Prediction models for ageing/deterioration

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Selective maintenance planning based on a Markovian approach

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ABSTRACT: Existing buildings within an aggressive environment suffer deterioration of their making up materials. This deterioration, over the time, can lead the system to a loss of performance. When the loss of performance is greater than prescribed threshold values it is considered as a *failure* and the system passes from current service state to another one characterized by a lower level of performance. However, maintenance and/or rehabilitation can improve structural performance. Both these interventions can be defined as transition stages and modeled as Markovian processes. In this paper applying a Markovian approach associated to a Monte Carlo simulation, the maintenance planning within decision-making analysis is assessed.

1 INTRODUCTION

The assessment of the life-cycle performance of deteriorating structures can be formulated as a reliability problem where a loss of performance greater than prescribed threshold values is considered as a *failure*. Therefore, when a failure occurs, the system moves from the current state to another one characterized by a lower level of performance. On the other hand, an operative maintenance and/or rehabilitation can improve structural performance. In this case the system may move from the current state to another one characterized by a higher level of performance. Anyway the *failure process* may be defined as a *transition process* through different service states due to environmental attacks and/or maintenance actions. Since the problem is affected by uncertainty, the assessment of the life-cycle performance of a deteriorated structure must base on a probabilistic analysis able to model the time-variant deterioration process over the time. To represent such a transition process, the Markovian system seems to be effective. However, each probabilistic approach requires databases obtained by monitoring actions. Usually these available monitoring databases are limited in time ad extension, facing to compromise the success of the probabilistic approach. Thence the probabilistic approach can be associated to a Monte Carlo simulation implemented with a suitable deterioration modeling. In this way the combined system is able to simulate a huge number of possible damage evolutions of a certain structural typology over the time.

In this paper, the deterioration affects structural members due to aggressive environment, is simulated using the damage model proposed by Biondini *et al.* (2008) and implemented into a computer code based on the Monte Carlo approach.

Using a Markovian approach based on the results obtained by the Monte Carlo simulation, the development of the structural system's state performance over the time is analyzed.

Afterwards referring to repair intervention applied just on heavily deteriorated elements, with a high probability of a state transition occurrence, some selective maintenance scenarios are investigated. Studying the effects of selective maintenance, the proposed method is applied on the case study of a steel truss structure.

2 DETERIORATION PROCESS AS A MARKOVIAN TRANSITION PROCESS

The deterioration process can be seen as a loss of performance affecting a material system. Whenever deterioration reaches a given level, the system suffers a *"failure"* and a loss of performance. This phenomenon can be described as the transition of the *system* through different service states characterized by different levels of performance. Thus, the deterioration process (failure process) may be defined as a *transition process* (Garavaglia *et al*., 2004).

Every transition depends on:

- the magnitude of the attack (stress cycle);
- the system's capability to withstand this attack.

Both these parameters depend on a huge number of time dependent random variables (r.v.); therefore it is right to consider the transition process as a stochastic process. The r.v. *service lifetime* τ_i is suitable to describe this process. It is defined as: "the waiting time spent by the material in the performance state i before a transition" (Garavaglia *et al.* 2004).

From this point of view the reliability function $R(x)$ can be defined as:

$$R(t) = \Pr\left\{ \tau_i > t \right\}. \tag{1}$$

The (1) represents the probability that a transition by state i into a next state doesn't occur by waiting time τ_i (Cox, 1962); it can be described by a survive function of the r.v. τ_i.

If the deterioration process is assumed to be a state transition, a stochastic dynamic process can be assumed in order to represent it. Biondini & Garavaglia (2005) have shown that the Markov Renewal Process (MRP) (Howard, 1971, Limnios & Oprisan, 2001) seems to be suitable to describe such a dynamic process. The MRP can describe the development of the system's life among different service states with different waiting times; it is also able to take in account the age t_0 of the system, i.e. the time already spent by the system into the current service state before the prevision is made. This aspect is very important in reliability analysis when maintenance must be planned.

2.1 *Some remarks on Markov Renewal Processes*

A MRP is completely determined if we know its initial laws (Limnios & Oprisan, 2001):

1. initial conditions: *initial state*, i.e. the state occupied by the system at the starting time of the prevision. Defining the time t_0 already spent by the system into the initial state before the starting time of the prevision is also needed (Howard, 1971).
2. $\Pr(\text{initial state} = i) = a_i$ for every $i \in E$, this probability distribution represents the probability that initial state will be i.
3. $\Pr(\text{next state } j, \tau_{ij} \le t \mid \text{previous state } i) = p_{ij} F_{ij}(t)$ for every $t \in (0, +\infty)$ and $i,j \in E$. (2)

Assuming i as present state, the (2) describes the probability that transition into the next state j, occurs by time t; within it p_{ij} are the transition probabilities of the Markov chain and $F_{ij}(t)$ are the distribution functions associated to waiting times in state i before moving to state j.

4. $$\Pr(\tau_1 \le t_1, \ldots, \tau_i \le t_i \mid \text{current state}(i-1)) = \prod_{i=1}^{n} F_{(i-1),i}(t_i) \quad \forall\, n \ge 0 \quad \text{and} \quad t_i \ge 0 \tag{3}$$

This probability is the marginal distribution function of the waiting times τ_i describing the probability of transition into each state i by time t_i.

If the MR process is defined as shown in points 1-3, assuming i as present state and t_0 as the time already passed by the system into i-state, the probability to move into *state j,* can be defined as follows:

$$\Pr(\text{state } j, \tau_{ij} \le t_0 + \Delta t \mid \text{state } i, \tau_{ii} > t_0), \tag{4}$$

where, i is the state of the present performance; j is the state of next lower performance; τ_{ij} is the waiting time spent by the system into the state i before moving into j, under the limiting condition that no transition is already happened (defined through the condition: $\text{state } i, \tau_{ij} > t_0$); Δt is the discrete time in which the prediction should be obtained.

If the distribution functions $F_{ij}(t)$ are defined and $\tau_{ij}=t_0$, the probability (4) can assume the following form:

$$\mathrm{P}_{\Delta t|t_0}^{(ij)} = \frac{\left[F_{ij}(t_0 + \Delta t) - F_{ij}(t_0) \right] p_{ij}}{\sum_{k=1}^{s} \left[1 - F_{ik}(t_0) \right] p_{ik}}. \tag{5}$$

The probability p_{ij} can be obtained by experimental observations trough the ratio:

$$p_{ij} = \frac{\text{observed transitions from } i \text{ to } j}{\text{observed transitions from } i}. \tag{6}$$

Of course, probability (5) can be evaluated both for the system and for each member composes the system; in the following member the probability will be indicated with: $\mathrm{P}_{\Delta t|t_0}^{(m,ij)}$ where m is the number identifying the structural member analyzed.

2.2 *Deterioration process as Markovian Process*

Different deterioration processes connected with environmental aggressive attacks can affect each single member of a structure, until they reach the failure threshold value of the whole system.

The failure of a member or of the whole system is connected to the loss of performance. Assuming the stress value σ recorded/evaluated at every monitoring as risk parameter and considering it as the ratio between the internal forces and the deteriorated area of the elements' cross sections, using an appropriate computer code for structural analysis the performance decrease can be evaluated. Hereafter this parameter is assumed as random variable in the deterioration process. In order to model the time evolution of such variable within a Markov renewal modeling, the following assumptions are introduced (Biondini & Garavaglia, 2005):

– The structure as well as each component member are undamaged at the initial time t_0=0.

– At each instant t the members are considered to occupy a given state $i,$ with $i > 0$, when $\sigma_i \le \sigma \le \sigma_{(i+1)}$, where σ_i and $\sigma_{(i+1)}$ are the lower and upper thresholds respectively, which characterize the state i.

– The member performance moves from a state i, $i > 0$, to another state j, with $j > i$, characterized by a lower level of performance, σ_i, $\sigma_i < \sigma_j$, during a time interval τ_{ij}. Of course, condition $j < i$, with $\sigma_i > \sigma_j$, is also possible if some maintenance is operated.

In MRP each waiting time τ_{ij} must be modeled by choosing an appropriate PDF. This is not a simple choice and it can be "not unique". It should be made on the basis of physical knowledge of phenomenon and on the basis of the characteristic distributions in their tails where, usually, no much data are recorded.

The physical knowledge, based on experimental evidence, suggests that the deterioration of building materials increases following a characteristic time-dependent law due to aggressive environment. Therefore, considering the increasing hazard rate functions $\lambda(t)$ over the time, the deteriorated material's behavior can be model by using distributions satisfying this tendency. Distributions obeying to this law are, for example, the Weibull distributions where, if $t\rightarrow\infty$, $\lambda(t)$ tends to an infinite value too, and Gamma distributions where, if $t\rightarrow\infty$, $\lambda(t)$ tends to an asymptotic value. In this paper, starting from the previous considerations Weibull distributions are chosen in order to model structural members deterioration. The parameters involved in Weibull distributions are estimated by the maximum likelihood criterion using a computer code where the routine ROSE of IMSL Fortran Library, involved the Rosenbrock's optimization method, was implemented.

3 DETERIORATION MODELING

As already introduced, the uncertainty affecting the deterioration process of building materials suggests a probabilistic approach in the modeling. The hereafter proposed method is a MR process that, as each probabilistic approach, requires a huge dataset built on the basis of monitoring actions. Usually, monitoring actions are limited in space and time and this can compromise the right modeling of the phenomenon investigated. To overcome this problem the probabilistic approach can be associated with a Monte Carlo simulation able to simulate the deterioration evolution for several structures with same typology. However Monte Carlo simulation requires an appropriate deterioration model. Biondini *et al*. (2008) proposed a suitable damage model that seems to be effective in order to simulate the deterioration process investigated. This model has been implemented into our computer code for structural analysis and Monte Carlo simulation.

3.1 *Damage Index*

Following Biondini *et al.* (2008), by denoting Θ a generic material property, the material deterioration over time t can be evaluated as follows:

$$\Theta(t) = \Theta_0[1 - \delta(t)] \tag{7}$$

where "0" denotes the initial undamaged state, and the deterioration over time t is measured by a time-variant damage index $\delta=\delta(t)\in[0;1]$. The following damage model is assumed (Biondini *et al.* 2008):

$$\delta(\tau) = \begin{cases} \omega^{1-\rho}\tau^{\rho} & , \ \tau \le \omega \\ 1-(1-\omega)^{1-\rho}(1-\tau)^{\rho} & , \ \omega < \tau < 1 \\ 1 & , \ \tau \ge 1 \end{cases} \tag{8}$$

where $\tau = t/T_C$, T_C is the normalized time instant when the failure threshold δ=1 is reached and ω and ρ are shape parameters of the damage curve (Fig. 1).

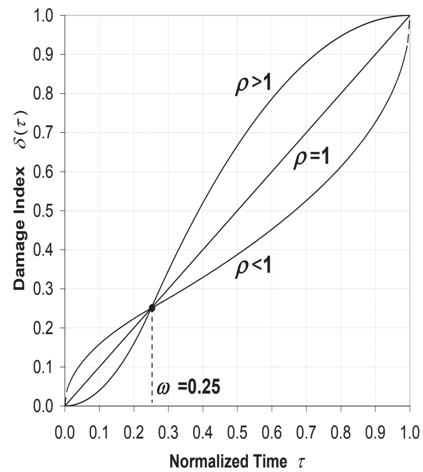


Figure 1. Damage index δ vs normalized time $\tau = t$/TC. Damage laws for ω=0.25 and varying ρ (by Biondini *et al.*, 2008).

The damage parameters ρ and ω must be chosen according to the actual damage process development. Damage rates may be associated with the aggressiveness of the environment, as well as with the level of acting stress.

The following linear relationship is assumed (Biondini *et al.*, 2008):

$$\rho = \rho_a + (\rho_b - \rho_a)\xi \tag{9}$$

$$\omega = \omega_a + (\omega_b - \omega_a)\xi \tag{10}$$

where the subscript "*a*" refers to damage associated with environmental aggression, and the subscript "*b*" refers to damage associated with loading effects and ξ refers to ratio between the level of acting stress and the limit state value.

Thereby the proposed damage law is able to represent damage mechanisms induced by environmental deterioration, like carbonation of concrete, corrosion of steel or material fatigue. These mechanisms are usually present and interacting. Thence basing on experimental observations and/or laboratory accelerated test data, a proper calibration of the damage parameters is required.

4 CASE STUDY: STEEL TRUSS

In this paper the MR approach is applied to evaluate the transition process of each structural member of a simple case study. The aim is the schedule of maintenance actions on single or multiple members based on the probable instant of transition between states evaluated with the MRP.

The attention is pointed out truss systems subjected to deterioration process involving a reduction of both the cross-sectional area A and material strength $\bar{\sigma}$ of each structural member. Without any loss of generality, in this study it is assumed that such properties suffer from the same damage process:

$$A(t) = A_0[1-\delta(t)] \tag{11}$$

$$\bar{\sigma}(t) = \bar{\sigma}_0[1-\delta(t)] \tag{12}$$

where "0" denotes the initial undamaged state, and the damage rates may be associated with the aggressiveness of the environment, as well as with the level of acting stress σ with respect to the material strength $\bar{\sigma}$, or $\xi = \sigma / \bar{\sigma}$. The damage model assumed is the model in Eq. (8).

The approach is applied on a statically indeterminate truss system with members' area A=2500mm^2 and total volume V=0.0723m^3. The allowable material strength is $\bar{\sigma} = 140$ MPa. Buckling failures are assumed to be avoided. The structure is subjected to a set of forces F=25 kN as shown in Figure 2. The initial value of member cross-sectional area A and of material strength $\bar{\sigma}$, as well as the force F, are assumed as deterministic.

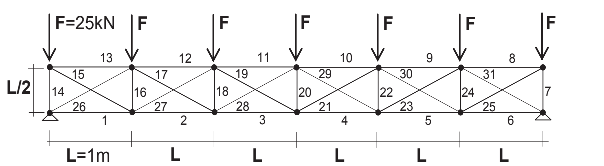


Figure 2. Statically indeterminate truss system with members area A=2500 mm^2 and total volume V=0.0723 m^3.

Each probabilistic approach requires several experimental data usually difficult to collect. In this study, the samples were obtained using the Monte Carlo simulation. In this simulation process the damage parameters ρ_a, ρ_b, ω_a, ω_b, and T_C, are modeled as random variables with prescribed probability distribution. Each distribution is chosen on the basis of physical knowledge of phenomena investigated and the modeled is made starting from mean and standard deviation (Ciampoli, 1999, Gupta & Lawsirirat, 2005).

During its life the structure as well as its single members, move into different service state; the transitions can occur state by state, otherwise can involve two or more states, which means that a transition can involve two or more states for each step. Thence the choice of the state size is a tricky issue that can compromise the modeling. The following three different states are assumed: *State 1* relates to a low damage, *State 2* relates to moderate damage and *State 3* relates to heavy damage. The states' bounders are chosen on the basis of expert judgment and are collected in Table 1.

Table 1 State bounders.

State	σ_{min} (MPa)	σ_{max} (MPa)	Damage level
State 1	>0.00	≤1100	Low (undamaged)
State 2	>1100	≤1250	Moderate
State 3		>1250	Heavy

For each simulation cycle, the time-variant structural analysis is carried out by updating step-by-step the stiffness matrix of the deteriorated structural members. Referring to the limit state i, $|\sigma| \leq \sigma^i_{\max}$, the member's transition happens when the member reaches the upper bound $\sigma^i_{\max}$ of the state i. At every simulation, the transition time for each member is recorded and it is defined as the waiting time τ_{ij} spent by the member into state i before moving into a successive state j.

Considering the limit state $(|\sigma| - \bar{\sigma}) = 0$, the system failure is reached when the number of failed members is able to activate a certain mechanism. The time in which the failure occurs is called *failure time*. While whenever a member comes to a given damage threshold, the time related to the reaching of this damage state is called *crossing time*.

The approach here presented is applied to probabilistically investigate the next transition of each member of the system in Figure 2, from the current state i since t_0, with t_0>0 to the state j in the next interval Δt. Therefore using a population of 5000 samples of the same structural system a MC simulation has been built. Each sample was affected by random deterioration process following the damage law (8); the members' crossing times connected with each damage threshold and the failure time of the whole system have been recorded and modeled with Weibull distributions.

Using Eq. (5) the probability $\mathrm{P}^{(m,ij)}_{\Delta t|t_0}$ are evaluated for each m members of the system (m=31) (Fig. 3). Considering new buildings, when $\mathrm{P}^{(m,ij)}_{\Delta t|t_0}$, with $i \neq j$, reaches a value major or equal to r*10^{-4} (r=positive number) the risk of transition from i to j is assumed upcoming; therefore maintenance could be planned in instants close to which the probability is recorded (Fig. 4).

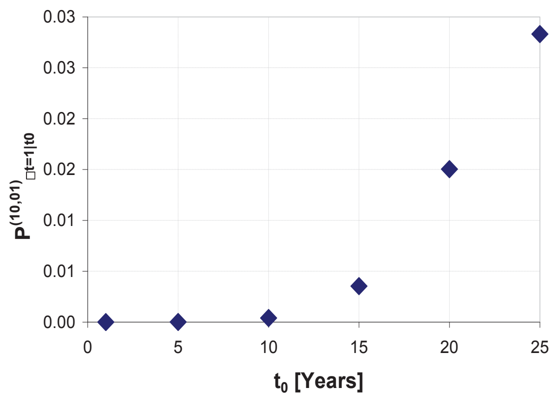


Figure 3. Member 10: Cumulative probability of transition $\mathrm{P}^{(m,ij)}_{\Delta t|\,t_0}$ (i=0, j=1, Δt=1year) at different t_0.

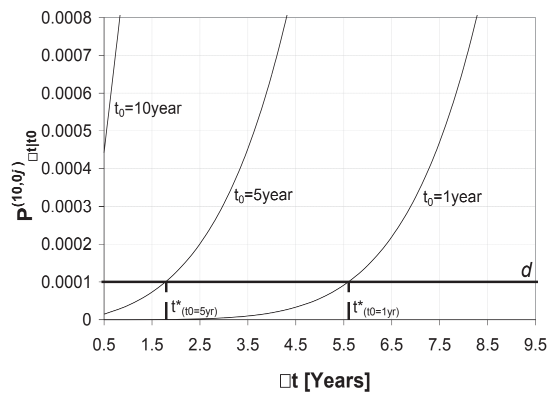


Figure 4. Member 10: Probability of transition $\mathrm{P}^{(m,ij)}_{\Delta t|\,t_0}$ vs Δt (i=0, vs. $\forall\, j$) evaluated at different t_0. Damage d=1*10^{-4} is the damage threshold; instants t^* are the instants when the member exceeds d if the time t_0 spent into state i=0 is 5 years or 10 years respectively.

5 MAINTENANCE PLANNING BASED ON MRP EVIDENCE

The maintenance of existing building can be approached as follows:

a) *Replacing* all damaged members: the initial system reliability is wholly restored and future maintenance can be applied at constant intervals.
b) *Repairing* all damaged members: the initial system reliability is almost restored, future maintenance could be required at every closer interval.
c) *Replacing/repairing* several damaged members: the initial system reliability is just partially restored.

The application of the MRP on each member of a system can lead the decision on *when* and *where* to operate maintenance, therefore, it's interesting to investigate possible selective maintenance scenarios on the basis of MRP results and the economic implications.

In this paper are presented the first results of a selective maintenance approached with the MRP and focused on the replacement of the deteriorated members only.

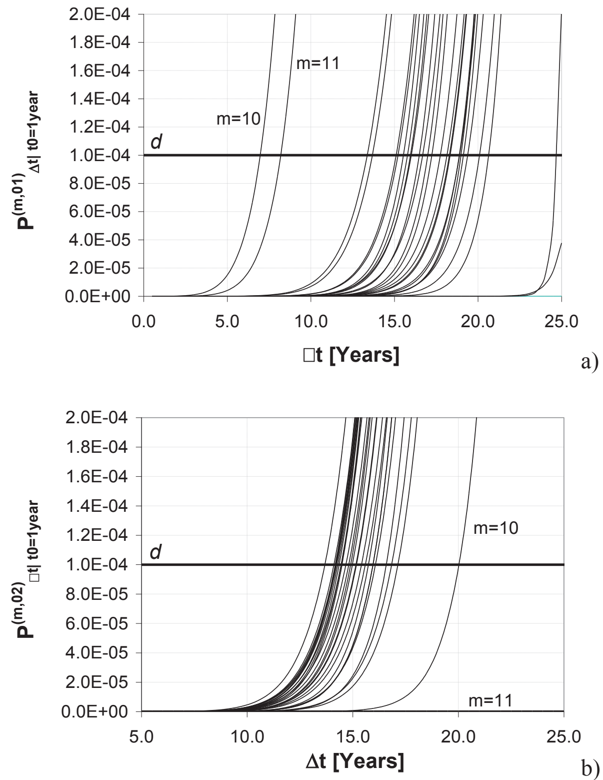


Figure 5. Probability of transition $\mathrm{P}^{(m,ij)}_{\Delta t|\,t_0}$ vs Δt evaluated on the 31 members of the system with t_0=1 year: a) transition 01, from initial *state* 0 to *state* 1; b) transition 02, from initial *state* 0, directly to the *state* 2.

The Markovian approach applied on 5000 has pointed out the possibility of moving through contiguous states or directly in states with higher damage also for the same element of a structure (Fig. 5); this happens because the 5000 systems are subjected to a random deterioration always different and the behavior of the members changes time by time.

Figure 5 shows the tendency of each member of the system to move into deterioration state 2 within the first 17 years of life. Therefore assuming the 17th year as warning threshold of time, maintenance should be operate before this instant.

Evaluating the waiting times spent by the system into the initial state 0 before the next state transition and assuming as dangerous the threshold d=1*10^{-4}, Figure 6 suggests to make a first selective maintenance in advance on several members, and afterwards a second one on the whole system.

On the bases of results shown in Figure 6, different selective maintenance scenarios can be proposed (Fig. 7). Furthermore each scenario must be economically evaluated too. The economical investigation of different maintenance scenario will be the future step of this research.

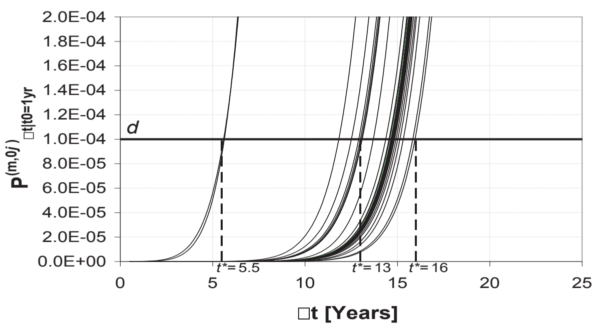


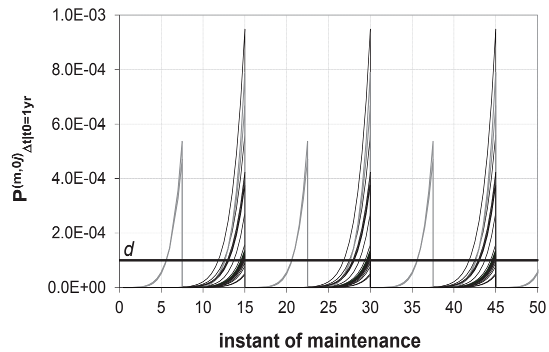
Figure 6. Probability of transition $\mathrm{P}^{(m,ij)}_{\Delta t|\,t_0}$ vs Δt evaluated on the 31 members of the system with t_0=1 year and transition 0j from initial *state* 0 to *state j* (j=1,2). This figure describes the probability of transition from initial state versus one of the future damage states. Instant t* represents the possible instant of selective maintenance only on the members with probability of transition higher that d (on the left side of t*).

5.1 *Selective maintenance: first results and conclusion remarks*

As a first application of the procedure proposed, two selective maintenance scenarios are investigated. The maintenance is scheduled assuming t_0=1 year: considering a one-year-old system, just the replacement of deteriorated elements, without analyzing the system after the first replacement, is assumed. This is the mainly critical issue within the initial application phase. Since the new replacement might extend the waiting time of the next maintenance with cost effectiveness, comparing with results obtained by an analysis where replacements of new members are taken into account, the results here shown surely ensure structural safety but probably less economic benefit.

Figure 7 shows two possible selective maintenance scenarios: scenario a) maintenance is renewed each Δt=7.5 years with two different actions: Δt=7.5 years replacing of two members (10 and 11), Δt=15 years overall replacing of system's members; scenario b) maintenance is renewed each Δt=8.5 years with two different actions: Δt=8.5 years replacing of 10 members (3, 4, 5, 10, 11, 19, 21, 23, 28 and 29), Δt=17 years overall replacing of system's members.

Comparing the two scenarios, scenario b) presents the replacement of more members respect to scenario a); in fact, the low but increasing $\mathrm{P}^{(m,ij)}_{\Delta t|\,t_0}$ after the maintenance instant chosen, suggests an in advance replacement. During the service life (usually assumed of 50 years), scenario b) leads to obtain 5 partial maintenance actions and only 2 on the whole system; while scenario a) leads to 6 partial maintenances and 3 overall maintenances.



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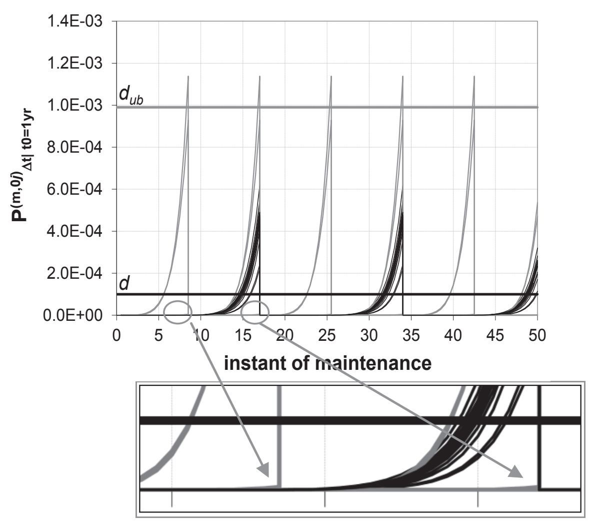
Scenario a)



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
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Scenario b)

Figure 7. Two possible selective maintenance scenarios. Considering maintenance instants, the figures show the system's leaning towards initial level of performance when the replacement of damaged elements is carried out. Tables show which members are replaced at any maintenance. Circles identify members with a low probability P to cross d before Δt=8.5 years and Δt=17 years respectively. The sudden increasing of P after Δt suggests an in advance replacement.

In this paper an approach to decision making support within maintenance and replacing strategies is proposed. This combined method evaluates when and where is necessary to act. The authors are aware that these are just preliminary results. Thence it will be important to verify the economic implications of the actions investigated as well as comparing this approach with others among literature.

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