On the Impact of Regression Technique to Airtightness Measurements Uncertainties

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Abstract

The Ordinary Least Square (OLS) method is often used in the estimation of buildings' airtightness. However, recent research demonstrated that this regression technique neglects a significant part of uncertainty. This paper investigates two alternative methods: Weighted Line of Organic Correlation (WLOC) and Iterative Weighted Least Square (IWLS). A series of 30 repeated tests were performed on a newly-constructed apartment in Brussels (Belgium). Real uncertainty and its estimation were compared for the three regression methods. In-situ observations showed that the statistical significance of differences between regression techniques depends on the considered pressure difference. Theoretical calculations showed that uncertainties estimated with IWLS and WLOC were higher than OLS. Lastly, comparing calculations and in-situ observations confirmed that IWLS and WLOC provide a better uncertainty estimation than OLS. Therefore, WLOC or IWLS should be preferred to OLS, especially in the presence of wind. WLOC is expected to be more robust than IWLS since it does not imply an iterative procedure. Along with ongoing work investigating other sources of errors, these findings contribute to a deeper understanding of uncertainty in fan pressurization tests.

Keywords: Uncertainty calculation; Airtightness Measurements; Fan Pressurization Test; Air Infiltration; Linear Regression Techniques; Weighted Line of Organic Correlation; Iterative Weighted Least Square; Ordinary Least Square.

Highlights

- At low- and high-pressure differences, OLS has higher uncertainty than IWLS and WLOC.
- Unlike OLS, IWLS and WLOC methods takes into account uncertainty in pressure differences.
- Uncertainty estimated using IWLS or WLOC is more reliable than using OLS.

1. Introduction

Excessive air infiltration in buildings can lead to several inconveniences, such as an increase of heating and cooling loads [1, 2], a reduction of acoustic insulation [3], or a decrease of the hygrothermal performance of insulation materials [4]. The rate of air infiltration in a building is determined by its airtightness [5], whose direct measurement is an issue of major concern considering that, according to the literature, no predictive model can yet replace it [6, 7].

Although the fan pressurization test (also called blower door test) has long been used by building professionals to measure airtightness [4], there is an important lack of knowledge when dealing with the uncertainty of the procedure. Over the last years, several European countries have imposed airtightness tests on new buildings and large retrofit interventions [8]. Therefore, reducing the uncertainty of these tests has become a key subject in this domain. In addition, it is essential that the reliability of airtightness measurements can be assessed. The uncertainty provides a direct information on the deviation of a result from the real value of the measured parameter. Without this information, results cannot be compared to each other or to reference values provided in regulations, specifications or standards [9]. Recently, various studies have been focusing on repeatability (i.e., successive measurements carried out under the same conditions) and reproducibility (i.e., successive measurements taken under changing conditions) of the test [10-14]. Research has also investigated several sources of uncertainty [15, 16] among which wind is likely to be the most challenging, since it can cause different types of errors in both envelope pressure and air flow rate measurements.

As recent studies from the authors have demonstrated, wind speed and direction can have a strong impact on uncertainty in fan pressurization measurements, making the envelope pressure uncertainty non-negligible [17, 18]. This implies that the most used regression technique, the Ordinary Least Square (OLS) method, might not be appropriate for fan pressurization measurements. The OLS method consists in minimizing the distance in the y-direction between the measurements and the fitted line. This method neglects the uncertainty in the x-direction, which is the pressure difference measurement in the fan pressurization test. In this context, Sherman and Palmiter had already questioned the relevance of the Ordinary Least Square (OLS) regression method in 1995 [15]. In 2012, Okuyama and Onishi suggested the use of the Iterative Weighted Least Square (IWLS) regression method [19] and, in 2017, Delmotte suggested the use of the Weighted Line of Organic Correlation (WLOC) [18]. The IWLS method consists in minimizing the distance between the measurements and the fitted line in the x- and y-directions, while considering uncertainties on both axes in weighting these measurements. Since this minimization has no analytical solution, the IWLS method induces an iterative process for the determination of the weights and the slope of the fitting line. On the other hand, the WLOC method, which has an analytical solution, consists in minimizing the product of the distances between the x- and y- coordinates of the measurement points and the fitting line, when both variables are weighted. Since the inadequacy of OLS has been pointed out in the literature [15], the impact of a change of method requires detailed investigation.

Based on previous research [15-20], this paper aims to demonstrate that changing the regression technique in the fan pressurization test protocol allows taking into account the effect of wind in the uncertainty, influencing both the observed (measured) and calculated (estimated) uncertainty of the airtightness measurement. This is achieved by applying different regression techniques on the same dataset and comparing results both practically and theoretically. In this paper, the terms *observed* and *calculated* refer respectively to the practical and the theoretical estimations of uncertainty. Observed uncertainty is determined by repeating multiple times the same measurement and looking at the variation of the results. Calculated uncertainty is estimated by formulating hypotheses about the sources of error and then propagating them through the different steps of the procedure for the estimation of building airtightness. The observed uncertainty is, to this date, much more reliable than its calculated value and should, therefore, be considered as a better indicator of the real uncertainty. However, due to time constraints, it is hardly possible to determine this uncertainty *in-situ*, hence making the calculated uncertainty a more appropriate option. Since consideration of both is essential, observed and calculated uncertainties are discussed in this paper.

The paper is articulated in four parts: the first section illustrates the methodologies for theoretical calculation and *in-situ* observation of uncertainty, and the protocol for their determination; in the second part, the results obtained by applying both methods on a series of 30 tests in repeatability conditions are presented; the third section discusses the implications of the results in practice and research, and illustrates the limitations of the study; the paper is concluded by a summary of the findings and the further work that is needed in the field of uncertainties in airtightness measurements.

2. Methodology

Airtightness measurements using fan pressurization tests are increasingly required in specifications and regulations worldwide. A fan pressurization test consists in measuring the airflow induced by a pressure difference between two sides of the building envelope. A fan placed at one of the building openings (usually a door or a window) induces this pressure difference. This measurement procedure is regulated by the international standard ISO 9972:2015 [21]. A linear regression is then performed between multiple couples of pressure difference/airflow measurements to determine two characteristics of the building: its flow exponent (n, the slope of the regression line) and its leakage coefficient (C, related to the intercept of the regression line). The airflow at a given pressure difference (based on the requirement or specification) can be deduced from the regression line, as a result of the fan pressurisation test.

To be reliable and comprehensive, any measurement should be provided with an indication of its uncertainty in order to estimate the ‘doubt’ about its effective accuracy [9]. The uncertainty is generally expressed as the expected standard deviation that would be observed for a large number of repeated measurements. As previously illustrated, the uncertainty can be defined both practically (observed) and theoretically (calculated)

Each uncertainty definition method presents some advantages but also suffers from some drawbacks. While calculated uncertainty allows understanding how different sources of uncertainty affect the variable of interest, observed uncertainty works as a "black-box method" where the operator gets an uncertainty value without a fundamental characterization of the processes and phenomena behind it. However, observed uncertainty incorporates all sources of errors experienced during the testing period under the given conditions, while theoretical uncertainty requires assumptions to be made for each of them. One of the most relevant differences lies in the time needed for the uncertainty determination. Observed uncertainty requires multiple identical tests, which is time consuming and, therefore, hardly done in practice. Conversely, since calculated uncertainty only requires hypotheses made prior to the measurement, the calculation is performed simultaneously with the fan pressurization test, without requiring any additional time.

This paper studies both calculation and observation methods for uncertainty estimation and compares them for three regression techniques. On the one hand, analysing theoretical calculations allows understanding the impact of neglecting some sources of uncertainty on the measurement protocol. On the other, investigating observed uncertainty provides a reference value for the uncertainty, which is more reliable than a theoretical estimation due to the lack of knowledge on the sources of uncertainty. Lastly, comparing both methods allows quantifying and understanding the "gap" between a theoretical and a practical approach.

2.1. Observed Uncertainties

In practice, a good estimate of uncertainty (i.e., $u_p(X)$) on a given variable is represented by the standard deviation (i.e., $s_p(X)$) of this variable measured repeatedly [9]. Therefore, observed uncertainty can be determined by repeating the same protocol for estimation of the variable and then computing the standard deviation (i.e., accounting for the amount of dispersion) of all these estimations. The sources of error considered in this estimate depend on the measurement conditions. For example, if estimations are made under various representative wind patterns, the error related to changes in wind conditions will be included in the determination of the observed uncertainty. Repeatability and reproducibility conditions refer to successive measurements carried out, respectively, under equal and changing circumstances. Perfect repeatability and reproducibility conditions are, in turn, the lower and upper limits of the observed uncertainty. In this study, the main experimental settings (i.e., operator and equipment) were kept constant between the tests. However, since wind conditions changed between measurements, these can be considered as near-repeatability conditions [10, 11].

A series of 30 fan pressurization measurements was performed for this study. In the field of building airtightness measurements, no previous research has defined an appropriate sample size for robust statistical analysis. Therefore, the number of tests repeated during a measurement period is an arbitrary trade-off between the duration of the testing and the uncertainty on the standard deviation (i.e., the capability of the standard deviation to estimate the unknown true uncertainty) that is given by $(2[n-1])^{-0.5}$, where n is the number of measurements performed. For 30 measurements, the uncertainty in $s_p(X)$ is 13.1% of the standard deviation calculated [9]. A short testing period (15 days) was chosen to avoid any impact of ageing and seasonal variations on airtightness, to limit the difference of outside temperature between tests, and to respect site constraints.

2.1.1. Measurement Protocol

Each test consisted in the measurement of 20 airflow-pressure difference couples between 10 and 100 Pa in both pressurization and depressurization. For both cases, these measurements were processed according to ISO 9972:2015 standard [21] to obtain $\ln(\Delta p_e)/\ln(q_e)$ couples. In practice, measurements made according to ISO 9972:2015 standard have five steps:

1. Measure $(\Delta p_m\ ;\ q_m)$ couples;
2. Transform measured couples to envelope couples $(\Delta p_e\ ;\ q_e)$, including inside-outside temperature difference and zero-flow pressure;
3. Compute $(\ln(\Delta \mathrm{p_e})\,;\ln(\mathrm{q_e}))$ couples;
4. Perform linear regression to obtain air flow exponent (n) and coefficient ($\mathrm{C_{env}}$);
5. Compute the variable of interest (e.g., $\mathrm{Q_{50}}$, $\mathrm{n_{50}}$ or $\mathrm{Q_4}$, depending on the case).

Where Δp_m and q_m are the measured pressure difference and air flow rate, Δp_e and q_e are the envelope pressure difference and airflow rate. Q_{50} and Q_4 are respectively the airflow rate at 50 and 4 Pa, while $\mathrm{n_{50}}$ is the air change rate per hour at 50 Pa.

In this work, for each test, step 4 was made three times: once for each linear regression (i.e., OLS, IWLS and WLOC methods), to derive three sets of regression parameters. Each set was therefore used to determine the air flow exponent (n) and air leakage coefficient (C_L) of the building (step 4) and, consequently, the relation between air flow rate through the envelope (Q) and the pressure difference (Δp) (step 5) either in depressurization or pressurization (Equation 1). Lastly, the air flow rate through the building envelope at a given pressure difference was computed as the average of pressurization and depressurization airflows at that pressure difference.

Equation 1

$$Q = C_L(\Delta p)^n$$

2.1.2. Measured Equipment and Weather Data

Pressure differences and air flow rates were recorded in October 2017 using a DG-700 pressure gauge (last calibrated four months before the tests) from The Energy Conservatory (TEC). According to the manufacturer, the accuracy in pressure measurement of the DG-700 is the greatest between $\pm$ 1% of the reading and $\pm$ 0.15 Pa, and it has a resolution of 0.1 Pa [23]. Regarding air flow rate measurements, the accuracy of the DG-700 in combination with a TEC BlowerDoor Model 4 is the greatest between $\pm$ 4% of the reading and $\pm$ 1.7 m³/h.

During the tests, a weather station (Ahlborn FMD 760) placed on the roof above the measurement site recorded the external air temperature, the wind speed and the wind direction every 10 seconds. The weather station provided 10-minute averages for the external air temperature (T_{ext}), the mean wind speed ($\bar{v}_w$) and the mean wind direction. It also provided the maximum wind speed measured during these 10-minute periods. A thermometer (Testo 417) was used to measure the internal air temperature (T_{int}) before each test with a maximum permissible error of $\pm$ 0.5 K and a resolution of 0.1 K. Table 1 shows, for each weather variable, the minimum, maximum, mean and standard deviation of all the values recorded by the weather station (10-minute samples) and the thermometer (before each test) during the 15-day testing period. Wind direction was mostly from the west and southwest.

	Min	Max	Mean	Standard Deviation
T_{ext} [°C]	8.8	24.3	14.2	2.77
T_{int} [°C]	20.5	22.0	21.3	0.57
$\bar{v}_w$ [m/s]	0	3.8	1.3	0.75
$\max(v_w)$ [m/s]	0.3	8.3	3.3	1.57

Table 1 – Minimum, maximum, mean and standard deviation of weather data measured during the testing period.

2.1.3. Measurement Site

The series of tests was performed on a newly built (2017) apartment within a period of 15 days in October 2017. The apartment was a masonry construction of 228 m³ with a floor area of 90 m² located on the second floor of a 3-storey building in Brussels (Belgium). Only two perimetral walls were exposed to the outside (Figure 1). Over recent years, masonry apartments have represented a large share of new residential constructions in Brussels.

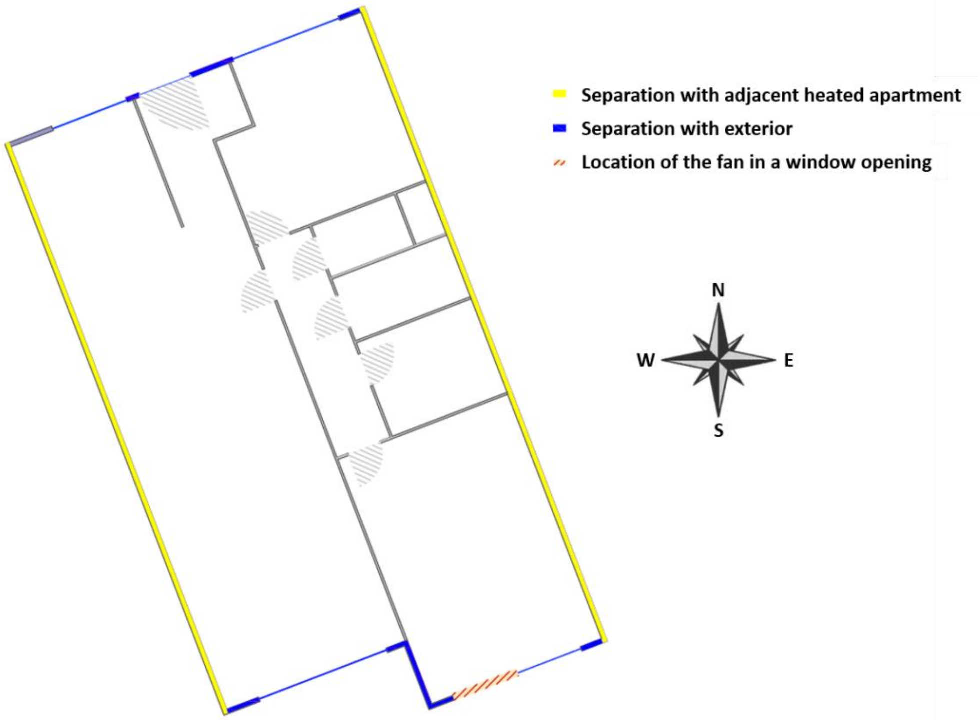

Figure 1 – Plan of the tested apartment, including separation type and fan location.

There were no airtightness requirements for the apartment itself, but the whole building (i.e., all the apartments pressurized simultaneously) had to reach a maximum air change rate of 0.6 h^{-1} at 50 Pa. The preparation of the apartment during the tests was consistent with method 1 described in ISO 9972:2015 [21].

2.2. *Theoretical Calculation of Uncertainty*

In the current state of knowledge, observed uncertainties are much more reliable than calculated values. However, they are hardly determined in practice when performing a fan pressurization test since the procedure required can be demanding in terms of time and resources. On the contrary, calculated uncertainty – that is performed simultaneously with the fan pressurization test – can be more suitable for the estimation of uncertainty in the context of real practice.

Uncertainty can be computed theoretically by determining all possible sources of uncertainty and then propagating them from the step they occur to the last one (i.e., the determination of the variable of interest). These sources of uncertainty can be divided in three categories: precision, bias (systematic error), and modelization errors. Precision and bias errors are, respectively, random and fixed variations of the measured values around the true quantity that is unknown. Modelization errors come from assumptions behind the estimation method.

2.2.1. Sources of Error Considered in Calculations

Precision and bias errors derive mainly from equipment, for both pressure and airflow measurements. In fact, the value read during the measurement can be different from the “true quantity”, which is unknown, because of the accuracy and resolution of the instruments utilized. The equipment manufacturer often provides information about accuracy and resolution. These can be translated into uncertainty depending on the way errors are expressed. Specific guidelines can be used to deduce the uncertainty from the information provided [9]. Precision and bias errors occur also when measuring internal and external temperatures, or when estimating building volume or envelope area based on building plans. In addition to these inaccuracies, an important modelization error can be due to short-term variations in wind speed and direction. In practice, the pressure induced by weather conditions (i.e., zero-flow pressure) is measured before and after the fan pressurization test and is generally assumed constant during the test. However, previous studies by the authors have shown that this is a questionable assumption that comes along with an important uncertainty [17, 18].

Although they are important in the context of airtightness estimation and deserve attention, this study did not focus on other bias errors than those already accounted for in the accuracy provided by the manufacturer. In addition, only the modelization error due to zero-flow pressure approximation was considered. This is because these were the only bias and modelization errors identified in the repeatability conditions applied in this study. For example, the location of pressure taps could be considered as a source of error in the calculation. However, since the location is kept constant between tests, this error would not be observed in the repeatability test. Therefore, comparing calculated and observed uncertainty would only be relevant if both took into account the same sources of errors.

The source of errors considered in this study are quantified and illustrated in the third part of this paper (section 3.2.1). These include: equipment accuracy for pressure and airflow measurements; equipment resolution for pressure measurement; equipment accuracy and resolution for temperature measurements; and, modelization error due to the zero-flow pressure hypothesis. Once sources of error are identified, one must propagate them from their occurrence to the variable of interest. In this study, the uncertainty calculation was performed for each of the 30 tests in both pressurization and depressurization, and it was then propagated to average air flow rate.

2.2.2. Uncertainty Propagation

Error propagation occurs when one variable is obtained from others: when a response variable y is obtained from explanatory variables (x_i, x_j, …), the uncertainty in the response variable ($u(y)$) is deduced from uncertainties in explanatory variables ($u(x_i)$, $u(x_j)$, …). This propagation is computed with Equation 2, where N is the number of explanatory variables, f is the functions linking response and explanatory variables and $r(x_i, x_j)$'s are the correlation coefficients between each explanatory variable [9].

Equation 2

$$u(y) = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} u(x_i)\right)^2 + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left(\frac{\partial f}{\partial x_i}\right) \left(\frac{\partial f}{\partial x_j}\right) u(x_i)\, u(x_j)\, r(x_i, x_j)}$$

The operator performing a fan pressurization test is interested in the uncertainty in the air flow rate and its derived quantities. In this context, each step of the protocol can be associated to various sources of error that propagate from step to step:

1. Measure (Δp_m ; q_m) couples;
2. Transform measured couples to envelope couples (Δp_e ; q_e), including inside-outside temperature difference and zero-flow pressure;
3. Compute ($\ln(\Delta \mathrm{p_e})$; $\ln(\mathrm{q_e})$) couples;
4. Perform linear regression to obtain air flow exponent (n) and coefficient ($\mathrm{C_{env}}$);
5. Compute the variable of interest (e.g., $\mathrm{Q_{50}}$, $\mathrm{n_{50}}$ or $\mathrm{Q_4}$, depending on the case).

Therefore, the uncertainty in measurements (i.e., pressure difference, air flow rate and temperature) must be propagated to the logarithm of the envelope pressure difference and airflow (i.e., $\ln(\Delta p_e)$ and $\ln(q_e)$), which are then used to perform the linear regression. Delmotte investigated in depth these multiple propagations and developed the relevant equations in previous research [20]. These equations are not discussed in this paper. However, in the study of Delmotte, only the uncertainty due to equipment accuracy and resolution was considered. That work was completed with another study by Delmotte where values for the uncertainty due to the zero-flow pressure hypothesis were suggested [18].

In this paper, the uncertainty due to the zero-flow pressure hypothesis was assumed uncorrelated with the uncertainty due to the accuracy and resolution of the equipment. This seems a reasonable hypothesis since the characteristics of the equipment are not expected to vary when testing conditions are within the operating range. Therefore, the total uncertainty in pressure measurement can be computed using Equation 3:

Equation 3

$$u_p(\Delta p) = \sqrt{u_{p,m}^2(\Delta p) + u_{p,m}^2(\Delta p_{0,a}) + u_{p,m}^2(\Delta p_{0,m})}$$

With, $u_p(\Delta p)$ the uncertainty in envelope pressure, $u_{p,m}(\Delta p)$ the uncertainty in pressure difference measurement due to the equipment, $u_{p,m}(\Delta p_{0,a})$ the uncertainty in pressure difference measurement due to short-term variations in wind-induced pressure, and $u_{p,m}(\Delta p_{0,m})$ the uncertainty in zero-flow pressure measurement due to the equipment.

Then, the uncertainty in air flow exponent (n) and air flow coefficient (C_{env}) must be computed. They depend on the uncertainties in $\ln(\Delta p_i)$ and $\ln(q_i)$, but also on the regression technique used. The choice of regression technique depends on its objective and on the uncertainty in the variables used to perform the regression [22]. Three different regression techniques are described in this section. The equations regarding regression parameters estimation and uncertainty calculation are provided in the Appendix.

In airtightness measurements, the most used method for linear regression is the Ordinary Least Square (OLS) method. OLS aims at minimizing the differences between the regression line and the *y* coordinate of the points used to perform the regression (i.e., the vertical difference between regression line and measurements). This method is expected to provide a reliable calculation of parameter uncertainty when only one variable is subject to constant errors [22]. These assumptions (i.e., only one variable with errors and constant errors) are questioned by several authors [17-19]. The uncertainty in regression parameters with the OLS method can be computed on the basis of two approaches: based on calculated uncertainty in the *y*-variable; or, based on the distance between the regression line and the *y* coordinate of the measurements (see Appendix for calculation details). The uncertainty in *y*-variable is calculated based on assumptions regarding sources of uncertainty in the air flow rate measurement (e.g., error in the equipment, see section 2.2.1). The second approach is often used in practice since it is easy to compute and does not require assumptions from the operator. In this study, OLS-1 refers to a calculation based on sources of error and OLS-2 to a calculation based on the distance between the regression line and the *y*-coordinates of the measurements.

The constant uncertainty assumption can be removed by weighting the measurements that are subject to errors (i.e., $w_i x_i$ or $v_i y_i$ instead of x_i or y_i). These weights depend on the uncertainty in each measurement. Therefore, one can use the Weighted Least Square (WLS) instead of the OLS method to consider non-constant uncertainties. This improved method is not discussed here since it still assumes that the uncertainty in the x variable is negligible. However, the weighting of variables can be used with any method to remove the constant uncertainty constraint.

In 2013, Okuyama and Onishi suggested the use of the Iterative Weighted Least Square (IWLS) method in the context of building airtightness [19]. ISO/TS 28037:2010 developed the mathematics related to this method [24]. Unlike OLS and WLS, the IWLS method allows for non-constant uncertainty in both variables. This method consists in minimizing the sum of distances between the line and the x and y coordinates, when both variables are weighted based on their uncertainty [25]. Since the minimization of this sum has no analytical solution, an iterative approach must be used to solve the problem [26].

The Weighted Line of Organic Correlation (WLOC) method was suggested in 2017 [18]. Similar to IWLS, WLOC allows for non-constant uncertainties in both variables. The method consists in minimizing the product of the distances between the x and y coordinates and the line, when both variables are weighted. Although equations are more complex than IWLS, WLOC does not demand an iterative procedure. Therefore, it requires fewer computing resources and, contrary to the iterative process, it does not present any risk of non-convergence.

Each regression technique provides uncertainty in regression parameters (i.e., $u_{th}(n)$ and $u_{th}(c)$) and the correlation coefficient between both parameters (i.e., $r_{th}(n,c)$). These values are then combined with Equation 4 [20] to obtain the uncertainty in air flow rate at a given pressure in pressurization or depressurization (i.e., $u_{th}(Q_{\Delta p\pm})$). In this equation, T is the recorded temperature (external and internal temperature for depressurization and pressurization, respectively), T_0 is the reference temperature (i.e., 293.15 K) and $u_{th}(T)$ is the uncertainty in temperature measurement. In this equation, c refers to the intercept of the regression line with the y axis and is directly used in the calculations. However, the following section of this paper refers to the air flow coefficient C_{env} (that is defined as e^c) instead of c. This is because c is never mentioned in measurement protocols and is only useful when dealing with uncertainty calculations.

Equation 4

$$u_{th}(Q_{\Delta p\pm}) = \sqrt{\begin{gathered}\left(e^c\,\Delta p^n\left(\frac{T_0}{T}\right)^{1-n}\ln\left(\Delta p\frac{T}{T_0}\right)u_{th}(n)\right)^2+\left(e^c\,\Delta p^n\left(\frac{T_0}{T}\right)^{1-n}u_{th}(c)\right)^2+\\ \left(\frac{e^c(a-1)}{T}\Delta p^n\left(\frac{T_0}{T}\right)^{1-n}u_{th}(T)\right)^2+\,2\left(e^c\,\Delta p^n\left(\frac{T_0}{T}\right)^{1-n}\right)^2\ln\left(\Delta p\frac{T}{T_0}\right)u_{th}(n)\,u_{th}(c)\,r(n,c)\end{gathered}}$$

Lastly, since the total air flow rate at a given pressure is obtained by averaging airflow in pressurization and depressurization at that pressure, uncertainty in $Q_{\Delta p}$ is obtained with Equation 5.

Equation 5

$$u_{th}(Q_{\Delta p}) = \sqrt{\frac{u_{th}^2(Q_{\Delta p-})}{4} + \frac{u_{th}^2(Q_{\Delta p+})}{4}}$$

3. Results

3.1. Observed Uncertainty

3.1.1. Air Flow Rate at 5, 50 and 100 Pa

Table 2 presents the mean values of air flow rate based on 30 tests, at pressure differences of 5, 50 and 100 Pa, in both pressurization and depressurization, and using the OLS, IWLS and WLOC methods.

	OLS (in m³/h)			IWLS (in m³/h)			WLOC (in m³/h)		
	$Q+$	$Q-$	Q_m	$Q+$	$Q-$	Q_m	$Q+$	$Q-$	Q_m
5 Pa	55.0	46.7	50.9	54.8	47.1	51.0	55.8	46.7	51.3
50 Pa	280.0	225.7	252.8	280.5	225.6	253.1	280.3	225.6	252.9
100 Pa	457.3	363.1	410.2	458.7	361.8	410.2	455.9	362.1	409.0

Table 2 – Air flow rate (in m³/h) at 5, 50 and 100 Pa in pressurization ($Q+$), depressurization ($Q-$) and average (Q_m) for OLS, IWLS and WLOC methods.

Since the distribution of these data was not significantly different from normal (no $p_{val} \leq 0.05$ for the Shapiro-Wilk tests), and since the same set of data was used to compare different regression techniques, paired t-tests were used to evaluate the statistical significance of differences in means. The tests showed that, in depressurization, none of the differences were statistically significant. In pressurization, there was a statistically significant difference between OLS and WLOC at 5 Pa ($t = -2.38$, $p_{val} = 0.024$) and 50 Pa ($t = -7.59$, $p_{val} < 0.001$), and between OLS and IWLS at 50 Pa ($t = -3.59$, $p_{val} = 0.001$). In average, there was a statistically significant difference between OLS and IWLS at 50 Pa ($t = -2.82$, $p_{val} = 0.009$) and between OLS and WLOC at 50 Pa ($t = -3.62$, $p_{val} = 0.001$). However, none of these differences was found to have an effect size of substantive magnitude (Cohen's $d < 0.2$) according to Cohen's thresholds [27]. Coherent with previous research from the authors [15], Cohen's d was used as an estimation of the magnitude (effect size) of the differences detected since the standard deviation of different groups were roughly the same (p_{val} from F-tests are lower than 0.05 for all comparisons) [28]. The thresholds proposed by Cohen [29] were used to infer the practical relevance of effects as these benchmarks are appropriate to use in the absence of previous knowledge in the area [15].

3.1.2. Regression Parameters

Table 3 presents the mean values and standard deviations of the regression parameters for different regression techniques in both pressurization and depressurization.

	OLS		IWLS		WLOC	
	Mean	Deviation	Mean	Deviation	Mean	Deviation
n_+	0.71	0.033	0.71	0.021	0.70	0.023
n_-	0.69	0.039	0.68	0.016	0.68	0.017
$C_{env,+}$	17.60	2.58	17.43	1.79	17.92	1.96
$C_{env,-}$	15.48	2.40	15.65	1.02	15.56	1.14

Table 3 – Mean and standard deviation of the regression parameters for the 30 tests in both pressurization and depressurization, using three regression techniques.

Paired t-tests showed that, in pressurization, the mean values of both n and C_{env} were significantly different between OLS and WLOC ($t = 2.48$, $p_{val} = 0.019$ for n and $t = -2.56$, $p_{val} = 0.016$ for C_{env}). In depressurization, no statistically significant differences were detected in the comparisons. To estimate the effect size, Glass's Δ was used instead of Cohen's d since this coefficient can be utilised when groups have different standard deviation but the same sample size [28]. None of the comparisons was found having an effect of substantive magnitude according to Cohen's benchmark (d< 0.2) [27]. Regarding the standard deviation, in depressurization, the difference between OLS and IWLS was found to be statistically significant for both n and C_{env} ($F = 6.04, p_{val} < 0.001$ and $F = 5.77, p_{val} < 0.001$). Significant differences in standard deviation were also detected between OLS and WLOC ($F = 5.05$, $p_{val} < 0.001$ and $F = 4.74$, $p_{val} < 0.001$). In pressurization, the difference between OLS and IWLS was found to be statistically significant for both n and C_{env} ($F = 2.33$, $p_{val} = 0.026$ and $F = 2.13$, $p_{val} = 0.046$), but not for the comparison between OLS and WLOC.

3.1.3. Standard Deviation of Air Flow Rate at a Given Pressure Difference

Information on the standard deviation of regression parameters cannot be used to draw conclusions on the standard deviation of air flow rate because of the strong correlation between n and C_{env} [20]. To obtain these values, the standard deviation of the air flow rate at each pressure difference needs to be estimated (Figure 2).

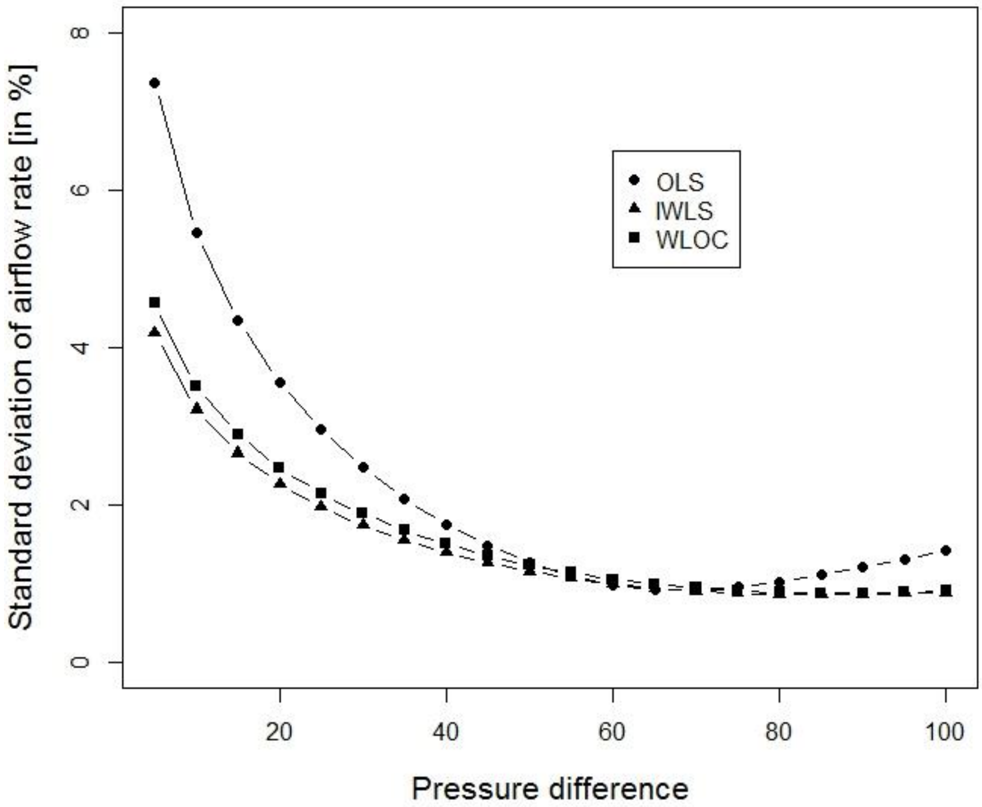

Figure 2 – Standard deviation of air flow rate (in %) at multiple pressure differences for the OLS, IWLS and WLOC methods.

Figure 2 shows that the standard deviation of the air flow rate around the centroid of the pressure measurement (in this case, around 50 Pa) is equivalent irrespective of the regression technique. In addition, the figure shows that using the OLS method leads to a higher standard deviation for air flow rates estimated away from the centroid pressure differences than using IWLS or WLOC.

Table 4 presents the standard deviation of air flow rates at 5, 50 and 100 Pa used to draw the figure. These values show that averaging pressurization and depressurization seems to reduce the standard deviation (i.e., standard deviation of Q_m seems lower than standard deviation of Q_+ and Q_- separately). This was expected since Equation 2 shows that, when making the mean of two quantities with similar uncertainty, the uncertainty of the mean is lower than the uncertainty of individuals. Furthermore, looking at pressurization and depressurization separately shows that the uncertainty in IWLS and WLOC is lower than OLS when air flow rates are estimated away from the centroid pressure difference. However, results differ between modes. This is because standard deviations are expressed in %, and the air flow rates in pressurization and depressurization are different. Moreover, wind conditions cannot be kept identical between pressurization and depressurization tests.

	OLS			IWLS			WLOC		
	Q_+	Q_-	Q_m	Q_+	Q_-	Q_m	Q_+	Q_-	Q_m

5 Pa	9.40%	9.39%	7.37%	6.72%	4.14%	4.18%	7.19%	4.60%	4.56%
50 Pa	2.04%	1.85%	1.27%	1.91%	1.81%	1.16%	2.00%	1.83%	1.23%
100 Pa	1.29%	2.95%	1.42%	1.16%	2.08%	0.89%	1.16%	2.08%	0.90%

Table 4 – Standard deviation of air flow rate (in %) for depressurization, pressurization and average at 5, 50 and 100 Pa for OLS, IWLS and WLOC methods.

3.2. Theoretical Calculation of Uncertainty

3.2.1. Uncertainties in Envelope Pressure Difference ($ln(\Delta p_i)$) and in Air Flow Rate through the Building Envelope ($ln(q_i)$)

Manufacturers often provide values for accuracy and resolution of their measurement equipment. These are usually expressed as maximum permissible error or 95% confidence interval. This study assumed that the information provided by the manufacturer (section 2.1.2) corresponded to the maximum permissible error for both the manometer and the thermometer. In 2013, Delmotte proposed equations to determine the uncertainty in $\mathrm{x_i} = \ln(\Delta p_i)$ and $y_i = \ln(q_i)$ based on the uncertainty in pressure difference, air flow rate, and temperature measurements [20]. Another study by Prignon et al. suggested that the uncertainty due to zero-flow pressure approximation is a constant value (the same for all the couples measured during a test) that depends on the standard deviation and the duration of the zero-flow pressure measurement (Table 5) [17]. These values were considered in the calculations performed in this study.

	30 s	60 s	90 s	120 s
$\sigma(\Delta p_0) < 1$	0.45 Pa	0.44 Pa	0.43 Pa	0.42 Pa
$1 < \sigma(\Delta p_0) < 2$	0.91 Pa	0.81 Pa	0.80 Pa	0.80 Pa
$\sigma(\Delta p_0) > 2$	1.52 Pa	1.46 Pa	1.39 Pa	1.39 Pa

Table 5 – Uncertainty in zero-flow pressure approximation ($u(\Delta p_{0,a})$) as a function of the duration and the standard deviation ($\sigma(\Delta p_0)$) of zero-flow pressure measurements [17].

3.2.2. Error Propagation

The equations presented in the Appendix were used on each test to compute the uncertainty in regression parameters and in air flow rate at a given pressure. Table 6 presents the mean values of regression parameters and the mean of their calculated uncertainty for both pressurization and depressurization. These results show that the calculated uncertainties in regression parameters are similar between IWLS and WLOC, and are higher than the OLS-2 and OLS-1 methods.

	OLS-1		OLS-2		IWLS		WLOC	
	Val	Unc	Val	Unc	Val	Unc	Val	Unc

n_+	0.71	0.007	0.71	0.011	0.71	0.012	0.70	0.013
n_-	0.69	0.006	0.69	0.010	0.68	0.012	0.68	0.013
$C_{env,+}$	17.60	0.44	17.60	0.60	17.43	0.85	17.92	0.91
$C_{env,-}$	15.48	0.37	15.48	0.55	15.65	0.79	15.56	0.84

Table 6 – Value and calculated uncertainty of both regression parameters for the theoretical case in both pressurization and depressurization for the four cases (OLS-1, OLS-2, IWLS and WLOC).

Due to the high correlation between regression parameters [20], it is important to draw the conclusions based on the uncertainty of the air flow rate. Figure 3 presents the mean calculated uncertainty in airflow rate at multiple pressure differences, for different regression techniques.

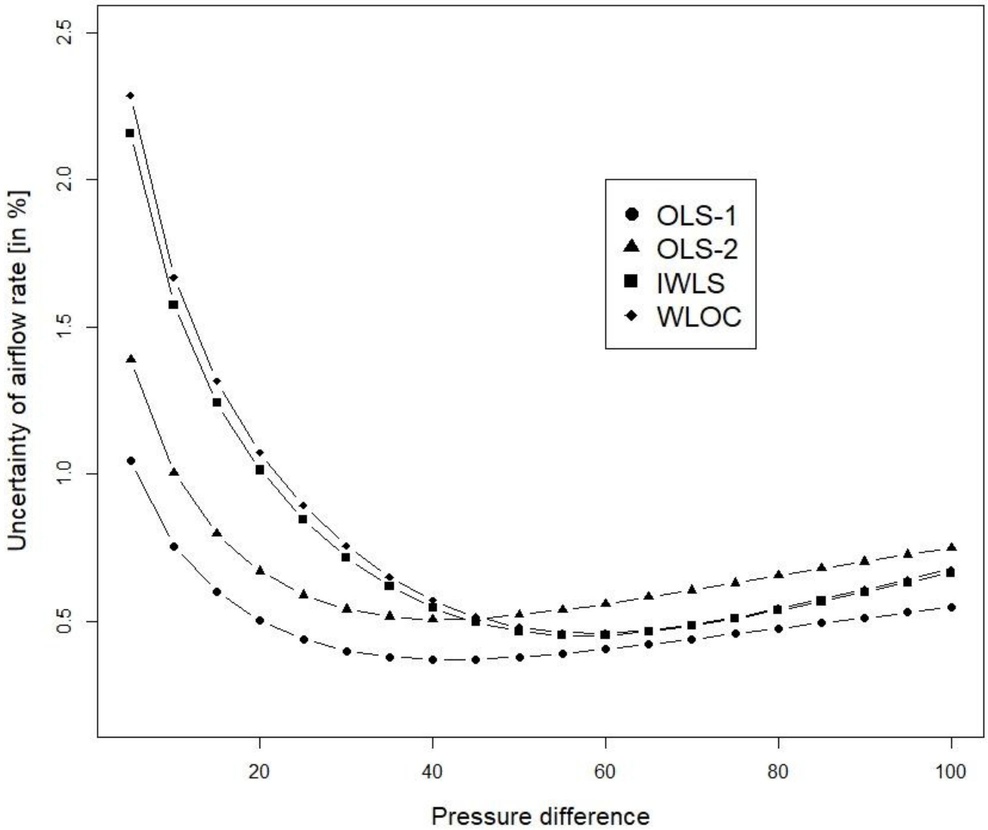

Figure 3 – Calculated uncertainty of air flow rate (in %) at multiple pressure differences for OLS-1, OLS-2, IWLS and WLOC methods.

The figure shows that the IWLS and WLOC methods provide close results in terms of uncertainty calculation and are higher than the OLS-1 and OLS-2 methods. Paired t-tests show that all the comparisons (i.e., OLS-1 vs OLS-2, OLS-1 vs IWLS, OLS-1 vs. WLOC, OLS-2 vs. IWLS, OLS-2 vs. WLOC and IWLS vs WLOC) are highly statistically significant. However, when comparing IWLS and WLOC, all the computed Glass's Δ were below 0.2, meaning an effect of non-substantive magnitude according to Cohen's benchmarks. In addition, the graph presents a gap between the OLS-1 and OLS-2 lines, this highlighting an issue either in the suitability of the OLS method or in the assumptions about uncertainties in $\ln(q_e)$. This observation is confirmed by measuring the effect size since all Glass's Δ values are higher than 3.5 (large effect for values higher than 0.8 according to Cohen's benchmarks).

3.3. *<u>Comparison between In-situ Observation and Theoretical Calculation of Uncertainty</u>*

3.3.1. Comparison of Regression Parameters

Figure 4 and Figure 5 show, for n and C_{env} respectively, the comparison between the calculated (solid lines) and observed (dotted lines) uncertainty in pressurization (grey zones) and depressurization (white zones) for each regression technique. Calculated uncertainties are mean averages of uncertainties calculated for 30 tests, while observed uncertainties are the standard deviation of the regression parameters obtained for repeated tests.

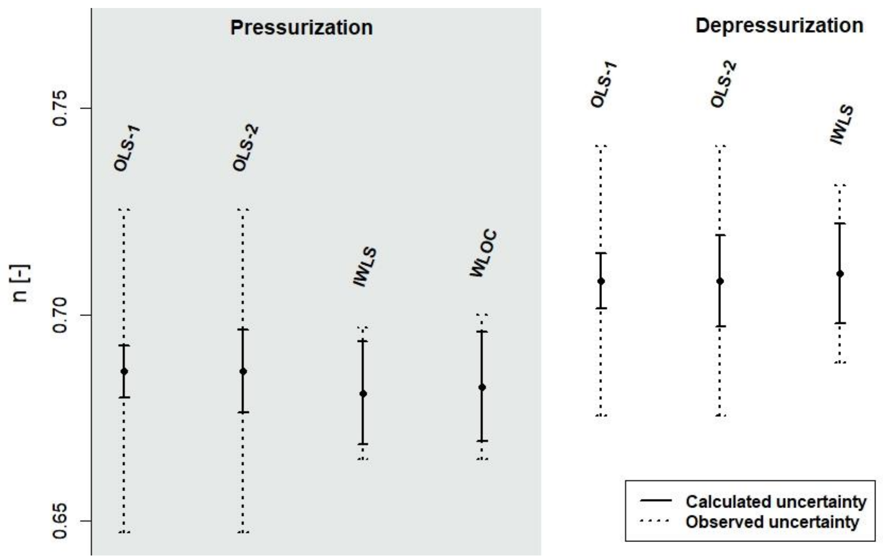

Figure 4 – Comparison between calculated (solid lines) and observed (dotted lines) uncertainty in pressurization and depressurization for n values.

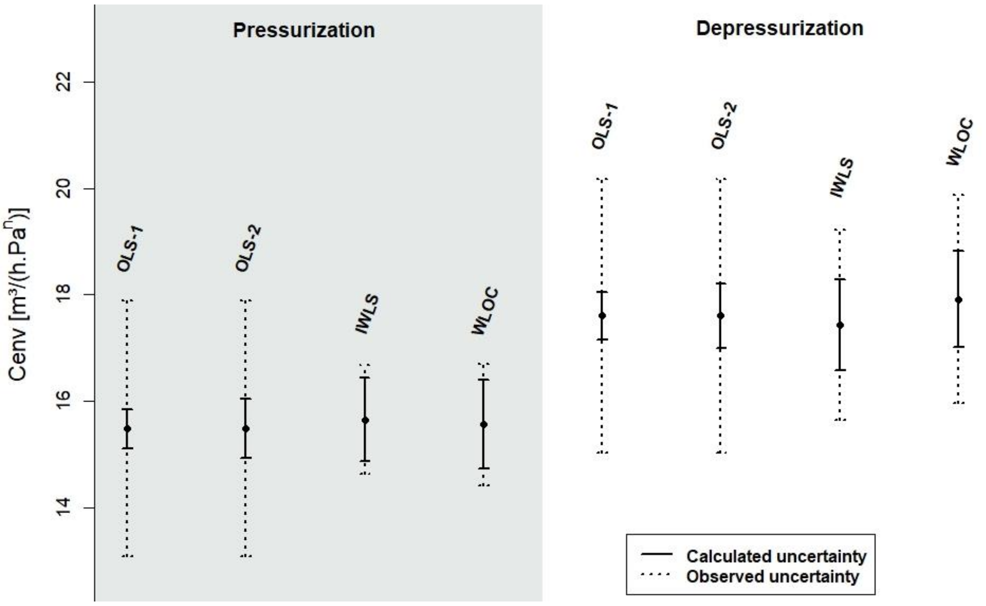

Figure 5 – Comparison between calculated (solid lines) and observed (dotted lines) uncertainty in pressurization and depressurization for C_{env} values.

The comparison shows that the calculated uncertainties with the IWLS and WLOC methods fit better the observed uncertainty than when using the OLS method, whatever the mode (i.e., pressurization and depressurization) and the regression parameter. In addition, it shows that, even in the best case, there is still a gap between calculated and measured uncertainty, meaning that assumptions on uncertainty are not yet representing the reality.

3.3.2. Comparison of Air Flow Rate at Multiple Pressure Differences

Figure 6 shows the uncertainty observed with OLS, IWLS and WLOC, and the uncertainties calculated with the OLS-1, OLS-2, IWLS and WLOC methods.

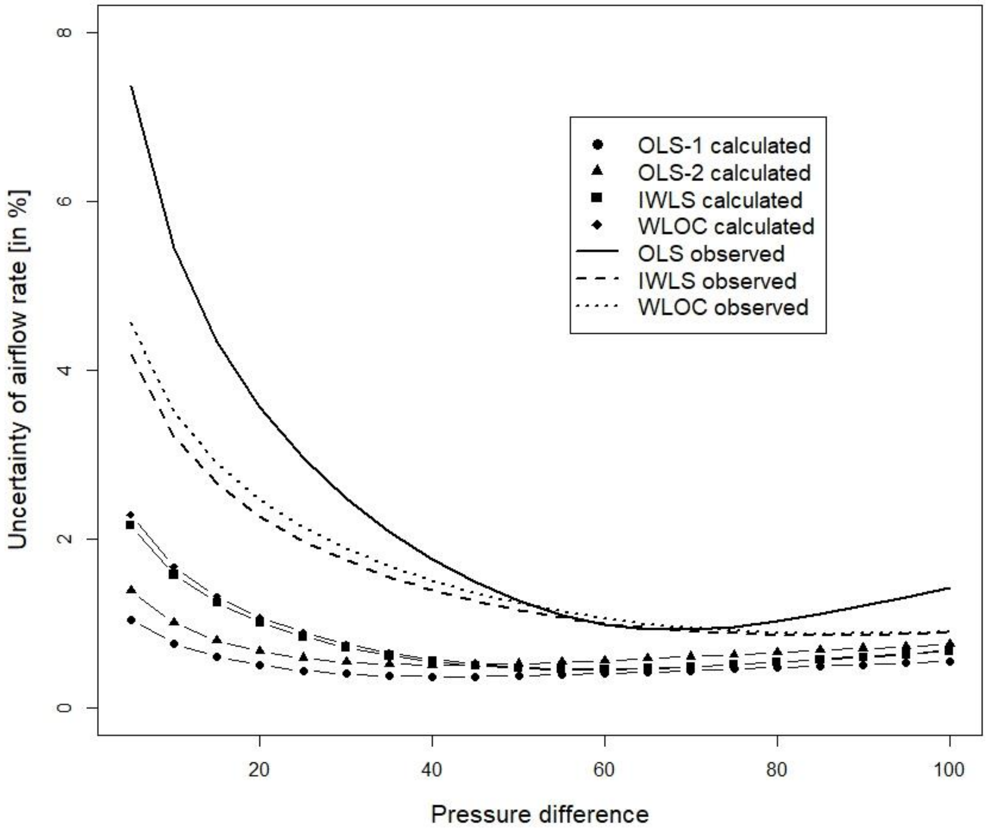

Figure 6 – Observed and calculated uncertainties in airflow rate as a function of pressure difference for different regression techniques.

This comparison shows that, regardless of the regression technique utilised, the calculated uncertainty always underestimates the observed uncertainty, especially away from the centroid. In fact, the differences between calculated and observed uncertainties at 5, 50 and 100 Pa are 6.3%, 0.7% and 0.9% for OLS-1, and 6.0%, 0.6% and 0.7% for OLS-2. These differences are lower when looking at IWLS and WLOC: at 5, 50 and 100 Pa these are, respectively, 2.0%, 0.7% and 0.2% for IWLS, and 2.3%, 0.7% and 0.2% for WLOC. Table 7 provides the example of a virtual case ($C = 20$ [m³/(h.Pa^n)], $n = 0.67$ [-]) in order to visualize the relevance of these results in the determination of airflow values.

	at 5 Pa	at 50 Pa	at 100 Pa
Observed OLS	59 ± 4.33	275 ± 3.49	438 ± 6.21
Calculated OLS-1	59 ± 0.63	275 ± 1.57	438 ± 2.28
Calculated OLS-2	59 ± 0.81	275 ± 1.84	438 ± 3.15
Observed IWLS	59 ± 2.46	275 ± 3.19	438 ± 3.89
Calculated IWLS	59 ± 1.28	275 ± 1.27	438 ± 3.02
Observed WLOC	59 ± 2.68	275 ± 3.38	438 ± 3.94
Calculated WLOC	59 ± 1.33	275 ± 1.46	438 ± 3.06

Table 7 – Visualization of the results presented in Figure 6 applied to a virtual case ($C = 20$ and $n = 0.67$).

The better fit between observed and calculated uncertainty using the IWLS and WLOC methods was expected since these methods allow to consider uncertainty in pressure difference, including the uncertainty due to the zero-flow pressure hypothesis that is known to have an important impact on the reliability of fan pressurization measurements.

Figure 7 presents, for each test, the air flow rate at 5 (a, c and e) and 50 (b, d and f) Pa and their calculated expanded uncertainty for the OLS (a and b), IWLS (c and d) and WLOC (e and f) methods. These graphs allow to properly visualise the impact of changing the regression technique on the variability of the results and the estimation of their uncertainty. On the graphs: the crosses represent the airflows determined for each test; the error bars provide the calculated expanded uncertainty; and, the coloured zones show the observed expanded uncertainty applied on the average value of the 30 estimated airflows. The expanded uncertainty is obtained by multiplying the uncertainty by a coverage factor (k). This defines a confidence interval ($y - k.u(y)\,;y + k.u(y)$), which may be expected to include large part (depending on the coverage factor) of the distribution of values that could be attributed to the measured quantity. The coverage factor used in Figure 7 is 2, and it corresponds to a confidence interval of 95% [9]. If calculated and observed uncertainty are close, the size of the observed (i.e., coloured zone) and calculated (i.e., error bars) expanded uncertainty should be of the same order of magnitude.

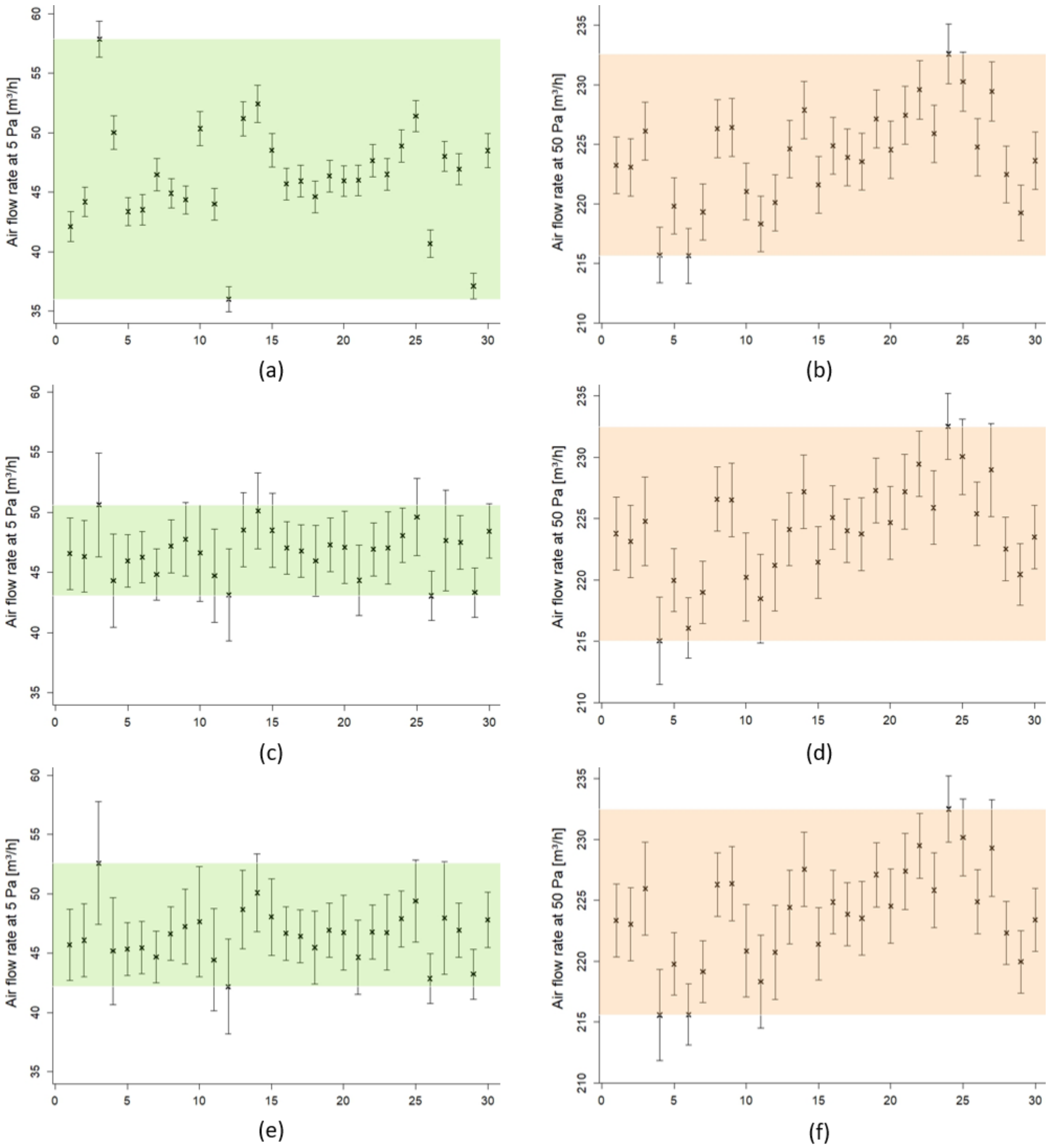

Figure 7 – Values, for each test, of the air flow rate at 5 (a, c and e) and 50 Pa (b, d and f) and calculated expanded uncertainty (k=2) for OLS (a and b), IWLS (c and d) and WLOC (e and f) methods.

Figure 7 confirms previous findings by illustrating that, at 50 Pa, the results of the three methods are equivalent and all underestimate the real uncertainty. For most tests, the 95% confidence interval calculated for each test is far from including 95% of the test results as it should for a robust estimation of the uncertainty. However, at 5 Pa there are strong differences between the OLS, IWLS and WLOC methods. In fact, real confidence intervals (coloured zones) are much larger when using OLS with respect to the IWLS and WLOC methods. In addition, the 95% confidence intervals (error bars) calculated with IWLS and WLOC include much more test results than when computed with the OLS method. Consequently, the IWLS and WLOC methods provide a much better estimation of the uncertainty than OLS at low pressure difference. However, even with IWLS and WLOC calculations at 5 Pa, the calculated expanded uncertainty is still underestimating the observed 95% confidence interval.

4. Discussion

The limitations of the OLS regression technique in the context of airtightness measurements have long been recognised in the literature [15]. Research on other regression methods has intensified over the last years [10, 19, 20], assuming further relevance in light of recent studies that quantified the uncertainty in pressure difference due to the zero-flow pressure approximation [16, 17]. Consistent with these scientific developments, this paper sought to quantify the impact of changing regression technique when including the uncertainty due to the zero-flow pressure hypothesis in the calculations.

The first part of the results (Section 3.1 – observed uncertainty) showed that changing the regression method has no effect on the mean average value of the air flow rate, whatever the pressure difference. However, it has an impact on the standard deviation of the air flow rate when the pressure difference is away from the centroid of the measurements. Also, the regression method has a statistically significant impact on the standard deviation of regression parameters. In fact, using the IWLS or WLOC methods rather than OLS leads to lower standard deviation in regression parameters and air flow rates at pressure differences away from the centroid of the measurements. This suggests that the OLS regression technique could still be used when looking at the airflow rate at 50 Pa, but only if the pressure sequence is chosen appropriately (i.e., having 50 Pa as the centroid of the measurements). In all other cases, other methods are likely to provide outcomes that are more robust. On these basis, the first part of the results presented in this paper can have some important implications in practice, especially in countries where airflow at low- or high-pressure difference is used in the regulations [30].

The second part of the results (Section 3.2 – calculated uncertainty) provided evidence that the uncertainty in $\ln(\Delta p_e)$ should not be neglected, especially on windy days where the uncertainty due to the zero-flow pressure hypothesis is important. This strengthens the view, shared by many authors, that the OLS method is not appropriate for determining the uncertainty in airtightness estimation [15, 18-20]. Indeed, by neglecting the uncertainty in pressure difference, the OLS method disregards an important part of uncertainty due to variations in wind speed, which are known to have a relevant impact on uncertainty in airtightness estimation [15].

The third part of the results (Section 3.3 – uncertainty comparison) showed that, in terms of regression parameters and air flow rate, the IWLS and WLOC methods provide a more robust estimation of uncertainty than OLS-1 and OLS-2. However, although the uncertainty calculated with the IWLS and WLOC methods fits better the ‘real’ uncertainty, they are still underestimating its ‘true’ value.

On the basis of these findings, the authors suggest that the IWLS and WLOC methods should be preferred over the OLS method since they can offer more reliable results, except when looking at values of air flow rate around the centroid of pressure measurements. This can have an important impact in practice since most simulation software use the OLS-2 method to obtain values for regression parameters and for an estimation of their uncertainty from measurements. In addition, the WLOC method should be preferred over IWLS in order to avoid an iterative process, hence enabling a more robust computation procedure (i.e., no risk of non-convergence) and requiring fewer computing resources.

Before generalizing the results obtained in this research, some limitations should be acknowledged. First, the data analysed in this paper were obtained from only one apartment that cannot represent the entire building stock. Since uncertainty components vary from one building to another, a change of configuration could also alter the results of the analysis. Even if trends in the findings are expected to remain the same, their magnitude could be affected. Also, this study identified only part of the sources of error that could be encountered during a fan pressurization test. Other sources that can lead to uncertainty (e.g., the position of inside and outside pressure taps) were not considered in this study.

While this paper has discussed the impact of regression techniques on precision errors, it is important to notice that such techniques do not reduce bias error. Since it is not possible to calibrate on-site airtightness tests, bias is supposed to be limited by respecting the requirements of measurement standards (e.g. ISO 8872:2015 [21]). This issue is still open for more research. Also, since the flow exponent is not constant, and depends on the pressure difference [15, 31], it can be expected that giving more weight to high pressure measurements (IWLS and WLOC regression techniques) may introduce bias error. This issue is linked to the very basic choice of the flow modelling and it was not investigated in this study.

Lastly, in this study authors have investigated two alternative regression methods (i.e., IWLS and WLOC) related to fan pressurization tests that have been already studied in the literature and that are consistent with the procedures required by current standards [21]. However, other methods could be explored to calculate and/or reduce the uncertainty resulting from fan pressurization measurements. Among these are the Weighted Least Normal Square (WLNS) method, and the application of Bayesian statistics. The WLNS method consists in minimizing the normal distance (i.e., the perpendicular line) between the measurements and the regression line [22]. Conversely, the use of Bayesian statistics involves a change of approach in the determination of measurement uncertainties. Conventional *frequentist* inference uses methods such as maximum likelihood estimation (MLE) to produce parameter estimates and standard errors, and test the statistical significance of parameters (e.g., null-hypothesis testing) or set a confidence interval around an estimate [32]. Under this approach, model parameters are regarded as fixed quantities and their values are estimated from random variables (i.e., data). Conversely, in *Bayesian* inference, the data are treated as fixed and the model parameters are considered as random. Bayesian methods consider each unknown parameter to have an associated *posterior probability distribution* that describes the uncertainty about population parameter values. The target of the analysis consists in estimating the posterior distribution of parameters as the normalised product of a *prior distribution*, reflecting previous knowledge, and the *likelihood* provided by the data. This allows assigning probabilities to parameters, making the inferential framework more intuitive [33]. The Bayesian approach is thus markedly different from classical statistics, where the population parameter has one (unknown) value. Bayesian statistics considers a probability distribution of possible values for each unknown parameter and the uncertainty is expressed by assigning to it a prior probability distribution of possible values that is independent from the data. Once data have been collected, the prior is combined with the likelihood to produce a posterior distribution that, typically, has smaller variance than the prior, since observing the data reduced the uncertainty on the possible population values [34]. In the context of fan pressurization measurements, for example, Bayesian statistics could be used to determine probability distributions, instead of single values, for flow exponent and leakage coefficients.

5. Conclusion

This study investigated how a change in regression technique can affect the estimation of building airtightness and its uncertainty. Analysis of the measured uncertainty, the calculated uncertainty, and their comparison led to the following main findings:

- The IWLS, WLOC and OLS methods provide the same mean average value for the air flow rate, whatever the pressure difference. The three methods result in the same standard deviation (i.e., observed uncertainty) for the air flow rate at pressure differences close to the centroid of the measurement (in this case, 50 Pa).
- Compared to OLS, the IWLS and WLOC methods reduce the standard deviation of n in pressurization (from 0.033 to 0.021 for IWLS, and to 0.023 for WLOC) and in depressurization (from 0.039 to 0.016 for IWLS, and to 0.017 for WLOC). They also reduce the standard deviation of C_{env} in both pressurization (from 0.143 to 0.098 for IWLS, and to 0.106 for WLOC) and depressurization (from 0.158 to 0.066 for IWLS, and to 0.072 for WLOC).
- The IWLS and WLOC methods lead to lower uncertainty than OLS for the air flow rate at a pressure away from the centroid of the measurements (4.18% and 4.56% instead of 7.37% at 5 Pa, and 0.89% and 0.90% instead of 1.42% at 100 Pa).
- Theoretical calculations using IWLS and WLOC methods provide a better estimate of the ‘reference uncertainty’ (i.e., the observed uncertainty) than using the OLS method. In this study, the differences between calculated and observed uncertainty of air flow rate estimated at 5, 50

and 100 Pa, were: 6.3%, 0.7% and 0.9% for OLS-1; 6.0%, 0.6% and 0.7% for OLS-2; 2.0%, 0.7% and 0.2% for IWLS; and 2.3%, 0.7% and 0.2% for WLOC.

Based on these findings, the authors suggest that IWLS or WLOC should be preferred over the most used OLS method. However, some constraints were identified in this study, highlighting relevant further work needed in the field of uncertainty in fan pressurization measurements. In fact, to generalize the findings, similar studies should be performed on buildings that experience different wind exposures and air flow rates of different magnitude. Then, other sources of errors (e.g., the position of inside and outside pressure taps) should also be investigated both in theory (calculations) and in practice (repeated tests) due to the gap that is still detected between observed and calculated uncertainty. This would allow to define a more complete and relevant model to deepen our fundamental understanding of uncertainty in fan pressurization tests. Further issues (i.e., bias in leakage exponent and in airflow at low pressure due to a change in regression technique) and methods for parameter estimation (i.e., WLNS, Bayesian statistics, etc.), not included in this study, could be relevant for future research.

Acknowledgment

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Appendix – Determination of Slope (a) and Intercept (b) for Different Regression Techniques

This Appendix provides the equations used to determine the regression parameters (i.e., slope a and intercept b), the uncertainty of these parameters (i.e., $u(a)$, $u(b)$), and their correlation coefficient (i.e., $r(a,b)$) for the three methods studied in this paper. In the interest of simplification, $\sum x_i$ is used for $\sum_{i=1}^{N} x_i$. When relevant (i.e., for methods considering uncertainty in both variables), the calculations were developed assuming that uncertainties in both variables were uncorrelated. This hypothesis is acceptable since uncertainty in $\ln(q)$ is almost constant. Table 8 summarizes the symbols used in the equations of this Appendix and their definition.

Symbol	**Definition**
a and b	Parameters of interest, respectively the slope and the intercept.
$u(a)$ and $u(b)$	Uncertainties related to each parameter.
$r(a,b)$	Correlation coefficient between both parameters.
$(x_i; y_i)$	x and y coordinates for the i_{th} measurement. For a fan pressurization test, these correspond to $\ln(\Delta p_i)$ and $\ln(q_i)$ respectively.
N	Number of measurement used when performing the linear regression.
s	Standard error of the regression.
$w(x_i)$ and $w(y_i)$	Weights applied to the x and y values of the i_{th} measurement. These depends on the source of errors considered.
W_i	General weight applied on each measurement considering that both coordinates are subject to uncertainties.
U_i and V_i	Distance between the coordinate for the i_{th} measurement and the average of all measurements, respectively for x- and y- coordinates.
β	This parameter has no specific definition; York et al. used it to simplify equations [25].
v_i and w_i	Weights applied to the x and y values of the i_{th} measurement. These depends on the source of errors considered.
Δx and Δy	These parameters have no specific definitions; Delmotte used them to simplify equations [18].
$u(a+b)$	Uncertainty of the sum of parameters of interest.

Table 8 – Symbols used in equations for OLS, IWLS and WLOC, and their definition

A1 – Ordinary Least Square Method

The OLS method consists in finding the best straight line that minimizes the differences between the regression line and the y-coordinate of the points used to perform the regression (i.e., the vertical difference between regression line and measurements). In order to be valid, the OLS formulas require uncertainties on the x and y coordinates to be respectively negligible and constant. When performing the OLS method, the regression parameters are given by Equation 6 and Equation 7.

Equation 6

$$a = \frac{N \sum x_i y_i - \sum x_i \sum y_i}{N \sum x_i^2 - (\sum x_i)^2}$$

Equation 7

$$b = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{N \sum x_i^2 - (\sum x_i)^2}$$

With this method, uncertainty in regression parameters a and b is theoretically computed with Equation 8 and Equation 9 respectively, where N is the number of points used for the regression, x_i is the x-axis value for each point (i.e. $\ln(\Delta p_i)$), and s^2 is the variance. It can be computed in two ways. Either it is equal to 1 over the square of the uncertainty in the y-axis (i.e., $1/u^2(\ln(q_i))$, or it is computed as the sum of the square of the distance between the regression line and the y coordinate of the points. The correlation coefficient is given by Equation 10.

Equation 8

$$u(a) = \sqrt{\frac{N\, s^2}{N \sum {x_i}^2 - (\sum x_i)^2}}$$

Equation 9

$$u(b) = \sqrt{\frac{s^2\, \sum x_i^2}{N \sum {x_i}^2 - (\sum x_i)^2}}$$

Equation 10

$$r(a,b) = \frac{-\sum x_i}{\sqrt{N\, \sum x_i^2}}$$

<u>A2 – Iterative Weighted Least Square Method</u>

The IWLS method consists in finding the best straight line that minimizes the sum of the distances between the line and the x and y coordinates, when both variables are weighted based on their uncertainty. Since the minimization of this sum has no analytical solution, an iterative approach must be used to solve the problem. Equations in this section are taken from the work of York et al. (2004) that developed the unified equations for the iterative weighted least square regression method [25]. When performing the IWLS method, the weights are computed first with Equation 11 and Equation 12.

Equation 11

$$w(x_i) = \frac{1}{u^2(x_i)}$$

Equation 12

$$w(y_i) = \frac{1}{u^2(y_i)}$$

Then a value for the slope (a) is assumed, and Equation 13 to Equation 19 are repeated until the change in slope between two iterations corresponds to the desired accuracy.

Equation 13

$$W_i = \frac{w(x_i)w(y_i)}{w(x_i)+a^2 w(y_i)}$$

Equation 14

$$\bar{x} = \frac{\sum_{i=1}^{N} W_i x_i}{\sum_{i=1}^{N} W_i}$$

Equation 15

$$\bar{y} = \frac{\sum_{i=1}^{N} W_i y_i}{\sum_{i=1}^{N} W_i}$$

Equation 16

$$U_i = x_i - \bar{x}$$

Equation 17

$$V_i = y_i - \bar{y}$$

Equation 18

$$\beta_i = W_i \left[\frac{U_i}{w(y_i)} + \frac{aV_i}{w(x_i)}\right]$$

Equation 19

$$a = \frac{\sum_{i=1}^{N} W_i \beta_i V_i}{\sum_{i=1}^{N} W_i \beta_i U_i}$$

Lastly, once the desired accuracy is reached, the second regression parameter is estimated using Equation 20.

Equation 20

$$b = \bar{y} - a\bar{x}$$

Using this method, uncertainties in regression parameters are computed after the last iteration with Equation 21 and Equation 22, and the correlation coefficient is given by Equation 23.

Equation 21

$$u(a) = \sqrt{\frac{1}{\sum W_i (\beta_i - \bar{\beta})^2}}$$

Equation 22

$$u(b) = \sqrt{\frac{1}{\sum W_i} + \left(\bar{x} + \bar{\beta}\right)^2 u^2(a)}$$

Equation 23

$$r(a,b) = -\frac{\frac{\sum W_i x_i}{\sqrt{\sum W_i}}}{\sqrt{\sum W_i x_i^2}}$$

<u>A3 – Weighted Line of Organic Correlation</u>

The WLOC method consists in finding the best straight line that minimizes the product of the distances between the x and y coordinates and the line, when both variables are weighted. Although equations are more complex than with IWLS, WLOC does not demand an iterative procedure. Equations in this section come from the work of Delmotte [18]. Similar to IWLS, the first steps in the Weighted Line of Organic Correlation (WLOC) method consists in computing weights for both variables with Equation 24 and Equation 25.

Equation 24

$$v_i = \frac{1}{u(x_i)}$$

Equation 25

$$w_i = \frac{1}{u(y_i)}$$

Then, the weights are directly used to compute both regression parameters with Equation 26 and Equation 27.

Equation 26

$$a = \frac{\sqrt{\sum v_i w_i \sum v_i w_i y_i^2 - \left(\sum v_i w_i y_i\right)^2}}{\sqrt{\sum v_i w_i \sum v_i w_i x_i^2 - \left(\sum v_i w_i x_i\right)^2}}$$

Equation 27

$$b = \frac{\sum v_i w_i y_i - a \sum v_i w_i x_i}{\sum v_i w_i}$$

Uncertainty in the slope is given by Equation 28, Equation 29 and Equation 30.

Equation 28

$$u(a) = \sqrt{\sum \left(\frac{\partial a}{\partial x_i}\right)^2 \frac{1}{v_i^2} + \left(\frac{\partial a}{\partial y_i}\right)^2 \frac{1}{w_i^2}}$$

Equation 29

$$\frac{\partial a}{\partial x_i} = \sqrt{\frac{\Delta y}{\Delta x}}\, \frac{\left(\sum v_j w_j x_j\right) v_i w_i - \left(\sum v_j w_j\right) v_i w_i x_i}{\Delta x}$$

Equation 30

$$\frac{\partial a}{\partial y_i} = \sqrt{\frac{\Delta y}{\Delta x}}\, \frac{\left(\sum v_j w_j\right) v_i w_i y_i - \left(\sum v_j w_j y_j\right) v_i w_i}{\Delta y}$$

Uncertainty in the intercept is given by Equation 31, Equation 32 and Equation 33.

Equation 31

$$u(b) = \sqrt{\sum \left(\frac{\partial b}{\partial x_i}\right)^2 \frac{1}{v_i^2} + \left(\frac{\partial b}{\partial y_i}\right)^2 \frac{1}{w_i^2}}$$

Equation 32

$$\frac{\partial b}{\partial x_i} = a \left(\frac{-v_i w_i}{\sum v_j w_j} - \frac{\left(\sum v_j w_j x_j\right)^2 v_i w_i}{\Delta x \sum v_j w_j} + \frac{\left(\sum v_j w_j x_j\right) v_i w_i x_i}{\Delta x} \right)$$

Equation 33

$$\frac{\partial b}{\partial y_i} = \frac{v_i w_i}{\sum v_j w_j} - \frac{\left(\sum v_j w_j x_j\right) v_i w_i y_i}{\sqrt{\Delta x}\sqrt{\Delta y}} + \frac{\left(\sum v_j w_j x_j\right)\left(\sum v_j w_j y_j\right) v_i w_i}{\sqrt{\Delta x}\sqrt{\Delta y} \sum v_j w_j}$$

In all these equations, Δx and Δy are given by Equation 34 and Equation 35 respectively.

Equation 34

$$\Delta x = \sum v_i w_i \sum v_i w_i x_i^2 - \left(\sum v_i w_i x_i\right)^2$$

Equation 35

$$\Delta y = \sum v_i w_i \sum v_i w_i y_i^2 - \left(\sum v_i w_i y_i\right)^2$$

Lastly, the correlation coefficient is given by Equation 36 where $u^2(a+b)$ is given by Equation 37.

Equation 36

$$r(a, b) = \frac{u^2(a+b) - u^2(a) - u^2(b)}{2\, u(a)\, u(b)}$$

Equation 37

$$u^2(a+b) = \sum \left(\frac{\partial a}{\partial x_i} + \frac{\partial b}{\partial x_i}\right)^2 \frac{1}{v_i^2} + \left(\frac{\partial a}{\partial y_i} + \frac{\partial b}{\partial y_i}\right)^2 \frac{1}{w_i^2}$$

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