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DETECTING TOO LONG HOSPITAL STAYS: COMPARISON OF NONPARAMETRIC AND PARAMETRIC FRONTIER APPROACHES

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COMPARISON OF NONPARAMETRIC AND PARAMETRIC FRONTIER APPROACHES

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Abstract

Since 1995, length of stay by Diagnosis Related Groups is taken into account in Belgian hospital's financing. Usually, trimming rules are used before the estimation of the mean length of stay by pathology. In this study we propose to take into account the characteristics of the patients in order to highlight hospital stays with discrepancy between the severity and the resources the patient consumes. A deterministic parametric frontier model is used so that the efficiencies estimated, regarding the length of stay, allow to rank hospital stays, after exclusion of the too efficient stays. The results coming from parametric and nonparametric frontier models are compared.

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1. Introduction

Frontier models used to measure the productivity of firms in industry seem promising for evaluation of the activities of health-care providers. Hospital efficiencies has been examined by several authors (Clément et al, 1996; Grosskopf et al, 1987, 1997[a]; Linna et al, 1998; Morey et al, 1995; Zuckerman et al, 1994) . Special activities of hospitals like teaching and research have also been analysed by Grosskopf et al (1997[a]) and Linna et al (1998). But utilisation of frontier models is not limited to hospitals. Sexton et al (1989) have studied the performance of the Veterans Administration medical centres that function as both hospitals and nursing homes. Kooreman (1994) and Vitiliano and Toren (1994) have focused on the performance or costs of nursing homes. Recently, more attention has converged on physicians or specific types of care, as well as characteristics of patients and quality of care (Chilingerian, 1995; Puig-Junoy, 1998; Thanassoulis et al, 1995).

In the health care sector, outputs are more difficult to define than in other industries, such as manufacturing. For example, no clear measure of improved health status exists (Grosskopf and Valdmanis, 1987). Typically outputs are estimated by the number of inpatient admissions, or discharges, the number of outpatient visits, the number of ambulatory or acute surgical procedures and perhaps average length of stay or inpatient-days (Grosskopf et al, 1987, 1997[a]; Zuckerman et al, 1994). Thanassoulis et al (1995) have also included the number of babies at risk of death and the number of satisfied mothers in order to account for the quality of prenatal care. When analysing physician efficiency, outputs are sometimes measured by the number of low and high severity cases or by the number of days surviving at hospital and the surviving discharge status (Chilingerian, 1995; Puig-Junoy, 1998).

When evaluating the cost efficiency of hospitals or nursing homes, a stochastic translog or Cobb-Douglas frontier cost function is frequently used (Linna et al, 1998; Vitaliano and Toren, 1994; Zuckerman et al, 1994). To compare shadow prices to reimbursement rate, Clément et al (1996) use the input distance function estimated from a translog model solved by linear programming. Data envelopment analysis is often used to estimate technical or allocative efficiency. Sometimes in a second step efficiency scores are investigated via loglinear regression, tobit or probit models or compared between different groups by Kruskal and Wallis tests or F-tests (Chilingerian, 1995; Grosskopf et al, 1987, 1997[a]; Kooreman, 1994; Puig-Junoy, 1998; Sexton et al, 1989; Thanassoulis, 1995).

Our application of frontier modelling is somewhat different and is motivated by recent changes in hospital financing in Belgium. In order to penalise hospitals with excessive length of stay (LOS), the Belgian Ministry of Health estimates an average LOS by Diagnosis Related Group (DRG) after removing outliers (denoted "budget outlier" hereafter). The bounds for the budget outlier definition are delimited by rules based on the quartile and the interquartile range. In 1998, the lower bound was $\exp\left[\ln Q_1 - 2*\left(\ln Q_3 - \ln Q_1\right)\right]$ and the upper bound was $Q_3 + 2*\left(Q_3 - Q_1\right)$. When LOS is not included in this interval, the hospital stay is not taken into account in hospital's financing. Nevertheless, if LOS is greater than the upper bound but if there is only one ill system, LOS takes the value of the upper bound. In 1999, the lower bound was $\exp\left[\ln Q_1 - 2*\left(\ln Q_3 - \ln Q_1\right)\right]$ and 2 upper bounds were calculated: $Q_3 + 2*\left(Q_3 - Q_1\right)$ and $Q_3 + 4*\left(Q_3 - Q_1\right)$. When LOS is not included in the greater interval, the hospital stay is not taken into account in hospital's financing. When LOS varies between the 2 upper bounds, LOS takes the value of the first upper bound. Similar rules are frequently used in this type of analysis.

In this paper, the use of deterministic frontier models is proposed to define the "efficiency score" of each hospital stay in order to weight the total expenses, length of stay, or the length of stay and the medical fees given the severity of the patient. The method offers the possibility to rank hospital stays taking into account the individual characteristics of the patient. This can be viewed as an exploratory data analysis where 'budget' outliers could be detected among the less efficient stays. The analysis is not limited to the length of stay or the total expenses, called univariate approaches in the next sections, but considers also a multivariate approach when studying both length of stay and medical fees simultaneously. Translog model is used to model the distance to the frontier in the univariate approach. In the multivariate approach, the Shephard distance function is estimated also by a translog model. Deterministic models are very sensitive to 'efficient outlier' since, in a certain sense, they envelop all the data points. We will then use an adaptation of the method developed by P.Wilson (1995) before definitive estimation of hospital stay's efficiencies. In the first part of the paper, the parametric frontier model is described, then we present the data and the methods. At the end of the paper, the results are provided and discussed. These results are compared to those obtained in the same framework with nonparametric frontier models (Beguin, 2001).

2. Theoretical background

2.1. Definition of a frontier

The objective is to find the minimal achievable level of the variable of interest, say y. According the case, y will represent either length of stay or total expenses or both length of stay and medical fees. The original idea is to find this minimal level taking into account the characteristics of the patients such as age, number of ill systems, length of stay in intensive care unit (ICU), which can be considered as “proxy” for measuring the severity score of the patient. Also other qualitative characteristics of the patient such as sex and admission in emergency could be taken into account. The minimal achievable level of the variable of y is considered as a “frontier” of a possibility set which is the set of attainable values of y given the individual characteristics of the patient. For a specific level of the patient’s characteristics, the minimal value of y delimits the frontier enveloping all the possible values. The efficiency of an hospital stay will then be defined as the distance between the observed value of y and the frontier.

This type of problem appears in productivity analysis when seeking for the minimal sets of inputs of a firm, which allow to produce a given level of outputs. The production set is the set of achievable combination of inputs and outputs and the frontier of production defines the set of efficient production plans. Eventually, the technical efficiency of a firm is measured by the distance of this firm to the obtained frontier. However, the vocabulary used in the context of productivity is not well adapted to patient nor to the problem here. It will be modified as follows: the term ‘production set’ has to be replaced by ‘possibility set’, ‘frontier of production’ by ‘frontier of possibilities’, ‘input’ by ‘variable to minimise’ (y-variables), 'output' by 'individual severity score' (x-variables) and the 'technical efficiency' by 'efficiency'.

The set Ψ of possibilities is the set of attainable points (y, x). It is defined by $\Psi = \left\{ (y,x) \in \Re_{+}^{p+q} \,\middle|\, (y,x) \text{ are attainable} \right\}$ where y are the quantities to minimise (total expenses, length of stay and/or medical fees) and x are the individual severity scores of the patient. $\forall\, x \in \Psi$, the section of possible values of y is the set defined as $Y(x) = \left\{ y \in \Re_{+}^{p} \,\middle|\, (y,x) \in \Psi \right\}$ and the efficient boundary is a subset of $Y(x)$: $\partial Y(x) = \left\{ y \,\middle|\, y \in Y(x), \theta y \notin Y(x),\ \forall\, 0 < \theta < 1 \right\}$. The measure of the efficiency of a stay (y_k, x_k) is expressed by $\theta_k = \min\left\{ \theta : \theta\, y_k \in Y(x_k) \right\}$. It is the distance from (y_k, x_k) to the efficient boundary $\partial Y(x_k)$. If $\theta_k = 1$, the unit (y_k, x_k) is considered as being 'efficient'. The efficiency score $\theta_k < 1$ represents the feasible proportionate reduction of y (total expenses, length of stay and/or medical fees) given the characteristics x_k of the patient in order to achieve the minimal attainable level of y_k given x_k.

The frontier of possibilities Ψ is unknown and can be estimated by non parametric models, such as data envelopment analysis (DEA) or free disposal hull (FDH), or by parametric models such as Cobb Douglas or translog. For a general overview of these aspects, one may consult Simar and Wilson (2000) or Grosskopf (1996). In the case of a deterministic frontier, all $(y_{it}, x_{it}) \in \Psi$, $\forall\, it$ where i is the index for stays and t the index for hospitals. In the next sections, we develop a parametric model for both the univariate and the multivariate cases by using the translog formulation.

2.2. A linear deterministic frontier model: the univariate case

When y is univariate, we can use the standard linear model $y_{it} = \beta_o + x_{it}^{'}\beta + v_{it}$, where $\beta_o + x_{it}^{'}\beta$ represents the frontier of possibilities, v_{it} the deviation of the decision unit to the frontier where $v_{it} \geq 0$. Ordinary least squares method provides $\hat{\beta}_o$ *and* $\hat{\beta}$, estimators of β_o *and* β , then corrected ordinary least squares (COLS) allows to calculate $\hat{\hat{\beta}}_o = \hat{\beta}_o + \min_{it} \hat{\varepsilon}_{it}$ for taking into account the positiveness of the residuals. Indeed, $y_{it} = \hat{\beta}_o + \hat{\beta}' x_{it} + \hat{\varepsilon}_{it}$ which can be written as $y_{it} = \hat{\beta}_o + \min_{it} \hat{\varepsilon}_{it} + \hat{\beta}' x_{it} + \hat{\varepsilon}_{it} - \min_{it} \hat{\varepsilon}_{it}$. Now $y_{it} = \hat{\hat{\beta}}_o + \hat{\beta}' x_{it} + \hat{\hat{v}}_{it}$, where $\hat{\hat{v}}_{it} = \hat{\varepsilon}_{it} - \min_{it} \hat{\varepsilon}_{it}$, are positive and represent the distance between the decision unit and the frontier of possibilities. Finally, $y_{it} = \hat{\hat{y}}_{it,frontier} + \hat{\hat{v}}_{it}$, where $\hat{\hat{y}}_{it,frontier} = \hat{\hat{\beta}}_o + \hat{\beta}' x_{it}$ represents the minimal value of y for a patient with characteristics x_{it} and $\hat{\hat{v}}_{it}$ returns the excess of y_{it} with respect of the optimal frontier (Greene, 1980; Simar, 1992).

The linear model chosen in our application is the translog model. The translog model can be viewed as a second order approximation of a "true" model, while the Cobb-Douglas is a particular case where all the second order terms vanishes (Christensen and Greene, 1976). The model is written as:

$$\log y_{it} = \beta_o + \sum_{m=1}^{p} \beta_m \log x_{it}^m + 0.5 \sum_{m=1}^{p} \sum_{n=1}^{p} \beta_{mn} \log x_{it}^m \log x_{it}^n + v_{it} \qquad \text{, where } v_{it} \geq 0 \text{ and}$$

m indexes the x-variables. OLS gives the estimation of $\hat{\beta}_o, \hat{\beta}_m, \hat{\beta}_{mn}$ *and* $\hat{\varepsilon}_{it}$, and COLS estimates $\hat{\hat{\beta}}_o = \hat{\beta}_o + \min_{it} \hat{\varepsilon}_{it}$ and $\hat{\hat{v}}_{it} = \hat{\varepsilon}_{it} - \min_{it} \hat{\varepsilon}_{it}$, where $\hat{\beta}_{mn} = \hat{\beta}_{nm}$. As the model is expressed in log ($\log y_{it} = \log \hat{\hat{y}}_{it,frontier} + \hat{\hat{v}}_{it}$), one can deduce $y_{it} = \hat{\hat{y}}_{it,frontier} * e^{\hat{\hat{v}}_{it}}$. So the ratio $\frac{y_{it}}{\hat{\hat{y}}_{it,frontier}} = e^{\hat{\hat{v}}_{it}}$ is greater or equal to 1 since $\hat{\hat{v}}_{it} \geq 0$.

2.3. Multivariate model

In this part, the distance functions of Shephard is used because it allows for multiple input (Clément et al, 1996; Shephard, 1970). The Shephard input distance function may be defined as follows (see Shephard, 1970): $D(y_k, x_k) = Max\left\{\gamma : \frac{y_k}{\gamma} \in Y(x_k)\right\}$. It is the inverse of the efficiency measure as defined in section 2.1. For any point in the set of possibilities $D(y_k, x_k) \geq 1$ and $D(y_k, x_k) = 1$ for a frontier point. In a parametric approach, the distance function is modelized by a translog function as shown below in equation (1). Due to the property of homogeneity in degree +1 in y of the distance function, it is possible to transform the model so that one of the input will be described in terms of linear function of all outputs and the other 'scaled' inputs. The procedure was proposed by Grosskopf et al. (1997[b]) and can be summarised as follows. First we modelize the Shephard distance function by a translog function:

$$\begin{aligned} D(y_{it}, x_{it}) \quad = \quad & \beta_o + \sum_j \beta_j \log\left(y_{it}^j\right) + 0.5 \sum_j \sum_k \beta_{jk} \log\left(y_{it}^j\right) * \log\left(y_{it}^k\right) \\ & + \sum_j \sum_m \rho_{jm} \log\left(y_{it}^j\right) * \log x_{it}^m \\ & + \sum_m \lambda_m \log x_{it}^m + 0.5 \sum_m \sum_n \lambda_{mn} \log x_{it}^m \log x_{it}^n + v_{it} \quad . \qquad (1) \end{aligned}$$

We know that $\forall k > 0, D\left(y_{it}, x_{it}\right) = k * D\left(\frac{y_{it}}{k}, x_{it}\right)$. If $k = y_{it}^1$, then $D\left(y_{it}, x_{it}\right) = y_{it}^1 * D\left(\frac{y_{it}}{y_{it}^1}, x_{it}\right)$. When a point $\left(y_{it}, x_{it}\right)$ is on the frontier of possibilities, $1 = y_{it}^1 * D\left(\frac{y_{it}}{y_{it}^1}, x_{it}\right)$ and the log transformation gives $0 = \log\left[y_{it}^1 * D\left(\frac{y_{it}}{y_{it}^1}, x_{it}\right)\right]$. Finally, one can deduce that $\log y_{it}^1 = -\log\left[D\left(\frac{y_{it}}{y_{it}^1}, x_{it}\right)\right]$. If a point $\left(y_{it}, x_{it}\right)$ is not on the frontier of possibilities, $1 \leq y_{it}^1 * D\left(\frac{y_{it}}{y_{it}^1}, x_{it}\right)$ and $0 \leq \log\left[y_{it}^1 * D\left(\frac{y_{it}}{y_{it}^1}, x_{it}\right)\right]$. So that for any point $\left(y_{it}, x_{it}\right)$ in the possibility set, we have $0 = \log y_{it}^1 + \log\left[D\left(\frac{y_{it}}{y_{it}^1}, x_{it}\right)\right] - v_{it}$ where $v_{it} \geq 0$.

This can also be written as $$\log y_{it}^1 = -\log\left[D\left(\frac{y_{it}}{y_{it}^1}, x_{it}\right)\right] + v_{it} \qquad (2)$$

where v_{it} represents the distance to the frontier. The estimation of $\hat{\hat{v}}_{it}$ by the frontier model provides $\hat{\hat{v}}_{it} = \log\left[y_{it}^1 * \hat{\hat{D}}\left(\frac{y_{it}}{y_{it}^1}, x_{it}\right)\right] = \log \hat{\hat{D}}\left(y_{it}, x_{it}\right)$. So the distance of a point to the estimated frontier is $\hat{\hat{D}}\left(y_{it}, x_{it}\right) = e^{\hat{\hat{v}}_{it}}$, the same expression as for the univariate case.

Using(1) and (2), the translog model may be written as follows:

$$\begin{aligned}
\log y_{it}^1 \quad = \quad & -\beta_o - \sum_j \beta_j \log\left(\frac{y_{it}^j}{y_{it}^1}\right) - 0.5 \sum_j \sum_k \beta_{jk} \log\left(\frac{y_{it}^j}{y_{it}^1}\right) * \log\left(\frac{y_{it}^k}{y_{it}^1}\right) \\
& - \sum_j \sum_m \rho_{jm} \log\left(\frac{y_{it}^j}{y_{it}^1}\right) * \log x_{it}^m \\
& - \sum_m \lambda_m \log x_{it}^m - 0.5 \sum_m \sum_n \lambda_{mn} \log x_{it}^m \log x_{it}^n + v_{it} , \qquad (3)
\end{aligned}$$

where $\hat{\beta}_o, \hat{\beta}_j, \hat{\beta}_{jk}, \hat{\rho}_{jm}, \hat{\lambda}_m, \hat{\lambda}_{mn}$ are estimated by OLS regression providing the OLS residuals $\hat{\varepsilon}_{it}$ and COLS estimates are $\hat{\hat{\beta}}_o = \hat{\beta}_o + \min_{it} \hat{\varepsilon}_{it}$ and $\hat{\hat{v}}_{it} = \hat{\varepsilon}_{it} - \min_{it} \hat{\varepsilon}_{it}$. Coelli (2000) analyses the consistency of the OLS estimates under various general assumptions. As the $\log \frac{y_{it}^1}{y_{it}^1} = 0$, the parameters $\hat{\beta}_1, \hat{\beta}_{1,k}, \hat{\rho}_{1,m}$ are not estimated through model (1), $\forall k$ and $\forall m$. But to satisfy the condition of homogeneity in degree +1 in y, one has $\sum_j \hat{\beta}_j = 1$, $\sum_j \sum_k \hat{\beta}_{jk} = 0$ and $\sum_j \sum_m \hat{\rho}_{jm} = 0$. So for j equal to 1, the parameters can be evaluated by $\hat{\hat{\beta}}_1 = 1 - \sum_{j \neq 1} \hat{\beta}_j$, $\hat{\hat{\beta}}_{1,k} = 0 - \sum_{j \neq 1} \hat{\beta}_{jk}$ and $\hat{\hat{\rho}}_{1m} = 0 - \sum_{j \neq 1} \hat{\rho}_{jm}$.

3. Data and Methods

The objective is to work with data routinely collected in hospitals. At each patient discharge a minimal basic medical data set is collected. These data contain information like age, sex, admission in emergency, destination of the patient after the hospital stay, length of stay, length of stay in intensive care unit (ICU), diagnoses, and procedures performed. Diagnoses and procedures are coded with the ICD-9-CM classification system allowing grouping of hospital stays in Diagnosis Related Groups (DRGs) (Fetter et al, 1980). As hospitals are not financed by global budgets but partly by a fixed day-price and partly by payment for each medical act, it is possible for each hospital stay to establish a summary of the bill. So the medical minimum basic data set of a hospital stay can be linked with the summary of the bill.

The database used in this study contains data from 34 hospitals collected during the second semester of 1988. The DRG 122 (Acute myocardial infarction, discharge alive, with complicated cardiovascular diagnosis) is retained for further analysis. As there is no deceased patient at discharge in this DRG, censoring difficulties must not be taken into account in the statistical models. For this pathology, the variables analysed are total expenses, length of stay, medical fees and the characteristics of the patient: age, sex, admission in emergency, number of ill systems and length of stay in ICU.

In order to compare the results of the parametric approach with those of the nonparametric application, a translog model is used as functional form for each level of the explanatory dichotomous variables, admission in emergency and sex (Beguin, 2001). In the univariate case, the dependent variable is the length of stay or the total expenses and the explanatory variables are age, number of ill system and length of stay in intensive care unit (ICU). In the multivariate approach, applying the method described above, length of stay corresponds to y_{it}^{1} and is used as dependent variable in the OLS regression. Medical fees divided by length of stay refers to $\frac{y_{it}^{2}}{y_{it}^{1}}$ and is introduced in list of the explanatory variables of this regression along with age, number of ill system and length of stay in ICU. The frontier is estimated in a second step by COLS in the univariate approach and by COLS with correction for the homogeneity condition in the multivariate case.

The results are expressed in terms of efficiencies. The efficiency of each hospital stay is estimated by reference to the frontier of the corresponding level of the dichotomous variables, admission in emergency and sex. In order to give a global view of the efficiency levels, we provide some descriptive statistics of the efficiency of all stays in table 2.

We know that in deterministic frontier models, the estimators are very sensitive to too efficient stays, called 'efficiency outliers', because the estimated model envelops all the observed data points. P.Wilson (1995) proposes a method to detect influential observations in data envelopment analysis. In Beguin (2001), this method is adapted for large data sets. The same method is used here to detect too efficient hospital stays. In nonparametric frontier models, a subset of hospital stays are estimated as being efficient and lie on the estimated frontier. This is not the case with parametric models where often only one hospital stay lies on the estimated frontier. Therefore in this parametric approach, we define as a potential efficiency outlier each hospital stay with an efficiency greater than the percentile 95 value of the efficiency distribution. Then we apply the procedure proposed in Beguin (2001): we remove all potential efficiency outlier from the sample once at a time. The frontier is estimated with the N-1 remaining stays. The efficiency of all hospital stays are estimated from this new frontier. By this way, the efficiency of the removed stay may be greater than 1. The impact of the removed stay on the efficiency of the other stays can be measured by the number of stays with a modified efficiency and by the mean of the difference of the efficiencies. In our application below, the most efficient stays with the highest impact on the efficiency of the other stays and/or with the greatest increase of their own efficiency are excluded. We have chosen the 1% with the highest impact. It is of course useful to analyse these stays in detail. In particular, we confront the severity level of these too efficient stays with the resources they consume

The efficiencies of the remaining stays are estimated once again after the exclusion of all too efficient stays. All stays with a efficiency lower than the first percentile are classified as 'inefficient'. As for too efficient stays, the description of these stays confronts the resources they consume to the severity level of the patient. The capacity of both methods, parametric and nonparametric, to identify the same inefficient stays is also evaluated in our application.

4. Results

The sample contains 1643 hospital stays, 74.7% of patients are males and 74.9% are admitted in emergency. The mean of the explicative variables is 61.6 year for age (median 62), 1.2 for the number of ill system (median 1) and 2.1 for the number of days in ICU (median 0). For total expenses, the average is 118050BF (median 92921BF), the average of length of stay is 11.2 days (median 11 days) and the average of medical fees is 18477BF (median 6457BF). The variables number of ill systems and length of stay in ICU are positively and significantly correlated with total expenses, length of stay and medical fees; age is only correlated with length of stay.

Table 1 shows the parameter estimates of the OLS regression for each level of the dichotomic variables admission in emergency and sex when analysing both length of stay and medical fees. Some are insignificant. The interactions age-number of ill system and age-number of days in ICU are never significant in the regression models estimated for length of stay and medical fees. Moreover, some parameters take a negative value that is sometimes significant. The OLS regressions with length of stay or total expenses as dependent variables present the same kind of results. Except for the interaction between age and the number of ill systems, the parameters are at least significant in one of the 4 regressions when studying total expenses or length of stay.

For the whole sample, the mean of the efficiencies provided by these deterministic models varies between 0.0638 for total expenses, 0.1355 for length of stay and 0.0327 for medical fees and length of stay (table 2). Removing the too efficient stays, globally 1% of the sample, increases the efficiency of the other stays as shown in table 2. This augmentation is the most important for total expenses but the values of the efficiencies remain quite low, particularly for the multivariate model. Excluding all stays with an efficiency greater than the P95 value doesn't improve the efficiency of the other stays when studying total expenses or length of stay. The improvement is greater in the multivariate approach: the mean efficiency becomes 0.1801 and the median 0.1006. But this implies the exclusion of a greater number of stays and is not done in the following. The too efficient hospital stays are characterised by total expenses, length of stay or medical fees lower than the median value of the corresponding variable in the entire sample, whereas one, two or three of their variables reflecting severity show a value greater than the median.

It is also interesting to describe the most inefficient stays: we analyse the hospital stays with an efficiency lower than the P1 value. They are characterised by total expenses, length of stay or medical fees higher than the median value of the corresponding variable in the entire sample, whereas their variables reflecting severity show a value lower than the median. To be too efficient or too inefficient reflect some inconsistency between resources and severity variables and identify hospital stays needing further scrutiny in order to verify the quality of the data. Perhaps it would be necessary to exclude these stays in case of financing by pathology.

It is of course important to stress the economic consequence of removing the extreme stays. Removing the too efficient stays corresponds to a reduction of 0.26% of the total expenses, 0.27% of days of hospitalisation and 0.21% of medical fees, which is economically not important. However, 1% of hospital stays classified as too inefficient corresponds to 2.6% of total expenses, 3.1% of hospitalisation days and in the multivariate approach, 3.6% of medical fees and 3.8% of hospitalisation days. So removing these inefficient stays for computing the financing of an hospital could change it more significantly. If the proportion of inefficient stays excluded is 2%, these proportions become respectively 3.9%, 4.8%, 5.5% and 6.4%.

Another study describes the efficiencies of the same hospital stays estimated by a nonparametric frontier model (FDH) (Beguin, 2001). Positive and significant correlations are found when comparing the parametric and nonparametric estimations of efficiencies (table 3). The correlations between parametric and nonparametric efficiencies are higher for models concerning length of stay. Generally, the too efficient stays of the parametric models are too efficient or efficient stays in the FDH models. Only half of the too efficient stays in the nonparametric models are too efficient or efficient in the parametric models. Table 4 shows the distribution in the nonparametric model of the efficiencies of the too efficient stays of the parametric model and inversely. Table 5 presents the same results for the inefficient stays. Globally, for length of stay and total expenses, inefficient stays in one model are inefficient or have an efficiency lower than the P25 in the opposite model. For multivariate model, results are more contradictory. Indeed, 4 hospital stays inefficient in parametric model are efficient in nonparametric model.

The correlations between the dependent variables, total expenses, length of stay, medical fees, and the parametric and nonparametric efficiencies are shown in table 6. In both parametric and nonparametric models, the correlations between efficiencies and length of stay are higher than between efficiencies and the other dependent variables. But this phenomenon is particularly true in the parametric model.

5. Discussion

Our objective is to propose a method to highlight hospital stays that are to be considered as ‘outliers’ or even excluded from a global budget. The frontier models are used in order to rank hospital stays taking into account the medical characteristics of the patients routinely collected and available in hospital’s databases. The efficiencies obtained reflect this ranking.

In the regression models used before the estimation of the efficiencies, some parameters take a negative value while others do not reach a significant value. This is also observed by other authors working with translog model (Dor and Farley, 1996; Linna et al, 1998; Vita, 1990). Some of them propose a Box-Cox transformation of the explicative variables in order to avoid a negative value for a parameter or impose restrictions to the translog model in order to diminish the number of parameters (Linna et al, 1998; Vita, 1990). But, they report that this problem is not so severe when the objective is to estimate efficiencies (Linna et al, 1998). Nevertheless negative values of the parameters put some questions on the validity of the hypothesis of free disposability needed when using the nonparametric estimation (Simar and Wilson, 2000).

The efficiencies estimated show quite low values. This is partly explained by the fact that variables used to measure severity don’t completely describe the medical patient’s reality. Moreover medical severity of a patient is not always correlated to high expenses. The increase of the efficiencies after exclusion of the too efficient stays is not surprising as the deterministic models are very sensitive to outliers. But as the decision unit in this analysis is the hospital stay and not the hospital, deterministic models seem more appropriate. Therefore, hospital stays considered as too efficient are highlighted before a final estimation of efficiencies. The method used to point out too efficient stays is derived from the method developed for nonparametric model by P.Wilson (1995). As the results of parametric models are compared to those of nonparametric models, the same method is used to highlight the too efficient stays in parametric frontier model.

The correlations between parametric and nonparametric efficiencies are not specially high but they increase for length of stay. Some authors publish contradictory results on this topic (Banker et al, 1986; Bjurek and Hjalmarsson, 1990; Ferrier and Lovell, 1990; Gong and Sickles, 1992; Thanassoulis, 1993). Ferrier and Lovell (1990) find no significant correlations between efficiencies of banks estimated by a stochastic parametric cost frontier or by a non stochastic nonparametric cost frontier. The main reasons invoked are noise interpreted as inefficiency in the nonparametric frontier, the parametric structure imposed by the econometric approach, estimation of mean value by the econometric model. These arguments can be extended also to our application, except for noise as we only work with deterministic models. Gong and Sickles (1992) note that if the functional form of the stochastic model is close to data, the parametric model outperforms DEA. He proposes to choose DEA model when the divergence between the functional form and data increases or when the correlations between efficiencies and inputs raise. In our analysis, the dependent variable may be considered as the classical input of the frontier model. As the correlation between efficiencies and LOS are relatively high, perhaps for this aspect, nonparametric model should be preferred. Banker et al (1986) mention a broad agreement between efficiencies estimated by a DEA model or by a deterministic translog model but the results are shown by categories of efficiencies and no coefficient of correlation are reported. So, comparisons with the results of this analysis are difficult. The use of both parametric and nonparametric estimation of efficiencies offers to possibility to verify the hypothesis and validity of both methods. The negative parameter's values in parametric model allow to check the hypothesis of free disposability of the nonparametric model. The analysis of correlations between efficiencies and input gives an idea of the adequacy of parametric models.

In conclusion, a method to point out 'budget' outliers using parametric deterministic frontier model in order to take into account the characteristics of the patient is proposed. The models used are described. The method developed by P.Wilson to detect influencial observations when defining the frontier is used to highlight too efficient stays. The too efficient and inefficient stays are described. Finally, the results of parametric models are compared to those obtained when applying nonparametric deterministic frontier to the same set of data.

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Table 1. Parameters of the regression models for medical fees and length of stay (multivariate model)

Variables	No emergency, Women		No emergency, Men		Emergency, Women		Emergency, Men	
	β	p	β	p	β	p	β	p
log fees / log LOS	4.2503	0.1264	3.7991	0.0001	6.0450	0.0002	1.4921	0.0012
(log fees / log LOS)^2	-0.1308	0.0053	-0.2089	0.0001	-0.2187	0.0001	-0.0913	0.0001
*(log fees / log LOS)*log age*	-0.6653	0.2520	-0.0819	0.5618	-0.6949	0.0122	-0.1522	0.1097
*(log fees / log LOS)*log nb ill syst*	0.1983	0.6803	-0.5912	0.0013	-0.2323	0.4318	-0.0329	0.7904
*(log fees / log LOS)*log nb days ICU*	0.1153	0.2574	0.1254	0.0087	0.2286	0.0001	0.2197	0.0001
log age	-32.3633	0.1735	7.0220	0.0983	9.4917	0.2519	6.2186	0.0301
log nb days ICU	-0.4776	0.8544	-1.0373	0.3034	-0.9790	0.3537	-1.0110	0.0235
log nb ill syst	-12.7738	0.5922	0.7523	0.8885	1.6646	0.7933	1.6233	0.5298
(log age)^2	4.4759	0.1010	-0.7455	0.1355	-0.3599	0.6732	-0.5485	0.0734
(log nb days ICU)^2	0.0736	0.5149	-0.0183	0.8421	-0.0456	0.2259	-0.0686	0.0042
(log nb ill syst)^2	1.1100	0.4800	4.2886	0.0006	2.0263	0.0476	0.3420	0.4373
*log age * log nb days ICU*	-0.0367	0.9447	0.1996	0.3739	-0.1633	0.4133	-0.0566	0.5256
*log age * log nb ill syst*	2.1690	0.6658	-0.5406	0.6308	-0.7363	0.5204	-0.2904	0.5327
*log nb ill syst * log nb days ICU*	-0.3587	0.5048	-0.6645	0.0089	0.1758	0.2800	-0.2465	0.0119

Table 2. Description of efficiencies

Variables to minimise	*N*	*Q25*	*Median*	*Q75*	*Mean*	*Std Deviation*
Total Expenses (univariate)	*1643*	*0.0103*	*0.0177*	*0.0688*	*0.0638*	*0.1021*
Length of stay (LOS) (univariate)	*1643*	*0.0579*	*0.0874*	*0.1590*	*0.1355*	*0.1332*
Medical fees and LOS (multivariate)	*1643*	*0.0024*	*0.0100*	*0.0297*	*0.0327*	*0.0835*
After removal of the too efficient stays						
Total Expenses (unioutput)	*1624*	*0.1141*	*0.1854*	*0.2557*	*0.2009*	*0.1204*
Length of stay (LOS) (unioutput)	*1624*	*0.0823*	*0.1232*	*0.2068*	*0.1681*	*0.1355*
Medical fees and LOS (multioutput)	*1625*	*0.0065*	*0.0343*	*0.0860*	*0.0766*	*0.1208*

Table 3: Correlations between parametric and nonparametric efficiencies

	Pearson Correlation		*Spearman Correlation*	
	r	*p*	*r*	*p*
Total expenses				
Complete data (4 frontiers)	*0.37*	*0.0001*	*0.39*	*0.0001*
Length of stay				
Complete data (4 frontiers)	*0.55*	*0.0001*	*0.71*	*0.0001*
LOS and Medical fees				
Complete data (4 frontiers)	*0.28*	*0.0001*	*0.23*	*0.0001*

Table 4: Distribution of efficiencies of the too efficient stays in the opposite model

Too efficient stays	*Too eff*	*Eff*	*4°Quart*	*3° Quart*	*2° Quart*
Total expenses					
			Non parametric efficiencies		
Parametric model (*n* = 19)	*4*	*10*	*4*		*1*
			Parametric efficiencies		
Non parametric model (*n* = 19)	*4*	*5*	*7*	*2*	*1*
Length of stay					
			Non parametric efficiencies		
Parametric model (*n* = 19)	*5*	*14*			
			Parametric efficiencies		
Non parametric model (*n* = 17)	*5*	*2*	*6*	*3*	*1*
LOS - Medical fees					
			Non parametric efficiencies		
Parametric model (*n* = 18)	*6*	*12*			
			Parametric efficiencies		
Non parametric model (*n* = 17)	*6*	*3*	*3*	*4*	*1*

Table 5: Distribution of inefficient stay's efficiencies in the other model

Inefficient stays	*Ineff*	*1° Quart*	*2°Quart*	*3° Quart*	*4° Quart*	*Efficient*
Total expenses						
			Nonparametric efficiencies			
Parametric model (n = 16)		*9*	*7*			
			Parametric efficiencies			
Nonparametric model (n = 16)		*16*				
Length of stay						
			Nonparametric efficiencies			
Parametric model (n = 15)	*7*	*5*	*2*			
			Parametric efficiencies			
Nonparametric model (n = 17)	*8*	*9*				
LOS - Medical fees						
			Nonparametric efficiencies			
Parametric model (n = 16)	*1*	*5*	*4*	*2*		*4*
			Parametric efficiencies			
Nonparametric model (n = 22)	*1*	*15*	*5*	*1*		

Table 6: Correlations between efficiencies and dependent variables

	Pearson Correl.	*p*	*Spearman Correl.*	*p*
Non Parametric models				
Total expenses	*-0.06*	*0.0115*	*0.01*	*0.7517*
Length of stay	*-0.14*	*0.0001*	*-0.29*	*0.0001*
LOS and Medical fees	*-0.07 (LOS)*	*0.0024*	*-0.17 (LOS)*	*0.0001*
	-0.11 (Med fees)	*0.0001*	*-0.18 (Med fees)*	*0.0001*
Parametric models				
Total expenses	*-0.19*	*0.0001*	*-0.39*	*0.0001*
Length of stay	*-0.51*	*0.0001*	*-0.77*	*0.0001*
LOS and Medical fees	*-0.34 (LOS)*	*0.0001*	*-0.78 (LOS)*	*0.0001*
	-0.20 (Med fees)	*0.0001*	*-0.31 (Med fees)*	*0.0001*