

A new track for unifying general relativity with quantum field theories

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Abstract

In the perspective of unifying quantum field theories with general relativity, the equations of the internal dynamics of the vacuum and mass structures of a set of interacting particles are proved to be in one-to-one correspondence with the equations of general relativity.

This leads to envisage a high value for the cosmological constant, as expected theoretically.

Chapter 1

Introduction

It is generally believed that a convincing theory of quantum gravity [Ash], [Alv], [Dew], [Wal], [Whe] will emerge from some unification of quantum field theory with general relativity.

This objective motivated the creation of the standard perturbative approaches based on Feynman perturbation theory for graviton modes of the form $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ where $h_{\mu\nu}$ refers to a small excitation of a flat metric [Pag], [E-G-H], [Hoo2]. But, these models failed because they are perturbatively nonrenormalizable [Nie], [Gro], [D-V].

In order to overcome these problems, string theories [G-S-W], [Pol], [Wit2] and loop quantum gravity [R-S], [Smo], [Bae] were created in such a way that point like models of elementary particles be replaced by one-dimensional models.

In spite of an intense activity in this field and evident successes, the contact with experiment still remain nebulous and, furthermore, the conceptual framework of (super)string theories has not reached the expected maturity degree.

So, as G. 't Hooft [Hoo1] pointed out, it might be that quantum gravity could not be solved without revising the principles of quantum theory and, especially, at the Planck scale.

As a matter of fact, it is commonly admitted that Lorentz invariance could be broken at the Planck length due to the strong fluctuations of the space-time at this scale [H-S-L-S], [Wit1].

To go beyond these difficulties, it was envisaged in [Pie3] and in [Pie5] **to enlarge the conceptual framework of quantum field theories** in order that the new proposed quantum model takes into account the quantum Physics below the Planck scale and corresponds to quantum field theories at the scale for which they were developed [Ati].

But, to become acceptable, the enlargement of the conceptual frame of quantum field theories has to imply some unification of these with the equations of general relativity.

The problem is that general relativity is a classical theory having resisted until now to a convincing quantization.

It is thus the aim of this paper to reconsider the matter in question on new basis at the light of what can bring close together general relativity and quantum field theories.

What characterizes general relativity with respect to a possible quantization is:

1. The influence of the space-time curvature on the matter and vice versa.
2. The existence of the cosmological constant λ interpreted by Y. Zel'dovich [Zel1] as a medium tied to the vacuum polarization of the QFT (quantum field theory).

Reciprocally, one of the main features of QFT which could be used to quantize GR (General relativity) consists in the creation of (pairs of) particles from the vacuum energy which reacts backwards on the curvature of the space-time: this means that the energy of the created particles is taken away from the space-time itself [Dew].

In this respect, as the quantum level is essentially that of elementary particles, it was envisaged [Pie1] by the author to set up equations describing the (internal) dynamics of a set of elementary particles in such a way that these equations be isomorphic to the equations of general relativity with respect to a metric contraction given by a suitable compactification from the quantum level to the classical level and associated with the condition $\hbar \rightarrow 0$.

To fulfil these conditions **in order to bridge the gap between QFT and GR**, it was envisaged that **the expanding space-time**, to which the cosmological constant of GR may correspond, **could constitute the fundamental structure of the QFT vacuum shared out amongst the considered elementary particles.**

In this context, every elementary particle would then be characterized by an internal expanding space-time structure constituting its own vacuum from which its matter shell could be generated due to the strong fluctuations of the internal space-time of its internal vacuum.

Indeed, at this scale which likely corresponds to the Planck scale, the fluctuations of the space-time generate local strong curvatures which are responsible for the generation of degenerate singularities allowing by versal deformations and blowups of these to create two external contracting enveloping structures above the particle internal vacuum in such a way that the most external structure is interpreted as its mass shell.

So, an important part of the vacuum of QFT would be of space-time nature distributed amongst **the internal vacua of the elementary particles** constituting massless parti-

cles **potentially able to generate their mass shells** due to the strong fluctuations of their internal space-time vacua.

Thus, the attempt of unifying QFT with GR leads us to **envisage elementary particles endowed with internal structures which must be of space-time type and quantized**.

The first step, developed in chapter 1, then consists in **generating mathematically space and time internal fields that must be really quantized**: this can be only realized algebraically by assuming that **quanta must have an algebraic structure given by algebraic closed subsets** characterized by a Galois extension degree N , $N \in \mathbb{N}$.

As extensively developed in [Pie5], the internal fields of elementary particles have a **twofold nature** due to the fact that **every elementary particle has to be viewed as an elementary bisemiparticle** composed of a left semiparticle, localized in the upper half space, and of a right symmetric (co)semiparticle, localized in the lower half space in such a way that:

- a) the right semiparticle, dual to the left semiparticle, is observed as projected on the latter and is thus normally unobservable.
- b) the product of the internal structure of the right semiparticle by its left equivalent gives rise to a “working interaction space” responsible for the electric charge and the magnetic moment.

In this respect, the internal space-time structure of the vacuum of a bisemiparticle will be composed of an internal time field and of an internal space field having a bilinear nature and being localized in orthogonal spaces.

The time internal field (as well as the space internal field) is then given mathematically by a (bisemi)sheaf of differentiable bifunctions over an algebraic bilinear semigroup: a bifunction is defined as the product of a right function localized in the lower half space by its symmetric left equivalent and the considered algebraic bilinear semigroup is $GL_2(F_{\overline{v}} \times F_v)$ covering its linear correspondent [Pie2] and defined over the product $(F_{\overline{v}} \times F_v)$ of the set of pairs $\{F_{\overline{v}_{\mu, m_{\mu}}}, F_{v_{\mu, m_{\mu}}}\}$ or right and left real ramified algebraic subsets having a structure at μ quanta of degree

$$[F_{\overline{v}_{\mu, m_{\mu}}} : K] = [F_{v_{\mu, m_{\mu}}} : K] = \mu \cdot N$$

over a global number field K of characteristic 0 .

The representation space $\text{Repsp}(GL_2(F_{\overline{v}} \times F_v))$ of $GL_2(F_{\overline{v}} \times F_v)$ then consists in the product, right by left, of **two symmetric towers of conjugacy class representatives**

in such a way that the functional representation subspaces of the products of **the corresponding right and left compactified conjugacy class representatives behave like harmonic oscillators**. This leads us to consider that **the functional (modular) representation space of $\mathrm{GL}_2(L_{\bar{v}} \times L_v)$** , where $L_{\bar{v}}$ (resp. L_v) results from the compactification of $F_{\bar{v}}$ (resp. F_v), **can be interpreted as a time internal field of an elementary bisemiparticle**.

The corresponding internal space field can be obtained from the internal time field throughout a smooth (bi)endomorphism based on Galois antiautomorphisms as developed in proposition 2.7. The end of chapter 1 deals with **the compactification of the conjugacy class representatives of the “time” and “space” bilinear algebraic semigroups** leading to continuous upper and lower 4-dimensional (semi)manifolds of space-time.

So, **the consideration of algebraic quanta at a sub-Planckian scale allows to generate time and space quantized internal fields of the internal vacua** of elementary particles likely at the Planck scale.

These time and space internal fields are products, right by left, of time and space semifields restricted to the lower and upper half spaces and described mathematically by (semi)sheaves of differentiable functions over the appropriate algebraic bilinear semigroups.

Due to the strong fluctuations of space-time at this level, these differentiable functions become afflicted by degenerate singularities of which versal deformations and blowups allow **to generate two covering “middle-ground” and “mass” fields of space and time**.

The internal vacuum of an elementary (bisemi)particle is thus described mathematically by a space and time internal field and by an enveloping “middle-ground” space and time field. A process of versal deformations and blowups of the singularities on the “middle-ground” space and time fields is responsible for the creation or **generation of the enveloping “mass” fields of space and time, phenomenon corresponding to the creation of this (bisemi)particle from its own vacuum**.

The generation of the middle-ground and mass fields of an elementary particle from its internal space and time fields constitute the contents of chapter 3, which allows to formulate **the equations of the internal dynamics of a bisemifermion**.

On the other hand, the unification of quantum field theory with general relativity, leading to a coherent quantum gravity theory, requires **a new interpretation of the equations of general relativity at the Planck scale**.

These objectives will be reached if it is taken into account that:

1. an **initial state mathematically well-defined** must be introduced “at the beginning” in such a way that:

- (a) it corresponds to some **universal structure** in the perspective of the Langlands program by a one-to-one correspondence with its automorphic representation.
 - (b) it be of (bilinear) space-time nature.
 - (c) its **space-time structure be totally quantized** from an algebraic point of view.
 - (d) it gives rise to the fundamental structure of the **internal vacua of the elementary particles**.
 - (e) it be directly **connected to the (small) value of the cosmological constant Λ** [Wei2].
2. **the interactions between elementary particles** must appear naturally (i.e. without divergences) in the model by considering:
- (a) bilinear interactions between pairs of semiparticles as it was developed in chapter 5 of [Pie5].
 - (b) a unification of electro-magnetism with gravitation as it was hoped by Einstein [Ein2].

In this new context, a special importance is thus given to the structure of elementary particles whose space-time nature is quantized and less to the evolution of states of these particles as currently done in quantum field theories and (super)string theories.

The idea then consists in building up a quantum gravity theory at the level of elementary particles in such a way that:

1. **the space-time structure**, constituting the “initial” state **of each elementary particle**, corresponds to its internal vacuum structure of expanding nature from which its mass structure of contracting nature can be generated dynamically from the singularities on its internal vacuum structure submitted to strong fluctuations at the Planck scale.
2. **the set of equations of vacuum and mass structure of elementary particles is in one-to-one correspondence with the equations of general relativity** if:
 - the big points, describing the quanta, at the Planck scale are viewed as ordinary points at the classical level by means of a metric contraction based on a condition equivalent to $\hbar \rightarrow 0$.

- the one-dimensional components of the time and space structures of the elementary particles, considered as flow lines, are compactified to give rise to a four dimensional manifold endowed with a Riemann geometry.

The equations of general relativity, generated in that manner at the Planck scale, are characterized by a **high value of the cosmological constant**, corresponding to the set of internal vacua of elementary particles, in contrast with the standard interpretation of these equations where the cosmological constant, being very small [Edd], is used to adjust the geometry in order that it be on a average flat.

Chapter 4 is then devoted to the set up of the equations of the internal dynamics of a set of interacting particles and to the proof of their equivalence with the equations of general relativity in the perspective of unifying quantum field theories with general relativity.

All developments of this paper refer to the preprint “Algebraic quantum theory” [Pie5].

Chapter 2

The space-time structure of the internal vacua of elementary (bisemi)fermions

A first step will then consist in recalling the algebraic quantized space-time structure of the vacuum of elementary particles, under the circumstances the elementary (stable) fermions, i.e.

- the leptons e^- , μ^- , τ^- and their neutrinos.
- the quarks u^+ , d^- , s^- , c^+ , b^- , t^+ .

2.1 An elementary bisemifermion

As developed in [Pie1] and [Pie3], **an elementary fermion must be viewed as an elementary bisemifermion:**

- composed of (the product of) a left semifermion, localized in the upper half space, and of a right semifermion, localized in the symmetric lower half space.
- centred on an emergence point allowing the transfer of quanta from the time structures of a semifermion to their space structures and vice-versa.
- to which it can be associated a “working space” composed of (tensor) products between right and left (time and space) internal structures, respectively of a right and of a left semifermion.

2.2 “Initial state” of a bisemifermion

The “initial state” of a bisemifermion, corresponding to its most internal vacuum structure, must be chosen to be:

- a) of mathematical nature, and, consequently, of space-time type.
- b) discrete, and, thus quantized.

The only way to satisfy these two conditions is that the internal vacuum structure of a bisemifermion be **of algebraic type**.

And, as the bilinear nature of a bisemifermion must be taken into account, an algebraic bilinear semigroup over a set of products, right by left, of symmetric algebraic subsets characterized by increasing Galois extension degrees was introduced in [Pie1] in such a way that a natural automorphic representation corresponds to it by means of a Langlands global correspondence [Pie2].

2.3 Algebraic bilinear semigroup over symmetric splitting fields

- Let K be a global number field of characteristic 0 and let $K[x]$ denote a polynomial ring composed of a set of pairs of polynomials $\{P(x), P(-x)\}$, x being a time or space variable.

The real and complex algebraic extensions of K , noted respectively $F_{\overline{v}-v}$ and $F_{\overline{\omega}-\omega}$, are assumed to be **symmetric splitting fields** $F_{\overline{v}-v} = F_{\overline{v}} \cup F_v$ and $F_{\overline{\omega}-\omega} = F_{\overline{\omega}} \cup F_{\omega}$ composed of right extension semifields $F_{\overline{v}}$ and $F_{\overline{\omega}}$ and of left extensions semifields F_v and F_{ω} in one-to-one correspondence [Pie1].

- **The real ramified algebraic subsets** are assumed to be generated from **irreducible** (one-dimensional) **algebraic closed subsets** characterized by a Galois extension degree $[F_{v_{\mu}} : K] \equiv [F_{\overline{v}_{\mu}} : K] = N$, $1 \leq \mu \leq q \leq \infty$, equal to N and **interpreted as space or time quanta**.

The real ramified algebraic subsets $F_{v_{\mu}}$ (resp. $F_{\overline{v}_{\mu}}$) are indexed in equivalence classes characterized by their ranks (i.e., their Galois extension degrees) which are integers modulo N :

$$[F_{v_{\mu}} : K] = * + \mu \cdot N \quad (\text{resp.} \quad [F_{\overline{v}_{\mu}} : K] = * + \mu \cdot N)$$

where:

- $F_{v_\mu} \subset F_v$, $1 \leq \mu \leq q \leq \infty$ (resp. $F_{\bar{v}_\mu} \subset F_{\bar{v}}$, $1 \leq \mu \leq q \leq \infty$).
- $*$ denotes an integer inferior to N .

The ranks will be generally chosen to be equal to 0 modulo N . So, we get **a tower of classes of real ramified algebraic subsets**:

$$F_{v_1} \subset \cdots \subset F_{v_\mu, m_\mu} \subset \cdots \subset F_{v_q, m_q}$$

$$(\text{resp. } F_{\bar{v}_1} \subset \cdots \subset F_{\bar{v}_\mu, m_\mu} \subset \cdots \subset F_{\bar{v}_q, m_q}),$$

where F_{v_μ, m_μ} (resp. $F_{\bar{v}_\mu, m_\mu}$) is the m_μ -th representative of the μ -th equivalence class, in such a way that **all representatives of the μ -th equivalence class have a structure at μ quanta** of degree N , since their extension degrees (or ranks) are

$$[F_{v_\mu, m_\mu} : K] = \mu \cdot N \quad (\text{resp. } [F_{\bar{v}_\mu, m_\mu} : K] = \mu \cdot N).$$

It must be noted that the real ramified algebraic subsets are included, in the sense of [Pie3], into the corresponding complex ramified algebraic subsets.

- The next step consists in considering an algebraic group over the product, right by left, of corresponding equivalent representatives of

$$F_v = \{F_{v_1}, \dots, F_{v_\mu, m_\mu}, \dots, F_{v_q, m_q}\}$$

$$\text{and of } F_{\bar{v}} = \{F_{\bar{v}_1}, \dots, F_{\bar{v}_\mu, m_\mu}, \dots, F_{\bar{v}_q, m_q}\}$$

to take into account the bilinear (or twofold) structure of the elementary bisemiparticles as noticed in section 2.2.

Let then $T_2(F_v)$ (resp. $T_2^t(F_{\bar{v}})$) denote the (semi)group of upper (resp. lower) triangular matrices of order 2 over F_v (resp. $F_{\bar{v}}$).

An algebraic bilinear general semigroup

$$\text{GL}_2(F_{\bar{v}} \times F_v) \equiv T_2^t(F_{\bar{v}}) \times T_2(F_v)$$

can be introduced so that:

- a) the product $(F_{\bar{v}} \times F_v)$ is taken over the set of corresponding pairs $\{F_{\bar{v}_\mu, m_\mu}, F_{v_\mu, m_\mu}\}_{v_\mu, m_\mu}$ of right and left real ramified algebraic subsets.
- b) $\text{GL}_2(F_{\bar{v}} \times F_v)$ has the Gauss bilinear decomposition:

$$\text{GL}_2(F_{\bar{v}} \times F_v) = [D_2(F_{\bar{v}}) \times D_2(F_v)][UT_2^t(F_{\bar{v}}) \times UT_2(F_v)]$$

where:

- $D_2(\cdot)$ is the subgroup of diagonal matrices.
- $UT_2(\cdot)$ (resp. $UT_2^t(\cdot)$) is the subgroup of upper (resp. lower) unitriangular matrices.
- c) $GL_2(F_{\bar{v}} \times F_v)$ covers its linear equivalent $GL_2(F_{\bar{v}} - F_v)$.
- d) the modular representation space $\text{Repsp}(GL_2(F_{\bar{v}} \times F_v))$ of $GL_2(F_{\bar{v}} \times F_v)$ is given by the tensor product $M_R(F_{\bar{v}}) \otimes M_L(F_v)$ of a right $T_2^t(F_{\bar{v}})$ -semimodule $M_R(F_{\bar{v}})$ by a left $T_2(F_v)$ -semimodule $M_L(F_v)$.
- e) the μ -th conjugacy class representative $GL_2(F_{\bar{v}_{\mu, m_{\mu}}} \times F_{v_{\mu, m_{\mu}}})$ of $GL_2(F_{\bar{v}} \times F_v)$, with respect to the product, right by left, $(F_{\bar{v}_{\mu}^1} \times F_{v_{\mu}^1})$ of irreducible algebraic closed subsets of rank N , has for representation the $GL_2(F_{\bar{v}_{\mu, m_{\mu}}} \times F_{v_{\mu, m_{\mu}}})$ -subbisemimodule $(M_{F_{\bar{v}_{\mu, m_{\mu}}}} \otimes M_{F_{v_{\mu, m_{\mu}}}})$.

2.4 The compactification of the algebraic bilinear semi-group $GL_2(F_{\bar{v}} \times F_v)$

- The problem is that the conjugacy class representatives of $GL_2(F_{\bar{v}} \times F_v)$ are not locally compact.

Now, the two elements $M_{F_{\bar{v}_{\mu, m_{\mu}}}}$ and $M_{F_{v_{\mu, m_{\mu}}}}$ of each conjugacy class representative $GL_2(F_{\bar{v}_{\mu, m_{\mu}}} \times F_{v_{\mu, m_{\mu}}})$ of $GL_2(F_{\bar{v}} \times F_v)$ rotate in opposite senses giving them a spin orientation according to [Pie5].

The rotation of the elements $M_{F_{\bar{v}_{\mu, m_{\mu}}}}$ and $M_{F_{v_{\mu, m_{\mu}}}}$ of the conjugacy class representatives leads to a compactification of them as developed by W. Fulton and R. McPherson [F-M] and adapted in the present case in [Pie2].

- **The compactification of $M_{F_{v_{\mu, m_{\mu}}}}$ (resp. $M_{F_{\bar{v}_{\mu, m_{\mu}}}}$) given by the map:**

$$\gamma_{M_{F_{v_{\mu, m_{\mu}}}}}^c : M_{F_{v_{\mu, m_{\mu}}}} \longrightarrow M_{L_{v_{\mu, m_{\mu}}}} \quad (\text{resp.} \quad \gamma_{M_{F_{\bar{v}_{\mu, m_{\mu}}}}}^c : M_{F_{\bar{v}_{\mu, m_{\mu}}}} \longrightarrow M_{L_{v_{\mu, m_{\mu}}}})$$

results from:

- a) a compactification of the μ irreducible algebraic closed subsets, i.e. of the μ quanta, of $M_{F_{v_{\mu, m_{\mu}}}}$ (resp. $M_{F_{\bar{v}_{\mu, m_{\mu}}}}$) by a sequence of blowups on the algebraic points of each irreducible subset transforming it into an irreducible completion centred on a point in the upper (resp. lower) half space according to the maps:

$$\gamma_{v_{\mu}^1}^c : F_{v_{\mu}^1} \longrightarrow L_{v_{\mu}^1} \quad (\text{resp.} \quad \gamma_{\bar{v}_{\mu}^1}^c : F_{\bar{v}_{\mu}^1} \longrightarrow L_{\bar{v}_{\mu}^1})$$

where:

- the closed irreducible algebraic subset $F_{v_\mu^1}$ (resp. $F_{\bar{v}_\mu^1}$) is one of the μ quanta of $M_{F_{\bar{v}_\mu, m_\mu}}$ (resp. $M_{F_{v_\mu, m_\mu}}$).
 - $L_{v_\mu^1}$ (resp. $L_{\bar{v}_\mu^1}$) is an irreducible completion of rank N , corresponding to a compactified quantum.
- b) The connectedness of the set of the μ irreducible completions put from end to end alter their compactifications in such a way that they generate a one-dimensional closed string $M_{L_{v_\mu, m_\mu}}$ (resp. $M_{L_{\bar{v}_\mu, m_\mu}}$).
- By this way, each compactified quantum $L_{v_\mu^1}$ (resp. $L_{\bar{v}_\mu^1}$) on the conjugacy class representative $M_{L_{v_\mu, m_\mu}}$ (resp. $M_{L_{\bar{v}_\mu, m_\mu}}$), which is a closed string, can be viewed as one of its μ **quantum “big points”** centred on the compactification points of the blowups as described in (a).

2.5 The compactified algebraic bilinear semigroup $\text{GL}_2(L_{\bar{v}} \times L_v)$

- So, the compactification of all the conjugacy class representatives $(M_{F_{\bar{v}_\mu, m_\mu}} \otimes M_{F_{v_\mu, m_\mu}})$ of the bilinear algebraic semigroup $\text{GL}_2(L_{\bar{v}} \times L_v)$ transforms it into the bilinear algebraic semigroup $\text{GL}_2(L_{\bar{v}} \times L_v)$ where

$$L_v = \{L_{v_1}, \dots, L_{v_\mu, m_\mu}, \dots, L_{v_q, m_q}\}$$

$$(\text{resp. } L_{\bar{v}} = \{L_{\bar{v}_1}, \dots, L_{\bar{v}_\mu, m_\mu}, \dots, L_{\bar{v}_q, m_q}\})$$

is the set of completions corresponding to the set F_v (resp. $F_{\bar{v}}$) of real ramified algebraic subsets.

- The μ -th conjugacy class representative $\text{GL}_2(L_{\bar{v}_\mu, m_\mu} \times L_{v_\mu, m_\mu})$ of $\text{GL}_2(L_{\bar{v}} \times L_v)$, with respect to the product, right by left, $(L_{\bar{v}_\mu^1} \times L_{v_\mu^1})$ of irreducible completions of rank N , has for representation the $\text{GL}_2(L_{\bar{v}_\mu, m_\mu} \times L_{v_\mu, m_\mu})$ -subbisemimodule $(M_{L_{\bar{v}_\mu, m_\mu}} \otimes M_{L_{v_\mu, m_\mu}})$.
- But, as the elements $M_{L_{\bar{v}_\mu, m_\mu}}$ and $M_{L_{v_\mu, m_\mu}}$ of $\text{GL}_2(L_{\bar{v}} \times L_v)$ rotate in opposite senses as noticed in section 2.4, it is more exactly the Lie algebra $\mathfrak{gl}_2(L_{\bar{v}} \times L_v)$ of the bilinear algebraic semigroup $\text{GL}_2(L_{\bar{v}} \times L_v)$ which would be considered.

2.6 Proposition

The discrete structure, generated by the algebraic bilinear semigroup $\text{GL}_2(F_{\bar{v}} \times F_v)$:

- corresponds to the time structure of the internal vacuum of an elementary (bisemi)-fermion.
- has been transformed by the compactification map:

$$\gamma_{F_{\bar{v}} \times F_v}^c : \text{Repsp}(\text{GL}_2(F_{\bar{v}} \times F_v)) \longrightarrow \text{Repsp}(\text{GL}_2(L_{\bar{v}} \times L_v))$$

into a corresponding **locally compact quantized time structure**.

Proof.

1. It was proved in [Pie5] and in [Pie3] that $\text{Repsp}(\text{GL}_2(F_{\bar{v}} \times F_v))$ corresponds to the time structure of the internal vacuum of an elementary bisemifermion because its conjugacy class representatives $(M_{F_{\bar{v}\mu}, m_\mu} \otimes M_{F_{v\mu}, m_\mu})$ are isomorphic, under the compactifications $\gamma_{M_{F_{\bar{v}\mu}}}^c \times \gamma_{M_{F_{v\mu}}}^c$, to products of pairs of strings $(M_{L_{\bar{v}\mu}, m_\mu} \otimes M_{L_{v\mu}, m_\mu})$ behaving like harmonic oscillators. And, a (bisemi)sheaf of differentiable (bi)functions on it is a (time) field as it will be seen.
2. As $M_{F_{v\mu}, m_\mu}$ (resp. $M_{F_{\bar{v}\mu}, m_\mu}$) is composed of μ quanta, its compactified equivalent $M_{L_{v\mu}, m_\mu}$ (resp. $M_{L_{\bar{v}\mu}, m_\mu}$) will also have a structure at μ quanta set out in a continuous way. ■

2.7 Proposition

Under the composition of maps $\gamma_{t_{R \times L} \rightarrow r_{R \times L}} \circ E_{R \times L}$, where:

- $E_{R \times L} : \text{Repsp}(\text{GL}_2(L_{\bar{v}} \times L_v)_t) \rightarrow \text{Repsp}(\text{GL}_2^*(L_{\bar{v}}^* \times L_v^*)_t) \oplus \text{Repsp}(\text{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_t)$ is a smooth biendomorphism
- $\gamma_{t_{R \times L} \rightarrow r_{R \times L}}$ sends “time” complementary bistructures into orthogonal “space” bistructures,

the “time” representation space $\text{Repsp}(\text{GL}_2(L_{\bar{v}} \times L_v)_t)$ can be transformed into:

- a) a reduced “time” representation space $\text{Repsp}(\text{GL}_2^*(L_{\bar{v}}^* \times L_v^*)_t)$ over a set of reduced completions $(L_{\bar{v}}^* \times L_v^*)$.

- b) a complementary “space” representation space $\text{Repsp}(\text{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_r)$ over a complementary set of completions $(L_{\bar{v}}^I \times L_v^I)$, localized in a space perpendicular to $\text{Repsp}(\text{GL}_2^*(L_{\bar{v}}^* \times L_v^*)_t)$.

Proof.

1. It was seen in [Pie5] that the “time” representation space $\text{Repsp}(\text{GL}_2(L_{\bar{v}} \times L_v)_t)$, submitted to Galois antiautomorphisms on its right and left parts, can be decomposed into the direct sum of two non connected “time” representation spaces of bilinear algebraic semigroups:

$$E_{R \times L} : \text{Repsp}(\text{GL}_2(L_{\bar{v}} \times L_v)_t) \longrightarrow \text{Repsp}(\text{GL}_2^*(L_{\bar{v}}^* \times L_v^*)_t) \oplus \text{Repsp}(\text{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_t)$$

corresponding to a smooth biendomorphism $E_{R \times L}$ in such a way that the biquanta (i.e. products of pairs of corresponding quanta on the same conjugacy class representatives $(M_{L_{\bar{v}\mu}, m_\mu} \otimes M_{L_{v\mu}, m_\mu})$), extracted from $\text{Repsp}(\text{GL}_2^*(L_{\bar{v}}^* \times L_v^*)_t)$ by Galois antiautomorphisms, be used to build up a new non connected complementary “time” representation space $\text{Repsp}(\text{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_t)$.

2. Under the map

$$\gamma_{t_{R \times L} \rightarrow r_{R \times L}} : \text{Repsp}(\text{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_t) \longrightarrow \text{Repsp}(\text{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_r)$$

the complementary “time” representation space $\text{Repsp}(\text{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_r)$ is sent, out of the origin, biquantum by biquantum, into the orthogonal space generating by this way the complementary representation space $\text{Repsp}(\text{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_r)$ of “space”.

3. Thus, the composition of maps:

$$\begin{aligned} \gamma_{t_{R \times L} \rightarrow r_{R \times L}} \circ E_{R \times L} : \text{Repsp}(\text{GL}_2(L_{\bar{v}} \times L_v)_t) \\ \longrightarrow \text{Repsp}(\text{GL}_2^*(L_{\bar{v}}^* \times L_v^*)_t) \oplus \text{Repsp}(\text{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_r) \end{aligned}$$

transforms the “time” representation space $\text{Repsp}(\text{GL}_2(L_{\bar{v}} \times L_v)_t)$ into the reduced “time” representation space $\text{Repsp}(\text{GL}_2^*(L_{\bar{v}}^* \times L_v^*)_t)$ and into the complementary “space” representation space $\text{Repsp}(\text{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_r)$.

As a consequence, **the “time” structure of the internal vacuum** of an elementary (bisemi)fermion, given by the “time” representation space $\text{Repsp}(\text{GL}_2(L_{\bar{v}} \times L_v)_t)$, **can be transformed** partially, or completely, **into a “space” structure**, given by the complementary “space” representation space $\text{Repsp}(\text{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_r)$ **and this “space-time” structure is quantized.** ■

2.8 Bisemisheaf of differentiable (bi)functions on $\mathrm{GL}_2(L_{\bar{v}} \times L_v)$

- Let $\phi_L(M_{v_\mu})$ (resp. $\phi_R(M_{\bar{v}_\mu})$) denote a complex-valued differentiable function over the μ -th real conjugacy class representative $M_{L_{v_\mu}}$ (resp. $M_{L_{\bar{v}_\mu}}$) of $T_2(L_v)$ (resp. $T_2^t(L_{\bar{v}})$) $\subset \mathrm{GL}_2(L_{\bar{v}} \times L_v)$ and let $\phi_R(M_{\bar{v}_\mu}) \otimes \phi_L(M_{v_\mu})$ be the corresponding bifunction on the conjugacy class representative $(M_{L_{\bar{v}_\mu}} \otimes M_{L_{v_\mu}})$ of $\mathrm{GL}_2(L_{\bar{v}} \times L_v)$.
- The set $\{\phi_L(M_{v_{\mu,m_\mu}})\}_{\mu,m_\mu}$ (resp. $\{\phi_R(M_{\bar{v}_{\mu,m_\mu}})\}_{\mu,m_\mu}$) of $\mathbb{C}$ -valued differentiable functions, localized in the upper (resp. lower) half space and defined over the $T_2(L_v)$ (resp. $T_2^t(L_{\bar{v}})$)-semimodule $M_L(L_v)$ (resp. $M_R(L_{\bar{v}})$), constitutes the set $\Gamma(\phi_L(M_L(v)))$ (resp. $\Gamma(\phi_R(M_R(L_{\bar{v}})))$) of sections of **the semisheaf of rings $\phi_L(M_L(L_v))$** (resp. **$\phi_R(M_R(L_{\bar{v}}))$**), also noted $\widetilde{M}_L(L_v)$ (resp. $\widetilde{M}_R(L_{\bar{v}})$).
- And the set $\{\phi_R(M_{\bar{v}_{\mu,m_\mu}}) \otimes \phi_L(M_{v_{\mu,m_\mu}})\}_{\mu,m_\mu}$ of differentiable bifunctions over the $\mathrm{GL}_2(L_{\bar{v}} \times L_v)$ -bisemimodule $M_R(L_{\bar{v}}) \otimes M_L(L_v)$ constitutes the set of bisections of **the bisemisheaf of rings $\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v))$** . ■

2.9 Proposition

The bisemisheaf $\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v))$ of differentiable (bi)functions is a physical string field.

Proof.

- The bisections $\phi_R(M_{\bar{v}_{\mu,m_\mu}}) \otimes \phi_L(M_{v_{\mu,m_\mu}})$ of the bisemisheaf $\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v))$ are $\mathbb{C}$ -valued differentiable bifunctions on the conjugacy class representatives $(M_{\bar{v}_{\mu,m_\mu}} \otimes M_{v_{\mu,m_\mu}})$ which are (tensor) products of two symmetric (closed) strings at μ quanta in such a way that $(M_{\bar{v}_{\mu,m_\mu}} \otimes M_{v_{\mu,m_\mu}})$ (and also $\phi_R(M_{\bar{v}_{\mu,m_\mu}}) \otimes \phi_L(M_{v_{\mu,m_\mu}})$) behave like harmonic oscillators [Pie1].
- So, the set $\{\phi_R(M_{\bar{v}_{\mu,m_\mu}}) \otimes \phi_L(M_{v_{\mu,m_\mu}})\}_{\mu,m_\mu}$ of bisections of the bisemisheaf $\phi_R(M_R(L_{\bar{v}})) \otimes \phi_L(M_L(L_v))$ is a physical string field composed of packets of products of symmetric closed strings characterized by increasing numbers of quanta and behaving like harmonic oscillators. ■

2.10 Getting a compact 4-dimensional semimanifold of space-time

1. A smooth linear **compact 3-dimensional semispace** M_τ^I (resp. $M_{\bar{\tau}}^I$), restricted to the upper (resp. lower) half space, can be constructed by considering in the first place the compactification C_L^r (resp. C_R^r) of the conjugacy class representatives M_{v_μ, m_μ}^I (resp. $M_{\bar{v}_\mu, m_\mu}^I$) of the “space” bilinear algebraic semigroup $\text{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_r$ in the following way:

- (a) The real conjugacy class representatives $\{M_{v_\mu, m_\mu}^I\}_{m_\mu}$ (resp. $\{M_{\bar{v}_\mu, m_\mu}^I\}_{m_\mu}$), m_μ varying, for each class μ , are compactified into a surface in such a way that they coincide with the complex equivalent $M_{\omega_\mu}^I$ (resp. $M_{\bar{\omega}_\mu}^I$):

$$\bigcup_c M_{v_\mu, m_\mu}^I \subseteq M_{\omega_\mu}^I \quad (\text{resp.} \quad \bigcup_c M_{\bar{v}_\mu, m_\mu}^I \subseteq M_{\bar{\omega}_\mu}^I)$$

as described in [Pie3], where $\bigcup_c$ denotes the “compact unions” or compactification.

- (b) The set of these surfaces $\{M_{\omega_\mu}^I\}_\mu$ (resp. $\{M_{\bar{\omega}_\mu}^I\}_\mu$), μ varying, $1 \leq \mu \leq q \leq \infty$, are also compactified into a 3-dimensional volume $M_{\tau_L}^I$ (resp. $M_{\tau_R}^I$), which is assumed to be of “space type”, in such a way that $M_{\tau_L}^I$ (resp. $M_{\tau_R}^I$) be foliated by the surfaces $M_{\omega_\mu}^I$ (resp. $M_{\bar{\omega}_\mu}^I$).
2. Consider then the complementary **“time” bilinear algebraic semigroup** $\text{GL}_2(L_{\bar{v}} \times L_v)_t$ from which $\text{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_r$ was generated according to proposition 2.7.
As for $\text{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_r$, the left (resp. right) linear conjugacy class representatives M_{v_μ, m_μ} (resp. $M_{\bar{v}_\mu, m_\mu}$) of the “time” bilinear algebraic semigroup $\text{GL}_2^*(L_{\bar{v}}^* \times L_v^*)_t$ are one-dimensional, corresponding then physically to “time” strings.
 3. If the set $\{M_{v_\mu, m_\mu}\}_{m_\mu, \mu}$ (resp. $\{M_{\bar{v}_\mu, m_\mu}\}_{m_\mu, \mu}$) of the **“time” conjugacy class representatives are glued together** under the map C_L^{t-r} (resp. C_R^{t-r}) **“above” the 3-dimensional volume $M_{\tau_L}^I$ (resp. $M_{\tau_R}^I$) of space**, then we get a compactified 4-dimensional semispace of space-time $M_{\tau_L}^{T-S}$ (resp. $M_{\tau_R}^{T-S}$).
 4. **In summary**, let:

$$\bullet \quad C_L^r : \{M_{v_\mu, m_\mu}^I\}_{\mu, m_\mu} \rightarrow M_{\tau_L}^I = \bigcup_C M_{\omega_\mu}^I$$

$$(\text{resp.} \quad C_R^r : \{M_{\bar{v}_\mu, m_\mu}^I\}_{\mu, m_\mu} \rightarrow M_{\tau_R}^I = \bigcup_C M_{\bar{\omega}_\mu}^I)$$

be the compactification of the one-dimensional conjugacy class representatives of space into a 3-dimensional semispace.

- $C_L^{t-r} : M_{\tau_L}^I \cup \{M_{v_{\mu}, m_{\mu}}\}_{\mu, m_{\mu}} \rightarrow M_{\tau_L}^{T-S} = M_{\tau_L}^I \cup \{M_{v_{\mu}, m_{\mu}}\}_{\mu, m_{\mu}}$
 (resp. $C_R^{t-r} : M_{\tau_R}^I \cup \{M_{\bar{v}_{\mu}, m_{\mu}}\}_{\mu, m_{\mu}} \rightarrow M_{\tau_R}^{T-S} = M_{\tau_R}^I \cup \{M_{\bar{v}_{\mu}, m_{\mu}}\}_{\mu, m_{\mu}}$)

be the compactification of the one-dimensional conjugacy class representatives of time with the 3-dimensional compact semispace $M_{\tau_L}^I$ (resp. $M_{\tau_R}^I$).

5. A **4-dimensional left (resp. right) semimanifold** $\mathcal{M}_L^{T-S}$ (resp. $\mathcal{M}_R^{T-S}$) is then defined from the 4-dimensional semispace $M_{\tau_L}^{T-S}$ (resp. $M_{\tau_R}^{T-S}$) as a collection of charts $(M_{\tau_{\mu}}^{T-S}, \phi_{\mu})$ (resp. $(M_{\bar{\tau}_{\mu}}^{T-S}, \bar{\phi}_{\mu})$), where:

- $M_{\tau_{\mu}}^{T-S}$ (resp. $M_{\bar{\tau}_{\mu}}^{T-S}$) are the compactified 4-dimensional semispaces obtained from the compactification of the linear conjugacy class representatives of $\text{GL}_2(L_{\bar{v}} \times L_v)_t$ and of $\text{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_r$.
- ϕ_{μ} (resp. $\bar{\phi}_{\mu}$) are one-to-one maps of $M_{\tau_{\mu}}^{T-S}$ (resp. $M_{\bar{\tau}_{\mu}}^{T-S}$) to open sets in the upper (resp. lower) half of $\mathbb{R}^4$,

such that

- (a) $\mathcal{M}_L^{T-S} = \bigcup_{\mu} M_{\tau_{\mu}}^{T-S}$ (resp. $\mathcal{M}_R^{T-S} = \bigcup_{\mu} M_{\bar{\tau}_{\mu}}^{T-S}$).
- (b) if $M_{\tau_{\mu}}^{T-S} \cap M_{\tau_{\nu}}^{T-S}$ (resp. $M_{\bar{\tau}_{\mu}}^{T-S} \cap M_{\bar{\tau}_{\nu}}^{T-S}$) is non-empty, then the map

$$\phi_{\mu} \circ \phi_{\nu}^{-1} : \phi_{\nu}(M_{\tau_{\mu}}^{T-S} \cap M_{\tau_{\nu}}^{T-S}) \longrightarrow \phi_{\mu}(M_{\tau_{\mu}}^{T-S} \cap M_{\tau_{\nu}}^{T-S})$$

 (resp. $\bar{\phi}_{\mu} \circ \bar{\phi}_{\nu}^{-1} : \bar{\phi}_{\nu}(M_{\bar{\tau}_{\mu}}^{T-S} \cap M_{\bar{\tau}_{\nu}}^{T-S}) \longrightarrow \bar{\phi}_{\mu}(M_{\bar{\tau}_{\mu}}^{T-S} \cap M_{\bar{\tau}_{\nu}}^{T-S})$)

is a map of open upper (resp. lower) half subset of $\mathbb{R}^4$ to an open (resp. lower) half subset of $\mathbb{R}^4$.

6. The composition of maps:

$$C_L^{t-r} \circ C_L^r : \{M_{v_{\mu}, m_{\mu}}\}_{\mu, m_{\mu}} \cup \{M_{v_{\mu}, m_{\mu}}^I\}_{\mu, m_{\mu}} \longrightarrow M_{\tau_L}^{T-S}$$

$$(\text{resp. } C_R^{t-r} \circ C_R^r : \{M_{\bar{v}_{\mu}, m_{\mu}}\}_{\mu, m_{\mu}} \cup \{M_{\bar{v}_{\mu}, m_{\mu}}^I\}_{\mu, m_{\mu}} \longrightarrow M_{\tau_R}^{T-S}),$$

compactifying one-dimensional conjugacy class representatives of space with one-dimensional conjugacy class representatives of time into a compact 4-dimensional upper (resp. lower) semispace $M_{\tau_L}^{T-S}$ (resp. $M_{\tau_R}^{T-S}$) of space-time, **allows to get a classical continuous (Riemann) 4-dimensional semispace from locally compact one-dimensional conjugacy class representatives of space-time at the Planck scale, the compactification map** $C_L^{t-r} \circ C_L^r$ (resp. $C_R^{t-r} \circ C_R^r$) **corresponding to a change of scale.**

2.11 Proposition

The product $(C_R^{t-r} \circ C_R^r) \times (C_L^{t-r} \circ C_L^r)$ of the 4-dimensional compactifications introduced in section 2.10.6. sends the direct sum of the representation space $\text{Repsp}(\text{GL}_2^*(L_v^* \times L_v^*)_t)$ of the reduced “time” bilinear algebraic semigroup $\text{GL}_2^*(L_v^* \times L_v^*)_t$ and of the representation space $\text{Repsp}(\text{GL}_2^I(L_v^I \times L_v^I)_r)$ of the complementary “space” bilinear algebraic semigroup $\text{GL}_2^I(L_v^I \times L_v^I)_r$ into the product $(\mathbf{M}_{\tau_R}^{T-S} \otimes \mathbf{M}_{\tau_L}^{T-S})$ of the 4-dimensional compact lower semispace $\mathbf{M}_{\tau_R}^{T-S}$ by its upper equivalent $\mathbf{M}_{\tau_L}^{T-S}$ according to:

$$\begin{aligned} (C_R^{t-r} \circ C_R^r) \times (C_L^{t-r} \circ C_L^r) : \quad & \text{Repsp}(\text{GL}_2^*(L_v^* \times L_v^*)_t) \oplus \text{Repsp}(\text{GL}_2^I(L_v^I \times L_v^I)_r) \\ & \longrightarrow (\mathbf{M}_{\tau_R}^{T-S} \otimes \mathbf{M}_{\tau_L}^{T-S}) \end{aligned}$$

in such a way that:

- each point of $(\mathbf{M}_{\tau_R}^{T-S} \otimes \mathbf{M}_{\tau_L}^{T-S})$ is in fact a bipoint characterized by a metric tensor g .
- each bipoint of $(\mathbf{M}_{\tau_R}^{T-S} \otimes \mathbf{M}_{\tau_L}^{T-S})$ is in one-to-one correspondence with a bipoint of $\text{Repsp}(\text{GL}_2^*(L_v^* \times L_v^*)_t) \oplus \text{Repsp}(\text{GL}_2^I(L_v^I \times L_v^I)_r)$ so that $(C_R^{t-r} \circ C_R^r) \times (C_L^{t-r} \circ C_L^r)$ is an isomorphism.

Proof.

1. the fact that $(C_R^{t-r} \circ C_R^r) \times (C_L^{t-r} \circ C_L^r)$ is a (bi)isomorphism results from its compactifying nature introduced in section 2.10.
2. Each point $\mathcal{P}$ of $(\mathbf{M}_{\tau_R}^{T-S} \otimes \mathbf{M}_{\tau_L}^{T-S})$ is a bipoint $\mathcal{P}_R \times \mathcal{P}_L$, product of a point $\mathcal{P}_L \in \mathbf{M}_{\tau_L}^{T-S}$ and of a point $\mathcal{P}_R \in \mathbf{M}_{\tau_R}^{T-S}$.

A metric tensor g at $\mathcal{P} \equiv \mathcal{P}_R \times \mathcal{P}_L$ has for components $g_a^b = g(E_a, E^b)$ (or g_{ab}) with respect to basis vectors $E_a \in \mathbf{M}_{\tau_R}^{T-S}$ and $E_b \in \mathbf{M}_{\tau_L}^{T-S}$, $a = t, x, y, z$, $b = t, x, y, z$, in such a way that $g(E_a, E^b)$ is a scalar product between basis vectors.

3. The compactified bisemisphere $\mathbf{M}_{\tau_R}^{T-S} \otimes \mathbf{M}_{\tau_L}^{T-S}$ can be understood if it is realized that each bipoint $(\mathcal{P}_R \times \mathcal{P}_L)$ of it is in one-to-one correspondence with a bipoint $(p_R \times p_L)$ belonging to the product, right by left, of two **one-dimensional** symmetric conjugacy class representatives of $\text{Repsp}(\text{GL}_2^*(L_v^* \times L_v^*)_t)$ or of $\text{Repsp}(\text{GL}_2^I(L_v^I \times L_v^I)_r)$, if it is referred to section 2.10. ■

Chapter 3

The equations of the internal dynamics of a bisemifermion

3.1 Singularizations on the internal vacuum semisheaves

- It was seen in proposition 2.7 that **the internal vacuum space-time structure** of an elementary (bisemi)fermion is given by the direct sum of the representation spaces of the bilinear algebraic semigroups $\mathrm{GL}_2(L_{\bar{v}} \times L_v)_t$ and $\mathrm{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_r$ according to:

$$(M_{ST_R}^T \otimes M_{ST_L}^T) \oplus (M_{ST_R}^S \otimes M_{ST_L}^S) = \mathrm{Repsp}(\mathrm{GL}_2(L_{\bar{v}} \times L_v)_t) \oplus \mathrm{Repsp}(\mathrm{GL}_2^I(L_{\bar{v}}^I \times L_v^I)_r)$$

where:

- $M_{ST_L}^T \equiv \mathrm{Repsp}(T_2(L_{\bar{v}} \times L_v)_t)$ (resp. $M_{ST_R}^T \equiv \mathrm{Repsp}(T_2^t(L_{\bar{v}})_t)$)
represents the algebraic time structure of the internal vacuum of a left (resp. right) semifermion.
- $M_{ST_L}^S \equiv \mathrm{Repsp}(T_2^I(L_v^I))$ (resp. $M_{ST_R}^S \equiv \mathrm{Repsp}(T_2^{t,I}(L_v^I))$)
represents the algebraic space structure of the internal vacuum of a left (resp. right) semifermion.

- **The semisheaf of differentiable functions** on $M_{ST_L}^T$ (resp. $M_{ST_R}^T$) and on $M_{ST_L}^S$ (resp. $M_{ST_R}^S$) is noted respectively $\widetilde{M}_{ST_L}^T$ (resp. $\widetilde{M}_{ST_R}^T$) and $\widetilde{M}_{ST_L}^S$ (resp. $\widetilde{M}_{ST_R}^S$).
The bisemisheaves $(\widetilde{M}_{ST_L}^T \otimes \widetilde{M}_{ST_R}^T)$ and $(\widetilde{M}_{ST_L}^S \otimes \widetilde{M}_{ST_R}^S)$ are, respectively, a “time” and a “space” string field of the internal vacuum of an elementary bisemifermion.

- As these internal vacuum string fields have a spatial extension of the order of the Planck length, they are submitted to strong fluctuations generating **degenerate**

singularities on the sections (or strings) of these, as described mathematically in [Pie4].

Taking into account that the time (and space) left and right semisheaves are symmetrical by construction and localized in small open balls, it is reasonable to assume that the singularities are generated symmetrically on the corresponding sections respectively in the upper and lower half spaces.

- By this way, it is easy to understand that **the metric tensor** g_a^b at each bipoint $(\mathcal{P}_R \times \mathcal{P}_L)$ of a bisection $(\widetilde{M}_{\bar{v}_\mu, m_\mu}^T \otimes \widetilde{M}_{v_\mu, m_\mu}^T)$ (also noted $\phi_R(M_{\bar{v}_\mu, m_\mu}) \otimes \phi_L(M_{v_\mu, m_\mu})$ in proposition 2.9) of $(\widetilde{M}_{ST_R}^T \otimes \widetilde{M}_{ST_L}^T)$ or of a bisection $(\widetilde{M}_{\bar{v}_\nu, m_\nu}^S \otimes \widetilde{M}_{v_\nu, m_\nu}^S)$ of $(\widetilde{M}_{ST_R}^S \otimes \widetilde{M}_{ST_L}^S)$ is not constant but varies according to the singularities on the sections.

3.2 Versal deformation and its blowup

- Under a strong external perturbation, a **degenerate singularity** of multiplicity inferior or equal to 3 is assumed to be generated on each section $\widetilde{M}_{v_\nu, m_\nu}^S$ (resp. $\widetilde{M}_{\bar{v}_\nu, m_\nu}^S$) of the “space” semisheaf $\widetilde{M}_{ST_L}^S$ (resp. $\widetilde{M}_{ST_R}^S$).
- Then, a **versal deformation** of $\widetilde{M}_{ST_L}^S$ (resp. $\widetilde{M}_{ST_R}^S$) can be envisaged as given by the fibre bundle:

$$\begin{aligned} D_{S_L} : \quad & \widetilde{M}_{ST_L}^S \times \theta_{S_L} \longrightarrow \widetilde{M}_{ST_L}^S \\ (\text{resp. } D_{S_R} : \quad & \widetilde{M}_{ST_R}^S \times \theta_{S_R} \longrightarrow \widetilde{M}_{ST_R}^S) \end{aligned}$$

where the fibre $\theta_{S_L} = \{\theta_1(\omega_L^1), \theta_2(\omega_L^2), \theta_3(\omega_L^3)\}$ (resp. $\theta_{S_R} = \{\theta_1(\omega_R^1), \theta_2(\omega_R^2), \theta_3(\omega_R^3)\}$) is composed of the set of three sheaves of the base S_L (resp. S_R) of the versal deformation in such a way that the ω_L^i (resp. ω_R^i , $1 \leq i \leq 3$), are the generators of the base of the quotient algebra.

- We refer to [Pie4] for more complete developments of the versal deformation and of its blowup succinctly recalled in the following.
- **The blowup of the versal deformation** allows to generate a semisheaf $\widetilde{M}_{MG_L}^S$ (resp. $\widetilde{M}_{MG_R}^S$) covering the space internal vacuum semisheaf $\widetilde{M}_{ST_L}^S$ (resp. $\widetilde{M}_{ST_R}^S$) by means of the spreading-out isomorphism:

$$SOT_L = \tau_{v_{\omega_L}} \circ \pi_{S_L} \quad (\text{resp. } SOT_R = \tau_{v_{\omega_R}} \circ \pi_{S_R})$$

where:

$$- \quad \pi_{s_L} : \quad \widetilde{M}_{ST_L}^S \times \theta_{S_L} \longrightarrow \widetilde{M}_{ST_L}^S \oplus \theta_{S_L}$$

$$(\text{resp.} \quad \pi_{s_R} : \quad \widetilde{M}_{ST_R}^S \times \theta_{S_R} \longrightarrow \widetilde{M}_{ST_R}^S \oplus \theta_{S_R})$$

is an endomorphism, based on a Galois antiautomorphism [Pie5], which disconnects the fibre θ_{S_L} (resp. θ_{S_R}) from $\widetilde{M}_{ST_L}^S$ (resp. $\widetilde{M}_{ST_R}^S$).

- $\tau_{v_{\omega_L}}$ (resp. $\tau_{v_{\omega_R}}$) is the projective map:

$$\tau_{v_{\omega_L}} : \quad \text{TAN}(\theta_{S_L}) \longrightarrow \theta_{S_L} \quad (\text{resp.} \quad \tau_{v_{\omega_R}} : \quad \text{TAN}(\theta_{S_R}) \longrightarrow \theta_{S_R})$$

of the vertical tangent bundle $T_{v_{\omega_L}}$ (resp. $T_{v_{\omega_R}}$) sending θ_{S_L} (resp. θ_{S_R}) into the total tangent space $\text{TAN}(\theta_{S_L})$ (resp. $\text{TAN}(\theta_{S_R})$).

By this way, the three functions $\omega_L^i(v_{\nu, m_\nu})$ (resp. $\omega_R^i(\bar{v}_{\nu, m_\nu})$) of the base of the versal deformation are projected above each section $\widetilde{M}_{v_{\nu, m_\nu}}^S$ (resp. $\widetilde{M}_{\bar{v}_{\nu, m_\nu}}^S$) in the vertical tangent space.

Being glued together, these three functions generate the sections $\widetilde{M}_{MG_{v_{\nu, m_\nu}}}^{S_p}$ (resp. $\widetilde{M}_{MG_{\bar{v}_{\nu, m_\nu}}}^{S_p}$) of a semisheaf $\widetilde{M}_{MG_L}^{S_p}$ (resp. $\widetilde{M}_{MG_R}^{S_p}$) (called middle ground) which covers the internal vacuum semisheaf.

If the numbers of quanta on the sections $\widetilde{M}_{MG_{v_{\nu, m_\nu}}}^{S_p}$ (resp. $\widetilde{M}_{MG_{\bar{v}_{\nu, m_\nu}}}^{S_p}$) of the **middle ground semisheaf** $\widetilde{M}_{MG_L}^{S_p}$ (resp. $\widetilde{M}_{MG_R}^{S_p}$) are equal to the numbers of quanta on the sections $\widetilde{M}_{v_{\nu, m_\nu}}^S$ (resp. $\widetilde{M}_{\bar{v}_{\nu, m_\nu}}^S$) (rewritten according to $\widetilde{M}_{ST_{v_{\nu, m_\nu}}}^S$ (resp. $\widetilde{M}_{ST_{\bar{v}_{\nu, m_\nu}}}^S$)) of the internal vacuum semisheaf $\widetilde{M}_{ST_L}^S$ (resp. $\widetilde{M}_{ST_R}^S$), then these sections $\widetilde{M}_{MG_{v_{\nu, m_\nu}}}^{S_p}$ (resp. $\widetilde{M}_{MG_{\bar{v}_{\nu, m_\nu}}}^{S_p}$) are open strings covering the closed strings $\widetilde{M}_{ST_{v_{\nu, m_\nu}}}^S$ (resp. $\widetilde{M}_{ST_{\bar{v}_{\nu, m_\nu}}}^S$) of $\widetilde{M}_{ST_L}^S$ (resp. $\widetilde{M}_{ST_R}^S$).

3.3 Proposition

The inverse of the projective map $\tau_{v_{\omega_L}}$ (resp. $\tau_{v_{\omega_R}}$) of the tangent bundle $T_{v_{\omega_L}}$ (resp. $T_{v_{\omega_R}}$) of the spreading-out isomorphism is the elliptic operator:

$$DT_{L;MG}^S = \left\{ i \frac{\hbar_{MG}}{c_{t \rightarrow r;MG}} \frac{\partial}{\partial x}, i \frac{\hbar_{MG}}{c_{t \rightarrow r;MG}} \frac{\partial}{\partial y}, i \frac{\hbar_{MG}}{c_{t \rightarrow r;MG}} \frac{\partial}{\partial z} \right\}$$

$$\left(\text{resp.} \quad DT_{R;MG}^S = \left\{ -i \frac{\hbar_{MG}}{c_{t \rightarrow r;MG}} \frac{\partial}{\partial x}, -i \frac{\hbar_{MG}}{c_{t \rightarrow r;MG}} \frac{\partial}{\partial y}, -i \frac{\hbar_{MG}}{c_{t \rightarrow r;MG}} \frac{\partial}{\partial z} \right\} \right),$$

where the constants $\hbar_{MG}$ and $c_{t \rightarrow r; MG}$ are defined in chapter 3 of [Pie5], sending the semi-sheaf $\widetilde{M}_{MGL}^S$ (resp. $\widetilde{M}_{MGR}^S$) into the perverse semisheaf $\widetilde{M}_{MGL}^{Sp}$ (resp. $\widetilde{M}_{MGR}^{Sp}$) according to:

$$DT_{L;MG}^S : \widetilde{M}_{MGL}^S \longrightarrow \widetilde{M}_{MGL}^{Sp} \quad (\text{resp.} \quad DT_{R;MG}^S : \widetilde{M}_{MGR}^S \longrightarrow \widetilde{M}_{MGR}^{Sp}).$$

Proof. The elliptic operator $DT_{L;MG}^S$ (resp. $DT_{R;MG}^S$) maps the semisheaf $\widetilde{M}_{MGL}^S$ (resp. $\widetilde{M}_{MGR}^S$) into its perverse equivalent $\widetilde{M}_{MGL}^{Sp}$ (resp. $\widetilde{M}_{MGR}^{Sp}$) since this latter belongs to the derived category of string semifields shifted in the three geometrical dimensions of space, a string semifield being given by a semisheaf, for example $\widetilde{M}_{MGL}^S$ (resp. $\widetilde{M}_{MGR}^S$). ■

3.4 Generation of perverse mass semisheaves $\widetilde{M}_{M_L}^{Sp}$ and $\widetilde{M}_{M_R}^{Sp}$

- As the singularities on the sections of the internal vacuum semisheaf $\widetilde{M}_{STL}^S$ (resp. $\widetilde{M}_{STR}^S$) have a multiplicity inferior or equal to 3, the functions $\omega_L^i(v_{\nu, m_\nu})$ (resp. $\omega_r^i(\overline{v}_{\nu, m_\nu})$) of the quotient algebra of the versal deformation of $\widetilde{M}_{STL}^S$ (resp. $\widetilde{M}_{STR}^S$) can have degenerate singularities of multiplicity one.
- So, the middle ground semisheaf $\widetilde{M}_{MGL}^S$ (resp. $\widetilde{M}_{MGR}^S$), of which sections $\widetilde{M}_{MGv_\nu, m_\nu}^S$ (resp. $\widetilde{M}_{MG\overline{v}_\nu, m_\nu}^S$) are the functions $\omega_L^i(v_{\nu, m_\nu})$ (resp. $\omega_R^i(\overline{v}_{\nu, m_\nu})$), $1 \leq i \leq 3$, glued together, can undergo a versal deformation and a blowup of it according to:

$$\begin{aligned} SOT_L^{(MG)} \circ D_{S_L}^{(MG)} : \widetilde{M}_{MGL}^S &\longrightarrow \widetilde{M}_{MGL}^S \oplus \widetilde{M}_{M_L}^{Sp} \\ (\text{resp.} \quad SOT_R^{(MG)} \circ D_{S_R}^{(MG)} : \widetilde{M}_{MGR}^S &\longrightarrow \widetilde{M}_{MGR}^S \oplus \widetilde{M}_{M_R}^{Sp}) \end{aligned}$$

where:

$$\begin{aligned} - \quad D_{S_L}^{(MG)} : \widetilde{M}_{MGL}^S \times \theta_{S_L}^{(MG)} &\rightarrow \widetilde{M}_{MGL}^S \\ (\text{resp.} \quad D_{S_R}^{(MG)} : \widetilde{M}_{MGR}^S \times \theta_{S_R}^{(MG)} &\rightarrow \widetilde{M}_{MGR}^S) \end{aligned}$$

is the versal deformation of $\widetilde{M}_{MGL}^S$ (resp. $\widetilde{M}_{MGR}^S$).

$$\begin{aligned} - \quad SOT_L^{(MG)} : \widetilde{M}_{MGL}^S \times \theta_{S_L}^{(MG)} &\rightarrow \widetilde{M}_{MGL}^S \oplus \widetilde{M}_{M_L}^{Sp} \\ (\text{resp.} \quad SOT_R^{(MG)} : \widetilde{M}_{MGR}^S \times \theta_{S_R}^{(MG)} &\rightarrow \widetilde{M}_{MGR}^S \oplus \widetilde{M}_{M_R}^{Sp}) \end{aligned}$$

in such a way that $\widetilde{M}_{M_L}^{S_p}$ (resp. $\widetilde{M}_{M_R}^{S_p}$) is the perverse mass semisheaf generated from $\widetilde{M}_{M_L}^S \equiv \theta_{S_L}^{(MG)}$ (resp. $\widetilde{M}_{M_R}^S \equiv \theta_{S_R}^{(MG)}$) by the map:

$$DT_{L;M}^S : \widetilde{M}_{M_L}^S \longrightarrow \widetilde{M}_{M_L}^{S_p} \quad (\text{resp.} \quad DT_{R;M}^S : \widetilde{M}_{M_R}^S \longrightarrow \widetilde{M}_{M_R}^{S_p})$$

so that

$$DT_{L;M}^S = \left\{ i \frac{\hbar}{c_{t \rightarrow r;M}} \frac{\partial}{\partial x}, i \frac{\hbar}{c_{t \rightarrow r;M}} \frac{\partial}{\partial y}, i \frac{\hbar}{c_{t \rightarrow r;M}} \frac{\partial}{\partial z} \right\}$$

$$\left(\text{resp.} \quad DT_{R;M}^S = \left\{ -i \frac{\hbar}{c_{t \rightarrow r;M}} \frac{\partial}{\partial x}, -i \frac{\hbar}{c_{t \rightarrow r;M}} \frac{\partial}{\partial y}, -i \frac{\hbar}{c_{t \rightarrow r;M}} \frac{\partial}{\partial z} \right\} \right)$$

corresponds to the inverse of the projective map

$$\tau_{v_{\omega_L}}^{(M)} : \text{TAN}(\theta_{S_L}^{(MG)}) \longrightarrow \theta_{S_L}^{(MG)} \quad (\text{resp.} \quad \tau_{v_{\omega_R}}^{(M)} : \text{TAN}(\theta_{S_R}^{(MG)}) \longrightarrow \theta_{S_R}^{(MG)})$$

of the vertical tangent bundle of the blowup of the versal deformation of the middle ground semisheaf $\widetilde{M}_{M_{GL}}^S$ (resp. $\widetilde{M}_{M_{GR}}^S$).

- **The sections $\widetilde{M}_{M_{v\nu},m\nu}^{S_p}$ (resp. $\widetilde{M}_{M_{\bar{v}\nu},m\nu}^{S_p}$) of the perverse “mass” semisheaf $\widetilde{M}_{M_L}^{S_p}$ (resp. $\widetilde{M}_{M_R}^{S_p}$) cover the corresponding sections of the “middle ground” and “internal vacuum” semisheaves $\widetilde{M}_{M_{GL}}^{S_p}$ (resp. $\widetilde{M}_{M_{GL}}^{S_p}$) and $\widetilde{M}_{S_{TL}}^S$ (resp. $\widetilde{M}_{S_{TR}}^S$):** they are open strings if the numbers of quanta on their sections are inferior or equal to the number of quanta on the sections of the “middle ground” semisheaves $\widetilde{M}_{M_{GL}}^{S_p}$ (resp. $\widetilde{M}_{M_{GR}}^{S_p}$).

3.5 Embedding of “internal vacuum”, “middle ground” and “mass” semisheaves of space

- So, the “middle ground” and “mass” semisheaves of space $\widetilde{M}_{M_{GL}}^S$ (resp. $\widetilde{M}_{M_{GR}}^S$) and $\widetilde{M}_{M_L}^S$ (resp. $\widetilde{M}_{M_R}^S$) can be generated by versal deformations and blowups of these from the “internal vacuum” semisheaf $\widetilde{M}_{S_{TL}}^S$ (resp. $\widetilde{M}_{S_{TR}}^S$) of space leading to the **embedding**:

$$\widetilde{M}_{S_{TL}}^S \subset \widetilde{M}_{M_{GL}}^S \subset \widetilde{M}_{M_L}^S \quad (\text{resp.} \quad \widetilde{M}_{S_{TR}}^S \subset \widetilde{M}_{M_{GR}}^S \subset \widetilde{M}_{M_R}^S).$$

- The perverse “middle ground” and “mass” semisheaves $\widetilde{M}_{MG_L}^{S_p}$ (resp. $\widetilde{M}_{MG_R}^{S_p}$) and $\widetilde{M}_{M_L}^{S_p}$ (resp. $\widetilde{M}_{M_R}^{S_p}$), belonging to the derived category of string semifields, **are of contracting nature**, while the “internal vacuum” semisheaf of space $\widetilde{M}_{ST_L}^S$ (resp. $\widetilde{M}_{ST_R}^S$) **is of expanding nature**: so, it was assumed in [Pie5] that the 3D-differential operator

$$T_{L;ST}^S = \left\{ i \frac{\hbar_{ST}}{c_{t \rightarrow r;ST}} dx, i \frac{\hbar_{ST}}{c_{t \rightarrow r;ST}} dy, i \frac{\hbar_{ST}}{c_{t \rightarrow r;ST}} dz \right\}$$

$$\left(\text{resp. } T_{R;ST}^S = \left\{ -i \frac{\hbar_{ST}}{c_{t \rightarrow r;ST}} dx, -i \frac{\hbar_{ST}}{c_{t \rightarrow r;ST}} dy, -i \frac{\hbar_{ST}}{c_{t \rightarrow r;ST}} dz \right\} \right)$$

applies to all the sections of $\widetilde{M}_{ST_L}^S$ (resp. $\widetilde{M}_{ST_R}^S$) according to:

$$T_{L;ST}^S : \widetilde{M}_{ST_L}^S \longrightarrow \widetilde{M}_{ST_L}^{S_p} \quad (\text{resp. } T_{R;ST}^S : \widetilde{M}_{ST_R}^S \longrightarrow \widetilde{M}_{ST_R}^{S_p})$$

where $\widetilde{M}_{ST_L}^{S_p}$ (resp. $\widetilde{M}_{ST_R}^{S_p}$) was written abusively as a perverse semisheaf.

Similarly, we have the following embedding

$$\widetilde{M}_{ST_L}^{S_p} \subset \widetilde{M}_{MG_L}^{S_p} \subset \widetilde{M}_{M_L}^{S_p} \quad (\text{resp. } \widetilde{M}_{ST_R}^{S_p} \subset \widetilde{M}_{MG_R}^{S_p} \subset \widetilde{M}_{M_R}^{S_p})$$

between “perverse” semisheaves.

3.6 Middle ground and mass semisheaves of time

- Similarly, as the “middle ground” and “mass” semisheaves of space are generated by versal deformations and blowups from the “internal vacuum” semisheaf of space $\widetilde{M}_{ST_L}^S$ (resp. $\widetilde{M}_{ST_R}^S$), the “middle ground” and “mass” semisheaves of time $\widetilde{M}_{MG_L}^T$ (resp. $\widetilde{M}_{MG_R}^T$) and $\widetilde{M}_{M_L}^T$ (resp. $\widetilde{M}_{M_R}^T$) can be generated:
 - either by versal deformations and blowups from the “internal vacuum” semisheaf of time $\widetilde{M}_{ST_L}^T$ (resp. $\widetilde{M}_{ST_R}^T$).
 - or from the respective semisheaves of space by the composition of maps:

$$\gamma_{r_L \rightarrow t_L}^{(MG)} \circ E_L^{(MG)} : \widetilde{M}_{MG_L}^S \longrightarrow \widetilde{M}_{MG_L}^T$$

$$\text{and } \gamma_{r_L \rightarrow t_L}^{(M)} \circ E_L^{(M)} : \widetilde{M}_{M_L}^S \longrightarrow \widetilde{M}_{M_L}^T$$

$$(\text{resp. } \gamma_{r_R \rightarrow t_R}^{(MG)} \circ E_R^{(MG)} : \widetilde{M}_{MG_R}^S \longrightarrow \widetilde{M}_{MG_R}^T$$

$$\text{and } \gamma_{r_R \rightarrow t_R}^{(M)} \circ E_R^{(M)} : \widetilde{M}_{M_R}^S \longrightarrow \widetilde{M}_{M_R}^T),$$

as described in proposition 2.7, where $E_L^{(MG)}$ and $E_L^{(M)}$ (resp. $E_R^{(MG)}$ and $E_R^{(M)}$) are endomorphisms based on Galois antiautomorphisms.

As for the semisheaves of space, we have **for the semisheaves of time the embedding**:

$$\widetilde{M}_{ST_L}^T \subset \widetilde{M}_{MG_L}^T \subset \widetilde{M}_{M_L}^T \quad (\text{resp. } \widetilde{M}_{ST_R}^T \subset \widetilde{M}_{MG_R}^T \subset \widetilde{M}_{M_R}^T)$$

as well as for the perverse semisheaves of time:

$$\widetilde{M}_{ST_L}^{T_p} \subset \widetilde{M}_{MG_L}^{T_p} \subset \widetilde{M}_{M_L}^{T_p} \quad (\text{resp. } \widetilde{M}_{ST_R}^{T_p} \subset \widetilde{M}_{MG_R}^{T_p} \subset \widetilde{M}_{M_R}^{T_p}).$$

These perverse semisheaves of time result form the morphisms:

$$\begin{aligned} T_{L;ST}^T : \quad & \widetilde{M}_{ST_L}^T \longrightarrow \widetilde{M}_{ST_L}^{T_p}, \\ DT_{L;MG}^T : \quad & \widetilde{M}_{MG_L}^T \longrightarrow \widetilde{M}_{MG_L}^{T_p}, \\ DT_{L;M}^T : \quad & \widetilde{M}_{M_L}^T \longrightarrow \widetilde{M}_{M_L}^{T_p}, \end{aligned}$$

where

$$T_{L;ST}^T = i \frac{\hbar_{ST}}{c_{t \rightarrow r; ST}} dt_0, \quad DT_{L;MG}^T = i \frac{\hbar_{MG}}{c_{t \rightarrow r; MG}} \frac{\partial}{\partial t_0}, \quad DT_{L;M}^T = i \frac{\hbar_M}{c_{t \rightarrow r; M}} \frac{\partial}{\partial t_0}$$

(the right cases are handled similarly).

3.7 Proposition

Let $(\widetilde{M}_{ST_R}^{T-S} \otimes \widetilde{M}_{ST_L}^{T-S})$ be the “internal vacuum” bisemisheaf or string field of an elementary bisemifermion.

*Then, by versal deformations and blowups of these, **the “internal vacuum” string field can generate the two covering “middle ground” and “mass” string fields of space-time***

$$(\widetilde{M}_{MG_R}^{T-S} \otimes \widetilde{M}_{MG_L}^{T-S}) \quad \text{and} \quad (\widetilde{M}_{M_R}^{T-S} \otimes \widetilde{M}_{M_L}^{T-S})$$

leading to the embedding:

$$(\widetilde{M}_{ST_R}^{T-S} \otimes \widetilde{M}_{ST_L}^{T-S}) \subset (\widetilde{M}_{MG_R}^{T-S} \otimes \widetilde{M}_{MG_L}^{T-S}) \subset (\widetilde{M}_{M_R}^{T-S} \otimes \widetilde{M}_{M_L}^{T-S}).$$

Proof.

- According to section 3.1, the “internal vacuum” string field is given by:

$$(\widetilde{M}_{ST_R}^T \otimes \widetilde{M}_{ST_L}^T) \oplus (\widetilde{M}_{ST_R}^S \otimes \widetilde{M}_{ST_L}^S)$$

where $(\widetilde{M}_{ST_R}^T \otimes \widetilde{M}_{ST_L}^T)$ (resp. $(\widetilde{M}_{ST_R}^S \otimes \widetilde{M}_{ST_L}^S)$) is the time (resp. space) string field. But, this “internal vacuum” string field of space time is not complete, because it does not allow interactions between the time string field and the space string field.

- A more general approach would consider the following “internal vacuum” string field:

$$\begin{aligned} (\widetilde{M}_{ST_R}^{T-S} \otimes \widetilde{M}_{ST_L}^{T-S}) &\equiv (\widetilde{M}_{ST_R}^T \oplus \widetilde{M}_{ST_R}^S) \otimes (\widetilde{M}_{ST_L}^T \oplus \widetilde{M}_{ST_L}^S) \\ &= (\widetilde{M}_{ST_R}^T \otimes \widetilde{M}_{ST_L}^T) \oplus (\widetilde{M}_{ST_R}^S \otimes \widetilde{M}_{ST_L}^S) \oplus (\widetilde{M}_{ST_R}^T \otimes \widetilde{M}_{ST_L}^S) \oplus (\widetilde{M}_{ST_R}^S \otimes \widetilde{M}_{ST_L}^T) \end{aligned}$$

constituting the **completely reducible non orthogonal bilinear representation space in bisemisheaves** [Pie4] of $\mathrm{GL}_{2(T+S)}(L_{\overline{v}} \times L_v)$ according to:

$$\begin{aligned} \mathrm{Repsp} \theta(\mathrm{GL}_{2(T+S)}(L_{\overline{v}} \times L_v)) \\ = \mathrm{Repsp} \theta(\mathrm{GL}_{2(T)}(L_{\overline{v}} \times L_v)) \oplus \mathrm{Repsp} \theta(\mathrm{GL}_{2(S)}(L_{\overline{v}}^I \times L_v^I)) \\ \oplus \mathrm{Repsp} \theta(T_{2(T)}^t(L_{\overline{v}}) \times T_{2(S)}(L_v^I)) \oplus \mathrm{Repsp} \theta(T_{2(S)}^t(L_{\overline{v}}^I) \times T_{2(T)}(L_v)) \end{aligned}$$

where

$$\begin{aligned} (\widetilde{M}_{ST_R}^T \otimes \widetilde{M}_{ST_L}^S) &\equiv \mathrm{Repsp} \theta(T_{2(T)}^t(L_{\overline{v}}) \times T_{2(S)}(L_v^I)) \\ \text{and } (\widetilde{M}_{ST_R}^S \otimes \widetilde{M}_{ST_L}^T) &\equiv \mathrm{Repsp} \theta(T_{2(S)}^t(L_{\overline{v}}^I) \times T_{2(T)}(L_v)) \end{aligned}$$

are mixed bisemisheaves over off diagonal bilinear representation spaces of “time-space” and “space-time” responsible for the electric charge of the considered (bisemi)-fermion as developed in [Pie5].

- Referring to the preceding sections, the “internal vacuum” bisemisheaf $(\widetilde{M}_{ST_R}^{T-S} \otimes \widetilde{M}_{ST_L}^{T-S})$ generates by versal deformations and blowups of these the “middle ground” and “mass” bisemisheaves $(\widetilde{M}_{MGR}^{T-S} \otimes \widetilde{M}_{MGL}^{T-S})$ and $(\widetilde{M}_{MR}^{T-S} \otimes \widetilde{M}_{ML}^{T-S})$ which are embedded as announced in this proposition. ■

3.8 Proposition

1. *The corresponding perverse bisemisheaves (or fields) $(\widetilde{M}_{ST_R}^{T_p-S_p} \otimes \widetilde{M}_{ST_L}^{T_p-S_p})$, $(\widetilde{M}_{MG_R}^{T_p-S_p} \otimes \widetilde{M}_{MG_L}^{T_p-S_p})$ and $(\widetilde{M}_{M_R}^{T_p-S_p} \otimes \widetilde{M}_{M_L}^{T_p-S_p})$ of the internal structure of an elementary bisemifermion give rise to the set of equations of its internal dynamics:*

$$\begin{aligned}
 & (\widetilde{M}_{ST_R}^{T_p-S_p} \oplus \widetilde{M}_{MG_R}^{T_p-S_p} \oplus \widetilde{M}_{M_R}^{T_p-S_p}) \otimes (\widetilde{M}_{ST_L}^{T_p-S_p} \oplus \widetilde{M}_{MG_L}^{T_p-S_p} \oplus \widetilde{M}_{M_L}^{T_p-S_p}) \\
 &= (\widetilde{M}_{ST_R}^{T_p-S_p} \otimes \widetilde{M}_{ST_L}^{T_p-S_p}) \oplus (\widetilde{M}_{MG_R}^{T_p-S_p} \otimes \widetilde{M}_{MG_L}^{T_p-S_p}) \oplus (\widetilde{M}_{M_R}^{T_p-S_p} \otimes \widetilde{M}_{M_L}^{T_p-S_p}) \\
 &\quad \oplus (\widetilde{M}_{ST_R}^{T_p-S_p} \otimes \widetilde{M}_{MG_L}^{T_p-S_p}) \oplus (\widetilde{M}_{ST_R}^{T_p-S_p} \otimes \widetilde{M}_{M_L}^{T_p-S_p}) \oplus (\widetilde{M}_{MG_R}^{T_p-S_p} \otimes \widetilde{M}_{ST_L}^{T_p-S_p}) \\
 &\quad \oplus (\widetilde{M}_{MG_R}^{T_p-S_p} \otimes \widetilde{M}_{M_L}^{T_p-S_p}) \oplus (\widetilde{M}_{M_R}^{T_p-S_p} \otimes \widetilde{M}_{ST_L}^{T_p-S_p}) \oplus (\widetilde{M}_{M_R}^{T_p-S_p} \otimes \widetilde{M}_{MG_L}^{T_p-S_p}) \\
 &= 0
 \end{aligned}$$

where the set of six mixed bisemisheaves $(\widetilde{M}_{ST_R}^{T_p-S_p} \otimes \widetilde{M}_{ST_L}^{T_p-S_p}) \dots (\widetilde{M}_{M_R}^{T_p-S_p} \otimes \widetilde{M}_{MG_L}^{T_p-S_p})$ generates the interactions between the right and left semisheaves of different levels “ST”, “MG” and “M”.

2. *If the interactions between the right and left internal semistructures “ST”, “MG” and “M” are assumed to be negligible, then the equations of the internal dynamics of a bisemifermion are:*

$$(\widetilde{M}_{ST_R}^{T_p-S_p} \otimes \widetilde{M}_{ST_L}^{T_p-S_p}) \oplus (\widetilde{M}_{MG_R}^{T_p-S_p} \otimes \widetilde{M}_{MG_L}^{T_p-S_p}) = -(\widetilde{M}_{M_R}^{T_p-S_p} \otimes \widetilde{M}_{M_L}^{T_p-S_p}) \quad (*)$$

Proof.

- This proposition is a direct consequence of the preceding sections.
- The interactions between the right and left internal semisheaves “ST”, “MG” and “M” are especially responsible for the internal angular momentum of the considered bisemifermion as developed in [Pie5].
- If the interactions between the right and left internal semisheaves are assumed to be negligible, then **the equations of the internal dynamics:**

$$(\widetilde{M}_{ST_R}^{T_p-S_p} \otimes \widetilde{M}_{ST_L}^{T_p-S_p}) \oplus (\widetilde{M}_{MG_R}^{T_p-S_p} \otimes \widetilde{M}_{MG_L}^{T_p-S_p}) = -(\widetilde{M}_{M_R}^{T_p-S_p} \otimes \widetilde{M}_{M_L}^{T_p-S_p}) \quad (*)$$

describe the generation of the matter field $(\widetilde{M}_{M_R}^{T_p-S_p} \otimes \widetilde{M}_{M_L}^{T_p-S_p})$ of a bisemifermion from its own vacuum fields given the lefthand side of (*). ■

3.9 Extended bilinear Hilbert spaces of internal structures

According to [Pie3], each “ ST ”, “ MG ” or “ M ” field in $(*)$ is an **operator valued string field** on the corresponding string field which defines an extended bilinear Hilbert space.

Indeed, if we apply a $(B_L \circ p_L)$ map on the string fields $(\widetilde{M}_{ST_R}^{T-S} \otimes \widetilde{M}_{ST_L}^{T-S})$, $(\widetilde{M}_{MG_R}^{T-S} \otimes \widetilde{M}_{MG_L}^{T-S})$ and $(\widetilde{M}_{M_R}^{T-S} \otimes \widetilde{M}_{M_L}^{T-S})$ in such a way that:

- p_L be a projective map projecting each right semifield on its corresponding left equivalent,
- B_L be a bijective isometric map sending each covariant element into its contravariant equivalent,

then they are transformed into:

$$\begin{aligned} (B_L^{(ST)} \circ p_L^{(ST)}) : \quad & \widetilde{M}_{ST_R}^{T-S} \otimes \widetilde{M}_{ST_L}^{T-S} \longrightarrow H_{ST}^+ = \widetilde{M}_{ST_{LR}}^{T-S} \otimes \widetilde{M}_{ST_L}^{T-S} \\ (B_L^{(MG)} \circ p_L^{(MG)}) : \quad & \widetilde{M}_{MG_R}^{T-S} \otimes \widetilde{M}_{MG_L}^{T-S} \longrightarrow H_{MG}^+ = \widetilde{M}_{MG_{LR}}^{T-S} \otimes \widetilde{M}_{MG_L}^{T-S} \\ (B_L^{(M)} \circ p_L^{(M)}) : \quad & \widetilde{M}_{M_R}^{T-S} \otimes \widetilde{M}_{M_L}^{T-S} \longrightarrow H_M^+ = \widetilde{M}_{M_{LR}}^{T-S} \otimes \widetilde{M}_{M_L}^{T-S} \end{aligned}$$

where H_{ST}^+ , H_{MG}^+ and H_M^+ are the extended bilinear Hilbert spaces of the internal vacuum, middle ground and mass structures of a bisemifermion if complete internal bilinear forms are defined on them.

3.10 Actions of bioperators on H_{ST}^+ , H_{MG}^+ and H_M^+

On the extended bilinear Hilbert spaces H_{ST}^+ , H_{MG}^+ and H_M^+ , we have the actions of the bioperators:

$$\begin{aligned} T_{R;ST}^{T-S} \otimes T_{L;ST}^{T-S} : \quad & \widetilde{M}_{ST_{LR}}^{T-S} \otimes \widetilde{M}_{ST_L}^{T-S} \longrightarrow \widetilde{M}_{ST_{LR}}^{T_p-S_p} \otimes \widetilde{M}_{ST_L}^{T_p-S_p} \\ DT_{R;MG}^{T-S} \otimes DT_{L;MG}^{T-S} : \quad & \widetilde{M}_{MG_{LR}}^{T-S} \otimes \widetilde{M}_{MG_L}^{T-S} \longrightarrow \widetilde{M}_{MG_{LR}}^{T_p-S_p} \otimes \widetilde{M}_{MG_L}^{T_p-S_p} \\ DT_{R;M}^{T-S} \otimes DT_{L;M}^{T-S} : \quad & \widetilde{M}_{M_{LR}}^{T-S} \otimes \widetilde{M}_{M_L}^{T-S} \longrightarrow \widetilde{M}_{M_{LR}}^{T_p-S_p} \otimes \widetilde{M}_{M_L}^{T_p-S_p} \end{aligned}$$

sending the internal vacuum, middle ground and mass bisemisheaves $\widetilde{M}_{ST_{LR}}^{T-S} \otimes \widetilde{M}_{ST_L}^{T-S}$, $\widetilde{M}_{MG_{LR}}^{T-S} \otimes \widetilde{M}_{MG_L}^{T-S}$ and $\widetilde{M}_{M_{LR}}^{T-S} \otimes \widetilde{M}_{M_L}^{T-S}$ into their perverse equivalents.

The bioperators are explicitly given by:

$$T_{R;ST}^{T-S} \otimes T_{L;ST}^{T-S} = \left(-i \frac{\hbar_{ST}}{c_{t \rightarrow r;ST}} \{s_{0R} dt_0; s_{xR} dx, s_{yR} dy, s_{zR} dz\} \right) \\ \otimes \left(-i \frac{\hbar_{ST}}{c_{t \rightarrow r;ST}} \{s_{0L} dt_0; s_{xL} dx, s_{yL} dy, s_{zL} dz\} \right)$$

$$DT_{R;MG}^{T-S} \otimes DT_{L;MG}^{T-S} = \left(-i \frac{\hbar_{MG}}{c_{t \rightarrow r;MG}} \left\{ s_{0R} \frac{\partial}{\partial t_0}; s_{xR} \frac{\partial}{\partial x}, s_{yR} \frac{\partial}{\partial y}, s_{zR} \frac{\partial}{\partial z} \right\} \right) \\ \otimes \left(-i \frac{\hbar_{MG}}{c_{t \rightarrow r;MG}} \left\{ s_{0L} \frac{\partial}{\partial t_0}; s_{xL} \frac{\partial}{\partial x}, s_{yL} \frac{\partial}{\partial y}, s_{zL} \frac{\partial}{\partial z} \right\} \right)$$

$$DT_{R;M}^{T-S} \otimes DT_{L;M}^{T-S} = \left(-i \frac{\hbar_M}{c_{t \rightarrow r;M}} \left\{ s_{0R} \frac{\partial}{\partial t_0}; s_{xR} \frac{\partial}{\partial x}, s_{yR} \frac{\partial}{\partial y}, s_{zR} \frac{\partial}{\partial z} \right\} \right) \\ \otimes \left(-i \frac{\hbar_M}{c_{t \rightarrow r;M}} \left\{ s_{0L} \frac{\partial}{\partial t_0}; s_{xL} \frac{\partial}{\partial x}, s_{yL} \frac{\partial}{\partial y}, s_{zL} \frac{\partial}{\partial z} \right\} \right)$$

where s_0 and (s_x, s_y, s_z) are the direction cosines of the unit vectors $\vec{s}_{t_0}$ and $\vec{s}_r$ referring to the spin and allowing to defined directional gradients $\vec{s}_{t_0} \frac{\partial}{\partial t_0}$ and $\vec{s}_r \nabla$.

(More concretely, the function $\phi(t_0)$ is said to be derivable at t_0 in the direction s if $\lim_{\varepsilon \rightarrow 0} \frac{\phi(t_0+\varepsilon s) - \phi(t_0)}{\varepsilon}$ exists).

3.11 Proposition

- Let $r = q + p$ be the number of algebraic conjugacy (or equivalence) classes of time and space, the indices of time and space varying separately according to $1 \leq \mu \leq q$ and $1 \leq \nu \leq p$ and commonly according to $1 \leq \sigma \leq r$.

- Let

$$\bullet \quad \left\{ \widetilde{M}_{ST_{v\sigma}, m_\sigma} = \widetilde{M}_{ST_{v\mu}, m_\mu}^T + \widetilde{M}_{ST_{v\nu}, m_\nu}^S \right\}_{\sigma=1}^r \subset \widetilde{M}_{ST_L}^{T-S} \\ \left(\text{resp.} \quad \left\{ \widetilde{M}_{ST_{\bar{v}\sigma}, m_\sigma} = \widetilde{M}_{ST_{\bar{v}\mu}, m_\mu}^T + \widetilde{M}_{ST_{\bar{v}\nu}, m_\nu}^S \right\}_{\sigma=1}^r \subset \widetilde{M}_{ST_R}^{T-S} \right) \\ \bullet \quad \left\{ \widetilde{M}_{MG_{v\sigma}, m_\sigma} = \widetilde{M}_{MG_{v\mu}, m_\mu}^T + \widetilde{M}_{MG_{v\nu}, m_\nu}^S \right\}_{\sigma=1}^r \subset \widetilde{M}_{MG_L}^{T-S} \\ \left(\text{resp.} \quad \left\{ \widetilde{M}_{MG_{\bar{v}\sigma}, m_\sigma} = \widetilde{M}_{MG_{\bar{v}\mu}, m_\mu}^T + \widetilde{M}_{MG_{\bar{v}\nu}, m_\nu}^S \right\}_{\sigma=1}^r \subset \widetilde{M}_{MG_R}^{T-S} \right)$$

$$\bullet \quad \left\{ \widetilde{M}_{M_{v\sigma}, m_\sigma} = \widetilde{M}_{M_{v\mu}, m_\mu}^T + \widetilde{M}_{M_{v\nu}, m_\nu}^S \right\}_{\sigma=1}^r \subset \widetilde{M}_{M_L}^{T-S}$$

$$\left(\text{resp.} \quad \left\{ \widetilde{M}_{M_{\bar{v}\sigma}, m_\sigma} = \widetilde{M}_{M_{\bar{v}\mu}, m_\mu}^T + \widetilde{M}_{M_{\bar{v}\nu}, m_\nu}^S \right\}_{\sigma=1}^r \subset \widetilde{M}_{M_R}^{T-S} \right)$$

be the set of sections of the space-time left (resp. right) semisheaves of the internal vacuum (“ST”), middle ground (“MG”) and mas (“M”) structures of a left (resp. right) semifermion.

• Let

$$\bullet \quad T_{ST_L} = -i \frac{\hbar_{ST}}{c_{t \rightarrow r; ST}} (\vec{s}_{t_0} dt_0 + \vec{s}_{r_x} dx + \vec{s}_{r_y} dy + \vec{s}_{r_z} dz)$$

$$\left(\text{resp.} \quad T_{ST_R} = +i \frac{\hbar_{ST}}{c_{t \rightarrow r; ST}} (\vec{s}_{t_0} dt_0 + \vec{s}_{r_x} dx + \vec{s}_{r_y} dy + \vec{s}_{r_z} dz) \right)$$

$$\bullet \quad DT_{MG_L} = -i \frac{\hbar_{MG}}{c_{t \rightarrow r; MG}} \left(\vec{s}_{t_0} \frac{\partial}{\partial t_0} + \vec{s}_{r_x} \frac{\partial}{\partial x} + \vec{s}_{r_y} \frac{\partial}{\partial y} + \vec{s}_{r_z} \frac{\partial}{\partial z} \right)$$

$$\left(\text{resp.} \quad DT_{MG_R} = +i \frac{\hbar_{MG}}{c_{t \rightarrow r; MG}} \left(\vec{s}_{t_0} \frac{\partial}{\partial t_0} + \vec{s}_{r_x} \frac{\partial}{\partial x} + \vec{s}_{r_y} \frac{\partial}{\partial y} + \vec{s}_{r_z} \frac{\partial}{\partial z} \right) \right)$$

$$\bullet \quad DT_{M_L} = -i \frac{\hbar_M}{c_{t \rightarrow r; M}} \left(\vec{s}_{t_0} \frac{\partial}{\partial t_0} + \vec{s}_{r_x} \frac{\partial}{\partial x} + \vec{s}_{r_y} \frac{\partial}{\partial y} + \vec{s}_{r_z} \frac{\partial}{\partial z} \right)$$

$$\left(\text{resp.} \quad DT_{M_R} = +i \frac{\hbar_M}{c_{t \rightarrow r; M}} \left(\vec{s}_{t_0} \frac{\partial}{\partial t_0} + \vec{s}_{r_x} \frac{\partial}{\partial x} + \vec{s}_{r_y} \frac{\partial}{\partial y} + \vec{s}_{r_z} \frac{\partial}{\partial z} \right) \right)$$

be the corresponding differential operators acting on these sets of sections.

Then, the equations (*) of the internal dynamics of a bisemifermion can be put in the equivalent form:

$$\left[T_{ST_R}(\widetilde{M}_{ST_{\bar{v}\sigma}, m_\sigma}) \otimes T_{ST_L}(\widetilde{M}_{ST_{v\sigma}, m_\sigma}) \right] + \left[DT_{MG_R}(\widetilde{M}_{MG_{\bar{v}\sigma}, m_\sigma}) \otimes DT_{MG_L}(\widetilde{M}_{MG_{v\sigma}, m_\sigma}) \right]$$

$$= - \left[DT_{M_R}(\widetilde{M}_{M_{\bar{v}\sigma}, m_\sigma}) \otimes DT_{M_L}(\widetilde{M}_{M_{v\sigma}, m_\sigma}) \right], \quad \forall \sigma, m_\sigma, \quad 1 \leq \sigma \leq r.$$

Proof. Indeed, it appears from the preceding developments that the actions of the bi-operators $(T_{ST_R} \otimes T_{ST_L})$, $(DT_{MG_R} \otimes DT_{MG_L})$ and $(DT_{M_R} \otimes DT_{M_L})$ on the corresponding bisections, as developed above, transform these into bisections of the corresponding perverse bisemisheaves and give rise to the equations (*) of the internal dynamics of a bisemifermion. ■

3.12 Corollary

Let $T_{ST_{L_R}}$, $DT_{MG_{L_R}}$ and $DT_{M_{L_R}}$ be the adjoint operators of T_{ST_R} , DT_{MG_R} and DT_{M_R} respectively:

$$T_{ST_{L_R}} = T_{ST_R}^\dagger, \quad DT_{MG_{L_R}} = DT_{MG_R}^\dagger \quad \text{and} \quad DT_{M_{L_R}} = DT_{M_R}^\dagger.$$

Let $\widetilde{M}_{\overline{ST}_{\overline{v}\sigma, m_\sigma}}$, $\widetilde{M}_{\overline{MG}_{\overline{v}\sigma, m_\sigma}}$ and $\widetilde{M}_{\overline{M}_{\overline{v}\sigma, m_\sigma}}$ be the sections of the semisheaves $\widetilde{M}_{ST_R}^{T-S}$, $\widetilde{M}_{MG_R}^{T-S}$ and $\widetilde{M}_{M_R}^{T-S}$ projected onto the sections of the corresponding left semisheaves according to:

$$\begin{aligned} B_L^{(ST)} \circ p_L^{(ST)} : \quad & \widetilde{M}_{ST_{\overline{v}\sigma, m_\sigma}} \longrightarrow \widetilde{M}_{\overline{ST}_{\overline{v}\sigma, m_\sigma}}, \\ B_L^{(MG)} \circ p_L^{(MG)} : \quad & \widetilde{M}_{MG_{\overline{v}\sigma, m_\sigma}} \longrightarrow \widetilde{M}_{\overline{MG}_{\overline{v}\sigma, m_\sigma}}, \\ B_L^{(M)} \circ p_L^{(M)} : \quad & \widetilde{M}_{M_{\overline{v}\sigma, m_\sigma}} \longrightarrow \widetilde{M}_{\overline{M}_{\overline{v}\sigma, m_\sigma}}. \end{aligned}$$

Then, the equations on the sections of the internal dynamics of a bisemifermion are transformed under the isomorphism $ID_{R \rightarrow L_R}$ into:

$$\begin{aligned} ID_{R \rightarrow L_R} : \quad & [T_{ST_R}(\widetilde{M}_{ST_{\overline{v}\sigma, m_\sigma}}) \otimes T_{ST_L}(\widetilde{M}_{ST_{v\sigma, m_\sigma}})] \\ & + [DT_{MG_R}(\widetilde{M}_{MG_{\overline{v}\sigma, m_\sigma}}) \otimes DT_{MG_L}(\widetilde{M}_{MG_{v\sigma, m_\sigma}})] \\ & = -[DT_{M_R}(\widetilde{M}_{M_{\overline{v}\sigma, m_\sigma}}) \otimes DT_{M_L}(\widetilde{M}_{M_{v\sigma, m_\sigma}})] \\ & \longrightarrow [(T_{ST_{L_R}} \times T_{ST_L})(\widetilde{M}_{\overline{ST}_{\overline{v}\sigma, m_\sigma}} \times \widetilde{M}_{ST_{v\sigma, m_\sigma}})] \\ & \quad + [(DT_{MG_{L_R}} \times DT_{MG_L})(\widetilde{M}_{\overline{MG}_{\overline{v}\sigma, m_\sigma}} \times \widetilde{M}_{MG_{v\sigma, m_\sigma}})] \\ & \quad = -[(DT_{M_{L_R}} \times DT_{M_L})(\widetilde{M}_{\overline{M}_{\overline{v}\sigma, m_\sigma}} \times \widetilde{M}_{M_{v\sigma, m_\sigma}})]. \end{aligned}$$

Proof. This isomorphism $ID_{R \rightarrow L_R}$ transforms the (tensor) product of the right action of DT_{M_R} on $\widetilde{M}_{M_{\overline{v}\sigma, m_\sigma}}$ by the left action of DT_{M_L} on $\widetilde{M}_{M_{v\sigma, m_\sigma}}$ into the biaction of $(DT_{M_{L_R}} \times DT_{M_L})$ onto the product of the sections $(\widetilde{M}_{\overline{M}_{\overline{v}\sigma, m_\sigma}} \times \widetilde{M}_{M_{v\sigma, m_\sigma}})$, and so on for the two other levels “ ST ” and “ MG ”.

And, the products of the following sections belong to the extended bilinear Hilbert spaces:

$$\widetilde{M}_{\overline{ST}_{\overline{v}\sigma, m_\sigma}} \times \widetilde{M}_{ST_{v\sigma, m_\sigma}} \in H_{ST}^+, \quad \widetilde{M}_{\overline{MG}_{\overline{v}\sigma, m_\sigma}} \times \widetilde{M}_{MG_{v\sigma, m_\sigma}} \in H_{MG}^+, \quad \widetilde{M}_{\overline{M}_{\overline{v}\sigma, m_\sigma}} \times \widetilde{M}_{M_{v\sigma, m_\sigma}} \in H_M^+,$$

according to section 3.9. ■

Chapter 4

Equivalence between the equations of general relativity and the equations of the internal dynamics of bisemiparticles

The aim of this chapter consists in generalizing the equations of the internal dynamics to a set of J (bisemi)particles, and, more particularly, to a set of J elementary (bisemi)fermions and in pointing out that they are in one-to-one correspondence with the equations of general relativity.

4.1 The internal structure of a set of bisemifermions

- It was seen in chapter 3 that the time or space string field of the internal vacuum, middle ground or mass structure of a bisemifermion is given by the representation space (in bisemisheaf) of the bilinear algebraic semigroup $GL_2(L_{\overline{v}} \times L_v)$.
- Generalizing to a set of J bisemifermions, we have to take into account the partition $2J = 2_1 + \dots + 2_i + \dots + 2_J$ of $2J$ in such a way that the “ ST ”, “ MG ” or “ M ” string field of time or space of them be given by **the completely reducible non orthogonal representation space $\text{Repsp}(GL_2(L_{\overline{v}} \times L_v))$** of the bilinear algebraic semigroup $GL_{2J}(L_{\overline{v}} \times L_v)$ of dimension $2J$, as introduced in [Pie2].

This representation space $\text{Repsp}(GL_{2J}(L_{\overline{v}} \times L_v))$ **decomposes non orthogonally**

according to:

$$\text{Repsp}(\text{GL}_{2J}(L_{\bar{v}} \times L_v)) = \bigsqcup_{i=1}^J \text{Repsp}(\text{GL}_{2_i}(L_{\bar{v}} \times L_v)) \bigsqcup_{i \neq j=1}^J \text{Repsp}(T_{2_i}^t(L_{\bar{v}}) \times T_{2_j}(L_v))$$

where the **off-diagonal representation spaces** $\text{Repsp}(T_{2_i}^t(L_{\bar{v}}) \times T_{2_j}(L_v))$ **are responsible for the generation of gravito-electro-magnetic fields of interaction** between bisemifermions as developed in [Pie5].

- Referring to Proposition 3.7, we see that the internal vacuum (“ ST ”) field structure of space-time of J interacting bisemifermions is given by the bisemisheaves:

$$\begin{aligned} (\widetilde{M}_{ST_{R_J}}^{T-S} \otimes \widetilde{M}_{ST_{L_J}}^{T-S}) &= \text{Repsp} \theta(\text{GL}_{2J(T+S)}(L_{\bar{v}} \times L_v)) \\ &= \bigsqcup_{i=1}^J (\widetilde{M}_{ST_{R_i}}^{T-S} \otimes \widetilde{M}_{ST_{L_i}}^{T-S}) \bigsqcup_{i \neq j=1}^J (\widetilde{M}_{ST_{R_i}}^{T-S} \otimes \widetilde{M}_{ST_{L_j}}^{T-S}). \end{aligned}$$

- By versal deformations and blowups of these, the “internal vacuum” string fields $(\widetilde{M}_{ST_{R_J}}^{T-S} \otimes \widetilde{M}_{ST_{L_J}}^{T-S})$ of a set of J interacting bisemifermions generate **the two covering “middle ground” and “mass” string fields of space-time**:

$$(\widetilde{M}_{MG_{R_J}}^{T-S} \otimes \widetilde{M}_{MG_{L_J}}^{T-S}) = \bigsqcup_{i=1}^J (\widetilde{M}_{MG_{R_i}}^{T-S} \otimes \widetilde{M}_{MG_{L_i}}^{T-S}) \bigsqcup_{i \neq j=1}^J (\widetilde{M}_{MG_{R_i}}^{T-S} \otimes \widetilde{M}_{MG_{L_j}}^{T-S})$$

and

$$(\widetilde{M}_{M_{R_J}}^{T-S} \otimes \widetilde{M}_{M_{L_J}}^{T-S}) = \bigsqcup_{i=1}^J (\widetilde{M}_{M_{R_i}}^{T-S} \otimes \widetilde{M}_{M_{L_i}}^{T-S}) \bigsqcup_{i \neq j=1}^J (\widetilde{M}_{M_{R_i}}^{T-S} \otimes \widetilde{M}_{M_{L_j}}^{T-S})$$

in such a way that we have the embeddings:

$$(\widetilde{M}_{ST_{R_J}}^{T-S} \otimes \widetilde{M}_{ST_{L_J}}^{T-S}) \subset (\widetilde{M}_{MG_{R_J}}^{T-S} \otimes \widetilde{M}_{MG_{L_J}}^{T-S}) \subset (\widetilde{M}_{M_{R_J}}^{T-S} \otimes \widetilde{M}_{M_{L_J}}^{T-S}).$$

4.2 Proposition

Let $(\widetilde{M}_{ST_{R_J}}^{T_p-S_p} \otimes \widetilde{M}_{ST_{L_J}}^{T_p-S_p})$, $(\widetilde{M}_{MG_{R_J}}^{T_p-S_p} \otimes \widetilde{M}_{MG_{L_J}}^{T_p-S_p})$ and $(\widetilde{M}_{M_{R_J}}^{T_p-S_p} \otimes \widetilde{M}_{M_{L_J}}^{T-S})$ be the perverse bisemisheaves of the internal structures “ ST ”, “ MG ” and “ M ” of a set of J interacting bisemifermions.

If the interactions between the right and left internal semifields “ ST ”, “ MG ” and “ M ” are negligible, then **the equations of the internal dynamics of this set of J bisemifermions are**:

$$(\widetilde{M}_{ST_{R_J}}^{T_p-S_p} \otimes \widetilde{M}_{ST_{L_J}}^{T_p-S_p}) \oplus (\widetilde{M}_{MG_{R_J}}^{T_p-S_p} \otimes \widetilde{M}_{MG_{L_J}}^{T_p-S_p}) = -(\widetilde{M}_{M_{R_J}}^{T_p-S_p} \otimes \widetilde{M}_{M_{L_J}}^{T-S}).$$

Proof.

- The perverse bisemisheaves $(\widetilde{M}_{ST_{R_J}}^{T_p-S_p} \otimes \widetilde{M}_{ST_{L_J}}^{T_p-S_p})$, $(\widetilde{M}_{MG_{R_J}}^{T_p-S_p} \otimes \widetilde{M}_{MG_{L_J}}^{T_p-S_p})$ and $(\widetilde{M}_{M_{R_J}}^{T_p-S_p} \otimes \widetilde{M}_{M_{L_J}}^{T-S})$ are obtained from the corresponding bisemisheaves $(\widetilde{M}_{ST_{R_J}}^{T-S} \otimes \widetilde{M}_{ST_{L_J}}^{T-S})$, $(\widetilde{M}_{MG_{R_J}}^{T-S} \otimes \widetilde{M}_{MG_{L_J}}^{T-S})$ and $(\widetilde{M}_{M_{R_J}}^{T-S} \otimes \widetilde{M}_{M_{L_J}}^{T-S})$ by the respective actions of the biopearators $(T_{ST_L} \otimes T_{ST_R})$, $(DT_{MG_L} \otimes DT_{MG_R})$ and $(DT_{M_R} \otimes DT_{M_L})$ introduced in section 3.11.
- The equations of the internal dynamics of a set of J bisemifermions are a generalization of the equations $(*)$ of proposition 3.8. $\blacksquare$

4.3 Proposition

- Let $r_i = q_i + p_i$, $1 \leq i \leq J$, be the numbers of algebraic conjugacy classes of time and space varying commonly according to $1 \leq \sigma_i \leq r_i$.

- Let

$$\begin{aligned} \{\widetilde{M}_{ST_{v\sigma_i, m\sigma_i}}\}_{\sigma_i=1}^{r_i} &\subset \widetilde{M}_{ST_{L_i}}^{T-S} & (\text{resp. } \{\widetilde{M}_{ST_{\bar{v}\sigma_i, m\sigma_i}}\}_{\sigma_i=1}^{r_i} &\subset \widetilde{M}_{ST_{R_i}}^{T-S}) \\ \{\widetilde{M}_{MG_{v\sigma_i, m\sigma_i}}\}_{\sigma_i=1}^{r_i} &\subset \widetilde{M}_{MG_{L_i}}^{T-S} & (\text{resp. } \{\widetilde{M}_{MG_{\bar{v}\sigma_i, m\sigma_i}}\}_{\sigma_i=1}^{r_i} &\subset \widetilde{M}_{MG_{R_i}}^{T-S}) \\ \text{and } \{\widetilde{M}_{M_{v\sigma_i, m\sigma_i}}\}_{\sigma_i=1}^{r_i} &\subset \widetilde{M}_{M_{L_i}}^{T-S} & (\text{resp. } \{\widetilde{M}_{M_{\bar{v}\sigma_i, m\sigma_i}}\}_{\sigma_i=1}^{r_i} &\subset \widetilde{M}_{M_{R_i}}^{T-S}) \end{aligned}$$

be the set of sections of the space-time left (resp. right) semisheaves of the “internal vacuum” (ST), “middle ground” (MG) and “mass” (M) structures of the J , $1 \leq i \leq J$, left (resp. right) considered semifermions.

Then, **the equations**

$$(\widetilde{M}_{ST_{R_J}}^{T_p-S_p} \otimes \widetilde{M}_{ST_{L_J}}^{T_p-S_p}) \oplus (\widetilde{M}_{MG_{R_J}}^{T_p-S_p} \otimes \widetilde{M}_{MG_{L_J}}^{T-S}) = -(\widetilde{M}_{M_{R_J}}^{T_p-S_p} \otimes \widetilde{M}_{M_{L_J}}^{T-S})$$

of the internal dynamics of a set of J bisemifermions are in one-to-one correspondence with the equations

$$\begin{aligned} [T_{ST_{R_i}}(\widetilde{M}_{ST_{\bar{v}\sigma_i, m\sigma_i}}) \otimes T_{ST_{L_j}}(\widetilde{M}_{ST_{v\sigma_j, m\sigma_j})}] \\ + [DT_{MG_{R_i}}(\widetilde{M}_{MG_{\bar{v}\sigma_i, m\sigma_i}}) \otimes DT_{ST_{L_j}}(\widetilde{M}_{MG_{v\sigma_j, m\sigma_j})}] \\ = -[DT_{M_{R_i}}(\widetilde{M}_{M_{\bar{v}\sigma_i, m\sigma_i}}) \otimes DT_{ST_{L_j}}(\widetilde{M}_{M_{v\sigma_j, m\sigma_j})}] \\ \forall \sigma_i, \sigma_j, 1 \leq \sigma_i, \sigma_j \leq r_i, r_j \text{ and } \forall i, j \quad 1 \leq i, j \leq J \leq \infty \end{aligned}$$

referring to the sections of the considered bisemisheaves.

Proof. The one-to-one correspondence between the two types of equations results from:

- a generalization of proposition 3.11 to a set of J bisemifermions.
- a homomorphism

$$H_{ST} : (\widetilde{M}_{ST_{R_J}}^{T_p-S_p} \otimes \widetilde{M}_{ST_{L_J}}^{T_p-S_p}) = \bigoplus_{i=1}^J (\widetilde{M}_{ST_{R_i}}^{T_p-S_p} \otimes \widetilde{M}_{ST_{L_i}}^{T-S}) \bigoplus_{i \neq j=1}^J (\widetilde{M}_{ST_{R_i}}^{T_p-S_p} \otimes \widetilde{M}_{ST_{L_j}}^{T-S}) \\ \longrightarrow [T_{ST_{R_i}}(\widetilde{M}_{ST_{\bar{v}\sigma_i, m\sigma_i}}) \otimes T_{ST_{L_j}}(\widetilde{M}_{ST_{v\sigma_j, m\sigma_j}})]$$

sending the sums $\bigoplus_{i=1}^J$ and $\bigoplus_{i \neq j=1}^J$ of bisemisheaves, referring respectively to diagonal and off diagonal interactions between right and left “ ST ” semistructures of the J bisemifermions, into the set $[T_{ST_{R_i}}(\widetilde{M}_{ST_{\bar{v}\sigma_i, m\sigma_i}}) \otimes T_{ST_{L_j}}(\widetilde{M}_{ST_{v\sigma_j, m\sigma_j}})]$ of the corresponding bisections of these bisemisheaves.

- two similar homomorphisms H_{MG} and H_M referring to the “ MG ” and “ M ” structures of the J bisemifermions. ■

4.4 Proposition

$$\begin{aligned} \bullet \text{ Let } B_L^{(ST)} \circ p_L^{(ST)} : \quad & \widetilde{M}_{ST_{\bar{v}\sigma_i, m\sigma_i}} \longrightarrow \widetilde{M}_{ST_{\bar{v}\sigma_i, m\sigma_i}} \\ B_L^{(MG)} \circ p_L^{(MG)} : \quad & \widetilde{M}_{MG_{\bar{v}\sigma_i, m\sigma_i}} \longrightarrow \widetilde{M}_{MG_{\bar{v}\sigma_i, m\sigma_i}} \\ \text{and } B_L^{(M)} \circ p_L^{(M)} : \quad & \widetilde{M}_{M_{\bar{v}\sigma_i, m\sigma_i}} \longrightarrow \widetilde{M}_{M_{\bar{v}\sigma_i, m\sigma_i}} \end{aligned}$$

be the maps projecting the sections $\widetilde{M}_{ST_{\bar{v}\sigma_i, m\sigma_i}}$, $\widetilde{M}_{MG_{\bar{v}\sigma_i, m\sigma_i}}$ and $\widetilde{M}_{M_{\bar{v}\sigma_i, m\sigma_i}}$ onto their left equivalents.

- Let $T_{ST_{L_{R_i}}}$, $DT_{MG_{L_{R_i}}}$ and $DT_{M_{L_{R_i}}}$ be the adjoint operators of $T_{ST_{R_i}}$, $DT_{MG_{R_i}}$ and $DT_{M_{R_i}}$ respectively:

$$T_{ST_{L_{R_i}}} = T_{ST_{R_i}}^\dagger, \quad T_{MG_{L_{R_i}}} = T_{MG_{R_i}}^\dagger \quad \text{and} \quad T_{M_{L_{R_i}}} = T_{M_{R_i}}^\dagger.$$

Then, the equations on the sections of the internal dynamics of a set of J bisemifermions are transformed under the isomorphisms $ID_{R_J \rightarrow L_{R_J}}$ into:

$$\begin{aligned}
ID_{R_J \rightarrow L_{R_J}} : \quad & [T_{ST_{R_i}}(\widetilde{M}_{ST\bar{v}\sigma_i, m\sigma_i}) \otimes T_{ST_{L_j}}(\widetilde{M}_{STv\sigma_j, m\sigma_j})] \\
& + [DT_{MG_{R_i}}(\widetilde{M}_{MG\bar{v}\sigma_i, m\sigma_i}) \otimes DT_{MG_{L_j}}(\widetilde{M}_{MGv\sigma_j, m\sigma_j})] \\
& = -[DT_{M_{R_i}}(\widetilde{M}_{M\bar{v}\sigma_i, m\sigma_i}) \otimes DT_{M_{L_j}}(\widetilde{M}_{Mv\sigma_j, m\sigma_j})] \\
& \longrightarrow [(T_{ST_{L_{R_i}}} \times T_{ST_{L_j}})(\widetilde{M}_{\overline{ST}\bar{v}\sigma_i, m\sigma_i} \times \widetilde{M}_{STv\sigma_i, m\sigma_i})] \\
& \quad + [(DT_{MG_{L_{R_i}}} \times DT_{MG_{L_j}})(\widetilde{M}_{\overline{MG}\bar{v}\sigma_i, m\sigma_i} \times \widetilde{M}_{MGv\sigma_j, m\sigma_j})] \\
& \quad = -[(DT_{M_{L_{R_i}}} \times DT_{M_{L_j}})(\widetilde{M}_{\overline{M}\bar{v}\sigma_i, m\sigma_i} \times \widetilde{M}_{Mv\sigma_j, m\sigma_j})] .
\end{aligned}$$

Proof. This proposition is a generalization of corollary 3.12 to a set of J bisemifermions. ■

4.5 The small value of the cosmological constant

The equations obtained under the isomorphism $ID_{R_J \rightarrow L_{R_J}}$ and describing the internal dynamics of a set of (bisemi)fermions are rather close to the equations of general relativity as it will be seen in the following sections.

But, one of the big problems of the equations of general relativity in connection with the phenomenology of quantum field theories consists in the small value given to the cosmological constant. Indeed, in order to avoid a static solution to his equations:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

where:

a) $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ with:

- $R_{\mu\nu}$ the Ricci tensor which is the contracted form of the Riemann-Christoffel curvature tensor $R^\lambda_{\mu\lambda\nu}$ by $R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}$,
- $R = g^{\mu\nu} R_{\mu\nu}$ the curvature scalar,
- $g_{\mu\nu}$ the metric tensor of space-time,

b) $T_{\mu\nu}$ is the symmetric energy-momentum tensor and G is the gravitational constant,

Einstein introduced in these a new term $\lambda g_{\mu\nu}$, where **λ is the cosmological constant**, leading to [Ein2], [Ein3]:

$$\lambda g_{\mu\nu} + G_{\mu\nu} = 8\pi G T_{\mu\nu} .$$

The equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ describe how matter, given by $T_{\mu\nu}$, generates gravitational forces, characterized by the tensor $G_{\mu\nu}$, by means of the curvature of the space-time; indeed, the gravitational field is assumed to be represented by the metric tensor itself.

In this context, Zel'dovich [Zel1], [Zel2] envisaged **the possible connection between the vacuum energy density of quantum field theories and the Einstein's cosmological constant λ** in such a way that the author proposed in [Pie1] to describe the vacuum energy density ρ_{vac} by

$$\rho_{\text{vac}} \simeq \rho_{\text{vac}}^{(ST)} + \rho_{\text{vac}}^{(MG)} \simeq \lambda_{\text{eff}} / 8\pi G$$

where:

- the internal vacuum energy density $\rho_{\text{vac}}^{(ST)}$ could correspond to $\lambda/8\pi G$,
- $\rho_{\text{vac}}^{(MG)}$ would correspond to the middle ground energy density “ MG ”.

The revised equations of general relativity can then take the form:

$$G_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{\lambda}{8\pi G} g_{\mu\nu} \right)$$

in such a way that $\lambda g_{\mu\nu}$ could correspond to the internal vacuum energy density $\rho_{\text{vac}}^{(ST)}$ which would behave like an ideal fluid with negative pressure $\rho_{\text{vac}}^{(ST)} = -p^{(ST)}$ [Pee], [Lem].

Considerations on the expansion of the universe allowed Weinberg [Wei1] to show that the effective cosmological constant λ_{eff} can be related to the Hubble constant H_0 by:

$$|\lambda_{\text{eff}}| \leq H_0^2$$

and, also, that:

$$|\rho_{\text{vac}}| \leq 10^{-29} \text{g/cm}^3 \simeq 10^{-47} \text{Gev} .$$

So, the present expansion rate of the universe viewed throughout the curvature of space-time by means of the Friedmann's model [Frie] leads to envisage very small values for the vacuum energy density and the related cosmological constant [Wei1].

This expectation small value of the cosmological constant λ must be understood in the frame of general relativity describing gravity by the curvature of space-time by noticing that **the square root of λ^{-1} is a “distance” referring to the domain where the vacuum energy density $\rho_{\text{vac}}^{(ST)}$ alters the geometry of space-time by its gravitational effects** [Abb] in such a way that the curvature be on an average null.

4.6 New interpretation of the general relativity equations

Thus, the only way to go beyond this problem of the small value of λ is to take into account a new interpretation of the equations of general relativity [Ein1]

$$\lambda g_{\mu\nu} + G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

as developed in [Pie5] and [Pie3].

This new context is especially characterized by:

- a) the fact that **the gravitational potential is no more** assumed to be **described by the metric tensor $g_{\mu\nu}$** and the gravity is thus not explicitly described by the tensor $G_{\mu\nu}$ when the context of curved space-time geometry remains.
- b) **the composition of “matter” given by the three embedded structures $ST \subset MG \subset M$** at the elementary particle level, as developed from the beginning of chapter 3, in such a way that:

- the terms $\lambda g_{\mu\nu} + G_{\mu\nu}$ refer to the vacuum structure of matter as it clearly appears from section 4.5 that, when the energy stress tensor of matter $T_{\mu\nu}$ (corresponding to the level “ M ”) is null, then the revised equations of GR in the vacuum are:

$$\lambda g_{\mu\nu} + G_{\mu\nu} = 0 .$$

- – the term $\lambda g_{\mu\nu}$ would be associated to the “ ST ” internal vacuum substructures;
- the term $G_{\mu\nu}$ would correspond to the “ MG ” middle ground vacuum substructures;
- the term $8\pi G T_{\mu\nu}$ would correspond to the “ M ” mass boundary structures.

It then appears that the vacuum, considered at the elementary particle level, must be composed of:

1. the “internal vacuum” substructures “ ST ” of which nature is of space-time type having a dynamical and expansive aspect.
2. the “middle ground” substructures generated from the corresponding internal “ ST ” substructures and having a contracting aspect allowing to confine “ ST ” substructures inside the elementary particles.

This elementary particle vacuum then corresponds to the vacuum of the quantum field theories allowing to generate the particle masses [Par] (i.e. the “ M ” boundary structures, in the context of AQT, from the degenerate singularities on the “middle ground” substructures) and would be associated with the map:

$$G_M : \quad \lambda g_{\mu\nu} + G_{\mu\nu} = 0 \longrightarrow \lambda g_{\mu\nu} + G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

in the frame of general relativity equations.

This new interpretation of general relativity, connecting its vacuum energy with that of quantum field theories, was foreboded by Sakharov [Sak] who claimed that gravity is not a fundamental quantum field but an induced quantum effect caused by an interaction of quantum vacuum fluctuations with space-time curvature.

In other terms, the inhomogeneity of vacuum fluctuations induces the Riemann space-time geometry [Gli], [Ban] which does not describe ontologically gravity.

4.7 Compactification of the “ ST ”, “ MG ” and “ M ” structures

It remains to prove that a one-to-one correspondence, given by the isomorphism $I_{QT \rightarrow GR}$, exists between the equations of general relativity and the string field equations on the sections of the internal dynamics of a set of (bisemi)fermions (or, more generally, of a set of (bisemi)particles) as given in proposition 4.4:

$$\begin{aligned} I_{QT \rightarrow GR} : \quad & [(T_{ST_{L_{R_i}}} \times T_{ST_{L_j}})(\widetilde{M}_{\overline{ST}_{\overline{\sigma}_i, m_{\sigma_i}}} \times \widetilde{M}_{ST_{\overline{\sigma}_j, m_{\sigma_j}}})] \\ & + [(DT_{MG_{L_{R_i}}} \times DT_{MG_{L_j}})(\widetilde{M}_{\overline{MG}_{\overline{\sigma}_i, m_{\sigma_i}}} \times \widetilde{M}_{MG_{\overline{\sigma}_j, m_{\sigma_j}}})] \\ & = -[(DT_{ML_{R_i}}} \times DT_{ML_j})(\widetilde{M}_{\overline{M}_{\overline{\sigma}_i, m_{\sigma_i}}} \times \widetilde{M}_{M_{\overline{\sigma}_j, m_{\sigma_j}}})] \\ & \longrightarrow \lambda g_{\mu\nu} + G_{\mu\nu} = 8\pi G T_{\mu\nu} . \end{aligned}$$

To reach this objective, it is necessary to:

1. compactify the diagonal perverse semisheaves $(T_{ST_{L_{R_i}}} \widetilde{M}_{\overline{ST}_{\overline{\sigma}_i, m_{\sigma_i}}})$ and $(T_{ST_{L_i}} \widetilde{M}_{ST_{\overline{\sigma}_i, m_{\sigma_i}}})$ of the internal vacua “ ST ” of the “ J ” considered bisemifermions as it was done in section 2.10 in order to obtain four-dimensional (1 dimension of time and 3 dimensions of space) perverse right (resp. left) compactified semisheaves according to the

compactification map:

$$\begin{aligned} c_{ST_R}^{t-r} : \quad T_{ST_{L_{R_i}}} \widetilde{M}_{\overline{ST}_{\overline{v}_{\sigma_i}, m_{\sigma_i}}} &\longrightarrow T_{ST_{L_{R_i}}} \widetilde{M}_{\overline{ST}_{\overline{v}_{\sigma_i}, m_{\sigma_i}}}^c \\ (\text{resp. } c_{ST_L}^{t-r} : \quad T_{ST_{L_i}} \widetilde{M}_{ST_{v_{\sigma_i}, m_{\sigma_i}}} &\longrightarrow T_{ST_{L_i}} \widetilde{M}_{ST_{v_{\sigma_i}, m_{\sigma_i}}}^c) . \end{aligned}$$

2. compactify similarly the diagonal perverse semisheaves of the right and left “middle ground” (MG) and “mass” (M) structures of the considered bisemifermions.
3. bring together the “ J ” bisemifermions in such a way that the off-diagonal bisemisheaves

$$(T_{ST_{L_{R_i}}} \widetilde{M}_{\overline{ST}_{\overline{v}_{\sigma_i}, m_{\sigma_i}}}^c \times T_{ST_{L_j}} \widetilde{M}_{ST_{v_{\sigma_j}, m_{\sigma_j}}}^c) , \quad \forall i \neq j ,$$

of interaction of the “ ST ” structures, but also of the “ MG ” and “ M ” structures, have a continuous character.

4.8 Proposition

An isomorphism $I_{QT \rightarrow GR}^c$ exists between the string field equations of the internal dynamics of a set of compactified bisemiparticles and the equations of general relativity:

$$\begin{aligned} I_{QT \rightarrow GR}^c : \quad & [(T_{ST_{L_{R_i}}} \times T_{ST_{L_j}})(\widetilde{M}_{\overline{ST}_{\overline{v}_{\sigma_i}, m_{\sigma_i}}}^c \times \widetilde{M}_{ST_{v_{\sigma_j}, m_{\sigma_j}}}^c)] \\ & + [(DT_{MGL_{R_i}} \times DT_{MGL_j})(\widetilde{M}_{\overline{MG}_{\overline{v}_{\sigma_i}, m_{\sigma_i}}}^c \times \widetilde{M}_{MG_{v_{\sigma_j}, m_{\sigma_j}}}^c)] \\ & = -[(DT_{ML_{R_i}} \times DT_{ML_j})(\widetilde{M}_{\overline{M}_{\overline{v}_{\sigma_i}, m_{\sigma_i}}}^c \times \widetilde{M}_{M_{v_{\sigma_j}, m_{\sigma_j}}}^c)] \\ & \longrightarrow \lambda g_{\mu\nu} + G_{\mu\nu} = 8\pi G T_{\mu\nu} \end{aligned}$$

at the conditions that [Pie1]:

- a) the sections of the right and left semisheaves “ ST ”, “ MG ” and “ M ”, which are in fact “ ST ”, “ MG ” and “ M ” strings, be viewed as families of geodesics which must be interpreted as the flow lines of a fluid.
- b) the directional gradients used in the left hand side of $I_{QT \rightarrow GR}^c$ be replaced by covariant derivatives on the right hand side.

Proof.

1. At first, the conditions a) will be precised.

The sections of the semisheaves “ MG ” and “ M ” are assumed to correspond respectively to families of geodesics $\mathcal{P}_{MG}(\lambda_{MG}, n_{MG})$ and $\mathcal{P}_M(\lambda_M, n_M)$, where:

- λ_{MG} and λ_M are affine parameters telling where we are on a given geodesics,
- n_{MG} and n_M are selector parameters allowing to distinguish one geodesics from the next [M-T-W],

if these sections have been desingularized in order that the tangent vectors

$$\vec{u}_{MG} = \frac{\partial \mathcal{P}_{MG}}{\partial \lambda_{MG}} \quad \text{and} \quad \vec{u}_M = \frac{\partial \mathcal{P}_M}{\partial \lambda_M}$$

be parallel on the corresponding geodesics.

Note that the vectors

$$\vec{n}_{MG} \equiv \frac{\partial \mathcal{P}}{\partial n_{MG}} \quad \text{and} \quad \vec{n}_M \equiv \frac{\partial \mathcal{P}}{\partial n_M}$$

measure the separation between points with the same values λ_{MG} and λ_M on neighbouring geodesics.

2. Then, the terms “ ST ”, “ MG ” and “ M ” will be shown to be in one-to-one correspondence on the left and on the right of $I_{QT \rightarrow GR}^c$.

- (a) The term $(T_{ST_{L_{R_i}}} \times T_{ST_{L_j}})(\widetilde{M}_{ST_{\overline{\sigma}_i, m_{\sigma_i}}}^c \times \widetilde{M}_{ST_{v_{\sigma_j}, m_{\sigma_j}}}^c)$, describing the internal vacuum structures “ ST ” at the level of elementary particles and being of expanding space-time nature (which can be seen by the differential nature of the bioperator $(T_{ST_{L_{R_i}}} \times T_{ST_{L_j}})$ acting on H_{ST}^+ according to section 3.10), must correspond to the term $\lambda g_{\mu\nu}$ of GR if it is taken into account that:

- this term $\lambda g_{\mu\nu}$ is not very well shaped, all the information having been smashed in the cosmological constant λ .
- the internal vacuum energy density $\rho_{\text{vac}}^{(ST)}$, to which $\lambda/8\pi G$ corresponds, must be of expanding space-time nature: for this reason, the space-time differential operator $(T_{ST_{L_{R_i}}} \times T_{ST_{L_j}})$ was chosen and not a directional gradient bioperator of the type $(DT_{ST_{L_{R_i}}} \times DT_{ST_{L_j}})$ as introduced in proposition 3.11.

(b) The term $(DT_{MG_{L_{R_i}}} \times DT_{MG_{L_j}})(\widetilde{M}_{MG_{\bar{v}\sigma_i, m\sigma_i}}^c \times \widetilde{M}_{MG_{v\sigma_j, m\sigma_j}}^c)$, describing the “middle ground” vacuum structures “ MG ” of elementary particles and being of contracting space-time nature, is in one-to-one correspondence with the term $G_{\mu\nu}$ of the general relativity equations if:

- the conditions (a) and (b) of this proposition are taken into account.
- it is noted that each right section $\widetilde{M}_{MG_{\bar{v}\sigma_i, m\sigma_i}}^c$ has been projected onto each left section $\widetilde{M}_{MG_{v\sigma_j, m\sigma_j}}^c$ in such a way that they are confounded: they can then be rewritten according to:

$$(\widetilde{M}_{MG_{\bar{v}\sigma_i, m\sigma_i}}^c \times \widetilde{M}_{MG_{v\sigma_j, m\sigma_j}}^c) \longrightarrow \widetilde{M}_{MG_{v\sigma_{i-j}, m\sigma_{i-j}}}^c .$$

So, to the set $\{\widetilde{M}_{MG_{v\sigma_{i-j}, m\sigma_{i-j}}}^c\}$ of bisections of “ MG ” will correspond a family $\mathcal{P}_{MG_{L_{R-L}}}(\lambda_{MG}, n_{MG})$ of products of left geodesics, localized in the upper half space, by projected symmetric right geodesics localised in the lower half space, in such a way that to the term $(DT_{MG_{L_{R_i}}} \times DT_{MG_{L_j}})(\widetilde{M}_{MG_{v\sigma_{i-j}, m\sigma_{i-j}}}^c)$ will correspond the term $\vec{\nabla}_{\vec{u}_{MG}} \cdot \vec{\nabla}_{\vec{u}_{MG}} \cdot \vec{n}_{MG}$.

$\vec{\nabla}_{\vec{u}_{MG}} \vec{n}_{MG}$ is the covariant derivative of the vector field $\vec{n}_{MG}$ along a product, right by left, $\mathcal{P}_{MG_{L_{R-L}}}(\lambda_{MG})$ of symmetric geodesics with tangent vector

$$\vec{u}_{MG} = \frac{\partial \mathcal{P}_{MG}}{\partial \lambda_{MG}} ,$$

and $\vec{\nabla}_{\vec{u}_{MG}} \vec{\nabla}_{\vec{u}_{MG}} \vec{n}_{MG}$ is the corresponding relative acceleration.

As the relative acceleration of geodesics allows to define the Riemann curvature tensor [M-T-W] by:

$$\vec{\nabla}_{\vec{u}_{MG}} \vec{\nabla}_{\vec{u}_{MG}} \vec{n}_{MG} + \text{Riemann}(\dots \vec{u}_{MG}, \vec{n}_{MG}, \vec{u}_{MG}) = 0$$

which leads to the components of the tensor of Riemann $R_{\mu\lambda\nu}^\lambda$ in a coordinate basis.

Due to the antisymmetric property of $R_{\mu\lambda\nu}^\lambda$ [Wei1], there are only two tensors which can be generated by contraction from $R_{\mu\lambda\nu}^\lambda$: it is the Ricci tensor $R_{\mu\nu} \equiv R_{\mu\lambda\nu}^\lambda$ and the curvature scalar $R = R_\mu^\mu$ [Dar].

So, the only tensor which can be formed from $R_{\mu\nu}$ and R is the tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ which is thus in one-to-one correspondence with the “ MG ” term on the left hand side of $I_{QT \rightarrow GR}^c$.

- (c) Finally, the term $(DT_{M_{L_{R_i}}} \times DT_{M_{L_j}})(\widetilde{M}_{\overline{M}v\sigma_i, m\sigma_i}^c \times \widetilde{M}_{Mv\sigma_j, m\sigma_j}^c)$, describing the “mass” structures of elementary particles and being also of contracting space-time nature, is in one-to-one correspondence with the tensor $T_{\mu\nu}$ of the equations of general relativity.

Indeed, if it is taken into account that the “mass” term “ M ” and the “middle ground” term “ MG ” on the left of $I_{QT \rightarrow GR}^c$ have the same structure, it is immediate to associate with $(DT_{M_{L_{R_i}}} \times DT_{M_{L_j}})(\widetilde{M}_{\overline{M}v\sigma_{i-j}, m\sigma_{i-j}}^c)$ the “mass” curvature tensor of Riemann $R_{\mu\lambda\nu}^\lambda$ from which the “mass” tensor

$$G_{\mu\nu}^{(M)} = R_{\mu\nu}^{(M)} - \frac{1}{2}g_{\mu\nu}^{(M)}R^{(M)}$$

can be formed.

Considering the Einstein equations

$$G_{\mu\nu} = 8\pi GT_{\mu\nu} ,$$

it is evident that:

- (1) the tensor $G_{\mu\nu}^{(M)}$ is equal to the “mass” energy-momentum tensor $8\pi GT_{\mu\nu}$.
- (2) to the mass term on the left hand side of $I_{QT \rightarrow GR}^c$ corresponds the tensor $T_{\mu\nu}$. ■

4.9 Proposition

In the context of the new interpretation of the equations of general relativity describing the dynamics of the generation of the mass shells of the elementary particles from their vacuum structures, we have that:

1. *the cosmological constant will likely have a high value and will be noted λ_{ST} .*
2. *the internal vacuum structure “ ST ” itself (without the generated “ MG ” and “ M ” structure) at the elementary particle level is probably responsible for the dark energy.*

Proof.

1. The cosmological constant λ_{ST} , referring now to the internal vacuum structures of elementary particles, is now directly related to the Planck scale; consequently, λ_{ST} will have a high value.

2. The dark energy, being of expanding space-time nature, will probably correspond to sets of elementary particles endowed only with their internal vacuum structures “ ST ”. ■

4.10 The algebraic quantum theory is a quantum gravity theory

The algebraic quantum theory [Pie5], recalled at the beginning of this chapter for a set of interacting bisemiparticles, is a quantum gravity theory.

Indeed, the gravity is no more introduced ontologically as resulting from the curvature of space-time but from the diagonal interactions between right and left semisheaves “ ST ”, “ MG ” and “ M ” belonging to different bisemiparticles as it was developed in [Pie5]. We refer thus to the preprint “Algebraic quantum theory” [Pie5] for a description of quantum gravity as resulting from the diagonal interactions between pairs of semiobjects.

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