

Early universe tomography with CMB and gravitational waves

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We discuss how one can reconstruct the thermal history of the Universe by combining cosmic microwave background (CMB) measurements and gravitational wave (GW) direct detection experiments. Assuming various expansion eras to take place after the inflationary reheating and before big bang nucleosynthesis, we show how measurements of the GW spectrum can be used to break the degeneracies associated with CMB data, the latter being sensitive to the total amount of cosmic expansion only. In this context, we argue that the expected constraints from future CMB and GW experiments can probe a scenario in which there exists late-time entropy production in addition to the standard reheating. We show that, for some cases, combining data from future CMB and GW direct detection experiments allows the determination of the reheating temperature, the amount of entropy produced and the temperature at which the standard radiation era started.

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I. INTRODUCTION

Our understanding of the evolution of the Universe is now becoming clearer owing to precise cosmological observations such as cosmic microwave background, large scale structure, type Ia supernovae and others. From such observations, we can obtain information about the current energy budget and the history of the Universe. In particular, the evolution after the time of big bang nucleosynthesis (BBN) to the present is relatively well understood. On the other hand, one can also probe the evolution during inflation since cosmic density fluctuations, which can be probed by cosmic microwave background (CMB) and large scale structure, are considered to be initially generated during that epoch.

Compared to the above-mentioned eras, the evolution, or thermal history, of the Universe during the period after inflation to BBN is relatively unexplored, certainly due to the lack of associated cosmological observables. Although, in the standard scenario, the Universe is considered to be radiation dominated until BBN (precisely speaking, until the radiation-matter equality epoch) after the inflaton reheating, the thermal history can be more complicated. In theories beyond the standard model of particle physics such as in supersymmetric models and string theory, there can exist some scalar fields (other than the inflaton, as for instance the moduli field) that are long-lived and can

dominate the energy density of the Universe. Their decay may also produce a huge amount of entropy thereby influencing the early universe history.

In the light of these considerations, it would be worth investigating how one could probe the thermal history during these epochs. In fact, some authors have investigated this issue by using observations of CMB [1–3] and direct detection of gravitational waves (GWs) in Refs. [4–7], while using the combination of both has been pushed forward in Ref. [8]. From CMB observations, we can probe the amplitude of the primordial scalar and tensor fluctuations as well as their scale dependencies around the so-called pivot scale. Notice that the time at which such a reference scale exited the Hubble radius during inflation depends on the amount of cosmic expansion from Hubble exit to the present times, which of course includes all of the above-mentioned postinflationary eras. As a result, by measuring the primordial power spectra in a given inflationary model, we can obtain information on the amount of the total cosmic expansion, i.e., the integrated thermal history since the end of inflation. This can be also applicable in GW direct detection experiments through measurements of the amplitude and scale dependence of tensor fluctuations. Not only that, direct detection of GWs could be used to probe the background evolution as the shape of the GW's spectrum is very sensitive to it. Thus, in the inflationary framework, detection/nondetection of GWs can give invaluable information on the thermal history of the Universe thereby allowing a “tomography” of these eras.

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In this paper, we investigate this issue by complementing observations of the CMB and GWs, paying particular attention to the period from the end of inflation to BBN. For this purpose, we first recall how one can constrain the thermal history of the Universe from these experiments. Although we have not detected any gravitational waves yet, CMB observations, such as those from the Wilkinson Microwave Anisotropy Probe (WMAP) [9], are precise enough to already give some constraints within some inflationary models [3]. However, in the near future, a direct detection of GWs could be achieved for some inflationary models that would allow us to combine both CMB and GW experiments. To see this in an explicit manner, we investigate the projected constraints on the thermal history of the Universe from future observations of CMB such as CMBpol and the PLANCK satellite combined with future direct detection GW experiments such as big bang observer (BBO) [10], DECI-hertz Interferometer Gravitational wave Observatory (DECIGO) [11] and Ultimate-DECIGO [12].

The organization of this paper is as follows. In the next section, we give a brief description on how CMB and GWs can probe or constrain the thermal history after inflation and justify their complementarity. We also give the current constraints on the thermal history within the so-called large field model of inflation coming from CMB using WMAP data. Then in Sec. III, we present our forecasts derived from a Fisher matrix analysis based on the above-mentioned future CMB and GW experiments. We conclude in the last section.

II. CMB AND GWS AS A PROBE OF THE THERMAL HISTORY OF THE UNIVERSE

In this section, we describe how one can probe the thermal history of the Universe with CMB and GW observations in the context of inflationary cosmology.

We assume that inflation is the origin of both scalar and tensor perturbations around the Friedmann-Lemaître-Robertson-Walker metric

$$ds^2 = -a^2(1 + 2\Phi)d\eta^2 + a^2[(1 - 2\Psi)\delta_{ij} + h_{ij}]dx^i dx^j, \quad (1)$$

where Ψ and Φ are the Bardeen potential and h_{ij} is the transverse and traceless spin two fluctuations. If inflation is driven by a slowly rolling scalar field ϕ , the quantum fluctuations of the field-metric system generate an almost scale invariant power spectrum for both kinds of perturbation. The observable quantities are those which are conserved on super-Hubble scales, that is, the comoving curvature for scalar perturbations, which reads in the longitudinal gauge,

$$\zeta(\eta, \mathbf{x}) \equiv \Psi(\eta, \mathbf{x}) + \mathcal{H} \frac{\delta\phi(\eta, \mathbf{x})}{\phi'}, \quad (2)$$

where $\mathcal{H} \equiv aH$ is the conformal Hubble parameter and a prime denotes derivatives with respect to the conformal

time η . The tensor modes h_{ij} are themselves gauge invariant and conserved on super-Hubble scales. It is convenient to decompose them on their two polarization states h_λ in Fourier space as

$$h_{ij}(\eta, \mathbf{x}) = \sum_{\lambda=+, \times} \int \frac{dk^3}{(2\pi)^{3/2}} h_\lambda(\eta, \mathbf{k}) \varepsilon_{ij}^\lambda e^{i\mathbf{k} \cdot \mathbf{x}}, \quad (3)$$

where ε_{ij}^λ are the polarization tensors satisfying $\varepsilon_{ij}^\lambda \varepsilon_{\lambda'}^{ij} = 2\delta_{\lambda\lambda'}$.

At first order in the slow-roll formalism, the primordial power spectrum for the scalars is given by [13]

$$\begin{aligned} \mathcal{P}_\zeta &\equiv \frac{k^3}{2\pi^2} |\zeta|^2 \\ &\simeq \frac{H_*^2}{8\pi^2 M_{\text{pl}}^2 \epsilon_{1*}} \left[1 - 2(C + 1)\epsilon_{1*} - C\epsilon_{2*} \right. \\ &\quad \left. - (2\epsilon_{1*} + \epsilon_{2*}) \ln \frac{k}{k_*} \right], \end{aligned} \quad (4)$$

while the tensor spectrum (sum of polarization included) reads

$$\mathcal{P}_h \equiv \frac{2k^3}{\pi^2} |h|^2 \simeq \frac{2H_*^2}{\pi^2 M_{\text{pl}}^2} \left[1 - 2(C + 1)\epsilon_{1*} - 2\epsilon_{1*} \ln \frac{k}{k_*} \right]. \quad (5)$$

In these equations, $M_{\text{pl}}^2 = 1/(8\pi G)$ stands for the reduced Planck mass, $C = -2 + \ln 2 + \gamma_E \simeq 0.73$ with γ_E being the Euler constant, and ϵ_i are the slow-roll parameters which are defined as¹

$$\epsilon_1 \equiv -\frac{d \ln H}{dN}, \quad \epsilon_2 \equiv \frac{d \ln \epsilon_1}{dN}, \quad (8)$$

where $N \equiv \ln a$ is the number of e-folds. The spectral index defined as $n_s - 1 = d \ln \mathcal{P}_\zeta / d \ln k$ is given by

$$n_s = 1 - 2\epsilon_{1*} - \epsilon_{2*}. \quad (9)$$

The tensor-to-scalar ratio r , which is usually used to quantify the amplitude of the tensor mode, is given by

$$r \equiv \frac{\mathcal{P}_h}{\mathcal{P}_\zeta} = 16\epsilon_{1*}. \quad (10)$$

¹In the literature, the slow-roll parameters defined using the potential for the inflaton $V(\phi)$ are also used, which are given by $\epsilon_V = (1/2)M_{\text{pl}}^2 (V'/V)^2$ and $\eta_V = M_{\text{pl}}^2 (V''/V)$ with a prime representing the derivative with respect to ϕ . The correspondence between $\{\epsilon_V, \eta_V\}$ and $\{\epsilon_1, \epsilon_2\}$ at leading order in slow-roll parameters is

$$\epsilon_1 = \epsilon_V, \quad \epsilon_2 = 2\epsilon_V - 2\eta_V. \quad (6)$$

With these parameters, the spectral index and the tensor-to-scalar ratio are respectively given by

$$n_s = 1 - 6\epsilon_{V*} + 2\eta_{V*}, \quad r = 16\epsilon_{V*}. \quad (7)$$

An asterisk “*” indicates that the quantities have to be evaluated at the time when the pivot mode k_* crossed the Hubble radius during inflation, i.e., the solution of

$$k_* = a(\eta_*)H(\eta_*). \quad (11)$$

Here, we neglect the running of the spectral index, which will not affect our results for CMB. However, we note that such truncation can give large deviation for the power spectrum from the one that obtained exact numerical calculation in some models [14].

A. CMB constraints on the postinflationary universe history

1. Standard scenario

The power spectrum functional forms of Eqs. (4) and (5) are usually compared to the current CMB data to constrain the slow-roll (Hubble flow) parameters, or equivalently the spectral index and tensor-to-scalar ratio [1,9,15–20]. This is done by choosing a pivot scale k_* in the observable range, typically $k_* = 0.05 \text{ Mpc}^{-1}$. However, if one assumes an inflationary model, there is much more to say. Indeed, the tensor-to-scalar ratio, the spectral index and all other observable quantities are completely determined by the inflaton potential $V(\phi)$. As discussed earlier, they have to be evaluated at the time η_* , which can be determined by solving Eq. (11). In order to obtain η_* , it is compulsory to make assumptions on the subsequent thermal history of the Universe, i.e., including at least the reheating and preheating stages. In terms of the number of e-folds during inflation, the physical pivot wavenumber is given by

$$\frac{k_*}{a} = \frac{k_*}{a_0} (1 + z_{\text{end}}) e^{N_{\text{end}} - N}, \quad (12)$$

where z_{end} is the redshift at which inflation ended, after N_{end} e-folds. As shown in Refs. [1–3,21], a convenient way to calculate z_{end} is to introduce the so-called “reheating parameter”:

$$R_{\text{rad}} \equiv \frac{a_{\text{end}}}{a_{\text{reh}}} \left(\frac{\rho_{\text{end}}}{\rho_{\text{reh}}} \right)^{1/4}. \quad (13)$$

The quantity R_{rad} encodes all of our ignorance of the subsequent thermal evolution after the end of inflation and quantifies any deviations from a pure radiation era. In fact, by assuming instantaneous transition between the inflationary epoch to inflaton oscillating era and inflaton oscillating to radiation dominated eras, one has

$$1 + z_{\text{end}} = \frac{1}{R_{\text{rad}}} \left(\frac{\rho_{\text{end}}}{\bar{\rho}_{\gamma_0}} \right)^{1/4}, \quad (14)$$

where ρ_{end} is the energy density of the universe at the end of inflation and $\bar{\rho}_{\gamma_0}$ is the energy density of radiation today, eventually rescaled by any change in the number of gravitating relativistic degrees of freedom. In terms of the cosmological parameters today,

$$\bar{\rho}_{\gamma_0} = \mathcal{Q}_{\text{reh}} \rho_{\gamma_0} = 3 \mathcal{Q}_{\text{reh}} \frac{H_0^2}{M_{\text{pl}}^2} \Omega_{r_0}, \quad (15)$$

where we have defined

$$\mathcal{Q}_{\text{reh}} \equiv \frac{g_{\text{reh}}}{g_0} \left(\frac{g_{s0}}{g_{\text{reh}}} \right)^{4/3}. \quad (16)$$

Here, g_s and g , respectively, denote the number of entropic and energetic relativistic degrees of freedom at the epoch of interest. H_0 and Ω_{r_0} are the Hubble parameter and radiation density parameter today.

As shown in Ref. [21], using energy conservation, Eq. (13) can be recast into two other strictly equivalent forms:

$$\begin{aligned} \ln R_{\text{rad}} &= \frac{1}{4} (N_{\text{reh}} - N_{\text{end}}) (3\bar{w}_{\text{reh}} - 1) \\ &= \frac{1 - 3\bar{w}_{\text{reh}}}{12(1 + \bar{w}_{\text{reh}})} \ln \left(\frac{\rho_{\text{reh}}}{\rho_{\text{end}}} \right), \end{aligned} \quad (17)$$

with $\bar{w}_{\text{reh}}$ standing for the *mean* equation of state parameter during the inflaton oscillating era. Using R_{rad} , Eq. (11) is solved for the e-fold time $\Delta N_* \equiv N_* - N_{\text{end}}$, verifying [21]

$$\begin{aligned} \Delta N_* &= -\ln R_{\text{rad}} + N_0 - \frac{1}{4} \ln \left(\frac{H_*^2}{M_{\text{pl}}^2 \epsilon_{1*}} \right) \\ &\quad + \frac{1}{4} \ln \left(\frac{3}{\epsilon_{1*}} \frac{V_{\text{end}}}{V_*} \frac{3 - \epsilon_{1*}}{3 - \epsilon_{\text{end}}} \right), \end{aligned} \quad (18)$$

where the constant N_0 stands for

$$N_0 \equiv \ln \left(\frac{k_*/a_0}{\bar{\rho}_{\gamma_0}^{1/4}} \right). \quad (19)$$

Let us emphasize that the right-hand side of Eq. (18) usually depends on ΔN_* itself, but in a completely algebraic way once the model, i.e., $V(\phi)$, is specified. It can also be simplified further by using $\epsilon_{1*} \ll 1$, $\epsilon_{\text{end}} = 1$ and Eq. (4) as

$$\begin{aligned} \Delta N_* &= -\ln R_{\text{rad}} + N_0 - \frac{1}{4} \ln (8\pi^2 \mathcal{P}_*) \\ &\quad - \frac{1}{4} \ln \frac{r}{72} + \frac{1}{4} \ln \frac{V_{\text{end}}}{V_*}. \end{aligned} \quad (20)$$

As an example, we have plotted in Fig. 1 the predicted values for ϵ_{1*} , ϵ_{2*} , as well as the spectral index n_s and tensor-to-scalar ratio r for the chaotic inflation models with $V(\phi) \propto \phi^p$. We show the cases of $p = 1, 2, 3, 4, 6$, and 10 . For each p , there is a range of possible ΔN_* since the reheating should occur from the end of inflation to the BBN epoch, whose values are indicated in the figure at both end points and for each case of p . Along with the theoretical predictions, we also show 1σ and 2σ confidence intervals associated with the WMAP7 data [3,22,23] and Hubble Space Telescope (HST) data [24]. From the figure, one can

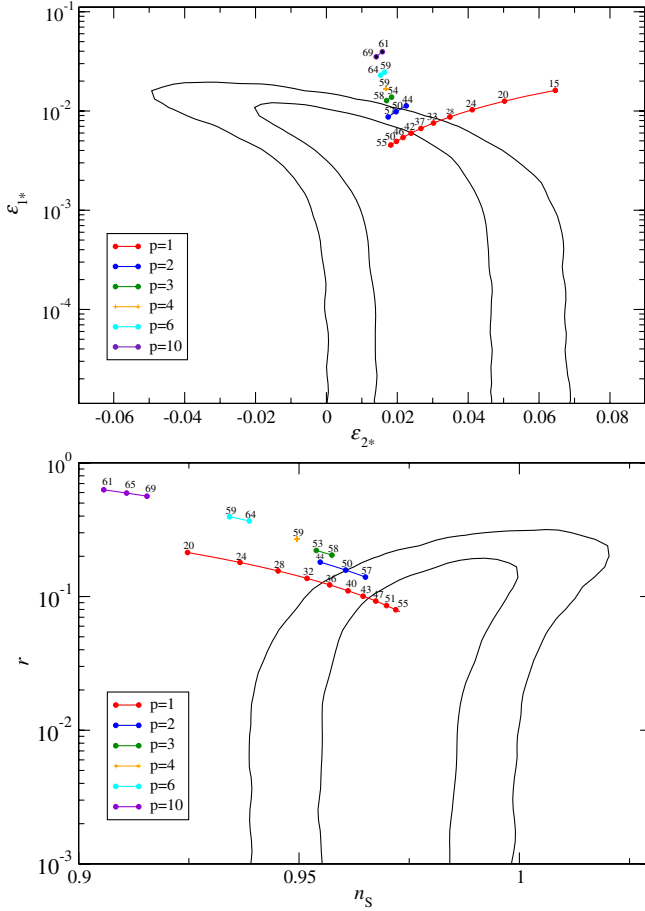


FIG. 1 (color online). WMAP7 constraints in the plane $(\epsilon_{1*}, \epsilon_{2*})$ (top) and (n_s, r) (bottom) compared with the large field model predictions obtained by solving Eq. (18) for various monomial potentials $V(\phi) \propto \phi^p$. The annotated values are those of $|\Delta N_*|$ and they range for a reheating occurring as low as BBN to an instantaneous reheating after inflation.

easily see that inflation models with $p \geq 3$ are excluded by current data. Even for the cases with $p = 2$ and 1, there is a lower bound on $|\Delta N_*|$ to be consistent with WMAP7, which can be translated into the constraints on the thermal history of the Universe.

2. Nonstandard scenarios

Up to here, we have assumed the “standard” scenario in which once the reheating from the inflaton is completed, the Universe becomes radiation dominated until the radiation-matter equality $z_{\text{eq}} \sim 10^4$. However, this standard scenario could be modified. For instance, one could assume that, inserted after the reheating era, there is a phase of evolution dominated by a gravitating source X , characterized by an equation of state parameter w_X . As shown in Ref. [21], one can define a parameter R_X exactly as in Eq. (13) by

$$R_X \equiv \frac{a_{\text{xini}}}{a_{\text{xend}}} \left(\frac{\rho_{\text{xini}}}{\rho_{\text{xend}}} \right)^{1/4}, \quad (21)$$

for which it is immediate to verify that Eq. (17) also applies using the mean value $\bar{w}_X$ and by the replacement “end $\rightarrow$ xini,” “reh $\rightarrow$ xend.” From this definition, one can check that all equations are unchanged, and in particular Eq. (18), by replacing R_{rad} with $R_{\text{rad}} R_X$. Assuming another Y era to take place just after the X era and before the radiation dominated era, we would reach exactly the same conclusions by replacing $R_{\text{rad}} R_X$ with $R_{\text{rad}} R_X R_Y$. In other words, CMB can only constrain the overall thermal history and only feels those parameters, $R_{\text{rad}} R_X R_Y \dots$, multiplied. Let us also notice that the correction coefficient entering Eq. (15) is now given by $\mathcal{Q}_{\text{yend}}$ instead of $\mathcal{Q}_{\text{reh}}$.

As a well motivated example, and the one we will be discussing in Sec. III, such a situation is typical of scenarios in which a late-decaying massive scalar field, denoted

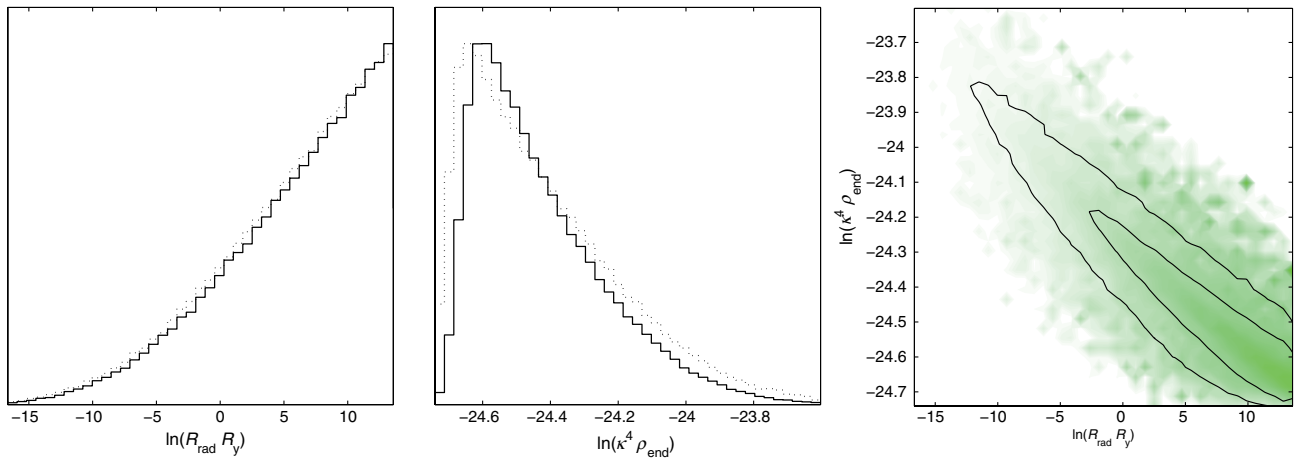


FIG. 2 (color online). Marginalized posterior probability distributions (solid line) and mean likelihood (dotted line) from WMAP7 and HST data for massive inflation with $p = 2$. The right figure shows the one- and two-sigma confidence intervals of the two-dimensional marginalized posterior in the plane $(R_{\text{rad}} R_Y, \kappa^4 \rho_{\text{end}})$.

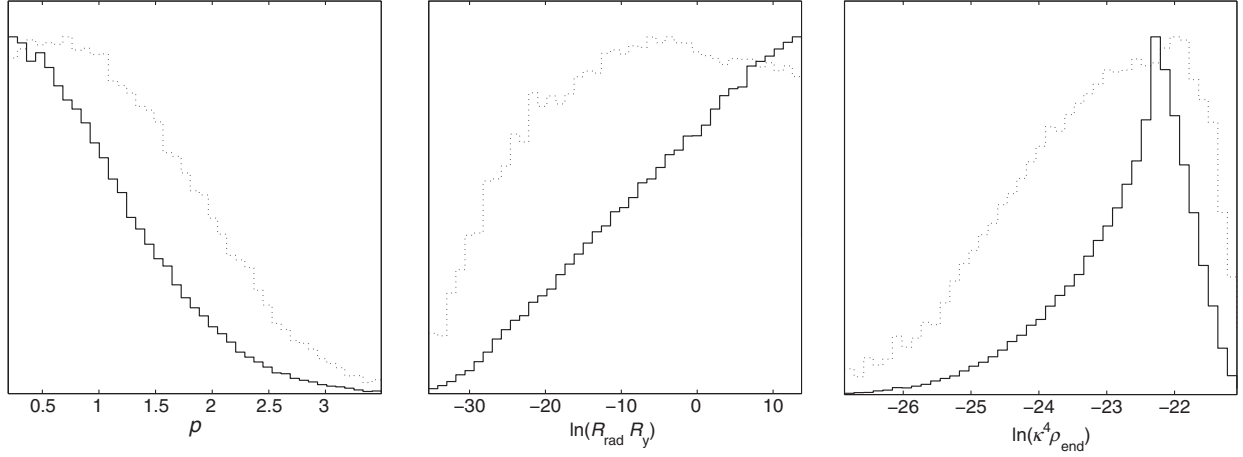


FIG. 3. Marginalized posterior probability distributions (solid line) and mean likelihood (dotted line) from WMAP7 and HST data for the large field potential with a power law exponent p being varied in $0.2 < p < 5$.

as σ hereafter, produces a large amount of entropy well after the inflaton reheating. In that case, the X era is a short radiation dominated era standing just after inflaton reheating and before the σ domination. For such a scenario, one has $R_x = 1$ whereas, the Y era would precisely correspond to the field domination era having $w_y = 0$ such that R_y can only take negative values (quadratic potential). In Figs. 2–4 we have represented the WMAP7 constraints on the combination $R_{\text{rad}} R_x R_y$ (for $R_x = 1$) for various large field models, either massive as the scenario we are interested in or, for any values of p , the power law exponent of the inflaton potential. The method we have used is the same as in Ref. [3] and we do not repeat the details here. As already shown in Fig. 1, large values of $|\Delta N_*|$, which corresponds

to $R_{\text{rad}} R_y < 0$, give bigger r and more red-tilted spectral index n_s . Hence smaller values of $R_{\text{rad}} R_y$ are disfavored, which means that the current observations already give some constraints on the thermal history of the Universe. The posteriors for ρ_{end} are also depicted and since it is essentially determined by the amplitude of the curvature perturbation, it is well bounded [3].

In some literature, when one considers a late-time entropy production scenario, the parameter F is adopted to quantify the amount of entropy production, instead of R_y , and is defined as

$$F \equiv \frac{s_{\text{yend}} a_{\text{yend}}^3}{s_{\text{yini}} a_{\text{yini}}^3}. \quad (22)$$

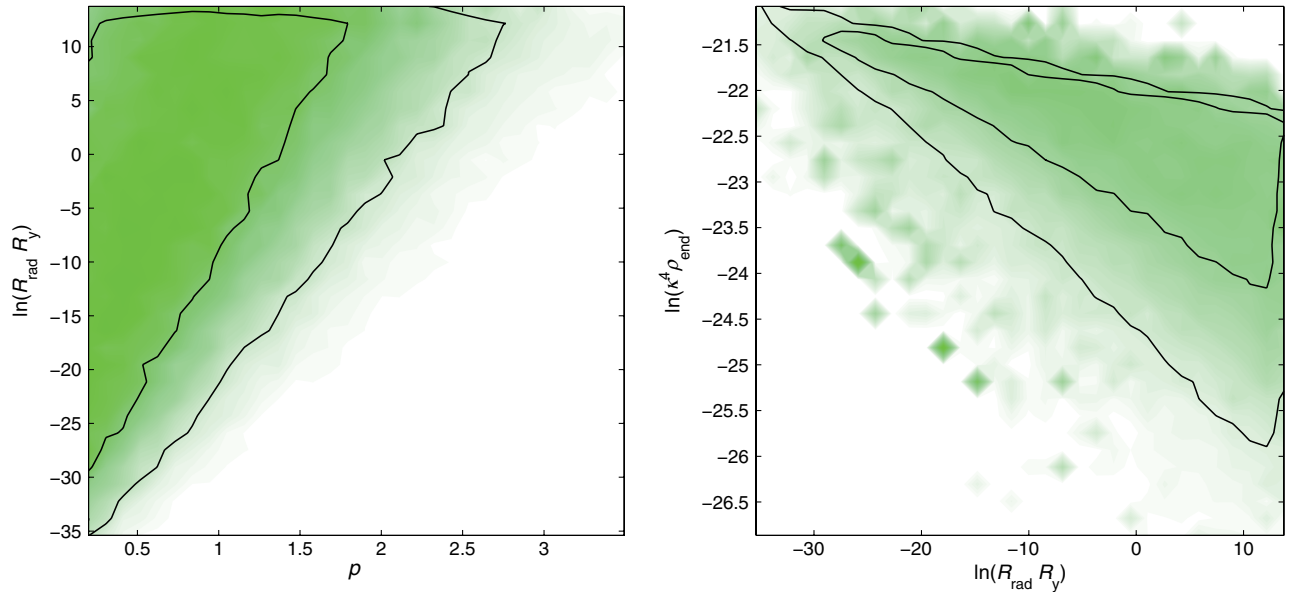


FIG. 4 (color online). Two-dimensional posteriors with 1σ and 2σ confidence regions from WMAP7 and HST data in the $(p, R_{\text{rad}} R_y)$ plane (left) and in the $(R_{\text{rad}} R_y, \kappa^4 \rho_{\text{end}})$ plane (right).

The subscript “yini” and “yend” indicate that the quantities are the ones evaluated at the time when the Y era starts and ends. Contrary to the definition of the R_{rad} and R_x parameters which only require that total energy density is conserved, the definition of F is convenient if thermalization is achieved. In our scenario, at the beginning of the Y era, and also just after its end, the Universe is assumed to be radiation dominated, and if thermalized, the entropy density is dominated by relativistic species. In that situation, it is straightforward to show that

$$R_y = F^{-1/3} \left(\frac{Q_{\text{yini}}}{Q_{\text{yend}}} \right)^{1/4}, \quad (23)$$

where Q is defined as in Eq. (16) for the epochs of interest.

B. Stochastic gravitational waves background

As discussed in the previous section, CMB can constrain the amount of entropy production but this will remain completely degenerated with reheating from the inflaton as the only quantities appearing in the determination of ΔN_* are the product of R parameters such as $R_{\text{rad}} R_x R_y$. As we show below, this is not the same for stochastic gravitational waves of inflationary origin: they feel these parameters in a different way which can be used to break the degeneracies, thereby performing the tomography of the history of the Universe. In particular, direct detection experiments such as BBO and DECIGO, which probe the frequency range $f \sim \mathcal{O}(1)$ Hz, would give new and complementary information with respect to CMB.

In order to discuss the amplitude of stochastic GWs, one usually uses the spectrum of the energy density of GWs normalized by the critical energy density ρ_{crit} . From the pseudo-stress tensor, assuming a stochastic background in which spatial and time averages are identical, one gets [25,26]

$$\begin{aligned} \Omega_{\text{gw}} &\equiv \frac{1}{\rho_{\text{crit}}} \frac{d\rho_{\text{gw}}}{d \ln k} = \frac{1}{12} \left(\frac{k}{aH} \right)^2 \frac{k^3}{\pi^2} \sum_{\lambda} |h_{\lambda}|^2 \\ &= \frac{1}{12} \left(\frac{k}{aH} \right)^2 \mathcal{P}_h^{(\text{obs})}(k). \end{aligned} \quad (24)$$

The observed power spectrum $\mathcal{P}_h^{(\text{obs})}$ at the time of interest (e.g., today) is given by Eq. (5) times a transfer function encoding the evolution of the sub-Hubble modes

$$\mathcal{P}_h^{(\text{obs})}(k) = \mathcal{P}_h(k) \mathcal{T}^2(k). \quad (25)$$

The transfer function $\mathcal{T}$ can be evaluated analytically since, after inflation, the tensor perturbations are decoupled from other sources and their equation of evolution in Fourier space is given by

$$h'' + 2\mathcal{H}h' + k^2 h = 0, \quad (26)$$

where for simplicity the polarization index has been dropped. Assuming for the moment that the background

is dominated by a fluid having a constant equation of state parameter w , the solution reads

$$h(\eta, k) \propto (k\eta)^{(3w-3)/(6w+2)} J_{\frac{3-3w}{6w+2}}(k\eta). \quad (27)$$

For super-Hubble modes ($k\eta \ll 1$) we recover that $h(\eta, k)$ is constant while for sub-Hubble wave numbers one has

$$|h(\eta, k)|^2 \propto_{k\eta \gg 1} a^{-2}. \quad (28)$$

Since h stays constant on super-Hubble scales, the transfer function is determined only by the era at which a given mode k reenters the Hubble radius. Every mode k will then be damped till the present time by a factor $(a_k/a_0)^2$ where a_k is the solution of

$$k = a_k H(a_k), \quad (29)$$

and the scale factor today will be taken as unity $a_0 = 1$.

1. Radiation and matter eras

We start with the case for which the mode reenters during the radiation era. Assuming an instantaneous transition, the result can be approximated by squaring Eq. (29) and using the first Friedmann-Lemaître equation, $H^2 = \rho/(3M_{\text{pl}}^2)$. One gets

$$a_k^2 = \frac{3M_{\text{pl}}^2 k^2}{\rho(a_k)} = \frac{3M_{\text{pl}}^2 k^2}{\rho_{\text{eq}} a_{\text{eq}}^4} a_k^4, \quad (30)$$

where $\rho \propto a^{-4}$ is used. The solution reads

$$a_k = a_{\text{eq}} \frac{k_{\text{eq}}}{k} = \frac{1}{1+z_{\text{eq}}} \frac{k_{\text{eq}}}{k}, \quad (31)$$

with

$$k_{\text{eq}} \equiv \frac{1}{\sqrt{3}M_{\text{pl}}} \frac{\sqrt{\rho_{\text{eq}}}}{1+z_{\text{eq}}} \simeq H_0 \frac{\Omega_{m_0}}{\sqrt{\Omega_{r_0}}}. \quad (32)$$

In the rightmost side, Ω_{m_0} and Ω_{r_0} are the matter and radiation density parameters today and $\rho_{\text{eq}} \simeq 2\rho_{\gamma_0}/a_{\text{eq}}^4$ is the energy density at equality. For all modes entering the Hubble radius in the radiation era, Eq. (31) immediately gives the transfer function for $k > k_{\text{eq}}$ [27]:

$$\mathcal{T}_{\text{rad}}(k) \simeq H_0^2 \Omega_{m_0} \frac{k_{\text{eq}}}{k} \simeq \frac{H_0 \sqrt{\Omega_{r_0}}}{k}. \quad (33)$$

Similarly, starting from Eq. (29), the matter era solution reads

$$\mathcal{T}_{\text{mat}}(k) \simeq \frac{H_0^2 \Omega_{m_0}}{k^2} \Theta(k < k_{\text{eq}}), \quad (34)$$

where $\Theta(x)$ is the Heaviside step function. In fact, considering a mixture of radiation and matter, one would find

$$\mathcal{T}_{\text{eq}}(k) \simeq \frac{H_0^2 \Omega_{m_0}}{2k^2} \left[1 + \sqrt{1 + 4 \left(\frac{k}{k_{\text{eq}}} \right)^2} \right], \quad (35)$$

which gives back Eqs. (33) and (34) in the appropriate limits. Although not specified, the above transfer functions are unity for super-Hubble modes, i.e., $k < H_0^{-1}$. Combined with Eq. (24) we recover the well-known result [26] that Ω_{gw} is constant for $k > k_{\text{eq}}$ and decays as $k^{-2} \propto f^{-2}$ for modes entering during matter domination.

The same line of reasoning can be applied to the post-inflationary universe assuming the nonstandard history. Just after reheating we assume the Universe is in an X era (radiation dominated in our scenario), and then becomes Y dominated, e.g., driven by an oscillating scalar field, which finally decays into radiation.

2. Nonstandard postinflationary eras

If an observable mode today entered the Hubble radius during the Y era, Eq. (29) could be dealt exactly as the radiation-era case, provided w_y remains constant. The equivalent of Eq. (30) now reads

$$a_k^2 = \frac{3M_{\text{pl}}^2 k^2}{\rho_{\text{yend}} a_{\text{yend}}^{3+3w_y}} a_k^{3+3w_y}, \quad (36)$$

whose solution can be recast into

$$a_k = a_{\text{yend}} \left(\frac{k_{\text{yend}}}{k} \right)^{2/(1+3w_y)}. \quad (37)$$

This expression can be further simplified by remarking that

$$a_{\text{yend}} = a_0 \mathcal{Q}_{\text{yend}}^{1/4} \left(\frac{\rho_{\gamma_0}}{\rho_{\text{yend}}} \right)^{1/4}, \quad (38)$$

since the end of the Y era matches with the beginning of the standard radiation dominated era. The wave number k_{yend} corresponds to a mode entering the Hubble radius just at the end of the Y era, i.e., at the beginning of the radiation era:

$$k_{\text{yend}} \equiv \frac{a_{\text{yend}}}{\sqrt{3}M_{\text{pl}}} \sqrt{\rho_{\text{yend}}} = \frac{1}{\sqrt{3}M_{\text{pl}}} (\tilde{\rho}_{\gamma_0} \rho_{\text{yend}})^{1/4}, \quad (39)$$

where the last equality comes from Eq. (38). At this point, Eq. (37) shows that the quantities k_{yend} and w_y are observable and completely determined by the measurement of $\Omega_{\text{gw}}(k)$. As a result, we should try to express all quantities in terms of them, and in particular the redshift at which the Y era ended. From Eq. (38), one gets

$$1 + z_{\text{yend}} = \sqrt{3} \frac{k_{\text{yend}} M_{\text{pl}}}{\tilde{\rho}_{\gamma_0}^{1/2}}. \quad (40)$$

We finally get the transfer function during the Y era:

$$\mathcal{T}_y(k) \simeq \frac{\tilde{\rho}_{\gamma_0}^{1/2}}{\sqrt{3}k_{\text{yend}}M_{\text{pl}}} \left(\frac{k_{\text{yend}}}{k} \right)^{2/(1+3w_y)} \Theta(k > k_{\text{yend}}) + \Theta(k < k_{\text{yend}}). \quad (41)$$

It remains now to deal with the modes entering the Hubble radius before, i.e., either during X domination or during reheating. The calculations are again the same although the scale factor at the end of the X era reads

$$a_{\text{xend}} = a_{\text{yend}} \frac{a_{\text{xend}}}{a_{\text{yend}}} = a_{\text{yend}} R_y \left(\frac{\rho_{\text{yend}}}{\rho_{\text{yini}}} \right)^{1/4}, \quad (42)$$

where Eq. (21) has been used for $a_{\text{xend}}/a_{\text{yend}}$. One can further simplify this expression by using Eq. (38) to get

$$a_{\text{xend}} = R_y \left(\frac{\tilde{\rho}_{\gamma_0}}{\rho_{\text{xend}}} \right)^{1/4}. \quad (43)$$

Defining k_{xend} the wave number of a mode entering the Hubble radius at the end of the X era, we have

$$k_{\text{xend}} \equiv \frac{a_{\text{xend}}}{\sqrt{3}M_{\text{pl}}} \sqrt{\rho_{\text{xend}}} = \frac{R_y}{\sqrt{3}M_{\text{pl}}} (\tilde{\rho}_{\gamma_0} \rho_{\text{xend}})^{1/4}, \quad (44)$$

such that the corresponding redshift can be expressed in terms of observable quantities as

$$1 + z_{\text{xend}} = \sqrt{3} \frac{k_{\text{xend}} M_{\text{pl}}}{R_y \tilde{\rho}_{\gamma_0}^{1/2}}. \quad (45)$$

The transfer function during this era is again given by $|a_k|$ and reads

$$\mathcal{T}_x(k) \simeq \frac{R_y^2 \tilde{\rho}_{\gamma_0}^{1/2}}{\sqrt{3}k_{\text{xend}}M_{\text{pl}}} \left(\frac{k_{\text{xend}}}{k} \right)^{2/(1+3w_x)} \Theta(k > k_{\text{xend}}) + \Theta(k < k_{\text{xend}}). \quad (46)$$

Finally, in order to deal with modes entering the Hubble radius during reheating (the inflaton oscillation dominated era), we similarly express a_{reh} in terms of a_{xend} to get

$$a_{\text{reh}} = R_x R_y \left(\frac{\tilde{\rho}_{\gamma_0}}{\rho_{\text{reh}}} \right)^{1/4}. \quad (47)$$

Again, the wave number crossing the end of reheating is given by

$$k_{\text{reh}} \equiv \frac{a_{\text{reh}}}{\sqrt{3}M_{\text{pl}}} \sqrt{\rho_{\text{reh}}} = \frac{R_x R_y}{\sqrt{3}M_{\text{pl}}} (\tilde{\rho}_{\gamma_0} \rho_{\text{reh}})^{1/4}, \quad (48)$$

and the redshift at which reheating ended reads

$$1 + z_{\text{reh}} = \sqrt{3} \frac{k_{\text{reh}} M_{\text{pl}}}{R_x R_y \tilde{\rho}_{\gamma_0}^{1/2}}. \quad (49)$$

Therefore, the transfer function during reheating is

$$\mathcal{T}_{\text{reh}}(k) \simeq \frac{R_x^2 R_y^2 \tilde{\rho}_{\gamma_0}^{1/2}}{\sqrt{3}k_{\text{reh}}M_{\text{pl}}} \left(\frac{k_{\text{reh}}}{k} \right)^{2/(1+3w_{\text{reh}})} \Theta(k > k_{\text{reh}}) + \Theta(k < k_{\text{reh}}), \quad (50)$$

which shows the influence of R_x and R_y , making them observable in the gravitational wave spectrum. However, contrary to CMB, they are no longer degenerated with R_{rad} . Conversely, using both CMB and Ω_{gw} , one expects to be able to disambiguate the effects of R_{rad} , R_x and R_y . The same reasoning could be extended to another Z era inserted somewhere.

To summarize, one finally gets

$$\Omega_{\text{gw}}(k) \simeq \frac{k^2}{12H_0^2} \mathcal{T}_{\text{eq}}^2(k) \mathcal{T}_y^2(k) \mathcal{T}_x^2(k) \mathcal{T}_{\text{reh}}^2(k) \mathcal{P}_h(k), \quad (51)$$

where the three transfer functions are given by Eqs. (35), (41), (46), and (50) and their respective pivot wave numbers by Eqs. (32), (39), (44), and (48). All of those four wave numbers therefore correspond to frequencies of the gravitational wave f_{eq} , f_{yend} , f_{xend} and f_{reh} .

III. FORECASTS ON THERMAL HISTORY FROM FUTURE CMB AND GW EXPERIMENTS

In this section, we discuss how and to what extent we can probe the thermal history of the Universe with future CMB and direct detection of GW experiments.

A. An illustrative scenario

For illustrative purposes, we consider in the following the specific scenario mentioned earlier. Inflation is driven by a massive field ϕ having the potential $V(\phi) = (1/2)m^2\phi^2$ with m being the inflaton mass. Furthermore there also exists another scalar field, denoted as σ that comes to dominate the Universe at later times. After inflation, ϕ oscillates around the minimum of its potential and the reheating (inflaton oscillation dominated) era is matterlike [28] till the Universe thermalizes and becomes radiation dominated (the X era). The reheating temperature T_{reh} , associated with the energy density ρ_{reh} , refers to the time at which the reheating era ends and the radiation X era starts. Then the field σ begins to oscillate at some epoch and starts the σ oscillation dominated era, referred to as the Y era in the previous section. We moreover assume that σ decays after it dominates the Universe to start the usual radiation dominated period. Thus, the thermal history of the Universe proceeds as follows: inflation $\rightarrow$ reheating. The final radiation dominated era continues until the radiation-matter equality, just before the recombination epoch. As discussed in the previous section, depending on the duration of each era, the predictions for CMB and GW spectra are different, from which we can probe the thermal history.

Under these hypothesis, we can express all the quantities of the previous section in terms of temperatures and frequencies. With $R_x = 1$, assuming that $g = g_s = g_*$ at the time of “yend,” “xend,” and “reh” and using Eqs. (22), (39), (44), and (48), one obtains

$$\begin{aligned} f_{\text{yend}} &\approx 0.2 \text{ Hz} \left(\frac{g_{\text{yend}}}{100} \right) \left(\frac{T_\sigma}{10^7 \text{ GeV}} \right), \\ f_{\text{xend}} &\approx 0.2 \text{ Hz} \left(\frac{g_{\text{yend}}}{100} \right) \left(\frac{T_\sigma}{10^7 \text{ GeV}} \right) F^{2/3}, \\ f_{\text{reh}} &\approx 0.2 \text{ Hz} \left(\frac{g_{\text{yend}}}{100} \right) \left(\frac{T_{\text{reh}}}{10^7 \text{ GeV}} \right) F^{-1/3}, \end{aligned} \quad (52)$$

where T_σ is the temperature at which σ decays into radiation.

For CMB constraints, one can also simplify the quantity $R_{\text{rad}}R_y$ in terms of temperatures as

$$\begin{aligned} \ln(R_{\text{rad}}R_y) &= \frac{1}{3} \ln \frac{T_{\text{reh}}}{M_{\text{pl}}} - \frac{1}{3} \ln F - \frac{1}{12} \ln \frac{\rho_{\text{end}}}{M_{\text{pl}}^4} \\ &+ \frac{1}{12} \ln \left(\frac{\pi^2}{30} \frac{g_{\text{reh}} g_{\text{yend}}}{g_{\text{xend}}} \right), \end{aligned} \quad (53)$$

where $\bar{w}_{\text{reh}} = 0$ has been used. Since T_{reh} is assumed to be bigger than T_σ , we have the expected hierarchy $f_{\text{reh}} > f_{\text{xend}} > f_{\text{yend}}$. For the effective degrees of freedom, we assume those of the standard model of particle physics, i.e., $g_{\text{reh}} = g_{\text{xend}} = g_{\text{yend}} = 106.75$ in the following analysis. Notice that this is certainly not verified but the influence of g_* remains very small in the final forecasts, the sensible quantities being logarithmic [see Eq. (53)].

Some example plots for the GW spectrum are shown in Fig. 5. The GW spectrum is obtained by numerically integrating Eq. (26), using the WKB approximation ($h \propto e^{\pm i k \eta} / a$) for the oscillating phase. To see how the spectrum shape depends on the reheating temperatures and F , we show several cases for these parameters. For reference, the sensitivity frequency bands for BBO/Fabry-Perot

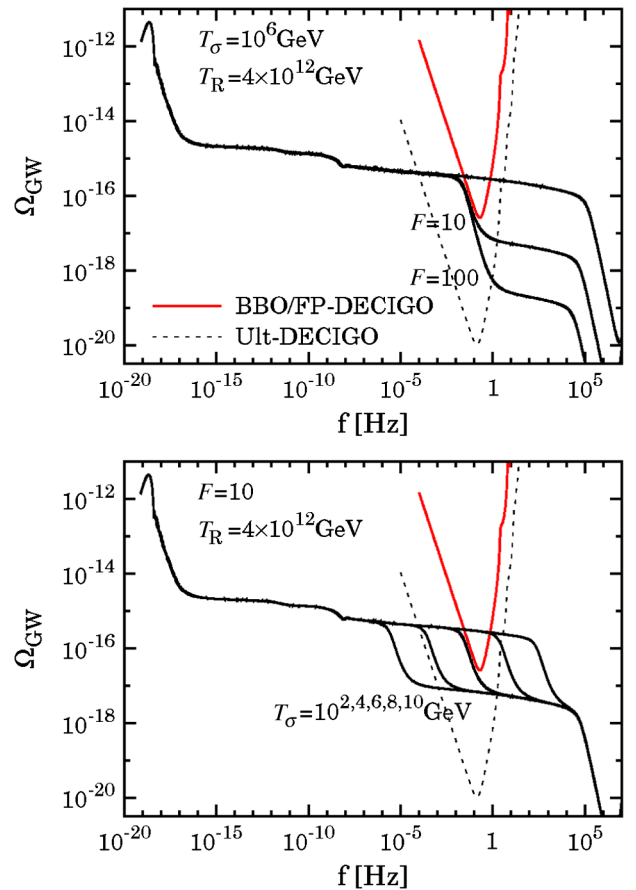


FIG. 5 (color online). GW spectrum for several values of F ranging from $F = 0$ to $F = 100$ at fixed T_σ (top). The bottom panel shows the spectra at fixed $F = 10$ and for various values of T_σ . The reheating temperature T_{reh} from the inflaton is fixed at $T_{\text{reh}} = 10^{12}$ GeV in these figures.

(FP)-DECIGO and Ultimate-DECIGO are also depicted (for the specifications of these experiments, see Table II). In some cases, the change of the spectrum can be traced with future experiments, in particular, for Ultimate-DECIGO. In such a case, the parameters such as the reheating temperatures and F can be well determined, which is going to be studied by using Fisher matrix analysis in the following.

B. Fisher matrix analysis

To forecast constraints from future experiments, we use a Fisher matrix analysis for both CMB [31] and GW direct detection [32]. Under the assumption of a Gaussian likelihood, the Fisher matrix is given by the second derivative of the log-likelihood with respect to the parameters p_i at the likelihood maximum,

$$\mathcal{F}_{ij} = -\left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial p_i \partial p_j} \right\rangle, \quad (54)$$

and its inverse gives marginalized 1σ error of the parameter of interest,

$$\sigma(p_i) = \sqrt{(\mathcal{F}^{-1})_{ii}}. \quad (55)$$

1. CMB experiment

In the case of CMB experiment, we assume that the Fisher matrix for both CMBpol and PLANCK is given by

$$\mathcal{F}_{ij} = \sum_{\ell=2}^{\ell_{\max}} \sum_{XX', YY'} \frac{\partial C_{\ell}^{XX'}}{\partial p_i} (\text{Cov}_{\ell}^{-1})_{XX'YY'} \frac{\partial C_{\ell}^{YY'}}{\partial p_j}, \quad (56)$$

where X and X' are summed over the CMB temperature, E -mode polarization, and B -mode polarization ($X = T, E, B$). The covariance matrix is given by

$$\begin{aligned} (\text{Cov}^{-1})_{TTTT} &= \frac{2}{(2l+1)f_{\text{sky}}} (C_{\ell}^{TT} + w_T^{-1} B_{\ell}^{-2}), \\ (\text{Cov}^{-1})_{EEEE} &= \frac{2}{(2l+1)f_{\text{sky}}} (C_{\ell}^{EE} + w_P^{-1} B_{\ell}^{-2}), \\ (\text{Cov}^{-1})_{BBBB} &= \frac{2}{(2l+1)f_{\text{sky}}} (C_{\ell}^{BB} + w_P^{-1} B_{\ell}^{-2}), \\ (\text{Cov}^{-1})_{TETE} &= \frac{2}{(2l+1)f_{\text{sky}}} [(C_{\ell}^{TE})^2 + (C_{\ell}^{TT} + w_T^{-1} B_{\ell}^{-2}) \\ &\quad \times (C_{\ell}^{EE} + w_P^{-1} B_{\ell}^{-2})], \\ (\text{Cov}^{-1})_{TTEE} &= \frac{2}{(2l+1)f_{\text{sky}}} (C_{\ell}^{TE})^2, \\ (\text{Cov}^{-1})_{TTTE} &= \frac{2}{(2l+1)f_{\text{sky}}} C_{\ell}^{TE} (C_{\ell}^{TT} + w_T^{-1} B_{\ell}^{-2}), \\ (\text{Cov}^{-1})_{EETE} &= \frac{2}{(2l+1)f_{\text{sky}}} C_{\ell}^{TE} (C_{\ell}^{EE} + w_P^{-1} B_{\ell}^{-2}), \end{aligned} \quad (57)$$

where $w_{(T,P)}^{-1} = 4\pi\sigma_{(T,P)}^2/N_{\text{pix}}$ is the variance of the noise temperature per pixel (in μK^2). For simplicity, we have

TABLE I. Instrument parameter values for CMB experiments.

Experiment	f_{sky}	Center frequency (GHz)	θ (FWHM arcmin)	σ_T (μK)	σ_P (μK)
Planck [33]	0.65	70	14	12.8	18.2
		100	10	6.8	10.9
		143	7.1	6.0	11.4
		217	5.0	13.1	26.7
CMBpol [34]	0.65	100	4.2	0.87	1.18
		150	2.8	1.26	1.76
		220	1.9	1.84	2.60

assumed a Gaussian beam $B_{\ell} \approx \exp[-\ell(\ell+1)\sigma_b^2/2]$ with $\sigma_b = \theta/\sqrt{8\ln 2}$ being the beam width. In Table I, we list the values of the observed fraction of the sky f_{sky} , the temperature noise per pixel σ_T , the polarization noise per pixel σ_P and the Gaussian beam width $\theta^2 = 4\pi/N_{\text{pix}}$ for Planck [33] and CMBpol [34] experiments that are adopted to derive our forecasts. The different frequency channels are combined according to $w_{(T,P)} B_{\ell}^2 = \sum_{\nu} w_{(T,P)}^{\nu} (B_{\ell}^{\nu})^2$, where ν refers to each channel component [35]. Finally, the maximum multipole value has been set to $\ell_{\max} = 2000$.

Hereafter, we assume a flat Λ CDM Universe and set the fiducial cosmological parameters to the WMAP7 mean values [9].

2. GW direct detection

For the GW direct detection experiments, we will be considering three future experiments, FP-DECIGO, BBO and Ultimate-DECIGO [10–12]. FP-DECIGO is planned to be a Fabry-Perot Michelson interferometer, while the BBO experiment will use time-delay interferometry (TDI). Although they use different technology, in this paper, we do not distinguish these experiments, since they have similar sensitivity. The Fisher matrix for GW direct detection is given by [32]

$$\begin{aligned} \mathcal{F}_{ij} &= \left(\frac{3H_0^2}{10\pi^2} \right)^2 2t_{\text{obs}} \\ &\quad \times \sum_{(I,J)} \int_0^{\infty} df \frac{|\gamma_{IJ}(f)|^2 \partial_{p_i} \Omega_{\text{gw}}(f) \partial_{p_j} \Omega_{\text{gw}}(f)}{f^6 S_I(f) S_J(f)}, \end{aligned} \quad (58)$$

where Ω_{gw} is given in Eq. (24) and t_{obs} is the observation time. The subscripts I and J refer to independent signals obtained at each detector, or observables generated by combining the detector signals. For a BBO-like experiment, the summation runs over the TDI channel output index ($I = A, E, T$) [32]. The overlap reduction $\gamma_{IJ}(f)$ for TDI variables is calculated following the procedure of Ref. [29]. For the cross term ($I \neq J$), we have $\gamma_{IJ}(f) = 0$. The noise spectrum $S_{I,J}(f)$ is given by

TABLE II. Instrument parameters for GW experiments.

Experiments	$S_{\text{shot}} [(L/\text{km})^{-2} \text{Hz}^{-1}]$	$S_{\text{accel}} [(2\pi f/\text{Hz})^{-4} (2L/\text{km})^{-2} \text{Hz}^{-1}]$	L (km)
BBO/FP-DECIGO [29]	2×10^{-40}	9×10^{-40}	5×10^4
Ultimate-DECIGO [30]	9×10^{-44}	9×10^{-44}	5×10^4

$$\begin{aligned}
S_A(f) &= S_E(f) = 8\sin^2(\hat{f}/2)\{(2 + \cos \hat{f})S_{\text{shot}} \\
&\quad + 2[3 + 2\cos \hat{f} + \cos(2\hat{f})]S_{\text{accel}}\}, \\
S_T(f) &= 2(1 + 2\cos \hat{f})^2[S_{\text{shot}} + 4\sin^2(\hat{f}/2)S_{\text{accel}}], \quad (59)
\end{aligned}$$

where $\hat{f} = 2\pi Lf$. The values of the arm length L , the shot noise S_{shot} and the radiation pressure noise S_{accel} have been reported in Table II for BBO/FP-DECIGO and Ultimate-DECIGO, respectively. One may be concerned about the noise contamination from white dwarf binaries at frequencies below ~ 0.1 Hz, and introduce a low-frequency cutoff to the integral. However, it may be possible to remove it by identifying all binaries and subtracting their contributions from data streams [36–38]. In this paper, we assume that this is the case and do not introduce such a low-frequency cutoff.

Although we focus on the inflationary GW background in this paper, it should also be noted that there may be other possible GW signals in the sensitivity range of BBO and DECIGO. Such examples include for instance first order phase transition [39–43], preheating [44–48], particle production [49] and cosmic strings [50–54]. These GWs may be significant in some cases but they will not be considered in the following.

C. Forecasts on early universe history

Now we investigate the constraint on the thermal history of the Universe using the Fisher matrix method discussed in the previous section. For this purpose, we focus on the reheating temperature T_{reh} associated with the inflaton, the temperature associated with the second field T_σ and the amount of late-time entropy production F (or equivalently R_y).

1. Unobservable spectral features

In this section, we have performed a Fisher analysis to obtain the expected constraints from Planck/CMBpol for CMB and BBO/DECIGO for GW direct detection. As a fiducial model for the analysis, we consider two cases with $g_{* \text{reh}}^{1/4} T_{\text{reh}} \sim 10^{16}$ GeV (corresponding to $N \equiv -\Delta N_* \sim 57$) and $g_{* \text{reh}}^{1/4} T_{\text{reh}} \sim 10^9$ GeV (corresponding to $N \sim 52$), assuming that there is no late-time entropy production, i.e., $F = 1$.

For the above fiducial models, there are no spectral signatures in the sensitivity range of the GW direct detection experiments and the Fisher analysis remains insensitive to any information coming from the spectral shapes. As a result, one expects some degeneracies to

occur between the model parameters. In Figs. 6 and 7, we have represented the 2σ allowed regions in the plane (T_{reh}, F) and (m, F) , respectively, for several combinations of those future data.

There are two cases to consider according to the values of T_σ . In Fig. 6, we have also represented the region where the signal-to-noise ratio is $S/N > 5$ for BBO/FP-DECIGO as the dark (blue) shaded region. As studied in Fig. 5, a late time entropy production would induce a suppression of the GW spectrum amplitude at frequencies higher than f_{yend} , which corresponds to T_σ in Eq. (52). For BBO/FP-DECIGO, the suppression occurs in the sensitivity region

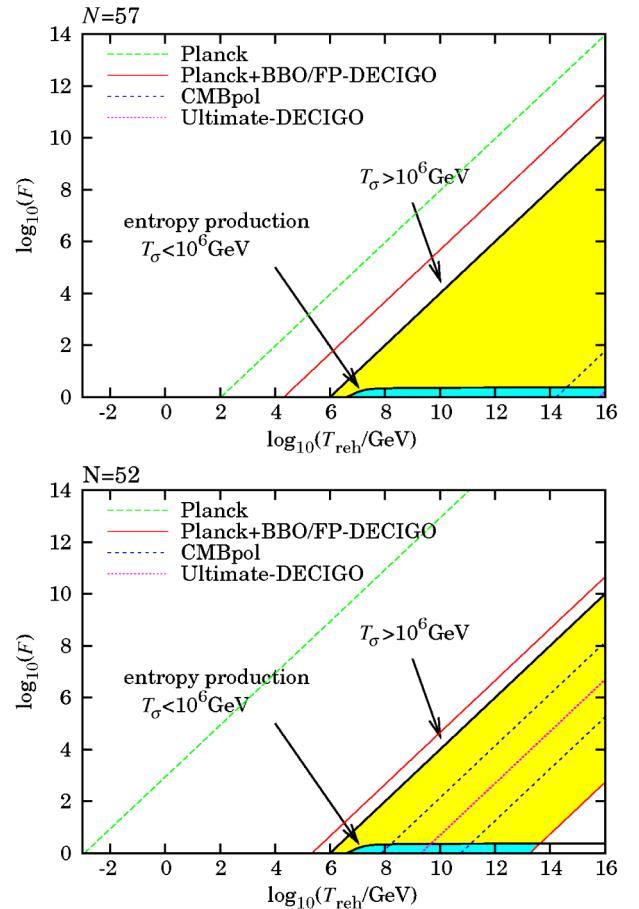


FIG. 6 (color online). Future constraints on the (T_{reh}, F) plane from CMB (Planck and CMBpol) and/or direct detection of GWs (BBO/FP-DECIGO and Ultimate-DECIGO). The light shaded region traces the 2σ confidence interval of the two-dimensional marginalized posterior under the Fisher matrix analysis. The dark (blue) shaded region traces the parameter space in which the signal-to-noise ratio $S/N > 5$ (see text).

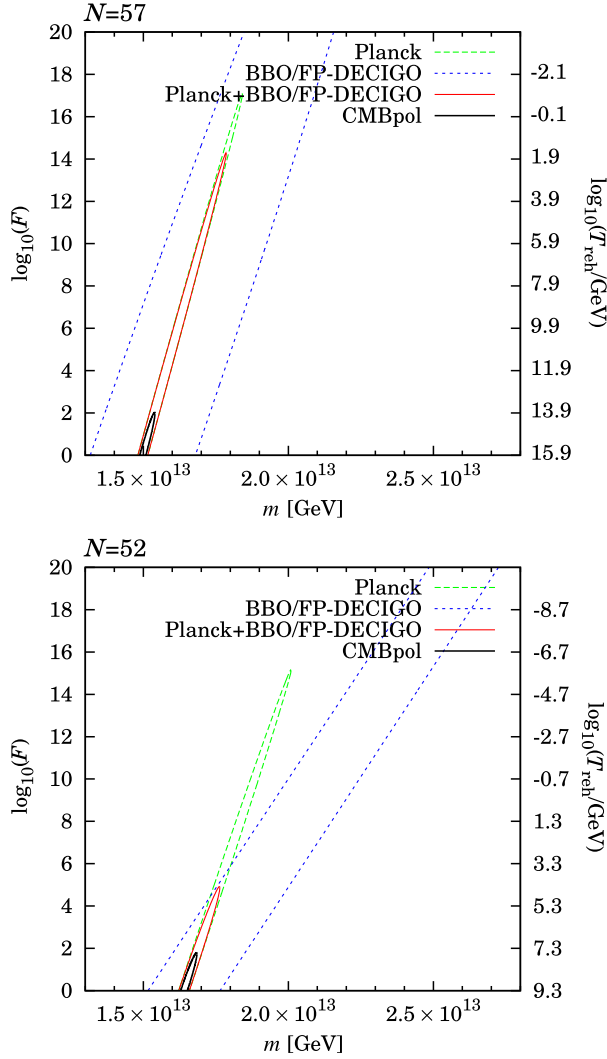


FIG. 7 (color online). Future constraint in the plane (m, F) from CMB (Planck and CMBpol) and GWs (BBO/FP-DECIGO), for the two fiducial models having $T_{\text{reh}} \approx 10^9$ GeV ($N = 52$) and $T_{\text{reh}} \approx 10^{16}$ GeV ($N = 57$).

when $T_\sigma < 10^6$ GeV. In that situation, a direct detection of the primordial GWs by BBO/FP-DECIGO would immediately yield a strong upper bound for $F \lesssim 2.4$ (see Fig. 6). At the same time, since the reheating also induces a suppression of the GW spectrum amplitude, direct detection would readily exclude $T_{\text{reh}} < 10^6$ GeV for the same reason.

In the opposite situation, i.e., the case with $T_\sigma > 10^6$ GeV, the spectrum is suppressed at higher frequencies and beyond the sensitivity range of the experiment. Therefore, the amplitude of the observable GW spectrum remains mostly independent of the values of F . In this case, direct detection of GWs still allows a large parameter space represented as the light (yellow) shading in Fig. 6. Within this region, the upper limit of F , equivalently the lower limit of T_{reh} , is imposed by the consideration that reheating should end before the late-time entropy production begins.

In other words, $f_{\text{xend}} < f_{\text{reh}}$ gives $T_\sigma F^{2/3} < T_{\text{reh}} F^{-1/3}$ using Eq. (52), and given $T_\sigma > 10^6$ GeV, we get $T_{\text{reh}}/F > 10^6$ GeV. Since the constraints on F are completely degenerated with the one on T_{reh} , we have also reported the values of T_{reh} along the right vertical axis of Fig. 7.

Regarding CMB constraints, there is also a degeneracy between T_{reh} and F , which is clear from Eq. (53). However, the inflaton mass m can be probed through the amplitude of primordial curvature perturbation while n_s and r are related to F and T_{reh} . On the other hand, the constraints from GW experiments basically come from their sensitivity to r and n_T . Since in the single field inflation model these two quantities are related by $n_T = -r/8$ (at leading order in slow-roll parameters), m and F are constrained in a different way compared to CMB. In addition, CMB and GW direct detection experiments measure slow-roll parameters' quantities at different scales. As a result, the direction of this degeneracy differs, thereby showing the complementarity of these observables.

2. Detection of spectral features

For Ultimate-DECIGO, the suppression region of the GW signal occurs at $T_\sigma < 10^3$ GeV. However, there are large possibilities that the extreme sensitivity of Ultimate-DECIGO enables us to measure some of the spectral features. In that case, the detection of GWs would provide a precise determination of the parameters [55]. In the following, we investigate in more detail the determination of the thermal history parameters by Ultimate-DECIGO in combination with CMBpol.

In Fig. 9, we show the constraints on the parameters T_{reh} and F expected from Ultimate-DECIGO together with ones expected from CMBpol. Three different fiducial models have been considered:

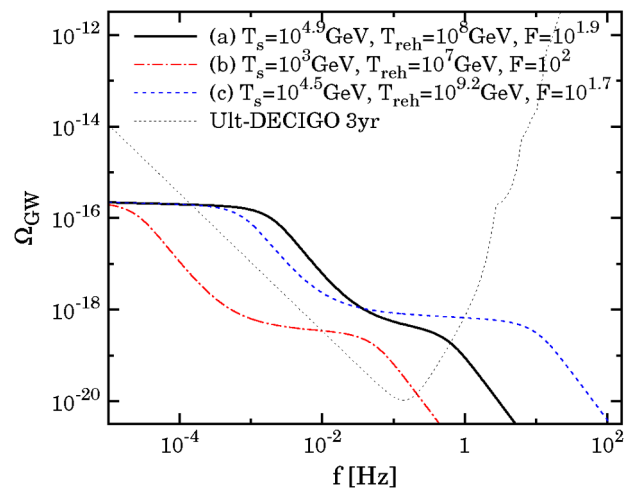


FIG. 8 (color online). Spectra of the three fiducial models (a), (b), and (c) together with the sensitivity region of the Ultimate-DECIGO GW experiment (see text). The corresponding forecasts are represented in Fig. 9.

- (i) Both transition frequencies are inside the sensitivity frequency band of the experiment. We use the fiducial values $T_\sigma = 10^{4.9}$ GeV, $T_{\text{reh}} = 10^8$ GeV and $F = 10^{1.9}$.

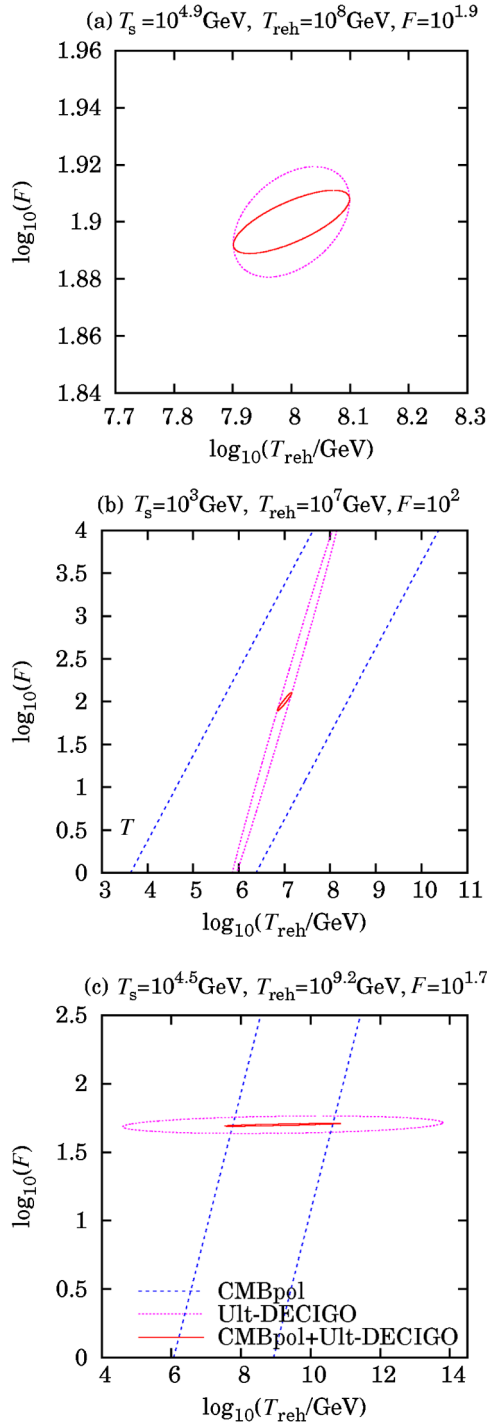


FIG. 9 (color online). Predicted constraints in the (T_{reh}, F) plane for the three different fiducial models (a), (b), and (c). The dotted and dashed lines show the marginalized 2σ confidence contours expected from Ultimate-DECIGO and CMBpol, respectively. The solid line is the combined constraints. See also Fig. 10.

- (ii) The frequency f_{xend} is outside the sensitivity range. As such an example, we consider the case with $T_\sigma = 10^3$ GeV, $T_{\text{reh}} = 10^7$ GeV and $F = 10^2$.
- (iii) The frequency f_{reh} is outside the sensitivity range, which occurs for the fiducial values $T_\sigma = 10^{4.5}$ GeV, $T_{\text{reh}} = 10^{9.2}$ GeV and $F = 10^{1.7}$.

The spectra of these three models have been represented in Fig. 8. Here, we take T_{reh} , F , T_σ and m as free parameters and the predicted constraints are obtained by marginalizing over the remaining parameters. For CMBpol, the other cosmological parameters are also marginalized.

In the case (a), all features of the reheating and late-time entropy production are within the GW sensitivity range. Since, in this case, Ultimate-DECIGO alone can well determine all the parameters, we do not show the constraint from CMBpol alone (in fact, the 1σ contour from CMBpol ends up being outside the range of the figure). However, it should be noted here that adding CMB data improves the constraint, as seen from the figure.

For the case (b), the damping of the GW spectrum within the sensitivity zone is due to the inflationary reheating and T_{reh} can be inferred. On the other hand, we find that GW measurements alone cannot determine the value of F , since the feature of late-time entropy production is now outside the region of detectability. However, if we combine CMB data, the constraint can be improved as seen from the second panel of Fig. 9. To illustrate this more clearly, we show in Fig. 10 the marginalized constraints for case (b) in the (m, F) plane. Constraints from direct detection have a strong parameter degeneracy between m and F , since they both cause suppression of the amplitude of GWs at the maximum sensitivity region of Ultimate-DECIGO. However, CMB data such as CMBpol are strongly sensitive to the value of m and greatly help to break the degeneracy.

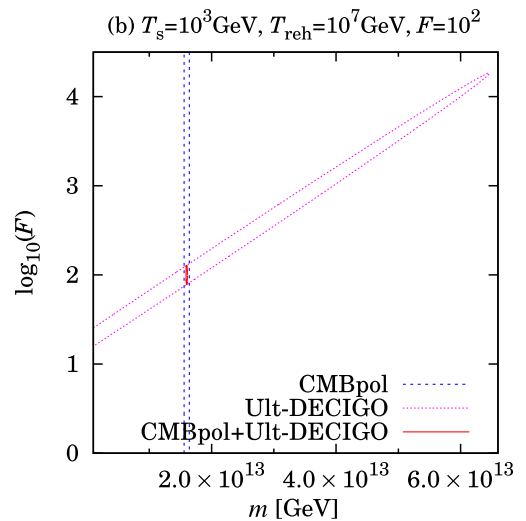


FIG. 10 (color online). Predicted constraints for the fiducial model (b) in the (m, F) plane for Ultimate-DECIGO, CMBpol and both (same convention as in Fig. 9).

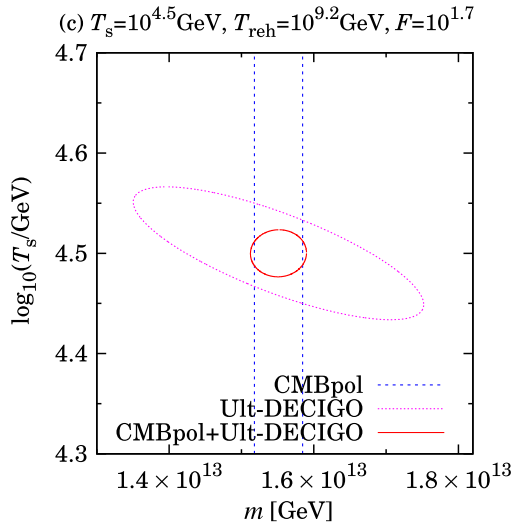


FIG. 11 (color online). Predicted constraints for the fiducial model (c) in the (m, T_σ) plane for Ultimate-DECIGO, CMBpol and both (same convention as in Fig. 9).

Therefore, although CMBpol data alone only provide constraints on F and T_{reh} with large uncertainties (see middle panel of Fig. 9), the data can significantly tighten the constraints from Ultimate-DECIGO through improving the determination on m .

In the case (c), the reheating frequency is outside the sensitivity region such that the value of T_{reh} can no longer be well determined by GW measurements. Still T_σ and F can be well probed instead. As a result, the constraint from Ultimate-DECIGO lies parallel to the T_{reh} axis (see Fig. 9). However, as repeatedly emphasized above, CMB probes the parameters differently from GWs, thus in combination with CMBpol, T_{reh} can again be inferred and this ends up with breaking its degeneracy with F . Furthermore, in Fig. 11, we have also depicted the expected constraints in the (m, T_σ) plane, which also clearly illustrates the complementarity of observations of CMB and GWs.

IV. SUMMARY

We have discussed how one can probe the thermal history of the Universe from the era just after the end of inflation until the BBN epoch. In any given inflationary

models, the spectral index and the tensor-to-scalar ratio are related to the e-fold number at which a reference scale exited the Hubble radius during inflation; thus in turn, they give a constraint on the total expansion of the scale factor since the above-mentioned epoch. By assuming chaotic inflation, we have presented how the CMB is constraining the thermal history when there is an epoch of late-time entropy production, as, for instance, a scenario where an oscillating scalar field dominates the Universe at some epoch, and then it decays.

In the future, direct detection GW experiments are expected to provide new cosmological probes, in addition to more precise CMB experiments such as Planck and CMBpol. Although CMB data can be used to determine the total amount of the cosmic expansion, they are only sensitive to the integrated thermal history and an epoch of late-time entropy production remains fully degenerated with the standard inflationary reheating era. This is not the case for GW experiments as they are precisely sensitive to the transition between these epochs. In particular, for the scenario with late-time entropy production, the GW spectrum bends twice at some transition frequencies (see Figs. 5 and 8). We have shown that if one, or more, frequencies do not lie in the GW experiment sensitivity region, CMB experiments can still greatly help to break the degeneracy. As shown in this paper, future experiments of GWs and CMB are complementary in performing a tomography of the thermal history between the end of inflation and BBN. As such, they are expected to play a major role in our understanding of the whole history of the Universe.

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