

Artificial intelligence for the analysis and design of complex bridge structures

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ABSTRACT: A structure can be defined complex if its behavior is influenced from nonlinearities, uncertainties and/or interactions. These aspects increase the complexity of the numerical model that one has to consider to reproduce the actual behavior of the structure. Therefore, many analyses must be developed to investigate particular aspects of the structural behavior of these kinds of structures and the time of analysis may be very expensive. The artificial intelligence theory provide a series of techniques (evolutive procedures, fuzzy theory, neural networks...) that can be used conveniently in order to improve the robustness of the analyses.

1 INTRODUCTION

During the last quarter of century many authors as Lewis (1989) or Blockley (1980), have shown that the activity of designer is not a linear process: for example, in his book, Lewis emphasizes the numerous design cycles needed to define the design problem, the specific criteria and the alternatives, in order to find an optimal solution.

On a different side, exploring the field of the psychology, Arielli (2003) examines the way of thinking of human mind and put in evidence as the design process is not a deductive activity but it is a pure abductive reasoning. The *abduction* is a form of reasoning that has been defined as *creative* and it has been deeply analyzed from the philosopher Peirce (1948). In general, the conclusion of an abductive reasoning is a hypothesis. This conclusive hypothesis can connect information notes in new knowledge and the truth of the hypothesis can be established through a process of confutation (Popper 1962).

In addition, Habermas (1973) distinguishes two types of abductive reasoning:

- the *explanatory abduction*, in which an observed fact is explained through a law that it is already known;
- the *innovative abduction* where the observed fact brings to the creation of a general law.

In the design process, the designer knows a suitable final state and he has to think something able to produce this final state. In this process, one has different kind of knowledge and the designer uses many laws or rules to obtain the solution of the problem.

If the designer uses standard law and common knowledge, the abduction is similar to the explanatory abduction. However, if the design is original and the designers used their creativity to reach a new final state, the reasoning developed belongs to the class of the innovative abduction (Arielli 2003).

Long suspension bridges are complex structures because their behavior is influenced from nonlinearities, uncertainties and interactions (Bontempi et al. 2004-A, Bontempi et al. 2004-B). The analysis and design of these structures can involve innovative knowledge and reasoning (Figure 1). For this reason, it is of great importance to give attention at the followings aspects: the problem decomposition, the conceptual design phase, the sensitivity analysis. In addition, the use of different numerical approaches is the unique way to govern the errors and the approximations due to numerical modeling or a human factor (Downes 1995, Sgambi et al. 2004).

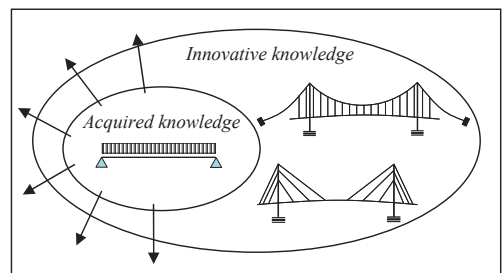


Figure 1. Acquired knowledge (or explanatory abduction) and innovative knowledge (or innovative abduction).

2 NUMERICAL MODELS AND THE REPRESENTATION OF THE REALITY

An accurate numerical analysis is fundamental for a proper design of complex structures. A numerical model has the purpose to reproduce the correct mechanical behavior of the structure. However, the numerical representation is not the reality (Bateson 1979). Usually, one adopts a series of hypothesis during the construction of the numerical representation. These hypotheses influence the accuracy of the numerical response.

One considers the deck of a suspension bridge. The behavior of this substructure can be described with a series of beam elements, or using shells or solids elements (Figure 2). Clearly, passing from the beam model to shell model, one has a notable increment of information available from the numerical model. One can conclude that the various type of model can result more or less concrete according to the quantity of information that is possible to extract from its. Therefore, the shell model is a more concrete representation than the beam model.

A second aspect about the numerical model concerns the extension of the model. A numerical model can represent the whole structure or only a part of it, according to the purpose that the numerical analyst wants to reach. For example, if one is interested to a local behavior of a part of the structure, one can build a partial (not complete) model such as represented in Figure 2.

Often the computational resources are limited. In this case, the choice of a partial model is forced in reason to increase the concreteness of the model. In addition, a complete and concrete model can be not suitable. In fact, from a great model derive a great quantity of information and probably, many of this information are unessential respect to design problem. The great quantity of useless information increases the possibility of human errors, amplify the loss of precision and expand the times of calculation. Therefore, it is preferable, in the study of complex structures, to adopt various models, with different degree of concreteness and completeness according to the purpose desired (Figure 3).

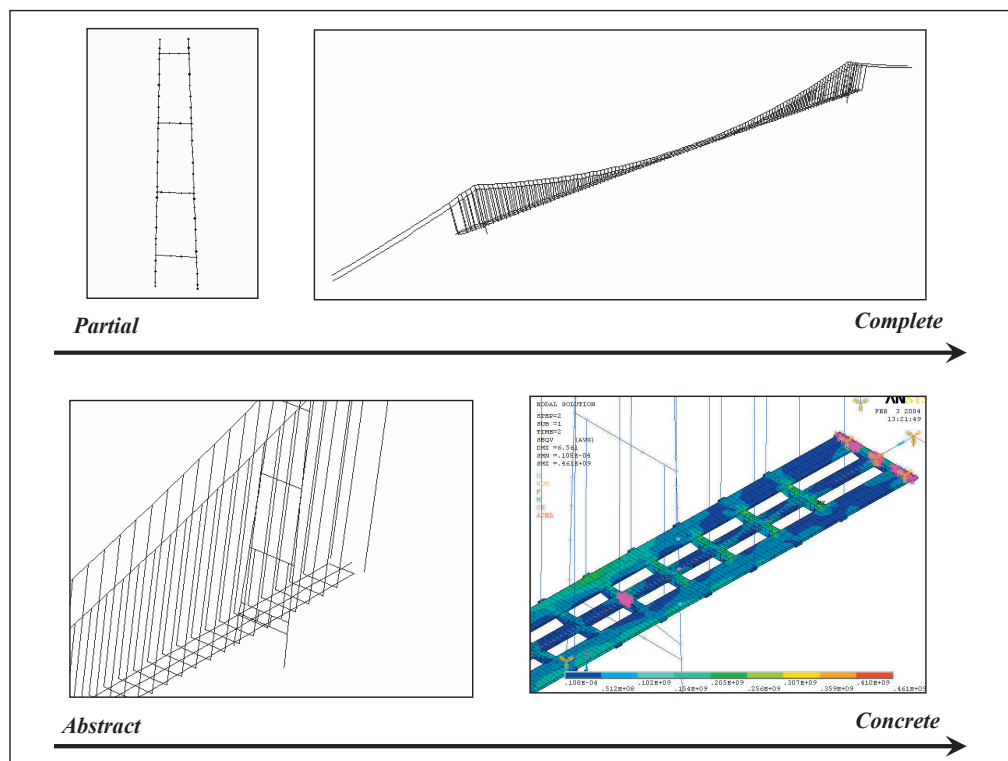


Figure 2. The two dimensions of the numerical representation: concreteness and completeness.

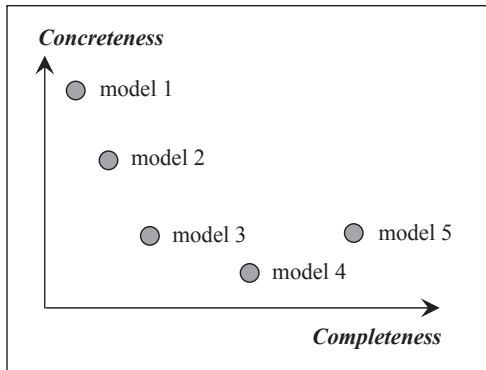


Figure 3. Different levels of the numerical modeling.

In the analysis of a complex structure, it is suitable to use local and global models with different degree of concreteness.

A local model can be built with refined discretizations and more accurate mathematical formulations. It is able to represent in a proper way the local behavior of the element but, geometrically not represent the whole structure so it is not able to give information respect the global behavior of the structure.

Contrarily, a global model is able to describe the general behavior of the structure, but it is not able to provide information on the local behavior of the single element or substructure.

Clearly, both the local and the global models are strong reductions of the reality (Bateson 1979). The necessity to use various representations with different level of detail derives from the difference of information that every model can give. Unfortunately, the information present in the local model usually it is not present in the global model and vice versa.

The information produced by a local modeling can be used to improve the response of the global model using a multigrid finite element analysis (Casciaro 1998). In the multigrid analysis the local and the global model are built separated but the interface condition between the models permit the swap of the knowledge from the local to global model and from the global to local model.

In Figure 4 a possible multigrid approach for the study of the bridge deck behavior is presented. The local model of two fields of the bridge deck, developed using a refined shell mesh, is coupled with the global model developed using beam elements.

Otherwise, the refinement of the global model with knowledge present in the local model can be formulated as an identification problem. The solution of the identification problem can be performed using soft computing methods as neural networks (the paragraph 4.1 explain this approach).

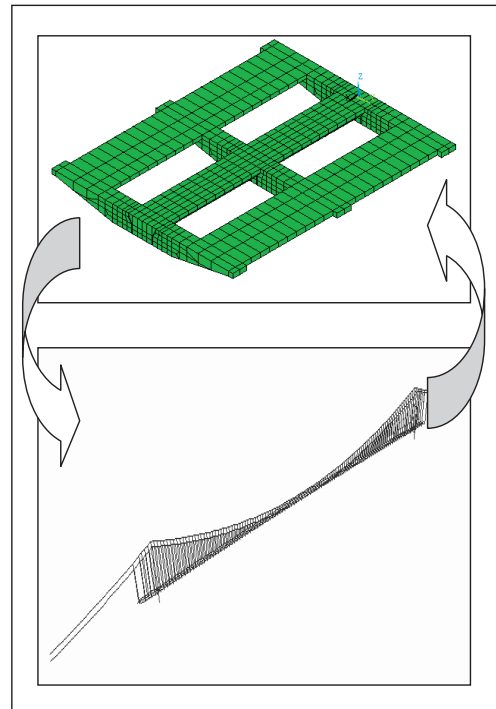


Figure 4. A possible multigrid approach for the study of the bridge deck behavior.

3 ARTIFICIAL INTELLIGENCE AND SOFT-COMPUTING METHODS

The artificial intelligence is the branch of computer science concerned with making computer behave like human. The term *artificial intelligence* was coined in 1956 by John McCarthy at the Massachusetts Institute of Technology. Today the artificial intelligence methods are applied with profit in varied fields: game playing, speech recognition, understanding the natural language, computer vision, expert systems, heuristic classification.

Among the methods developed in the area of the artificial intelligence, they are some techniques that are classified as *soft-computing* methods. The concept of soft-computing was introduced by Zadeh in 1964. Soft-computing is a whole methodologies directed to exploit the tolerance to the imprecision and the uncertainty to achieve tractability and robustness with low computational cost. The model of reference for the soft-computing is the human mind (Zadeh 1964).

These methods of analysis are therefore suitable for the treatment of problems, which an exact solution does not exist, or exists but it is very complex.

The principal constituents of the soft-computing are: artificial neural networks, evolutionary algorithms and fuzzy logic. One remarks that soft-computing is not a mixture of methods, but different methods can contribute with different methodologies to face a unique problem. The methodologies are therefore complementary and not competitive. This has an important consequence: a problem can often be resolved more effectively using a combination of the listed methods rather than the single separated methods. Following, one gives a brief description of the most important techniques of soft-computing and their employment in the civil engineering field.

3.1 Artificial neural networks

The artificial neural network is a system of elaboration of the information inspired to the biological neural network. For their characteristics of robustness, flexibility and for their ability to generalize, the artificial neural networks are been used in varied fields of the computer science: compression dates, the noise reduction, recognition of signals, phonetic typewriters, recognition of the characters.

Nevertheless, the artificial neural network was been used also in the systems of control, in the financial analysis, in medicine, in the neurosciences, and in the psychology. In the last years the artificial neural network is also been applied with success to the fields of the civil engineering and design (Cauvin 1998, Flood 2001, Hong et al. 2002).

3.2 Evolutionary algorithms

The evolutionary algorithms are heuristic algorithms of searching and they include: the evolutionary programming, the evolutionary strategies, the classificatory systems and the genetic algorithms. In particular genetic algorithms are inspired by Darwin's theory of evolution.

Genetic algorithm is an example of stochastic evolutionary procedure where an initial random population evolves in order to maximize a fitness function. It is important to notice that during their evolution the genetic algorithm acquires knowledge about the problem definition. Genetic algorithms are adaptive; they are able to interact with a mutable environment.

In the last years, many applications are also developed in the field of the civil engineering. In particular the genetic algorithms have been used in the field of the structural optimization, in the allocation of the resources for construction problems and in the optimization of road and water networks (Senouci and Eldin 2004, Yang and Soh 1997, Biondini 2000, Tolson et al. 2004).

3.3 Fuzzy logic

The fuzzy logic allows to reproduce the approximate reasoning of the human mind. The term *fuzzy logic*

was introduced by Zadeh (Zadeh 1965). However the idea of a third logical state contrasted to the ambivalent logic (true or false) of Aristotle was to the philosopher Greek Plato or, in modern times, to the philosopher Lukasiewicz.

The first applications of the fuzzy logic were in the engineering fields of the control systems, in the systems of decisional support, in the natural language, in the recognition of forms and in other various fields. Also in the civil engineering, the fuzzy logic finds notable importance in problems that require analyses in presence of uncertainty. Some of the major applications in civil engineering concern the control techniques, the structural reliability and the treatment of the uncertainties in the materials (Biondini et al. 2000, Provenzano and Bontempi 2000, Savoia 2002).

4 APPLICATIONS

In the following paragraphs one present an application of these methods to study the behavior of long suspension bridges. The design problem of these structures presents a great complexity for all the aspects involved in the design phases. However, it is possible to use soft-computing methods for improving the knowledge and the robustness of the resolution. Every soft-computing method presented, can be interpreted as a method for facing the engineering problem from a specific point of view, in particular one can use (see Figure 5):

- Artificial neural networks for improving the knowledge about the numerical model (Arangio 2004, Sgambi 2005).
- Evolutionary algorithms for improving the robustness of the analysis (Sgambi et al. 2004).
- Fuzzy methodologies for handling the uncertainties involved in the analyses (Sgambi and Bontempi 2004).

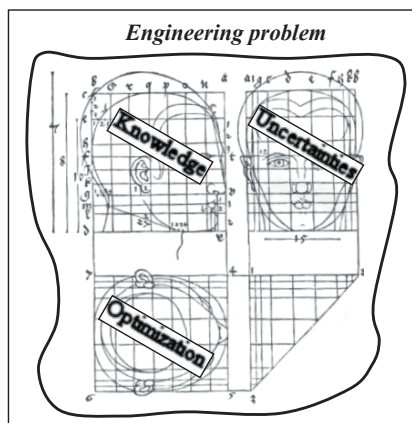


Figure 5. Different points of view of an engineering problem.

4.1 Artificial neural networks

Neural networks can be used to increment the knowledge of the global numerical model of the structure. In fact, one can improve the behaviour of the beams model defining in an appropriate way the mechanical properties of beams element. One can solve an identification problem to define the mechanical properties of the beam elements considering the deformability of a more accurate local model. The neural networks are powerful methods to develop nonlinear regression, so they are suitable to solve the identification problems.

In this study, a portion of the bridge deck is modeled using a refined mesh of shell elements developed by ANSYS. This model and the same beam model are reported in the Figure 7. The mechanical properties of the beam model are found to have equality on the first ten mode shapes between the two models. This is a typical inverse problem where the result (deformability) is known but the structure (mechanical properties) is unknown.

The data used for the training of the neural network has been gotten through the resolution of a direct problem of the structural analysis. One has fixed a specific range of variability for the mechanical characteristics to assign at the various sections and one has built a sequence of test cases. Using the structural code ADINA the modal characteristics (frequencies and modal shape) correspondents to every test case has been investigated. In this way 64 test cases was created. One has used 50 test cases for the training of the neural network and 14 test cases for the validation phase.

Many different networks were tested to define the optimal neural network. For every network the total error on the 64 test cases was evaluated and plotted in graphs (Figure 6).

Examining the behavior of the neural network tested was possible to define the optimal topology and the optimal number of training epochs. The optimal network for this problem is defined with 10 neurons in the hidden layer with random strategy of training based on 5000 epochs.

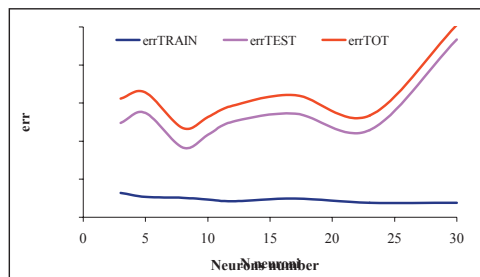


Figure 6. Error in the training (*errTRAIN*) and test phases (*errTEST*); *errTOT* represent the sum of the errors.

After having identified the type of optimal network topology for the resolution of the problem, the vector with the 10 values of frequency selected was presented at the neural network. As a result one obtains the suitable mechanical properties.

In this study one has considered only 3 mechanical properties of the beam model (the thickness of the three different elements). The Figure 7 shown the resemblance on the principal mode shape between the two models considered. In Table 1 the first 10 frequencies of the two models are reported.

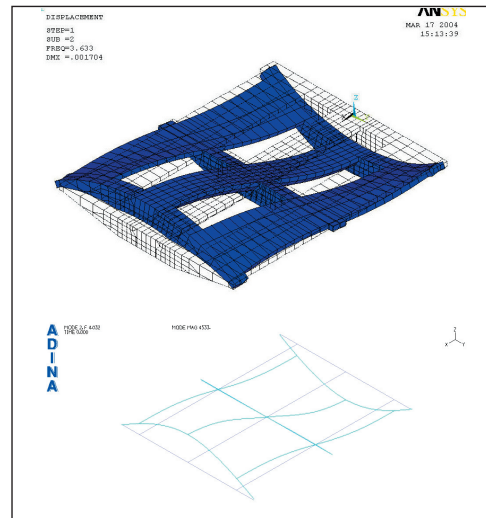


Figure 7. First shape mode for the two models considered.

Mode number	Beams model (ADINA)	Shells model (ANSYS)
1	2.871	2.778
2	4.032	3.663
3	5.640	5.065
4	5.729	5.881
5	6.423	5.818
6	7.735	7.142
7	7.942	7.248
8	8.010	7.062
9	8.270	7.648
10	9.876	9.901

Table 1. The first 10 frequencies of the two models.

In the procedure performed, there is the possibility of a further improvement of the beam model using more variables in the identification problem. In fact, assuming more variables one has able to transmit more knowledge from the shells model to the beams model.

4.2 Evolutionary algorithms

The use of genetic algorithms in the structural analyses can increase notably the reliability of the results. The genetic algorithms are example of stochastic evolutionary procedures where an initial random population evolves in order to maximize a fitness function. One can defines as variables the loads position while as a fitness function one can define the maximum transversal slope on the bridge deck (or other static or cinematic parameter). During the evolution of the process, the genetic algorithm explores the load combinations that maximize this specific parameter. During this process, the genetic algorithm explores many load combinations near the local and global maximum points. In this way, it is possible to obtain reliable envelope diagrams for the deformability parameters considered (Sgambi 2005).

The steps necessary to perform a genetic analysis are:

- set the variables of the problem and the numerical representation of its;
- set the fitness function;
- define the initial population (first generation in the Figure 8) and the genetic operator parameters;
- perform the analysis.

In the present study, one has individualized sixteen variables: 1 variable for the train position, 2 variables for each roadway vertical line load (one has four roadway line load listed in heavy and light traffic for each roadway), 2 variables for the wind load, 5 variables to define the acceleration or the deceleration of the traffic. The dimension fo the population was fixed to 100 individuals. The value of the variables for the inizial popouluation was initialized with random process.

One has performed the genetic analysis for various cases. In particular one has considered as a fitness function the following parameters:

- For the bridge deck:
 - the vertical displacement (positive and negative);
 - the longitudinal slope;
 - the transversal slope.
- For the cable:
 - the axial stress.
- For the bridge tower:
 - the stress state involved from the axial action and the two bending moments.

For each genetic analysis 100 generations was performed for a total of 10000 load combinations considered. It is important the possibility of automatize the analysis using a commercial code (ADINA, ANSYS or LUSAS for example) coupled with a in house made program (Sgambi 2004).

The Figure 8 shows the convergence of the position of the railway load to the value that maximizes the fitness function.

In Figure 9 the nondimensional loads position that maximize the transversal slope is raffigured, while in Table 1 the numerical results for each case analyzed are reported (One observe that the load position values are divided by the bridge length).

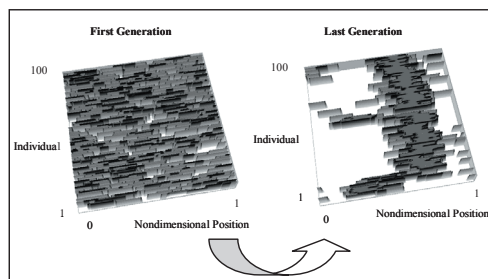


Figure 8. Distribution of the location of the train loads for the first and the last generation.

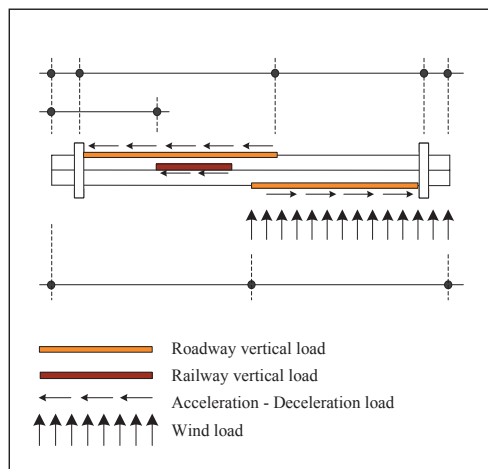


Figure 9. Nondimensional loads position.

Variable	Max Displ. (negative)		Max Displ. (positive)		Max Long. Slope	
Train	0.22	D	0.22	D	0.05	D
Carriage 1	0.03 to 0.50	D	0.03 to 0.50	D	0.05 to 0.59	D
Carriage 2	0.03 to 0.50	A	0.03 to 0.50	D	0.05 to 0.59	A
Wind load	0.50 to 1.00	---	0.50 to 1.00	---	0.50 to 1.00	---
Variable	Max Trans. Slope		Max Stress on the cable		Max Stress on the tower	
Train	0.29	D	0.42	A	0.42	A
Carriage 1	0.05 to 0.56	D	0.00 to 1.00	A	0.00 to 1.00	A
Carriage 2	0.50 to 0.95	D	0.00 to 1.00	D	0.00 to 1.00	D
Wind load	0.50 to 1.00	---	null	---	0.00 - 1.00	---

Table 2. Position of the traffic load. (One notes that the symbol D means deceleration while A means acceleration. All position are referred a left side of the bridge).

4.3 Fuzzy logic

In a complex structure as a long suspension bridge, many parameters about the mechanical properties or the load input are affected from imprecision. In these cases, fuzzy analysis can be performed to increment the robustness of the results.

A very complex problem about the analysis of the long bridge behavior is the definition of the input seismic motion. The intensity and the direction of the seismic action are examples of unknown seismic parameter that influence the result of analysis. Clearly, the performance evaluation of this kind of structure cannot be evaluated in deterministic way, using a low number of seismic simulations. A probabilistic or fuzzy approach has to be adopted to perform reliable analyses (Sgambi & Bontempi 2004).

In order to represent and investigate the uncertainties importance both for the seismic intensity and for the seismic direction, one has developed a fuzzy approach to the seismic event. Fuzzy sets and fuzzy logic are, in fact, means for representing, manipulating and utilizing uncertain information.

An uncertain input parameter can be fuzzified using a triangular (or another shape) membership function. Various types of membership functions are commonly used in fuzzy theory. The choice of the shape depends on the specific application. In this paper, one has utilized a triangular function to fuzzifier the three seismic intensities. One can assign to this function the label of *input membership function*. The fuzzy response of the bridge is evaluated in some measure points. In particular, one has considered:

- the tension in the principal cable near the anchorage zone (1 – TC) and in the middle of the bridge (6 – TC),
- the tension in the left side hanging (4 – TH),
- the longitudinal displacement of the deck at the dilatation joint (3 – LD),
- the transversal displacement of the deck close the tower (5 – TD) and in the middle of the bridge (7 – TD),
- the vertical displacement of the deck close the tower (5 – VD) and in the middle of the bridge (7 – VD).

The output parameters are linked to the input parameters by the structural behavior. Here is assigned to this relation the label *translation function* (Figure 10). In fact, one can use this function to reproduce the uncertainties on the output variables. It is possible to transform the input interval in the output interval sectioning the input membership functions for specified degree values of the membership function (α -cuts) and using the outputs of the translation function (Figure 10). The output membership function is built repeating this procedure for different values of the input membership function.

Table 3 summarizes the results of the analysis in terms of defuzzificated value with center of gravity rule, maximum value and increment related to the non fuzzy analysis. A more exhaustive explanation about this argument is presented in (Sgambi & Bontempi 2005).

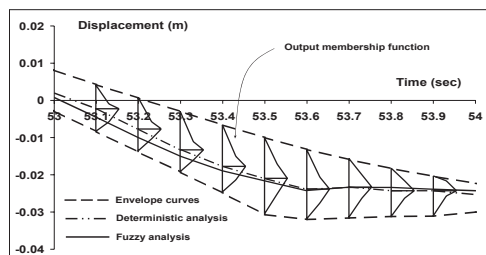


Figure 11. Qualitative image of the fuzzy response construction (Sgambi & Bontempi 2005).

Output parameters	Def. value	Max. value	Incr.
1-TC	12020	15780	31%
3-LD	0.81	1.0	23%
4-TH	213	286	34%
5-TD	0.081	0.090	11%
5-VD	0.053	0.070	32%
7-TD	0.64	0.83	30%
7-VD	0.62	0.84	35%

Table 3. Variation interval in the fuzzy response curves (displacements = m, forces = Ton).

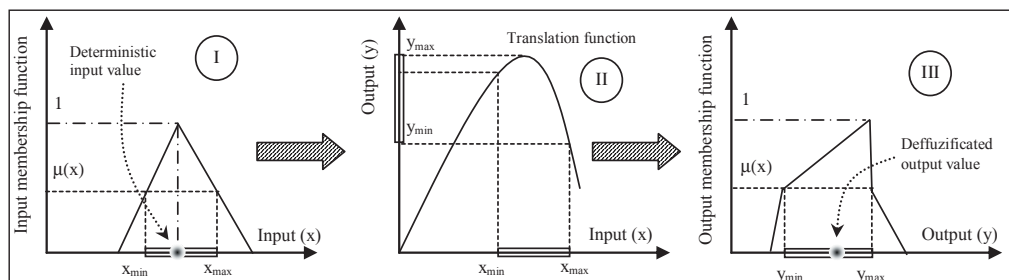


Figure 10. Estimate of the output interval from the input interval.

5 CONCLUSIONS

This paper considers the analysis and design problem of complex structures as long suspension bridge. In particular one presents the applicability of the soft-computing method to improve the reliability of the analyses. Neural networks, genetic algorithms, fuzzy analyses can be performed to handling the uncertainties involved in the problem and to get a more exhaustive knowledge about the structural behavior.

Today the conceptual design and the performance based design play central roles in the design phase of complex structures. The exploration of the design space and the evaluation of the design alternatives have to be the more accurate possible. For this reason, the methods of soft-computing coupled with modern commercial codes (as ADINA, ANSYS, LUSAS or other) become powerful methodologies at the service of the designers.

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