

together with a specially adapted version of the recursive reduction procedure given in [4].

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Stable sets of matrices and efficient approximations of the joint spectral radius

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(joint work with Yurii Nesterov)

The joint spectral radius of a set of matrices is a measure of the maximal asymptotic growth rate that can be obtained by forming long products of matrices taken from the set. This quantity appears in a number of application contexts but is notoriously difficult to compute and to approximate. We describe a procedure for approximating the joint spectral radius of a finite set of matrices with arbitrary high accuracy. Our approximation procedure is polynomial in the size of the matrices once the number of matrices and the desired accuracy are fixed.

Let $\{A_1, \dots, A_m\}$ be some set of square matrices with real entries. To the finite sequence $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_k) \in \{1, \dots, m\}^k$ we associate the corresponding matrix product

$$A_\sigma = A_{\sigma_k} \cdots A_{\sigma_2} A_{\sigma_1}.$$

With this notation, the joint spectral radius is defined by

$$\rho(A_1, \dots, A_m) = \limsup_{k \rightarrow +\infty} \max_{\sigma \in \{1, \dots, m\}^k} \|A_\sigma\|^{1/k}.$$

Our approximation procedures for the joint spectral radius provide approximations of relative accuracy $1 - \epsilon$ in time polynomial in $n^{(\ln m)/\epsilon}$, where m is the number of matrices and n is their size. These bounds are close from optimality since we show that, unless P=NP, no approximation algorithm is possible that provides a relative accuracy of $1 - \epsilon$ and runs in time polynomial in n and $1/\epsilon$.

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Controllability for nonlinear behaviors

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(joint work with Wolfgang Kliemann)

The analysis of differential equations and, more generally, dynamical systems, via the time shift on a space of trajectories is a classical approach going at least back to the work of Bebutov [1] in 1940 and has fostered the development of topological dynamics, compare Sell [3]. In control theory, the analysis of input- and output-functions has a long tradition. A new paradigm, called the behavioral approach to control, has been introduced by Willems [4] considering systems interacting with the environment without making a difference between inputs and outputs. However, so far, this latter theory has essentially been restricted to an algebraic framework. The present talk aims at the analysis of behaviors via topological dynamics of the time shift. It turns out that the basic notion of controllability does not directly lend itself to such an analysis. However, a weakened version, chain controllability, is intimately related to the classical notion of chain transitivity in topological dynamics (see, e.g., Robinson [2]). Then we provide a sufficient condition which allows us to show that, generically, chain controllability implies controllability.

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