Ekman pumping over a sloping bottom: stating the obvious... or not?

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Introduction

The overall objective of the present working note is to derive an analytical expression of the vertical velocity outside the ocean bottom Ekman layer that causes water to be injected into or pumped out of the bottom boundary layer, a process that will be referred to — adequately or not — as bottom Ekman pumping.

Ekman pumping is an expression commonly used to refer to the vertical motion taking place in the upper part of the ocean and caused by the curl of the surface wind stress (e.g. Cushman-Roisin 1994). Hereinafter the simplest version of the relevant theoretical developments are briefly revisited and an attempt is made to extend them to vertical motion occurring in the vicinity of the ocean bottom. When doing so, one is faced with two distinct difficulties: the slope of the bottom cannot be ignored (whereas that of the ocean surface can) and the bottom Ekman transport depends on the vertical eddy diffusivity (whereas that due to the wind stress does not). Splitting the vertical velocity into upsloping and upwelling contributions (Deleersnijder 1989) is of some help, but is probably insufficient for convincing results to be derived.

It must be born in mind that all of the flows considered in this working note are time-independent.

Surface Ekman pumping

For Ekman flows on the *f*-plane, there exists an intrinsic vertical length scale

$$\Lambda = \sqrt{\frac{2\nu_t}{|f|}} \quad , \tag{1}$$

where f is the Coriolis factor (which is positive in the northern hemisphere and negative in the southern one) and ν_t is the (kinematic) eddy viscosity. For oceans at mid-latitudes, the values $|f| \approx 10^{-4}\ \mathrm{s}^{-1}$ and $\nu_t \approx 10^{-2}\ \mathrm{m}^2\,\mathrm{s}^{-1}$ are presumably appropriate, leading to $\Lambda \approx 10\,\mathrm{m}$. As is well known, the processes of interest in the framework of the Ekman theory essentially develop in a layer adjacent to the ocean surface a few Λ in height. If the depth of the ocean is much larger, then it is may be safely assumed that the water column is infinitely deep. Thus, if z denotes the vertical coordinate, increasing upward, that is zero at the ocean-atmosphere interface, the domain of interest is defined by the inequalities $-\infty < z < 0$ (Figure 1).

Let $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$ denote orthonormal vectors; the first two of them are horizontal, whilst the third one is vertical and points upward. The velocity reads

$$\mathbf{v} = \underbrace{u\,\mathbf{e}_x + v\,\mathbf{e}_y}_{=\mathbf{u}} + w\,\mathbf{e}_z \quad , \tag{2}$$

where $\mathbf{u}$ is the horizontal velocity. The Boussinesq approximation is assumed to hold valid, implying that the divergence of the velocity is negligible. In other words, the continuity equation simplifies to

$$\nabla \bullet \mathbf{v} = \nabla_h \bullet \mathbf{u} + \frac{\partial w}{\partial z} = \underbrace{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}}_{=\nabla_h \bullet \mathbf{u}} + \frac{\partial w}{\partial z} = 0 \quad , \tag{3}$$

where

$$\nabla_h = \nabla - \mathbf{e}_z \frac{\partial}{\partial z} = \mathbf{e}_x \frac{\partial}{\partial x} + \mathbf{e}_y \frac{\partial}{\partial y} \tag{4}$$

is the “horizontal del operator”.

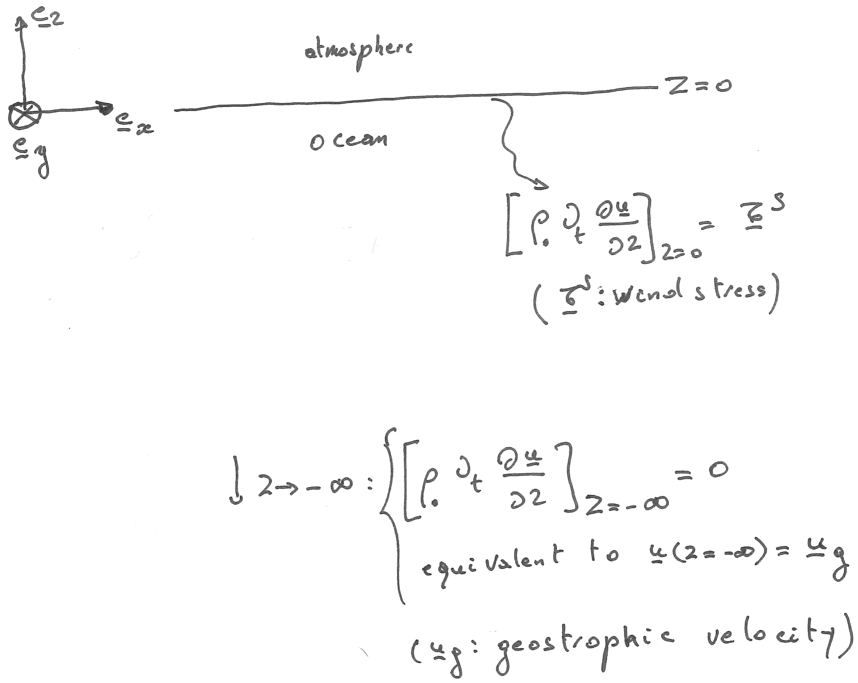

Figure 1. Illustration of the geometry and boundary conditions of the model used to study Ekman pumping (in the ocean surface layers). The depth of the water column is assumed to be infinite.

Since the flow is assumed to be at a steady state and the Rossby number is expected to be small, the horizontal momentum equation is reduced to

$$f \mathbf{e}_z \times \mathbf{u} = -\frac{1}{\rho_\bullet} \nabla_h p + \frac{\partial}{\partial z}\left(\nu_t \frac{\partial \mathbf{u}}{\partial z} \right) \quad , \tag{5}$$

where the constant $\rho_\bullet$ denotes the reference density (Boussinesq approximation) and p is the pressure. Then, the geostrophic velocity, $\mathbf{u}_g$, is such that the Coriolis force and the pressure force balance eachother, leading to

$$f\,\mathbf{e}_z \times \mathbf{u}_g \;=\; -\frac{1}{\rho_\bullet}\nabla_h p \quad\Rightarrow\quad \mathbf{u}_g \;=\; \frac{1}{\rho_\bullet f}\,\mathbf{e}_z \times \nabla_h p \;\;. \tag{6}$$

It is readily seen that the geostrophic velocity is divergence free:

$$\nabla_h \bullet \mathbf{u}_g \;=\; \nabla_h \bullet \left(\frac{1}{\rho_\bullet f}\,\mathbf{e}_z \times \nabla_h p \right) \;=\; -\frac{1}{\rho_\bullet f}\mathbf{e}_z \bullet \underbrace{\left(\nabla_h \times \nabla_h p\right)}_{=0} \;=\; 0 \;\;. \tag{7}$$

As usual, the vertical momentum equation is simplified to the hydrostatic equilibrium (mainly because the aspect ratio is assumed to be small) and, in addition, stratification effects are considered negligible (in the top layer of the ocean). Thus, the pressure obeys

$$0 \;=\; -\frac{1}{\rho_\bullet}\frac{\partial p}{\partial z} \;-\; g \;\;, \tag{8}$$

where g is the gravitational acceleration. As a consequence, the geostrophic velocity is independent of the vertical coordinate:

$$\frac{\partial \mathbf{u}_g}{\partial z} \;=\; \frac{\partial}{\partial z}\left(\frac{1}{\rho_\bullet f}\,\mathbf{e}_z \times \nabla_h p \right) \;=\; \frac{1}{\rho_\bullet f}\,\mathbf{e}_z \times \nabla_h \frac{\partial p}{\partial z} \;=\; \frac{1}{\rho_\bullet f}\,\mathbf{e}_z \times \underbrace{\nabla_h(-\rho_\bullet g)}_{=0} \;=\; 0 \;\;. \tag{9}$$

It is convenient to introduce the deviation of the horizontal velocity with respect to the geostrophic velocity:

$$\hat{\mathbf{u}} \;=\; \mathbf{u} \;-\; \mathbf{u}_g \;\;. \tag{10}$$

Substituting (10) into continuity equation (3) and momentum equation (5), taking into account properties (7) and (9), one obtains

$$\nabla_h \bullet \hat{\mathbf{u}} \;+\; \frac{\partial w}{\partial z} \;=\; 0 \tag{11}$$

and

$$f\,\mathbf{e}_z \times \hat{\mathbf{u}} \;=\; \frac{\partial}{\partial z}\left(\nu_t \frac{\partial \hat{\mathbf{u}}}{\partial z} \right) \;\;. \tag{12}$$

The ocean surface is assumed to be horizontal and impermeable, leading to the boundary condition

$$\left[w\right]_{z=0} = \;0 \;\;. \tag{13}$$

The horizontal momentum is to be solved under the following boundary conditions:

$$\left[\rho_\bullet \nu_t \frac{\partial \mathbf{u}}{\partial z} \right]_{z=0} \;=\; \left[\rho_\bullet \nu_t \frac{\partial \hat{\mathbf{u}}}{\partial z} \right]_{z=0} \;=\; \boldsymbol{\tau}^s \tag{14}$$

and

$$\left[\mathbf{u}\right]_{z=-\infty} = \;\mathbf{u}_g \quad\Rightarrow\quad \left[\hat{\mathbf{u}}\right]_{z=-\infty} = \;0 \tag{15a}$$

where $\boldsymbol{\tau}^s = \tau_x^s\,\mathbf{e}_x + \tau_y^s\,\mathbf{e}_y$ denotes the surface wind stress. Since the horizontal velocity tends to a finite limit as $z \to -\infty$, its vertical gradient must tend to zero as $z \to -\infty$, implying that boundary condition (15a) is equivalent to

$$\left[\rho_\bullet \nu_t \frac{\partial \mathbf{u}}{\partial z} \right]_{z=-\infty} \;=\; \left[\rho_\bullet \nu_t \frac{\partial \hat{\mathbf{u}}}{\partial z} \right]_{z=-\infty} \;=\; 0 \;\;. \tag{15b}$$

If the eddy viscosity is a constant, the solution of differential equation (12) under boundary conditions (14)-(15) is readily seen to be

$$\mathbf{u} \;=\; \mathbf{u}_g \;+\; \frac{\Lambda e^{z/\Lambda}}{2\rho_\bullet \nu_t}\left[\left(\sin\frac{z}{\Lambda}+\cos\frac{z}{\Lambda}\right)\boldsymbol{\tau}^s \;+\; \sigma\left(\sin\frac{z}{\Lambda}-\cos\frac{z}{\Lambda}\right)\mathbf{e}_z\times\boldsymbol{\tau}^s\right] \quad , \tag{16}$$

where σ denotes the sign of the Coriolis factor ($f=\sigma|f|$), i.e. σ is equal to unity in the northern hemisphere and is -1 in the southern one. However, as will be seen, it is not necessary to know the velocity profile to determine the vertical velocity that leads to Ekman pumping. In fact, this vertical velocity may be evaluated even if the eddy viscosity is not a constant, in which case there is no simple solution to (12).

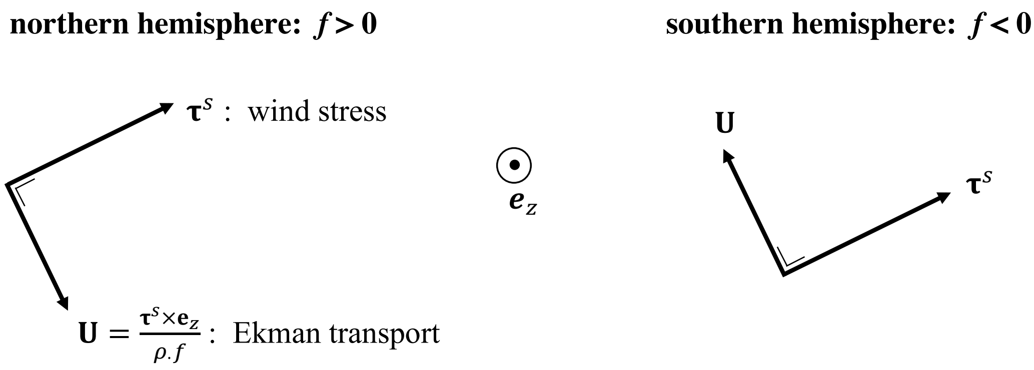

Figure 2. Illustration of the direction of the Ekman transport **U** induced in the top layers of the ocean by the wind stress; **U** is to be evaluated by means of (19). This figure is taken from Deleersnijder 2014.

To obtain the Ekman pumping vertical velocity, a necessary first step is to evaluate the Ekman transport, which is defined to be

$$\mathbf{U} \;\equiv\; \int_{-\infty}^{0} \hat{\mathbf{u}}\, dz \quad . \tag{17}$$

This (horizontal) transport ($\mathrm{m^2\,s^{-1}}$) may be calculated without knowing the horizontal velocity profile. To do so, one has to integrate horizontal momentum equation (12) and take into account boundary conditions (14) and (15):

$$\underbrace{f\,\mathbf{e}_z\times\int_{-\infty}^{0}\hat{\mathbf{u}}\,dz}_{\substack{=\mathbf{U}\\ \text{see (17)}}} \;=\; \underbrace{\int_{-\infty}^{0}\frac{\partial}{\partial z}\left(\nu_t\frac{\partial\hat{\mathbf{u}}}{\partial z}\right)dz}_{=\frac{\boldsymbol{\tau}^s-0}{\rho_\bullet},\ \text{see (14)-(15)}} \quad\Rightarrow\quad f\,\mathbf{e}_z\times\mathbf{U} \;=\; \frac{\boldsymbol{\tau}^s}{\rho_\bullet} \quad . \tag{18}$$

This expression transforms to (Figure 2)

$$\mathbf{U} \;=\; \frac{1}{\rho_\bullet f}\,\boldsymbol{\tau}^s\times\mathbf{e}_z \quad . \tag{19}$$

Integrating continuity equation (11) over the height of the water column yields

$$0 = \int_{-\infty}^{0} \nabla_h \bullet \hat{\mathbf{u}}\, dz + \int_{-\infty}^{0} \frac{\partial w}{\partial z} dz = \nabla_h \bullet \underbrace{\int_{-\infty}^{0} \hat{\mathbf{u}}\, dz}_{=\mathbf{U}} + \underbrace{\int_{-\infty}^{0} \frac{\partial w}{\partial z} dz}_{\underbrace{[w]_{z=0}}_{=0} - w_{-\infty}} = \nabla_h \bullet \mathbf{U} - w_{-\infty} \qquad (20)$$

so that

$$w_{-\infty} = \nabla_h \bullet \mathbf{U} \quad . \qquad (21)$$

Combining (19) and (21) leads to the well-known formula (Figure 3)

$$\boxed{w_{-\infty} = \frac{1}{\rho_\bullet f} \mathbf{e}_z \bullet (\nabla_h \times \boldsymbol{\tau}^s) = \frac{1}{\rho_\bullet f} \left(\frac{\partial \tau_y^s}{\partial x} - \frac{\partial \tau_x^s}{\partial y} \right)} \qquad (22)$$

where x and y are horizontal coordinates. The vertical velocity that prevails below the Ekman layer is proportional to the curl of the surface wind stress and is independent of the eddy viscosity profile, which, in most cases, cannot be determined in a reliable manner. Therefore, relation (22) is robust and trustworthy. As will be seen below, so good a result cannot be obtained for the bottom layer, mainly because the vertical velocity above the bottom Ekman layer critically depends on the eddy viscosity profile.

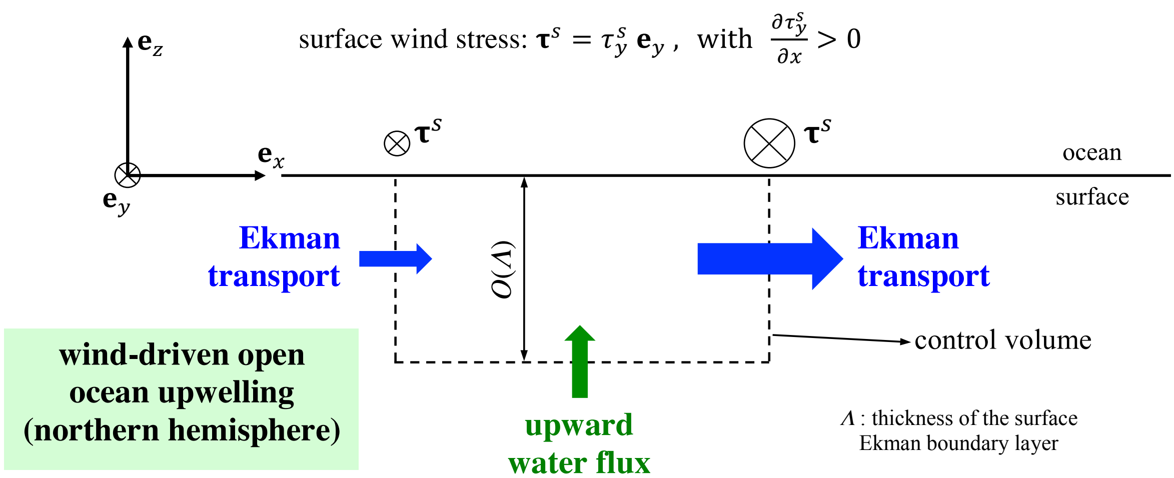

Figure 3. Illustration of wind-induced open ocean upwelling in the northern hemisphere (f>0). As indicated by formula (22), it is the variation of the wind stress in the direction normal to the wind stress that drives this type of upwelling. This figure is taken from Deleersnijder (2014).

Bottom layer

The order of magnitude of the thickness of the bottom Ekman layer thickness, Λ, is estimated by means of expression (1), in which the eddy viscosity is not necessarily of the same order of magnitude as that used for the surface Ekman layer. If the height of the water column is much larger than Λ (that is generally the case, since the viscosity near the ocean bottom is seldom larger than that prevailing in the top layers), then the water column height may be regarded as infinite. Assuming that the ocean bottom is located at $z = -\xi(x,y)$, the domain of interest is defined by the inequalities $-\xi < z < \infty$, where z is the vertical coordinate, increasing upward (Figure 4). It is convenient to introduce a new set of space coordinates:

$$\tilde{x} = x \ , \quad \tilde{y} = y \ , \quad \tilde{z} = z + \xi(x,y) \ . \tag{23}$$

In the transformed space, whose coordinates are identified by a tilde, the water column is defined by $0 < \tilde{z} < \infty$, i.e. the ocean bottom is located at $\tilde{z} = 0$. The relevant differential operators transform as follows:

$$\nabla_h \ = \ \tilde{\nabla}_h \ + \ \nabla_h \xi \, \frac{\partial}{\partial \tilde{z}} \quad , \tag{24}$$

$$\frac{\partial}{\partial z} \ = \ \frac{\partial}{\partial \tilde{z}} \quad , \tag{25}$$

where

$$\tilde{\nabla}_h \ = \ \mathbf{e}_x \frac{\partial}{\partial \tilde{x}} \ + \ \mathbf{e}_y \frac{\partial}{\partial \tilde{y}} \tag{26}$$

may be viewed as the horizontal del operator of the transformed space. Dimensionless vector $\nabla_h \xi$ points in the direction of steepest descent of the ocean bottom.

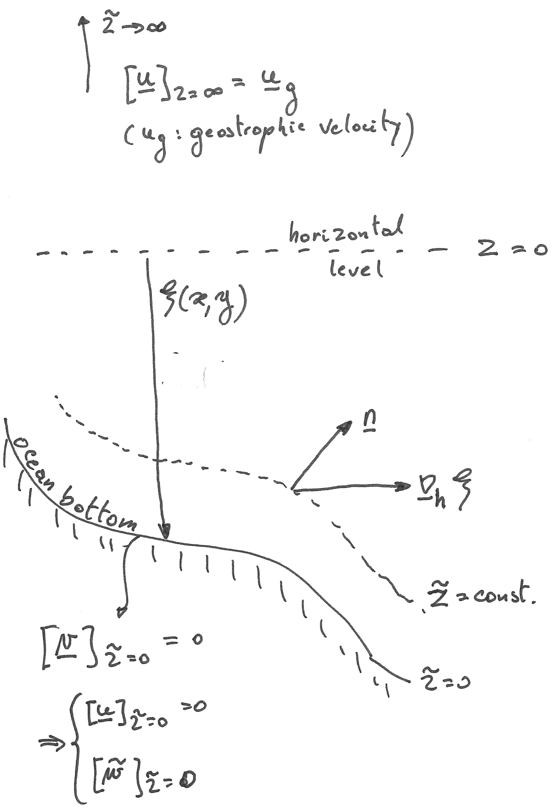

Figure 4. Illustration of the geometry and boundary conditions of the model used to study vertical motion in the vicinity of the bottom of the ocean. The height of the water column is assumed to be infinite.

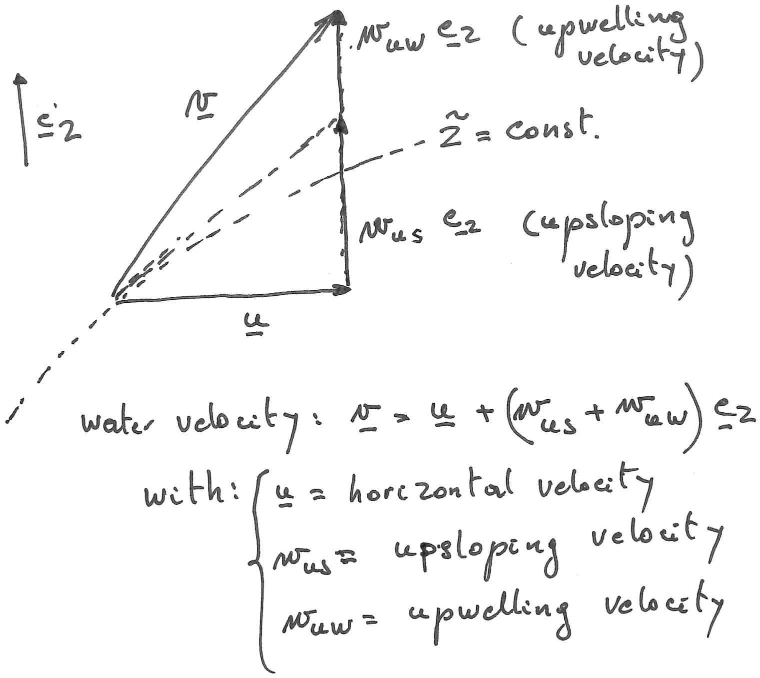

Figure 5. Illustration of the upsloping (w_{up}) and upwelling (w_{uw}) vertical velocities. The upsloping velocity is that allowing a water parcel to remain on an iso-$\tilde{z}$ surface (i.e. at the same distance to the ocean bottom). The vertical velocity with which a water parcel crosses an iso-$\tilde{z}$ surface is the upwelling velocity.

When a new vertical coordinate is resorted to, it is customary to introduce a modified velocity defined as the material derivative of the new vertical coordinate (see, for instance, Deleersnijder and Ruddick, 1992, and references therein). Accordingly, the new vertical velocity is defined to be

$$\tilde{w} \;=\; \mathbf{u} \bullet \underbrace{\nabla_h \tilde{z}}_{=\nabla_h \xi} \;+\; w \underbrace{\frac{\partial \tilde{z}}{\partial z}}_{=1} \;=\; \mathbf{u} \bullet \nabla_h \xi \;+\; w \quad . \tag{27}$$

The unit vector that is normal to an iso-$\tilde{z}$ surface reads

$$\mathbf{n} \;=\; \frac{\nabla_h \xi + \mathbf{e}_z}{\sqrt{\left|\nabla_h \xi\right|^2 + 1}} \quad . \tag{28}$$

If the velocity of a fluid parcel is $\mathbf{u} - \left(\mathbf{u} \bullet \nabla_h \xi\right)\mathbf{e}_z$, then this fluid parcel remains on the same iso-$\tilde{z}$ surface, since $\left[\mathbf{u} - \left(\mathbf{u} \bullet \nabla_h \xi\right)\mathbf{e}_z\right] \bullet \mathbf{n}$ is identically zero. Therefore, the vertical velocity,

$$w \;=\; -\,\mathbf{u} \bullet \nabla_h \xi \;+\; \tilde{w} \quad , \tag{29}$$

may be viewed as the sum of the upsloping velocity (Deleersnijder 1989, 1994), $w_{us} = -\mathbf{u} \bullet \nabla_h \xi$, i.e. the vertical velocity that would allow a fluid to remain on an iso-$\tilde{z}$ surface, and the velocity at which the iso-$\tilde{z}$ surfaces are crossed, $w_{up} = \tilde{w}$. The latter may be termed upwelling velocity (Deleersnijder 1989, 1994), for it is the vertical velocity that is not directly induced by the bottom topography (Figure 5). Presumably, the upwelling velocity provides a better measure of the true upwelling than the actual vertical velocity. Accordingly, the Ekman pumping velocity that will be determined below is $\tilde{w}_\infty \equiv [\tilde{w}]_{\tilde{z}=\infty}$ rather than the actual vertical velocity outside the bottom Ekman layer, $w_\infty = [w]_{z=\infty}$. Clearly, the water velocity $\mathbf{v}$ is the sum of all the abovementioned velocities, i.e. $\mathbf{v} = \mathbf{u} + (w_{us} + w_{uw})\mathbf{e}_z$ (Figure 5).

In the transformed space, continuity equation (3) modifies to

$$\tilde{\nabla}_h \bullet \mathbf{u} + \frac{\partial \tilde{w}}{\partial \tilde{z}} = 0 \tag{30}$$

or

$$\tilde{\nabla}_h \bullet \hat{\mathbf{u}} + \frac{\partial \tilde{w}}{\partial \tilde{z}} = 0 \quad , \tag{31}$$

since the geostrophic velocity is unaffected by the coordinate transformation under consideration. The horizontal momentum equation and the relevant boundary conditions are as follows:

$$f\,\mathbf{e}_z \times \hat{\mathbf{u}} = \frac{\partial}{\partial z}\left(\nu_t \frac{\partial \hat{\mathbf{u}}}{\partial z}\right) \quad , \tag{32}$$

$$[\hat{\mathbf{u}}]_{z=0} = -\,\mathbf{u}_g \quad , \qquad [\hat{\mathbf{u}}]_{z=\infty} = 0 \quad , \tag{33}$$

and

$$[\tilde{w}]_{z=0} = 0 \quad . \tag{34}$$

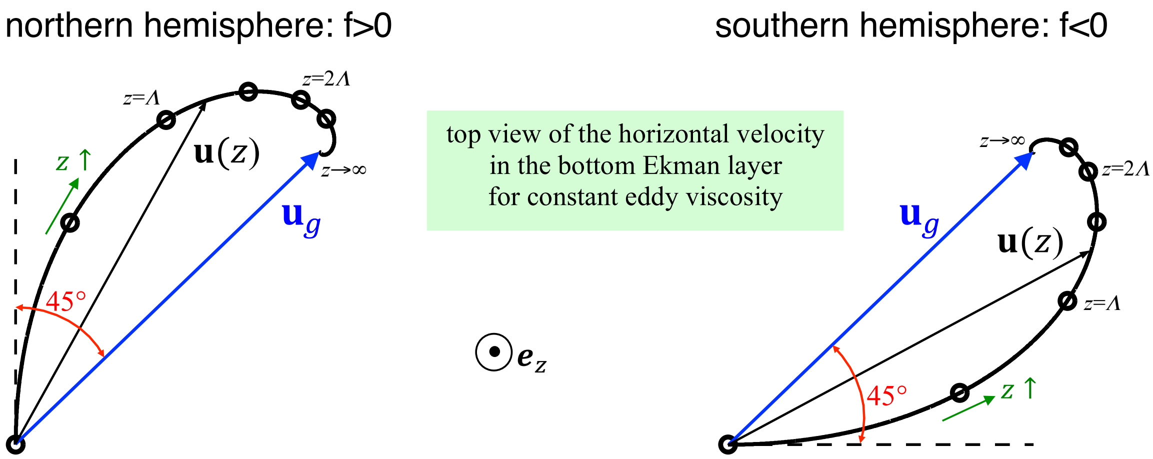

Figure 6. Illustration of the horizontal velocity in the bottom Ekman layer (bottom Ekman spiral) in the northern (f>0) and southern (f<0) hemispheres. The graphs are derived from analytical solution (35b), which is obtained under the crucial assumption that the eddy viscosity is a constant. This figure is taken from Deleersnijder (2014).

For the bottom Ekman layer, it is not possible to derive the value of $\tilde{w}_\infty$ without solving explicitly the horizontal momentum equation. To do so, the eddy viscosity is assumed to be a constant, which is a rather crude assumption. Then, the following result is obtained:

$$\hat{\mathbf{u}} \;=\; -e^{-\tilde{z}/\Lambda}\cos\frac{\tilde{z}}{\Lambda}\mathbf{u}_g \;+\; \sigma\, e^{-\tilde{z}/\Lambda}\sin\frac{\tilde{z}}{\Lambda}\mathbf{e}_z\times\mathbf{u}_g \tag{35a}$$

or

$$\mathbf{u} \;=\; \mathbf{u}_g \;-\; e^{-\tilde{z}/\Lambda}\cos\frac{\tilde{z}}{\Lambda}\mathbf{u}_g \;+\; \sigma\, e^{-\tilde{z}/\Lambda}\sin\frac{\tilde{z}}{\Lambda}\mathbf{e}_z\times\mathbf{u}_g \quad . \tag{35b}$$

In the vicinity of the ocean bottom, the horizontal velocity points to the left of the geostrophic velocity in the northern hemisphere (bottom Ekman spiral) and is to the right of $\mathbf{u}_g$ in the southern hemisphere (Figure 6). As is seen below, this feature is key to understanding vertical motion in the neighbourhood of the ocean bottom.

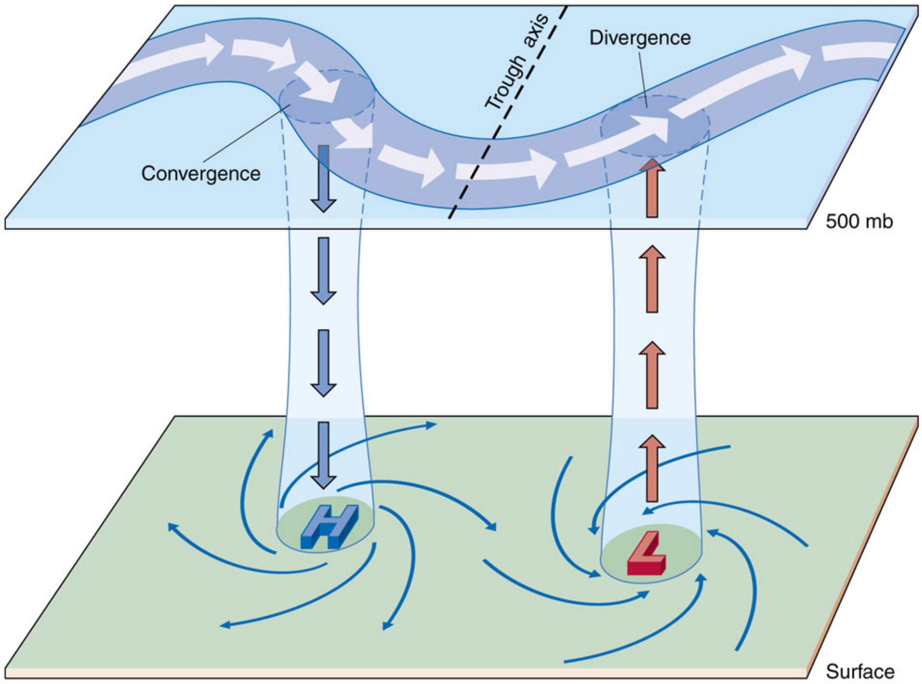

Figure 7. Illustration[1] of vertical motion developing in atmospheric low (H) and high (H) pressure systems. Near the Earth's surface, in a low pressure zone, the horizontal velocity tends to convergence toward the centre of the system, leading to updraft. The opposite takes place in a high pressure system.

Combining (31) and (35) yields

$$\tilde{\nabla}_h\bullet\hat{\mathbf{u}} \;+\; \frac{\partial\tilde{w}}{\partial\tilde{z}} \;=\; \underbrace{-\;\sigma\, e^{-\tilde{z}/\Lambda}\sin\frac{\tilde{z}}{\Lambda}\mathbf{e}_z\bullet(\nabla_h\times\mathbf{u}_g)}_{=\tilde{\nabla}_h\bullet\hat{\mathbf{u}}} \;+\; \frac{\partial\tilde{w}}{\partial\tilde{z}} \;=\; 0 \quad . \tag{36}$$

Integrating this expression over the height of the water column and taking into account boundary condition (34) leads to

[1] Source: http://www.geog.ucsb.edu/~joel/g110_w08/lecture_notes/midlat_upper/agburt10_07.jpg, last viewed on 1st September 2016

$$-\,\sigma\underbrace{\int_0^{\infty}\left(e^{-\tilde{z}/\Lambda}\sin\frac{\tilde{z}}{\Lambda}\right)d\tilde{z}}_{=\Lambda/2}\;\mathbf{e}_z\bullet(\nabla_h\times\mathbf{u}_g)\;+\;\underbrace{\frac{\partial\tilde{w}}{\partial\tilde{z}}}_{=\tilde{w}_\infty}\;=\;0\quad. \tag{37}$$

Then, using the definition of the geostrophic velocity (6), the Ekman pumping velocity is readily seen to be

$$\tilde{w}_\infty\;=\;\frac{\sigma\Lambda}{2}\,\mathbf{e}_z\bullet(\nabla_h\times\mathbf{u}_g)\;=\;\frac{\sqrt{\nu_t}}{\sqrt{2}\rho_\bullet|f|^{3/2}}\nabla_h^2\,p \tag{38}$$

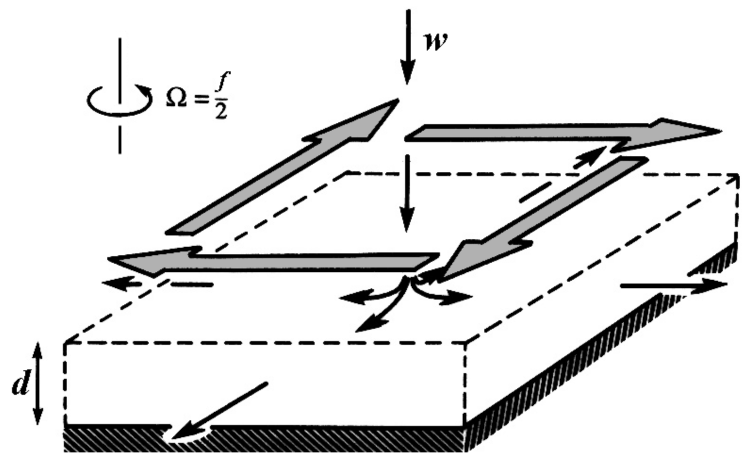

Figure 8. Schematic illustration (for the northern hemisphere) of the divergence in the bottom Ekman layer due to the curl of the geostrophic velocity, which, in the present case, leads to downwelling. The height *d* in the figure is equivalent to Λ in the present working note. This is figure 5-3 of Cushman-Roisin (1994).

If the geostrophic flow is of a cyclonic nature ($\nabla_h^2\,p > 0$), the upwelling velocity is positive. Otherwise, for an anticyclonic situation, ($\nabla_h^2\,p < 0$), $\tilde{w}_\infty$ is negative. This is mainly because the transformed-space divergence of the horizontal velocity and the Laplacian of the pressure have opposite sign in the vicinity of the bottom ($\tilde{z} < \pi\Lambda/2$):

$$\tilde{\nabla}_h\bullet\mathbf{u}\;=\;\tilde{\nabla}_h\bullet\hat{\mathbf{u}}\;=\;-\,\sigma\,e^{-\tilde{z}/\Lambda}\sin\frac{\tilde{z}}{\Lambda}\mathbf{e}_z\bullet(\nabla_h\times\mathbf{u}_g)\;=\;-\,\frac{1}{\rho_\bullet|f|}\,e^{-\tilde{z}/\Lambda}\sin\frac{\tilde{z}}{\Lambda}\nabla_h^2 p \tag{39}$$

In other words, for a cyclonic (anticyclonic) situation, the flow exhibits a convergence near the bottom, leading to positive (negative) upwelling velocity irrespective of the bottom slope. Such qualitative results are well known to meteorologists, though they do not seem to take into account the slope of the Earth's surface in their mathematical developments and illustrations(Figure 7). It is noteworthy, however, that the outstanding textbook of Cushman-Roisin obtained similar results, though the mathematical developments did not have recourse to a coordinate transformation (Figure 8).

By combining (29) and (38), one obtains the following vertical velocity outside the bottom

Ekman layer:

$$w_\infty = -\mathbf{u}_g \cdot \nabla_h \xi + \tilde{w}_\infty = \underbrace{-\mathbf{u}_g \cdot \nabla_h \xi}_{\text{upsloping}} + \underbrace{\frac{\sqrt{\nu_t}}{\sqrt{2}\rho_\bullet |f|^{3/2}} \nabla_h^2 p}_{\text{upwelling}} \tag{40}$$

The main strength of expression (40) is that the contribution due to the slope of the ocean bottom and the "true" upwelling velocity are clearly separated. That the sign of the "true" upwelling velocity is directly related to the cyclonic or anticyclonic nature of the geostrophic flow is well known, at least in meteorology. Unfortunately, relation (38) is based on the working hypothesis that the eddy viscosity is a constant, which is presumably too crude an approximation. The eddy viscosity definitely varies in the vertical direction and its order of magnitude depends on the norm of the geostrophic velocity. Unfortunately, so far, I have been unable to take into account these constraints in a simple manner.

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