Network Coding for the Multiple Access Relay Channel using Lattices

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***Abstract*—We[1] consider the problem of two transmitters would like to communicate with a destination with the help of a half-duplex relay. In this work, we are presenting the advantages of using nested lattices for the AWGN channels. The sources map their messages using lattice code and then broadcast them to the relay and the destination. The relay receives two independent symbols at the same channel. The relay either combines the two symbols using lattice modulo and then decode or decode the two symbols separately also using modulo lattice, then forwards the new symbol to the destination. The destination tries to recover the two messages using different decoding strategies. One of the strategies is to recover two linear equations in function of the two received symbols with integer coefficients then, solve these equations to recover the two messages. The integer coefficients need to be optimally selected to reduce the noise at the receivers. The other strategy is to use successive decoding at the relay and the destination. This strategy outperforms the first when two integer coefficients are zero. The strategies are discussed and compared with the traditional DF (Decode and Forward). The simulation results show the advantages of using lattice codes and the improvement in rates for certain regimes.**

I. Introduction and System Model

Relay was introduced to broaden coverage, to enhance capacity or to improve robustness of a system. Conventional unstructured or random codes have been used before along with DF (Decode and Forward), AF (Amplify and Forward) and others [1] [2]. However, structured or linear codes have an advantage over the random codes with respect to the performance. The reason is that the relay is able to decode the sum of the messages, over an additive channel, with no rate loss and without the need of joint decoding of the messages [3].

Nested lattices have been proved to achieve the AWGN channel capacity with lattice encoding and decoding for a single user [4]. Researchers have been using nested lattices for the two-way relay channels, and achieved an exchange rate of $1/2\log(1/2+SNR)$ [5] [6]. Moreover, in [7] they showed the advantages of using lattices compared with DF and AF. In [8] [9], they have used structured codes to build up a set of linear equations at the receiver for multiple users. The relay is only recovering a linear functions of codewords instead of the individual codewords. They showed how structure codes can be useful in network information theory and how to achieve better rates than CF and DF.

In this work, we consider a MARC (Multiple Access Relay channel) based on structured codes. The system consists of two transmitters (A and B), a receiver (destination D) and a transceiver (relay R). All nodes use lattices to encode their messages. We consider a nested lattices which means the lattice Λ (the coarse lattice) is nested in Λ_1 (the fine lattice) i.e. $\Lambda \subseteq \Lambda_1$ [4]. Let the fundamental Voronoi regions of the lattices Λ and Λ_1, be $\mathcal{V}(\Lambda)$ and $\mathcal{V}(\Lambda_1)$. Each user encodes his message $\mathrm{w_j}$, $j \in \{a,b\}$ and maps it to a fine lattice point $\mathrm{v_j} \in \{\Lambda_1 \cap \mathcal{V}\}$. Further, let U_i (dither), $i \in \{a,b,r\}$, be a random independent variable uniformely distributed over $\mathcal{V}$. We assume that the dither U_i is available at all nodes. Hence the transmitted vector $\mathrm{x_i}$, $i \in \{a,b,r\}$, can be written as,

$$\mathrm{x_i} = \frac{(\mathrm{v_i} - U_i)\mathbf{mod}\Lambda}{\beta_i} \tag{1}$$

where β_i, ($\beta_i \in \mathbb{R}$), is the normalization factor to be determined. The factor β_i is used to ensure that the constraint of the total power transmitted by node i is satisfied. Assume the second moment of Λ is P_Λ [4]. We have $P_{\Lambda_i} = P_\Lambda$ where $P_{\Lambda_i} = \beta_i^2 P_i$, and P_i is the power constraint available at each node.

The transmission time consists of two time slots. During the first time slot, node A and B broadcast their symbols to the relay and the destination. The destination receives two different signals from node A and B. Then it will recover and decode a linear combination of the two messages given as $t_a\mathrm{v_a} + t_b\mathrm{v_b}$, where t_a and $t_b \in \mathbb{Z}$. During the second time slot, the relay performs network coding operation using modulo lattices and transmits a new symbol, $\mathrm{x_r}$, to the destination. Where $\mathrm{v_r}$ is a linear combination of the two vectors $\mathrm{v_a}$ and $\mathrm{v_b}$, given as $\mathrm{v_r} = k_a\mathrm{v_a} + k_b\mathrm{v_b}$, where k_a and $k_b \in \mathbb{Z}$. The relay decodes $\mathrm{v_r}$ then forwards it to the destination. Finally, the destination retrieves the two messages using the two equations decoded during the two time slots.

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Let us denote the channel by h_{ad}, h_{bd}, the sources-destination channel gains, by h_{ar}, h_{br} the sources-relay channel gains, and by h_{rd} the relay-destination channel gain. The noise samples $\mathrm{z_l}$, $l \in \{r, d_1, d_2\}$, at the relay and at the destination are all zero mean circular gaussian, white and uncorrelated with a variance P_N.

We will analyze the system model mentioned above using lattices. Then we will allocate the values of β_i, $i \in \{a, b, r\}$, and the lattices coefficients t_a, t_b, k_a, and k_b in a way to maximze the sum rate at the destination. At optimum the lattice coefficients values are those the same given by the channels [9], [10]. However, the integer constraint on the lattice coefficients is necessary, as only integer combinations of lattice points will be a lattice point [9], [10], [11]. As the channel coefficients are non-integer, the lattice coefficients must have a value in such a way two different linear equations at the destination are recovered and the noise is minimized. Then, the two messages sent by the nodes A and B can be recovered.

We will compare the results with the same system model but using random codes and the relay operates in Decode and Forward mode [12]. We show that at some channel coefficients lattices outperforms random codes, and that the complexity regarding which decoding order should be used is simplified. Moreover, we show that for certain regimes it is better to apply successive decoding at the relay and the destination rather than recovering two linear equations using lattice codes.

II. Decode and Forward

In this section, we present the DF scheme for the MARC. During the first time slot, node A and B broadcast their symbols to the relay and the destination. The destination and the relay receive two different signals from node A and B. The relay decodes either one or both symbols, and then forwards either one or both to the destination during the second time slot. The achievable sum rate using DF is [12],

$$\begin{aligned} R_{DF} &= \max_{\eta_1,\eta_2,\rho_1,\rho_2} \Big[\bar{\eta}_1\bar{\eta}_2 R_1 + \eta_1\bar{\eta}_2 R_2 \\ &+ \bar{\eta}_1\eta_2 R_3 + \eta_1\eta_2 R_4\Big] \end{aligned} \tag{2}$$

where R_p, $(p \in \{1, 2, 3, 4\})$, corresponds to the sum rate at the destination for different decoding order of the signals from nodes A and B at the relay and the destination. The indicators η_1 ($\bar{\eta}_1 = 1 - \eta_1$) and η_2, whose value should be 0 or 1, are to select the most appropriate decoding order. The ρ_1 and ρ_2 indicate whether node A , node B, both nodes or none is relayed. The rate R_1 is given by:

$$\begin{aligned} R_1 &= \Big[\bar{\rho}_1\bar{\rho}_2 r_{1,1} + \rho_1\bar{\rho}_2 r_{1,2} \\ &+ \bar{\rho}_1\rho_2 r_{1,3} + \rho_1\rho_2 r_{1,4}\Big] \end{aligned} \tag{3}$$

where

$$\begin{aligned} r_{1,1} &= \mathcal{C}\left(\frac{P_a|h_{ad}|^2}{P_N} + \frac{P_b|h_{bd}|^2}{P_N}\right) \\ r_{1,2} &= \min\left\{\mathcal{C}\left(\frac{P_a|h_{ar}|^2}{P_N}\right), \mathcal{C}\left(\frac{P_a|h_{ad}|^2}{P_N} + \frac{P_r|h_{rd}|^2}{P_N}\right)\right\} \\ &+ \min\left\{\mathcal{C}\left(\frac{P_b|h_{br}|^2}{P_N + P_a|h_{ar}|^2}\right), \mathcal{C}\left(\frac{P_b|h_{bd}|^2}{P_N + P_a|h_{ad}|^2}\right)\right\} \\ r_{1,3} &= \min\left\{\mathcal{C}\left(\frac{P_b|h_{br}|^2}{P_N + P_a|h_{ar}|^2}\right),\right. \\ &\left.\mathcal{C}\left(\frac{P_b|h_{bd}|^2}{P_N + P_a|h_{ad}|^2} + \frac{P_r|h_{rd}|^2}{P_N}\right)\right\} + \mathcal{C}\left(\frac{P_a|h_{ad}|^2}{P_N}\right) \\ r_{1,4} &= \min\left\{\mathcal{C}\left(\frac{P_a|h_{ar}|^2}{P_N}\right), \mathcal{C}\left(\frac{P_a|h_{ad}|^2}{P_N} + \frac{P_{r_a}|h_{rd}|^2}{P_N}\right)\right\} \\ &+ \min\left\{\mathcal{C}\left(\frac{P_b|h_{br}|^2}{P_N + P_a|h_{ar}|^2}\right),\right. \\ &\left.\mathcal{C}\left(\frac{P_b|h_{bd}|^2}{P_N + P_a|h_{ad}|^2} + \frac{P_{r_b}|h_{rd}|^2}{P_N + P_{r_a}|h_{rd}|^2}\right)\right\}. \end{aligned} \tag{4}$$

given $\mathcal{C}(x) = \frac{1}{4}\log_2(1 + x)$, P_{r_a} and P_{r_b} ($P_{r_a} + P_{r_b} \le P_r$) are the power assigned to transmit the symbols of node A and B at the relay, and $r_{p,f}$, $(f \in \{1, 2, 3, 4\})$, is the sum rate at the destination for a defined decoding order p and different relayed source. The rate for different p can be obtained in a similar manner we will not include them due to limited space.

III. Lattices Rate Analysis

In this section, we derive the achievable sum rate at the destination using lattices. Let P_{a_1} and P_{b_1} be the power used by user A and B, where $P_{a_1} \le P_a$, $P_{b_1} \le P_b$ and $P_{a_1}\beta_{a_1}^2 = P_{b_1}\beta_b^2 = P_\Lambda$. The power P_{a_1} and P_{b_1} are assigned to transmit the lattice symbols $\mathrm{x_{a_1}}$ and $\mathrm{x_{b_1}}$. The power left $P_{g_2} = P_g - P_{g_1}$, $g \in \{a, b\}$ (at either of the source nodes) is assigned to transmit the symbol $\mathrm{x_{g_2}}$, where $\mathrm{x_{g_2}}$ is encoded using a Gaussian codebook.

Let us assume that user A has enough power to transmit $\mathrm{x_a} = \mathrm{x_{a_1}} + \mathrm{x_{a_2}}$. The transmitted symbol $\mathrm{x_a}$ contains two symbols $\mathrm{x_{a_1}}$ encoded using lattice code and $\mathrm{x_{a_2}}$ encoded using gaussian codebook. User B transmits $\mathrm{x_b} = \mathrm{x_{b_1}}$, where $\mathrm{x_{b_1}}$ encoded using lattice code.

During the first time slot, node A and B broadcast their symbols $\mathrm{x_a}$ and $\mathrm{x_b}$ to the relay and destination node. The relay node receives the signal $\mathrm{y_r}$ and the destination node receives the signal $\mathrm{y_{d_1}}$, which are summarized as follow:

$$\begin{aligned} \mathrm{x_a} &= \mathrm{x_{a_1}} + \mathrm{x_{a_2}} = \frac{(\mathrm{v_{a_1}} - U_{a_1})\,\mathbf{mod}\Lambda}{\beta_{a_1}} + \mathrm{x_{a_2}} \\ \mathrm{x_b} &= \frac{(\mathrm{v_b} - U_b)\,\mathbf{mod}\Lambda}{\beta_b} \\ \mathrm{y_r} &= h_{ar}\mathrm{x_a} + h_{br}\mathrm{x_b} + \mathrm{z_r} \\ \mathrm{y_{d_1}} &= h_{ad}\mathrm{x_a} + h_{bd}\mathrm{x_b} + \mathrm{z_{d_1}} \end{aligned} \tag{5}$$

At the relay node, $\mathrm{x_{a_2}}$ is first decoded, and deducted from the received signal $\mathrm{y_r}$, and we get,

$$\mathrm{y'_r} = h_{ar}\mathrm{x_{a_1}} + h_{br}\mathrm{x_b} + \mathrm{z_r} \tag{6}$$

The received signal at the relay is transformed to the following modulo lattice additive channel signal:

$$\begin{aligned}\mathrm{Y'_r} &= (\alpha_r \mathrm{y'_r} + k_{a_1}U_{a_1} + k_bU_b)\mathbf{mod}\Lambda \\ &= (\alpha_r(h_{ar}\mathrm{x_{a_1}} + h_{br}\mathrm{x_b} + \mathrm{z_r}) + k_{a_1}U_{a_1} + k_bU_b)\mathbf{mod}\Lambda\end{aligned}$$

where α_r is the minimum mean square error factor that minimizes the effective noise [4], k_{a_1} and k_b are the integer lattice coefficients.

$$\begin{aligned}\mathrm{Y'_r} &= \Big(\alpha_r(h_{ar}\mathrm{x_{a_1}} + h_{br}\mathrm{x_b} + \mathrm{z_r}) - \beta_{a_1}k_{a_1}\mathrm{x_{a_1}} - \beta_b k_b \mathrm{x_b} \\ &+ \beta_{a_1}k_{a_1}\mathrm{x_{a_1}} + \beta_b k_b\mathrm{x_b} + k_{a_1}U_{a_1} + k_bU_b\Big)\mathbf{mod}\Lambda \\ &= \Big(\mathrm{k_{a_1}v_{a_1}} + \mathrm{k_bv_b} + h_{ar}\mathrm{x_{a_1}}(\alpha_r - \frac{\beta_{a_1}k_{a_1}}{h_{ar}}) \\ &+ h_{br}\mathrm{x_b}(\alpha_r - \frac{\beta_b k_b}{h_{br}}) + \alpha_r\mathrm{z_r}\Big)\mathbf{mod}\Lambda \\ &= \Big(\mathrm{v_{r_1}} + \mathrm{z_{r_{eff}}}\Big)\mathbf{mod}\Lambda\end{aligned} \tag{7}$$

where $\mathrm{z_{r_{eff}}} = \Big(h_{ar}\mathrm{x_{a_1}}(\alpha_r - \frac{\beta_{a_1}k_{a_1}}{h_{ar}}) + h_{br}\mathrm{x_b}(\alpha_r - \frac{\beta_b k_b}{h_{br}}) + \alpha_r\mathrm{z_r}\Big)\mathbf{mod}\Lambda$ and $\mathrm{v_{r_1}} = (\mathrm{k_{a_1}v_{a_1}} + \mathrm{k_bv_b})\mathbf{mod}\Lambda$ is a lattice point in the fine lattice [5].

The power of the effective noise $\mathbf{E}[|\mathrm{z_{r_{eff}}}|^2]$ can be calculated as follows:

$$\begin{aligned}\mathbf{E}[|\mathrm{z_{r_{eff}}}|^2] &= \alpha_r^2P_N + |h_{ar}|^2P_{a_1}(\alpha_r - \frac{\beta_{a_1}k_{a_1}}{h_{ar}})^2 \\ &+ |h_{br}|^2P_b(\alpha_r - \frac{\beta_b k_b}{h_{br}})^2\end{aligned} \tag{8}$$

The optimal value of α_r is calculated by minimizing the power of the effective noise, and it is given by:

$$\alpha_r = \frac{|h_{ar}|^2P_{a_1}\frac{\beta_{a_1}k_{a_1}}{h_{ar}} + |h_{br}|^2P_b\frac{\beta_b k_b}{h_{br}}}{|h_{ar}|^2P_{a_1} + |h_{br}|^2P_b + P_N} \tag{9}$$

Thus, $\mathrm{v_{r_1}}\mathbf{mod}\Lambda$ can be decoded at the relay by using a rate R_{sr} given as [5] [8],

$$R_{sr} = 1/4\log_2\left(\frac{|\beta_{a_1}|^2P_{a_1}}{\mathbf{E}[|\mathrm{z_{r_{eff}}}|^2]}\right). \tag{10}$$

By assumption the normalized second moment of the lattice equal to $\frac{1}{2\pi e}$ as $n \to \infty$ (n is the length of $\mathrm{x_i}$) [4]. After decoding $\mathrm{v_{r_1}}$ the relay transmits $\mathrm{x_r}$ as,

$$\begin{aligned}\mathrm{x_r} &= \mathrm{x_{r_1}} + \mathrm{x_{r_2}} \\ &= \frac{(\mathrm{v_{r_1}} - U_{r_1})\mathbf{mod}\Lambda}{\beta_{r_1}} + \mathrm{x_{r_2}}\end{aligned} \tag{11}$$

where $\beta_{r_1} = \sqrt{\beta_{a_1}^2P_{a_1}/P_{r_1}}$ is the normalization factor at the relay, P_{r_1} and P_{r_2} ($P_{r_1} + P_{r_2} \le P_r$) are the power assigned to transmit the symbols $\mathrm{x_{r_1}}$ and $\mathrm{x_{r_2}}$ at the relay.

At the destination node, $\mathrm{x_{a_2}}$ is first decoded, and deducted from the received signal $\mathrm{y_{d_1}}$, and we get,

$$\mathrm{y'_{d_1}} = h_{ad}\mathrm{x_{a_1}} + h_{bd}\mathrm{x_b} + \mathrm{z_{d_1}} \tag{12}$$

The received signal at the destination is transformed to the following modulo lattice additive channel signal:

$$\mathrm{Y'_{d_1}} = \Big(\mathrm{t_{a_1}v_{a_1}} + \mathrm{t_bv_b} + \mathrm{z_{d_{1eff}}}\Big)\mathbf{mod}\Lambda \tag{13}$$

where $\mathrm{z_{d_{1eff}}} = \Big(h_{ad}\mathrm{x_{a_1}}(\alpha_{d_1} - \frac{\beta_{a_1}t_{a_1}}{h_{ad}}) + h_{bd}\mathrm{x_b}(\alpha_{d_1} - \frac{\beta_b t_b}{h_{bd}}) + \alpha_{d_1}\mathrm{z_{d1}}\Big)\mathbf{mod}\Lambda$.

During the second time slot the destination receives,

$$\mathrm{y_{d_2}} = h_{rd}\mathrm{x_r} + \mathrm{z_{d_2}} \tag{14}$$

The received signal at the destination is transformed to the following modulo lattice additive channel signal, after deducting $\mathrm{x_{r_2}}$ out of $\mathrm{y_{d_2}}$,

$$\begin{aligned}\mathrm{Y'_{d_2}} &= (\alpha_{d_2}\mathrm{y'_{d_2}} + m_{r_1}U_{r_1})\mathbf{mod}\Lambda \\ &= \Big(m_{r_1}\mathrm{v_{r_1}} + \mathrm{z_{d_{2eff}}}\Big)\mathbf{mod}\Lambda\end{aligned} \tag{15}$$

where $\mathrm{z_{d_{2eff}}} = \Big(h_{rd}\mathrm{x_{r_1}}(\alpha_{d_2} - \frac{\beta_{r_1}m_r}{h_{rd}}) + \alpha_{d_2}\mathrm{z_{d2}}\Big)\mathbf{mod}\Lambda$, where α_{d_2} can be calculated as follows,

$$\alpha_{d_2} = \frac{|h_{rd}|^2P_{r_1}m_{r_1}\beta_{r_1}/h_{rd}}{|h_{rd}|^2P_{r_1} + P_N} \tag{16}$$

and the total noise at the destination can be written as,

$$\mathbf{E}[|\mathrm{z_{d_{2eff}}}|^2] = \frac{m_{r_1}^2\beta_{a_1}^2P_{a_1}P_N}{|h_{rd}|^2P_{r_1} + P_N} \tag{17}$$

and then we can calculate the maximum achievable rate from the relay to the destination as,

$$R_{RD} = 1/4\log_2\left(\frac{|\beta_{a_1}|^2P_{a_1}}{\mathbf{E}[|\mathrm{z_{d_2eff}}|^2]}\right) = 1/4\log_2\left(\frac{1}{m_{r_1}^2} + \frac{|h_{rd}|^2P_{r_1}}{m_{r_1}^2P_N}\right)$$

This leads us to that m_{r_1}, at optimum, should be equal to 1 for any value of h_{rd}.

The destination can recover the two messages sent by the sources, by decoding $\hat{Y}_{d_1}$ and $\hat{Y}_{d_2}$. Hence, $\mathrm{\hat{v}_{a_1}}$ and $\mathrm{\hat{v}_b}$.

Thus, $\mathrm{\hat{w}_{a_1}}$, $\mathrm{\hat{w}_{a_2}}$ and $\mathrm{\hat{w}_b}$ can be decoded at the destination by using a sum rate R_{DL} given as,

$$\begin{aligned}R_{DL} &= \max_{\beta_i,k_i,t_i,\alpha_l} 1/2 * \min\left\{\log_2\left(\frac{|\beta_{a_1}|^2P_{a_1}}{\mathbf{E}[|\mathrm{z_{r_{eff}}}|^2]}\right)\right. \\ &, \left.\log_2\left(\frac{|\beta_{a_1}|^2P_{a_1}}{\mathbf{E}[|\mathrm{z_{d_{1eff}}}|^2]}\right), \log_2\left(\frac{|\beta_{a_1}|^2P_{a_1}}{\mathbf{E}[|\mathrm{z_{d_2eff}}|^2]}\right)\right\} \\ &+ \min\left\{\mathcal{C}\left(\frac{|h_{ar}|^2P_{a_2}}{|h_{ar}|^2P_{a_1} + |h_{br}|^2P_b + P_N}\right)\right. \\ &, \left.\mathcal{C}\left(\frac{|h_{rd}|^2P_{r_2}}{|h_{rd}|^2P_{r_1} + P_N} + \frac{|h_{ad}|^2P_{a_2}}{|h_{ad}|^2P_{a_1} + |h_{bd}|^2P_b + P_N}\right)\right\}\end{aligned} \tag{18}$$

subject to that $\beta_i \neq 0$, $\alpha_l \neq 0$, the lattice coefficients $(t_{a_1}, t_b, k_{a_1}, k_b)$ are integer, and the matrix $\mathbf{M}$ is full row rank,

$$\mathbf{M} = \begin{pmatrix} t_{a_1} & t_b \\ k_{a_1} & k_b \end{pmatrix}.$$

The optimization of (18) is non-convex. In the following section we will discuss some methods used to calculate the values of β_is and the lattice coefficients. After getting the appropriate values, the optimization of (18) is straightforward with respect to α_ls.

IV. Discussions

As mentioned before, the lattice coefficients $(t_{a_1}, t_b, k_{a_1}, k_b)$ must be integer, and the matrix $\mathbf{M}$ must be full row rank.

One method to get the lattice coefficients is to round the channel coefficients and then assign their values to the lattice coefficients. However the noise will be high if the rounding loss was high. To reduce the rounding loss, we will preprocess the signals at the sources side (multiplying the signal by a factor $1/\beta_i$) in such a way to make the channel coefficients seen by the receivers as close as to integer value. At most we can make two coefficients equal to $'1'$ by assigning the β_i values equal to those of the channel coefficients (h_{ir} or h_{id})(this means that the transmitter nodes must have full CSI (Channel State Information)). This will reduce the other channel coefficients, and might make the rounding equal to zero. In another way, we can make β_{a_1} to take the value of h_{ad} or h_{ar}, same with β_b. If we let $\beta_{a_1} = h_{ad}$ and and assume $h_{ad} > h_{ar}$, this means that $t_{a_1} = 1$ and $k_{a_1} = 0$. In parallel, we have $h_{br} > h_{bd}$ and $\beta_b = h_{br}$, this means that $t_b = 0$ and $k_b = 1$. With these coefficients we can retrieve the two messages sent by the nodes at a sum rate equal to,

$$\begin{aligned} R_{DL_1} &= 2 * \min\left\{\mathcal{C}\left(\frac{|h_{ad}|^2 P_{a_1}}{|h_{bd}|^2 P_b + P_N}\right), \mathcal{C}\left(\frac{|h_{rd}|^2 P_{r_1}}{P_N}\right)\right. \\ &, \left.\mathcal{C}\left(\frac{|h_{br}|^2 P_b}{|h_{ar}|^2 P_{a_1} + P_N}\right)\right\} \\ &+ \min\left\{\mathcal{C}\left(\frac{|h_{ar}|^2 P_{a_2}}{|h_{ar}|^2 P_{a_1} + |h_{br}|^2 P_b + P_N}\right)\right. \\ &, \left.\mathcal{C}\left(\frac{|h_{rd}|^2 P_{r_2}}{|h_{rd}|^2 P_{r_1} + P_N} + \frac{|h_{ad}|^2 P_{a_2}}{|h_{ad}|^2 P_{a_1} + |h_{bd}|^2 P_b + P_N}\right)\right\} \end{aligned} \tag{19}$$

As $|h_{br}|^2 P_b = |h_{ad}|^2 P_{a_1} = P_\Lambda$, if $|h_{bd}|^2 P_b > |h_{ar}|^2 P_{a_1}$ this makes $\mathcal{C}\left(\frac{|h_{ad}|^2 P_{a_1}}{|h_{bd}|^2 P_b + P_N}\right)$ the bottleneck and eventually decrease the rate for node B. However, the destination does not need the received signals during the first time slot to decode the message of node B. We can do better by using successive decoding at the relay and the destination, where the sum rate is,

$$\begin{aligned} R_{DL_2} &= \min\left\{\mathcal{C}\left(\frac{|h_{br}|^2 P_b}{|h_{ar}|^2 P_a + P_N}\right), \mathcal{C}\left(\frac{|h_{rd}|^2 P_{r_1}}{P_N}\right)\right\} \\ &+ \mathcal{C}\left(\frac{|h_{ad}|^2 P_{a_1}}{|h_{bd}|^2 P_b + P_N}\right) \\ &+ \mathcal{C}\left(\frac{|h_{ad}|^2 P_{a_2}}{|h_{ad}|^2 P_{a_1} + |h_{bd}|^2 P_b + P_N}\right) \end{aligned} \tag{20}$$

As we will see in Figure 1, using lattices, the successive decoding is better than recovering two linear equations for certain regimes like the one discussed above. A suboptimal method can be used to calculate the best value for β_is is by transforming the channel coefficients at the relay and the destination as close as possible to integer value. This can be done by minimizing $h_{id}/\beta_i - \lfloor h_{id}/\beta_i \rfloor + h_{ir}/\beta_i - \lfloor h_{ir}/\beta_i \rfloor$ over β_i, ($\lfloor 1.01 \rfloor = 1$). It is suboptimal because it does not maximize the rate at the destination neither minimize the noise at each node.

V. Simulation Results

In this section, we present the simulation results. We assume that the channel coefficients are independent and randomly generated according to a zero-mean circular gaussian distribution with a variance equal to 25 for the sources-destination, and 100 for the sources-relay link and the relay-destination links. We assume all the nodes have full CSI knowledge. We assign the values of the power as $P_a = 15dB$, $P_b = 15dB$, $P_r = 20dB$, and the noise P_N varies between $0dB$ and $20dB$.

In figure 1, we show how the successive decoding (DL2) is better than recovering two linear equations (DL1), where the situation is that h_{ar} and h_{bd} are very weak ($h_{bd} > h_{ar}$), and h_{ad}, h_{br}, h_{rd} are very good. As we see both are still lower than DF, in fact this is one of the situations, which makes the use of lattices and network coding weak and give the advantages to random coding using DF.

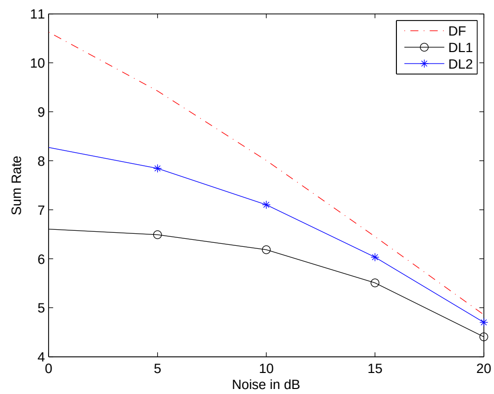

Fig. 1. Comparison of the sum rate at the destination.

On the contrary as shown in figure 2, in the other situations lattices will outperform DF, if the βs and lattice coefficients were well assigned.

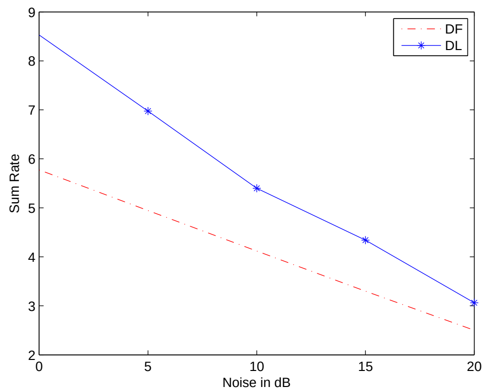

Fig. 2. Comparison of the sum rate at the destination.

Finally, figure 3 shows the sum rate obtained by averaging over 200 channel realizations for each value of the SNR. Where we are comparing the Lattice methods (DL) with decode and forward (DF), and (UL) is the maximum value obtained at each channel realization and SNR between DL and DF.

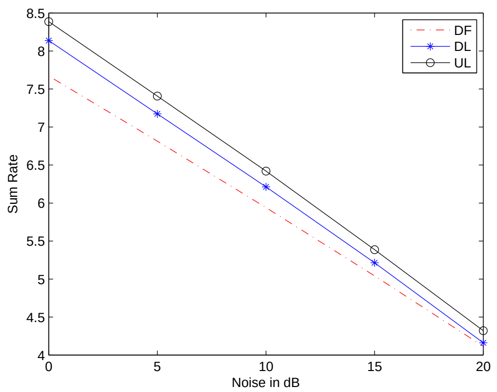

Fig. 3. Comparison of the average sum rate at the destination, for 200 channel realizations.

VI. Conclusion

In this paper, we presented the MARC based on structure codes. We showed for certain regimes that using network coding at the relay, by applying lattice modulo, the performance of the system is improved over DF strategy. We also showed using lattices the rate can be improved by using successive decoding rather than recovering two linear equations when two of the coefficients are zero.

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