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Modeling the dependence of conditional correlations on volatility

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Abstract

Several models have been developed to capture the dynamics of the conditional correlations between time series of financial returns, but few studies have investigated the determinants of the correlation dynamics. A common opinion is that the market volatility is a major determinant of the correlations. We extend some models to capture explicitly the dependence of the correlations on the volatility of the market of interest. The models differ in the way by which the volatility influences the correlations, which can be transmitted through linear or nonlinear, and direct or indirect effects. They are applied to different data sets to verify the presence and possible regularity of the volatility impact on correlations.

Keywords: volatility effects, conditional correlation, DCC, Markov switching.

JEL classification: C32, C58

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1 Introduction

Several studies have tried to uncover the determinants of correlations between time series of financial returns, but few of them are based on econometric models that relate explicitly the correlations to their determinants. This is an important topic, because the increase in correlation is linked to the phenomenon of contagion (Forbes and Rigobon, 2002), so the possibility to forecast the correlations is crucial in the study of spillover effects between financial markets, as well as of portfolio choice, hedging, and option pricing.

Some of these studies link the correlations in financial markets to economic and geographical variables (see, for example, Karolyi and Stulz, 1996, and Bayoumi et al., 2007), or to the market trend. In particular, Longin and Solnik (2001), using monthly data, notice that correlations tend to increase in correspondence with negative returns, arguing that correlations tend to increase in bear markets but not in bull markets. Modeling the multivariate distribution tails, they derive the distribution of extreme correlations for a wide class of return distributions and their finding is that high volatility per se does not seem to lead to an increase in conditional correlations, whereas correlations are mainly affected by the market trend.

Most empirical studies conclude that stock market correlations between countries increases in crash times (see, for example, Forbes and Chinn, 2004). Knif et al. (2005), using a logit functional relation between conditional correlation and conditional volatility, provide evidence that increases in the stock market volatility at the local and national level push the correlations up between stock market returns. Many authors notice that the correlations among international markets tend to increase when stock returns fall precipitously (see King and Wadhwani, 1990, Solnik et al., 1996, Chesnay and Jondeau, 2001, Ang and Bekaert, 2002). Ramchand and Susmel (1998), by using a SWARCH model (Hamilton and Susmel, 1994) with weekly data, obtain that the correlations between US and other stock markets are 2 to 3.5 times higher when the US stock market is in a high volatility state rather than a low one. Similarly, using a bivariate GARCH model, Longin and Solnik (1995) found that the episodes of abnormal volatility of the U.S. stock market are the main determinant of increasing stock market correlations. Boudt et al. (2012), analyzing series of US deposit bank holding companies, notice a significant better performance of regime switching models when switching probabilities are time-varying and driven by the VIX index[1] rather than constant. The common feeling is that in a time series context, volatility is a major determinant of correlations, in particular in presence of high volatility regimes.

These considerations are also implicitly present in other works. For example, Engle and Figlewski (2012) develop factorial models with time-varying correlations driven by shocks correlated across stocks, finding a strong relation between the changes in the implied volatilities of the stocks and the VIX index. In another study, Otranto (2012), analyzing the Italian market, finds a strong relationship between similar conditional variance structure of two assets and their conditional correlation.

The one-factor ARCH of Engle et al. (1990), with the market return as factor, directly implies that the conditional volatility of the market return is the single determinant of the

[1]The VIX index is a kind of implied volatility index for the 30-day options on the Standard and Poor's 500 index, introduced by the Chicago Board Options Exchange in 1993 (see CBOE, 2003, for details).

conditional correlations between the individual assets. The factor double ARCH introduces idiosyncratic volatilities as additional determinants, and the factor DCC (Colacito et al., 2011) enriches this structure to make it more flexible. See Engle (2009, Ch. 8) for a review.

Accepting the idea that the volatility affects the degree of correlation, several questions arise to introduce this effect in an econometric model of correlations:

1. Are the correlations sensitive to the volatility itself or to the regime (low or high) of the volatility?

2. Given a model that incorporates in some way a volatility effect, what is the marginal impact of the volatility on the correlation between two assets? Is this effect varying with the level of volatility?

3. Does the volatility affect the unconditional correlation (long-run effect) or the conditional correlation (short-run effect)?

4. Does the volatility help in forecasting the correlations between assets?

In this paper we try to answer to these questions by proposing several models able to capture in different ways the dependence of the conditional correlations of a set of financial time series on the volatility of the market of interest. In particular we extend the Dynamic Conditional Correlation (DCC) model of Engle (2002) in different ways: by including the volatility, or its regime, as an additive independent variable, or by including its effect in the value of the scalar coefficients of the DCC. The effect of changes in the regime of the volatility is also considered by extending the Regime Switching Dynamic Correlation (RSDC) model of Pelletier (2006) to include the effect of the volatility, or the regime of volatility, in the transition probabilities.

The new models are applied to two data sets. A detailed analysis is provided for a case with three assets, in which the volatility is represented by the VIX index; the explanation of this case is useful to illustrate in detail the main characteristics of the proposed models and the results. Then we extend the analysis to a data set constituted by the thirty assets composing the Dow Jones industrial index.

The paper is structured as follows. In the next section we describe the models proposed, whereas in Section 3 we detail the results relative to the example of three assets. In Section 4 we describe the results relative to the larger data set and in Section 5 we conclude the paper with some remarks. The Appendices provide the estimation results for the univariate models relative to the conditional variances and the volatility indices (first-step estimation) and explicit formulas of the marginal impact of the volatility on the conditional correlations in each model.

2 Volatility Dependent Conditional Correlation Models

Several existing models for dynamic conditional correlations can be extended to include some form of dependence on the volatility of a certain market, measured with a known indicator (such as the VIX). We consider a common structure for all the models, represented by the conditional variance-covariance matrix. More precisely, let $\boldsymbol{r}_t$ denote a $(n \times 1)$ vector of returns (with mean equal to zero) of n time series ($t = 1, \ldots, T$). Let us indicate with $\boldsymbol{H}_t$ the conditional covariance matrix of $\boldsymbol{r}_t$. It can be represented as

$$\boldsymbol{H}_t = \boldsymbol{S}_t \boldsymbol{R}_t \boldsymbol{S}_t, \tag{2.1}$$

where $\boldsymbol{S}_t$ is a diagonal matrix containing the conditional standard deviations and $\boldsymbol{R}_t$ is a time-varying positive definite matrix of correlations. Following Engle (2002), we can estimate the elements of $\boldsymbol{H}_t$ in two steps: in the first step we estimate the parameters of $\boldsymbol{S}_t$ (call them $\boldsymbol{\theta}_V$) using n univariate models for the conditional variances (for example, simple GARCH models); in the second step we estimate the parameters present in $\boldsymbol{R}_t$ (call them $\boldsymbol{\theta}_R$), conditioning on the estimate of $\boldsymbol{\theta}_V$. This is possible because the full log-likelihood function can be split into the sum of the following components:

$$\begin{aligned} L(\boldsymbol{\theta}_V) &= -\textstyle\sum_{t=1}^{T} \left[\log(|\boldsymbol{S}_t|) + 0.5\boldsymbol{u}_t'\boldsymbol{u}_t\right], \\ L(\boldsymbol{\theta}_R|\boldsymbol{\theta}_V) &= -\textstyle\sum_{t=1}^{T} \left[\log(|\boldsymbol{R}_t|) + 0.5\boldsymbol{u}_t'\boldsymbol{R}_t^{-1}\boldsymbol{u}_t\right], \end{aligned} \tag{2.2}$$

where $\boldsymbol{u}_t = \boldsymbol{S}_t^{-1}\boldsymbol{r}_t$ is the vector of *degarched* returns.

We are interested to develop models for $\boldsymbol{R}_t$ including a dependence on the volatility index. We call this class of models the Volatility Dependent Conditional Correlation (VDCC) models.

We consider two possibilities: 1) the level of volatility (call it v_t) affects the conditional correlations; 2) the regime of the volatility (high or low) affects the conditional correlations.

The second approach requires to explicit a regime switching model for the volatility. Formally we consider the $(n+1)$ vector $\boldsymbol{w}_t = (\boldsymbol{r}_t'; v_t)'$, where the last element v_t follows a proper dynamics, for example a Markov Switching (MS) ARIMA with two regimes; we label with 1 the regime of high volatility, 0 the regime of low volatility. We assume that v_t is not correlated with each element of $\boldsymbol{r}_t$, so the conditional correlation matrix of $\boldsymbol{w}_t$ is given by:

$$\boldsymbol{R}_t^* = \begin{bmatrix} \boldsymbol{R}_t & \boldsymbol{0} \\ \boldsymbol{0} & 1 \end{bmatrix}.$$

Let ζ_t denote the regime of v_t at time t, with $\zeta_t = 1$ to indicate the high volatility regime and $\zeta_t = 0$ the low one. The conditional covariance matrix of $\boldsymbol{w}_t$ is given by

$$\boldsymbol{H}_t^* = \boldsymbol{S}_t^* \boldsymbol{R}_t^* \boldsymbol{S}_t^*,$$

where

$$\boldsymbol{S}_t^* = \begin{bmatrix} \boldsymbol{S}_t & \boldsymbol{0} \\ \boldsymbol{0} & \sigma_{\zeta_t} \end{bmatrix}$$

and σ_{ζ_t} is the switching standard deviation of v_t in correspondence with the regime ζ_t.

Notice that the likelihood function can be split again in two parts; it is sufficient to substitute in (2.2) $\boldsymbol{S}_t$ with $\boldsymbol{S}_t^*$ and $\boldsymbol{R}_t$ with $\boldsymbol{R}_t^*$. The first likelihood contains the parameters of the n univariate GARCH models and the MS-ARIMA, and the second one the parameters relative to the correlation part, conditional on the first group of parameters. Moreover, in the first part, we obtain the filtered one step ahead probabilities ($Pr(\zeta_t = i|\mathbf{\Psi}_{t-1})$), the same updated at the current time ($Pr(\zeta_t = i|\mathbf{\Psi}_t)$), and the smoothed ones $Pr(\zeta_t = i|\mathbf{\Psi}_T)$, where $\mathbf{\Psi}_t$ represents the information available at time t (see Hamilton, 1990). The expected value of the regime at time t, conditional on the information available at the same time t, is equal to $E_t(\zeta_t) = \sum_{i=0}^{1} iPr(\zeta_t = i|\mathbf{\Psi}_t) = Pr(\zeta_t = 1|\mathbf{\Psi}_t)$. We use $E_t(\zeta_t)$ as a conditioning variable to represent the effect of the regime of volatility on the conditional correlations. Note that if we label with 0 the high volatility regime and 1 the low volatility regime, $E_t(\zeta_t)$ will change, but the corresponding coefficients of the model adopted to measure its effect would adjust to that.

2.1 Smooth Transition VDCC

A first model that can be included in the VDCC family was introduced by Silvennoinen and Teräsvirta (2012), who suppose that the conditional correlation matrix changes along the time according to a smooth function f_t ranging in the interval $[0, 1]$. We suppose that f_t is a function of x_t, where x_t is either the volatility v_t or its expected regime $E_t(\zeta_t)$. This model, known as the Smooth Transition Conditional Correlation (STCC) model, is given by:

$$\begin{aligned} &\boldsymbol{R}_t = \boldsymbol{R}_l f_t + \boldsymbol{R}_h(1 - f_t), \\ &f_t = [1 + \exp(-\gamma(x_{t-1} - c))]^{-1}, \text{with } \gamma > 0. \end{aligned} \tag{2.3}$$

The matrices $\boldsymbol{R}_l$ and $\boldsymbol{R}_h$ are positive definite correlation matrices. The former corresponds to the low correlation regime and the latter to the high one, since f_t is increasing in x. As said in the Introduction, several authors distinguish the effect of volatility on correlations for periods of turmoil (high volatility) and quietness (low volatility). This is consistent with the presence of two regimes, which can be included in a model for conditional correlations; model (2.3) is a way to represent this hypothesis.

This structure ensures that $\boldsymbol{R}_t$ is a positive definite correlation matrix at each point in time. We call model (2.3) the STCC-V model when $x_t = v_t$ and the STCC-R model when $x_t = E_t(\zeta_t)$ (V stands for volatility, R for regime).

2.2 VDCC with Markov Switching

A different way to consider the presence of regimes is to model the conditional correlations with MS dynamics. Pelletier (2006) proposes a Regime Switching for Dynamic Correlations (RSDC) model by specifying

$$\boldsymbol{R}_t = \boldsymbol{R}_{s_t}, \tag{2.4}$$

where s_t is a discrete unobservable random variable, ranging in $[1, k]$, for which he hypothesizes a Markovian dynamics, with p_{ij} representing the probability to switch from regime i at time $t-1$ to regime j at time t. We consider two regimes ($k = 2$), labeled again l (low correlation regime) and h (high correlation regime).

A simple extension of the RSDC model includes the volatility as a determinant of the regime of the correlations. We propose two cases that correspond to two alternative specifications of the transition probability matrix of the RSDC model.

The first case is implemented by specifying a time-varying transition probability matrix, as in Filardo (1994). In practice, we parameterize p_{ij} using a logistic function. Considering a two-regime MS model, the transition probabilities are expressed as:

$$\begin{array}{l} p_{ii,t} = \frac{exp(\theta_{0,i}+\theta_{1,i}v_{t-1})}{1+exp(\theta_{0,i}+\theta_{1,i}v_{t-1})} \\ p_{ij,t} = 1 - p_{ii,t} \qquad i = l, h, \qquad i \neq j \end{array} \tag{2.5}$$

We call this model the Time-Varying transition probability (TV-RSDC) model. Notice that TV-RSDC nests RSDC, by constraining $\theta_{1,h} = \theta_{1,l} = 0$. A simple Wald test can be used for checking this joint hypothesis, or individual t-tests to verify the hypothesis $\theta_{1,i} = 0$ for each i can be used to detect the presence of the volatility effect in each regime.

To represent the second case, the one of dependence on the regime of the volatility (instead of the volatility itself as in the first case), we can introduce an effect of the regime of volatility on the regime of correlations. We hypothesize different Markov chains driving the regimes of $\boldsymbol{R}_t$ and v_t. The basic idea is similar to the one proposed by Otranto (2005) who introduces the Multi-Chain MS model, but its estimation is simpler. We assume that the elements of the transition probability matrix relative to the regime of $\boldsymbol{R}_t$ depend on the volatility regime indicator variable ζ_t:

$$p_{ij|m} = Pr(s_t = j|s_{t-1} = i, \zeta_{t-1} = m).$$

The advantage of this specification is the fact that it is possible to distinguish the effect of the volatility on the regime of correlations during high and low volatility regimes, obtaining time-varying transition probabilities in the following way:

$$p_{ij,t} = \sum_{m=0}^{1} p_{ij|m} Pr(\zeta_{t-1} = m|\boldsymbol{\Psi}_{t-1}). \tag{2.6}$$

The constant probability model is not nested in this new model, but we can test with a simple Wald test if

$$H_0 : p_{ij|0} = p_{ij|1}.$$

If the null is accepted, we obtain evidence in favor of a constant probability model, represented by the RSDC model. We call this model the Double Chain (DC-RSDC) model.

2.3 Dynamic VDCC

The most widespread model for conditional correlations is the DCC model of Engle (2002), where the time-varying correlation matrix $\boldsymbol{R}_t$ is obtained by the following equations (we use the consistent specification of Aielli, 2009):

$$\begin{array}{l} \boldsymbol{R}_t = \tilde{\boldsymbol{Q}}_t^{-1} \boldsymbol{Q}_t \tilde{\boldsymbol{Q}}_t^{-1}, \\ \boldsymbol{Q}_t = (1 - a - b)\boldsymbol{R} + a\tilde{\boldsymbol{Q}}_{t-1}\boldsymbol{u}_{t-1}\boldsymbol{u}'_{t-1}\tilde{\boldsymbol{Q}}_{t-1} + b\boldsymbol{Q}_{t-1}, \\ \tilde{\boldsymbol{Q}}_t = \text{diag}(\sqrt{q_{11,t}} \sqrt{q_{22,t}}, \ldots, \sqrt{q_{nn,t}}), \end{array} \tag{2.7}$$

where a and b are unknown non negative scalar coefficients constrained by $a + b < 1$. A natural way to extend the DCC model for our purpose is to consider an additive effect of the volatility, or its expected regime, on the conditional correlations. Thus we modify the second equation of (2.7) in the following way:

$$\boldsymbol{Q}_t = (1 - a - b - g\bar{x})\boldsymbol{R} + a\tilde{\boldsymbol{Q}}_{t-1}\boldsymbol{u}_{t-1}\boldsymbol{u}_{t-1}^{'}\tilde{\boldsymbol{Q}}_{t-1} + b\boldsymbol{Q}_{t-1} + gx_{t-1}, \tag{2.8}$$

where $\bar{x}$ is the mean of x_t and g is a parameter; the constant is modified to do correlation targeting. We call this model the DCC with Additive Volatility Effect (DCC-AVE) when $x_t = v_t$ and the DCC with Additive volatility Regime Effect (DCC-ARE) if $x_t = E_t(\zeta_t)$.

The additive effect is not the only possibility to include the effect of volatility in the DCC model; a non-linear effect is obtained by rendering the coefficients a and b in (2.7) time-varying and dependent on the level of volatility or its expected regime. In this case the second equation of (2.7) is:

$$\boldsymbol{Q}_t = (1 - a_t - b_t)\boldsymbol{R} + a_t\tilde{\boldsymbol{Q}}_{t-1}\boldsymbol{u}_{t-1}\boldsymbol{u}_{t-1}^{'}\tilde{\boldsymbol{Q}}_{t-1} + b_t\boldsymbol{Q}_{t-1}, \tag{2.9}$$

where $a_t = a_0 + a_1 f_{a,t}$ and $b_t = b_0 + b_1 f_{b,t}$ are functions of x_t for each t, with the constraint $a_t + b_t < 1$ for all t. In our experiments we have obtained good results adopting logistic functions, such as:

$$\begin{aligned} f_{a,t} &= \frac{exp(\theta_{a,0}+\theta_{a,1}x_{t-1})}{1+exp(\theta_{a,0}+\theta_{a,1}x_{t-1})}, \\ f_{b,t} &= \frac{exp(\theta_{b,0}+\theta_{b,1}x_{t-1})}{1+exp(\theta_{b,0}+\theta_{b,1}x_{t-1})}. \end{aligned} \tag{2.10}$$

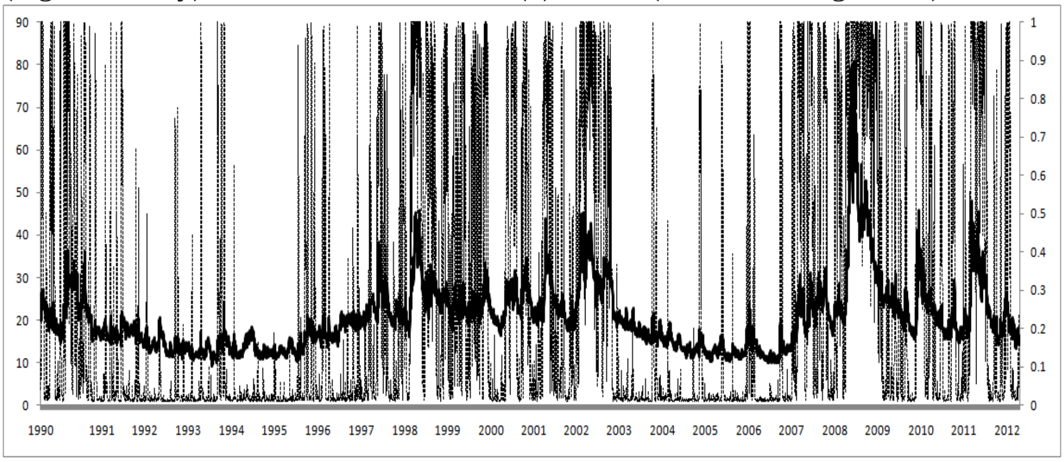
We call model (2.9) the DCC with Time-Varying coefficients depending on Volatility (DCC-TVV) if in (2.10) $x_t = v_t$, and the DCC with Time-Varying coefficients depending on the Regime of volatility (DCC-TVR) if $x_t = E_t(\zeta_t)$.

3 Empirical Evidence for Three Stocks

We perform a comparison of the models, with the purpose to answer to the questions listed in the Introduction, using a data set of daily returns relative to three of the most frequently traded stocks at the New York Stock Exchange included in the S&P500 index: Ford (labeled with F), Hewlett-Packard (H), IBM (I). They are also three of the five series analyzed by Silvennoinen and Teräsvirta (2012) to illustrate their STCC model, with different time span, preliminary transformations and transition variable. The data span the period from January, 2 1990 to September 17, 2012 (source Yahoo Finance). The closing price data are transformed into returns for a total of 5725 observations for each time series. The volatility index adopted is the VIX, its sample mean is 20.5, and its standard deviation 8.2.

For each series of returns we have estimated the univariate GARCH(1,1) model obtaining the corresponding degarched returns, whereas for the VIX series (divided by 100) we have estimated a MS-AR(2) model with two regimes to obtain the variable $E_t(\zeta_t)$. The results relative to this first estimation step, corresponding to the maximization of the first equation of (2.2), are shown in Appendix 1. In Figure 1 we show the VIX series with the corresponding filtered updated probabilities of the state of high volatility. We notice that the filtered probabilities follow the dynamics of the VIX, capturing not only the highest peaks, but also several jumps in the VIX index identifying strong relative increases in the volatility.

Figure 1: VIX series (continuous line and left axis) and filtered probabilities of state 1 (high volatility) obtained with a MS-AR(2) model (dotted line, right axis)



3.1 In-sample Comparison

Our first purpose is to check if the models for the analysis of conditional correlations can be extended to include the effect of the volatility and how this effect enters in the correlation dynamics. We have estimated all the models previously defined, in addition to the models that do not include the volatility effects, including the Constant Conditional Correlation (CCC) model of Bollerslev (1990), which corresponds to the DCC model in (2.7) with $a = b = 0$. Summarizing, we have estimated eleven different models:

- Three Conditional Correlation (CC) Models without volatility effects: CCC, RSDC, DCC;
- Two Smooth Transition VDCC models: STCC-V, STCC-R;
- Two VDCC models with Markov Switching: TV-RSDC, DC-RSDC;
- Four Dynamic VDCC models: DCC-AVE, DCC-ARE, DCC-TVV, DCC-TVR

The estimation results are shown in Table 1. The corresponding log-likelihood functions and model choice criteria are in Table 2. For the DCC-TVV and DCC-TVR models, the coefficient b_t is found constant, so we have suppressed the coefficient b_1 and the nuisance parameters $\theta_{b,0}$ and $\theta_{b,1}$. The coefficients r_{ij} $(i, j = F, H, I)$ are the elements of the constant correlation matrices $\boldsymbol{R}$ between assets i and j in the different DCC models, and the indices h and l distinguish the elements of $\boldsymbol{R}_h$ and $\boldsymbol{R}_l$ in (2.3) and (2.4).

The bad performance of the STCC models in terms of likelihood function is clear (and perhaps surprising). The worst performance achieved by the CCC model is not surprising since the dynamic correlations cannot be constant over the long sample that we use. With the exception of the STCC-V model,[2] all the VDCC models have significant (at conventional levels) coefficients of the volatility or its expected regime, implying that the volatility (or its regime) has an impact on the dynamics of the conditional correlations.

[2]The estimate of γ, equal to 39.2, suggests a swift transition mechanism. It is common in ST models that the estimate of γ is insignificant when it is large. See Teräsvirta (1994).

Table 1: Parameter estimates of CC and VDCC models

No Volatility Effect										
	r_{FH}	r_{FI}	r_{HI}	a	b					
CCC	0.28	0.28	0.42							
	(0.02)	(0.01)	(0.02)							
DCC	0.29	0.31	0.43	0.008	0.988					
	(0.01)	(0.01)	(0.01)	(0.002)	(0.003)					
	$r_{FH,h}$	$r_{FI,h}$	$r_{HI,h}$	$r_{FH,l}$	$r_{FI,l}$	$r_{HI,l}$	p_{hh}	p_{ll}		
RSDC	0.61	0.60	0.79	0.10	0.10	0.21	0.76	0.77		
	(0.04)	(0.05)	(0.03)	(0.02)	(0.02)	(0.03)	(0.04)	(0.03)		
Smooth Transition VDCC										
	$r_{FH,h}$	$r_{FI,h}$	$r_{HI,h}$	$r_{FH,l}$	$r_{FI,l}$	$r_{HI,l}$	γ	c		
STCC-V	0.38	0.44	0.60	0.24	0.21	0.35	39.24	0.25		
	(0.04)	(0.05)	(0.04)	(0.03)	(0.03)	(0.03)	(33.09)	(0.02)		
STCC-R	0.56	0.61	0.87	0.17	0.16	0.26	1.45	1.03		
	(0.08)	(0.11)	(0.09)	(0.05)	(0.04)	(0.05)	(0.10)	(0.27)		
VDCC with Markov Switching										
	$r_{FH,h}$	$r_{FI,h}$	$r_{HI,h}$	$r_{FH,l}$	$r_{FI,l}$	$r_{HI,l}$	$\theta_{0,h}$	$\theta_{1,h}$	$\theta_{0,l}$	$\theta_{1,l}$
TV-RSDC	0.60	0.60	0.79	0.09	0.09	0.20	0.14	4.06	1.81	-3.78
	(0.04)	(0.05)	(0.03)	(0.03)	(0.02)	(0.03)	(0.07)	(0.82)	(0.23)	(1.42)
	$r_{FH,h}$	$r_{FI,h}$	$r_{HI,h}$	$r_{FH,l}$	$r_{FI,l}$	$r_{HI,l}$	$p_{hh\|0}$	$p_{hh\|1}$	$p_{ll\|0}$	$p_{ll\|1}$
DC-RSDC	0.60	0.59	0.78	0.09	0.09	0.20	0.67	0.91	0.75	0.74
	(0.04)	(0.05)	(0.03)	(0.03)	(0.02)	(0.03)	(0.05)	(0.03)	(0.04)	(0.05)
Dynamic VDCC										
	r_{FH}	r_{FI}	r_{HI}	a_0	a_1	b	g	$\theta_{a,0}$	$\theta_{a,1}$	
DCC-AVE	0.10	0.11	0.27	0.010		0.979	0.011			
	(0.00)	(0.00)	(0.00)	(0.001)		(0.002)	(0.001)			
DCC-ARE	0.18	0.19	0.34	0.010		0.972	0.009			
	(0.00)	(0.00)	(0.00)	(0.001)		(0.003)	(0.001)			
DCC-TVV	0.25	0.25	0.38	0.000	0.017	0.979		-3.06	15.46	
	(0.00)	(0.00)	(0.00)	(0.000)	(0.002)	(0.004)		(0.67)	(3.06)	
DCC-TVR	0.24	0.23	0.37	0.000	0.023	0.977		-1.23	2.52	
	(0.00)	(0.00)	(0.00)	(0.000)	(0.002)	(0.002)		(0.13)	(0.18)	

Robust standard errors in parentheses.

Table 2: Log-Likelihood, AIC and BIC of CC and VDCC models

Model	Log-Lik	AIC	BIC
CCC	-7676.25	2.685	2.688
DCC	-7609.78	2.662	2.668
RSDC	-7383.69	2.584	2.593
STCC-V	-7621.25	2.667	2.676
STCC-R	-7644.20	2.675	2.684
TV-RSDC	**-7363.51**	**2.578**	**2.589**
DC-RSDC	-7366.05	2.579	2.590
DCC-AVE	-7607.20	2.661	2.668
DCC-ARE	-7598.99	2.659	2.666
DCC-TVV	-7604.71	2.661	2.671
DCC-TVR	-7601.18	2.660	2.669

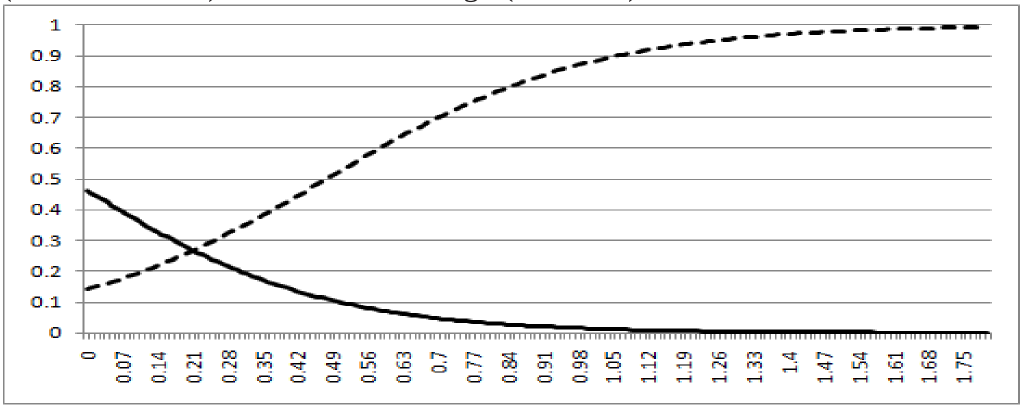
From Table 2, we notice that the VDCC models with MS have much smaller AIC and BIC values than the other VDCC models. The RSDC model has the third rank among all models according to the criteria values. Notice that the log-likelihood value of the TV-RSDC is 20 units larger than the corresponding value of the RSDC. For DC-RSDC, the increase is 17. In both cases, the increase is quite significant for two additional parameters.

The Wald tests favor the existence of the volatility effect in the TV-RSDC models, in particular in the case of high volatility: the p-value for the hypothesis $\theta_{1,h} = 0$ is less than 0.001, for the hypothesis $\theta_{1,l} = 0$ it is equal to 0.035. In the case of DC-RSDC model, the Wald test rejects the null hypothesis $p_{hh|0} = p_{hh|1}$ (p-value less than 0.01), whereas it does not reject the null hypothesis $p_{ll|0} = p_{ll|1}$ (p-value equal to 0.17). These results suggest that the high volatility regime impacts the probability to change the regime of the conditional correlations, whereas the low volatility periods do not seem to have any effect. Considering that in the DC-RSDC model the probability to stay in the regime of high correlation is higher when the volatility is high (0.91) than low (0.67), whereas in the low correlation case it is almost constant (0.75 versus 0.74), this result is consistent with the works finding that correlations increases in turmoil periods (see Introduction). This fact can be appreciated also for the TV-RSDC model with a graphical analysis of the probability to change regime.

In Figure 2 we plot the logistic functions, defined in equation (2.5): $p_{hl,t}$ (probability to switch from the high to the low correlation regime; continuous line) and $p_{lh,t}$ (probability to switch from the low to high; dash line) as functions of the volatility level. Notice that the probability $p_{hl,t}$ decreases quickly when the volatility increases and reaches virtually zero when the VIX index is at 100 or more; the opposite behavior can be observed for $p_{lh,t}$, with a slower convergence to 1. In the data set used, the minimum value of the VIX index is 9.31 and the maximum 80.86 (the scale is divided by 100 on the graph).

The four DCC-type VDCC models hardly improve the AIC and BIC values of the DCC model; there are even two cases (DCC-TVV and DCC-TVR) where the BIC increases (by 0.003 and 0.001 respectively). Judging by likelihood ratios, however, these VDCC models improve significantly the DCC model: the p-values of the $\chi^2(1)$ statis-

Figure 2: TV-RSDC model: probability to switch from high to low correlation regime (continuous line) and from low to high (dash line), as functions of VIX/100



tics are 0.02 (DCC-AVE) and less than 0.0001 (DCC-ARE); those of the $\chi^2(3)$ are 0.02 (DCC-TVV) and less than 0.001 (DCC-TVR). For the last two, the p-values are probably underestimated, since the volatility expected regime is a generated regressor.

Another interesting result in Table 1 is that, comparing each VDCC with MS model with the corresponding model without volatility effects, the estimates of the constant terms of the correlation matrices hardly change and the main difference is in the dynamic coefficients. Some differences can be noted in the Dynamic VDCC models with respect to DCC, in particular in the DCC-AVE and DCC-ARE, probably due to the presence of the g coefficient in the constant part. In practice, TV-RSDC and DC-RSDC provide a similar inference on the level of the correlations in each regime as RSDC, and the difference is just in the capability of the two VDCC models to capture the effect of the volatility through the volatility regime variable. In the Dynamic VDCC models the presence of the volatility (regime of volatility) modifies the level of correlation and the b coefficient is lower with respect to the DCC case.

The previous interpretations can be related to the results of the evaluation of the in-sample forecasting performance for the correlation matrices obtained with the eleven models.[3] We have applied the Model Confidence Set (MCS) approach of Hansen et al. (2003); in particular we have used the Quasi-Likelihood (QL) loss function with the semi-quadratic statistic (TSQ); see Clements et al. (2009) for details. The results are presented in Table 3. We can notice that the MCS approach excludes first the CCC model, then the STCC models, next the DCC models, and the final group is the one of MS models. The RSDC and DC-RSDC models show a significant improvement respect to TV-RSDC at the test size of 5% (but not at 1%). We have repeated the MCS procedure only for the STCC models and only for the DCC models. In both cases there is no significant difference in the performance within these two groups in terms of the QL loss function.

Our conclusion is that the VDCC models show a better performance (AIC and BIC) than the corresponding models without volatility effect. The volatility effect is statisti-

[3]For the MS models, all the estimates of the correlation coefficients are conditional on the full information available, so the estimated correlations are given by $\boldsymbol{R}_t = \boldsymbol{R}_h Pr(s_t = h|\boldsymbol{\Psi}_T) + \boldsymbol{R}_l Pr(s_t = l|\boldsymbol{\Psi}_T)$.

Table 3: MCS results for individual in-sample forecasts, using the Quasi-Likelihood loss function and the TSQ statistics

Model	p-value
CCC	0.000
STCC-R	0.000
STCC-V	0.000
DCC-AVE	0.000
DCC-TVV	0.000
DCC-ARE	0.000
DCC-TVR	0.000
DCC	0.000
TV-RSDC	0.023
DC-RSDC	0.094
RSDC	1.000

The first row represents the first model removed, down to the best performing model in the last row.

cally significant, except in the STCC-V model. In terms of in-sample forecasting, the VDCC models and their counterparts without volatility effect do not seem to differ significantly. In other terms, it seems that the volatility or its expected regime is a relevant determinant of the periods of high and low correlations, but it does not increase the in-sample fitting significantly. More precisely, the dynamics of the conditional correlations seems subject to abrupt changes, given the better performance of the MS models with very different matrices $\boldsymbol{R}_h$ and $\boldsymbol{R}_l$. These results are consistent with those of Boudt et al. (2012), who apply a particular regime switching model to a set of four US deposit bank holding companies over the period 1994-2011 and find evidence in favor of time-varying switching probabilities driven by the VIX index, with a constant correlation within the regimes, as in our VDCC RSDC models.

3.2 Marginal Impact of Volatility

Given our conclusion in favor of a statistically significant effect of the volatility on the conditional correlations, it is interesting to investigate how the correlations change in reaction to a given variation of the volatility, in the different models. This type of marginal impact can be calculated by evaluating the derivative of each conditional correlation function with respect to v_{t-1} or to $E_{t-1}(\zeta_{t-1})$. Clearly, the interpretation is different: in the first case we evaluate the impact of the increase of 100 points in volatility on the correlations (recall that $v_t = VIX/100$); in the second case we evaluate the impact of a change in the regime of volatility (from low to high or high to low).

The formulas of the marginal impact for each model are provided in Appendix 2, where it is also underlined that the marginal impact in the STCC-V, TV-RSDC and DCC-TVV models depends on the value of v_{t-1}, in the STCC-R and DCC-TVR on the value of $E_{t-1}(\zeta_{t-1})$, in the DC-RSDC model on the value of $Pr(\zeta_{t-1})$, whereas it is constant for

Table 4: Marginal impact of volatility and regime of volatility on conditional correlations

υ_{t-1}	variation	STCC-V			TV-RSDC			DCC-TVV		
		r_{FH}	r_{FI}	r_{HI}	r_{FH}	r_{FI}	r_{HI}	r_{FH}	r_{FI}	r_{HI}
0.093 (min)	0.1	0.001	0.002	0.002	0.064	0.064	0.073	0.003	0.003	0.002
0.147 (Q1)	0.1	0.009	0.015	0.016	0.070	0.070	0.080	0.004	0.004	0.003
0.190 (Q2)	0.1	0.042	0.070	0.074	0.073	0.072	0.083	0.005	0.005	0.004
0.240 (Q3)	0.1	0.132	0.220	0.232	0.073	0.072	0.084	0.004	0.004	0.003
0.809 (max)	0.1	0.000	0.000	0.000	0.009	0.009	0.010	0.000	0.000	0.000
	variation	DCC-AVE								
	0.1	0.001	0.001	0.001						
$E_{t-1}(\zeta_{t-1})$	variation	STCC-R			DCC-ARE			DCC-TVR		
0	1	0.084	0.100	0.133	0.006	0.006	0.005	0.007	0.007	0.005
1	-1	-0.141	-0.167	-0.222	-0.006	-0.006	-0.005	-0.008	-0.008	-0.006
$Pr(\zeta_{t-1} = 1)$	variation	DC-RSDC								
0	1	0.093	0.091	0.104						
1	-1	-0.181	-0.178	-0.203						

DCC-AVE and DCC-ARE. In Table 4 we show the results for the MS and the dynamic VDCC models for these cases:

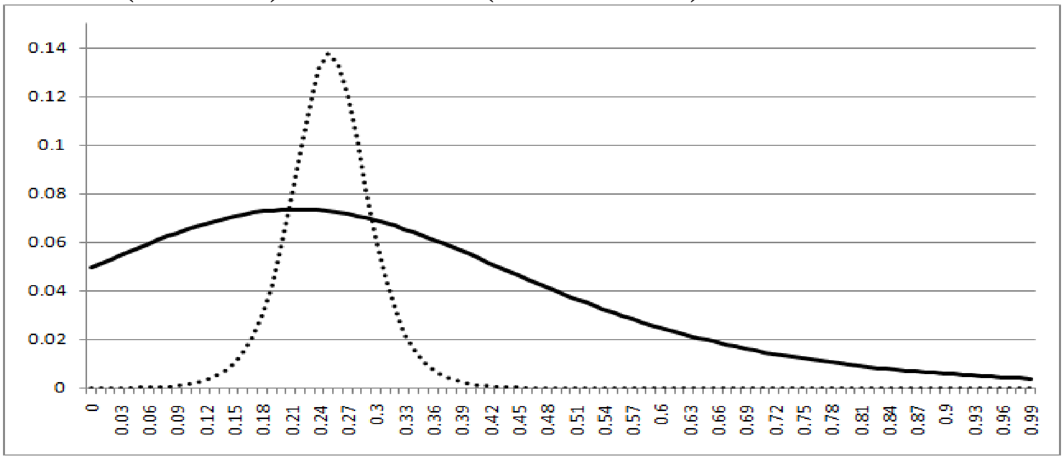
- for υ_{t-1} we report the impact of a 0.1 point increase of the variable, i.e. 10 points of the VIX (about one standard deviation), at the minimum, at the three quartiles and at the maximum values of the VIX index in the period considered;
- for $E_{t-1}(\zeta_{t-1})$ we consider the impact of the regime of volatility by a change of the variable from 0 to 1 and a change from 1 to 0;
- in the DC-RSDC case, we consider the impact of a change of $Pr(\zeta_{t-1})$ from 0 to 1 and from 1 to 0.

The first clear evidence derived from Table 4 is that the marginal impact of the volatility (or regime of volatility) is visible in the STCC and MS models, whereas it is very small in the DCC models. The explanation is obvious: the DCC models include the dependence of the conditional correlations on the past values of the degarched returns and the matrix $\mathbf{Q_t}$, whereas in the STCC and MS models the dynamics is only relative to the change in regime of the correlation matrix.[4] It is interesting to underline that the time series correlations between the elements of the $\mathbf{Q_t}$ matrix and the υ_t index are rather high: 0.58 for the element referred to (F,H), 0.62 for (F,I), 0.55 for (H,I), so it seems clear that the presence of the autoregressive dynamics obscures the volatility effect.

For the TV-RSDC case, we can notice that the marginal effect of volatility increases until the third quartile, whereas it is very low when the level of volatility is at its maximum

[4]Theoretically, it is possible to add a similar MS dynamics to the VDCC models, but this creates a path dependence problem (see, for example, Bauwens et al., 2010), due to the presence of the lagged value of $\mathbf{Q_t}$, which renders ML estimation impossible. We have tried a MS DCC model with only the ARCH term (i.e. imposing $b = 0$) but the corresponding coefficient (a) is not significantly different from zero.

Figure 3: Marginal impact of the volatility on the correlation between F and H using STCC-V (dotted line) and TV-RSDC (continuous line)



because it is likely that the correlation is already in the high regime. In the DC-RSDC and STCC-R cases, notice that switching from regime 1 to regime 0, the correlations decrease by about 0.2. and this is twice larger (in absolute value) than when the volatility switches from a quiet regime to a turmoil (0 to 1).

The STCC-V model shows a particular behavior; in fact the marginal impact of the volatility is very strong in correspondence of the third quartile (with increases of more than 0.2 for the pairs T-I and H-I), whereas it is low for the other cases. In Figure 3 it is possible to analyze more in detail the behavior of the marginal impact function of the STCC-V model, compared with the marginal impact function of the TV-RSDC model (we show the results only for the pair F-H, but the behavior of the other two pairs is very similar). We can notice that the effect of volatility for STCC-V is concentrated around values of the VIX index between 20 and 30, whereas it is tiny in the other cases; on the other hand the TV-RSDC model shows a lower impact of volatility on correlation spread out on a much wider set of volatility values.[5]

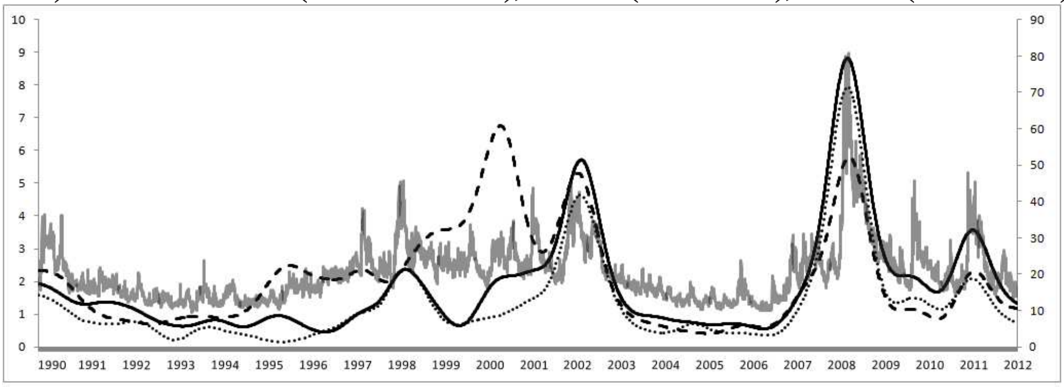
Our conclusion is that the marginal effect of the volatility, or its regime, is evident when we consider an indirect effect, working through the regime of correlations. The direct marginal effect in DCC models, and the indirect effect through the time-varying coefficient of the DCC are tiny.

3.3 Long-run and short-run effects

Bauwens et al. (2012) have distinguished between long-run and short-run correlations in the analysis of financial markets, introducing the multiplicative DCC (mDCC) model. Given the presence of a volatility effect, we wonder if the volatility impacts both kinds of correlations or just one of them. For this purpose, following the three-step procedure of Bauwens et al. (2012), we first estimate the long-run covariance matrix of the three assets, then we model the conditional variance of the standardized returns with univariate

[5]Since the observed values of the VIX between 20 and 30 represent only 34% of the observed sample, this explains the lack of significance of the estimate of γ, see also footnote 2.

Figure 4: VIX index (gray line and right axis) and estimated long-run covariances (left axis) between F and H (continuous line), F and I (dotted line), H and I (dashed line)



GARCH models, and finally we apply the eight VDCC models proposed in this work to the degarched residuals. [6]

In this approach the matrix $\boldsymbol{H}_t$ is decomposed as

$$\boldsymbol{H}_t = \boldsymbol{\Sigma}_t^{1/2} \boldsymbol{G}_t \boldsymbol{\Sigma}_t^{1/2}, \tag{3.1}$$

where $\boldsymbol{G}_t$ follows a VDCC process. The unconditional covariance matrix $\boldsymbol{\Sigma}_t$ is estimated non-parametrically using the Nadaraya-Watson kernel estimator (see Bauwens et al., 2012, for details):[7]

$$\hat{\boldsymbol{\Sigma}}_t = \frac{\sum_{i=1}^{T} K_h(\frac{i}{T} - \frac{t}{T}) \boldsymbol{r}_i \boldsymbol{r}_i'}{\sum_{i=1}^{T} K_h(\frac{i}{T} - \frac{t}{T})} \tag{3.2}$$

where $K_h(\cdot) = \frac{1}{h} K(\frac{\cdot}{h})$, $K(\cdot)$ is a kernel function, h is a positive bandwidth. In our application the nonparametric procedure provides h equal to 0.015, with a very smoothed dynamics of the elements of $\hat{\boldsymbol{\Sigma}}_t$.

In the second step we calculate the 'long-run standardized' returns $\boldsymbol{\xi}_t = \hat{\boldsymbol{\Sigma}}_t^{-1/2} \boldsymbol{r}_t$, and then we estimate the conditional variances of the elements of $\boldsymbol{\xi}_t$ using n univariate GARCH models. Finally we apply the VDCC models to the degarched (long-run standardized) returns. Our finding is that the volatility (or the regime of volatility) does not have any effect on these degarched returns, so our conclusion is that the volatility effects are relevant for the long-run correlation matrix.[8] This is confirmed by a direct look at Figure 4. The relationship between each estimated long-run covariance series and the VIX index is clear; the correlations between the time series of covariances and VIX are equal to 0.74 (F-H), 0.75 (F-I) and 0.65 (H-I).

More evidence can be provided by fitting VDCC-type models to the estimated long-run covariances. We show the results for the models that performed best in the previous

[6]In the third step Bauwens et al. (2012) adopt the DCC model.

[7]We have tried to generalize this approach using kernel with different bandwidth parameters for each variance and for the covariances, but the comparison with the original estimator with a common bandwidth does not provide relevant differences.

[8]To save space we do not show all the estimation results of the third step.

in-sample comparison, the MS VDCC models. We consider the k-dimensional vector $\hat{\boldsymbol{\chi}}_t$ ($k = n(n-1)/2$, so $k = 3$ in this section) containing the covariances of the upper triangular part of $\hat{\boldsymbol{\Sigma}}_t$. The corresponding RSDC model is defined as

$$\hat{\boldsymbol{\chi}}_t = \boldsymbol{\varsigma}_{s_t} + \boldsymbol{\epsilon}_t, \tag{3.3}$$

where $\boldsymbol{\varsigma}_{s_t}$ is a vector with elements $r_{ij,s_t}\hat{\sigma}_{i,t}\hat{\sigma}_{j,t}$, with r_{ij,s_t} the unknown correlation coefficients (to be estimated) and $\hat{\sigma}_{i,t}$ and $\hat{\sigma}_{j,t}$ the square root of the variances contained in the diagonal of $\hat{\boldsymbol{\Sigma}}_t$. The vector of disturbances $\boldsymbol{\epsilon}_t$ is hypothesized to follow a multivariate Normal distribution with zero mean and a diagonal covariance matrix $\boldsymbol{\Omega} = \mathrm{diag}(\omega_1, \ldots, \omega_\mathrm{k})$. The choice to model the long-run covariances and not the long-run correlations is motivated by the possibility to use the Normal distribution without constraints. The hypothesis of diagonal covariance matrix of the disturbances is made for the sake of simplicity, but it could be easily removed. As in the RSDC model, we suppose that s_t is a latent discrete variable representing the regime of the correlations, changing according to a Markov chain, with transition probability matrix represented by one of the RSDC, TV-RSDC or DC-RSDC specifications. In Table 5 we show the estimation results for the long-run covariance matrix and for the short-run MS VDCC models for the degarched (long-run standardized) residuals.

The first obvious result is that the correlation coefficients do not vary by changing the model (in particular in the long-run case) and that in the short-run case there is a clear change of regime only for the correlation between H and I. In the long-run case all the models show a very high persistence in the same regime (probability to stay in the same regime near to 1), but the volatility seems to have a significant effect, as shown by the Wald statistics; in particular, in the DC-RSDC model, a change in the regime of volatility seems necessary to obtain a change in the regime of correlations (otherwise the probability to stay in the regime of high (low) correlation with high (low) volatility is equal to 1). In the short-run case, the Wald tests carry out a strong evidence in favor of the acceptance of the null of no volatility effect. It is important to underline that $\boldsymbol{\Sigma}_t$ is an unconditional covariance matrix, whereas in the previous experiments we have modeled the conditional correlations with VDCC models. The lesson we learn from this experiment is that the volatility has an effect on the long-run covariance matrix and not on the short-run one.

3.4 Out-of-sample Comparison

To verify if volatility helps to forecast correlations we have performed a comparison in terms of out-of-sample forecasts. The last 400 observations were deleted from the sample and then we have re-estimated the models, adding up iteratively one observation and obtaining 400 one-step ahead forecasts for each model. Of course, to perform this experiment, we have calculated also the one-step ahead forecasts for the three univariate GARCH models, providing the degarched returns, and the one-step ahead forecasts of the probabilities of the regimes of high and low volatility, using the MS-AR(2) model, to be used in the VDCC models depending on the regime of volatility.

The out-of-sample forecasts are compared again using the MCS approach, analogously to the in-sample case; the results are shown in Table 6. The procedure does not

Table 5: Parameter estimates of Markov Switching models for the long-run covariances and Markov Switching (short-run) VDCC models for the degarched standardized returns (with standard errors in parentheses), and Wald statistics

Long-run Correlation Coefficients									
	$r_{FH,h}$	$r_{FI,h}$	$r_{HI,h}$	$r_{FH,l}$	$r_{FI,l}$	$r_{HI,l}$			
RSDC	0.41	0.44	0.57	0.21	0.18	0.43			
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)			
TV-RSDC	0.41	0.44	0.57	0.21	0.18	0.43			
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)			
DC-RSDC	0.41	0.44	0.57	0.21	0.18	0.43			
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)			
Short-run Correlation Coefficients									
	$r_{FH,h}$	$r_{FI,h}$	$r_{HI,h}$	$r_{FH,l}$	$r_{FI,l}$	$r_{HI,l}$			
RSDC	-0.07	-0.09	0.44	0.03	0.05	-0.20			
	(0.03)	(0.03)	(0.08)	(0.02)	(0.03)	(0.04)			
TV-RSDC	-0.04	-0.08	0.43	0.02	0.05	-0.20			
	(0.05)	(0.05)	(0.07)	(0.05)	(0.04)	(0.04)			
DC-RSDC	-0.06	-0.09	0.43	0.02	0.05	-0.20			
	(0.02)	(0.03)	(0.07)	(0.02)	(0.04)	(0.04)			
Long-run Variance of Disturbances									
	ω_{FH}	ω_{FI}	ω_{HI}						
RSDC	0.14	0.12	0.21						
	(0.00)	(0.00)	(0.01)						
TV-RSDC	0.14	0.12	0.21						
	(0.00)	(0.00)	(0.01)						
DC-RSDC	0.14	0.12	0.21						
	(0.00)	(0.00)	(0.00)						
Long-run Probability Coefficients									
RSDC		TV-RSDC				DC-RSDC			
p_{hh}	p_{ll}	$\theta_{0,h}$	$\theta_{1,h}$	$\theta_{0,l}$	$\theta_{1,l}$	$p_{hh\|0}$	$p_{hh\|1}$	$p_{ll\|0}$	$p_{ll\|1}$
0.999	0.999	6.38	1.50	9.84	-14.90	0.998	1.000	1.000	0.994
(0.000)	(0.000)	(0.009)	(0.34)	(0.05)	(0.25)	(0.000)	(0.000)	(0.000)	(0.000)
p-values of the Wald statistics									
		$H_0: \theta_{1,h} = 0$		$H_0: \theta_{1,l} = 0$		$H_0: p_{hh\|0} = p_{hh\|1}$		$H_0: p_{ll\|0} = p_{ll\|1}$	
		0.00		0.00		0.00		0.00	
Short-run Probability Coefficients									
RSDC		TV-RSDC				DC-RSDC			
p_{hh}	p_{ll}	$\theta_{0,h}$	$\theta_{1,h}$	$\theta_{0,l}$	$\theta_{1,l}$	$p_{hh\|0}$	$p_{hh\|1}$	$p_{ll\|0}$	$p_{ll\|1}$
0.658	0.782	1.68	-3.48	2.34	-3.93	0.776	0.558	0.855	0.713
(0.073)	(0.078)	(1.08)	(3.81)	(1.23)	(4.68)	(0.095)	(0.108)	(0.084)	(0.084)
p-values of the Wald statistics									
		$H_0: \theta_{1,h} = 0$		$H_0: \theta_{1,l} = 0$		$H_0: p_{hh\|0} = p_{hh\|1}$		$H_0: p_{ll\|0} = p_{ll\|1}$	
		0.36		0.40		0.15		0.11	

Table 6: MCS results for individual out-of-sample forecasts, using the Quasi-Likelihood loss function and the TSQ statistics

Model	p-value
STCC-R	0.206
CCC	0.638
STCC-V	0.649
RSDC	0.714
TV-RSDC	0.768
DCC-ARE	0.655
DCC-RSDC	0.839
DCC-AVE	0.592
DCC-TVR	0.996
DCC	0.865
DCC-TVV	1.000

The first row represents the first model removed, down to the best performing model in the last row.

identify any relevant difference among the models, which do not differ in their out-of-sample forecasting performance. This results is not so unexpected, in particular referring to the findings of Hansen (2010). In this paper it is shown that the in-sample and out-of-sample fits are strongly negatively related and that good in-sample fit translates into poor out-of-sample fit, not only in expectation, but one-to-one. In practice Hansen (2010) shows that more complexity is added to a model the better will the model fit the data in-sample, while the contrary tends to be true out-of-sample. This finding is consistent with our results.

4 Evidence for Thirty Stocks

A more interesting scenario is one in which a large number n of assets is analyzed, and we shall proceed with $n = 30$ in Section 4.2. In this case, we are confronted with the problem of estimating correlation matrices of dimension 30×30 in all models. Therefore, we must impose some restrictions on the parameter space to make the estimations feasible. We explain how we do this in Section 4.1, where we also test the restrictions on the trivariate data set of the previous section. This motivates our choice of restricted models in the application to thirty stocks.

4.1 Restricted Models

Dynamic VDCC models

In the Dynamic VDCC models, we substitute the sample correlation matrix $\bar{\boldsymbol{R}}$ for the parameter matrix $\boldsymbol{R}$. This is known as ”correlation targeting” (or tracking) and amounts to use a method-of-moments estimator for $\boldsymbol{R}$. Thus we estimate the remaining parameters after imposing $\boldsymbol{R} = \bar{\boldsymbol{R}}$ and we test (using the LR statistic) if the restricted DCC model is not rejected with respect to the unrestricted model.

RSDC and STCC models

A possible solution is the one proposed by Pelletier (2006) for the RSDC models, replacing $\boldsymbol{R}_h$ with the sample correlation matrix $\bar{\boldsymbol{R}}$, and $\boldsymbol{R}_l$ with the identity matrix $\boldsymbol{I}_n$; they represent the two extreme cases of maximum correlation and absence of correlation, respectively. In the case of RSDC models, the conditional correlation matrix is then parameterized in the following way:

$$\begin{aligned} &\boldsymbol{R}_t = \boldsymbol{R}_{s_t},\ s_t = h, l, \\ &\boldsymbol{R}_h = \bar{\boldsymbol{R}}, \quad \boldsymbol{R}_l = \bar{\boldsymbol{R}}\lambda_l + \boldsymbol{I}_n(1-\lambda_l), \\ &\lambda_l \in [0,1], \end{aligned} \tag{4.1}$$

where λ_l is the only unknown switching coefficient. Our viewpoint is that this parameterization is too restrictive because the high correlation matrix is equal to the sample correlation, which can be considered as the mean of the correlations in the period analyzed.

A more coherent model lets the high correlation matrix surpass the sample one, with

$$\begin{aligned} &\boldsymbol{R}_h = \bar{\boldsymbol{R}}\lambda_h + \boldsymbol{I}_n(1-\lambda_h), \quad \boldsymbol{R}_l = \bar{\boldsymbol{R}}\lambda_l + \boldsymbol{I}_n(1-\lambda_l), \\ &\lambda_l \in [0,1],\ \lambda_h \in [1, 1/\bar{r}_{max}], \end{aligned} \tag{4.2}$$

where $\bar{r}_{max}(>0)$ is the maximum correlation coefficient in $\bar{\boldsymbol{R}}$. We call the latter proposal the "2-λ" RSDC model, and (4.1) the "1-λ" RSDC model, since it fixes $\lambda_h = 1$.

In the same spirit, a third restricted RSDC model has an equi-correlated matrix in the the high correlation regime.

$$\begin{aligned} &\boldsymbol{R}_h = \boldsymbol{R}_M\lambda_h + \boldsymbol{I}_n(1-\lambda_h), \quad \boldsymbol{R}_l = \bar{\boldsymbol{R}}\lambda_l + \boldsymbol{I}_n(1-\lambda_l) \\ &\lambda_l \in [0,1],\ -\tfrac{1}{|\bar{r}_M|} \leq \lambda_h \leq \tfrac{1}{|\bar{r}_M|},\ \lambda_h \geq \lambda_l, \end{aligned} \tag{4.3}$$

where $\boldsymbol{R}_M$ is a correlation matrix having all its off-diagonal elements equal to the mean of all the sample correlations, denoted by $\bar{r}_M$ (with $\bar{r}_M > -1/n$). We call this model the "high equi-correlation" (hec) RSDC model. This can be motivated by the observation that in a high correlation state, the correlations tend to be close to 1 and thus to be similar.

To compare the unrestricted and the three restricted RSDC models, we proceed according to the following steps:

1. Estimate the three restricted RSDC models and choose the best-fitting one.

2. Compare the inference on the regime using the smoothed probabilities derived from the selected restricted model and the unrestricted model; a simple way is a graphical analysis of the absolute differences between the two sets of probabilities

3. Using the Wald statistic, test the null

$$\frac{r_{FH,i}}{\bar{r}_{FH}} = \frac{r_{FI,i}}{\bar{r}_{FI,i}} = \frac{r_{HI,i}}{\bar{r}_{HI}}, \qquad i = h, l,$$

Table 7: Estimates of the parameters of the restricted RSDC models and criteria

	Model		
Parameters	1-λ	2-λ	*hec*
λ_h	1	1.871	2.462
		(0.046)	(0.081)
λ_l	0.000	0.414	0.649
	(0.000)	(0.076)	(0.058)
p_{hh}	0.966	0.785	0.603
	(0.004)	(0.032)	(0.063)
p_{ll}	0.647	0.765	0.832
	(0.041)	(0.041)	(0.023)
Log-Lik	-7613.80	-7391.93	-7446.11
AIC	2.673	2.586	2.604
BIC	2.666	2.590	2.609

Robust standard errors in parentheses.

where $\bar{r}_{ij}$ indicates the (i, j) element of the sample correlation matrix $\bar{\boldsymbol{R}}$. These tests serve to check if all the correlation coefficients within the same regime decrease in the same proportion. If the parameterization selected is (4.3), when $i = h$ the denominator of each element under the null is given by $\bar{r}_M$.

For the STCC model, we can also use the 1-λ, 2-λ and hec specifications of $\boldsymbol{R}_h$ and $\boldsymbol{R}_l$ in (2.3).

Application to the three stocks

Table 7 shows the estimation results for the three restricted RSDC models, with the corresponding log-likelihood, AIC and BIC values. The three models provide very different values of the λ coefficients and transition probabilities. In particular in the 1-λ case, the parameter λ_l is estimated to be 0 (at a boundary of its admissible values), so that the two correlations matrices correspond to the sample correlation and the identity matrix.[9] This result sounds as an alarm, indicating that the choice of a restricted model is a crucial step when it must be adopted. In terms of log-likelihood values and other criteria, the preference for the 2-λ rmodel is quite clear, so we adopt it for the comparison with the unrestricted model. To ease the comparison, we recall in Table 8 the estimates of both models, with other results useful for the comparison.

The estimates of the correlation coefficients are very similar in regime l, whereas there is a difference of 0.08 in $r_{FH,h}$ and $r_{FI,h}$. The transition probabilities are very sim-

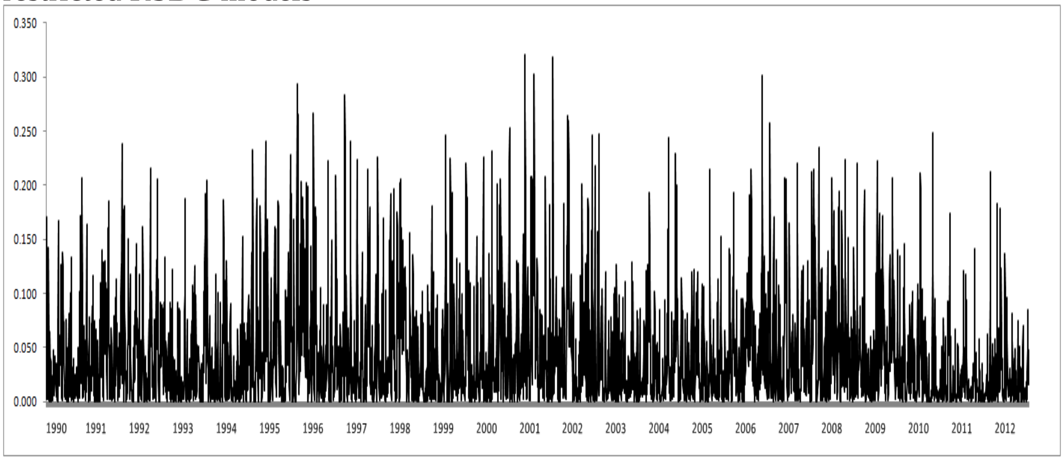
[9] Pelletier (2006) evaluates the goodness of the 1-λ model in a 4-variate case with two regimes, by comparing the values of the estimated correlations in the restricted and unrestricted models, and by performing a LR test of the restrictions. He finds that there is an ordering in the magnitude of the correlations across the different regimes (also in the unrestricted case all the correlations decrease when switching from the regime of high correlation to the one of low correlation), but the LR test rejects the restrictions imposed under the null (statistic equal to 16.48, to be compared to a chi-squared with 4 degrees of freedom).

Table 8: Comparison of restricted and unrestricted RSDC models

	Parameter Estimates							
Model	$r_{FH,h}$	$r_{FI,h}$	$r_{HI,h}$	$r_{FH,l}$	$r_{FI,l}$	$r_{HI,l}$	p_{hh}	p_{ll}
Unrestricted	0.61	0.60	0.79	0.10	0.10	0.21	0.76	0.77
Restricted	0.52	0.52	0.79	0.12	0.12	0.17	0.79	0.76
	Log-Likelihood Criteria							
Model			Log-Lik	AIC	BIC			
Unrestricted			-7383.7	2.58	2.59			
Restricted			-7391.9	2.59	2.59			
	Absolute differences in smoothed probabilities							
		Min	$Q_{0.25}$	$Q_{0.50}$	$Q_{0.75}$	Max		
		0.000	0.009	0.025	0.058	0.320		
	Wald test for proportional decreasing in correlations							
				Statistic	p-value			
Regime h				8.55	0.01			
Regime l				6.62	0.04			

The estimates of the correlations of the restricted model are obtained by computing (4.2) at the "2-λ" estimates of λ_h and λ_l reported in Table 7, using the known value of $\bar{\boldsymbol{R}}$.

Figure 5: Absolute differences between the smoothed probabilities from the 2-λ and unrestricted RSDC models



ilar, supporting the choice of the 2-λ model. Both models provide a similar inference on the regime, as is revealed by a graphical inspection of the absolute differences between the smoothed probabilities (see Figure 5). We notice that most differences are very small, and the maximum difference is 0.3. Moreover, observing also the five summary numbers of the absolute differences reported in Table 8, it is clear that many of the absolute differences are less than 0.1; in particular the values higher than 0.1 correspond to the 10.4$\%$ of the set of absolute differences, the values higher than 0.2 to 1.4$\%$, and the values higher than 0.3 to 0.07$\%$.

In the last part of Table 8 we show the results of the Wald tests to check a proportional decrease in the correlations within each regime. The two statistics, distributed as chi-squared with 2 degrees of freedom, indicate that the null hypothesis is not rejected at the 0.01 size. Given the similarity of the inferences on the regime and the results of these tests, we can conclude that the restricted 2-λ RSDC model is a sufficiently good approximation of the unrestricted model.

We have estimated the restricted model also in the DCC case. The LR test to compare it with the unrestricted model in (2.7) does not reject the restrictions, since the test statistic, equal to 1.108, has a p-value (calculated for a chi-squared with 3 degrees of freedom) of 0.775.

To save space, we do not report the not so interesting results for the STCC models.

Application to other subsets of stocks

As a check for robustness, we have applied the procedure for comparing the restricted models and the unrestricted one to 100 subsets of three stocks, and to 100 subsets of seven stocks.[10] These subsets were selected randomly from the 30 stocks composing the Dow Jones index, described in subsection 4.2.

For the RSDC models and the 100 groups of three stocks, the BIC selects the 2-λ model in 46 cases, the hec in 54 cases, and the 1-λ model in no case. If we include the unrestricted model in the comparison, the numbers are 26 for the latter, 32 for 2-λ, 42 for hec, and zero for 1-λ. For the 100 groups of seven stocks, the BIC selects the 2-λ model in 54 cases, the hec in 46 cases, and the 1-λ model in no case. If we include the unrestricted model in the comparison, the numbers are 29 for the latter, 40 for 2-λ, 31 for hec, and zero for 1-λ.[11]

We also did the comparisons of the DCC models. In the case of the groups of three stocks, using a test size of 0.01, the LR test does not reject the null of equality of the constrained and unconstrained model in 97 out of 100 cases. For the 100 groups of seven stocks, the non-rejection holds for all groups. Whether for groups of three or of seven stocks, the BIC always selects the restricted model.

These results tell us that the results for the three stocks presented above are not specific to the choice of the three stocks. They also tell us that for the higher dimension, the

[10] Increasing the dimension above seven renders the estimations too difficult. With seven stocks, the number of correlations is 28 in $\boldsymbol{R}_h$ and the same in $\boldsymbol{R}_l$ in the unrestricted RSDC model.

[11] If we use LR tests, the null of equality between the best restricted model and the unrestricted model is not rejected in 41 cases for groups of three, but it is rejected in all cases for groups of seven. There is thus some discrepancy between the BIC and LR results.

same type of results seem to hold, although we are limited to a dimension that is still much lower than the dimension used in the next subsection. Thus the procedure we have proposed for selecting a restricted RSDC model and we want to apply for thirty stocks can be considered to be sensible.

4.2 The Dow Jones Components

We have applied eleven models to the 30 stocks composing the Dow Jones for the period 2 January 2002-23 May 2012 (2617 daily returns for each series), using as measure of volatility the VXD index,[12] an indicator constructed following the same methodology as to obtain the VIX, but referred to as real-time prices of options on the Dow Jones Industrial Average.[13] For the Markov Switching models and the STCC models we have used the three restricted models (1-λ, 2-λ and hec). As in the trivariate case, the 2-λ model is clearly better in terms of AIC and BIC, so we have adopted it in the following experiments.[14] In the STCC case, in particular STCC-R, the estimated smooth transition function varies in a small range: it is almost constant (values between 0.88 and 1) for STCC-R, whereas, in the STCC-V case, its range is [0.64;1].

In Table 9 we show the estimation results and the log-likelihood evaluation of the eleven models. A certain consistency with the results obtained in the three asset case emerges immediately: the STCC models show high values of AIC and BIC with a non significant γ coefficient in STCC-R; the RSDC models show the best performance in terms of AIC and BIC, with a significant increase in the likelihood of the VDCC versions; the effect of the volatility in the probabilities of the regimes are significant at 5% in the high correlation case, whereas the low volatility case does not seem affected by them; in DCC-TVV and DCC-TVR, only the a coefficient is time-varying. The parameters λ_h and λ_l are the same in the three Markov Switching models; this implies a correlation in state h between 0.24 and 0.90 and in regime l between 0.07 and 0.26 (of course each correlation increases when switching from l to h). State h is much more persistent than state l; for example, in the fixed probability case of the RSDC model, the expected duration[15] of regime h is 8.77 and 1.61 for regime l. A novelty is due to the very good performance, in terms of likelihood values, of the Dynamic VDCC models depending on the regime of the volatility with respect to the analogous models using the VXD index; on the other hand DCC-AVE does not show a clear improvement with respect to the simple DCC, with a tiny g coefficient. Notice also that the λ_h coefficient of STCC-R is only slightly larger than 1, whereas $\lambda_l = 0$. Considering that the smooth transition function provides values almost constant and near to 1, the STCC-R model is very similar to the CCC model.

The MCS procedure in terms of in-sample forecasting favors again the MS models (Table 10), which do not show significant differences between them. STCC models and CCC are the first models excluded from the procedure, then the DCC group. If we apply the MCS procedure without the MS models, we detect a clear separation between

[12] Available at the web site http://www.cboe.com/micro/vxd/.

[13] We have also used alternative estimators of the volatility, such as the realized kernel volatility, but, for this kind of data, the VXD seems to perform better.

[14] Notice that two of the three stocks of the trivariate example, H and I, are also included in this data set.

[15] In MS models the expected duration of regime i is given by $1/(1 - p_{ii})$.

Table 9: Parameter estimates of CC and VDCC models for the Dow Jones data set and corresponding log-likelihood, AIC and BIC

			No Volatility Effect						
							Log-Lik	AIC	BIC
CCC							-18343.1	14.04	14.04
	a	b					Log-Lik	AIC	BIC
DCC	0.005	0.979					-17986.3	13.77	13.77
	(0.000)	(0.001)							
	p_{hh}	p_{ll}	λ_h	λ_l			Log-lik	AIC	BIC
RSDC	0.89	0.38	1.09	0.32			-16780.5	12.85	12.86
	(0.01)	(0.03)	(0.00)	(0.03)					
			Smooth Transition VDCC						
	γ	c	λ_h	λ_l			Log-lik	AIC	BIC
STCC-V	9.95	0.04	1.11	0.47			-18177.7	13.92	13.93
	(1.58)	(0.03)	(0.02)	(0.03)					
STCC-R	3.68	-0.52	1.09	0.00			-18182.7	13.92	13.93
	(2.49)	(0.35)	(0.02)	(0.00)					
			VDCC with Markov Switching						
	$\theta_{0,h}$	$\theta_{1,h}$	$\theta_{0,l}$	$\theta_{1,l}$	λ_h	λ_l	Log-lik	AIC	BIC
TV-RSDC	1.01	5.32	-0.34	-1.02	1.09	0.32	**-16766.4**	**12.84**	**12.85**
	(0.36)	(1.85)	(0.98)	(6.60)	(0.00)	(0.03)			
	$p_{hh\|0}$	$p_{hh,1}$	$p_{ll\|0}$	$p_{ll,1}$	λ_h	λ_l	Log-lik	AIC	BIC
DC-RSDC	0.86	0.95	0.39	0.24	1.09	0.32	-16768.7	**12.84**	**12.85**
	(0.01)	(0.02)	(0.05)	(0.31)	(0.00)	(0.04)			
			Dynamic VDCC						
	a	b	g				Log-lik	AIC	BIC
DCC-AVE	0.005	0.978	0.001				-17985.2	13.77	13.77
	(0.000)	(0.001)	(0.000)						
DCC-ARE	0.006	0.944	0.016				-17843.5	13.66	13.67
	(0.000)	(0.003)	(0.001)						
	a_0	a_1	b	$\theta_{a,0}$	$\theta_{a,1}$		Log-lik	AIC	BIC
DCC-TVV	0.000	0.010	0.975	-3.79	18.51		-17875.7	13.69	13.70
	(0.000)	(0.000)	(0.001)	(0.14)	(0.69)				
DCC-TVR	0.000	0.030	0.970	-2.38	2.09		-17810.8	13.64	13.65
	(0.00)	(0.001)	(0.001)	(0.04)	(0.04)				

Robust standard errors in parentheses. The RSDC and STCC models use the 2-λ version, see (4.2).

Table 10: Dow Jones components: MCS results for individual in-sample forecasts, using the Quasi-Likelihood loss function and the TSQ statistics

Model	p-value
STCC-V	0.000
CCC	0.000
STCC-R	0.000
DCC	0.000
DCC-AVE	0.000
DCC-TVV	0.000
DCC-TVR	0.000
DCC-ARE	0.000
TV-RSDC	0.245
RSDC	0.090
DC-RSDC	1.000

The first row represents the first model removed, down to the best performing model in the last row.

the group of constant (or quasi-constant) correlation models and the models of the DCC group, the latter showing a similar performance, as in the trivariate case.

The marginal impacts of the volatility and regime of volatility seem less strong than in the trivariate case for each estimated model. In Table 11 the average of the volatility (or regime of volatility) of the 435 correlations and the corresponding standard deviations are shown. The standard deviations are small in most cases, indicating that these mean impacts are quite similar for all the correlations. In general, the behavior of the models with marginal impact depending on the level of volatility is opposite to what we find in the trivariate case, in the sense that the STCC-V and TV-RSDC models show a decreasing marginal effect when volatility increases; again these variations are moderate for the MS model. DC-RSDC shows a weak impact of a change in regime on the volatility. The DCC models, with a practically null marginal effect of volatility and regime of volatility, confirm the feeling that the autocorrelated dynamics of the conditional correlations capture most of the volatility effect on correlations.

For this data set we have also performed the estimation of the long-run correlation matrix using the Nadaraya-Watson kernel estimator; the estimated bandwidth coefficient is 0.035. In this case we cannot repeat the estimations made in the trivariate case based on the comparison of MS models and applied to long-term covariances and degarched standardized residuals, to evaluate analytically the presence of the volatility effect. Actually, the estimation of MS models as (3.3) is not feasible due the large number of parameters involved. On the other hand the RSDC models for the standardized degarched returns cannot be estimated with the parameterization adopted; in fact, the sample correlation matrix of the degarched residuals is approximately an identity matrix and the matrices multiplying of λ_h and λ_l in (4.1) are therefore the same. Anyway, the visual inspection of the graphs of the correlations between the VXD index and each long-run and short-run covariance time series provides some heuristic conclusions. In Figure 6 we show the cor-

Table 11: Dow Jones components: Marginal impact average* of volatility and regime of volatility on conditional correlations

v_{t-1}	variation	mean	st.dev.	mean	st.dev.	mean	st.dev.
		STCC-V		TV-RSDC		DCC-TVV	
0.093 (min)	0.1	0.060	0.054	0.027	0.054	0.001	0.000
0.138 (Q1)	0.1	0.050	0.010	0.024	0.005	0.002	0.000
0.179 (Q2)	0.1	0.040	0.008	0.022	0.004	0.003	0.000
0.234 (Q3)	0.1	0.028	0.006	0.018	0.004	0.003	0.000
0.746 (max)	0.1	0.000	0.000	0.002	0.000	0.000	0.000
	variation	DCC-AVE					
	0.1	0.000	0.000				
$E_{t-1}(\zeta_{t-1})$	variation	STCC-R		DCC-ARE		DCC-TVR	
0	1	0.179	0.036	0.010	0.001	0.003	0.000
1	-1	-0.006	0.001	-0.010	0.001	-0.009	0.001
$Pr(\zeta_{t-1} = 1)$	variation	DC-RSDC					
0	1	0.041	0.008				
1	-1	-0.035	0.007				

*Average of the impacts for the 435 correlations between the 30 stocks.

Figure 6: Dow Jones Components: Correlation between the VXD index and the estimated long-run covariances (continuous line) and between the VXD index and the rolling short-time covariances (dotted line)

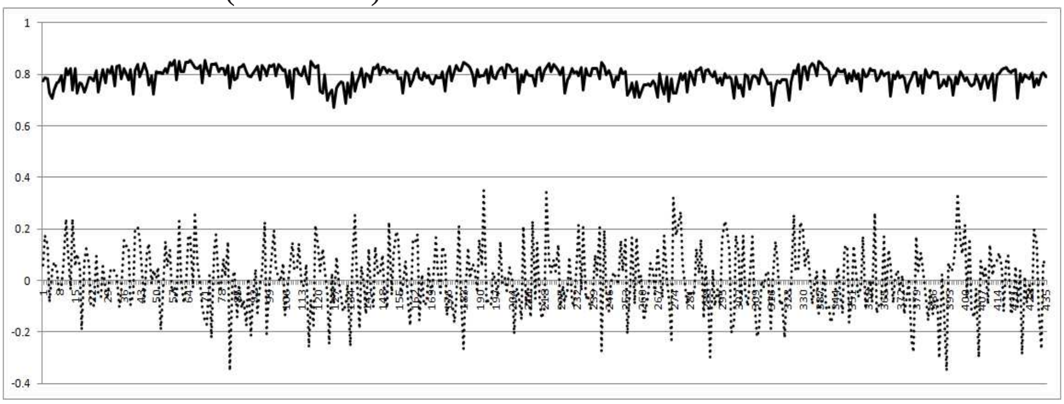


Table 12: Dow Jones components: MCS results for individual out-of-sample forecasts, using the Quasi-Likelihood loss function and the TSQ statistics

Model	p-value
STCC-V	0.044
STCC-R	0.084
DCC-ARE	0.061
CCC	0.077
TV-RSDC	0.114
RSDC	0.635
DC-RSDC	0.282
DCC	0.070
DCC-AVE	0.028
DCC-TVV	0.637
DCC-TVR	1.000

The first row represents the first model removed, down to the best performing model in the last row.

relations between each of the the 435 estimated long-run covariance time series and the VXD index (continuous line), as well as the correlations between the 130-terms rolling short term covariance time series of the degarched standardized residuals and the VXD index (dotted line). A strong relationship is clear between the long-run covariances and the VXD index since the correlations are ranging in the interval [0.67-0.85]. On the contrary, the correlations between the short-term covariances and the VXD index are weak, lying in ([-0.35;0.35]), with 88.5% of them in [-0.2,0.2].

Finally, we have performed the out-of-sample forecasts; to reduce the computational burden, we have considered only the last 100 observations as testing set, re-estimating all the models when a new observation is added. The MCS approach (Table 12) shows again a very similar performance among the eleven models, considering a 1% size test; increasing the size to 10%, we can exclude until the CCC model in the sequence shown in the Table. Thus it is confirmed that the MCS approach selects a large group of models with similar out-of-sample forecasting performance. This group includes models without volatility effect, such as RSDC and DCC. In substance, the conclusion seems to be that volatility does not help in out-of-sample forecasting in this data set.

In general, our feeling is that the results obtained in the previous simple trivariate case are confirmed also in this 30-variate experiment, in spite of the need to parameterize or to estimate differently the unconditional correlation matrices.

5 Concluding Remarks

The idea of this work is to include the effect of the volatility in the widespread used models for dynamic conditional correlations (calling them VDCC models), using two data sets (a 3-variate and a 30-variate model) to verify the presence of this effect, the marginal contribution to the changes in conditional correlations, the time-horizon of this effect (short or long run effect) and the out-of-sample performance. The choice of a large and a small data set is made to distinguish the case in which we need a suitable reparameterization of the unconditional correlation matrix, which involves a dimensionality problem, from the unrestricted case. In our experiments we notice that the reparameterization called 2-λ provides a good approximation of the unrestricted model. In general, dealing with large data sets, a good practice would be to perform a comparison between the restricted and unrestricted models on one (or more) subsample of the full data set.

The results of the small and the large data sets are very similar; in particular we find that VDCC models show a better performance than the corresponding models without volatility effect. The volatility effect is statistically significant, except in the STCC models, which seem very sensitive to the choice of the variable driving the smooth function. In particular, the presence of regimes seems evident, both in volatility and correlation dynamics. The influence of the regime of volatility on the regime of correlations seems consistent with the findings of many authors, that assess that the turmoil periods (characterized from high volatility regimes) cause an increase in the correlation levels and dynamics.

In terms of in-sample forecasting, the VDCC models and their counterparts without volatility effect do not seem to differ significantly. In other terms, it seems that the volatility or its expected regime is a relevant determinant of the periods of high and low correlations, but it does not increase the in-sample fitting significantly.

The marginal effects of the volatility is more evident in the RSDC models, whereas the DCC family shows significant but tiny effects, probably because part of the volatility effects is included in the GARCH dynamics of this kind of models.

The volatility seems to have a long-run effect and not a short-run effect; this result was obtained using the multiplicative approach proposed by Bauwens et al. (2012), which is very simple because based on a nonparametric approach, avoiding the estimation of additional parameters in our models. An alternative approach to distinguish between long-run and short-run correlations is the DCC-MIDAS of Colacito et al. (2011), in which it should be possible to insert directly the effect of volatility in the long-run dynamics.

In terms of out-of-sample performance, the presence of volatility does not seem to improve it; anyway it is important to notice that all the models show a similar performance, with or without switching, with or without dynamics, etc. This is consistent with the theoretical and empirical results of Hansen (2010). In practice it seems that the specification of the model is not a crucial choice if the purpose is the out-of-sample forecasting, or, more in general, that the forecast of an unobserved component, as the conditional correlation, based on the estimation of other unobserved components, as the conditional variances and covariances, is a difficult task.

Finally, it would be necessary to check if the specification of the model for the conditional variances is a crucial or not. In our experiments we have used simple GARCH(1,1) models, but of course we could use several other specifications. We have made some experiments with GJR-GARCH models (Glosten et al., 1993) and the results are not affected.

Table 13: Estimates of the parameters of GARCH(1,1) (for three stocks) and MS-AR(2) (for the VIX index).

Series	GARCH parameters							
	μ	c	a	b				
Ford	0.020	0.085	0.063	0.923				
	(0.020)	(0.010)	(0.009)	(0.010)				
HP	0.069	0.093	0.048	0.938				
	(0.016)	(0.031)	(0.017)	(0.020)				
IBM	0.075	0.038	0.080	0.913				
	(0.017)	(0.007)	(0.018)	(0.017)				
	MS-AR parameters							
VIX	κ_0	κ_1	ϕ_1	ϕ_2	$\sigma_{\eta,0}$	$\sigma_{\eta,1}$	p_{00}	p_{11}
	1.410	2.462	1.426	-0.508	0.927	3.154	0.973	0.925
	(0.060)	(0.185)	(0.013)	(0.009)	(0.061)	(0.296)	(0.003)	(0.010)

Robust standard errors in parentheses.

Appendix 1: First-step Estimation Results

In the first-step estimation, to provide the estimates of the conditional variances $h_{i,t}$ for each series $i = 1, \dots, n$, we have considered GARCH(1,1) models for the returns, i.e.

$$h_{i,t} = c_i + a_i(r_{i,t-1} - \mu_i)^2 + b_i h_{i,t-1},$$

where $r_{i,t}$ represents the return of series i at time t. If v_t is also considered in the first-step estimation, we hypothesize that it follows a MS-AR(2) model, written as

$$v_t = \kappa_{\zeta_t} + \phi_1 v_{t-1} + \phi_2 v_{t-2} + \sigma_{\eta,\zeta_t}\eta_t,$$

where κ_{ζ_t} and σ_{η,ζ_t} are switching coefficients that change according to the state $\zeta_t \in \{0, 1\}$, and η_t are i.i.d. N(0,1) disturbances.[16] The state ζ_t is driven by an ergodic Markov Chain with unknown coefficients p_{ii} ($i = 0, 1$), representing the probability $Pr(\zeta_t = i|\zeta_{t-1} = i)$, and $p_{ij} = 1 - p_{ii} = Pr(\zeta_t = j|\zeta_{t-1} = i)$ for $i \neq j$. The first-step coefficients present in the first equation of (2.2), are

$$\boldsymbol{\theta}_V = (\mu_1, c_1, a_1, b_1, \dots, \mu_n, c_n, a_n, b_n, \kappa_0, \kappa_1, \phi_1, \phi_2, \sigma_{\eta,0}, \sigma_{\eta,1}, p_{00}, p_{11})'.$$

If the VDCC model used does not depend on the regime of volatility, the MS-AR coefficients are not included in $\boldsymbol{\theta}_V$.

In Tables 13 and 14 we show the estimates of the GARCH and MS-AR coefficients for the data sets analyzed in this paper.

Appendix 2: Marginal Impacts of Volatility on Correlation

We provide the formulas to calculate the marginal impact of the volatility for each model considered. The variable x_t represents the volatility or its expected regime, recalling that,

[16] We prefer to consider the intercept as switching coefficient and not the mean of v_t because the first is more sensitive to small jumps in the level of the volatility series.

Table 14: Estimates of the parameters of GARCH(1,1) (for Dow Jones components) and MS-AR(2) (for the VXD index).

Parameters	Stocks									
	AA	AXP	BA	BAC	CAT	CSCO	CVX	DD	DIS	GE
μ	0.02	0.07	0.09	0.03	0.09	0.02	0.09	0.05	0.07	0.03
	(0.01)	(0.02)	(0.02)	(0.02)	(0.03)	(0.06)	(0.02)	(0.02)	(0.02)	(0.02)
c	0.14	0.04	0.09	0.06	0.19	0.25	0.06	0.04	0.08	0.04
	(0.02)	(0.01)	(0.01)	(0.01)	(0.03)	(0.06)	(0.00)	(0.01)	(0.01)	(0.00)
a	0.08	0.12	0.10	0.13	0.08	0.08	0.10	0.08	0.10	0.12
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)
b	0.90	0.88	0.88	0.87	0.88	0.87	0.88	0.90	0.88	0.88
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)
	HD	HPQ	IBM	INTC	JNJ	JPM	KFT	KO	MCD	MMM
μ	0.08	0.09	0.06	0.05	0.04	0.06	0.06	0.06	0.08	0.04
	(0.02)	(0.03)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)	(0.07)
c	0.06	0.17	0.06	0.05	0.05	0.03	0.10	0.03	0.03	0.11
	(0.01)	(0.05)	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.02)
a	0.09	0.09	0.12	0.05	0.15	0.10	0.14	0.11	0.07	0.09
	(0.02)	(0.03)	(0.02)	(0.01)	(0.02)	(0.01)	(0.02)	(0.02)	(0.01)	(0.02)
b	0.89	0.88	0.86	0.94	0.82	0.90	0.82	0.88	0.92	0.86
	(0.02)	(0.04)	(0.02)	(0.01)	(0.02)	(0.01)	(0.02)	(0.02)	(0.01)	(0.02)
	MRK	MSFT	PFE	PG	T	TRV	UTX	VZ	WMT	XOM
μ	0.00	0.03	-0.02	0.04	0.05	0.06	0.09	0.03	0.01	0.08
	(0.00)	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	(0.03)	(0.02)
c	0.29	0.05	0.11	0.07	0.06	0.07	0.04	0.03	0.05	0.07
	(0.12)	(0.02)	(0.03)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
a	0.06	0.06	0.10	0.11	0.11	0.10	0.09	0.09	0.08	0.09
	(0.03)	(0.02)	(0.04)	(0.03)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)
b	0.86	0.92	0.86	0.84	0.87	0.88	0.89	0.90	0.89	0.87
	(0.06)	(0.02)	(0.04)	(0.03)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.01)
	MS-AR parameters									
VXD	κ_0	κ_1	ϕ_1	ϕ_2	$\sigma_{\eta,0}$	$\sigma_{\eta,1}$	p_{00}	p_{11}		
	1.35	2.68	1.42	-0.50	0.92	3.60	0.98	0.94		
	(0.21)	(0.43)	(0.06)	(0.04)	(0.05)	(0.47)	(0.00)	(0.02)		

Robust standard errors in parentheses.

in spite of the same formula, the interpretation is different. The marginal impact is defined as the first derivative of the conditional correlation function at time t with respect to x_{t-1}.

Smooth Transition VDCC models

The conditional correlation between series i and series j is given by

$$r_{ij,t} = r_{ij,h} f_t + r_{ij,l}(1 - f_t),$$
$$f_t = (1 + exp(-\gamma(x_{t-1} - c)))^{-1}.$$

The marginal impact is given by:

$$(r_{ij,h} - r_{ij,l}) \frac{exp[-\gamma(x_{t-1} - c)]\gamma}{\{1 + exp[-\gamma(x_{t-1} - c)]\}^2}.$$

Notice that the marginal impact depends on the value of x_{t-1}.

Markov Switching VDCC models

The conditional correlation between series i and j is given by

$$r_{ij,t} = r_{ij,s_t},$$

where s_t is the regime of the correlation, assuming the labels h and l. Since s_t is unobservable, we can use the expected value of the conditional correlation, given by

$$r_{ij,h} Pr(s_t = h|\mathbf{\Psi}_T) + r_{ij,l}(1 - Pr(s_t = h|\mathbf{\Psi}_T)),$$

where the smoothed probability $Pr(s_t = h|\mathbf{\Psi}_T)$ depends on x_t. The computation of the analytic derivative of the smoothed probabilities would be prohibitive, being based on filtering and smoothing. A possible solution is to replace it with the ergodic probability

$$\pi_h = \frac{1 - p_{ll,t}}{2 - p_{hh,t} - p_{ll,t}},$$

where, using the TV-RSDC specification:

$$p_{ii,t} = \frac{exp(\theta_{0,i} + \theta_{1,i} v_{t-1})}{1 + exp(\theta_{0,i} + \theta_{1,i} v_{t-1})} \qquad i = h, l,$$

whereas, using DC-RSDC:

$$p_{ii,t} = p_{ii|0}(1 - Pr(\zeta_t = 1)) + p_{ii|h} Pr(\zeta_t = 1) \qquad i = h, l.$$

Let $\partial p_{ii,t}$ the derivative of $p_{ii,t}$ with respect to x_{t-1}; then the marginal impact of x_{t-1} in a MS VDCC model is given by

$$(r_{ij,h} - r_{ij,l}) \frac{-\partial p_{ll,t} - (-\partial p_{hh,t} - \partial p_{ll,t})(1 - \partial p_{ll,t})}{(2 - p_{hh,t} - p_{ll,t})^2}.$$

In particular, when we are using a TV-RSDC model:

$$\partial p_{ii,t} = \frac{exp(\theta_{0,i} + \theta_{1,i} v_{t-1})}{[1 + exp(\theta_{0,i} + \theta_{1,i} v_{t-1})]^2},$$

whereas if the model is DC-RSDC:

$$\partial p_{ii,t} = p_{ii|1} - p_{ii|0}.$$

Again the two marginal effects depend on the values of v_{t-1} and $Pr(\zeta_t = 1)$ respectively. In the second case it is interesting to evaluate the marginal effect when $Pr(\zeta_t = 1) = 0$ (in this case the derivative measures the marginal effect corresponding to a change of $Pr(\zeta_t = 1)$ from 0 to 1) and $Pr(\zeta_t = 1) = 1$ (the derivative, multiplied by -1, then measures the marginal effect when $Pr(\zeta_t = 1)$ changes from 1 to 0).

Dynamic VDCC

In a DCC framework the conditional correlation at time t between series i and series j is given by:

$$r_{ij,t} = \frac{q_{ij,t}}{q_{ii,t}^{1/2} q_{jj,t}^{1/2}},$$

where $q_{ij,t}$ is the element (i, j) of the $\boldsymbol{Q}_t$ matrix.

The marginal effect of x_{t-1} on the conditional correlation is then

$$\frac{\partial q_{ij,t}(1 - 0.5 q_{ii,t}^{-1} q_{ij,t} - 0.5 q_{jj,t}^{-1} q_{ij,t})}{q_{ii,t}^{1/2} q_{jj,t}^{1/2}}$$

where $\partial q_{ij,t}$ is the derivative of $q_{ij,t}$ with respect to x_{t-1}. This is the only element that changes using alternative dynamic VDCC models.

In the case of DCC-AVE and DCC-ARE:

$$\partial q_{ij,t} = g\left(1 - \frac{1}{T} r_{ij}\right).$$

Notice that the marginal effect is constant along the time.

In the case of DCC-TVV and DCC-TVR:

$$\partial q_{ij,t} = \partial a_t u_{i,t-1} u_{j,t-1} q_{i,t-1} q_{j,t-1} + \partial b_t q_{ij,t-1},$$

where

$$\partial a_t = a_1 \frac{exp(\theta_{a,0} + \theta_{a,1} x_{t-1})\theta_{a,1}}{[1 + exp(\theta_{a,0} + \theta_{a,1} x_{t-1})]^2}, \qquad \partial b_t = b_1 \frac{exp(\theta_{b,0} + \theta_{b,1} x_{t-1})\theta_{b,1}}{[1 + exp(\theta_{b,0} + \theta_{b,1} x_{t-1})]^2}.$$

In this case there is a dependence on the value of x_{t-1}.

The derivative depends also on the value of $\boldsymbol{Q}_{t-1}$ and, only in the time-varying parameter case, on the value of $\boldsymbol{u}_{t-1}$. We fix them to their average value in the sample considered.

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