $\frac{\Lambda^{k,t_1}_{s_n,d_{un}}+\Lambda^{k,t_2}_{s_n,d_{un}}}{\Lambda^{k,t_1}_{s_n,d_{un}}\Lambda^{k,t_2}_{s_n,d_{un}}}$ or from $\frac{1+\alpha^k_{un}}{\Lambda^k_{s_n,r_{jn}}\alpha^k_{un}}$. With the direct mode subcarrier, the water lever is $\frac{2}{\gamma_n}$. While with the relayed subcarrier, the water lever is $\frac{1}{\gamma_n}$.

We now propose the optimal MSSA algorithm. Let us first assume that the optimal transmission mode of each subcarrier is decided. $u^k_{opt}(d)$ is introduced to denote the optimal user allocated for a direct mode subcarrier k. $u^k_{opt}(r)$ and $j^k_{opt}(r)$ are introduced to denote the optimal user and RS allocated for a relay aided mode subcarrier k. According to the waterfilling feature of the PA algorithm, we now have:

$$u^k_{opt}(d) = \arg\max_{u} \frac{\Lambda^{k,t_1}_{s_n,d_{un}} + \Lambda^{k,t_2}_{s_n,d_{un}}}{\Lambda^{k,t_1}_{s_n,d_{un}}\Lambda^{k,t_2}_{s_n,d_{un}}}, \ \forall k \in S_n(d), \tag{6.54}$$

$$(u^k_{opt}(r), j^k_{opt}(r)) = \arg\max_{u,j} \frac{1+\alpha^k_{un}}{\Lambda^k_{s_n,r_{jn}}\alpha^k_{un}}, \ \forall k \in S_n(r). \tag{6.55}$$

With $u^k_{opt}(d)$ and $(u^k_{opt}(r), j^k_{opt}(r))$ calculated for each subcarrier k, we now propose the optimal mode selection method for the subcarrier. Let us define

$$c^k_{2,n} = \frac{\Lambda^k_{s_n,r_{jn}}\alpha^k_{u'n}}{1+\alpha^k_{u'n}} - \frac{\Lambda^{k,t_1}_{s_n,d_{un}} + \Lambda^{k,t_2}_{s_n,d_{un}}}{2}, \tag{6.56}$$

where indices u and (u', j) are assigned with $u^k_{opt}(d)$ and $(u^k_{opt}(r), j^k_{opt}(r))$ respectively. When $c^k_{2,n} < 0$, we certainly have $R^k_{un,1} > R^k_{ujn,2}$, meaning that the direct mode should be chosen for this subcarrier k. On the contrary, when $c^k_{2,n} > 0$, we define $F_1(P^k_n) = P^k_n \frac{\Lambda^{k,t_1}_{s_n,d_{un}}+\Lambda^{k,t_2}_{s_n,d_{un}}}{2} + (P^k_n)^2 \frac{\Lambda^{k,t_1}_{s_n,d_{un}}\Lambda^{k,t_2}_{s_n,d_{un}}}{4} + \frac{(\Lambda^{k,t_1}_{s_n,d_{un}}-\Lambda^{k,t_2}_{s_n,d_{un}})^2}{4\Lambda^{k,t_1}_{s_n,d_{un}}\Lambda^{k,t_2}_{s_n,d_{un}}} - P^k_n \frac{\Lambda^k_{s_n,r_{jn}}\alpha^k_{u'n}}{1+\alpha^k_{u'n}}$. If $F_1(P^k_n) < 0$, we certainly have $R^k_{un,1} < R^k_{ujn,2}$, meaning that the relay aided mode should be chosen for this subcarrier k. Otherwise, we still have $R^k_{un,1} > R^k_{ujn,2}$, meaning that the direct mode should be chosen for this subcarrier k. Let us further denote $F_1(P^k_n) = (P^k_n)^2 c^k_{1,n} - P^k_n c^k_{2,n} + \frac{c^k_{3,n}}{4c^k_{1,n}} < 0$, where

$$c^k_{1,n} = \frac{\Lambda^{k,t_1}_{s_n,d_{un}}\Lambda^{k,t_2}_{s_n,d_{un}}}{4}, \tag{6.57}$$

$$c^k_{3,n} = \frac{(\Lambda^{k,t_1}_{s_n,d_{un}} - \Lambda^{k,t_2}_{s_n,d_{un}})^2}{4}. \tag{6.58}$$

Here index u is assigned with $u^k_{opt}(d)$. Note that $F_1(P^k_n)$ is a quadratic function. $c^k_{1,n} > 0, c^k_{2,n} > 0, c^k_{3,n} > 0$, the discriminant of $F_1(P^k_n)$ is $(c^k_{2,n})^2 - 4c^k_{1,n}\frac{c^k_{3,n}}{4c^k_{1,n}} = (c^k_{2,n})^2 - c^k_{3,n}$ and the roots of $F_1(P^k_n)$ are $\frac{c^k_{2,n}+\sqrt{(c^k_{2,n})^2-c^k_{3,n}}}{2c^k_{1,n}} > 0$ and $\frac{c^k_{2,n}-\sqrt{(c^k_{2,n})^2-c^k_{3,n}}}{2c^k_{1,n}} > 0$. Thus $F_1(P^k_n) < 0$ is valid only for $(c^k_{2,n})^2 > c^k_{3,n}$ and

$$\begin{aligned}\frac{c^k_{2,n} - \sqrt{(c^k_{2,n})^2 - c^k_{3,n}}}{2c^k_{1,n}} &< P^k_n \\ &< \frac{c^k_{2,n} + \sqrt{(c^k_{2,n})^2 - c^k_{3,n}}}{2c^k_{1,n}}.\end{aligned} \tag{6.59}$$

Considering (6.56) and (6.58), $(c^k_{2,n})^2 > c^k_{3,n}$ is equivalent to $c^k_{4,n} > c^k_{5,n}$, where

$$c^k_{4,n} = \frac{\Lambda^k_{\mathrm{s}_n,\mathrm{r}_{jn}}\alpha^k_{u'n}}{1+\alpha^k_{u'n}} \tag{6.60}$$

$$c^k_{5,n} = \max\{\frac{\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}}}{2}, \frac{\Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}}{2}\}. \tag{6.61}$$

Here indices u and (u', j) are assigned with $u^k_{opt}(d)$ and $(u^k_{opt}(r), j^k_{opt}(r))$ respectively.

As depicted by Algorithm 6.3, we now propose an optimal RA algorithm inspired from [16] for each cell n given the value of $\mathbf{\Lambda}^k_n = [\Lambda^{k,t_1}_{\mathrm{s}_n,\mathrm{d}_{un}}, \Lambda^{k,t_2}_{\mathrm{s}_n,\mathrm{d}_{un}}, \Lambda^k_{\mathrm{s}_n,\mathrm{r}_{jn}}, \Lambda^k_{\mathrm{r}_{jn},\mathrm{d}_{un}}]^T$. Specifically, we first obtain $u^k_{opt}(d)$ and $(u^k_{opt}(r), j^k_{opt}(r))$ by (6.54) and (6.55) respectively. Then we calculate $c^k_{4,n}, c^k_{5,n}, \forall k$. If $c^k_{4,n} \leq c^k_{5,n}$, the direct mode should be chosen for the subcarrier k. Otherwise, the relay aided mode is still an option and tentatively chosen for this subcarrier k. Then with the tentative MSSA results, we carry out the proposed waterfilling PA algorithm, as depicted in (6.52) and (6.53). With the tentative PA results, we further check the power criterion (6.59) for each relayed subcarrier. If (6.59) is fulfilled, we keep the mode for the relayed subcarrier. Out of all subcarriers violating (6.59), we only change the mode of the subcarrier k which gives rise to the largest rate increase when switched to the direct mode. The PA algorithm is applied again. This procedure is iterated until none of the relayed subcarrier violates (6.59).

We now propose the modified IWF algorithm, which consists of two stages as depicted in Algorithm 2. Specifically, we first set $m' = 0$ and initialize the

Algorithm 6.3 Local RA Algorithm For Cell n

1: **Initialize:** Set $m = 0$. $\forall k$, find $u_{opt}^k(d)$ and $(u_{opt}^k(r), j_{opt}^k(r))$ with the given value of $\mathbf{\Lambda}_n^k = [\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1}, \Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}, \Lambda_{\mathrm{s}_n,\mathrm{r}_{jn}}^{k}, \Lambda_{\mathrm{r}_{jn},\mathrm{d}_{un}}^{k}]^T$ by (6.54) and (6.55) respectively. Calculate $c_{4,n}^k$ and $c_{5,n}^k$. If $c_{4,n}^k \leq c_{5,n}^k$, set $k \in S_n(d)$. Otherwise, set $k \in S_n(r)$. Obtain the initial MSSA results $\mathbf{A}_n^m$ and $\mathbf{B}_n^m$.

2: **repeat**

3: $m = m + 1$;

4: **The PA stage:** Compute the optimized PA results $\mathbf{P}_n^m$ of (6.38) by (6.52) and (6.53) with the tentative MSSA results $\mathbf{A}_n^{m-1}$ and $\mathbf{B}_n^{m-1}$;

5: **The MSSA adjusting stage:** $\forall k \in S_n(r)$, check if $\mathbf{P}_n^{k,m}$ satisfies (6.59). If it is true, keep $k \in S_n(r)$ and calculate the corresponding rate $R_n^{(k)}$ with $\mathbf{A}_n^{m-1}$, $\mathbf{B}_n^{m-1}$ and $\mathbf{P}_n^m$. Otherwise, set $k \in S_n(d)$ and calculate $R_n^{(k)}$ after recomputing the PA results by (6.52) and (6.53). Finally, set $k_0 \in S_n(d)$, where $k_0 = \arg \max\limits_{k \in S_n(r)} R_n^{(k)}$ and update the tentative MSSA results $\mathbf{A}_n^m$ and $\mathbf{B}_n^m$.

6: **until** (6.59) is fulfilled, $\forall k \in S_n(r)$

7: **Output:** $\mathbf{A}_n^m$, $\mathbf{B}_n^m$ and $\mathbf{P}_n^m$.

PA results either by the UPA algorithm or with zeros. During the CCI updating stage of iteration m', $\mathbf{\Lambda}_n$ of each cell n can be calculated with both the local power $\mathbf{P}_n^{(m'-1)}$ and the received CCI power $\mathbf{Q}_n^{(m'-1)}$ at each station of the cell. Note that, in practice, the received CCI power values of a cell n can be estimated at each local receiver of the cell. Thus the CCI updating stage can be carried out autonomously in each cell. During the RA stage, the proposed Algorithm 6.3 is applied in each cell to calculate its local RA results. The two stages are carried out iteratively until the iteration number m' exceeds a prescribed value M'. Note that, as both stages can be executed autonomously in each cell, the proposed modified IWF algorithm is autonomous. Although it seems intractable to derive conditions under which the Modified-IWF algorithm provably converges, the convergence is always observed in our experiments. As will be illustrated in Section V, the Modified-IWF algorithm is recommended as it provides a good trade-off between the performance and the complexity.

Algorithm 6.4 Overall RA Optimization Algorithm

1: **Initialize:** set $m' = 0$ and initialize $\mathbf{P}^{(m')} = \{\mathbf{P}_n^{(m')}, \mathbf{P}_{-n}^{(m')}\}$ either by the UPA algorithm or with zeros;
2: **repeat**
3: $m' = m' + 1$;
4: **The CCI updating stage:** Estimate the received CCI power ($\mathbf{Q}_n^{(m'-1)}$) at each subcarrier of each receiver in each cell. Compute $\mathbf{\Lambda}_n^{(m')}, \forall n$ with $\mathbf{P}_n^{(m'-1)}$ and $\mathbf{Q}_n^{(m'-1)}$, where $\mathbf{\Lambda}_n^{(m')} = [[\mathbf{\Lambda}_n^{1,(m')}]^T, ..., [\mathbf{\Lambda}_n^{K,(m')}]^T]^T, \mathbf{\Lambda}_n^k = [\Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_1}, \Lambda_{\mathrm{s}_n,\mathrm{d}_{un}}^{k,t_2}, \Lambda_{\mathrm{s}_n,\mathrm{r}_{jn}}^{k}, \Lambda_{\mathrm{r}_{jn},\mathrm{d}_{un}}^{k}]^T$;
5: **The RA stage:** $\forall n$, compute $\mathbf{A}_n^{(m')}$, $\mathbf{B}_n^{(m')}$ and $\mathbf{P}_n^{(m')}$ using Algorithm 6.3 with $\mathbf{\Lambda}_n^{(m')}$;
6: **until** $m' = M'$
7: **Output:** $\mathbf{A}^{(m')}$, $\mathbf{B}^{(m')}$ and $\mathbf{P}^{(m')}$.

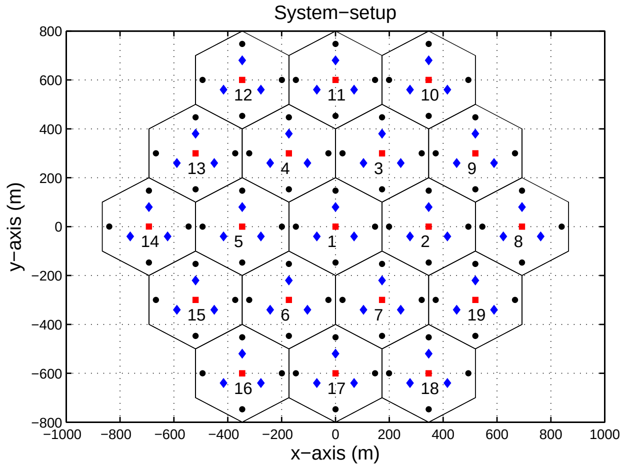


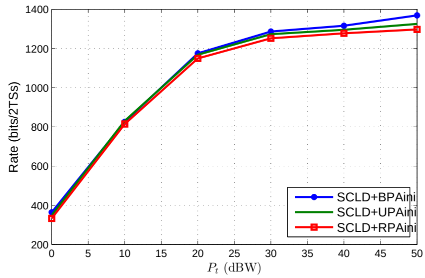
Figure 6.1 A random system setup.

6.4 Numerical experiments

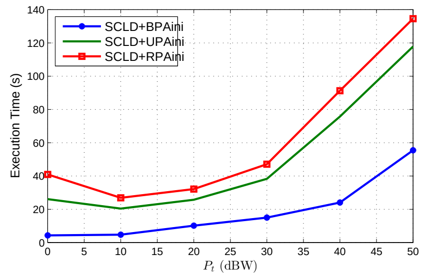
In this section, we evaluate the performances of two proposed RA algorithms. Both the convergence and the effectiveness of the SCLD based RA algorithm and the modified IWF algorithm will be illustrated. For the convergence, sum rates w.r.t. iterations are first presented with different initialization methods for one particular channel realization. Then for the effectiveness, results averaged over many channel realizations are provided and discussed for the proposed algorithms.

6.4.1 System setup

As shown in Figure 6.1, a multi-cell OFDMA system with $N = 19$ coordinated cells and $K = 32$ available subcarriers is considered for illustration. Each cell contains $U = 4$ MSs and $R = 3$ RSs. Red squares, blue diamonds and black dots denote the BSs, RSs and MSs, respectively. An 8-tap delay line model is introduced to randomly generate the channel impulse response (CIR) of each link. Here each tap obeys a circular complex gaussian distribution with zero mean and variance as σ_i^2. We further assume $\frac{\sigma_i^2}{\sigma_{i+1}^2} = e^3$ and $\sum\limits_i \sigma_i^2 = d^{-3}$. Finally, we set $P_T^n = P_T, \forall n$, $\sigma^2 = -80dBm$, $\lambda_0 = K$, $\mu_0 = 0.5$,



(a) Average sum rates



(b) Average execution time

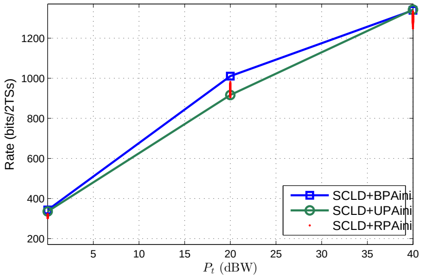
Figure 6.2 Average performances w.r.t. different power constraints with 7 cells.

$\alpha_\lambda = K/4$, $\alpha_\mu = 0.02$, $\epsilon_1 = R_{ini}/500$, $\epsilon_2 = 1$, $\epsilon_3 = R_{ini}/500$ and $M' = 15$. Here R_{ini} denotes the sum rate calculated with the initial RA results at the beginning of each iteration.

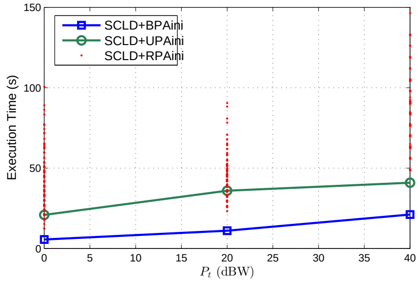
6.4.2 Results for the SCLD based RA algorithm with three initialization methods

For the proposed SCLD based RA algorithm, three initialization PA algorithms (RPA, UPA and BPA) are introduced. Specifically, RPA corresponds to the random power allocation. Each power value of each subcarrier is randomly drawn from a uniform distribution in $[0, 1]$ and normalized to achieve a total power value equal to P_T. UPA corresponds to the uniform power allocation algorithm and BPA corresponds to applying the optimal PA algorithm proposed in [16] when the CCI is always set to 0.

In order to compare the performance of the proposed SCLD based RA algorithm using three initialization methods, we set $N = 5$ and generate 200



(a) Average sum rates



(b) Average execution time

Figure 6.3 Average performances w.r.t. different power constraints with 7 cells.

random realizations of channels. Different total power constraints are set up ranging from 0 dBm to 50 dBm. The three PA algorithms (RPA, UPA and BPA) are applied to initialize the proposed SCLD based RA algorithm. As shown in Figure 6.2, the average sum rates are similar with the three initialization algorithms under all power constraints. The execution time with BPA however is much lower than those with UPA and RPA.

We further generate three realizations of channels, each of which corresponds to a specific total power constraint (0dBm/20dBm/40dBm). For each channel realization, we randomly generate 500 initial points using the RPA algorithm and compare the performance with those obtained when the initial points are generated by the UPA and the BPA. As shown in Figure 6.3, the sum rates are similar with the three initialization algorithms under all power constraints. The execution time with BPA however is much lower than those with UPA and RPA.

Thus in the following simulations, we will initialize the proposed SCLD based RA algorithm with BPA.

6.4.3 Results for a random realization of channels

In order to illustrate the convergence of our proposed algorithms, we set $N = 10$ and randomly generate a set of channels when $P_T = 40dBm$. As shown in Figure 6.4, for the proposed SCLD based RA algorithm, the total rates keep increasing continuously and converge smoothly to the final rate. For the modified IWF algorithm, the total rates keep increasing with slight fluctuations. After convergence, the total rate is increased by around 37%, 34% and 27% using the CA proposed in chapter 4, the SCLD based RA algorithm and the modified IWF algorithm, respectively. Note that the modified IWF algorithm converges very fast. As will be further illustrated, the modified IWF algorithm with one step outer iteration performs similarly as the modified IWF algorithm with 15 iterations.

6.4.4 Results averaged over channel distribution

To illustrate the effectiveness of the proposed two RA algorithms, we set $N = 3$ and generate 100 random realizations of channels. Different total

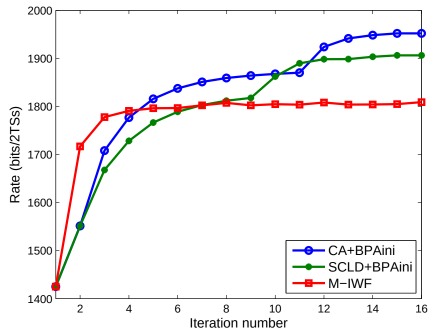


Figure 6.4 Rate results for the random realization of channels.

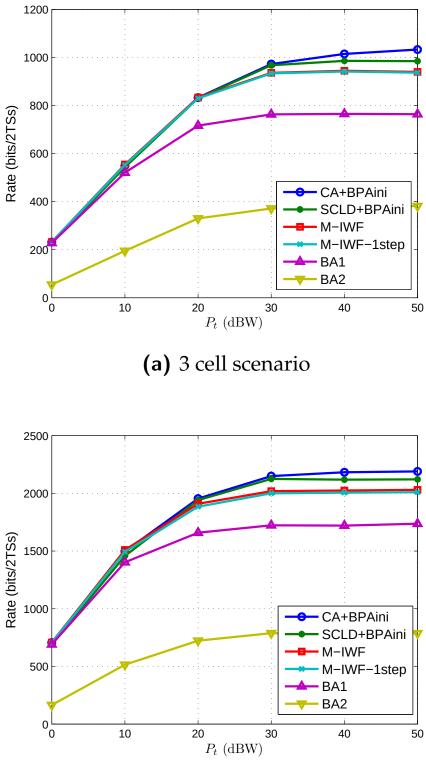
power constraints are set up ranging from 0 dBm to 50 dBm. For the purpose of comparison, we introduce two benchmark algorithms (BAs). Here BA1 corresponds to the optimal RA algorithm proposed in [16], when the CCI is always set to 0. BA 2 corresponds to the algorithm when the power is uniformly allocated and the RS and MS are randomly selected for each subcarrier.

As shown in Figure 6.5 (a), the two proposed algorithms outperform the two benchmark algorithms. Note that, at low SNR scenarios, both the modified IWF algorithm and BA 1 perform similarly as the CA and the SCLD based RA algorithm. This is because both the modified IWF algorithm and BA 1 are near-optimal when the CCI is small. The sum rate of the proposed SCLD based RA algorithm is quite close to that of the CA.

We further set $N = 10$ and generate 100 random realizations of channels. As shown in Figure 6.5 (b), the same conclusion can be found as that shown in Figure 6.5 (a). Thus the effectiveness of the proposed RA algorithms is illustrated.

To illustrate the complexity and effectiveness of the proposed two RA algorithms, we set $P_T = 40dBm$ and generate a series of system setups with different cell numbers ranging from 2 to 18. For each system setup, average execution times and performances of the CA, the SCLD based RA algorithm and the modified IWF algorithm are presented over 20 random realizations of channels.

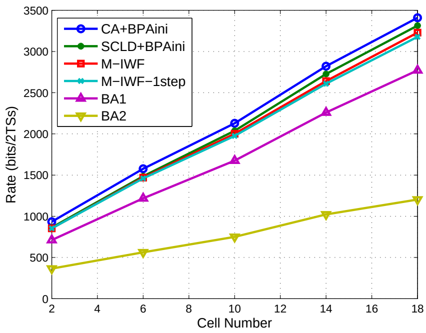
As shown in Figure 6.6, both the SCLD based RA algorithm and the modified IWF algorithm outperform the BAs, especially when the cell number is quite large. The SCLD based RA algorithm results in sum rates similar to those



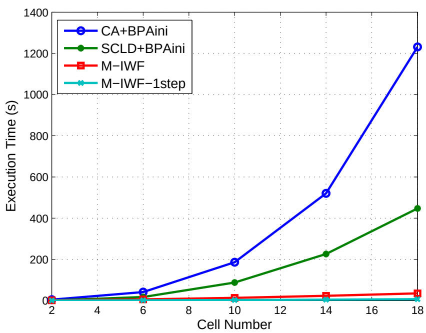
(a) 3 cell scenario

(b) 10 cell scenario

Figure 6.5 Average performances w.r.t. different power constraints.



(a) Average sum rates

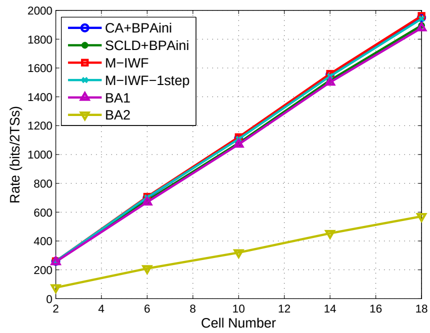


(b) Average execution time

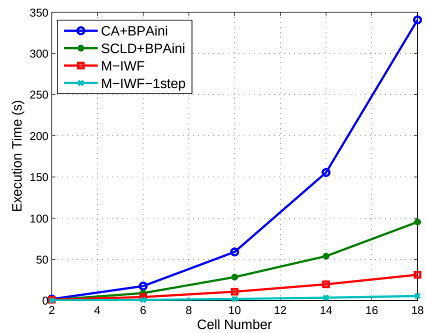
Figure 6.6 Average performances w.r.t. different cell numbers when $P_T = 40dBm$.

obtained with the CA, while its execution time is much less than that of the centralized one. Thus the SCLD based RA algorithm provides good tradeoff between performance and complexity, meaning that it is much more practical than the centralized one.

Also, the modified IWF algorithm results in sum rates close to those obtained with the SCLD based RA algorithm, while it is autonomous and consumes less execution time than the SCLD based RA algorithm. Thus the modified IWF algorithm is more preferred for practical implementation. Note that the convergence of the modified IWF algorithm is always found in simulations, although it is not theoretically guaranteed. More interestingly, the modified IWF algorithm with one step outer iteration performs similarly as the modified IWF algorithm with 15 iterations.



(a) Average sum rates



(b) Average execution time

Figure 6.7 Average performances w.r.t. different cell numbers when $P_T = 5dBm$.

We further set $P_T = 5dBm$. As shown in Figure 6.7, both the SCLD based RA algorithm and the modified IWF algorithm perform similarly as the CA and slightly better than the BA1. This is because both the modified IWF algorithm and BA 1 are near-optimal when the CCI is small. Note that the modified IWF algorithm is autonomous and less complex than the SCLD based RA algorithm. Thus, it is recommended for practical implementation.

Conclusions and Future Work

7

In this chapter, we summarize the contributions of this thesis and provide some idea about future work.

7.1 Summary of contributions

In this thesis, we have studied rate maximized RA problems in multi-cell OFDMA downlink systems with CCI.

In Chapter 2, some basic preliminaries and mathematical backgrounds have been provided.

In Chapter 3, we have overviewed different signal models used for formulating RA problems in multi-cell OFDMA downlink systems with CCI. Four signal models were summarized, which bring the same RA problem with formulations of different complexities. For comparison, we considered the classic sum rate maximized RA problem in multi-cell OFDMA downlink systems with per-station individual power constraints. Applying the four signal models, exact formulations of the considered RA problem were proposed. Properties of the different formulations were unveiled and compared. Based on existing RA methods, suggestions about choosing a suitable signal model and problem formulation were given. Finally, an interesting property of the recommended formulations was unveiled.

In Chapter 4, we have considered multi-cell DF relay-aided OFDMA downlink systems, in which all sources and relays are coordinated by a central controller for RA. The improved subcarrier-pair based opportunistic DF relaying protocol proposed and studied in [8] and [23] was applied. This protocol has a HSE in the sense that all unpaired subcarriers were utilized for data transmission during the second TS. In particular, the sum (over all cells and all destinations) rate maximized problem with a total power constraint in each cell was formulated. To solve this problem, an iterative RA algorithm was proposed to optimize MSSAP and PA in an alternate way. As for the MSSAP stage of each iteration, the formulated problem was decoupled into subproblems with the tentative PA results. Each subproblem was solved by using the optimal results of a LAP, which can be solved by the Hungarian Algorithm in polynomial time. As for the PA stage of each iteration, an algorithm based on SCGP was proposed to optimize PA in polynomial time with the tentative MSSAP results. The proposed algorithm was COA based and therefore can reach a local optimum in polynomial time. Finally, the convergence and effectiveness of the proposed algorithm, the impact of relay position and total power on the system performance as well as the benefits of using SP and the HSE protocol were illustrated through numerical experiments.

In Chapter 5, we have considered the same system as that of Chapter 4, except that SP was not considered here. The problem of maximizing the WSMR with per-cell total power constraints was formulated and its per-cell maximum fairness property was proven. An iterative RA algorithm was proposed to optimize MSSA and PA alternatively. Each iteration was composed of the MSSA stage and the PA stage. During the MSSA stage, the original problem was decoupled into MILPs with the tentative PA results, which can be solved by typical MILP solvers. To solve the MILPs more efficiently in polynomial time, a RR-MSSA algorithm and a DR-MSSA algorithm were further proposed. During the PA stage, an algorithm based on SCGP was designed to optimize PA with the tentative MSSA results. The convergence and the per-cell user fairness of the proposed RA algorithm were proven. Finally, the performance of the RA algorithm and the benefits of using OR and the HSE protocol were illustrated through numerical experiments.

In Chapter 6, we have considered the same system as that of Chapter 5, except that multiple DF RSs were placed in each cell. The problem considered was the maximization of the system sum rate with a total power constraint in each cell. An iterative semi-distributed RA algorithm was first proposed to optimize MSSA and PA alternatively. During the MSSA stage, the problem

Table 7.1 A summary of main chapters in this thesis

	Chapter 4	Chapter 5	Chapter 6
RS number	1	1	J
With SP	yes	no	no
Objective	sum rate, all cells (centralized)	WSMR (centralized)	sum rate, all cells/each cell (centralized/distributed)
Algorithm	CA	CA	SCLD based, Modified IWF

Table 7.2 A summary of algorithms studied in this thesis

	based on	implementation	converge
CA	SCA	centralized	yes
SCLD based algo.	SCA and dual	semi-distributed	yes
Modified IWF algo.	competitive game	autonomous	unproved

was decoupled into subproblems which can be solved distributively in linear time. During the PA stage, an algorithm based on SCLD was designed to optimize PA with the tentative MSSA results. The convergence of the SCLD based RA algorithm was theoretically guaranteed and an local optimum was reached after convergence. To solve the formulated problem autonomously, a modified IWF algorithm was further proposed. Specifically, each cell autonomously optimized its own sum rate with the estimated power values of the received interferences from the other cells. An optimum algorithm was proposed to solve the local RA problem in each cell. Through numerical experiments, the convergence of the two proposed algorithms as well as their benefits compared with the CA proposed in Chapter 4 were illustrated.

A summary of the three main chapters in this thesis is given in Table 7.1. Also, a summary of the algorithms studied in this thesis is given in Table 7.2.

7.2 Future work

In this section, some new directions are provided to extend the scope of this thesis.

- In this thesis, for a relay aided transmission, the signal received at the destination during the first TS is discarded, which costs a waste of useful information. To improve it, we may consider combining signals received during two TSs. Different combining schemes may be investigated. Taking CCI into account, the combining scheme used in multicell scenario is more challenging than that used in single cell scenario. Thus the RA algorithm design would be more interesting.

- In Chapter 5, only the per-cell maximum user fairness is considered. In future, other utility functions with other fairness considerations may be investigated.

- In Chapter 6, the modified IWF algorithm is quite promising for practical implementation. However, its convergence and performance are illustrated only through numerical experiments. In future, it would be interesting to further study its necessary conditions of convergence. As the modified IWF algorithm is game theory based, it is also interesting to further investigate the existence and features of the Nash Equilibrium.

- In this thesis, per-cell sum power constraints are considered with the assumption of perfect CSI in the controller. In future, we may study RA problems with per-device power constraints using imperfect CSI.

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