

# Discontinuous Galerkin finite element modelling of geophysical and environmental flows

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*Certains imaginent le doctorat comme un long et pénible travail solitaire. Long, ça l'est certainement ; et du travail, il y en a beaucoup ! Mais pénible n'est pas un bon qualificatif. Entreprendre un doctorat est certes une tâche ardue et très exigeante, mais elle est surtout passionnante. Finalement solitaire : non, je ne qualifierais pas mon doctorat de solitaire, car j'ai été très entouré par de nombreuses personnes, que je tiens à remercier chaleureusement.*

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# Model variables and acronyms

## Model variables

### SLIM 3D equations

|                    |                                                |                              |
|--------------------|------------------------------------------------|------------------------------|
| $\mathbf{u}$       | Horizontal velocity vector                     | $[\text{m s}^{-1}]$          |
| $w$                | Vertical velocity                              | $[\text{m s}^{-1}]$          |
| $\eta$             | Sea surface elevation                          | $[\text{m}]$                 |
| $p$                | Hydrostatic pressure                           | $[\text{N m}^{-2}]$          |
| $\rho$             | Water density                                  | $[\text{Kg m}^{-3}]$         |
| $\rho_0$           | Reference water density                        | $[\text{Kg m}^{-3}]$         |
| $\rho'$            | Water density deviation                        | $[\text{Kg m}^{-3}]$         |
| $\bar{\mathbf{u}}$ | Vertically averaged horizontal velocity vector | $[\text{m s}^{-1}]$          |
| $f$                | Coriolis factor                                | $[\text{s}^{-1}]$            |
| $g$                | Gravitational acceleration                     | $[\text{m s}^{-2}]$          |
| $\nu$              | Vertical viscosity                             | $[\text{m}^2 \text{s}^{-1}]$ |
| $\nu_h$            | Horizontal viscosity                           | $[\text{m}^2 \text{s}^{-1}]$ |
| $\kappa$           | Vertical diffusivity                           | $[\text{m}^2 \text{s}^{-1}]$ |
| $\kappa_h$         | Horizontal diffusivity                         | $[\text{m}^2 \text{s}^{-1}]$ |
| $h$                | Bathymetry                                     | $[m]$                        |
| $H$                | Total water column height                      | $[m]$                        |

### SLIM 3D mesh adaptation

|          |                                    |                     |
|----------|------------------------------------|---------------------|
| $w_m$    | Moving mesh velocity               | $[\text{m s}^{-1}]$ |
| $w^*$    | Adaptive moving mesh velocity      | $[\text{m s}^{-1}]$ |
| $e$      | Adaptive error function            | $[1]$               |
| $f_e$    | Adaptive background error function | $[1]$               |
| $\tau_e$ | Relaxation parameter               | $[s]$               |

## Acronyms

|       |                                                    |                                             |
|-------|----------------------------------------------------|---------------------------------------------|
| ALE   | Arbitrary Lagrangian Eulerian                      | p. <a href="#">38</a>                       |
| CFSR  | Climate Forecast System Reanalysis                 | pp. <a href="#">21</a> , <a href="#">70</a> |
| CLM   | Community Land Model                               | p. <a href="#">97</a>                       |
| COSMO | Consortium for Small-scale Modelling               | p. <a href="#">97</a>                       |
| DG    | Discontinuous Galerkin                             | pp. <a href="#">5</a> , <a href="#">43</a>  |
| FLAKE | Freshwater Lake Model                              | pp. <a href="#">95</a> , <a href="#">97</a> |
| GBR   | Great Barrier Reef                                 | pp. <a href="#">15</a> , <a href="#">65</a> |
| GOS   | Global Optimal Solution                            | p. <a href="#">18</a>                       |
| GOTM  | General Ocean Turbulence Model                     | p. <a href="#">37</a>                       |
| HF    | High Frequency                                     | p. <a href="#">16</a>                       |
| PAN   | Panchromatic                                       | p. <a href="#">32</a>                       |
| PIV   | Particle Image Velocimetry                         | p. <a href="#">18</a>                       |
| SLIM  | Second-generation Louvain-la-Neuve Ice-ocean Model | pp. <a href="#">5</a> , <a href="#">20</a>  |
| TPXO  | TOPEX/Poseidon Global Inverse Solution             | pp. <a href="#">21</a> , <a href="#">70</a> |
| UTC   | Coordinated Universal Time                         | p. <a href="#">17</a>                       |

# CHAPTER 1

## Introduction

*Jacques-Yves Cousteau once said: “Nous oublions que le cycle de l’eau et le cycle de la vie ne font qu’un”<sup>1</sup>. All the developments and the work carried out during the preparation of this thesis are pointless if we forget the importance of water.*

Water bodies of all sizes are essential for life on Earth. Oceans cover about 70% of the surface of the planet, they are a major player in the weather and climate. Continental seas cover a much smaller surface ( $\sim 8\%$  of ocean surface) and volume ( $< 1\%$  of total volume) but have a key biological role: at the interface between land and deep ocean, they are responsible for 19 to 28% of the global ocean primary production (Yool and Fasham, 2001). They may have been the first locations where organic life appeared and developed (Dyer, 1997). Lakes have, depending on their size, a local or regional impact on the ecology and the weather, and estuaries and rivers connect the land to coastal waters, transporting organic and inorganic matter.

Water is also crucial for humans. It plays an essential role in our health, food production, transport, etc. In return, human activities have a large and increasing impact on the aquatic ecosystems. Agriculture, fisheries, deforestation, damming, waste disposal, industrial pollution and transportation, among other things, are a constant pressure on the coastal zone (Crossland et al., 2005). The population living within 60 km of all coastlines is estimated at 60% of the world population (Rao et al., 2008). The population density in the coastal zone is more than three times larger than in the inland area.

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<sup>1</sup>“We forget that the water cycle and the life cycle are one.”

The hydrodynamics of those water bodies, that we define as geophysical and environmental flows, directly affects populations of different species and concentrations of organic and inorganic matter along with pollutants. This circulation has an important multiscale nature: from seconds to years and from millimetres to thousands of kilometres, all of them being important for the water body ecology. Understanding this hydrodynamics can help to apprehend the impact of humans on the water ecosystems on the one hand, and to decrease the consequences of a number of natural and man-made hazards such as tsunamis, floods, oil spills, etc., on the other hand.

While in situ measurement, radars, satellite imagery and other forms of observation can provide accurate information about the water state, this information is generally very localised, partial and expensive to obtain. Moreover, observations can only provide information about the previous and current state of the water. In turn, numerical modelling is a much cheaper tool. It is able to simulate and predict the hydrodynamics and the different reaction processes. But cheap does not mean simple: formulating the right equations, discretising them, initialising the model, taking into account available information, etc., is a real challenge. Those models are complex, and none of them is able to resolve all the scales of the hydrodynamics. They are constantly under development. Unresolved scales need to be parameterised, data need to be assimilated, models have to be calibrated. But development is not limited to the oceanographical physics: new numerical schemes are developed for a better accuracy, new implementations are achieved to improve the efficiency, etc. All of those physical, empirical, mathematical, numerical and computational developments aim to obtain models able to draw the closest representation to the marine flows.

## 1.1 Hydrodynamic numerical modelling

Geophysical numerical models have been developed since the 1940s and the advent of electronic computers ([Cushman-Roisin and Beckers, 2011](#)). The first numerical marine model was simulating regional storm surges and tide propagations in a two-dimensional domain ([Hansen, 1956](#)). Only ten years later, the first ocean general circulation model was developed ([Bryan and Cox, 1967](#); [Bryan, 1969](#)). For decades, different models emerged, each time more complex but all based on structured grids using a finite difference scheme. Important developments, often inspired by those achieved in atmospheric modelling, were carried out for those grids, such as the distribution of the variables over the grid points ([Arakawa and Lamb, 1977](#)) or the type of vertical coordinates ([Griffies et al., 2000](#), see also Chapter 5).

For various applications, it can be important to increase the grid resolution in sub-regions of the domain. While classical structured grids do not enable such refinement, nested grids were developed to obtain different resolutions

at different locations (Galperin and Mellor, 1990; Spall and Holland, 1991). However, the nesting technique suffers from marked drawbacks. Indeed, connecting two grids of different resolutions without introducing noise or signal reflection and conserving fluxes is not straightforward (Hanert, 2004; Debreu and Blayo, 2008). Moreover, building an arbitrary resolution does not allow a lot of freedom. In very complex regions, nested grids are unlikely to be used (Lambrechts et al., 2008).

Another limitation of the structured grids is the staircase representation of the coastline, which exerts spurious form stress on boundary currents (Adcroft and Marshall, 1998; Legrand et al., 2000). The latter problem can be somehow circumvented using curvilinear grids (Androsov et al., 1995), but again those grids suffer from a lack of flexibility and are not very well suited for complex geometry domains.

The aforementioned limitations of structured grids motivated the development and emergence of a second generation of marine models, based on unstructured meshes, which allow to locally refine and to align the grid with the coastline (Fig. 1.1). Those models are not based on finite difference schemes anymore but on other methods that enable the use of unstructured grids, such as finite volume or finite element methods.

The first ocean circulation model using an unstructured mesh was probably the finite element model of Fix (1975). Lynch and Gray (1979) developed a 2D finite element model for tide propagation in shallow waters. While unstructured grid methods offer the previously mentioned advantages, an important drawback is inherent to those methods: they have a much lower computational efficiency as compared to structured methods (Danilov et al., 2008). Indeed, while the topology of structured grid is rather simple, with for example the mesh node having index  $(i, j)$  connected to nodes  $(i - 1, j)$ ,  $(i + 1, j)$ ,  $(i, j - 1)$  and  $(i, j + 1)$ , the unstructured meshes topology is much more complex, which makes the computational implementation much less efficient. Therefore, unstructured mesh models were initially mostly limited to tidal and coastal applications, for which their advantages overcome their limitations. Since the seventies, a number of unstructured mesh models have been developed (for a review, see e.g. Pain et al., 2005), mostly using the finite element or finite volume methods.

During the last twenty-five years, a large variety of models were developed, for large scales, shelf seas, estuaries and lakes. Some models are very similar to what is developed for different engineering applications: the domain is discretised into a set of unstructured tetrahedra. The equations are then solved using the finite element method. This framework is used for the FEOM model (Danilov et al., 2004) and the ICOM/Fluidity model (Imperial College Ocean Model<sup>2</sup> Piggott et al., 2005), which moreover includes adaptive meshes

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<sup>2</sup><http://fluidityproject.github.io/>

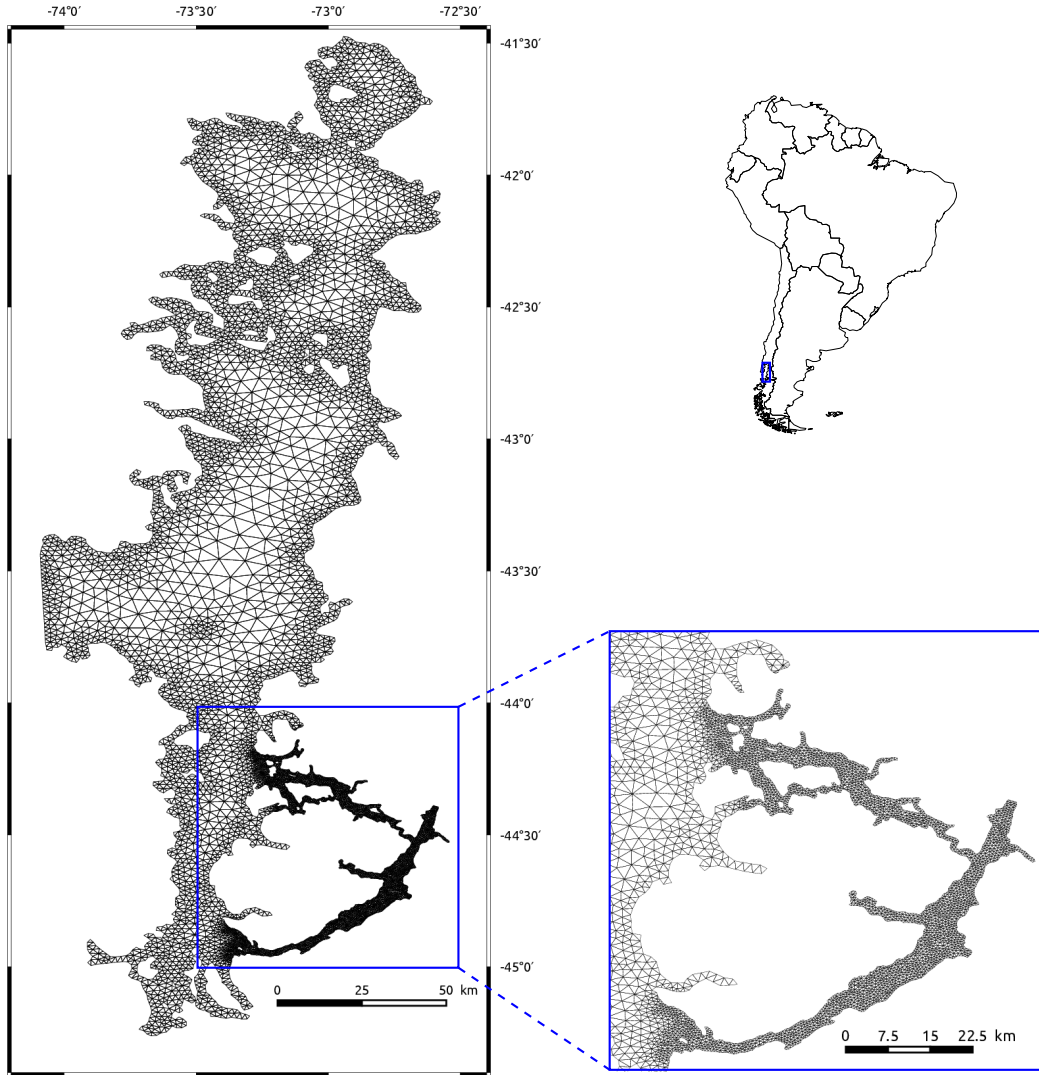


Fig. 1.1: Puyuhuapi fjord, Chile. Example of unstructured mesh composed of around 13,000 triangular elements. The mesh was generated using the GMSH software (<http://gmsh.info>).

in its modelling. However, most of the models use a mesh built as an unstructured 2D horizontal mesh extruded vertically to form structured vertical columns of prisms.

Among the models simulating large scale circulation or both small and large scale hydrodynamics, the majority opts for the finite volume method, such as MPAS (Model for Prediction Across Scales<sup>3</sup>, [Skamarock et al., 2012](#); [Ringler et al., 2013](#)), the model of Alfred Wegener Institute ([Danilov, 2012](#)),

<sup>3</sup><https://mpas-dev.github.io/>

and MITgcm (MIT General Circulation Model<sup>4</sup>, [Marshall et al., 1997](#)), which has the particularity of solving non-hydrostatic equations. The finite element method, which has a higher order of discretisation than finite volumes, is used by FESOM (Finite Element Sea Ice-Ocean Model, the next version of FEOM, [Wang et al., 2008](#)) or TELEMAC (open TELEMAC-MASCARET<sup>5</sup>, [Janin et al., 1992](#); [Hervouet, 2007](#)), which was first developed for coastal flows and is now widely used by a large scientific community.

Other models were developed specifically for coastal flows. The model FVCOM (Finite Volume Coastal Ocean Model<sup>6</sup>, [Chen et al., 2003](#)) is based on finite volumes while ADCIRC (ADvanced CIRCulation<sup>7</sup>, [Luettich Jr et al., 1992](#)) and SELFIE (Semi-implicit Eulerian–Lagrangian Finite Element<sup>8</sup>, [Zhang and Baptista, 2008](#)) are based on finite elements. A challenge for those models is to remain stable and robust on the one hand, and accurate on the other hand. Numerical dissipation is often a drawback of the model, such as FVCOM ([Danilov, 2013](#)) or SELFIE ([Kärnä et al., 2015](#)). It is worth mentioning that research is undertaken to compare the different spatial discretisation techniques. To do so, a large flexibility of the model is helpful: a novel project in marine modelling is Thetis<sup>9</sup>, an unstructured grid ocean modelling framework, which enables a large variety of discretisations.

A particular version of the finite element method is the discontinuous Galerkin formulation. This method, which combines some features from finite elements and others from finite volumes, has also been used in coastal modelling, such as in the model developed in Ohio State University ([Conroy, 2014](#)), or in the SLIM model, developed at Université catholique de Louvain, Belgium.

## 1.2 SLIM, a discontinuous Galerkin finite element model

This thesis focuses on the development of SLIM, the Second-generation Louvain-la-Neuve Ice-ocean Model<sup>10</sup>, which uses the discontinuous Galerkin (DG) finite element method to solve one-, two- or three-dimensional continuity, momentum and reactive transport equations. SLIM aims to simulate geophysical and environmental flows with a strong multiscale character, in domains characterised by a complex geometry, such that the use of unstructured meshes ([Hanert et al., 2004](#)) has a large impact. The project started in 1999. The goal was

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<sup>4</sup><http://mitgcm.org/>

<sup>5</sup><http://www.opentelemac.org/>

<sup>6</sup><http://fvcom.smast.umassd.edu/fvcom/>

<sup>7</sup><http://adcirc.org/>

<sup>8</sup>[http://www.stccmop.org/knowledge\\_transfer/software/selfe](http://www.stccmop.org/knowledge_transfer/software/selfe)

<sup>9</sup><http://thetisproject.org/>

<sup>10</sup><http://www.climate.be/slim>

to use the finite element method to take advantage of unstructured meshes in oceanography. First, a 2D model was developed, solving the shallow water equations. Different finite element pairs were evaluated (Hanert et al., 2003, 2005; Le Roux et al., 2005), before opting for the DG formulation (Lambrechts et al., 2008).

The DG method is very attractive especially for advection-dominated problems (Bernard et al., 2007; Cockburn et al., 2000; Adjerid et al., 2002; Bassi and Rebay, 1997). It ensures local conservation, which may be a critical issue in ocean modelling. Also, the compactness of the stencil has considerable advantages for efficient high-order parallel implementation (Seny et al., 2013, 2014). Moreover, monotonic advection schemes can be worked out by means of slope limiters (Cockburn, 2003; Aizinger, 2011; Kärnä et al., 2013). Finally, the DG formulation provides a good estimation of the discretisation error, based on the solution discontinuities at the element interfaces (Ainsworth, 2004; Bernard et al., 2007).

For applications on domains such as the Scheldt estuary and tidal river network in Belgium and the Netherlands (de Brye et al., 2010; Gourgue et al., 2013; de Brauwere et al., 2009; De Brauwere et al., 2011; de Brauwere et al., 2014; Elskens et al., 2014) and the Mahakam river and delta in Indonesia (de Brye et al., 2011; Sassi et al., 2011, 2012; Pham Van et al., 2016), a one-dimensional version of SLIM was developed for the upstream part of the river network, for which the flow equations of the 2D model are averaged over the section of the rivers (de Brye et al., 2010).

For applications in which the water column is not well mixed, typically where there is strong vertical shear or stratification (Delandmeter et al., 2015), the 2D vertically-averaged modelling is not acceptable anymore. The three-dimensional version of the model was designed to deal with such flows. The model solves the 3D hydrostatic Navier-Stokes Reynolds-averaged equations under the Boussinesq approximation, using the DG finite element formulation. The spatial discretisation of the three-dimensional baroclinic free-surface marine model is derived on a mesh of prisms extruded in the vertical direction from an unstructured two-dimensional mesh of triangles or quads.

SLIM 3D has been under development for 10 years. This work resulted promising (White et al., 2008a; Blaise et al., 2010; Comblen et al., 2010; Kärnä et al., 2013), but several limitations still remained before the beginning of this thesis. Moreover, the model was mainly applied to theoretical and idealised applications. What is the potential of SLIM 3D in modelling actual complex flow dynamics? How can the model design be improved to increase simulation stability and result accuracy? How to evaluate this accuracy on real applications? What development can be achieved taking into account that the model uses the finite element method? More specifically, can the DG formulation be exploited for further improvement?

## 1.3 Scope of this thesis

The objective of this thesis is to improve the simulation of geophysical and environmental flows in complex settings by means of DG finite element models in general, and especially SLIM 3D. Two key aspects of the model development are addressed in this thesis: the model validation by means of high-resolution satellite imagery and the improvement of the SLIM 3D accuracy and stability for complex hydrodynamics configurations.

### 1.3.1 Modelled velocity validation using high-resolution satellite imagery

High-resolution models such as SLIM are able to resolve flow features with a length scale of the order of a hundred metres, such as small lee eddies or unsteady wakes, but it is difficult to evaluate the accuracy of such modelling. The classical validation tools for such models include current-metres and sea-level moorings, HF radar and satellite imagery. This validation is always limited, especially concerning the velocity field.

In 2015, two very high-resolution satellite images (with a 2-m resolution, Fig. 1.2) were obtained for a sub-region of the Great Barrier Reef, Australia, taken only 54 seconds apart. This resolution combined with the short time interval between the two pictures is unprecedented in oceanography.

Chapter 2 presents how, using a cross-correlation technique or an optical-flow method, it is possible to extract the velocity field from such imagery, at a resolution of 64 m. This new validation technique is used to evaluate the 2D version of SLIM on modelling tidal eddies in the wake of coral islands in the Great Barrier Reef where the satellite pictures were taken. Such usage of satellite high-resolution imagery has a great potential in coastal flows dynamics, where small scale patterns can have an impact on the local marine ecology.

### 1.3.2 3D modelling of baroclinic flows

This thesis further develops the model in order to increase its stability and accuracy, and to add a number of features to the code.

Different improvements were achieved during this thesis in the framework of an “application-oriented” strategy: the model was already operational before the beginning of this thesis, it has been updated in order to be used for different types of applications. The most relevant parts of the numerical improvements are described in Chapter 3. Those include a new formulation for the internal pressure gradient, which strongly reduces the spurious velocity in a stratified flow, and a new computation of the moving mesh velocity, which ensures both tracer conservation and consistency, while only conservation was preserved in the previous formulation. Moreover, the new split between the 2D and the

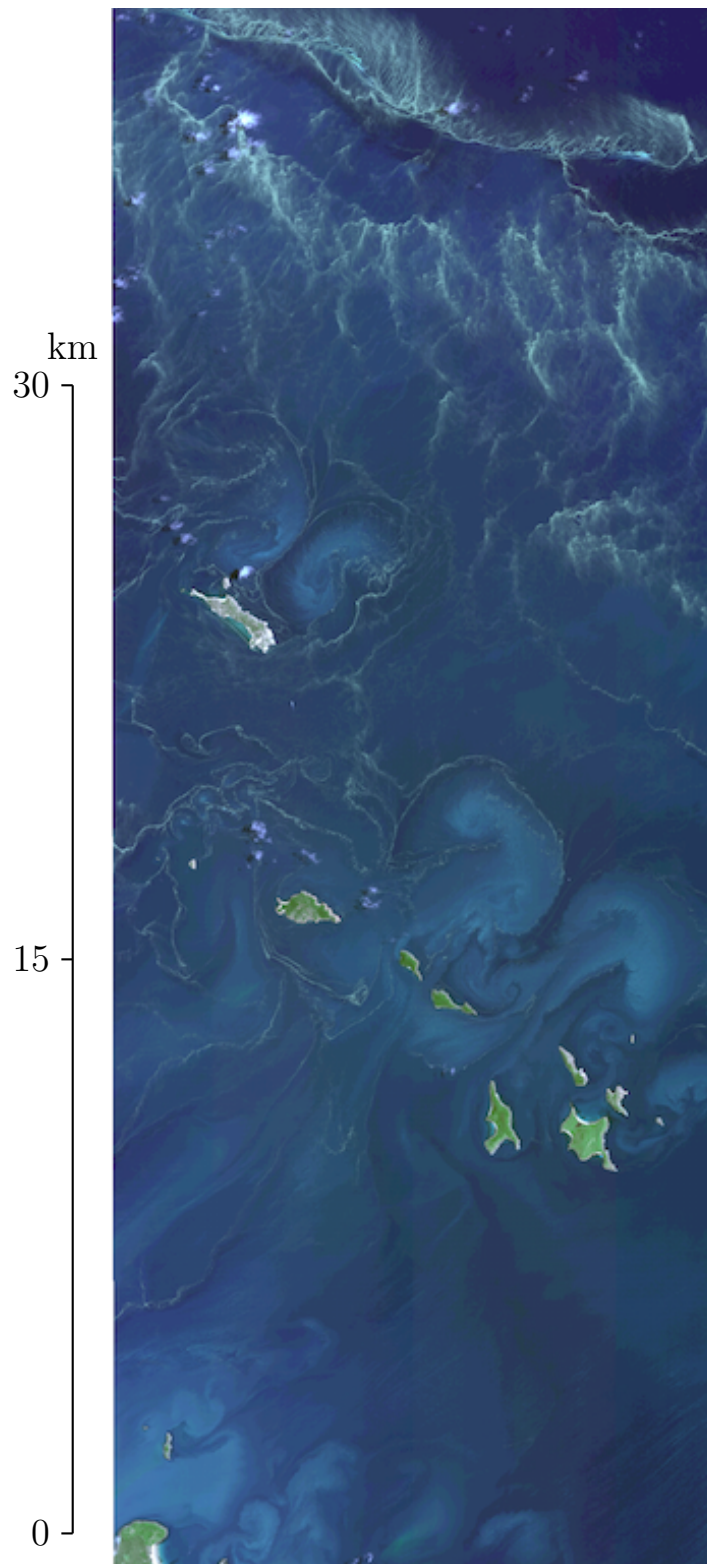


Fig. 1.2: Satellite image of a sub-region of the central Great Barrier Reef, Australia, from WorldView-3 satellite.

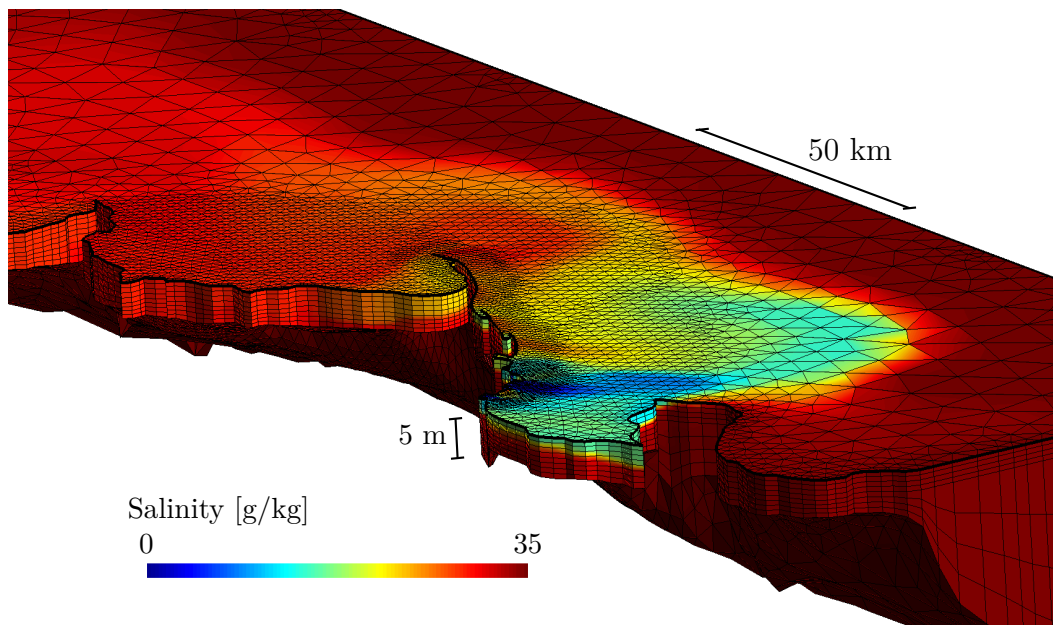


Fig. 1.3: Snapshot of a SLIM 3D simulation of the Burdekin River plume, in the Great Barrier Reef, Australia.

3D mode of the model is described, quadrilateral and periodic meshes are introduced and a different slope limiter is implemented.

Two important applications of this thesis are the modelling of the Burdekin River plume in the Great Barrier Reef, and the modelling of the thermocline oscillations in Lake Tanganyika, in the East African Rift.

The Burdekin River is the largest supplier of sediment to the Great Barrier Reef lagoon. The objective of the Burdekin project is to evaluate where, when and how the suspended sediment exported by the river is transported. To do so, a fine sediment module is implemented in SLIM 3D and a one-year simulation of the Burdekin River is carried out (Fig. 1.3), forced by the surface wind stress, river flow, sediment concentration and tidal elevation and velocity on the open boundaries. This study, presented in Chapter 4, highlights the deposition areas of the Burdekin sediment and the different regimes of the river, explaining the importance of each of them.

A key outcome of this thesis is the development of a vertically adaptive mesh for DG finite element methods. This new feature is aimed at using a mesh able to dynamically adapt its vertical resolution in order to better capture strong shear or stratification. For DG methods, the resolution is controlled by the density and velocity vertical discontinuities at the inter-element horizontal interfaces. The details of the method are laid out in Chapter 3.

The objective of adaptive meshes is to obtain an accurate modelling at a

relatively low computational cost, by reducing the total number of elements but keeping a fine vertical resolution of the mesh in the regions of interest. This method is particularly interesting to model oscillations of a sharp thermocline. Indeed, a very fine vertical resolution can be set along the interface while a low resolution is kept elsewhere. In Chapter 5, the method is applied to simple benchmarks simulating oscillations of a sharp interface, before simulating the oscillations of the Lake Tanganyika thermocline. For the latter, a comparison with measurements of the temperature vertical profile shows the good behaviour of the model. Important features of the lake, such as its stratification, thermocline oscillations, outcropping events or observed Kelvin waves are reproduced by SLIM 3D. Combined with available input data for surface wind stress and heat flux, the model shows promising results and is ready for achieving a large variety of studies of the lake hydrodynamics and ecology.

## Supporting publications

- P. Delandmeter**, S. Lewis, J. Lambrechts, E. Deleersnijder, V. Legat and E. Wolanski, 2015, The transport and fate of riverine fine sediment exported to a semi-open system, *Estuarine, Coastal and Shelf Science*, 167, 336 – 346  
doi: [10.1016/j.ecss.2015.10.011](https://doi.org/10.1016/j.ecss.2015.10.011)
- P. Delandmeter**, J. Lambrechts, G.O. Marmorino, V. Legat, E. Wolanski, J.-F. Remacle, W. Chen and E. Deleersnijder, 2017, Submesoscale tidal eddies in the wake of coral islands and reefs: satellite data and numerical modelling, *Ocean Dynamics*, 67, 897 – 913  
doi: [10.1007/s10236-017-1066-z](https://doi.org/10.1007/s10236-017-1066-z)
- P. Delandmeter**, J. Lambrechts, V. Legat, V. Vallaey, J. Naithani, W. Thiery, J.-F. Remacle and E. Deleersnijder, A fully consistent and conservative 3D discontinuous Galerkin finite element model using a vertically adaptive mesh, with an application to the oscillations of the thermocline of a deep, elongated lake, *Geoscientific Model Development*, (submitted)
- V. Vallaey, T. Kärnä, **P. Delandmeter**, J. Lambrechts, A. M. Baptista, E. Deleersnijder and E. Hanert, Seamless numerical modeling of the Columbia River estuary-to-ocean continuum: skill assessment and physical validation of the plume dynamics, *Ocean Modeling*, (in preparation)

## Presentations at international conferences

- SLIM, A baroclinic discontinuous Galerkin finite element model for marine flows, *Mathematical and Computational Issues in the Geosciences, SIAM GS13*, Padova, Italy – June 17-20, 2013
- Preliminary results of a 3D baroclinic model of the Burdekin River Plume, *Joint Numerical Sea Modelling Group Conference, JONSMOD 2014*, Bruxelles, Belgium – May 12-14, 2014

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- A 3D baroclinic model of the Burdekin River Plume, *Multiscale (Un)-structured Mesh Numerical Ocean Modelling, IMUM 2014*, Lisbon, Portugal – August 25-27, 2014
- 3D modelling of the transport and fate of riverine fine sediment exported to a semi-enclosed system, *European Geosciences Union General Assembly, EGU 2015*, Vienna, Austria – April 12-17, 2015
- Influence of the vertical discretisation of SLIM 3D on a river plume modelling, *Multiscale (Un)-structured Mesh Numerical Ocean Modelling, IMUM 2015*, Portland, USA – September 28-30, 2015
- An adaptive vertical coordinate system for a coastal flows discontinuous Galerkin finite element model, *Joint Numerical Sea Modelling Group Conference, JONSMOD 2016*, Oslo, Norway – May 10-12, 2016
- Comparison of a satellite-derived high-resolution current map and numerical modelling of submesoscale eddies in a shallow-water domain, *48th International Liege Colloquium on Ocean Dynamics Submesoscale Processes*, Liège, Belgium – May 23-27, 2016
- Modelling the oscillations of the thermocline in a lake by means of a fully consistent and conservative 3D finite-element model with a vertically adaptive mesh, *European Geosciences Union General Assembly, EGU 2017*, Vienna, Austria – April 23-28, 2017



# Submesoscale tidal eddies in the wake of coral islands and reefs: satellite data and numerical modelling

This chapter reproduces the following article<sup>1</sup>:

**P. Delandmeter**, J. Lambrechts, G.O. Marmorino, V. Legat, E. Wolanski, J.-F. Remacle, W. Chen and E. Deleersnijder, 2017, Submesoscale tidal eddies in the wake of coral islands and reefs: satellite data and numerical modelling, *Ocean Dynamics*, 67, 897 – 913  
[doi: 10.1007/s10236-017-1066-z](https://doi.org/10.1007/s10236-017-1066-z)

Interaction of tidal flow with a complex topography and bathymetry including headlands, islands, coral reefs and shoals create a rich submesoscale field of tidal jets, vortices, unsteady wakes, lee eddies and free shear layers, all of which impact marine ecology. A unique and detailed view of the submesoscale variability in a part of the Great Barrier Reef lagoon, Australia, that includes a number of small islands was obtained by using a “stereo” pair of 2-m resolution visible-band images that were acquired just 54 seconds apart by the WorldView-3 satellite. Near-surface current and vorticity were extracted at

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<sup>1</sup>Only minor modifications were made.

a 50-m resolution from those data using a cross-correlation technique and an optical-flow method, each yielding a similar result.

The satellite-derived data are used to test the ability of two-dimensional version of SLIM to reproduce the details of the currents in the region. The model succeeds in simulating the large scale ( $> 1$  km) current patterns, such as the main current and the width and magnitude of the jets developing in the gaps between the islands. Moreover, the order of magnitude of the vorticity and the occurrence of some vortices downstream of the islands are correctly reproduced. The smaller scales ( $< 500$  m) are resolved by the model, although various discrepancies with the data are observed. The smallest scales ( $< 50$  m) are unresolved by both the model and image-derived velocity fields.

This chapter shows that high-resolution models are able to a significant degree to accurately simulate the currents close to a rugged coast. Very high-resolution satellite oceanography stereo images offer a new way to obtain snapshots of currents near a complex topography that has reefs, islands and shoals, and is a potential resource that could be more widely used to assess the predictive ability of coastal circulation models.

## 2.1 Introduction

Tidal flows have been studied for decades, from global (Lyard et al., 2006; Carrere et al., 2015) to small scales such as narrow fjords or straits (Eliassen et al., 2001). While the main processes controlling the tidal dynamics are globally understood, it is still challenging to simulate correctly the tidal currents in regions with a complex bathymetry. Sea surface elevation is generally not extremely sensitive to the details of the model (Thomas et al., 2014), but modelling satisfactorily the velocity field is much more challenging. Furthermore, it is precisely the velocity that affects the transport and fate of waterborne particles of interest to environmental implications, such as plastic debris (Critchell and Lambrechts, 2016), fine sediment (Lambrechts et al., 2010; Delandmeter et al., 2015), turtle hatchlings (Hamann et al., 2011) and fish and coral larvae (Wolanski and Kingsford, 2014; Thomas et al., 2014; Wolanski, 2016).

For various ecological applications, it is then crucial to model the different phenomena generated by the tides. Tidal currents occur over a wide range of space and time scales, from a few metres to hundreds of kilometres and from a few seconds to various weeks (Wolanski et al., 2003a; Lambrechts et al., 2008). Submesoscale processes and features such as vortices, free-shear layers and jets in the wake of small islands or reefs affect significantly the transport of tracers and particles (Wolanski et al., 1984, 1996; Lambrechts et al., 2008; Mantovanelli et al., 2012; Wolanski and Elliott, 2015). Typically, observed submesoscale eddies have a size varying from 10-20 m, which last a few minutes at most, to about 1 km, which live for a few hours until the tide reverses.

The Great Barrier Reef (GBR) lies on the north-eastern continental shelf of Australia (Fig. 2.1a). It contains over 2500 reefs, in a band 2600 km long and 200 km wide. The topography and bathymetry of the region are complex and control the tidal circulation. The zone of islands shown in Figure 2.1d has an area of  $13.5 \text{ km} \times 9.3 \text{ km}$  and is the focus of the present study.

To model accurately the complex tidal currents in such regions, high-resolution models are necessary (Wolanski et al., 2003a). Falconer et al. (1986) studied the model sensitivity to the bathymetry representation, for which a high resolution is then crucial; Coutis and Middleton (2002) highlighted the importance of the island geometry on controlling the wake formation downstream this island; Munday et al. (2010) went even further showing the potential of adaptive models. The simulation of such complex flows is now frequently done using finite element models, such as FINEL 2D (Dam et al., 2007), TELEMAC (Jones and Davies, 2008) or SLIM (Lambrechts et al., 2008). The key feature of these models is the possibility to deal with unstructured meshes. They allow for a correct representation of a complex bathymetry with narrow straits and gaps, small islands or reefs (Legrand et al., 2006; Lambrechts et al., 2008). Furthermore, they give the possibility to substantially increase the resolution in regions of interest (Hanert et al., 2004). SLIM has been used

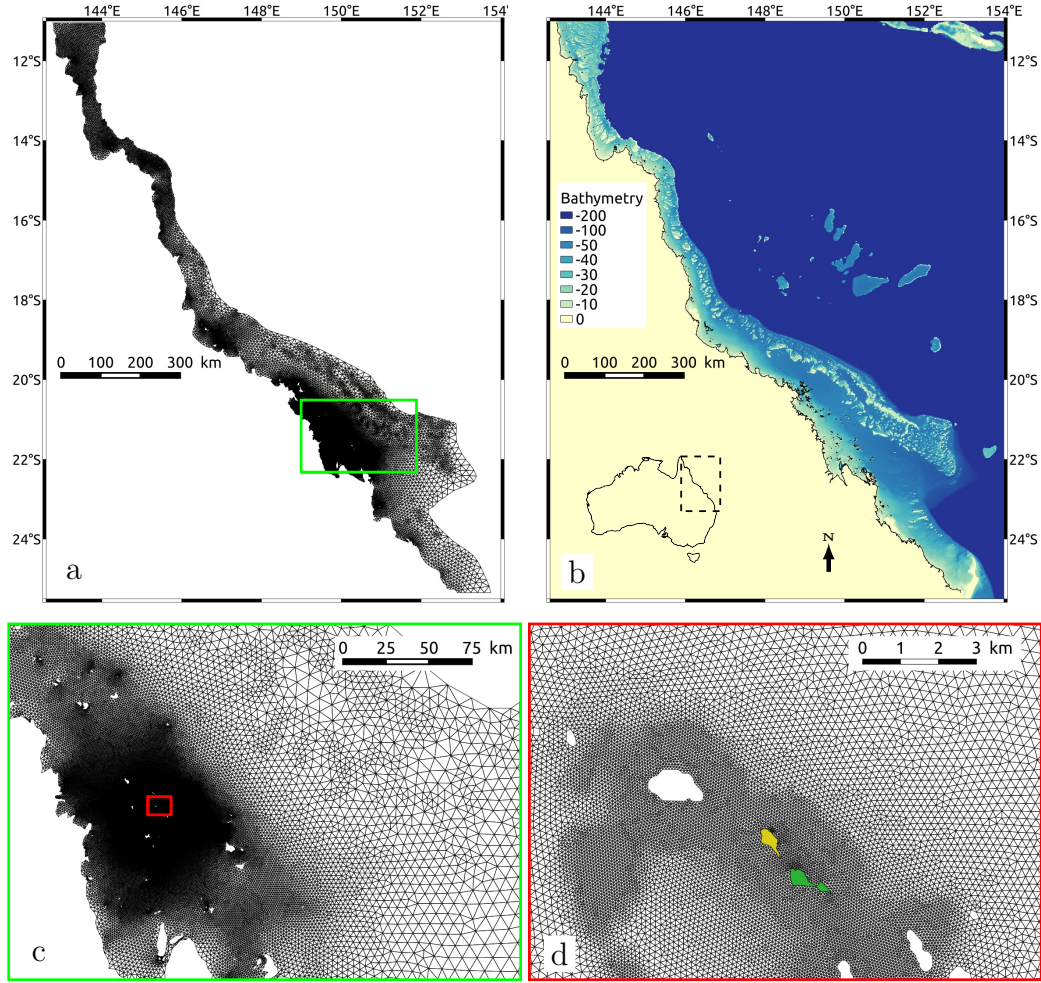


Fig. 2.1: Great Barrier Reef mesh (a,c,d) and bathymetry (b). The mesh used by the model SLIM to simulate the submesoscale processes around Beverlac (in yellow) and Hull (in green) Islands is composed of  $\sim 110,000$  DG triangles and its size varies between 50 m and 20 km. The mesh covers the whole GBR lagoon. The  $13.5 \text{ km} \times 9.3 \text{ km}$  zoomed area (d) is the focus of the present study.

extensively in the GBR hydrodynamics modelling (Lambrechts et al., 2008, 2010; Hamann et al., 2011; Thomas et al., 2014, 2015; Delandmeter et al., 2015; Critchell and Lambrechts, 2016; Grech et al., 2016).

Validation of such models has been attempted in a limited way, and over relatively small areas, through the use of current-meter and sea-level moorings (Thomas et al., 2014) and high-frequency (HF) radar (Young et al., 1994). The use of satellite imagery for direct validation of a modelled velocity field is hampered by relative low spatial resolution (of the order of 1 km) and a typically long time interval between repeat views of a scene. For example, MODIS (Moderate Resolution Imaging Spectroradiometer) ocean colour imagery from

the Terra and Aqua satellites has  $\sim 1$  km resolution and a repeat interval of about 3 h ([modis.gsfc.nasa.gov](http://modis.gsfc.nasa.gov)), which is longer than the time scale of the dominant M2 tide. At the other end of the spectrum of satellite imagery resolution are the WorldView-2 and -3 satellites (<http://www.digitalglobe.com/about/our-constellation>), which have very high-resolution imagery (2-m pixels for colour imagery) and a repeat interval of the order of 1 min for those cases where data are collected in “along-track stereo” mode. Data collected in that mode are typically used to construct digital elevation models of land features, and rarely does one find a case revealing oceanographic phenomena.

The objective of the present study is thus two-fold: to examine the utility of high-resolution data and the predictive capability of a state-of-the-art coastal model to predict submesoscale features. Section 2.2 details the satellite images used and how they were processed by means of both a Particle Image Velocimetry (PIV) method and an optical-flow method, the so-called Global Optimal Solution (GOS), to compute velocity maps. It also gives details about the two-dimensional version of SLIM. Section 2.3 explains the submesoscale patterns evolution during the tidal cycle, it validates the satellite data processing and it describes the numerical simulation results. Those results are discussed in Section 2.4 where the skills of SLIM on submesoscale eddy simulations in a rugged bathymetry are evaluated, before concluding in Section 2.5.

## 2.2 Methods

### 2.2.1 Satellite data and velocity algorithms

WorldView-3 imagery of an area of the GBR lagoon was acquired on 27 February, 2015, at 0010 UTC (10:10AM local time). The imagery consists of an in-line stereo pair, having a time interval between the two images of just 54 s. The data from each image has a spatial resolution of 2 m in each of eight visible and near-infrared bands, and 0.5 m in a single panchromatic band. An inner-field area (Fig. 2.1d), containing many complex submesoscale patterns, was chosen for the detailed study. Colour-stretched versions of the inner-field area, shown in Figure 2.2, emphasize patterns of re-suspended sediment (bright-blue areas) and near-surface aggregations of algae (white filaments). The algae in the study are most likely the cyanobacteria *Trichodesmium* as it is common to the study area and the only algae to form locally large-scale, dense surface aggregations (Furnas, 1992; McKinna et al., 2011; McKinna, 2015). Extensive aggregations of *Trichodesmium* are most favoured in warm water and during extended periods of calm winds. Over the 10 h prior to the satellite acquisition, wind speeds measured near the study area were less than  $3 \text{ m s}^{-1}$ , and water temperatures were above  $28.4^\circ\text{C}$ . There were no nearby measurements of stratification, but the water column is generally well mixed (Furnas, 1992; Wolanski, 1994). Flow towards the north-east can be inferred from linear sediment bands in the gaps between islands, connecting to large downstream vortices,

and giving the appearance of large “mushroom” shapes. White areas in the images are clouds, and adjacent dark areas are cloud shadows. The azimuth viewing angle is necessarily different for the two images ( $332^\circ$  vs  $271^\circ$ ); note the larger distance between clouds and their shadows in Figure 2.2 ( $t_1$  image). And, because of parallax, the location of a cloud in one image is considerably different in the other; the net effect being that the total area contaminated by clouds is approximately doubled when both images are considered together.

Velocity maps were extracted from those images using both a PIV technique and a GOS method, which are described below. To account for contamination by clouds and cloud shadows, as well as by a small amount of boat traffic, a mask was manually created to remove contaminated pixels from the velocity calculations. As the processing tools uses only gray-scale imagery, a single colour wavelength band needed to be chosen for analysis. The yellow band was used (wavelength range of 590 to 630 nm), as this was judged to have the best signal-to-noise ratio for sediment and algae patterns.

### Particle Image Velocimetry (PIV) method

Currents are derived from the stereo image pairs using a normalized cross-correlation algorithm (Tseng et al., 2012), which is implemented as a PIV “plugin” to the ImageJ processing and analysis package (<https://imagej.nih.gov/ij/>). In this algorithm an  $n \times n$  pixel-sized interrogation window in the  $t_1$  image is compared against a larger search area in the  $t_2$  image. The difference in centre positions of the interrogation window and its best match within the search area is then selected as the displacement vector; the displacement vector divided by the time step yields the velocity vector. A threshold (0.6, in this study) is used to distinguish high from low image correlations; a low correlation, resulting from a small interrogation window presenting insufficient features (or, image “texture”), which tends to yield an erroneous vector, can thus be easily filtered out and replaced by an interpolated value. Each successive interrogation window is displaced in the  $x$  and  $y$  directions by  $n/2$  pixels; the resulting velocity vectors thus have a physical spacing equal to  $n/2 \times 2$  m. In the present study,  $n = 64$ , so that the velocity maps have a resolution of 64 m. Higher resolutions maps were computed, obtaining consistent results with the one shown in this paper, but they were contaminated by noise related to artefacts in the satellite imagery and are not shown in this paper.

### Global Optical Solution (GOS) method

The GOS method (Chen, 2011) uses the conservation of a passive tracer to compute the velocity field. It assumes that the total (Lagrangian) derivative of the tracer is equal to zero for a short time interval, as represented by the following integrated form of the tracer-conservation equation:

$$I(\mathbf{r} + \mathbf{u}\Delta t, t_1) = I(\mathbf{r}, t_0), \quad (2.1)$$

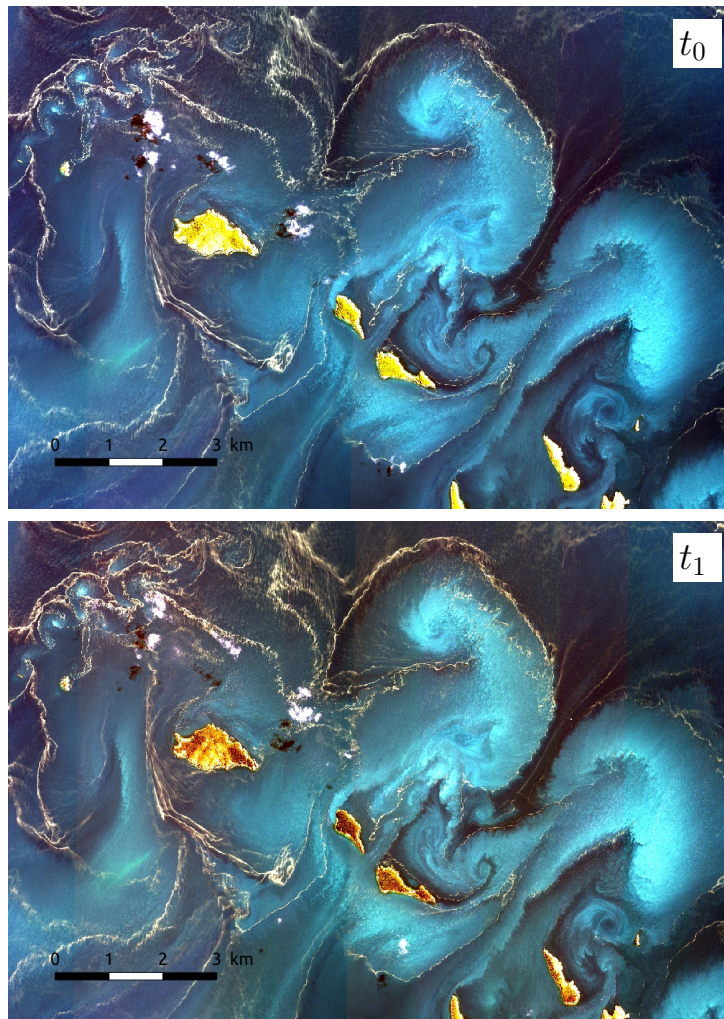


Fig. 2.2: WorldView-3 satellite imagery acquired on 27 February, 2015, at 0010 UTC (10:10AM local time). The  $t_0$  picture was taken 54 s before the  $t_1$  one. Shown are colour-stretched versions, using red, green, and blue wavelength bands.

where  $I$  is tracer intensity,  $\Delta t = t_1 - t_0$  is the time difference between the two images and  $\mathbf{r}$  and  $\mathbf{u}$  are the position and velocity vectors at  $t_0$  for a particular image pixel. The solution for  $\mathbf{u}$  is an under-constrained problem. To tackle it,  $\mathbf{u}$  is computed on a relative coarse grid and bi-linearly interpolated on all the pixels. In the present application, the coarse grid spacing is 200 m, while the pixel size is 2 m. The velocity  $\mathbf{u}$  is eventually derived as the result of a non-linear optimization problem, minimizing the error in Equation 2.1. In this study, the tracer is the intensity of light scattered from re-suspended sediment and near-surface algae. These materials do not necessarily provide a passive tracer, because it is possible for the light intensity to vary with angle of view. Sun glint can affect both GOS and PIV calculations; fortunately, good view angles coupled with a low sea state resulted in minimal contamination.

Note that each method (PIV and GOS) requires suitable small-scale “texture”, or trackable features, to derive valid estimates of velocity and vorticity; and each may fail in areas having poor texture.

## 2.2.2 Numerical modelling

### SLIM

The Second-generation Louvain-la-Neuve Ice-ocean Model (SLIM), a discontinuous Galerkin finite element model for shallow waters (Lambrechts et al., 2008; Thomas et al., 2014; Critchell et al., 2015; Le Bars et al., 2016), was used to simulate the hydrodynamics of the region. The main feature of the finite element method is the possibility to deal with unstructured meshes. This is particularly important to model the GBR, which is characterized by a complex topography and strong velocity gradients. Figure 2.1 shows the mesh used by SLIM for this study. Even though the simulation focuses on the inner-field area, the mesh includes all of the GBR lagoon. The eastern boundary was defined as the 200 m isobath, which corresponds to the shelf break. The mesh resolution was increased close to the coast, the reefs and in the region of interest, with a size varying between 50 m and 20 km, to generate a final grid of  $\sim 110,000$  triangles. The 50 m mesh resolution in the inner-field was chosen to be comparable to the PIV and GOS data resolution.

The two-dimensional version of SLIM solves the depth-averaged shallow water equations, to compute the water elevation  $\eta$  and the depth-averaged current velocity  $\bar{\mathbf{u}} = (\bar{u}, \bar{v})$ :

$$\frac{\partial \eta}{\partial t} + \nabla_h \cdot (H \bar{\mathbf{u}}) = 0, \quad (2.2)$$

$$\begin{aligned} \frac{\partial \bar{\mathbf{u}}}{\partial t} + (\bar{\mathbf{u}} \nabla_h) \cdot \bar{\mathbf{u}} = & -g \nabla_h \eta - f \mathbf{e}_z \wedge \bar{\mathbf{u}} - C_D |\bar{\mathbf{u}}| \bar{\mathbf{u}} + \frac{\boldsymbol{\tau}}{\rho H} \\ & + \frac{1}{H} \nabla_h \cdot \left( H \nu \left( \nabla_h \bar{\mathbf{u}} + (\nabla_h \bar{\mathbf{u}})^T \right) \right), \end{aligned} \quad (2.3)$$

where  $H$  is the total water column height and the physical parameters  $g$ ,  $f$ ,  $\rho$  and  $\boldsymbol{\tau}$  are the gravitational acceleration, the Coriolis parameter, the density and the surface wind stress, respectively.  $\nabla_h$  is the horizontal gradient operator,  $\mathbf{e}_z$  is the vertical unit vector pointing upward,  $\wedge$  is the vector product operator and the superscript  $T$  stands for the transpose matrix operator. The bottom friction is parameterised as a quadratic expression, with the parameters:

$$C_D = g/(C^2 H), \quad C = H^{1/6}/n. \quad (2.4)$$

$C$  and  $n$  are called the Chezy and the Manning coefficients, respectively. The eddy viscosity  $\nu$  is obtained from the Smagorinsky parameterisation (Smagorin-

sky, 1963; Pham Van et al., 2014):

$$\nu = \alpha \Delta_x^2 \sqrt{2 \left( \frac{\partial \bar{u}}{\partial x} \right)^2 + 2 \left( \frac{\partial \bar{v}}{\partial y} \right)^2 + \left( \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right)^2}, \quad (2.5)$$

with  $\alpha$  a non-dimensional coefficient and  $\Delta_x^2$  the local mesh size. The model setup used for the simulation is identical to that of Thomas et al. (2014), with  $\alpha = 0.01$  and  $n = 2.5 \times 10^{-2} \text{ s m}^{-1/3}$ . Moreover,  $C_D$  is multiplied by a factor of 10 over shallow reefs.

## Bathymetry

The bathymetry data used comes from the Project 3DGBR dataset v4 (Beaman, 2010), and has a resolution of 100 m (Fig. 2.1). This 4th version shows various differences comparing to the 3rd version, used in Thomas et al. (2014) and Delandmeter et al. (2015). The bathymetry was smoothed in such a manner that it does not vary from more than 50% between two nodes of an element. This smoothing operation preserves better the original bathymetry than diffusion operators used in previous studies. The error introduced by the smoothing operation is smaller than 20% on more than 88% of the domain. This error is mainly introduced close to the reefs far away from the region of interest, where the mesh size is large. In the inner-field region, shown on Figure 2.3, the error is smaller than 0.1% on more than 99% of the domain.

## Open boundaries

The open boundaries are the interface between the GBR and the Coral Sea (Fig. 2.1), and these were forced with tides using the OSU TOPEX/Poseidon Global Inverse Solution TPXO 7.2 dataset (Egbert and Erofeeva, 2002; Delandmeter et al., 2015), with a spatial resolution of  $0.25^\circ$ . Furthermore, the East Australian Current was accounted for by imposing a constant flow between  $15.0^\circ\text{S}$  and  $17.6^\circ\text{S}$ , as in Thomas et al. (2014).

## Wind

Wind data, having a temporal resolution of 6 h and a spatial resolution of  $0.3^\circ \times 0.3^\circ$ , were obtained from the NCEP Climate Forecast System Reanalysis (Saha et al., 2010). The wind was weak between February 20th and 27th, with an average speed of  $7.3 \text{ m s}^{-1}$ . The wind speed was smaller than the average speed more than half of the time. It blew at more than  $12 \text{ m s}^{-1}$  during 9% of the time, with a maximum speed of  $15 \text{ m s}^{-1}$ . Measurements at Hardy Reef and Heron Islands showed wind speeds less than  $3 \text{ m s}^{-1}$  over seven hours prior to the satellite overpass. Model results (not shown) that include wind forcing revealed that wind has no major influence on the velocity field; consequently, wind forcing was not used to obtain the results shown in this paper.

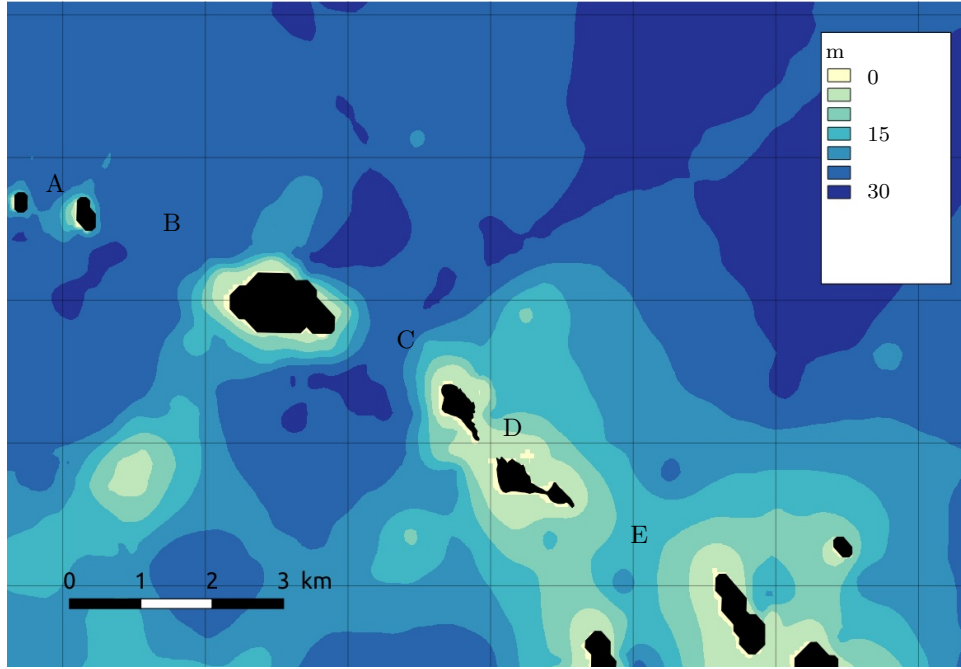


Fig. 2.3: Bathymetry used for the numerical simulation. The map displays the inner-field area, which corresponds to Figure 2.1d. The full bathymetry is also displayed in Figure 2.1b. This inner-field area is used for all the results displayed in the following figures. The gaps between the different islands are tagged with letters from A to E.

## Analysis tools

To quantify the errors between PIV and GOS methods, a coefficient of determination  $r^2$  is computed. To do so, in the inner-field region, the components  $u_{//}$  and  $u_{\perp}$  are defined as the flow aligned and perpendicular to the mean tidal flow, respectively. Then PIV and GOS data are interpolated on a structured grid and grid nodes are numbered from 1 to  $N$ . Defining the GOS velocities  $u_{//}^i$  and  $u_{\perp}^i$  and the PIV velocities  $\hat{u}_{//}^i$  and  $\hat{u}_{\perp}^i$  at node  $i$ , the coefficients of determination  $r_{//}^2$  and  $r_{\perp}^2$  read:

$$\begin{aligned}
 \bar{u}_{//} &= \frac{1}{N} \sum_{i=1}^N \hat{u}_{//}^i & \bar{u}_{\perp} &= \frac{1}{N} \sum_{i=1}^N \hat{u}_{\perp}^i \\
 \text{SSE}_{//} &= \sum_{i=1}^N (u_{//}^i - \hat{u}_{//}^i)^2 & \text{SSE}_{\perp} &= \sum_{i=1}^N (u_{\perp}^i - \hat{u}_{\perp}^i)^2 \\
 \text{SST}_{//} &= \sum_{i=1}^N (\bar{u}_{//} - \hat{u}_{//}^i)^2 & \text{SST}_{\perp} &= \sum_{i=1}^N (\bar{u}_{\perp} - \hat{u}_{\perp}^i)^2 \\
 r_{//}^2 &= 1 - \frac{\text{SSE}_{//}}{\text{SST}_{//}} & r_{\perp}^2 &= 1 - \frac{\text{SSE}_{\perp}}{\text{SST}_{\perp}}. \tag{2.6}
 \end{aligned}$$

SSE and SST are the sum of squared error and the sum of squared total, respectively. The closer  $r^2$  to unity, the better the matching.

As there are only two satellite images, only a single velocity “snapshot” could be constructed. It is then impossible to follow the path of particles during a tidal cycle. To compare simulation results and data, velocity is assessed comparing “instantaneous streamlines” obtained from data and from the velocity field at the same instant. The vertical component of the relative vorticity, defined as  $\zeta = \partial v / \partial x - \partial u / \partial y$ , is also computed. Throughout this paper, it will be referred to as vorticity, for simplicity.

## 2.3 Results

### 2.3.1 Parameter sensitivity

The model is sensitive to its parameters. In this study, those parameters were chosen identical to [Thomas et al. \(2014\)](#). The parameters for the bottom friction and the eddy diffusivity are:

$$n = 2.5 \times 10^{-2} \text{ s m}^{-1/3}, \quad \alpha = 10^{-2}. \quad (2.7)$$

In preliminary simulations, the sensitivity to those parameters was evaluated. Increasing by a factor 3 the Manning coefficient ( $n$ ) reduces by 1/3 the jet speeds in the inner-field, contributing to a large decrease of the submesoscale activity. A similar result is obtained by increasing the eddy viscosity parameter ( $\alpha$ ) by a factor 100.

### 2.3.2 Summary of the tidal cycle

The GBR hydrodynamics was simulated for eight days, starting on February 20th, 2015. Figure 2.4 shows the dynamics in the inner-field region during part of the tidal cycle, on February 26th and 27th. At 1930 UTC, the tidal currents reversed and began to propagate towards north-east. The current strength increased and vortices developed on both sides of each jet. The jets were the strongest around 2230 UTC then began to weaken, but vortices still developed, even when the tidal current reversed (0230 UTC to 0330 UTC). They were then dissipated while the tide began to propagate in the opposite direction (0430 UTC) in a new half a cycle.

### 2.3.3 Coefficients of determination and scatterplots

Velocities computed by the PIV and GOS methods are closely matched. The coefficients of determination obtained are:

$$r_{//}^2 = 0.926, \quad r_{\perp}^2 = 0.913. \quad (2.8)$$

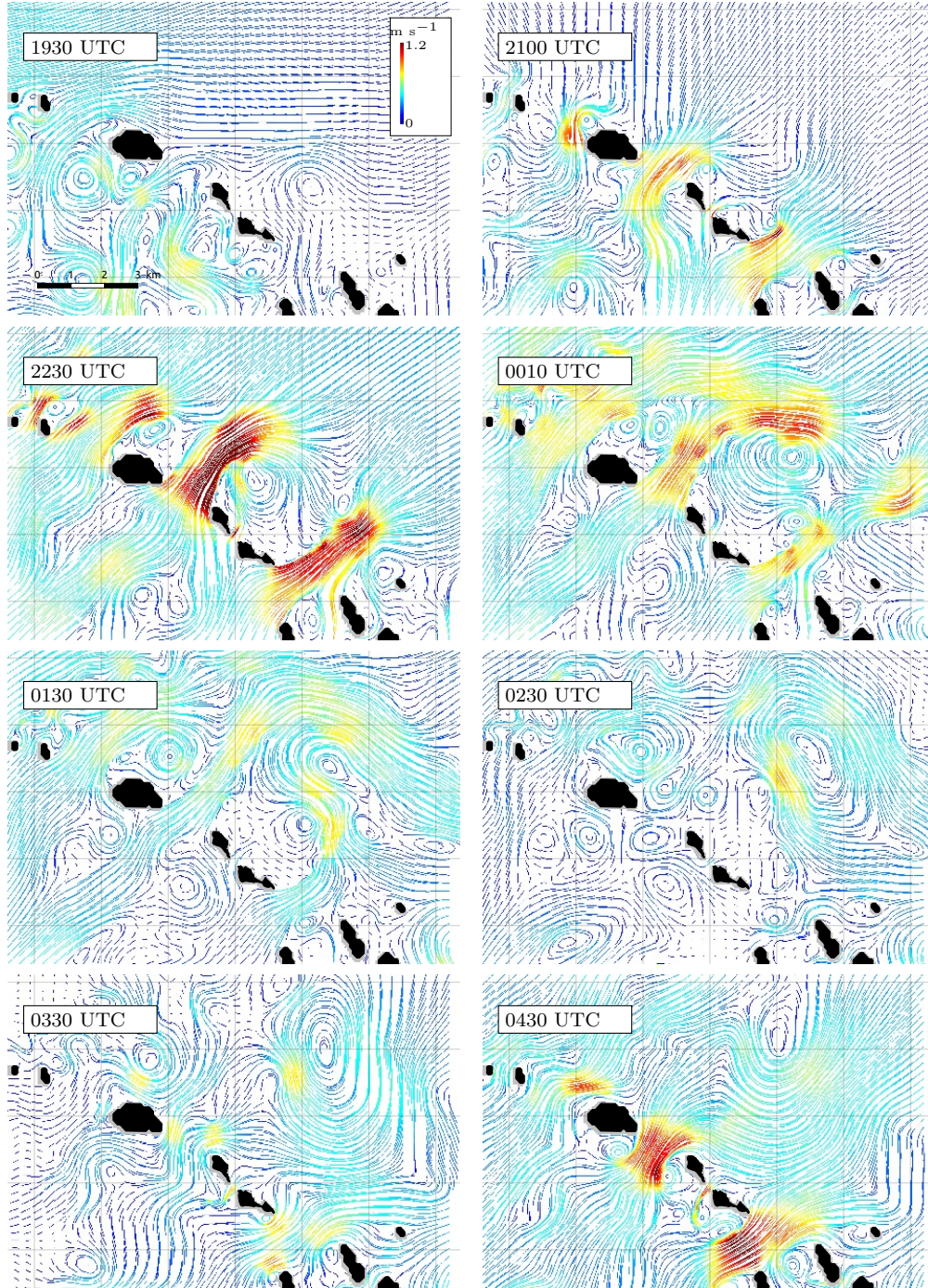


Fig. 2.4: Evolution of the tidal current in the inner-field region, from February 26th 1930 UTC until February 27th 0430 UTC.

The coefficient of determination is a good tool to compare PIV and GOS methods which process the same data, i.e. the same patterns. However, it cannot be used to compare two datasets in which there is a possible space shift between the patterns, as it is the case when comparing SLIM and data. SLIM results give a negative value for both  $r_{//}^2$  and  $r_{\perp}^2$ . According to the  $r^2$  method, it means that SLIM is worse than simply computing the averaged velocity over the domain, which by construction gives  $r^2 = 0$ . This simply illustrates how the method cannot be used to compare datasets with possible shift, although it works fine to compare PIV and GOS methods.

PIV, GOS and SLIM results are also compared through the use of scatterplots. Figure 2.5 shows an example for the parallel flow component. The PIV/GOS comparison gives almost a perfect match, which is not unexpected as those estimates derive from the same image data. The PIV/SLIM comparison, however, is rather widely dispersed; this can derive from either actual errors in modelling or small spatial shifts between PIV and SLIM velocity patterns. The main information given by the PIV/SLIM scatterplot is the  $\sim 0.1 \text{ m s}^{-1}$  shift in the flow parallel component. There are a number of possible explanations for this discrepancy. One of them is the use of an incorrect bottom stress coefficient. Another one is the vertical shear, which is missing from the two-dimensional approach: the vertically-averaged SLIM flow naturally underestimates the surface flow, since the vertical profile of the jet is logarithmic. Finally, an unseasonal increase in the South Equatorial Current or the influence of the Capricorn eddy in the southern GBR, which is not included in the model (Kingsford and Wolanski, 2008; Mao and Luick, 2014), may affect the results.

Since PIV and GOS methods show the same patterns, only the PIV method results are compared to SLIM, for simplicity.

### 2.3.4 Wind effect on the PIV velocities

Because the colour patterns detected by the satellite derive in part from material close to the sea surface, the possible extent of wind drift on the derived velocity field should be considered. Figure A.2 from the appendix compares the result as given in Figure 2.7 with the case for which there is a wind-drift current, defined as 2% of the wind velocity (Critchell and Lambrechts, 2016). The wind does not change the global dynamics, but the mean flow is rotated counter-clockwise by about  $5^\circ$ .

### 2.3.5 Sea surface elevation validation

The satellite provides no information about tidal elevation. To validate the tidal evolution over time, SLIM sea-surface elevation is compared to output from tidal prediction software ([www.mrtides.com](http://www.mrtides.com)) at Middle Island Anchorage station ( $21^\circ 39'S$ ,  $150^\circ 15'E$ ), which is located 42 km south-east of Hull Island.

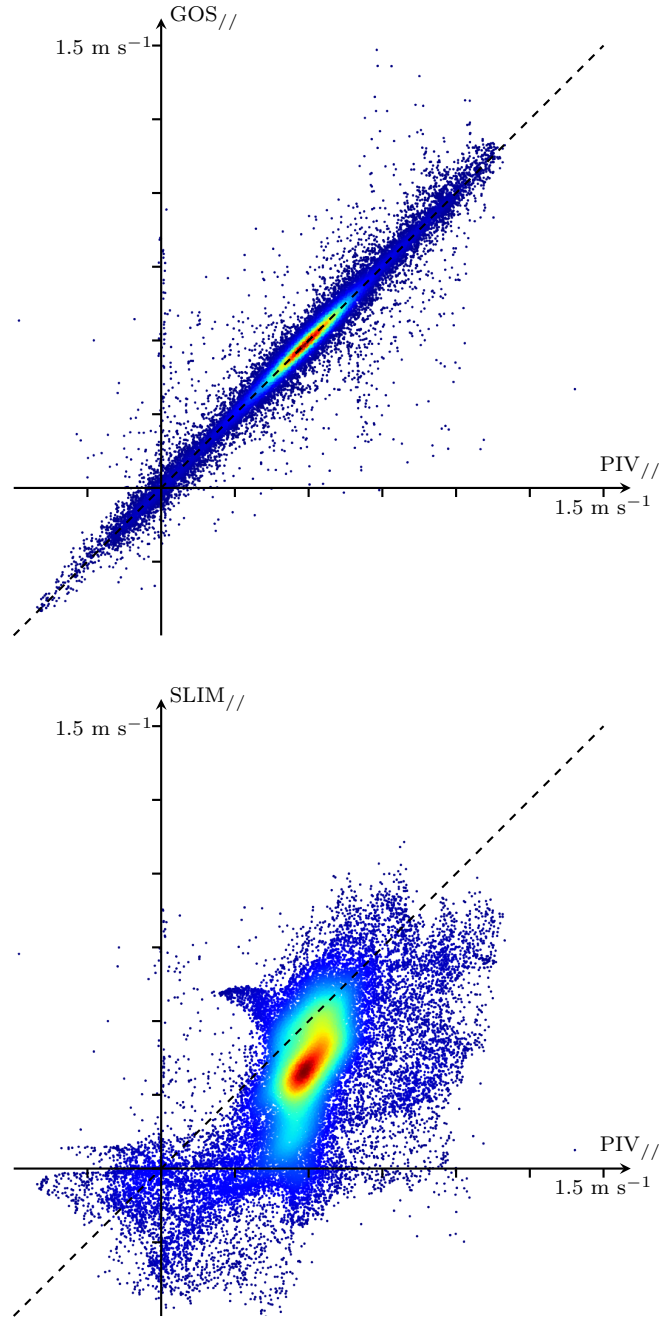


Fig. 2.5: Scatterplot comparing the parallel flow component of the PIV and GOS data (top panel) and PIV and SLIM data (bottom panel). While the PIV/GOS comparison gives almost a perfect match, the PIV/SLIM comparison is more dispersed. A shift is observed in the PIV/SLIM comparison. The colourmap indicates the point density.

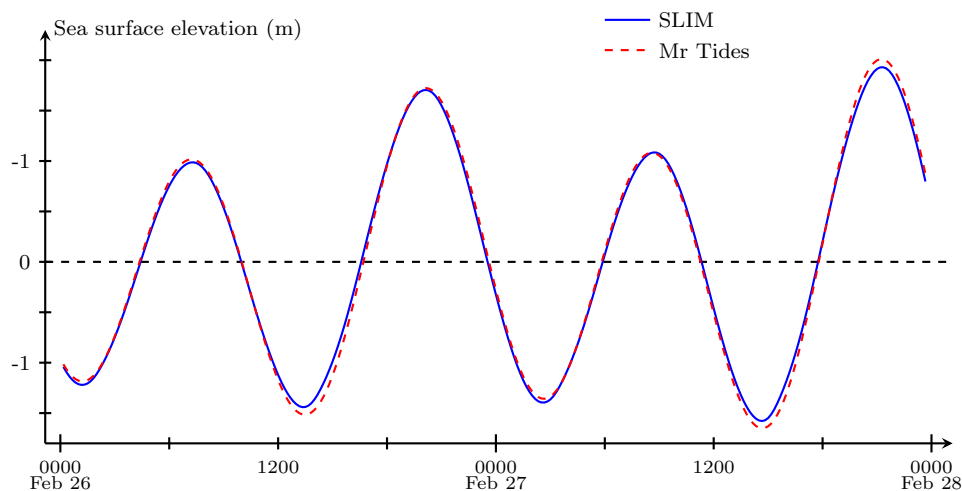


Fig. 2.6: Comparison between sea surface elevation from the Mr Tides software and SLIM results at Middle Island Anchorage station. The maximum relative error is 3.8%.

A comparison over four tidal cycles shows a maximum relative error of 3.8% (Fig. 2.6).

### 2.3.6 PIV and SLIM spatial comparison

Over the large scale, the main tidal current has the correct direction and speed, as observed upstream of the islands. At smaller scales, however, some differences occur; for example, the vortices downstream of the islands tend to be larger in the model than in the data. Figure 2.7 compares streamlines computed from the satellite data (PIV method) and from the SLIM results. The main patterns observed on the data (top map) are reproduced by the model: three large jets, developing from island-gaps B, C and E appear in both SLIM and PIV maps. The modelled jets widths are similar to the PIV jets; and the speeds are close, except for jet E, which is slower than in PIV data. Jet B is weaker and wider than the two others on both maps, although it is partially masked in the PIV map by the presence of clouds in the area (see Fig. 2.2).

The streamlines follow similar paths downstream of gap A and in the south-eastern corner of the maps, where small-scale vortices (diameters  $< 1$  km) appear. However, many vortices observed in the model upstream of the islands are not present in the PIV map.

On the PIV map, jet D develops only  $\sim 800$  m downstream of gap D (see Fig. 2.7, top map), which is physically impossible. The jet is generated by the gap, it should start at the gap or even upstream of it, not downstream of it where there is nothing anymore to generate it. This error arises from a lack of trackable small-scale features in the region of the gap, presumably as

the result of enhanced mixing throughout the water column in that shallow area. Jet D is essentially missing from the model. This may be the result of “upstream blocking” by the residual tidal eddies found in the model upstream of the Hull and Beverlac Islands.

Figure 2.8 shows the vorticity as computed from PIV (top map) and SLIM (bottom map). The first observation is that the data vorticity magnitude within the free shear layers developing downstream of the different islands is reproduced by the model. The “vortex street” of cyclonic and anti-cyclonic vortices downstream of the two north-western islands (around gap A) is correctly reproduced by the model, with the correct vorticity magnitude, similar free shear layer width, and the embedded patterns. Those good results are also present in the south-eastern corner of the inner-field, but the widths of other free shear layers are not as accurate as in above-mentioned areas. While the different vortices downstream of gap C detach one from each other in the SLIM results, such activity is not apparent in the PIV map. Finally, as already observed with the streamlines maps, there is some submesoscale activity observed upstream of the islands in the model but not in the data. However, all the vortices observed in the model in this area are not absent from data. For example, the negative and positive vortices, located 3 km south of gap B also appear in the PIV data.

## 2.4 Discussion

### 2.4.1 Satellite data and processing

High-resolution satellite data were processed by both the PIV and GOS methods, leading to similar results. The completely different approach of the two methods and the similarity of the output inspires confidence in the results. However, regions with lack of suitable texture can be affected by a loss of accuracy, such as gap D. A jet should develop near the entrance to the gap, not only 800 m downstream (Fig. 2.7, top map). This is an example of a localized error. Elsewhere, the small-scale patterns of algae and re-suspended sediment enable the generation of a physically realistic velocity field.

### 2.4.2 Modelling

The quality of the simulation is evaluated through the comparison of the patterns observed on the streamlines and vorticity maps (Figs. 2.7 and 2.8). SLIM succeeds in globally reproducing the correct speeds of both the main flow and the jets, and vorticity patterns similar to those observed. One difference is found for the modelling of jet E. While in the data it is a constant speed large jet extending on  $\sim 5$  km, the jet speed is spatially oscillating in the model. This difference in behaviour might be the consequence of various factors. Firstly, the parameterisations of the unresolved smaller scales are not

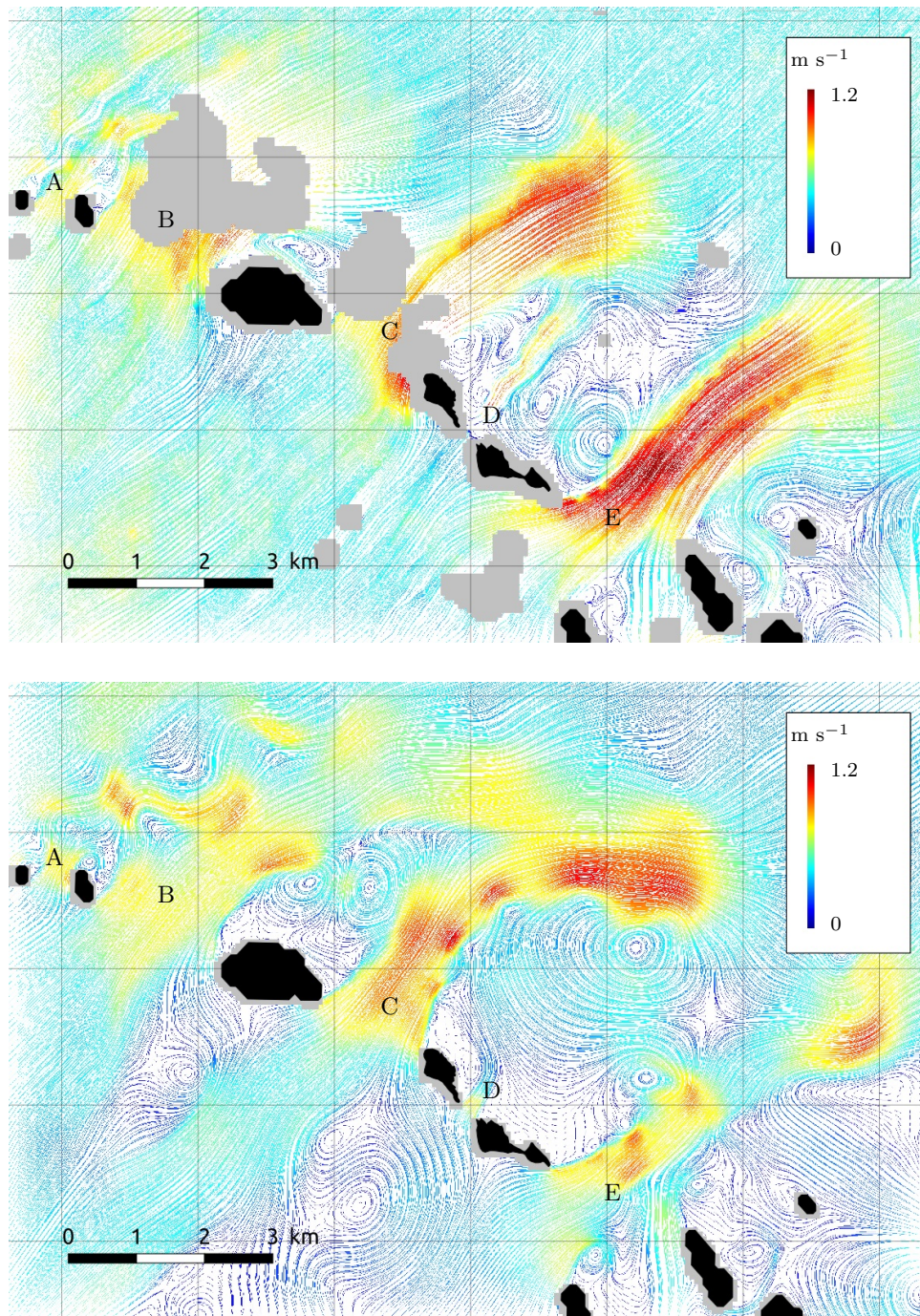


Fig. 2.7: Streamlines computed from the velocity field obtained with the PIV method (top map) and with SLIM (bottom map). Grey areas in the PIV map indicate where contaminating effects such as clouds are masked. Grey areas in the SLIM map result from the projection from the unstructured grid to the structured grid of PIV map. The colour shows the magnitude of the velocity field.

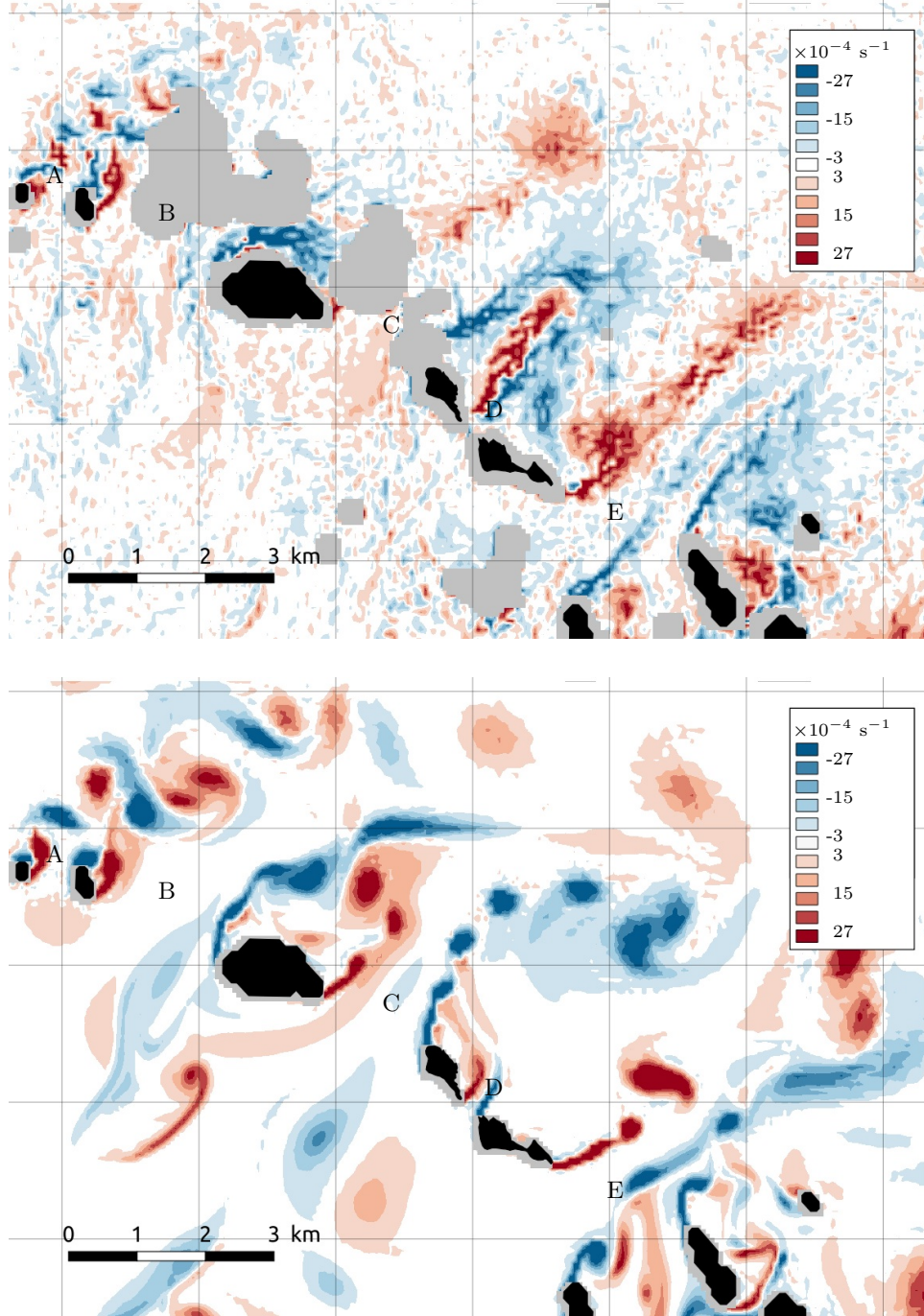


Fig. 2.8: Vorticity computed from the velocity field obtained with the PIV method (top map) and with SLIM (bottom map).

perfect. Secondly, the roughness of the sea floor, which has a large impact on the dynamics in shallow waters, is not known precisely. Thirdly, errors in bathymetry can drastically affect the jet speed. Jet E speed decreases suddenly 3 km downstream of gap E, before re-accelerating 1 km further away. This change corresponds to sudden changes in the bathymetry. But computing the transport conservation in the jet shows that the bathymetry variations are not sufficient to explain the jet speed oscillations. Eventually, the interactions between the jet and the surroundings vortices should be taken into account. In the PIV data, jet E is much ( $\sim 50\%$ ) stronger than in SLIM, its strength potentially prevents it from being affected by surroundings vortices while it is not the case for the modelled jet. The overall problem could then come from the jet being weaker since the beginning, at gap E.

Vortices are observed upstream of the islands in the SLIM results. Those vortices are generated by the bathymetry patterns in that area. Some of those appear in the PIV map, such as around the shallower area 3 km south of gap B, but other are missing completely. This difference could be a consequence of errors in the bathymetry input. Those vortices have a large importance, since they weaken the transport towards north-east, affecting the strength of jets D and E.

The vorticity inside the modelled free shear layers is unresolved. This result is not unexpected: besides the fact that the mesh size used prevents the model from resolving such small scales, those small size patterns are presumably non-hydrostatic and cannot be resolved by SLIM. However, even if the free shear layer is not fully resolved, it does not mean that the total vorticity flux is wrong, since the vorticity flux is not affected by the details in the free shear layer. It only depends on the velocity at the layer boundary. This result is detailed in Appendix A.1.

Despite the aforementioned differences, the model performs correctly to simulate the submesoscale patterns observed on PIV data. Indeed, the jets developing from the different gaps have globally the correct extent, width and speed, except jet E. The model is not too diffusive, since too large a diffusion would decrease the velocity gradient and show incorrect vorticity magnitudes, which is not the case in the model results. The model does represent correctly the main vortices on both sides of the jets developing from the different gaps. Figure 2.7 shows that vortices do not appear at the same location at the same moment, and the vortex developing from gap D is much larger in the model than in the data. But the vorticity in the wake of an island is highly unsteady (see the modelled vortices evolution in Figure 2.4), it would be hard to represent exactly the vortices size and timing.

### 2.4.3 Sub-grid scale processes and ecological impact

The satellite imagery reveals the presence of small eddies in the free shear layers (Fig. A.3). Field and laboratory studies of free shear layers at other sites but at similar spatial scales, as well the analogy with field studies of the small eddies, suggest that small eddies of 10-20 m in diameter are continuously produced in the free shear layer and dissipated within a few minutes (Onishi and Nishimura, 1980; Onishi, 1984; Wolanski and Elliott, 2015). These small eddies generate a very strong up-welling in their centre, up to  $0.1 \text{ m s}^{-1}$  (Wolanski, unpublished data), and a strong down-welling along their outer edges and this increases the vertical eddy viscosity within the free shear layer; essentially the free shear layers provide a viscous outer boundary to the island wakes, thereby partially isolating them from waters on the outside. In turn, as shown by Wolanski et al. (1996), this generates an upwelling in the centre of the island wake and a downwelling on the outer edges, while the strength of the eddy in the island wake is only slightly altered. The satellite images and the depth-averaged model are of course unable to assess the magnitude of the vertical motions.

The eddy-rich free shear layers, with a size varying from a few hundred metres to a few kilometres, have important ecological implications because they control the formation of patches of plankton, which can remain stable until the tide reverses (Hammer and Hauri, 1977). Also, the isolating effect of the free shear layers should decrease the export of waterborne larvae originating from the island and thus enhances self-recruitment (Nakano, 1957; Wolanski and Hamner, 1988; Signell and Geyer, 1991; Wolanski, 1993; Wolanski et al., 1996; Obst et al., 1996; White and Wolanski, 2008). The eddies present in the studied area are tidally generated and have a short lifespan, but their long-term implications are unknown. A starting point to address this last would be an understanding of the velocity field on the scale of the free shear layer eddies. So, one idea is to redo the analysis using the PAN imagery (zoom shown in Fig. A.3), which has a spatial resolution of 0.5 m. The problem here is time resolution: the time-scale of the eddies is comparable to the image time interval of 54 s; thus, the velocity field on the scale of the smallest eddies cannot be obtained from existing imagery. It is suggested, therefore, that higher-resolution model studies, perhaps using idealized bathymetry, be performed to study how such eddy-rich shear layers may affect the ecology around islands and reefs.

A better understanding of the eddy and free shear layer dynamics could be obtained by comparing the eddies observed in this study with smaller scale engineering applications or larger scale geophysical studies. At a smaller scale, Huang et al. (1995) studied the wake dynamics downstream of a porous medium, similar to a set of various solid body wakes, such as the present multiple islands. The small eddies generated directly downstream of the porous medium interact and merge together further away, analogously to what is observed in Figure 2.7. Other studies focused on the suppression of the vortices

downstream of a solid body by the placement of a second body downstream of the first one (Strykowski and Sreenivasan, 1990; Sakamoto et al., 1991). Such a setting is similar to the small islands set in the south-eastern corner of the map in Figures 2.7 and A.1. At a larger scale, the interaction of multiple island wakes (Dong and McWilliams, 2007; Caldeira and Sangrà, 2012) is also important, where eddies generated by small islands lead to the destabilisation of the free shear layer of a neighbouring larger island. Smaller and larger scale applications have then many similarities and should not be neglected for a study of the wake dynamics.

## 2.5 Conclusions and perspectives

A unique pair of 2 m-resolution satellite images, taken only 54 seconds apart, was obtained in the central Great Barrier Reef. Using two different approaches, a cross-correlation method (PIV) and an optical-flow method (GOS), a velocity field was obtained. It shows, with a high resolution, a rich submesoscale activity, including many small and large jets and vortices of different sizes. PIV and GOS methods give closely matching results, with however some errors in regions with lack of suitable texture, such as in the narrow gap between Beverlac and Hull Islands.

The unstructured-mesh, finite-element model SLIM was used to simulate the full Great Barrier Reef lagoon hydrodynamics, with an increased resolution, up to 50 m, in the Beverlac and Hull region (Fig. 2.1d). Results obtained are in overall agreement with the PIV and GOS data, even if discrepancies appear between the smallest-scale patterns. The global submesoscale activity, such as the extent, width and speed of the jets, and the magnitude of vortices at both sides of each jet and each island, is in accordance with PIV and GOS data. The model is not able to simulate exactly every single vortex position and size. It is concluded that these shortcomings must be related to the model assumptions and the accuracy of the forcing and bathymetry, both of which have a much coarser resolution than the size of the missed patterns.

High-resolution satellite images combined with the PIV or the GOS methods to build a velocity map are a new way to observe spatially complex submesoscale processes in shallow water regions. They also give much more information than current-metres and elevation moorings, for a much better validation of models. They are a potential resource that could be more widely used to assess the predictive ability of coastal circulation models. SLIM is generally successful in this validation, showing similar patterns to those in the satellite images

## 2.6 The way forward

This study has shown that a stereo-pair of high-resolution satellite images can capture the complexity of the flow around islands and reefs in shallow continental shelf waters, that include jets, eddies, shear layers and stagnation zones. The satellite-derived velocity field was used to assess if oceanographic models are able to reproduce that complexity. To some degree the model is successful, but discrepancies occur. These are unlikely to be due to the two-dimensional modelling approach, as three-dimensional models do not simulate the phenomena any better ([Wolanski et al., 2003a](#)). It is suggested that the lessons from engineering fluid mechanics may be useful here, including non-hydrostatic models; though even those models would not resolve all the complexity of this flow, especially if the resolution and the accuracy of the bathymetry and the forcing is not markedly increased. A better knowledge of the sea bed would also be necessary to improve the bottom friction parameterisation. Significant improvement may only result from the assimilation of high-resolution field data, such as surface currents measured by a high-frequency radar. The limit will remain the parameterisation of sub-grid-scale eddies, even in the absence of islands or reefs ([Robinson, 2012](#)).

## Acknowledgements

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# Numerical development in SLIM 3D

## 3.1 Introduction

SLIM 3D had already been under development for various years before this thesis began. [White et al. \(2008a\)](#) developed the first three-dimensional model of the SLIM project. At that time, the model was still barotropic and the finite element discretisation was linear continuous in the horizontal direction, conforming for the sea surface elevation, the vertical velocity and the passive tracers, and non-conforming for the horizontal velocity. The vertical discretisation was linear discontinuous for all the fields. [Blaise et al. \(2010\)](#) and [Comblen et al. \(2010\)](#) modified the model in various aspects. On the numerical aspect, they changed the space discretisation of all the fields to the current discontinuous Galerkin formulation and they developed an implicit-explicit (IMEX) Runge–Kutta time integration method. On the physical aspect, they added the baroclinity to the model. Eventually, [Kärnä et al. \(2013\)](#) updated the model by adding flux-limiters and changing the temporal scheme to a split-explicit one.

During this thesis, SLIM 3D has evolved from the [Kärnä et al. \(2013\)](#) version, improving its stability and accuracy, but the main features of the code did not change. This chapter summarizes the main equations which do not differ from the previous version of the model. Then, the modifications provided to SLIM 3D are detailed. Those modifications can be split into three categories: the manipulation of the equations, the equation discretisation and the development of new features. The first category concerns principally the

new way of splitting the equations between 2D and 3D modes. The internal pressure gradient is also computed differently to increase the stability. The main development in the equation discretisation is the computation of the moving mesh velocity. Finally, different features were added to the model, such as a vertically adaptive mesh, the handling of a discontinuous bathymetry, a new slope limiter and hexahedral and periodic meshes.

## 3.2 SLIM 3D: a hydrodynamic model

### 3.2.1 Basic equations

SLIM 3D solves the hydrostatic Boussinesq equations on a three-dimensional domain  $\Omega$ . The equations are defined in a Cartesian coordinate system  $\mathbf{x} = x \mathbf{e}_x + y \mathbf{e}_y + z \mathbf{e}_z$ , where  $x$  and  $y$  are the horizontal coordinates, and  $z$  is the vertical coordinate.  $\mathbf{e}_x$  and  $\mathbf{e}_y$  are the two horizontal unit vectors which are perpendicular and  $\mathbf{e}_z$  is the vertical unit vector, pointing upwards. While the lateral boundary  $\Gamma_l$  and the bottom boundary  $\Gamma_b$  are fixed, the surface boundary  $\Gamma_s$  is varying in time. The vertical coordinate varies between  $z = -h$  at the bottom boundary,  $h$  being the bathymetry, and  $z = \eta$  at the surface, with  $\eta$  the free surface elevation (Fig. 3.1).  $\Gamma_{s,0}$  is the reference surface boundary, equal to  $\Gamma_s$  when the free surface elevation is zero.

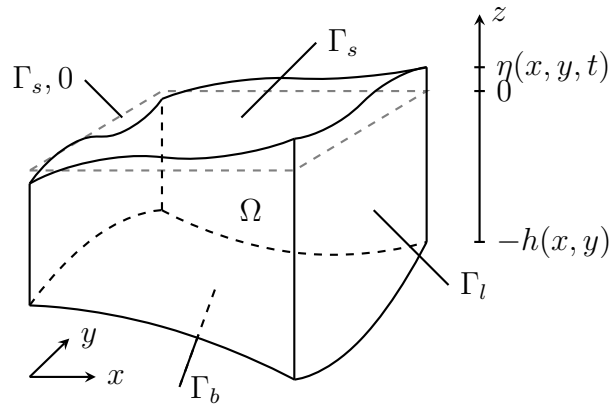


Fig. 3.1: Illustration of the domain on which SLIM 3D solves the hydrostatic Boussinesq equations. The boundary  $\Gamma_{s,0}$  is located at  $\eta = 0$  and refers to the reference fixed mesh.

The main unknowns are the horizontal velocity  $\mathbf{u} = u \mathbf{e}_x + v \mathbf{e}_y$ , the vertical velocity  $w$ , the pressure  $p$ , the salinity  $S$  and the temperature  $T$ . The

3D hydrostatic Boussinesq equations read:

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla_h \cdot (\mathbf{u}\mathbf{u}) + \frac{\partial (w\mathbf{u})}{\partial z} = \nabla_h \cdot \left( \nu_h (\nabla_h \mathbf{u} + (\nabla_h \mathbf{u})^T) \right) + \frac{\partial}{\partial z} \left( \nu \frac{\partial \mathbf{u}}{\partial z} \right) - f \mathbf{e}_z \wedge \mathbf{u} - \frac{1}{\rho_0} \nabla_h p, \quad (3.1)$$

$$\nabla_h \cdot \mathbf{u} + \frac{\partial w}{\partial z} = 0, \quad (3.2)$$

$$\frac{1}{\rho_0} \nabla_h p = g \nabla_h \eta + \frac{g}{\rho_0} \nabla_h \int_z^\eta (\rho - \rho_0) d\zeta, \quad (3.3)$$

$$\frac{\partial S}{\partial t} + \nabla_h \cdot (\mathbf{u}S) + \frac{\partial (wS)}{\partial z} = \nabla_h \cdot (\kappa_h \nabla_h S) + \frac{\partial}{\partial z} \left( \kappa \frac{\partial S}{\partial z} \right), \quad (3.4)$$

$$\frac{\partial T}{\partial t} + \nabla_h \cdot (\mathbf{u}T) + \frac{\partial (wT)}{\partial z} = \nabla_h \cdot (\kappa_h \nabla_h T) + \frac{\partial}{\partial z} \left( \kappa \frac{\partial T}{\partial z} \right). \quad (3.5)$$

The equation for the pressure  $p$  results from the hydrostatic hypothesis. The density  $\rho$  is given by the state equation  $\rho = \hat{\rho}(S, T)$  (Jackett et al., 2006), while  $\rho_0$  is the reference density, which is constant. The symbol  $\nabla_h$  stands for the horizontal derivative operator, such that  $\nabla_h \cdot \mathbf{u} = \partial u / \partial x + \partial v / \partial y$ . The material parameters are the Coriolis factor  $f$ , the horizontal and vertical viscosities  $\nu_h$  and  $\nu$  and the horizontal and vertical diffusivities for  $\kappa_h$  and  $\kappa$ . Horizontal viscosity  $\nu_h$  follows the Smagorinsky parameterisation (Smagorinsky, 1963; Blaise et al., 2007). Vertical eddy viscosity  $\nu$  and diffusivity  $\kappa$  can be determined from the  $\kappa$ - $\epsilon$  turbulence closure model from GOTM (Burchard et al., 1999) coupled to SLIM 3D (Kärnä et al., 2012).

### 3.2.2 Boundary conditions

The impermeability boundary condition is:

$$\mathbf{u} \cdot \mathbf{n}_h = 0, \quad \mathbf{x} \in \Gamma_l, \quad (3.6)$$

$$w + \mathbf{u} \cdot \nabla_h h = 0, \quad \mathbf{x} \in \Gamma_b, \quad (3.7)$$

$$w - \mathbf{u} \cdot \nabla_h \eta = \frac{\partial \eta}{\partial t}, \quad \mathbf{x} \in \Gamma_s, \quad (3.8)$$

where  $\mathbf{n}_h$  is the horizontal vector normal to the lateral boundary  $\Gamma_l$ , which is strictly vertical. One important consequence of Equation 3.8 is that the domain  $\Omega$  is moving, since the surface boundary  $\Gamma_s$  located at  $z = \eta(x, y, t)$  is varying in time. The bottom friction and the wind stress imply additional

bottom and surface boundary conditions, respectively:

$$\nu \frac{\partial \mathbf{u}}{\partial z} = \frac{\boldsymbol{\tau}_b}{\rho_0}, \quad \mathbf{x} \in \Gamma_b, \quad (3.9)$$

$$\nu \frac{\partial \mathbf{u}}{\partial z} = \frac{\boldsymbol{\tau}_s}{\rho_0}, \quad \mathbf{x} \in \Gamma_s, \quad (3.10)$$

with  $\boldsymbol{\tau}_b$  a quadratic function of the velocity, defined in [Kärnä et al. \(2013\)](#) and  $\boldsymbol{\tau}_s$  the wind stress.

### 3.2.3 Pressure gradient

Equation 3.3 is the classical formulation of the pressure gradient under the hydrostatic Boussinesq approximation ([Kärnä et al., 2013](#)). However, to increase the stability of the model, it is preferable to compute the horizontal gradient before the vertical integral of the density deviation. Equation 3.3 is then rewritten in the following way:

$$\frac{1}{\rho_0} \nabla_h p = g \nabla_h \eta + \frac{g}{\rho_0} \int_z^\eta \nabla_h \rho' d\zeta + \frac{g}{\rho_0} \rho'|_\eta \nabla_h \eta, \quad (3.11)$$

where  $\rho' = \rho - \rho_0$  is the density deviation. The consequences of this change are studied in Section 3.3.7.

### 3.2.4 Moving mesh and ALE formulation

Since the free surface elevation varies in time, the surface boundary and the whole domain move in the vertical direction as well. Equations 3.1, 3.4 and 3.5 are written in the Eulerian coordinates and are valid in the continuously moving domain  $\Omega_t$ . However, when the domain and the equations are discretised in order to be solved, the discrete domain does not move continuously and those equations are not valid anymore. The Arbitrary Lagrangian Eulerian (ALE) formulation ([Formaggia and Nobile, 2004](#); [White et al., 2008b](#); [Blaise et al., 2010](#); [Kärnä et al., 2013](#)) is ideal to deal with such moving mesh. It enables to compute the equations on the ALE coordinates, in a fixed domain. The principle of this formulation, is defined in a general framework in [Formaggia and Nobile \(2004\)](#). Hereinafter, it is explained simply in the case of the vertically moving mesh.

Considering the reference fixed domain  $\tilde{\Omega}$ ,  $\mathcal{A}$  is an invertible mapping from  $\tilde{\Omega}$  to  $\Omega_t$  (similar to [Deleersnijder and Ruddick, 1992](#)):

$$\mathcal{A} : \tilde{\Omega} \longrightarrow \Omega_t, \quad \mathcal{A}(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{t}) = (x, y, z, t), \quad (3.12)$$

$$\begin{cases} x = \tilde{x} \\ y = \tilde{y} \\ z = z(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{t}) \\ t = \tilde{t}. \end{cases}$$

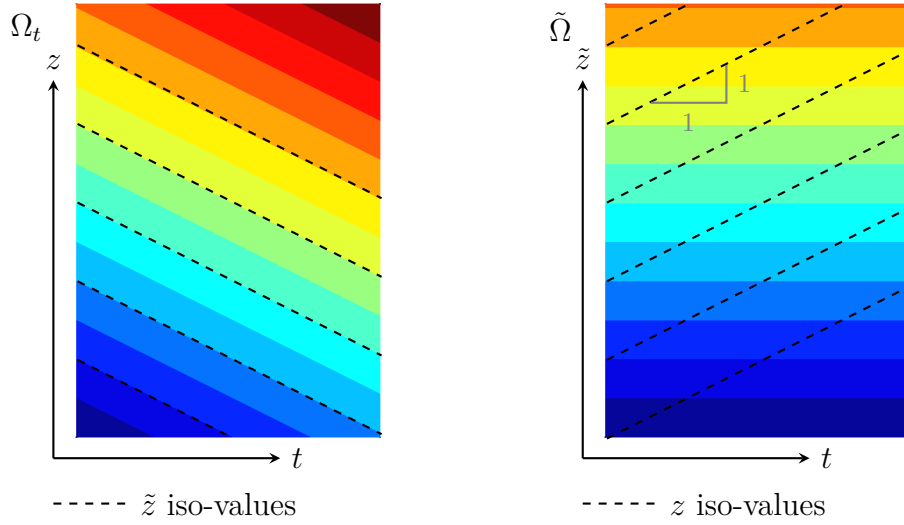


Fig. 3.2: Temporal evolution of a field  $f$  for the following configuration:  $\partial f / \partial z = -1$  and  $\partial f / \partial t = w_m = 1$ , which results in  $\partial f / \partial \tilde{t} = 0$ . Left panel shows the result in (a part of) the moving domain  $\Omega$  while the right panel shows the result in (a part of) the reference domain  $\tilde{\Omega}$ .

The Jacobian of the invertible mapping is  $J = \partial z / \partial \tilde{z}$ . Furthermore, the vertical moving mesh velocity  $w_m$ , which denotes the movement of the moving domain  $\Omega_t$  with respect to the fixed domain  $\tilde{\Omega}$ , reads in the ALE framework:

$$w_m = \frac{\partial z}{\partial \tilde{t}}. \quad (3.13)$$

Similarly to a material derivative, the ALE derivative simply reads as:

$$\frac{\partial \cdot}{\partial \tilde{t}} = \frac{\partial \cdot}{\partial t} + w_m \frac{\partial \cdot}{\partial z}. \quad (3.14)$$

Equation 3.14 is illustrated on Figure 3.2. Temperature Equation 3.5 can now be re-written in the (non-conservative) ALE formulation:

$$\frac{\partial T}{\partial \tilde{t}} + \nabla_h \cdot (\mathbf{u}T) + \frac{\partial (wT)}{\partial z} - w_m \frac{\partial T}{\partial z} = rhs_T, \quad (3.15)$$

with  $rhs_T$  the right hand side of Equation 3.5. To obtain the conservative ALE formulation, it is necessary to derive the Jacobian:

$$\begin{aligned} \frac{\partial J}{\partial \tilde{t}} &= \frac{\partial}{\partial \tilde{t}} \frac{\partial z}{\partial \tilde{z}} \\ &= \frac{\partial}{\partial \tilde{z}} \frac{\partial z}{\partial \tilde{t}} \\ &= \frac{\partial w_m}{\partial \tilde{z}} \\ &= \frac{\partial w_m}{\partial z} \frac{\partial z}{\partial \tilde{z}} = J \frac{\partial w_m}{\partial z}. \end{aligned}$$

The conservative ALE derivative then becomes:

$$\frac{1}{J} \frac{\partial(JT)}{\partial \tilde{t}} = \frac{\partial T}{\partial \tilde{t}} + \frac{\partial w_m}{\partial z} T,$$

and the conservative ALE equation is obtained for the temperature Equation 3.15:

$$\frac{1}{J} \frac{\partial(JT)}{\partial \tilde{t}} + \nabla_h \cdot (\mathbf{u}T) + \frac{\partial((w - w_m)T)}{\partial z} = rhs_T. \quad (3.16)$$

Equations 3.1 and 3.4 are rewritten similarly:

$$\frac{1}{J} \frac{\partial(J\mathbf{u})}{\partial \tilde{t}} + \nabla_h \cdot (\mathbf{u}\mathbf{u}) + \frac{\partial((w - w_m)\mathbf{u})}{\partial z} = rhs_{\mathbf{u}}, \quad (3.17)$$

$$\frac{1}{J} \frac{\partial(JS)}{\partial \tilde{t}} + \nabla_h \cdot (\mathbf{u}S) + \frac{\partial((w - w_m)S)}{\partial z} = rhs_S, \quad (3.18)$$

with  $rhs_{\mathbf{u}}$ ,  $rhs_S$  the right hand sides of Equations 3.1 and 3.4, respectively.

### 3.2.5 2D - 3D time split

The model equations are now defined and could be discretised in order to be solved. However, an important double issue still needs to be solved. On the one hand, the range of time scales of the hydrodynamics is very wide, from gravity waves propagating at a speed of the order of  $100 \text{ m s}^{-1}$ , to slower flow, with time scales varying from days to weeks or months. On the other hand, the model is quite complex and is computationally expensive to solve. Because the time integration is explicit, it is necessary to resolve the gravity waves propagation, leading to a maximum time step of the order of a few seconds. This problem would make the code extremely slow in terms of computational time.

The good news is that the fast propagating gravity waves are a 2D horizontal phenomenon. The issue can then be solved by splitting the equations between a 2D mode and a 3D mode. The 2D mode is composed of two equations: the continuity and the momentum conservation. The first is obtained by integrating the 3D continuity Equation 3.2 using the bottom and surface boundary conditions (Eqs. 3.7, 3.8). The second equation results from the vertical integration of Equation 3.1. In Kärnä et al. (2013), the 2D mode was solved explicitly using an Adams-Bashforth scheme. This resulted in a small (but cheap) time step. Then the 2D mode was low-passed filtered in order to avoid signal aliasing.

In this work, the 2D equations are firstly simplified: only the pressure gradient and the Coriolis factor are kept in the momentum conservation equation,

resulting in the following linear system:

$$\frac{\partial \eta}{\partial t} = -\nabla_h \cdot (H \bar{\mathbf{u}}), \quad (3.19)$$

$$\frac{\partial \bar{\mathbf{u}}}{\partial t} = -f \mathbf{e}_z \wedge \bar{\mathbf{u}} - g \left( 1 + \frac{\rho'|\eta|}{\rho_0} \right) \nabla_h \eta, \quad (3.20)$$

with the vertically averaged horizontal velocity  $\bar{\mathbf{u}} = \frac{1}{H} \int_{-h}^{\eta} \mathbf{u} \, dz$  and  $H = h + \eta$  the total water column depth. This 2D mode is solved implicitly in time, using the same time step as the 3D mode.

### 3.3 SLIM 3D discretisation

The following discretisation for SLIM 3D is built in order to strictly preserve both the tracer conservativity and the consistency of the scheme. Tracer conservativity consists in preserving the global mass of the tracer through the entire simulation. Tracer consistency is the property of keeping a constant tracer at its constant value while advected. Some effort has already been carried out in that purpose in previous versions of the model, but obtaining both properties at a numerical level was not obtained before using SLIM 3D. Basically, the main changes were achieved in the computation of the moving mesh velocity and in the time integration scheme.

This section does not rewrite the complete discretisation of the model, which is available in [Kärnä et al. \(2013\)](#), but focuses on the development achieved to obtain conservativity and consistency. Firstly, the time integration scheme is summarized. Then, the finite element framework and notations are introduced. Eventually, the finite element discretisation of the fields which are not computed as in [Kärnä et al. \(2013\)](#) are developed.

#### 3.3.1 Time integration

The principle of the time integration scheme is the following (see Figure 3.3 and Algorithm 1 for illustration). The only unknowns which exist at the beginning of an iteration are the mesh coordinates ( $\mathbf{x}^n$ ), the horizontal velocity ( $\mathbf{u}^n$ ), the salinity ( $S^n$ ) and the temperature ( $T^n$ )<sup>1</sup>. Firstly, the 2D mode is updated, from which the velocity  $\bar{\mathbf{u}}^+$  and the elevation  $\eta^+$  are obtained. Time  $t^+$  is illustrated in Figure 3.3 between  $t^n$  and  $t^{n+1}$ , but it cannot be interpreted simply as the mean of  $t^n$  and  $t^{n+1}$ . It is just corresponding to the situation after solving part of the hydrodynamic equations. Then the mesh is updated, and new coordinates  $\mathbf{x}^+$  are obtained using a continuous function of the elevation ( $F_{CG}$ ). This mesh is thus continuous<sup>2</sup>. The horizontal 3D velocity  $\mathbf{u}^+$  is

<sup>1</sup>It is noteworthy that if the vertical viscosity and diffusivity follow the  $k$ - $\epsilon$  parameterisation,  $k$  and  $\epsilon$  are advected similarly to the salinity and the temperature.

<sup>2</sup>This property is not strictly necessary, but it simplifies some aspects of the computation on the current code architecture, such as the parallel computation.

obtained, corresponding to the vertical distribution of  $\mathbf{u}^n$ , but with a vertical average equal to  $\bar{\mathbf{u}}^+$ . Using the continuity equation, the vertical velocity  $w^+$  is computed. The sea surface elevation is now computed from the surface vertical velocity, obtaining  $\eta^{n+1}$ . Note that  $\eta^+$  and  $\eta^{n+1}$  are not the same, because the interface terms are not solved equally in the 2D mode and the 3D mode. With the new sea surface elevation, the new mesh is obtained, which is necessarily discontinuous. Otherwise, the continuity equation would be broken at the surface. The moving mesh velocity is computed using the mesh at  $t^n$  and  $t^{n+1}$ . This operation is essential to preserve the consistency, and will be detailed later. Eventually,  $S$ ,  $T$  and  $\mathbf{u}$  are updated. While the two tracers are updated from  $t^n$ , the horizontal velocity is updated from  $t^+$ , since the barotropic pressure terms and the vertically averaged Coriolis term, which were in the 2D wave equation, are removed from  $F_{\mathbf{u}}$ , the right hand side of the 3D horizontal momentum equation. The vertical advection and the vertical diffusion of  $F_{\mathbf{u}}$ ,  $F_S$  and  $F_T$  are solved implicitly, while the other terms are solved explicitly. This difference is due to the fact that solving an equation on the global mesh would be too expensive, while the implicit solving of the vertical advection and the vertical diffusion is cheap since it only includes nodes of a local column.

**Input:**  $\mathbf{x}^0, \mathbf{u}^0, S^0, T^0$

**for**  $n=0; n < N; n++$  **do**

$$M^{2D}(\eta^+, \bar{\mathbf{u}}^+) = M^{2D}(\eta^n, \bar{\mathbf{u}}^n) + \Delta t F_{2D}(\eta^+, \bar{\mathbf{u}}^+) \quad (3.21)$$

$$\mathbf{x}^+ = F_{CG}(\eta^+) \quad (3.22)$$

$$\mathbf{u}^+ = \mathbf{u}^n - \bar{\mathbf{u}}^n + \bar{\mathbf{u}}^+ \quad (3.23)$$

$$w^+ = F_w^+(\mathbf{u}^+, \eta^+) \quad (3.24)$$

$$\eta^{n+1} = \eta^n + \Delta t (w^+|_{\eta} - \mathbf{u}^+|_{\eta} \cdot \nabla_h \eta^+) \quad (3.25)$$

$$\mathbf{x}^{n+1} = F_{DG}(\eta^{n+1}) \quad (3.26)$$

$$w_m^+ = F_{w_m}^+(\mathbf{x}^n, \mathbf{x}^{n+1}) \quad (3.27)$$

$$M^{n+1} S^{n+1} = M^n S^n + \Delta t F_S^+(S^n, S^{n+1}, \mathbf{u}^+, w^+, w_m^+, \eta^+) \quad (3.28)$$

$$M^{n+1} T^{n+1} = M^n T^n + \Delta t F_T^+(T^n, T^{n+1}, \mathbf{u}^+, w^+, w_m^+, \eta^+) \quad (3.29)$$

$$M^{n+1} \mathbf{u}^{n+1} = M^+ \mathbf{u}^+ + \Delta t F_{\mathbf{u}}^+(\mathbf{u}^+, \mathbf{u}^{n+1}, w^+, w_m^+, S^{n+1}, T^{n+1}) \quad (3.30)$$

**end**

**Algorithm 1:** Time integration scheme of SLIM 3D. Different variables are not yet defined at this stage, and will be defined in the following sections.

The explicit part of the 3D equations update is solved using a first order Euler time scheme. The order of the time scheme could be increased to im-

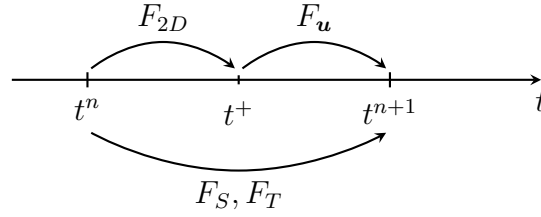


Fig. 3.3: Time integration scheme. First the 2D wave equation is solved, then the 3D equations are updated.

prove the temporal accuracy of the method. The choice of this simple time scheme was done for a simpler development of the tracer consistency (see Section 3.3.6). However, it is noteworthy that in the current applications, the error was dominated by spatial error rather than temporal error.

### 3.3.2 Discontinuous Galerkin finite element discretisation

The reference boundary surface  $\Gamma_{s,0}$  is meshed into a set of linear triangles or quadrilaterals  $\mathcal{T}$ . Each triangle or quad is then extruded from the surface to the bottom, creating a set of prisms  $\mathcal{P}$ . Those prisms are either triangular prisms or hexahedra<sup>3</sup>. The classical DG finite element discretisation is applied.

For the 2D mode, the piecewise discontinuous polynomial function space  $\mathbb{W}$  is the set of shape functions  $\phi_i^Q$ , with  $i$  varying between 1 and the number of nodes per element (3 for the triangles and 4 for the quads) and  $Q$  varying between 1 and the number of elements in  $\mathcal{T}$  (Fig. 3.4). They are piecewise linear for triangular elements and piecewise bi-linear for quadrilaterals. Every  $\phi_i^Q$  is defined such that it is equal to 0 everywhere except on element  $Q$  where it is a linear function equal to 1 on the  $i^{\text{th}}$  node and 0 on the other nodes, located at the element vertices. The shape functions are then discontinuous on the element interfaces.

For the 3D mode, the piecewise discontinuous polynomial function space  $\mathbb{V}$  is the set of shape functions  $\phi_i^K$ , which are bi- or tri- linear, for extruded triangles and quads, respectively,  $K$  being the index of the prism. Figure 3.5 illustrates the position of the nodes. All the fields are then approximated by interpolating their discontinuous nodal values. For example, the approximation of the temperature is given by:

$$\tilde{T}(\mathbf{x}) = \sum_{K,i} T_i^K \phi_i^K(\mathbf{x}) = T_i^K \phi_i^K(\mathbf{x}), \quad (3.31)$$

using the Newton notation for the second equality.  $\tilde{T}(\mathbf{x})$  is then the approximated solution function and  $T_i^K$  is the approximated solution vector.

<sup>3</sup>A discussion about the hexahedra is made in Section 3.4.

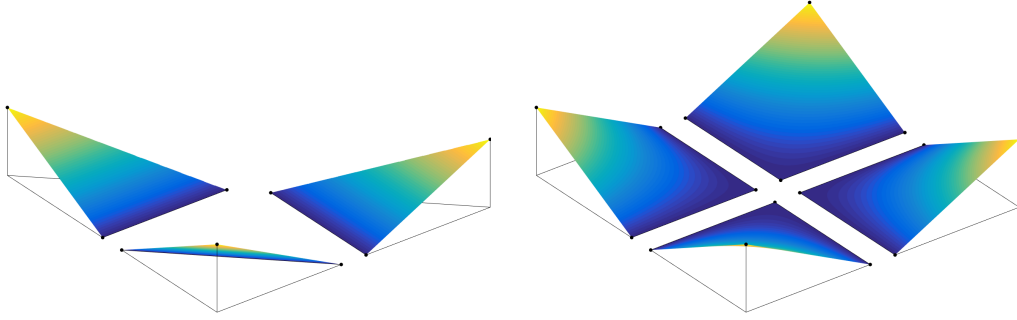


Fig. 3.4: Piecewise polynomial shape functions used for the 2D mode on the triangular (left) or quadrilateral (right) elements.

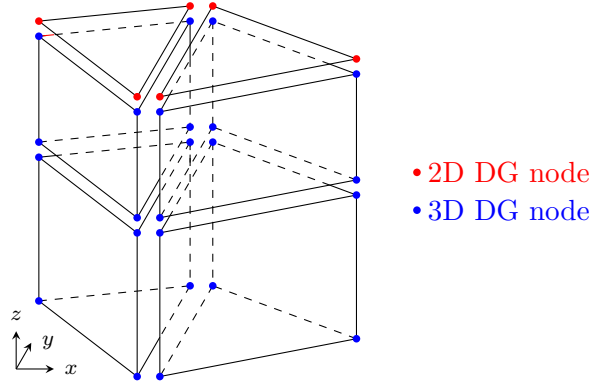


Fig. 3.5: 3D DG mesh. On this simple mesh, two triangles were extruded to form two prisms each. The nodes from both the 2D and the 3D mesh are represented.

The interface of the 2D element  $Q$  is defined  $\mathcal{J}(Q) = \{Q \cap Q' | Q' \in \mathcal{T}\}$ , with  $\mathbf{n}_h$ , the normal to the interface, which is pointing outward. A similar notation is defined for the 3D mesh, with the interface  $\mathcal{I}(K) = \{K \cap K' | K' \in \mathcal{P}\}$ , with the outward pointing normal  $\mathbf{n}$ .  $\mathcal{I}(K)$  can be split between the lateral (vertical) interface  $\mathcal{I}_l(K)$ , the top interface  $\mathcal{I}_{top}(K)$  and the bottom interface  $\mathcal{I}_{bot}(K)$ . Finally,  $\mathcal{I}_h(K) = \mathcal{I}_{top}(K) \cup \mathcal{I}_{bot}(K)$ . It is noteworthy that  $\mathcal{I}_h(K)$  is generally not horizontal.

For the sake of clarity, a simple notation is used hereinafter. First, all the fields are the approximated fields and the tilde sign is removed from the notation which appears in Equation 3.31. The weak form of the equation is written for one element  $K$  or  $Q$ . Finally, the different integrals are written in

a simplified notation. For the 3D equations, they read:

$$\langle \cdot \rangle = \int_K \cdot d\mathbf{x}, \quad K \in \mathcal{P} \quad (3.32)$$

$$\langle\langle \cdot \rangle\rangle = \sum_{\mathcal{I}} \int_{\mathcal{I}} \cdot dA, \quad \mathcal{I} \in \mathcal{I}(K) \quad (3.33)$$

$$\langle\langle \cdot \rangle\rangle_h = \sum_{\mathcal{I}} \int_{\mathcal{I}} \cdot dA, \quad \mathcal{I} \in \mathcal{I}_h(K), \quad (3.34)$$

with  $dA$  an infinitesimal area of the  $\mathcal{I}$  interface. A notation similar to Equation 3.34 is used for  $\mathcal{I}_l(K)$ ,  $\mathcal{I}_{bot}(K)$  or  $\mathcal{I}_{top}(K)$ . For the 2D equations, the integrals read:

$$\langle\langle \cdot \rangle\rangle = \int_Q \cdot d\mathbf{x}_h, \quad Q \in \mathcal{T} \quad (3.35)$$

$$\langle\langle\langle \cdot \rangle\rangle\rangle = \sum_{\mathcal{J}} \int_{\mathcal{J}} \cdot dS, \quad \mathcal{J} \in \mathcal{J}(Q), \quad (3.36)$$

with  $dS$  an infinitesimal length of the  $\mathcal{J}$  interface.

### 3.3.3 2D mode

The DG weak formulation of the right hand side of Equations 3.19 and 3.20, which corresponds to Equation 3.21 in the time scheme algorithm, is computed. For simplicity,  $F_{2D}$  is split into its  $\eta$  component and its  $\bar{\mathbf{u}}$  component:  $F_{2D} = (F_{2D,\eta}, F_{2D,\bar{\mathbf{u}}})$ .  $F_{2D,\eta}$  can be written as:

$$F_{2D,\eta} = \langle\langle (\eta + h) \bar{\mathbf{u}} \cdot \nabla_h \phi_j \rangle\rangle - \langle\langle\langle (\mathbf{n}_h \cdot \bar{\mathbf{u}}^* (\eta^* + h^*) + f_\eta^{LF}) \phi_j \rangle\rangle\rangle, \quad (3.37)$$

with  $h^* = \min(h_L, h_R)$  the minimum between the left and the right values on the interface, the mean values  $\bar{\mathbf{u}}^* = (\bar{\mathbf{u}}_L + \bar{\mathbf{u}}_R)/2$  and  $\eta^* = (\eta_L + \eta_R)/2$  and the Lax-Friedrichs flux  $f_{\eta,i}^{LF} = \sqrt{gH^*}(\eta_L - \eta_R)/2$ , with  $H^* = h^* + \eta^*$ .

The computation of  $F_{2D,\bar{\mathbf{u}}}$  is slightly different. First, the weak form is obtained as the scalar product between the vectorial Equation 3.20 and the vector  $\phi_j$ . Then, it is necessary to approximate the gradient of the surface density deviation:

$$\rho'_{\text{grad},\eta} = \rho'|_{\eta,i} \nabla_h \phi_i.$$

With this approximation,  $F_{2D,\bar{\mathbf{u}}}$  reads:

$$\begin{aligned} F_{2D,\bar{\mathbf{u}}} = & -f \langle\langle (\mathbf{e}_z \wedge \bar{\mathbf{u}}) \cdot \phi_j \rangle\rangle + g \langle\langle \left(1 + \frac{\rho'|_{\eta}}{\rho_0}\right) \eta \nabla_h \cdot \phi_j \rangle\rangle \\ & + \frac{g}{\rho_0} \langle\langle \eta \rho'_{\text{grad},\eta} \cdot \phi_j \rangle\rangle - \langle\langle\langle \left( \left(1 + \frac{\rho'^*|_{\eta}}{\rho_0}\right) \eta \mathbf{n}_h + \mathbf{f}_{\bar{\mathbf{u}}}^{LF} \right) \cdot \phi_j \rangle\rangle\rangle. \end{aligned} \quad (3.38)$$

Again,  $\rho'^*|_\eta$  is the mean between the right and left values, and the Lax-Friedrichs flux is  $\mathbf{f}_u^{LF} = \sqrt{gH^*}(\bar{\mathbf{u}}_L - \bar{\mathbf{u}}_R)/2$ .

Finally, to define the mass matrix  $M_{2D}$  in Equation 3.21, the weak formulation of the left hand side of Equation 3.19 is computed. But unlike in previous equations where the explicit interpolation of the nodal values were hidden for the sake of clarity, it is here detailed:

$$\left\langle\left\langle \frac{\partial \eta}{\partial t} \phi_j \right\rangle\right\rangle = \frac{\partial}{\partial t} \left\langle\left\langle \phi_i \phi_j \right\rangle\right\rangle \eta_i = M_{2D,\eta,ij} \frac{\partial \eta_i}{\partial t}. \quad (3.39)$$

### 3.3.4 Vertical velocity

The vertical velocity is obtained by integrating from bottom to top the continuity Equation 3.2. It is the 3D equivalent of the sea surface elevation in the 2D mode. It is important that when integrating the vertical velocity equation, the sea surface elevation equation is re-obtained. Consequently, the weak formulation of the continuity equation uses the same interface values and the same Lax-Friedrichs flux as Equation 3.37 and reads:

$$\begin{aligned} -\left\langle w \frac{\partial \phi_j}{\partial z} \right\rangle &= -\left\langle\left\langle w^d \phi_j n_z \right\rangle\right\rangle_h + \left\langle \mathbf{u} \cdot \nabla_h \phi_j \right\rangle \\ &\quad - \left\langle\left\langle \left( \mathbf{u}^* \cdot \mathbf{n}_h + \frac{f_\eta^{LF}}{H^*} \right) \phi_j \right\rangle\right\rangle_l - \left\langle\left\langle \mathbf{u}^* \cdot \mathbf{n}_h \phi_j \right\rangle\right\rangle_h, \end{aligned} \quad (3.40)$$

with  $w^d$  the value of the lower element.

An important comment must be pointed out. Since the stabilising Lax-Friedrichs flux is the same in the 3D continuity equation and the 2D sea surface elevation equation, one could think that recomputing  $\eta^{n+1}$  using the surface vertical velocity as it is done in Equation 3.25 should not be necessary. One could also think that  $\eta^{n+1}$  should be equal to  $\eta^*$ , but this is not the case. Indeed, to obtain this equality, the vertical integral of all the lateral fluxes must be equal to the lateral 2D fluxes. The 2D fluxes are computed using  $\eta^* = (\eta^L + \eta^R)/2$ , and this geometry cannot be re-obtained exactly in the 3D mesh. It is why  $\eta^{n+1}$  is recomputed using a L2 projection of Equation 3.25.

### 3.3.5 Tracer equations

In the following weak formulation of the temperature Equation 3.16, the right hand side, which corresponds to the diffusive terms, is neglected. A full description of the discretisation of those terms is given in Kärnä et al. (2013).

The weak formulation of the temperature equation reads then:

$$\begin{aligned}
\frac{d}{dt} \langle JT \phi_j \rangle &= \langle JT \mathbf{u} \cdot \nabla_h \phi_j \rangle + \left\langle \left\langle JT^* \left( \mathbf{u}^* \cdot \mathbf{n}_h + \frac{f_\eta^{LF}}{H^*} \right) \phi_j \right\rangle \right\rangle_l \\
&+ \left\langle \left\langle JT^* (\mathbf{u}^* \cdot \mathbf{n}_h) \phi_j \right\rangle \right\rangle_h - \left\langle JT (w - w_m) \frac{\partial \phi_j}{\partial z} \right\rangle \\
&+ \left\langle \left\langle JT^* (w^d - w_m^d) n_z \phi_j \right\rangle \right\rangle_h = 0,
\end{aligned} \tag{3.41}$$

where  $T^*$  is the temperature value on the upstream element of the interface and all the integrals are computed over the fixed domain  $\tilde{\Omega}$ .

Using the first order explicit Euler scheme, and integrating the right and side of Equation 3.29 at time  $t^+$ , Equation 3.41 reads:

$$\begin{aligned}
&\frac{\langle T^{n+1} \phi_j \rangle^{n+1} - \langle T^n \phi_j \rangle^n}{\Delta t} \\
&- \langle T^n \mathbf{u}^+ \cdot \nabla_h \phi_j \rangle^+ + \left\langle \left\langle T^{*,n} \left( \mathbf{u}^{*,+} \cdot \mathbf{n}_h + \frac{f_\eta^{LF}}{H^{*,+}} \right) \phi_j \right\rangle \right\rangle_l^+ \\
&+ \left\langle \left\langle T^{*,n} (\mathbf{u}^{*,+} \cdot \mathbf{n}_h) \phi_j \right\rangle \right\rangle_h^+ - \left\langle T^n (w^+ - w_m^+) \frac{\partial \phi_j}{\partial z} \right\rangle^+ \\
&+ \left\langle \left\langle T^{*,n} (w^{d,+} - w_m^{d,+}) n_z \phi_j \right\rangle \right\rangle_h^+ = 0.
\end{aligned} \tag{3.42}$$

In this case, the integrals are computed on the moving domain, with some integrals computed on the mesh at time step  $n$  while other are computed at time step  $n + 1$ , and  $+$ . In this case, the Jacobian  $J$  cancels. The mesh on which the integral is computed is denoted by the superscript sign after the integral closing bracket.

### 3.3.6 Moving mesh velocity and consistency

In Kärnä et al. (2013), the discrete moving mesh velocity  $w_m$  is obtained by interpolating Equation 3.13 at the mesh nodes. However, this approach breaks the tracer consistency. Here, the mesh velocity is constructed from the mesh new position in a way that ensures tracer consistency at a discrete level. The consistency of the method relies on the compatibility of the tracer Equation 3.41 with the continuity Equation 3.40. Multiplying Equation 3.40 by the Jacobian and inserting it in Equation 3.41 in which the temperature is assumed to be constant, the weak formulation of the moving mesh velocity is obtained:

$$\frac{d}{dt} \langle J \phi_j \rangle = - \left\langle J w_m \frac{\partial \phi_j}{\partial z} \right\rangle + \left\langle \left\langle J w_m^d n_z \phi_j \right\rangle \right\rangle_h. \tag{3.43}$$

Using the Euler time scheme, the same operation is done based on Equation 3.42, obtaining:

$$\frac{\langle \phi_j \rangle^{n+1} - \langle \phi_j \rangle^n}{\Delta t} = - \left\langle w_m^+ \frac{\partial \phi_j}{\partial z} \right\rangle^+ + \left\langle \left\langle w_m^{d,+} n_z \phi_j \right\rangle_h \right\rangle^+. \quad (3.44)$$

It is important to add that in the weak formulation of the continuity equation, all the fields are evaluated at time step +, and the integration is also computed on the mesh at this time step to be compatible with the temperature equation, such that the consistency is guaranteed.

Starting from the bottom boundary condition  $w_m|_{-h} = 0$ , the equality can be integrated element by element from bottom to top to obtain the moving mesh velocity. The first term of the left hand side is integrated on the new mesh. For more complex temporal schemes, the right hand side must be computed on the same meshes as for the tracer and continuity equations.

Using this method, the tracer equation reduces by construction to the continuity equation if the tracer is constant, and then the consistency property holds. The error in the tracer consistency is of the order of  $10^{-12}$ , 6 orders of magnitude better than for the previous moving mesh velocity formulation.

### 3.3.7 Internal pressure horizontal gradient computation

The computation of the internal pressure gradient is a classical problem of models using terrain following meshes. Since the iso- $\sigma$  surfaces are generally not horizontal, it is difficult to maintain a stratified flow in a domain with steep bottom slope. This problem has been extensively studied and documented (e.g. Haney, 1991; Deleersnijder and Beckers, 1992b; Stelling and Van Kester, 1994; Mellor et al., 1994, 1998).

The internal pressure gradient is defined as:

$$\frac{1}{\rho_0} \nabla_h p_{\text{int}} = \frac{g}{\rho_0} \nabla_h \underbrace{\int_z^\eta \rho' d\zeta}_r, \quad (3.45)$$

with  $r$  called the baroclinic head. While in  $z$ -coordinates on a  $z$ -level mesh the computation of the internal pressure horizontal gradient is trivial, it is not the case for  $\sigma$ -coordinates, defined as  $\sigma = (z - \eta)/H$ , for which two terms are involved:

$$\frac{1}{\rho_0} \nabla_h p_{\text{int}} = \frac{1}{\rho_0} \nabla_h^* p_{\text{int}} - \frac{\sigma}{\rho_0 H} \frac{\partial p_{\text{int}}}{\partial \sigma} \nabla_h^* H, \quad (3.46)$$

where  $\nabla_h^*$  is the horizontal gradient operator along a constant  $\sigma$ . While the left hand side of Equation 3.46 is equal to 0 in a stratified flow, both terms of the right hand side can be quite large. Considerable effort was made to reduce the

errors in the computation of both terms in finite difference schemes and so to limit the spurious pressure gradient (e.g. [Thiem and Berntsen, 2006](#); [Berntsen and Oey, 2010](#); [Berntsen et al., 2015](#)). For the case of vertically adaptive meshes, [Gräwe et al. \(2015\)](#) showed that the internal pressure gradient problem is reduced thanks to the horizontal smoothing of the mesh.

The pressure gradient formulation in SLIM 3D is different from the finite difference schemes. Even if the mesh is composed of  $\sigma$ -levels, the equations are in  $z$ -coordinates. The fact that the levels are not horizontal is handled by the finite element formulation. However, having steep slope also induces difficulties. The weak formulation of the horizontal gradient of a field  $f$  reads:

$$\left\langle \nabla_h f \cdot \phi_j \right\rangle = - \left\langle f \nabla_h \cdot \phi_j \right\rangle + \left\langle\left\langle f \phi_j \cdot \mathbf{n}_h \right\rangle\right\rangle_h + \left\langle\left\langle f \phi_j \cdot \mathbf{n}_h \right\rangle\right\rangle_l. \quad (3.47)$$

Since first order shape functions are used in the finite element formulation, only first order polynomials are interpolated exactly at the integration points in order to compute the integrals of the right hand side of Equation 3.47.

In the previous version of SLIM 3D ([Kärnä et al., 2013](#)), the internal pressure gradient was computed by first integrating the density deviation  $\rho'$ , and then its horizontal gradient, following Equation 3.45. But even for a linear  $\rho'$ ,  $r$  is a quadratic function that cannot be represented exactly. Those integration errors generate a spurious noise in the gradient. The new approach consists in computing the horizontal derivative before the vertical integration:

$$\frac{1}{\rho_0} \nabla_h p_{\text{int}} = \frac{g}{\rho_0} \underbrace{\int_z^\eta \nabla_h \rho' d\zeta}_{\mathbf{q}} + \frac{g}{\rho_0} \rho'|_\eta \nabla_h \eta. \quad (3.48)$$

The equation for  $\mathbf{q}$  reads then:

$$\frac{\partial \mathbf{q}}{\partial z} = \nabla_h \rho', \quad (3.49)$$

and is solved similarly to the continuity equation, but integrating from top to bottom:

$$\begin{aligned} - \left\langle\left\langle \mathbf{q}^u \cdot \phi n_z \right\rangle\right\rangle_{\text{top}} + \left\langle \mathbf{q} \cdot \frac{\partial \phi}{\partial z} \right\rangle &= \left\langle\left\langle \mathbf{q}^u \cdot \phi n_z \right\rangle\right\rangle_{\text{bot}} + \left\langle \rho' \nabla_h \cdot \phi \right\rangle \\ &\quad - \left\langle\left\langle \rho'^* \phi \cdot \mathbf{n}_h \right\rangle\right\rangle_l - \left\langle\left\langle \rho'^u \phi \cdot \mathbf{n}_h \right\rangle\right\rangle_h, \end{aligned} \quad (3.50)$$

where  $\rho'^*$  is the mean of the values of  $\rho'$  on both sides of the interface, and  $\mathbf{q}^u$  is the value on the upper element of the interface.

Using Equation 3.50, the computation of the gradient of a linearly stratified sea does not involve a quadratic field, and the computed internal pressure gradient is zero, up to the machine accuracy (Fig. 3.6).

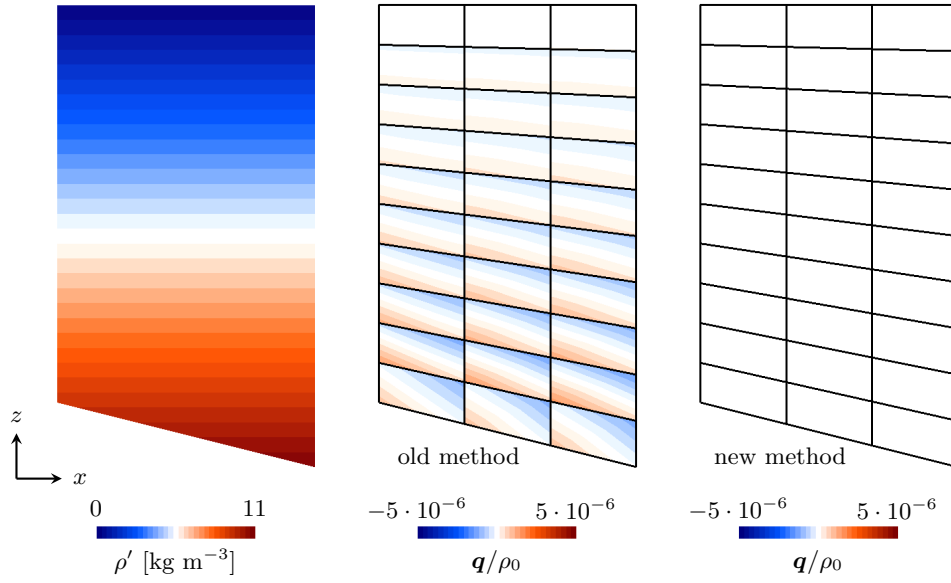


Fig. 3.6: Comparison between the baroclinic pressure gradient computed with the old method (from SLIM 3D previous version) and the current method, in a linearly stratified water column. With the new version, the pressure gradient error is of the order of the machine error.

### 3.3.8 $z$ -level meshes

As explained hereinabove, SLIM 3D solves the different equations using Cartesian coordinates. Even if in the previous version of the code, all the simulations were done using  $\sigma$ -meshes, with a constant number of levels, this is not a fundamental prerequisite for the model. However, to allow the usage of  $z$ -level meshes, some special care must be taken with the boundary at the vertical walls created where the bathymetry is not constant. This section explains what is done at those vertical walls, called “vertical bottoms”.

The real problem is not only located at those walls, it is a fundamental problem of SLIM 3D. The equations solved by the model are hydrostatic, which means that the only pressure taken into account in the model is the pressure of the water column mass. But physically, if the current pushes against a wall, it generates a pressure against this wall, which in reaction applies some pressure in the opposite direction. But the model does not solve this pressure equation. Two situations can occur:

- A vertically well-mixed current pushes the water towards the wall. Then the sea level increases close to the wall and the sea level gradient generates a pressure in the opposite direction. Then there is no fundamental problem with the 2D part of the model.

- But if on one half of the vertical column, the water is going towards the wall and on the other half it is going in the opposite direction, the vertically averaged current is null, and no sea surface elevation gradient is created. There is then potentially a shock against the wall. Indeed, the model only sees the wall at the nodes located there. The other nodes do not see the pressure at all, and this creates a shock in the element touching the wall. This problem cannot be solved increasing the mesh resolution; it would be even worse, since the limit layer, with a thickness of one element, will be even smaller. Additional diffusion helps, but it does not fix the problem.

This is particularly a problem for “vertical bottoms”, the vertical walls created by the  $z$ -level meshes, as illustrated in Figure 3.7. The velocity normal to the wall should be null at those boundaries. Thanks to the boundary condition, the velocity is weakly forced to be zero. But this is not enough to prevent from instabilities. Two features are implemented:

- For the computations between the 2D velocity  $\bar{\mathbf{u}}$  and 3D velocity  $\mathbf{u}$ , the velocity normal to the wall is assumed to be null. In practice, it means that the component of  $\bar{\mathbf{u}}$  in the direction normal to the vertical bottom is integrated only above the vertical bottom, and that  $\mathbf{u}^+$  is computed by adding the 2D velocity only above the vertical bottom.
- It is currently necessary to force strongly the velocity to be null at the vertical bottoms, after the computation of the horizontal momentum equation. This forcing is done before the computation of the vertical velocity, in order not to break the water mass conservation. For some applications with a strong Coriolis effect, forcing to zero the velocity tangent to the vertical bottom increases the stability.

### 3.4 Periodic meshes and hexahedral elements

Neither theoretical development nor detailed explanation about the implementation of the periodic meshes and hexahedral elements is given in this section. Hence, what is the point of this section? The idea is simply to highlight the importance of periodic meshes and hexahedral elements for the development of a 3D code.

Many issues occur during the development of a complex code such as SLIM 3D, including stability problems due to the internal pressure gradient, the vertical velocity or the wall boundaries. Those problems are usually difficult to address and then to solve on complex actual geometries and should be evaluated on simpler ones. For most of them, the use of a 2D “ $x$ - $z$ ” model, with

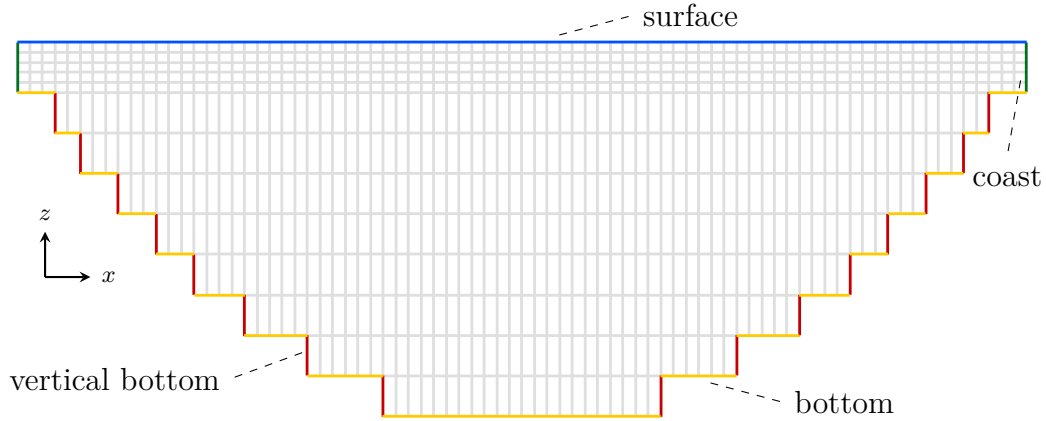


Fig. 3.7: Vertical cut of a  $z$ -level mesh. The vertical bottom boundary is drawn in red.

one horizontal and one vertical dimension, is much easier to: (1) implement a potential solution, (2) run it fast and (3) evaluate easily its effectiveness.

However, the first goal is still to develop the 3D code. Developing a complete 2D “ $x$ - $z$ ” model is not a solution. Rather than generalizing the 3D model to reduce the horizontal direction to a single dimension, the choice was made to shrink the third  $y$ -direction to one single element. The use of prisms, i.e. extruded triangles, to compose the mesh is not good anymore, since it produces asymmetrical behaviour in the  $y$ -direction. The code is then modified to be able to deal with extruded quads, forming hexahedra. Those modifications are simple technical changes in the code.

The “ $x$ - $z$ ” model does not necessarily imply that no circulation occurs in the  $y$ -direction. It only requires that the flow is homogeneous in this direction. In particular, if the model would be composed of one single element in the  $y$ -direction located between two walls, test cases including a Coriolis effect could not be evaluated. Periodic meshes were then developed for such cases. The development of such meshes is a technical task, including the three following steps: (1) to build a mesh such that the nodes and elements are located at the same place on the two interfaces to connect, (2) to find the mapping between the elements and nodes of the two interfaces, (3) to reconnect the interfaces.

The development of periodic meshes enables the simulation of many applications such as coastal upwelling (on  $x$ - $z$  model) or reentrant channel (on a 3D mesh). It also helps to remove noise in many  $x$ - $z$  applications for which the model is sensitive to waves reflecting on the two walls perpendicular to the  $y$ -direction. It is why periodic meshes are systematically used for all the  $x$ - $z$  applications.

## 3.5 Vertical adaptive levels

### 3.5.1 A model with vertical columns

SLIM 3D has been developed using a finite element method, enabling the use of unstructured meshes. This geometrical flexibility gives many advantages (Hanert et al., 2004). On the one hand, it allows the model to represent in detail complex topographies such as narrow straits, small islands or reefs. On the other hand, it gives the possibility to increase naturally the resolution in regions of interest.

However, the mesh is only unstructured in the horizontal direction. The vertical direction is composed of vertically aligned prisms (if the 2D mesh is composed of triangles) or hexahedra (extruded from 2D quads). Structured columns in the vertical direction do not necessarily mean there is no freedom to adjust the vertical resolution. The thickness of prisms can still be adjusted, to increase the vertical resolution, as it is illustrated in Figure 3.8 for a simple river plume test case. In this application, freshwater is pushed in at  $1 \text{ m s}^{-1}$  at the left boundary of the domain, which is initially filled with salt water. The test case compares simulations with a coarse mesh with 5  $\sigma$ -levels, another coarse mesh with 5 levels, but with an increased resolution close to the surface, and a reference fine mesh with 20  $\sigma$ -levels. The results after 15 hours show that for the first mesh, the lack of vertical resolution at the surface not only reduces the vertical accuracy but completely affects the plume dynamics. In this case, the plume is thicker and consequently moves more slowly than expected. The plume front is also much more diffused horizontally. A change of resolution in the vertical affects then all the directions.

### 3.5.2 Adapting the mesh

As aforementioned, designing a mesh adapted to the application enhances substantially the modelling quality. But for various applications, an a priori mesh cannot be designed. Moreover, a good mesh should be able to evolve according to some criteria. Mesh adaptation is not a new field in ocean modelling using finite element methods. While h-adaptive methods (Bernard et al., 2007) adapt the mesh size during the simulation, p-adaptive methods adapt the space discretisation order. Both methods can be combined to form hp-adaptive methods (Eskilsson, 2011; Blaise and St-Cyr, 2012). However, those methods have a non-negligible cost, such as re-meshing, mesh topology modification or projection of the solution field from the old mesh to the new one. While those h- and p- adaptations have shown good results for the aforementioned studies, the efficiency of such methods for 3D models is still an open question.

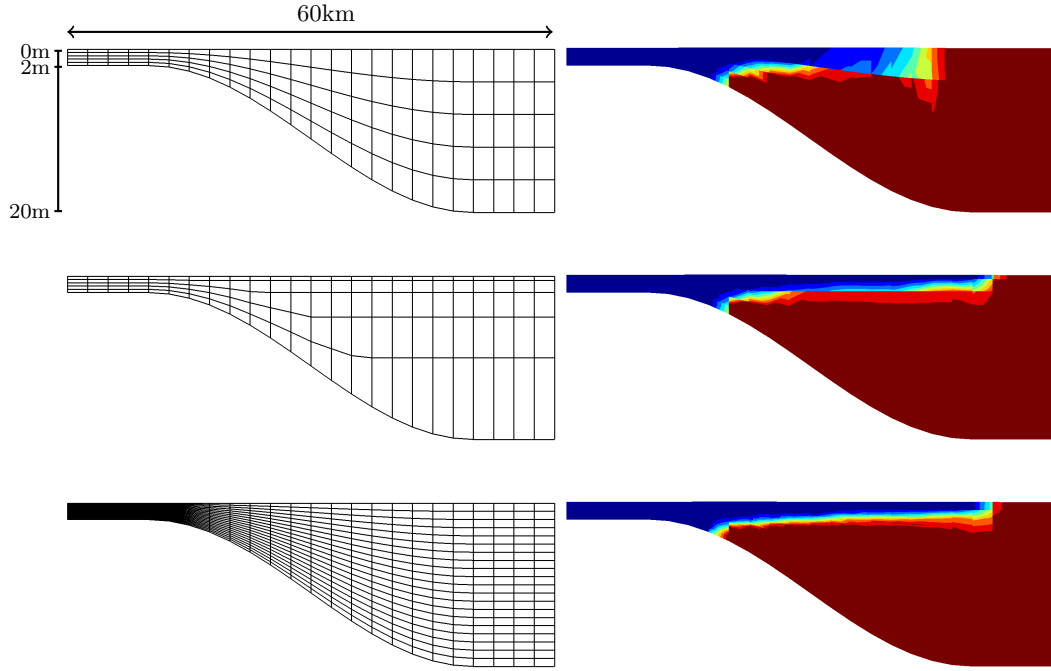


Fig. 3.8: Comparison between three different vertical resolution meshes to simulate a river plume dynamics in a simple 2D “ $x$ - $z$ ” channel.

Another solution is to adapt the mesh only vertically, by means of r-adaptivity. The principle of vertical r-adaptivity is to move the mesh nodes in the vertical direction, without affecting the mesh topology. This method has less freedom than previous ones, but it has the main advantage of not increasing the computational cost of the model. Indeed, the mesh does already move vertically to adapt to the free surface elevation, and this adaptation is taken into account with the ALE formulation. In SLIM 3D, the mesh moves then in the vertical direction for two reasons: to follow the free surface ([Campin et al., 2004](#)) and to obtain the desired vertical resolution.

To obtain accurate results at a reasonable computational cost, the mesh vertical resolution should be high in areas with strong stratification or shear, and low elsewhere. In the current version of the model, the refinement is achieved as a function of the stratification only. The shear is thus ignored, but it could be taken into account similarly to what is developed hereinafter for the stratification. The mesh velocity from Equation 3.13 reads:

$$w_m = \frac{h + z}{h + \eta} \frac{\partial \eta}{\partial t} + w^*. \quad (3.51)$$

The first term of the right hand side is the mesh velocity due to the free surface movement while the second term is the mesh adaptive velocity.

The mesh resolution variation results from a diffusion process of the Eulerian vertical coordinate  $z(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{t})$ . In contrast to [Burchard and Beckers](#)

(2004) and Hofmeister et al. (2010), the diffusion process is not defined in the continuous domain but directly on the discrete mesh. The mesh is made up of vertically extruded triangles or quads to form columns of prisms. As a consequence, the mesh also consists of vertical columns of nodes, which are the vertices of the prisms. Those nodes are connected by vertical segments. For each column composed of  $n$  vertical segments, the nodes are labelled with an index  $i$  varying between 0 at the bottom and  $n$  at the top. Segment  $i + 1/2$  joins nodes  $i$  and  $i + 1$ . The objective of the mesh adaptation is to distribute the nodes of a column such that:

$$h_{i+1/2} e_{i+1/2} = \text{constant}, \quad (3.52)$$

where  $h_{i+1/2} = (z_{i+1} - z_i)$  is the segment height and  $e_{i+1/2}$  is a relevant measure of the segment error. A finite difference diffusion equation is implemented:

$$\begin{cases} w_i^* &= \kappa_{z\ i+1/2} (z_{i+1} - z_i) - \kappa_{z\ i-1/2} (z_i - z_{i-1}), \quad \forall i = 1 \dots n-1, \\ w_0^* &= w_n^* = 0. \end{cases} \quad (3.53)$$

The mesh “diffusivity”  $\kappa_z$  (which has the physical unit  $\text{s}^{-1}$ ) is then defined by:

$$\kappa_{z\ i+1/2} = \frac{e_{i+1/2}^2 + f_{e\ i+1/2}}{\tau_e}. \quad (3.54)$$

The dependency in  $e^2$  ensures that, at a given level, if the upper segment error is larger than the lower segment error, the diffusion process will tend to reduce the upper segment size, and vice versa. The background diffusivity  $f_e/\tau_e$  allows for manual control of the mesh resolution independently of the discretisation error. The time  $\tau_e$  is a relaxation parameter controlling the speed of the adaptivity process.

The main difference between the original approach and the implementation in SLIM 3D is the definition of the error density  $e^2$ . In Hofmeister et al. (2010), a function of shear and stratification is used. In DG finite element methods, the discretisation error converges at the same rate as the inter-element discontinuities (jumps) of the solution (Ainsworth, 2004; Bernard et al., 2007). The error density is then defined as a function of the vertical jumps:

$$e_{i+1/2}^2 = \frac{[\rho]_i^2 + [\rho]_{i+1}^2}{(\Delta\rho)^2}, \quad (3.55)$$

where  $[\rho]_i$  refers to the maximum jump, between all the upper DG values adjacent to continuous node  $i$  and lower DG values adjacent to this same node.

This diffusion algorithm is valid for meshes with both  $\sigma$ - and  $z$ -levels. On a mesh with  $\sigma$ -levels, the number of levels is constant over the entire mesh and the bathymetry is continuous (Fig. 3.9a). On a mesh with  $z$ -levels, the number of levels is not constant and the bathymetry is discontinuous (Fig. 3.9b).

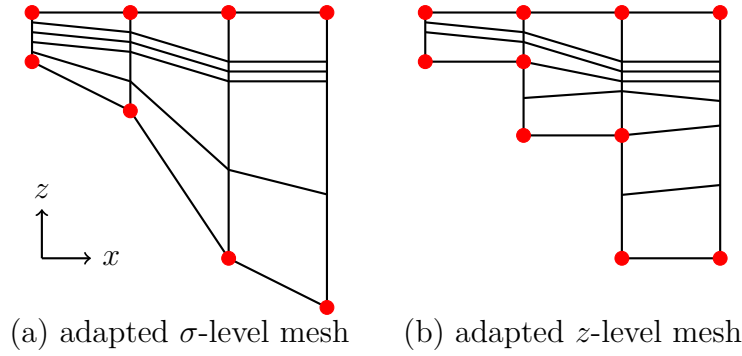


Fig. 3.9: Comparison between an adapted  $\sigma$ -level mesh (a) and an adapted  $z$ -level mesh (b). The nodes marked with a red dot are fixed during the mesh vertical diffusion process, which is independent of the movement necessary to accommodate to the motion of the free surface (Eq. 3.51).

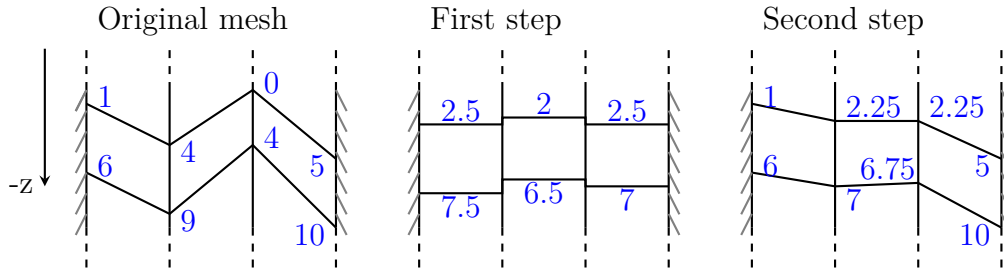


Fig. 3.10: Illustration of the horizontal smoothing algorithm on a 2D “ $x$ - $z$ ” mesh. First, for each element, the upper and lower nodal values of the vertical coordinate  $z$  are set to their mean value, then  $z$  is projected onto a continuous field.

The discrete mesh is updated by interpolating the function defined in Equation 3.51 at the element vertices. Then the  $z$ -coordinates of the vertices are smoothed in the horizontal direction, except on the mesh lateral boundaries. This smoothing is achieved with a simple 2-step algorithm (Fig. 3.10). First, for each element of the mesh, the  $z$ -coordinates of the upper and lower nodes are set to the mean value of those upper and lower nodes, respectively. The  $z$  field is now discontinuous. Second,  $z$  is projected onto a continuous field, such that the mean value of every vertex is preserved. This simple algorithm smooths the  $z$ -coordinates in the horizontal direction. Eventually,  $z$  is corrected such that all the elements have a thickness between appropriate minimal and maximal values. This correction is achieved by a double loop over each segment column. First, looping from bottom to top, the top node of each segment is moved if necessary. Second, looping from top to bottom, the bottom node of each segment is moved.

## 3.6 Anisotropic slope limiters

Slope limiters were introduced to prevent local maxima and minima from increasing and decreasing in time, respectively (Kärnä et al., 2013). The principle of such limiters is simple: the solution on all the nodes is shifted from its original value towards a combination between this value and the element mean solution, such that all nodal values are within a specified range. While the use of limiters was necessary in the earlier versions of the SLIM 3D code to maintain stability, the current version is more stable and the limiter is mainly used for its original purpose: preventing overshoots, although it still helps for stability, enabling the use of larger time steps.

The specified range in which the solution is limited can be defined in different ways. In the classical formulation of Cockburn and Shu (1998), the (discontinuous) nodal values from an element have to remain between the minimum and the maximum of the mean values of the elements adjacent to this element (Fig. 3.11a). In the formulation of Kuzmin (2010) and Aizinger (2011), the values of the discontinuous nodes mapping to the same continuous nodes remain between the minimum and the maximum of the mean values of the elements adjacent to this continuous node (Fig. 3.11b).

Different methods exist to ensure this limitation: the single parameter limiter, the advanced limiter and the anisotropic limiter. Kärnä et al. (2013) was using the first two methods with the minimum and maximum values defined by Kuzmin (2010) and Aizinger (2011). Here, the third algorithm is developed, using the minimum and maximum values of Cockburn and Shu (1998). This section summarizes the different methods available.

### 3.6.1 Minimum/maximum values

The two extrema methods are illustrated on Figure 3.11 and defined here. Given a scalar field  $T$ , the nodal values on element  $K$  are defined  $T_i^K$ , and the mean value of  $T$  on element  $K$  is defined as

$$\bar{T}^K = \frac{\sum_i V_i^K T_i^K}{\sum_i V_i^K},$$

with  $V_i^K$  the nodal volumes defined as  $V_i^K = \langle \phi_i \rangle$ .

#### Cockburn and Shu limiter

Let define  $\mathcal{P}_{\partial K}$  the subset of  $\mathcal{P}$  containing all the elements adjacent to  $K$ . The minimum and maximum value for node  $i$  of element  $K$  are:

$$T_{i,K}^{\text{Min}} = \min_{K' \in \mathcal{P}_{\partial K}} \bar{T}^{K'}, \quad T_{i,K}^{\text{Max}} = \max_{K' \in \mathcal{P}_{\partial K}} \bar{T}^{K'}. \quad (3.56)$$

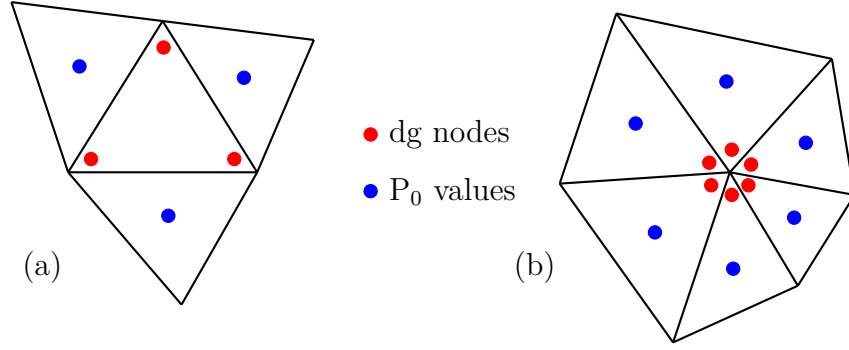


Fig. 3.11: Slope limiter extrema from Cockburn and Shu (1998) (a) and from Kuzmin (2010) and Aizinger (2011) (b). The nodal values (red dots) are limited to be between the minimum and the maximum of the adjacent mean values (blue dots).

It is noteworthy that the minimum and maximum values are the same for all the nodes of a single element.

### Kuzmin and Aizinger limiter

Let define  $\mathcal{P}_{\chi(i,K)}$  the subset of  $\mathcal{P}$  containing all the elements whose one of the nodes is the same as the  $i^{\text{th}}$  node of element  $K$  in the continuous mesh. The minimum and maximum allowed values are defined as:

$$T_{i,K}^{\text{Min}} = \min_{K' \in \mathcal{P}_{\chi(i,K)}} \bar{T}^{K'}, \quad T_{i,K}^{\text{Max}} = \max_{K' \in \mathcal{P}_{\chi(i,K)}} \bar{T}^{K'}. \quad (3.57)$$

This ensures that all the discontinuous nodes mapping to the same node of the mesh share the same minimum and maximum values.

### 3.6.2 Limiter routine

With the minimum and maximum values defined hereinabove, the limiter goal is to define the limited values  $\tilde{T}_i^K$ , such that

$$T_{i,K}^{\text{Min}} \leq \tilde{T}_i^K \leq T_{i,K}^{\text{Max}} \quad (3.58)$$

$$\tilde{\bar{T}}^K = \bar{T}^K. \quad (3.59)$$

Figure. 3.12 illustrates the limiter principle in a 1D case.

### Single-parameter limiter

The simplest choice for this limitation is to control each element with a single parameter  $\lambda$  using the convex combination between the original value and the element mean value (Kuzmin, 2010):

$$\tilde{T}_i^K = \lambda T_i^K + (1 - \lambda) \bar{T}^K, \quad (3.60)$$

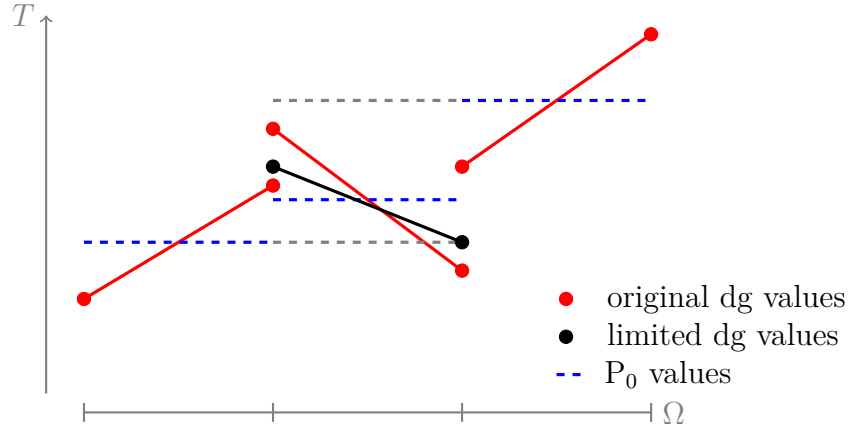


Fig. 3.12: The inner element is limited in order to stay between the minimum and the maximum of the mean values of its neighbourhood.

with  $\lambda \in [0, 1]$  defined per element. This method is mass conservative by construction. While  $\lambda = 1$  keeps the original values,  $\lambda = 0$  sets the tracer value constant on the element. To limit as little as possible the solution but still hold the constraints from Equation 3.58,  $\lambda$  is defined as:

$$\lambda = \max(0, \min(1, \min_i \lambda_i)) \quad (3.61)$$

$$\lambda_i = \begin{cases} \frac{T_{i,K}^{\text{Max}} - \bar{T}^K}{T_i^K - \bar{T}^K} & \text{if } T_i^K > T_{i,K}^{\text{Max}} \\ \frac{\bar{T}^K - T_{i,K}^{\text{Min}}}{\bar{T}^K - T_i^K} & \text{if } T_i^K < T_{i,K}^{\text{Min}} \\ 1 & \sim \end{cases} \quad (3.62)$$

### Advanced limiter

The second limiter was introduced by Kärnä et al. (2013). Its goal is to not change all the nodal values of the element with one single controlled parameter, but to minimise the nodal value modification.  $\tilde{T}_i^K$  is the solution of a local optimization problem:

$$\underset{\tilde{T}_l^K}{\text{minimize}} \quad \sum_l \|\tilde{T}_l^K - T_l^K\|^2, \quad (3.63)$$

under the constraints from Equations 3.58 and 3.59.

### Anisotropic limiter

The single-parameter and the advanced limiters were defined above in a general framework. However, in SLIM 3D, the limiters are only used on 3D fields: the horizontal velocity and the tracers. The anisotropic limiter can be applied only

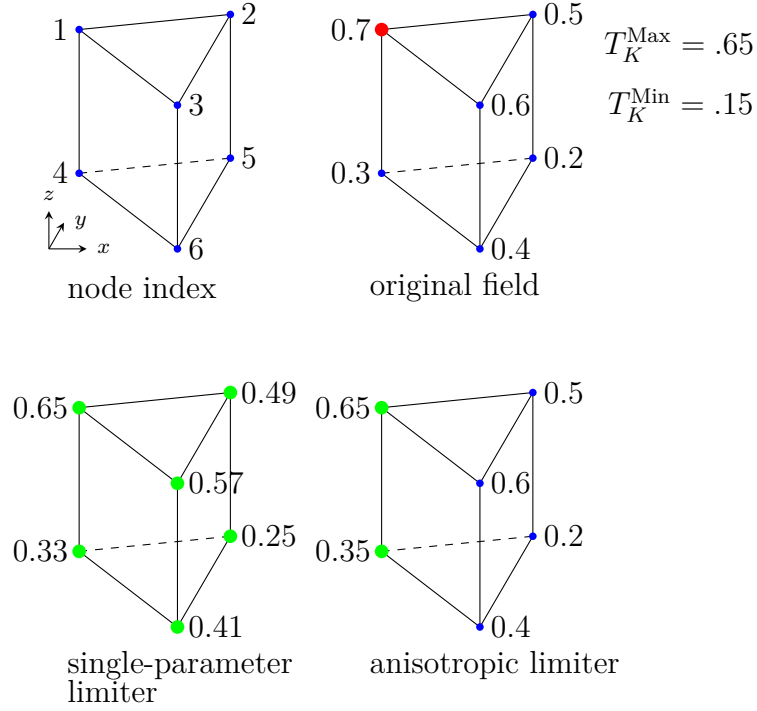


Fig. 3.13: Comparison between the single-parameter limiter (bottom left) and the anisotropic limiter (bottom right). It is observed that the original field (top right) is only modified on two nodes for the anisotropic limiter, while it is affecting all the nodes for the single-parameter limiter.

in this configuration. The idea is to simply limit first in the vertical direction before limiting globally on the element. This limiter is compared to the single-parameter limiter in Figure 3.13. On this figure,  $T_K^{\max}$  and  $T_K^{\min}$  are obtained by the Cockburn and Shu (1998) method and is constant on the element. The first node is slightly larger than  $T_K^{\max}$ . While the single-parameter limiter affects all the nodes to correct this overshoot, the anisotropic limiter only affects the other single node on the vertical. The limiter is then simply a triple-1D vertical limiter, defined as:

$$\tilde{T}_i^{K*} = \lambda_i T_i^K + (1 - \lambda_i) \frac{T_i^K + T_{i+3}^K}{2}, \quad i = 1, 2, 3, \quad (3.64)$$

$$\tilde{T}_{i+3}^{K*} = \lambda_i T_{i+3}^K + (1 - \lambda_i) \frac{T_i^K + T_{i+3}^K}{2}, \quad i = 1, 2, 3, \quad (3.65)$$

with the  $\lambda_i$  defined similarly as  $\lambda$  in Equations 3.61 and 3.62. The  $\tilde{T}_i^{K*}$  limited values are not necessarily within the allowed range. It is why they are limited another time using the single-parameter limiter, this second limiting operation being inactive in many cases.

This limiter was developed and implemented in a previous version of the code, somewhere in between the version Kärnä et al. (2013) and the actual

version described in this chapter. In the modelling of a river plume, it was observed that this limiter was less altering the original field than the other limiters. However, in the actual version of the code, which is more stable, the three limiters have a lower impact on the solution and limit it in a similar way. While Figure 3.13 illustrates the advantage of the anisotropic limiter, this limiter does not induce better results in real applications using the current version of the model.

## 3.7 Conclusions

The main numerical features of the model, which are the DG finite element formulation and the split between 2D and 3D modes, were developed before the start of this PhD thesis and were conserved in the model. The numerical development aimed to improve the accuracy and stability of SLIM 3D in order to use it on realistic applications.

Modifications were achieved in three categories: the continuous equations, the discrete equations and the development of new features. Those developments were carried out with a precise objective. The development of quad meshes and periodic meshes enabled to run simulations in an exact 2D “ $x$ - $z$ ” configuration. Those simple runs greatly simplified the analysis of the consequences of any update of the model. The new formulation of the internal pressure gradient and the  $z$ -levels increased the model stability, making possible complex simulations such as the one of Burdekin River mouth dynamics (see Chapter 4). The development of vertical adaptive coordinates and the consistent moving mesh enabled to reduce drastically the number of elements and the simulation cost for some situations (see Chapter 5).

Although this chapter is mostly descriptive and does not contain a lot of results, it is the necessary basis for the results presented in the next chapters. A large part of this thesis was dedicated to the development of the model numerics, and made the simulation of the following complex flow dynamics possible.



## The transport and fate of riverine fine sediment exported to a semi-open system

This chapter reproduces the following article<sup>1</sup>:

**P. Delandmeter**, S. Lewis, J. Lambrechts, E. Deleersnijder, V. Legat and E. Wolanski, 2015, The transport and fate of riverine fine sediment exported to a semi-open system, *Estuarine, Coastal and Shelf Science*, 167, 336 – 346  
doi: [10.1016/j.ecss.2015.10.011](https://doi.org/10.1016/j.ecss.2015.10.011)

Understanding the transport and fate of suspended sediment exported by rivers is crucial for the management of sensitive marine ecosystems. Sediment transport and fate can vary considerably depending on the geophysical characteristics of the coastal environment. Fine sediment transport is studied in a setting in between “open” (uninterrupted coasts) and “semi-enclosed” (bays) coastal systems, namely a “semi-open” system of shallow coastal water with long ( $\sim 20$  km) stretches of open coasts separated by capes and headlands. The case study is the large, seasonal Burdekin River that discharges to a wide continental shelf containing headlands and shallow embayments adjacent to the Great Barrier Reef, Australia.

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<sup>1</sup>Only minor modifications were made.

For this study, a new three-dimensional fine sediment module for the unstructured-mesh SLIM 3D hydrodynamic model was developed. The complex topography and bathymetry of this semi-open system was a real challenge for SLIM 3D. It is for this application that the new pressure gradient computation (defined in Section 3.3.7) and the anisotropic limiter (defined in Section 3.6) were developed.

In this chapter, the model is successfully validated against available field data. The results are compared to previous studies on the Burdekin River sediment transport and differences are analysed. The relatively small quantity of validation data does not allow to draw accurate conclusions on the exact path of the sediment, but conclusions on the most important processes of the sediment transport and the main patterns can be drawn.

Wind direction and speed during river floods largely control the dynamics and the fate of the fine sediment. Most of the riverine fine sediment load is deposited near the river mouth; the remaining sediment is transported further afield in a riverine freshwater plume; that sediment can reach sensitive marine ecosystems and should be a priority for management. During the rest of the year, when the river flow has ceased, wind-driven resuspension events redistribute the deposited sediment within embayments but generate negligible longshore transport. This study suggests that semi-open systems trap most of the riverine fine sediment, somewhat like semi-enclosed systems.

## 4.1 Introduction

Increased loads of riverine suspended sediment due to watershed development (Syvitski et al., 2005; Milliman and Farnsworth, 2011) cause considerable impacts on sensitive marine ecosystems including coral reefs (Fabricius, 2005; Bartley et al., 2014) and seagrass meadows (Restrepo et al., 2006). These impacts include sedimentation (Weber et al., 2012; Flores et al., 2012; Cabaço et al., 2008), elevated turbidity reducing photic depth (Fabricius et al., 2005, 2013, 2014; Petus et al., 2014), and stressors associated with pollutants attached to sediment such as nutrients, pesticides and trace metals (Weber et al., 2006; Harrington et al., 2005).

The understanding of the delivery, transport and final fate of terrestrial sediment is crucial to predict the impacts of sediment on marine ecosystems. The fate of riverine sediment exported to the ocean depends on the width and depth of the continental shelf, the presence of an offshore canyon and the geomorphology of the coastline. The different regions can be differentiated between open and semi-enclosed systems. In open systems the exported river sediment moves long distances (hundreds to thousands of kilometres) along or across the continental shelf. In these cases, the fate of the sediment exported from the river is influenced by the offshore bathymetry. For example, the Eel (Puig et al., 2003, California) and Sepik (Kuehl et al., 2004, Papua New Guinea) Rivers discharge sediment across a narrow and steep shelf, where this sediment is then transported through a canyon to the deep ocean. In contrast, the Yangtze (Liu et al., 2007), Amazon (Froidefond et al., 1988) and Mekong (Xue et al., 2010) Rivers discharge to a wide and shallow shelf where the sediment is transported longshore by coastal currents and is largely deposited close to the coastline, but far from the river mouth. For semi-enclosed systems such as the La Sa Fua River (Wolanski et al., 2003b, Guam), nearly 100% of the river sediment is deposited into a sheltered embayment near the mouth, although large storms (i.e. typhoons) can flush sediment out of the bay. Somewhere in between open and semi-enclosed systems, one defines semi-open systems, where the open coast is cut by numerous capes and headlands. The bays are more open to the sea than in semi-enclosed systems. There is consequently more interaction between the bays and the open sea. But the capes and headlands prevent coastal currents from transporting the sediment over long distances. An example of such a system is the Great Barrier Reef (GBR), where riverine sediment is discharged to a wide and shallow continental shelf containing numerous headlands and embayments. The sediment transport and fate in such a system are less well understood.

The Burdekin River (Fig. 4.1) is the largest supplier of suspended sediment to the Great Barrier Reef off north-eastern Australia with an average annual contribution of approximatively 4 Mt (1 Mt is  $10^9$  kg) per year, or 28% of the total sediment export to the GBR (Kroon et al., 2012). Burdekin River

sediment loads have increased by a factor of 5 to 10 since European settlement ( $\sim 1850$ ) (Bartley et al., 2014). The annual loads are highly variable from year to year, due to the rainfall variability in its dry tropical watershed, with a range from  $4 \cdot 10^{-3}$  to 15 Mt per year, between 1987 and 2010 (Kuhnert et al., 2012). A recent study found a significant relationship between Burdekin discharge, total phosphorus and suspended sediment loads with photic depth in Cleveland Bay (Fabricius et al., 2014), implying that the river has a considerable influence on the health and biodiversity of coral reefs (Roff et al., 2013; Clark et al., 2014) and seagrass meadows (Devlin et al., 2013; Petus et al., 2014) in the region. Indeed, several studies have shown that land use change and climatic variability in the Burdekin catchment are recorded in coral cores (Isdale et al., 1998; McCulloch et al., 2003; Hendy et al., 2003; Lewis et al., 2007; Lough, 2007).

The vast majority of suspended sediment exported from the Burdekin River is delivered during elevated flow conditions of short duration (i.e. commonly a few weeks) which coincide with NE Australia's summer wet season (December to April). These elevated Burdekin River flows form freshwater plumes in the GBR lagoon which can be traced hundreds of kilometres from the river mouth (King et al., 2001; Devlin and Brodie, 2005). While the movements of freshwater and suspended sediment in Burdekin flood plumes have been well documented (Wolanski and Jones, 1981; Wolanski and van Senden, 1983; Devlin and Brodie, 2005; Bainbridge et al., 2012), the fate of the newly delivered suspended sediment is disputed.

Sediment budgeting work initially suggested that the majority of suspended sediment loads ( $\sim 90\%$ ) delivered from the Burdekin River was primarily deposited close to the river mouth (Devlin and Brodie, 2005; Bainbridge et al., 2012) before being resuspended and moved hundreds of kilometres northward and longshore with the prevailing coastal wind-driven currents into Bowling Green, Cleveland and Halifax Bays (Woolfe and Larcombe, 1998; Orpin et al., 2004). However, recent research using newly collected and dated sediment cores has challenged this hypothesis and concluded that "very little" sediment delivered from the Burdekin River moves past Upstart Bay (Lewis et al., 2014). Under this scenario, most of the sediment transport outside Upstart Bay occurs during times of high river discharge. These conflicting conclusions considerably influence the management strategies for controlling sediment erosion in the Burdekin watershed as the emphasis for catchment prioritisation needs to be placed on the sediment that moves longer distances and has the potential to impact sensitive marine ecosystems (Bartley et al., 2014). Specifically, if the sediment that is transported past Upstart Bay occurs only during large river flow events, then the subsequent long shore drift process becomes unimportant, and management should target the fine ( $< 15.6 \mu\text{m}$ ) organic rich material that travels large distances in Burdekin River flood plumes (Bainbridge et al., 2012). Hence the key research questions are as follows: where is

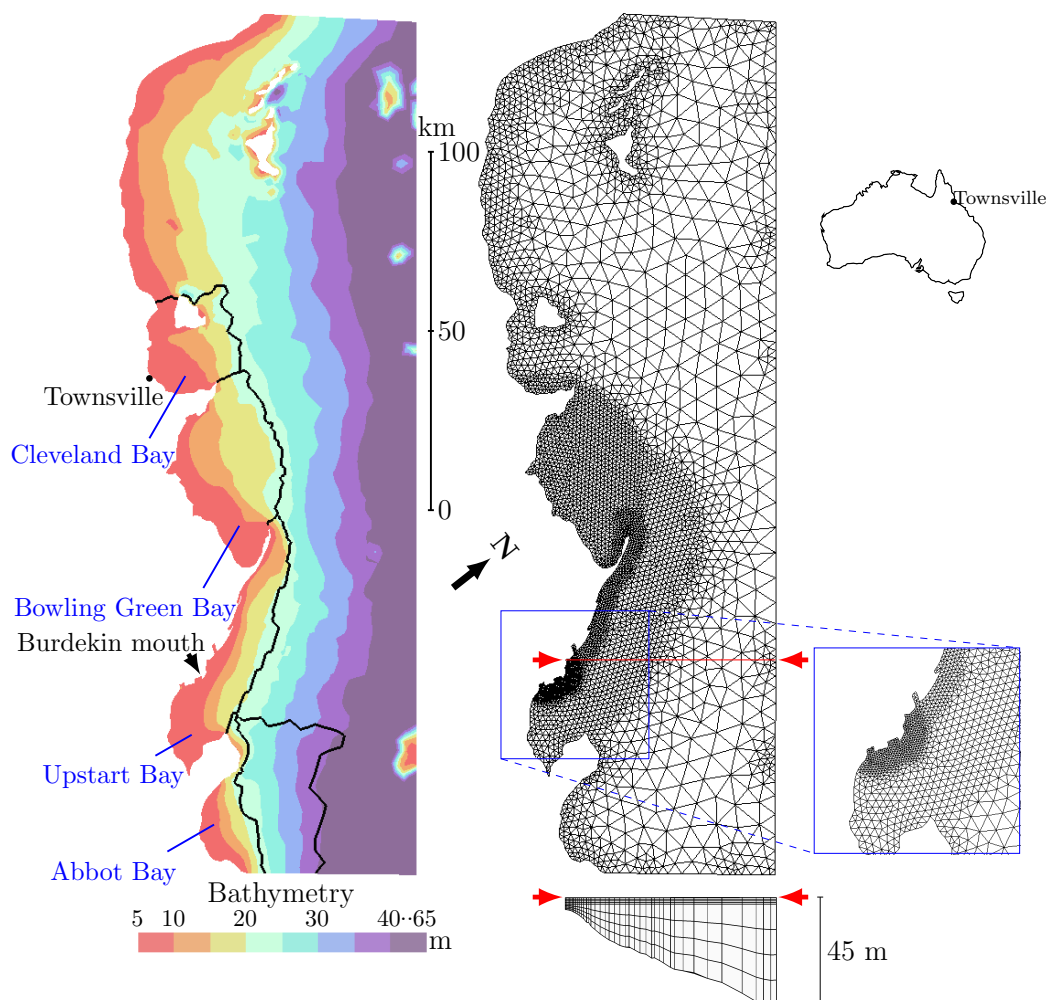


Fig. 4.1: Burdekin Region. Left: In order to compute sediment budgets, five zones were defined: Upstart, Bowling Green, Cleveland and Abbot Bays, and the middle-shelf offshore Abbot Bay. Right: Unstructured Mesh used by SLIM 3D: top view (top) and vertical cut (bottom). The mesh was discretised into  $\sim 8,000$  horizontal elements, whose size varies between 500 m and 7 km, and extruded in 8 vertical levels. The mesh horizontal resolution is increased close to the coastline, in shallow waters, and in Upstart and Bowling Green Bays. The first four top vertical levels have a constant depth, the others are adapted to the bathymetry as  $\sigma$ -levels.

the suspended sediment discharged from the Burdekin River deposited in the marine environment and when does this occur?

This study used numerical modelling to examine the delivery, transport and fate of suspended sediment delivered from the Burdekin in the 2007 calendar year. Section 4.2 describes the numerical model used in this study: SLIM 3D, and details the different data used. Section 4.3 validates the model, explains the sediment transport process and gives the budget of the sediment delivered by the Burdekin River. These results are discussed in Section 4.4

before concluding in Section 4.5.

## 4.2 Methods

### 4.2.1 Numerical modelling

SLIM 3D is a baroclinic discontinuous Galerkin finite element model for coastal flows (Kärnä et al., 2013; Blaise et al., 2010; Comblen et al., 2010). This model enables the use of unstructured grids, which greatly enhances the quality of simulations in the case of a domain with complex topography and large velocity and density variations, such as the Burdekin region. SLIM 3D has already been applied to the Rhine River plume, in an idealised set up (Kärnä et al., 2013). However, it has never been used in such a complex region. A 2D sediment module coupled with the 2D version of SLIM has already been used in many studies (Lambrechts et al., 2010; Gourgue et al., 2013; Elskens et al., 2014; de Brauwere et al., 2014; Pham Van et al., 2014), but there was no 3D sediment module in SLIM 3D prior to this study.

The fine sediment module is the 3D extension of the 2D module used by Lambrechts et al. (2010) in a study of the sediment budget in Cleveland Bay, NE Australia. The key aspects of this model of flocculating fine sediment are as follows (Fig. 4.2):

1. The suspended sediment concentration is represented as a passive tracer that is transported by the water currents. At the open boundaries, sediment leaves the domain with currents pointing outward, but there is no inflow of sediment with currents pointing inward.
2. Over the whole domain, a vertical settling velocity is added to the hydrodynamical vertical velocity. This velocity is proportional to the sediment concentration, as it is characteristic of cohesive sediment (Lambrechts et al., 2010). The sediment plume is then much smaller than the fresh-water plume.
3. At the bottom boundary, i.e. on the sea floor, the suspended sediment can settle to become deposited sediment if the weather conditions are calm. Otherwise, the deposited sediment erodes to become suspended sediment. This erosion is due to either wave induced fluidisation of the mud, or entrainment.

Model equations are detailed in Appendix B.1. The sediment model is sensitive to key parameters such as the settling velocity. In this study, the parameters are directly adapted from those of Lambrechts et al. (2010), which were calibrated in the same region (Cleveland Bay).

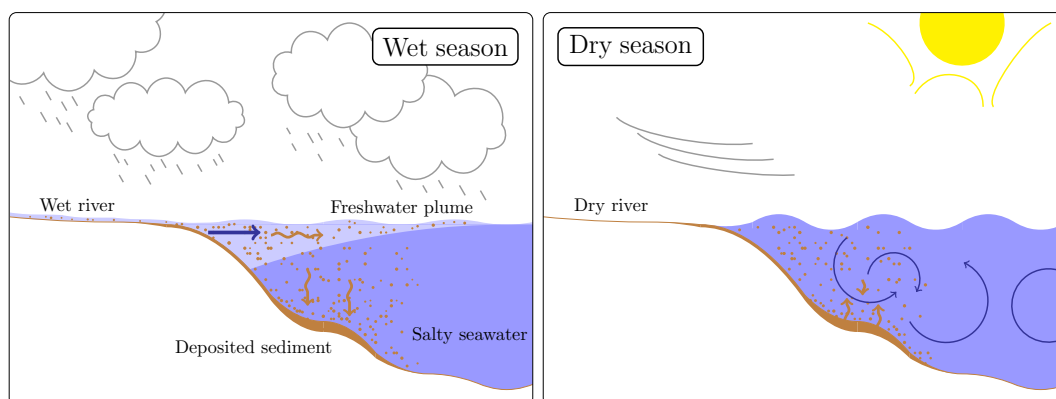


Fig. 4.2: Schematic representation of the processes involved in the transport of the sediment: flood event during wet season (left) and resuspension event during dry season (right).

In order to draw budgets and analyse the proportion of suspended and deposited sediment in the different bays, the region was divided into the five following zones: Upstart, Bowling Green, Cleveland and Abbot Bays (Fig. 4.1), extending to the 22 m isobath (Larcombe and Carter, 2004; Belperio, 1983), where the majority of the terrigenous sediment deposition and resuspension occurs (Orpin et al., 1999), and the middle-shelf region offshore Abbot Bay extending from the 22 m to the 40 m isobath.

The region was discretised into  $\sim 8,000$  triangular elements, whose size varies between 500 m and 7 km. The horizontal mesh size is a function of the distance to the coast and the bathymetry: the closer to the coast and the shallower, the thinner the mesh. Moreover, the resolution is increased in Upstart and Bowling Green Bays, where the sediment was expected to go. Those horizontal elements were extruded to build 8 vertical levels in all the domain. In order to resolve correctly the freshwater plume, the vertical discretisation is thinner close to the surface. The first four top levels are at constant depth, which is important to preserve the vertical profile of the plume. The next four levels are adapted to the bathymetry as  $\sigma$ -levels. The four thin top levels are necessary to resolve correctly the plume dynamics. Without them, the plume would be too thick and would not extend far enough from the river mouth. The bottom levels have a coarser resolution in deeper areas. However the sediment is not deposited in the deepest regions where the vertical resolution is the worst. The loss of resolution in mid-depth regions where sediment is deposited does not seem to affect the results. Therefore the 3D mesh has  $\sim 65,000$  prismatic elements (Fig. 4.1). The horizontal mesh was generated using GMSH software (Geuzaine and Remacle, 2009).

### 4.2.2 Data acquisition

To either run the model or validate the simulation results, various data were necessary. This section gives details on the data used in this study.

**Bathymetry** The bathymetry is a smoothed linear interpolation from the Project 3DGBR dataset, with a resolution of 100 m (Beaman, 2010). The smoothing was achieved with a diffusion operator. It removed the unresolved features of the bathymetry. The error introduced by this operation was smaller than 5% on more than 97% of the domain. Error was mainly introduced at the reefs close to the north-east open boundary.

**Wind** Wind data were taken from the NCEP Climate Forecast System Re-analysis (Saha et al., 2010), with a temporal resolution of 6 hours, and a spatial resolution of  $0.3^\circ \times 0.3^\circ$ . The wind shadow observed downwind in the Cleveland, Bowling Green and Upstart Bays are not resolved in these data. Wind shadows of 8 km were added to NCEP wind data in order to reproduce the pattern of wind resuspension observed in the data. The main winds in this region are the trade winds, blowing from the south-east. While they generally do not exceed  $5 \text{ m s}^{-1}$ , they can occasionally reach more than  $10 \text{ m s}^{-1}$ .

**Tides** At open sea boundaries, tidal forcing was applied using the OSU TOPEX/ Poseidon Global Inverse Solution TPXO 7.2 dataset (Egbert and Erofeeva, 2002), with a spatial resolution of  $1/4^\circ$ . The tidal range in the region is about 3m.

**Burdekin river flow and sediment loads** The water flow discharged at the Burdekin River mouth was forced using the daily data measured at the 120006B Clare station, 40 km upstream from the river mouth (DNRM, 2014). The volumetric flow rate was under  $100 \text{ m}^3 \text{ s}^{-1}$  most of the year, except for a large flood event in February, during which the flow rate exceeded  $8000 \text{ m}^3 \text{ s}^{-1}$  (Fig. 4.3). A much smaller wet event occurred in July. The sediment concentration at the river was imposed using data collected at Inkerman Bridge,  $\sim 26$  km downstream of Clare Station (Bainbridge et al., 2007) (Fig. 4.3). The load of sediment discharged by the river in 2007 is typical of an average year ( $\sim 6$  Mt). Every year, the river is almost dry all the time except for a big flood event during summer.

**Photoc depth** The photic depth is a measure of water clarity. The fine sediment concentration predicted by the model was compared with photic depth data determined from satellite observation (Weeks et al., 2012), with a spatial resolution of  $1 \text{ km} \times 1 \text{ km}$ . The photic depth was converted to sediment concentration following Wolanski and Elliott (2015):  $C = 6600 \zeta^{-1.12}$ , where  $C$  is the sediment concentration ( $\text{mg l}^{-1}$ ) and  $\zeta$  is the photic depth (m).

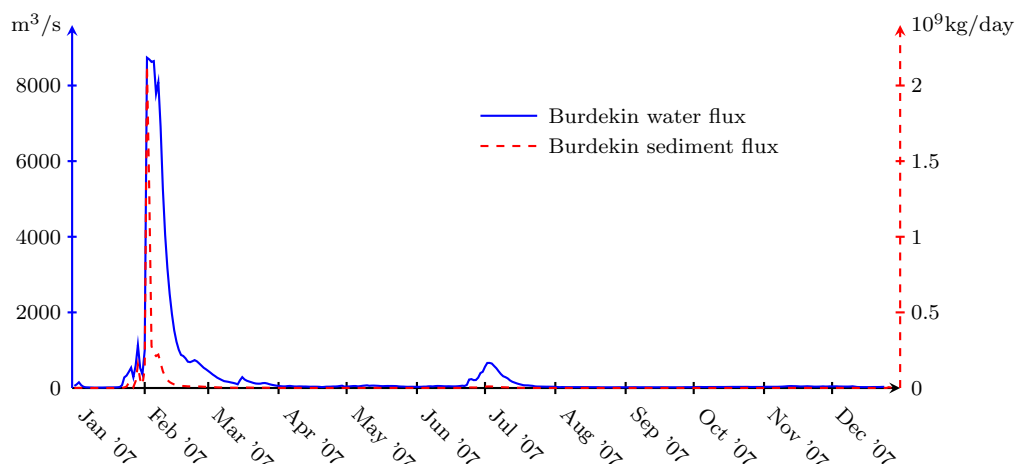


Fig. 4.3: Burdekin River water and sediment discharge at Clare station for 2007. In blue: volumetric flow rate (with scale on the left); in dashed red: river sediment flux (with scale on the right).

**Plume suspended sediment and salinity** During the flood event, sea surface suspended sediment concentration and salinity measurements were taken at various locations on February 6th and 7th, 2007 ([Bainbridge et al., 2007](#)). Those data were used for model validation.

**Cyclone Yasi wind conditions** To study the transport of sediment under cyclone conditions, the wind conditions during Tropical Cyclone Yasi were used. Yasi was a category 5 tropical cyclone that struck the Burdekin region on February 3rd, 2011. During three days, 10 minute-averaged wind speed was above  $10 \text{ m s}^{-1}$ , and during a few hours, it was above  $20 \text{ m s}^{-1}$ , blowing most of the time towards the west and north-west. Data used in this study were 10 minute-averaged wind speed measured at Hardy Reef platform, from the Australian Institute of Marine Science ([AIMS, 2014](#)). Other data necessary for the “cyclone condition” run, such as initial conditions, river flow and tides were taken running the model in April 2007 (conditions after the flood event).

**2007 year data** Apart from Cyclone Yasi data, all the data were taken during the 2007 calendar year. This year was chosen because it represents well an average year and for the large amount of data available. January 1st and December 31st are the initial and end times of the simulation. This allows the model to be in regime when the flood event begins.

## 4.3 Results

### 4.3.1 Sediment dynamics quick summary

The regional hydrodynamics and sediment dynamics were simulated from January 1st, 2007 to December 31st, 2007 with SLIM 3D. The flood event began in early February. Most of the sediment (67%) settled within Upstart Bay. Driven by the northerly wind (Fig. 4.4), the remaining part drifted to Abbot Bay, and further away. Later, during south-easterly (SE) wind the sediment was pushed northwards along the coast. After the flood event, the sediment mass released at the Burdekin mouth was negligible. During a few subsequent SE wind events (Fig. 4.7), settled sediment was resuspended and pushed northwards. Part of it was deposited just to the north of Cape Bowling Green, since this region is sheltered from SE winds.

### 4.3.2 Flood season modelling results

Figure 4.4 shows the comparison between the simulation results and satellite images of the plume during the flood event of February 2007. At this time, a slight northerly wind ( $3\text{--}5\text{ m s}^{-1}$ ) pushed the plume southward. For February 11, vertical cuts of the simulated sediment concentration and salinity are shown on Figure 4.5. The model closely matched the outer extent of the flood plume, although the finer-scale features of the primary plume (i.e. close to the river mouth) shown by the true color satellite image were not reproduced by the model (Fig. 4.4). The true color satellite image did not provide a quantitative measure of salinity or suspended sediment concentration. The measurements of sediment concentration and salinity from the surface of the plume compared reasonably well with those produced by SLIM 3D with some exceptions (Fig. 4.6). The latter occurred mostly near the river mouth where measured suspended sediment concentrations and salinity were highly variable.

There is no data about the vertical distribution of salinity and suspended sediment. However, a wrong vertical salinity profile would perturb the horizontal plume velocity, which in turn would deteriorate the sea surface salinity and sediment concentration. Similarly correct deposition areas could not be obtained without a plausible vertical sediment distribution. Moreover, the salinity and sediment concentration vertical distributions presented on Figure 4.5 are in qualitative agreement with the 2011 Burdekin Plume samples presented in [Bainbridge et al. \(2012\)](#): beyond 11 km from the river mouth, the salinity plume is still clearly marked, with a large difference between the surface and 5 m depth salinity. The sediment roughly follows the salinity dynamics for the first 9 km but is mixed on the vertical from the 11th km.

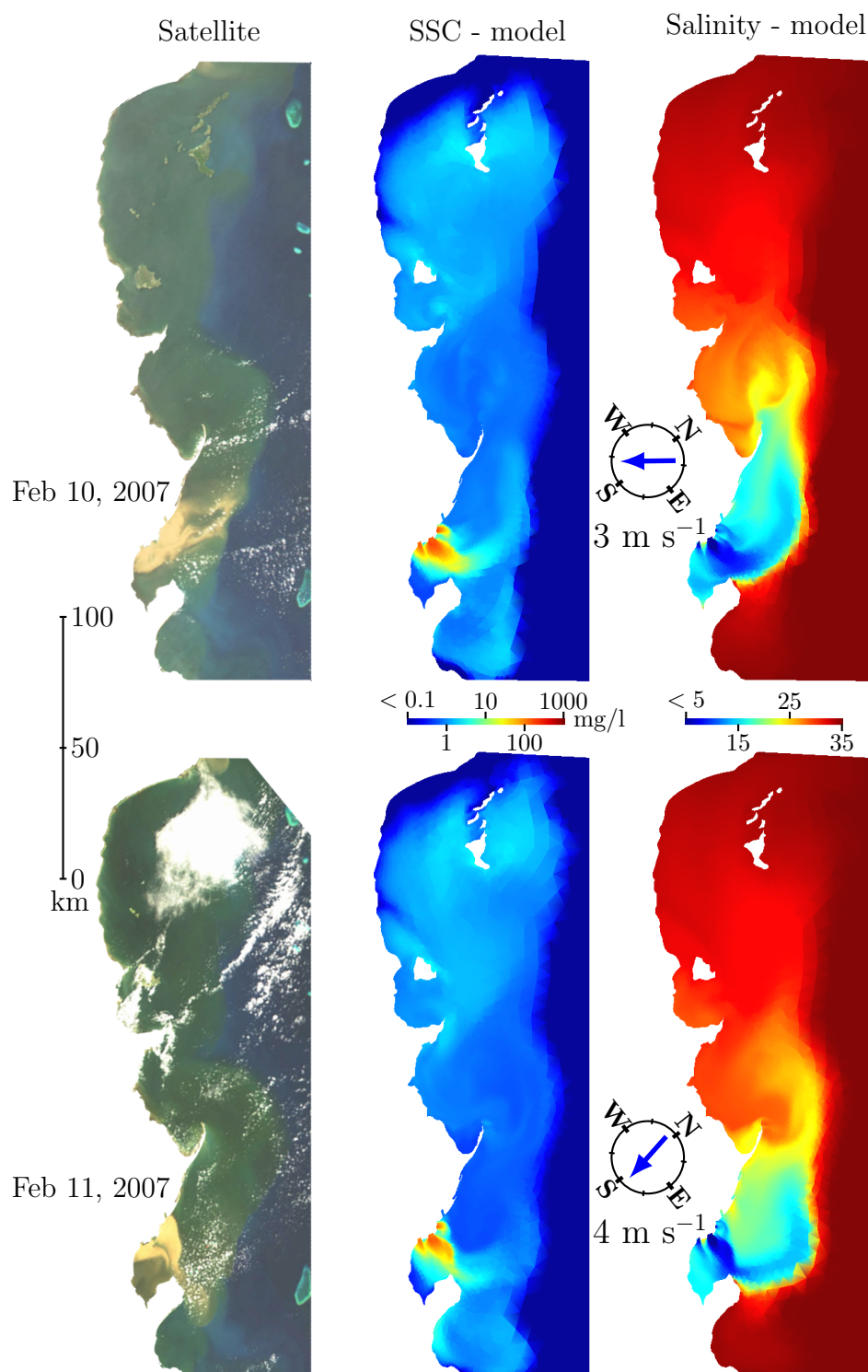


Fig. 4.4: Comparison between true color satellite images, sea surface suspended sediment concentration (SSC) and sea surface salinity during the flood event on February 10 (top) and February 11 (bottom). Wind speed and direction are shown in the compass.

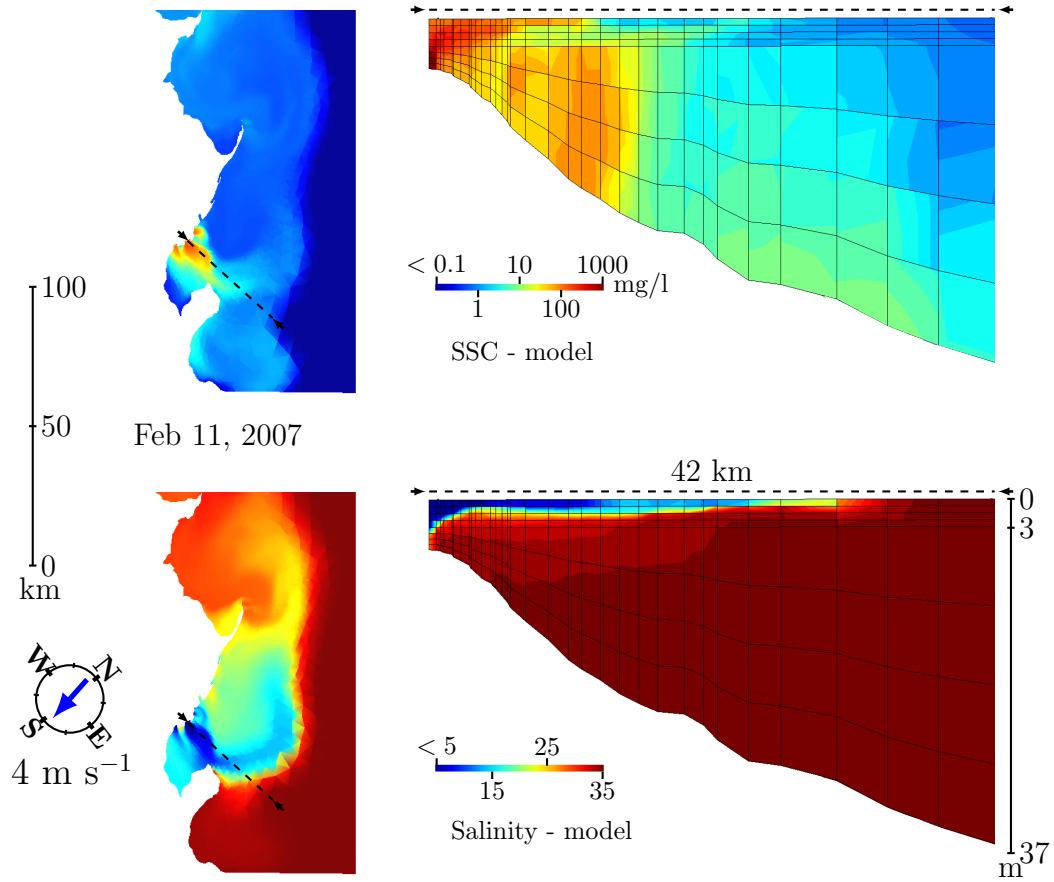


Fig. 4.5: Vertical cut of suspended sediment concentration (top) and salinity (bottom) for the flood event on February 11 (same event as that represented at the bottom of Figure 4.4). Left: sea surface suspended sediment concentration and salinity, respectively. The cut location is illustrated with a dashed line. Right: vertical cuts. Wind speed and direction are shown in the compass.

### 4.3.3 Dry season modelling results

Under the larger resuspension events which occurred in 2007, SLIM 3D outputs were comparable to the processed photic depth data. In particular the model was able to reproduce the width and magnitude of the sediment resuspension event that occurred during the strong SE (the prevalent winds in the region) winds ( $10 \text{ m s}^{-1}$ ) on April 8-10 (Fig. 4.7 (left)) as well as several other SE wind events that occurred in 2007 (similar to Fig. 4.7 (left), not shown.). The model performed reasonably well for a comparatively weaker northerly wind ( $\sim 5 \text{ m s}^{-1}$ ) which occurred in October (Fig. 4.7 (right)).

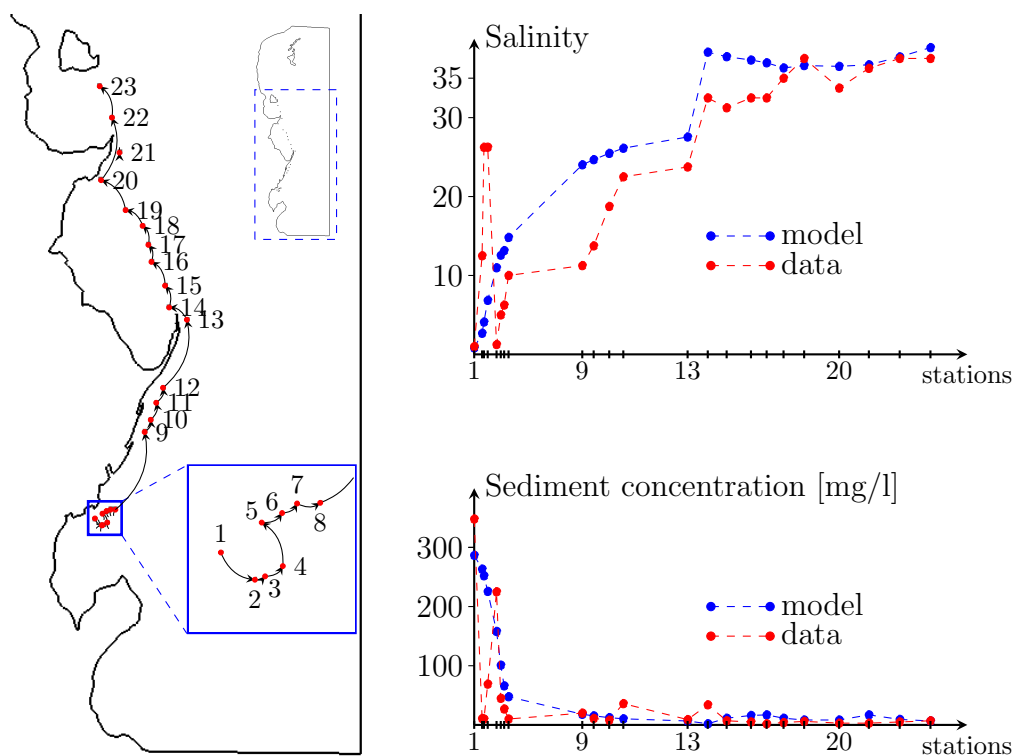


Fig. 4.6: Comparison between measured (Bainbridge et al., 2007) and simulated sea surface salinity and suspended sediment concentration. The map on the left shows the location of the points where measurements were taken between Feb 6 and 7 (See Table B.4 in B.2 for details on the measure locations and time). On the right, the model and data are compared for sea surface salinity (top) and sea surface sediment concentration (bottom).

#### 4.3.4 Global sediment budget

The model computed the fate of the 6 Mt of suspended sediment delivered from the Burdekin River in 2007. The model predicted that 86% of the suspended sediment load delivered by the Burdekin River was deposited within the modelling domain, with another 2% of the load remaining in suspension in the area. The remaining 12% of the load left the modelling domain through the open boundaries. 67% of the load delivered from the Burdekin was deposited and retained within Upstart Bay. The remaining deposited sediment load occurred in Abbot Bay, on the inner-shelf (6%) and the middle-shelf (7%), Bowling Green (5%) and Cleveland (0.09%) Bays (Fig. 4.8 (left)). 1% of the sediment was deposited in the domain outside one of the five previously defined zones.

The sediment deposited in the bays was infrequently resuspended during strong wind events in the dry season when the river flow was negligible (Fig. 4.8 (right)). During these wind-driven resuspension events, the sediment was not transported from bay to bay, but was slightly redistributed within the bays.

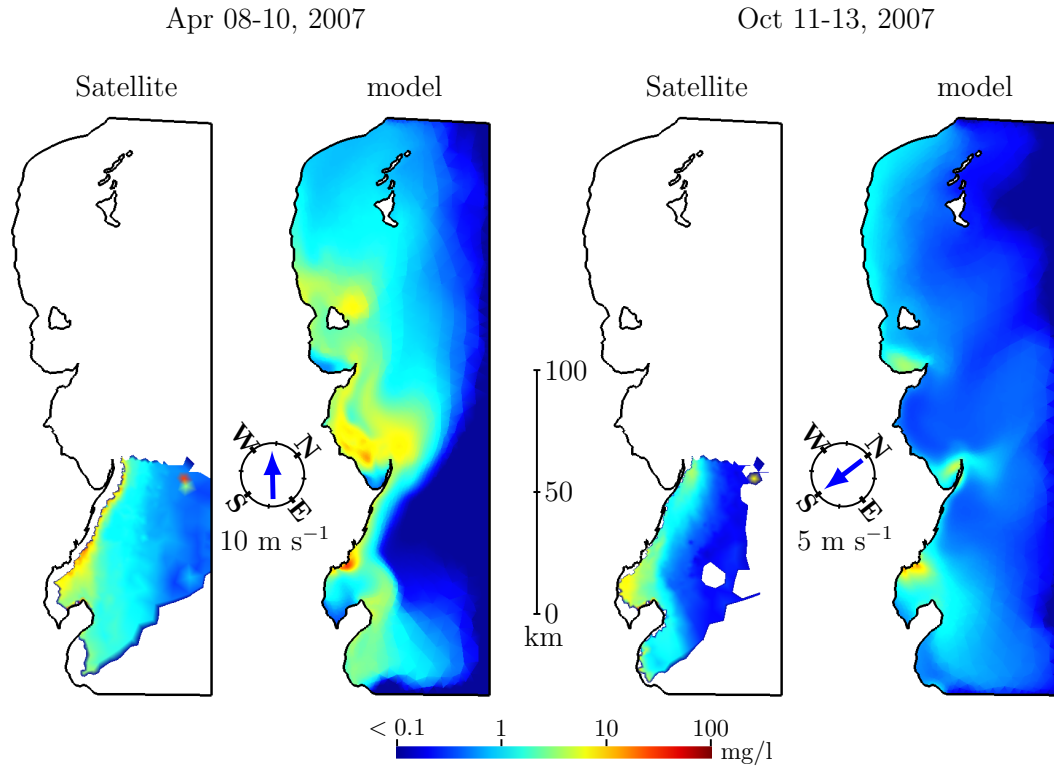


Fig. 4.7: Comparison between surface sediment concentration from processed satellite data (1st and 3rd map) and SLIM 3D simulation (2nd and 4th map) for the April 8-10 period (left) and the October 11-13 period (right), respectively. Satellite data are shown where available ([Weeks et al., 2012](#)). Wind speed and direction are shown in the compass.

The sediment accumulation from the 2007 Burdekin discharge event varied between 10 and 50 mm at the mouth of the Burdekin River in Upstart Bay (Fig. 4.9 (right)), and varied between 1 and 3 mm offshore near Cape Bowling Green. Deposition up to 1 mm in thickness was modelled to have occurred in parts of Abbot Bay.

#### 4.3.5 Sediment transport under cyclone conditions

Figure 4.10 shows the comparison between the deposited sediment prior to and after Tropical Cyclone Yasi. The results show all the deposited sediment left Abbot Bay: 99.9% of the deposited sediment in the inner-shelf part of the Bay left it, and 79% left the middle-shelf part of the Bay. The sediment from Upstart Bay was pushed northwards along the coast, and around 20% of it moved to Bowling Green Bay, where most (75%) of it remained in suspension at the end of simulation, and 25% was re-deposited north of Cape Bowling Green, in a similar area to the main depositional zone in this bay.

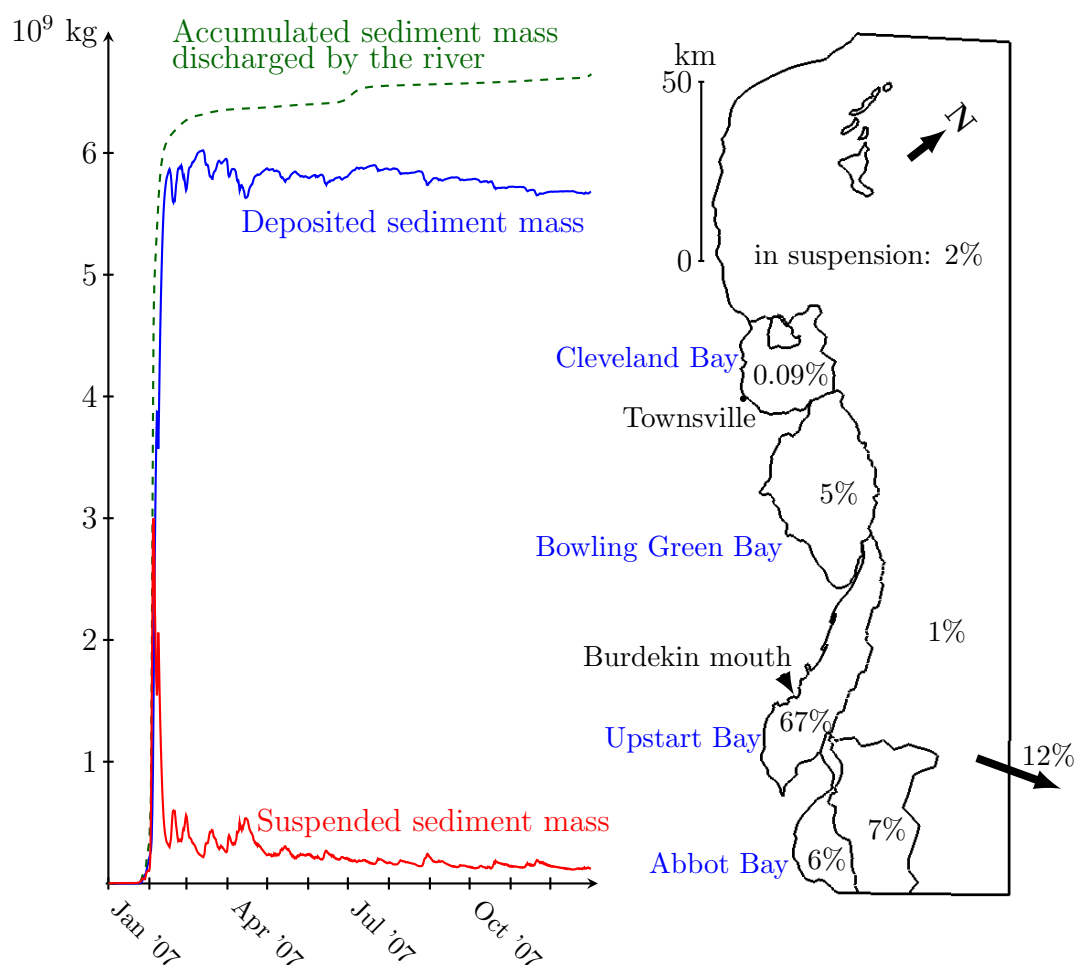


Fig. 4.8: Burdekin River sediment mass budget. Left: in blue: total mass deposited on the sea bed in 2007; in red: total suspended sediment mass. The brown dashed line represents the accumulated sediment mass released by the Burdekin River. Right: map of the sediment budget distribution at the end of the year. Except for the 2% which are still in suspension and the 12% which left the modelling domain, all the numbers refer to deposited sediment.

## 4.4 Discussion

### 4.4.1 Model

The flood event was reliably simulated by SLIM 3D. The model was verified reasonably well with both the satellite data from the 10-11th of February and the suspended sediment concentration and salinity dataset from the 6-7th of February (Figs. 4.4 and 4.6). The plume is driven by the vertical salinity profile and the sediment deposition and resuspension fluxes at the bottom are functions of the sediment vertical distribution. Those two vertical features, essential to understand the sediment dynamics, are captured explicitly

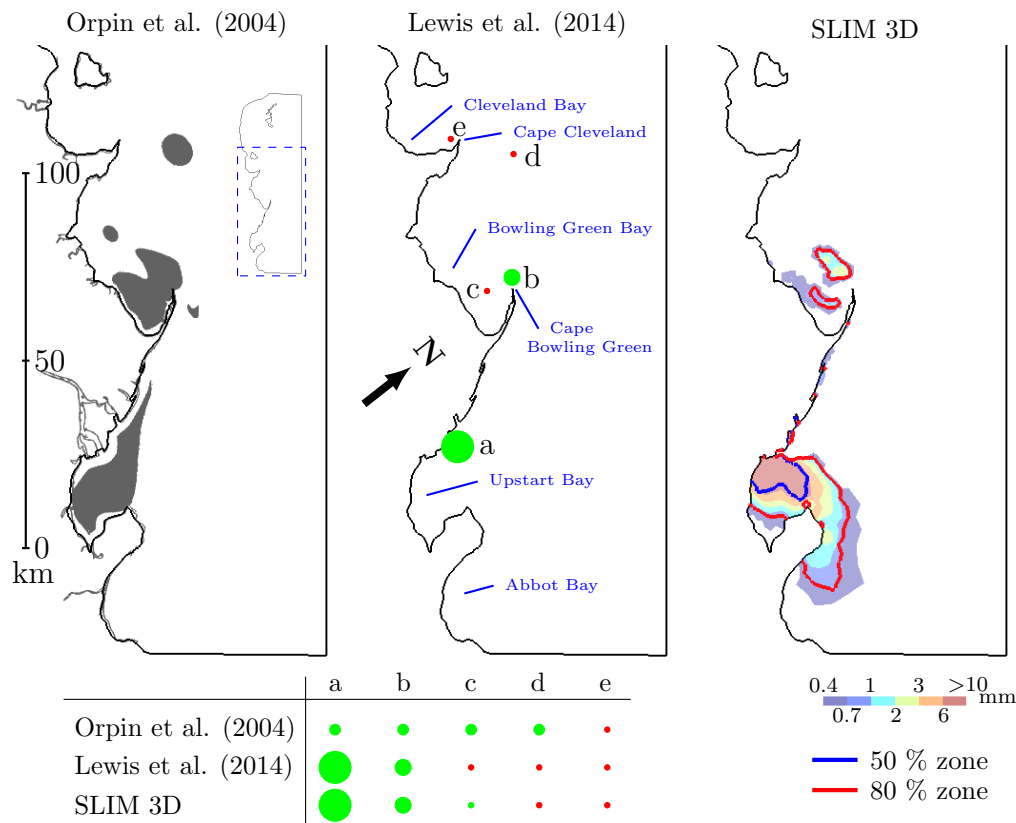


Fig. 4.9: Deposited sediment at the end of year 2007 modelled by SLIM 3D, and compared to core data from Orpin et al. (2004) and Lewis et al. (2014). On the left map, the grey zone represents the deposition region defined by Orpin et al. (2004). On the central map, five sediment accumulation rates have been measured by Lewis et al. (2014) (points ‘a’ to ‘e’). Large green circles represent high accumulation rates, medium green circles represent smaller rates and red circles represent negligible rates. On the right map, the thickness of the sediment deposit at the end of 2007 is shown, and blue and red lines represent the boundary of the zones where 50% and 80% of the sediment is deposited, respectively. The table compares the deposition areas of the three studies (the size of the circles for Orpin et al. (2004) is subjective, since accumulation rate is not quantified).

while more parameterisations would be required in a 2D model. In this semi-open system the use of an unstructured grid was necessary to increase the resolution in the deposition areas and to capture the coastlines. The discontinuous Galerkin finite element approach allowed for an accurate modelling of the transport processes and an easy implementation on a parallel computer cluster.

However, the model could not reproduce the fine small-scale features observed, particularly close to the river mouth. But the spatial and temporal

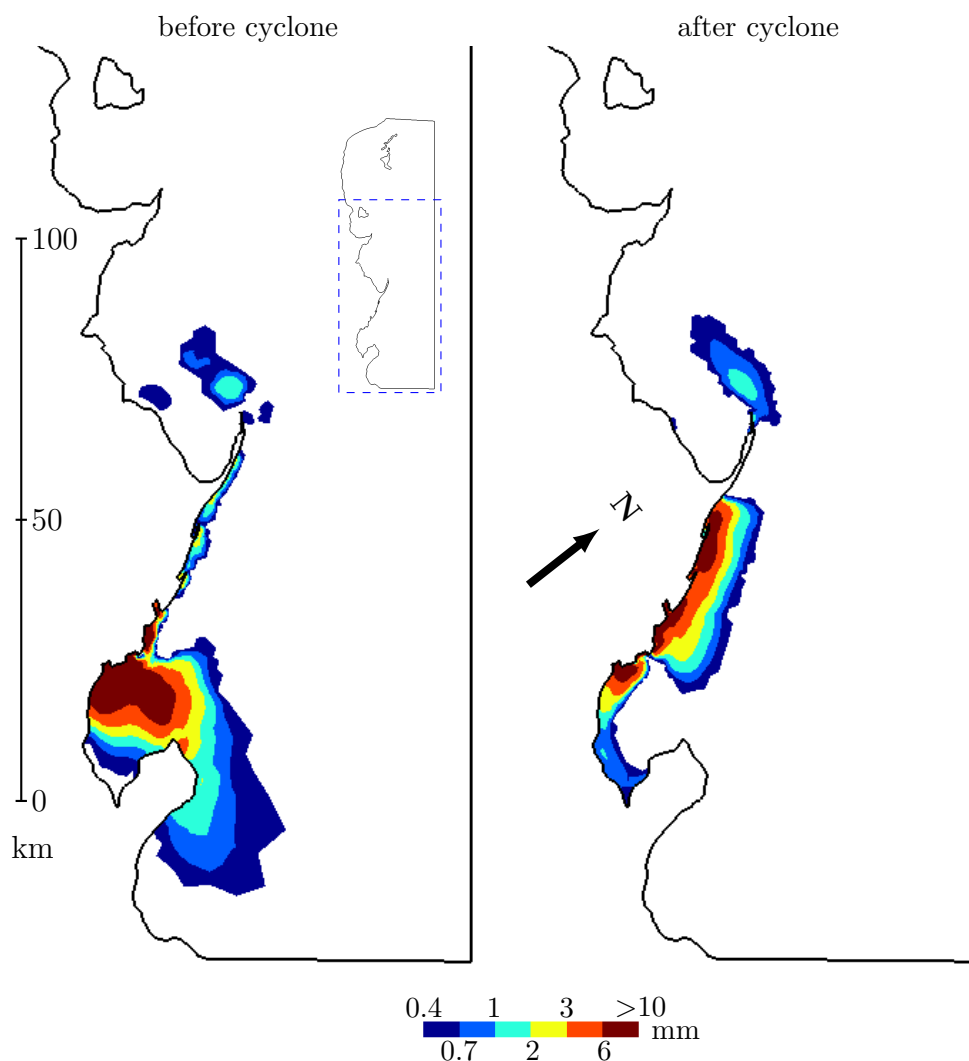


Fig. 4.10: Cyclone effect on sediment fate. Situation prior to (left panel) and after (right panel) the cyclone, respectively.

resolution of flood plumes are highly dynamic: large changes in salinity and suspended sediment concentration can occur over a scale of a few metres, i.e. a length scale that the model cannot resolve. Furthermore, the river discharge can also change rapidly in time and hence the daily resolution used in the data input to the model would not be able to capture those changes. Despite these fine scale discrepancies, which cannot be avoided using a model at this scale, SLIM 3D performed very well in characterising the salinity and suspended sediment dynamics in the flood plume over the period examined.

### 4.4.2 Burdekin sediment dynamics

The model showed that during the dry season, when river flow is negligible, strong winds are necessary to resuspend and transport sediment in coastal waters. It suggested that sediment fluxes from bay to bay are negligible. The finding that 67% of the 6 Mt of suspended sediment delivered in the 2007 flow event remains in Upstart Bay provides the first quantification of the partitioning and fate of sediment delivered from the Burdekin River (Fig. 4.8) and agrees with the assertion of Lewis et al. (2014) that “most” sediment is retained within the bay. This value is much higher than that suggested by Orpin et al. (2004) who estimated that only 10% of the Burdekin sediment load is deposited in Upstart Bay, although that study was limited by the chronology of sediment cores.

The model highlighted key depositional areas of the suspended sediment delivered from the Burdekin River that closely resemble some of the areas previously identified by Orpin et al. (2004) who collected benthic sediment grab samples (see Fig. 4.9 (left)). However, the model suggested that other mud-rich areas highlighted in Orpin et al. (2004), such as the southern inshore area of Bowling Green Bay and the middle of Bowling Green Bay, are currently zones of minor deposition (less than  $1 \text{ mm year}^{-1}$ ). Also, the area to the east of Cape Cleveland does not appear to be a zone of current sediment deposition from the Burdekin River. Indeed a sediment core collected from this location (point ‘e’ on Fig. 4.9) only revealed a thin (7 cm) veneer of Holocene sediment (Lewis, unpublished data). Furthermore, Abbot Bay was not sampled by Orpin et al. (2004) and should be the target for further investigation based on our SLIM 3D results. The calculations of Orpin et al. (2004) are likely to be relevant over the longer Holocene period (past 7000 years), during which the Burdekin River discharged into both Bowling Green and Upstart Bays and constructed at least 13 different delta lobes (Fielding et al., 2006). However, the findings of Orpin et al. (2004) do not conform for the past 1000 years, since the Burdekin channel last avulsed to discharge into Upstart Bay, which also led to the formation of the Cape Bowling Green sand spit which now differentiates Bowling Green and Upstart Bays (Fielding et al., 2006).

The modelling results provide clear guidance on the depositional areas for recent suspended sediment delivered from the Burdekin River and hence provide locations where the collection of sediment cores should be targeted. The accumulation rates modelled for the 2007 Burdekin River flow event matched closely with those found in the available sediment core data calculated over longer timescales (100 to 200 years, (Lewis et al., 2014)) and provide some validation of the model. Indeed, Lewis et al. (2014) measured an average accumulation rate of  $25 \text{ mm year}^{-1}$  over the past 100 years in Upstart Bay (point ‘a’ in Fig. 4.9). While the position of the sediment core (point ‘a’ in Fig. 4.9) is within a zone identified by the model to have little accumulation, it is less than 6 km from the modelled 25 mm zone. The peak discharge period in the

2007 Burdekin River flood coincided with a northerly wind which may account for this slight variation in the sediment depositional area. Maximum sediment accumulation in Bowling Green Bay occurred to the west of the tip of Cape Bowling Green and was modelled to be of the order of 1 to 3 mm. This value is lower, but of the same order of magnitude of the average accumulation in a sediment core ( $9 \text{ mm year}^{-1}$  over the past 200 years: point ‘b’ in Fig. 4.9) collected from this area (Lewis et al., 2014). Accumulation rates calculated in sediment cores from the southern inshore area of Bowling Green Bay (point ‘c’ in Fig. 4.9) and the west of Cape Cleveland (point ‘e’ in Fig. 4.9) show negligible recent sediment deposition. In these areas the model suggests accumulation of less than 0.3 mm and 0.1 mm, respectively, of the sediment delivered in the 2007 Burdekin flow event.

It is noteworthy that the sediment load delivered from the Burdekin River over the past hundred years has changed with land use changes in the catchment (Lewis et al., 2007), climatic/hydrological changes (Lough et al., 2015) and dam construction (Bainbridge et al., 2014). Hence, the long-term accumulation data in the sediment cores cannot be directly compared to the modelling results for one year. The long-term loads based on  $\sim 25$  years of monitoring data (Kuhnert et al., 2012) suggest that the mean load over this time is in the order of 4 Mt, which is thought to have increased by a factor 5 to 10 since catchment modification from the 1850s; in fact the Burdekin Falls Dam constructed in 1987 traps a large proportion of sediment (Bainbridge et al., 2014) and hence the loads delivered to the end of river over a period prior to 1987 would likely have been larger than today. In that regard, the annual mean sediment loads delivered from the Burdekin River over the past hundred years are likely to be of the order of 2 to 8 Mt and so the accumulation rate data in the cores were expected to be within an order of magnitude of the modelled sediment deposition rate.

### 4.4.3 Management prioritisation

The results from SLIM 3D provide information relevant to the management prioritisation of suspended sediment delivered from the Burdekin River. Specifically the model shows that the Burdekin sediment that travels past Upstart Bay into areas of sensitive marine ecosystems such as seagrass meadows and coral reefs occur during flood plumes. Hence it is this sediment in the flood plumes that needs to be better characterised and sourced to a specific catchment area or soil type. Bainbridge et al. (2012) showed that the suspended sediment in the Burdekin flood plumes becomes more organic-rich as the plume moves past Upstart Bay and is composed of clay and fine silt material ( $< 15.6 \mu\text{m}$ ). While SLIM 3D cannot model the biological material formed in the plume, such as bacterial mucus and phytoplankton, it does reveal the fate of the “mineral” sediment which is likely coupled with this biological material (Bainbridge et al., 2012). Further insights into the transport and fate of sus-

pendent sediment from the Burdekin River will be gained by modelling larger flow and sediment load events as well as wind-driven resuspension events that occur at different times of the year to examine the movement and compaction potential of this newly delivered sediment. Future studies could also model different sediment types (e.g. different particle sizes and different densities) to examine if there is a compositional difference between the sediment retained within Upstart Bay and the sediment transported further afield.

#### 4.4.4 Semi-open systems

The present study provides insights on the sediment deposition and fate for semi-open systems. During the wet season, most of the sediment is deposited close to the river mouth (Fig. 4.9). A long distance sediment transport occurs within freshwater plumes during flood events but remains limited. During the dry season, while regular wind events flush the sediment out of the exposed coastal areas, only exceptional events, such as cyclones, are able to move the sediment out of the bays (Fig. 4.10).

Whilst the widely accepted long-shore drift process (e.g. Woolfe and Lacombe (1998); Lambeck and Woolfe (2000); Orpin et al. (2004)) is critical for sand movement along beaches (e.g. Pringle (1986); Muller et al. (2006)) in open systems, it is unlikely to be a major process in the transport of mud-sized ( $< 63 \mu\text{m}$ ) sediment in semi-open systems. Such systems are sheltered and differ from open settings which allow sediment to be transported hundreds to thousands of kilometres via longshore coastal wind-driven currents (e.g. Mekong and Amazon Rivers). However, those systems cannot be regarded as semi-enclosed systems, since a large fraction of the sediment mass leaves the bay of the river mouth.

### 4.5 Conclusions

SLIM 3D was used to model the transport and fate of suspended sediment delivered by a large tropical river to a semi-open system located on a shallow and wide continental shelf. The model reproduced key sediment depositional areas identified in previous studies and was able to closely reproduce the patterns and concentrations of suspended sediment and salinity measured in both the flood plume and during large resuspension events. Furthermore, the modelled sediment accumulation rates closely matched the longer-term sediment accumulation rates measured in sediment cores from the region. The model reveals that for such a system, most of the sediment is deposited and retained in the bay near the river mouth while the suspended sediment that travels longer distances is transported within freshwater plumes during flood events. Wind resuspension processes only redistribute the sediment within the embayments and do not move it from bay to bay, although the sediment is flushed out of

the exposed coastal areas. Cyclones are able to move the sediment out of the bays.

It is suggested that for these semi-open systems, the management of the sediment transported long distances in the flood plumes should be the priority for protecting sensitive marine ecosystems. Future studies should examine sediment transport and fate for other similar systems with different characteristics such as a narrower or deeper shelf.

## Acknowledgements

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# Modelling lake thermocline oscillations using a vertically adaptive mesh

This chapter reproduces the following article<sup>1</sup>:

**P. Delandmeter**, J. Lambrechts, V. Legat, V. Vallaes, J. Naithani, W. Thiery, J.-F. Remacle and E. Deleersnijder, A fully consistent and conservative 3D discontinuous Galerkin finite element model using a vertically adaptive mesh, with an application to the oscillations of the thermocline of a deep, elongated lake, *Geoscientific Model Development*, (submitted)

The discontinuous Galerkin finite element method is well suited to the modelling, with a relatively small number of elements, of three-dimensional flows exhibiting strong velocity or density gradients. Its performance can be highly enhanced by having recourse to r-adaptivity. In Section 3.5, a vertical adaptive mesh method was developed for SLIM 3D. This method, originally designed for finite difference schemes, is based on the vertical diffusion of the mesh nodes, with the diffusivity controlled by the density jumps at the mesh element interfaces.

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<sup>1</sup>Important structure modifications were brought. In particular, the description of the vertical adaptive mesh method was merged with the general SLIM 3D description in Chapter 3.

In this chapter, a brief review of vertical discretisation schemes used in marine flow modelling is first carried out. The vertical adaptive method is then evaluated on idealised benchmarks, simulating the oscillations of a sharp thermocline accurately, while a fixed mesh fails to preserve this sharp interface. Then, preliminary simulations of the Lake Tanganyika hydrodynamics are undertaken. For those runs, no vertical diffusivity is applied to the temperature field, and a uniform wind stress is forced at the lake surface. Adaptive and fixed meshes are compared on this set-up for both a 2D “ $x$ - $z$ ” and a 3D models, and the ability of the adaptive method to conserve the thermocline is evaluated. Finally, the relevance of the vertical adaptivity technique is assessed by simulating Lake Tanganyika thermocline oscillations. The results are compared to measured vertical profiles of temperature, showing similar stratification and outcropping events.

The simple benchmarks show the clear benefit of using adaptive meshes to increase the accuracy of the model in presence of a moving sharp horizontal interface. Moreover, the Lake Tanganyika simulation also proves the stability of the method. In this chapter, the simulations on Lake Tanganyika are achieved to illustrate the effectiveness of the mesh adaptivity, not to do a large investigation of the lake hydrodynamics. However, the model is ready and the tools and input data are available to realise such a comprehensive study.

## 5.1 Vertical discretisation in marine models

The vertical discretisation strategy of marine models has evolved drastically during the last five decades. The first models were using  $z$ -coordinates (e.g. [Bryan, 1969](#)), discretising the ocean into fixed horizontal levels, resulting in a stepwise representation of the ocean bottom. Later, other discretisations were developed, mainly inspired by the progress in atmospheric modelling, for which coordinates based on the pressure field were used ([Sutcliffe, 1947](#); [Eliassen, 1949](#); [Phillips, 1957](#)). In oceanography, such development led to  $\rho$ - or  $\sigma$ - coordinates ([Freeman et al., 1972](#); [Owen, 1980](#); [Bleck and Boudra, 1986](#); [Nihoul et al., 1986](#); [Blumberg and Mellor, 1987](#)). For the  $\rho$ -coordinates, the vertical coordinate is based on the density field. This method is well suited to tracer transport in the ocean interior, which occurs mainly along isopycnal surfaces ([Griffies et al., 2000](#)). The  $\sigma$ -coordinates place a constant number of levels evenly spaced in the vertical column. This method enables a smooth representation of the bottom topography, which is particularly appropriate for coastal applications. The discretisation of the internal pressure gradient constitutes a major difficulty of the vertical coordinate systems for which none of the iso-coordinate surface is horizontal, such as the  $\sigma$ -coordinate system. Since the iso- $\sigma$  surfaces are generally not horizontal, it is difficult to maintain a vertically-stratified water body at rest in a domain with a steep bottom slope. This problem has been extensively studied and documented (e.g. [Haney, 1991](#); [Deleersnijder and Beckers, 1992b](#); [Stelling and Van Kester, 1994](#); [Mellor et al., 1994, 1998](#)).

Later on, the general  $s$ -coordinate system was developed ([Gerdes, 1993a,b](#); [Song and Haidvogel, 1994](#)), which was also inspired by generalised coordinates in atmospheric modelling ([Kasahara, 1974](#)). The  $s$ -coordinates are able to arbitrarily combine  $z$ -,  $\rho$ - and  $\sigma$ - coordinates. With this technique, a single grid is able to cope with  $z$ -levels close to the ocean surface,  $\rho$ -levels in the interior and  $\sigma$ -levels close to the bottom. Those different methods have their own strengths and weaknesses, which are discussed in details in [Griffies et al. \(2000\)](#) for large scale applications. The modeller is free to build an a-priori optimal mesh, depending on the application.

The  $s$ -coordinates reach their limits when the optimal vertical distribution of the mesh nodes should vary in space and time. This is why [Burchard and Beckers \(2004\)](#) proposed a non-uniform grid system that adapts the resolution by moving the nodes during the simulation so as to minimise a suitably-defined error measure ([Hanert et al., 2006, 2007](#)). This is referred to as r-adaptive method, i.e. the mesh nodes are moved without modifying the mesh topology. [Hofmeister et al. \(2010\)](#) implemented this type of vertical movement of the mesh in a three-dimensional model. This method reduces the numerical mixing and the errors in the pressure gradient computation, enabling realistic simulations of large inflows in the Baltic sea ([Hofmeister et al., 2011](#)). The

price of the reduced numerical mixing is the computation of the mesh velocity. Dealing with the mesh movement within the hydrodynamics equations has negligible extra cost. Indeed, free-surface models do already move the mesh to take into account the surface motion (e.g. [Shchepetkin and McWilliams, 2005](#)).

It is noteworthy that this r-adaptive method only moves the nodes in the vertical direction. In contrast, the models using 3D hr-adaptation (e.g. [Piggott et al., 2005, 2008](#); [Hill et al., 2012](#)) follow a completely different approach, based on tetrahedral meshes unstructured in the three directions.

To the best of author's knowledge, the vertically adaptive coordinate method has only been applied to structured grid models before being developed for SLIM 3D. In Chapter 3, the method, based on the vertical diffusion of the mesh nodes, was defined and applied to SLIM 3D. Unlike the original method of [Burchard and Beckers \(2004\)](#) and [Hofmeister et al. \(2010\)](#), the diffusion process is controlled by the density vertical jumps at the mesh element interfaces. The following sections validate the method on simple test cases and evaluate its efficiency by simulating the dynamics of the thermocline of Lake Tanganyika.

## 5.2 Simple validation benchmarks

The model is evaluated on simple test cases. First the internal seiche benchmark is used to assess the model ability to preserve a sharp interface. Second, a convergence analysis is performed on this benchmark. Third, the accuracy of the steady-state thermocline position under constant wind stress modelling is evaluated.

### 5.2.1 Internal seiche

The first test to evaluate the adaptive coordinate system is the internal seiche modelling of [Hofmeister et al. \(2010\)](#). This two-layer benchmark bears similarities with the dynamics of the thermocline of Lake Tanganyika. It is fully defined in [Hofmeister et al. \(2010\)](#). The objective is to simulate the oscillations of the interface in a long (64-km long) and shallow (20-m deep) channel. In contrast to Lake Tanganyika, the density is here a function of salinity, not of temperature. Also, since the original test case is defined using a two-layer model, there is no vertical mixing. The goal is therefore to diffuse the interface as little as possible.

For this application, the error measure used to diffuse vertically the mesh is a function of the vertical jumps in the density field, with a small background error  $f_e = 10^{-5}$ . The time constant is set to  $\tau_e = 100$  s, and the minimum

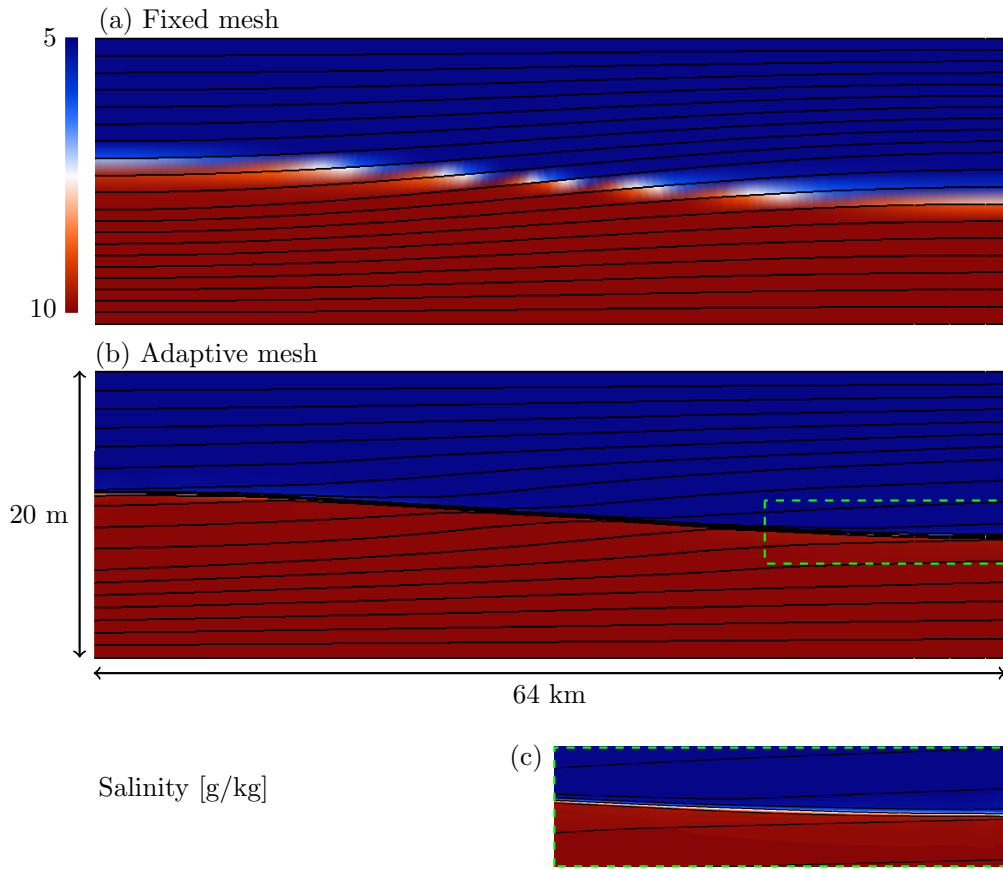


Fig. 5.1: Comparison between the internal seiche modelling with (a) fixed and (b) adaptive meshes. While the first one induces important numerical mixing at the interface, the interface remains sharp in the second one. (c) Zoom of the area delimited by the green rectangle in (b), showing two levels with the highest vertical permitted resolution on both sides of the interface. The full lines represent the model levels.

and maximum heights of an element are set to 0.1 and 1.5 m, respectively (see Chapter 3 for model details).

Figure 5.1 compares the salinity vertical profile after half an oscillation for a fixed mesh (a) and the adaptive mesh (b), both with 20 levels. For both runs, the initial mesh is set such that it captures perfectly the interface initial position. While the fixed mesh induces large numerical mixing in one case, the mesh adapts and follows perfectly the interface owing to the vertical adaptivity in the other case.

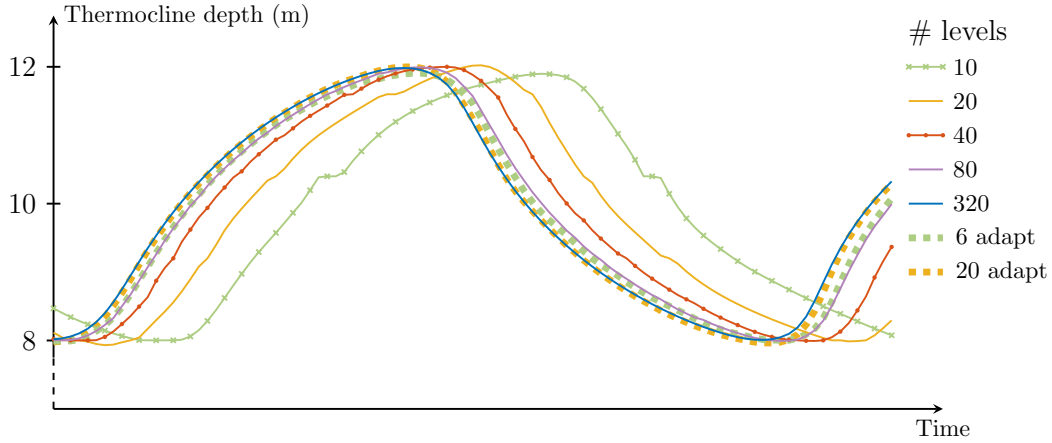


Fig. 5.2: Convergence analysis for the internal seiche modelling. The graph shows the third oscillation of the seiche, at which moment the results are beginning to diverge from the high-resolution solution.

### 5.2.2 Convergence analysis

To evaluate the model accuracy, a convergence analysis is performed for the internal seiche. The evolution of the interface depth at the right boundary of the domain is compared for different simulations using a number of fixed levels, varying between 10 and 320, which induces a level thickness varying between 2 m and 6.25 cm, using the same time step for all the simulations ( $\Delta t = 60$  s). Two simulations are also shown using adaptive meshes, with 6 and 20 levels, respectively. In the coarse-resolution simulations, the interface is diffused (Fig. 5.1), and the seiche oscillates too slowly compared to the 320-level run. While the first oscillation is rather well captured by all the simulations, the coarse-resolution simulations miss the correct dynamics during the next oscillations. After two oscillations, the simulation with 20 fixed levels fails to reproduce the dynamics (Fig. 5.2). In contrast, the simulation with 20 adaptive levels is as accurate as the simulation with 320 fixed levels. The minimal number of adaptive levels for an acceptable simulation is 6. With this number, two thin levels stick to the upper part of the interface and two other on the lower part, while the two last levels cover the remaining part of the domain (one on the top and one on the bottom). The 6-level simulation with the adaptive method thereby produces results as good as the simulation with 80 fixed levels. Considering that the CPU time of the simulations is proportional to the number of levels (Fig. 5.3) and that the computational overhead of adaptive levels is negligible, the adaptive method is about 16 times faster for a similar accuracy.

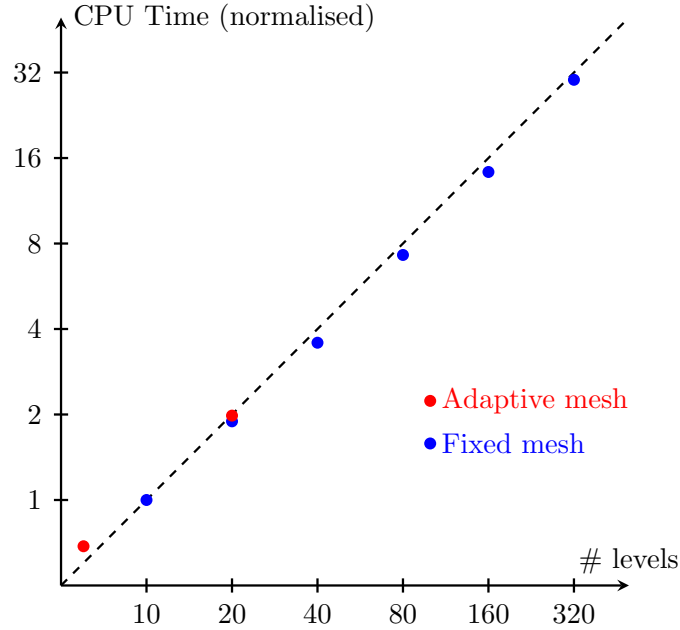


Fig. 5.3: CPU time of the simulations for a different number of levels (using a loglog scale). CPU time are normalised by the 10 fixed levels simulation time. As expected, CPU time is proportional to the number of levels. Moreover, the computational overhead of adaptive levels is negligible.

### 5.2.3 Steady-state thermocline position under constant wind stress

To assess the model, the equilibrium position of the thermocline in a lake under a constant wind stress is evaluated in a 2D “ $x$ - $z$ ” domain. The lake is composed of an upper thin layer, the epilimnion, and a lower deep and heavier layer, the hypolimnion. The two layers are separated by the thermocline. The position of the thermocline is approximated using the analytical solution of a 1D two-layer model, which simulates the epilimnion and hypolimnion vertically averaged velocity and the thermocline depth (Cushman-Roisin and Beckers, 2011). At the steady-state equilibrium, the pressure gradient due to the slope of the thermocline depth  $d$  is balanced by the wind stress:

$$\epsilon g d \frac{\partial d}{\partial x} = \frac{\tau_x}{\rho}, \quad (5.1)$$

where  $\epsilon = (\rho - \rho_0)/\rho_0$  is the relative density difference between the upper and the lower layers,  $\rho$  and  $\rho_0$  being the epilimnion and hypolimnion densities, respectively.  $g$  is the gravitational acceleration and  $\tau_x$  the wind stress. The solution of Equation 5.1 reads:

$$d = \sqrt{A + \frac{2\tau_x}{\rho\epsilon g}x}, \quad (5.2)$$

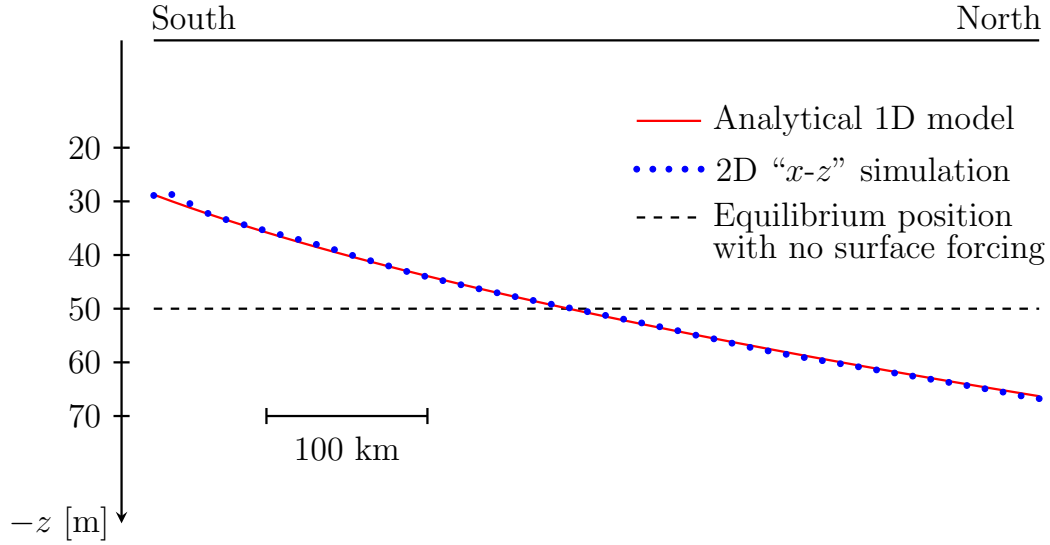


Fig. 5.4: Comparison between the analytical steady state thermocline profile using the 1D two-layer approximation and the 2D “ $x$ - $z$ ” simulation, under a constant wind stress. The dashed line represents the thermocline position without wind stress.

where constant  $A$  is such that the total epilimnion volume is conserved:

$$\int_0^L \sqrt{A + \frac{2\tau_x}{\rho\epsilon g}x} \, dx = d_0 L. \quad (5.3)$$

$d_0$  is the initial epilimnion height (when the thermocline is horizontal) and  $L$  is the length of the lake.

The thermocline depth is simulated for a wind stress of  $0.02 \text{ N/m}^2$ , for which there is no outcropping. The simulation starts with the thermocline located at  $d_0 = 50 \text{ m}$  and the model runs until a steady-state is arrived at, resulting in an average difference of  $0.5\%$  and a maximum difference of  $2.5\%$  between the 2D “ $x$ - $z$ ” simulation and the 1D analytical solution (Fig. 5.4). The maximum difference occurs close to the southern tip of the lake, where the thermocline is slightly diffused close to the model boundary.

## 5.2.4 Discussion

### Internal seiche and convergence analysis

The internal seiche test case is the typical application for which adaptive coordinates are necessary. The strong discontinuity cannot be preserved using a fixed mesh, which introduces large numerical mixing at the interface (Fig. 5.1a). The adaptive method introduced in this paper results in a smooth alignment of the levels in the vicinity of the interface (Fig. 5.1b,c). The method reaches its goal for this benchmark, which is to have one level on each side of the interface with the maximal resolution. It is noteworthy that the numerical

mixing at the interface affects the complete dynamics of the lake oscillations. An unresolved interface leads to oscillations with too large a period (Fig. 5.2). The convergence analysis quantifies the benefit of mesh adaptation, with a gain of the order of 16 in the number of elements and in computation time for a similar error. The decrease of one order of magnitude in the number of elements is similar to the result obtained by Bernard et al. (2007) with a h-adaptive DG finite element method.

One drawback of the method is that it is necessary to manually set-up the adaptation parameters. In the case of this application, the objective is to maintain the discontinuity at the interface, such that the error is a function of the vertical jumps in the density field. The background error function is a small function just big enough to avoid that a small error in the density field perturbs the mesh smoothness. Eventually, the time relaxation  $\tau_e$  must be small to obtain a fast adaptation. However, if  $\tau_e$  is too small compared to the simulation time step, the smoothness of the moving mesh can be affected.  $\tau_e$  typically has a value of the order of magnitude of the simulation time step.

### Steady-state thermocline position

The thermocline slope under a weak constant wind stress test case results in a thermocline slope similar to the analytical solution under the 1D two-layer approximation, with a small deviation close to the southern boundary (Fig. 5.4). This difference is most likely due to the hydrostatic assumption of the model. Indeed, in the overturning circulation, there is an increase of the pressure close to the wall, but this increased pressure is not captured by the model. As a consequence, the boundary layer is not captured by the model and small errors appear in the area irrespective of the horizontal resolution close to the wall. This is not a problem for the 1D model which does not model the overturning circulation within the epilimnion.

For a small, constant wind stress, the analytical 1D solution and the 2D “ $x$ - $z$ ” model simulation match very well (Fig. 5.4). However, for stronger stresses, the epilimnion height decreases such that the two-layer model hypothesis no longer holds. In this situation, the 2D “ $x$ - $z$ ” model simulates an outcropping of the hypolimnion layer while the 1D model retains a thin epilimnion layer, as it cannot represent outcropping by construction.

## 5.3 Thermocline oscillations of Lake Tanganyika

Lake Tanganyika, in East Africa, is the largest of the African Great Lakes in terms of water volume and the second largest in term of surface (Ogutu-Ohwayo et al., 1997). It is very long ( $\sim 650$  km), narrow ( $\sim 50$  km wide on average) and deep, with an average depth of 570 m and a maximum depth

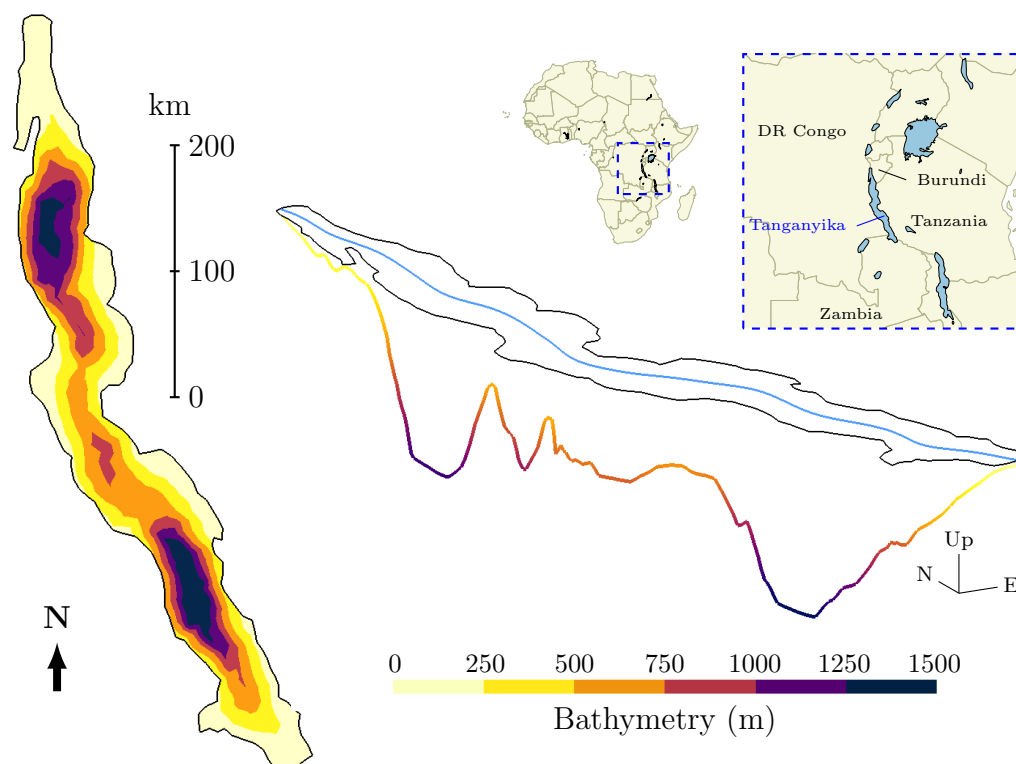


Fig. 5.5: Lake Tanganyika bathymetry: isobaths (left) and perspective view of bottom profile along the main axis of the lake (right).

of 1470 m (Fig. 5.5), making it the second deepest lake in the world. The lake is shared by four countries: Burundi, Democratic Republic of the Congo, Tanzania and Zambia. It is an extremely important source of clean water and dietary proteins in the region where fish represents between 50% and 70% of the animal proteins consumed by the people (Ogutu-Ohwayo et al., 1997). In the late nineties, the population depending on the lake for their income or their water or food consumption was estimated to ten millions (Mölsä et al., 1999). The lake is also a key transport artery in the region. Any threat to its ecology would thus have large medical, social and economical impacts.

A number of studies have been carried out on Lake Tanganyika since the end of the 19th century. A large review about the lake geology, hydrodynamics and ecology edited by Coulter et al. (1991) gave a comprehensive bibliography of those lake studies. The present study does only focus on hydrodynamics and not directly on the lake ecology. However, the water chemical composition is influenced by the turbulence and the mixed layer depth (Plisnier et al., 1999; Gourgue et al., 2011). This last one also impacts some fish species abundance (Plisnier et al., 2009). Vertical mixing has also consequences on primary production (Naithani et al., 2007). Eventually, some patterns in cholera outbreaks may also be linked to the hydrodynamics (Plisnier et al., 2015). The hydrodynamics is then an important actor of the lake ecology and health.

Schematically, the waters of the lake exhibit two layers separated by a thermocline (Coulter and Spigel, 1991; Naithani and Deleersnijder, 2004). The shallow upper layer, called the epilimnion, is relatively warm (24-28°C, Coulter and Spigel, 1991; Naithani et al., 2003) and has a typical depth of about 50 m. Below the thermocline, the deep bottom layer, called the hypolimnion, is composed of cooler water ( $\sim 23.5^\circ\text{C}$ ). Forced by the surface wind stress, the thermocline oscillates. There are two main seasons in the region. During the dry season, approximately from April or May to September, strong south-easterly wind blows along the main axis of the lake (Docquier et al., 2016). The wind pushes the epilimnion water towards the North, causing upwelling at the southern tip and downwelling at the northern tip (Fig. 5.6), and resulting in a thermocline tilted towards the North. The wind oscillations are characterised by a period of 3 to 4 weeks (Naithani et al., 2002), which is of the same order of magnitude as the period of the first free mode of oscillation of the thermocline (Naithani et al., 2003), giving rise to quasi-resonance. Thus, during the abovementioned season, the thermocline oscillations are essentially a direct response to the wind forcing, i.e. large-amplitude, forced oscillations (Gourgue et al., 2011). The thermocline is deep and sharp at the northern tip but becomes diffuse at the southern tip (Coulter and Spigel, 1991). During strong wind events, the oscillations are so large that the thermocline outcrops. Besides the hydrodynamical effect, the mixed layer also deepens during the dry season. This mixing is due to evaporative-driven cooling (Thiery et al., 2014a,b). On the other hand, during the wet season (from October to March), the wind stress is significantly smaller, leading to thermocline oscillations that may be viewed as progressively decaying internal seiches.

Different modelling studies were performed on the lake. Using the 1D vertical FLake model and meteorological observations, Thiery et al. (2014a) studied the temperature vertical profile at various locations.

Using a 2D reduced-gravity model, which simulates the epilimnion vertically averaged velocity and thermocline depth, Naithani et al. (2003) studied those thermocline oscillations. For this study, the wind stress forcing was uniform. Using the same model, the presence of internal Kelvin waves in the lake was highlighted (Naithani and Deleersnijder, 2004), which showed that the importance of the Coriolis force is not negligible as initially stated (Coulter and Spigel, 1991). Gourgue et al. (2011) decomposed the modelled thermocline oscillations into their free and forced components.

However, the reduced-gravity model has many limitations. The model sees the lake as a superposition of two layers, the hypolimnion and the epilimnion on top of it. The interface between those layers, the thermocline, has a null thickness, and is impermeable. Apart from being unable to simulate what is the velocity vertical profile and thus the overturning circulation within the upper layer, the model cannot simulate the diffusion of the thermocline or outcropping situations.

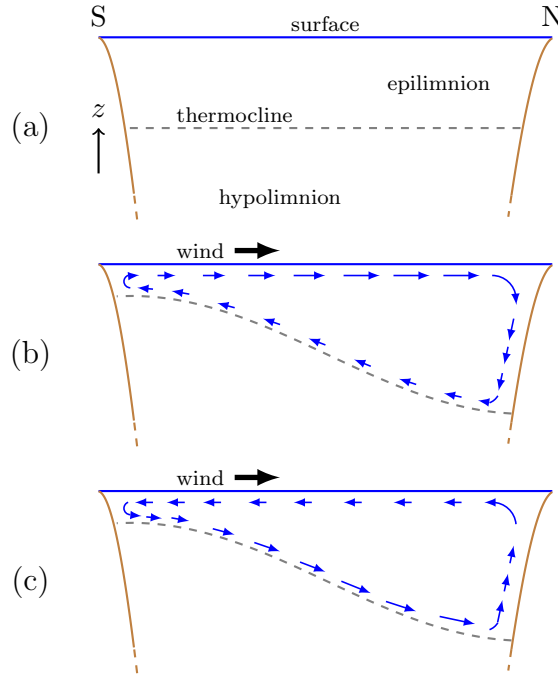


Fig. 5.6: Schematic lake configuration with no wind stress (a) and with the south-easterly wind during the dry season in line with the classical circulation (b, [Mortimer, 1961](#)) such that the surface water follows the wind, or according to the non-classical circulation (c, [Verburg et al., 2008](#)) with reversed currents.

A few 3D modelling studies were conducted on the lake. [Huttula \(1997\)](#) used a 3D barotropic model to simulate the transport of sediment in sub-regions of the lake. [Podsetchine et al. \(1999\)](#) used the same model to study the lake response to different wind stress regimes. Their simulations were focused on the diurnal cycle of the water velocity. They did not aim to model the seasonal variability of the current and the thermocline dynamics, for which the 3D model must be baroclinic.

[Verburg et al. \(2008\)](#) studied the overturning circulation in the lake. In contrast to the classical overturning circulation in a two-layer lake under constant wind stress (Fig. 5.6, [Mortimer, 1961](#)), they proposed a reversed circulation with the deepest water of the epilimnion following the wind and the surface water flowing in the opposite direction. [Verburg et al. \(2011\)](#) quantified the conditions for which they proposed a counter-wind surface circulation, driven by the surface heat flux distribution of the lake. They used a 3D model ([Hodges et al., 2000](#)) to assess their hypothesis by simulating the lake dynamics in 1996.

Using SLIM 3D and the vertical adaptive coordinates, such that the mesh resolution is increased in the thermocline region, the lake overturning circulation is studied, using another forcing dataset. The results are briefly compared to the one obtained by [Verburg et al. \(2011\)](#).

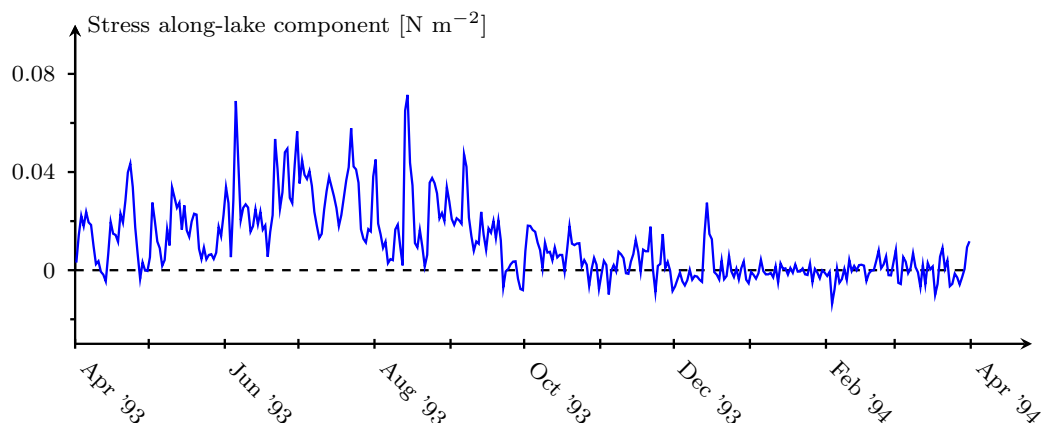


Fig. 5.7: Along-lake component of the daily-averaged wind stress measured in Mpulungu, at the southern tip of the lake. Positive values mean northwestward blowing wind.

### 5.3.1 Data and model set-up

#### Wind forcing

Two wind datasets are available: a time series of measurements at one location, and a spatial wind map obtained from a model.

Wind speed and direction were measured every hour from April 1993 to August 1994 (Huttula et al., 1999) in Mpulungu, at the southern tip of the Lake ( $8^{\circ}45'S$ ;  $31^{\circ}6'E$ ). These wind speed observations were used for earlier studies using a 2D reduced-gravity model (Naithani et al., 2003, 2007; Naithani and Deleersnijder, 2004; Gourgue et al., 2007, 2011). Figure 5.7 shows the daily-averaged wind stress along the main axis of the lake.

On the other hand, non-uniform wind data were obtained from the COSMO-CLM<sup>2</sup> model, which couples the non-hydrostatic regional climate model COSMO-CLM version 4.8 to the Community Land Model version 3.5 (CLM3.5) and the Freshwater Lake model (FLake; Davin and Seneviratne, 2012). The COSMO-CLM<sup>2</sup> model was recently applied in its tropical configuration (Akkermans et al., 2014; Panitz et al., 2014) to assess the two-way interactions between the African Great Lakes and the surrounding climate (Thiery et al., 2015, 2016; Docquier et al., 2016) as well as evaluate natural hazards in the region (Jacobs et al., 2016a,b). The climate simulations were conducted at a horizontal resolution of  $0.0625^{\circ}$  ( $\sim 7$  km) for the period 1996 - 2008 and provide near-surface wind fields at a temporal resolution of 3 h.

Figure 5.8 shows the component of the surface wind stress along the main axis of the lake at three locations. While the wind is mostly blowing northwestward during the dry season (April - September), it is much weaker and does not have a dominant direction during the wet season. The wind is weaker

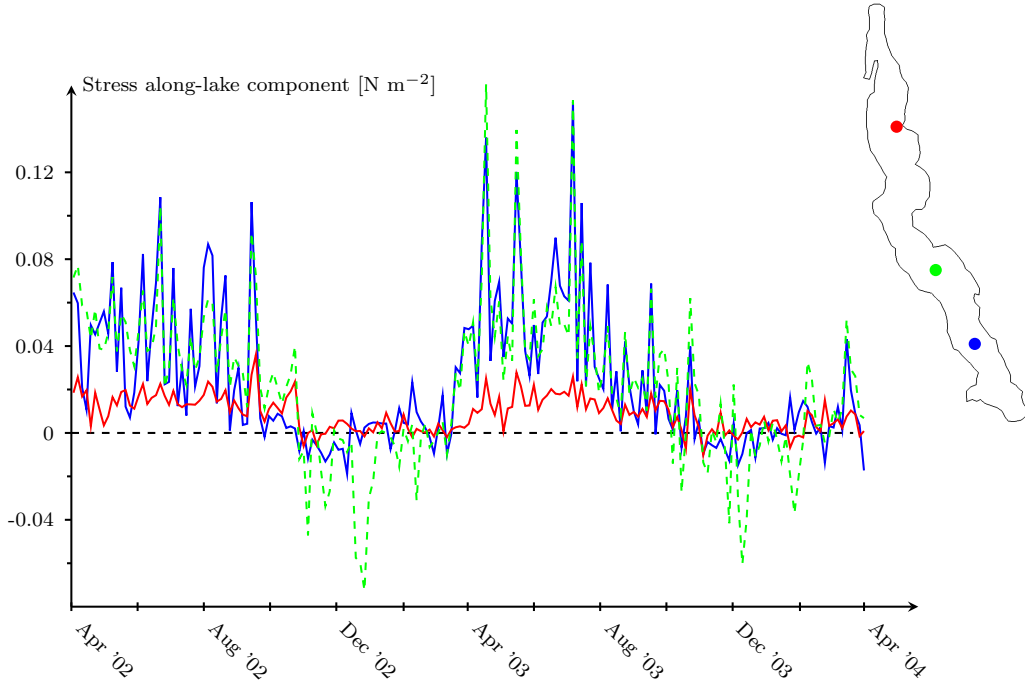


Fig. 5.8: Along-lake component of the daily-averaged wind stress, at the northern, central and southern part of the lake, between April 2002 and 2004, as modelled by the regional climate model COSMO-CLM<sup>2</sup>.

in the northern part of the lake (Docquier et al., 2016).

### Model set-up

Two configurations of the model are run. In the first configuration, aimed to analyse the effect of adaptive coordinates, no vertical diffusivity is applied to the temperature field, such that the modelled thermocline should remain sharp. The vertical viscosity is determined from the  $\kappa$ - $\epsilon$  turbulence closure model implemented in GOTM (Burchard et al., 1999) and coupled to SLIM 3D (Kärnä et al., 2012). Moreover, the homogeneous wind stress from Mpulungu measurement is applied at all the lake surface and no surface heat flux is applied.

In the second set-up, the vertical diffusivity and viscosity are taken into account. They are determined from the  $\kappa$ - $\epsilon$  turbulence closure model. The spatial wind from COSMO-CLM<sup>2</sup> is applied. The surface heat flux is parameterised by adding a relaxation term, in the upper layer of the lake. The relaxation term is defined as:

$$f_{\text{relax}} = \frac{\max(z_r + z, 0)}{z_r \tau_r} (T_{\text{ref}} - T), \quad (5.4)$$

with  $T_{\text{ref}}$  the reference temperature,  $z_r$  the depth of the relaxation zone and  $\tau_r$  the relaxation time parameter. The surface reference temperature comes

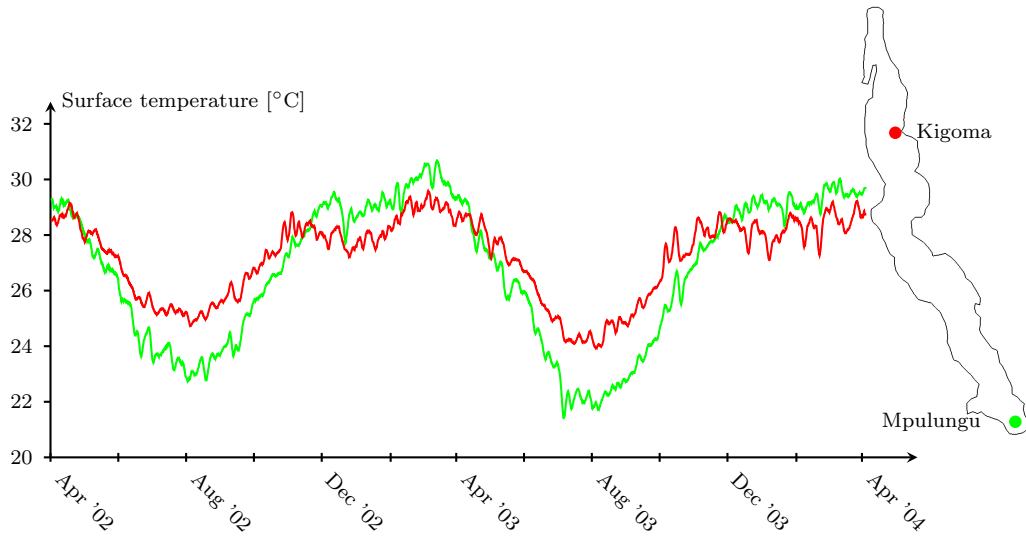


Fig. 5.9: Time series of the surface temperature at Kigoma (red curve) and Mpulungu (green curve), as modelled by COSMO-CLM<sup>2</sup> (Davin and Seneviratne, 2012).

from the same data set as the wind data, i.e. from the COSMO-CLM<sup>2</sup> model. Furthermore, the depth of the relaxation zone is set to  $z_r = 12$  m, which corresponds to a typical value for the photic depth (Descy et al., 2006), which is used as a proxy for the water column influenced by solar radiation and other heat fluxes. The relaxation time is equal to  $\tau_r = 10$  days, which was obtained after calibration. Figure 5.9 illustrates the evolution of this temperature at two locations: Kigoma, in the northern basin, and Mpulungu, at the southern tip of the lake. It is observed that at both locations the surface temperature drops during the dry season.

### Lake temperature vertical profile

Within the framework of the CLIMLAKE project (Descy et al., 2006), the water temperature was measured at Kigoma and Mpulungu (see location on Fig. 5.9) between 2002 and 2004, at vertical intervals of 20 metres between 0 and 100 m deep. This vertical profile temporal series is used for model validation. It is noteworthy that the surface temperature at Kigoma and Mpulungu from the CLIMLAKE in situ measurements exhibit significant discrepancies with the data from the COSMO-CLM<sup>2</sup> data set, which are used as input data for the model simulations.

## 5.3.2 Results

### Lake Tanganyika simulation without vertical diffusion

Lake Tanganyika hydrodynamics is simulated using the first model configuration. The model ability to preserve a sharp thermocline is assessed for

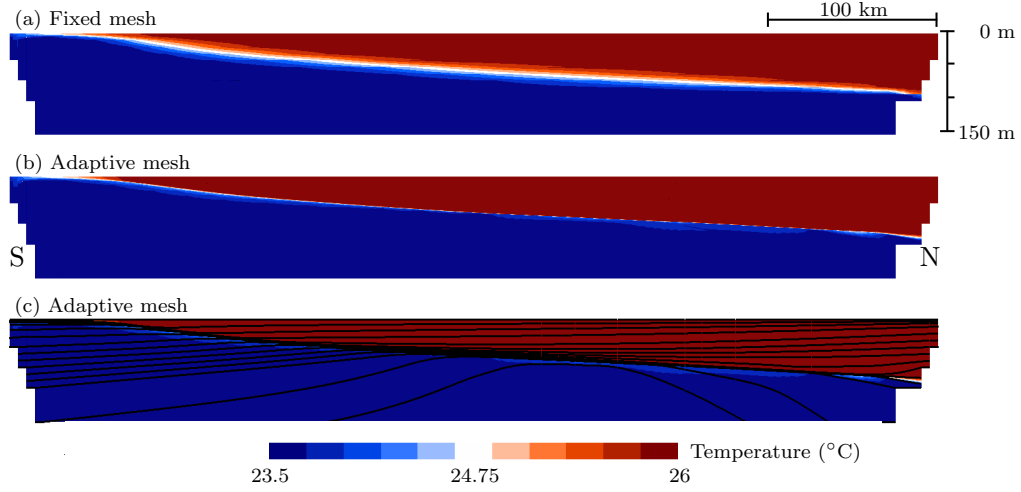


Fig. 5.10: Temperature profile using the uniform wind on July 8th 1993, with the 2D “ $x$ - $z$ ” model, with a fixed mesh (a) and an adaptive mesh (b,c) having an equal number of elements. (c) also shows the levels position for the moving mesh simulation.

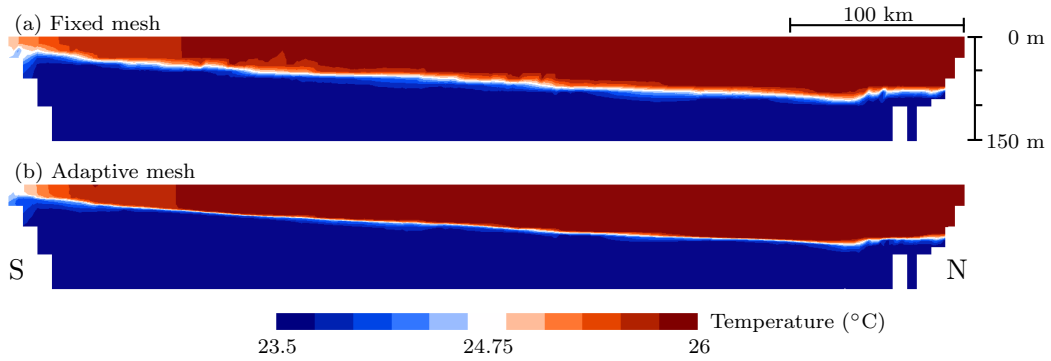


Fig. 5.11: Temperature profile using the uniform wind on July 8th 1993, with the 3D model, with a fixed mesh (a) and an adaptive mesh (b) having an equal number of elements. The temperature profile is given along the lake main-axis.

simulations with and without the vertically adaptive mesh. First, the lake dynamics is simulated with a simple 2D “ $x$ - $z$ ” mesh, representing the south-north thermocline position (Fig. 5.10). Then, it is simulated using a full 3D mesh, and the thermocline position along the main axis is extracted from the results (Fig. 5.11). For both 2D “ $x$ - $z$ ” and 3D simulations, a sharp thermocline is maintained using an adaptive mesh, while a fixed mesh results in the blurring of the thermocline. Again this confirms that SLIM 3D with vertically adaptive mesh is able to simulate the thermocline dynamics much more accurately than without adaptation.

## Lake Tanganyika simulation

Lake Tanganyika dynamics is simulated from December 2000 to April 2004 with the second model configuration, using a simple horizontal mesh of  $\sim 1,000$  triangles, extruded to form  $\sim 13,000$  triangular prisms. The horizontal levels are initially located 0, 2 and 5 m, then every 10 m down to 100 m. The next levels are located at 150, 200, 300, 500, 700, 950, 1200, 1400 and 1500 m. The mesh adaptation is driven by the vertical jump in the density field, with  $\tau_e = 1$  h. The background error is set to  $f_e = 10^{-5}$  at the surface, and is then defined such that if there is no vertical jump, the mesh stays at its initial position. Indeed, with a constant background error, the mesh would adapt to reach a situation with the same depth for all the levels at each vertical column. The first 16 months of the simulation are used as a spin-up period, after which the results are analysed.

The temporal evolution of the vertical profile of the temperature is analysed at Mpulungu (Fig. 5.12) and Kigoma (Fig. 5.13). The model performs well to reproduce most of the observed features of the lake. Outcropping events are observed at Mpulungu in July and August of both 2002 and 2003 (Fig. 5.12b), such as observed in situ (Fig. 5.12c). However, the 2003 outcropping lasts much longer in the model than in the data, due to the surface input forcing. The model reproduces the evolution of the 26°C isotherm during December 2002 - April 2003 period, but fails during the same period of the following year.

At Kigoma, the modelled temperature matches the observations better than at Mpulungu. The 26°C isotherm profile follows the data profile, evolving from 10 to 60 metres deep during the December 2002 - July 2003 period, then from 10 to 50 m from November 2003 to April 2004. This 26°C isotherm outcrops during the dry season strong wind period. A similar observation can be done for the cooler temperatures. The surface temperature is too high during almost the entire year, probably due to the surface heat flux being biased by the input data.

The thermocline profile along the main axis shows the different regimes of the lake dynamics during the year 2003. At the end of the wet season (March 1st, Fig. 5.14a), the lake is strongly stratified and the thermocline is approximately horizontal. Then, the dry season begins with strong southeasterly winds (May 15th), the surface water flows towards the north (Fig. 5.15b) and the thermocline is tilted (Fig. 5.14b). At the end of the dry season, there is a marked outcropping at the southern part of the lake (Fig. 5.14c), the wind weakens and the water circulation reverses (Fig. 5.15c).

Figure 5.16 shows the 26°C isotherm distribution on (a) March 1st and (b) May 15th, and the 25°C isotherm on August 3rd (c). Figure 5.16a shows that the temperature is not homogeneous in the direction perpendicular to the main lake axis. East-west temperature gradients may change sign during different

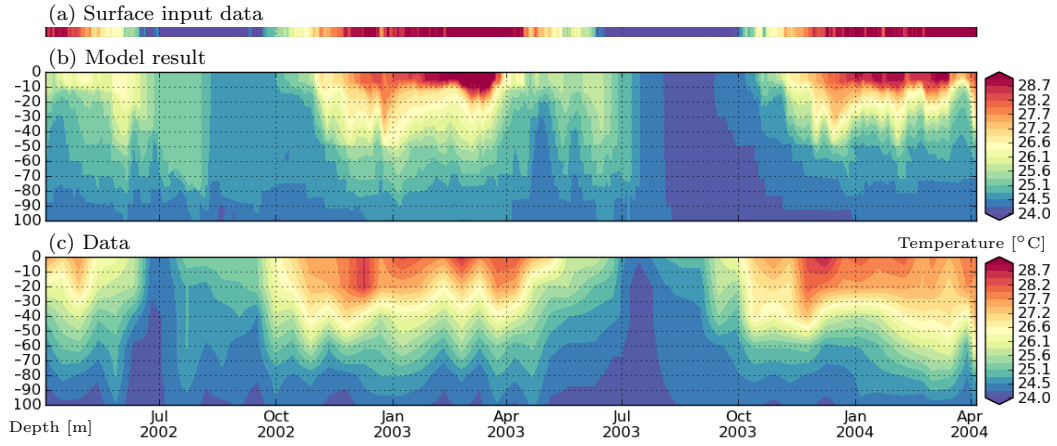


Fig. 5.12: Lake water temperature ( $^{\circ}\text{C}$ ) at Mpulungu as predicted by the model (b) and from in situ observations (c). Panel (a) shows the surface temperature used to force the temperature flux at the lake surface.

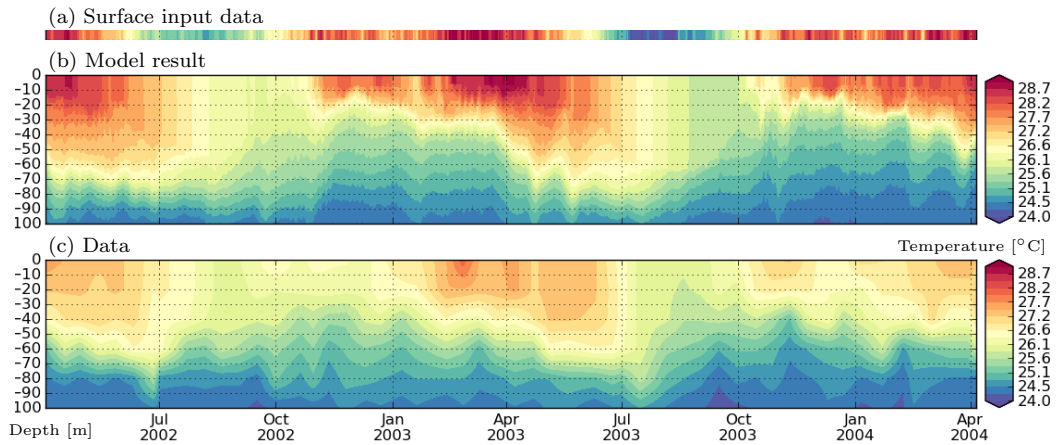


Fig. 5.13: Lake water temperature ( $^{\circ}\text{C}$ ) at Kigoma as predicted by the model (b) and from in situ observations (c). Panel (a) shows the surface temperature used to force the temperature flux at the lake surface.

periods of the year. This suggests that internal Kelvin waves are travelling along the lake boundary. This temperature gradient switch can occur within a few days in the southern basin of the lake. Moreover, during the dry season, large outcroppings of water cooler than  $26^{\circ}\text{C}$  (May 15th) and  $25^{\circ}\text{C}$  (August 3rd) are observed in the southern basin.

### 5.3.3 Discussion

While the simple benchmarks in previous section are 2D “ $x$ - $z$ ” applications, the Lake Tanganyika 3D modelling is more difficult due to the complex coastline, spatial wind patterns and Coriolis effect. Despite this challenge, SLIM 3D re-

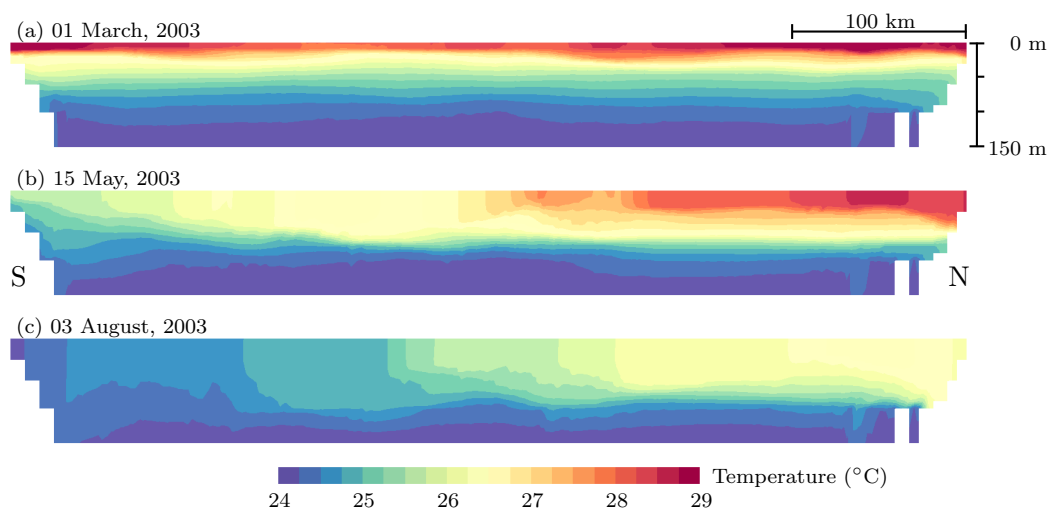


Fig. 5.14: South-north temperature transect on March 1st (a), May 15th (b) and August 3rd (c), 2003. Only the 150 upper metres vertical profile are displayed on the figure.

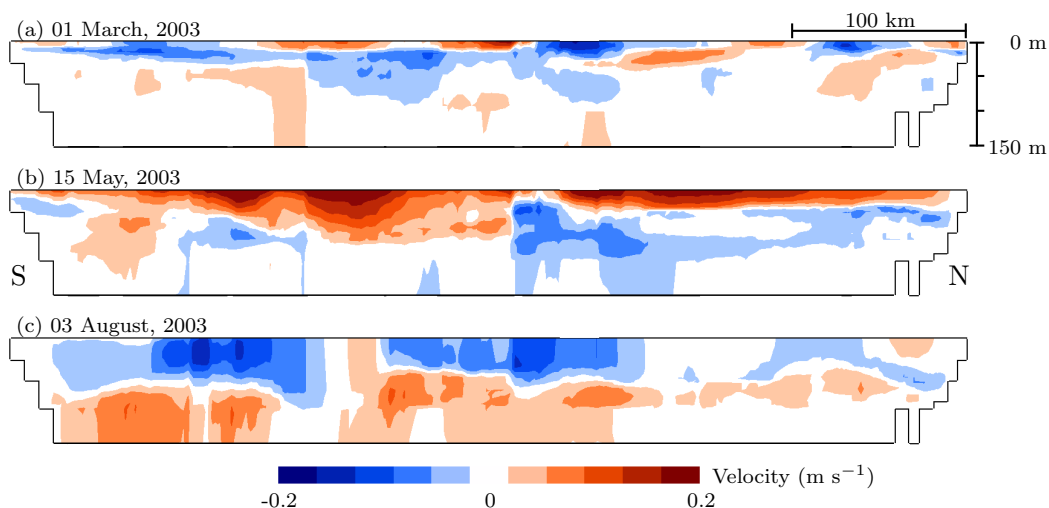


Fig. 5.15: South-north velocity transect on March 1st (a), May 15th (b) and August 3rd (c), 2003. Only the 150 upper metres vertical profile are displayed on the figure. Positive value means northward current.

alistically represents thermocline oscillations with the adaptive mesh, although a larger relaxation time parameter  $\tau_e$  is necessary to maintain a smooth mesh. In the preliminary simulations, the model runs without vertical diffusivity. In this configuration, a sharp thermocline is maintained under weak wind stress conditions. For a stronger wind that generates outcropping, the thermocline is slightly diffused near the southern tip of the lake. Indeed, when outcropping occurs, the thermocline is vertical at the border of the outcropping region, and its sharpness depends on the horizontal resolution, which is fixed, indepen-

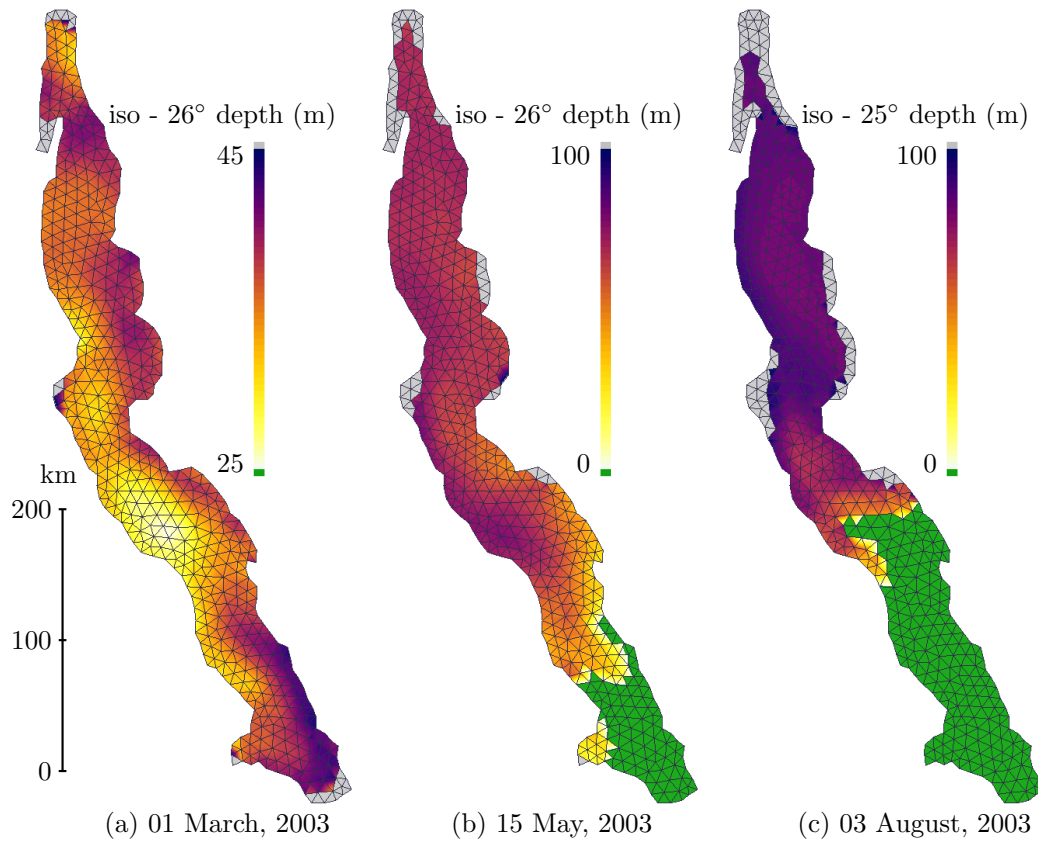


Fig. 5.16: Isotherm depths in Lake Tanganyika on March 1st (26°C), May 15th (26°C) and April 3rd (25°C). It is important to note that range of the color bar is not the same through the different maps. Regions in gray represent areas where the bottom water is warmer than the limit temperature. Green regions represent outcropping zones.

dently of the vertical adaptation. During outcropping, the vertical thermocline is thus diffused horizontally. After the wind stress decreases, the thermocline comes back to its original position, but the numerical mixing introduced during the outcropping period cannot be cancelled. However, the thermocline diffusion is much weaker with the adaptive mesh than using a fixed mesh (Figs. 5.10 and 5.11).

For the actual Tanganyika runs, vertical diffusivity and viscosity are taken into account. While data scarcity limits a complete validation, the comparison between the model and the available data indicates a good representation of the dynamics of Lake Tanganyika by SLIM 3D. The surface temperature used in the relaxation boundary condition does not match well with the vertical profiles available at Mpulungu and Kigoma, so the modelled vertical profile is biased by construction. But a comparison can still be achieved based on the modelled and observed patterns, such as stratification and thermocline position.

The south-north lake transects of Figure 5.14 are consistent with the results shown on Figure 8 from Verburg et al. (2011), which modelled the 1996 lake dynamics. In May, which corresponds to the beginning of the strong wind season, the 26.5°C isotherm outcrops at the middle of the lake in both studies, while the 25.5°C isotherm outcrops only at the southern tip of the lake. The agreement between SLIM 3D and Verburg et al. (2011) is also good at the end of this wind season, which corresponds to beginning of August in 2003 and July in 1996.

The southward surface current, going in the opposite direction to the wind suggested by Verburg et al. (2011) is also observed in SLIM results, although this current is most likely due to the weakening of the wind stress and pressure gradient pushing the water mass back to its equilibrium position. This comparison with the results of Verburg et al. (2011) for the year 1996 motivates to investigate further the lake circulation, for a longer period using weather forcings from the COSMO-CLM<sup>2</sup> model (Davin and Seneviratne, 2012). This will enable to study the occurrence of events during which there is a reversed overturning circulation.

The presence of internal Kelvin waves in the lake, which was first simulated by means of a 2D reduced-gravity model (Naithani and Deleersnijder, 2004) and then demonstrated using scaling arguments supported by laboratory and field investigations (Antenucci, 2005) is also shown with the 3D modelling (Fig. 5.16). Those waves are more visible during the wet season when the thermocline is almost horizontal.

## 5.4 Conclusions

The efficiency of the adaptation method defined in Chapter 3 was tested on simple benchmarks consisting in preserving a sharp interface between two layers of different densities. While the fixed mesh diffuses the interface and produces global errors in the hydrodynamics, the adaptive mesh is able to preserve the interface profile, by aligning thin levels along it. The DG formulation with the mesh adaptation controlled by the vertical jumps preserves the expected field discontinuity with minimal mixing. The necessary manual configuration of the adaptation parameters remains a limitation.

A new formulation for the computation of the mesh vertical velocity, both conservative and consistent, was developed for the adaptive mesh. It is noteworthy that this formulation solves the tracer consistency problem with and without adaptation.

The adaptation was then evaluated by modelling the oscillations of the Lake Tanganyika thermocline. First, a simulation was run without vertical diffusivity and a uniform wind stress, showing the good behaviour of the adaptive mesh. Then, a full simulation of the lake dynamics was performed and com-

pared to time series of vertical temperature profile in the south and the centre of the lake. Overall the outcropping events and the stratification observed on the data are well reproduced by the model. The remaining differences are partially due to discrepancies between the data used to force the surface heat flux and the validation data.

During the 2-year simulation, the along-axis velocity shows similar patterns to the results from [Verburg et al. \(2011\)](#). To understand better the interactions between the wind velocity, the surface heat flux and the water dynamics, additional simulations would be necessary. They could be achieved using the full data set available from the COSMO-CLM<sup>2</sup> model and the adaptive mesh model SLIM 3D. The model has a strong potential for different applications about the lake hydrodynamics, such as the impact of inter-annual variability and climate change.

## Acknowledgements

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## Conclusions and perspectives

Numerical modelling is an essential tool to study geophysical and environmental flows. It is of course useful for forecasting, but it also enables to test assumptions on the processes driving these flows, to conduct sensitivity analyses with respect to some of the parameters, etc. Numerical modelling is used in countless other applications, in which observations and measurements reach the limits of their potential.

My research work mainly consisted in contributing to two aspects of numerical modelling: the model validation and the development of a 3D DG finite element model for geophysical and environmental flows.

While model development was clearly my main research project, the importance of validation should not be neglected. Indeed, it measures the gap between the physical reality and its modelled representation. Using validation, a model is categorised into one of the following three groups: (1) it is not able to resolve a given phenomenon, (2) it is able to resolve it, (3) it is able to resolve it and to reproduce accurately the observed dynamics. Depending on the objective of the modeller, a model can be evaluated as successful if it belongs to group (2) or (3). The difficulty of fitting the validation data depends on the nature of this data. While the sea surface elevation evolves relatively slowly in time and space, temperature and salinity also have relatively slow time scales, but potentially exhibit large gradient. The most difficult challenge for a model is to fit velocity data, which evolve rapidly in time and show strong spatial gradients. Since the velocity field has complex spatial patterns, obtaining a high-resolution map of the velocity instead of a simple time series measured at a single point is a strong tool for validation. HF radars are able to build such maps in the immediate vicinity of the coast, but their action range is limited.

In other areas, the use of velocity maps derived from satellite imagery has a very strong potential. The necessity of such data is substantial. Since there is currently no systematic generation of satellite stereo images, it is not yet a full success, but when such data will become easily available, it will be of a great help to validate high-resolution models.

Modelling marine flows is not straightforward and the numerical methods are under constant development. Such development concerns various aspects of the physical, engineering and computational sciences. Numerically, new methods which are less dissipative, more stable, or enabling larger time steps emerge. Computationally, the methods are implemented in order to maximise the efficiency of the computer power. Physically, new parameterisations and new ways to assimilate phenomena or data are developed. For multiscale flows occurring in a complex topography, unstructured mesh models have proved their high potential and efficiency. The SLIM project takes place in this framework. SLIM aims to build a one-, two- and three- dimensional discontinuous Galerkin finite element model for geophysical and environmental flows, using unstructured meshes. The model has been applied to a number of different applications, showing its ability to deal with multiscale problems.

I worked on the SLIM 3D development in order to make it operational for complex applications. An important achievement was the stabilisation of the code, necessary for the development stage. In order to apply SLIM 3D to a baroclinic flow in a region with complex topography and bathymetry, the internal pressure gradient computation, which originally led to spurious velocity, needed to be modified; to develop a discontinuous bathymetry, necessary for the construction of  $z$ -levels, stability required a consistent connection between the 2D and the 3D modes; etc. After stabilising the model, more specific features could be developed for different applications, such as new boundary conditions, a sediment module and an adaptive mesh. The current version of the model is a stable code to which a developer can easily add new functionalities and assess their quality.

While the development of SLIM 3D was the clear objective of this thesis, the benefits for other models are also important. This work showed that vertical adaptive coordinates can reduce significantly the computational cost of finite element methods while maintaining a similar accuracy. This method could also be adapted to finite volume models following the same process. The discontinuous field which is a consequence of the discontinuous Galerkin formulation gave a natural and simple error estimation ideal for the moving mesh algorithm, particularly when strong gradients appear in the solution field. The discontinuous formulation also enabled to perform simple cheap operations, such as the horizontal smoothing of the mesh distribution or slope limiters, which are applied on a compact stencil. Therefore, the different numerical developments achieved in this thesis could be considered while developing new finite volume or finite element marine models.

The following sections detail more specific conclusions relative to the different projects I worked on.

## 6.1 HR satellite data and model validation

Chapter 2 presented how high-resolution satellite images could be used to obtain a comprehensive map of the surface flow velocity field. The 2-m resolution of the pair of images is unprecedented in oceanography. Combined by means of the particle image velocimetry (PIV) technique or the global optimal solution (GOS) method, those images led to a high-resolution velocity map resolving the tidal eddies and vortices downstream of small islands. This technique has an important potential to study flows at a high resolution, mostly to calibrate or validate numerical models. Here, the satellite velocity and vorticity maps were compared to a SLIM 2D simulation in the region of Beverlac and Hull islands, in the Great Barrier Reef (GBR) in Australia. The satellite imagery and model abilities to resolve patterns of different scales were evaluated.

In this study, we worked in collaboration with G. Marmorino and W. Chen, from the Naval Research Laboratory, USA. They processed the satellite images and produced the velocity maps using the PIV and GOS methods. My contribution consisted in modelling the GBR with a focus on the Beverlac and Hull area, and analysing and comparing the satellite-derived data with SLIM results.

The satellite velocity map, with a resolution of 64 m, reproduced tidal jets, eddies and other vorticity patterns larger than  $\sim 100$  m. Smaller patterns such as the small vortices in the free shear layers, which were observed on the satellite pictures, were not resolved by the velocity maps. This is due to the 64-m resolution of the map. To increase this resolution, given that the satellite pictures have a 2-m resolution, it would be necessary to reduce the 54-s time lapse between the two pictures rather than to increase further the image resolution. Indeed, the time scale of the  $\sim 10$ -m eddies in the free shear layers is of the order of a minute, and is thus too small to be resolved by the current stereo image.

SLIM 2D was successful in representing the different patterns of the flows up to a 100-m resolution. It is noteworthy that, while the modelled larger patterns ( $> 1$  km) were in good agreement with the satellite-derived results, the smaller patterns obtained from the satellite data and the numerical modelling had similar features but did not match accurately. There are numerous sources for those differences: errors in the topography and bathymetry representation; errors in the input forcing such as the wind stress or the boundary conditions; errors in the parameterisations such as the bottom drag coefficient; simplified assumptions on the flow such as the 2D depth-averaged approach or the hydrostatic hypothesis. But at a resolution of tens of metres and in presence of

non-hydrostatic processes, it is important to note that solving all those problems will not save the modelling. Data assimilation would be necessary for an accurate modelling of the hydrodynamics at this scale.

Satellite stereo imagery reveals a large variety of phenomena of different scales, which can be used to validate high-resolution models. It is noteworthy that the satellite used here, World-View 3, is not initially designed for oceanography. While in the future, special efforts should be made in obtaining those images using a smaller time lapse between the two pictures, the first priority is to collect more data similar to those used for this study. It is crucial to apply this technology to different regions and set-ups, in order to evaluate how robust and efficient this new tool is.

## 6.2 Burdekin River sediment transport

To simulate the transport and fate of the Burdekin River fine sediment, stabilising SLIM 3D for complex topography and bathymetry was necessary. The main developments undertaken in the model for this application were the change of formulation for the internal pressure horizontal gradient and a new slope limiter. They are described in Chapter 3. The SLIM 3D simulation of the Burdekin River plume hydrodynamics and sediment transport, detailed in Chapter 4, confirmed the sediment deposition areas from [Lewis et al. \(2014\)](#). Moreover, it particularly highlighted what the settling process is. Unlike previous studies, it showed that the key event controlling sediment transport is the flood season, rather than the wind events occurring during the dry season.

For this study, a simple sediment module was implemented into SLIM 3D. This module is the straightforward 3D extension of a sediment module used for a 2D study of the sediment transport in Cleveland Bay, 100 km north of the Burdekin River mouth. This model has the advantage of being already calibrated. However, if more data were available, it should be improved. In particular, the erosion rate is a function of the wind velocity in the 2D model. This function is a proxy for the waves and, hence, for the bottom turbulence. This proxy should not be used in the 3D model. The erosion rate should be a function of the turbulent kinetic energy.

This model could also be used to study the transport of different types of sediment. Very fine sediment ( $< 4\mu\text{m}$ ) flocculates and is then able to travel long distances before settling ([Bainbridge et al., 2012](#)), as the floc density only slightly differs from that of salt water. A model representing a larger diversity of sediment types would help to build a more comprehensive representation of the Burdekin sediment transport.

In the current version of the sediment code, it is noteworthy that the sediment accumulation on the sea bed does not affect the bathymetry and, hence, does not have any impact on the hydrodynamics. This does not have signifi-

cant impact in the GBR where the deposition rate is quite low. But in other applications, it could be relevant. Modifying the bathymetry itself is no difficult task, but two important constraints cannot be circumvented. First, this bathymetry modification must be achieved in a conservative way, such that no mass is lost. Second, SLIM 3D currently needs a bathymetry minimal value in order to remain stable. Strong bathymetry modifications in shallow areas would then need a wetting and drying algorithm that manages small water columns or even dry areas.

## 6.3 Thermocline oscillations of Lake Tanganyika

In Chapter 5, SLIM 3D was used to simulate thermocline oscillations in a lake. Following the development of a vertically adaptive mesh for SLIM 3D, described in Chapter 3, simple benchmarks simulating the oscillations of a sharp interface were run in a 2D “ $x$ - $z$ ” domain. Those benchmarks assessed the ability of the mesh to adapt such that the sharp interface is not smeared out during the simulation. The adaptation was controlled by the water column stratification. Through those applications, the adaptive mesh approach was validated. It showed that for a similar accuracy, the number of elements and thus the computational cost could be reduced by a factor of about 16.

The model was then used to simulate the Lake Tanganyika hydrodynamics. The model was forced by surface wind stress and surface temperature. The results compared well with key features of the lake dynamics, such as dry and wet season stratification, outcropping events or internal Kelvin wave propagation (Naithani and Deleersnijder, 2004). Moreover, the results were compared to a previous study of Verburg et al. (2011). Although Verburg simulated the 1996 lake dynamics and SLIM 3D reproduced the 2002 - 2004 period, the results could be compared by analysing the similarities between the temperature and velocity profiles obtained from both studies.

Verburg et al. (2008) proposed a new model of the Lake Tanganyika overturning circulation, in which the surface water flows against the wind, due to the surface heat flux patterns. Verburg et al. (2011) modelled such circulation during the end of the 1996 wind season. In very specific situations, SLIM 3D also reproduced it, but the modelling results do not allow to draw strong conclusions on Verburg et al. (2008) theory. A study over a longer period would help to analyse this phenomenon and its frequency. Using SLIM 3D and the COSMO-CLM<sup>2</sup> wind and surface temperature data, which are available for a 13-year period, a comprehensive study of the lake overturning circulation can be achieved.

To improve the model quality, a more complex parameterisation of the surface heat flux could be implemented. For the moment, this flux is a simple

relaxation to the surface temperature data applied in the surface layer. But different types of heat flux are available among the COSMO-CLM<sup>2</sup> data and could be used for a better modelling.

Finally, it is noteworthy that while the adaptive mesh method shows excellent results for thermocline oscillations modelling, it still suffers from some limitations. First, the adaptation process needs to be manually calibrated in order to obtain a smooth mesh with an acceptable vertical resolution distribution. Second, the adaptive mesh has only been used for interface oscillation dynamics and a simple 2D “ $x$ - $z$ ” idealised river plume modelling. To obtain a more robust method, it is necessary to apply it to different set-ups. In particular, an application in which the mesh resolution increases in various parts of the water column would test the method robustness. After the method evaluation on those set-ups, an automated value for the time relaxation controlling the adaptation could be developed.

## 6.4 General perspectives

For the fulfilment of this PhD thesis, I worked on the development of SLIM 3D, a complex 3D numerical model for geophysical and environmental flows. In the previous sections, features that would improve the model were mentioned. Besides those changes, there are many larger developments that could lead to a better modelling. But the countless possible challenges and the finite size of the SLIM team require to select what would be the most important ones.

During the last ten years, new numerical methods have constantly been implemented. In its conclusions, [Kärnä \(2012\)](#) proposed the development of a new multirate integration method and to increase the order of the DG basis functions. The current P1 DG finite element approach combines advantages of both finite volumes and finite elements. But it also has the limitations of both methods: a relatively low order similar to finite volumes and the computational cost of finite elements. However, other models such as Thetis are specifically designed to investigate what is the best finite element discretisation for marine models. Such development must not necessarily be done with SLIM.

The main question to address before developing further the model is thus to define what we expect from it.

The next challenge for SLIM 3D should be to further increase the capability of the model to simulate complex hydrodynamics. In particular, SLIM aims to simulate the land-sea continuum, which is the main objective of the project.

To fulfil it, it is necessary to connect the one-, two- and three- dimensional components of the model. The 1D model will be used to model the river network composing the catchment area of an important estuary. The 2D model will be used in the sea where stratification is negligible and the water current

is vertically homogeneous, and the 3D model will be used in the estuary, the region of fresh water influence and areas where the circulation has an important vertical distribution. Special efforts should be made in implementing a smooth connection between the different model components.

Finally, when the model is implemented, the work is not finished. A hard task is, even before running the model, to obtain accurate input data. First, topographical data are necessary to create the mesh: coastlines definition and bathymetry. Second, input data such as wind forcing, river flows and salinity, surface heat fluxes and open sea boundary conditions greatly impact the simulation. The modelling errors due to problems with the data can often be larger than the errors due to the model hypotheses or the numerical dissipation. Assuming the availability of data, assimilating it in the modelling process could have a significant impact on the result accuracy. The available data should also be used to parameterise the unresolved processes in order to improve the model quality without increasing the complexity of the model physical equations.

Such development should be achieved keeping an “application-oriented” strategy.





## Supplementary material for Chapter 2

### A.1 Total vorticity flux in the free shear layer

In this section, it is shown that the total vorticity flux in the lee of a backward facing step computed using two different finite element methods, respectively a P1 continuous Galerkin method and a P1 discontinuous Galerkin method (such as SLIM), is insensitive to the numerical method and mesh resolution if computed correctly. This appendix is inspired from [Deleersnijder \(2016\)](#) and D.P. Marshall (2006, personal communication to E. Deleersnijder), who studied the sensitivity of the total vorticity flux to various finite difference numerical schemes, obtaining the same conclusions as for the finite element schemes detailed hereinafter.

Let assume a flow propagating alongside a wall (see Fig. [A.1](#)), with a velocity  $u$ . This velocity is zero on the wall ( $y = 0$ ). A boundary layer develops in the  $y$ -direction. The vorticity is  $\omega = -\partial u / \partial y$  and the total vorticity flux is:

$$\Phi = \int_0^\infty u \omega \, dy = -\frac{u_\infty^2}{2}. \quad (\text{A.1})$$

#### CG formulation

The 1D domain is discretised into linear elements. On element  $e$ , the two nodes are numbered  $e$  and  $e + 1$  and the finite element solution is  $\hat{u} = \sum_{i=0}^1 U_{e+i} \phi_i^e$ , where  $\phi_0^e$  and  $\phi_1^e$  are the piecewise linear shape functions, equal to 0 everywhere except at node  $e$  and  $e + 1$ , respectively.  $\hat{\phi}_0$  and  $\hat{\phi}_1$  are the shape

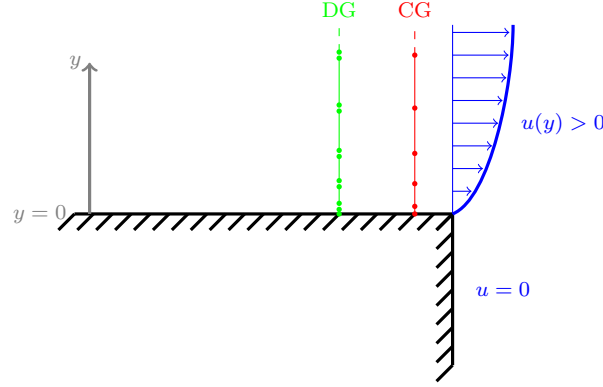


Fig. A.1: Vorticity generated in a boundary layer. Simple 1D test case.

functions on the parent element. The approximated vorticity is defined as  $\hat{\omega} = -\sum_{i=0}^1 U_{e+i} \partial \phi_i^e / \partial y$ . The approximated vorticity flux  $\hat{\Phi}$  is then:

$$\hat{\Phi} = -\sum_{e=0}^{\infty} \int_{\Omega_e} \hat{u} \frac{\partial \hat{u}}{\partial y} dy \quad (\text{A.2})$$

$$= -\sum_{e=0}^{\infty} \int_{\Omega_e} \sum_{i=0}^1 \sum_{j=0}^1 U_{e+i} \phi_i^e U_{e+j} \frac{\partial \phi_j^e}{\partial y} dy \quad (\text{A.3})$$

$$= -\sum_{e,i,j} U_{e+i} U_{e+j} \int_{\hat{\Omega}_e} \hat{\phi}_i \frac{\partial \hat{\phi}_j}{\partial \xi} d\xi \quad (\text{A.4})$$

$$= -\sum_{e=0}^{\infty} \left( \frac{U_{e+1}^2}{2} - \frac{U_e^2}{2} \right) \quad (\text{A.5})$$

$$= -\frac{U_{\infty}^2}{2}. \quad (\text{A.6})$$

## DG formulation

Again, the domain is discretised into linear elements. On element  $e$ , the two nodes are numbered  $2e$  and  $2e+1$ . This time, the vorticity can not be defined as for the CG formulation, since the nodes are discontinuous at the element interfaces. This discontinuity has to be accounted for properly. On element  $e$ , vorticity  $W_{2e}$  and  $W_{2e+1}$  are computed resolving:

$$\sum_{j=0}^1 W_{2e+j} \int_{\omega_e} \phi_j^e \phi_i^e dy = \sum_{j=0}^1 U_{2e+j} \int_{\omega_e} \phi_j^e \frac{\partial \phi_i^e}{\partial y} dy - [u^* \phi_i], \quad i = 0, 1. \quad (\text{A.7})$$

The only way to obtain the correct vorticity flux is by defining the following  $u^*$  at the interfaces as the mean of the two nodal values:

$$u^* = \frac{U_{\text{top}} + U_{\text{bot}}}{2}, \quad (\text{A.8})$$

where  $U_{\text{top}}$  and  $U_{\text{bot}}$  are the nodal values above and below the interface, respectively. Now the vorticity flux can be computed easily:

$$\hat{\Phi} = \sum_{e,i,j} U_{2e+i} W_{2e+j} \int_{\Omega_e} \phi_i^e \phi_j^e dy \quad (\text{A.9})$$

$$= \sum_{e=0}^{\infty} \frac{1}{2} \left[ - (U_{2e+1} U_{2e+2}) + (U_{2e-1} U_{2e}) \right] \quad (\text{A.10})$$

$$= -\frac{U_{\infty}^2}{2}. \quad (\text{A.11})$$

Provided that the velocity at the element interfaces is computed correctly and that the velocity outside the boundary layer is correct, the DG finite element method computes accurately the vorticity flux, even if the local velocity in the boundary layer is under-resolved or even wrong.

## A.2 Extra figures

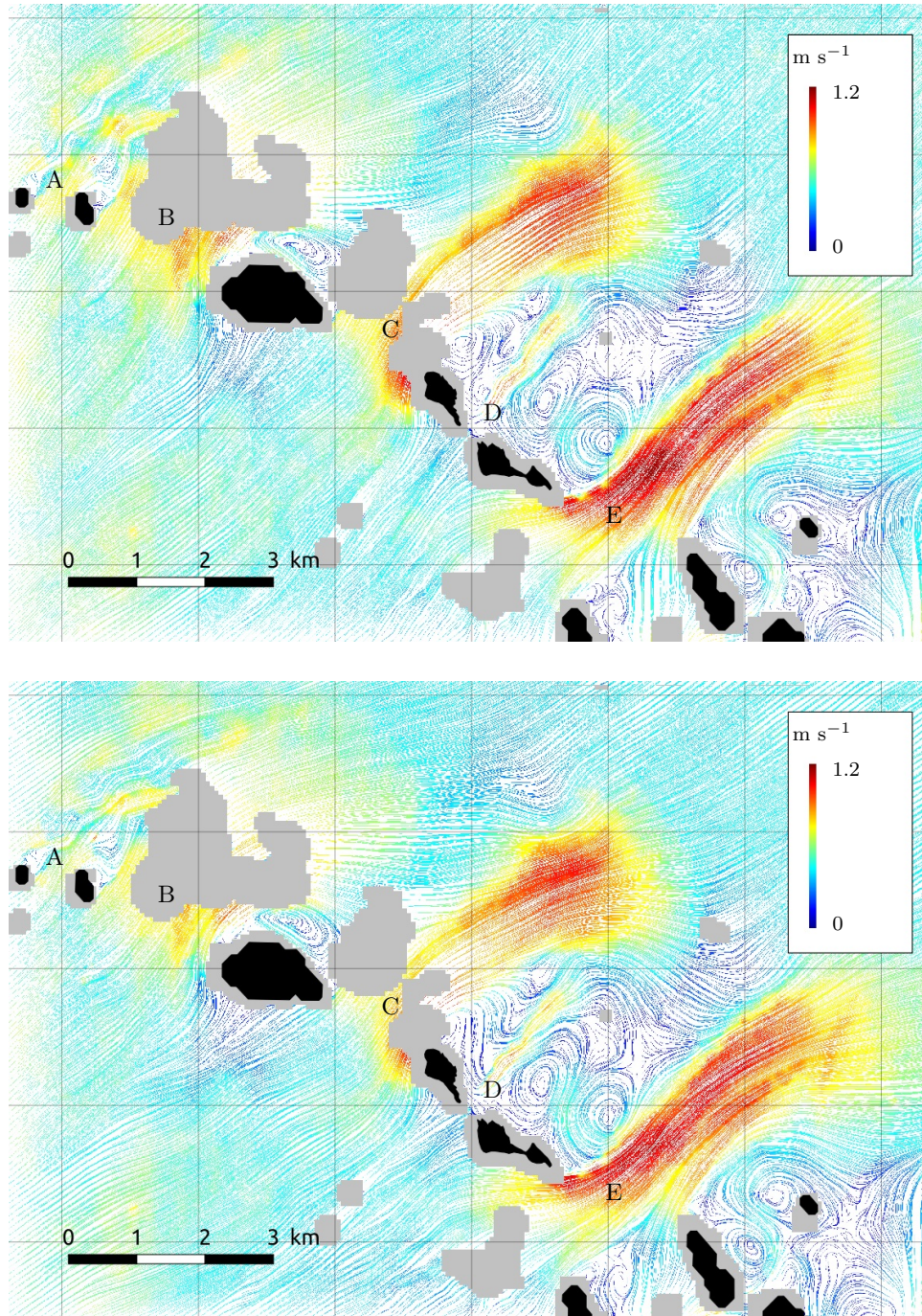


Fig. A.2: Effect of a 2% wind shift on the velocity map of PIV data. While top map shows the original velocities computed with the PIV method (same as in Fig. 2.7), the bottom map removes a 2% contribution of the wind velocity to the PIV velocity.

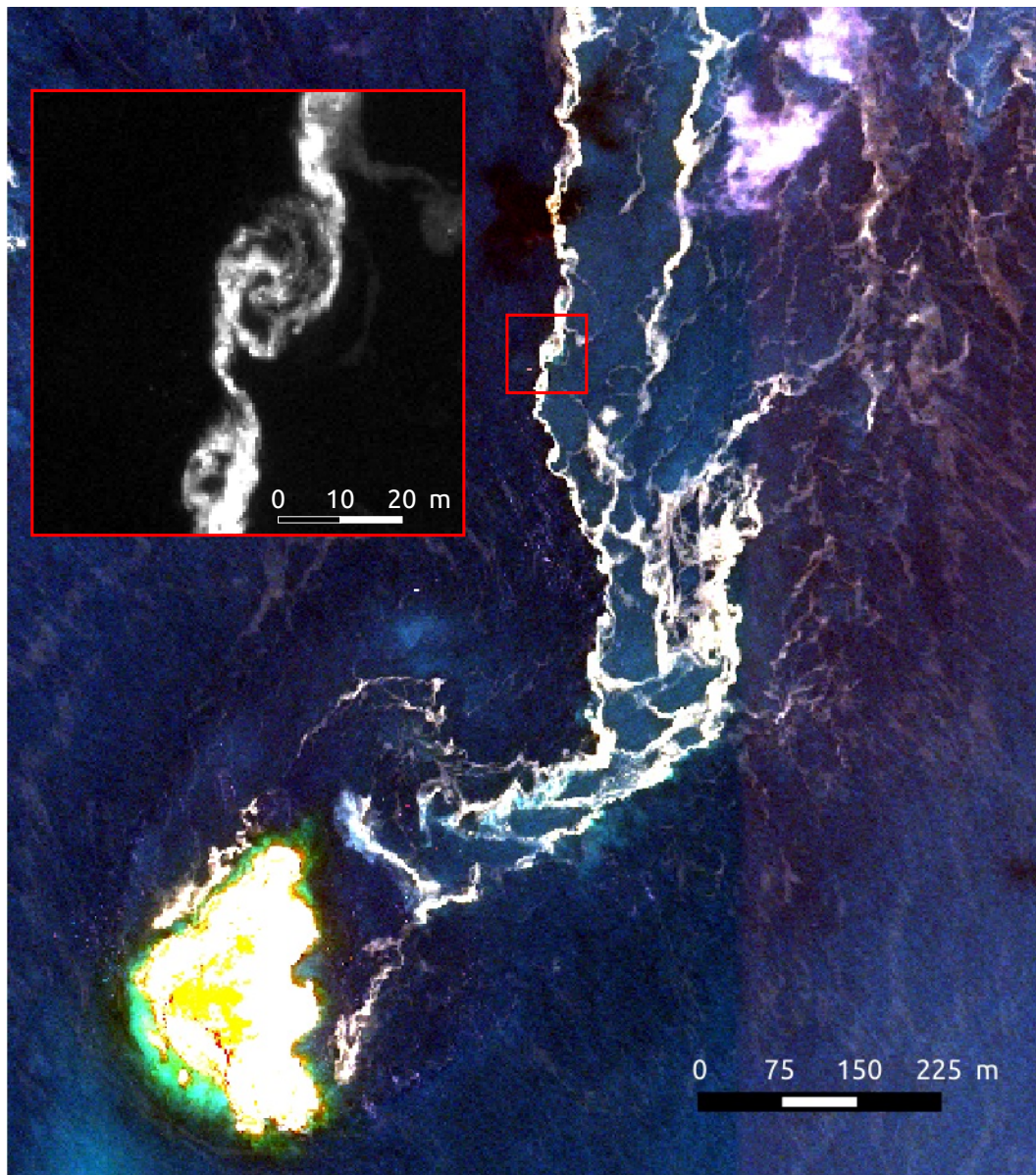


Fig. A.3: Small eddies in the free shear layer downstream of Renou Island, located on the east side of gap A (Fig. 2.3). Inset shows 0.5 m resolution panchromatic imagery within a zoomed area (red box). The pan data reveal clockwise swirls of algae, consistent with shear-induced eddies of about 10-m diameter.



# Supplementary material for Chapter 4

## B.1 Sediment model in SLIM 3D

The sediment model is a 3D extension of the model developed by [Lambrechts et al. \(2010\)](#). The sediment concentration  $C$  is a passive tracer which is advected and diffused. The main unknowns are:

| variable | definition                                                       |
|----------|------------------------------------------------------------------|
| $C$      | suspended sediment Concentration ( $\text{mg l}^{-1}$ )          |
| $D$      | sediment Deposit concentration ( $\text{mg m}^{-2}$ )            |
| $E$      | Erosion and deposition flux ( $\text{mg m}^{-2} \text{s}^{-1}$ ) |

Table B.1: Main unknowns of the sediment model

Using the variables and parameters defined in Tables [B.2](#) and [B.3](#), the

governing equations are:

$$\begin{aligned} \text{On } \Omega : \quad & \frac{\partial C}{\partial t} + \nabla_h \cdot (\mathbf{u}C) + \frac{\partial((w + w_s)C)}{\partial z} = \nabla_h \cdot (\kappa_h \nabla_h C) + \frac{\partial}{\partial z} \left( \kappa \frac{\partial C}{\partial z} \right), \\ \text{On } \Gamma_b : \quad & -\kappa \frac{\partial C}{\partial z} = E, \end{aligned} \quad (\text{B.1})$$

$$\text{On } \Gamma_b : \quad \frac{\partial D}{\partial t} = E, \quad (\text{B.2})$$

$$\begin{aligned} E = a_1 C w_s \max(1 - \beta^2, 0) - \delta_{D>0} & \left( a_2 \max(\beta^4 - 1, 0) \right. \\ & \left. + a_3 \left( \frac{w}{w_0} \right)^3 \beta^4 \exp(-f_0 \omega^2 (H + \eta)/g - f_1) \right), \end{aligned} \quad (\text{B.3})$$

where  $\Omega$  is the domain and  $\Gamma_b$  the bottom boundary. Sediment depot  $D$  is only defined at  $\Gamma_b$ . At the open boundaries, sediment leaves the domain with currents pointing outward, but there is no inflow of sediment with currents pointing inward. In Equation B.3, the first term on the right hand side is the deposition flux, the second term is the erosion flux due to entrainment, and the last term is due to wave induced fluidisation of the mud.

The settling vertical velocity  $w_s$  is defined by:

$$w_s = -\delta_{w < w_0} \min(a_0 C, w_{s0}). \quad (\text{B.4})$$

| variable           | definition                                                         |
|--------------------|--------------------------------------------------------------------|
| $w_s$              | settling vertical velocity ( $\text{m s}^{-1}$ )                   |
| $\mathbf{w}$       | wind velocity: $\mathbf{w} = (w_x, w_y)$ ( $\text{m s}^{-1}$ )     |
| $w$                | wind speed: $w = \ \mathbf{w}\ $ ( $\text{m s}^{-1}$ )             |
| $\mathbf{u}_b$     | bottom water velocity ( $\text{m s}^{-1}$ )                        |
| $u_b$              | bottom water speed: $u_b = \ \mathbf{u}_b\ $ ( $\text{m s}^{-1}$ ) |
| $\beta$            | $u_b/u_0$                                                          |
| $g$                | gravitational acceleration ( $\text{m s}^{-2}$ )                   |
| $\delta_{w < w_0}$ | $= 1$ is $w < w_0$ , else 0                                        |
| $\delta_{D > 0}$   | $= 1$ is $D > 0$ , else 0                                          |

Table B.2: Notations for the equations resolved by the sediment model

| variable | definition               | value                                                                                  |
|----------|--------------------------|----------------------------------------------------------------------------------------|
| $w_{s0}$ | settling speed threshold | $2 \cdot 10^{-4} \text{ m s}^{-1}$                                                     |
| $w_0$    | wind speed threshold     | $10 \text{ m s}^{-1}$                                                                  |
| $\omega$ | wave frequency           | $1.57 \text{ Hz}$                                                                      |
| $a_0$    |                          | $10^{-5} \text{ m l mg}^{-1} \text{ s}^{-1}$                                           |
| $a_1$    |                          | $10^3 \text{ l m}^{-3}$                                                                |
| $a_2$    |                          | $1 \text{ mg m}^{-2} \text{ s}^{-1}$                                                   |
| $a_3$    |                          | $a_{30} + (a_{31} - a_{30}) \left(0.5 - \frac{\arctan(10\alpha)}{2\arctan(10)}\right)$ |
| $\alpha$ | wind orientation factor  | $\alpha = (\mathbf{u}_b \cdot \mathbf{w}) / (u_b w)$                                   |
| $a_{30}$ |                          | $6.86 \cdot 10^3 \text{ mg m}^{-2} \text{ s}^{-1}$                                     |
| $a_{31}$ |                          | $3.528 \cdot 10^4 \text{ mg m}^{-2} \text{ s}^{-1}$                                    |
| $f_0$    |                          | $0.905$                                                                                |
| $f_1$    |                          | $-0.0207$                                                                              |

Table B.3: Values of the different parameters used in the sediment model

## B.2 Flood data

| Site ID | date          | latitude | longitude | Salinity |         | SSC (mg/l) |         |
|---------|---------------|----------|-----------|----------|---------|------------|---------|
|         |               |          |           | data     | SLIM 3D | data       | SLIM 3D |
| 1       | Feb 06, 13:30 | 19°40.9' | 147°37.5' | 1        | 1       | 348        | 287     |
| 2       | Feb 06, 14:10 | 19°40.6' | 147°38.6' | 10       | 2       | 11         | 263     |
| 3       | Feb 06, 14:25 | 19°40.4' | 147°38.7' | 21       | 3       | 10         | 252     |
| 4       | Feb 06, 14:40 | 19°39.9' | 147°38.8' | 21       | 5       | 69         | 226     |
| 5       | Feb 06, 15:20 | 19°39.6' | 147°37.6' | 1        | 9       | 225        | 158     |
| 6       | Feb 06, 15:35 | 19°39.1' | 147°37.7' | 4        | 10      | 45         | 101     |
| 7       | Feb 06, 15:40 | 19°38.6' | 147°37.7' | 5        | 11      | 27         | 66      |
| 8       | Feb 06, 15:55 | 19°38.2' | 147°38.1' | 8        | 12      | 10         | 48      |
| 9       | Feb 06, 16:40 | 19°29.6' | 147°32.7' | 9        | 19      | 20         | 18      |
| 10      | Feb 06, 16:50 | 19°28.2' | 147°31.9' | 11       | 20      | 11         | 15      |
| 11      | Feb 06, 17:05 | 19°26.5' | 147°30.7' | 15       | 20      | 8          | 12      |
| 12      | Feb 06, 17:25 | 19°24.8' | 147°29.8' | 18       | 21      | 36         | 10      |
| 13      | Feb 06, 17:50 | 19°17.5' | 147°24.9' | 19       | 22      | 9          | 7       |
| 14      | Feb 07, 7:20  | 19°18.2' | 147°22.2' | 26       | 31      | 34         | 2       |
| 15      | Feb 07, 7:45  | 19°17.0' | 147°19.8' | 25       | 30      | 7          | 11      |
| 16      | Feb 07, 8:13  | 19°16.5' | 147°16.4' | 26       | 30      | 5          | 16      |
| 17      | Feb 07, 8:35  | 19°15.5' | 147°14.4' | 26       | 30      | 2          | 17      |
| 18      | Feb 07, 8:53  | 19°14.7' | 147°12.1' | 28       | 29      | 4          | 11      |
| 19      | Feb 07, 9:10  | 19°15.2' | 147°9.3'  | 30       | 29      | 6          | 8       |
| 20      | Feb 07, 9:30  | 19°15.2' | 147°4.3'  | 27       | 29      | 3          | 8       |
| 21      | Feb 07, 10:00 | 19°11.4' | 147°3.0'  | 29       | 29      | 3          | 17      |
| 22      | Feb 07, 10:20 | 19°9.7'  | 146°59.0' | 30       | 30      | 5          | 9       |
| 23      | Feb 07, 10:45 | 19°8.5'  | 146°54.8' | 30       | 31      | 7          | 7       |

Table B.4: Comparison between measured ([Bainbridge et al., 2007](#)) and simulated sea surface salinity and sea surface suspended sediment concentration.

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