Strategic Fertility, Education Choices and Conflicts in Deeply Divided Societies

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Strategic Fertility, Education Choices, and Conflicts in Deeply Divided Societies

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Abstract

Fertility becomes a strategic choice when having a larger population helps to gain power. Minority groups might find it optimal to promote high fertility among their members – this is known as the "weapon of the womb" argument. If, in addition, parents have to invest resources to educate their children, a higher fertility for strategic motives might reduce their investment. Indonesian census data dispel this view, as minority religious groups do not invest less in education. If anything, they invest more in education, as well as in their number of children. This finding is consistent with human capital being an input to appropriation. Solving for the Nash equilibrium of a game between two groups with two strategic variables, we derive the condition under which the minority group displays a higher investment in both the quantity and quality of children. The material cost of conflict involved through the weapon of the womb mechanism is mitigated when human capital enters the contest function.

JEL Classification Numbers: D74, J13, J15
Keywords: fertility, quality-quantity trade-off, minorities, conflict, population engineering, human capital, Nash equilibrium, Indonesia

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1 Introduction

In their analysis on how lowering fertility can help developing countries to grow, economists have mostly considered individual incentives to have children: a higher opportunity cost for educated parents (Becker and Lewis 1973 ,de la Croix and Doepke 2003, Becker, Cinnirella, and Woessmann 2010, Vogl 2016a), a higher return to education of children (Galor and Weil 2000), higher income and the possibility of inter-generational transfers (Córdoba and Ripoll 2016), a lower cost of contraception (Bhattacharya and Chakraborty 2017), and changing gender-specific opportunities (Voigtländer and Voth 2013). Beyond these mechanisms, a few authors have shown that external social norms affect individual preferences to have children (Spolaore and Wacziarg 2014, Daudin, Franck, and Rapoport 2019). When social norms matter, the logic of the social group influences the behavior of individuals.

In this paper, we consider that fertility responds to the logic of the social group to which the individual belongs. When the number of groups is small, doing so introduces a game theoretic dimension to fertility behavior, which stems from the strategic interactions between groups in order to gain power. This new approach in the literature sheds light on how institutions and demographics interact in societies divided along religious or ethnic dimensions.

Our model is based on the idea of "People as Power" (Yuval-Davis 1996), according to which the population weight of a religious or ethnic group determines its power to appropriate resources when property rights are not perfectly defined. When there are increasing returns to the effect of demographic weight on political power, there is an incentive for each group to increase fertility, so as to benefit from more power in the future (de la Croix and Dottori 2008, Janus 2013). In such a context, the effect of religious and ethnic divisions on a country's economic performance is not only related to the effect of these divisions on the quality of governance and the cost of civil wars (Toft 2005, Alesina and La Ferrara 2005), but may also involve a population race between the groups.

The promotion of fertility may however backfire against the group which favors it or the country as a whole. One of such adverse side effects may lie in education. If people or groups face a Beckerian quality-quantity trade-off (Doepke 2015), encouraging fertility may be at the expense of the quality of future generations. Intuitively, this adverse effect can be mitigated if political power not only depends on the number of people, but is also a function of their quality (human capital).

We first investigate whether our intuition that the size of one's group may reflect on fertility, but also on education. We consider a large fragmented country: Indonesia. Indonesia is divided into hundreds of counties called regencies, for which we have detailed demographic data thanks to censuses and other surveys, including religious affiliation. A majority of regencies are homogenous along the religious dimension, but, in some of them, various religions coexist. In line with a strategic motive behind fertility behavior, we find that the number of children born from women is a decreasing function of the share of their religious group in the regency. More surprisingly, we also find that the education level of young adults is also decreasing in the share of their group.

In a second step, we develop a theoretical framework to model the strategic motives behind fertility and education choices, and determine the conditions under which religious or ethnic division favors fertility at the expense of education. These conditions likely depend not only on the production and contest technologies in the model, but also on the initial composition of the country and the weight of the different groups. The model accordingly captures the following features. Two groups decide on their fertility and play a Nash game. The total output is divided between them according to their power. Power depends on the relative population size. The output increases with labor (population), but with decreasing marginal returns. In this context, we can show that at the Nash equilibrium, the fertility of each group is a decreasing function of its relative size.

When the model is extended to include human capital as a factor of production that requires an investment in each child, the strategic motive to increase fertility plays against human capital accumulation, reflecting the usual quality-quantity trade-off. Finally, when political power is made dependent on the human capital of each group, the latter result can be reversed, and groups may compete by increasing both their fertility and education spending. This reversal arises when the elasticity of power to human capital is large enough.

We also examine the cost of conflict by comparing economic outcomes in the *anarchic* environment we have considered so far to the same outcomes in a *peaceful* environment. The latter is defined as a situation in which both parties have successfully found an agreement in which the resource is shared according to the parties' relative size in the previous period. We emphasize two new channels through which conflict affects income per capita: a decrease in the share of the cake (due to a higher population size) and a decrease/increase in human capital (depending on the elasticity of power to human capital). The cost of conflict induced by strategic fertility is reduced (and even negative) when the elasticity of power to human capital is sufficiently high, the returns to education are strong, and the decreasing marginal returns to labor are low.

Our results speak to two different literatures. First, they pertain to the field of family macroeconomics and development (Doepke and Tertilt 2016), by introducing multi-dimensional strategies at the core of fertility and education choices. They also link institutional failure with demographics, as the strategic motives we highlight are made possible by the assumption that property rights are not perfectly defined and enforced. Second, our results speak to the literature on the economics of conflict. A theoretical literature has focused on the trade-off between appropriation (which requires producing "guns") and productive activities, studying implications for economic outcomes (i.e. cost of conflict and distribution of resources); see Garfinkel and Skaperdas (2007a) for a survey. Compared to this literature, we consider a different conflict technology in which fertility and/or education rather than physical capital are used as inputs to appropriation. This framework allows us to highlight the role of population size in situations where groups compete for a resource. We are also able to study the impact of conflict on population dynamics and human capital.

Our view on education and human capital as an input in the appropriation technology is quite new. The literature on conflict generally considers physical capital, not human capital, as an input. We claim that a high-skilled population can more easily use a more advanced war technology. A historical example of the importance of education is provided by the novel tactics implemented by the Prussian army in the nineteenth century, which required a high level of education at all levels and actively involved independent decision making by the lower ranks. This has led historians to claim that the outcome of the battle of Sedan in 1870, won against the French army, was determined by Prussian elementary school teachers (Nipperdey 1994, p531). The effect of human capital may go through other channels as well. A more educated ethnic group will more easily construct a version of history which heightens the role of that group at the expense of others (Bush and Saltarelli 2000). Minorities may also be excluded from power and influence through lack of access to the language of power and government (Graham-Brown 1994).

The paper is organized as follows. Section 2 shows how individual fertility and education choices are correlated with religious group sizes across Indonesian regencies. Section 3 elaborates the theory. Section 4 concludes.

2 The Minority Hypothesis in Indonesia and the Effect on Education

2.1 Context and description of the data

Indonesia is a very diverse country with about 700 languages and dialects spoken. There is a literature on the effect of ethnic diversity on various outcomes in Indonesia, including the prevalence of community organizations (Okten and Osili 2004), the pattern of public good provision (Bandiera and Levy 2011), and individual social capital (Mavridis 2015). Accounting for diversity also matters for assessing policy interventions, as in the case of the school building program of the 1970s, for which a positive impact on female education is only observed among girls from ethnic groups that traditionally engage in monetary bride price payments at marriage (Ashraf et al. 2015).

Beyond ethnicity and language, religion is also salient issue in Indonesia. Gaduh (2012) finds that individuals are more cooperative and trusting of their community members in more religiously homogeneous communities. Chen (2010) shows that Islam plays the role of an ex-post insurance mechanism by looking at the link between religious intensity and economic distress in the aftermath of the Indonesian financial crisis of 1997. Chen (2006) further illustrates how the economic crisis exacerbated violent conflicts along religious cleavages.[1] Bazzi, Koehler-Derrick, and Marx (2018) describe how rural elites have used Islamic institutions to circumvent land reforms, highlighting further the role of religion in power struggles in Indonesia. Finally, examining six south-Asian countries, including Indonesia, de la Croix and Delavallade (2018) find that religion matters for fertility behavior, and in particular that while Catholicism is the most pro-child religion (increasing total spending on children), followed by Buddhism, Islam has a strong pro-birth component (redirecting spending from quality to quantity).

We focus on religion affiliation as defining cultural groups. We pool extracts of the Indonesian censuses of 1971 (0.5%), 1980 (5%), 1990 (0.5%), 2000, and 2010 (both 10% samples), which we complement with the large Intercensal Population Surveys of 1976 (0.2%), 1985 (0.4%), and 1995 (0.4%) as made available by IPUMS International.[2] IPUMS provides information on 269 regencies with consistent boundaries across all waves for which we can compute the shares of the various religions. Figure 1, which is drawn using the 2010 census, shows that Indonesia is dominated by Islam, but significant other groups exist and even are the majority on certain islands. We disregard any issue linked to intermarriage across religious groups both in the empirical analysis and in the theoretical model, as we observe that only about 5% of married couples in our sample are in a religiously non-homogenous marriage.

In Table 1, we show some summary statistics by regency, including religious composition, average education level, and child mortality rate.[3] These statistics are based on the population aged 18-55. There is quite a bit of variation in religious group shares, with for example 25%

[1]See also Schiller and Garang (2002) for a sociological account of the inter-ethnic violence outbreaks of the period.

[2]Minnesota Population Center. Integrated Public Use Microdata Series, International: Version 7.0 [dataset]. Minneapolis, MN: IPUMS, 2018. https://doi.org/10.18128/D020.V7.0. The Intercensal Population Survey of 2005 is dropped as information on religious affiliation is lacking.

[3]The child mortality rate is computed following Baudin, de la Croix, and Gobbi (2018) as one minus the share of surviving children among those ever born.

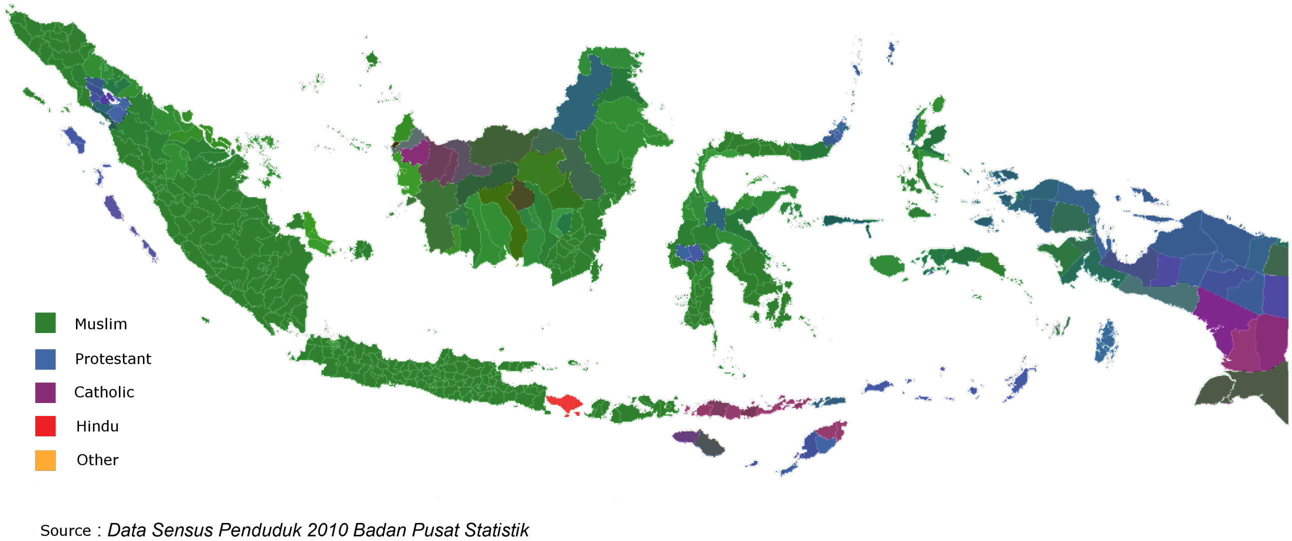


Figure 1: Religious Affiliations in the Indonesian 2010 Census

of the regencies having less than 80% of Muslim population, while another 25% are over 99% Muslim.

Notice that the mean of the shares of religious groups is taken across regencies, not individuals. At the country level, we have in our sample close to 88% of Muslims, 6% of Protestants, 3% of Catholics, 2% of Hindus, and 1% of Buddhists. Confucianism and no religion are negligible categories. The fact that the share of Muslims is lower across regencies than over individuals can be explained by the fact that regencies with fewer Muslims than average are smaller in size; and conversely for Hindus or Catholics.

Population shares are overall remarkably stable over time, with the exception of Buddhism and Confucianism that seem to decline. The distribution of those shares has not drastically changed over time either. This lack of time variation has a consequence in terms of empirical strategy as we are unable to include regency or even province fixed effects as they would essentially remove all the meaningful variation in group size that we aim to exploit.[4] Conversely, there are strong trends in educational attainments, with a doubling of the number of years of schooling over the period, and in child mortality, which has been divided by four.

To analyze fertility, we consider the sample of all women aged 45-59. Taking an age range

[4]Another reason not to include geographical fixed effects is that there exists a dynamic relation between fertility and group size in the future, as ceteris paribus, higher fertility groups tend to increase in size. Focusing on the time variation for identification, as is the case when using geographical fixed effects, magnifies this issue. When we do include geographical fixed effects, either alone or interacted with religious affiliation, the coefficient on group size actually never turns out significant, with the exception of the fertility regression with province fixed effects in which the coefficient is positive and significant. We interpret this result as a situation in which the effect of fertility on group size dominates that of group size on fertility.

Table 1: Summary Statistics for regencies (N=269)

Statistic	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
			% Buddhist				
1971-1976	1.2	4.4	0.0	0.0	0.0	0.2	48.1
1980-1985	1.1	3.3	0.0	0.0	0.1	0.6	42.7
1990-1995	1.0	2.6	0.0	0.0	0.1	0.5	26.9
2000-2010	0.7	1.7	0.0	0.0	0.1	0.4	13.3
			% Hindu				
1971-1976	3.5	17.3	0.0	0.0	0.0	0.0	100.0
1980-1985	3.3	15.8	0.0	0.0	0.0	0.2	99.5
1990-1995	3.6	17.0	0.0	0.0	0.0	0.2	99.6
2000-2010	3.1	15.2	0.0	0.0	0.1	0.3	98.8
			% Muslim				
1971-1976	82.6	29.4	0.0	82.1	97.0	100.0	100.0
1980-1985	81.8	28.5	0.3	79.2	95.6	99.0	100.0
1990-1995	82.6	29.0	0.1	84.8	96.2	99.0	100.0
2000-2010	83.1	27.6	0.9	84.0	96.3	98.9	100.0
			% Catholic				
1971-1976	3.3	12.0	0.0	0.0	0.2	2.2	100.0
1980-1985	4.4	14.5	0.0	0.1	0.5	2.0	97.5
1990-1995	5.1	15.5	0.0	0.3	0.8	2.9	97.3
2000-2010	4.6	14.4	0.0	0.2	0.6	2.1	91.6
			% Protestant				
1971-1976	6.9	17.5	0.0	0.0	0.6	3.9	100.0
1980-1985	8.2	17.3	0.0	0.5	1.3	6.2	95.5
1990-1995	7.3	17.4	0.0	0.3	1.2	4.6	96.0
2000-2010	8.1	17.3	0.0	0.4	1.2	6.5	88.0
			% Confucianist				
1971-1976	2.4	8.5	0.0	0.0	0.0	0.9	94.6
1980-1985	1.2	4.8	0.0	0.0	0.0	0.4	51.4
1990-1995	0.4	2.5	0.0	0.0	0.0	0.0	27.2
2000-2010	0.3	1.6	0.0	0.0	0.0	0.1	19.7
			Years of schooling				
1971-1976	3.8	1.7	0.1	2.6	3.6	4.8	8.5
1980-1985	4.9	1.6	1.2	3.8	4.6	5.8	10.3
1990-1995	6.5	1.7	1.9	5.4	6.3	7.5	12.9
2000-2010	7.8	1.7	3.0	6.6	7.7	8.8	12.1
			% Child mortality				
1971-1976	17.6	6.4	0.0	13.1	17.3	22.2	40.7
1980-1985	13.8	4.1	4.2	10.7	13.5	16.5	29.4
1990-1995	8.2	3.5	1.1	5.4	7.8	10.4	22.2
2000-2010	4.5	2.2	1.0	2.8	4.1	6.0	14.7

larger than the spell across years of observation allows us to balance cohorts mainly covered by smaller surveys with observations from larger censuses at a different age. Of course the oldest and youngest cohorts in the whole sample will be under-represented. We obtain a little over 3 million observations, whose characteristics are shown in Table 2. The median woman is aged slightly over 50, has just below five years of education, and gave birth to just over four children, of which 3.65 were still alive at the time of the survey. Close to 80% of women are currently married, 42% live in an urban area, and 88% reside in their region of birth.

To analyze education, we consider the sample of all individuals aged 26-32 instead, of which we observe close to 6.5 million individuals. The rationale for looking at this age range is to analyze the educational outcomes of individuals who could plausibly be the offspring of the women in the fertility sample. Moreover, looking at the educational attainment of individuals 26 and above provides a measure of completed education, which is a better measure of voluntary investment in human capital than educational attainment or school attendance measured at earlier ages.[5] Individuals in this sample are 29 years of age on average, 8 of which were devoted to schooling. 47% of them live in urban areas and 94% reside in their region of birth. The higher level of educational attainment and urban residence in the education sample reflects rising trends over time in those variables. The slightly larger share of migrants in the older fertility sample instead is probably due to the de facto longer horizon over which migration is measured, which compares current residence to region of birth.

Table 2: Summary statistics at the individual level

Variable	**Mean**	**(Std. Dev.)**
Fertility sample		
Children ever born	4.19	(2.52)
Children surviving	3.65	(2.05)
Currently married (%)	79	(0.41)
Age	50.75	(4.21)
Urban status (%)	42	(0.49)
Years of schooling	4.78	(4.2)
Residing in province of birth	0.88	(0.32)
Number of observations	3,027,999	
Education sample		
Years of schooling	8.21	(4.12)
Age	28.96	(1.94)
Urban status (%)	47	(0.5)
Residing in province of birth	0.94	(0.24)
Number of observations	6,385,620	

[5]We analyze educational attainment for individuals aged 12 to 17 and 18 to 25 in Appendix C and find consistently signed coefficients, though less significant.

2.2 The variation of fertility and education with group size

We now regress the number of surviving children on a series of usual explanatory variables, to which we add the population share of the religious group each woman belongs to (henceforth, group size). Both the total number of children ever born and the number of surviving children should respond to group size according to the mechanism we shed light on. However, since we look at a developing country where child mortality still crucially affects fertility outcomes, we believe the number of surviving children is a less noisy measure of completed fertility. In Appendix C, we perform a robustness check using the number of children ever born as dependent variable and find consistent results. We include deciles of group size as our main explanatory variable to allow the effect to be non-linear.

The results for different specifications are shown in Figure 2a. Since fertility is a count variable, all models are run using a Poisson regression. The coefficients are therefore not interpretable as marginal effects, but the economic magnitude of the effect is shown in Figure 2b instead. Using OLS instead of Poisson does not alter the conclusions.

The first model only contains controls for year of birth, urban status, and survey year. The second model adds religious affiliation, the mother's educational attainment, and the average educational attainment in the regency. The rationale for including religious affiliation is to avoid relying on cross-religion differences in group size, but rather to focus on the within-religion variation. The mother's education is our best estimate of the opportunity cost of time for women. Although it is endogenous to fertility choices, we include it here as a control in order to ensure that group size has a direct effect on fertility, not only mediated through the mother's education. The average education in the regency is included in order to proxy the level of development, which could be systematically linked to group size and affect fertility but is not the mechanism we want to capture.

Lastly, the third model includes controls for child mortality in the regency, as well as marital status. Both variables are endogenous to fertility choices as well, but have been used in the demographic literature to track "proximate" determinants of fertility. We therefore include them in this specification to make sure that the mechanism we propose does not go through either of these channels exclusively. Model (4) repeats the specification in model (3), but excludes migrants, that is women who do not reside in their region of birth, from the sample. The rationale for this exercise is to ensure that our mechanism is not driven by selective migration.

We find a robustly significant negative association between one's religious group size and the number of surviving children in all specifications. All deciles below the median have a significantly higher fertility than the deciles above, which do not differ significantly from each other.

The bottom 10% however do not seem to behave according to our mechanism, as they display a lower fertility than the second decile. To a lesser extent, the top decile seems to deviate from the otherwise monotonic relationship between fertility and group size, as it has a slightly larger fertility than other deciles above the media. The difference is however not statistically significant. For the remaining 80% of the distribution, however, the negative relationship between group size and fertility applies. Here, we find the standard result of the minority status hypothesis (Chabé-Ferret and Melindi Ghidi 2013). The full table is available in Appendix B, where all coefficients are very much in line with the literature.

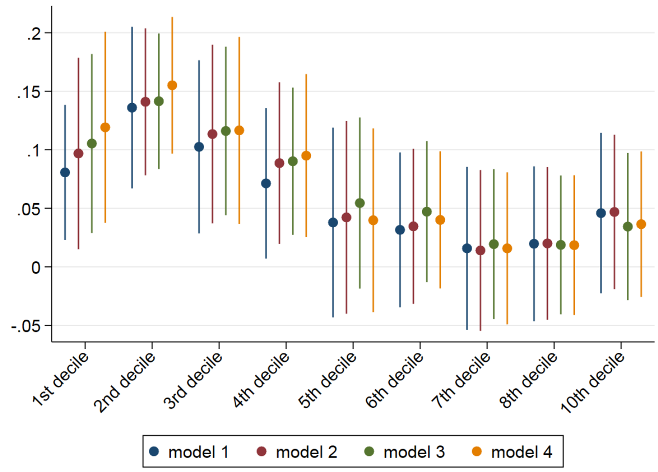
Figure 2b tells us about the magnitude of the effect by showing the predicted counts of fertility by group size decile. Deciles 5 and 6 just around the median have a very similar predicted fertility of 3.6 children. Switching to decile 4 is associated with an increase in fertility of 0.2 child, while the maximum effect is found by switching to decile 2 where fertility reaches about 4 children per women. These magnitudes are economically and demographically meaningful as they represent respectively 5 and 10% of the overall average fertility in Indonesia. Sustaining such fertility differentials could bring a society divided 40-60% into two groups to parity in about five generations.

Figure 2c presents the results of the education regression. As in Figure 2a, model (1) only controls for year of birth, urban status, and survey year. Model (2) adds religious affiliation and child mortality in the regency. Including child mortality allows us to control for the level of development in the regency. We choose not to control for mean educational attainment as it could capture the effect of the size of the dominant religious group and bias our estimates towards zero. Finally, model (3) excludes migrants from the sample.

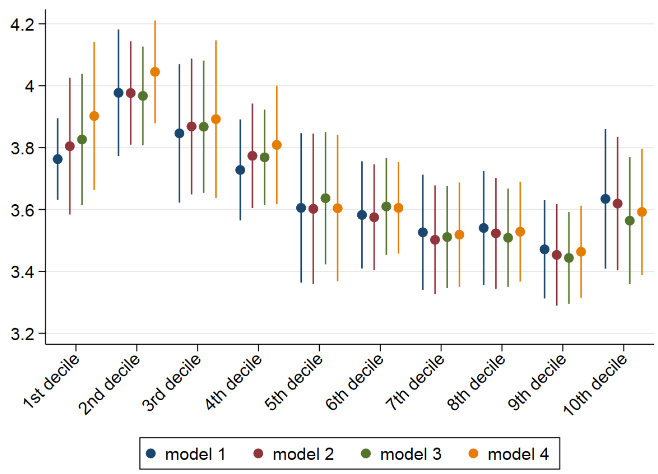
The results in Figure 2c show a negative correlation between group size in the regency and individual educational attainment. The size of the coefficient increases when we include controls and exclude migrants. Using the standard deviation in the dependent variable to get a sense of the magnitude of the effect, we obtain that a one-standard-deviation increase in group size is associated to a decrease in educational attainment of 0.35 to 0.45 years of schooling, which represents between 7 and 9% of the baseline.

Together with the result about fertility, this seems at odds with the quality-quantity trade-off approach. Here, minority groups do not seem to substitute quality of children for quantity. Rather, they invest more heavily in both dimensions.

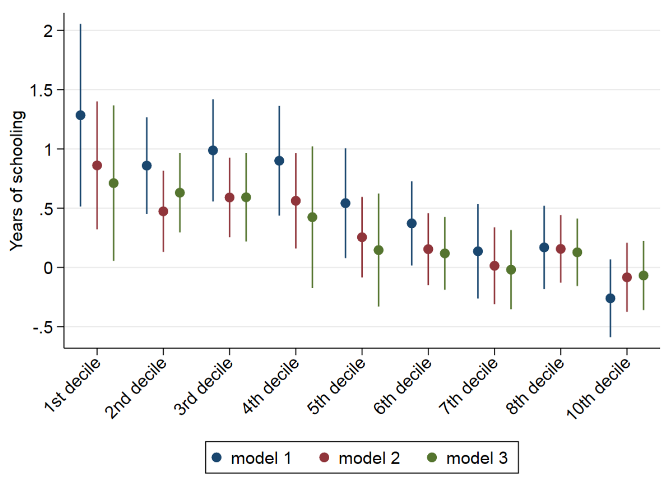
One may be concerned by the large number of observations with respect to the number of covariates included, which could artificially inflate the statistical significance of the coefficients. To address this issue, we perform a test inspired by Chabé-Ferret and Gobbi (2018). We collapse



(a) Coefficients on group size deciles in the fertility regression - Poisson
Source: Indonesian Census, waves 1971-2010



(b) Predicted count for group size deciles in the fertility regression - Poisson
Source: Indonesian Census, waves 1971-2010



(c) Coefficients on group size deciles in the education regression - OLS
Source: Indonesian Census, waves 1971-2010

our sample at the regency/religion/year level and therefore obtain much smaller samples of 6,243 and 7,583 observations respectively for the fertility and education regressions. The results presented in Appendix C show a very consistent picture with the baseline, with if anything a stronger magnitude in the education regression.[6]

3 Theory

We now analyze how fertilily and education are decided in a context where two groups compete to appropriate resources whose property rights are not perfectly defined.

3.1 A Model of Strategic Fertility and Education Choices

We consider an overlapping-generation economy in which individuals live for three periods. During the first period, children get some education from their parents. In the second period, adults work, support their elderly parents' consumption, choose fertility, and educate their children. During the third period, elderly agents consume what is provided by their adult children. The population consists of two groups a and b. All individuals belong to either group $i \in \{a, b\}$, and there is no possibility of changing group membership. Group i consists of N_t^i adults at time t. The utility of household $j = \{1, \cdots, N_t^i\}$ of group i at time t is given by

$$U_t^{ij} = c_t^{ij} + \beta d_{t+1}^{ij} - \frac{\lambda}{2}\left(n_t^{ij}\right)^2, \tag{1}$$

where c_t^{ij} (resp. d_{t+1}^{ij}) is the consumption of household j in group i in period t (resp. $t+1$), and $n_t^{ij} \in [0, \bar{n}]$ is the number of children (the upper bound on the number of children captures physiological constraints). The preference parameters are $\beta > 0$, the discount factor and $\lambda > 0$, the factor affecting the utility cost of child-rearing.

Each adult supports their elderly parents with a fraction τ of their income y_t^i, which is the same for all group members. The adult's budget constraint is:

$$c_t^{ij} = (1-\tau)y_t^i - \gamma\, n_t^{ij} e_t^{ij}, \tag{2}$$

where e_t^{ij} denotes the education spending per child, and $\gamma > 0$ is the cost of this spending. The

[6] Since fertility is no longer a count when the sample is collapsed at the regency/religion/year level, we perform this robustness check using OLS instead of Poisson and confirm that the magnitude of the effect is very much in line with the baseline results.

elderly adult's budget constraint is:

$$d_{t+1}^{ij} = \tau n_t^{ij}\, y_{t+1}^{i}. \tag{3}$$

It is thus proportional to the number of children they had, and to the income these children will have when adult.

Given the absence of Inada conditions, the following additional constraints must be imposed:

$$c_t^{ij},\ d_{t+1}^{ij} \geq 0. \tag{4}$$

For d_{t+1}^{ij}, this is automatically verified by (3). For c_t^{ij}, this condition imposes that the spending on the children not exceed the first period income. The latter is a predetermined variable. We will see later that the choices of n_t^{ij} and e_t^{ij} do not depend on y_t^i. Hence, the condition $c_t^{ij} \geq 0$ amounts to imposing a lower bound on the predetermined y_t^i. Note also that when the condition $c_t^{ij} \geq 0$ holds, we must have $e_t^{ij} < \big((1-\tau)y_t^i - c_t^{ij}\big)/\gamma n_t^{ij}$, that is e_t^{ij} is lower than some level $\bar{e}$ since $n_t^{ij} < \bar{n}$.

The human capital of household j of group i at time $t+1$ is positively affected by the spending in education at time t, e_t^{ij}:

$$h_t^{ij} = \left(e_t^{ij}\right)^{\rho}, \quad \rho \in [0,1]. \tag{5}$$

Households decide on their consumption path, while they are assumed to obey a social norm as far as fertility and education are concerned. Group i's norms are denoted by n_t^i and e_t^i; the following constraints are imposed on the household:

$$n_t^{ij} = n_t^i \quad \forall j = \{1, \cdots, N_t^i\}, \quad \forall i\{a, b\}, \tag{6}$$

$$e_t^{ij} = e_t^i \quad \forall j = \{1, \cdots, N_t^i\}, \quad \forall i\{a, b\}, \tag{7}$$

Plugging constraints (2), (3), (5), (6), and (7) into (1), the indirect utility of the representative household of group i is:

$$U_t^i = (1-\tau)y_t^i - \gamma\, n_t^i e_t^i + \beta\tau n_t^i\, y_{t+1}^i - \frac{\lambda}{2}\left(n_t^i\right)^2, \tag{8}$$

To determine future income, the way in which total output is produced and shared between the groups must be defined. The total production in each period t is an increasing function of total human capital, given by

$$Y_t = (h_t^a N_t^a + h_t^b N_t^b)^{(1-\alpha)}, \quad \alpha \in [0,1]. \tag{9}$$

Property rights are assumed not to be perfectly enforced, so that groups compete for the appropriation of produced wealth Y_t. Each group might claim the rights to a share of the wealth. Let us denote by Π_t^a group a's claim over total wealth. Individual income is thus given by:

$$y_t^a = \Pi_t^a \frac{Y_t}{N_t^a} \text{ and } y_t^b = (1 - \Pi_t^a) \frac{Y_t}{N_t^b} \tag{10}$$

In this setting, claims can only be settled by the threat of conflict. Following the literature on the economics of conflict (Garfinkel and Skaperdas 2007b), we model the conflict over Y_t as a "winner-take-all contest": Π_t^i is the probability that group i wins the fight over Y_t and is able to appropriate the whole resource. Departing from classical models where Π_t^i depends on capital through weapon production, we assume that a group's power increases with both the size of its population, N_t^i, and its level of human capital, h_t^i. More precisely, the power of group $i \in \{a, b\}$ is given by:

$$\Pi_t^a = \Pi^a(N_t^a, N_t^b, h_t^a, h_t^b) = \begin{cases} \frac{(h_t^a)^\mu N_t^a}{(h_t^a)^\mu N_t^a + \left(h_t^b\right)^\mu N_t^b}, & \mu \in [0,1], \quad h_t^i \neq 0 \text{ and } N_t^i \neq 0 \quad \forall i \in \{a,b\}, \\ \frac{N_t^a}{N_t^a + N_t^b}, & \text{if } h_t^i = 0 \text{ and } N_t^i \neq 0 \quad \forall i \in \{a,b\}, \\ \frac{(h_t^a)^\mu}{(h_t^a)^\mu + \left(h_t^b\right)^\mu}, & \text{if } h_t^i \neq 0 \text{ and } N_t^i = 0 \quad \forall i \in \{a,b\}, \\ \frac{1}{2}, & \text{if } h_t^i = 0 \text{ and } N_t^i = 0 \quad \forall i \in \{a,b\}, \end{cases} \tag{11}$$

which satisfies the different axioms stated in Skaperdas (1996).[7]

Finally, the evolution of the population in group $i \in \{a, b\}$ follows

$$N_{t+1}^i = n_t^i N_t^i.$$

The group decides on the norm regarding fertility and education before the bargaining begins. According to Bookman (2002), "An inter-ethnic war of numbers is taking place in numerous locations. The goal of this war of numbers is to increase the economic and political power of an ethnic group relative to other groups, and the method by which this is achieved entails the

[7] More precisely, note that we extend the power (or contest) function in the following way:

$$\Pi^a(0, 0, h^a, h^b) = \lim_{N^a = N^b \to 0} \Pi^a = \frac{(h_t^a)^\mu}{(h_t^a)^\mu + \left(h_t^b\right)^\mu},$$
$$\Pi^a(N^a, N^b, 0, 0) = \lim_{e^a = e^b \to 0} \Pi^a = \frac{N_t^a}{N_t^a + N_t^b},$$
$$\Pi^a(0, 0, 0, 0) = \lim_{N^a = N^b = h^a = h^b \to 0} \Pi^a = \frac{1}{2}.$$

increase in size of one population relative to the others. Most ethnic groups in multinational states across the globe are engaged in this activity in varying degree, ..."

One may wonder whether it is not too extreme to assume that groups control their members' fertility and education. In our framework, there is no division between the subjects who are involved in bargaining (the groups) and those who actually have children (all agents are perfectly equal within the groups). This holds if the number of children is in fact to some extent a social norm conveyed by religion.

Notice from Equations (12)-(13) that the parameters λ and γ can be normalized without loss of generality, so as to have

$$\beta\tau = 1.$$

The problem of the group is maximizing the utility of its representative member. We define $x_t = N_t^a/N_t$. Recognizing that y_t^i is predetermined, and using constraints (9), (10), and (11) into (8), the payoff function for group a is written

$$\begin{aligned} V_t^a(n_t^a, n_t^b, e_t^a, e_t^b, x_t) = \\ n_t^a \left(\frac{(e_t^a)^{\mu\rho}\, n_t^a x_t}{(e_t^a)^{\mu\rho}\, n_t^a x_t + \left(e_t^b\right)^{\mu\rho} n_t^b (1-x_t)} \right) \frac{\left((e_t^a)^{\rho}\, n_t^a x_t + \left(e_t^b\right)^{\rho} n_t^b (1-x_t)\right)^{(1-\alpha)}}{n_t^a x_t {N_t}^{\alpha}} \\ - \gamma\, n_t^a e_t^a - \frac{\lambda}{2} \left(n_t^a\right)^2, \end{aligned}$$

which can be simplified as:

$$V_t^a(n_t^a, n_t^b, e_t^a, e_t^b, x_t) = \frac{N^{-\alpha} n_t^a\ (e_t^a)^{\mu\rho}\ \left((e_t^a)^{\rho}\, n_t^a x_t + \left(e_t^b\right)^{\rho} n_t^b (1-x_t)\right)^{(1-\alpha)}}{(e_t^a)^{\mu\rho}\, n_t^a x_t + \left(e_t^b\right)^{\mu\rho} n_t^b (1-x_t)} - \gamma\, n_t^a e_t^a - \frac{\lambda}{2} \left(n_t^a\right)^2. \tag{12}$$

Similarly for group b:

$$V_t^b(n_t^b, n_t^a, e_t^b, e_t^a, 1-x_t) = \frac{N^{-\alpha} n_t^b\ \left(e_t^b\right)^{\mu\rho}\ \left((e_t^a)^{\rho}\, n_t^a x_t + \left(e_t^b\right)^{\rho} n_t^b (1-x_t)\right)^{(1-\alpha)}}{(e_t^a)^{\mu\rho}\, n_t^a x_t + \left(e_t^b\right)^{\mu\rho} n_t^b (1-x_t)} - \gamma\, n_t^b e_t^b - \frac{\lambda}{2} \left(n_t^b\right)^2. \tag{13}$$

The postive effect of fertility due to a higher number of children to support elderly parents is completely offset by the negative effect due to the decrease in per capita income. When choosing fertility, parents compare the benefit from both the increase in production and the rise in group power to the cost of raising children. The benevolent planner of group i solves

$$\max_{n_t^i \in [0,\bar{n}], e_t^i [0,\bar{e}]} V_t^i(n_t^i, n_t^{-i}, e_t^i, e_t^{-i}, x_t^i), \tag{14}$$

where $x_t^i = x_t$ when $i = a$ and $x_t^i = 1 - x_t$ when $i = b$.

Lemma 1 *Suppose that $\rho \leq \frac{1}{2}$. There exist some thresholds $\bar{\lambda}$, $\bar{\gamma}$ such that if $\lambda > \bar{\lambda}$ and $\gamma > \bar{\gamma}$, there exists a (best response) function $\mathbf{B}_{x_t} : [0,\bar{n}] \times [0,\bar{e}] \to [0,\bar{n}] \times [0,\bar{e}]$ given by*

$$\begin{aligned} \mathbf{B}_{x_t}(n_t^b, e_t^b) &= \arg\max_{n_t^a, e_t^a} V_t^a(n_t^a, n_t^b, e_t^a, e_t^b, x_t), \\ \mathbf{B}_{1-x_t}(n_t^a, e_t^a) &= \arg\max_{n_t^b, e_t^b} V_t^b(n_t^b, n_t^a, e_t^b, e_t^a, 1 - x_t). \end{aligned}$$

Proof. See Appendix D.1. ■

Utility maximisation takes place within a two-player game in which each group i simultaneously chooses their vector of strategy $(e_t^i, n_t^i) \in [0,\bar{n}] \times [0,\bar{e}]$ taking the other group's strategy (e_t^{-i}, n_t^{-i}) as given.

Definition 1 (Nash equilibrium of period t) *A pure-strategy Nash equilibrium of period t is a strategy profile $(n_t^{a\star}, n_t^{b\star}, e_t^{a\star}, e_t^{b\star}) = (n^a(x_t), n^b(x_t), e^a(x_t), e^b(x_t))$ with $n^i : [0,1] \to [0,\bar{n}]$ and $e^i : [0,1] \to [0,\bar{e}]$ such that for all $i \in \{a, b\}$,*

$$V_t^i(n_t^{i\star}, n_t^{-i\star}, e_t^{i\star}, e_t^{-i\star}, x_t^i) \geq V_t^i(n_t^i, n_t^{-i\star}, e_t^i, e_t^{-i}, x_t^i) \quad \forall (n_t^{i\star}, e_t^{i\star}) \in [0,\bar{n}] \times [0,\bar{e}].$$

A Nash equilibrium can also be defined in terms of the best response functions as a strategy profile $(n_t^{a\star}, n_t^{b\star}, e_t^{a\star}, e_t^{b\star}) = (n^a(x_t), n^b(x_t), e^a(x_t), e^b(x_t))$ such that

$$\begin{cases} \mathbf{B}_{x_t}(n_t^{b\star}, e_t^{b\star}) = \arg\max_{n_t^a, e_t^a} V_t^a(n_t^a, n_t^{b\star}, e_t^a, e_t^{b\star}, x_t), \\ \mathbf{B}_{1-x_t}(n_t^{a\star}, e_t^{a\star}) = \arg\max_{n_t^a, e_t^a} V_t^a(n_t^{a\star}, n_t^b, e_t^{a\star}, e_t^b, 1 - x_t). \end{cases}$$

Proposition 1 *For any $x_t \in [0,1]$, (i) there exists a pure-strategy Nash equilibrium, and (ii) the symmetric strategy profile $(n^a(x_t), n^a(1-x_t), e^a(x_t), e^a(1-x_t))$ is a pure-strategy Nash equilibrium.*

Proof. See Appendix D.2. ■

Note that the term symmetry is used to refer to a symmetry of strategic choices with respect to $x = 1/2$ (and not to an equality of choices at equilibrium).

3.2 Strategic Behavior and Groups' Demographic Weight

We first analyze how strategic fertility and education depend on the demographic weights of the two groups. We successively consider three cases. First, we solve a model in which only fertility decisions affect utility (i.e. $\gamma = 0$ and $\rho = 0$). Second, we examine a case in which education impacts the production function only (i.e. $\mu = 0$). Third, we consider the more general case in which education affects both the production and contest functions.

3.2.1 Case without Education ($\rho = 0$)

When $\rho = 0$, education has no effect on human capital. As education is costly, it is trivial to show that optimal education is always zero. Hence, the problem is simplified into a problem of fertility choice. For group a, resp. b, the previous program is written:

$$\max_{n_t^a} n_t^a \frac{(n_t^a x_t + n_t^b(1-x_t))^{(1-\alpha)}}{n_t^a x_t + n_t^b(1-x_t)} N_t^{-\alpha} - \frac{\lambda}{2}(n_t^a)^2$$
$$\max_{n_t^b} n_t^b \frac{(n_t^a x_t + n_t^b(1-x_t))^{(1-\alpha)}}{n_t^a x_t + n_t^b(1-x_t)} N_t^{-\alpha} - \frac{\lambda}{2}(n_t^b)^2$$

Each group chooses its fertility level n_t^i taking the other group's fertility as given. The first order conditions are:

$$N_t^{-\alpha}\left((1-\alpha)\frac{n_t^a x_t(n_t^a x_t + n_t^b(1-x_t))^{-\alpha}}{n_t^a x_t + n_t^b(1-x_t)} + \frac{n_t^b(1-x_t)(n_t^a x_t + n_t^b(1-x_t))^{(1-\alpha)}}{(n_t^a x_t + n_t^b(1-x_t))^2}\right) = \lambda n_t^a, \quad (15)$$

$$N_t^{-\alpha}\left((1-\alpha)\frac{n_t^b(1-x_t)(n_t^a x_t + n_t^b(1-x_t))^{-\alpha}}{n_t^a x_t + n_t^b(1-x_t)} + \frac{n_t^a x_t(n_t^a x_t + n_t^b(1-x_t))^{(1-\alpha)}}{(n_t^a x_t + n_t^b(1-x_t))^2}\right) = \lambda n_t^b. \quad (16)$$

Fertility positively impacts utility by increasing consumption in the second period of life through two channels, represented by the two positive terms on the left-hand side. First, it increases total production at time $t+1$. Second, it increases the power of the group. The marginal cost of fertility is on the right-hand side.

Given Proposition 1, this system of equations admits a solution. Our main result is now:

Proposition 2 (Minority-Group Status Result)
At the Nash equilibrium, the fertility of group $i = a, b$ is decreasing with the share of group i in the population.

Proof: See Appendix D.3.■

Our framework generates substitutability between fertility decisions and group shares. An increase in group share affects the benefit from fertility through several channels. First, a rise in group share positively impacts the benefit of increasing total output. Second, it increases the cost due to the decrease in output per capita. Third, the rise in group share increases the benefit associated to greater power. Since power depends on both groups' fertility, the other group responds to the decrease in its own population by changing its fertility choice. This strategic adaptation lowers the benefit associated to greater power so that the negative effect due to the decrease in output per capita outweighs the two positive effects (increase in total output and greater power), and incentives to higher fertility are reduced.

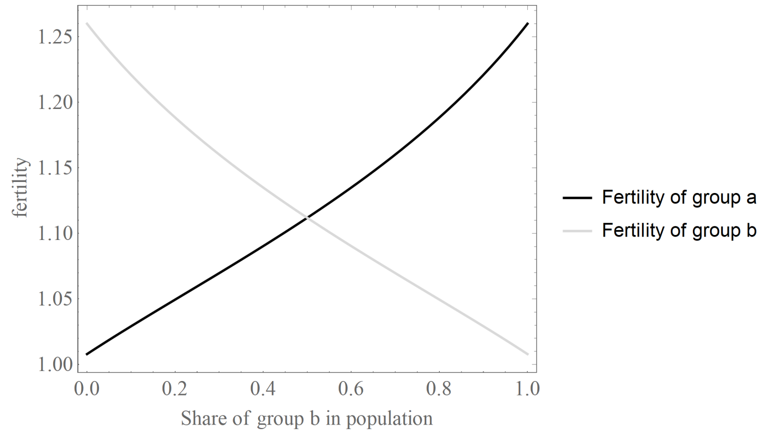
Examples of group policies aiming to encourage fertility as a response to a lower population share are given in Bookman (2002): "A passive pronatalist policy may simply entail urging people to procreate (such as the call on Jews worldwide by their religious leaders in response to falling birth rates), while an active policy may entail direct monetary compensation (as in Italy under Mussolini), financial stimuli (as in Singapore during the 1970s), and prohibition of birth control (as in Romania under Ceausescu). The Palestinian leadership spoke of their population's demographic weapon against the Israeli, counting on their high birth rates to alter the demographic balance in the Middle East. Indeed Chairman Arafat is said to have referred to the *weapon of the womb*."

A numerical example illustrates this result. Assume $\alpha = 0.8$, $\lambda = 0.25$. Figure 3 shows how the fertility of two groups varies when the share of group 2 goes from 0 to 1.

In the above result, we did not relate the cost of having children to the human capital of the parents. If, on the contrary, we assume that the opportunity cost of having children is higher for parents with high human capital, our parameter λ should be a positive function of y. It would follow that fertility should be decreasing in the human capital of the parents, in line with the regression results presented in Section 2.

3.2.2 Case with Education in the Production Function ($\rho > 0$, $\mu = 0$)

Assume now that agents can also invest in quality, i.e. education for their children. Education increases human capital (i.e. $\rho \neq 0$), which positively impacts aggregate production. The



Note: $\alpha = 0.8$, $\lambda = 0.25$, $\rho = 0$, $\gamma = 0.05$, $\mu = 0$

Figure 3: Fertility as a Function of Group Size

maximization program for the representative agent of group a is (for group b, the symmetric reasoning applies):

$$\max_{n_t^a, e_t^a} \left(\frac{n_t^a x_t}{n_t^a x_t + n_t^b(1-x_t)} \right) \frac{\left((e_t^a)^\rho n_t^a x_t + \left(e_t^b\right)^\rho n_t^b (1-x_t)\right)^{(1-\alpha)}}{x_t N_t^{\alpha}} \\ -\gamma\, n_t^a e_t^a - \frac{\lambda}{2} \left(n_t^a\right)^2,$$

The first-order conditions for group a are written

$$\frac{x_t}{n_t^a x_t + n_t^b(1-x_t)} \Pi_{t+1}^b N^{-\alpha} \frac{(n_t^a x_t (e_t^a)^\rho + n_t^b(1-x_t))(e_t^b)^\rho)^{(1-\alpha)}}{x_t} \\ + (1-\alpha)\Pi_{t+1}^a N^{-\alpha} (n_t^a x_t (e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho)^{-\alpha} (e_t^a)^\rho = \gamma e_t^a + \lambda n_t^a, \quad (17)$$

$$\rho \Pi_{t+1}^a N^{-\alpha} (1-\alpha)(n_t^a x_t (e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho)^{-\alpha} n_t^a e_t^{a\rho-1} = \gamma n_t^a, \quad (18)$$

Similar conditions apply, *mutatis mutandis*, for group b. As in the previous case, fertility positively impacts utility by both increasing total production and increasing power. Education only affects production. The properties of the Nash equilibrium can now be stated.

Proposition 3 (Quality-quantity trade-off) *Suppose that $\rho \leq \frac{1}{2}$. At the Nash equilibrium, (i) education of group i is increasing with the share of group i in the population, and (ii) there exists a threshold $\tilde{x} \in \mathbb{R}^+$ such that $\forall x_t < \tilde{x}$, the fertility of group a (resp. b) is decreasing (resp. increasing) with the share of group a.*

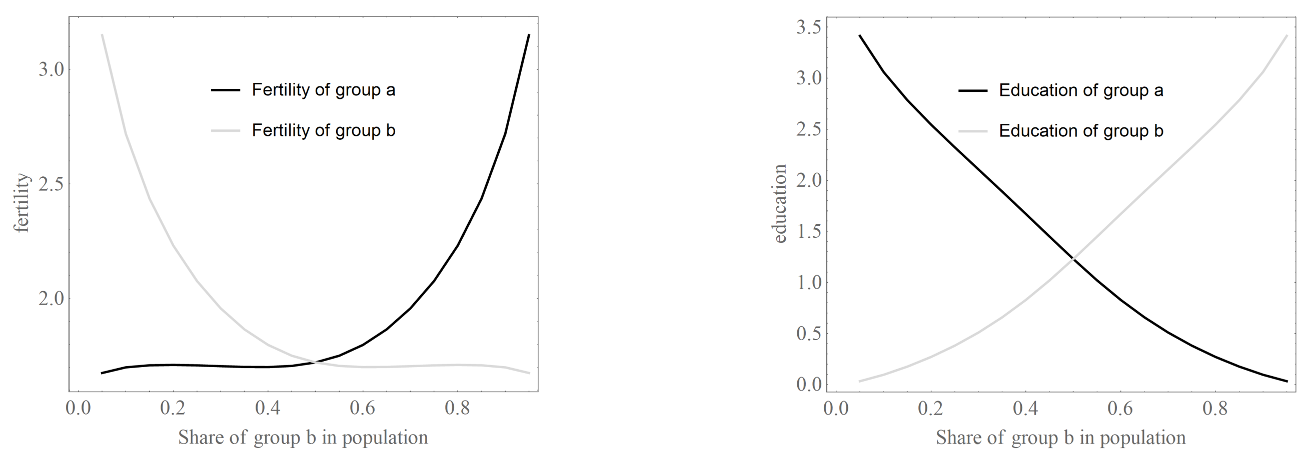
Proof: See Appendix D.4.■

Education of group i is increasing with the share of group i in the population, i.e. education and group share are complements. Contrary to fertility, education affects utility by increasing total output only. An increase in group share affects the benefit of increasing total output by (i) increasing the number of workers who benefit from education, (ii) increasing the power of the group, and (iii) decreasing output per capita. The first two effects always outweigh the third one so that a rise in group share positively affects incentives to educate children.

Proposition 3 also states that when the share of group i is low enough, type-i agents' fertility is decreasing with the share of group i. There is a quality-quantity trade-off.

As in Section 3.2.1, a rise in the population share affects the marginal benefit of fertility through different forces. It increases the benefit of rising total output and of greater power, while also increasing the cost due to lower income per capita. Yet, unlike Section 3.2.1, the benefit of rising output is affected by the level of education. When the population share is low (i.e. lower than the threshold $\tilde{x}$) and since education and the population share are complements, education is low, meaning that the benefit of rising output is low. Also, due to the adaptation of the other group in response to a change in their own population share, the benefit from greater power is low. Hence, the negative effect of the increase in population share due to the decrease in output per capita outweighs the two other positive effects, and the marginal benefit from fertility decreases.

A numerical example illustrates this result. Assume $\alpha = 0.8$, $\lambda = 0.25$, $\rho = 0.5$, and $\gamma = 0.1$. Figure 4 shows how the fertility and education of the two groups vary when the share of group b goes from 0 to 1.



Note: $\alpha = 0.8$, $\lambda = 0.25$, $\rho = 0.5$, $\gamma = 0.1$, $\mu = 0$

Figure 4: Fertility and Education as a Function of Group Size when Education Enters the Production Function ($\rho > 0$)

3.2.3 Case with Education in the Contest Function ($\rho > 0,\ \mu > 0$)

We now consider the case in which education also affects the contest function, i.e. when $\mu > 0$. The maximization program is written

$$\max_{n_t^a, e_t^a} n_t^a (e_t^a)^{\rho\mu} \frac{(n_t^a x_t (e_t^a)^\rho + n_t^b (1-x_t)(e_t^b)^\rho)^{(1-\alpha)}}{n_t^a x_t (e_t^a)^{\rho\mu} + n_t^b (1-x_t)(e_t^b)^{\rho\mu}} N_t^{-\alpha} - \gamma n_t^a e_t^a - \frac{\lambda}{2}(n_t^a)^2$$

The first-order conditions for group a are given by

$$\begin{aligned}(e_t^a)^{\rho\mu} \Pi_{t+1}^b N_t^{-\alpha} & \frac{\left(n_t^a x_t (e_t^a)^\rho + n_t^b (1-x_t)(e_t^b)^\rho\right)^{(1-\alpha)}}{n_t^a x_t (e_t^a)^{\rho\mu} + n_t^b (1-x_t)(e_t^b)^{\rho\mu}} \\ & + \Pi_{t+1}^a N_t^{-\alpha}(1-\alpha) \left(n_t^a x_t (e_t^a)^\rho + n_t^b (1-x_t)(e_t^b)^\rho\right)^{-\alpha} (e_t^a)^\rho = \gamma e_t^a + \lambda n_t^a\end{aligned}$$

$$\begin{aligned}\rho\mu (e_t^a)^{\rho\mu-1} n_t^a x_t \Pi_{t+1}^b N_t^{-\alpha} & \frac{\left(n_t^a x_t (e_t^a)^\rho + n_t^b (1-x_t)(e_t^b)^\rho\right)^{(1-\alpha)}}{n_t^a x_t (e_t^a)^{\rho\mu} + n_t^b (1-x_t)(e_t^b)^{\rho\mu}} \\ & + \rho \Pi_{t+1}^a (1-\alpha) N_t^{-\alpha} \left(n_t^a x_t (e_t^a)^\rho + n_t^b (1-x_t)(e_t^b)^\rho\right)^{-\alpha} n_t^a (e_t^a)^{\rho-1} = \gamma n_t^a\end{aligned}$$

Similar conditions apply, *mutatis mutandis*, for group b.

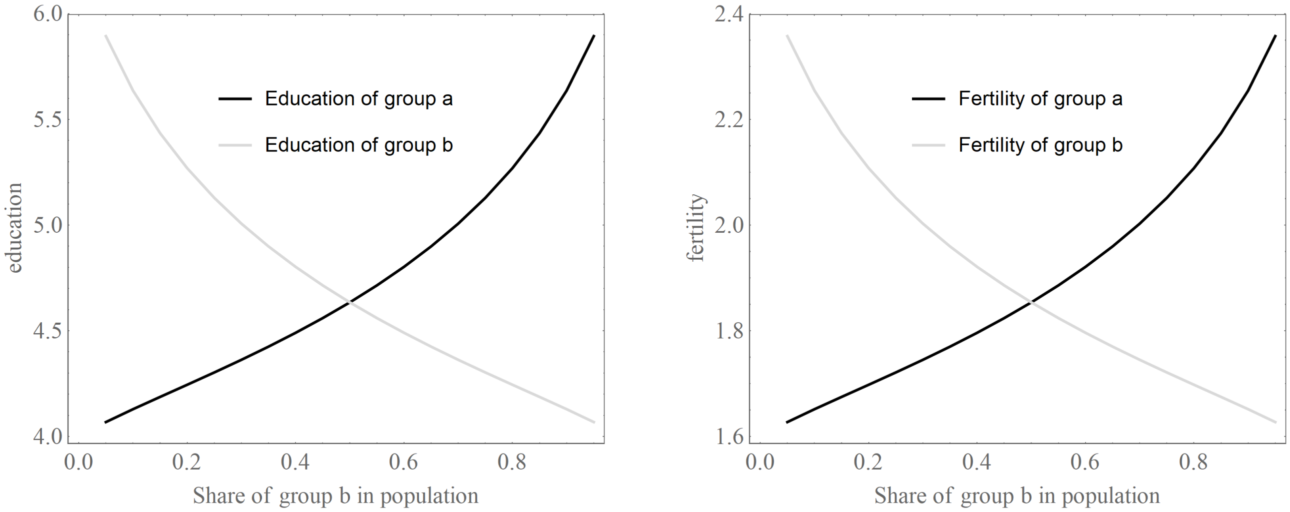
With power depending on education, the model becomes more involved. We can however still determine the properties of the equilibrium in the case where $\mu = 1$. The usual negative correlation between fertility and education does not hold anymore.

Proposition 4 (Reversed quality-quantity trade-off)
For $\mu = 1$, both the fertility and education of group i are decreasing with the share of group i in the population at the Nash equilibrium.

Proof. See Appendix D.5. ■

When human capital has a strong positive impact on power, the result of the previous case is reversed. Although fertility still increases with the size of the competing group, education is not affected negatively. On the contrary, it is increasing too, and appears complementary to fertility in increasing power.

A numerical example illustrates this result. Assume $\alpha = 0.8$, $\lambda = 0.25$, $\rho = 0.5$, $\gamma = 0.1$, and $\mu = 1$. Figure 5 shows how the fertility and education of the two groups vary when the share of group b goes from 0 to 1.



Note: $\alpha = 0.8$, $\lambda = 0.25$, $\beta\tau = 0.8$, $\rho = 0.5$, $\gamma = 0.1$, $\mu = 1$

Figure 5: Fertility and Education as a Function of Group Size when Education Enters the Contest Function ($\mu > 0$)

Propositions 3 and 4 seem to imply that there is a threshold value of μ, say $\bar{\mu}$, above which the quantity-quality trade-off does not hold anymore. Although we cannot prove it formally, since the Nash equilibrium cannot be computed for $\mu \in (0, 1)$, this is what we find in the numerical example as reported in Appendix E.

At this stage of the paper, we have learned that if human capital has a sufficiently strong effect on the probability of winning a conflict, smaller groups might invest in both quantity and quality relatively more. This is true at the pure-strategy Nash equilibrium. It implies that in contexts where the appropriation technology does not depend too heavily on human capital, the usual quality-quantity trade-off is observed, while this is no longer the case when human capital matters. An example of the latter can be found when military technology requires that high-skilled soldiers use high-tech weapons. In a more peaceful context, one can think of civil servants who control the bureaucracy and are nominated based on exams.

3.3 The Dynamics of Group Size and Education

In the case without education, and in the case where human capital has a strong effect on political power, there is a negative relationship between the share of each group and its fertility, which implies that population dynamics converge towards $N_t^a = N_t^b$.

If, in the long-run, fertility is larger than one, the population grows unboundedly. This poses a technical difficulty: given the decreasing returns to labor assumed in (9), it implies that the income per person tends towards zero. This leads to the violation of the positivity constraint on consumption (4) at some point in time. This problem can however be easily solved by assuming strong enough deterministic technical progress in the production function.

3.4 The Cost of Conflict

We now analyze the impact of conflict on economic outcomes, i.e. fertility, education, and income by comparing these variables in two different frameworks: the *anarchic* environment and the *peaceful* environment. The *anarchic* environment corresponds to the situation considered so far in which aggregate resources are subject to conflict and the probability of winning is endogenous. The *peaceful* environment is defined as a situation in which parties have successfully found an agreement in which the resources are divided between the two groups depending on their share in the total population in the previous period. It is as if the power of group $i \in \{a, b\}$ were given by

$$\bar{\Pi}_t^a = \frac{N_{t-1}^a}{N_{t-1}^a + N_{t-1}^b}, \tag{19}$$

which is predetermined at time t. Note that the game in the peaceful environment is similar to a pure public good game in which agents choose their contribution, taking other agents' contribution as given.

In the classical literature on conflict (Garfinkel and Skaperdas 2007a), the anarchic environment involves some costs: it decreases aggregate resources since it implies a trade-off between increasing aggregate resources and producing guns for appropriation. In the present framework, there is no such trade-off. Fertility both increases power and production. Nevertheless, conflict can still generate some costs by negatively affecting income per capita. First, a larger population positively impacts total resources, but reduces the share of the cake. Second, a larger population may negatively affect total income by decreasing education.

Let us determine economic choices under the peaceful situation. Consider the general case in which $\mu > 0$ and $\rho > 0$. An agent of type i chooses n_t^i, e_t^i solving

$$\max_{n_t^i, e_t^i} n_t^i \bar{\Pi}_{t+1}^i N_t^{-\alpha} \frac{(n_t^i x_t (e_t^i)^\rho + n_t^j (1 - x_t)(e_t^j)^\rho)^{(1-\alpha)}}{n_t^i x_t} - \gamma n_t^i e_t^i - \frac{\lambda}{2}(n_t^i)^2.$$

The first-order conditions are given by

$$\Pi_{t+1}^i N_t^{-\alpha}(1-\alpha) \frac{\left(n_t^i x_t (e_t^i)^\rho + n_t^j (1 - x_t)(e_t^j)^\rho\right)^{-\alpha}}{x_t} (e_t^i)^\rho x_t = \gamma e_t^i + \lambda n_t^i, \tag{20}$$

and

$$\rho \Pi_{t+1}^{i}(1-\alpha)N_t^{-\alpha}\frac{\left(n_t^i x_t (e_t^i)^{\rho} + n_t^j (1-x_t)(e_t^j)^{\rho}\right)^{-\alpha}}{x_t} n_t^i (e_t^i)^{\rho-1} x_t = \gamma n_t^i. \tag{21}$$

The Nash equilibrium of this public good game exists:

Proposition 5 (Nash Equilibrium in a peaceful environment) *For any $x_t \in [0,1]$, (i) there exists a pure-strategy Nash equilibrium, and (ii) the symmetric strategy profile $(n^a(x_t), n^a(1-x_t), e^a(x_t), e^a(1-x_t))$ is a pure-strategy Nash equilibrium.*

Proof. The proof relies on the same arguments as those used in the proof of Proposition 1. ■

Theoretical predictions on the peaceful vs anarchic environment can now be derived in the case of a symmetry between the two groups.

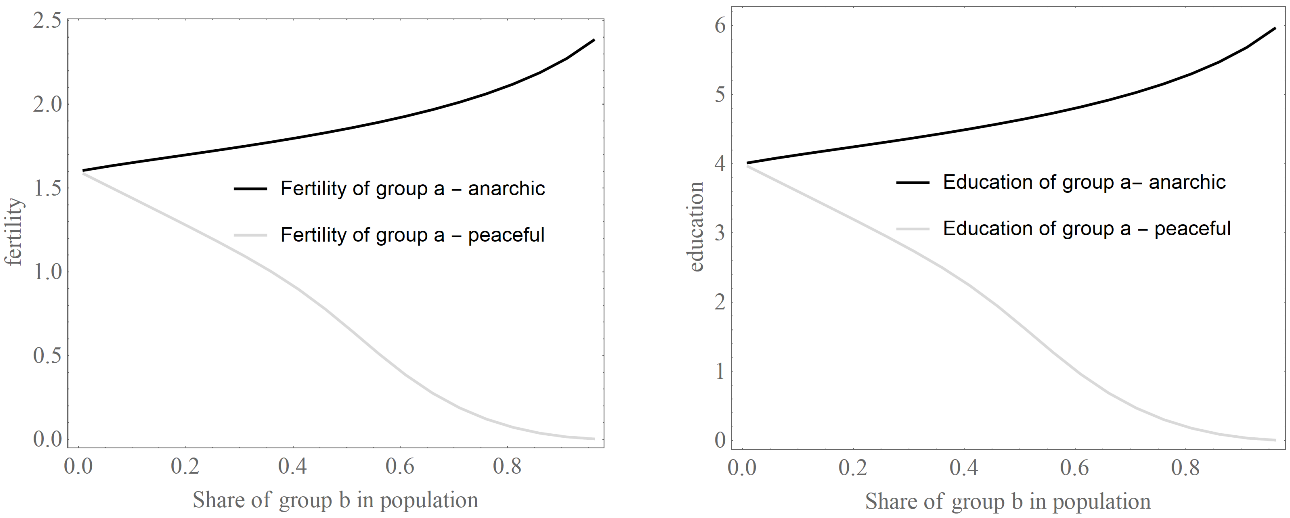
Proposition 6 (Fertility, education, and the cost of conflict) *Suppose that $N_t^a = N_t^b$.*
(i) Case $\rho = 0$, $\mu = 0$. In the anarchic environment, fertility is higher and income per capita is lower.
(ii) Case $\rho > 0$, $\mu = 1$. Both fertility and education are higher in the anarchic environment than in the peaceful environment. Income per capita in $t+1$ is higher in the anarchic environment if and only if

$$\rho > \alpha/(1-\alpha).$$

In both cases (i) and (ii), in an anarchic environment, agents have higher incentives for fertility since fertility positively impacts appropriation in such an environment. In case (i), where education does not impact power, the rise in fertility negatively affects income per capita. In this case, due to decreasing returns to labor, the population increase outweighs the rise in total production.

In addition, when education also positively impacts power, compared to the peaceful environment, the anarchic environment creates more incentives to educate children since it affects the share of total resources which agents appropriate.

Figure 6 displays the fertility and education of group a as a function of the share of group b in the population in the anarchic and peaceful environments. Contrary to the case of the anarchic environment, in the peaceful environment, fertility increases with the share of one's group in the population. There are complementarities between fertility choice and group share. An increase in group share affects the benefit from fertility through the same channels: (i) it



Note: $\alpha = 0.8$, $\lambda = 0.25$, $\rho = 0.5$, $\gamma = 0.1$, $\mu = 1$

Figure 6: Fertility and Education: Comparison between Anarchic and Peaceful Environments

positively impacts the benefit of increasing total output, (ii) it increases the cost due to the decrease in output per capita, and (iii) it increases the benefit associated to greater power. Yet, contrary to what happens in the anarchic environment, the other players cannot respond to a change in the population share. Therefore, the positive effect that the rise in population share has through greater power is high, so that the two positive effects outweigh the negative one. Hence, the rise in the population share increases the marginal benefit from fertility. One can also see that while the education of group a increases with the share of group b in the anarchic environment, it decreases with the share of that group in the peaceful environment.

In both types of environments, the quality-quantity trade-off disappears. In the first case, as shown in Proposition 4, it is due to a strong impact of education on power (i.e. $\mu = 1$). In the second case, the quality-quantity trade-off breaks due to complementarities: as the share of group i increases, the share of resources that an agent of type i appropriates increases, and the incentives to increase resources by investing in children's education are greater. Finally, the increase in both education and fertility due to the anarchic environment has an ambiguous impact on income per capita. There are two opposite effects. As stated above, the rise in fertility negatively impacts income per capita because the increase in total production is outweighed by the rise in population, so that the share of resources which each agent receives decreases. However, the rise in education unambiguously increases production per capita. When the impact of education on human capital is high compared to decreasing returns in production, i.e. $\rho > \alpha/(1-\alpha)$, the positive effect of education on output dominates the dilution effect of higher fertility, and income per capita increases.

4 Conclusion

When social norms were introduced as determinants of individual fertility, the workhorse model of the economics of fertility gained a new dimension. Still, these norms were considered exogenous in most cases. In this paper, we go one step further, modelling the idea that the social norm itself may respond to an economic logic, one of competition between groups. In addition to endogenizing fertility norms, we also take into account the education choice made by parents, and acknowledge that this choice might also be subject to endogenous norms.

We have modelled a deeply divided society in which two groups compete for resources, assuming that property rights are not fully defined and/or enforced. We have analyzed whether a "population race" between the two groups would translate into lower education, as would be predicted by the standard quality-quantity trade-off model. We have shown that when human capital enters the appropriation function as an important enough factor, the smaller group will invest more in both fertility and education than the majority group.

The paper has also shown that the predictions of the theory on how fertility and education are correlated with group size can be found in Indonesian censuses. We have shown that, at the regency level, smaller religious groups tend to have more children, but also tend to invest more in their education. The correlations between group size and outcomes are not small. For fertility, the mothers who belong to a median-size group, compared to all regencies, have a predicted net fertility of 3.6 surviving children. Switching to women belonging to smaller groups (decile 2 of the distribution of group shares), fertility reaches about 4 children per women. For education, the women in decile 2 invest more in education by half a year compared to the median.

Our results have implications for thinking about how institutions, demographics, and development interact. If institutions were of the highest quality, there would be no need for groups to enhance their investment in children to obtain a larger share of future resources, and the standard of living of current generations would likely be higher.

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A Descriptive Statistics

In Table 3, we look at how fertility and socio demographic characteristics vary with group size in the raw data. Group size is not normally distributed with a median of 0.96, which is very close to the maximum value of 1 and a long left tail. Before starting to include controls, there actually exists a U-shaped relationship between fertility and group size, with both the first and fourth quartiles having about 0.2 more children. The same pattern persists, though not as marked, for surviving children. Looking at how quartiles differ, it appears that individuals belonging to larger groups tend to be less urban, less educated and less mobile. Turning to the education sample, a larger group size is associated with a lower educational attainment and also a less urban and less mobile population.

Table 3: Summary statistics, by quartile of religious group size

	1st quartile	2nd quartile	3rd quartile	4st quartile
Fertility sample				
Share of own religious group in regency	0.58	0.94	0.98	0.99
	(0.02)	(0.00)	(0.00)	(0.00)
Children ever born	4.30	4.11	4.09	4.27
	(0.09)	(0.08)	(0.09)	(0.10)
Children surviving	3.84	3.64	3.54	3.60
	(0.08)	(0.06)	(0.07)	(0.07)
Currently married	0.80	0.80	0.79	0.78
	(0.00)	(0.00)	(0.01)	(0.01)
Age	50.76	50.69	50.80	50.76
	(0.03)	(0.04)	(0.03)	(0.03)
Urban status	0.57	0.51	0.32	0.27
	(0.05)	(0.05)	(0.03)	(0.02)
Years of schooling	6.05	5.15	4.15	3.74
	(0.20)	(0.24)	(0.18)	(0.18)
Share of native population	0.79	0.81	0.95	0.98
	(0.03)	(0.03)	(0.01)	(0.00)
Number of observations	757,266	767,145	748,504	754,687
Education sample				
Share of own religious group in regency	0.55	0.92	0.98	0.99
	(0.02)	(0.00)	(0.00)	(0.00)
Years of schooling	9.06	9.10	7.66	7.03
	(0.24)	(0.19)	(0.14)	(0.11)
Share of female	0.50	0.50	0.50	0.51
	(0.00)	(0.00)	(0.00)	(0.00)
Age	28.93	28.94	28.97	28.99
	(0.01)	(0.01)	(0.01)	(0.00)
Urban status	0.59	0.65	0.35	0.28
	(0.05)	(0.05)	(0.04)	(0.02)
Share of native population	0.75	0.77	0.91	0.97
	(0.03)	(0.03)	(0.01)	(0.00)
Number of observations	1,612,798	1,584,136	1,604,849	1,583,837

B Full Regression Tables

The effect of education on fertility is hump-shaped (Vogl 2016b, Baudin, de la Croix, and Gobbi 2018): women with some or completed primary education have more children than those with less than primary or no schooling (reference category). Then, having more than primary is correlated with fewer children. Not surprisingly, being single or never married is correlated with lower fertility. Higher educational attainment in the regency goes with higher fertility, possibly reflecting a positive income effect. A higher child mortality in the regency correlates with more surviving children per individual woman (See (Angeles 2010; Lawson, Alvergne, and Gibson 2012)). In terms of religious affiliation, Buddhists, Hindus and non religious people tend to have fewer children than Muslims, Christians and Confucianists.

In the education regression, we retrieve as well classic results from the literature. Women get over half a year of schooling less than men and schooling is much more advanced in urban areas. Non religious and Confucianists are lagging behind other religions. Finally child mortality correlates with low educational attainment.[8]

[8] See for instance Pamuk, Fuchs, and Lutz (2011, Breierova and Duflo (2004, Chou et al. (2010)

Table 4: Fertility - Poisson Regression Results - Full Table

The dependent variable is the number of surviving children	(1)	(2)	(3)	(4)
Share of own religious group in regency				
(baseline: 9th decile)				
1st decile	0.081***	0.097**	0.105***	0.119***
	(0.029)	(0.042)	(0.039)	(0.042)
2nd decile	0.136***	0.141***	0.141***	0.155***
	(0.035)	(0.032)	(0.030)	(0.030)
3rd decile	0.103***	0.113***	0.116***	0.117***
	(0.038)	(0.039)	(0.037)	(0.041)
4th decile	0.071**	0.089**	0.090***	0.095***
	(0.033)	(0.035)	(0.032)	(0.036)
5th decile	0.038	0.042	0.055	0.040
	(0.041)	(0.042)	(0.037)	(0.040)
6th decile	0.032	0.035	0.047	0.040
	(0.034)	(0.034)	(0.031)	(0.030)
7th decile	0.016	0.014	0.019	0.016
	(0.036)	(0.035)	(0.033)	(0.033)
8th decile	0.020	0.020	0.019	0.019
	(0.034)	(0.033)	(0.030)	(0.030)
10th decile	0.046	0.047	0.034	0.036
	(0.035)	(0.034)	(0.032)	(0.032)
Urban	-0.033***	-0.014	-0.010	-0.003
(baseline: non-urban)	(0.012)	(0.012)	(0.011)	(0.011)
Less than primary		-0.032***	-0.026***	-0.021***
(baseline: no schooling)		(0.007)	(0.007)	(0.006)
Some primary		0.084***	0.078***	0.079***
		(0.005)	(0.005)	(0.006)
Primary completed		0.038***	0.027***	0.026***
		(0.007)	(0.007)	(0.007)
Lower secondary (general)		0.007	-0.008	-0.008
		(0.008)	(0.008)	(0.009)
Lower secondary (technical)		0.237***	0.224***	0.226***
		(0.015)	(0.014)	(0.017)
Secondary (general) completed		-0.115***	-0.135***	-0.130***
		(0.010)	(0.010)	(0.010)
Some college		-0.139***	-0.137***	-0.110**
		(0.037)	(0.035)	(0.054)
Secondary (technical) completed		-0.070***	-0.085***	-0.087***
		(0.011)	(0.011)	(0.012)
Post secondary (technical)		-0.177***	-0.199***	-0.179***
		(0.013)	(0.013)	(0.011)
University completed		-0.232***	-0.256***	-0.240***
		(0.012)	(0.011)	(0.011)

Table 5: Fertility - Poisson Regression Results - Full Table (cont'd)

No religion		-0.207***	-0.221***	-0.239***
(baseline: Muslim)		(0.058)	(0.057)	(0.064)
Buddhist		-0.033	-0.044	-0.050
		(0.039)	(0.038)	(0.045)
Hindu		-0.162***	-0.162***	-0.173***
		(0.037)	(0.035)	(0.042)
Catholic		-0.001	-0.008	-0.002
		(0.035)	(0.033)	(0.037)
Protestant		0.028	0.023	0.014
		(0.041)	(0.039)	(0.042)
Confucianist		0.082**	0.071*	0.050
		(0.037)	(0.037)	(0.041)
Mean years of schooling in the regency		-0.000	0.016*	0.020**
		(0.008)	(0.009)	(0.009)
Married/in union			0.597***	0.664***
(baseline: single/never married)			(0.127)	(0.132)
Separated/divorced/spouse absent			0.243*	0.301**
			(0.126)	(0.132)
Widowed			0.488***	0.551***
			(0.127)	(0.132)
Unknown/missing marital status			-0.210	-0.144
			(0.128)	(0.132)
Child mortality			1.866***	1.931***
			(0.278)	(0.290)
Year of birth dummies	YES	YES	YES	YES
Census year dummies	YES	YES	YES	YES
Migrants included	YES	YES	YES	NO
Observations	3,027,999	3,027,999	3,027,999	2,671,908
Pseudo-R^2	0.012	0.015	0.021	0.020

Note: *p<0.1; **p<0.05; ***p<0.01. All standard errors are clustered at the regency level.

Table 6: Education - OLS Regression Results - Full Table

	The dependent variable is the number of years of schooling		
	(1)	(2)	(3)
Share of own religious group in regency *(baseline: 9th decile)*			
1st decile	1.284***	0.861***	0.711**
	(0.391)	(0.274)	(0.333)
2nd decile	0.859***	0.474***	0.631***
	(0.207)	(0.174)	(0.170)
3rd decile	0.988***	0.590***	0.592***
	(0.219)	(0.170)	(0.190)
4th decile	0.900***	0.562***	0.424
	(0.235)	(0.204)	(0.303)
5th decile	0.542**	0.255	0.147
	(0.235)	(0.173)	(0.242)
6th decile	0.372**	0.154	0.119
	(0.180)	(0.154)	(0.156)
7th decile	0.137	0.014	-0.019
	(0.202)	(0.165)	(0.170)
8th decile	0.169	0.157	0.128
	(0.178)	(0.145)	(0.145)
10th decile	-0.260	-0.083	-0.068
	(0.167)	(0.148)	(0.148)
Female *(baseline: male)*	-0.613***	-0.613***	-0.597***
	(0.023)	(0.023)	(0.026)
Urban *(baseline: non-urban)*	2.664***	2.518***	2.381***
	(0.118)	(0.106)	(0.105)
No religion *(baseline: Muslim)*		-2.533***	-3.059***
		(0.521)	(0.611)
Buddhist		-0.004	-0.004
		(0.307)	(0.354)
Hindu		0.067	0.069
		(0.261)	(0.260)
Catholic		0.142	-0.051
		(0.263)	(0.249)
Protestant		0.378	0.134
		(0.405)	(0.445)
Confucianist		-2.151***	-2.122***
		(0.292)	(0.332)
Child mortality		-19.501***	-20.413***
		(1.888)	(2.089)
Year of birth dummies	YES	YES	YES
Survey year dummies	YES	YES	YES
Migrants included	YES	YES	NO
Observations	6,385,620	6,385,028	5,429,536
R^2	0.259	0.269	0.259

Note: *p<0.1; **p<0.05; ***p<0.01. All standard errors are clustered at the regency level.

C Robustness

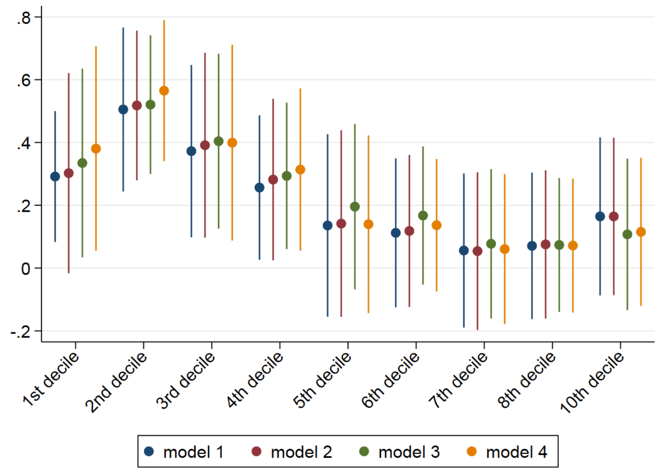
Fig 7a repeats the baseline fertility regressions using OLS rather than Poisson. It confirms that the results are not affected by this choice, as both significance and magnitude of the effect are kept intact.

Fig 7b and 7c replicate the baseline Poisson regressions, showing respectively coefficients and predicted counts, but using children ever born instead of surviving children as dependent variable. Again, results remain very similar, with the exception that predicted counts are shifted upwards as the number of children ever born is always higher that that of surviving children.

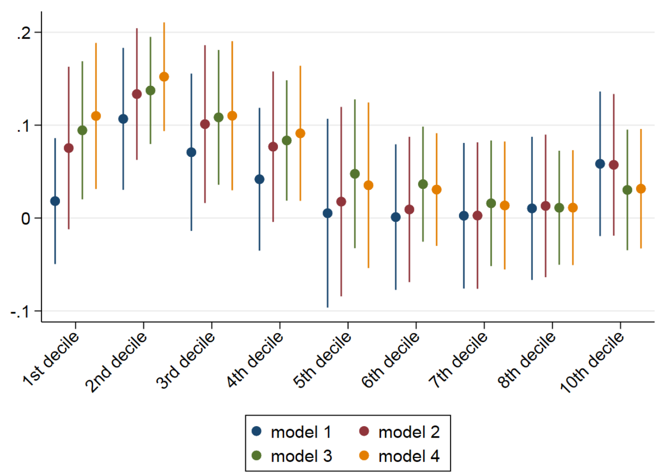
Fig 8a and 8b replicate the main education regression but changing the age groups we include, focusing respectively on 18-25 and 12-17 years old. The picture remains qualitatively the same for the older of the two groups, with lower significance though. There is virtually no relation between educational attainment and group size when looking at an even younger age. Indeed, years of schooling is a poor measure of investment in education for children age 12 to 17 in a country where education is compulsory until 16.

Fig 9a, 9b and 9c replicate the main specifications (except the last column excluding migrants) on a collapsed sample where the unit of observation is the religion group in a given regency in a given year. All covariates for these new observations are computed as the average of the individual characteristics comprised in those cells. We weight each new observation by the number of individuals in the cell. Results are very similar in the fertility regression, even though coefficients for lower deciles in model 1 are quite substantially higher than those in models 2 and 3, which was not the case in the baseline. This is probably due to controlling for average birth year rather than having dummies for each birth year in the baseline, which proves to be more flexible. This issue disappears when we include a more complete set of controls.

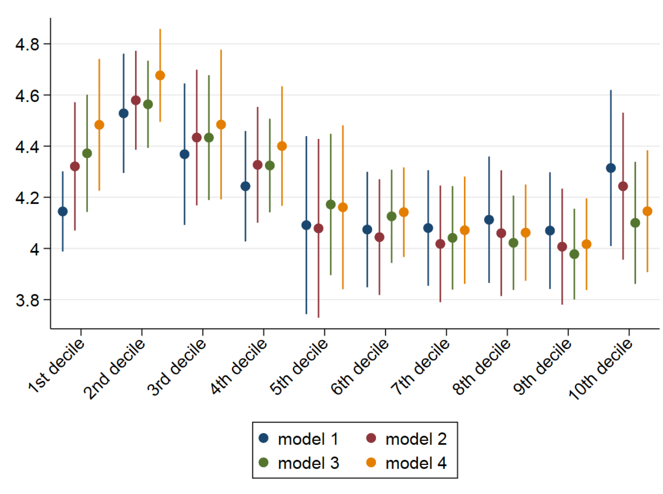
In the education regression, the general pattern is also similar, although we loose significance at the 5% level above the first decile.



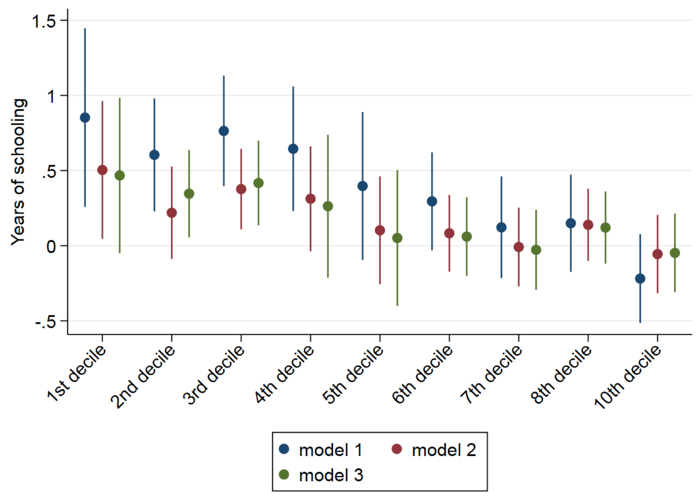
(a) Coefficients on deciles of group size in the fertility regression - OLS
Source: Indonesian Census, waves 1971-2010



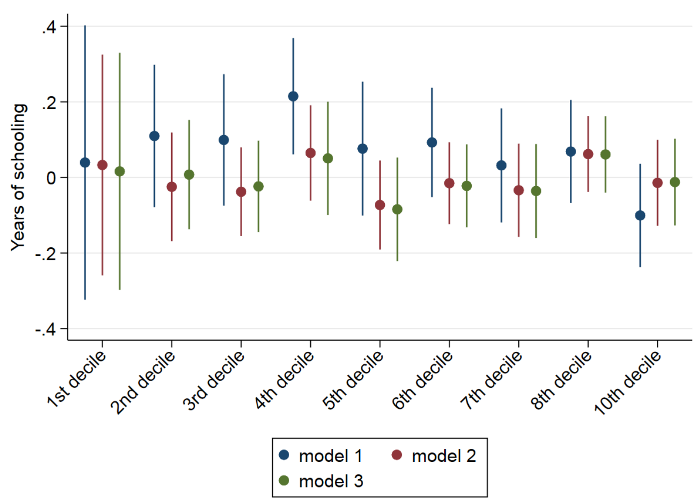
(b) Coefficients on deciles of group size in the fertility regression - Children ever born
Source: Indonesian Census, waves 1971-2010



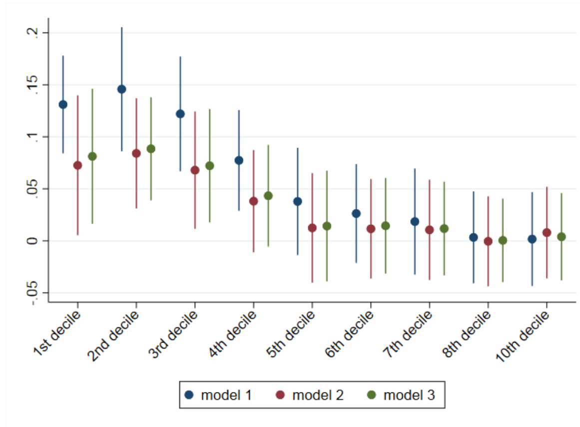
(c) Predicted count for deciles of group size in the fertility regression - Children ever born
Source: Indonesian Census, waves 1971-2010



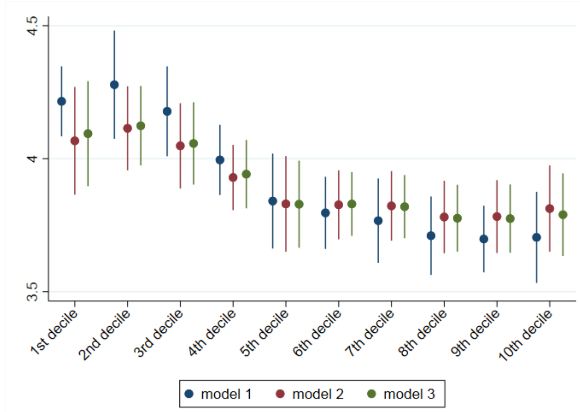
(a) Coefficients on deciles of group size in the education regression - 18-25 years old
Source: Indonesian Census, waves 1971-2010



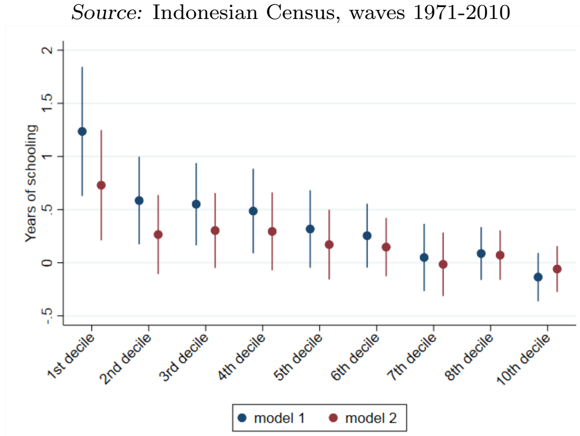
(b) Coefficients on deciles of group size in the education regression - 12-17 years old
Source: Indonesian Census, waves 1971-2010



(a) Coefficients on deciles of group size in the fertility regression - Collapsed sample
Source: Indonesian Census, waves 1971-2010



(b) Predicted count for deciles of group size in the fertility regression - Collapsed sample
Source: Indonesian Census, waves 1971-2010



(c) Coefficients on deciles of group size in the education regression - Collapsed sample
Source: Indonesian Census, waves 1971-2010

D Proofs of Propositions

D.1 Proof of Lemma 1

Lemma 1 is equivalent to say that for any given $(n_t^b, e_t^b, x_t) \in [0, \bar{n}] \times [0, \bar{e}] \times [0, 1]$, the function $V_t^a(n_t^a, n_t^b, e_t^a, e_t^b, x_t)$ admits a global maximum $(\hat{n_t^a}, \hat{e_t^a})$.

To show this for group a (then the symmetric reasoning applies for group b) we proceed in two steps:

Step 1: We show that

$$\begin{aligned}
&\forall e_t^a \in [0, \bar{e}], \exists! \tilde{n^a}_t \quad \text{such that} \quad \frac{\partial V_t^a}{\partial n_t^a}|_{n_t^a = \tilde{n^a}_t} = 0, \quad \text{with} \quad \frac{\partial V_t^{a2}}{\partial^2 n_t^a}|_{\tilde{n^a}_t} < 0, \\
&\forall n_t^a \in [0, \bar{n}], \exists! \tilde{e^a}_t \quad \text{such that} \quad \frac{\partial V_t^a}{\partial e_t^a}|_{n_t^a = \tilde{e^a}_t} = 0 \quad \text{with} \quad \frac{\partial V_t^{a2}}{\partial^2 e_t^a}|_{\tilde{e^a}_t} < 0.
\end{aligned}$$

Step 2: We apply Kakutani's fixed point theorem to functions determined in Step 1 and implicitely given by

$$\begin{aligned}
\frac{\partial V_t^a}{\partial n_t^a}|_{n_t^a = \tilde{n^a}_t} = 0, \\
\frac{\partial V_t^a}{\partial e_t^a}|_{n_t^a = \tilde{e^a}_t} = 0,
\end{aligned}$$

to show that there exists a couple $(\hat{n_t^a}, \hat{e_t^a})$ such that both equations hold.

Step 1: Consider $V_t^a(n_t^a, n_t^b, e_t^a, e_t^b, x_t)$ and $V_t^b(n_t^a, n_t^b, e_t^a, e_t^b, x_t)$ given by

$$V_t^a(n_t^a, n_t^b, e_t^a, e_t^b, x_t) = \frac{N^{-\alpha} n_t^a \; (e_t^a)^{\mu\rho} \; \left((e_t^a)^\rho \, n_t^a x_t + \left(e_t^b\right)^\rho n_t^b (1 - x_t) \right)^{(1-\alpha)}}{(e_t^a)^{\mu\rho} \, n_t^a x_t + \left(e_t^b\right)^{\mu\rho} n_t^b (1 - x_t)} - \gamma \, n_t^a e_t^a - \frac{\lambda}{2} \left(n_t^a\right)^2 . \tag{22}$$

$$V_t^b(n_t^a, n_t^b, e_t^a, e_t^b, x_t) = \frac{N^{-\alpha} n_t^b \; \left(e_t^b\right)^{\mu\rho} \; \left((e_t^a)^\rho \, n_t^a x_t + \left(e_t^b\right)^\rho n_t^b (1 - x_t) \right)^{(1-\alpha)}}{(e_t^a)^{\mu\rho} \, n_t^a x_t + \left(e_t^b\right)^{\mu\rho} n_t^b (1 - x_t)} - \gamma \, n_t^b e_t^b - \frac{\lambda}{2} \left(n_t^b\right)^2 . \tag{23}$$

To alleviate notations denote

$$
\begin{aligned}
D &\equiv n_t^a x_t (e_t^a)^{\rho\mu} + n_t^b (1-x_t)(e_t^b)^{\rho\mu}, \\
H &\equiv n_t^a x_t (e_t^a)^{\rho} + n_t^b (1-x_t)(e_t^b)^{\rho}, \\
\Pi^a &\equiv \frac{n_t^a x_t \ (e_t^a)^{\mu\rho}}{(e_t^a)^{\mu\rho} \, n_t^a x_t + \left(e_t^b\right)^{\mu\rho} n_t^b (1-x_t)}
\end{aligned}
$$

Let us perform $\frac{\partial V_t^a}{\partial n_t^a}$ and $\frac{\partial V_t^a}{\partial e_t^a}$. One obtains

$$
\frac{\partial V_t^a}{\partial n_t^a} = (1-\alpha) H^{-\alpha} \Pi^a \frac{\partial H}{\partial n_t^a} + H^{(1-\alpha)} \frac{\partial \Pi^a}{\partial n_t^a} - \gamma e_t^a - \lambda n_t^a,
$$

which can also be re-written as

$$
\begin{aligned}
\frac{\partial V_t^a}{\partial n_t^a} &= N^{-\alpha} \frac{H^{-\alpha}}{D} \left((e_t^a)^{\rho\mu} H + n_t^a (e_t^a)^{\rho\mu} (1-\alpha) \frac{\partial H}{\partial n_t^a} + n_t^a (e_t^a)^{\rho\mu} \frac{H}{D} \frac{-\partial D}{\partial n_t^a} \right) - \gamma e_t^a - \lambda n_t^a, \\
&= \frac{H^{-\alpha}}{D} \left((e_t^a)^{\rho\mu} H \left(1 - \Pi_{t+1}^a\right) + n_t^a (e_t^a)^{\rho\mu} (1-\alpha) \frac{\partial H}{\partial n_t^a} \right) - \gamma e_t^a - \lambda n_t^a.
\end{aligned}
$$

Furthermore, we have

$$
\frac{\partial V_t^a}{\partial e_t^a} = (1-\alpha) H^{-\alpha} \Pi^a \frac{\partial H}{\partial e_t^a} + H^{(1-\alpha)} \frac{\partial \Pi^a}{\partial e_t^a} - \gamma n_t^a,
$$

which re-writes as

$$
\begin{aligned}
\frac{\partial V_t^a}{\partial e_t^a} &= N^{-\alpha} \frac{H^{-\alpha}}{D} \left(\rho\mu (e_t^a)^{\rho\mu-1} n_t^a H + n_t^a (e_t^a)^{\rho\mu} (1-\alpha) \frac{\partial H}{\partial e_t^a} + n_t^a (e_t^a)^{\rho\mu} \frac{H}{D} \frac{-\partial D}{\partial e_t^a} \right) - \gamma n_t^a, \\
&= \frac{H^{-\alpha}}{D} \left(\rho\mu (e_t^a)^{\rho\mu-1} n_t^a H \left(1 - \Pi_{t+1}^a\right) + n_t^a (e_t^a)^{\rho\mu} (1-\alpha) \frac{\partial H}{\partial e_t^a} \right) - \gamma n_t^a.
\end{aligned}
$$

a. We now examine the existence of solutions for $\frac{\partial V_t^a}{\partial n_t^a} = 0$ and $\frac{\partial V_t^a}{\partial e_t^a} = 0$.

First, consider $\frac{\partial V_t^a}{\partial e_t^a}$.

$$\frac{\partial V_t^a}{\partial e_t^a} = 0,$$

$$\Leftrightarrow \frac{H^{-\alpha}}{D}\left(\rho\mu(e_t^a)^{\rho\mu-1}H\left(1-\Pi_{t+1}^a\right)+(e_t^a)^{\rho\mu}(1-\alpha)\frac{\partial H}{\partial e_t^a}\right)-\gamma=0 \quad \text{if} \quad n_t^a \neq 0.$$

First, at $e_t^a = 0$, one easily shows that $\forall(e_t^b, n_t^a, n_t^b) \in [0,\bar{e}] \times [0,\bar{n}] \times [0,\bar{n}]$, $\frac{\partial V_t^a}{\partial e_t^a} = +\infty$.

At $e_t^a = \bar{e}$, $\forall(e_t^b, n_t^a, n_t^b) \in [0,\bar{e}] \times [0,\bar{n}] \times [0,\bar{n}]$, the first term of the sum is bounded above. Hence we have

$$\lim_{\gamma \to +\infty} \frac{\partial V_t^a}{\partial e_t^a} = -\infty.$$

Hence, $\forall(e_t^b, n_t^a, n_t^b) \in [0,\bar{e}] \times [0,\bar{n}] \times [0,\bar{n}]$, there exists $\bar{\gamma} \geq 0$ such that $\forall \gamma > \bar{\gamma}$,

$$\frac{\partial V_t^a}{\partial e_t^a}|_{e_t^a=\bar{e}} < 0.$$

We deduce that $\forall(e_t^b, n_t^a, n_t^b) \in [0,\bar{e}] \times [0,\bar{n}] \times [0,\bar{n}]$, and $\forall \gamma > \bar{\gamma}$, there exists $\tilde{e_t^a}$ such that

$$\frac{\partial V_t^a}{\partial e_t^a}|_{e_t^a=\tilde{e_t^a}} = 0.$$

Now consider $\frac{\partial V_t^a}{\partial n_t^a}$.

Using a similar reasoning that for $\frac{\partial V_t^a}{\partial e_t^a}$, we can deduce that $\forall(e_t^b, e_t^a, n_t^b) \in [0,\bar{e}] \times [0,\bar{e}] \times [0,\bar{n}]$, there exists $\tilde{\lambda} \geq 0$ such that $\forall \lambda > \tilde{\lambda}$,

$$\frac{\partial V_t^a}{\partial n_t^a}|_{n_t^a=\bar{n}} < 0.$$

At $n_t^a = 0$, we have

$$\frac{\partial V_t^a}{\partial n_t^a}|_{n_t^a=0} = (1-\alpha)H^{-\alpha}\Pi^a\frac{\partial H}{\partial n_t^a} + H^{(1-\alpha)}\frac{\partial \Pi^a}{\partial n_t^a} - \gamma e_t^a.$$

There are two cases. First, if

$$(1-\alpha)H^{-\alpha}\Pi^a\frac{\partial H}{\partial n_t^a} + H^{(1-\alpha)}\frac{\partial \Pi^a}{\partial n_t^a} - \gamma e_t^a < 0,$$

which is equivalent to say that $e_t^a > k$, where k is a positive constant, then $\frac{\partial V_t^a}{\partial n_t^a} = 0$ may admit zero solution in n_t^a.

Or,

$$(1-\alpha)H^{-\alpha}\Pi^a \frac{\partial H}{\partial n_t^a} + H^{(1-\alpha)} \frac{\partial \Pi^a}{\partial n_t^a} - \gamma e_t^a < 0,$$

which is equivalent to say that $e_t^a < k$. In this case, we deduce that $\forall (e_t^b, e_t^a, n_t^b) \in [0, \bar{e}] \times [0, k] \times [0, \bar{n}]$, and $\forall \lambda > \tilde{\lambda}$, there exists $\tilde{n_t^a}$ such that

$$\frac{\partial V_t^a}{\partial n_t^a}|_{n_t^a = \tilde{n_t^a}} = 0.$$

b. Let us now examine the sign of the second derivatives of the function V_t^a.

Consider the case of $\frac{\partial V_t^{a2}}{\partial^2 n_t^a}$. One has

$$\frac{\partial V_t^{a2}}{\partial^2 n_t^a} = N^{-\alpha}(e_t^a)^{\rho\mu} \frac{H^{-\alpha}}{D} \times$$
$$\left(2(1-\alpha)\frac{\partial H}{\partial n_t^a} - \alpha n_t^a (1-\alpha) \left(\frac{\partial H}{\partial n_t^a}\right)^2 - 2\frac{H}{D}\frac{\partial D}{\partial n_t^a} - 2(1-\alpha)\frac{n_t^a}{D}\frac{\partial D}{\partial n_t^a}\frac{\partial H}{\partial n_t^a} + 2\frac{n_t^a}{D^2}\left(\frac{\partial D}{\partial n_t^a}\right)^2\right)$$
$$- \lambda.$$

$$\frac{\partial V_t^{a2}}{\partial^2 n_t^a} = -\alpha(1-\alpha)H^{-\alpha-1}\Pi^a \frac{\partial H}{\partial n_t^a}\frac{\partial H}{\partial n_t^a} + H^{(1-\alpha)}\frac{\partial^2 \Pi^a}{\partial n_t^{a2}} + 2(1-\alpha)H^{-\alpha}\frac{\partial \Pi^a}{\partial n_t^a}\frac{\partial H}{\partial n_t^a} - \lambda,$$

with

$$\frac{\partial^2 \Pi^a}{\partial n_t^{a2}} = \frac{-\left(e_t^a\right)^{2\mu\rho}\left(e_t^b\right)^{\mu\rho} n_t^b (1-x_t)}{\left((e_t^a)^{\mu\rho} n_t^a x_t + \left(e_t^b\right)^{\mu\rho} n_t^b (1-x_t)\right)^4}.$$

Suppose that $n^b = 0$.

(i) Either $n^a > 0$ and then, the term corresponding to the multiplication of first two lines is bouded above (remind that, in addition, n and e are bounded above). We hence have

$$\lim_{\lambda \to +\infty} \frac{\partial^2 V_t^a}{\partial n_t^{a2}} = -\infty.$$

Since $\frac{\partial V_t^{a2}}{\partial^2 n_t^a}$ is continuous in λ, we deduce that there exists $\tilde{\lambda}' \geq 0$ such that $\forall \lambda > \tilde{\lambda}'$, $\frac{\partial V_t^{a2}}{\partial^2 n_t^a} < 0$.

(ii) Or, $n^a = 0$. As for the contest function, we extend the above function by taking the value at the limit when $n^a = n^b \to 0$. At $n^a = n^b = n$, we have

$$\begin{aligned}\frac{\partial V_t^{a2}}{\partial^2 n_t^a} &= -\alpha(1-\alpha)n^{-\alpha-1}\left((e_t^a)^{\mu\rho} + \left(e_t^b\right)^{\mu\rho}\right)^{-\alpha-1}\frac{1}{2}(x_t)^2\left(e_t^a\right)^{2\rho} \\ &- n^{(-3-\alpha)}\left((e_t^a)^{\mu\rho} + \left(e_t^b\right)^{\mu\rho}\right)^{(1-\alpha)}\frac{\left(e_t^a\right)^{2\mu\rho}\left(e_t^b\right)^{\mu\rho} n_t^b(1-x_t)}{\left((e_t^a)^{\mu\rho}\, x_t + \left(e_t^b\right)^{\mu\rho}(1-x_t)\right)^4} \\ &+ 2(1-\alpha)n^{(-2-\alpha)}\left((e_t^a)^{\mu\rho} + \left(e_t^b\right)^{\mu\rho}\right)^{-\alpha}\frac{\left(e_t^a\right)^{\mu\rho}\left(e_t^b\right)^{\mu\rho} n_t^b(1-x_t)}{\left((e_t^a)^{\mu\rho}\, x_t + \left(e_t^b\right)^{\mu\rho}(1-x_t)\right)^2}x_t\left(e_t^a\right)^{\rho} \\ &- \lambda,\end{aligned}$$

which tends to $-\infty$ when $n \to 0$.

Hence, we deduce that $\forall n^b \in [0, \bar{n}]$, $\forall \lambda > \tilde{\lambda}'$,

$$\frac{\partial^2 V_t^a}{\partial n_t^{a2}} < 0, \quad \forall n_t^a \in [0, \bar{n}].$$

A similar reasoning allows to deduce that $\forall e^b \in [0, \bar{e}]$, there exists $\tilde{\lambda}''$ such that $\forall \lambda > \tilde{\lambda}''$,

$$\frac{\partial^2 V_t^a}{\partial n_t^{a2}} < 0, \quad \forall e_t^a \in [0, \bar{e}].$$

Finally, we have $\forall (n_t^b, e_t^b) \in [0, \bar{n}] \times [0, \bar{e}]$ and $\forall \lambda > \max\{\tilde{\lambda}', \tilde{\lambda}''\}$,

$$\frac{\partial^2 V_t^a}{\partial n_t^{a2}} < 0, \quad \forall (n_t^a, e_t^a) \in [0, \bar{n}] \times [0, \bar{e}].$$

Denote $\bar{\lambda} \equiv \max\{\tilde{\lambda}, \tilde{\lambda}', \tilde{\lambda}''\}$. Using our previous result we can deduce that

(i) $\forall (e_t^b, e_t^a, n_t^b) \in [0, \bar{e}] \times [k, \bar{e}] \times [0, \bar{n}]$, and $\forall \lambda > \bar{\lambda}$,

$$\frac{\partial V_t^a}{\partial n_t^a}\Big|_{n_t^a = \tilde{n_t^a}} < 0, \quad \forall n_t^a \in [0, \bar{n}].$$

(ii) $\forall (e_t^b, e_t^a, n_t^b) \in [0, \bar{e}] \times [0, k] \times [0, \bar{n}]$, and $\forall \lambda > \bar{\lambda}$, there exists a unique $\tilde{n_t^a}$ such that

$$\frac{\partial V_t^a}{\partial n_t^a}|_{n_t^a = \tilde{n_t^a}} = 0.$$

Consider the case $\frac{\partial^2 V_t^a}{\partial e_t^{a2}}$.

$$\begin{aligned}
\frac{\partial^2 V_t^a}{\partial e_t^{a2}} = & \left(-\alpha \frac{H^{-\alpha-1}}{D} \frac{\partial H}{\partial e_t^a} + \frac{H^{-\alpha}}{D^2} \frac{-\partial D}{\partial e_t^a}\right) \left(\rho\mu (e_t^a)^{\rho\mu-1} n_t^a H \left(1 - \Pi_{t+1}^a\right) + n_t^a (e_t^a)^{\rho\mu} (1-\alpha) \frac{\partial H}{\partial e_t^a}\right) \\
& + \frac{H^{-\alpha}}{D} \left(\rho\mu(\rho\mu - 1)(e_t^a)^{\rho\mu-2} n_t^a H + \rho\mu (e_t^a)^{\rho\mu-1} n_t^a \frac{\partial H}{\partial e_t^a}\right. \\
& + \rho\mu n_t^a (e_t^a)^{\rho\mu-1} (1-\alpha) \frac{\partial H}{\partial e_t^a} + n_t^a (e_t^a)^{\rho\mu} (1-\alpha) \frac{\partial^2 H}{\partial e_t^{a2}} + \rho\mu n_t^a (e_t^a)^{\rho\mu-1} \frac{H}{D} \frac{-\partial D}{\partial e_t^a} \\
& \left. + n_t^a (e_t^a)^{\rho\mu} \frac{\frac{\partial H}{\partial e_t^a}}{D} \frac{-\partial D}{\partial e_t^a} + n_t^a (e_t^a)^{\rho\mu} \frac{H}{D} \frac{-\partial^2 D}{\partial e_t^{a2}} + n_t^a (e_t^a)^{\rho\mu} \frac{H}{D^2} \left(\frac{-\partial D}{\partial e_t^a}\right)^2 \right).
\end{aligned}$$

This expression simplifies to

$$\begin{aligned}
\frac{\partial^2 V_t^a}{\partial e_t^{a2}} = & \left(-\alpha \frac{H^{-\alpha-1}}{D} \frac{\partial H}{\partial e_t^a} - \frac{H^{-\alpha}}{D^2} \frac{\partial D}{\partial e_t^a}\right) \left(n_t^a (e_t^a)^{\rho\mu} (1-\alpha) \frac{\partial H}{\partial e_t^a}\right) \\
& + \left(-\alpha \frac{H^{-\alpha-1}}{D} \frac{\partial H}{\partial e_t^a}\right) \left(\rho\mu (e_t^a)^{\rho\mu-1} n_t^a H \left(1 - \Pi_{t+1}^a\right)\right) \\
& + \frac{H^{-\alpha}}{D} \left[\rho\mu (e_t^a)^{\rho\mu-2} n_t^a \left((\rho\mu - 1) H + \rho (e_t^a)^{\rho} n_t^a x_t N_t\right)\right. \\
& (e_t^a)^{\rho+\rho\mu-1} (1-\alpha) \rho \left(n_t^a\right)^2 x_t N_t \left(\rho\mu + \rho - 1\right) \\
& \left. - \rho\mu (n_t^a)^2 x_t N_t \frac{H}{D} (e_t^a)^{2\rho\mu-2} - n_t^a (e_t^a)^{\rho\mu} \frac{\frac{\partial H}{\partial e_t^a}}{D} \frac{\partial D}{\partial e_t^a}\right] \\
& + \frac{H^{1-\alpha}}{D^2} (e_t^a)^{2\rho\mu-2} (n_t^a)^2 x_t N_t (\rho\mu)^2 (2\Pi_{t+1}^a - 1),
\end{aligned}$$

which can also writes as

$$\begin{aligned}\frac{\partial^2 V_t^a}{\partial e_t^{a2}} &= \left(-\alpha\frac{H^{-\alpha-1}}{D}\frac{\partial H}{\partial e_t^a} - \frac{H^{-\alpha}}{D^2}\frac{\partial D}{\partial e_t^a}\right)\left(n_t^a(e_t^a)^{\rho\mu}(1-\alpha)\frac{\partial H}{\partial e_t^a}\right)\\
&+ \left(-\alpha\frac{H^{-\alpha-1}}{D}\frac{\partial H}{\partial e_t^a}\right)\left(\rho\mu(e_t^a)^{\rho\mu-1}n_t^a H\left(1-\Pi_{t+1}^a\right)\right)\\
&+ \frac{H^{-\alpha}}{D}\left[\rho\mu(e_t^a)^{\rho\mu-2}n_t^a\left((\rho\mu-1)H+\rho(e_t^a)^{\rho}n_t^a x_t N_t\right)\right.\\
&(e_t^a)^{\rho+\rho\mu-1}(1-\alpha)\rho\left(n_t^a\right)^2 x_t N_t\left(\rho\mu+\rho-1\right)\\
&\left.-n_t^a(e_t^a)^{\rho\mu}\frac{\frac{\partial H}{\partial e_t^a}}{D}\frac{\partial D}{\partial e_t^a}\right]\\
&+ \frac{H^{1-\alpha}}{D^2}(e_t^a)^{2\rho\mu-2}(n_t^a)^2 x_t N_t(\rho\mu)\left((\rho\mu)(2\Pi_{t+1}^a-1)-1\right).\end{aligned}$$

The first terms of the first two lines are unambiguously negative. Now consider succesively the three terms in brackets (third, fourth and fifth lines). The first term of the sum is negative whenever

$$(\rho\mu-1)H+\rho(e_t^a)^{\rho}n_t^a N_t^a < 0.$$

Note that $H > (e_t^a)^{\rho}n_t^a N_t^a$, so that a sufficient condition for the above inequality to hold is

$$\begin{aligned}&(\rho\mu-1)+\rho<0,\\ &\Leftrightarrow \rho(\mu+1)-1<0,\end{aligned}$$

which is true whenever $\rho \leq 1/2$. This condition also implies that the second term of the sum in brackets (the fourth line) is negative. Finally the third term of the sum (the fifth line) is unambigously negative). Now consider the sixth line term. It is negative as long as

$$(\rho\mu)(2\Pi_{t+1}^a-1)-1<0,$$

which is true since $\rho\mu < 1$. We conclude that whenever $\rho \leq 1/2$, $\forall(n_t^a, e_t^a) \in [0,\bar{n}]\times[0,\bar{e}]$,

$$\frac{\partial^2 V_t^a}{\partial e_t^{a2}} < 0, \quad \forall(n_t^b, e_t^b) \in [0,\bar{n}]\times[0,\bar{e}].$$

Using our previous results, we deduce that if $\rho \leq 1/2$, $\forall(e_t^b, n_t^a, n_t^b) \in [0,\bar{e}]\times[0,\bar{n}]\times[0,\bar{n}]$, and

$\forall \gamma > \bar{\gamma}$, there exists a unique $\tilde{e_t^a}$ such that

$$\frac{\partial V_t^a}{\partial e_t^a}|_{e_t^a=\tilde{e_t^a}} = 0.$$

Finally, regarding the cross derivatives, one can easily show that we have

$$\frac{\partial^2 V_t^a}{\partial e_t^a \partial n_t^a} = \frac{\partial^2 V_t^a}{\partial n_t^a \partial e_t^a}.$$

From the above, we conclude that (i) when $\lambda > \bar{\lambda}$, (ii) $\gamma > \bar{\gamma}$, (iii) $\rho \leq 1/2$, $\forall (e_t^b, n_t^b) \in [0, \bar{e}] \times [0, \bar{n}]$,

$$\forall e_t^a \in [0, k], \exists ! \tilde{n^a}_t \quad \text{such that} \quad \frac{\partial V_t^a}{\partial n_t^a}|_{n_t^a=\tilde{n^a}_t} = 0, \tag{24}$$

$$\forall n_t^a \in [0, \bar{n}], \exists ! \tilde{e^a}_t \quad \text{such that} \quad \frac{\partial V_t^a}{\partial e_t^a}|_{n_t^a=\tilde{e^a}_t} = 0. \tag{25}$$

Denote

$$(\hat{n_t^a}, \hat{e_t^a}) = \text{argmax}_{(n_t^a, e_t^a)} V_t^a(n_t^a, e_t^a, n_t^b, e_t^b, x_t).$$

We show that $(\hat{n_t^a}, \hat{e_t^a})$ is such that $\hat{e_t^a} < k$. Suppose that $\hat{e_t^a} > k$, that is

$$(1-\alpha) H^{-\alpha} \Pi^a \frac{\partial H}{\partial n_t^a} + H^{(1-\alpha)} \frac{\partial \Pi^a}{\partial n_t^a} - \gamma \tilde{e^a}_t < 0,$$

so that $\text{argmax}_{n_t^a} V_t^a = 0$. It implies[9]

$$\frac{\partial V_t^a}{\partial e_t^a}|_{n_t^a=0} = n_t^a \left(\frac{H^{-\alpha}}{D} \left(\rho\mu (e_t^a)^{\rho\mu-1} H \left(1 - \Pi_{t+1}^a\right) + (e_t^a)^{\rho\mu} (1-\alpha) \frac{\partial H}{\partial e_t^a} \right) - \gamma \right) < 0,$$

But $\frac{\partial V_t^a}{\partial e_t^a} < 0$ in turn implies $\hat{e_t^a} = 0$ which contradicts the above. We deduce that $\hat{e_t^a}$ is such that

$$(1-\alpha) H^{-\alpha} \Pi^a \frac{\partial H}{\partial n_t^a} + H^{(1-\alpha)} \frac{\partial \Pi^a}{\partial n_t^a} - \gamma \hat{e_t^a} > 0,$$

that is $\hat{e_t^a} < k$.

Now we show that a solution to both equations (24) and (25) exists and is unique.

[9]Remind that we consider the extended function, i.e., $\frac{\partial V_t^a}{\partial e_t^a}|_{n_t^a=0} = \lim_{n_t^a \to} \frac{\partial V_t^a}{\partial e_t^a}$.

Equation (24) is equivalent to say that $\forall(e_t^b, n_t^b) \in [0, \bar{e}] \times 0, \bar{n}]$, $\forall e_t^a \in [0, \bar{e}]$, there exists a function $g : [0, \bar{n}] \to [0, \bar{e}]$, such that $\frac{\partial V_t^a}{\partial n_t^a}|_{e_t^a = g(n_t^a)} = 0$. Equation (25) is equivalent to say that $\forall(e_t^b, n_t^b) \in [0, \bar{e}] \times 0, \bar{n}]$, $\forall n_t^a \in [0, \bar{n}]$, there exists a function $h : [0, \bar{e}] \to [0, \bar{n}]$, such that $\frac{\partial V_t^a}{\partial e_t^a}|_{n_t^a = h(e_t^a)} = 0$.

Define $\mathbf{F} : [0, \bar{n}] \times [0, \bar{e}] \to [0, \bar{n}] \times [0, \bar{e}]$, the (multi-valued) function which at each couple (n_t^a, e_t^a) associates the couple $(g(n_t^a), h(e_t^a))$. Given that $\forall(e_t^b, n_t^b) \in [0, \bar{e}] \times [0, \bar{n}]$, $\frac{\partial^2 V_t^a}{\partial n_t^{a2}} < 0$ and $\frac{\partial^2 V_t^a}{\partial e_t^{a2}} < 0$, that is V_t^a is concave in n_t^a and e_t^a, we deduce that $\mathbf{F}$ is a convex value function. Also $\mathbf{F}$ has a closed graph since V_t^a is continous in both n_t^a and e_t^a. Using Kakutani's fixed point theorem we can deduce that $\forall(e_t^b, n_t^b) \in [0, \bar{e}] \times [0, \bar{n}]$, the map $\mathbf{F}$ has a fixed point $(\hat{n_t^a}, \hat{e_t^a})$, that is

$$\frac{\partial V_t^a}{\partial n_t^a}|_{(\hat{n_t^a}, \hat{e_t^a})} = 0,$$
$$\frac{\partial V_t^a}{\partial e_t^a}|_{(\hat{n_t^a}, \hat{e_t^a})} = 0.$$

Given the second derivatives, the hessien matrix of the map V_t^a (see step b.) is negative semidefinite.[10] We deduce taht $(\hat{n_t^a}, \hat{e_t^a})$ is a global maximum on $]0, \bar{n}[\times]0, \bar{e}[$. Given that

$$\frac{\partial V_t^a}{\partial e_t^a}|_{n_t^a = 0} > 0,$$
$$\frac{\partial V_t^a}{\partial e_t^a}|_{n_t^a = \bar{n}} < 0,$$
$$\frac{\partial V_t^a}{\partial e_t^a}|_{e_t^a = 0} = +\infty > 0,$$
$$\frac{\partial V_t^a}{\partial e_t^a}|_{e_t^a = \bar{e}} < 0,$$

and given the continuity of V_t^a in e_t^a and n_t^a, we deduce that $(\hat{n_t^a}, \hat{e_t^a})$ is a global maximum on $[0, \bar{n}] \times [0, \bar{e}]$.

Finally, this is equivalent to say that there exists a (best response) function $\mathbf{B}_{x_t} : [0, \bar{n}] \times [0, \bar{e}] \to [0, \bar{n}] \times [0, \bar{e}]$ such that

$$\mathbf{B}_{x_t}(n_t^b, e_t^b) = \text{argmax}_{n_t^a, e_t^a} V_t^a(n_t^a, n_t^b, e_t^a, e_t^b, x_t).$$

Note that V_t^a and V_t^b are symmetric functions, i.e., $V_t^a(n_t^a, n_t^b, e_t^a, e_t^b, x_t) = V_t^b(n_t^b, n_t^a, e_t^b, e_t^a, 1 - x_t)$. Hence, the symmetric reasoning applies for V_t^b and allows to deduce that there exists a $\mathbf{B}_{1-x_t}$:

[10] Remind that we showed that the cross derivatives are equal so that they have the same sign.

$[0,\bar{n}]\times[0,\bar{e}]\rightarrow[0,\bar{n}]\times[0,\bar{e}]$ such that

$$\mathbf{B}_{1-x_t}(n_t^a,e_t^a)=\text{argmax}_{n_t^b,e_t^b}V_t^b(n_t^b,n_t^a,e_t^b,e_t^a,1-x_t).$$

D.2 Proof of Proposition 1

(i) Since $\forall i\in\{a,b\}$, V_t^i is continuous and concave in n_t^i and e_t^i we can apply Kakutani's fixed point theorem to deduce that there exists a Pure strategy Nash equilibrium given by $(n_t^{a\star},n_t^{b\star},e_t^{a\star},e_t^{b\star})=(n^a(x_t),n^b(x_t),e^a(x_t),e^b(x_t))$ where $\forall i\in\{a,b\}$, $n^i:[0,1]\rightarrow[0,\bar{n}]$ and $e^i:[0,1]\rightarrow[0,\bar{e}]$.

(ii) Now we show that $n^a(x_t)=n^b(1-x_t)$ and $e^a(x_t)=e^b(1-x_t)$ is a Nash equilibrium.

Any strategies vector $(n_t^a,n_t^b,e_t^a,e_t^b)$ which solves

$$\begin{cases}(n_t^a,e_t^a)=\mathbf{C}_{x_t}(n_t^{b\star},e_t^{b\star}),\\(n_t^b,e_t^b)=\mathbf{C}_{1-x_t}(n_t^{a\star},e_t^{a\star}).\end{cases}$$

is a Nash equilibrium.

Given that definition, for each vector (n_t^a,e_t^a), the solution is the matrix $\big((n_t^a,e_t^a),\mathbf{C}_{1-x_t}(n_t^a,e_t^a)\big)$, for each vector (n_t^b,e_t^b), the solution is the matrix $\big((n_t^b,e_t^b),\mathbf{C}_{x_t}(n_t^b,e_t^b)\big)$. Then at the Pure strategy equilibrium,

(i) for any $x_t\in[0,1]$, the solution matrix is given by $\big((n^a(x_t),e^a(x_t)),\mathbf{C}_{1-x_t}(n^a(x_t),e^a(x_t))\big)$,

(ii) or by $\big((n^b(x_t),e^b(x_t)),\mathbf{C}_{x_t}(n^b(x_t),e^b(x_t))\big)$.

Note that (i) implies that the matrix $\big((n^a(1-x_t),e^a(1-x_t)),\mathbf{C}_{x_t}(n^a(1-x_t),e^a(1-x_t))\big)$ is also a Nash equilibrium which in turn implies that $n^b(x_t)=n^a(1-x_t)$ and $e^b(x_t)=e^a(1-x_t)$ is a Nash equilibrium.

D.3 Proof of Proposition 2

The first order condition associated to the maximization program of group a and b are respectively given by

$$N_t^{-\alpha}\left((1-\alpha)\frac{n_t^a x_t(n_t^a x_t+n_t^b(1-x_t))^{-\alpha}}{n_t^a x_t+n_t^b(1-x_t)}+\frac{n_t^b(1-x_t)(n_t^a x_t+n_t^b(1-x_t))^{(1-\alpha)}}{(n_t^a x_t+n_t^b(1-x_t))^2}\right)=\lambda n_t^a,$$

$$N_t^{-\alpha}\left((1-\alpha)\frac{n_t^b(1-x_t)(n_t^a x_t+n_t^b(1-x_t))^{-\alpha}}{n_t^a x_t+n_t^b(1-x_t)}+\frac{n_t^a x_t(n_t^a x_t+n_t^b(1-x_t))^{(1-\alpha)}}{(n_t^a x_t+n_t^b(1-x_t))^2}\right)=\lambda n_t^b.$$

These two equations can be re-written as

$$\begin{aligned}H_{t+1}^{-\alpha}\left(1-\alpha\Pi_{t+1}^a\right)&=\lambda n_t^a,\\ H_{t+1}^{-\alpha}\left(1-\alpha\Pi_{t+1}^b\right)&=\lambda n_t^b.\end{aligned}$$

At a Nash equilibrium, we have

$$\begin{aligned}&H_{t+1}^{-\alpha}\left(1-\alpha\Pi_{t+1}^b\right)=H_{t+1}^{-\alpha}\left(1-\alpha\Pi_{t+1}^a\right)\frac{n_t^b}{n_t^a},\\ \Leftrightarrow&(1-\alpha)*(2x-1)\frac{n_t^a}{n_t^b}+(1-x)-x\left(\frac{n_t^a}{n_t^b}\right)^2=0,\\ \Leftrightarrow&\frac{n_t^a}{n_t^b}=\frac{(1-\alpha)(2x-1)+\left((1-\alpha)^2(2x-1)^2+4x(1-x)\right)^{1/2}}{2x}\equiv f(x)\end{aligned}$$

Let us perform the derivative of f, we have

$$\begin{aligned}f^{'}(x)=x^{-2}\left(\frac{(1-\alpha)}{2}+\frac{1}{2}\left((1-\alpha)^2(1-\frac{1}{2x})^2+\frac{1}{x}-1\right)^{-1/2}(1-\alpha)^2(1-\frac{1}{2x})-1\right)&<0\\ \Leftrightarrow(1-\alpha)+\left((1-\alpha)^2(1-\frac{1}{2x})^2+\frac{1}{x}-1\right)^{-1/2}(1-\alpha)^2(1-\frac{1}{2x})&<2.\end{aligned}$$

We have $1-\alpha<1$ so that a sufficient condition for the above inequality to hold is

$$\begin{aligned}\left((1-\alpha)^2(1-\frac{1}{2x})^2+\frac{1}{x}-1\right)^{-1/2}(1-\alpha)^2(1-\frac{1}{2x})&<1,\\ \Leftrightarrow(1-\alpha)^4(1-\frac{1}{2x})^2<(1-\alpha)^2(1-\frac{1}{2x})^2+\frac{1}{x}&-1,\end{aligned}$$

which is true since $\frac{1}{x}-1<1$.

Hence we have $\frac{dn_t^a/n_t^b}{dx}<0$. Due to the symmetry of strategic choices (see Proposition 1, we deduce $\frac{dn_t^a}{dx}<0$ and (ii) $\frac{dn_t^b}{dx}>0$.

A symmetric reasoning leads to $\frac{dn_t^a/n_t^b}{dx}>0$ implies (i) $\frac{dn_t^a}{dx}>0$ and (ii) $\frac{dn_t^b}{dx}<0$.

D.4 Proof of Proposition 3

We first examine how the ratio $\frac{e_t^a}{e_t^b}$ varies with x_t, then how the ratio $\frac{n_t^a}{n_t^b}$ varies with x_t. Due to symmetry, we deduce how each strategic choice varies with x_t at the Nash equilibrium.

At the (symmetric) Nash equilibrium, we have

$$
\begin{aligned}
&\Pi_{t+1}^a \rho(1-\alpha) N_t{}^{-\alpha} \frac{(n_t^a x_t (e_t^a)^\rho + n_t^b (1-x_t)(e_t^b)^\rho)^{-\alpha}}{x_t} e_t^{a\rho-1} x_t \\
&\qquad = \Pi_{t+1}^b \rho(1-\alpha) N_t{}^{-\alpha} \frac{(n_t^a x_t (e_t^a)^\rho + n_t^b (1-x_t)(e_t^b)^\rho)^{-\alpha}}{(1-x_t)} \left(e_t^b\right)^{\rho-1} (1-x_t),
\end{aligned}
$$

that is

$$
\begin{aligned}
\Pi_{t+1}^a \left(e_t^a\right)^{\rho-1} &= \Pi_{t+1}^b \left(e_t^b\right)^{\rho-1}, \\
\Leftrightarrow \frac{n_t^a}{n_t^b} &= \left(\frac{x_t}{(1-x_t)}\right)^{-1} \left(\frac{e_t^a}{e_t^b}\right)^{1-\rho}.
\end{aligned}
$$

Using the two FOC together for group 1, we deduce that

$$
\begin{aligned}
&\frac{x_t \Pi_{t+1}^b}{n_t^a x_t + n_t^b(1-x_t)} N_t^{-\alpha} \frac{(n_t^a x_t (e_t^a)^\rho + n_t^b (1-x_t)(e_t^b)^\rho)^{(1-\alpha)}}{x_t} \\
&+ (1-\rho)\Pi_{t+1}^a N_t^{-\alpha}(1-\alpha) \frac{(n_t^a x_t (e_t^a)^\rho + n_t^b (1-x_t)(e_t^b)^\rho)^{-\alpha}}{x_t} (e_t^a)^\rho = \lambda n_t^a.
\end{aligned}
$$

Symetricaly, for group 2,

$$
\begin{aligned}
&\frac{(1-x_t)\Pi_{t+1}^a}{n_t^a x_t + n_t^b(1-x_t)} N_t^{-\alpha} \frac{(n_t^a x_t (e_t^a)^\rho + n_t^b (1-x_t)(e_t^b)^\rho)^{(1-\alpha)}}{(1-x_t)} \\
&+ (1-\rho)\Pi_{t+1}^b N_t^{-\alpha}(1-\alpha) \frac{(n_t^a x_t (e_t^a)^\rho + n_t^b (1-x_t)(e_t^b)^\rho)^{-\alpha}}{(1-x_t)} (e_t^b)^\rho (1-x_t) = \lambda n_t^b.
\end{aligned}
$$

Hence, we deduce

$$
\begin{aligned}
&\frac{n_t^b}{n_t^a} \frac{\Pi_{t+1}^b}{n_t^a x_t + n_t^b(1-x_t)} (n_t^a x_t (e_t^a)^\rho + n_t^b (1-x_t)(e_t^b)^\rho)^{(1-\alpha)} \\
&\qquad + (1-\rho)\Pi_{t+1}^a (1-\alpha)(n_t^a x_t (e_t^a)^\rho + n_t^b (1-x_t)(e_t^b)^\rho)^{-\alpha} (e_t^a)^\rho \\
&\qquad = \frac{\Pi_{t+1}^a}{n_t^a x_t + n_t^b(1-x_t)} (n_t^a x_t (e_t^a)^\rho + n_t^b (1-x_t)(e_t^b)^\rho)^{(1-\alpha)} \\
&\qquad\qquad + (1-\rho)\Pi_{t+1}^b (1-\alpha)(n_t^a x_t (e_t^a)^\rho + n_t^b (1-x_t)(e_t^b)^\rho)^{-\alpha} (e_t^b)^\rho,
\end{aligned}
$$

which using $\frac{n_t^a}{n_t^b} = \left(\frac{x_t}{1-x_t}\right)^{-1}\left(\frac{e_t^a}{e_t^b}\right)^{1-\rho}$ can be re-written as

$$\frac{e_t^a}{e_t^b}^{\rho}\left(\frac{(\frac{e_t^a}{e_t^b})^{-1}+1}{1+\frac{e_t^a}{e_t^b}^{(1-\rho)}}+(1-\alpha)(1-\rho)x_t\right) - \left(\frac{(\frac{e_t^a}{e_t^b})+1}{1+\frac{e_t^a}{e_t^b}^{(1-\rho)}}+(1-\alpha)(1-\rho)(1-x_t)\right) = 0.$$

Let us set $\frac{e_t^a}{e_t^b} \equiv X$, the above equation is equivalent to

$$\begin{aligned}&X^{-1+2\rho}(1+X^{-1}) + (1-\alpha)(1-\rho)x_t X^{\rho} + (1-\alpha)(1-\rho)x_t X^{-1+2\rho}\\ &-1-X-(1-\alpha)(1-\rho)(1-x_t)-(1-\alpha)(1-\rho)(1-x_t)X^{-(1-\rho)} \equiv \Phi(X,x_t) = 0.\end{aligned}$$

The implicit function theorem gives

$$\frac{dX}{dx_t} = -\frac{\partial\Phi/\partial x_t}{\partial\Phi/\partial X}.$$

One has

$$\frac{\partial\Phi}{\partial x_t} = (1-\alpha)(1-\rho)X^{\rho}\left(1+X^{\rho-1}+X^{-1}\right) > 0.$$

Let us examine the sign of $\partial\Phi/\partial X$.

$$\begin{aligned}\frac{\partial\Phi}{\partial X} =& -(1-2\rho)X^{-2(1-\rho)}\left(1+(1-\alpha)(1-\rho)x_t\right) - (1-\rho)X^{-(1-\rho)-1}\left(1-(1-\alpha)x_t\right)\\ &+(1-\alpha)(1-\rho)x_t X^{\rho-1} - 1.\end{aligned}$$

Note that since $\rho \leq \frac{1}{2}$, $\forall X \in \mathbb{R}^+$, $\frac{\partial\Phi}{\partial X} < F(X)$ where

$$F(X) = -(1-\rho)X^{-(1-\rho)-1}\left(1-(1-\alpha)x_t\right) + (1-\alpha)(1-\rho)x_t X^{\rho-1} - 1.$$

We have

$$
\begin{aligned}
&F(X) < 0, \\
\Leftrightarrow &-(1-\rho)\left(1-(1-\alpha)x_t\right) + (1-\alpha)(1-\rho)x_t X - X^{2-\rho} \equiv G(X) < 0.
\end{aligned}
$$

First, $G(0) = -(1-\rho)\left(1-(1-\alpha)x_t\right) < 0$. Furthermore,

$$
\begin{aligned}
G^{'}(X) &= (1-\alpha)(1-\rho)x_t - (2-\rho)(1-\rho)X^{1-\rho}, \\
G^{'}(X) &= -(2-\rho)(1-\rho)X^{-\rho} < 0
\end{aligned}
$$

so that the function G is concave and $G^{'}(0) < 0$. We deduce that $G(X) < 0\ \forall X \in \mathbb{R}^+$ which is equivalent to $F(X) < 0\ \forall X \in \mathbb{R}^+$, which implies $\partial\Phi/\partial X < 0\ \forall X \in \mathbb{R}^+$.

One finally deduces that

$$
\frac{dX}{dx_t} = -\frac{\partial\Phi/\partial x_t}{\partial\Phi/\partial X} > 0.
$$

Now let us examine $d\frac{n_t^a}{n_t^b}/dx_t$.

$$
\frac{d\frac{n_t^a}{n_t^b}}{dx_t} = \frac{X^{-\rho}}{x_t}\left(\frac{-X}{x_t} + (1-\rho)\frac{dX}{dx_t}\right)
$$

Note that since $\frac{\partial\Phi}{\partial X} < F(X)$, then $-\frac{\partial\Phi}{\partial X} > -F(X)$ and

$$
\frac{dX}{dx_t} < -\frac{\partial\Phi/\partial x_t}{-F(X)} = \frac{(1-\alpha)(1-\rho)X^{\rho}\left(1+X^{\rho-1}+X^{-1}\right)}{(1-\rho)X^{-(1-\rho)-1}\left(1-(1-\alpha)x_t\right) - (1-\alpha)(1-\rho)x_t X^{\rho-1} + 1}.
$$

$$
\begin{aligned}
&\frac{d\frac{n_t^a}{n_t^b}}{dx_t} < 0, \\
\Leftrightarrow &(1-\rho)\frac{dX}{dx_t}/X < 1/x_t
\end{aligned}
$$

since both $\frac{dX}{dx_t}$ and $1/x_t$ are continuous functions of x_t which is true if

$$\Leftrightarrow \frac{(1-\alpha)(1-\rho)^2 X^\rho \left(1+X^{\rho-1}+X^{-1}\right)}{(1-\rho)X^{-(1-\rho)}\left(1-(1-\alpha)x_t\right)-(1-\alpha)(1-\rho)x_t X^\rho + X} < 1/x_t,$$

$$\Leftrightarrow x_t(1-\alpha)(1-\rho)^2 X^\rho \left(1+X^{\rho-1}+X^{-1}\right) - (1-\rho)X^{-(1-\rho)}\left(1-(1-\alpha)x_t\right) + (1-\alpha)(1-\rho)x_t X^\rho - X < 0$$

When $x_t \to 0$ then $1/x_t$ tends to $+\infty$ so that the above inequality holds. Furthermore, the Left-hand side of the inequality is a continuously increasing function of x_t. The right-hand side is a continuously decreasing function of x_t. We deduce that there exists a unique $\tilde{x} \in \mathbb{R}^+$ such that $\forall x_t < \tilde{x}$, the inequality holds. This in turn implies that $\forall x_t < \tilde{x}$, $d\frac{n_t^a}{n_t^b}/dx_t < 0$.

Finally, using Proposition 1 (item (ii)), we deduce that $de_t^a/dx_t > 0$, $de_t^b/dx_t > 0$, $\forall x_t < \tilde{x}$, $dn_t^a/dx_t < 0$, $dn_t^a/dx_t > 0$.

D.5 Proof of Proposition 4

Suppose that $\mu = 1$. The first order conditions can be rewritten as

$$\frac{x_t(e_t^a)^\rho}{n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho}\Pi_{t+1}^b N_t^{-\alpha}\frac{(n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho)^{(1-\alpha)}}{x_t}$$
$$+\Pi_{t+1}^a N_t^{-\alpha}(1-\alpha)\frac{(n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho)^{-\alpha}}{x_t}(e_t^a)^\rho x_t = \gamma e_t^a + \lambda n_t^a$$

$$\frac{\rho\,(e_t^a)^{\rho-1}\, n_t^a x_t}{n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho}\Pi_{t+1}^b N_t^{-\alpha}\frac{(n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho)^{(1-\alpha)}}{x_t}$$
$$+\rho\Pi_{t+1}^a(1-\alpha)N_t^{1-\alpha}\frac{(n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho)^{-\alpha}}{x_t} n_t^a e_t^{a\rho-1} x_t = \gamma n_t^a$$

Equalizing, the second FOC for group 1 and 2, we obtain

$$\left(\frac{e_t^a}{e_t^b}\right)^{-(1-\rho)} = \frac{1-\alpha\Pi_{t+1}^b}{1-\alpha\Pi_{t+1}^a}$$

Also using the two FOC together (for group 1) we deduce that

$$\frac{x_t(e_t^a)^\rho}{n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho}\Pi_{t+1}^b N_t^{-\alpha}\frac{n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho)^{(1-\alpha)}}{x_t}$$
$$+\Pi_{t+1}^a N_t^{-\alpha}(1-\alpha)\frac{(n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho)^{-\alpha}}{x_t}(e_t^a)^\rho x_t = \frac{\lambda}{1-p}n_t^a$$

Equalizing, this equation for for group 1 and 2, we obtain

$$\left(\frac{e_t^a}{e_t^b}\right)^\rho = \frac{1-\alpha\Pi_{t+1}^b}{1-\alpha\Pi_{t+1}^a}\frac{n_t^a}{n_t^b}.$$

We deduce

$$\frac{e_t^a}{e_t^b} = \frac{n_t^a}{n_t^b}.$$

Hence, $\frac{n_t^{1*}}{n_t^{2*}}$ is implicitely given by

$$\left(\frac{n_t^{1*}}{n_t^{2*}}\right)^{-(1-\rho)} - \frac{1-\alpha\Pi_{t+1}^{2*}}{1-\alpha\Pi_{t+1}^{1*}} \equiv k(\frac{n_t^{1*}}{n_t^{2*}}) = 0,$$

where

$$\Pi_{t+1}^{1*} = \frac{1}{1+\frac{n_t^a}{n_t^b}^{-1-\rho}(1-x_t)},$$
$$\Pi_{t+1}^{2*} = \frac{1}{1+\frac{n_t^a}{n_t^b}^{1+\rho}x_t}$$

One has

$$\frac{d\frac{n_t^{1*}}{n_t^{2*}}}{dx_t} = -\frac{\frac{\partial k}{\partial x_t}}{\frac{\partial k}{\partial\frac{n_t^{1*}}{n_t^{2*}}}}.$$

One easily finds that

$$\frac{\partial k}{\partial x_t} < 0,$$
$$\frac{\partial k}{\partial\frac{n_t^{1*}}{n_t^{2*}}} < 0,$$

so that we deduce $\frac{d\frac{n_t^{1*}}{n_t^{2*}}}{dx_t} = \frac{d\frac{e_t^{1*}}{e_t^{2*}}}{dx_t} < 0$.

Finally, using Proposition 1 (item (ii)), we deduce that $de_t^a/dx_t < 0$, $de_t^b/dx_t < 0$, $dn_t^a/dx_<0$, $dn_t^b/dx_t < 0$.

D.6 Proof of Proposition 6

1. Peaceful environment and $x_t = \frac{1}{2}$. The first order conditions associated to the maximization program of agent i are given by

$$\frac{1}{2}N_t^{-\alpha}(1-\alpha)\frac{\left(n_t^i x_t (e_t^i)^\rho + n_t^j(1-x_t)(e_t^j)^\rho\right)^{-\alpha}}{x_t}(e_t^i)^\rho x_t = \gamma e_t^i + \lambda n_t^i$$

and

$$\rho\frac{1}{2}(1-\alpha)N_t^{-\alpha}\frac{\left(n_t^i x_t (e_t^i)^\rho + n_t^j(1-x_t)(e_t^j)^\rho\right)^{-\alpha}}{x_t}n_t^i(e_t^i)^{\rho-1}x_t = \gamma n_t^i$$

Let us skip time indexation. Using these two equations, one obtains the following relationship

$$e^i = \frac{\lambda}{\gamma}\frac{\rho}{1-\rho}n^i.$$

Also, one easily deduces (due to symmetry) $n^a = n^b$ and education $e^a = e^b$. One can replace n^b by n^a, e^b by e^a and finally e^a by $\frac{\lambda}{\gamma}\frac{\rho}{1-\rho}n^a$ in the second first order condition for agent a to find

$$n^{a\star\star} = \frac{(1-\alpha)}{2}^{\frac{1}{1-\rho+\alpha(1+\rho)}}\lambda^{-\frac{1-\rho(1-\alpha)}{1-\rho+\alpha(1+\rho)}}\rho^{\frac{\rho(1-\alpha)}{1-\rho+\alpha(1+\rho)}}\gamma^{-\frac{\rho(1-\alpha)}{1-\rho+\alpha(1+\rho)}}(1-\rho)^{\frac{\rho(1-\alpha)}{1-\rho+\alpha(1+\rho)}}N^{\frac{-\alpha}{1-\rho+\alpha(1+\rho)}}$$

2. Anarchic environment and $x = \frac{1}{2}$. Whatever the parameters ρ and μ we have $n^a = n^b$ and education $e^a = e^b$.

(i) Case $\mu = 0$, $\rho = 0$. Using Proof of Proposition one finds

$$\begin{aligned}
\lambda n^a &= H^{-\alpha}\left(1-\frac{\alpha}{2}\right),\\
\Leftrightarrow \lambda\left(n^a\right)^{1-\alpha} &= N^{-\alpha}\left(1-\frac{\alpha}{2}\right),\\
\Leftrightarrow n^{a\star} &= N^{\frac{-\alpha}{1+\alpha}}\left(1-\frac{\alpha}{2}\right)^{\frac{1}{1+\alpha}}\lambda^{-\frac{1}{1+\alpha}}.
\end{aligned}$$

At $\rho = 0$, we have

$$n^{a\star\star} = (1-\alpha)^{\frac{1}{1+\alpha}} \lambda^{-\frac{1}{1+\alpha}} N^{\frac{-\alpha}{1+\alpha}} 2^{-\frac{1}{1+\alpha}}.$$

We deduce $n^{a\star} > n^{a\star\star}$.

In the anarchic environment, income per capita is given by

$$\frac{N^{(1-\alpha)}}{N^a}(n^{a\star})^{-\alpha}.$$

In the peaceful environment, it is given by

$$\frac{N^{(1-\alpha)}}{N^a}(n^{a\star\star})^{-\alpha}.$$

Income per capita decreases with n^a so that income per capita is higher in the peaceful environment.

(ii) Case $\mu = 1$. Using the symmetry result the FOC conditions defined in subsection 3.2.3, we obtain the relationship

$$e^i = \frac{\lambda}{\gamma} \frac{\rho}{1-\rho} n^i.$$

Using the symmetry result and this relationship, into the FOC for agent i, one obtains

$$n^{a\star} = \frac{(2-\alpha)}{2}^{\frac{1}{1-\rho+\alpha(1+\rho)}} \lambda^{-\frac{1-\rho(1-\alpha)}{1-\rho+\alpha(1+\rho)}} \gamma^{-\frac{\rho(1-\alpha)}{1-\rho+\alpha(1+\rho)}} (1-\rho)^{\frac{\rho(1-\alpha)}{1-\rho+\alpha(1+\rho)}} N^{\frac{-\alpha}{1-\rho+\alpha(1+\rho)}}.$$

We deduce $n^{a\star} > n^{a\star\star}$ and $e^{a\star} > e^{a\star\star}$.

In the anarchic environment, income per capita is given by

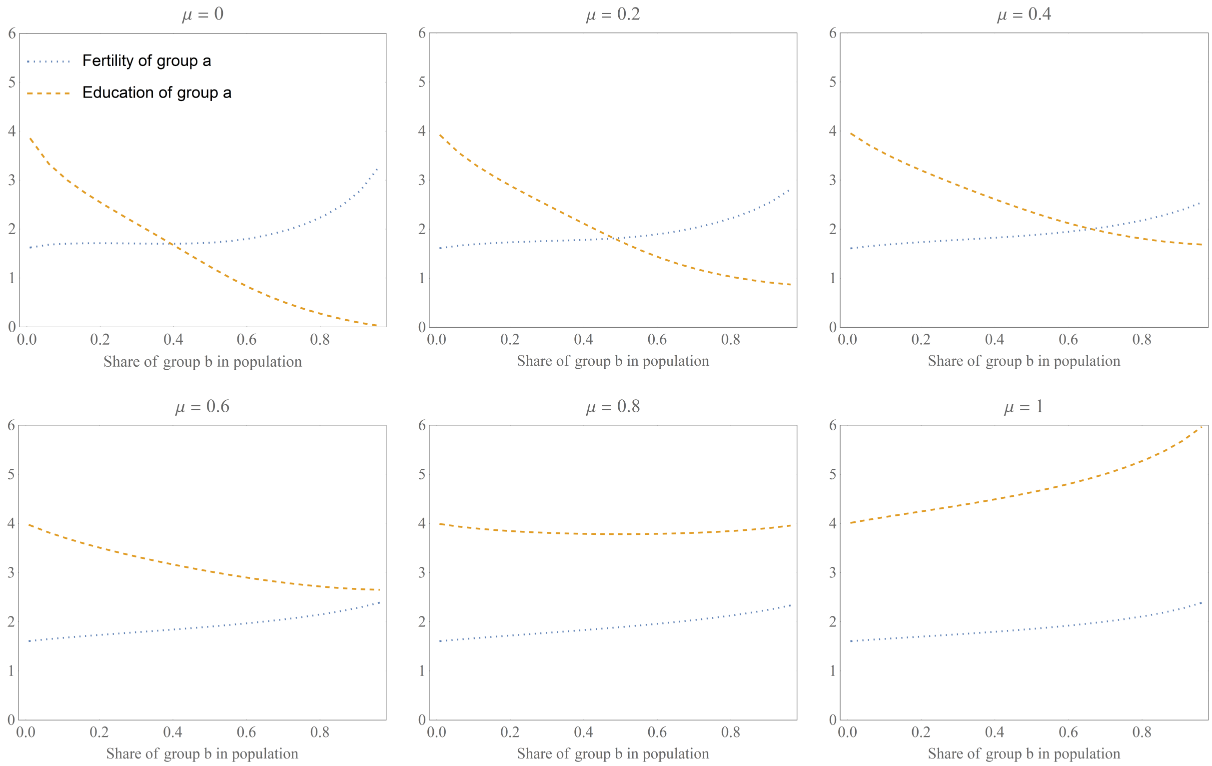
$$\frac{N^{(1-\alpha)}}{N^a}(n^{a\star})^{\rho(1-\alpha)-\alpha}.$$

In the peaceful environment, it is given by

$$\frac{N^{(1-\alpha)}}{N^a}(n^{a\star\star})^{\rho(1-\alpha)-\alpha}.$$

Income per capita increases with n^a if and only if $\rho > \alpha/(1-\alpha)$. Hence we deduce that income per capita is higher in the peaceful (resp. anarchic) environment if $\rho < \alpha/(1-\alpha)$ (resp. $\rho > \alpha/(1-\alpha)$).

E Role of μ for the Quantity Quality Tradeoff



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