

Fast and Accurate Simulation Technique for Large Irregular Arrays

Ha Bui-Van^{ib}, Jens Abraham, Michel Arts, Quentin Gueuning^{ib}, Christopher Raucy,
David González-Ovejero^{ib}, *Senior Member, IEEE*, Eloy de Lera Acedo,
and Christophe Craeye, *Senior Member, IEEE*

Abstract—A fast full-wave simulation technique is presented for the analysis of large irregular planar arrays of identical 3-D metallic antennas. The solution method relies on the macro basis function (MBF) approach and an interpolatory technique to compute the interactions between MBFs. The harmonic-polynomial model is established for the near-field interactions in a modified system of coordinates. For extremely large arrays made of complex antennas, two approaches assuming a limited radius of influence for mutual coupling are considered: one is based on a sparse matrix LU-decomposition and the other one on a tessellation of the array in the form of overlapping subarrays. The computation of all embedded element patterns is sped up with the help of the nonuniform fast Fourier transform algorithm. Extensive validations of this new approach, named HARP, are shown for arrays of log-periodic antennas envisaged for the low-frequency Square Kilometer Array (SKA) radio telescope. The analysis of SKA stations with such a large number of elements has not been treated yet in the literature. Validations include comparison with results obtained with commercial software and with experiments. The proposed method is particularly well suited to array synthesis, in which several orders of magnitude can be saved in terms of computation time.

Index Terms—Full-wave simulation, irregular antenna arrays, macro basis functions (MBFs), method of moments (MoM), Square Kilometer Array (SKA), wideband arrays.

I. INTRODUCTION

MODERN radars [1], massive MIMO [2], and radio astronomy arrays [3], [4] are in need of accurate modeling of their main electromagnetic properties. These are strongly impacted by mutual coupling and include the array impedance matrix and all embedded element patterns (EEPs),

i.e., the patterns of each individual element in the presence of all other antennas. This can become a challenging task, knowing that arrays can consist of hundreds or thousands of elements, as considered in the Square Kilometer Array (SKA) project [5], [6] and in modern radar systems [1].

The arrays can be regular as well as nonregular. Irregular arrays exhibit a series of advantages over regular arrays, namely the randomization of the sidelobes and of the mutual coupling effects [7]. Both of these factors allow the operation of the arrays over a much larger frequency band compared with traditional regular arrays [8], with a limited number of elements. The accurate knowledge of the antenna and array beams, as well as of the elements' impedance in the presence of mutual coupling [9] is essential for the design of these systems, i.e., for the design of the antenna elements, the array configuration and the electronics connected to the antennas, especially the first-stage amplification. The number of elements in the arrays, the geometrical complexity of the elements, and the electrical size of these systems represent a challenge regarding their computer modeling. In particular, the irregularity of the elements' positions in the array prevents us from using certain approximations, such as periodic boundary conditions possibly combined with corrections for truncation [10]. This paper proposes a solution based on two novel contributions: 1) a fast full-wave technique relied on the harmonic-polynomial representation of interactions between antennas with a new change of variables versus radial coordinate to accurately and efficiently simulate large irregular arrays of complex structures and 2) a tessellation scheme that exploits this fast simulation technique and the partition of the arrays into nonoverlapping subarrays for the analysis of extremely large arrays consisting of hundreds or thousands of elements. The exploitation of the nonuniform fast Fourier transform (NFFT) also vastly accelerates the computation of the EEP.

Integral equation techniques solved with the method of moments (MoM) are very convenient for antenna problems, since they inherently account for radiation. However, when one uses Gaussian elimination, the MoM solution has a complexity of $O((N_a N_e)^3)$, where N_a is the number of antennas in the array and N_e is the number of elementary basis functions per element. For large arrays of complex antennas, the multilevel fast multipole (MLFMA) technique [11] and its hybridization with other techniques (e.g., HOMoM-MLFMA [12]) are often exploited in an iterative scheme.

Manuscript received February 1, 2017; revised October 16, 2017; accepted February 3, 2018. Date of publication February 14, 2018; date of current version April 5, 2018. This work was supported by U.K. Grant ST/M001393/1. (Corresponding author: Ha Bui-Van.)

H. Bui-Van, Q. Gueuning, C. Raucy, and C. Craeye are with the Institute of Information and Communication Technologies, Electronics and Applied Mathematics, Université catholique de Louvain, 1348 Louvain-la-Neuve, Belgium (e-mail: buivanha@uclouvain.be; christophe.craeye@uclouvain.be).

J. Abraham was with the Astrophysic Group, Cavendish Laboratory, University of Cambridge, Cambridge CB2 1TN, U.K. He is now with the Department of Electronic Systems, Norwegian University of Science and Technology, 7491 Trondheim, Norway.

M. Arts is with ASTRON, 7991 PD Dwingeloo, The Netherlands.

D. González-Ovejero is with the Institut d'Électronique et de Télécommunications de Rennes (IETR, UMR CNRS 6164), Université de Rennes 1, 35042 Rennes, France (e-mail: david.gonzalez-ovejero@univ-rennes1.fr).

E. de Lera Acedo is with the Astrophysic Group, Cavendish Laboratory, University of Cambridge, Cambridge CB2 1TN, U.K.

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TAP.2018.2806222

Unfortunately, the calculation of the array impedance matrix and all EEPs in large finite arrays requires a new series of iterations for excitation at each antenna port. This often leads to an unaffordable computational effort for large arrays, besides the uncertainty related to the number of iterations. A noniterative technique is then favorable, provided that it can benefit from the proper compression of unknowns and from further accelerations. That is the case for techniques such as macro basis functions (MBF) [13], characteristic basis functions [14], [15], synthetic function expansion [16], [17], or entire/subentire-domain basis functions [18], [19].

MBF-type techniques achieve a compression of the MoM impedance matrix by replacing the original set of elementary basis functions by a new set of functions obtained through the solution of smaller problems. Once the MoM matrix size is reduced by means of the MBF technique, the $O((N_a N_e)^3)$ complexity for solving the system of equations is reduced to $O((N_a N_m)^3)$, where N_m is the number of MBFs and $N_m \ll N_e$. Nevertheless, the impedance matrix filling time remains $O((N_a N_e)^2)$, the same as for the original MoM, which is prohibitive for large arrays. To overcome this obstacle, different techniques have been proposed, such as the multipole expansion algorithm [20], [21], the interpolatory technique [22], the perturbation technique based on the domain Green's function [23] and its enhanced version using the adaptive cross approximation [24], or the combination of multipole, and Contour-FFT approaches for printed structures [25]. The multipole expansion, however, suffers from the instability of near-field interaction, which appears when antennas are too close to each other, i.e., for distances smaller than about one wavelength. For dense wideband arrays, such as those envisaged for the SKA, antennas are very close to each other at low frequencies, which requires additional efforts to explicitly calculate the near-field interactions. In this respect, the interpolatory technique presented in [22] is very effective in providing highly accurate solutions even in the near-field [26]. Moreover, MBF interactions are rapidly evaluated from a model, which is built beforehand and is independent of the array configuration. The method, renamed later as HARP [27], has shown a great capability and has been extended to the analysis of antennas made of wires, in order to model the SKA1-Low arrays [8], [28].

Preliminary results of HARP on SKA1-Low antennas have been presented in recent conference papers [26], [27], [29]. Nonetheless, limited details were presented about the technique itself and its performance. This paper further extends this method including a detailed description of the proposed technique, new improvements, and extended validation. HARP is now capable of simulating SKA stations over the whole frequency band (50–350 MHz) in a matter of minutes for each frequency point. From now on the name HARP refers to the whole method that includes all novel contributions described in this paper.

Additionally, two approaches are presented to exploit the fast simulation method for the analysis of extremely large arrays, namely, the sparse matrix approach and the tessellation approach. These methods make use of the so-called radius of influence (RI) concept, which is often applied to the analysis

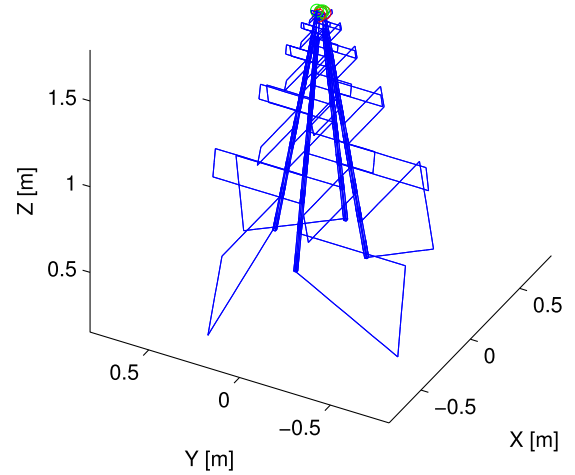


Fig. 1. Wire mesh of the SKALA having a footprint of $1.2 \times 1.2 \text{ m}^2$. The SKALA consists of four arms supported by four spines, with two orthogonal feeds (on top), providing two polarizations.

of large arrays [30]. The first approach calculates only the interaction between antennas whose relative distance is smaller than the RI, which leads to a sparse interaction matrix that can be effectively LU decomposed and solved exploiting sparse matrix libraries. In the tessellation approach, the full array is first partitioned into overlapping subarrays. The EEPs are retained only for the internal (or nonoverlapping) part of the subarray. The size of the internal part and the extended surrounding area of the subarray is determined so as to minimize the computational cost. As each subarray can be analyzed independently, this approach enables the simulation of very large arrays of complex elements on a normal desktop computer; it also facilitates parallel implementation. Finally, as explained earlier, the MBF approach allows the fast calculation of induced currents for excitation at each port. The calculation of the corresponding EEPs is then expressed as superposition of pattern multiplication problems, one per MBF [31], and the array factors calculation is accelerated using the NFFT [31]–[33].

In this paper, the SKA1-Low arrays are taken as a case study to extensively validate the performance of HARP. Upon its completion, the SKA1-Low instrument will consist of 512 stations, each with 256 log-periodic antennas [SKA log-periodic antenna (SKALA)] [8], [28] (see Fig. 1), arranged in a pseudorandom configuration, as shown in Fig. 2. With an ultrawideband characteristic (7:1 fractional bandwidth) and its complex structure, the modeling and simulation of the SKA1-Low frequency array presents a challenge as the array is quite dense at low frequencies, while at high frequencies, the antenna needs to be meshed precisely. The accurate modeling of the SKALA antenna has a strong impact on the results obtained with a given simulation software. The results obtained using HARP will be validated with those calculated using commercial software on the SKA station, and with measurements of the isolated elements and a small 3×3 array.

The remainder of this paper is organized as follows. In Section II, the fast full-wave technique and the strategies

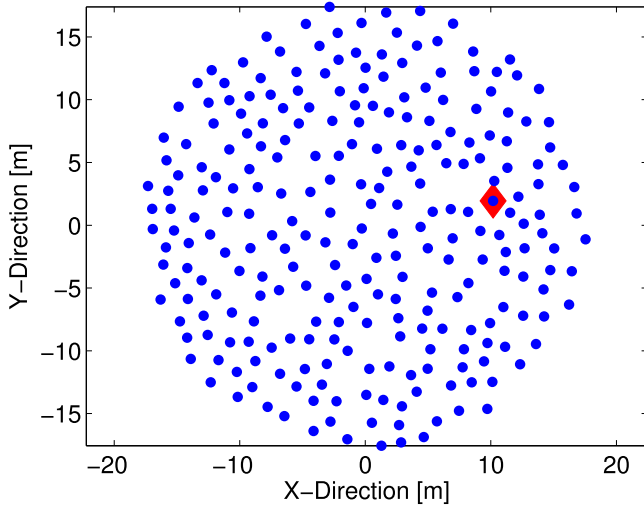


Fig. 2. Layout of an SKA1-low station consisting of 256 SKALA antennas with a minimum distance of 1.35 m.

to analyze extremely large arrays are detailed. In Section III, numerical results for the SKA1-Low case are presented, with data obtained using HARP, CST Studio [34], WIPL-D [35], and measurements for self-impedance and coupling in a small array. In Section IV, the performance of the tessellation scheme on very large arrays is examined and compared with those from the traditional RI approach on different arrays of SKALA antennas. Finally, the conclusion is drawn in Section V.

II. FAST FULL-WAVE TECHNIQUE

In this section, we will explain in detail the HARP method, and present improvements that enable HARP to analyze large random arrays made of complex radiators, such as the SKALA antenna. The tessellation approach, which exploits the fast interaction model, is also introduced; it enables the effective simulation of extremely large arrays. For the sake of clarity, the interpolatory method is first recalled. Interested readers are referred to [22] for detailed explanations regarding the interpolatory technique. In this paper, the MBFs are obtained using the "primary-and-secondary" approach [14], where the primary MBF corresponds to the current distribution on an isolated element and the secondary MBFs are the currents induced on a set of neighboring antennas by the element that supports the primary MBF.

A. Background of HARP

The interpolatory approach was proposed in [22] to quickly compute the interaction matrix between MBFs, which are the entries of the reduced MoM matrix. The interaction between a source MBF S on antenna n placed at the origin and a testing MBF T on antenna m , positioned at $(r_{mn}, \hat{\alpha})$, was first approximated as

$$Z_{TS}^{\text{app}}(r_{mn}, \hat{\alpha}) \approx -j\omega\mu \vec{F}_{T,m}^{\hat{\alpha},*} \cdot \vec{F}_{S,n}^{\hat{\alpha}} \frac{e^{-jk r_{mn}}}{4\pi r_{mn}} \quad (1)$$

where $\hat{\alpha}$ points from the reference point in the source antenna to the reference point in the testing antenna and r_{mn} represents

the interelement distance, ω is the angular frequency, μ is the free-space permeability, $\vec{F}_{T,m}^{\hat{\alpha}}$, $\vec{F}_{S,n}^{\hat{\alpha}}$ are the radiation patterns of the testing and source MBFs, respectively, and $(*)$ stands for complex conjugate. The actual interaction is then obtained as a sum of the above far-field approximation and a complementary part, which is represented using a harmonic-polynomial model. The model is built by first calculating explicitly the exact interaction between MBFs over a grid with a limited number of points. A mathematical representation of the difference between the exact and far-field approximated interactions is then constructed after phase correction and an appropriate change of variables. The first two steps can be expressed in a compact form as

$$B_{TS}(r_{mn}, \hat{\alpha}) = \frac{Z_{TS}^{\text{exact}}(r_{mn}, \hat{\alpha}) - Z_{TS}^{\text{app}}(r_{mn}, \hat{\alpha})}{e^{-jk r_{mn}}} \quad (2)$$

where $Z_{TS}^{\text{exact}}(r_{mn}, \hat{\alpha})$ is the exact interaction explicitly calculated on a predefined polar-radial grid. The numerator of (2) is the far-field subtraction operation, while the denominator stands for the phase correction step. The result of (2) is then modeled in a harmonic-polynomial form [22] with a proper change of variables, i.e., $d = 1/r^2$

$$B_{TS}(d, \hat{\alpha}) = \sum_{p=-P}^P e^{jpa} \sum_{q=0}^Q c_{pq} d^q \quad (3)$$

where P and Q are the orders of the Fourier Series and polynomial, respectively, and c_{pq} are coefficients calculated in the least-squares sense. Once the model has been built, the interaction between MBFs is rapidly calculated by adding up the far-field contribution and the complementary part evaluated from the model

$$Z_{TS}(r_{mn}, \hat{\alpha}) = Z_{TS}^{\text{app}}(r_{mn}, \hat{\alpha}) + B_{TS}(r_{mn}, \hat{\alpha})e^{-jk r_{mn}}. \quad (4)$$

B. Improved Model for MBF Interactions

Improvements have been introduced to the original version of HARP to capture the interaction of complex structures such as the SKALA element. The first enhancement appears at the change of variables step, where $d = 1/r$ is now implemented instead of $d = 1/r^2$ as in (3). This modification is physically interpreted as taking into account not only the contribution of $1/r^2$, but also of $1/r^3$ field components [36], which are non-negligible in the case of SKALA. Such a change of variables is also consistent with the multipole-based model in [26], where a Laurent series versus the r variable is obtained. The latter work, however, suffers from the stability issues in the very near field. Therefore, in this case, we followed [22] to build the interaction model using the new change of variables.

The improvement in the accuracy of HARP using $1/r$ change of variables is illustrated in Fig. 3, where the interaction between two primary MBFs is calculated for varying distance at 110 MHz. Both versions of HARP can capture the near distance interaction, while only the $1/r$ change of variables allows the HARP method to model much more accurately the interaction from very small to far distances. The same improvement is observed at other frequencies. This enhancement enables the proposed method to capture precisely

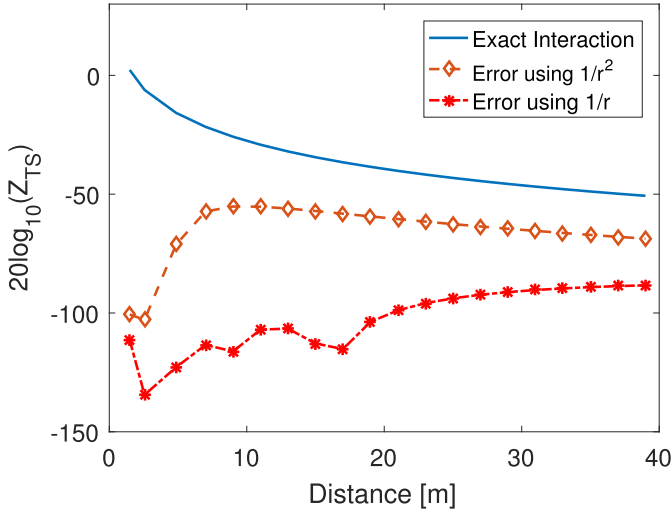


Fig. 3. Interaction between two primary MBFs on two SKALA antennas with a source antenna (S) at the origin and a test one (T) moving along the x -axis at 110 MHz. Error is evaluated between the exact interaction and the interaction obtained by HARP with $1/r^2$ and $1/r$ change of variable. Error is calculated as $20 \log_{10} |Z_{TS}^{\text{exact}} - Z_{TS}^{\text{HARP}}|$.

mutual coupling in large random arrays. It is noted that the interaction models are constructed using the same number of coefficients for both cases (i.e., $1/r$ and $1/r^2$ change of variables).

Additionally, improvements have been added to the implementation of HARP. The discrete Fourier transform approach has been used instead of a least-squares estimator to obtain the harmonic coefficients (c_{pq}). Also, the sampling points on the grid are determined by an appropriate sampling routine, i.e., a regular grid on angular variable and inversely regular along the radial distance (regular in $1/r$ scale). Finally, it is noticed that the interpolation from HARP model via (3) is carried out for every pair of MBFs using two summations. This step is accelerated by writing (3) as a vector-matrix-vector product in MATLAB [37] implementation (i.e., the left vector is a row-vector of Fourier series, the center matrix is the matrix c_{pq} , and the right vector is a column of polynomials order Q).

Regarding the computational aspect using HARP, the complexities for filling the reduced matrix and solving the system of equations are $O(N_a^2 N_m^2 S)$ and $O((N_a N_m)^3)$, respectively, where S is a small factor related to the calculation of (4) [22]. The number of MBFs is generally smaller by about two orders of magnitude than the number of basis functions used to describe the current on the antenna (related to meshing); the drastic reduction in the number of unknowns enables the acceleration of the analysis using HARP. In [22], it is shown that the interpolatory (or HARP) technique is more efficient than the multipole expansion [20]. Moreover, HARP is able to accurately evaluate the near-field interaction for closely packed arrays.

C. Fast Pattern Calculations: NFFT

The use of MBFs allows one to write the pattern of each antenna as a sum of MBF patterns multiplied by the corresponding coefficients for the MBF. Hence, the array

pattern can be expressed as

$$\vec{F}_{\text{array}}(\theta, \phi) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_m} c_{ij} \vec{F}_j(\theta, \phi) e^{jk(u_x x_i + u_y y_i)} \quad (5)$$

$$= \sum_{j=1}^{N_m} \vec{F}_j(\theta, \phi) \sum_{i=1}^{N_a} c_{ij} e^{jk(u_x x_i + u_y y_i)} \quad (6)$$

where (x_i, y_i) is the position of the i th antenna, $\hat{u} = (u_x, u_y, u_z)$ is a unit vector in the direction (θ, ϕ) of interest, $\vec{F}_j(\theta, \phi)$ is the pattern of MBF j , and c_{ij} is the coefficient for MBF j on antenna i . The swapping of the summations is possible since only one set of MBFs is used for all identical antennas of the array. As the MBF patterns in (6) are factored out, the second summation becomes the array factor, which can be rapidly evaluated using the N-FFT [31]–[33] for irregular arrays. It is noted that since the MBF patterns are smooth functions, they are evaluated over a much sparser grid in advance. The higher resolution patterns then can be rapidly obtained using 2-D interpolation on the grid used for the NFFT. This approach is also applied for the calculation of each EEP of the array, where the induced currents are quickly calculated for excitation at a given antenna port.

D. Analysis Schemes for Very Large Arrays

For extremely large arrays, comprising thousands of antennas, the direct implementation of HARP on the whole array would require too much memory and computation time. An effective approach exploiting HARP for such arrays is then needed. Generally, the so-called RI concept is often applied [23], [30] for the analysis of very large arrays; here the interactions are only considered when the separation between elements is smaller than the RI. This results in a sparse matrix for the reduced MoM system of equations. The filling time of this sparse matrix and the solution time of the system of equations are significantly reduced owing to the use of sparse operations. More precisely, sparse matrix libraries [38] are applied to obtain a sparse LU decomposition of the matrix. Fast solutions can then be computed for any excitation law. The approach, however, requires a large RI [30] in order to obtain a highly accurate pattern, which implies an increase of the computational cost.

An alternative technique is proposed as follows. It relies on the tessellation of the array into subarrays. The RI concept is then applied to each subarray individually. The idea is illustrated in Fig. 4, where the elements in the array are clustered into subarrays, i.e., hexagonal shapes with the inner nonoverlapping part of radius R_{in} (blue dashed line), and an extended area between the inner radius R_{in} and the outer one $R_{\text{in}} + R_{\text{ext}}$ (green area). To analyze the original array, each subarray is considered separately including the inner and the outer parts. The new interaction model described in Section II-B is then called to rapidly obtain the interaction matrix for a subarray. The MBF-currents and EEPs are calculated by exciting only elements within the inner part while terminating the elements in the outer part. It should be noted that only EEPs of elements within the inner part

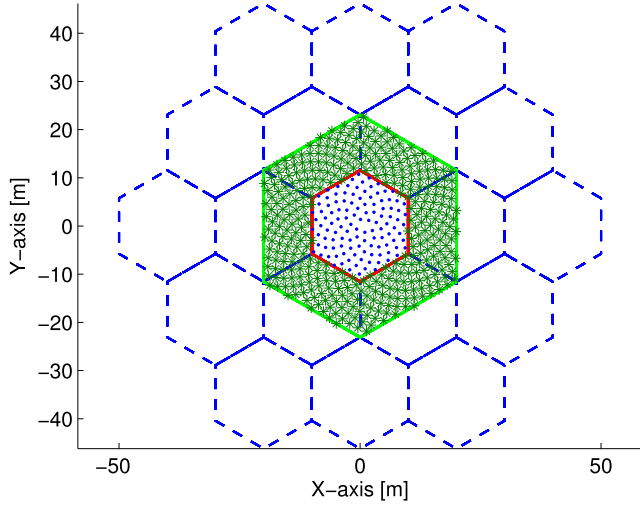


Fig. 4. Tesselation (subarray) approach: inner nonoverlapping part (blue dashed hexagonal) and an extended area limited to the green hexagonal.

are retained. The tessellation approach deals with subarrays having much smaller size than the whole array, which will ease the computational effort of solving the MoM system of equations. The use of an outer part with a width of R_{ext} at least equal to R_{in} helps to increase the accuracy of the results.

An optimal choice of R_{ext} can be made by analyzing the computational complexity of the approach. Assuming that a random array has a constant antenna density, the number of antennas on a subarray (inner and outer parts) will be proportional to $(R_{\text{in}} + R_{\text{ext}})^2$. The number of subarrays will be of order of N_a/R_{in}^2 . Following the calculation in [22], the computational complexity for filling the matrix can be expressed as a function of R_{in} :

$$C = \alpha \frac{N_a}{R_{\text{in}}^2} (R_{\text{in}} + R_{\text{ext}})^4 \quad (7)$$

where α is a constant taking into account all other constant factors such as the number of MBFs. C presents a minimum when $R_{\text{in}} = R_{\text{ext}}$. The same minimum can be found regarding the complexity of solving the system of equations. This gives a rule of thumb for the choice of R_{in} and R_{ext} so as to optimize the quality of the solution while minimizing the computational complexity.

One of the important advantages of HARP is the reusability of the interaction model, which is established only once and can be exploited to analyze any array made of the same element. This feature is very helpful for the tessellation approach, as in this scenario, several subarrays can be simulated separately, which straightforwardly allows parallelization. The implementation and evaluation of HARP using the sparse matrix and tessellation approaches will be studied in Section IV.

III. NUMERICAL AND EXPERIMENTAL VALIDATION

Preliminary data at a single frequency have been presented in recent conference papers [27], [29] with limited results on relatively small arrays and on an SKA1-Low station. In this paper, the performance of HARP is studied in depth on

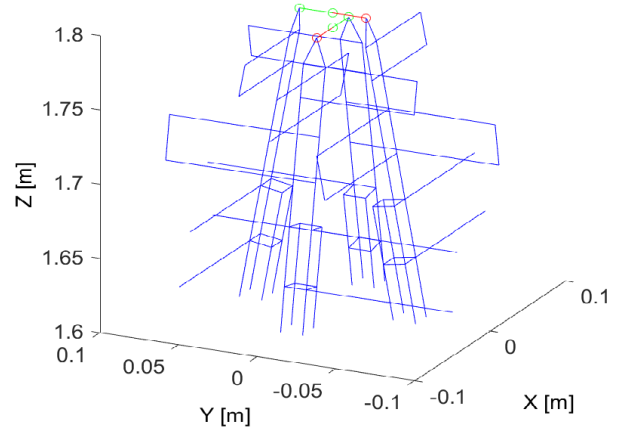


Fig. 5. Zoomed-in view of the upper part of the SKALA mesh implemented in this paper: two feed points on top and four cages at the bottom.

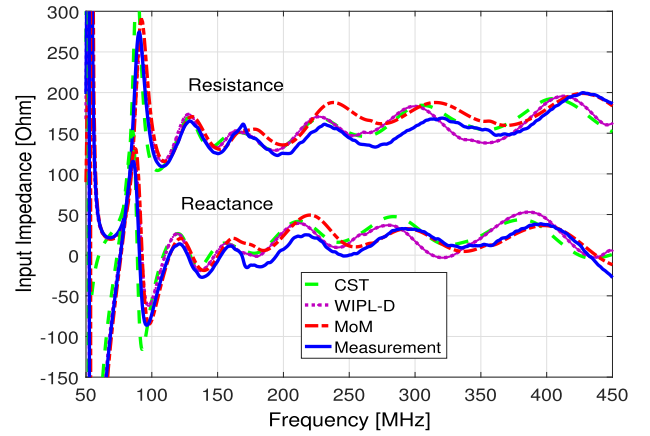


Fig. 6. Real and imaginary parts of the self-impedance of SKALA. Measurement with antenna standing on a 2 m square metallic ground.

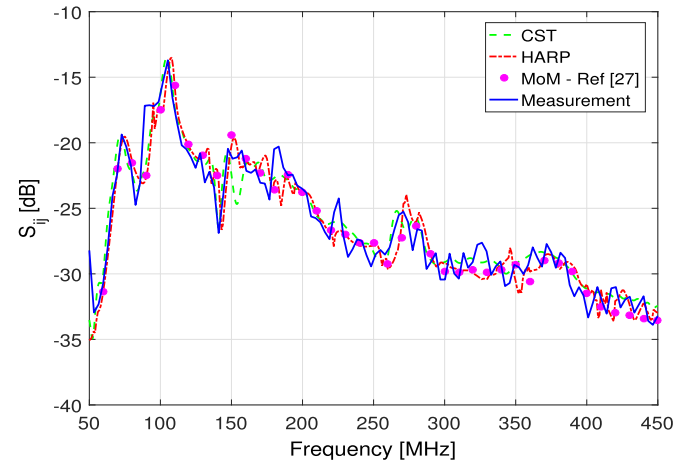


Fig. 7. Coupling coefficient in a 3×3 array, between the center element and middle of edge.

several examples, which include an isolated antenna, a small compact array, and the SKA station, as shown in Fig. 2, at 50, 110, 200, and 350 MHz, covering the whole SKA1-Low band. Simulated results from two commercial software, CST Microwave Studio [34] and WIPL-D [35], are taken as a reference to examine the performance of HARP, while the measured data are presented whenever they are available.

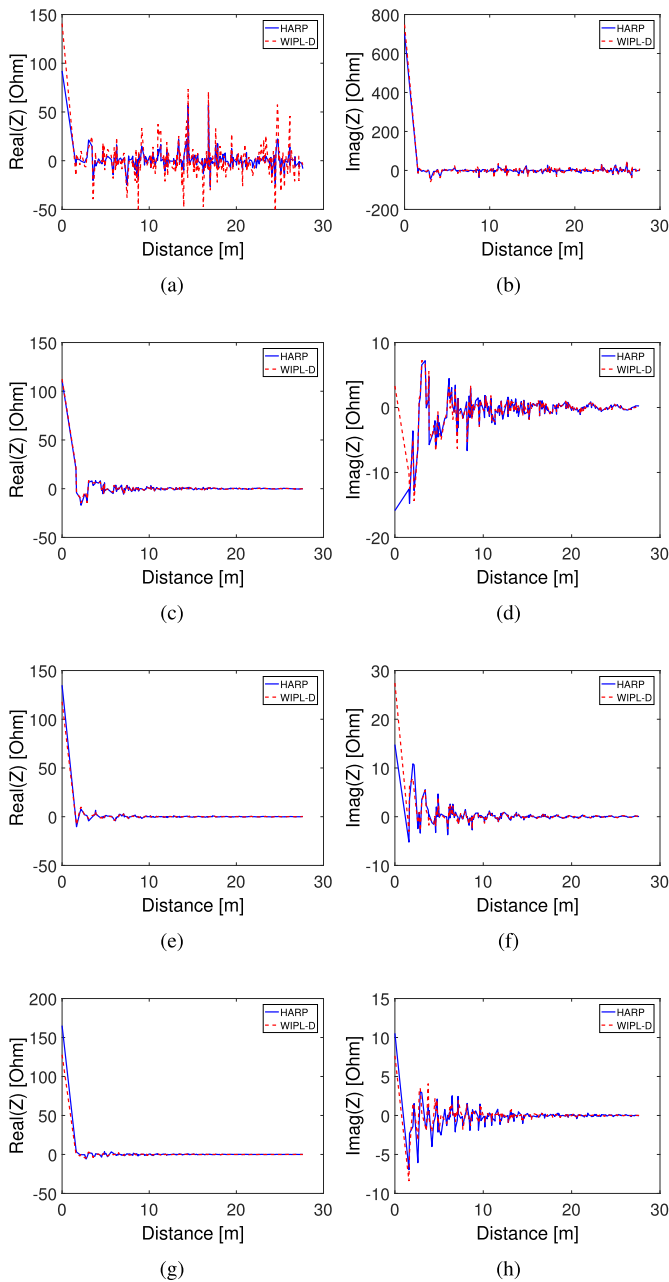


Fig. 8. Mutual impedance of a random antenna i in the SKA station (a red dot in Fig. 2) and other antennas j sorted by the distance to antenna i . (a) Real part at 50 MHz. (b) Imaginary part at 50 MHz. (c) Real part at 110 MHz. (d) Imaginary part at 110 MHz. (e) Real part at 200 MHz. (f) Imaginary part at 200 MHz. (g) Real part at 350 MHz. (h) Imaginary part at 350 MHz.

Different parameters are analyzed, including input impedance, coupling coefficients, EEPs, and array patterns. In all cases, the data for one polarization are shown, while the results for the other polarization have the same quality and are not shown. The SKALA model used in HARP consists of a wire mesh comprising 1218 basis functions. This HARP implementation is based on an MoM code that considers wires, with the Poklington approximation [39]. The 4 cm diameter tube that supports each arm does not satisfy that approximation in the upper part of the frequency range. Therefore, those tubes have been replaced by wire cages with a square cross section and a perimeter equal to that of the tube. Fig. 1 shows a global

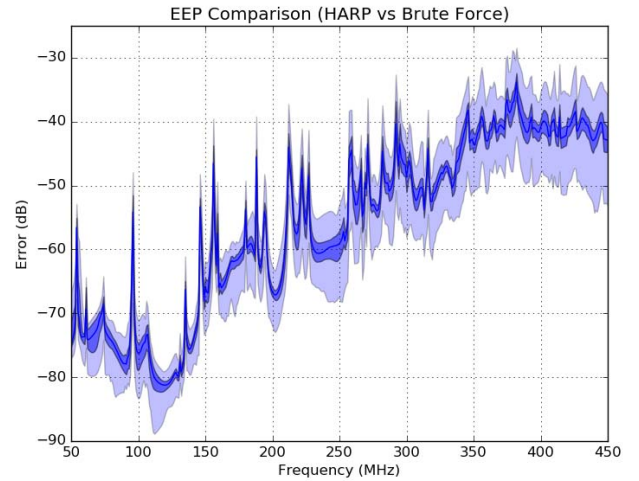


Fig. 9. Error of the EEPs between the HARP and the brute-force solution over all 16 antennas in all directions against frequency. The median, 25 % and 75 % percentiles as well as the minimum and maximum errors are shown by the blue line, the dark blue shaded, and the light blue shaded areas, respectively.

view of the structure, while Fig. 5 shows a zoom near the feed points and near the cage. The numbers of MBFs are 15, 20, 45, and 70 at 50, 110, 200, and 350 MHz, respectively (linearly scaled vs. frequency).

A. Self-Impedance and Mutual Impedance

Due to the complexity of the design, an accurate model of the SKALA in Fig. 1 is critical for analyzing the performance of the array, e.g., self-impedance and mutual impedance are required to study parameters such as noise coupling [40], [41]. The input impedance is then the first parameter to study. Fig. 6 shows the self-impedance simulated using different software, along with the measured data over the low-frequency band. A small deviation of the order of 20Ω is seen around 200–300 MHz between the simulations and the measurements. This can be attributed to the fact that different software packages model and mesh the SKALA differently, and that the SKALA in the measurement had a metallic ground limited to 2 m, whereas the simulations assume an infinite ground plane. These variations may also explain some of the differences among patterns in Sections III-B and III-C.

Another validation example carries on a 3×3 regular array of SKALAs with element spacing of 1.5 m. The coupling coefficient is calculated between the center antenna and another element at the middle of the edge, as shown in Fig. 7. This example is a repetition of an example in [27], where the solution of the traditional wire-MoM, without acceleration, is reported. Fig. 7 shows that results obtained using HARP and MoM are extremely close, which confirms the accuracy of the HARP method with respect to the traditional wire-MoM method. Moreover, the HARP and CST simulations follow well the measurement across the whole frequency band.

The mutual impedance is studied for the SKA station as well. Fig. 8 plots the real and imaginary parts of the mutual impedance Z_{ij} for different frequencies in an SKA station between an arbitrary antenna i (red dot in Fig. 2) and another antenna j running from 1 to 256; the obtained values are

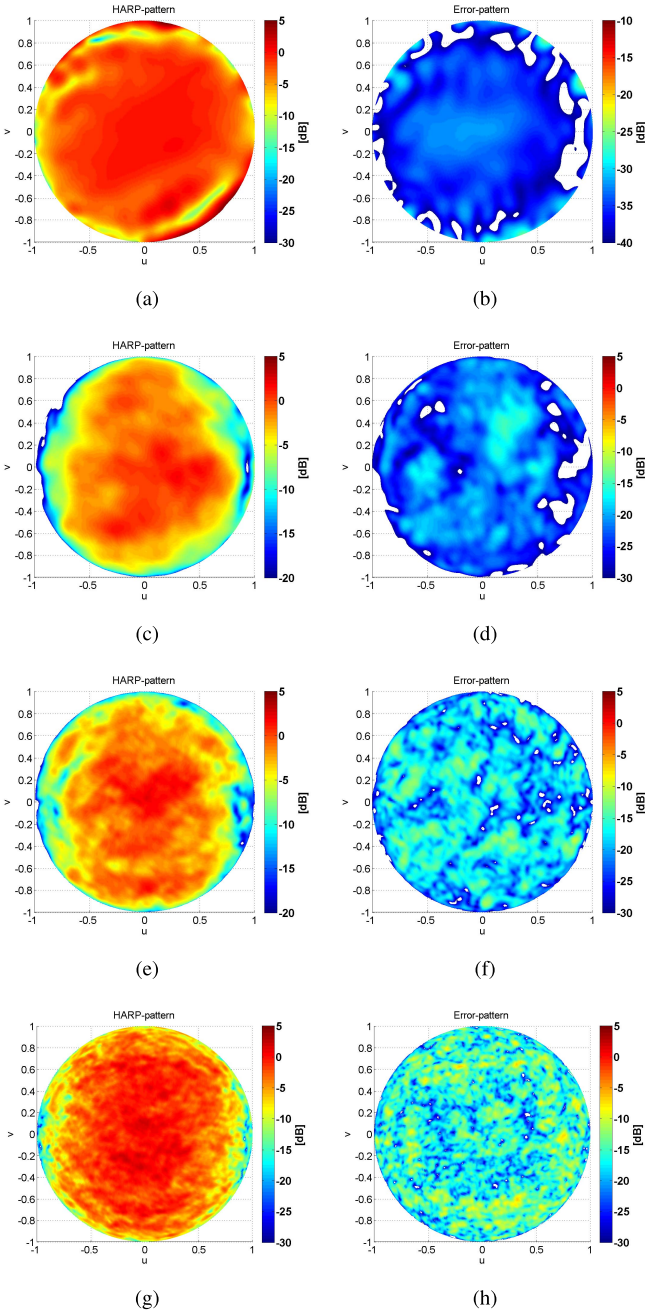


Fig. 10. EEP obtained by HARP and the difference with respect to WIPL-D of a random element in the SKA station (red dot in Fig. 2). (a) 50 MHz. (b) Error pattern 50 MHz. (c) 110 MHz. (d) Error pattern 110 MHz. (e) 200 MHz. (f) Error pattern 200 MHz. (g) 350 MHz. (h) Error pattern 350 MHz.

sorted versus the distance to antenna i . A good agreement is found between HARP and WIPL-D for coupling between antennas. The main difference regards the self-impedance, i.e., zero distance, which is also illustrated in Fig. 6. The results confirm the capability of the current version of HARP to capture the interaction between SKALA antennas in their array environment.

B. Embedded Element Patterns

In applications such as the SKA telescope, the EEPs are of primary interest as they allow one to form different beams

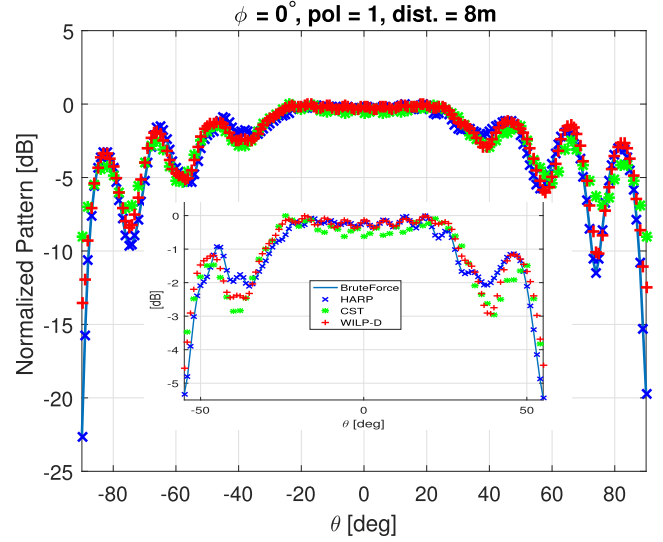


Fig. 11. $\phi = 0^\circ$ cut of one EEP in an array of two SKALAs at a distance of 8 m for the first polarization at 350 MHz. Inset: zoomed-in view of the main beam.

collecting signal from sky. An accurate and effective calculation of all EEPs is then required, while taking into account the mutual coupling. In this section, the quality of EEPs obtained using HARP is evaluated. A small testbed of 16 SKALA elements is first analyzed, where the EEPs of all elements are calculated at all frequencies with 1 MHz frequency steps. Then, the SKA station in Fig. 2 is simulated at four different frequencies, i.e., 50, 110, 200, and 350 MHz, using HARP and WIPL-D, and the calculated EEPs are compared.

1) *EEPs of a Small Array*: The performance of HARP has been tested against the brute-force MoM solution of a small array of 16 SKALA elements for the frequency band (50–450 MHz). The array corresponds to a testbed for SKA1-Low technology and is situated at the Mullard Radio Astronomy Observatory near Cambridge, U.K. [8]. This array with only 16 elements is in a pseudorandom configuration, such as that of SKA stations, and has the same expected element density, with an average interelement spacing of half-wavelength at 77 MHz. The error between the patterns is calculated as

$$\epsilon = \sqrt{|E_\theta - E_\theta^{\text{ref}}|^2 + |E_\phi - E_\phi^{\text{ref}}|^2} \quad (8)$$

with E and E^{ref} being the EEP calculated by HARP and brute-force MoM with the electric fields being normalized by the maximum of the isolated antenna pattern. This error is calculated in all directions for all involved antennas. The condensed result is shown in Fig. 9 by calculating the percentiles of the error and plotting them over frequency. The HARP method shows very good accordance to the brute-force MoM results over the whole frequency range. The error increases at the highest end of the band, but it is still below -30 dB.

2) *EEPs of SKA Station*: The performance of HARP on the SKA station is now validated with patterns simulated using the WIPL-D software [35]. The EEP of an arbitrary element (red dot in Fig. 2) obtained using HARP is taken as an example. Fig. 10 shows the EEPs at four different frequencies and

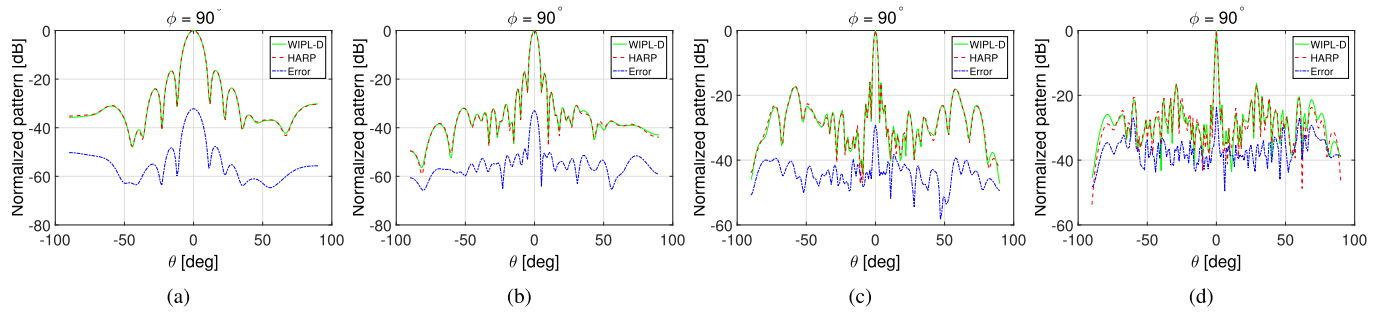


Fig. 12. Normalized radiation pattern of an SKA station in H-plane ($\phi = 90^\circ$) at different frequencies. (a) H-plane: $\phi = 90^\circ$, 50 MHz. (b) H-plane: $\phi = 90^\circ$, 110 MHz. (c) H-plane: $\phi = 90^\circ$, 200 MHz. (d) H-plane: $\phi = 90^\circ$, 350 MHz.

their differences with respect to WIPL-D, where the error patterns are computed using (8) with E and E^{ref} being patterns obtained using HARP and WIPL-D, respectively.

An excellent agreement is shown at low frequencies, where the difference level is found below -30 dB at 50 MHz. The variation becomes more visible as the frequency increases. A detailed study of the performance of different software at 350 MHz is carried out on an array of two antennas positioned along the x -axis with varying distance. The EEPs obtained using CST, WIPL-D, brute-force MoM, and HARP are plotted together in Fig. 11 for a separation equal to 8 m. For this two-element array, the EEPs are similar to the isolated pattern, and mutual coupling clearly appears as ripples in the main beam. The results show that while the mutual coupling is accounted for in all software packages, a small difference is observed in the EEPs. The reason is attributed to the difference in the geometrical modeling of the SKALA, i.e., the use of different meshes in each software, which leads to differences in the isolated pattern. At the highest frequency, a small difference in antenna modeling will result in a significant variation in the radiation patterns. This explains the discrepancies in the EEPs visible in Fig. 10(h), as well as the self-impedance behavior in Fig. 6. A stronger indicator that the discrepancies mainly come from the mesh is that the errors in the isolated pattern are almost as large as in the EEPs. Moreover, it is important to note that, in Fig. 11, patterns calculated using brute-force MoM and HARP overlap very well, which confirms the accuracy of the HARP method.

C. Array Patterns

In this section, the EEPs calculated using HARP and WIPL-D software are exploited to form the array pattern. A simple beamforming method is implemented with uniform excitation and scanning at broadside. Fig. 12 shows the radiation pattern of the SKA station at different frequencies in two principal planes calculated using HARP and WIPL-D. The two methods show a great agreement across the frequency band, as the difference in patterns, evaluated by (8), is only shown at a -30 dB level, except for the highest frequency, i.e., 350 MHz, where the error in the main beam is near -24 dB. As explained earlier, the visible difference in the patterns at higher frequency is due to the difference in modeling of the SKALA in the two software. To demonstrate the impact of the isolated element modeling, we show that it can be

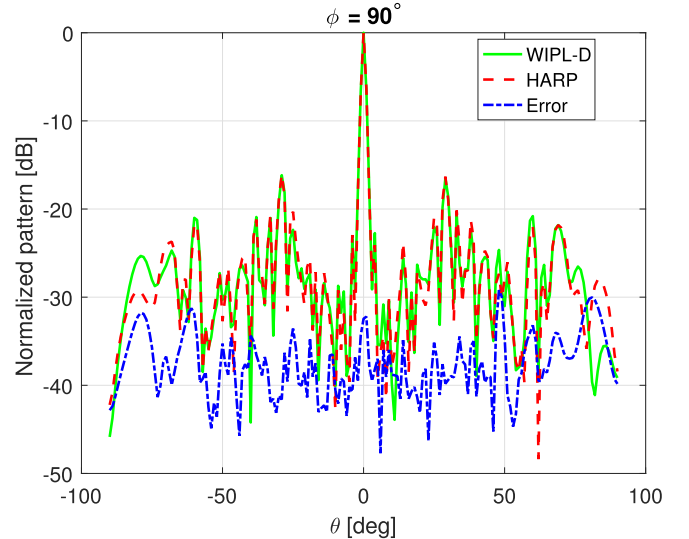


Fig. 13. Normalized radiation pattern of an SKA station at 350 MHz after the difference between isolated pattern is added to all EEPs; in H-plane ($\phi = 90^\circ$) cut.

TABLE I
COMPUTATIONAL TIME OBTAINING EEPs OF AN SKA1-LOW STATION AT 110 MHz

	CST	WIPL-D	HARP
Simulation Time	96 hours	97 hours	0.5 min.

partially undone by properly adding the difference of isolated patterns between HARP and WIPL-D into the EEPs, as shown in Fig. 13. A better agreement is clearly seen with respect to Fig. 12(d), especially near the main beam. This exercise illustrates that the difference between WIPL-D and HARP at highest frequency is mainly due to the modeling of the SKA antennas in these two software packages. Expectedly, this fine modeling difference becomes more visible at high frequency.

Finally, the performance of WIPL-D, CST, and HARP for the analysis of an SKA station at 110 MHz is detailed in Table I. A great time saving factor is achieved using the enhanced HARP, while providing highly accurate results, as demonstrated in Fig. 12. The CST simulations were carried out on a server with two processors and two cores each with 384 GB of RAM at the University of Cambridge, Cambridge, U.K., whereas the WIPL-D ones were performed on a Windows Server 2008 R2 Enterprise with an Intel Xeon X7550 processor and 125 GB RAM at ASTRON, Dwingeloo,

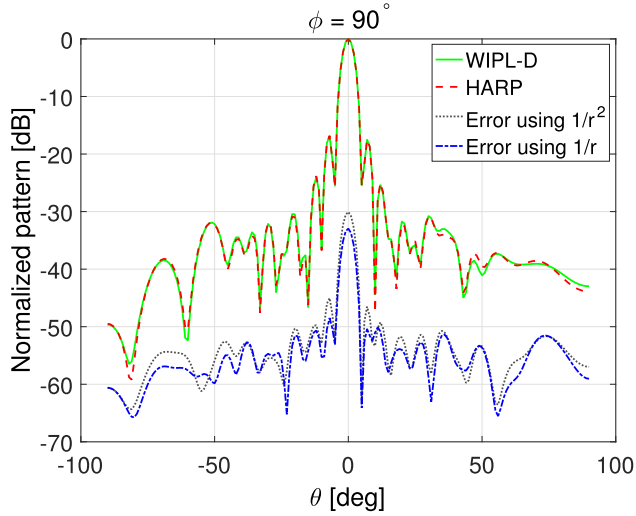


Fig. 14. Array pattern of the SKA station in $\phi = 90^\circ$ plane at 110 MHz and the errors of HARP (with respect to WIPL-D) using $1/r^2$ and $1/r$ change of variables.

TABLE II

ANALYSIS TIME FOR AN SKA STATION USING HARP AT 110 MHz (IN SECONDS)

Operations	This paper	Ref. [27]
Matrix Filling	10	350
Solving MoM equations	12.6	12.6
Calculation EEPs	6	1234
Total	28.6	1596.6

The Netherlands. All the simulations for HARP were carried out on a desktop computer with 16 GB RAM and Processor Intel Core *i7* – 3770 CPU 3.4 GHz at UCL. For HARP, the accounted time stands for matrix filling, solving the system of equations and calculation of all EEPs once the HARP model is built, as reported in Table II. The establishment of the HARP model takes about 3 h per frequency. It is important to recall that this preparation phase takes place only once and for all, regardless of the array configuration, and can be reused to analyze any array made of the same element. This feature is very beneficial for simulating SKA arrays, where hundreds of stations are planned, as well as to analyze extremely large arrays, as will be discussed in Section IV. It is worth noting that the previous version of HARP [27] took nearly half an hour for the same simulation. The reduction comes from the new implementation of the matrix filling as well as the use of NFFT pattern calculation as presented in Section II-C. The enhanced HARP also offered more accurate results as shown in Fig. 3, as well as in Fig. 14, where the errors in SKA station pattern using two versions of HARP are plotted. An improvement of 3 dB in the main beam and the first sidelobes is observed using $1/r$ change of variables. This enhancement is crucial to accurately analyze the array, especially in the upper frequency band.

IV. SIMULATION OF EXTREMELY LARGE ARRAYS

This section shows the performance of the sparse matrix and tessellation techniques on the analysis of very large arrays. The impact of the RI on the array patterns is studied on

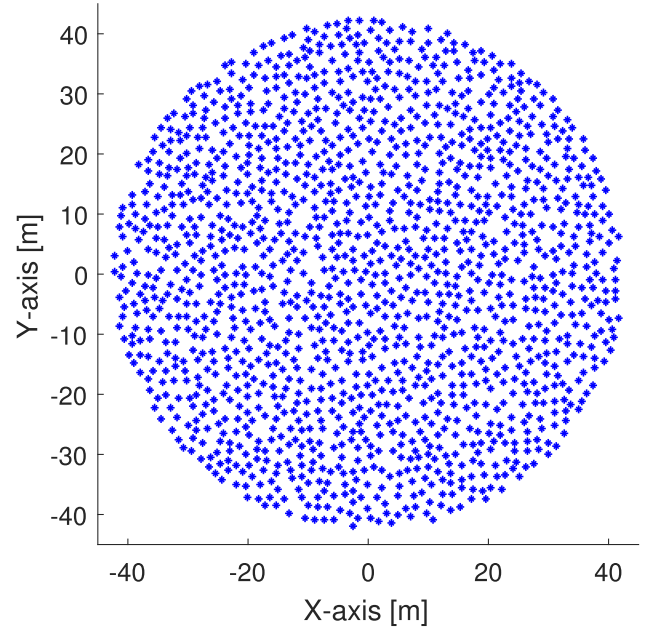


Fig. 15. Layout of 1500 SKALA element arrays with the minimum distance of 1.35 m.

TABLE III

COMPUTATIONAL COMPLEXITY BY SPARSE MATRIX AND TESSELLATION APPROACHES

	Sparse Matrix	Tessellation
Filling Matrix	$\mathcal{O}(N_a N_I N_m^2)$	$\mathcal{O}(N_a N_s N_m^2)$
Solving	$\mathcal{O}(N_a N_I^2 N_m^3)$	$\mathcal{O}(N_a N_s^2 N_m^3)$

where N_I and N_s are average number of elements within the RI and the subarray, respectively.

arrays of different sizes containing 256, 500, 800, 1000 and 1500 elements. The latter has a diameter of 85 m, as shown in Fig. 15, and the same array density has been considered for all the other arrays. The arrays are simulated at 110 MHz using HARP; EEPs and the array pattern are calculated. It is noted that the array of 1500 elements cannot be handled using CST nor WIPL-D with our current hardware, and the "exact solutions" considered in this section are obtained by simulating the full array without any tessellation (i.e., using only the methods described in Sections II-B and II-C).

A. Sparse Matrix Approach

Fig. 16 shows the exact patterns and the error for different RIs in E-plane ($\phi = 0^\circ$) for the 1500 elements array. The errors are evaluated using (8), where $\vec{E}(\theta, \phi)$ and $\vec{E}^{\text{ref}}(\theta, \phi)$ stand for the exact pattern and those obtained using the sparse array approach, respectively. It is clear that for larger values of the RI, the errors are smaller, since more elements are included in the RI region. It is also seen that, using only the isolated pattern, the error level is higher than the sidelobe level at far-out angles. Even for $\text{RI} = 30$ m, the error is close to some sidelobes. The results again show the strong effect of mutual coupling in a random array made of SKALA elements. Fig. 17 displays the convergence of error curves versus the RI for different array sizes [i.e., number of elements (NoE)] in the

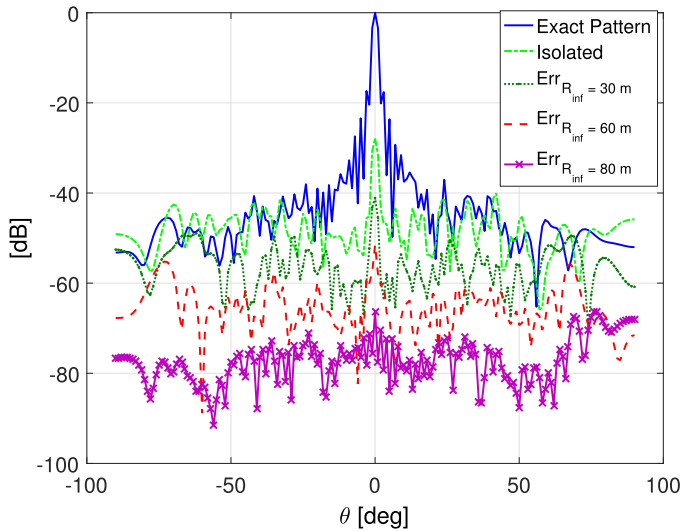


Fig. 16. Normalized radiation pattern of the array in Fig. 15 at 110 MHz in E-plane ($\phi = 0^\circ$) and errors for different RIs in two principal planes using the sparse matrix approach.

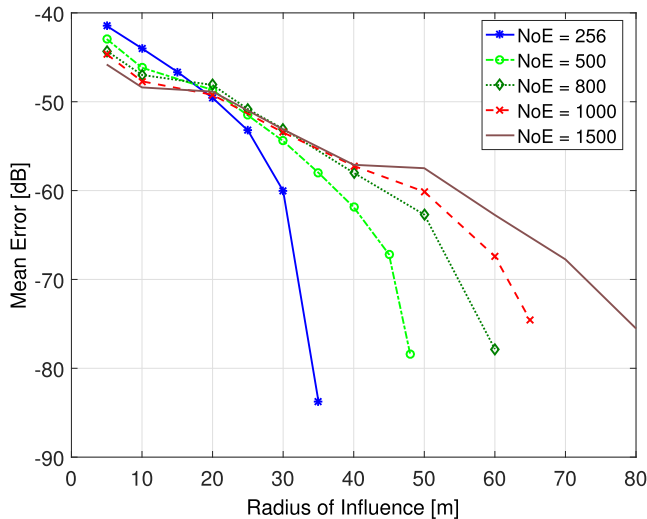


Fig. 17. Mean errors versus RI for different array sizes using the sparse matrix approach at 110 MHz.

arrays are indicated in the legends] at 110 MHz. It shows the trend of reducing errors for larger RI and one can observe that to achieve a given error level, larger arrays require a smaller RI (normalized to array size). This is probably due to the slow averaging effect of randomization of the array on mutual coupling. As the RI is smaller (with respect to array size) for large arrays, the interaction matrix will be sparser, which implies the capability to analyze large arrays using the RI concept. The computational complexity of the approach is detailed in Table III, where N_I is the average number of elements within the RI, which is proportional to RI^2 .

B. Tessellation Approach

Fig. 18 shows the radiation patterns and errors using the tessellation approach on the array of 1500 SKALA elements shown in Fig. 15 for $R_{in} = 10$ m and different R_{ext} values.

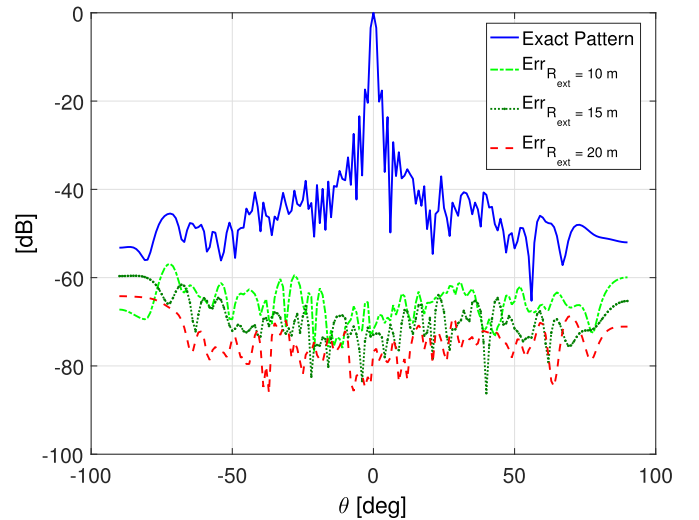


Fig. 18. Normalized radiation pattern of the array in Fig. 15 at 110 MHz in E-plane ($\phi = 0^\circ$) and errors using the tessellation approach in two principal planes.

It is interesting to see that tessellation is able to provide high-quality solutions with respect to the exact ones as the error levels are quite low even for small R_{ext} . Despite sharing the same RI idea as the sparse array approach, the tessellation seems to perform better. The reason lies in the fact that even if the sparse array approach only accounts for the interaction between elements inside the RI, the solution of the resulting system of equations still provides nonzero currents for elements outside of the RI region, while the excited elements remain within the RI. This "artificial" current contributes to the radiation patterns, while probably not being highly relevant nor accurate. Conversely, the tessellation approach only solves for the subarray and calculates the contribution of those elements inside the subarray.

Fig. 19 shows the convergence of the tessellation approach for different array sizes. As R_{ext} increases, the error decreases almost linearly in logarithmic scale. This allows one to obtain a given accuracy by finding a proper R_{ext} . Increasing R_{ext} , however, results in increasing the computational complexity, both for matrix filling and for solving the system of equations. An optimum choice of R_{ext} can be found following (7), as discussed in Section II-D; it favors an extended region comprising half of all neighboring inner hexagons. A detailed complexity of the tessellation approach is listed in Table III, where N_s is the average number of elements in a subarray, which is proportional to $(R_{in} + R_{ext})^2$ with $R_{in} = R_{ext}$.

By selecting N_I and N_s , i.e., choosing RI and R_{in} , respectively, one can obtain the same complexity for both methods. For extremely large arrays, N_I and N_s are at least one order of magnitude smaller than N_a , which results in a big saving factor compared to the direct solution over the whole array.

It is also interesting to see that the convergence curve in Fig. 19 is quite independent of the array size, which assures the performance of the tessellation approach for other (larger) arrays, and allows one to select R_{in} and R_{ext} in advance. Regarding the analysis, the tessellation approach offers the possibility of parallelizing the computation as the analysis of

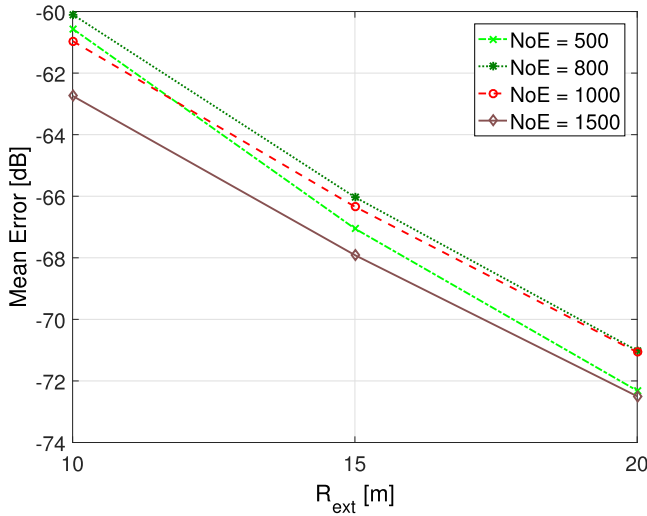


Fig. 19. Mean errors versus R_{ext} for different array sizes using the tessellation approach at 110 MHz.

each subarray is totally independent. Moreover, the tessellation is not limited by the array size as each subarray can be solved effectively with a present-day desktop computer, providing enough temporary memory RAM to store the reduced MoM matrix and solutions, e.g., 16 GB has been proven sufficient in all tested cases. For the largest array, i.e., consisting of 1500 SKALA antennas, the analysis took about 15 min using the tessellation approach to simulate the array and calculate all the EEPs.

V. CONCLUSION

A fast full-wave technique for the analysis of large arrays made of complex 3-D metallic antennas has been presented. The calculation of interactions between antennas has been accelerated using the HARP technique, where a nonexpensive model is built for MBF interactions. The reduced matrix is then rapidly filled using such model. Several improvements have been added to HARP to cope with the complexity of solving very large arrays. An experimental example of compact array and simulations of SKALA arrays have demonstrated the performance of HARP. Furthermore, the sparse matrix and the tessellation approach have been proposed for analyzing very large arrays of SKALA elements, offering a solution to the analysis of extremely large arrays, such as the SKA telescope, for which hundreds of stations are foreseen. The sparse matrix converged relatively slowly and required large RIs to achieve accurate result. However, the proposed tessellation approach showed excellent performance for all tested arrays. The approach enables the analysis of very large arrays on present-day desktop computers, while providing highly accurate results.

The simulation technique opens the possibility to integrate the full-wave simulation in an optimization procedure [42], which is capable of taking into account mutual coupling to optimize the performance of the arrays (e.g., in terms of sensitivity, including noise coupling). Moreover, the accurate EEPs obtained using HARP can be incorporated in the

calibration stage for modern radars and astronomical instruments such as the SKA. The use of the PMCHWT (Poggio–Miller–Chang–Harrington–Wu–Tsai) approach [43], combined with the surface equivalence principle [44], allows the extension of HARP to antennas comprising dielectric material.

ACKNOWLEDGMENT

The authors would like to thank the Square Kilometer Array (SKA) colleagues for numerous discussions about SKA station modeling.

REFERENCES

- [1] D. Vollbracht, "System specification for dual polarized low power X-band weather radars using phased array technology," in *Proc. Int. Radar Conf.*, Lille, France, Oct. 2014, pp. 1–6.
- [2] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186–195, Feb. 2014.
- [3] S. W. Ellingson, "Sensitivity of antenna arrays for long-wavelength radio astronomy," *IEEE Trans. Antennas Propag.*, vol. 59, no. 6, pp. 1855–1863, Jun. 2011.
- [4] E. de Lera Acedo, N. Razavi-Ghods, L. E. Garcia, P. Duffett-Smith, and P. Alexander, "Ultra-wideband aperture array element design for low frequency radio astronomy," *IEEE Trans. Antennas Propag.*, vol. 59, no. 6, pp. 1808–1816, Jun. 2011.
- [5] P. E. Dewdney, P. J. Hall, R. T. Schilizzi, and T. J. L. W. Lazio, "The square kilometre array," *Proc. IEEE*, vol. 97, no. 8, pp. 1482–1496, Aug. 2009.
- [6] *The Square Kilometer Array*. Accessed: Jan. 31, 2017. [Online]. Available: <https://www.skatelescope.org/>
- [7] N. Razavi-Ghods, E. de Lera Acedo, A. El-Makadema, P. Alexander, and A. Brown, "Analysis of sky contributions to system temperature for low frequency SKA aperture array geometries," *Experim. Astron.*, vol. 33, no. 1, pp. 141–155, 2012.
- [8] E. de Lera Acedo, N. Razavi-Ghods, N. Troop, N. Drought, and A. J. Faulkner, "SKALA, a log-periodic array antenna for the SKA-low instrument: Design, simulations, tests and system considerations," *Experim. Astron.*, vol. 39, no. 3, pp. 567–594, 2015.
- [9] C. Craeye and D. González-Ovejero, "A review on array mutual coupling analysis," *Radio Sci.*, vol. 46, no. 2, pp. 1–25, 2011.
- [10] A. Neto, S. Maci, G. Vecchi, and M. Sabbadini, "A truncated floquet wave diffraction method for the full wave analysis of large phased arrays. I. Basic principles and 2-D cases," *IEEE Trans. Antennas Propag.*, vol. 48, no. 4, pp. 594–600, Apr. 2000.
- [11] J.-M. Song and W. C. Chew, "Multilevel fast-multipole algorithm for solving combined field integral equations of electromagnetic scattering," *Microw. Opt. Technol. Lett.*, vol. 10, no. 1, pp. 14–19, Sep. 1995.
- [12] X. Zhao, Z. Lin, Y. Zhang, S.-W. Ting, and T. K. Sarkar, "Parallel hybrid method of HOMoM—MLFMA for analysis of large antenna arrays on an electrically large platform," *IEEE Trans. Antennas Propag.*, vol. 64, no. 12, pp. 5501–5506, Dec. 2016.
- [13] E. Suter and J. R. Mosig, "A subdomain multilevel approach for the efficient MoM analysis of large planar antennas," *Microw. Opt. Technol. Lett.*, vol. 26, no. 4, pp. 270–277, Aug. 2000.
- [14] S. J. Kwon, K. Du, and R. Mittra, "Characteristic basis function method: A numerically efficient technique for analyzing microwave and RF circuits," *Microw. Opt. Technol. Lett.*, vol. 38, pp. 444–448, Sep. 2003.
- [15] R. Maaskant, R. Mittra, and A. Tjihuis, "Fast analysis of large antenna arrays using the characteristic basis function method and the adaptive cross approximation algorithm," *IEEE Trans. Antennas Propag.*, vol. 56, no. 11, pp. 3440–3451, Nov. 2008.
- [16] L. Matekovits, V. A. Laza, and G. Vecchi, "Analysis of large complex structures with the synthetic-functions approach," *IEEE Trans. Antennas Propag.*, vol. 55, no. 9, pp. 2509–2521, Sep. 2007.
- [17] L. Matekovits, G. Vecchi, M. Bercigli, and M. Bandinelli, "Synthetic-functions analysis of large aperture-coupled antennas," *IEEE Trans. Antennas Propag.*, vol. 57, no. 7, pp. 1936–1943, Jul. 2009.
- [18] W. B. Lu, T. J. Cui, Z. G. Qian, X. X. Yin, and W. Hong, "Accurate analysis of large-scale periodic structures using an efficient sub-entire-domain basis function method," *IEEE Trans. Antennas Propag.*, vol. 52, no. 11, pp. 3078–3085, Nov. 2004.

- [19] X. Wang, D. H. Werner, and J. P. Turpin, "A fast analysis of scattering from large-scale finite periodic microstrip patch arrays arranged on a non-orthogonal lattice using sub-entire domain basis functions," *IEEE Trans. Antennas Propag.*, vol. 62, no. 5, pp. 2543–2552, May 2014.
- [20] C. Craeye, "A fast impedance and pattern computation scheme for finite antenna arrays," *IEEE Trans. Antennas Propag.*, vol. 54, no. 10, pp. 3030–3034, Oct. 2006.
- [21] D. González-Ovejero, F. Mesa, and C. Craeye, "Accelerated macro basis functions analysis of finite printed antenna arrays through 2D and 3D multipole expansions," *IEEE Trans. Antennas Propag.*, vol. 61, no. 2, pp. 707–717, Feb. 2013.
- [22] D. González-Ovejero and C. Craeye, "Interpolatory macro basis functions analysis of non-periodic arrays," *IEEE Trans. Antennas Propag.*, vol. 59, no. 8, pp. 3117–3122, Aug. 2011.
- [23] D. J. Ludick, R. Maaskant, D. B. Davidson, U. Jakobus, R. Mittra, and D. de Villiers, "Efficient analysis of large aperiodic antenna arrays using the domain Green's function method," *IEEE Trans. Antennas Propag.*, vol. 62, no. 4, pp. 1579–1588, Apr. 2014.
- [24] D. J. Ludick, R. Maaskant, D. B. Davidson, and U. Jakobus, "Accelerating the domain Green's function method through adaptive cross approximation," in *Proc. Int. Conf. Electromagn. Adv. Appl. (ICEAA)*, Palm Beach, FL, USA, 2014, pp. 636–639.
- [25] S. N. Jha and C. Craeye, "Contour-FFT based spectral domain MBF analysis of large printed antenna arrays," *IEEE Trans. Antennas Propag.*, vol. 62, no. 11, pp. 5752–5764, Nov. 2014.
- [26] Q. Gueuning, E. de Lera Acedo, E. Colin-Beltran, and C. Craeye, "Mutual coupling analysis of large irregular arrays: From multipole to interpolatory methods," in *Proc. 9th Eur. Conf. Antennas Propag.*, Lisbon, Portugal, 2015, pp. 1–4.
- [27] Q. Gueuning, C. Raucy, C. Craeye, E. Colin-Beltran, and E. de Lera Acedo, "Mutual coupling analysis in non-regular arrays of SKALA antennas with the HARP approach," in *Proc. AP-S*, 2015, pp. 1528–1529.
- [28] E. de Lera Acedo, "SKALA: A log-periodic antenna for the SKA," in *Proc. Int. Conf. Electromagn. Adv. Appl. (ICEAA)*, 2012, pp. 353–356.
- [29] H. B. Van, J. Abraham, Q. Gueuning, E. de Lera Acedo, and C. Craeye, "Further validation of fast simulation method at the element and array pattern levels for SKA," in *Proc. 10th Eur. Conf. Antennas Propag.*, Davos, Switzerland, Apr. 2016, pp. 1–4.
- [30] E. de Lera Acedo, N. Razavi-Ghods, D. G. Ovejero, R. Sarkis, and C. Craeye, "Compact representation of the effects of mutual coupling in non-regular arrays devoted to the SKA telescope," in *Proc. Int. Conf. Electromagn. Adv. Appl. (ICEAA)*, Turin, Italy, Sep. 2011, pp. 390–393.
- [31] C. Craeye, D. González-Ovejero, and X. Dardenne, "Efficient numerical analysis of arrays of identical elements with complex shapes," *J. Appl. Comput. Electromagn. Soc.*, vol. 24, no. 2, pp. 224–232, Apr. 2009.
- [32] A. Dutt and V. Rokhlin, "Fast Fourier transforms for nonequispaced data," *SIAM J. Sci. Comput.*, vol. 14, no. 6, pp. 1368–1393, 1993.
- [33] A. Capozzoli, C. Curcio, G. D'Elia, A. Lisenio, and P. Vinetti, "Fast CPU/GPU pattern evaluation of irregular arrays," *J. Appl. Comput. Electromagn. Soc.*, vol. 25, no. 4, pp. 355–372, Apr. 2010.
- [34] (2015). *CST Microwave Studio*. [Online]. Available: <https://www.cst.com/>
- [35] *WIPL-D—Electromagnetic Simulation Software Version 2016*. [Online]. Available: <http://www.wipl-d.com/>
- [36] A. Boag, E. Michielssen, and A. Brandt, "Nonuniform polar grid algorithm for fast field evaluation," *IEEE Antennas Wireless Propag. Lett.*, vol. 1, no. 1, pp. 142–145, 2002.
- [37] *MATLAB R2016a*, The MathWorks, Natick, MA, USA, 2016.
- [38] T. A. Davis, *Direct Methods for Sparse Linear Systems*. Philadelphia, PA, USA: SIAM, Sep. 2006. [Online]. Available: <http://dx.doi.org/10.1137/1.9780898718881>
- [39] C. Balanis, *Advanced Engineering Electromagnetics*. Hoboken, NJ, USA: Wiley, 1989.
- [40] C. Craeye, B. Parvais, and X. Dardenne, "MoM simulation of signal-to-noise patterns in infinite and finite receiving antenna arrays," *IEEE Trans. Antennas Propag.*, vol. 52, no. 12, pp. 3245–3256, Dec. 2004.
- [41] K. F. Warnick, M. V. Ivashina, R. Maaskant, and B. Woestenburg, "Unified definitions of efficiencies and system noise temperature for receiving antenna arrays," *IEEE Trans. Antennas Propag.*, vol. 58, no. 6, pp. 2121–2125, Jun. 2010.
- [42] T. Clavier *et al.*, "A global-local synthesis approach for large non-regular arrays," *IEEE Trans. Antennas Propag.*, vol. 62, no. 4, pp. 1596–1606, Apr. 2014.
- [43] A. J. Poggio, E. K. Miller, and R. Mittra, "Integral equation solutions of three-dimensional scattering problems," in *Computer Techniques for Electromagnetics*. Oxford, U.K.: Pergamon, 1973.
- [44] C. Craeye, T. Gilles, and X. Dardenne, "Efficient full-wave characterization of arrays of antennas embedded in finite dielectric volumes," *Radio Sci.*, vol. 44, no. 1, pp. 1–12, Feb. 2009.



Ha Bui-Van received the electronics and telecommunication engineering from the Hanoi University of Technology, Hanoi, Vietnam, in 2010, and the Ph.D. degree in electrical engineering from the Politecnico di Milan, Milan, Italy, in 2014.



In 2014, he joined the Politecnico di Turin, Turin, Italy, as a Research Assistant. Since 2014, he has been a Post-Doctoral Researcher with the Antenna Group, Institute of Information and Communication Technologies, Electronics and Applied Mathematics, Université catholique de Louvain, Louvain-la-Neuve, Belgium. His current research interests include the study of mutual coupling in antenna arrays, fast simulation techniques for large irregular arrays, including the Square Kilometer Arrays (SKA-low and SKA-mid), and array optimizations.

Jens Abraham received the Dipl.Ing. degree in electrical engineering from TU Dresden, Dresden, Germany, in 2015.

From 2015 to 2017, he was a Research Assistant with the Astrophysics Group, Cavendish Laboratory, University of Cambridge, Cambridge, U.K. Since 2017, he has been a Ph.D. Fellow with the Department of Electronics, Norwegian University of Science and Technology, Trondheim, Norway. His current research interests include measurements and modeling of massive MIMO radio channels.



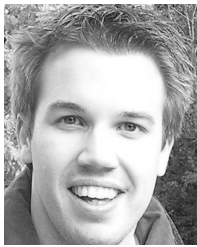
Michel Arts was born in Deurne, The Netherlands, in 1968. He received the M.Sc. degree in electrical engineering from the Eindhoven University of Technology, Eindhoven, The Netherlands, in 1992. From 1993 to 1995, he was involved in a post M.Sc. Program Information and Communication Technology with the Stan Ackermans Institute, Eindhoven University of Technology.

Since 1995, he has been an Antenna Design Engineer with ASTRON, the Netherlands Institute for Radio Astronomy, Dwingeloo, The Netherlands, where he was involved in several projects such as low frequency array, the phased array demonstrators such as thousand element array and electronic multibeam radio astronomy concept, and the phased array feed for the westerbork synthesis radio telescope. His current research interests include array modeling and phased array design for radio astronomy.



Quentin Gueuning received the B.Sc. and M.Sc. degrees from the Université catholique de Louvain (UCL), Louvain-La-Neuve, Belgium, in 2012 and 2014, respectively, where he is currently pursuing the Ph.D. degree with the Institute of Information and Communication Technologies, Electronics and Applied Mathematics (ICTEAM).

Since 2014, he has been a Research Assistant with ICTEAM, UCL. His current research interests include high-frequency and fast numerical methods for electromagnetic radiation.



Christopher Raucy received the master's degree in electro-mechanical engineering from the Ecole Polytechnique de Louvain, Louvain-la-Neuve, Belgium in 2011 and the Ph.D. degree in mathematics from the Ecole Mathématique de Louvain, Louvain-la-Neuve, with a focus on the Myrrha RFQ simulation with the MoM.

He was with the Université catholique de Louvain, Louvain-La-Neuve, where he was involved in the Square Kilometer Array in collaboration with the University of Cambridge, Cambridge, U.K. His current research interests include electronics and fundamental physics.



Eloy de Lera Acedo received the Ph.D. degree in multimedia and communication from Carlos III University, Madrid, Spain, in 2010.

His Ph.D. was co-supervised by the University of Cambridge, U.K., where he stayed the 2 final year. He is currently a Senior Research Fellow with the Astrophysics Group, Cavendish Laboratory, University of Cambridge, Cambridge, U.K., where he has developed his career on instrumentation for radio astrophysics. He leads the Novel Sensors Research Group, Cavendish Astrophysics, and the Antenna and LNA Work Package (AL WP) of the Aperture Array Design Consortium, which is developing the array stations for the Square Kilometer Array (SKA) telescope. Within the AL WP, he has led the design and development of SKALA, the SKA log-periodic antenna and its LNA. He has also led substantial pieces of work in other international projects such as the Hydrogen Epoch of Reionization Array for which he has led the design of the wideband feed within the Analog Team Group. His current research interests include ultrawideband array and antenna design, electromagnetic modeling, telescope calibration, and low-noise systems to the broader science case of 21 cm cosmology experiments.



David González-Ovejero (S'01–M'13–SM'17) was born in Ghent, Spain, in 1982. He received the master's degree in telecommunication engineering from the Universidad Politécnica de Valencia, Valencia, Spain, in 2005, and the Ph.D. degree in electrical engineering from the Université catholique de Louvain, Louvain-la-Neuve, Belgium, in 2012.

From 2006 to 2007, he was a Research Assistant with the Universidad Politécnica de Valencia. In 2007, he joined the Université catholique de Louvain, where he was a Research Assistant until

2012. From 2012 to 2014, he was a Research Associate with the University of Siena, Siena, Italy. In 2014, he joined the Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA, USA, where he was a Marie Curie Post-Doctoral Fellow. He was a Research Scientist with the French National Center for Scientific Research, Institut d'Electronique et de Télécommunications de Rennes, Rennes, France.

Dr. González-Ovejero was a recipient of a Marie Curie International Outgoing Fellowship from the European Commission 2013, the Sergei A. Schelkunoff Transactions Prize Paper Award from the IEEE Antennas and Propagation Society in 2016, and the Best Paper Award in Antenna Design and Applications at the 11th European Conference on Antennas and Propagation in 2017.



Christophe Craeye (M'98–SM'11) was born in Ronse, Belgium in 1971. He received the B.Ph. degree in electrical engineering and the Ph.D. degree in applied sciences from the Université catholique de Louvain (UCL), Louvain-la-Neuve, Belgium, in 1994 and 1998, respectively.

From 1994 to 1999, he was a Teaching Assistant with UCL, where he was involved in the radar signature of the sea surface perturbed by rain, in collaboration with NASA and ESA. From 1999 to 2001, he was a Post-Doctoral Researcher with the

Eindhoven University of Technology, Eindhoven, The Netherlands, where he was involved in wideband phased arrays devoted to the Square Kilometer Array radio telescope. In 1999, he was with the University of Massachusetts, Amherst, MA, USA. In 2001, he was with the Netherlands Institute for Research in Astronomy, Dwingeloo, The Netherlands. In 2002, he started an Antenna Research Activity with the Université catholique de Louvain, where he is currently a Professor. In 2011, he was with the Astrophysics and Detectors Group, University of Cambridge, Cambridge, U.K. His current research interests include finite antenna arrays, wideband antennas, small antennas, metamaterials, and numerical methods for fields in periodic media, with applications to communication and sensing systems, funded by Région Wallonne, European Commission, ESA, FNRS, and UCL.

Prof. Craeye was a recipient of the 2005–2008 Georges Vanderlinden Prize from the Belgian Royal Academy of Sciences in 2009. He was an Associate Editor of the IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION from 2004 to 2010 and the IEEE ANTENNAS AND WIRELESS PROPAGATION LETTERS from 2011 to 2017.