

$\eta - \eta'$ masses and mixing: a large N_c reappraisal

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Abstract

We reconcile the $1/N_c$ expansion with the observed $\eta - \eta'$ mass spectrum. The chiral corrections introduced for that purpose are natural and consistent with the octet–singlet mixing angle $\theta = -(22 \pm 1)^\circ$ extracted from phenomenology in the large N_c limit.

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1. Introduction

There is, nowadays, a considerable interest in weak decay processes involving η and η' mesons as final or intermediate states. New physics beyond the Standard Model is indeed regularly advocated to explain, for example, the unexpectedly large $B \rightarrow K \eta'$ branching ratio [1] or the sizable direct CP violation in $K \rightarrow \pi\pi$ amplitudes [2]. This might be legitimate if non-perturbative hadronic effects such as a gluonium component in η' or a destructive $\pi^0 - \eta$ mixing contribution were fully under control. As a matter of fact, the $q\bar{q}$ -gluonium mixing would vanish [3] and all the $\Delta S = 1$ hadronic matrix elements could be factorized [4] if the number of colors N_c turned out to be infinite...

Interestingly enough, the large N_c approximation [5] has been proven to provide a simple and quite successful theoretical framework for elucidating various non-perturbative phenomena in strong dynamics. For illustration, the Okubo–Zweig–Iizuka (OZI) rule, which qualitatively explains the $\rho - \pi$ suppression in ϕ decay and the $\omega - \rho^0$ mass difference, can be coherently derived in this approximation. The chiral symmetry breaking pattern is also understood from the observed mass spectrum of the pseudoscalar mesons. However, more recently, this useful framework has been challenged at the quantitative level due to its apparent failure to reproduce the well-measured η and η' masses [6].

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In this Letter, we argue that reasonable chiral corrections alone may reproduce the η and η' masses in the large N_c limit. A direct extraction of the octet–singlet mixing angle in this limit confirms the natural size of these corrections.

2. Georgi's mass inequality revisited

In the strict large N_c limit, only the color-singlet channel of the quark–antiquark planar interaction is attractive and QCD with three massless flavors (u, d, s) exhibits chiral symmetry breaking [7]. The $U(3)_L \times U(3)_R$ chiral symmetry of the fundamental QCD theory is indeed spontaneously broken down to $U(3)_{L+R}$ such that a full nonet of Goldstone bosons

$$\pi = \sum_{a=0}^8 \lambda_a \pi^a = \sqrt{2} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 + \frac{\eta_0}{\sqrt{3}} \end{pmatrix} \quad (1)$$

is naively expected. To lowest order in derivatives, the effective Lagrangian for the corresponding unitary field

$$U(x) = \exp\left(i \frac{\sqrt{2}\pi}{f}\right) \quad (2)$$

reads then

$$\mathcal{L}_\infty^{(p^2)} = \frac{f^2}{8} \langle \partial_\mu U \partial^\mu U^\dagger \rangle \quad (3)$$

where f is the weak decay constant of the pseudoscalar nonet. The bracket $\langle \dots \rangle$ stands for the trace over light flavors. The other possible kinetic term

$$\mathcal{L}_{1/N_c}^{(p^2)} = \epsilon_1 \frac{f^2}{8} \langle \partial_\mu U U^\dagger \rangle \langle \partial^\mu U^\dagger U \rangle \quad (4)$$

contains two traces. Such a flavor structure necessarily arises from QCD Feynman diagrams with two quark loops and is therefore suppressed by one power of $1/N_c$.

Explicit symmetry breaking terms have to be introduced to reproduce the observed mass spectrum of the light pseudoscalar mesons. Let us classify these breaking terms according to the momentum expansion in the large N_c approximation. In other words, at each order in p^2 , let us only retain the dominant term in the $1/N_c$ expansion.

At leading order (p^0), the first non-trivial term only arises at the $1/N_c$ level and breaks the flavor singlet axial $U(1)_A$ symmetry [8]:

$$\Delta \mathcal{L}_{1/N_c}^{(p^0)} = \frac{m_0^2}{4N_c} \frac{f^2}{8} \langle \ln U - \ln U^\dagger \rangle^2 = -\frac{1}{2} m_0^2 \eta_0^2. \quad (5)$$

This octet–singlet mass-splitting is responsible for the large η' mass [9].

At next-to-leading order (p^2), the single trace term accounts for the $SU(3)_{L+R}$ symmetry breaking:

$$\Delta \mathcal{L}_\infty^{(p^2)} = \frac{f^2}{8} r \langle m U^\dagger + U m \rangle. \quad (6)$$

Disregarding here the possibility of a tiny T violation, we identify m with the real diagonal quark mass matrix

$$m = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}. \quad (7)$$

In the large N_c limit, the observed pseudoscalar spectrum unambiguously determines the quark mass ratios [10] since the chiral transformation $m \rightarrow m + \alpha(\det m)m^{-1}$ [11] violates the OZI rule. Let us therefore consider the realistic isospin limit ($m_u = m_d \equiv \hat{m} \ll m_s$) and work in the quark basis

$$\pi = \sqrt{2} \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} \quad (8)$$

which is proving to be more convenient than the octet–singlet one (privileged by Eq. (1)) to discuss the η and η' masses. In this basis, we obtain $m_\pi^2 = r\hat{m}$ and $m_K^2 = \frac{r}{2}(m_s + \hat{m})$ for the charged mesons and the following mass matrix for the neutral ($u\bar{u}, d\bar{d}, s\bar{s}$) states:

$$M^2 = \frac{m_0^2}{N_c} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1+R \end{pmatrix} + m_\pi^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (9)$$

with

$$R = \frac{2N_c}{m_0^2} (m_K^2 - m_\pi^2). \quad (10)$$

This matrix includes both the $U(1)_A$ and the $SU(3)_{L+R}$ symmetry breaking terms in the large N_c limit. Its straightforward diagonalization leads to

$$m_\eta^2 = \frac{m_0^2}{6} \left(3 + R - \sqrt{9 - 2R + R^2} \right) + m_\pi^2, \quad m_{\eta'}^2 = \frac{m_0^2}{6} \left(3 + R + \sqrt{9 - 2R + R^2} \right) + m_\pi^2 \quad (11)$$

for $N_c = 3$. With this assignment for the mass eigenstates, we easily obtain

$$0 \leq \frac{m_\eta^2 - m_\pi^2}{m_{\eta'}^2 - m_\pi^2} \leq \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \simeq 0.268. \quad (12)$$

The lower bound, reached in the $m_0^2 \rightarrow \infty$ limit, corresponds to the octet approximation with the Gell-Mann–Okubo mass relation for $\eta = \eta_8$. The upper bound, saturated for $R = 3$, is a simple generalization of Georgi's inequality [6] for $\hat{m} \neq 0$. The latter requires *at least* 15% corrections from higher order terms in the effective Lagrangian to be compatible with the physical π , η and η' masses:

$$\left(\frac{m_\eta^2 - m_\pi^2}{m_{\eta'}^2 - m_\pi^2} \right)_{\text{exp}} = 0.313. \quad (13)$$

The $1/N_c$ mass corrections at the order (p^2) in the momentum expansion have already been advocated in [6,12]. Here we would like to emphasize that the large N_c limit at the order (p^4) in the momentum expansion may be enough to reproduce the $\eta - \eta'$ mass spectrum. In fact, this second possibility seems to be favored by the extraction of the $\eta - \eta'$ mixing from phenomenology as well as by the observed $SU(3)$ splitting among weak decay constants.

3. Mixing angle from phenomenology

In the large N_c limit, the physical η and η' states decouple from gluonium states and are slightly off from η_8 and η_0 , respectively, due to the $\mathcal{O}(p^2)$ $SU(3)$ breaking term in Eq. (6). Consequently, they are parameterized in terms of a single and small mixing angle θ associated with the diagonalization of the two-by-two mass matrix:

$$\eta = \eta_8 \cos \theta - \eta_0 \sin \theta, \quad \eta' = \eta_8 \sin \theta + \eta_0 \cos \theta. \quad (14)$$

At this level, a first estimate of this well-defined mixing angle can be obtained from Eq. (9):

$$\theta = -\frac{1}{2} \tan^{-1} \left[\frac{2\sqrt{2}R}{9-R} \right]. \quad (15)$$

For R going to infinity, θ is shifted by $\pi/2$ at the singular point $R = 9$ such that the corresponding renaming $\eta \rightarrow \eta'$ and $\eta' \rightarrow -\eta$ required by Eq. (14) is compatible with our assignment in Eq. (11). Indeed, in this rather formal limit, we revive the so-called $U(1)$ problem [13] with $\theta = \theta_{\text{ideal}} \simeq +35^\circ$. For $R = 1$, we get $\theta \simeq -10^\circ$ with a totally unrealistic $\eta - \eta'$ mass ratio. For the optimal value $R = 3$ (see Eq. (12)), we obtain $\theta \simeq -27^\circ$. So, a more precise determination of θ clearly requires a better fit of the $\eta - \eta'$ mass spectrum, or vice versa. Here we choose a phenomenological extraction of this angle in order to get an upper bound on the $\mathcal{O}(p^4)$ chiral corrections needed to reproduce Eq. (13).

The explicit breaking of the flavor singlet axial $U(1)_A$ symmetry manifests itself as an anomaly in the divergence of the associated current:

$$(\partial^\mu J_{5\mu}^0)_{\text{anomaly}} = \frac{3\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu}. \quad (16)$$

At the effective level, we obtain from Eqs. (3) and (5)

$$(\partial^\mu J_{5\mu}^0)_{\text{anomaly}} = -\sqrt{3} f m_0^2 \eta_0 \quad (17)$$

such that $\alpha_s G_{\mu\nu} \tilde{G}^{\mu\nu}$ is a clean probe of the singlet component η_0 in η and η' for OZI-suppressed processes [14].

Let us consider the well-measured OZI-suppressed processes, $J/\psi \rightarrow \eta\gamma$ and $J/\psi \rightarrow \eta'\gamma$ where the initial $c\bar{c}$ annihilates into one photon by emitting two gluons. From the definition of the mixing angle in Eq. (14), the amplitude ratio of these two processes can be written as

$$R_{J/\psi} = \frac{A(J/\psi \rightarrow \eta\gamma)}{A(J/\psi \rightarrow \eta'\gamma)} = \frac{\langle 0 | \alpha_s G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} | \eta \rangle}{\langle 0 | \alpha_s G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} | \eta' \rangle} = -\tan \theta \quad (18)$$

due to the relations in Eqs. (16) and (17). Using the current experimental value $\Gamma(J/\psi \rightarrow \eta\gamma)/\Gamma(J/\psi \rightarrow \eta'\gamma) = 0.200 \pm 0.023$ [15], we obtain:

$$\theta^{\text{exp}} = -(22 \pm 1)^\circ. \quad (19)$$

We can now estimate R from Eqs. (15) and (19). The corresponding mass ratio obtained from Eq. (11):

$$R = 2.3 \pm 0.1 \quad \Rightarrow \quad \left(\frac{m_\eta^2 - m_\pi^2}{m_{\eta'}^2 - m_\pi^2} \right) \simeq 0.26 \quad (20)$$

indicates that the required corrections for the $\eta - \eta'$ masses are in fact less than 20%.

We would like to emphasize that the phenomenological extraction of θ^{exp} presented in this section remains valid as long as no further $U(1)_A$ anomalous term arises in the effective Lagrangian. This turns out to be the case for our momentum expansion in the large N_c limit in which all the dominant breaking terms are single traces, except for the $\mathcal{O}(p^0)$ effective Lagrangian! Combining Eqs. (11) and (15) to eliminate the $SU(3)$ -breaking parameter R , we may equally express Eq. (18) in terms of the *theoretical* $\eta - \eta'$ masses:

$$R_{J/\psi} = \cot[\theta + \tan^{-1} \sqrt{2}] \left(\frac{m_\eta^2 - m_\pi^2}{m_{\eta'}^2 - m_\pi^2} \right). \quad (21)$$

This relation resembles the standard PCAC one (see e.g. [16]). Notice however, that a misuse of the *physical* $\eta - \eta'$ masses (see Eq. (13)) at this level would imply $\theta \simeq -17^\circ$ instead of Eq. (19). As we will see, higher order terms in p^2 do modify Eq. (21) but *not* Eq. (18), such that the phenomenological value of the mixing angle given in Eq. (19) is consistently obtained.

4. Chiral corrections

The symmetry breaking Lagrangian at $\mathcal{O}(p^4)$ admits three terms with a single trace over flavors:

$$\Delta\mathcal{L}_\infty^{(p^4)} = \frac{f^2}{8} \left[-\frac{r}{\Lambda^2} \langle m \partial^2 U^\dagger \rangle + \frac{r^2}{2\Lambda_1^2} \langle m U^\dagger m U^\dagger \rangle + \frac{r}{2\Lambda_2^2} \langle m U^\dagger \partial_\mu U \partial^\mu U^\dagger \rangle \right] + \text{h.c.} \quad (22)$$

The first and third terms modify the weak currents and induce the $SU(3)$ splitting between π and K decay constants [17]:

$$\frac{f_K}{f_\pi} - 1 = (m_K^2 - m_\pi^2) \left(\frac{1}{\Lambda^2} + \frac{1}{2\Lambda_2^2} \right). \quad (23)$$

From the observed value $f_K/f_\pi = 1.22 \pm 0.01$, we conclude then that Λ and Λ_2 have to be around 1 GeV, the expected scale for any cut-off of the QCD effective theory. The 20% corrections needed for the $\eta - \eta'$ masses are therefore just at hand! From the second and third terms in Eq. (22), we now obtain the following mass matrix for the neutral ($u\bar{u}$, $d\bar{d}$, $s\bar{s}$) states:

$$\tilde{M}^2 = \frac{\tilde{m}_0^2}{3} \begin{pmatrix} 1 & 1 & 1 - \tilde{\delta} \\ 1 & 1 & 1 - \tilde{\delta} \\ 1 - \tilde{\delta} & 1 - \tilde{\delta} & 1 + \tilde{R} - 2\tilde{\delta} \end{pmatrix} + M_\pi^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (24)$$

A quick glance from Eq. (9) to Eq. (24) displays the crucial appearance of a new parameter

$$\tilde{\delta} = \frac{M_K^2 - M_\pi^2}{\Lambda_2^2} \quad (25)$$

beyond the simple redefinition of R and m_0^2 :

$$\tilde{R} = \frac{6(M_K^2 - M_\pi^2)}{\tilde{m}_0^2} \left[1 + (M_K^2 - M_\pi^2) \left(\frac{2}{\Lambda_1^2} - \frac{1}{\Lambda_2^2} \right) \right], \quad (26)$$

$$\tilde{m}_0^2 = m_0^2 \left(1 - \frac{M_\pi^2}{\Lambda_2^2} \right). \quad (27)$$

Note that M_π and M_K stand now for the physical pion and kaon masses at $\mathcal{O}(p^4)$:

$$M_\pi^2 = m_\pi^2 \left[1 + m_\pi^2 \left(\frac{2}{\Lambda_1^2} - \frac{1}{\Lambda_2^2} \right) \right], \quad M_K^2 = m_K^2 \left[1 + m_K^2 \left(\frac{2}{\Lambda_1^2} - \frac{1}{\Lambda_2^2} \right) \right]. \quad (28)$$

From the diagonalization of the mass matrix in Eq. (24), we obtain

$$\frac{M_\eta^2 - M_\pi^2}{M_{\eta'}^2 - M_\pi^2} = \frac{3 + \tilde{R} - 2\tilde{\delta} - \sqrt{9 - 2\tilde{R} + \tilde{R}^2 - 4\tilde{R}\tilde{\delta} - 12\tilde{\delta} + 12\tilde{\delta}^2}}{3 + \tilde{R} - 2\tilde{\delta} + \sqrt{9 - 2\tilde{R} + \tilde{R}^2 - 4\tilde{R}\tilde{\delta} - 12\tilde{\delta} + 12\tilde{\delta}^2}}, \quad (29)$$

$$M_\eta^2 + M_{\eta'}^2 - 2M_\pi^2 = \frac{\tilde{m}_0^2}{3} (3 + \tilde{R} - 2\tilde{\delta}), \quad (30)$$

$$\theta = -\frac{1}{2} \tan^{-1} \left[\frac{2\sqrt{2}(\tilde{R} - 3\tilde{\delta})}{9 - \tilde{R} - 6\tilde{\delta}} \right]. \quad (31)$$

Using the observed values for the pseudoscalar masses $M_{\eta,\eta'}^2$ and the mixing angle θ^{exp} (see Eq. (19)), we can fix the parameters $\tilde{R}$, $\tilde{m}_0$ and $\tilde{\delta}$. Our main result is shown in Fig. 1. The horizontal line slowly curving around $\tilde{R} = 3$ and the quasi vertical lines are obtained from Eq. (29) and Eq. (31), respectively. A remarkable property

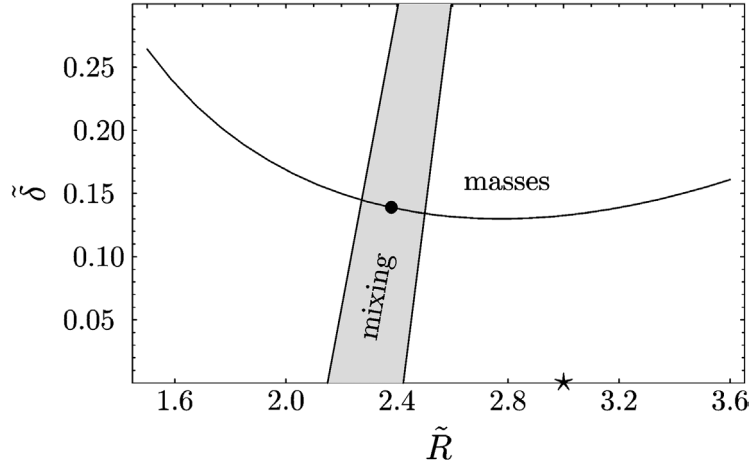


Fig. 1. Fit of the $\mathcal{O}(p^4)$ chiral corrections using the observed masses and mixing angle for η and η' . The line slowly curving around $\tilde{R} = 3$ is obtained from Eqs. (13) and (29) while the allowed shaded area is obtained from Eqs. (19) and (31) (left and right bounds correspond to $\theta = -21^\circ$ and $\theta = -23^\circ$, respectively). The black dot leads to Eq. (32). The star indicates the $\mathcal{O}(p^2)$ optimal fit [6] with $\theta \simeq -27^\circ$.

is that the constraints from the mass ratio and the mixing angle are quite independent of $\tilde{\delta}$ and $\tilde{R}$, respectively, at their intersection. Therefore, as the experimental errors for the radiative J/ψ decays are small, $\tilde{\delta}$ and $\tilde{R}$ are rather precisely determined from this complementary analysis. Using the central value for the mixing as well as Eq. (30), we obtain the reasonable values for the three parameters in $\tilde{M}^2$:

$$\tilde{\delta} \simeq 0.14, \quad \tilde{R} \simeq 2.4, \quad \tilde{m}_0 \simeq 0.83 \text{ GeV}. \quad (32)$$

As a result, all the cut-off parameters in the $\mathcal{O}(p^4)$ Lagrangian are fixed around 1 GeV;

$$\Lambda \simeq 1.2 \text{ GeV}, \quad \Lambda_1 \simeq 1.2 \text{ GeV}, \quad \Lambda_2 \simeq 1.3 \text{ GeV} \quad (33)$$

as it should be. Therefore, the masses and mixing can be quite naturally reproduced in the large N_c limit.

Note that the new, $\tilde{R}$ -independent, identity derived now from Eqs. (29) and (31) reads

$$-\tan \theta = \cot[\theta + \tan^{-1} \sqrt{2}] \left(\frac{M_\eta^2 - M_\pi^2 - \frac{2}{3} \tilde{m}_0^2 \tilde{\delta}}{M_{\eta'}^2 - M_\pi^2 - \frac{2}{3} \tilde{m}_0^2 \tilde{\delta}} \right). \quad (34)$$

By analogy with Eq. (21), we may rewrite

$$R_{J/\psi} = -\tan \theta = \cot[\tilde{\theta} + \tan^{-1} \sqrt{2}] \left(\frac{M_\eta^2 - M_\pi^2}{M_{\eta'}^2 - M_\pi^2} \right) \quad (35)$$

with $\theta \simeq -22^\circ$ but $\tilde{\theta} = -17^\circ$. This result confirms the need for a two-angle formalism [18,19], once one goes beyond PCAC to derive electroweak decay amplitudes.

5. $1/N_c$ corrections

Loops as well as tree-level multi-traces over flavors provide the $1/N_c$ corrections. One-loop corrections to the ratio f_K/f_π turn out to be numerically small if the renormalization scale associated with the chiral logarithms is chosen in the vicinity of the η mass. The large N_c limit adopted here legitimates this rule-of-thumb such that our successful understanding of the $\eta - \eta'$ masses and mixing in the large N_c limit is basically due to the Λ_2 term

in $\Delta\mathcal{L}_\infty^{(p^4)}$. This term could in principle be rotated away via a specific $\mathcal{O}(p^2)$ transformation

$$U \rightarrow U' = U + \frac{r}{4\Lambda_2^2} [m - Um^\dagger U] \quad (36)$$

on $\mathcal{L}_\infty^{(p^2)}$. Such a field redefinition preserves the unitarity of U up to $\mathcal{O}(p^4)$. It eliminates the Λ_2 term which causes tedious (and sometimes overlooked [12]) wave-function renormalizations and simply amounts to the substitutions

$$\frac{1}{\Lambda^2} \rightarrow \frac{1}{\Lambda^2} + \frac{1}{2\Lambda_2^2}, \quad (37)$$

$$\frac{1}{\Lambda_1^2} \rightarrow \frac{1}{\Lambda_1^2} - \frac{1}{2\Lambda_2^2} \quad (38)$$

in $\Delta\mathcal{L}_\infty^{(p^4)}$, as seen from Eqs. (23) and (28), respectively. However, acting simultaneously on $\Delta\mathcal{L}_{1/N_c}^{(p^0)}$, this chiral transformation would then require a $1/N_c$ -suppressed double trace term in the $\mathcal{O}(p^2)$ effective Lagrangian:

$$\Delta\mathcal{L}_{1/N_c}^{(p^2)} = \epsilon_2 \frac{f^2}{8} r \langle mU^\dagger - Um \rangle (\ln U - \ln U^\dagger) \quad (39)$$

to absorb its effect via another harmless substitution:

$$\epsilon_2 \rightarrow \epsilon_2 + \frac{m_0^2}{12\Lambda_2^2}. \quad (40)$$

The ϵ_2 term being forbidden in our large N_c limit, the field redefinition given in Eq. (36) would only amount to removing the Λ_2 -term given in Eq. (22) in favor of $\frac{m_0^2}{12\Lambda_2^2} \frac{f^2}{8} r \langle mU^\dagger - Um \rangle (\ln U - \ln U^\dagger)$ such that our results based on Eq. (24) are unchanged. The physical effect of the Λ_2 cut-off on the pseudoscalar mass matrix (Eq. (24)) is therefore a direct consequence of the expansion adopted here.

To summarize, the $1/N_c$ corrections to $\mathcal{O}(p^2)$ terms in Eqs. (4) and (39) are assumed to be negligible in our approach based on the hierarchy

$$\mathcal{O}(p^2, 1/N_c) \ll \mathcal{O}(p^4, \infty). \quad (41)$$

In an alternative combined expansion [20] in $p^2 = \mathcal{O}(\delta)$ and $1/N_c = \mathcal{O}(\delta)$, the ϵ_1 term would imply a wave-function renormalization of the η_0 field and, consequently, a global rescaling of the $q\bar{q}$ mass matrix. The ϵ_2 term considered in [18] is more problematic. Its contribution to the mass matrix could of course be absorbed into $\tilde{\delta}$, $\tilde{R}$ and $\tilde{m}_0$ (see Eq. (40)). But being not invariant under $U(1)_A$, it would definitely invalidate our phenomenological extraction of the mixing angle from OZI-suppressed processes. Consequently, while the constraint from masses displayed in Fig. 1 would remain the same, the one from mixing would simply disappear. So, from this point of view, $1/N_c$ corrections are not only unnecessary in reproducing the η and η' masses but they also generate an ambiguity for the size of the chiral corrections estimated in Eq. (33).

6. Conclusions

We have shown that the $\mathcal{O}(p^2)$ prediction for the η and η' masses indeed requires 15 ~ 20% of higher order corrections. In the large N_c limit, the $\mathcal{O}(p^4)$ corrections, which are welcome to explain the $SU(3)$ splitting of the f_K and f_π weak decay constants, naturally fill up this deficit if the octet–singlet mixing angle $\theta = -(22 \pm 1)^\circ$ consistently extracted from J/ψ OZI-suppressed decays is used. The large N_c approximation at each order in the momentum expansion provides therefore a simple and coherent description of the $\eta - \eta'$ mass spectrum and mixing. The $1/N_c$ expansion being trustworthy, we are now in a favorable position to constrain new physics from (electro-)weak processes involving η and η' mesons.

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