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Testing for the Interconnection channel

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Summary When modeling the dynamics of a large cross section of interdependent variables, the employment of small scale Vector Autoregressive (VAR) models augmented by few factors is the typical solution to avoid the proliferation of parameters in large heterogeneous VARs. The factors' are typically extracted as a linear combination of the variables under analysis. Their loadings/weights can be estimated or derived from economic literature, and are usually interpreted as interconnection channels. We propose a novel Likelihood Ratio Test procedure to empirically evaluate the chosen set of weights, and show that testing is fundamental for valid inferences. We exploit the intuition that, if the factors employed are empirically valid, no residual information from the cross section remains statistically significant. The proposed test is intuitive, easy to implement, and presents very good finite sample properties. In the empirical exercise, we test several interconnection channels for the sovereign bond market in the euro area.

Keywords: *Global VARs, FAVARs, Likelihood Ratio Test, Interdependence.*

1. INTRODUCTION

Markets are interconnected. Interconnections can arise from various sources, for example, common risk factors, shared resources, cross-border effects, spillovers, and many others. Modeling contemporaneous and dynamic dependencies across markets is fundamental for meaningful inferences. However, large heterogeneous Vector Autoregressive models (*VARs*) suffer from the so-called *curse of dimensionality* –the number of parameters to estimate close to, or higher than, the number of observations available. The employment of conditional small-scale *VARs* augmented by a weighted average/linear combination of foreign counterparts represents a typical solution to the proliferation of parameters.[1] Weights/loadings are either derived from economic intuition (e.g., trade shares or financial flows) or estimated (e.g., static or dynamic Principal Components). Forecasts, scenario analyses, Impulse Responses and shock transmission interpretations vary with the specific weighting scheme imposed.

In this paper, we propose a novel Likelihood Ratio test procedure for the specific set of weights employed. First, we show that only empirically valid weighting schemes effectively reduce the parameter space without distorting the reduced form *VAR* representation of the full cross section under analysis. Second, we derive an intuitive testable condition for the validity of the proposed weighting scheme. If the weighting scheme is empirically valid, no significant residual information from foreign counterparts is present. Consequently, we design a Likelihood Ratio test for the significance of the residual information coming from each counterpart. Third, in a simulation exercise featuring the typical cross-sectional

[1]The large Bayesian *VAR* literature (see, for example, Bańbura et al., 2010; Koop and Korobilis, 2013; Koop et al., 2019) focuses instead on assigning prior probability distributions on the *VAR* parameters.

and time series dimensions encountered in the literature, we prove that the test quickly converges to the asymptotic chi-square distribution. Finally, in the empirical application, we show how to implement the test to the debate around the interconnection channel across long-term sovereign bond spreads in the euro area during crises.

Our paper fills an important gap regarding specification tests in the analysis of cross-sectionally dependent time series in large heterogeneous *VARs* setting. Chudik and Pesaran (2011) divide existing solutions to the *curse of dimensionality* in large *VARs* into shrinking either the parameters' space, or the data. For example, the spatial dynamic literature (see, Elhorst et al., 2014, for an extensive literature review) proposes the shrinkage of the parameters' space by *ex ante* dividing markets into neighbours and non-neighbours. Non-neighbouring markets are assumed to carry negligible aggregate effects. Regarding the shrinkage of the data, the Factor Augmented *VAR* literature (see, for example, Bernanke et al., 2005; Stock and Watson, 2005, henceforth *FAVAR*) proposes to exploit the underlying assumptions that few common factors explain the dynamic behaviour of the large cross-section of variables under analysis. Therefore, they focus on the extraction of such factors to employ as relevant variables when modelling large cross-sections.

Chudik and Pesaran (2011) propose a third way, shrinking only part of the parameter space in the limit as the number of variables tends to infinity. Compared with spatial *VARs*, while the neighbourhood effects are fixed and do not change with the number of nodes, the coefficients of non-neighbouring units are only restricted to vanish when the cross section dimension grows to infinity. They show that the coefficients associated with neighbouring units can be consistently estimated by employing local cross-sectional averages augmented *VAR* models through Least Squares estimation, in the spirit of the Global *VAR* (*GVAR*) literature started by Pesaran et al. (2004). Compared with *FAVARs*, such framework can be effectively employed to obtain a full reduced form *VAR* representation of the cross-section, useful for structural analysis through, for example, *Generalized Impulse Response* analysis (Koop et al., 1996; Pesaran and Shin, 1998; Pesaran and Smith, 1998), or for analysing spillovers (see, for example, Crespo Cuaresma et al., 2019).

GVAR indeed builds on local Vector Autoregressive (*VAR*) models, augmented by the so-called *star* variable, that is, the weakly exogenous weighted average of foreign variables, resulting in local $VARX^*$ models (Harbo et al., 1998; Pesaran et al., 2000). Once estimated, local systems are solved simultaneously to obtain the large reduced form *VAR* representation of the world, unveiling the complex dynamic and contemporaneous interconnections among all the variables under analysis.

However, even if Chudik and Pesaran (2011) derive the regularity conditions under which *any* weighting scheme allows the consistent estimation of the local $VARX^*$ parameters, we prove that only an empirically valid specification of the weighting scheme allows the local estimated parameters to deliver a valid reduced form *VAR* representation.

Our paper contributes to the vast literature developing testing strategies for weights/factors selection underlying large cross section of variables. In the context of spatial modeling, Liu and Lee (2019) and Delgado and Robinson (2015) design specification tests for selecting weighting schemes among (non-nested) competitive alternatives. However, different channels might carry complementary information to jointly consider (see, for example, Debarsy et al., 2018). Moreover, such tests do not consider the validity of the set of weights proposed for testing. In the context of factor modeling, several testing strategies have been proposed, see, for example, Bai and Ng (2002), Onatski (2010), Ahn and Horenstein (2013), and Bai and Ng (2019). However, the selection of factors in the context of a dynamic high-dimensional framework remains challenging, in particular when variables under analysis exhibit heterogeneity not captured by factor pervasiveness (see, Chao and Swanson, 2022). Moreover, our paper speaks to the recent literature testing the validity of factors selected by exploiting the relevance of the remaining information in the cross section, see, for example, Fan et al. (2024) and Beyhum and Striaukas (2024). Lastly, we relate our idea of testing specific information coming from each variable to Chudik et al. (2018).

Our paper can be also closely linked to the $GVAR$ literature. Since the seminal contribution of Pesaran et al. (2004), $GVAR$ models have been employed in forecasting macroeconomic and financial variables (Pesaran et al., 2009; Greenwood-Nimmo et al., 2012), in credit risk modeling (Pesaran et al., 2006; Castrén et al., 2010), in identifying the interconnection between uncertainty and economic activity (Cesa-Bianchi et al., 2020), in analysing the macroeconomic effects of weather shocks (Cashin et al., 2017), as well as in studying international macroeconomic linkages (Dees et al., 2007; Favero et al., 2011; Crespo Cuaresma et al., 2019). For an extensive literature review, see Di Mauro and Pesaran (2013), Pesaran (2015), or Chudik and Pesaran (2016). Interestingly, both dynamic spatial and factor augmented *VARs* have been connected to $GVAR$ models. On the one hand, Elhorst et al. (2021) outline the structure, interpretation of spillovers, and the estimation correspondence between spatial and Global VAR models. On the other hand, Dees et al. (2007) show that $GVAR$ represent a suitable approximation of factor models. Furthermore, Candelon et al. (2022) show how to effectively interpret (static) factor loadings (see, for example, Bai et al., 2008) as cross-country interlinkages in a $GVAR$ fashion.

In such a framework, our paper fills a gap regarding specification tests. In fact, specification tests in $GVAR$ literature (see, for example Dees et al., 2007) regard parameters' stability and lag length decisions once the weighting scheme has been chosen. Cross-sectional dependence tests (Bailey et al., 2016) only guide the choice between sparse or dense interaction matrices, and outline the correct estimation procedure to follow (see, Elhorst et al., 2021). However, they do not propose a validation test, thus not distinguishing between empirically valid sparse or dense interconnections. Traditionally, the underlying assumption in $GVAR$ modeling is that the more two markets/countries interact in terms of trade, the more interconnected they are. However, such an assumption has been challenged in empirical literature. Lane and Shambaugh (2010) show how, in financial applications, weighting schemes based on financial asset exposures perform better than trade based ones. Favero and Missale (2012), Favero (2013), and Favero and Missale (2016) show how, in the context of sovereign yields modeling, fiscal fundamentals better reflect investors' expectations compared with traditional trade channels. Gross et al. (2018) modify $GVAR$ models including several different weighting schemes for macro-financial analyses. Moreover, Gross (2019) shows how different estimated $GVAR$ weights are when compared to trade based ones also in standard macroeconomic applications. Lastly, some papers adopt a time-varying approach to capture dynamic changes in the interconnection channels (see, for example, Cesa-Bianchi et al., 2012; Favero, 2013; Crespo Cuaresma et al., 2019).

Those studies illustrate how the choice of different weighting schemes could lead to different inferences, structural analyses, and forecasts. Hence, raising the key question of how to adequately determine $GVAR$ weights and/or test for the validity of the proposed ones. To the best of our knowledge, we are the first ones to design a test for the empirical validity of the proposed weighting scheme.

Let us introduce some notation. $\mathbb{R}$ indicates the set of real numbers, $\mathbb{N}$ the set of natural numbers, $\mathbb{Z}$ the set of integers. $\mathcal{N}$ indicates the Normal distribution, χ^2 indicates the Chi-square distribution , $\mathcal{U}$ indicates the Uniform distribution. Given a real valued $n \times n$ matrix A, let $\lambda_1(A), \lambda_2(A), \dots, \lambda_n(A)$ be the eigenvalues of such matrix. The spectral radius of A is defined as, $\varrho(A) \equiv \max\{|\lambda_1(A)|, \lambda_2(A), \dots, |\lambda_n(A)|\}$. $\|A\| \equiv \sqrt{\varrho(A'A)}$ indicates the spectral norm. Given a real valued $n \times p$ matrix B, we define the operators $P_{[B]} \equiv B(B'B)^{-1}B'$ and $M_{[B]} \equiv I_n - P_{[B]}$. The former is an orthogonal projector onto the space spanned by all the linear combinations of the columns of B. The latter projects onto the orthogonal space unspanned by B.

Given a process $\nu_{b,c}$ with $b \in \mathbb{N}$ and $c \in \mathbb{Z}$, $\nu_{b,c} \overset{i.d.}{\sim}$. indicates that the process is independently distributed according to a specific distribution (left as . in the previous notation), when the distribution is not specified, the mean and the variance will be specified. L indicates the lag operator such that $L^d\nu_{b,c} = \nu_{b,c-d}$ with $d \in \mathbb{Z}$, when the superscript d is omitted, we mean $L\nu_{b,c} = \nu_{b,c-1}$. $O(.)$ specifies the maximum order of a deterministic process, while $O_p(.)$ specifies the maximum order in probability of a random process.

The remainder of the paper is organized as follows, in Section 2 we list the assumptions under which large heterogeneous *VAR* can be expressed as a set of small scale *VARX** models, in Section 3 we summarize how to retrieve the global reduced form VAR representation once local *VARX** have been estimated, in Section 4 we describe the proposed Likelihood Ratio Test for the weighting scheme, in Section 5 we present the Monte Carlo experiment, in Section 6 we apply the test to the euro area long term sovereign bond market, and in Section 7 we present our conclusions.

2. FROM LARGE HETEROGENEOUS VAR TO LOCAL VARX*

Let us consider a system of N nodes, with N large, representing countries or regions or markets, indexed by $i = 1, 2, \dots, N$, observed over a certain period T, indexed by $t = 1, 2, \dots, T$. Each node features k_i local variables collected in the $k_i \times 1$ vector $Y_{i,t}$. We can collect all the node-specific variables in the $k \times 1$ vector $Y_t = (Y'_{1t}, \dots, Y'_{Nt})'$ with $k = \sum_{i=1}^N k_i$. The data generating process is assumed to be of the form,[2]

$$Y_t = \Phi Y_{t-1} + R\varepsilon_t, \tag{2.1}$$

the matrices Φ and R are of dimension $k \times k$, and capture dynamic and contemporaneous dependencies, respectively. The estimation of the elements of Φ is often unfeasible given the large number of parameters, larger than the available observations even with a moderate number of nodes and/or items.

Assumption 2.1. $\varepsilon_t \overset{i.d.}{\sim} (0, I_k)$.

Assumption 2.2. *R has bounded row and column matrix norms.*

Assumption 2.3. $\exists \epsilon \in \mathbb{R}$, *and* $0 < \epsilon < 1$, *such that,*

$$\|\Phi\| \leq 1 - \epsilon.$$

Assumptions 2.1 to 2.3 are sufficient conditions for the existence of the variance of Y_t even

[2]For ease of explanation, we abstract here from deterministic components and additional lags.

when the number of nodes and/or the number of items grows to infinity. Furthermore, Assumption 3 ensures that the polynomial $(I_k - \Phi L)$ is invertible, and that $\varrho(\Phi) \leq 1 - \epsilon$, sufficient condition for covariance stationarity (Chudik and Pesaran, 2011).

Let us introduce a set of m unobserved common factors, $f_t = (f'_{1,t}, f'_{2,t}, \dots, f'_{m,t})'$, such that,

$$(Y_t - \Gamma f_t) = \Phi(Y_{t-1} - \Gamma f_{t-1}) + R\varepsilon_t, \tag{2.2}$$

$m \in \mathbb{N}$ is fixed and unknown. $\Gamma = (\gamma_1, \gamma_2, \dots, \gamma_k)'$ is the $k \times m$ matrix of factor loadings.

ASSUMPTION 2.4. *f_t are covariance stationary, following the linear process,*

$$f_{s,t} = \sum_{l=0}^{\infty} \Psi_{s,l}(L^l)\varepsilon_{f_s,t}, \tag{2.3}$$

for $s = 1, 2, \dots, m$. The coefficients of the matrices $\Psi_{s,l}$ are absolute summable and not varying with k. Moreover, $\varepsilon_{f_s,t}$ are independently distributed, with constant variances $\sigma^2_{f_s}$ not varying with k, independently distributed from ε_t, and $\mathbb{E}(f_t f'_t)$ exists and is positive definite. Furthermore, $\mathbb{E}(\varepsilon^4_{f_s,t})$ and $\mathbb{E}(\varepsilon^4_{i,t})$ exist and are finite. Finally, each of the loadings collected in Γ are bounded, and do not vary with N.

Consider *any* set of weights $\tilde{W}_i$, useful for computing cross-sectional averages, referred to as *star* variables, as $Y^*_{it} = \tilde{W}'_i Y_t$, that satisfies the following *granularity* conditions,

$$\|\tilde{W}_i\| = O(N^{-1/2}), \tag{2.4}$$

$$\frac{\|\tilde{W}_{ij}\|}{\|\tilde{W}_i\|} = O(N^{-1/2}), \tag{2.5}$$

with $\tilde{W}_{ij}$ being the blocks in the partitioned form of $\tilde{W}_i = (\tilde{W}'_{i1}, \tilde{W}'_{i2}, \dots, \tilde{W}'_{iN})'$ (Chudik and Pesaran, 2016). Chudik and Pesaran (2011) prove that under such assumptions,

$$Y^*_{it} = \tilde{W}'_i Y_t = \Gamma^*_i f_t + \xi^*_{it}, \tag{2.6}$$

with $\Gamma^*_i = \tilde{W}'_i \Gamma$, then $||\mathbb{E}(\xi^*_{it}\xi'^*_{it})|| = O(N^{-1})$ and $||\xi^*_{it}|| = O_p(N^{-1/2})$. Unobserved common factors can then be approximated by cross-sectional averages, justifying the following form

$$Y_{i,t} = \Phi_i Y_{i,t-1} + \Lambda_{i0} Y^*_{i,t} + \Lambda_{i1} Y^*_{i,t-1} + \epsilon_{i,t}, \tag{2.7}$$

where $\epsilon_{i,t} \overset{i.i.d.}{\sim} (0, \Sigma_i)$ is the $k_i \times 1$ idiosyncratic residual term for node i with variance-covariance matrix Σ_i of dimension $k_i \times k_i$, Φ_i is the $k_i \times k_i$ matrix of lagged coefficients, and Λ_{i0} and Λ_{i1} are the $k_i \times k^*_i$ matrices collecting the coefficients associated with the cross sectional averages.

3. FROM LOCAL VARX* BACK TO REDUCED FROM VAR

The estimation takes place at the local level in (2.7), we shall discuss the estimation features in the next subsection. To obtain the *GVAR* representation of the global economy, we need to simultaneously solve for all the domestic variables. Therefore, by stacking all local Vector Autoregressive augmented models in (2.7) we obtain the equivalent representation,

$$Y_t = \tilde{\Phi} Y_{t-1} + \tilde{\Lambda}_0 Y^*_t + \tilde{\Lambda}_1 Y^*_{t-1} + \epsilon_t, \tag{3.8}$$

where $Y_t^* = ({Y^*}'_{1,t}, \ldots, {Y^*}'_{N,t})'$ is the $k^* \times 1$ vector collecting all the node specific *star variables*, with $k^* = \sum_{i=1}^N k_i^*$, $\tilde{\Phi} = diag(\Phi_1, \ldots, \Phi_N)$ is of dimension $k \times k$, $\tilde{\Lambda}_0 = diag(\Lambda_{10}, \ldots, \Lambda_{N0})$ and $\tilde{\Lambda}_1 = diag(\Lambda_{11}, \ldots, \Lambda_{N1})$ are of dimension $k \times k^*$, and $\epsilon_t = (\epsilon'_{1,t}, \ldots, \epsilon'_{N,t})'$ is the $k \times 1$ vector obtained by stacking all the country specific residual terms from (2.7).

Moreover, we can re-write the vector Y_t^* in terms of the local variables' vector Y_t as

$$Y_t^* = \tilde{W} Y_t, \tag{3.9}$$

with $\tilde{W} = (\tilde{W}'_1, \ldots, \tilde{W}'_N)'$ being $k^* \times k$-dimensional matrix.

Therefore, we can specify (3.8) in terms of the local variables as

$$Y_t = \tilde{\Phi} Y_{t-1} + \tilde{\Lambda}_0 \tilde{W} Y_t + \tilde{\Lambda}_1 \tilde{W} Y_{t-1} + \epsilon_t. \tag{3.10}$$

Re-arranging the terms we obtain

$$[I_k - \tilde{\Lambda}_0 \tilde{W}] Y_t = [\tilde{\Phi} + \tilde{\Lambda}_1 \tilde{W}] Y_{t-1} + \epsilon_t. \tag{3.11}$$

We can define $G = I_k - \tilde{\Lambda}_0 \tilde{W}$, and $H = \tilde{\Phi} + \tilde{\Lambda}_1 \tilde{W}$, and express (3.11) as

$$G Y_t = H Y_{t-1} + \epsilon_t. \tag{3.12}$$

The matrix G is generally full rank and hence non-singular (Pesaran et al., 2004).[3] Therefore, we can express the $GVAR$ representation of the world as the reduced form VAR

$$Y_t = G^{-1} H Y_{t-1} + G^{-1} \epsilon_t, \tag{3.13}$$

or, equivalently,

$$Y_t = \Pi Y_{t-1} + \zeta_t, \tag{3.14}$$

$$\Pi = G^{-1} H, \tag{3.15}$$

$$\zeta_t = G^{-1} \epsilon_t. \tag{3.16}$$

4. THE LIKELIHOOD RATIO TEST FOR $\tilde{W}$

In this subsection, we aim at proposing and justifying a new testing strategy for the following set of hypotheses,

$$H_0 : \tilde{W} = W,$$
$$H_1 : \tilde{W} \neq W,$$

with W being the specific weighting scheme employed by the researcher.

In order to show the testing procedure for the specific matrix W proposed, it is important to first introduce the estimation technique of the parameters that characterize (2.7). Pesaran et al. (2006), and Chudik and Pesaran (2011) prove that the Least Squares estimator is a consistent estimator of the parameters of interest.[4] Interestingly, Elhorst et al. (2021) shows that the granularity conditions (2.4) and (2.5) are equivalent to the

[3] The case of rank deficient matrix G is discussed in Pesaran (2015).

[4] In the case of co-integration between the star and domestic variables, it is possible to either proceed with Least Squares (LS) estimation on first-differences or through reduced rank regressions as prescribed by Harbo et al. (1998) and Pesaran et al. (2000).

following,

$$\lim_{N\to\infty} \frac{\sum_{j=1}^{N} w_{i,j}}{\sqrt{N}} = \infty. \tag{4.17}$$

for each i, where $w_{i,j}$ are the elements of the matrix $\tilde{W}_i$. In this context, Lee (2002) and Aquaro et al. (2021) show that the Least Squares estimator is the (quasi) Maximum Likelihood (ML) estimator of the stacked system in (3.8) with the Likelihood written in the following form,

$$lnL_T(\hat{\Theta}_r) = -\frac{N}{2}ln2\pi - \frac{1}{2}ln|\hat{\Sigma}_{\epsilon}| + ln|\hat{G}| - \frac{1}{2(T-1)}\sum_{t=2}^{T} \hat{\epsilon}_t' \hat{\Sigma}_{\hat{\epsilon}}^{-1} \hat{\epsilon}_t, \tag{4.18}$$

with $\hat{\Theta}_r = \{\hat{G}, \hat{H}, \hat{\Sigma}_{\hat{\epsilon}}\}$ being the LS estimates. If the errors ϵ_t are normally distributed, the LS is equivalent to ML Estimators. In the case of non-Gaussians errors, $\hat{\Theta}_r$ denotes the quasi ML Estimators (Aquaro et al., 2021, derive the consistency and asymptotic normality of such estimators).

PROPOSITION 4.1. *Under the null hypothesis of validity of W, the Maximum Likelihood Estimates of the parameters in* (2.1) *are not statistically different from those obtained by the GVAR procedure.*

PROOF. Given the correspondence between (2.7) and the reduced form *VAR* representation in (3.14), $\hat{\Theta}_r$ represents the set of (quasi) Maximum Likelihood Estimators of the equivalent Likelihood forms,

$$\begin{aligned} lnL_T(\hat{\Theta}_r) &= -\frac{N}{2}ln2\pi - \frac{1}{2}ln|\hat{\Sigma}_{\hat{G}^{-1}\hat{\epsilon}}| - \frac{1}{2(T-1)}\sum_{t=2}^{T} (\hat{G}^{-1}\hat{\epsilon}_t)' \hat{\Sigma}_{\hat{G}^{-1}\hat{\epsilon}}^{-1} (\hat{G}^{-1}\hat{\epsilon}_t) = \\ &= -\frac{N}{2}ln2\pi - \frac{1}{2}ln|\hat{\Sigma}_{\hat{u}}| - \frac{1}{2(T-1)}\sum_{t=2}^{T} \hat{u}_t' \hat{\Sigma}_{\hat{u}}^{-1} \hat{u}_t. \end{aligned} \tag{4.19}$$

Under H_0, the Least Squares estimates of the local parameters in (2.7) correspond to the (quasi) Maximum Likelihood estimates of the parameters expressed as in the stacked system in (3.8), which, in turn, implies that such estimates maximize the Likelihood of the reduced form (3.14), and therefore they are the (quasi) Maximum Likelihood Estimates of (2.1). □

However, testing the full $\tilde{W}$ matrix would be extremely challenging. In a classic Likelihood Ratio test framework, we would need to directly estimate the Maximum Likelihood parameters in (2.1) to use as the unrestricted benchmark. As mentioned before, Global VARs represent a solution to the *curse of dimensionality.* Therefore, even with a moderate number of nodes, the estimation of the unrestricted parameters in (2.1) would be unfeasible. Moreover, testing for the full interconnection channel, W, would hide important information to the researcher. Specifically, one channel might be adequate for some of the nodes (or some of the items) and not for others.

Therefore, we propose to take a different route. Under H_0, no additional information from foreign nodes is relevant for any $i \neq j$, the interconnection between any node i and

j is then fully captured by the weighted average $Y^*_{i,t}$, built using the proposed W, and the estimation procedure. We can extract the *residual* information coming from *each* foreign counterpart's item as $\tilde{Y}_{j,i,t} = M_{[X_i]}Y_{j,i,t}$, with $X_i = (Y'_{i,t-1}, Y^{*\prime}_{i,t}, Y^{*\prime}_{i,t-1})'$.

Consider the following auxiliary regression,

$$Y_{i,t} = \Phi_i Y_{i,t-1} + \Lambda_{i0} Y^*_{i,t} + \Lambda_{i1} Y^*_{i,t-1} + \gamma'_{ji,l}\tilde{Y}_{j,l,t-1} + \tilde{\epsilon}_{i,t}, \tag{4.20}$$

where $\tilde{Y}_{j,l,t-1}$ represents the residual information contained in the l^{th} item for country j with $j \neq i$, $\gamma_{ji,l}$ has dimension equal to k_i.

Proposition 4.2. *Testing for the validity of $\tilde{W}$ is equivalent to testing for the relevance of residual information coming from each foreign counterpart. That is, testing for,*

$$H_0 : \gamma_{ji,l} = \boldsymbol{0},$$
$$H_1 : \gamma_{ji,l} \neq \boldsymbol{0},$$

for every i and for each item l of each j counterpart, with $j \neq i$.

Proof. Let us assume that $\tilde{W} = W$ and $\gamma_{ji,l} \neq 0$ for any i and j and l. It is enough to recall that under H_0, and given *Proposition 1*, the global reduced form VAR representation implies that the parameters obtained by the $GVAR$ procedure correspond to the Maximum Likelihood parameters of the reduced form VAR as in (2.1), when $\tilde{W}$ is empirically valid. Therefore, adding to the regression setting the orthogonal information coming from any of the items of foreign nodes will not provide any statistically significant improvement in the log-Likelihood level. Conversely, if any statistically significant improvement was detected, it would imply that the estimates obtained through the $GVAR$ procedure, given $\tilde{W}$, are not corresponding to the unrestricted Maximum Likelihood estimates. Therefore, $W \neq \tilde{W}$. This would be impossible under H_0. □

Chudik and Pesaran (2011) show that the cross-section augmented least-squares estimator (CALS) is consistent for retrieving the parameters of (4.20). As proven before, under H_0 no significant improvement in the Likelihood Level is possible because of the remaining information from any of the j^{th} nodes (with $i \neq j$). We can therefore employ the following simple Likelihood Ratio Test statistics for every j^{th} node and every l^{th} item,

$$LR_{ij,l} = T(ln|\hat{\Sigma}_{\epsilon_i}| - ln|\hat{\Sigma}_{\tilde{\epsilon}_i}|), \tag{4.21}$$

distributed as a χ^2_m with $m = k_i$ degrees of freedom. Employing the auxiliary regression as in (4.20) we propose to effectively test each item for each foreign counterpart. However, the reasoning is analogous for testing jointly the weights employed for each foreign counterpart j. In such a case, it would be important to consider the auxiliary regression,

$$Y_{i,t} = \Phi_i Y_{i,t-1} + \Lambda_{i0} Y^*_{i,t} + \Lambda_{i1} Y^*_{i,t-1} + \Gamma_{ji}\tilde{Y}_{j,t-1} + \tilde{\epsilon}_{i,t}, \tag{4.22}$$

with Γ_{ji} of dimension $k_i \times k_j$ and the degrees of freedom would be $m = k_i \times k_j$. Lastly, the test remains feasible even when the number of foreign items/nodes grows to infinity, as conditional on the chosen weighting scheme, the remaining information in the cross section could be jointly tested employing a penalizing scheme such as LASSO. In such a case, if any (sparse) significance is detected, the space reduction imposed through the weighting scheme does not rule out the presence of additional relevant information.

5. MONTE CARLO RESULTS

To analyze the behavior of the validity test in finite samples, size and power analyses are proposed via Monte Carlo simulations. Regarding the size analysis, the aim is to provide evidence that indeed, $GVAR$ models are W-dependent restricted procedures to estimate global reduced form $VARs$. Therefore, testing for the residual information should quickly converge to the asymptotic significance level of the correspondent theoretical χ^2_m, under H_0. Regarding the power analysis, the aim is to verify that the presence of significant additional information contained in the cross section results in the rejection of the null hypothesis.

The typical cross-sectional and time-series dimensions encountered by the researcher are reflected in the several different Data Generating Processes ($DGPs$) proposed. DGP_1 corresponds to a 10-node-1-item $GVAR$ of dimensions similar to Favero (2013) or Candelon et al. (2022).[5] DGP_2 corresponds to a 10-node-5-item $GVAR$ of dimensions similar to Pesaran et al. (2004). DGP_3 is a stationary 26-node-5-item $GVAR$ of dimensions similar to Pesaran et al. (2009). The parameters to estimate are drawn from random uniform distributions to ensure the stability of the $GVAR$ form as in (2.7). Specifically, $\tilde{\Phi} \overset{iid}{\sim} \mathcal{U}(-0.4, 0.4)$, $\tilde{\Lambda}_0 \overset{iid}{\sim} \mathcal{U}(-0.8, 0.8)$, $\tilde{\Lambda}_1 \overset{iid}{\sim} \mathcal{U}(-0.8, 0.8)$. The elements of the matrices W are also drawn randomly from a uniform distribution, then normalized to respect the granularity conditions. The errors ϵ are drawn from a Normal Distribution with zero mean and unit variance. For both the size and the power analysis, we report the simulation results for testing $w_{ij,l}$ with $i = 1$, $j = 2$, and $l = 1$.[6]

Once the local parameters, the W matrix, and the errors are drawn, we generate $10,000$ replications per DGP of the reduced form VAR as in (3.13) to derive the empirical size of this test. We discard the first 100 observations to ensure that there is no initial value dependence. Several sample sizes are considered, specifically, $T = 100, 200, 300$. Tables 1 and 2 report the simulated size and the 5% critical values for the different $DGPs$ and sample sizes. The size is reported as the rejection frequencies when the asymptotic χ^2_m critical value for the significance level $\alpha = 5\%$ is employed. Specifically, in Table 1 we report the size for DGP_1, DGP_2, and DGP_3 with $m = 1$ and the critical value is 3.84, as we present the test for a single item. In Table 2, for DGP_2 and DGP_3 the degrees of freedom are equal to 11.07 as $m = 5$. The critical values reported correspond to the 500^{th} values of the ordered series of the computed Likelihood Ratios of the $10,000$ draws, per DGP and per sample size.

Tables 3 and 4 report the size-adjusted rejection frequencies when simulating the $DGPs$ under the alternative hypothesis. Specifically, we retain the parameters and the W matrices employed to derive the empirical size of the test. We simulate under the alternative specification as in (4.20). For comparability, the error regards only one of the elements of $\gamma_{ji,l}$. We consider three errors' dimensions, 0.1, 0.25, and 0.5.

The rejection frequencies for DGP_1 are very close the nominal size even when the sample size is very small. When the number of items and/or the number of nodes increases, the test becomes slightly oversized but reasonably close to the nominal size when increasing the sample size. The size-adjusted power is positively related to the sample size as expected and to the error dimension. Very interestingly, the power does not result

[5] They are going to be two of the reference papers for the empirical analysis.

[6] The results for the other items/nodes are quantitatively similar and available upon request from the authors.

particularly impacted by the increase of nodes and/or items. The result is even more interesting if we consider that the bias introduced regards only one of the jointly tested coefficients when the number of items is larger than one.

Table 1. Size analysis for a single item

		$T = 100$	$T = 200$	$T = 300$
DGP_1	Size	5.75%	5.35%	5.28%
	Critical Value (5%)	4.026	3.966	3.903
DGP_2	Size	6.98%	6.27%	5.89%
	Critical Value (5%)	4.537	4.204	4.092
DGP_3	Size	7.33%	6.11%	5.81%
	Critical Value (5%)	4.517	4.155	4.090

Note: This table shows the empirical size and critical values of the Likelihood Ratio Test proposed in (4.21), for the weight w_{ij}, with $i = 1$ and $j = 2$, for a single s^{th} item $y_{i,s,t}$, for several DGPs, under the null hypothesis $\tilde{W} = W$. DGP_1 corresponds to the 10-node-1-item stationary case, DGP_2 corresponds to the 10-node-5-item stationary case, and DGP_3 corresponds to the 26-node-5-item stationary case. Simulations are performed with sample size $T = 100, 200, 300$. For each sample size and DGP, $10,000$ simulations are performed for a sample size of $T + 100$. The first 100 observations are discarded to avoid potential bias due to initial value dependence. Size refers to the rejection frequencies, expressed as a % of the $10,000$ draws, employing the asymptotic critical value for the significance level $\alpha = 5\%$. The Critical Value (5%) refers to the 500^{th} ordered Likelihood Ratio value encountered for each DGP and sample size.

Table 2. Size analysis for a single weight

		$T = 100$	$T = 200$	$T = 300$
DGP_2	Size	10.25%	7.87%	6.53%
	Critical Value (5%)	13.331	12.383	11.837
DGP_3	Size	10.42%	7.67%	6.95%
	Critical Value (5%)	13.495	12.259	11.934

Note: This table shows the empirical size and critical values of the Likelihood Ratio Test proposed in (4.21), for the weight w_{ij}, , for all the items of $y_{i,t}$, with $i = 1$ and $j = 2$, for several DGPs, under the null hypothesis $\tilde{W} = W$. DGP_2 corresponds to the 10-node-5-item stationary case, and DGP_3 corresponds to the 26-node-5-item stationary case. Simulations are performed with sample size $T = 100, 200, 300$. For each sample size and DGP, $10,000$ simulations are performed for a sample size of $T + 100$. The first 100 observations are discarded to avoid potential bias due to initial value dependence. Size refers to the rejection frequencies, expressed as a % of the $10,000$ draws, employing the asymptotic critical value for the significance level $\alpha = 5\%$. The Critical Value (5%) refers to the 500^{th} ordered Likelihood Ratio value encountered for each DGP and sample size.

6. EMPIRICAL APPLICATION

In order to illustrate the importance of the proposed testing strategy, we are considering the sovereign bond market in the euro area as a natural experiment. Modeling long-term sovereign bond spreads over the German *Bund* has stimulated a long lasting debate

Table 3. Power analysis for a single item

	$T=100$			$T=200$			$T=300$		
Error	0.1	0.25	0.5	0.1	0.25	0.5	0.1	0.25	0.5
DGP_1	12%	56%	99%	20%	85%	100%	28%	96%	100%
DGP_2	14%	76%	100%	31%	98%	100%	47%	100%	100%
DGP_3	17%	77%	100%	37%	99%	100%	53%	100%	100%

Note: This table shows the size-adjusted power of the Likelihood Ratio Test proposed in (4.21), for the weight w_{ij}, with $i=1$ and $j=2$, for item l with $l=1$, for a single s^{th} item $y_{i,s,t}$, for several DGPs under the alternative hypothesis. DGP_1 corresponds to the 10-node-1-item stationary case, DGP_2 corresponds to the 10-node-5-item stationary case, and DGP_3 corresponds to the 26-node-5-item stationary case. Simulations are performed with sample size $T=100,200,300$. For each sample size and DGP, $10,000$ simulations are performed for a sample size of $T+100$. The first 100 observations are discarded to avoid potential bias due to initial value dependence. Three sets of errors are considered, 0.1, 0.25, and 0.5. We report the rejection frequencies, expressed as a % of the $10,000$ draws, employing the empirical critical value for the significance level $\alpha=5\%$ computed for the size analysis.

Table 4. Power analysis for a single weight

	$T=100$			$T=200$			$T=300$		
Error	0.1	0.25	0.5	0.1	0.25	0.5	0.1	0.25	0.5
DGP_1	12%	56%	99%	20%	85%	100%	28%	96%	100%
DGP_2	11%	55%	100%	18%	90%	100%	27%	99%	100%
DGP_3	11%	55%	99%	21%	92%	100%	32%	99%	100%

Note: This table shows the size-adjusted power of the Likelihood Ratio Test proposed in (4.21), for the weight w_{ij}, with $i=1$ and $j=2$, for item l with $l=1$ for several $DGPs$ under the alternative hypothesis. DGP_1 corresponds to the 10-node-1-item stationary case, DGP_2 corresponds to the 10-node-5-item stationary case, and DGP_3 corresponds to the 26-node-5-item stationary case. Simulations are performed with sample size $T=100,200,300$. For each sample size and DGP, $10,000$ simulations are performed for a sample size of $T+100$. The first 100 observations are discarded to avoid potential bias due to initial value dependence. Three sets of errors are considered, 0.1, 0.25, and 0.5. We report the rejection frequencies, expressed as a % of the $10,000$ draws, employing the empirical critical value for the significance level $\alpha=5\%$ computed for the size analysis.

regarding the degree of integration and thus the identification of an interconnection channel. We are here referring to two topical periods, the one preceding the euro area debt crisis, and the more recent COVID period.

Differences in long-term sovereign bond spreads' dynamics in the euro area have been traditionally modeled as solely dependent on country specific factors–such as debt- and deficit-to-GDP ratios– and global factors reflecting global risk aversion (see, for example, Laubach, 2009; Beber et al., 2009; Favero et al., 2010), as follows,

$$\begin{aligned}\Delta Y_{i,t} &= \beta_{i0} + \beta_{i1} Y_{i,t-1} + \beta_{i2}(Baa_{t-1} - Aaa_{t-1}) + \beta_{i3}\Delta(Baa_t - Aaa_t) \\ &\quad + \beta_{i4}(debt_{i,t} - debt_{bd,t}) + \beta_{i5}(def_{i,t} - def_{bd,t}) + e_{i,t}, \qquad (6.23)\end{aligned}$$

where $\Delta Y_{i,t} = Y_{i,t} - Y_{i,t-1}$, $Y_{i,t}$ is the 10-year sovereign yield spread between country i and German Bund (the usual reference in the euro area), $e_{i,t}$ is the white noise residual term for country i with finite variance, $Baa - Aaa$ represents the time varying global risk aversion as measured by the long term US $Baa - Aaa$ corporate bond spreads, $\Delta(Baa_t - Aaa_t) = (Baa_t - Aaa_t) - (Baa_{t-1} - Aaa_{t-1})$, debt- and deficit-to-GDP ratios are also included as explanatory variables in the model (*debt* and def, respectively), expressed as distances from the German values (*bd* indicates Germany). The deficit- and debt-to-GDP ratios are taken as a simple average of the actual values and the forecasts (as known at time t) published by the European Commission to reflect also the importance of the future fiscal outlook.

Favero (2013) proved that such models are, however, not able of explaining the unstable pattern of co-movement observed in the euro area government bond market. He proposes to augment the specification in (6.23) in a *GVAR* fashion. Specifically, he identifies $w_{ij,t}$ as time-varying weights corresponding to the distance between countries i and j at time t, in terms of differences in fiscal fundamentals. In other words, he considers the public debt-to-GDP ratio and deficit-to-GDP ratio of each country i. For each fiscal indicator, a distance is built as the absolute difference between the values of countries i and j normalized by the value imposed by the Maastricht criteria (i.e., 3% for deficit, and 60% for debt). It is then possible to build two distance matrices as follows,

$$
\begin{aligned}
w_{ji,t}^{k} &= \frac{w_{ji,t}^{*,k}}{\sum_{j\neq i} w_{ji,t}^{*,k}}, \quad w_{ji,t}^{*,k} = \frac{1}{\mathrm{dist}_{ji,t}^{k}}, \quad with \;\; k = \{def, debt\},\\
dist_{ji,t}^{def} &= |def_{j,t} - def_{i,t}|/3,\\
dist_{ji,t}^{debt} &= |debt_{j,t} - debt_{i,t}|/60,
\end{aligned}
$$

Favero (2013)'s model is thus the following,

$$
\begin{aligned}
\Delta Y_{i,t} = \beta_{i0} &+ \beta_{i1} Y_{i,t-1} + \beta_{i2}(Baa_{t-1} - Aaa_{t-1}) + \beta_{i3}\Delta(Baa_t - Aaa_t)\\
&+ \beta_{i4}(debt_{i,t} - debt_{bd,t}) + \beta_{i5}(def_{i,t} - def_{bd,t})\\
&+ \beta_{i6} Y_{i,t-1}^{*,debt} + \beta_{i7} Y_{i,t-1}^{*,def} + e_{i,t},
\end{aligned}
\tag{6.24}
$$

where $Y_t^{*,k} = \sum_{j\neq i} w_{ji,t}^{k} Y_{j,t}$ with $k = \{def, debt\}$. The intuition would be that the more two countries are similar in terms of fiscal fundamentals, the more interdependent they are.

Favero (2013) shows that the inclusion of the interconnection terms based on fiscal fundamentals significantly improves the goodness-of-fit compared with the standard model in (6.23). Moreover, from an economic perspective, such a weighting scheme captures the increased interconnection between Greece and other high-indebted euro area countries well before the outburst of the euro area sovereign debt crisis in 2010.

While Favero (2013) thoroughly proves that the inclusion of an interconnection term improves inferences compared to a model that does not include it, the specific interconnection channel proposed is not tested. For example, trade weights (Pesaran et al., 2004) or financial flows (Sgherri and Zoli, 2009) constitute potential alternative weighting schemes used in the literature. We implement our testing strategy to assess the empirical validity of the interconnection channel proposed by Favero (2013). The simple idea of the testing strategy is that if debt- and deficit-to-GDP ratios constitute valid interconnection channels, no additional information from any foreign country would be found significant. The Likelihood Ratio Test values for the different weights are reported in Table 5. In his Impulse Response analysis, Favero (2013) shows that the Portuguese long term spread turns out to be more interconnected with the Greek one, compared with standard specifications as in (6.23). Our test reveals that fiscal fundamentals represent a valid interconnection for the effect of Greece on Portugal, as no additional significant

Table 5. Testing for the Interconnection: Favero (2013)

	BG	ES	FN	FR	GR	IR	IT	NL	OE	PT
BG	.	12.88	**0.00**	**0.66**	7.61	8.08	19.46	**1.93**	7.49	**2.12**
ES	7.21	.	**1.37**	8.84	5.10	**2.25**	34.55	**0.67**	**1.41**	**2.27**
FN	7.38	6.25	.	**0.01**	**0.09**	7.22	**0.06**	**0.30**	**0.52**	**0.09**
FR	9.86	22.77	**2.39**	.	**1.20**	5.30	**3.53**	5.69	**0.66**	**1.36**
GR	8.21	**0.23**	**1.93**	**3.18**	.	**1.03**	7.93	**1.75**	39.62	32.60
IR	**2.63**	**0.06**	10.51	**1.38**	6.25	.	**1.85**	**1.23**	14.24	29.46
IT	**0.48**	**0.14**	**3.26**	**0.21**	**1.78**	**2.54**	.	**0.92**	**2.83**	5.36
NL	**3.30**	**1.74**	**1.03**	4.99	**0.85**	**0.23**	16.76	.	**5.95**	**0.19**
OE	4.88	10.21	**0.46**	**1.19**	**1.69**	12.14	**0.48**	**1.80**	.	5.54
PT	10.91	11.03	**1.88**	**2.49**	**0.52**	**1.73**	4.23	**0.39**	8.35	.

Note: This table displays the values of the Likelihood Ratio Test computed as in 4.21, for each weight employed in Favero (2013). The asymptotic critical value for a significance level of $\alpha = 5\%$ is 3.84. When the null hypothesis of validity of the interconnection channel is not rejected, the test statistic appears in bold. Original data are from January 2000 to December 2009 as in Favero (2013). The countries under analysis are Austria (OE), Belgium (BG), Finland (FN), France (FR), Greece (GR), Ireland (IR), Italy (IT), the Netherlands (NL), Portugal (PT), and Spain (ES). Frequency: Monthly. Source: *DataStream.*

information results relevant between the Greek and the Portuguese long term sovereign bond spreads. However, when analysing the other weights, results are mixed, as we find that several of the foreign counterparts still carry significant information after conditioning on the interconnection based on fiscal fundamentals.

More recently, a new perspective with regard to the interconnection channel in sovereign bond spreads inside the euro area has been proposed. Candelon et al. (2022) proposes to model long-term sovereign bond spreads in a *FAVAR* framework à la Bernanke et al. (2005), augmenting the local Autoregressive models with the first and the second (static) Principal Components extracted from the cross section. Interestingly, they analyse the loading of the second principal component, finding a sign heterogeneity between high- and low-indebted countries. They argue that this principal component measures the risk of fragmentation–the risk that, in response to an asymmetric shock to the sovereign bond market, some countries would experience rising yields, while others would observe decreasing yields. De Grauwe and Ji (2022) instead doubt the importance of such component, as at the onset of the COVID pandemic–a shock of similar magnitude to the Great Financial Crisis of 2007-2008–the euro area sovereign bond market did not experience a fragmented response. We therefore propose to test the information coming from the second principal component. Candelon et al. (2022) model sovereign bond spreads the following way,

$$Y_{i,t} = \beta_{i0} + \beta_{i1} Y_{i,t-1} + \beta_{i2} Y^*_{i,t} + \beta_{i3} Y^*_{i,t-1} + \beta_{i4} \hat{F}_{1,t} + \beta_{i5} \hat{F}_{1,t-1} + e_{i,t}, \tag{6.25}$$

where $Y_{i,t}$ is the 10-year sovereign yield spread between country i and the German Bund, $\hat{F}_{1,t}$ is the first principal component extracted from the cross section of long term sovereign bond spreads, and $Y^*_{i,t}$ represent the part of the second principal component that is unspanned by the local yield spread. The unspanned information from the second Principal Component for each market under analysis is extracted as in Bernanke et al. (2005). We test the additional information coming from the contemporaneous and lagged second principal component for each country under analysis. Therefore, (6.25) represents the auxiliary regression. If no fragmentation risk was relevant for the yield spreads dynamics, no additional information would be found significant in our Likelihood Ratio Test. The results are shown in Table 6.

Table 6. Testing of the Interconnection: De Grauwe and Ji (2022)

BG	ES	FN	FR	GR	IR	IT	NL	OE	PT
5.32	11.89	**1.58**	13.72	33.93	**3.34**	19.15	21.61	10.14	8.12

Note: This table displays the values of the Likelihood Ratio Test computed as in 4.21, for the part of the second principal component unspanned by the local sovereign bond spreads as in Candelon et al. (2022). The asymptotic critical value for a significance level of $\alpha = 5\%$ is 5.99. When the null hypothesis of validity of the interconnection channel is not rejected, the test statistic appears in bold. Original data are from March 2019 to March 2021 as in Candelon et al. (2022). The countries under analysis are Austria (OE), Belgium (BG), Finland (FN), France (FR), Greece (GR), Ireland (IR), Italy (IT), the Netherlands (NL), Portugal (PT), and Spain (ES). Frequency: Weekly. Source: *Refinitiv EIKON.*

Table 7. Testing for the Interconnection: Candelon et al. (2022)

	BG	ES	FN	FR	GR	IR	IT	NL	OE	PT
BG	.	**0.01**	**0.95**	**0.70**	**1.78**	**0.90**	**0.06**	**1.38**	**0.00**	**0.34**
ES	**0.01**	.	4.10	**2.36**	**0.35**	**0.10**	**0.60**	5.38	0.00	10.12
FN	**0.10**	**0.00**	.	5.82	**0.50**	**2.86**	**1.40**	16.49	**1.32**	**1.86**
FR	4.72	**1.44**	**1.88**	.	**0.37**	**3.70**	7.31	16.48	**1.95**	**0.54**
GR	**1.19**	**0.28**	**0.50**	**0.56**	.	**0.04**	**0.32**	**3.93**	**1.17**	**0.79**
IR	**0.01**	**2.29**	**0.17**	6.27	**0.18**	.	**0.24**	**0.32**	**0.75**	**1.32**
IT	**0.18**	**0.12**	**0.01**	**0.42**	6.10	5.01	.	4.37	**1.28**	**0.02**
NL	**1.14**	4.39	**1.04**	3.90	0.**16**	**0.36**	**3.18**	.	**3.47**	**1.53**
OE	4.94	7.16	12.08	**0.59**	**1.40**	7.34	**1.55**	18.29	.	11.25
PT	**0.90**	**0.01**	**2.68**	**1.37**	**0.68**	**0.83**	**3.25**	4.84	**0.06**	.

Note: This table displays the values of the Likelihood Ratio Test computed as in 4.21, for each weight employed in Candelon et al. (2022). The asymptotic critical value for a significance level of $\alpha = 5\%$ is 3.84. When the null hypothesis of validity of the interconnection channel is not rejected, the test statistic appears in bold.Original data are from March 2019 to March 2021 as in Candelon et al. (2022). The countries under analysis are Austria (OE), Belgium (BG), Finland (FN), France (FR), Greece (GR), Ireland (IR), Italy (IT), the Netherlands (NL), Portugal (PT), and Spain (ES). Frequency: Weekly. Source: *Refinitiv EIKON.*

There is enough empirical evidence to reject the assumption of no significant external information spanned by the second principal component for Spain, France, Greece, Italy, the Netherlands, Austria, and Portugal. Interestingly, Candelon et al. (2022) described Ireland as mildly or not impacted by the risk of fragmentation, and we indeed find not enough empirical evidence to justify the importance of the fragmentation risk factor (as measured by the unspanned second principal component) on long term Irish sovereign bond spreads. Furthermore, in Table 7 we report the test for additional information coming from each other bond spread after conditioning on the first and the second principal component as in Candelon et al. (2022). Apart from some exceptions, the test does not find enough empirical evidence for the significance of additional information coming from the majority of the foreign nodes.

7. CONCLUSIONS

Global markets and economies are interconnected. Therefore, modeling and identifying the dynamic dependencies across units is fundamental for meaningful inferences. However, large heterogeneous VARs suffer from the curse of dimensionality. Typically, one common way to shrink the parameter space is represented by weighting foreign information when modeling local system. However, existing literature proposes several possible weighting schemes, derived from economic intuition or estimated from the data. Inferences are often different, at times conflicting.

In this paper, we exploit the simple intuition that if the proposed weighting scheme is empirically valid, no residual information coming from foreign counterparts turns out to be significant when modeling locally. We therefore propose a novel Likelihood Ratio Test. Exploiting the correspondence between the local conditional models featuring the weighted average of foreign counterparts and the global reduced form VAR representation, we first prove that the parameters' estimates remain conditional on the weighting scheme proposed, secondly, we derive the Likelihood Ratio test for the weighting scheme as testing the significance of the residual information coming from each counterpart. In a simulation exercise, we prove that the test presents very good short sample properties. Finally, in the empirical application, we show how to employ the test to the debated case of the interconnection channel across long term sovereign bond spreads in the euro area during two topical periods, the one preceding the sovereign debt crisis and the more recent COVID one. While we partially confirm that debt- and deficit-to-GDP ratios explained the tightness of the interconnection between Greece and some euro area countries, we provide evidence that such a channel does not globally capture the observed dynamics in long term sovereign bond spreads. With regards to the COVID period, we instead find evidence for the importance of the fragmentation risk for most of the euro area countries under analysis.

The new test presented paves the way for a wide range of economic implementations. As an example, in Global VARs, testing the weighting scheme would preserve the intuitive interpretation of the proposed channel for the transmission of shocks among local markets while ensuring that observed data support the restrictions imposed by theory. Inferences based on such a transmission matrix would therefore be both theoretically sound and empirically valid. Most of the studies, even when focusing simply on local effects of shocks, need to account for common factors. Our test would ensure that no significant information distorts the final inferences.

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