

Bertrand competition and Cournot outcomes: further results

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Abstract

The model of Kreps and Scheinkman where firms choose capacities and then compete in price is extended to oligopoly. Further, capacity is an imperfect commitment device: firms can produce beyond capacities at an additional unit cost θ . When θ is larger than the Cournot price, the Cournot outcome obtains in the unique subgame perfect equilibrium. When θ decreases from the Cournot price towards zero, the whole range of prices, from Cournot to Bertrand, is obtained in equilibrium. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

According to the Bertrand paradox, ‘two is enough for competitive outcomes.’ This result, however, is well known to rely on the hypothesis of constant marginal cost and for this reason lacks any generality [this critique of Bertrand (1883) dates at least back to Edgeworth (1925)]. Still, this paradox is often contrasted with the Cournot outcomes and reconciling these two approaches has been the aim of many papers, the most spectacular being Kreps and Scheinkman (1983) (hereafter KS). Two firms invest in capacities and then compete in prices with constant marginal cost up to capacity and an arbitrarily large marginal cost above capacity. Investing in limited capacity has a strategic value because it amounts to committing not to be aggressive in the pricing game.

KS reconcile the Cournot and Bertrand approaches by showing that the Cournot outcome is the unique subgame perfect equilibrium of their game. This result has also been much criticised. In particular because of the particular rationing rule, the efficient one, retained for the analysis. Still, the

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fact that capacity commitment relaxes price competition and drives equilibrium outcomes towards Cournotian ones is much less controversial.

An open question remains: to what extent is the ‘rigid capacity’ assumption central for this result to hold? In other words, would smoothly increasing marginal cost (or weaker forms of quantitative constraints) lead to similar outcomes? This question is interesting from a theoretical point of view, but it also belongs to the class of generalisations that seem necessary to pursue robust empirical studies on oligopolistic markets. As argued, for instance by Tirole (1988) (Chapter 5, p. 244), “the Bertrand and Cournot models should not be viewed as two rival models giving contradictory predictions of the outcome of competition in a given market. (After all, firms almost always compete in prices.) Rather, they are meant to depict markets with different cost structures.” It seems fair, however, to say that a rigorous link between the shape of marginal cost and equilibrium price is still missing under strategic price competition. The main goal of the present paper consists precisely in showing how the whole range of prices, from Bertrand towards Cournot, can be sustained as equilibrium outcomes in oligopolistic industries, depending on the shape of marginal costs.

In this letter we extend the KS result to the case of imperfect commitment and many firms. Firms commit to capacities in a first stage and then compete in price in the second stage. In the second stage, they may produce beyond installed capacity, but have to incur for this an extra unit cost, denoted by θ . The subgame perfect equilibria of this game exhibit the following features. Capacity has its full commitment value whenever producing beyond capacity entails an additional unit cost at least equal to the corresponding Cournot price. The lower the value of the additional unit cost, the closer the price to the competitive benchmark.

2. Bertrand–Edgeworth competition and imperfect commitment

The game tree is identical to KS except for the oligopoly setting: $n \geq 2$ firms choose costly capacities and then compete in price in the market for an homogeneous product. Rationing, if any, is organised according to the efficient rationing rule. Under this rule, low pricing firms are served first and ties are broken evenly. The firm exhibiting the highest price is left with a residual demand (if any) which is simply defined as a function of the other firms’ aggregate capacity. The cost structure in the pricing game is borrowed from Dixit (1980). The marginal cost up to the capacity level is w.l.o.g. zero while it is some positive θ beyond (this jump typically measures the legal wage gap for overtime work). Thus, in our capacity-price game tree, firm i invests in capacity x_i at cost $c(x_i)$, then all firms compete in prices for the demand function $D(\cdot)$ with the same production cost structure

$$c'_i(q) = \begin{cases} 0, & \text{if } q \leq x_i, \\ \theta, & \text{if } q > x_i, \end{cases} \quad \text{for all } i \leq n.$$

The original KS model corresponds to an infinite θ ; still, any value larger than the zero-demand market clearing price $D^{-1}(0)$ would trivially yield their result.

As a benchmark we consider the basic model of Cournot oligopolistic competition among n firms having the same convex cost function $c(\cdot)$. The aggregate consumer demand is $D(p)$, its inverse $P(x)$. Let $x_{-i} \equiv \sum_{j \neq i} x_j$ be the total quantity produced by firm i ’s opponents. A nil production is clearly optimal if $x_{-i} > D(0)$, otherwise firm i ’s profit is $x_i P(x_i + x_{-i}) - c(x_i)$. We assume that $xP(x+z)$ is

concave in x for all z . Therefore, the profit maximising quantity $r_c(x_{-i})$ is the unique solution of $P(x+z) + x\dot{P}(x+z) - \dot{c}(x) = 0$ and satisfies

$$\frac{\partial r_c}{\partial z} = - \frac{\dot{P}(x+z) + x\ddot{P}(x+z)}{2\dot{P}(x+z) + x\ddot{P}(x+z) - \ddot{c}(x)} \in [-1; 0].$$

We denote $r(x_{-i})$ the best reply with zero production cost; it plays a central role in the price competition. The symmetric Cournot–Nash equilibrium is the solution $\bar{x} < D(0)/n$ of $x = r_c((n-1)x)$. With constant marginal cost c and demand $P(z) = 1 - z$, we obtain

$$r_c(z) = \frac{1 - c - z}{2}$$

and

$$\bar{x} = \frac{1 - c}{n + 1}.$$

We are now in a position to solve the two-stage game with imperfect capacity commitment and price competition. The particular shape of marginal costs affects the analysis of price subgames. An upward price deviation may be profitable when other firms are likely to ration consumers. This requires first that the demand addressed to them exceeds their aggregate capacity and second that they are not willing to meet demand beyond capacity, i.e. the price is below θ . Otherwise, the standard Bertrand analysis applies as shown in Lemma 1.

Lemma 1. *The price equilibrium is the pure strategy θ if $\sum_i x_i \leq D(\theta)$, the pure strategy $P(x_1 + x_{-1})$ if $\sum_i x_i > D(\theta)$ and $x_1 \leq r(x_{-1})$, a mixed strategy equilibrium otherwise.*

Proof. W.l.o.g. firm 1 has the largest capacity in the subgame following the play of $(x_i)_{i \leq n}$. Let $\underline{p}_i$ and $\bar{p}_i$ be the lower and upper bounds of firm i 's equilibrium distribution F_i . Let $\underline{H} \equiv \arg \min_{i \leq n} \underline{p}_i$, $\underline{p} \equiv \min_{i \leq n} \underline{p}_i$, $\bar{H} \equiv \arg \max_{i \leq n} \bar{p}_i$ and $\bar{p} \equiv \max_{i \leq n} \bar{p}_i$. The cardinal of a set E is denoted by $\#E$.

Existence of an equilibrium is guaranteed by Theorem 5 of Dasgupta and Maskin (1986). By lowering its price a firm always benefits from an increase in demand (this property is not influenced by our rationing rule); its payoff is therefore left lower semi-continuous (l.s.c.) in its price, thus weakly l.s.c. The sum of payoffs is u.s.c. because discontinuous shifts in demand occur only when two firms or more derive the same profit.

Claim 1. $\#H > 1$ or the equilibrium is the pure strategy θ .

If $\#H = 1$, some firm k enjoys demand $D(p_k)$ on $[\underline{p}; \min_{i \notin \#H} \underline{p}_i]$ and two cases occur:

- (i) $\underline{p} < \theta$. If firm k is constrained at $\underline{p}$ its revenue is the strictly increasing function $p_k x_k$, a contradiction to $\underline{p}$ being in the support of the equilibrium distribution F_k . Thus, $D(\underline{p}) < x_k$ and because $p_k D(p_k)$ is a non-constant function, it must be the case that $\underline{p}$ is the monopoly price $P(r(0))$. Since no other price (irrespective of what may play the other firms) can yield the

monopoly payoff, firm k must be playing the pure strategy $P(r(0))$, but then any other firm i undercuts it, contradicting the optimality of its own equilibrium strategy.

(ii) $\underline{p} \geq \theta$. We are contemplating the classical Bertrand price competition, the outcome of which is pricing at the marginal cost θ .

Claim 2. $\forall i \in \underline{H}, \Pi_i^* = \underline{p}x_i$.

- $\underline{p} < \theta$. If firm $i \in \underline{H}$ deviates to $\underline{p}^-$ (this is shorthand for $\underline{p} - \varepsilon$ where ε is a small positive real number) its demand may jump upward; in order for $\underline{p}$ to be in the support of an equilibrium distribution it must be the case that this does not happen, thus the payoff is continuous at $\underline{p}$. If the demand at $\underline{p}^-$ is $D(\underline{p}^-)$ it must be the case that $\underline{p}^- < P(r(0))$, otherwise firm i would deviate downward, thus $D(\underline{p}^-) > r(0)$, which is an upper bound on capacity investment as it is the optimal quantity with zero cost for a monopoly (in a subgame perfect equilibrium we can eliminate strictly dominated strategies in the first stage). Therefore, firm i is constrained at $\underline{p}$ and we get $\Pi_i^* = \underline{p}x_i$.
- $\underline{p} \geq \theta$. We saw that θ is the unique possible price equilibrium. The claim is even valid for all firms since sales beyond capacity neither generate losses nor benefits.

Claim 3. If $x_1 + x_{-1} \leq D(\theta)$, θ is the unique price equilibrium.

The only case we need to consider is $\underline{p} < \theta$. If firm i plays $\theta^- > \underline{p}$ then the other firms that are less expensive receive full demand, but serve only their capacities so that firm i receives more than $D(\theta) - x_{-i} \geq x_i$, thus $\Pi_i(\theta^-, F_{-i}) = \theta^- x_i > \Pi_i^*$, a contradiction.

From now on we study the case where $p^* \equiv P(x_1 + x_{-1}) < \theta$.

Claim 4. $\bar{H} = \bar{H}^A \cup \bar{H}^B$, where $j \in \bar{H}^A$ if $r(x_{-j}) < x_j$ and $j \in \bar{H}^B$ if $\bar{p} = p^*$.

Let $\Psi_j(p_j) \equiv p_j \min\{x_j, \max\{0, D(p_j) - x_{-j}\}\}$ be the payoff to firm j when it names a price $p_j > \max_{i \in \bar{H}} \{\bar{p}_i\}$. If $p_j \leq \max_{i \neq j} \{\bar{p}_i\}$ then firm j gets at least the payoff $\Psi_j(p_j)$, thus this function must be maximal at $\bar{p}$ to sustain this price as a member of the support of an equilibrium strategy. Firm j cannot be fully served at $\bar{p}^+$ for otherwise it would deviate upward, thus it will only sell units with zero marginal cost. Two cases can occur. If $\Psi_j(p_j) = p_j(D(p_j) - x_{-j})$ in the neighbourhood of $\bar{p}$; we study the alternate formulation of profits $yP(y + x_{-j})$. The arg max is $r(x_{-j})$, thus $\bar{p} = P(r(x_{-j}) + x_{-j})$ and since firm j is not constrained at $\bar{p}$, it must be true that $r(x_{-j}) < x_j$. Besides the equilibrium payoff in that case is $\Pi_j^* = R(x_{-j})$, where $R(x) \equiv r(x)P(r(x) + x)$. Using the envelope theorem we obtain $\dot{R}(x) = r(x)\dot{P}(r(x) + x) < 0$. If, on the other hand, $\Psi_j(p_j) = x_j p_j$ at $\bar{p}^-$ then the upper price is $\bar{p} = P(x_j + x_{-j})$ and we have $x_j \leq r(x_{-j})$.

Claim 5. If $\bar{H}^B \neq \emptyset$, the equilibrium is $p^* \equiv P(x_1 + x_{-1})$.

Observe that p^* guarantees the revenue p^*x_i to any firm i . Indeed, if all other firms are less expensive, they are served first but the residual demand addressed to firm i is precisely its capacity. If $\bar{H}^B \neq \emptyset$, the equilibrium must be the pure strategy p^* for all firms.

Claim 6. If $\bar{H}^B = \emptyset$, then $\bar{H} = \bar{H}^A = \{1\}$, the large capacity firm.

Let $j \in \bar{H} = \bar{H}^A$. If $x_1 > x_j$ then $x_{-1} < x_{-j} \Rightarrow r(x_{-1}) + x_{-1} < r(x_{-j}) + x_{-j}$ as $r'(z) > -1$. Hence, firm 1 obtains $R(x_{-1}) \leq \Pi_1^*$ by naming $P(r(x_{-1}) + x_{-1}) > P(r(x_{-j}) + x_{-j}) = \bar{p}$. We now prove that $x_1 R(\bar{m} + x_1) < x_j R(\bar{m} + x_j)$, where $\bar{m} = x_{-1-j}$. Let $\Theta(z) \equiv zR(\bar{m} + z) = zr(\bar{m} + z)P(\bar{m} + z + r(\bar{m} + z))$ so that $\Theta'(z) = (r(\bar{m} + z) - z)P(\bar{m} + z + r(\bar{m} + z))$ obtains by the envelope theorem. If $r(\bar{m} + x_j) < x_j$, then $\Theta' < 0$: we are done. Otherwise, $r(\bar{m} + x_j) > x_j$ implies that $r(\bar{m} + x) = x$ is solved for x^* greater than x_j since $r(\cdot)$ is decreasing. By the same token, $x^* > x_j$ implies that the solution y^* to $r(\bar{m} + x) = x_j$ has to be greater than x^* . Finally, $r(\bar{m} + y^*) = x_j < x_1$ implies $y^* < x_1$. This is crucial as the positiveness of Θ on $[x_j; x^*]$ will be offset by its negativeness on the large interval $[x^*; x_1]$ as we now show:

$$\begin{aligned} \Theta(x_1) - \Theta(x_j) &= \int_{x_j}^{x^*} \Theta' + \int_{x^*}^{x_1} \Theta' \leq \int_{x_j}^{x^*} \Theta' + \int_{x^*}^{y^*} \Theta' = \Theta(y^*) - \Theta(x_j) = y^* R(r^{-1}(x_j)) - x_j R(\bar{m} + x_j) \\ &= y^* x_j P(x_j + r^{-1}(x_j)) - x_j R(\bar{m} + x_j) = x_j (y^* P(x_j + \bar{m} + y^*) - R(\bar{m} + x_j)) < 0 \end{aligned}$$

by definition of $R(\cdot)$.

$$\Pi_j^* = R(x_{-j}) < \frac{x_j}{x_1} R(x_{-1}) \leq \frac{x_j}{x_1} \Pi_1^* < \Pi_1^* = \underline{p}_1 [F_{-1}(\underline{p}_1)(D(\underline{p}_1) - x_{-1}) + (1 - F_{-1}(\underline{p}_1))D(\underline{p}_1)].$$

If firm j plays $\underline{p}_1$ its demand is $F_{-1-j}(\underline{p}_1)(D(\underline{p}_1) - x_{-1}) + (1 - F_{-1-j}(\underline{p}_1))D(\underline{p}_1)$, which is larger than the demand of firm 1 because there is more weight on the monopolistic term, therefore $\Pi_j(\underline{p}_1, F_{-j}) > \Pi_j^*$, the desired contradiction. We have shown that $1 \in \bar{H}^A$. Since $\bar{p} = P(r(x_{-j}) + x_{-j})$ holds true for any $j \in \bar{H}^A$, members of $\bar{H}^A$ must have the same (largest) capacity. ■

Relying on Lemma 1, we may state our theorem which extends the result of KS to oligopoly and imperfect commitment. For n symmetric firms and efficient rationing, the Cournot outcome emerges as the subgame perfect equilibrium outcome of the capacity-pricing game as soon as the ex-post marginal cost θ is larger than the Cournot price.

Theorem 1.

- If $\theta \geq P(n\bar{x})$, the symmetric Cournot–Nash investment $\bar{x}$ followed by the price $P(n\bar{x})$ is the unique subgame perfect equilibrium of Γ .
- If $\theta < P(n\bar{x})$, there is a continuum of SPE of Γ who nevertheless satisfy $D(\theta) = \sum_{i \leq n} x_i$. They converge toward the Bertrand solution as θ tends to zero.

Proof. If $x_i \leq D(\theta) - x_{-i}$, the equilibrium of the pricing game played after $(x_i)_{i \leq n}$ is the pure strategy θ and firm i 's payoff in Γ is θx_i . If $D(\theta) - x_{-i} < x_i \leq r(x_{-i})$, the equilibrium is the pure strategy $P(x_{-i} + x_i)$ and firm i is paid $f(x_{-i}, x_i) = x_i P(x_{-i} + x_i) - c(x_i)$. This function is concave with a maximum at $r_c(x_{-i}) < r(x_{-i})$. If $x_i > r(x_{-i})$, the equilibrium is in mixed strategies and firm i earns $g(x_{-i}, x_i) = R(x_{-i}) - c(x_i)$.

Notice that $P(x_{-i} + x_i) = \theta$ when $x_i = D(\theta) - x_{-i}$ and $g(x_{-i}, r(x_{-i})) = f(x_{-i}, r(x_{-i}))$. Hence the first period payoff as a function of x_i is continuous. Moreover, at $x_i = D(\theta) - x_{-i}$, $\partial f / \partial x_i = \theta + x_i \dot{P}(x_{-i} + x_i) - \dot{c}(x_i) < \theta$ and the slope of g is steeper than that of f as the second period payoff becomes

constant. The payoff function is thus concave in x_i for any x_{-i} ; its average over the equilibrium distributions of the others firms is also concave, meaning that the best reply of firm i is always a pure strategy. Because this applies for all firms, the equilibrium is in pure strategies and satisfies $x_i = \max\{D(\theta) - x_{-i}, r_c(x_{-i})\}$ for all i .

If $D(\theta) - x_{-j} > r_c(x_{-j})$ for some x_j then $D(\theta) = x_{-j} + x_j$. From this we deduce that $D(\theta) = x_{-i} + x_i$ must hold for all other firms, so that $x_i = r_c(x_{-i}) > D(\theta) - x_{-i}$ is impossible. Hence the candidate equilibria are all vectors $(x_i)_{i \leq n}$ satisfying $D(\theta) = \sum_{i \leq n} x_i$ and $D(\theta) > r_c(x_{-j}) + x_{-j}$ for all j . The symmetric equilibrium $D(\theta)/n$ exists if it is larger than the Cournot candidate $\bar{x}$, i.e. if $\theta \leq P(n\bar{x})$, the Cournot price. Solving for $D(\theta) = r_c(y) + y$ yields a value y^* that circumvents the range of asymmetric equilibria; they are given by the set of constraints $\forall i \leq n, x_i \geq y^*$ in addition to $D(\theta) = \sum_{i \leq n} x_i$. Now if $\theta > P(n\bar{x})$, the equilibrium is unique. Let $\bar{m} \equiv x_{-1-2}$ (zero if $n = 2$). If $x_1 = r_c(\bar{m} + x_2)$ and $x_2 = r_c(\bar{m} + x_1)$, then x_1 and x_2 are solutions of $z = h(z) \equiv r_c(\bar{m} + r_c(\bar{m} + z))$. But since we previously showed that $0 > r'_c(z) > -1$ it must be the case that $h(z) = r'_c(\bar{m} + r_c(\bar{m} + z))r'_c(\bar{m} + z) < 1$, thus h has a unique fixed point so that $x_1 = x_2$. By repetition of the argument to all pairs, we conclude that the equilibrium is unique and symmetric: it is the Cournot quantity $\bar{x}$. ■

For a linear demand $D(p) = 1 - p$ and zero marginal cost the Cournot quantity is $1/3$. If the sum of quantities $x_1 + x_2$ is less than $D(\theta) = 1 - \theta$ then firms are not able to avoid Bertrand competition. It is only for large aggregate capacities that the price equilibrium result in Cournot payoffs. Thus for $\theta > 1/3$, very low capacity choices lead firms to invest more ($x_1 + x_2$ reach $D(\theta)$), then for larger capacities the Cournot competition applies and leads to the symmetric equilibrium choice of $1/3$. For $\theta < 1/3$, the area where Bertrand competition applies incorporates the previous equilibrium meaning that firms are induced to build more capacity because the fierce price competition yields too small margins. There is now a continuum of equilibria where firms share the market, but not too asymmetrically, as the Cournot best replies provide lower bounds on one's equilibrium capacity (condition $x_i \geq y^*$ above).

3. Conclusion

Many researches have aimed at reconciling Cournotian outcomes with the explicit price mechanism involved in the Bertrand model. These researches have been successful to the extent that they have been able to combine the two features of oligopolistic industries which are limited scales of production (increasing marginal costs) and price setting behaviour. The main challenge in this respect consists in dealing with the issue of quantitative constraints (non-constant marginal cost) at the price competition stage which tends to make it unprofitable for the firms to meet full demand. This in turn generates rationing possibilities which are at the heart of Edgeworth's critique.

The issue of rationing in pricing games is best understood by studying closely the allocation process of a non-competitive market with price-setting firms. Three stages are needed to correctly describe this process. In the first stage, firms name prices and consumers address demand to firms. In the second stage, firms decide on their sales and possibly ration consumers. In the third stage, rationed consumers possibly report their demand to non-rationing firms who may or may not accept them. In the case of homogeneous products and perfect display of prices, the low pricing firm receives all the demand at the end of the first stage. Under constant returns to scale, this firm is willing to meet any

demand level so that it always chooses to serve all consumers in the second stage; the third stage is then irrelevant. With decreasing returns to scale, it may not be optimal to meet full demand in the first stage. Our paper takes this possibility into account, contrarily to a recent stream of literature where rationing is ruled by assumption (cf., e.g., Dastidar, 1995, 1997; Maggi, 1996).

In our model the marginal cost structure varies from the Bertrand one to the Kreps and Scheinkman one through a single parameter measuring the commitment value of capacities. Within this framework we obtain the whole range of prices, from Bertrand to Cournot ones, as subgame perfect equilibrium outcomes.

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