

Admissible geometrical domains within the context of Graphic Statics for evaluating constitutive elements of structural robustness

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Abstract

Geometrical domains characterising the degrees of freedom of a structural system within the context of Maxwell's reciprocal representation of force and geometry are likely to provide indicators of the constitutive elements of structural robustness. Structural robustness is defined as the "insensitivity to local failure". This definition emphasises the structure's capacity for force redistribution and the possibility of finding alternative load paths in a structure. Features linked to resistance and the redistribution of forces are likely to be modelled by load path, struts and ties or thrust lines, close to geometrical thinking.

Since most methods proposed today for assessing the robustness of structures are based on probabilistic approaches, they are of limited interest for the design phase. Of the few approaches that have adopted a deterministic formulation, all provide a type of survey that is based on an in-depth analysis of the structure once it has been designed, according to specific scenarios. A central challenge should be to manage the issue of robustness earlier during the design process, or even to be able to interact with a model of the future structure in order to adjust the features of robustness. This paper explores the ability of geometrical domains to assess some of the relevant elements of structural robustness in terms of design in order to characterise the capacity of structures to redistribute forces. It compares indicators linked to the area of these domains. Based on case studies, this quantification of the structural provision is then compared with indices of deterministic and energetic criteria currently proposed in literature to quantify structural robustness.

Keywords: robustness, graphic statics, structural design, strut-and-tie modelling, thrust lines, force redistribution

1. Introduction

Most methods proposed today for assessing the robustness of structures are based on probabilistic approaches (Cacavo *et al.* [1]). Of the few that have adopted a deterministic formulation, all provide a type of survey that is based on an in-depth analysis of the structure once it has been designed, according to specific scenarios. A central challenge in structural design is to manage the issue of robustness earlier on in the design process, or even be able to interact with a model of the future structure in order to adjust features of robustness. This paper contains an overview of a geometrical approach for evaluating constitutive elements of structural robustness. This research is linked to modelling methods and the analysis and refinement of structural designs using geometrical tools almost exclusively, even if they are implemented by means of computers and dynamic geometry. This is a way of simplifying analyses and making them more visual, enabling the designer to interact with the structure during the early stages of its design.

The paper begins with a literature review of methods to assess robustness issues and explains how they have been interpreted in the context of the geometrical approaches taken. It then introduces the geometrical methods that provide the origin of this geometrical approach to robustness (Fivet and Zastavni [2]). A characterisation of two case studies is then presented to highlight the major features of the geometrical results when considering undamaged and damaged structures. Elements of deterministic and energetic approaches are then explored and compared with the geometrical assessment. The paper finally provides conclusions and future recommendations with regard to the advocated approach.

2. The issue of robustness in literature

A series of methods are proposed in literature to characterise robustness (Cacavo *et al.* [1]; Sorensen *et al.* [3]). Four major approaches can be identified: risk-based, probabilistic, deterministic and energetic approaches. Risk-based approaches and probabilistic approaches are adopted by specialists and require very specific methods. In short, probabilistic approaches can be evaluated by a reliability-based index linked to redundancy that compares the probability of ruptures as in Frangopol & Curley [4]:

$$RI = \frac{P_{f(damaged)} - P_{f(intact)}}{P_{f(intact)}} \quad (1)$$

Risk-based approaches are based on a comparison of direct and indirect risks (Baker, Schubert and Faber [5]):

$$I_{rob} = \frac{R_{Dir}}{R_{Dir} + R_{Ind}} \quad (2)$$

They are said to be of limited practical interest (Sorensen *et al.* [3]). This is certainly the case at the design stage. Deterministic and energetic approaches provide indicators produced by structural analyses. They will be used below as references in the assessment of the two case studies presented here.

2.1. Deterministic approaches

A deterministic approach is proposed by Frangopol and Curley [4] as the application of a reserve strength factor based on the Residual Influence Factor used in the offshore industry. It compares the structural capacity of intact and damaged structures where an element has been completely damaged.

A simple way of appropriating this approach is to compare the load capacity of damaged and intact structures according to chosen scenarios:

$$R = \frac{L_{intact}}{L_{intact} - L_{damaged}} \quad (3)$$

2.2. Energetic approaches

Energetic approaches classically consist in calculating the deformation energy (work of failure) of a structure led to failure (Smith [6]). It consists in integrating the space below the curve that characterises the stain-stress relationship of the structure up to the collapse point.

The energetic approach considered here was slightly different and was adapted from the deterministic approach of Starossek and Haberland [7,1]: $R = 1 - \max_j (E_{r,j}/E_{f,k})$

where $E_{r,j}$ is the energy released by the initial failure of an element j and available for the damage of the next structural element k , and $E_{f,k}$ is the energy required for the failure of the next structural element k . The appropriation of the method in this study consisted in dividing the structure into its elements. The most fragile element was researched as being the one that reduced stiffness most. For a model made of bars, the stiffness matrix was calculated and $Kx = f$ was solved. The deformation energy of the system was $1/2 x^T K x$. The structural elements i considered in the scenarios were removed and $K_i x_i = f$ was calculated. The difference in deformation energy in each structural element was calculated with the deformation energy before and after the element was removed.

3. Geometrical domains of available equilibriums

The originality of the approach depicted in this paper comes mainly from its use of graphical representations of solution spaces within reciprocal diagrams. An introduction to these concepts can be found in Deschuyteneer, Zastavni and Fivet [13] and Fivet and Zastavni [8].

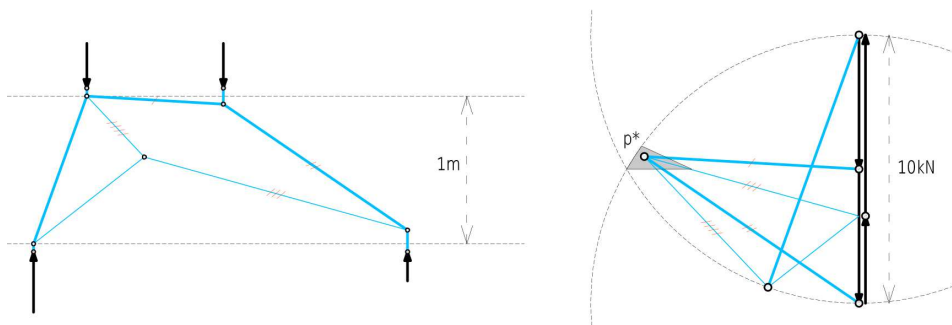


Figure 1: The shaded area in the force diagram (right) is the solution space of the node p^* such that the strut-and-tie network (left) is not more than 1 metre high and the magnitudes are below 10kN.

4. Geometrical approach to robustness

Robustness is defined as “insensitivity to local failure” (Starossek [9]). This definition emphasises the capacity of force redistribution in a structure. In other words, it is about the possibility of finding alternative load paths in a structure.

Under certain conditions, the geometrical domains presented above are a convenient tool for exploring the possible redistributions of forces in a strut-and-tie model or a model made of thrust lines, and hence for characterising its robustness. The first condition is the necessary aptitude in the structure for developing a plastic redistribution of forces so that the lower bound theorem of plastic design can be applied. Such behaviour is commonly assumed for steel frames, concrete frames, arches and shear walls, masonry structures, timber with screw or threaded rods etc.

4.1. Presentation and methods

According to the analysis of dimensions of robustness proposed by Knoll & Vogel [10, 11], five dimensions of robustness are likely to concern the design more directly: strength, second line of defence, multiple load paths and redundancy, stiffness considerations and post-buckling resistance.

Of the 16 strategies proposed by the Knoll and Vogel, not all of them are applicable simultaneously. Some of them are related to the ductility of the structure or its constitutive elements, making a link with the theorems of plastic design that provide the scope of application of the approach presented in this paper. Others are specific to the erection of the structure, its life and maintenance, or disruptive elements to be implemented.

Implementing the dimensions related to robustness during the design – geometry and dimensioning – means (inter)acting with the design, with key milestones mainly associated with the designer’s experience. Features linked to the resistance and redistribution of forces are likely to be modelled by load paths, struts and ties or thrust lines, close to geometrical thinking.

The key idea is to associate a load path made of struts, ties and/or thrust lines with the structure. In the context of constraint-based graphic statics, allowable stresses and spatial limits are likely to be represented by geometrical constraints applied to this load path. The extent to which a node of this load path is free to move can then be seen as a measure of the model’s capacity to redistribute loads. The assumption will be that the integral (in the mathematical meaning of a sum) of relevant characterising domains represents an interactive measure of the total level of robustness.

5. Application to study cases

The analysis of the geometrical domains characterising two structures is here performed for different scenarios of integrity: the whole intact structure, variations due to damages and variations of design geometry. The first set of scenarios referred to a comparison of the capacity of redistribution between the undamaged structure and damaged structures according to several scenarios. The second series, implementing geometrical variations, showed the influence of design choices on the capacity to redistribute loads and hence robustness. Here, the extent of geometrical domains was understood to be a constitutive dimension of the robustness. The two study cases analyse (1) a concrete shear wall with openings and (2) the *Ponte della Musica* in Rome, Italy.

5.1. Case study 1: Concrete shear wall

The first case study is adapted from the classic example developed by Schlaich, Schäfer and Jennewein [12]. The original example is a shear wall resting on its two extreme sides, including a square opening near to the left support and loaded by a vertical punctual force at around 3/5 of its length. In this analysis, to complicate this almost trivial matter and turning it into a real structural issue, the square opening became a door at the bottom of the structure and a new rectangular opening was added to the upper part of the structure. The loading scheme included seven punctual forces of varying magnitude.

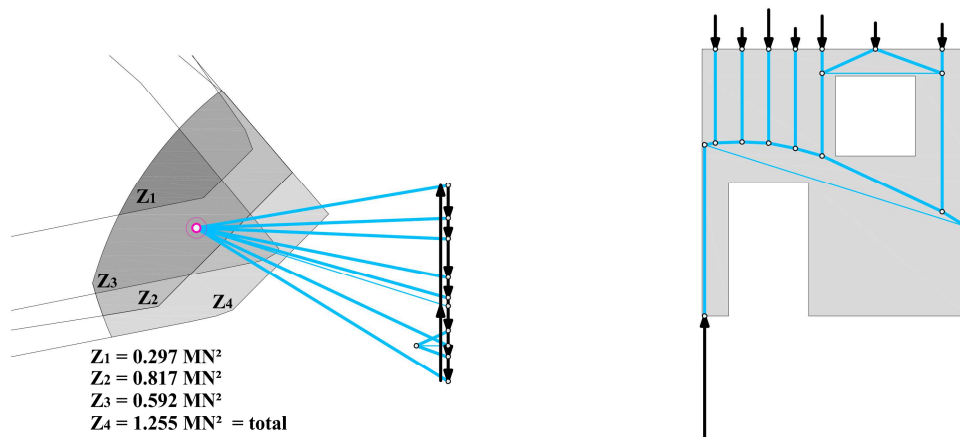


Figure 2: Shear wall and geometrical domains of the highlighted pole: damage of the column-support (Z3), damage of the support to the right of the shear wall (Z1), and displacement of the reactions under the wall in different parts of the wall (Z2, Z4). The scenario corresponding to the reaction forces as represented in the right-hand figure is linked to Z2 for a domain of 0,817 MN².

Struts and ties are modelled inside a non-Bernoulli shear wall (Fig. 2, right). The analysis of the structure as presented here is partial since it only considers a few characteristic typologies out of all the possible strut-and-tie models. The final global domain would be the sum of all the possible sub-domains for a given set of strut-and-tie models.

In this case, the set comprises the following structural mechanisms: vertical struts supported by a tied arch (Fig.2), inverted arch (Fig.3 a), three branching system (Fig.3 b) and fan-like suspended load path (Fig.3 c). The study of these various domains helps the modification of the structural configuration. For instance, new load paths can be allowed by adding reinforcement, which will consequently increase robustness.

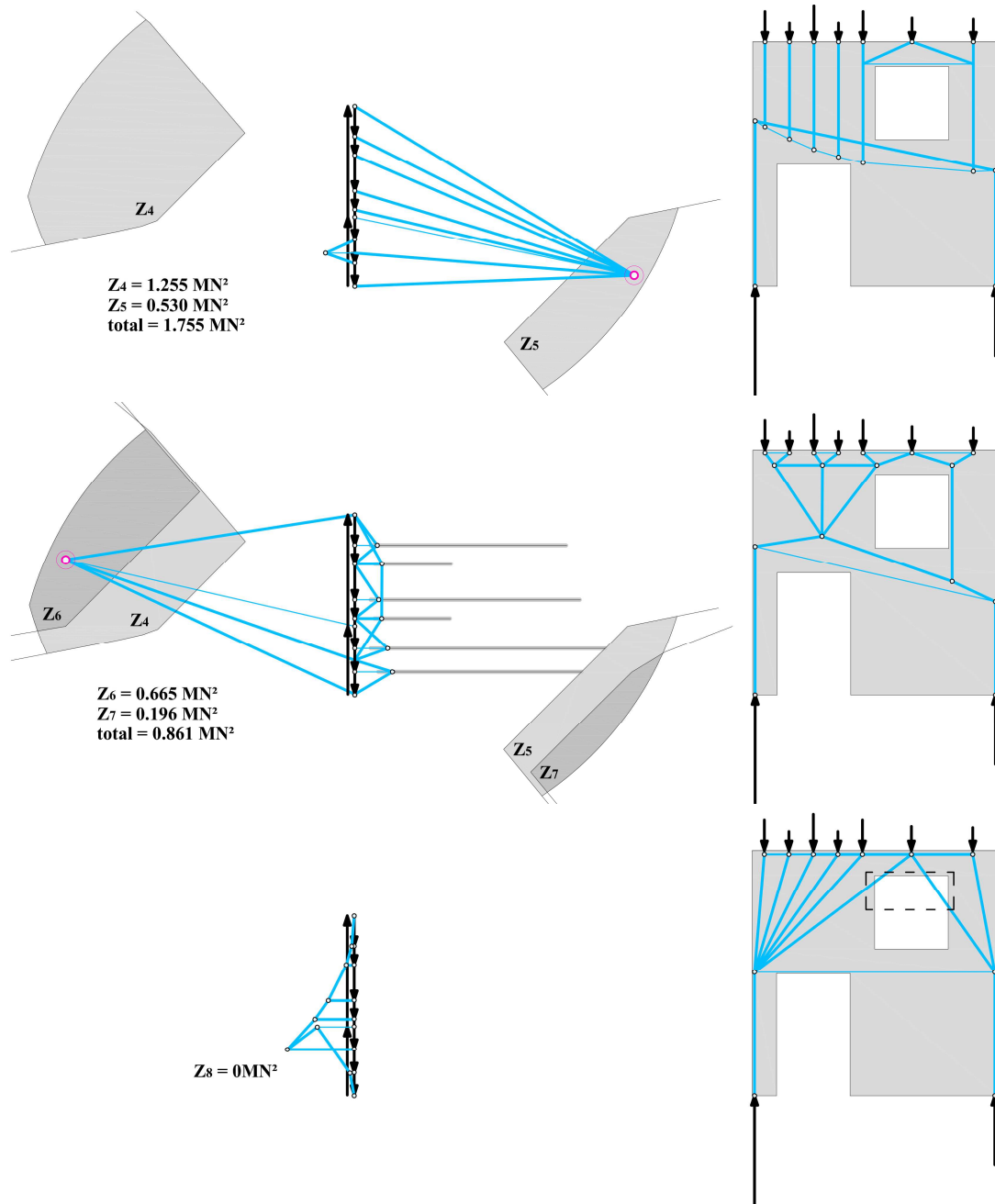


Figure 3: (a, b and c) domains resulting from various load-paths.

The analysis of the basic structural working – funicular compressed load path – has also been tested with an increased door in order to simulate changes of symmetry of the shear wall (Fig. 4). This can be considered as a design variant of positioning the edge of the door otherwise.

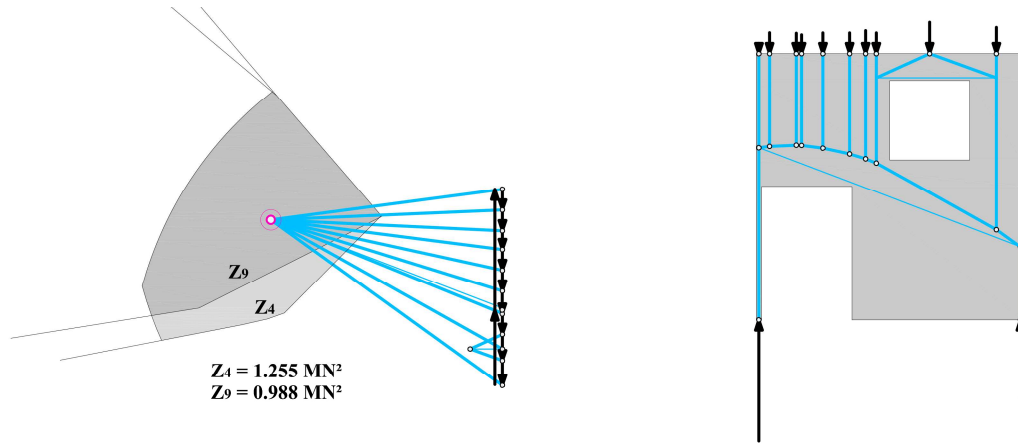


Figure 4: Shear wall and geometrical domain corresponding to the pole of the discharging arch when a variant in the design is taken with a thin column in the form diagram of Fig. 2

The geometrical analysis shows a sensible reduction of the possible variation of the pole in the case of a damaged support (Z3 only in Fig. 2) corresponding to 0,592 MN² compared to an undamaged arrangement (Z4 in Fig. 2). The geometrical Z4 domain characterises the possibility of finding variations in the drawing of the load path leading the forces to the support.

If, for instance, the supporting left column disappears, the domain moves. Similarly, if the width of the left column is modified (Fig. 4), the possible base geometrical domain of (Fig. 2) is displaced. The domain will be reduced in comparison to the base scenario of Z4. It is therefore a worse option for a robustness-oriented design. Diagrams may also show the cases where there is no solution according to the design constraints, or where the existence of a solution requires an increase in the magnitude of forces (constrained by the circular boundaries) and therefore a revision of the dimensioning.

In summary, a correlation can be shown between the aptitude of the structure to redistribute forces – comprised as an indicator of some constitutive dimensions of the robustness – and a geometrical characterisation of the permitted variations of the position of nodes constituting a strut-and-tie modelling of the structure.

5.2. Case study 2: Ponte della Musica

The *Ponte della Musica* in Rome (Fig. 5) was built in 2011 in Italy by the architect Kit Powell-Williams and engineers C. Lotti & Associati and BuroHappold. It is a hybrid typology between a steel arch bridge and a bow-string bridge with a clear span of 130m. The hangers are made of rigid steel profiles moving forwards towards the longitudinal central symmetrical axis of the structure. The bridge is used as a footbridge, but is likely to be used to carry buses and trams as well. The exercise consists in simulating the possible redistributions of forces in the arch according to different support conditions. Finally it compares these redistributions with those allowed when some of the hangers sustaining the deck are damaged.

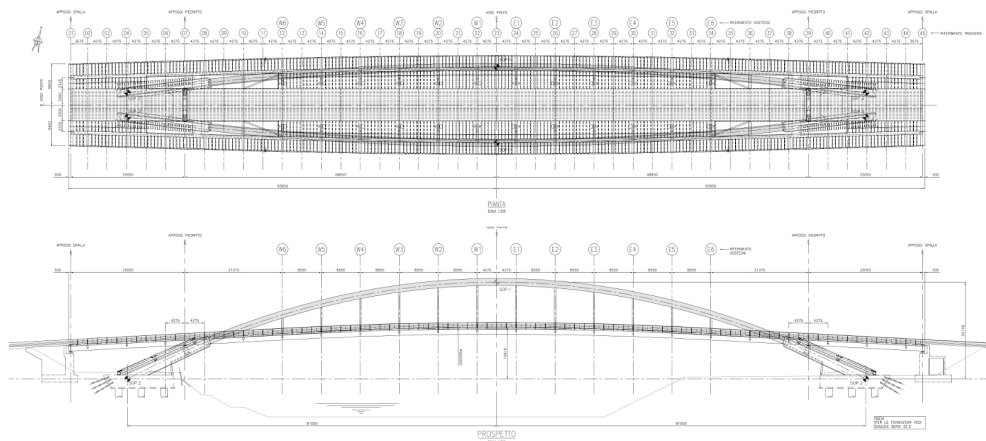


Figure 5: *Ponte della Musica*, Rome 2011, Powell-Williams, Lotti & Associati and Buro Happold

Firstly, the bridge's bending resistance is analysed. Bending forces are modelled as thrust lines, the off-centring of which is related to the magnitude of the axial compression forces (Fig. 6). Using graphic statics, the geometry of this thrust line is actually defined by a single point in the force diagram. The domain of this point consequently informs all the possible configurations of bending resistance. Other domains are then generated for altered structures in which hangers are damaged. In the first instance, the structural collaboration between hangers and the arch are neglected.

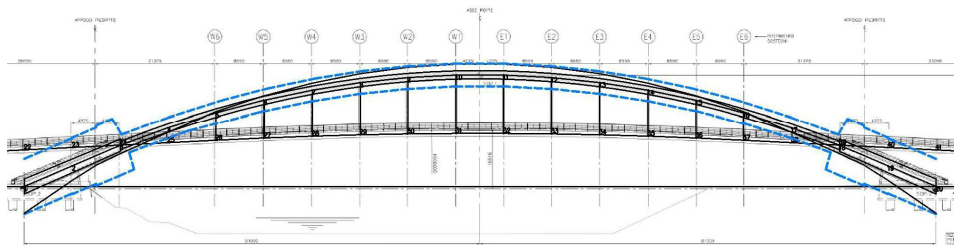


Figure 6: *Ponte della Musica*: resistance to bending forces shown in blue as off-centring, with a possible thrust line in the case of limited bending forces in the support.

The result of the analysis (Fig.7) shows a domain (Z1) of $18,8 \text{ MN}^2$ for the pole of the thrust line, defining the extent of possible geometries for the load path in the case of symmetrical loading. In the case of damage to four central hangers (Z2), this domain is reduced by 24 % to $14,4 \text{ MN}^2$, but still allows multiple load paths – and in the case of damage to three lateral hangers (Z3), this domain is only reduced by 9 % to $17,2 \text{ MN}^2$. This shows that this scenario reveals less damage with a view to redistributing forces. Under asymmetrical loadings, the intact structure (Z4) has the ability to redistribute load paths to an equivalent of $16,4 \text{ MN}^2$ (87 % of the symmetrical reference), *i.e.* slightly less than the reference maximum symmetrical loading. Where the four central hangers are damaged (Z5), the domain becomes $11,7 \text{ MN}^2$ (71 % of the asymmetrical reference and 62 % of the symmetrical reference) and in the case of damage to three lateral hangers (Z6), this domain is $16,35 \text{ MN}^2$ (99 % of the asymmetrical reference and 87 % of the symmetrical reference).

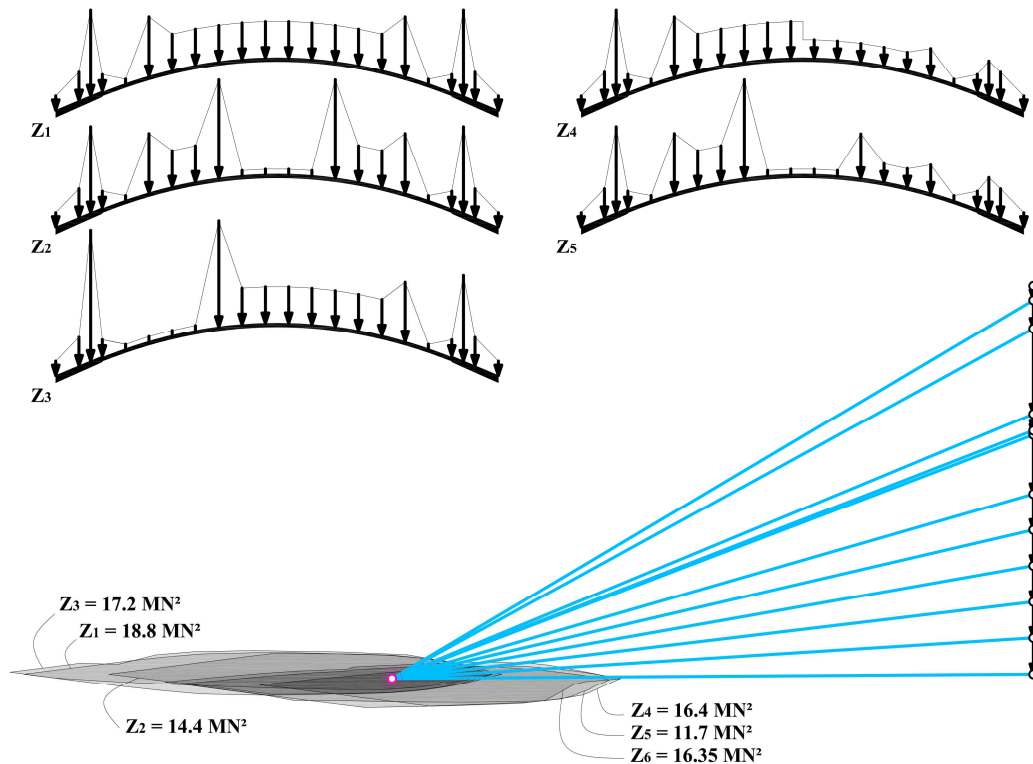


Figure 7: Pontede della Musica, domains for different loadings and levels of damage: intact with symmetrical load (Z1) and with asymmetrical load (Z4); 4 central hangers removed with symmetrical load (Z2) and with asymmetrical load (Z5); 3 lateral hangers removed with symmetrical load (Z3) and asymmetrical load (Z6).

The slightly different position of this asymmetrical domain demonstrates the ability of the structure to redistribute bending forces on both sides of the arch, enabling the reduction of the domain to be less significant.

6. Comparison of deterministic and energetic approaches to robustness

6.1. Deterministic approach: Ponte della Musica

A simulation was undertaken for the previous 6 different cases in order to obtain the ratio of service loadings that would lead to the failure of the structure. The structure is modelled as an arch for a maximum bending resistance of 29,6 MN.m (considering the axial compression force in the arch) in steel tubes and 146 MN.m in the concrete bases of the arch, with an elastic embedding in the foundation of 60 MN.m/degree enabling a plastic redistribution of forces. The results are given below:

<i>Multiplying coefficient</i>	Symmetrical Q	Asymmetrical Q
Intact	23,8	2,49
4 central hangers damaged	3,93	1,84
3 lateral hangers damaged	4,13	2,05

The factor for the symmetrical loading was quite large, leading to a complex comparison between the different cases. If the 74 % reduction between the damaged and undamaged structure in the asymmetrical case is taken as a temporary reference, the reduction is 77 % in the geometrical approach (symmetrical loads). However without more investigation the comparison cannot be maintained further between these different indices.

6.2. Energetic approach: Ponte della Musica

The energetic approach used here is a stiffness-based measure of robustness expressed in energies computed as described above in paragraph 2.2. The cases and the energies obtained for services loading are:

<i>Energies [MJ]</i>	Symmetrical Q	Asymmetrical Q
Intact	0,308	0,254
4 central hangers damaged	0,369	0,321

The cases and the energies obtained for yielding loading (as computed in 6.1) are:

<i>Energies [MJ]</i>	Symmetrical Q	Asymmetrical Q
Intact	174,56	1,58
4 central hangers damaged	5,82	1,09

During the service phase, the total asymmetric loading is half the symmetric total loading, so for a similar total loading, energies in the asymmetric case would be greater. As has been seen in 6.1, the damage occurs earlier if loading is asymmetrical, so the work is reduced. A comparison between the undamaged and damaged bridge shows that the energy of deformation is greater in the latter case, since there would be greater deformations between the start and the point when the failure occurs. Again, it is observed that both measures – the geometrical domain approach and the energetic approach – are not directly correlated, as they are not correlated to the deterministic approach shown in 6.1. Nevertheless, a similar evolution between different measures can be observed, but at variable scales. The impact of the asymmetrical loading is observed for the two approaches, but to a rather different extent. Indeed, one analysis refers to the complete structure (energetic approach) and the other to the extent of the possible redistribution of load paths. The parallel evolution of the indicators of both approaches cannot be sustained further without more extensive research and an attempt to correlate these measures more closely.

7. Comparison through direct results and indices

7.1. Comparison of the various results

Four types of values characterising the robustness are here analysed: a geometrical approach, a deterministic approach and two energetic approaches, one assessing the service state and the other assessing the state of yielding. These approaches compare very different types of magnitudes. The geometrical approaches compare only forces, redistributing load paths while loadings are maintained; the deterministic approach considers only forces, with load paths unchanged and loadings multiplied to reach yielding; energetic approaches consider forces and displacements, with load paths unchanged, one during service state, and the other multiplied loadings to reach the yielding state. This produces quite different values and extents as well.

7.2. Comparison through indices

Two families of indices are proposed in the literature:

Type A: $I[1 \dots \infty] = \frac{X_{\text{intact}}}{X_{\text{intact}} - X_{\text{damaged}}}$ according to Frangopol and Curley [4] or Baker, Schubert and

Faber [5] where X is the values to be compared. When energies growing with damage are considered, $X_{\text{damaged}} - X_{\text{intact}}$ replaces the denominator.

Indices of robustness		SLS Geometric	SLS Energetic	Yield limit Energetic	Yield limit Deterministic
Symmetrical loading	4 centre damaged	4.27	5.05	1.03	1.2
	3 side damaged	11.9	(lacking)	1.11	1.21
Asymmetrical loading	4 centre damaged	3.49	3.79	3.22	3.83
	3 side damaged	328	(lacking)	(lacking)	5.66

Type B: $I[0 \dots 1] = 1 - \frac{X_{\text{intact}} - X_{\text{damaged}}}{X_{\text{intact}}}$ according to Frangopol and Curley [4] with X the values to compare. When energies are considered the numerator becomes $X_{\text{damaged}} - X_{\text{intact}}$.

Indices of robustness		SLS Geometric	SLS Energetic	Yield limit Energetic	Yield limit Deterministic
Symmetrical loading	4 centre damaged	0.77	0.8	0.03	0.17
	3 side damaged	0.92	(lacking)	0.1	0.17
Asymmetrical loading	4 centre damaged	0.71	0.74	0.69	0.74
	3 side damaged	0.99	(lacking)	(lacking)	0.82

8. Conclusions and perspectives

This paper presents a geometrical approach to evaluate constitutive elements of structural robustness and compares it to other indices from literature. The geometrical approach proposed here proves to be

of interest during the design phase since it provides a qualitative summary of the possible load path redistributions. The analysis shows that the nature of the indices used in literature greatly differ since they are related to very different types of magnitudes. The geometrical index as proposed here is no exception. However the analysis shows that when put in the form of indices, their relative magnitude and evolution show similar evolutions according to the different parameters. This unexpected observation shows that their characterisation of the remaining robustness nevertheless appears to be convergent. However, further research must be carried to confirm the meaningfulness of these comparisons.

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