Strong Sustainability, Rent and Value-Added Sharing*

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Abstract

We reassess Ricardo's conjecture of a secular increase in rent in an endogenous growth model with an essential renewable material resouce and rising product quality. The model is consistent with the concept of strong sustainability and thus assumes that the resource productivity is bounded. Hence, only qualitative growth (i.e. a secular increase in the quality of final productions) may persist in the long run.

We analyse how the scarcity of the resource affects the rent level and the distribution of national income (between resource, capital and labour) in the short and long runs. In the long run, resource scarcity induces a distributive conflict opposing labour to capital and resource, a lower resource stock implying generally a lower labour share and higher capital and rent shares.

The transitory dynamics of the economy is analyzed numerically, starting from initial conditions characterized by a low capital stock and a large potential of technical progress. Even if initially the rent (share) may evolve non monotonically, simulations tend to confirm that Ricardo's conjecture emerges sooner or later during the transitory dynamics : except in the case of a very high dematerialization potential of final output, the rent share will raise as quantitative growth slows down.

Key words: growth, strong sustainability, rent, functional distribution of income, renewable resource, dematerialisation.

JEL: E25, D9, 044, Q0, Q56

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1 Introduction

Despite a relative eclipse between the 1960s and 1990s, the question of value added sharing has been a subject of intense study since the beginnings of economic science. Thus Ricardo (1817) described it as the principal problem of Political Economy. In an article whose title paraphrases this expression, Atkinson (2009) reviews several reasons why this issue remains important, including the fact that it enables a link to be established between macro- and micro-economic income growth. As Bentolila and Saint-Paul (2003) stress, this issue is also raised in public debate, and the changing wage share is often seen as an indicator of how the fruits of growth are shared between production factors.

For Ricardo, the question of value added sharing is inseparable from that of landowners' rent. The latter results from the farming of land of uneven fertility, and specifically from the yield differentials offered by farmed land in relation to marginal land (where the value of the harvest just covers the cost of production). Ricardo held that population growth would make it necessary to use ever less productive land, leading to ever higher rents. He therefore conjectured that the rent share in national income would tend to increase, while the profit share would tend to decrease, with a resultant fall in investment and economic growth[1].

Contrary to what Ricardo foresaw, the rent share in national income has in fact declined and is relatively low in industrialised countries (see especially Hill (2001)). This probably explains why rent, understood as the remuneration of non-produced factors of production such as land and natural resources, has gradually been neglected by the literature[2]. The question of the distribution of national income has thus become reduced essentially to that of the distribution between wages and profit, as can be seen from the work of Kalecki (1938) and Kaldor (1956) and from more recent work (see in particular Bentolita and Saint-Paul (op. cit.), Young (2011), and Zuleta and Young (2013)).

Hill (op. cit.) reviews various possible causes of the downward trend in the rent share that has characterised the industrialisation of Western countries. To these may be added the point that, if we are considering natural resources in general (and not specifically land), two processes may in the past have counteracted the increase in the rent share conjectured by Ricardo: firstly, natural resource-saving technological progress, and secondly, repeated discoveries of new deposits of resources. By reducing the demand for resources in the former case and reducing the marginal costs of their exploitation in the latter case, these two processes exert downward pressure on their price, and hence on the rent they are likely to yield. However, although these processes of innovation and discovery are far from complete, several arguments, put forward in particular by ecological economists, suggest that they have their limits: 1) for physical reasons, technical progress or substitution between natural resources and man-made factors will not make it possible to dispense with natural resources entirely; 2) the discovery of potentially exploitable new deposits (or reserves) occurs within the context of a stock of resources that is finite; 3) although certain resources are self-renewing by nature, their exploitation is limited by their rate of renewal, which is bounded.

Taking the arguments of the previous paragraph carefully into account, this article aims to reconsider Ricardo's conjecture (i.e. the possibility of an upward trend in the rent share in value added) in an economy with finite resources. In an endogenous growth model, we study how the relative scarcity of a renewable resource, which is essential to production, affects the rent associated with its exploitation and the distribution of value added (between resource, capital and labour)[3]. The model is consistent with the concept of strong sustainability and assumes that the

[1] For Ricardo, profit alone contributes to savings and hence investment, accumulation and growth. The disappearance of profit thus entails the disappearance of growth.

[2] Although rent is not completely ignored in some contributions, it is aggregated with other forms of property income (as opposed to labour income) or with other income from non-produced assets (as opposed to income from produced assets, i.e. accumulated capital). In certain recent contributions, especially the book by Askenazy et al. (2011) and the various contributions of n°61 of the Cahiers d'conomie Politique (Assous, 2011), rent is not explicitly distinguished. Atkinson (2009) classes profit and rent under income from capital.

[3] Unlike Hill (2001), who aims to throw light on past historical reality, this exercise is more forward-looking, and

productivity of the natural resource is bounded. Labour and capital must be allocated to three economic activities: resource extraction, final production and research. Technical progress is the result of research, and improves both the productivity of the factors and the quality of the final goods. As the resource stock is finite and resource-saving technical progress is limited, indefinite quantitative growth (in terms of the number of goods) is impossible; only indefinite qualitative growth (in terms of the quality of goods) may be possible (as in Fagnart and Germain (2011) or Krysiak (2006)).

The structure of the article is as follows. Section 2 presents the basic assumptions of our approach and its specific features, then details the model. Section 3 looks at its long-run equilibrium and how this equilibrium and value added sharing are affected by the scarcity of the natural resource. The analysis is initially analytical in the context of a particular case where the capital/labour ratio is identical in all activities, and is then extended numerically to the general case. Section 4 presents the results obtained dynamically, using a numerical approach. This analysis firstly relates to the case where the quantity of material contained in the economy-natural resource system is constant throughout the trajectory of the economy; we next consider a variant where a series of discoveries gradually increases the quantity of available resource. We then analyse the role played by the potential for technical progress in the emergence of Ricardo's conjecture. Finally, we study the consequences of stronger demographic pressure. The conclusion summarises the main findings and sets out three proposals for future research.

2 The model

We propose an endogenous growth model of a decentralised economy where final production uses an essential renewable natural resource. Our modelling is consistent with the concept of strong sustainability, which states that the possibilities for substituting a natural resource with man-made production factors are limited. This in turn means that the productivity of the natural resource is bounded from above. Our model thus differs from the neoclassical growth models with resources in the tradition of the pioneering contributions of Dasgupta-Heal (1974), Solow (1974) and Stiglitz (1974), which allow the productivity of natural resources to grow indefinitely and are based in this respect on the concept of weak sustainability. Growth models of neoclassical inspiration have been criticized by ecological economics on the grounds that these models ignore the laws of physics (in particular the laws of conservation and the second law of thermodynamics) or, more subtly, some of the constraints imposed by these laws on production processes and technology. Ecological economics thus emphasises that the result of unlimited growth in neoclassical models is based either on the assumption that growth is fuelled by sectors (especially R&D) that do not directly or indirectly require any resources (material or energy), or on the assumption that marginal resource productivity is asymptotically infinite. However, besides the fact that every human activity requires material and energy, various contributions (in particular Islam (1985), Anderson (1987) and Baumgärtner (2004)) have shown that the property of infinite marginal resource productivity would violate one of the above laws of physics. This further implies that both the possibilities for substituting natural resources with other factors and the prospects for technical progress in relation to a resource are bounded by these laws.

If these laws are taken strictly into account within the framework of a model with a renewable resource, Fagnart and Germain (2011) (henceforth FG) and Krysiak (2006) have shown that indefinite quantitative growth (in terms of number of goods) is impossible. FG shows that only indefinite qualitative growth (in terms of quality of goods) may be possible, and that quantitative growth can only be transitional. If the economy only had non-renewable resources, there would be no possibility of growth in the long term[4].

is based on the insight that the decline in the rent share that has been observed historically could be reversed in the future for the reasons outlined above.

[4]Note that this would also be the case in a single-sector neoclassical growth model with a technology of type $Y = F(K;L;R)$ (where Y is the output, K is capital, L is employment and R is a resource), *once one assumes* 1) limited substitutability between resource R and the man-made production factors $(K;L)$ and 2) technological

The model developed here enriches FG in several directions. FG offers only a single-sector model without a labour factor and with a free natural resource. As there are no wages or rent and capital is the only remunerated production factor, FG cannot be used to assess Ricardo's conjecture or to consider the changing distribution of value added during the transitional dynamics of an economy with finite resources. Here we introduce a primary sector which exploits and sells the resource and the labour factor in all productive activities.

Another limitation of FG is that a logarithmic utility function is assumed. This results in a constant saving rate whose value is independent of any issue related to the natural resource. We abandon this assumption here in favour of an iso-elastic utility function. The saving rate in this model is variable, and its value (which is one of the determinants of the distribution of value added) is affected by the abundance of the resource. It follows that this abundance also affects the ongoing research efforts of firms, which was not the case in FG.

The model's economy includes four types of markets: markets for final goods, the natural resource, labour and productive capital. There are two production sectors:

(i) Prior to final production, a *primary* sector consists of firms that exploit the natural resource (henceforth NR) in perfect competition; they sell it to firms in the final sector which process it. The rent from this primary sector is Ricardian in nature: each operator has access to the resource and these access points differ in their returns.

(ii) The *final* or manufacturing sector consists of firms in monopolistic competition. They have two activities: the *manufacture* of final goods and *research* aimed at increasing the quality of these goods. The goods produced have a resource content. Once consumed or used, they become waste, which is recycled naturally by the environment and feeds back into the resource stock[5]. Research activities that improve product quality also contribute, via an external effect, to the technical progress that increases the productivity of the different factors.

2.1 Technological assumptions related to the resource and other factors

The final firms use the resource to produce their output. Due to technical progress resulting from the dissemination of research efforts, the resource intensity of the manufacturing technology decreases over time. However, for the reason explained above, this process is bounded: complete dematerialisation of final production is deemed to be impossible, and the resource content of these production activities tends towards a strictly positive value limit. As production activities cannot be fully dematerialised, the same is also true of the productive capital resulting from this production: its material resource content may be reduced over time, but it remains bounded from below by a positive limit value. Due to the dependence of productive capital on the resource, all activities that require capital thus indirectly require the resource.

Let us suppose capital k and labour ℓ needed for all business operations. These can therefore be allocated to three different activities, indicated by e for extraction in the primary sector, f for manufacture in the final sector and r for research in the same sector. k_{zt} (or ℓ_{zt}), where $z = e, f, r$, denotes the capital (or labour) allocated to activity z. To simplify the presentation of the model, we will assume that the sectoral production functions have complementary factors. λ_{zt} will denote the labour/capital ratio in activity z at t:

progress bounded in terms of natural resource. The best the economy can hope for in the long term is then zero growth. Furthermore, if R was not renewable, the economy could not avoid decline, with the resource share in value added inevitably tending towards 1 (see Eriksson, 2013, Chap. 6). In this latter case, we would end up with a completely caricatured distribution of national income in a kind of extreme version of Ricardo's conjecture. Under the same assumptions of low substitution between R and $(K; L)$ and bounded technological progress, two merits of the modelling proposed here are that it leaves open the prospect of sustainable growth (though purely of a qualitative nature), and avoids offering a trivial answer to the question of the distribution of value added.

[5]This assumption of natural recycling is made for the sake of modelling simplicity, as it removes the need to describe a proper recycling activity. Given this assumption, the concept of NR used here refers to the great natural cycles of carbon, nitrogen, water and so on. *Sensu stricto*, however, it excludes metals and other non-renewable resources for which *natural* recycling is zero on a human scale.

$$\frac{\ell_{zt}}{k_{zt}} = \lambda_{zt} \text{ for } z = e, f, r. \tag{1}$$

The value of λ_{zt} is given in period t but decreases over time as a function of endogenous technical progress (cf.infra).

The choice of Leontief-type production functions is dictated by our desire to choose the simplest possible technology from among those consistent with the results of ecological economics. Ecological economics distinguishes between agents inputs (or factors) and transformed inputs: the former (capital and labour) perform the production process whereas the latter (material or energy) are transformed during this process. Given their radically different functions, there is strong complementarity between the two types of factor, and the elasticity of substitution between them is inevitably low (or even zero in the short term in many technologies). However, it is easier to allow more substitutability between agent factors (labour and capital). In this respect, we could therefore have made a more general assumption than the one used here, but it would have made the model and discussion more complicated. In addition, the technological choices represented by (1) do not prevent the substitutability between capital and labour *at the aggregate level*, through reallocations of capital and the workforce between sectors and activities. Moreover, the sectoral capital/labour ratios vary over time, as a function of endogenous technical progress (cf. infra).

2.2 Primary sector and operators rent

There is a continuum $[0, N]$ of access points to the resource (which is assumed to be homogeneous in quality). Each access point is controlled by an operator. The maximum quantity that can be extracted from an access point in period t depends on the overall stock of resource R_t available in t: it is the same for the N access points, and is R_t/N. However, resource access points do not all offer the same returns. They are classified in order of decreasing returns, with i indicating the access point that comes in i^{th} position. To extract a quantity x_{it} of NR in period t, operator i ($i \in [0, N]$) needs a quantity of capital given by

$$k_{eit} = \frac{x_{it}}{1 - \frac{i}{N}}. \tag{2}$$

The most efficient operator ($i = 0$) therefore has a capital/output ratio of 1. At the other extreme, the capital/output ratio of the least efficient operator ($i = N$) tends to infinity.

The market for the resource is competitive and the operators are price takers. Each operator i chooses its output x_{it} and input levels (k_{eit}, ℓ_{eit}) to maximise its income. Let P_{xt} be the sale price of the extracted NR, V_t the cost of use of a unit of productive capital and W_t the wage paid for one unit of work. Under (1) and (2), the unit cost of production of the operator with the highest return (for which $k_{eit} = x_{it}$) is:

$$U_{et} =_{def} V_t + \lambda_{et} W_t \tag{3}$$

and the rent of any operator i can be written

$$\theta_{it} = \left[P_{xt} - \frac{U_{et}}{1 - \frac{i}{N}} \right] x_{it}. \tag{4}$$

As the rent is linear at x_{it}, operators can be divided into two categories: those whose rent per unit is positive and which supply the maximum possible quantity R_t/N; those whose rent per unit is negative and whose supply is zero. The marginal operator, whose rent is zero, is indifferent between operating its site and closing it temporarily. This marginal operator, indicated as n_t,, is identified by the condition $\theta_{n_t,t} = 0$: hence,

$$P_{xt} = \frac{U_{et}}{1 - \frac{n_t}{N}} \qquad \text{or} \qquad n_t = N \left[1 - \frac{U_{et}}{P_{xt}} \right]. \tag{5}$$

As the operators $i \in [0, n_t]$ supply $x_{it} = R_t/N$ whereas the operators $i \in [n_t, N]$ supply $x_{it} = 0$, the total resource output will be

$$X_t = \int_0^{n_t} x_{it} di = n_t \frac{R_t}{N} = R_t \left[1 - \frac{U_{et}}{P_{xt}}\right], \tag{6}$$

with the last equality resulting from (5). The resource supply is increasing in the resource stock and the resource price but decreasing in the prices of the agent inputs (capital and labour). (6) can be written as a relationship between the price of the resource and the marginal operators unit cost of extraction:

$$P_{xt} = \frac{U_{et}}{1 - E_t} \qquad \text{with} \quad E_t =_{def} \frac{X_t}{R_t}. \tag{7}$$

E_t is the extraction rate of the resource. In view of (2), the capital stock mobilised by all active operators can be expressed as:

$$\begin{aligned} k_{et} = \int_0^{n_t} k_{eit} di = \frac{R_t}{N} \int_0^{n_t} \frac{di}{1 - \frac{i}{N}} &= R_t \ln \left(1 - \frac{n_t}{N}\right)^{-1} \\ &= R_t \ln (1 - E_t)^{-1} \end{aligned} \tag{8}$$

as $n_t/N = X_t/R_t (= E_t)$ in view of (5) and (6). Like the price of the NR, the quantity of capital used for extraction and the quantity of labour (in accordance with (1)) are therefore increasing and convex functions of the extraction rate E_t.

Finally, the extraction sector's total rent will be

$$\begin{aligned} \Theta_t = \int_0^{n_t} \theta_{it} di &= P_{xt} X_t - U_{et} k_{et} \\ &= P_{xt} R_t \left[E_t - [1 - E_t] \ln (1 - E_t)^{-1}\right], \end{aligned} \tag{9}$$

the latter equality arising from the substitution of X_t, U_{et} and k_{et} with expressions (6), (7) and (8). The rent is therefore an increasing function of the stock of NR, its price and the extraction rate.

2.3 The final production sector

Final production is carried out by a number of firms in monopolistic competition. These form a continuum in the interval $[0, 1]$. Firm $i \in [0, 1]$ is the only one to produce good i, whose price p_{it} and quality q_{it} it sets in each period. The market power enjoyed by each firm prompts it to invest in research in order to improve the quality of its output. Good i is used for consumption or investment. The quantity of good i intended for consumption in period t is denoted c_{it} and the quantity intended for investment is denoted d_{it}.

2.3.1 Final demand

We assume a horizontal differentiation model *à la* Dixit and Stiglitz (1977). The aggregates of consumption and investment are described by the following CES indices:

$$C_t = \left[\int_0^1 [\psi(q_{it}) c_{it}]^\alpha di\right]^{1/\alpha} \qquad \text{and} \qquad k_{t+1} = \left[\int_0^1 [\varphi(q_{it}) d_{it}]^\alpha di\right]^{1/\alpha} \tag{10}$$

where $0 < \alpha < 1$. Functions $\psi(q_{it})$ and $\varphi(q_{it})$ are positive, continuous and increasing in q_{it}: all other things being equal, better quality consumer goods contribute to a higher consumer index; similarly, better quality capital goods have greater productivity in terms of capital formation.

Appendix 7.1 details the derivation of the demand functions c_{it} and d_{it} and of the consumer and investment price indices. These latter are respectively

$$P_{ct} = \left[\int_0^1 \left[\frac{p_{it}}{\psi(q_{it})} \right]^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \quad \text{and} \quad P_{kt} = \left[\int_0^1 \left[\frac{p_{it}}{\varphi(q_{it})} \right]^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \tag{11}$$

where $\varepsilon =_{def} 1/[1-\alpha]$. Total demand for good i, $y_{it} = c_{it} + d_{it}$, is a decreasing function of its price and an increasing function of its quality. As Appendix 7.1 shows, the price elasticity of demand is $-\epsilon$, while the quality elasticity of demand is:

$$\frac{q_{it}}{y_{it}} \frac{\partial y_{it}}{\partial q_{it}} = (\epsilon - 1) \left[\frac{c_{it}}{y_{it}} \frac{q_{it}\psi'(q_{it})}{\psi(q_{it})} + \frac{d_{it}}{y_{it}} \frac{q_{it}\varphi'(q_{it})}{\varphi(q_{it})} \right]. \tag{12}$$

To simplify the presentation of the model, we now use the property of symmetry that characterises equilibrium in monopolistic competition[6] and will henceforth omit to use i to denote the variables describing the behaviour of a monopolistic firm and demand for its output.

2.3.2 Manufacturing technology

Manufacturing a quantity y_t in period t requires a quantity of material equal to

$$x_t = [\chi_t + \mu_t]\, y_t \tag{13}$$

where $\mu_t (> 0)$ is the quantity of material incorporated in the good and $\chi_t (> 0)$ is the quantity wasted during the production process. It is also necessary to use the following quantity of capital:

$$k_{ft} = \kappa_t y_t \tag{14}$$

with $\kappa_t > 0$. In accordance with (1), there is also a need for labour, the quantity of which is $\ell_{ft} = \lambda_{ft} k_{ft}$.

The coefficients $\chi_t, \mu_t, \kappa_t, \lambda_{ft}$ decrease over time as a function of endogenous technical progress induced by an external effect associated with firms' research activities (cf. infra).

2.3.3 Research, innovation and technical progress

Investing in research increases the output quality, which, at a given price, stimulates demand for that output. However, an innovative firm cannot keep its results to itself for more than one period: at the start of period t, all firms have the same knowledge (or quality) Q_{t-1}, which is a public legacy of all past research efforts. If a firm invests in research in period t, it increases the quality of its product above the average level Q_{t-1}, and has a competitive advantage during that period alone. Quality gains are generated deterministically: to increase the quality of its output y_t to a level $q_t > Q_{t-1}$, the firm must provide its research facility with a capital stock equal to

$$k_{rt} = h\left(\frac{q_t}{Q_{t-1}}\right) y_t \tag{15}$$

where $h(\cdot)$ is an increasing and convex function that satisfies $h(1) = 0$ (no research effort is required to maintain the existing quality level Q_{t-1}) and $h'(1) = 0$. In accordance with (1), the firm must also hire a number of researchers equal to $\ell_{rt} = \lambda_{rt} k_{rt}$.

Moreover, the research efforts have an external effect: at the end of period t, all results become public and firms then have free access to the quality level

$$Q_t = Q(q_{it}, i \in [0, 1]) \tag{16}$$

[6]The property that final firms take identical decisions results from the fact that (i) final goods appear symmetrically in consumption and capital indices and (ii) firms have access to the same technology (which means that the goods have the same material content).

where $Q(\cdot)$ is an increasing function of the individual research efforts.

The external effect associated with this average quality level achieved at the end of period t also engenders technical progress affecting all activities of the economy during period $t+1$. The technology coefficients χ_{t+1}, μ_{t+1}, κ_{t+1}, λ_{ft+1}, λ_{rt+1} and λ_{et+1} change negatively as a function of Q_t. In formal terms:

$$\chi_{t+1} = \chi(Q_t),\ \mu_{t+1} = \mu(Q_t),\ \kappa_{t+1} = \kappa(Q_t), \lambda_{zt+1} = \lambda_z(Q_t) \quad \text{for} \quad z = f, r, e. \tag{17}$$

$\chi(\cdot)$, $\mu(\cdot)$, $\kappa(\cdot)$ and $\lambda_z(\cdot)$ are monotonically decreasing functions of their argument. However, they do not tend to zero. The crucial assumption is made that *all* activities (including research) directly or indirectly require material, capital and labour. In other words, although there is no a priori limit to the improvement of the quality of goods produced, total dematerialisation of the output and of the production processes is impossible[7]. Consequently, the above technological coefficients are bounded from below by a number of strictly positive constants:

$$\lim_{Q\to+\infty} \chi(Q) = \underline{\chi} > 0,\ \lim_{Q\to+\infty} \mu(Q) = \underline{\mu} > 0,\ \lim_{Q\to+\infty} \kappa(Q) = \underline{\kappa} > 0,\ \lim_{Q\to+\infty} \lambda_z(Q) = \underline{\lambda}_z > 0 \tag{18}$$

with $z = f, r, e$. This reflects the fact that producing a unit of the final good will always require a non-infinitesimal quantity of material and capital; similarly, all activities (f, r, e) will always require a non-infinitesimal quantity of labour per unit output.

Likewise, it is assumed that:

$$\lim_{q\to+\infty} \varphi(q) = \overline{\varphi} < +\infty. \tag{19}$$

It is therefore impossible to produce an infinite stock of capital with a finite quantity of capital goods, which reflects the fact that the instruments used in the production process (tools, machinery and infrastructure) cannot be completely dematerialised. Consequently, research itself also indirectly requires material. For long-run equilibrium to exist, it is necessary that $\overline{\varphi} > \underline{\kappa} + \underline{\chi} + \underline{\mu}$, which we will assume.

2.3.4 Determining price, output and quality

In period t, each monopolistic firm chooses the levels of price p_t, quality q_t, output y_t and inputs k_{ft}, ℓ_{ft}, x_t that maximise its profit[8] $\Pi_t = p_t y_t - V_t\left[k_{ft} + k_{rt}\right] - W_t\left[\ell_{ft} + \ell_{rt}\right] - P_{xt} x_t$.

Let U_{zt}, with $z = f, r$, denote the *total* operating cost (i.e. including labour) of a unit of capital allocated to activity z. According to our assumptions (1), (14) and (15), we have

$$U_{zt} =_{def} V_t + \lambda_{ft} W_t \qquad \text{with } z = f, r. \tag{20}$$

The labour and capital cost of the production of y_t units of goods therefore amounts to $U_{ft}\kappa_t y_t$. Similarly, the labour and capital cost of the research facility of a firm of size y_t is $U_{rt} h\left(q_t/Q_{t-1}\right) y_t$. The profit function of period t can therefore be rewritten as follows:

$$\Pi_t \quad = \quad p_t y_t - \left[U_{ft}\kappa_t + U_{rt} h\left(\frac{q_t}{Q_{t-1}}\right) + P_{xt}\left[\chi_t + \mu_t\right]\right] y_t. \tag{21}$$

The firm's decision problem is represented by the maximisation of (21) relative to p_t and q_t subject to demand and provided that $q_t \geq Q_{t-1}$ (with Q_{t-1} being given). These choices of p_t and q_t also determine the activity level y_t and the levels of capital and labour allocated to production

[7]This assumption, which is consistent with the results cited earlier of Anderson (1987) and Baumgärtner (2004), seems eminently realistic: even services require material, energy and labour for their output, and the last of these factors itself requires material and energy for its survival and reproduction. Although technical progress and the reallocation of activities towards the service sector are undeniably likely to dematerialise aggregate output, the complete dematerialisation of that output or of the production process is therefore an unachievable abstraction.

[8]The fact that the firm rents capital and the assumption of public access to all research results at the end of period t (cf. (16)) make it possible to study the firms behaviour period by period.

and research. The first-order conditions for this maximisation lead to the following equations[9] (cf. Appendix 7.2):

$$p_t = \frac{1}{\alpha}\left[U_{ft}\kappa_t + U_{rt}h\left(\frac{q_t}{Q_{t-1}}\right) + P_{xt}\left[\chi_t + \mu_t\right]\right] \tag{22}$$

$$U_{rt}h'\left(\frac{q_t}{Q_{t-1}}\right)\frac{q_t}{Q_{t-1}} = \alpha p_t\left[\frac{c_t}{y_t}\frac{q_t\psi'(q_t)}{\psi(q_t)} + \frac{d_t}{y_t}\frac{q_t\varphi'(q_t)}{\varphi(q_t)}\right]. \tag{23}$$

Equation (22) describes the price behaviour, which is the product of the mark-up coefficient $1/\alpha(>1)$ and the marginal cost of production (the expression between square brackets[10]). Equation (23) describes the optimal research investment, which must result in the marginal cost and marginal income of an increase in quality being equal. The marginal income (which is proportional to the right-hand expression) derives from the increase in demand caused by the increase of q_t, while the marginal cost (which is proportional to the left-hand expression) derives from the cost of the factors involved in the increase of q_t.

2.4 Households

We will consider infinite-horizon households that supply their labour inelastically and save by accumulating the physical capital that they rent out to firms. Implicitly, there are three categories of households: the resource owners receive rent from the primary sector (Θ_t); the owners of the monopolistic firms receive the net profits of the final sector (Π_t); and the workers (numbering L) receive income from the labour that they supply inelastically (W_tL). The income from renting out capital to firms V_tk_t is shared between the households according to the capital they have accumulated. To keep the model relatively simple, however, we will assume that households preferences can be represented by homothetic and identical utility functions. This simplifying assumption means that the propensity to save is independent of the level and sources of income (wages, profits and rent), by contrast with the Kaldorian approach of Hill (2001). Under this assumption, we can describe the households' behaviour with reference to the behaviour of a single agent who receives the entire macro-economic income, consumes part of it and invests the rest in the form of physical capital.

As far as capital accumulation is concerned, we will assume (i) a one-period time to build and (ii) a lifespan of one period[11]. The investment of period t thus becomes part of the capital stock of the following period, k_{t+1}.

The budget constraint of period t is therefore expressed as $P_{ct}C_t + P_{kt}k_{t+1} = V_tk_t + W_tL + \Pi_t + \Theta_t$, where $P_{ct}C_t$ and $P_{kt}k_{t+1}$ denote the consumption expenditure and investment expenditure respectively. It will be recalled that the quantities k_{t+1} and C_t are the Dixit-Stiglitz-type indices given earlier and P_{kt} and P_{ct} denote the capital price and consumer price indices respectively.

Households' preferences are represented by the utility function $\sum_{t=1}^{T}\beta^t\mathcal{U}(c_t)$ where $0<\beta<1$ is the time discount factor, T is the time horizon (which may be infinite) and

$$\mathcal{U}(C_t) = \begin{cases} \frac{\sigma}{\sigma-1}C_t^{\frac{\sigma-1}{\sigma}} & \text{if } \sigma>0 \text{ and } \sigma\neq 1 \\ \ln(C_t) & \text{if } \sigma = 1, \end{cases}$$

where σ is the intertemporal elasticity of substitution in consumption. The intertemporal con-

[9] The property $h'(1)=0$ ensures that $q_t \geq Q_{t-1}$ is non-binding.

[10] Producing an additional unit requires a capital surplus (the total operating cost of which is the sum of the first two terms) and a material surplus (the cost of which is the third term).

[11] This assumption of a unit depreciation rate avoids multiple generations of capital which differ from one another in terms of material content because they have been manufactured using final goods which are ever less resource-intensive. If capital depreciation were extended over several periods, it would be necessary to use a capital generation model which would be much more complex than the one developed here, but which would not fundamentally change the results obtained.

sumption profile must satisfy the following condition:

$$\left[\frac{C_{t+1}}{C_t}\right]^{\frac{1}{\sigma}} = \beta\frac{1+\rho_{t+1}}{P_{ct+1}/P_{ct}} \qquad \text{où} \qquad \rho_{t+1} =_{def} \frac{V_{t+1}}{P_{kt}} - 1. \tag{24}$$

ρ_{t+1} denotes the rate of return on capital (here on the renting out of capital) in period $t+1$.

2.5 Equilibrium

2.5.1 Economic equilibrium

At the point of symmetrical equilibrium in the final sector, (16) is reduced to $Q_t = Q(q_t)$. For the sake of simplicity, we may assume that $Q_t = q_t$. When the consumer price index P_{ct} is normalised to 1, the relations between the price indices and the price p_t of the final goods become respectively:

$$1 = P_{ct} = \frac{p_t}{\psi(q_t)} \qquad \text{and} \qquad P_{kt} = \frac{p_t}{\varphi(q_t)} = \frac{\psi(q_t)}{\varphi(q_t)}. \tag{25}$$

The relations (10) can be written simply as:

$$C_t = \psi(q_t)c_t \qquad \text{and} \qquad k_{t+1} = \varphi(q_t)d_t. \tag{26}$$

Henceforth, we use the lower-case letters w_t, v_t, p_{xt} and u_{zt} where $z = e, f, r$ to denote the real value of the variables W_t, V_t, P_{xt} and U_{zt} when these are deflated by the price of the final goods $p_t = \psi(q_t)$: $w_t =_{def} W_t/\psi(q_t)$, $p_{xt} =_{def} P_{xt}/\psi(q_t)$ and $u_{zt} =_{def} U_{zt}/\psi(q_t)$, for $z = e, f, r$. The real unit costs associated with the use of the capital-labour mix can be written as:

$$u_{zt} = v_t + \lambda_z(q_{t-1})w_t \tag{27}$$

where v_t, the real rental price of capital, is:

$$\begin{aligned} v_t =_{def} V_t/\psi(q_t) &= \frac{V_t}{P_{kt-1}}\frac{P_{kt-1}}{\psi(q_t)} \\ &= \frac{1+\rho_t}{\psi(q_t)}\frac{\psi(q_{t-1})}{\varphi(q_{t-1})}. \end{aligned} \tag{28}$$

The last of these equalities follows from the definition of ρ_t (see 24) and the expression of P_{kt} in (25).

The equilibrium conditions of the different markets are written as follows:

- **Natural resource market equilibrium**: The resource demand is $X_t = x_t$ (given by (13)) and the supply-demand equilibrium is therefore given by

$$x_t = [\chi(q_{t-1}) + \mu(q_{t-1})]\, y_t \tag{29}$$

where supply x_t satisfies the optimality condition

$$p_{xt} = \frac{u_{et}}{1 - \frac{x_t}{R_t}}. \tag{30}$$

- **Equilibrium in the final goods markets**: the supply of each monopolistic firm satisfies the demand for consumer and investment goods arising from its choices of price and quality. We have

$$y_t = c_t + \frac{k_{t+1}}{\varphi(q_t)}, \tag{31}$$

with the firms' pricing behaviour and research effort being described as follows:

$$\alpha = u_{ft}\kappa(q_{t-1}) + u_{rt}h\left(\frac{q_t}{q_{t-1}}\right) + p_{xt}\left[\chi(q_{t-1}) + \mu(q_{t-1})\right] \tag{32}$$

$$u_{rt}h'\left(\frac{q_t}{q_{t-1}}\right)\frac{q_t}{q_{t-1}} = \alpha\left[\frac{c_t}{y_t}\frac{q_t\psi'(q_t)}{\psi(q_t)} + \left[1 - \frac{c_t}{y_t}\right]\frac{q_t\varphi'(q_t)}{\varphi(q_t)}\right], \tag{33}$$

while the intertemporal profile of household consumption satisfies the condition

$$\left[\frac{\psi(q_{t+1})c_{t+1}}{\psi(q_t)c_t}\right]^{\frac{1}{\sigma}} = \beta\left[1+\rho_{t+1}\right]. \tag{34}$$

- Equilibrium in the capital and labour markets: the rental price of capital and wages adjust so that firms rent a capital stock $k_{ft}+k_{rt}+k_{et}$ equal to that accumulated by households k_t and hire a number of workers $\ell_{ft}+\ell_{rt}+\ell_{et}$ equal to the labour supply L:

$$\begin{aligned} k_t &= \kappa(q_{t-1})y_t + h\left(\frac{q_t}{q_{t-1}}\right)y_t + R_t\ln\left(1-E_t\right)^{-1} & (35)\\ L &= \lambda_f(q_{t-1})\kappa(q_{t-1})y_t + \lambda_r(q_{t-1})h\left(\frac{q_t}{q_{t-1}}\right)y_t + \lambda_e(q_{t-1})R_t\ln\left(1-E_t\right)^{-1}. & (36)\end{aligned}$$

2.5.2 The material cycle

After consumption or scrapping, the final goods generate waste materials that are released into the environment. These are naturally recycled and re-enter the stock of NR. At any moment, the state of this stock therefore depends negatively on the quantity extracted by the primary sector (the outflow) and positively on the quantity of waste recycled by the environment (the inflow). The economy and the NR thus form a material cycle which is deemed to be closed, i.e. not to be involved in any exchange with the outside. Under the principle of conservation of matter, the amount of material M contained in the economy-NR system is constant. At the start of each period t,, this material is either in the form of NR or incorporated into the installed capital stock, or in other words (cf. Appendix 7.3):

$$M = R_t + \mu(q_{t-2})\frac{k_t}{\varphi(q_{t-1})}. \tag{37}$$

In total, we therefore have a dynamic system of nine equations (29) to (37) with nine unknown variables $y_t, c_t, R_t, x_t,\ k_{t+1}, q_t, \rho_t, p_{xt}, w_t$. The initial conditions are k_1, q_0, q_{-1} and the terminal conditions are $c_T = y_T$ et $k_{f,T+1},\ k_{r,T+1},\ k_{e,T+1} = 0$.

2.5.3 Value added sharing

The macro-economic value added $p_t y_t$ is divided between the wage bill $W_t L$, the extraction sector's rent Θ_t and the remuneration of the capital factor, understood here as the sum of the rental income $V_t k_t$ and the net profits of the monopolistic sector Π_t. We will denote as γ_{xt}, γ_{Kt} and γ_{Lt} the resource share, capital share and labour share in the value added respectively. We have (see Appendix 7.4):

$$\begin{aligned} \gamma_{xt} = \frac{\Theta_t}{p_t y_t} &= \left[\chi_t+\mu_t\right]p_{xt}\left[1+\frac{1-E_t}{E_t}\ln\left(1-E_t\right)\right] & (38)\\ &= \left[\chi_t+\mu_t\right]u_{et}\,G(E_t) \quad \text{avec} \quad G(E_t) =_{def} \frac{1}{1-E_t}+\frac{\ln\left(1-E_t\right)}{E_t}. & (39)\end{aligned}$$

It is easy to verify that for $E_t\in[0,1]$, the expression in E on the right side of (38) is increasing in E; likewise, $G(E)$ is such that $G(0)=0$ and $G'(E_t)>0$. The resource share thus increases as

- the price of the resource p_{xt} increases;
- the production technology in place is more resource-intensive (i.e. is characterized by a higher value of $\chi_t+\mu_t$);

- the resource extraction rate E_t increases: a higher extraction rate tends to increase the rent associated with the exploitation of the resource because it takes the efficiency of the marginal operator further away from that of the most efficient operator and thus increases the gap between the price of the resource (which is also the unit cost of the marginal operator) and the unit cost of the most profitable operator.

Of course, p_{xt} is not independent of E_t (cf. (30)) and the value of E_t is itself dependent on the resource-intensity of the technology. All other things being equal, more resource-intensive technology will imply a higher value of E_t and so too of p_{xt}, all of which will contribute to a higher resource share in the value added. With regard to Ricardo's conjecture, it can therefore be seen that on a growth path in which pressures on the resource intensify so that its extraction rate and price increase, the resource share will increase unless technical progress makes it possible to lower the technology's resource intensity $\chi_t + \mu_t$ quickly enough. Hence, in a growing economy, Ricardo's conjecture is all the more likely that the potential for dematerialising final output and its production process is limited. We will return to this point in Section 4.

The total capital share in value added at t is (cf. Appendix 7.4)

$$\begin{aligned}\gamma_{Kt} = \frac{\Pi_t}{p_t y_t} + \frac{V_t k_t}{p_t y_t} &= 1 - \alpha + v_t \frac{k_t}{y_t} \\ &= 1 - \alpha + v_t \left[\kappa_t + h\left(\frac{q_t}{q_{t-1}}\right) + \frac{\chi_t + \mu_t}{E_t} \ln\left(1 - E_t\right)^{-1} \right]. \end{aligned} \tag{40}$$

As the price elasticity of final demand is constant, the share of the monopolistic sector's *net* profit in value added $p_t y_t$ is constant $(1-\alpha)$). The share of the rental income $V_t k_t$ will increase as the real rental price increases and as the various activities (production, research and extraction) become more capital-intensive. In particular, an increase in the extraction rate makes this activity more capital-intensive and therefore increases γ_{Kt}, all other things being equal.

The labour share complements the other two shares: $\gamma_{L_t} = 1 - \gamma_{Kt} - \gamma_{xt}$. Given the impact of E_t on γ_{xt} and γ_{Kt}, greater pressure on the resource will generate a less favourable added-value share for labour, all other things being equal.

2.6 Long-run equilibrium

The technological progress resulting from research activities means that the functions κ,λ_f, λ_r, λ_e, μ, χ (or φ) decrease (or increase) over time and *ultimately* reach their limit values. The economy has then reached its long-run equilibrium which, as in Fagnart-Germain (2011), is a growth path characterised by the constancy of the volume variables (y, c, k) or material variables (R, x) and by the increase at a constant rate of the quality level q. Let $\hat{q}$ denote the growth factor for quality on the long-term path:

$$\left(\frac{q_t}{q_{t-1}}\right)_{long-term} =_{def} \widehat{q}.$$

Let $\hat{C}$ denote the growth factor for the consumption index along this path:

$$\widehat{C} =_{def} \left(\frac{C_t}{C_{t-1}}\right)_{long-term} = \frac{\psi(q_t)}{\psi(q_{t-1})}.$$

The equality on the right arises because along a steady-state growth path, the growth of the consumption index is only related to that of quality. For what follows, we assume that

$$\psi(q) = q^{\eta}, \quad \text{with} \quad 0 < \eta \leq 1.$$

In this case, $\widehat{C} = \widehat{q}^{\eta}$ and the steady-state expression of the condition describing consumption-smoothing behaviour (34) is written as a positive relationship between the quality growth rate $\hat{q}$ and the rate of return on capital ρ:

$$\widehat{q}^{\frac{\eta}{\sigma}} = \beta\left[1 + \rho\right]. \tag{41}$$

Intuitively, a higher quality growth rate means a steeper growth path for the consumption index, which is only compatible with consumption smoothing if the return rate ρ is higher. Indeed, a higher quality growth rate requires a greater research effort and more capital investment: the return on capital must thus increase to motivate households to make this investment.

Below, we rewrite the steady-state expressions of the equilibrium conditions (29), (31), (35), (36) and of the material conservation equation (37) by re-expressing the level variables as variables per worker. Thus, let $\tilde{y}$ denote the material output per worker: $\tilde{y} =_{def} y/L$. Likewise, the variables $\tilde{c}$, $\tilde{k}$ and $\tilde{R}$ denote the per capita values of the variables c, k, R. We have:

$$\begin{aligned}
\tilde{y} &= \tilde{c} + \frac{\tilde{k}}{\overline{\varphi}} && (42)\\
E\tilde{R} &= \left[\underline{\chi} + \underline{\mu}\right] \tilde{y} && (43)\\
\tilde{k} &= \underline{\kappa}\tilde{y} + h\left(\widehat{q}\right)\tilde{y} + \tilde{R}\ln\left(1-E\right)^{-1} && (44)\\
1 &= \underline{\lambda}_f\underline{\kappa}\tilde{y} + \underline{\lambda}_r h\left(\widehat{q}\right)\tilde{y} + \underline{\lambda}_e\tilde{R}\ln\left(1-E\right)^{-1} && (45)\\
\frac{M}{L} &= \tilde{R} + \underline{\mu}\frac{\tilde{k}}{\overline{\varphi}}. && (46)
\end{aligned}$$

Finally, the steady-state expressions of the optimality conditions for p and q ((32) and (33)) are

$$\begin{aligned}
\alpha &= u_f\underline{\kappa} + u_r h\left(\widehat{q}\right) + u_e\frac{\underline{\chi} + \underline{\mu}}{1-E} && (47)\\
\alpha\,\eta\,\frac{\tilde{c}}{\tilde{y}} &= u_r h'\left(\widehat{q}\right)\widehat{q} && (48)
\end{aligned}$$

where (47) uses (30), (48) makes use of the fact that the weighted sum of the right side of (23) tends toward the product of the elasticity of $\psi(q)$ and of $c/y = \tilde{c}/\tilde{y}$ in the long term; the steady-state values of the unit cost variables (27) are given by $u_z = v + \underline{\lambda}_z w$ (with $z = f, r, e.$) where the steady-state level of the real rental price of capital (obtained by introducing (41) in (28)) is

$$v = \frac{(1+\rho)}{\hat{q}^{\eta}}\frac{1}{\bar{\varphi}} = \frac{(1+\rho)^{1-\sigma}}{\beta^{\sigma}\bar{\varphi}}. \qquad (49)$$

It should be pointed out that the system's variables only depend on the inputs of materials M and labour L through their ratio M/L, which we will now designate as m: $m =_{def} M/L$. It follows that doubling M and L leaves m unchanged and does not affect the solution of the steady-state system ((41)-(48): in particular, the value of $\tilde{y}$ is unchanged, implying that $y = L\tilde{y}$ doubles like M and L. The returns to scale of the material output y relative to the inputs M and L are therefore constant. It follows that the returns to scale relative to each input (the other being constant) are decreasing.

3 Comparative statics in relation to $m = M/L$

We will now consider how the long-run equilibrium of the model is affected by the relative abundance of the material resource. There are two ways to interpret this comparative statics exercise. On the one hand, it compares the steady-state equilibria of two economies that differ from one another only in the level of relative abundance of the material resource. On the other hand, it can be seen as an exercise used to identify the macro-economic consequences of stabilising the population size at higher or lower levels, given the limited nature of the material resource. Since the quantity M of material is an unchangeable given throughout human history, considering higher or lower values of m thus amounts to considering different steady-state population levels for the economy.

The study is conducted in two stages: first analytically in the context of a particular case where the different activities are characterised by an identical capital/labour ratio, and then numerically for the general case.

3.1 The analytical approach in a particular case

To facilitate the analytical exploration of the steady state, it is assumed that the capital/labour ratio is homogeneous in the different activities, i.e. that

$$\underline{\lambda}_f = \underline{\lambda}_r = \underline{\lambda}_e = \underline{\lambda}. \tag{50}$$

One important consequence of (50) is that the unit cost associated with the use of the capital-labour mix is identical in all three activities (production, research and extraction) as shown by the expression (27): $u_z = v + \underline{\lambda}_z w =_{def} u$ for $z = e, f, r$. This implies the following

Lemme 1 *:*

If the material resource is relatively more abundant with respect to the labour input, i.e. if m is higher,

1. *it is also relatively more abundant with respect to the capital input;*
2. *the steady-state level of the available resource stock per capita $\tilde{R}$ is also higher.*

In formal terms,

$$\frac{d(\tilde{k}/m}{dm}\left[or\ \frac{d(k/m)}{dm}\right] < 0, \ \ \frac{d\tilde{R}}{dm} > 0 \qquad and,\ if\ dm > 0, \qquad \frac{d\tilde{R}}{\tilde{R}} > \frac{d\tilde{m}}{\tilde{m}}. \tag{51}$$

Proof:

1. We can rewrite the equilibrium condition of the capital market as an equation that gives the capital/output ratio (thus dividing (44) by $\tilde{y}$). Using (43), this gives us

$$\frac{\tilde{k}}{\tilde{y}} = \underline{\kappa} + h(\widehat{q}) + \frac{\underline{\chi} + \underline{\mu}}{E}\ln(1-E)^{-1}. \tag{52}$$

By doing the same for the equilibrium condition of the labour market, we obtain

$$\frac{1}{\tilde{y}} = \lambda\left[\underline{\kappa} + h(\widehat{q}) + \frac{\underline{\chi} + \underline{\mu}}{E}\ln(1-E)^{-1}\right]. \tag{53}$$

Dividing (52) by (53) gives $\tilde{k} = 1/\lambda$: capital per capita therefore does not depend on m under assumption (50). It follows that $\tilde{k}/m$ is smaller when m is larger.

2. As $\tilde{k}$ does not depend on m, (46) implies that $d\tilde{R} = dm$ and, if d$m > 0$,

$$\frac{d\tilde{R}}{\tilde{R}} = \frac{dm}{m - \underline{\mu}\tilde{k}/\varphi} > \frac{dm}{m}.$$

■

Point 2 of Lemma 1 is a consequence of the fact that the quantity of material embodied in the capital equipment in use $\left(\underline{\mu}\tilde{k}/\overline{\varphi}\right)$ does not depend on m in a steady-state situation: an increase in m therefore results in the same increase in the level of available resource $\tilde{R}$.

3.1.1 The impact of m on rates $(E, s, \hat{q}, \rho)$ and levels $(\tilde{y}, \tilde{x}, \tilde{c}, u)$

Henceforth, we use s to denote the steady-state saving rate: $s =_{def} \tilde{k}/(\overline{\varphi}\tilde{y})$ (and hence $\tilde{c}/\tilde{y} = 1 - s$ via (31)[12]). Under assumption (50), the steady-state system can be reduced to a system of three equations with three unknown quantities E, s and $\widehat{q}$ (see Appendix 7):

$$\begin{aligned} s\bar{\varphi} &= \underline{\kappa} + h\left(\widehat{q}\right) - \frac{\underline{\chi} + \underline{\mu}}{E} \ln\left(1 - E\right) && (54) \\ E &= \frac{1}{s} \frac{\underline{\chi} + \underline{\mu}}{m\underline{\lambda}\bar{\varphi} - \underline{\mu}} && (55) \\ \eta\left[1 - s\right] \left[\underline{\kappa} + h\left(\widehat{q}\right) + \frac{\underline{\chi} + \underline{\mu}}{1 - E}\right] &= h'\left(\widehat{q}\right)\widehat{q}. && (56) \end{aligned}$$

The analysis of this system leads to the following proposition:

Proposition 1 *:*

1. *A greater abundance of the material resource per capita ($dm > 0$) implies lower steady-state levels of the resource extraction rate and the saving rate:*

$$sign\left(\frac{dE}{dm}\right) = sign\left(\frac{ds}{dm}\right) < 0.$$

2. *A less severe resource constraint ($dm > 0$) is reflected in a higher level of final material output per capita, of intermediate resource consumption per capita and of final material consumption per capita. It also implies a higher real unit cost u associated with the use of the capital-labour mix. In formal terms,*

$$sign\left(\frac{d\tilde{y}}{dm}\right) = sign\left(\frac{d\tilde{x}}{dm}\right) = sign\left(\frac{d\tilde{c}}{dm}\right) = sign\left(\frac{du}{dm}\right) > 0. \qquad (57)$$

3. *If the productivity of the capital goods is not too low and the saving rate is not too high, a less severe resource constraint ($dm > 0$) implies a lower growth rate in the quality of final output and the rate of return on capital. In formal terms, if $\bar{\varphi} > 3[\underline{\chi} + \underline{\mu}]$, in any equilibrium that satisfies*

$$s < \frac{2}{3}, \qquad (58)$$

one has

$$sign\left(\frac{d\widehat{q}}{dm}\right) = sign\left(\frac{d\rho}{dm}\right) < 0.$$

Proof: See Appendices 7.6.1, 7.6.2 and 7.6.3 for Points 1, 2 and 3 respectively. ■

Point 2 is intuitive and requires no special comment. Regarding Point 1, the negative relationship between m and E results from two contradictory forces: (i) a negative effect associated with the increase in $\tilde{R}$ which results from the fact that at constant output, E decreases if $\tilde{R}$ increases and (ii) a positive effect associated with the increase in material output made possible by the greater resource abundance. The first effect necessarily outweighs the second, since $\tilde{R}$ increases proportionally more than m (see Lemma 1), whereas $\tilde{y}$ (and hence $\tilde{x}$) increases proportionally less

[12] The saving rate defined in this way corresponds to the fraction of primary income that households save (i.e. use for the accumulation of capital), and $1 - s$ corresponds to the fraction of primary income used for final consumption: given the value of the ratios P^k/p and P^C/p (see (25)),

$$\frac{P^k k}{py} = \frac{k}{\bar{\varphi}y}\left(= \frac{\tilde{k}}{\bar{\varphi}\tilde{y}}\right) \qquad \text{et} \qquad \frac{P^C C}{py} = \frac{P^C}{p}\frac{\psi(q)c}{y} = \frac{c}{y}\left(= \frac{\tilde{c}}{\tilde{y}}\right).$$

than m[13]. As a more abundant resource leaves $\tilde{k}$ unchanged (cf. Lemma 1), it inevitably implies a fall in the saving rate (or an increase of $\tilde{c}/\tilde{y}$): the fall in the extraction rate reduces the capital intensity of the extraction activity and thus the share of the output used for investment (savings).

Examination of (48) (which can be rewritten as $\alpha\eta[1-s] = uh'(\hat{q})\hat{q}$) reveals why an increase in m has two effects in opposite directions on the growth of quality $\hat{q}$: the increase in u makes research efforts more costly and thus reduces the incentive for research. Conversely, the decline in the saving rate (and thus the increase in $1-s$) reinforces this incentive. This is because, in the steady state, the only remaining benefit of investment in research is an improvement in the quality of consumer goods, and the incentive for such an investment becomes stronger as the share of consumption in total demand increases or as the saving rate falls: the fall in s associated with the increase in m therefore increases the marginal income from research investment (the right side of (48)) and therefore tends to stimulate research efforts when all else is equal. Under conditions $\bar{\varphi} > 3[\underline{\chi} + \underline{\mu}]$ and (58) (a condition that is confirmed once the long-run equilibrium is calibrated against empirically observed saving rates), the first effect necessarily outweighs the second.

Finally, note that although the aggregate value of capital per capita does not change with m under assumption (50), a reallocation of capital is nonetheless observed between the different activities. As $dm > 0$ implies a higher material output per capita, more capital (per capita) must be allocated to final production. This results in a decrease in the capital per capita allocated to research and extraction ($k_e + k_r$).

3.1.2 The impact of m on the real prices (w, v, p_x)

As u is an increasing function of m (see Proposition 1), it follows that real wages w and/or the real rental price of capital v are higher when the resource is more abundant. We can formulate the following proposition:

Proposition 2 *Under conditions $\varphi > 3[\underline{\chi} + \underline{\mu}]$ and (58), a greater abundance of the resource per capita ($dm > 0$) implies*

- *a real rental price of capital v that is lower if the intertemporal elasticity of substitution in consumption is less than one, and higher if it is greater than one:*

$$\frac{dv}{dm} \begin{cases} < 0 & \textit{if} \quad \sigma < 1 \\ = 0 & \textit{if} \quad \sigma = 1 \\ > 0 & \textit{if} \quad \sigma > 1; \end{cases}$$

- *higher real wages if $\sigma \leq 1$ or if σ is greater than but in the vicinity of 1.*

- *The abundance of the resource has an ambiguous impact on its real price p_x.*

Proof: See Appendix 7.6.4. ■

The role of σ in the impact of m on v can be understood by considering (49). Under conditions $\bar{\varphi} > 3[\underline{\chi} + \underline{\mu}]$ and (58), a higher m implies lower values of ρ and $\hat{q}$, with contradictory effects on v: decreasing ρ tends to reduce v; decreasing $\hat{q}$ does the opposite. But ρ and $\hat{q}$ are linked by (41): to be compatible with consumer behaviour, a reduction in the steady-state growth factor of consumption ($\hat{q}^{\eta}$) calls for a proportional variation in $[1+\rho]^{\sigma}$ and hence a more (respectively less) than proportional reduction in $1+\rho$ when σ is below (respectively above) than 1. The real rental price of capital is thus decreasing in m when $\sigma \leq 1$, as $\hat{q}^{\eta}$ (or indeed $[1+\rho]^{\sigma}$) will decrease in such a case proportionally less than $1+\rho$. Wherever v decreases (i.e. when $\sigma \leq 1$), w must inevitably increase. It is not impossible for wages to rise too when $\sigma > 1$ and when v is also increasing, but this can only be formally demonstrated in the vicinity of $\sigma = 1$. Intuitively, this wage increase

[13] As we have explained, the returns to scale of the material output are constant relative to (M, L) and hence diminishing relative to each input when the other is constant.

reflects from the fact that the greater abundance of the resource relative to the available labour force results in a greater tension between the demand for and supply of labour.

The ambiguity in the impact of m on p_x results from two conflicting effects of m on the marginal operators extraction cost: a higher m lowers E but raises the unit cost u. The first effect does not necessarily outweigh the second (as the numerical simulations confirm).

3.1.3 The impact of m on the steady-state distribution of value added

The distributive shares of the different factors can be written as follows (see Appendix 7.7.1):

$$\begin{aligned}
\gamma_x &= \alpha \frac{\left[\underline{\chi}+\underline{\mu}\right] G(E)}{\left[\underline{\chi}+\underline{\mu}\right] G(E)+\bar{\varphi} s} \leq \alpha && (59)\\
\gamma_K &= 1-\alpha+\frac{[1+\rho]^{1-\sigma}}{\beta^{\sigma}} s && (60)\\
\gamma_L &= \alpha \frac{\bar{\varphi} s}{\left[\underline{\chi}+\underline{\mu}\right] G(E)+\bar{\varphi} s}-\frac{[1+\rho]^{1-\sigma}}{\beta^{\sigma}} s. && (61)
\end{aligned}$$

Since $G'(E) > 0$ and the saving rate is, in view of (55), a decreasing function of E, the distributive share of resource γ_x is increasing in E. The capital share, γ_K, is an increasing function of the saving rate; it is increasing or decreasing in the rate of return on capital depending on whether σ is less than or greater than 1.

The following proposition sets out how the relative abundance of the material resource affects value added sharing among its three components.

Proposition 3 *:*

1. *If the extraction rate of the resource is not too close to 1, the resource share in value added is a decreasing function of its relative abundance m. In formal terms, in any steady-state equilibrium where E satisfies the condition,*

$$\frac{2}{E}\frac{\underline{\chi}+\underline{\mu}}{m\lambda\varphi-\underline{\mu}}+\frac{1+\phi_{h'}}{\eta}\frac{\frac{\varphi}{m\lambda\varphi-\underline{\mu}}-\frac{[G(E)]^2}{EG'(E)}}{\frac{\varphi}{m\lambda\varphi-\underline{\mu}}+G(E)}>1 \qquad où\ \phi_{h'}=\hat{q}\frac{h''(\hat{q})}{h'(\hat{q})}>0, \tag{62}$$

   one has $d\gamma_x/dm < 0$.

2. *When $\sigma \leq 1$, in any equilibrium satisfying (58), a greater abundance of the resource reduces the capital share in value added and, provided E satisfies (62), increases the labour share in value added: $d\gamma_K/dm < 0$ et $d\gamma_L/dm > 0$.*

   This will also necessarily be so if σ is greater than but close to 1.

Proof: Point (1) is demonstrated in Appendix 7.7.2. Point (2) follows from Proposition 1 and (60): s and ρ are lower when m is higher. When $\sigma \leq 1$, this also implies a lower value of γ_K; hence, γ_L is inevitably higher (since γ_K and γ_x are both lower). When $\sigma > 1$, the negative impact of m on both s and ρ has contradictory effects on γ_K. But by continuity, and at least in the vicinity of 1, the positive effect of a reduction of ρ on γ_K cannot outweigh the negative effect of the decrease of s, since the values of derivatives $\mathrm{d}s/\mathrm{d}m$, $\mathrm{d}\hat{q}/\mathrm{d}m$ and $\mathrm{d}\rho/\mathrm{d}m$ are finite at $\sigma = 1$. ■

It is easily checked that (62) will be satisfied if E is not too close to 1

The second point of the proposition states that the capital share is inevitably a decreasing function of resource abundance when $\sigma \leq 1$: a higher m decreases v (see Proposition 2) and $\tilde{k}/\tilde{y}$ and hence $v\tilde{k}/\tilde{y}$. When $\sigma > 1$, v is increasing in m, but γ_K can still decrease if v does not increase too much, i.e. proportionally less than the decrease in $\tilde{k}/\tilde{y}$. We can therefore see that, at least for values of σ that are less than or not too much greater than 1, the distributive conflict associated with a more or less acute resource scarcity necessarily sets up an opposition between labour and the other two factors: in a steady-state equilibrium, a less abundant resource implies a distribution of value added that is less favourable to labour and more favourable to the owners of resources and capital.

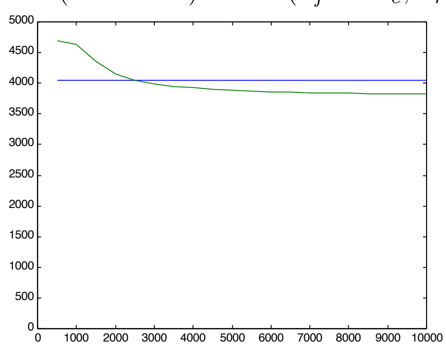
3.2 Analysis of the general case

In the general case, where the capital/labour ratio differs according to the activity, obtaining analytical results is not easy. In particular, $\tilde{k}$ now depends on m, but the connection between $\tilde{k}$ and m cannot be established analytically: $\tilde{k}$ may increase or decrease when m increases, depending on the degree of capital- and labour-intensity of the extraction activity relative to that of other activities.

However, the results obtained from the formal analysis conducted in case (50) have some general validity. We have studied numerically the impact of m on the steady-state equilibrium in an economy where the different extraction, production and research activities do not have the same capital/labour ratio. This numerical analysis[14] failed to identify any configuration of parameters in which a higher value of m would have led to a higher value for the extraction rate or the saving rate. What is more, the increasing or decreasing nature of the relationships described in Propositions 1-3 in the particular case could not be invalidated by the numerical analysis of the more general case.

Moreover, the simulations confirm that, depending on the circumstances, $\tilde{k}$ can be an increasing or a decreasing function of m. Figure 1.1 compares the impact of m on $\tilde{k}$ in a case where the labour/capital ratio is identical in all three activities and a case where it is higher in the production activity than in the research and extraction activities ($\underline{\lambda}_f > \underline{\lambda}_r > \underline{\lambda}_e$, with $\underline{\lambda}_f = 3\underline{\lambda}_e$ and $\underline{\lambda}_r = 2\underline{\lambda}_e$). If M increases, capital is reallocated from research and extraction activities towards production. This reallocation leaves the capital stock unchanged in the case that satisfies (50), but when production is more labour-intensive than the other activities and the labour supply is fixed, the reallocation of capital is inevitably accompanied by a decline in total capital as shown in the figure.

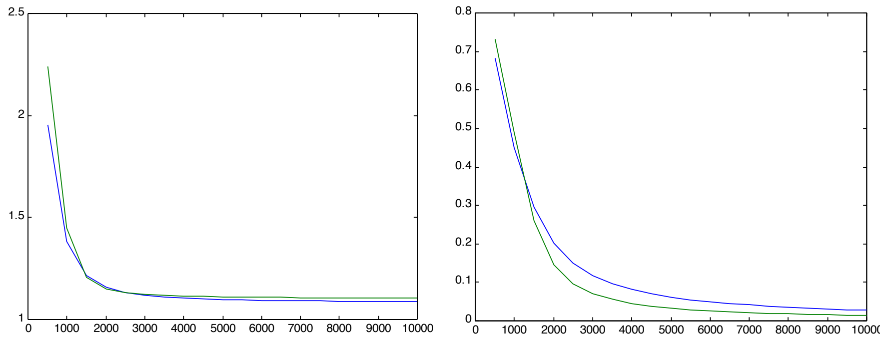
Figure 1.1: Stationnary impact of m on k
identical λ's (blue curve) versus ($\lambda_f = 3\lambda_e$, $\lambda_r = 2\lambda_e$)



The comparison of the two above-mentioned cases is illustrated in Figures 1.2 and 1.3 with respect to the quality growth rate and the rent share in value added. These figures confirm for the general case the results obtained in the particular case. The simulations also confirm that p_x can be an increasing or decreasing function of m, as mentioned in Proposition 2.

[14]The exercise consisted of varying M over a wide range around its reference value, using multiple configurations for the values of the other parameters in the model. As L is fixed, varying M is obviously equivalent to varying m.

Figures 1.2 - 1.3: Stationnary impact of m on $\hat{q}$ (left pannel) and γ_x (right pannel) identical λ's (blue curve) versus ($\lambda_f = 3\lambda_e$, $\lambda_r = 2\lambda_e$)



4 Dynamic analysis

The analysis of the models dynamics was performed numerically. As the model is highly schematic, the numerical values of the simulated variables are of little interest in themselves: attention will focus instead on the overall shape of their trajectory. It should also be borne in mind that our exercise is meant to be forward-looking (as the economy described here only uses a renewable resource) and does not aim to replicate historical facts or patterns relating to the distribution of value added[15].

4.1 The convergence of an initially undercapitalised economy towards its steady-state path

The first numerical exercise illustrates the growth path of an economy whose initial state is characterised by a capital stock well below its steady-state level and by the prospect of technical progress in terms of dematerialisation and economies of labour and capital. In particular, the potential for dematerialisation of output is still one-third of its original value. The population (the labour supply) and the stock of material are exogenous and constant (like all other exogenous factors in the model).

Given the potential for technical progress and the initial undercapitalisation of the economy, the level of material output y_0 is well below its steady-state level and the economy initially has multiple growth drivers: investment in physical capital, improvements in the productivity of labour and capital, the reduction of the dependence of final output on the resource, improvements in quality of final output and so on.

Figure 2.1 illustrates the trajectory of material output (in blue): it increases monotonically and tends towards a steady-state level for the reasons we have explained in the theoretical analysis: in several dimensions (improvements in the productivity of labour and capital, the reduction of the dependence of output on the resource), technical progress is limited, and once the variables λ_{zt}, χ_t, μ_t, κ_t reach their steady-state level $\underline{\lambda_z}$, $\underline{\chi}$, $\underline{\mu}$, $\underline{\kappa}$, the growth of material output tails off. Only quality growth remains ($\hat{q} > 1$) as shown in Figure 2.4.

Figure 2.2 shows that the saving rate decreases monotonically to its steady-state level. As we started with an initial value of capital well below its long-run level, capital is initially the factor that most limits the economy, in contrast to the population and the resource, which are relatively abundant. The incentive for the growth of capital and material output is therefore very strong, as the high saving rate values illustrate. Gradually, as the economy grows, capital becomes less scarce and its remuneration and the saving rate decrease.

[15] Specifically, we do not seek to ensure that the models transitional dynamics display a relatively stable value added distribution. Such stability is still often regarded as a stylised fact of modern growth, although some would dispute this: see in particular Solow (1958), Krämer (2010) and Young (op. cit.).

In such a simulation, the rate of extraction of the resource is subject to two opposing influences. On the one hand, technical progress makes the production process less dependent on the resource ($\chi_t + \mu_t$ decreases), which tends to reduce extraction, all other things being equal. On the other hand, material growth tends to increase resource requirements and the intensity of extraction: E_t tends to rise due to the increase in the material extracted *and* because of the decrease in R_t (due to the accumulation of material in the installed capital)[16]. Unless we assume the potential for dematerialisation to be very great (i.e. very low values of $\underline{\chi}$ and $\underline{\mu}$: see Section 4.3), the second of the two effects described above overrides the first and the extraction rate increases during the phase of material output growth, as shown in blue in Figure 2.3. The increase in extraction costs that accompanies the growth of E_t imposes an increasing constraint on the material dimension of economic growth, which ends up being zero as we have seen.

Figure 2.4 shows the quality growth rate, which is the only source of sustainable growth in the model. With a delay of one period, it is identical to the rate of technical progress in the various activities. In the simulation considered here (in blue), it is a decreasing function of time: as capital is the limiting factor at the start of the trajectory, it restricts the number of goods that can be manufactured. The incentive to invest in research in order to increase the quality of these goods is therefore strong, and the greatest quality improvements are observed at the start of simulation. We should point out that a non-monotonic evolution is possible in certain parameter configurations, as the growth factor $\widehat{q}_t$ can increase again when material growth ceases.

Figures 2.5 and 2.6 show the overall upwards trend in real wages and the real price of the resource in the simulation considered here. As the economy grows, the demand for these two factors increases, pushing their real price up. However, the change in the relative demand for the resource and for labour depends on the potential for technical progress, which affects them throughout the transitional dynamics: the change in real wages relative to rent therefore depends on this too.

The changes in the resource and labour shares are described in Figures 2.7 and 2.8 respectively: in the scenario considered here, they reflect the relative increase in the scarcity of labour and resources as the economy grows (whereas at the start of the simulation, it is capital that is the limiting factor). Examination of (38) shows that γ_x undergoes conflicting pressures during the transitional dynamics. On the one hand, the increasing dematerialisation of output tends to reduce it, especially as it also tends to reduce p_{xt} and E_t, all other things being equal. On the other hand, the growth of material output resulting from the accumulation of capital and the various forms of technical progress stimulates demand for the resource and pushes E_t and p_{xt} upwards. Unless we assume a very high dematerialisation potential (see Subsection 4.3), the second of the two effects outweighs the first throughout the transitional dynamics, and γ_x tends to increase, as shown here.

Figure 2.1: Material output

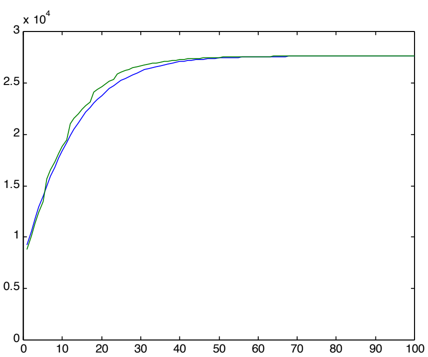
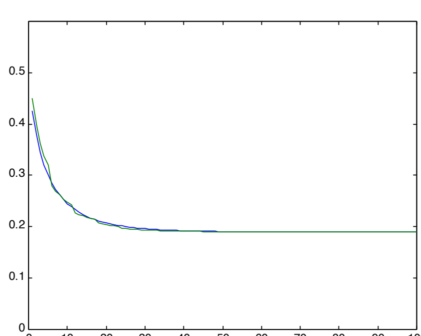


Figure 2.2: Saving rate



[16] However, this last effect makes little difference in the context of the simulation. This is because the material contained in the capital changes very little, since the accumulation of material in the capital is counteracted by the dematerialisation of output and of capital (through the reduction in the coefficient μ_t). R_t therefore does not decrease by much.

Figure 2.3: Extraction rate E_t

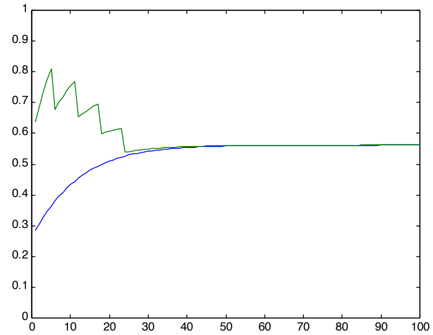


Figure 2.4: Quality growth rate$\hat{q}_t$

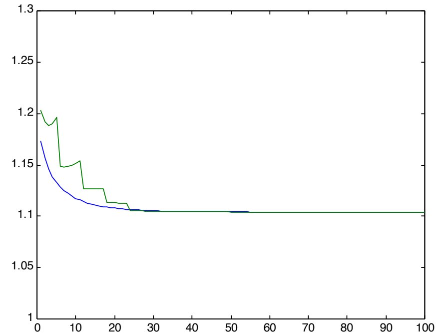


Figure 2.5: Rent p_{xt}

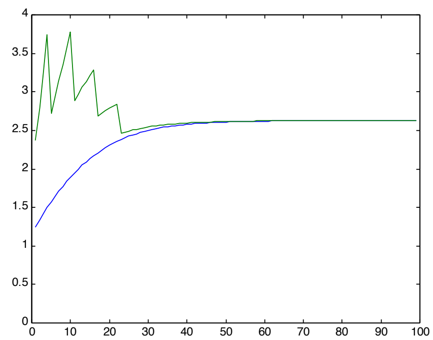


Figure 2.6: Real wages w_t

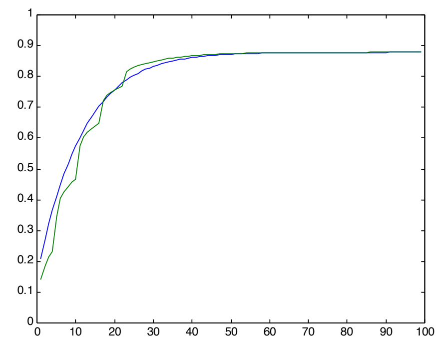


Figure 2.8 shows that the labour share also describes a monotonic upward trend[17]. Depending on the potential for technical progress that affect the resource and labour, the growth of γ_{Lt} may, however, be countered by that of γ_{xt}, as we shall see in Exercises 4.3 and 4.4.

Figure 2.7: Resource share γ_{xt}

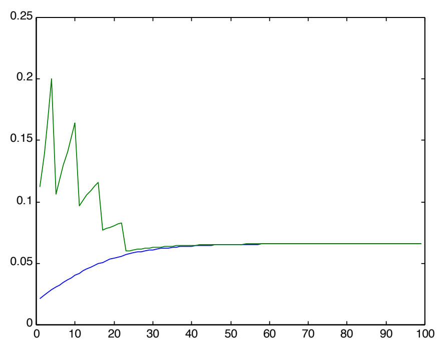
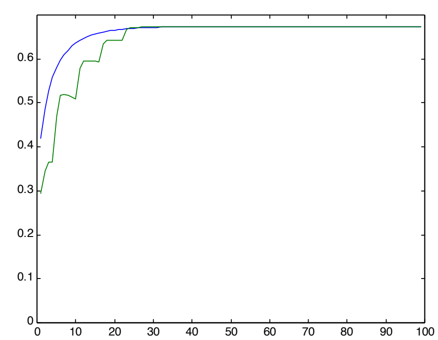


Figure 2.8: Labour share γ_{Lt}



[17]This trend is due to our technological assumptions and the type of simulation we are considering (in which the economy is initially undercapitalised): as the possibilities for substitution between labour and capital are limited, capital accumulation and the resultant economic growth are accompanied by an increase in the wage share in value added.

4.2 Discoveries of NR deposits

In the reference simulation, a monotonic upward development is observed in the real price of the resource (as shown in Figure 2.5) and in γ_x (as shown in Figure 2.7), both in blue. Such developments are not consistent with the downward trend in real prices of resources and the rent share that has characterised the industrialisation of Western countries (Hill, 2001). Caution is obviously required in interpreting the trends arising from the simulations considered here if we are seeking to make comparisons with historical trends observed over the last two centuries. The economy in the model only uses one renewable resource and its growth is not based in any way on the use of non-renewable resources, unlike what has broadly been the case over the last two centuries. However, one may wonder how the resource price and rent share develop when the available resource is only gradually discovered.

Whereas in the previous simulation the *total* resource stock is available from the start, we now consider an alternative simulation in which only a fraction of the resource stock is initially available, the rest becoming progressively extractable as a result of a series of discoveries occurring at regular intervals. At the end of this process of discovery, the economy has the same resource stock as in the initial simulation. The values of the other exogenous factors are also exactly the same in both simulations and both economies have the same steady state. The green curves in the preceding figures show the trajectory of the simulated variables in this scenario of progressive discoveries. The zigzags arise from the sudden emergence of discoveries.

As Figure 2.1 and Figure 2.2 show, the change in material output and in the saving rate respectively is similar in the two simulations. This is because capital is initially the most limiting factor, so that the initially lower availability of the resource ultimately has little effect on material output. However, the incentive to invest in quality is greater in the economy which is less well-endowed with material resources (Figure 2.4).

The initial trajectories of the extraction rate, the real price of the resource and γ_x are very different from those in the initial simulation. In the case of the variant with discoveries, the rent (Figure 2.5) and γ_x (Figure 2.7) start with values much higher than those in the reference case, because the material stock is smaller. Whenever a new deposit is discovered, the stock increases and the rent and γ_x undergo a sudden downturn. Between two discoveries, the rent and γ_x start rising again due to increase in demand driven by economic growth. These abrupt changes are enough to cause a downward *trend* in γ_x during the discovery phase. Once the discoveries have ceased, γ_x begins to increase again monotonically towards its equilibrium value. Depending on the configuration of parameters, the trend of the price resource (across its zigzags) may be hump-shaped as in the simulation described here, or rising (when the long-run value of γ_x is high).

Although the curves describing the change of real wages and of γ_L (Figures 2.6 and 2.8) show the same trend in both simulations, the relative gaps are important in the early discovery phases, with wages and γ_L lower in the initially less well-resourced economy. Each discovery episode has a beneficial effect on wages and γ_L. Comparison between the levels of γ_x and γ_L within the two simulations shows that the higher value of γ_x during the first part of the trajectory in the initially less well-resourced economy is achieved mainly at the expense of γ_L: the problem of greater scarcity in the initially less well-resourced economy mainly affects the distribution of value added between workers and resource-owners.

4.3 The role of the potential for technological progress in relation to the resource

This section examines the role of the varying degree of dependence of output on the resource in the development of the resources price and its distributive share. During the transitional dynamics, material output becomes ever less resource-intensive the further its dematerialisation can be pushed: starting from given initial conditions, a strong dematerialisation potential means a lower asymptotic value of $\underline{\chi} + \underline{\mu}$, enabling the economy to achieve a higher steady-state output level y, with the same resource stock. We will now conduct a sensitivity analysis relating to this potential for dematerialisation.

Imagine for a moment that there is no technical progress that makes dematerialisation possible, and no discovery of new deposits. We can understand that in these circumstances economic growth associated with the accumulation of capital and with other sources of technical progress will inevitably be accompanied by an increase in resource extraction and in the resource price. This is because an increase in material output would involve placing ever-increasing demand on the same resource stock: this would result in an increase of E_t, and hence of extraction costs and the price of the resource. As equation (38) shows, the increase in E_t and p_{xt} where $\chi+\mu$ is constant would mean that the resource share in value added would also inevitably increase throughout the transitional dynamics.

Resource-saving technical progress that therefore reduces $\chi_t + \mu_t$ exerts a force in the opposite direction to the changes we have just described. It enables the same level of economic growth to be achieved making less use of the resource than in the absence of technical progress, and this will mean a smaller increase in its rate of extraction, price and value added share (38). But this technical progress will also generate a rebound effect: it will stimulate economic growth, which will in turn affect the extraction and price of the resource and partly offset the effect of technical progress on these variables. If the technical progress is strong enough, (38) shows that it can lower the value added resource share even though growth is accompanied by an increase in the extraction of the resource and its price. If it were even stronger, so that economic growth could be accompanied by a decrease in extraction, the transitional dynamics could even be accompanied by a fall in the resource price as well as in the rent share.

The three figures 3.1 to 3.3 below show a sensitivity analysis of changes in the rate of extraction of the resource, its price and its value added share according to the potential for output dematerialisation. The highest curve in each of the three graphs shows the changes in these variables in a simulation where the dematerialisation potential is two times *less* than in the reference simulation detailed in 4.1, which assumes a dematerialisation potential of one-third. The reference simulation corresponds to the blue curve (the second highest curve in these first three figures). The four lower curves describe the transitional dynamics of E_t and γ_{xt} for dematerialisation potentials of 53%, 60%, 80% and 93% of the same initial value of $\mu_0 + \chi_0$. It can be seen that the steady-state values for the extraction rate, resource price and resource share become lower as the dematerialisation potential increases.

Figure 3.1: Extraction rate E_t

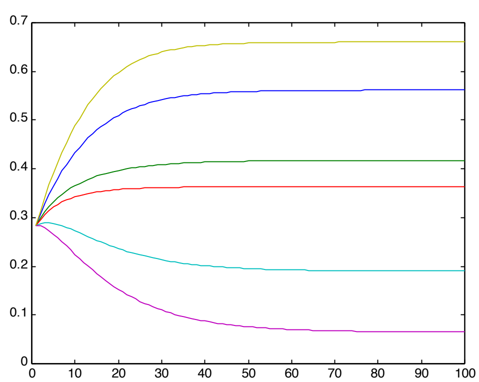
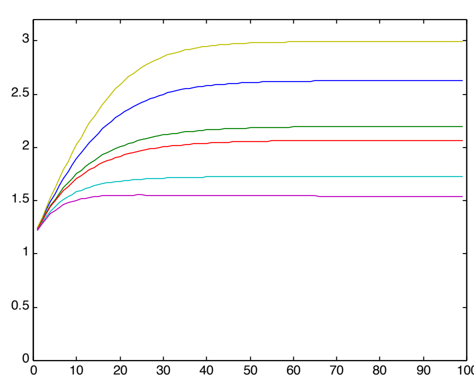


Figure 3.2: Resource price p_{xt}



The impact of the dematerialisation potential on the shape of these curves is consistent with the intuition that we have outlined above. The lower the dematerialisation potential, the greater the increase in the extraction rate, the price of the resource and its value added share during the transitional dynamics. With a sufficiently high dematerialisation potential, the extraction rate and the resource share may show a downward trend: this first arises with a dematerialisation potential of 60% for the resource share (Figure 3.3) and of 80% for the extraction rate (Figure 3.1)[18]. The

[18] Ultimately, the unrealistic case of total dematerialisation ($[\underline{\chi} + \underline{\mu}] \to 0$) would lead to a zero extraction rate (as

upward trend in the resource price (Figure 3.2) remains in all cases, but becomes weaker as the dematerialisation potential rises (even showing a slight hump in the case of a dematerialisation potential of more than 90%).

Figure 3.3: Resource share γ_{xt}

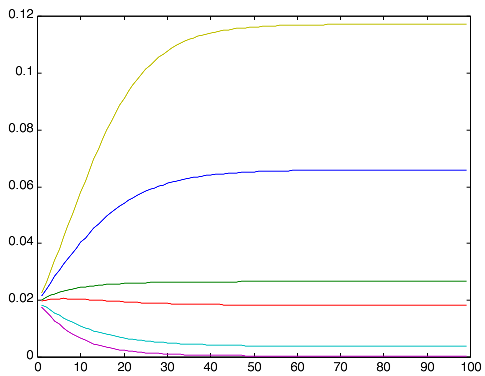
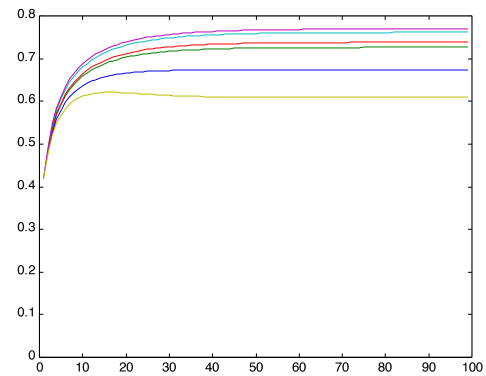


Figure 3.4: Labour share γ_{Lt}



This sensitivity analysis therefore shows that *a growing economy (in which the resource deposit discovery phase has ceased) only fails to confirm Ricardo's conjecture if the potential for the dematerialisation of output is high enough.* In the simulations described here, a sufficient condition for this to occur is that technical progress is strong enough to generate an *absolute decoupling* of resource consumption and economic growth, so that the consumption of the material resource tends to fall throughout the transitional dynamics. It should be emphasised that up to now, although the empirical evidence may be favourable to the thesis of a *relative* decoupling (in some parts of the world at least), the facts do not support the thesis of an absolute decoupling (see e.g. Laurent (2011)).

It is also worth noting that what matters here is not the absolute level of the potential for technical progress with regard to the resource so much as the relative value of the potential for technological progress affecting the resource and labour. Thus, the dematerialisation potential needed to defy Ricardo's conjecture will be all the greater as the potential for improvements in labour productivity rises. The steady-state expression (55) shows that at a given savings rate, the value of E becomes higher as the labour intensity of activities is lower: for the same dematerialisation potential, higher labour productivity (lower $\underline{\lambda}$) stimulates productive activity and resource extraction, and hence increases E and, via E, p_x and γ_x.

Figure 3.4 illustrates the change in the labour share in value added: the lowest curve shows the economy where the dematerialisation potential is the weakest, while the higher curves correspond to ever higher levels of dematerialisation. Comparison between the levels of γ_x and γ_L in the different scenarios (Figures 3.3 and 3.4) shows here again that tensions associated with the sharing of value added arise primarily between labour and resource: from one scenario to another, an increase in the steady-state value of either of these usually corresponds to a decrease in the value of the other. If the potential for dematerialising final output is low (relative to possible improvements in labour productivity), the higher tension over resource extraction means that the increase of γ_L is countered by that of γ_x, as the labour share in value added may decrease during the phase in which material growth comes to an end (as in the lowest curve in Figure 3.4).

4.4 The effect of varying degrees of demographic pressure

The last set of simulations examines the consequences of demographic pressure. It compares two initially identical economies which experience population growth and progressive discoveries of resources. In both cases, population growth continues beyond the end of the discovery phase, but

(55) shows), zero rent and, via (59), a zero resource share in the value added.

the population of one of the economies (the blue one in the figures) stabilises before that of the other (the green one in the figures). With a larger steady-state population (50% greater), the green economy therefore has a lower steady-state ratio m: relative to labour, the resource constraint is relatively more severe in the green economy.

The higher value of L enables the green economy to achieve a higher level of material output than in the blue economy (Figure 3.1), but this obviously implies a higher resource extraction rate (Figure 3.2). However, the greater increase of L for the same steady-state value of M implies an increase in material output that is proportionally less than that of employment.

According to the analysis in Section 3, the lower value of m in the green economy implies higher steady-state values in the saving rate and in quality improvements (Figures 3.3 and 3.4). In the green economy, stronger demographic pressure reinforces the incentive to look for quality gains, and is responsible for a non-monotonic evolution of $\hat{q}_t$.

The rent and the resource share in value added increase more in the economy where population pressure on the resource is stronger (Figures 3.5 and 3.6). This greater increase in rent eventually has an effect on wages and γ_L (Figures 3.7 and 3.8), which follow a bell-shaped development curve in the economy with stronger population growth. As in Exercise 4.2, the conflict over the distribution of value added mainly occurs between resource-owners and wage-earners.

Figure 3.1: Material output

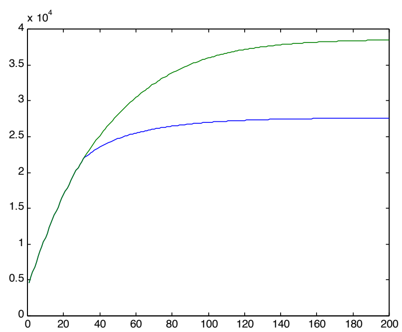


Figure 3.2: Extraction rate E_t

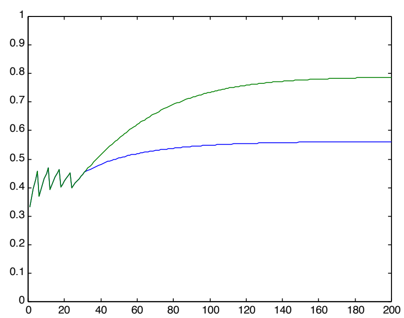


Figure 3.3: Saving rate

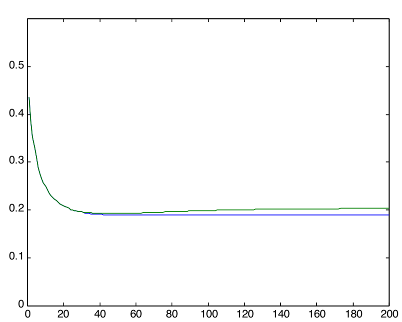


Figure 3.4: $\hat{q}_t$

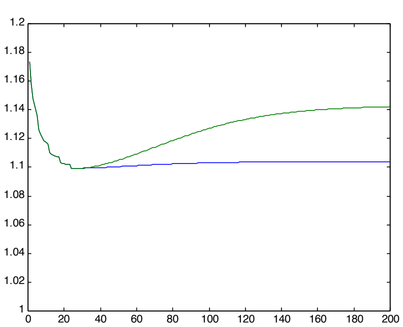


Figure 3.5: Rent

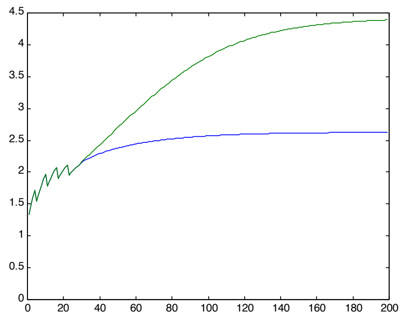


Figure 3.6: Resource share γ_x

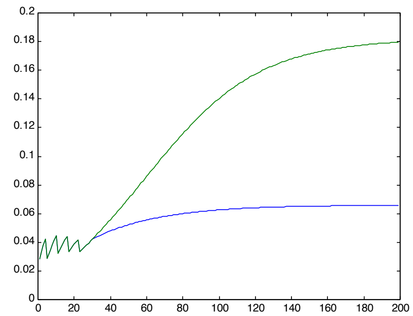


Figure 3.7: Real wages

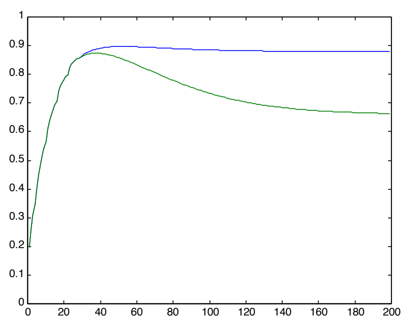
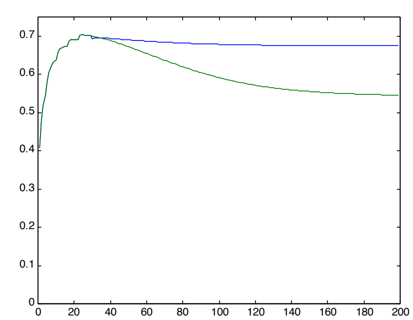


Figure 3.8: Labour share γ_L



5 Conclusion

In an endogenous growth model with a renewable natural resource, we have studied how resource scarcity affects the economy's growth trajectory, the rent associated with the use of the resource and the functional distribution of value added. One particular feature of our model is that it is based on the concept of strong sustainability, reflecting the fact that, for physical reasons, technical progress and/or substitution between natural factors and man-made factors are limited. The productivity of the resource is therefore necessarily bounded, which amounts to saying that it is impossible to completely dematerialise the final output. As in Fagnart-Germain (2011), it follows that quantitative growth (in terms of number of goods produced) can only be a transitional phenomenon, and that only qualitative growth (in terms of quality of goods) may be possible indefinitely.

We have proposed a Ricardo-inspired modelling of the sector that extracts the resource. The rent from this activity turns out to be an increasing function of the rate of resource extraction, which in this way affects the functional distribution of value added, both during the transitional dynamics and in the long run. Regarding the long run, we studied how resource scarcity influenced the steady-state growth path. Greater abundance of the resource leads to a higher steady-state level of material output per capita, while making a reduction in the equilibrium extraction rate possible. This makes the extraction activity less capital-intensive and allows a decrease in the macro-economic saving rate. This decrease is further accentuated by the fact that the better-resourced economy also *ultimately* invests less in research, with the steady-state growth rate of output quality being reduced accordingly. The decrease in the extraction rate also changes the distribution of value added to the advantage of the labour factor: where the resource is more abundant, the distributive shares of the resource and of capital are lower, while wages and the labour share are higher. *Conversely,* a lower per capita level of resource will change the distribution of value added to the detriment of the labour factor.

The dynamic analysis focused on the growth path of an economy whose initial state is characterised by a capital stock well below its steady-state level and the prospect of significant technological progress. The numerical simulations show that material output increases monotonically in this case and tends towards its steady-state level. In this simulation, the initially low capital acts as the limiting factor in the economy at the start of the trajectory, in contrast with the population and the resource, which are relatively abundant. Given its scarcity, the incentive to accumulate capital is strong, as the high saving rate values show. Gradually, as the economy grows, capital becomes less scarce and its remuneration decreases. Conversely, the relative scarcity of labour and material increases.

In the reference simulation, in which the available resource stock is known and accessible right from the initial state, economic growth is accompanied by increasing tensions over the resource: as its marginal productivity is limited, its real price and distributive share rise throughout the transitional dynamics, thus confirming Ricardo's conjecture. If the natural resource stock is only gradually discovered, the simulations show that successive discoveries may lead to a downward trend in the price of the resource and its distributive share during the discovery phase. But once the discoveries cease, the price and distributive share of the resource start to grow again, monotonically, towards their equilibrium value. Moreover, this equilibrium value will be higher (and the upward movement towards it more pronounced) when the transitional dynamics is accompanied by strong population pressure or greater gains in labour productivity. When the steady-state rent share is high, the simulations also show that the distributive conflict between resource-owners and workers is exacerbated during the transitional dynamics, and the labour share in value added may decrease when material growth ceases.

Our analysis therefore suggests that the historically observed decline in the resource share in value added could be reversed in the future: under the strong sustainability hypothesis, a growing economy where the phase of resource deposit discovery has ceased can only escape Ricardo's conjecture if the potential dematerialisation of output is sufficiently great, and in particular if it makes possible an absolute decoupling of economic growth and material consumption of resources. We also emphasised that what matters here is not so much the absolute level of the dematerialisation potential as the relative value of the potential for technological progress affecting the resource on the one hand and labour on the other. Thus, the dematerialisation potential necessary to escape Ricardo's conjecture is greater where the potential gain in labour productivity is itself high.

The model outlined in this article could be developed in various ways that would make it possible to assess the robustness of the results in a richer context. Three avenues for research particularly deserve to be mentioned. The first concerns the endogenisation of the population, and in particular that of the demographic transition. The second concerns the introduction of non-renewable natural resources. Although these obviously cannot play a role in the long run, their gradual depletion is nonetheless likely to affect the distribution of value added during the transitional phase, and in particular to exacerbate the distributive conflict during this phase. Finally, following Kaldor (op. cit.), one could consider a model in which individuals have a propensity to save that depends on their sources (and levels) of income and in which the functional distribution of national income therefore has additional macro-economic consequences.

6 References

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7 Appendices

7.1 Investment and consumption demands for each final good

We derive here the consumption demand for final good i. The investment demand can be derived in the same way. For a given level of the consumption index C_t, the household chooses the vector of final goods $(c_{it}, i \in [0,1])$ that minimizes the cost of C_t, i.e.

$$\min_{c_{it}} \int_0^1 p_{it} c_{it} di \text{ s.t. } C_t = \left[\int_0^1 \left[\psi(q_{it}) c_{it} \right]^\alpha di \right]^{1/\alpha}$$

The Lagrangian of this problem writes as:

$$\mathcal{L}_t = \int_0^1 p_{it} c_{it} di + \nu_t \left[\left[\int_0^1 \left[\psi(q_{it}) c_{it} \right]^\alpha di \right]^{\frac{1}{\alpha}} - C_t \right]$$

where ν_t is the multiplier associated to the constraint of the minimization problem. The first-order condition for good i leads to:

$$\begin{aligned} p_{it} &= \nu_t \frac{1}{\alpha} \left[\int_0^1 \left[\psi(q_{it}) c_{it} \right]^\alpha di \right]^{\frac{1}{\alpha}-1} \alpha \left[\psi(q_{it}) c_{it} \right]^{\alpha-1} \psi(q_{it}) \\ &= \nu_t C_t^{1-\alpha} \psi^\alpha(q_{it}) c_{it}^{\alpha-1} \end{aligned} \tag{63}$$

Multiplying the last equality by c_{it} gives

$$p_{it} c_{it} = \nu_t C_t^{1-\alpha} \left[\psi(q_{it}) c_{it} \right]^\alpha .$$

Summing over i's this last expression implies:

$$\int_0^1 p_{it} c_{it} di = \nu_t C_t^{1-\alpha} \int_0^1 \left[\psi(q_{it}) c_{it} \right]^\alpha di = \nu_t C_t^{1-\alpha} C_t^\alpha = \nu_t C_t .$$

The multiplier is thus the price index P_{ct} associated to the vector of consumption goods and such that $P_{ct} C_t = \int_0^1 p_{it} c_{it} di$. Consequently, (63) implies the following consumption demand for good i:

$$c_{it} = \psi^{\varepsilon-1}(q_{it}) \left[\frac{p_{it}}{P_{ct}} \right]^{-\varepsilon} C_t \qquad \text{where} \quad \varepsilon = 1/\left[1-\alpha\right]. \tag{64}$$

After substituting c_{it} with (64) into $P_{ct} C_t = \int_0^1 p_{it} c_{it} di$, one obtains the expression of the consumption price index:

$$\begin{aligned} P_{ct} C_t &= \int_0^1 p_{it} \psi^{\varepsilon-1}(q_{it}) \left[\frac{p_{it}}{P_{ct}} \right]^{-\varepsilon} C_t di \\ &\Rightarrow P_{ct}^{1-\varepsilon} = \int_0^1 \psi^{\varepsilon-1}(q_{it}) \left[p_{it} \right]^{1-\varepsilon} di, \end{aligned}$$

which leads to the expression given in (11).

Proceding similarly for investment, one shows easily that the investment demand for good i is

$$d_{it} = \varphi^{\varepsilon-1}(q_{it}) \left[\frac{p_{it}}{P_{kt}}\right]^{-\varepsilon} k_{t+1}$$

with the investment price index P_{kt} given in (11).
The total demand for good i is then

$$\begin{aligned} y_{it} &= c_{it} + d_{it} \\ &= \psi^{\varepsilon-1}(q_{it}) \left[\frac{p_{it}}{P_{ct}}\right]^{-\varepsilon} C_t + \varphi^{\varepsilon-1}(q_{it}) \left[\frac{p_{it}}{P_{kt}}\right]^{-\varepsilon} k_{t+1}. \end{aligned} \tag{65}$$

The price elasticity of demand is thus simply equal to $-\epsilon$. Moroever,

$$\begin{aligned} \frac{\partial y_{it}}{\partial q_{it}} &= \frac{\partial c_{it}}{\partial q_{it}} + \frac{\partial d_{it}}{\partial q_{it}} \\ &= [\varepsilon - 1]\,\psi^{\varepsilon-2}(q_{it})\psi'(q_{it}) \left[\frac{P_{ct}}{p_{it}}\right]^{\varepsilon} C_t + [\varepsilon - 1]\,\varphi^{\varepsilon-2}(q_{it})\varphi'(q_{it}) \left[\frac{P_{kt}}{p_{it}}\right]^{\varepsilon} k_{t+1} \\ &= [\varepsilon - 1] \left[\frac{\psi'(q_{it})}{\psi(q_{it})} c_{it} + \frac{\varphi'(q_{it})}{\varphi(q_{it})} d_{it}\right], \end{aligned}$$

which leads to the expression of the elasticity of y_{it} with respect to q_{it} given in (12).

7.2 Price, quality, production and factor demand of a monopolistic firm

Problem (21) leads to the following first-order conditions for period t:

$$\frac{\partial \Pi_t}{\partial p_t} = y_t + \left[p_t - \left[U_{ft}\kappa_t + U_{rt} h\left(\frac{q_t}{Q_{t-1}}\right) + P_{xt}\left[\chi_t + \mu_t\right]\right]\right] \frac{\partial y_t}{\partial p_t} = 0 \tag{66}$$

$$\frac{\partial \pi_t}{\partial q_t} = \left[p_t - \left[U_{ft}\kappa_t + U_{rt} h\left(\frac{q_t}{Q_{t-1}}\right) + P_{xt}\left[\chi_t + \mu_t\right]\right]\right] \frac{\partial y_t}{\partial q_t} - U_{rt} h'\left(\frac{q_t}{Q_{t-1}}\right) \frac{y_t}{Q_{t-1}} = 0. \tag{67}$$

Multiplying (66) by p_t/y_t gives

$$p_t = -\frac{p_t}{y_t}\frac{\partial y_t}{\partial p_t} \left[p_t - \left[U_{ft}\kappa_t + U_{rt} h\left(\frac{q_t}{Q_{t-1}}\right) + P_{xt}\left[\chi_t + \mu_t\right]\right]\right].$$

As the price elasticity of demand is $-\epsilon$, this expression may be rewritten as:

$$[\epsilon - 1]p_t = \epsilon \left[U_{ft}\kappa_t + U_{rt} h\left(\frac{q_t}{Q_{t-1}}\right) + P_{xt}\left[\chi_t + \mu_t\right]\right], \tag{68}$$

which is equivalent to (22).
Multiplying (67) by q_t/y_t gives

$$\left[p_t - \left[U_{ft}\kappa_t + U_{rt} h\left(\frac{q_t}{Q_{t-1}}\right) + P_{xt}\left[\chi_t + \mu_t\right]\right]\right] \frac{q_t}{y_t}\frac{\partial y_t}{\partial q_t} = U_{rt} h'\left(\frac{q_t}{Q_{t-1}}\right) \frac{q_t}{Q_{t-1}}.$$

Using (68), the expression between square brackets on the left side may be shown to be equal to p_t/ϵ and the equation may be rewritten as (23).

7.3 The dynamics of the NR

The quantity of NR extracted is sold to firms in the final sector. To produce y_t, a final firm needs quantity x_t of NR (given by (29)): one fraction of the material ($\mu(q_{t-1})y_t$) is incorporated into the goods produced and the other ($Z_{ft} = \chi(q_{t-1})y_t$) ends up as waste. These goods are in turn sold to households in the form of consumer and capital goods. The consumption goods are consumed immediately and contribute to the waste stream $Z_{ct} = \mu(q_{t-1})y_t$ at the end of period t. The material in the capital goods is incorporated into the capital stock operating during $t+1$. By contrast, the capital stock used during t is scrapped at the end of this period and contributes to the waste stream $Z_{kt} = \mu(q_{t-2})\frac{k_t}{\varphi(q_{t-1})}$ [19].

The waste generated in t by production, consumption and capital scrapping (Z_{ft}, Z_{ct} and Z_{kt} respectively) is assumed to be fully and immediately recycled by nature, so that it re-enters the stock of NR at the start of $t+1$.

Given the foregoing, the dynamics of the NR can be described as follows:

$$\begin{aligned}
R_{t+1} - R_t &= Z_{ft} + Z_{ct} + Z_{kt} - x_t \\
&= \chi(q_{t-1})y_t + \mu(q_{t-1})c_t + \mu(q_{t-2})\frac{k_t}{\varphi(q_{t-1})} - \left[\chi(q_{t-1}) + \mu(q_{t-1})\right] y_t \\
&= -\mu(q_{t-1})\frac{k_{t+1}}{\varphi(q_t)} + \mu(q_{t-2})\frac{k_t}{\varphi(q_{t-1})},
\end{aligned}$$

where use has been made of (31) and (29). This equation can also be written

$$R_{t+1} + \mu(q_{t-1})\frac{k_{t+1}}{\varphi(q_t)} = R_t + \mu(q_{t-2})\frac{k_t}{\varphi(q_{t-1})}, \tag{69}$$

which expresses the fact that the material in the economy-nature system is conserved between any two periods t and $t+1$: the amount of material in the economy-nature system is constant over time. Let M be that constant. (69) then leads to (37).

7.4 Factor shares in value added during period t

Given (9), the resource share in aggregate value added in t writes as:

$$\begin{aligned}
\frac{\Theta_t}{p_t y_t} &= p_{xt}\frac{R_t}{y_t}\left[E_t - [1-E_t]\ln(1-E_t)^{-1}\right] \\
&= p_{xt}\frac{\chi_t + \mu_t}{E_t}\left[E_t - [1-E_t]\ln(1-E_t)^{-1}\right] \\
&= u_{et}\frac{\chi_t + \mu_t}{1-E_t}\left[1 - \frac{1-E_t}{E_t}\ln(1-E_t)^{-1}\right].
\end{aligned}$$

The second equality follows from (29) and implies (38). The third follows from (7) and leads straightforwardly to (39).

Using (21) and (22), one verifies easily that the share of monopolistic profits in value added is a constant: $\Pi_t/p_t y_t = 1 - \alpha$.

The total share of capital in value added is thus equal to

$$\begin{aligned}
\gamma_{Kt} = \frac{\Pi_t}{p_t y_t} + \frac{V_t k_t}{p_t y_t} &= 1 - \alpha + \frac{V_t}{P_{kt-1}}\frac{P_{kt-1}}{p_t}\frac{k_t}{y_t} \\
&= 1 - \alpha + \frac{1+\rho_t}{\psi(q_t)}\frac{\psi(q_{t-1})}{\varphi(q_{t-1})}\frac{k_t}{y_t}
\end{aligned}$$

where the last equality follows from the definition of ρ_t (see 24) and (25).

[19] The material contained in the stock k_t comes from the capital goods used for its installation: $k_t/\varphi(q_{t-1})$ units of these goods were needed. As they were manufactured during $t-1$, each unit of good has a material content of $\mu_{t-1} = \mu(Q_{t-2}) = \mu(q_{t-2})$.

7.5 Reduction of the stationary system

Using $s =_{def} \tilde{k}/\overline{\varphi}\tilde{y}$ in (52), one obtains (54). As $\tilde{R} = \left[\underline{\chi} + \underline{\mu}\right] \tilde{y}/E$ (see 43), (46) implies

$$\frac{m}{\tilde{y}} = \frac{\underline{\underline{\chi + \mu}}}{E} + s\underline{\mu}. \tag{70}$$

The ratio between this equation and (53) writes as follows:

$$m = \frac{\frac{\underline{\underline{\chi + \mu}}}{E} + s\underline{\mu}}{\underline{\lambda}_f \underline{\kappa} + \underline{\lambda}_r h\left(\widehat{q}\right) - \underline{\lambda}_e \frac{\underline{\underline{\chi + \mu}}}{E} \ln\left(1 - E\right)}. \tag{71}$$

Under assumption (50), (71) and (54) imply

$$m = \frac{\frac{\underline{\underline{\chi + \mu}}}{E} + s\underline{\mu}}{\underline{\lambda}\left[\underline{\kappa} + h\left(\widehat{q}\right) - \frac{\underline{\underline{\chi + \mu}}}{E} \ln\left(1 - E\right)\right]} = \frac{\frac{\underline{\underline{\chi + \mu}}}{E} + s\underline{\mu}}{\underline{\lambda} s \overline{\varphi}},$$

which leads to (55).
Furthermore, (47) and (48) imply respectively:

$$\alpha = u_f \underline{\kappa} + u_r h\left(\widehat{q}\right) + u_e \frac{\underline{\underline{\chi + \mu}}}{1 - E} \tag{72}$$

$$\alpha\eta\left[1 - s\right] = u_r h'\left(\widehat{q}\right)\widehat{q}. \tag{73}$$

Under assumption (50), $u_z = u$ and the ratio between (73) and (72) gives

$$\eta\left[1 - s\right] = \frac{h'\left(\widehat{q}\right)\widehat{q}}{\underline{\kappa} + h\left(\widehat{q}\right) + \frac{\underline{\underline{\chi + \mu}}}{1 - E}},$$

from which one obtains (56).

7.6 Proof of Propositions 1 and 2

7.6.1 Sign of $ds/dm, dE/dm$

To study the impact of m on the stationary equilibrium, let us differentiate the system (54-56):

$$\bar{\varphi} s \frac{ds}{s} = h'(\hat{q})\hat{q}\frac{d\hat{q}}{\hat{q}} + \left[\underline{\chi} + \underline{\mu}\right] G(E)\frac{dE}{E} \tag{74}$$

$$\frac{dE}{E} = -\frac{ds}{s} - \frac{m\underline{\lambda}\bar{\varphi}}{m\underline{\lambda}\bar{\varphi} - \underline{\mu}}\frac{dm}{m} \tag{75}$$

$$-\frac{s}{1 - s}\frac{ds}{s} = -\frac{\frac{\underline{\underline{\chi + \mu}}}{1 - E}}{\underline{\kappa} + h\left(\widehat{q}\right) + \frac{\underline{\underline{\chi + \mu}}}{1 - E}}\frac{E}{1 - E}\frac{dE}{E} + \left[1 + \phi_{h'} - \eta[1 - s]\right]\frac{d\hat{q}}{\hat{q}} \tag{76}$$

where $\phi_{h'} = \hat{q}\frac{h''(\hat{q})}{h'(\hat{q})} \geq 0$ is the elasticity of $h'(\hat{q})$ to $\hat{q}$.
From (74) and (56), one obtains that

$$\frac{d\hat{q}}{\hat{q}} = \frac{\bar{\varphi} s}{\eta[1 - s]\left[\underline{\kappa} + h\left(\widehat{q}\right) + \frac{\underline{\underline{\chi + \mu}}}{1 - E}\right]}\frac{ds}{s} - \frac{\left[\underline{\chi} + \underline{\mu}\right] G(E)}{\eta[1 - s]\left[\underline{\kappa} + h\left(\widehat{q}\right) + \frac{\underline{\underline{\chi + \mu}}}{1 - E}\right]}\frac{dE}{E}. \tag{77}$$

Note that under assumption (50), (54) may be rewritten as follows,

$$\begin{aligned}\bar{\varphi}s &= \underline{\kappa} + h\left(\widehat{q}\right) + \frac{\underline{\chi} + \underline{\mu}}{1-E} - \frac{\underline{\chi} + \underline{\mu}}{1-E} - \frac{\underline{\chi} + \underline{\mu}}{E}\ln\left(1-E\right) \\ &= \underline{\kappa} + h\left(\widehat{q}\right) + \frac{\underline{\chi} + \underline{\mu}}{1-E} - \left[\underline{\chi} + \underline{\mu}\right] G(E).\end{aligned}$$

In other words,

$$\underline{\kappa} + h\left(\widehat{q}\right) + \frac{\underline{\chi} + \underline{\mu}}{1-E} = \bar{\varphi}s + \left[\underline{\chi} + \underline{\mu}\right] G(E). \tag{78}$$

In order to shorten the forthcoming mathematical expressions, let us introduce the following notation:

$$\Delta =_{def} \bar{\varphi}s + \left[\underline{\chi} + \underline{\mu}\right] G(E) \tag{79}$$

Using (78) and substituting $d\hat{q}/\hat{q}$ with (77) in (76), one obtains, after some manipulations,

$$-\left\{\frac{s}{1-s} + \left[\frac{1+\phi_{h'}}{\eta[1-s]} - 1\right]\frac{\bar{\varphi}s}{\Delta}\right\}\frac{ds}{s} = -\left\{\frac{1}{\Delta}\frac{\underline{\chi} + \underline{\mu}}{1-E}\frac{E}{1-E} + \left[\frac{1+\phi_{h'}}{\eta[1-s]} - 1\right]\frac{\left[\underline{\chi} + \underline{\mu}\right] G(E)}{\Delta}\right\}\frac{dE}{E}.$$

Inserting (75) in this last expression, one obtains finally a relationship between the rates of variation of s and m:

$$-\Lambda\frac{ds}{s} = \left\{\frac{1}{\Delta}\frac{\underline{\chi} + \underline{\mu}}{1-E}\frac{E}{1-E} + \left[\frac{1+\phi_{h'}}{\eta[1-s]} - 1\right]\frac{\left[\underline{\chi} + \underline{\mu}\right] G(E)}{\Delta}\right\}\frac{m\underline{\lambda}\bar{\varphi}}{m\underline{\lambda}\bar{\varphi} - \underline{\mu}}\frac{dm}{m}, \tag{80}$$

$$\text{where } \Lambda = \frac{s}{1-s} + \left[\frac{1+\phi_{h'}}{\eta[1-s]} - 1\right] + \frac{1}{\Delta}\frac{\underline{\chi} + \underline{\mu}}{1-E}\frac{E}{1-E} > 0.$$

As $\Lambda > 0$, (80) establishes a negative relationship between the change in s and the change in m. Using (80), one may rewrite (75) as a relationship between the rates of variation of E and m:

$$\frac{dE}{E} = \left\{\frac{1}{\Lambda}\left[\frac{1}{\Delta}\frac{\underline{\chi} + \underline{\mu}}{1-E}\frac{E}{1-E} + \left[\frac{1+\phi_{h'}}{\eta[1-s]} - 1\right]\frac{\left[\underline{\chi} + \underline{\mu}\right] G(E)}{\Delta}\right] - 1\right\}\frac{m\underline{\lambda}\bar{\varphi}}{m\underline{\lambda}\bar{\varphi} - \underline{\mu}}\frac{dm}{m}$$

or, equivalently,

$$\Lambda\frac{dE}{E} = \left\{\frac{1}{\Delta}\frac{\underline{\chi} + \underline{\mu}}{1-E}\frac{E}{1-E} + \left[\frac{1+\phi_{h'}}{\eta[1-s]} - 1\right]\frac{\left[\underline{\chi} + \underline{\mu}\right] G(E)}{\Delta} - \Lambda\right\}\frac{m\underline{\lambda}\bar{\varphi}}{m\underline{\lambda}\bar{\varphi} - \underline{\mu}}\frac{dm}{m}.$$

Using the expressions of Λ and Δ, one can simplify the right side of this equality and one gets

$$\Lambda\frac{dE}{E} = \left\{-\frac{s}{1-s} + \left[\frac{1+\phi_{h'}}{\eta[1-s]} - 1\right]\left[\frac{\left[\underline{\chi} + \underline{\mu}\right] G(E)}{\Delta} - 1\right]\right\}\frac{m\underline{\lambda}\bar{\varphi}}{m\underline{\lambda}\bar{\varphi} - \underline{\mu}}\frac{dm}{m}.$$

Finally, using the definition of Δ to simplify the last term between square brackets [...], on has

$$\Lambda\frac{dE}{E} = -\left\{\frac{s}{1-s} + \left[\frac{1+\phi_{h'}}{\eta[1-s]} - 1\right]\frac{\varphi s}{\Delta}\right\}\frac{m\underline{\lambda}\bar{\varphi}}{m\underline{\lambda}\bar{\varphi} - \underline{\mu}}\frac{dm}{m}. \tag{81}$$

Because both Λ and the term between square brackets in (81) are positive, E is thus a decreasing function of m.

7.6.2 Sign of $d\tilde{y}/dm, d\tilde{x}/dm, d\tilde{c}/dm, du/dm$

7.6.1 has established that $s =_{def} \tilde{k}/\overline{\varphi}\tilde{y}$ is decreasing in m. So is the stationary capital/output ratio $\tilde{k}/\tilde{y}$: because $\tilde{k}$ is independant of m (cfr. Lemma 1), a higher value of m implies a higher value of $\tilde{y}$ and thus also of $\tilde{x}$ (proportional to $\tilde{y}$) and $\tilde{c}$ (which varies by the same amount as $\tilde{y}$).

From (72) and assumption (50), one obtains that

$$u = \alpha \left[\underline{\kappa} + h\left(\widehat{q}\right) + \frac{\underline{\chi} + \underline{\mu}}{1-E} \right]^{-1}. \tag{82}$$

Using (78), one rewrites (82) as

$$u = \frac{\alpha}{\bar{\varphi}s + \left[\underline{\chi} + \underline{\mu}\right] G(E)}. \tag{83}$$

u is thus a decreasing function of s and E, which are both decreasing in m as shown in 7.6.1: u is thus increasing in m.

7.6.3 Sign of $d\hat{q}/dm$ and $d\rho/dm$

(77) shows that

$$sign\left\{\frac{d\hat{q}}{\hat{q}}\right\} = sign\left\{\bar{\varphi}s\frac{ds}{s} - \left[\underline{\chi} + \underline{\mu}\right] G(E)\frac{dE}{E}\right\}.$$

Given (80) and (81) and given that $\Lambda > 0$, one also has that

$$sign\left\{\frac{d\hat{q}}{\hat{q}}\right\} = sign\left\{-\bar{\varphi}s\left[\frac{1}{\Delta}\frac{\underline{\chi} + \underline{\mu}}{1-E}\frac{E}{1-E} + \left[\frac{1+\phi_{h'}}{\eta[1-s]} - 1\right]\frac{\left[\underline{\chi} + \underline{\mu}\right] G(E)}{\Delta}\right] ...\right.$$

$$\left. ... + \left[\underline{\chi} + \underline{\mu}\right] G(E)\left[\frac{s}{1-s} + \left[\frac{1+\phi_{h'}}{\eta[1-s]} - 1\right]\frac{\varphi s}{\Delta}\right]\right\} \cdot sign\left\{\frac{dm}{m}\right\},$$

which, using the definition of Δ, simplifies as

$$sign\left\{\frac{d\hat{q}}{\hat{q}}\right\} = sign\left\{\left[\underline{\chi} + \underline{\mu}\right] G(E)\frac{s}{1-s} - \frac{\bar{\varphi}s}{\Delta}\frac{\underline{\chi} + \underline{\mu}}{1-E}\frac{E}{1-E}\right\} \cdot sign\left\{\frac{dm}{m}\right\}.$$

In the case of an increase in m, the sign of the variation of $\hat{q}$ will be the same as the one of the first term at the right-hand-side of the last equation: $\hat{q}$ will thus decrease if

$$\left[\underline{\chi} + \underline{\mu}\right] G(E)\frac{s}{1-s} < \frac{\bar{\varphi}s}{\Delta}\frac{\underline{\chi} + \underline{\mu}}{1-E}\frac{E}{1-E}$$

or if

$$\frac{G(E)}{1-s} < \frac{\bar{\varphi}}{\Delta}\frac{E}{(1-E)^2}$$

or (using the definition of Δ) if

$$s + \frac{\underline{\chi} + \underline{\mu}}{\bar{\varphi}}G(E) < (1-s)\frac{E}{G(E)(1-E)^2}.$$

A sufficient condition for this inequality to be verified is $s < 2/3$ and $(\chi + \mu) < \varphi/3$. In this case indeed, one verifies easily that

$$s + \frac{\underline{\chi} + \underline{\mu}}{\bar{\varphi}}G(E) < \frac{2}{3} + \frac{1}{3}G(E) < \frac{1}{3}\frac{E}{G(E)(1-E)^2} < (1-s)\frac{E}{G(E)(1-E)^2}.$$

The second inequality is true for any $E \in]0,1]$ because $\frac{E}{G(E)(1-E)^2} - G(E)$ reaches a minimum of 2 when $E = 0$ and is increasing in E.

As ρ and $\hat{q}$ are positively related (cfr. equation 41), there is also a negative relationship between the relative variation of m and the one of ρ.

7.6.4 Sign of $dv/dm, dw/dm, dp_x/dm$

The link between v and m (through the negative impact of m on $\hat{q}$) follows straightforwardly from (49). Furthermore, as $u = v + \lambda w$ and because u is an increasing function of m, w is necessarily higher when v decreases or remains constant. By continuity, this is also the case when v does not increase too much and thus at least when σ is higher than 1 and in the neighborhood of 1.

Equation (30) implies that

$$p_x = \frac{u}{1-E}. \tag{84}$$

The evolution of the price of the resource is the result of two contradictory effects, one positive (and linked to the increase in u), and the other negative (linked to the decrease in E). It is not possible to establish analytically which effect dominates.

7.7 Stationary distribution of value added

7.7.1 Expression of the stationary shares

Under assumption (50), $\gamma_{L_t} = w_t L_t / y_t = w_t \lambda_t k_t / y_t$, so that

$$\gamma_{K_t} + \gamma_{L_t} = 1 - \alpha + v_t \frac{k_t}{y_t} + w_t \lambda_t \frac{k_t}{y_t} = 1 - \alpha + u_t \frac{k_t}{y_t}.$$

It follows that

$$\gamma_{x_t} = 1 - (\gamma_{K_t} + \gamma_{L_t}) = \alpha - u_t \frac{k_t}{y_t}.$$

In a stationary state:

- $k/y = \varphi s$ and one thus obtains that $\gamma_x = \alpha - \varphi s u$. Replacing u by (83) in this expression, one obtains (59).
- (24) and (26) imply that $\psi(q_t)/\psi(q_{t-1}) = \beta^\sigma [1+\rho]^\sigma$ and $\varphi(q_{t-1})$ tends to $\bar{\varphi}$. The stationary expression of γ_K is thus (60).

The expression of the stationary share of labour in the added value (γ_L) is obtained as the complementary of $\gamma_x + \gamma_K$.

7.7.2 Proof of point 1 of Proposition 3

From the stationary expression (59) of γ_x, one gets that

$$\frac{d\gamma_x}{\gamma_x} = \frac{\bar{\varphi} s}{\Delta} \left[\phi_G \frac{dE}{E} - \frac{ds}{s} \right] \qquad \text{with} \quad \phi_G = E \frac{G'(E)}{G(E)} \geq 1.$$

ϕ_G is the elasticity of $G(E)$ with respect to E and is increasing in E. One knows that a higher m implies lower values of E and s. It will also imply a lower value of γ_x if the term between square brackets in the above expression is negative, i.e. if

$$\phi_G \left[-\frac{dE}{E} \right] > \left[-\frac{ds}{s} \right]. \tag{85}$$

Let us identify under which condition this will be the case when $dm/m > 0$. Using (80) and (81),

$$\begin{aligned} &\phi_G \left[-\frac{dE}{E} \right] - \left[-\frac{ds}{s} \right] \\ &= \frac{1}{\Lambda} \left\{ \phi_G \frac{s}{1-s} + \left[\frac{1+\phi_{h'}}{\eta[1-s]} - 1 \right] \frac{\phi_G \varphi s - \left[\underline{\chi} + \underline{\mu} \right] G(E)}{\Delta} - \frac{1}{\Delta} \frac{\underline{\chi} + \underline{\mu}}{1-E} \frac{E}{1-E} \right\} \frac{m \underline{\lambda} \bar{\varphi}}{m \underline{\lambda} \bar{\varphi} - \underline{\mu}} \frac{dm}{m}. \end{aligned} \tag{86}$$

As Λ is positive, (86) will be positive if the term between curly brackets $\{...\}$ at the right side of this expression is positive. This will be the case if

$$\phi_G s + \Lambda \left[\frac{1+\phi_{h'}}{\eta} - [1-s] \right] \frac{\phi_G \varphi s - \left[\underline{\chi} + \underline{\mu}\right] G(E)}{\Delta} > \frac{1}{\Delta} \frac{\underline{\chi} + \underline{\mu}}{1-E} \frac{E}{1-E} [1-s]. \tag{87}$$

Note that

$$\frac{\underline{\chi} + \underline{\mu}}{1-E} \frac{E}{1-E} = \frac{\underline{\chi} + \underline{\mu}}{(1-E)^2} \frac{G(E)E}{G(E)} = [\underline{\chi} + \underline{\mu}] G(E) [1 + \phi_G].$$

After inserting this last expression in right-hand-side of (87) and rearranging terms, one can rewrite the inequality (87) as

$$\phi_G s + \left[\frac{1+\phi_{h'}}{\eta} - [1-s] \right] \frac{\phi_G \varphi s}{\Delta} > \frac{1+\phi_{h'}}{\eta} \frac{[\underline{\chi} + \underline{\mu}] G(E)}{\Delta} + [1-s][1+\phi_G - 1] \frac{\left[\underline{\chi} + \underline{\mu}\right] G(E)}{\Delta} \tag{88}$$

or (using the definition of Δ to simplify the terms in $[1-s]$)

$$\phi_G s + \frac{1+\phi_{h'}}{\eta} \frac{\phi_G \varphi s}{\Delta} > \frac{1+\phi_{h'}}{\eta} \frac{[\underline{\chi} + \underline{\mu}] G(E)}{\Delta} + [1-s]\phi_G. \tag{89}$$

Dividing this inequality by ϕ_G, on obtains finally

$$2s + \frac{1+\phi_{h'}}{\eta} \frac{\bar{\varphi} s - \left[\underline{\chi} + \underline{\mu}\right] G(E)/\phi_G}{\Delta} > 1 \tag{90}$$

where $\bar{\varphi} s - \left[\underline{\chi} + \underline{\mu}\right] G(E)/\phi_G \geq 0$ because (54) implies that

$$\bar{\varphi} s > \left[\underline{\chi} + \underline{\mu}\right] \frac{\ln[1-E]^{-1}}{E} \qquad \text{where} \qquad \frac{\ln[1-E]^{-1}}{E} \geq 1 \geq \frac{[G(E)]^2}{EG'(E)} = \frac{G(E)}{\phi_G} \geq 0,$$

for $0 \leq E \leq 1$.

Using (55) and the definition of Δ, one can rewrite (90) as

$$2 \frac{\underline{\chi} + \underline{\mu}}{m\lambda\bar{\varphi} - \underline{\mu}} \frac{1}{E} + \frac{1+\phi_{h'}}{\eta} \frac{\frac{\bar{\varphi}}{m\lambda\bar{\varphi} - \underline{\mu}} - G(E)/\phi_G}{\frac{\varphi}{m\lambda\varphi - \underline{\mu}} + G(E)} > 1, \tag{91}$$

which is equivalent to (62). The value of the product of the two last fractions at the left-hand-side of (91) decreases from $\frac{1+\phi_{h'}}{\eta} > 1$ when $E = 0$ towards 0 when $E = 1$. The condition (91) will thus be satisfied if E is not too close to 1.