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A NOTE ON THE DISCRETE SPECIFICATION OF THE STATE-ADJUSTMENT MODEL

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A Note on the Discrete Specification of the State-Adjustment Model*

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This note investigates the various ways of specifying the state adjustment model in discrete time. It is shown that the value of the structural parameters depends on the discrete approximation although the estimating form remains unchanged. Consequently, it is important that analysis involving those parameters be carried out using estimates obtained in a mutually consistent specification of the model ; failures to do so may result in erroneous conclusions.

1. *The state-adjustment model.*

The state adjustment model consists in one behavioral equation :

$$q(t) = \alpha + \beta s(t) + \gamma y(t), \tag{1}$$

and one state equation :

$$\dot{s}(t) = (1 - \mu)q(t) - \nu s(t). \tag{2}$$

The basic assumption - equation (1) - is that current demand for a commodity, $q(t)$, depends on its stock (state), $s(t)$, and current income (total expenditure), $y(t)$, other variables such as prices being omitted for simplicity. The state can be interpreted as the psychological stock of habits, mainly for non-durables, or the physical stock of durables. For the latter β is expected to be negative while in case of habit formation β has to be positive.

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The evolution of the state is determined by (2) where ν is a constant depreciation rate. The factor $(1-\mu)$ is rationalized on the grounds that, on the one hand, durables are depreciated once bought[1] and, on the other hand, that only a proportion of new flows may enter the stock of habits.

As pointed out by Winder (1971) and Weiserbs (1972), the system (1)-(2) can be presented in the form of the standard stock-adjustment model :

$$\dot{s}(t) = \phi[\tilde{s}(t) - s(t)], \tag{3}$$

the tilda denoting a long-run value. The long-run equilibrium is defined by the constancy of all variables, so that $\dot{s} = 0$ and

$$\tilde{q}(t) = \frac{\nu}{1-\mu}\ \tilde{s}(t).$$

It then follwos that

$$\tilde{s}(t) = \frac{(1-\mu)\alpha}{\nu-\beta(1-\mu)} + \frac{(1-\mu)\gamma}{\nu-\beta(1-\mu)}\ y(t), \tag{4}$$

which substituted in (3) yields

$$\phi = \nu - \beta(1-\mu). \tag{5}$$

From (1) and (4), one can easily deduce that the short-run and the long-run derivatives with respect to income are respectively γ and

$$\tilde{\gamma} = \frac{\gamma\nu}{\phi}\ . \tag{6}$$

The stock-adjustment model has been introduced in the literature to explain demand for durables[2]. The presentation in terms of a state-adjustment,

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[1] The reader may observe μ in trying to trade in a car the same day he bought it.

[2] See Phlips (1974) for a clear survey of the pioneering works of Chow (1960) and Stone and Rowe (1960).

allowing its application to non-durables, ignoring μ, is due to Houthakker and Taylor (1966) [in short H-T].

2. *The discrete specification.*

For empirical purposes, one has to express the system (1)-(2) in discrete time.

As for the behavioral equation, Chow (1960), Stone and Rowe (1960)[3], Mattei (1971) and most models of investment assume that expenditures for a period t result from a unique decision taken at the beginning of the period given the stock held at the end of period t-1 :

(B1) $$q_t = \alpha + \beta s_{t-1} + \gamma y_t.$$

On the other hand, H-T use a linear approximation of (1) which was shown by Winder (1971) to amount to writing (1) as

(B2) $$q_t = \alpha + \beta \bar{s}_t + \gamma y_t$$

where $\bar{s}_t = \frac{1}{2}(s_t + s_{t-1})$.

Turning now to the state equation, the common specification in investment model (and also in Chow) does not allow current purchases to depreciate :

(S1) $$\Delta s_t = q_t - \nu s_{t-1},$$

where $\Delta s_t = s_t - s_{t-1}$.

In Stone's and Rowe's (1960) model (in short S-R), depreciation is supposed to affect not only the existing stock at the beginning of the period but also purchases made during the period :

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[3] In Chow and Stone and Rowe, (1) is specified in the long-run only, to define $\tilde{s}$, but their estimating equation implies (B1), not (B2).

(S2) $\Delta s_t = (1-\mu)q_t - \nu s_{t-1}.$

The trouble with (S2) is that one parameter among μ, ν and β is underdetermined. To overcome this difficulty, Mattei (1971) assumed that purchases are spread out throughout the period so that, with a linear depreciation within the period, $\mu = 1/2\ \nu$. Then (S2) becomes

(S3) $\Delta s_t = (1-\frac{\nu}{2})q_t - \nu s_{t-1}.$

However, this restriction on μ is unnecessary, at least at this stage of the analysis. The parameters of interest are ϕ, γ and the depreciation rate. Since the estimating form involves only these (and α), a restriction on μ is supererogatory[4].

H-T also use a linear approximation to write (2) in discrete time. However they concentrate on the behavior of the stock within the period :

(S4) $\Delta s_t = q_t - \nu \bar{s}_t$.

If the H-T's approximation is used on (2) not ignoring μ, one obtains

(S5) $\Delta s_t = (1-\mu)q_t - \nu \bar{s}_t$

It is interesting to note that when μ is set equal to $1/2\ \nu$ as in (S2), i.e :

(S6) $\Delta s_t = (1-\frac{\nu}{2})q_t - \nu \bar{s}_t,$

new flows and the stock held at the beginning of the period are depreciated at the same rate at the end of the period (see table 1).

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[4]From the estimates of ν and ϕ, one can deduce the value of $\beta(1-\mu)$ and $\tilde{\gamma}$ (cfr. table 2). The problem of estimating separately β and μ may be solved by the estimating procedure if a disturbance term is introduced in, and only in, the state equation [see Garcia dos Santos (1972)].

Obviously, (S1) and (S3) are special cases of (S2) and similarly for (S4) and (S6) with respect to (S5).

3. *The effects on the structural parameters.*

It should be emphasized that each discrete specification of the state equation involves a different interpretation of the depreciation parameters. Denote the rate at which s_{t-1} and q_t are depreciated at the end of period t respectively by δ and δ'. Then δ and δ' are to be computed as follows :

	δ	δ'	restriction on μ
(S1)	ν	0	$\mu = 0$
(S2)	ν	μ	–
(S3)	ν	$\frac{1}{2}\nu$	$\mu = \frac{1}{2}\nu \to \delta' = \frac{1}{2}\delta$
(S4)	$2\nu(\nu+2)^{-1}$	$\nu(\nu+2)^{-1}$	$\mu = 0 \to \delta' = \frac{1}{2}\delta$
(S5)	$2\nu(\nu+2)^{-1}$	$(\nu+2\mu)(\nu+2)^{-1}$	–
(S6)	$2\nu(\nu+2)^{-1}$	$2\nu(\nu+2)^{-1}$	$\mu = \frac{1}{2}\nu \to \delta' = \delta$

table 1

This explains why models using (S4)[5] obtain high depreciation rates. For example, in Phlips'(1972) estimation of a dynamic linear expenditure system, the estimated rate of depreciation was .92 for automobiles and .62 for furnitures. In fact, those numbers are estimates of ν. Phlips' explanation that these high values reflect the influence of advertising and other phenomenas which tend to increase the economic depreciation does make sense. However the δ implied by these estimates, respectively .64 and .47, are less troublesome.

[5]See for instance, the dynamic demand systems estimated by H-T (1970), Phlips (1972), (1974) and Taylor and Weiserbs (1972).

On the other hand, the specification of the behavioral equation will modify the estimates of α, γ and ϕ.

When (B1) is used, implied partial adjustment is

(7) $$\Delta s_t = \phi(\tilde{s}_t - s_{t-1}),$$

whereas it is

(8) $$\Delta s_t = \phi(\tilde{s}_t - \bar{s}_t)$$

with (B2). Writing(8) in terms of (7) leads to

(9) $$\Delta s_t = \frac{2\phi}{2+\phi} (\tilde{s}_t - s_{t-1}),$$

since $\bar{s}_t = \frac{1}{2} \Delta s_t + s_{t-1}$. As an example, if Phlips (1972) had chosen (B1) instead of (B2), the ϕ he estimated for automobiles would have been .735 instead of his actual 1.163 !

The stock effect is determined by ν and ϕ and consequently depends on the specifications chosen for both the state equation and the behavioral equation. This is shown in table 2.

Combining the (Bi) discrete specifications of the behavioral equation (1) and the (S_j) of the state equation (2), gives twelve possible models (BiSj) which encompass those in the literature : (B1S1) is the usual discrete specification (investment models),(B1S2) represents the S-R approach and (B1S3) the Mattei's while (B2S4) is the H-T model. Give what has been said about the (Bi), we need only to analyze (B1S2), (B1S5), (B2S2) and (B2S5). One might choose to consider only (B1S2) and (B2S5) on the grounds that models (B1S5) and (B2S2) apply a different kind of approximation to (1) and (2). The choice then becomes a simple alternative between a direct discrete specification, (S1B2), and a discrete approximation of the continuous time model, (B2S5).

However we shall proceed with all four models to show whether a difference in the value of a structural parameters is due to (Bi) or (Sj). It is interesting to see that only the two "consistent models" (B1S2) and (B2S5) reproduce the same formula for the stock effect as the continuous time model [equation (3)] :

	$\beta(1-\mu)$ and β^*	β^{**}
(B1S2)	$\nu-\phi$	$\frac{2(\nu-\phi)}{2-\nu}$
(B1S5)	$\nu-\phi(1+\frac{\nu}{2})$	$\frac{2\nu-\phi(2+\nu)}{2-\nu}$
(B2S2)	$\nu-\phi(1-\frac{\nu}{2})$	$\frac{2\nu}{2-\nu}-\phi$
(B2S5)	$\nu-\phi$	$\frac{2(\nu-\phi)}{2-\nu}$

table 2

In table 2, β^* is the value of β when $\mu = 0$ [(S1) and (S4)] and β^{**} when $\mu = \frac{\nu}{2}$ [(S3) and (S6)].

4. *An empirical illustration*

The foregoing will be illustrated by two examples referring to a durable (clothing) and a non-durable (food).

All (BiSj) models lead to the same estimating equation

$$q_t = K_0 + K_1 q_{t-1} + K_2 \Delta y_t + K_3 y_{t+1} + u_t, \tag{10}$$

where the K_n are regression coefficients. Their solution in terms of the structural parameters differs according to the model chosen. The regression results of H-T (1966) for the U.S. consumption data 1929-64 :

clothing : $\hat{q}_t = 17.6 + .124\ q_{t-1} + .076\ \Delta y_t + .017\ y_{t-1}$;

food : $\hat{q}_t = 29.1 + .604\ q_{t-1} + .113\ y_t + .053\ y_{t-1}$;

yield :

	clothing				food			
	(B1S2)	(B1S5)	(B2S2)	(B2S5)	(B1S2)	(B1S5)	(B2S2)	(B2S5)
ϕ	.376	.376	.463	.463	.396	.396	.493	.493
γ	.076	.076	.083	.083	.113	.113	.108	.108
$\tilde{\gamma}$	.046	.046	.046	.046	.134	.134	.134	.134
ν	.227	.256	.227	.256	.468	.611	.468	.611
α	77.6	77.6	84.7	84.7	62.1	62.1	59.3	59.3
$\beta(1-\mu),\beta^*$	-.149	-.168	-.183	-.207	.072	.095	.090	.118
β^{**}	-.168	-.193	-.207	-.237	.095	.136	.118	.170

table 3

It should be emphasized that each model has a different implicit value of the state [for clothing, s_t in 1961 varies from $ 427 (B2S4) to $ 558 (B1S1)].

Only the long run derivative with respect to income is invariant whatever the specification. However, the short-run effect, γ, is higher with (B2) than with (B1) in case of stock adjustment (durable) and lower in case of habit formation (non-durable). In other words, the short-run income elasticity for clothing is increase by 9 % when (B2) is chosen instead of (B1) while it drops by 5 % for food.

5. *Concluding remarks.*

The choice of a discrete specification of the state-adjustment model reduces to one between the direct discrete specification (B1S2) and the discrete approximation of the continuous time model (B2S5). The first one leads to simpler relationships between the K_k and the structural parameters and, above all, presents an easier interpretation of these parameters. All in all, it has much to be recommended.

The preceeding discussion is equally valid for complete dynamic demand systems where the consumer is told to maximize u(q,s) subject to his budget constraint. All discrete specifications yield the same estimating form. This, however, will not be true in an intertemporal approach where the functional to be maximized is $\int_0^\infty e^{-rt}\, u(q,s)dt$. Therefore in this case, the choice of the discrete form is even more important.

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