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Well-being measurement with reference consumption

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Abstract

We axiomatically study how to measure well-being when individuals have heterogeneous preferences and consumption bundles are evaluated by comparison with some reference bundle (a poverty line bundle, an index of needs, the average consumption in a reference group, etc.). Our key requirement is that well-being remains constant if the reference bundle changes towards a bundle that the individual deems equivalent. Accounting for preferences may lead two individuals with identical consumption and reference bundles to have unequal well-being levels. To limit this inequality, we study four axioms based on the lattice structure of the set of indifference contours. We define and characterize five measures. One prominent measure corresponds to the ratio between equivalent incomes at consumption and at reference, where prices are chosen to minimize the difference between these incomes.

Keywords: well-being measurement, heterogeneous preferences, reference, lattice

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Statements and Declaration.

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1 Introduction

The classical tool in economics for conducting welfare analysis is the social welfare function, which typically aggregates individual well-being levels. Defining a social welfare function thus requires making decisions about two key aspects: 1) how to define well-being, and 2) how to aggregate it. While the literature on how to aggregate individual well-being is well-developed and has many surveys (see, for instance, d'Aspremont and Gevers 2002; Bossert and Weymark 2004), the literature on how to define well-being is less consensual. We contribute to this body of work by proposing a framework in which individual well-being is relative, meaning that it depends not only on an individual's consumption and preferences but also on a reference consumption level. This framework generalizes the poverty measurement literature, where well-being, or equivalently, an individual's contribution to poverty, is evaluated in reference to a poverty line bundle.¹

The best-known theory of well-being in poverty measurement is Deaton (1979)'s distance function, which was axiomatically characterized by Decancq et al. (2019); alternative approaches include those by Blackorby and Donaldson (1987) and Dimri and Maniquet (2020). According to this function, an individual's well-being is measured by the fraction of the poverty line bundle to which they are indifferent. However, when relative poverty analysis requires comparing individuals facing different poverty line bundles, due to spatial or temporal differences, two individuals with identical consumption and preferences may end up with different well-being levels. Specifically, if one of these individuals faces a poverty line bundle containing more of every good than another individual, the former will be considered to have lower well-being than the latter. This is a natural property, which suggests the following generalization: if two individuals with identical consumption and preferences face different poverty line bundles, the one facing the bundle they both prefer should be considered to have lower well-being (or a higher contribution to aggregate poverty). In other words, it is the level of preference satisfaction at the poverty line bundle that matters, not the bundle itself. Unfortunately, Deaton's distance function fails to meet this criterion.

Our contribution extends beyond poverty measurement, by developing a theory of well-being that depends on a reference bundle. This reference bundle can be interpreted in at least two additional ways. First, it can be viewed as capturing needs. For example, if the observation units are households of different types, the choice of reference bundles may reveal possible economies of scale for different goods. While the reference quantity of food may be proportional to household size, the quantity of transportation or housing goods may be less than proportional. Well-being could then be measured based on household consumption, household preferences, and this needs index. Comparing the well-being of singles and couples would not require identifying individual consumption and preferences within a couple, a notably difficult task.

Second, the reference bundle can be seen as an aspiration bundle, defined by a social observer to represent the reference consumption against which an individual

¹Many developments in the poverty measurement literature begin the analysis at the level of incomes, where the poverty threshold is expressed as an income level rather than a bundle of goods. However, as soon as individuals face different prices (for example, due to regional differences in the cost of goods considered otherwise equivalent) comparing incomes requires deflating them, typically by calculating the regional money value of a common reference bundle.

is normatively allowed to compare themselves. Well-being would then vary based on aspirations, which are defined by characteristics (e.g., age, region, education, gender) deemed normatively relevant by the social observer. This approach aligns well-being measurement with empirical evidence showing that individuals compare themselves to others, and their subjective well-being is influenced by the consumption of those around them. However, in this normative exercise, the reference is chosen by the social observer, so that two individuals with normatively equal characteristics are assigned the same reference bundle.

In this paper, we axiomatically characterize several new well-being measures. These measures can be used to assess poverty, compare the well-being of households with different needs, or conduct welfare analysis that accounts for the other-regarding nature of individual well-being. Because we distinguish between how and what to aggregate from the outset, these well-being measures can be integrated into various social welfare functions, such as utilitarian, egalitarian, Gini, or Atkinson functions.

The axioms we explore are based on three main ideas. The first idea is that well-being is easier to measure when preferences are homothetic, i.e., representable by a utility function that is homogeneous of degree 1. In this case, it is natural to measure well-being by referencing the ratio of utility at consumption to utility at the reference level. We elaborate on this idea and propose several axioms, which are not overly restrictive, as both existing and new measures satisfy them.

The second idea builds on the lattice structure of the set of lower (resp. upper) contours equipped with union (resp. intersection) partial order.² We define four axioms, requiring consistency between the well-being in two different situations, their supremum and their infimum. While lattice structures are common in measurement theory (see, for instance, [Kreps 1979](#); [Hougaard and Keiding 1998](#); [Christensen et al. 1999](#); [Chambers and Miller 2014b,a](#)), we demonstrate that our model does not allow us to use some known results. We prove that these two ideas lead us to choose a specific good and define well-being by referring to the bundles containing only this good, which individuals find equivalent to their consumption and reference bundles. However, we argue that such a measure is valid only if this good is a composite good encompassing all essential dimensions of consumption.

The third idea is that the most relevant preference information should pertain directly to the consumption and reference bundles themselves. To capture this, we require that well-being tends to one as consumption approaches the reference bundle. The measure described above violates this requirement. The axioms we define based on this idea force us to define well-being as the ratio of equivalent income at consumption to that at the reference.

Equivalent income is the minimum amount of money required, at a given price vector, to achieve a given satisfaction level. A central question concerning equivalent income is the choice of the reference price vector. Whereas [Blackorby et al. \(1993\)](#), among many others, insist on the arbitrariness of the choice of a reference price vector, a long literature has also shown that ethical principles can guide this choice (see, e.g., the survey in Chapter 4 of [Fleurbaey and Blanchet 2013](#)). We propose a new solution

²A set equipped with a partial order is a lattice if for any two elements of this set there exists a smallest element that is larger than both, their supremum, and a largest element that is smaller than both, their infimum.

to this issue by showing that the social observer must choose the price vector that either maximizes or minimizes the ratio of equivalent incomes at consumption and reference. As a result, the reference price vector depends on the reference bundle.

One may object to the fact that we solve the question of choosing the reference price vector at the cost of introducing an exogenous reference bundle, which is formally a similar object. This is correct, but we believe that the distributive consequences of choosing a reference bundle are more transparent for practitioners than those of choosing a reference price. A natural illustration comes from the poverty literature, where many studies start from the definition of a basic-needs or poverty bundle and then translate it into a poverty line representing the cost of such a bundle (Ravallion 1992; Deaton and Zaidi 2002; Mancini and Vecchi 2022). The identification of a poverty bundle is typically based on ‘objective’ criteria and recommendations from national and international organisations (see, for example, Mahrt et al. 2022; Hirvonen et al. 2020; Herforth et al. 2022; Allen 2017). Importantly, these criteria and recommendations may depend on household characteristics such as the ages of the household members and location (urban/rural area), leading to a multiplicity of poverty (and, in our framework, reference) bundles.³

Our analysis highlights a key trade-off between the preference domain over which we measure well-being and the axiomatic properties that the measure can satisfy. The well-being measures we consider satisfy different combinations of our axioms and are defined on different preference domains. Thus, the choice of a measure must take into account the preferences of the population of interest. Importantly, over the general domain of rational, continuous, convex, and monotonic preferences, only one measure satisfies our desirable properties: the minimal equivalent income distance (meid) well-being measure (see Definition 3 below).

The paper is organized as follows. In Section 2, we discuss some relationship between our undertaking and related literature. In Section 3, we introduce the preliminary notation and our definition of the welfare measure. In Section 4, we define and discuss the measures of well-being that will be justified by our axiomatic analysis. In Section 5, we introduce the main axioms and give the axiomatic characterizations of our well-being measures. In Section 6, we provide concluding remarks. The Appendix contains the proofs of our main results, additional details on our measures, and possible extensions of our framework.

2 Related literature

The works that come closest to our undertaking are Fleurbaey and Maniquet (2017b, 2018, 2019)’s recent axiomatic analysis of well-being measurement. The main axioms studied in these contributions are similar to the axioms based on the lattice structure of lower and upper contour sets that we use in this paper. Two sets of well-being measures are characterized in these papers, and they bear some similarity with the measures that we obtain here. The first set is a generalization of Deaton (1979)’s

³Beyond the poverty literature, several studies introduce individual reference bundles based on explicit normative principles. Examples include Berg and Piacquadio (2022) and Ooghe et al. (2023) in the optimal income tax framework, and De Rock and Moramarco (2025) and Almás et al. (2011) in multidimensional and unidimensional fairness assessments, respectively.

distance function: well-being is measured as the fraction of a reference bundle to which the individual finds their consumption equivalent. This reference bundle, however, remains unspecified.⁴ The first measures that we obtain in this paper also suggests to measure well-being by reference to the fraction of some bundles to which individuals find their consumption and reference equivalent. There is a major difference, however. Even if we make well-being depend on a reference bundle, well-being is not measured as the fraction of that bundle the individual is indifferent to. On the contrary, well-being is measured by considering bundles on the axes of the consumption set, that is bundles that only contain quantities of one good.

The second set of well-being measures characterized in [Fleurbaey and Maniquet \(2017b, 2018, 2019\)](#) bear some similarity with the other measures that we obtain here. Indeed, they suggest to measure well-being as equivalent income, computed at some reference price vector, that is left unspecified. This clearly shows a contribution of this paper; when a reference bundle is clearly identified, the issue of how to determine the relevant price vector is solved: it should be the one that either maximizes or minimizes the equivalent income ratio at consumption over reference.

We explained, in the previous section, that the reference bundle with respect to which well-being is measured can be interpreted in three ways. This connects our paper with three branches of the literature. We come back to each of them in this section to contrast our approach with related ones.

Many proposals have been made to measure poverty in a way that respects preferences. Most of them are based on [Samuelson \(1974, 1977\)](#)'s notion of equivalent income (also called money-metric utility). If reference prices are fixed, then a higher level of satisfaction is always associated with a higher equivalent income. However, the question of how to choose the reference prices remains largely unanswered (see [Fleurbaey and Blanchet 2013](#), for a longer discussion); this is one of the reasons why this concept has received less attention recently. By solving the problem of choosing the reference price vector, we hope to restore the interest in this concept. Another solution is the [Deaton \(1979\)](#)'s distance function, mentioned above. Other solutions include [Blackorby and Donaldson \(1987\)](#)'s welfare ratios and [Dimri and Maniquet \(2020\)](#)'s individualised price equivalent incomes. None of these solutions satisfies the axioms we define below.

The classical approach to correcting well-being measures to family needs consists in applying an equivalence scale to household incomes. Among the many shortcomings of this approach, one is that it does not take into account the interactions between preferences and economies of scale: households in which members have a strong preference for goods that benefit most from economies of scale are intuitively better at transforming household resources into well-being, and this is not captured by the equivalence scale approach. Our approach is a way to take it into account.

Another shortcoming of the equivalence scale approach, which we do not address here, is associated with the assumption of household (as opposed to individual) preferences. [Browning et al. \(2013\)](#) suggests a way of establishing comparability across households by referring to individual preferences and asking the question: How much

⁴In [Van der Linden \(2018\)](#), [Deaton \(1979\)](#)'s distance function is the individual welfare measure of a Paretian egalitarian social welfare order. In this case, the reference bundle is unique and set to the mean bundle.

income would an individual living alone need to achieve the same standard of living as if they were married? It is interesting to notice that the identifying assumption they make is that individuals have the same preferences if they remain single or if they marry. This is exactly the assumption that we can get rid of in our approach: differences in household size are captured by the reference consumption, and our well-being measures provide ways to construct comparability across households even if preferences differ between single and married people.

Concerning our contribution toward the measurement of well-being when individuals are affected by what happens around them, this brings us close to the literature on other-regarding preferences. In positive economics, since the last decade of the twentieth century, it has been widely documented that other-regarding preferences and distributional concerns affect choices (Kirchsteiger 1994; Fehr et al. 1998, among others), preferences (see, for example, Fehr et al. 1993; Fehr and Schmidt 1999; Bolton and Ockenfels 2000; Sobel 2005; Dufwenberg et al. 2011) and subjective well-being (Clark et al. 2008; Clark and Senik 2010; Clark 2018).

However, the fact that individuals compare themselves to others does not mean that the evaluation of social policies *should* recognize that individuals may suffer from an improvement in the situation of others. Sen (1970), for example, has argued against respecting individual preferences when they are defined in terms of the actions of others. On the other hand, feeling socially included may require having resources comparable to those with whom one interacts, and this feeling may be considered respectable from a normative point of view. By sticking to self-centered preferences and by introducing a reference bundle as an argument of the well-being function, we propose a way of combining Sen’s criticism with the objective of measuring well-being by comparison with others.

In our measure, however, well-being decreases when the reference bundle improves. This is at odds with the experimental literature on other-regarding preferences, in which a richer individual derives well-being from seeing poorer individuals thrive. Nonetheless, and it is the stance that we take in this paper, the *fact* that human beings behave this way does not necessarily mean that normative well-being measures *should* be increasing in reference when consumption is above it. Our undertaking is consistent with this literature only when the reference consumption is preferred to the actual consumption. We come back to this point in Appendix C where we discuss how our results extend to the case in which actual consumption is preferred.

If one agrees to take the resources obtained by (some) others as a normatively relevant argument for individual well-being, two strategies are possible. The first strategy is to accept other-regarding preferences as normatively compelling, including preferences that capture that individual satisfaction decreases when the consumption and satisfaction of others increase. If the normative analysis is restricted to Pareto efficiency, then Dufwenberg et al. (2011) has identified the necessary and sufficient restrictions on the domain of preferences for the second fundamental theorem to hold. Consequently, under these restrictions, it does not matter whether or not other-regarding preferences are accepted as normatively compelling. If welfare analysis goes beyond Pareto efficiency, e.g. if the social planner maximizes an inequality-averse social welfare function, then two individuals with identical personal consumption, reference

consumption and self-centered preferences (i.e. defined only in terms of their own consumption) may be treated differently, and the individual who receives the greatest compensation may be the one who suffers most from seeing those around them better off. In this paper, we start from the stance that this is normatively unattractive.⁵

The second strategy retains [Dufwenberg et al. \(2011\)](#)’s preference restrictions, but replaces the other-regarding part of the preferences with an objective reference consumption amid the arguments of the welfare measure. By objective, we mean that it is chosen by the social observer and not by the individuals themselves. It can be the average consumption of the country, the region or a reference group. In doing so, the social observer recognizes that individual well-being may fall if the consumption and well-being of others rise, but that the well-being of two individuals in the same situation and with the same reference consumption will fall in the same way, regardless of the difference in the sensitivity of one’s own well-being to the well-being of others. We adopt the second strategy.

Our idea that adding a reference consumption to the model of well-being measurement and sticking to self-centered preferences is a way to capture other-regarding preferences is well illustrated when we compare this model with recent developments in the literature on relative poverty measurement. Indeed, [Ravallion and Chen \(2011\)](#), [Ravallion \(2020\)](#), [Ravallion and Chen \(2017\)](#) and [Decerf et al. \(2025\)](#), for instance, require the poverty line to be consistent with preferences. These preferences are other-regarding. They are increasing in own income and decreasing in the reference income, so that the poverty line is the income level that, given the reference consumption in the country (which may be the mean, median or a more elaborated function of the distribution) maintains the utility of a representative agent constant. Even if we do not model it here, our reference consumption may perfectly well be derived from the distribution in this way, for some representative preferences that take social comparisons into account. This is what our axiom of Normalization achieves: the individual welfare associated to consuming the reference bundle is the same independently of the latter, exactly like the utility level of the reference preferences is constant at preference-consistent poverty lines.

3 Notation and definitions

We assume that there are K divisible and cardinally measurable goods, $K \geq 2$. The (closed) consumption set is $\mathcal{X} = \mathbb{R}_+^K$. The agents have complete, transitive, continuous, convex and monotonic preferences over $\mathcal{X}$. We denote the set of such preferences by $\mathcal{R}$.⁶ For any two bundles $z, z' \in \mathcal{X}$ we denote the weak preference of z over z' as $z R z'$ (z is at least as good as z'), strict preference of z over z' as $z P z'$ (z is strictly preferred to z'), and indifference as $z I z'$ (z is as good as z'). For any $z \in \mathcal{X}$ and $R \in \mathcal{R}$, we denote

⁵[Decerf and Van der Linden \(2016\)](#) circumvent this unattractive conclusion by imposing a very mild resource equality requirements, which many social welfare functions satisfy. Therefore, individuals who suffer most from seeing those around them better off may receive a larger or a lower compensation than those who do not suffer from social comparison as a function, for instance, of the preferences of even other individuals.

⁶At the risk of stressing the obvious, our restriction to preferences (and, therefore, the absence of utility functions) illustrates the fact that our well-being measures will only depend on ordinal and non-comparable information on individual choices. All information regarding happiness, subjective satisfaction, etc., is considered to be normatively irrelevant. An immediate consequence of this starting point is that we avoid the “happy peasants and miserable millionaires” paradox.

the lower, upper and indifference (closed) contours as follows $L(z, R) = \{z' \in \mathcal{X} | zRz'\}$, $U(z, R) = \{z' \in \mathcal{X} | z'Rz\}$, $I(z, R) = U(z, R) \cap L(z, R)$.

For reasons that will be explained below, after Definition 2, we restrict our attention to the subdomain $\mathcal{R}^c \subset \mathcal{R}$ of preferences having the property that all lower contour sets are bounded, so that all indifference contours have a non-empty intersection with each axis, and moreover, we require this intersection to contain only one bundle.⁷ Formally, for $k \in \{1, \dots, K\}$, let $A^k \subset \mathcal{X}$ denote the k -th axis, that is, $x \in A^k \Leftrightarrow x_\ell = 0, \forall \ell \in \{1, \dots, K\} \setminus \{k\}$. We say that $R \in \mathcal{R}^c$ if and only if for all $x \in \mathcal{X}$, $k \in \{1, \dots, K\}$, there exists $z^k \in \mathcal{X}$ such that $I(x, R) \cap A^k = \{z^k\}$.

Let $\mathcal{R}^H \subset \mathcal{R}^c$ denote the subdomain of homothetic preferences, that is preferences satisfying the property $xRx' \Leftrightarrow \lambda xR\lambda x'$ for all $x, x' \in \mathcal{X}$, $\lambda > 0$. Let $\mathcal{U}^H$ be the set of all homogeneous of degree one utility functions representing homothetic preferences. Moreover, for any indifference contour set $I(z, R)$ we call $R_{I(z, R)}$ the homothetic preferences we generate by homothetic transformations of $I(z, R)$ for any positive real number.⁸

We are interested in measuring well-being as a function of what an agent consumes, $x \in \mathcal{X}$, her preferences $R \in \mathcal{R}^c$, and a reference bundle, $y \in \mathcal{X}$. We refer to arguments (x, R, y) as situations.

In this simple model, no good plays a special role. As a result, no argument can be based on necessities versus luxuries, on survival considerations, on what is required for feeling included in society, etc. This has two consequences, First, we assume that the reference consumption is chosen by the social observer, but we don't know why a specific reference has been chosen. Second, this model is not appropriate to discuss issues of absolute poverty measurement, which would require that the absolute poverty line represents the satisfaction of some specific needs for the individuals with likely implications on the preferences above and below the threshold. Moreover, in the axioms that we impose on our well-being measures, making y vary turns out to be essential. To say it differently, the axiomatization results we develop below would not be valid in a model in which y is fixed and the same for all individuals.

A *well-being measure* is a function $W : \mathcal{X} \times \mathcal{R}^c \times \mathcal{X} \rightarrow \mathbb{R}$ that satisfies:

- (i) *Continuity (Cont)*: for all $R \in \mathcal{R}^c$, $W(\cdot, R, \cdot)$ is continuous in its first and third arguments;
- (ii) *Normalization (N)*: for all $y \in \mathcal{X}$, $W(y, R, y) = 1$;
- (iii) *Consumption Monotonicity (Mx)*: for all $x, x', y \in \mathcal{X}$ and $R, R' \in \mathcal{R}^c$, if $I(y, R) = I(y, R')$ and $L(x, R) \cap U(x', R') = \emptyset$, then $W(x', R', y) > W(x, R, y)$;
- (iv) *Homotheticity (H)*: for all $x \in \mathcal{X}$ and $\lambda, \mu > 0$, if $R, R' \in \mathcal{R}^H$, then $W(x, R, \lambda x) \geq W(x, R', \lambda x) \Leftrightarrow W(\mu x, R, \lambda x) \geq W(\mu x, R', \lambda x)$;
- (v) *Reference Rescaling (Ry)*: for all $x \in \mathcal{X}$, $R, R' \in \mathcal{R}^c$, $\alpha, \lambda \in \mathbb{R}_{++}$ such that either $1 < \alpha < \lambda$ or $\lambda < \alpha < 1$, if $I(\alpha x, R) = I(\alpha x, R')$, $I(\lambda x, R) = I(\lambda x, R')$ and

⁷Observe that this last requirement implies that preferences in $\mathcal{R}^c$ are strictly monotonic. This restriction is not necessary in empirical applications, where preferences are typically assumed to be homothetic and well-behaved. Moreover, most of our results hold in larger preference domains. We come back to this throughout the text.

⁸If $I(z', R)$ is an homothetic transformation of $I(z, R)$ for a scalar $\lambda \in \mathbb{R}_+$, then $x \in I(z, R)$ if and only if $\lambda x \in I(z', R)$. Notice that $R \in \mathcal{R}^c$ implies $R_{I(z, R)} \in \mathcal{R}^c$ and $R = R_{I(z, R)}$ if and only if $R \in \mathcal{R}^H$.

$I(\lambda x, R)$ is homothetic to $I(\alpha x, R)$, then

$$W(x, R, \alpha x) = W(x, R', \alpha x) \Leftrightarrow W(x, R, \lambda x) = W(x, R', \lambda x);$$

- (vi) *Consumption Rescaling (Rx)*: for all $x \in \mathcal{X}$, $R, R' \in \mathcal{R}^c$, $\alpha, \lambda \in \mathbb{R}_{++}$ such that either $1 < \alpha < \lambda$ or $\lambda < \alpha < 1$, if $I(\alpha x, R) = I(\alpha x, R')$, $I(\lambda x, R) = I(\lambda x, R')$ and $I(\lambda x, R)$ is homothetic to $I(\alpha x, R)$, then

$$W(\alpha x, R, x) = W(\alpha x, R', x) \Leftrightarrow W(\lambda x, R, x) = W(\lambda x, R', x).$$

Continuity does not require explanation. Normalization introduces a key feature of well-being measures with reference consumption, that is, well-being is measured in a relative sense. Consequently, two individuals that consume their reference will have the same well-being even if one is consuming strictly more than the other.

Consumption Monotonicity captures the natural intuition that well-being increases with consumption. It compares well-being when the indifference contours through the reference bundle, and the reference bundles, are the same in two situations. If it is the case that the consumption bundle in one situation is strictly preferred to another consumption bundle by both preferences, and if, moreover, all consumption bundles indifferent to the former are also strictly preferred to any bundle indifferent to the latter by both preferences, then well-being is larger in the former situation. Observe that this condition on the preferences at the two contemplated consumption bundles is equivalent to requiring that the lower contour at one bundle does not intersect the upper contour at the other.⁹

Applying Consumption Monotonicity to $R' = R$ leads to

$$W(x', R, y) > W(x, R, y) \Leftrightarrow x'Px.$$

If we add Continuity, we get

$$W(x', R, y) \geq W(x, R, y) \Leftrightarrow x'Rx,$$

so that $W(\cdot, R, y)$ is a utility representation of R for a given y .¹⁰

The next three properties all build on the special case of homothetic preferences and of homothetic transforms of indifference contours.

Homotheticity follows the idea that the concept of relative well-being has a clear interpretation when preferences are homothetic and consumption is a fraction of the reference bundle. In this case, it is natural to assume that all the relevant information regarding well-being is contained in these fractions. Homotheticity requires that if the well-being associated to some preferences is lower than that associated to other preferences when consumption is some fraction of the reference, then this comparison should also hold for all situations obtained by multiplying both consumption and

⁹Consumption Monotonicity is reminiscent of the Nested Contour axiom in [Fleurbaey and Maniquet \(2017b\)](#).

¹⁰Observe that [Blackorby and Donaldson \(1987\)](#)'s welfare ratios do not satisfy even the weakest version of this property. We view it as a major drawback of this well-being measure. [Dimri and Maniquet \(2020\)](#)'s individualised price equivalent incomes, on the contrary, does satisfy this axiom.

reference by the same number. Observe that when Normalization is also imposed, Homotheticity is equivalent to the following stronger property (Normalization implies equality for the case $\mu = \lambda$, which then extends to all other values of λ) :

- (v) *Homotheticity (H)*: for all $x \in \mathcal{X}$ and $\lambda > 0$, if $R, R' \in \mathcal{R}^H$, then $W(x, R, \lambda x) = W(x, R', \lambda x)$;

From now on, we stick to this second definition.

Reference Rescaling refers to the case in which the reference bundle is a fraction of consumption, two individuals have different indifference contours at consumption but the same indifference contour at reference. The property says that if these two individuals have the same well-being in these situations, then they should also have the same well-being if we move the reference away from consumption in such a way that the new reference is still proportional to consumption and individual contours at the new reference are an homothetic transform of the contours at the initial reference. Consumption Rescaling applies the same property to rescaling of indifference contours at consumption.

The central requirement for the well-being measures we study in this paper is the following Reference Preference. It captures the natural intuition that well-being decreases with reference, by considering that a higher reference is one that the individual herself prefers. Formally,

Reference Preference (Py) - For all $x, y, y' \in \mathcal{X}$ and $R \in \mathcal{R}^c$, if $y I y'$ then $W(x, R, y) = W(x, R, y')$, if $y P y'$ then $W(x, R, y) < W(x, R, y')$.

This is the only basic property that is not satisfied by Deaton (1979)'s distance function, and we can see this paper as an inquiry into well-being measures that are different from the distance function precisely because they are monotonic in reference. To recall, we can define the distance function as follows:

$$W_{Deaton}(x, R, y) = \omega \Leftrightarrow \omega y \in I(x, R).$$

If preferences R are not homothetic, then we can find $x, x' \in \mathcal{X}$, $\lambda > 0$ such that $x I x'$ whereas $\lambda x P \lambda x'$. By definition, $W_{Deaton}(x, R, \lambda x) = W_{Deaton}(x, R, \lambda x')$, violating Reference Preference.

We complete this section by noting an important consequence of the basic properties above. When $W : \mathcal{X} \times \mathcal{R}^c \times \mathcal{X} \rightarrow \mathbb{R}_+$ satisfies Consumption Monotonicity, Reference Preference, Continuity and Homotheticity, it has the property that $W(x, R, \lambda x) = W(x', R', \lambda x')$ for all x and x' and all homothetic preferences R and R' , as stated in the following lemma, which we prove, like all lemmas and theorems, in the Appendix.

Lemma 1. *If $W : \mathcal{X} \times \mathcal{R}^c \times \mathcal{X} \rightarrow \mathbb{R}_+$ satisfies Consumption Monotonicity (Mx), Reference Preference (Py), Continuity (Cont) and Homotheticity (H), then for all $x, x' \in \mathcal{X}$ and $R, R' \in \mathcal{R}^H$, $\lambda > 0$: $W(x, R, \lambda x) = W(x', R', \lambda x')$.*

Let $R \in \mathcal{R}^H$ and $u \in \mathcal{U}^H$. Lemma 1, applied to $R' = R$, implies that

$$W(x, R, y) = f\left(\frac{u(x)}{u(y)}\right)$$

for some strictly increasing real-valued function f . Indeed, if $y = \lambda x$, then $u(y) = \lambda u(x)$ and $W(x, R, y) = f(\lambda^{-1})$. In the general case, there exists λ such that $y I \lambda x$, so that $u(y) = \lambda u(x)$ as well and, again, $W(x, R, y) = f(\lambda^{-1})$.

In the aggregation of individuals' well-being levels into a measure of social welfare, f would be crucial. For instance, its curvature would measure the degree of inequality aversion of the social welfare function. As we do not build social welfare functions in this paper, for the sake of simplicity of the exposition, we assume that $f(u) = u$. Nothing in what we say below depends on this assumption. Any f function can be used to aggregate the well-being measures we obtain here. This will allow us to skip mentioning after each result that the relevant well-being measure is ordinally equivalent to this or that well-being measure satisfying the $f(u) = u$ property. We come back to the possible shapes of f in the conclusion.

4 Well-Being Measures

In this section, we define the well-being measures that will turn out to be axiomatically characterized in the next section.

Loosely speaking, each measure consists in measuring the distance between indifference contours at consumption and at reference. The first three measures defined below use a notion of distance that relates to the homothetic transformation of one indifference contour that is tangent to the other, whereas the following measures use a notion of distance based on the intersection of indifference contours and axes of the consumption set.

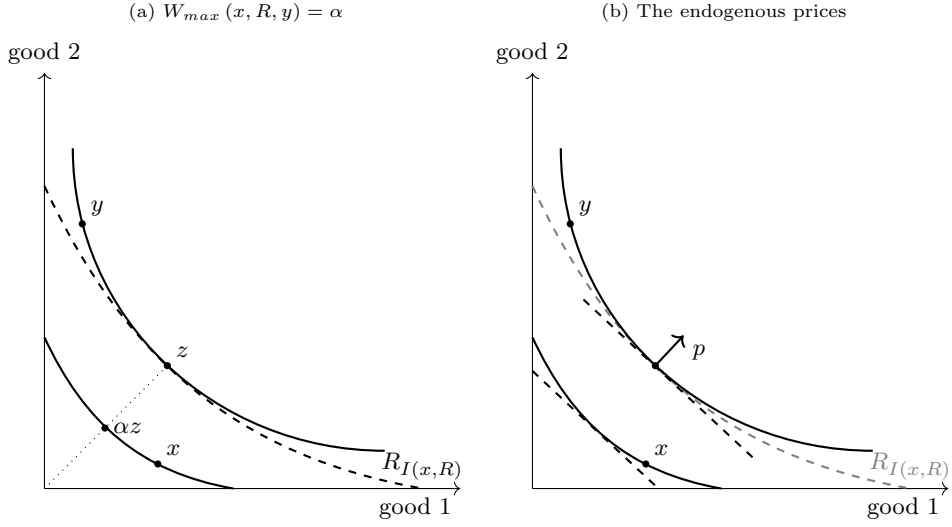
There are three equivalent ways to define our first measure. The most intuitive one defines it as the scale by which the indifference contour through consumption has to be homothetically transformed so as to become tangent from below to the indifference contour at reference. This is illustrated in Fig. 1a. The dashed indifference curve is the minimum (homothetic) rescaling of $I(x, R)$ that contains a bundle indifferent to y : bundle z . Consumption x is indifferent to αz and reference y is indifferent to z . Well-being is then equal to α .

This measure is also the scale by which the indifference contour through reference has to be homothetically transformed so as to become tangent from above to the indifference contour at consumption. In that case, the tangency between the transform of the indifference contour at y and the indifference contour at x would take place at αz .

The measure is also equal to the ratio between the equivalent income¹¹ at x and that at y , when prices are chosen in such a way as to maximize well-being. This is

¹¹Equivalent income at a given bundle and for a given price vector is the minimal income needed to reach the utility level corresponding to that bundle at these prices. It was introduced by McKenzie (1957) under the name of the expenditure function and renamed money-metric utility by Samuelson (1974). It was called equivalent income by King (1983), which has become the dominant name in welfare analysis after Adler and Fleurbaey (2016).

Figure 1: The maximum well-being measure.



illustrated in Fig. 1b, in which p is the price vector at which the equivalent income ratio is maximal. Unsurprisingly, tangencies of the equivalent budget lines to the indifference contours take place at αz and z .

Definition 1. The maximum well-being measure is the function $W_{max} : \mathcal{X} \times \mathcal{R}^c \times \mathcal{X} \rightarrow \mathbb{R}_+$ such that, for all $(x, R, y) \in \mathcal{X} \times \mathcal{R}^c \times \mathcal{X}$,

$$W_{max}(x, R, y) = \frac{u(x)}{\min_{y' \in U(y, R)} u(y')}$$

for $u \in \mathcal{U}^H$ representing $R_{I(x, R)}$.

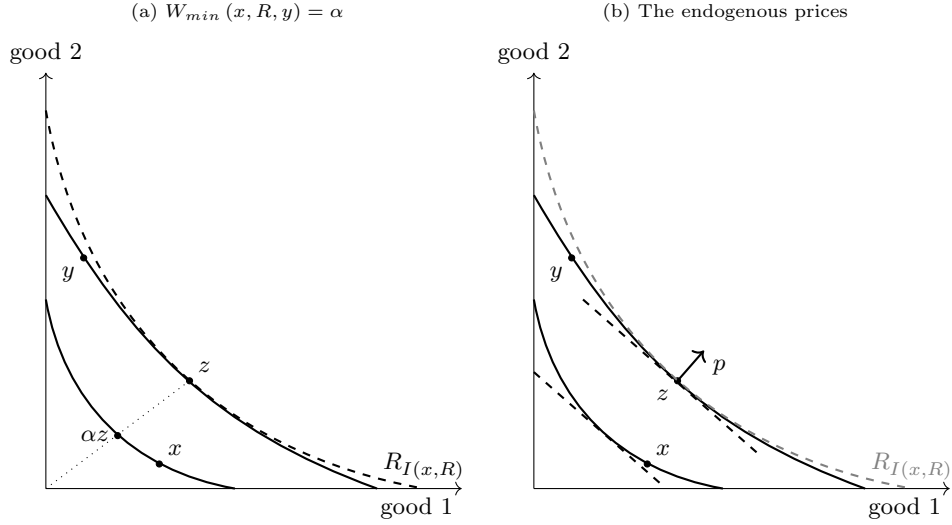
The next measure is the dual of the previous one. Well-being is the ratio between the equivalent income at x and that at y , when prices are chosen in such a way as to minimize well-being.

Definition 2. The minimum well-being measure is the function $W_{min} : \mathcal{X} \times \mathcal{R}^c \times \mathcal{X} \rightarrow \mathbb{R}_+$ such that, for all $(x, R, y) \in \mathcal{X} \times \mathcal{R}^c \times \mathcal{X}$,

$$W_{min}(x, R, y) = \frac{u(x)}{\max_{y' \in L(y, R)} u(y')}$$

for $u \in \mathcal{U}^H$ representing $R_{I(x, R)}$.

Figure 2: The minimum well-being measure.



It is straightforward to check that $W_{min}(x, R, y) = (W_{max}(y, R, x))^{-1}$. This implies that both measures are well-defined and continuous on the same domain. The need for a domain restriction for W_{min} is evident if we consider the example of an individual with linear indifference curve through x and Leontief indifference curve at y : in this case there is no rescaling of the indifference contour through consumption that is tangent from above to the indifference contour through reference. We choose to study these measures over the $\mathcal{R}^c$ domain. Observe that what we have called tangencies in our informal definitions may take place at corner bundles. It is easy to check that the absence of a unique supporting price vector does not prevent well-being from being uniquely defined.

Building on the definitions of W_{max} and W_{min} , we can define a third measure that avoids the domain shortcoming we just discussed. It consists of identifying the minimal homothetic rescaling of the lower indifference contour so as to become tangent from below to the upper indifference contour, independently of which contour corresponds to consumption or reference. Equivalently, it looks at the minimal distance, in terms of equivalent incomes, between consumption and reference. Note that it consists in applying W_{max} when $y R x$ and W_{min} when $x R y$. This measure has the advantage of being defined over the entire domain $\mathcal{R}$ of preferences.

Definition 3. *The minimal equivalent income distance well-being measure is the function $W_{meid} : \mathcal{X} \times \mathcal{R} \times \mathcal{X} \rightarrow \mathbb{R}_+$ such that, for all $(x, R, y) \in \mathcal{X} \times \mathcal{R} \times \mathcal{X}$,*

$$W_{meid}(x, R, y) = \begin{cases} \frac{u(x)}{\min_{y' \in U(y, R)} u(y')} & \text{if } x R y \\ \frac{u(x)}{\max_{y' \in U(y, R)} u(y')} & \text{if } y R x \end{cases}$$

for $u \in \mathcal{U}^H$ representing $R_{I(x,R)}$.

Even if W_{meid} seems an hybrid combination of W_{max} and W_{min} , it has a more intuitive interpretation, based on its minimal distance operation, which leads to the following formal property: $W_{meid}(y, R, x) = (W_{meid}(x, R, y))^{-1}$. Notice that W_{max} nor W_{min} this property.

In our axiomatic analysis, we will obtain a characterization of W_{meid} based on the characterizations of W_{max} and W_{min} , although with slightly different axioms. However, given that W_{meid} has a more intuitive construction (it always takes the price vector that minimizes the distance between the two relevant indifference contours) and is well defined over the largest domain, we consider that it is the one to highlight.

Our fourth well-being measure is actually parameterized by the name of one good, which plays a special role in the measure. The well-being measure works like this: we look at the intersection with the k -th axis of the indifference contours at consumption and at reference and we take the ratio between the two. It is easy to check that $\mathcal{R}^c$ is again the precise domain over which these K well-being measures are well-defined.

For $k \in \{1, \dots, K\}$ and $(x, R) \in \mathcal{X} \times \mathcal{R}^c$, let $A^k(I(x, R))$ denote the intersection of $I(x, R)$ with the k -th axis, and let $a^k(I(x, R))$ denote the k -th coordinate of $A^k(I(x, R))$.

Definition 4. The k -th axis well-being measure is the function $W_k : \mathcal{X} \times \mathcal{R}^c \times \mathcal{X} \rightarrow \mathbb{R}_+$ such that, for all $(x, R, y) \in \mathcal{X} \times \mathcal{R}^c \times \mathcal{X}$,

$$W_k(x, R, y) = \frac{a^k(I(x, R))}{a^k(I(y, R))}$$

Similarly to Definitions 1 and 2, a k -th axis measure coincides with the ratio between equivalent income at consumption and reference; in this case, however, the multiplicity of supporting prices is the norm, more than a special case.

The W_k measures suffer from an important drawback. The domain restriction to $\mathcal{R}^c$ is a strong one. It implies that whatever the good we select, say housing, and whatever the consumption bundle of an individual, there exists an indifferent bundle that only contains housing. This seems implausible, except if one composite good is constructed by gathering all the necessities. Building on this observation, we propose a fifth well-being measure, which we call *necessity-equivalent well-being measure*, that is defined on a broader preference domain. Let good 1 denote this composite good. The new measure is a special case of W_k , namely W_1 , and is well defined on the broader domain of preferences ($\mathcal{R}^1$) for which all indifference curves intersect the composite-good axis. To keep the exposition concise, we discuss this measure in detail in Appendix B.

We complete this section with a discussion about why W_{max} and W_{min} deserve to be called the maximum and minimum measures. We begin with this theorem, proved in the Appendix.

Theorem 1. For any well-being measure W , all $x, y \in \mathcal{X}$, $R \in \mathcal{R}^c$,

$$W_{min}(x, R, y) \leq W(x, R, y) \leq W_{max}(x, R, y).$$

In empirical applications, it is possible that Theorem 1 is all we need to compare distributions of well-being, if the difference between W_{max} and W_{min} is small. If the estimation of preferences is parametric, then W_{max} and W_{min} may moreover take very simple shapes (and, therefore, W_{meid} as well). For instance, if estimated preferences are of the Stone-Geary type, that is they are represented by utility functions¹²

$$u^{a,b}(x) = \prod_{i=1}^K (x_i - b_i)^{a_i},$$

with $-b \in \mathbb{R}_{++}^K$, $a_i \geq 0$ and $\sum_{i=1}^K a_i = 1$, it is intuitive that the price vector leading to W_{max} is the supporting vector along the ray of bundles proportional to $-b$ and the one that leads to W_{min} supports bundles on one of the axis. Simple computations yield (assuming W_{min} is computed by reference to the price that supports consumption on the good-1 axis, so that $W_{min} = W_{nec}$)

$$W_{max}(x, R^{a,b}, y) = \frac{\prod_{i=1}^K \left(\frac{x_i - b_i}{-b_i} \right)^{a_i} - 1}{\prod_{i=1}^K \left(\frac{y_i - b_i}{-b_i} \right)^{a_i} - 1}$$

$$W_{min}(x, R^{a,b}, y) = \frac{(x_1 - b_1) \prod_{i=2}^K \left(\frac{x_i - b_i}{-b_i} \right)^{\frac{a_i}{a_1}} + b_1}{(y_1 - b_1) \prod_{i=2}^K \left(\frac{y_i - b_i}{-b_i} \right)^{\frac{a_i}{a_1}} + b_1}.$$

When the difference between $W_{min}(x, R, y)$ and $W_{max}(x, R, y)$ is large, evaluating well-being requires to go beyond Theorem 1. The axioms we introduce in the next section will allow us to achieve this task.

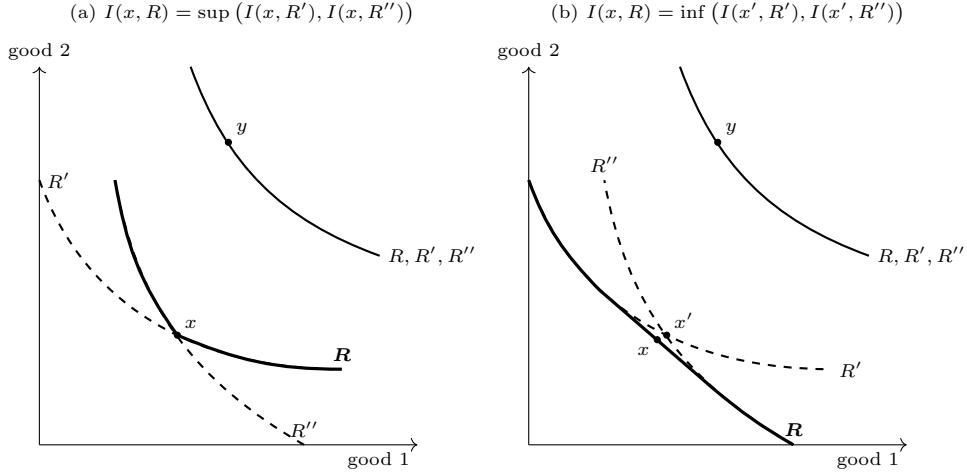
5 Axiomatic analysis

Taking individuals' preferences into account in the measurement of well-being implies the possibility that two individuals consuming identical bundles and having the same reference have different well-being levels. This may be viewed as a drawback of the approach (a drawback that is shared by all normative analysis that impose Pareto efficiency or any other way of respecting individual preferences). The best way to fight this drawback consists in minimizing well-being inequality *at identical or similar bundles*. Our axioms impose bounds on well-being inequality while still respecting individual preferences.

These axioms build on the lattice structure of the set of indifference contours, or, to present it differently, on the lattice structure of the set of upper contour sets equipped with the inclusion partial order. This is represented in Figures 3a and 3b. In Fig. 3a, the indifference curve at x for preferences R is the supremum of indifference

¹²Parameters b are sometimes presented as minimal consumption, beyond which preferences are behaving like Cobb-Douglas preferences. The case $b \geq 0$, though, is problematic. Indeed, for bundles x such that $0 \leq x_k < b_k$ for some $k \in \{1, \dots, K\}$, preferences are not monotonic. Observe that the condition $-b \in \mathbb{R}_{++}^K$ is also the one that guarantees that $R^{a,b} \in \mathcal{R}^c$.

Figure 3: Supremum and infimum contour at consumption.



curves at x of preferences R' and R'' . Observe that, by Consumption Monotonicity, any indifference curve above $I(x, R)$ would be associated to a strictly higher well-being than the one at x of preferences R' and R'' (assuming they have the same indifference curves at y , as it is the case in the figure). Therefore, well-being at x with preferences R is either equal or strictly higher than with preferences R' or R'' . Our first axiom, Supremum Contour at Consumption, imposes an upper bound on the well-being at x for preferences R : it should be *equal* to the maximum between well-being at x with preferences R' and R'' .

This upper bound can be further justified by the following reasoning. Because $I(x, R)$ is the supremum of $I(x, R')$ and $I(x, R'')$, any bundle that an individual with preferences R deems equally good as x is also deemed equally good as x by an individual with preferences R' or R'' , which gives us a reason not to claim that well-being at x with preferences R is strictly higher than well-being at x with both preferences R' and R'' .

We can apply the same idea by looking at indifference contours at reference across situations in which the indifference contours at consumption are the same. We obtain a lower bound on well-being. We call this second axiom Supremum Contour at Reference.

Supremum Contour at Consumption (Sup- x) - For all $x, y \in \mathcal{X}$ and $R, R', R'' \in \mathcal{R}^c$, if $L(x, R) = [L(x, R') \cup L(x, R'')]$ and $I(y, R) = I(y, R') = I(y, R'')$, then $W(x, R, y) = \max \{W(x, R', y), W(x, R'', y)\}$.

Supremum Contour at Reference (Sup- y) - For all $x, y \in \mathcal{X}$ and $R, R', R'' \in \mathcal{R}^c$, if $L(y, R) = [L(y, R') \cup L(y, R'')]$ and $I(x, R) = I(x, R') = I(x, R'')$, then $W(x, R, y) = \min \{W(x, R', y), W(x, R'', y)\}$.

Let us now build the dual axioms by looking at infimum of two indifference contours. In Fig. 3b, the indifference curves at x for preferences R is the infimum of indifference curves at x' of preferences R' and R'' . We see that convexity of R' and R'' prevents x' from belonging to the infimum of $I(x', R')$ and $I(x', R'')$. If we consider that x is *similar* to x' , we can again create a lower bound on well-being inequality by imposing that the well-being at x for an individual with preferences R be exactly equal to the minimum between the well-being of both an individual with preferences R' and an individual with preferences R'' at x' . We call this axiom Infimum Contour at Consumption.

Again, there is an additional motivation for imposing this requirement. Any bundle that an individual with preferences R considers at least as good as x , is the convex combination between a bundle that an individual with preferences R' considers at least as good as x' and a bundle that an individual with preferences R'' considers at least as good as x' . As a result, the indifference contour of R through x can be considered as intermediary between the indifference contours of R' and R'' through x' , so that the well-being at x with preferences R cannot be claimed to be strictly lower than the two others.

A similar axiom can be defined if we fix indifference contours at consumption and we look at infimum contours at reference. We call this axiom Infimum Contour at Reference.¹³

Let the *ch* operator refer to the convex hull of its argument.

Infimum Contour at Consumption (Inf-x) - For all $x, x', y \in \mathcal{X}$ and $R, R', R'' \in \mathcal{R}^c$, if $U(x, R) = \text{ch}[U(x', R') \cup U(x', R'')]$ and $I(y, R) = I(y, R') = I(y, R'')$, then $W(x, R, y) = \min\{W(x', R', y), W(x', R'', y)\}$.

Infimum Contour at Reference (Inf-y) - For all $x, y, y' \in \mathcal{X}$ and $R, R', R'' \in \mathcal{R}^c$, if $U(y, R) = \text{ch}[U(y', R') \cup U(y', R'')]$ and $I(x, R) = I(x, R') = I(x, R'')$, then $W(x, R, y) = \max\{W(x, R', y'), W(x, R'', y')\}$.

As already mentioned in the Introduction, imposing consistency between the well-being at two situations and at their supremum or their infimum is reminiscent to several works on measurement. In particular, [Fleurbaey and Maniquet \(2017b\)](#), in their work on well-being measurement without reference consumption, deduce from a supremum (resp. an infimum) axiom that there should exist evaluation preferences so that well-being is measured by reference to the best (resp. worst) bundle of these evaluations preferences in the lower (resp. upper) contour at the contemplated situation. We cannot apply these results here, for two reasons. First, our measures depend on both contours at consumption and at reference. This prevents the domain of admissible contours to be rich enough so that their result applies (contours at consumption

¹³Out of our four main axioms, [Dimri and Maniquet \(2020\)](#)'s individualized price equivalent incomes only satisfies Inf-x. The measures we discuss in this paper satisfy at least two out of the four main axioms, so that we don't view this measure as a prominent one in this paper. On a different note, the Deaton measure, which is excluded from our paper because it fails to satisfy Reference Preference, satisfies Supremum Contour at Consumption and Supremum Contour at Reference. A more complete axiomatic study of the Deaton measure can be found in [Maniquet and Moramarco \(2022\)](#).

and reference cannot intersect). Second, we have restricted our domain to preferences with bounded lower contours.

Consequently, our axioms have weaker implications – which we state and prove in Lemma 5 in the Appendix – than the corresponding axioms of Fleurbaey and Maniquet (2017b). As a result, there exist measures that satisfy all of them: the k -th axis well-being measure W_k . We then reach our first characterization result.

Theorem 2. 1) *Well-being measures W_k , $k \in \{1, \dots, K\}$, satisfy Supremum Contour at Consumption (Sup- x), Supremum Contour at Reference (Sup- y), Infimum Contour at Consumption (Inf- x) and Infimum Contour at Reference (Inf- y) over the domain $\mathcal{X} \times \mathcal{R}^c \times \mathcal{X}$.*

2) *Conversely, if a well-being measure $W : \mathcal{X} \times \mathcal{R}^c \times \mathcal{X} \rightarrow \mathbb{R}$ satisfies any three axioms among Supremum Contour at Consumption (Sup- x), Supremum Contour at Reference (Sup- y), Infimum Contour at Consumption (Inf- x) and Infimum Contour at Reference (Inf- y), then there exists $k \in \{1, \dots, K\}$ such that $W = W_k$.*

A corollary of Theorem 2 is the characterization of the *necessity-equivalent well-being measure* over the larger domain $\mathcal{R}^1$. The formal statement and the corresponding proof are reported in Appendix B.

There are reasons, however, to be unsatisfied with the W_k measures, even if they are restricted to one essential good. Indeed, by making well-being depend on the intersection of indifference contours and the axes, these measures have the drawback of letting welfare judgements depend on information about preferences over bundles that are far from individuals' consumptions and references. This difficulty is likely to be echoed in empirical works. Preferences are typically estimated from consumption data that are not close to the axes (where only one good is consumed). This is why we now define two additional axioms, which force well-being to be measured on the basis of preferences close to consumption and reference, at least when they get closer to each other. In the context of poverty measurement, for instance, if the consumption of an individual converges towards the poverty line, it is natural to require that the contribution of this individual to poverty converges to zero, independently of the preferences over other bundles. The following axiom formalizes this idea, when the reference bundle is preferred to the consumed one.

Convergence from Below (Cb) - For all $y \in \mathcal{X}$ and $R \in \mathcal{R}^c$, if there exists a sequence $(x^n, R^n)_{n \in \mathbb{N}} \in \mathcal{X} \times \mathcal{R}^c$ such that, for all $n \in \mathbb{N}$, (i) $I(y, R^n) = I(y, R)$, (ii) $L(x^n, R^n) \cap U(x^{n+1}, R^{n+1}) = \emptyset$, (iii) $y P x^n$, and (iv) $x^n \rightarrow y$, then $W(x^n, R^n, y) \rightarrow 1$.

We also consider the dual case in which the consumed bundle is preferred to the reference one.

Convergence from Above (Ca) - For all $y \in \mathcal{X}$ and $R \in \mathcal{R}^c$, if there exists a sequence $(x^n, R^n)_{n \in \mathbb{N}} \in \mathcal{X} \times \mathcal{R}^c$ such that, for all $n \in \mathbb{N}$, (i) $I(y, R^n) = I(y, R)$, (ii) $L(x^{n+1}, R^{n+1}) \cap U(x^n, R^n) = \emptyset$, (iii) $x^n P y$, and (iv) $x^n \rightarrow y$, then $W(x^n, R^n, y) \rightarrow 1$.

Well-being measures W_k and W_1 do not satisfy neither of these two properties. Well-being measures W_{max} and W_{meid} satisfy only one of them, whereas W_{meid} satisfies both of them.

Before stating the theorems, we introduce two last axioms, which precisely impose some consistency between the way to measure well-being when yRx and when xRy . They are strengthening of the properties of Reference Rescaling and Consumption Rescaling that are contained in the definition of a well-being measure. Reference Rescaling and Consumption Rescaling are defined in a way that preserves the relative position of consumption and reference bundles. For example, when consumption is preferred to the reference, the rescaling does not invert this relation. We drop this position restriction in the following axioms, which we call Rescaling Across Consumption and Rescaling Across Reference. These axioms are satisfied by all our well-being measures as well as Deaton's distance function, and, eventually, they play a limited role in the following results.

Rescaling Across Consumption (RAx) - Let $x \in \mathcal{X}$, $R, R' \in \mathcal{R}^C$, $\alpha, \lambda \in \mathbb{R}_{++}$, if $I(\alpha x, R) = I(\alpha x, R')$ and $I(\lambda x, R)$ and $I(\lambda x, R')$ are homothetic to $I(\alpha x, R)$, then

$$W(x, R, \alpha x) = W(x, R', \alpha x) \Leftrightarrow W(x, R, \lambda x) = W(x, R', \lambda x).$$

Rescaling Across Reference (RAy) - Let $y \in \mathcal{X}$, $R, R' \in \mathcal{R}^c$, $\alpha, \lambda \in \mathbb{R}_{++}$, if $I(\alpha y, R) = I(\alpha y, R')$ and $I(\lambda y, R)$ and $I(\lambda y, R')$ are homothetic to $I(\alpha y, R)$, then

$$W(\alpha y, R, y) = W(\alpha y, R', y) \Leftrightarrow W(\lambda y, R, y) = W(\lambda y, R', y).$$

We are now equipped to state our next results.

Theorem 3. 1) *The well-being measure W_{max} satisfies Supremum Contour at Consumption (Sup- x), Infimum Contour at Reference (Inf- y), Convergence from Below (Cb), Rescaling Across Consumption (RAx) and Rescaling Across Reference (RAy) over the domain $\mathcal{X} \times \mathcal{R}^c \times \mathcal{X}$.*

2) *Conversely, if a well-being measure $W : \mathcal{X} \times \mathcal{R}^c \times \mathcal{X} \rightarrow \mathbb{R}$ satisfies Convergence from below (Cb) and either Supremum Contour at Consumption (Sup- x) or Infimum Contour at Reference (Inf- y), then for all $x, y \in \mathcal{X}$ and $R \in \mathcal{R}^c$ such that yRx , $W(x, R, y) = W_{max}(x, R, y)$.*

3) *If, in addition, W satisfies either Rescaling Across Consumption (RAx) or Rescaling Across Reference (RAy), then $W = W_{max}$.*

The previous theorem, besides characterizing W_{max} , provides a powerful result for contexts, like poverty measurement, in which the reference bundle is above the consumption bundle. It is customary, in poverty measurement, to impose a focus axiom according to which the contribution to poverty of individuals who prefer their consumed bundle to the poverty line is normalized to zero. Therefore, the only situations for which one needs to compare individual well-beings are those in which the reference bundle is preferred to the consumed one. In such a framework, one of the two contour

axioms is sufficient to characterize this measure which, following Theorem 1, coincides with the smallest equivalent income poverty gap.

Theorem 4. 1) *The well-being measure W_{min} satisfies Supremum Contour at Reference (Sup-y), Infimum Contour at Consumption (Inf-x), Convergence from Above (Ca), Rescaling Across Consumption (RAx) and Rescaling Across Reference (RAy) over the domain $\mathcal{X} \times \mathcal{R}^c \times \mathcal{X}$.*

2) *Conversely, if a well-being measure $W : \mathcal{X} \times \mathcal{R}^c \times \mathcal{X} \rightarrow \mathbb{R}$ satisfies Convergence from above (Ca) and either Supremum Contour at Reference (Sup-y) or Infimum Contour at Consumption (Inf-x), then for all $x, y \in \mathcal{X}$ and $R \in \mathcal{R}^c$ such that xRy , $W(x, R, y) = W_{min}(x, R, y)$.*

3) *If, in addition, W satisfies either Rescaling Across Consumption (RAx) or Rescaling Across Reference (RAy), then $W = W_{min}$.*

Theorem 4 is particularly useful in frameworks in which individuals are likely to consume bundles they prefer to the reference. This is the case if the reference bundle is the minimum requirement for a life to be worth living. In the theory of critical level utilitarianism (see Blackorby and Donaldson 1984; Blackorby et al. 1995; Bossert et al. 2023, for example), social welfare is defined as the sum of the differences between individuals' utility at consumption and their utility at the minimum bundle for their life to be worth living. This theory is developed in a welfarist framework (that is when cardinal and comparable utility functions are supposed to be known). Theorem 4 can be seen as complementing this theory by building a way to go from possibly heterogenous preferences and reference bundles to such a social welfare function.

The following theorem completes the axiomatic analysis by characterizing W_{meid} . As already mentioned, this measure is an hybrid combination of W_{min} and W_{max} with the advantage of being defined over the entire domain $\mathcal{R}$ of preferences.

Theorem 5. 1) *The well-being measure W_{meid} satisfies Supremum Contour at Consumption (Sup-x) when yRx , Infimum Contour at Reference (Inf-y) when yRx , Supremum Contour at Reference (Sup-y) when xRy , Infimum Contour at Consumption (Inf-x) when xRy , Convergence from Above (Ca) and Convergence from Below (Cb) over the domain $\mathcal{X} \times \mathcal{R} \times \mathcal{X}$.*

2) *Conversely, if a well-being measure $W : \mathcal{X} \times \mathcal{R} \times \mathcal{X} \rightarrow \mathbb{R}$ satisfies Convergence from Above (Ca), Convergence from below (Cb), either Supremum Contour at Consumption (Sup-x) when yRx or Infimum Contour at Reference (Inf-y) when yRx , and either Supremum Contour at Reference (Sup-y) when xRy or Infimum Contour at Consumption (Inf-x) when xRy , then $W = W_{meid}$.*

Our axiomatic study allowed us to identify new ways of measuring well-being. We have already mentioned how W_{meid} can be seen as the prominent one, particularly if we wish to abandon the restriction of the preference domain. It is worth mentioning, however, that contrary to W_k , W_{min} and W_{max} are well-defined on a (much) larger domain than $\mathcal{R}^c$ (the domain of preferences for which lower contours are bounded). This is the case, for instance, of the general domain of homothetic preferences, which

includes CES preferences, on which W_{max} and W_{min} are well-defined and coincide (and, therefore, are equal to W_{meid}). W_{max} and W_{min} continue to satisfy all the axioms listed in Theorems 3 and 4 on the more general domain on which they are well defined.

Theorems 3, 4 and 5 remain valid even when extending the preference domain to include non-convex ones.¹⁴ Let us call $\mathcal{R}^{cnc}$ the domain of non-convex preferences with bounded lower contour sets. This is all the more surprising that such a domain extension enlarges the family of measures satisfying our main axioms. For instance, the following family of measures, which generalizes W_k , satisfy all Supremum and Infimum axioms if we enlarge the domain to $\mathcal{R}^{cnc}$: fix a bundle $r \in \mathcal{X}$. For any bundle $x \in \mathcal{X}$ and preferences $R \in \mathcal{R}^{cnc}$, let $r(x, R)$ be defined by: $r(x, R) = w \Leftrightarrow xIwr$, that is, $r(x, R)$ denotes the fraction of bundle r to which x is indifferent according to R . The relevant family of measures, W_r , are defined by $W_r(x, R, y) = \frac{r(x, R)}{r(y, R)}$.¹⁵

We complete this section with a summary of the axiomatic comparison between our measures. Table 1 lists the axioms that each well-being measure satisfies.

Table 1: Lists of axioms satisfied by the well-being measures

Measure	W_k	W_1	W_{max}	W_{min}	W_{meid}
Pref. Domain	$\mathcal{R}^c$	$\mathcal{R}^1$	$\mathcal{R}^c$	$\mathcal{R}^c$	$\mathcal{R}$
Axiom					
Sup-x yRx	+	+	+		+
Sup-x xRy	+	+	+		
Sup-y yRx	+	+		+	
Sup-y xRy	+	+		+	+
Inf-x yRx	+	+		+	
Inf-x xRy	+	+		+	+
Inf-y yRx	+	+	+		+
Inf-y xRy	+	+	+		
Cb			+		+
Ca				+	+
RAx	+	+	+	+	
RAy	+	+	+	+	

Description: The table reports, for each well-being measure (first row), the preference domain over which it is defined (second row) and the axioms it satisfies. For each axiom, when relevant, we distinguish between the case in which the reference bundle is above or below the consumption one. The symbol + means that the measure satisfies the given axiom under the considered restriction on the bundles. Empty cells mean that the axiom is not satisfied by the given measure.

Note: $\mathcal{R}^1$ is the domain of preferences with all indifference curves intersecting at least the good 1 axis. $\mathcal{R}^c \subset \mathcal{R}^1 \subset \mathcal{R}$; see Appendix B for details.

¹⁴ W_{max} and W_{min} still require preferences to have bounded lower contour sets. Their interpretation in terms of equivalent incomes, however, is no longer valid when preferences are non convex.

¹⁵Fleurbaey and Maniquet (2019) study the domain of non-convex preferences in their analysis of well-being measurement.

6 Conclusion

We have explored how to measure well-being when individuals differ in their consumption, preferences and reference. Our main objective was to identify appealing well-being measures that avoid the drawback of Deaton’s distance function, so that what matters at the reference is the satisfaction level, not the reference bundle per se.

In this conclusion, we comment on some features of the well-being measures that end up being characterized.

First, we comment on the role of income in these measures. Prices that individuals actually face in their consumption do not play any role in any of our measures. This is not surprising, as actual incomes do not lead to well-being comparisons that are consistent with individual preferences as soon as individuals face different prices. Equivalent incomes, on the contrary, play a crucial role here. The maximum (resp. minimum) well-being measures that we introduce in this paper defines well-being as the ratio between equivalent income at consumption and at reference, when the price vector that is used to compute equivalent income is the one that maximizes (resp. minimizes) well-being.

All in all, this means that measuring well-being using relative equivalent incomes receives some normative justification, provided prices are endogenous to preferences in the appropriate way. The reader may consider such endogeneity somehow *artificial*, as it follows from the choice of the reference bundle. It is nevertheless the case that justifying a specific reference bundle can be easier than choosing a price vector. Coming back to the issue of measuring poverty, it is easier to quantify the minimum amount of, say bread, an individual needs rather than the market price he has to pay for it. In this sense, our framework can help researchers in providing stronger normative bases for defining the price vector at which to compute equivalent incomes.

Our second comment has to do with the cardinalization of our measures. As explained after Lemma 1 above, we must have $W(x, R, y) = f\left(\frac{u(x)}{u(y)}\right)$ for all homothetic preferences R , with u being the homogenous of degree 1 utility function representing R . That leaves f unspecified. Of course, f does not matter when we make *inter-personal comparisons of levels* of well-being, but f will matter if we want to compare *differences* in well-being. This refers to the well-being inequality aversion that the social welfare function should embody. The final choice of f may itself be the topic of an axiomatic study. This is what [Fleurbaey and Maniquet \(2018\)](#) studies when there is no reference bundle. By reference to the solution they give to this problem, we can suggest the natural solution consisting of using a linear f and an inequality averse aggregator (that is one that satisfies the Pigou-Dalton principle in well-being).

Our third comment is related to the idea, which we defended in the Introduction, that the reference bundle is defined by the social evaluator. As a consequence, only self-centered preferences are relevant for welfare analysis. As it has been largely documented in the behavioral economics literature, though, individuals’ actual preferences are other-regarding. This raises the question of how to go from actual to self-centered preferences. Other-regarding preferences, however, are often assumed to be separable in own and others’ consumption, as in [Dufwenberg et al. \(2011\)](#), so that the relevant self-centered preferences are obtained by simply disregarding their social arguments.

Finally, our analysis has been developed in a classic and standard framework with non-satiation and convex preferences. Future research may focus on extending the current analysis to more specific contexts, in line with [Fleurbaey and Maniquet \(2019\)](#), or in exploring the consequences of other axioms in a larger preference domain.

Appendix A Proofs

A.1 Lemma 1

Proof Let $x, x' \in \mathcal{X}$, $R, R' \in \mathcal{R}^H$, $\lambda > 0$. Let $R^1, R^2 \in \mathcal{R}^H$ be two preferences such that: xI^1z , $x'I^2z$ and $I(z, R^1) \cap I(z, R^2) = \{z\}$. By H, $W(x, R, \lambda x) = W(x, R^1, \lambda x)$. By Mx, $W(x, R^1, \lambda x) = W(z, R^1, \lambda x)$. By Py, $W(z, R^1, \lambda x) = W(z, R^1, \lambda z)$. By H, $W(z, R^1, \lambda z) = W(z, R^2, \lambda z)$. By Mx and Py again, $W(z, R^2, \lambda z) = W(x', R^2, \lambda x')$. By H, $W(x', R^2, \lambda x') = W(x', R', \lambda x')$. Gathering the equalities, $W(x, R, \lambda x) = W(x', R', \lambda x')$, the desired outcome. $\square$

A.2 Theorem 1

Proof Let us start with the first inequality.

By contradiction, let W be such that $W(x, R, y) < W_{\min}(x, R, y)$ for some situation $(x, R, y) \in \mathcal{X} \times \mathcal{R}^c \times \mathcal{X}$. Let $R' \in \mathcal{R}^H$ and $\omega \in \mathbb{R}$ be such that: (i) $I(y, R') = I(y, R)$ and (ii) $I(\omega y, R')$ is tangent from below to $I(x, R)$. By H, $W(\omega y, R', y) = \omega$ which, by Definition 2, coincides with $W_{\min}(x, R, y)$. Gathering all this, we have

$$W(x, R, y) < \omega = W(\omega y, R', y) = W_{\min}(x, R, y) \quad (\text{A1})$$

By Cont, the first inequality in eq.(A1) implies that there exists $x' \in \mathcal{X}$ such that $x'Px$ and $W(x', R, y) = \omega$. Observe that, by (ii) we must have $U(x', R) \cap L(\omega y, R') = \emptyset$, so that (i) and Mx imply

$$\omega = W(x', R, y) > W(\omega y, R', y) \quad (\text{A2})$$

A contradiction. The proof of the second inequality follows the same logic, with $I(\omega y, R')$ tangent from above to $I(x, R)$ and xPx' . $\square$

A.3 Intermediary lemmas

We now prove a series of intermediary results, which will be used in the proofs of the next theorems. First, Consumption Monotonicity and Continuity imply that only the indifference contours at consumption and at reference matter; that is, well-being measures are not allowed to be sensitive to changes in preferences that do not modify these two indifference contours. We formalize this property into the following axiom.

Unchanged Contour Independence (UCI) - For all $x, y \in \mathcal{X}$ and $R, R' \in \mathcal{R}^c$, if $I(x, R) = I(x, R')$ and $I(y, R) = I(y, R')$, then $W(x, R, y) = W(x, R', y)$.

Lemma 2. *If W satisfies Continuity (Cont) and Consumption Monotonicity (Mx), then it satisfies Unchanged Contour Independence (UCI).*

Proof Let $x, y \in \mathcal{X}$ and $R, R' \in \mathcal{R}^c$ be such that $I(x, R) = I(x, R')$ and $I(y, R) = I(y, R')$. Let $x^n \in \mathcal{X}$, $n \in \mathbb{N}$, be a sequence of bundles so that $x^n P' x$ for all $n \in \mathbb{N}$ and $\lim_{n \rightarrow \infty} x^n = x$. By Mx, $W(x^n, R', y) > W(x, R, y)$ for all $n \in \mathbb{N}$. By Cont, $W(x, R', y) \geq W(x, R, y)$. Similarly, with a sequence $x^n \in \mathcal{X}$ such that $x P' x^n$ for all $n \in \mathbb{N}$ and $\lim_{n \rightarrow \infty} x^n = x$, we get $W(x, R', y) \leq W(x, R, y)$. Gathering the two inequalities, we obtain $W(x, R', y) = W(x, R, y)$, the desired outcome. $\square$

When a well-being measure satisfies Reference Preference, using Lemma 2, the relevant information about individual situations are a pair of non intersecting indifference contour sets. In this sense, $W(x, R, y)$ can be written as $W(I(x, R), I(y, R))$. Therefore, abusing notation, we will let W be a function of two indifference contours belonging to the set $\mathcal{I}^c$ of all admissible indifference contours. For $J \in \mathcal{I}^c$, $R \in \mathcal{R}^c$, we write $J \in R$ to denote that for all $x, x' \in J$, x is indifferent to x' according to R . For $I, J \in \mathcal{I}^c$, $\lambda > 0$, we write $I = \lambda J$ if I is the homothetic transformation of J by a factor λ . We also write $I \prec_B J$ (resp., $I \prec^A J$) if I is tangent to J from below (resp., above). Finally, we let $U(I)$ (resp. $L(I)$) denote the upper (resp. lower) contour of all bundles on I .

We proceed with an immediate consequence of the Homotheticity and Rescaling properties of a well-being measure.

Lemma 3. *For all $R \in \mathcal{R}^c$, all $x, y \in \mathcal{X}$, all $\lambda > 0$,*

$$W(x, R, y) = W(\lambda x, R^\lambda, \lambda y)$$

for $R^\lambda \in \mathcal{R}^c$ such that $I(\lambda x, R^\lambda) = \lambda I(x, R)$ and $I(\lambda y, R^\lambda) = \lambda I(y, R)$.

Proof Let W be a well-being measure. By Lemma 2, W satisfies UCI, so that, by Py, we can reduce any $W(x, R, y)$ into $W(X, Y)$ for $X = I(x, R)$ and $Y = I(y, R)$.

We need to prove that for all $X, Y \in \mathcal{I}$, all $\lambda > 0$, $W(\lambda X, \lambda Y) = W(X, Y)$. If X is homothetic to Y , then the claim follows from H and Lemma 1. Let us assume that X is not homothetic to Y . Let $w = W(X, Y)$. We have four cases to consider.

Case 1: $\lambda < 1$ and $w < 1$. By H, $W(X, w^{-1}X) = w$. Therefore, $W(X, Y) = W(X, w^{-1}X)$. By Rx, $W(\lambda X, Y) = W(\lambda X, w^{-1}X)$. By H, $W(\lambda X, w^{-1}X) = \lambda w$, so that $W(\lambda X, Y) = \lambda w$. By H, $W(\lambda wY, Y) = \lambda w$, so that $W(\lambda X, Y) = W(\lambda wY, Y)$. By Ry, $W(\lambda X, \lambda Y) = W(\lambda wY, \lambda Y)$. By H, $W(\lambda wY, \lambda Y) = w$, so that $W(\lambda X, \lambda Y) = w$, the desired outcome.

The three other cases - (2) $\lambda > 1$ and $w < 1$, (3) $\lambda < 1$ and $w > 1$, (4) $\lambda > 1$ and $w > 1$ - are treated in a similar way, where the order of the change from X into λX and Y into λY must guarantee that $\lambda X \cap Y = \emptyset$ when, like in case 1, X is transformed first, and $\lambda Y \cap X = \emptyset$ when Y is transformed first (cases 2 or 3). $\square$

The proofs below use the following Reference Monotonicity axiom, which is the proper adaptation of Consumption Monotonicity: if the indifference contours at consumption are identical in two situations and the indifference contour at one reference is everywhere higher than in the other situation (in the sense that the lower contour of the latter does not intersect with the upper contour of the former), then well-being is lower.

Reference Monotonicity (My) - For all $x, y, y' \in \mathcal{X}$ and $R, R' \in \mathcal{R}^c$, if $I(x, R) = I(x, R')$ and $L(y, R) \cap U(y', R') = \emptyset$, then $W(x, R, y) > W(x, R', y')$.

Lemma 4. *If $W : \mathcal{X} \times \mathcal{R}^c \times \mathcal{X} \rightarrow \mathbb{R}_+$ satisfies Continuity (Cont), Consumption Monotonicity (Mx) and Reference Preference (Py), then it satisfies Reference Monotonicity (My).*

Proof By Lemma 2, W satisfies UCI. Let $x, y, y' \in \mathcal{X}$ and $R, R' \in \mathcal{R}^c$ be such that $I(x, R) = I(x, R')$ and $L(y, R) \cap U(y', R') = \emptyset$. Let $R'' \in \mathcal{R}^c$ be such that $I(x, R'') = I(x, R)$, $I(y, R'') = I(y, R)$ and $I(y', R'') = I(y', R')$. By UCI, $W(x, R'', y) = W(x, R, y)$ and $W(x, R'', y') = W(x, R', y')$. By Py, $W(x, R'', y) > W(x, R'', y')$. Gathering these (in)equalities, we get $W(x, R, y) > W(x, R', y')$, the desired outcome. $\square$

The following lemma formalizes the consequence of the Supremum and Infimum Contour axioms for the measurement of well-being in $\mathcal{R}^c$. It is important to observe that we cannot derive the same consequence as Fleurbaey and Maniquet (2017b), but we almost replicate the proof of Chambers and Miller (2014a). In words, for a fixed consumption (resp. reference) indifference contour, our axioms allow us to define nested subsets of $\mathcal{X}$ which evaluate the reference (resp. consumption) indifference contours they contain. Those subsets have different shapes, depending on whether they are in the upper or the lower contour of the fixed consumption (resp. reference) and may not cover the whole consumption set $\mathcal{X}$. It is worth underlining that Lemma 5 does not require Rescaling Across Consumption nor Rescaling Across Reference.

Lemma 5. *Let $W : \mathcal{X} \times \mathcal{R}^c \times \mathcal{X} \rightarrow \mathbb{R}$ be a well-being measure.*

- (A) *If W satisfies Supremum Contour at Consumption (Sup- x), then for all $Y \in \mathcal{I}^c$, $w \in \mathbb{R}_+$, $w \neq 1$, there exists $L_w^Y \subset \mathcal{X}$ such that for all $X \in \mathcal{I}^c$ such that $X \cap Y = \emptyset$, $W(X, Y) \leq w \Leftrightarrow L(X) \subseteq L_w^Y$.*
- (B) *If W satisfies Supremum Contour at Reference (Sup- y), then for all $X \in \mathcal{I}^c$, $w \in \mathbb{R}_+$, $w \neq 1$, there exists $L_w^X \subset \mathcal{X}$ such that for all $Y \in \mathcal{I}^c$ such that $Y \cap X = \emptyset$, $W(X, Y) \geq w \Leftrightarrow L(Y) \subseteq L_w^X$.*
- (C) *If W satisfies Infimum Contour at Consumption (Inf- x), then for all $Y \in \mathcal{I}^c$, $w \in \mathbb{R}_+$, $w \neq 1$, there exists a closed and convex $U_w^Y \subset \mathcal{X}$ such that for all $X \in \mathcal{I}^c$ such that $X \cap Y = \emptyset$, $W(X, Y) \geq w \Leftrightarrow U(X) \subseteq U_w^Y$.*
- (D) *If W satisfies Infimum Contour at Reference (Inf- y), then for all $X \in \mathcal{I}^c$, $w \in \mathbb{R}_+$, $w \neq 1$, there exists a closed and convex $U_w^X \subset \mathcal{X}$ such that for all $Y \in \mathcal{I}^c$ such that $Y \cap X = \emptyset$, $W(X, Y) \leq w \Leftrightarrow U(Y) \subseteq U_w^X$.*

Proof of (A) Let $Y \in \mathcal{I}^c$ and $w \in \mathbb{R}_+$, $w \neq 1$. Let $L_w^Y \subset \mathcal{X}$ be defined by

$$L_w^Y = \bigcup_{X \in \mathcal{I}^c, W(X, Y) \leq w} L(X).$$

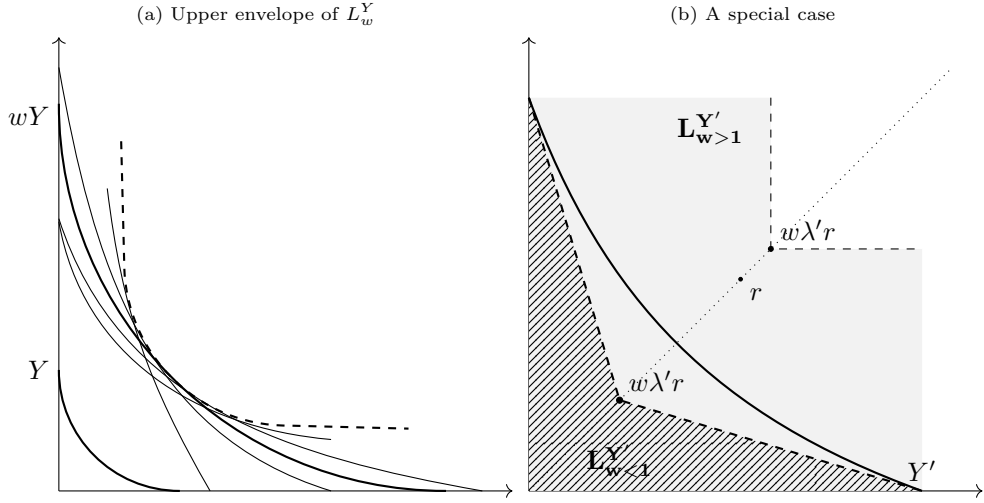
See Figure A1a for an illustration, in which the dashed line is an example of the upper envelope of L_w^Y . Observe that the constraint $X \in \mathcal{I}^c$ implies that we have a restricted set of X 's, with the consequence that L_w^Y is not necessarily compact.¹⁶ For instance, let $r \in \mathcal{X}$, and let W be defined by

$$W(X, Y) = \frac{\max\{\lambda | \lambda r \in L(X)\}}{\max\{\lambda | \lambda r \in L(Y)\}}.$$

Let $Y' \in \mathcal{I}^c$ be such that $\max\{\lambda | \lambda r \in L(Y')\} = \lambda'$ and $(y'_k)_{k \in \{1, \dots, K\}}$ are the intersections between Y' and the K axes. Then, for $w < 1$, $L_w^{Y'} = \mathcal{X} \setminus \text{ch}\{(y'_k), w\lambda' r\} \cup \{w\lambda' r\}$, which is

¹⁶In words, since preferences must be rational, X cannot intersect Y .

Figure A1: Union of lower contours.



bounded but not closed. For $w > 1$, $L_w^{Y'} = \mathcal{X} \setminus \{y | y \gg w\lambda'r\}$, which is closed but not bounded (see Figure A1b for an example). This prevents us from following the proof of [Fleurbaey and Maniquet \(2017b\)](#).

By construction, for all $X' \in \mathcal{I}^c$, $W(X', Y) \leq w \Rightarrow L(X') \subseteq L_w^Y$. Let us assume $L(X') \subseteq L_w^Y$ for some $X' \in \mathcal{I}$. Assume, by way of contradiction, that $W(X', Y) > w$. Then, by continuity of W with respect to consumption, there exists $\lambda < 1$ such that $W(\lambda X', Y) > w$. Observe that, since $L(\lambda X')$ is compact and $L(\lambda X') \subset L(X')$, its frontier is bounded away from the frontier of L_w^Y . As a result,

$$L(\lambda X') \subset \bigcup_{X \in \mathcal{I}^c, W(X, Y) \leq w} \text{int}(L(X)),$$

that is $\text{int}(L(X))$, for all $X \in \mathcal{I}^c$ such that $W(X, Y) \leq w$, are an open cover of $L(\lambda X')$, which is compact. Therefore, there exists a finite sequence $X^1, X^2, \dots, X^N$, such that $W(X^n, Y) \leq w$ for all $n \in \{1, \dots, N\}$, and

$$L(\lambda X') \subset \bigcup_{n \in \{1, \dots, N\}} \text{int}(L(X^n)), \quad (\text{A3})$$

that is $\text{int}(L(X^n))$ are an open subcover of $L(\lambda X')$. Let $X^* = \vee_{n \in \{1, \dots, N\}} X^n$.¹⁷ By Sup-x, $W(X^*, Y) \leq w$. By (A3), $L(\lambda X') \subset L(X^*)$, which, together with $W(\lambda X', Y) > w$, contradicts Mx. $\square$

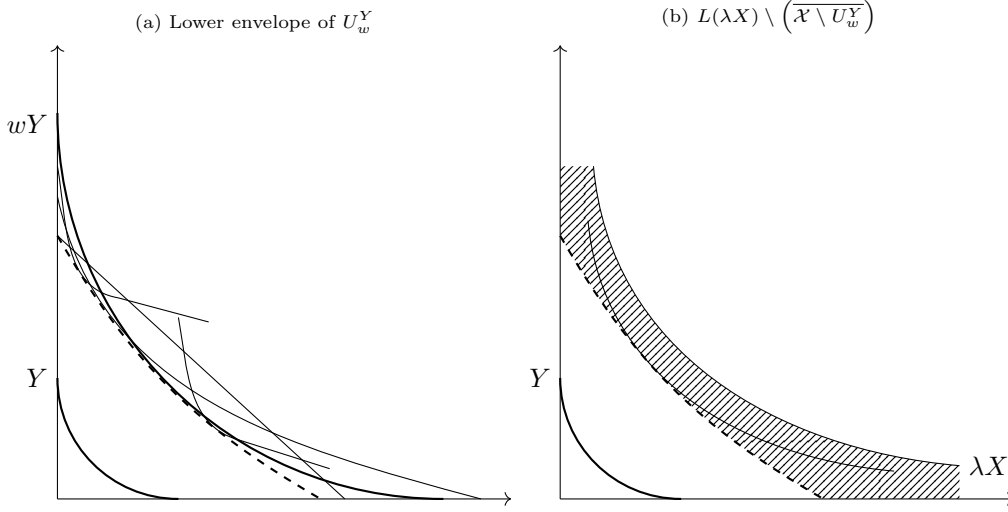
Proof of (C) Let $Y \in \mathcal{I}^c$ and $w \in \mathbb{R}$, $w \neq 1$. Let $U_w^Y \subset \mathcal{X}$ be defined by

$$U_w^Y = \bigcup_{X \in \mathcal{I}^c, W(X, Y) \geq w} U(X).$$

Note that, by H, $W(wY, Y) = w$, so that $U(wY) \subseteq U_w^Y$. Therefore, $\mathcal{X} \setminus U_w^Y$ is bounded. Also note that U_w^Y is the closure of $\bigcup_{X \in \mathcal{I}^c, W(X, Y) > w} U(X)$. Therefore, U_w^Y is closed.

¹⁷We refer here to the standard convention of denoting $\vee$ the join or supremum.

Figure A2: Union of lower contours.



Assume it is not convex. Then there exist $x, x' \in \mathcal{X}$ and $X^x, X^{x'} \in \mathcal{I}^c$ such that $W(X^x, Y) \geq w$, $W(X^{x'}, Y) \geq w$, $x \in X^x$, $x' \in X^{x'}$ and $\lambda x + (1 - \lambda)x' \notin U_w^Y$. Let $X^{xx'} \in \mathcal{I}^c$ be defined by $U(X^{xx'}) = \text{ch}(U(X^x) \cup U(X^{x'}))$. By construction, $\lambda x + (1 - \lambda)x' \in U(X^{xx'})$. By Inf-x, $W(X^{xx'}, Y) = \min\{W(X^x, Y), W(X^{x'}, Y)\} \geq w$, so that $U(X^{xx'}) \subseteq U_w^Y$, a contradiction proving that U_w^Y is convex.

We need to prove that $W(X, Y) \geq w \Leftrightarrow U(X) \subseteq U_w^Y$. By construction, for all $X' \in \mathcal{I}$, $W(X', Y) \geq w \Rightarrow U(X') \subseteq U_w^Y$. Let us prove the converse relationship. Let us assume $U(X') \subseteq U_w^Y$ for some $X' \in \mathcal{I}$. Assume, by way of contradiction, that

$$W(X', Y) < w. \quad (\text{A4})$$

Then, by continuity of W with respect to consumption, there exists $\lambda > 1$ such that $W(\lambda X', Y) < w$. Observe that, as also shown by the dashed line in Figure A2b, the lower frontier of $U(\lambda X')$ is bounded away from the lower frontier of U_w^Y (that is $U(\lambda X') \cap \overline{\mathcal{X} \setminus U_w^Y} = \emptyset$, where $\overline{A}$ stands for the closure of A). As a result, there exists a finite sequence $X^1, X^2, \dots, X^N$, such that $W(X^n, Y) \geq w$ for all $n \in \{1, \dots, N\}$, and

$$U(\lambda X') \subset \text{int} \left(\text{ch} \left(\bigcup_{n \in \{1, \dots, N\}} U(X^n) \right) \right). \quad (\text{A5})$$

Let $X^* = \bigwedge_{n \in \{1, \dots, N\}} X^n$. By Inf-x, $W(X^*, Y) \geq w$. By (A5), $U(\lambda X') \subset U(X^*)$, which, together with $W(\lambda X', Y) < w$, contradicts Mx. $\square$

Proof of (B) and (D) The results are obtained permuting X and Y in the previous proofs. $\square$

We are now endowed with the necessary notions to prove Theorems 2 to 4.

A.4 Theorem 2

Proof Let $k \in \{1, \dots, K\}$. In the terminology of lemma 5, W_k satisfies Sup-x with

$$L_w^Y = \{x \in \mathcal{X} | x_k \leq w a^k(Y)\}$$

such that $W(X, Y) \leq w$ if and only if $\{L(X) \cap A^k\} \subseteq L_w^Y$. Similarly, we can define

$$\begin{aligned} L_w^X &= \{x \in \mathcal{X} | x_k \leq w^{-1} a^k(X)\} \\ U_w^Y &= \mathcal{X} \setminus \{x \in A^k | x_k < w a^k(Y)\} \\ U_w^X &= \mathcal{X} \setminus \{x \in A^k | x_k < w^{-1} a^k(X)\} \end{aligned}$$

to show that W_k satisfies the other three axioms.

To prove the converse, let us assume that W satisfies Sup-x and Inf-x. A similar proof holds if we start from Sup-y and Inf-y. First, observe that if, for some $Y \in \mathcal{I}^c$ and some $w \in \mathbb{R}_+$, $L_w^Y = \{x \in \mathcal{X} | x_k \leq w a^k(Y)\}$, then $U_w^Y = \{x \in A^k | x_k \geq w a^k(Y)\}$, otherwise we have two competing ways of measuring $W(X, Y)$.

Claim 1: Let $Y \in \mathcal{I}^c$. There exists $k \in \{1, \dots, K\}$ such that for all $X, X' \in \mathcal{I}^c$, if $W(X, Y) = W(X', Y)$, then $A^k(X) = A^k(X')$.

Let $\mathcal{K}^0 \subseteq \{1, \dots, K\}$ be defined by: $k \in \mathcal{K}^0$ if and only if there exist $X, X' \in \mathcal{I}^c$ such that

$$W(X, Y) = W(X', Y) \text{ and } (1) A^k(X) = A^k(X'), (2) A^{k'}(X) \neq A^{k'}(X') \forall k' \neq k.$$

We need to prove that $|\mathcal{K}^0| = 1$. Assume $\mathcal{K}^0 = \emptyset$. Then there exist $X^1, X'^1, X^2 \in \mathcal{I}^c$, as illustrated in Fig. A3a, such that

$$\begin{aligned} A^1(X^1) &= A^1(X^2) \\ A^2(X'^1) &= A^2(X^2) \end{aligned}$$

and for all $X \in \mathcal{I}^c$ such that $U(X^1) \subset \text{int}(U(X)) \subset U(X) \subset \text{int}(U(X'^1))$:

$$L(X \wedge X^2) \cap U(X \vee X^2) = \emptyset. \quad (\text{A6})$$

By Cont and Mx,

$$W(X^1, Y) \geq W(X^2, Y) \geq W(X'^1, Y).$$

By the assumption that $\mathcal{K}^0 = \emptyset$, $W(X^1, Y) \neq W(X^2, Y) \neq W(X'^1, Y)$, so that $W(X^1, Y) > W(X^2, Y) > W(X'^1, Y)$. By Cont, there exists $X^* \in \mathcal{I}$ such that

$$U(X^1) \subset \text{int}(U(X^*)) \subset U(X^*) \subset \text{int}(U(X'^1))$$

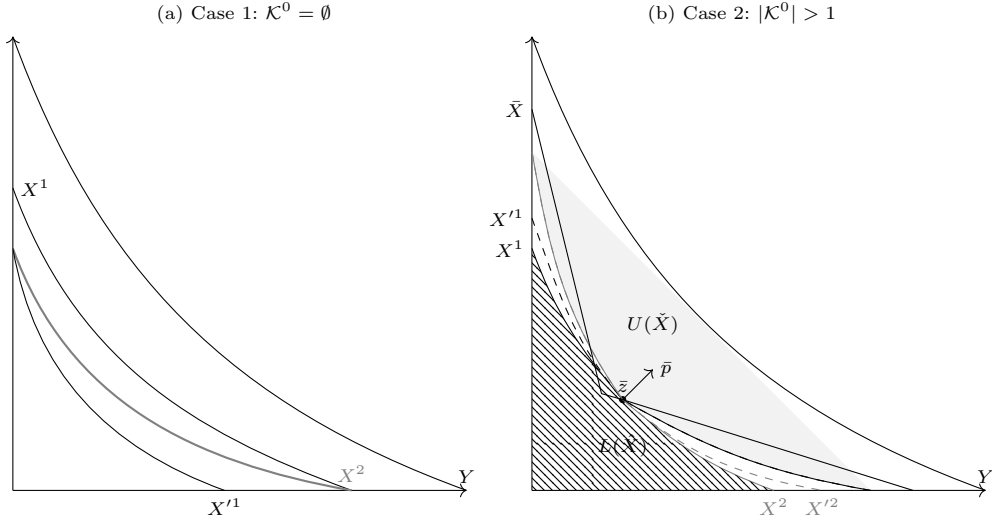
and $W(X^*, Y) = W(X^2, Y)$. By Sup-x and Inf-x, $W(X^* \vee X^2, Y) = W(X^* \wedge X^2, Y)$, which, given Eq. A6, violates Mx.

Assume $|\mathcal{K}^0| > 1$. Let us assume, wlog, that $\mathcal{K}^0 = \{1, \dots, K^0\}$. Then, by Rx, we can find $X^1, X'^1, \dots, X^{K^0}, X'^{K^0} \in \mathcal{I}^c$ such that $W(X^1, Y) = W(X'^1, Y) = \dots = W(X^{K^0}, Y) = W(X'^{K^0}, Y) = w$, with, for all $k \in \mathcal{K}^0$: $A^k(X^k) = A^k(X'^k)$ and $A^{k'}(X^k) \neq A^{k'}(X'^k) \forall k' \neq k$. By Sup-x, $W(X^1 \vee X'^1 \vee \dots \vee X^{K^0} \vee X'^{K^0}, Y) = w$, and by Inf-x, $W(X^1 \wedge X'^1 \wedge \dots \wedge X^{K^0} \wedge X'^{K^0}, Y) = w$. In a similar way as above, if $L(X^1 \wedge X'^1 \wedge \dots \wedge X^{K^0} \wedge X'^{K^0}) \cap U(X^1 \vee X'^1 \vee \dots \vee X^{K^0} \vee X'^{K^0}) = \emptyset$, then we get a violation of Mx and the claim is proven. Let us assume that $L(X^1 \wedge X'^1 \wedge \dots \wedge X^{K^0} \wedge X'^{K^0}) \cap U(X^1 \vee X'^1 \vee \dots \vee X^{K^0} \vee X'^{K^0}) \neq \emptyset$ (see Figure A3b for an example). That implies the existence of $\bar{z} \in X^1 \cap X'^1 \cap \dots \cap X^{K^0} \cap X'^{K^0}$ and a price vector $\bar{p} \in \mathbb{R}_{++}^K$ that supports all $X^k, X'^k, k \in \mathcal{K}^0$ at $\bar{z}$. Let

$$\begin{aligned} \hat{X} &= X^1 \wedge X'^1 \wedge \dots \wedge X^{K^0} \wedge X'^{K^0} \\ \check{X} &= X^1 \vee X'^1 \vee \dots \vee X^{K^0} \vee X'^{K^0}. \end{aligned}$$

Let $\bar{X} \in \mathcal{I}$ be such that

Figure A3: Illustration of the proof of Theorem 2



- $\bar{z} \in \bar{X}$,
- there exists $\tilde{z} \in \tilde{X}$ such that (i) $\bar{p}\tilde{z} < \bar{p}\bar{z}$, and (ii) there exist $1 < \hat{\alpha} < \check{\alpha}$ such that $\hat{\alpha}\tilde{z} \in \hat{X}$ and $\check{\alpha}\tilde{z} \in \check{X}$,¹⁸ and
- $A^k(\bar{X}) \geq A^k(\tilde{X})$ for all $k \in \mathcal{K}^0$.

Note that

$$L(\hat{X} \wedge \tilde{X} \wedge \bar{X}) \cap U(\hat{X} \vee \tilde{X} \vee \bar{X}) = \emptyset,$$

which implies $W(\bar{X}, Y) \neq w$. Also note that for all $\lambda > 0$,

$$L(\hat{X} \wedge \tilde{X} \wedge \lambda\bar{X}) \cap U(\hat{X} \vee \tilde{X} \vee \lambda\bar{X}) = \emptyset. \quad (\text{A7})$$

By Cont, there exists $\bar{\lambda}$ such that $W(\bar{\lambda}\bar{X}, Y) = w$. Then, by Sup-x et Inf-x, we get

$$W(\hat{X} \wedge \tilde{X} \wedge \bar{\lambda}\bar{X}, Y) = W(\hat{X} \vee \tilde{X} \vee \bar{\lambda}\bar{X}, Y) = w,$$

which, in view of A7, violates Mx.

Claim 2: There exists $k \in \{1, \dots, K\}$ such that for all $Y \in \mathcal{I}$, all $X, X' \in \mathcal{I}$, if $W(X, Y) = W(X', Y)$, then $A^k(X) = A^k(X')$.

Assume not. That is, assume there exist $Y, Y' \in \mathcal{I}$ and $k, k' \in \{1, \dots, K\}$ such that claim 1 holds with k for Y and with k' for Y' . By Rx and Ry, we know that Y and Y' cannot be homothetic to each other, because if they were homothetic to each other, then L_w^Y (resp. U_w^Y) should be homothetic to $L_w^{Y'}$ (resp. $U_w^{Y'}$) so that $k = k'$. Also, by Ry again, we may assume that $Y \cap Y' \neq \emptyset$, because otherwise we can rescale Y so that it crosses Y' . Then we can find $X, X' \in \mathcal{I}$ such that $W(X, Y) = W(X', Y') = w$ whereas $A^k(X) < A^k(X')$ and $A^{k'}(X') < A^{k'}(X)$, so that $W(X', Y) > w$ and $W(X, Y') > w$. By Sup-y, $W(X, Y \vee Y') = w = W(X', Y \vee Y')$, violating claim 1. If we want to use Inf-y rather than Sup-y, then we can find $X, X' \in \mathcal{I}$ such that $W(X, Y) = W(X', Y') = w$ whereas $A^k(X) > A^k(X')$ and $A^{k'}(X') > A^{k'}(X)$, so that $W(X', Y) < w$ and $W(X, Y') < w$. By Inf-y, $W(X, Y \wedge Y') = w = W(X', Y \wedge Y')$, violating claim 1. $\square$

¹⁸To maintain a clean illustration, $\tilde{z}$ does not appear in Fig. A3b but can be any point in $L(\tilde{X}) \cap \bar{X}$.

A.5 Theorems 3 and 4

In order to prove Theorem 3, we need to prove the following consequence of our convergence axioms. Convergence from Above and Convergence from Below are defined for fixed reference bundle, which is the natural definition: we evaluate well-being when the reference is known whereas the consumption of the individuals may vary. It turns out that similar axioms can be defined for fixed consumptions and a converging reference, and these axioms are equivalent to Convergence from Above and Convergence from Below.

Convergence of the Reference from Above (CRa) - For all $x \in \mathcal{X}$ and $R \in \mathcal{R}^c$, if there exists a sequence $(y^n, R^n)_{n \in \mathbb{N}} \in \mathcal{X} \times \mathcal{R}^c$ such that, for all $n \in \mathbb{N}$, (i) $I(x, R^n) = I(x, R)$, (ii) $L(y^{n+1}, R^{n+1}) \cap U(y^n, R^n) = \emptyset$, (iii) $y^n P x$, and (iv) $y^n \rightarrow x$, then $W(x, R^n, y^n) \rightarrow 1$.

Convergence of the Reference from Below (CRb) - For all $x \in \mathcal{X}$ and $R \in \mathcal{R}^c$, if there exists a sequence $(y^n, R^n)_{n \in \mathbb{N}} \in \mathcal{X} \times \mathcal{R}^c$ such that, for all $n \in \mathbb{N}$, (i) $I(x, R^n) = I(x, R)$, (ii) $L(y^n, R^n) \cap U(y^{n+1}, R^{n+1}) = \emptyset$, (iii) $x P y^n$, (iv) and $y^n \rightarrow x$, then $W(x, R^n, y^n) \rightarrow 1$.

Lemma 6. *If a well-being measure W satisfies Convergence from Below (Cb), then it satisfies Convergence of the Reference from Above (CRa). If a well-being measure W satisfies Convergence Above (Ca), then it satisfies Convergence of the Reference from Below (CRb).*

Proof We prove the first part of the lemma. The second part can be proved in a similar way. Observe that W satisfies the property of Lemma 3. Let sequence $(y^n, R^n)_{n \in \mathbb{N}} \in \mathcal{X} \times \mathcal{R}^c$ be such that, for all $n \in \mathbb{N}$, (i) $I(x, R^n) = I(x, R)$, (ii) $L(y^{n+1}, R^{n+1}) \cap U(y^n, R^n) = \emptyset$, (iii) $y^n P x$, and (iv) $y^n \rightarrow x$. By Py, we can assume, w.l.o.g., that for all $n \in \mathbb{N}$, there exists $\lambda_n > 0$ such that $y^n = \lambda_n x$. Indeed, if it is not true, then we can replace y^n in the sequence with $\lambda^n x$ such that $\lambda^n x \in I(y^n, R^n)$ because, by Py, $W(x, R^n, y^n) = W(x, R^n, \lambda^n x)$. Observe that $y^n \rightarrow x$ is equivalent to $\lambda^n \rightarrow 1$.

By Lemma 3, $W(x, R^n, \lambda^n x) = W((\lambda^n)^{-1}x, (R^n)^{(\lambda^n)^{-1}}, x)$.

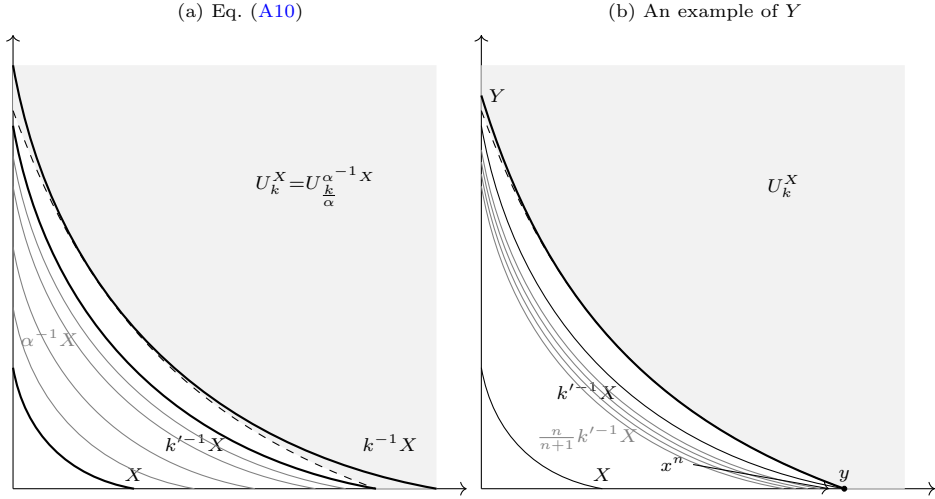
Therefore, $\lim_{n \rightarrow \infty} W(x, R^n, y^n) = \lim_{n \rightarrow \infty} W((\lambda^n)^{-1}x, (R^n)^{(\lambda^n)^{-1}}, x)$.

By Cb, $W((\lambda^n)^{-1}x, (R^n)^{(\lambda^n)^{-1}}, x) \rightarrow 1$, so that $W(x, R^n, y^n) \rightarrow 1$, the desired outcome. $\square$

Before we prove Theorem 3, we state the following lemma, of which we omit the straightforward proof. It implies that we can use either Rescaling Across Consumption or Rescaling Across Reference in the last parts of Theorems 3 and 4.

Lemma 7. *W satisfies Rescaling Across Consumption (RAx) and Consumption Rescaling (Rx) if and only if it satisfies Reference Rescaling (Ry) and Rescaling Across Reference (RAy).*

Figure A4: Illustration of the proof of Theorem 3



Theorem 3

Proof It is easy to check that $W_{\max}$ satisfies the axioms. In the terminology of Lemma 5, $W_{\max}$ satisfies the axioms with $L_w^Y = L(wY)$ and $U_w^X = U(w^{-1}X)$.

Let us prove that if W satisfies Cb and Inf-y then $W = W_{\max}$ whenever reference is above consumption.

By Lemma 5 (D), for all $X \in \mathcal{I}^c$, $w \in \mathbb{R}_+$, $w \neq 1$, there exists closed and convex $U_w^X \subset \mathcal{X}$ such that for all $Y \in \mathcal{I}^c$ such that $X \cap Y = \emptyset$, $W(X, Y) \leq w \Leftrightarrow U(Y) \subseteq U_w^X$. Let us fix $X \in \mathcal{I}^c$.

We need to show that for all $w < 1$, $U_w^X = U(w^{-1}X)$. Assume not. Then there exists $k < 1$ such that

$$U_k^X \neq U(k^{-1}X). \quad (\text{A8})$$

Note that, by H, $W(X, k^{-1}X) = k$, so that by Lemma 5 (D), $U(k^{-1}X) \subseteq U_k^X$ and U_k^X is closed. Together with Eq. A8, this implies, using Cont, that there exists $k' > k$ such that

$$k'^{-1}X \asymp_B U_k^X. \quad (\text{A9})$$

By Rx,

$$\forall \alpha \in]1, k'[: U_{\frac{k}{\alpha}}^{\alpha^{-1}X} = U_k^X. \quad (\text{A10})$$

See Figure A4a for an example.

Let $y \in \mathcal{X}$ be such that $y \in L(k^{-1}X) \cap U_k^X$. There exists $Y \in \mathcal{I}^c$ such that: $U(Y) \subset U_k^X$; $U(\lambda Y) \not\subset U_k^X$, for all $\lambda < 1$; $k'^{-1}X \asymp_B Y$ and $k'^{-1}X \cap Y = \{y\}$. See Figure A4b for an example. Because $U(Y) \subset U_k^X$ and $y \in Y$, y being at the boundary of U_k^X , $W(X, Y) = k$. Therefore, by H and Rx,

$$W(\alpha^{-1}X, Y) = \frac{k}{\alpha}, \forall 1 < \alpha < k'. \quad (\text{A11})$$

Let sequence $(x^n, R^n)_{n \in \mathbb{N}} \in \mathcal{X} \times \mathcal{R}^c$ be defined by, for all $n \in \mathbb{N}$: $I(y, R^n) = Y$, $L(x^n, R^n) = L(\frac{n}{n+1}k'^{-1}X)$, $x^n = \frac{n}{n+1}y$.

Note that $W(x^n, R^n, y) = W(\frac{n}{n+1}k'^{-1}X, Y)$. By Eq. A11, $W(\frac{n}{n+1}k'^{-1}X, Y) = \frac{n}{n+1}\frac{k}{k'}$. Therefore, $W(x^n, R^n, y) \rightarrow \frac{k}{k'} < 1$, violating Cb. That proves that for all $w < 1$, $U_w^X = U(w^{-1}X)$.

The proof that if W satisfies Cb and Sup-x then $W = W_{max}$ whenever the reference is above consumption is similar to the proof above, provided we replace Cb with CRa, which is possible, by Lemma 6.

Finally, to prove the second statement of the theorem, we need to prove that the previous property holds for all $w > 1$. This amounts to prove that for all $Y, Y' \in \mathcal{I}^c$ such that $Y \succ^A w^{-1}X$ and $Y' \succ^A w^{-1}X$ for some $w > 1$, $W(X, Y) = W(X, Y')$. This follows immediately from the fact that it holds for $w < 1$ and W satisfies RAx (or RAY, given Lemma 7). $\square$

Theorem 4

Proof The proof is similar to the one of Theorem 3. For the sake of completeness, we prove here only one part of the second statement. Let W satisfy Sup-y and Ca. By Lemma 5 (B), for all $X \in \mathcal{I}^c$, $w \in \mathbb{R}_+$, $w \neq 1$, there exist $L_w^X \subset \mathcal{X}$ such that, for all $X \in \mathcal{I}^c$ such that $X \cap Y = \emptyset$, $W(X, Y) \geq w \iff L(Y) \subseteq L_w^X$. Remember that L_w^X is not necessarily compact.

Fix $X \in \mathcal{I}^c$. Observe that the statement is proven if the following statement is true. For all $w > 1$, if $L(Y) \subseteq L_w^X$ then $L(Y) \subseteq L(w^{-1}X)$. Notice that it must be $L(w^{-1}X) \subseteq L_w^X$. Assume that our claim is wrong. Then there exist $Y \in \mathcal{I}^c$ such that $L(Y) \subseteq L_w^X$ and $\text{int}(U(w^{-1}X)) \cap L(Y) \neq \emptyset$.

Since $L(Y)$ is compact, there exist $w' < w$ such that $w'^{-1}X \succ^A Y$. Let $y \in \mathcal{X}$ be such that $y \in Y \cap w'^{-1}X$ and $(x^n, R^n)_{n \in \mathbb{N}} \in \mathcal{X} \times \mathcal{R}^c$ be a sequence defined by, for all $n \in \mathbb{N}$: $I(y, R^n) = Y$, $L(x^n, R^n) = L(\frac{n+1}{n}w'^{-1}X)$, $x^n = \frac{n+1}{n}y$. Notice that $W(x^n, R^n, y) = W(\frac{n+1}{n}w'^{-1}X, Y)$. By Ca, $W(\frac{n+1}{n}w'^{-1}X, Y) \rightarrow 1 = W(w'^{-1}X, w'^{-1}X)$. Therefore, by Rx, $W(\frac{n+1}{n}X, Y) \rightarrow W(X, w'^{-1}X) = w'$: a contradiction to $L(Y) \subseteq L_w^X$. $\square$

A.6 Theorem 5

Before we prove Theorem 5, we need to modify Lemma 5 to account for the unrestricted preference domain. Let $\mathcal{I}$ denote the set of indifference curves belonging to some preference in $\mathcal{R}$.

Lemma 8. *Let $W : \mathcal{X} \times \mathcal{R} \times \mathcal{X} \rightarrow \mathbb{R}$ be a well-being measure.*

- (A) *If W satisfies Supremum Contour at Consumption (Sup-x) when yRx and Convergence from Below (Cb), then for all $Y \in \mathcal{I}$, $w \in \mathbb{R}_+$, $w < 1$, there exists $L_w^Y \subset \mathcal{X}$ such that for all $X \in \mathcal{I}^c$ such that $L(X) \cap U(Y) = \emptyset$, $W(X, Y) \leq w \iff L(X) \subseteq L_w^Y$.*
- (B) *If W satisfies Supremum Contour at Reference (Sup-y) when xRy and Convergence from Above (Ca), then for all $X \in \mathcal{I}^c$, $w \in \mathbb{R}_+$, $w \neq 1$, there exists $L_w^X \subset \mathcal{X}$ such that for all $Y \in \mathcal{I}^c$ such that $Y \cap X = \emptyset$, $W(X, Y) \geq w \iff L(Y) \subseteq L_w^X$.*
- (C) *If W satisfies Infimum Contour at Consumption (Inf-x) when xRy and Convergence from Above (Ca), then for all $Y \in \mathcal{I}$, $w \in \mathbb{R}_+$, $w > 1$, there exists a*

- closed and convex $U_w^Y \subset \mathcal{X}$ such that for all $X \in \mathcal{I}^c$ such that $U(X) \cap L(Y) = \emptyset, W(X, Y) \geq w \Leftrightarrow U(X) \subseteq U_w^Y$.
- (D) If W satisfies Infimum Contour at Reference (Inf-y) when xRy and Convergence from Below (Cb), then for all $X \in \mathcal{I}^c$, $w \in \mathbb{R}_+$, $w \neq 1$, there exists a closed and convex $U_w^X \subset \mathcal{X}$ such that for all $Y \in \mathcal{I}^c$ such that $Y \cap X = \emptyset$, $W(X, Y) \leq w \Leftrightarrow U(Y) \subseteq U_w^X$.

Proof of (A) Let $Y \in \mathcal{I}$ and $w \in \mathbb{R}_+$, $w < 1$. Let $\mathcal{I}_w^Y = \{X \in \mathcal{I} | W(X, Y) \leq w\}$ and let $L_w^Y \subset \mathcal{X}$ be defined by

$$L_w^Y = \bigcup_{X \in \mathcal{I}_w^Y} L(X).$$

Let $\bar{L}_w^Y \subset \mathcal{X}$ be the closure of L_w^Y . Observe that by construction $L_w^Y \cap U(Y) = \emptyset$, whereas it is not necessarily the case that $\bar{L}_w^Y \cap U(Y) = \emptyset$. The case in which $\bar{L}_w^Y \cap U(Y) \neq \emptyset$ is the difficult case, in which known proofs do not apply, and it was solved in Lemma 5 thanks to the compactness of all lower contours, a property we no longer have in $\mathcal{R}$.

Let $D \subset \mathcal{X}$ be a countable and dense subset of $\bar{L}_w^Y$ such that $D \subset L_w^Y$. We write $D = \{x_n, n \in \mathbb{N}\}$. Let a sequence $\{L_n, n \in \mathbb{N}\}$ be such that for all $n \in \mathbb{N}$, $x_n \in L_n$ and $L_n = L(I_n)$ for some $I_n \in \mathcal{I}_w^Y$. Let the sequence $\{K_n, n \in \mathbb{N}\}$ be defined by $K_0 = L_0$ and $K_n = L_n \cup K_{n-1}$. By Sup-x,

$$W(\partial K_n, Y) \leq w, \tag{A12}$$

where ∂K denotes the upper frontier of K . Observe that

$$\lim_{n \rightarrow \infty} K_n = \bar{L}_w^Y.$$

Indeed, if it were not true, then there would be $x \in D$ such that $x \notin K_n$ for all $n \in \mathbb{N}$, a contradiction.

We have to distinguish between two cases.

Case 1: $\bar{L}_w^Y \cap U(Y) \neq \emptyset$. Let $z \in \bar{L}_w^Y \cap U(Y)$. Let $R_n \in \mathcal{R}$ be such that $\partial K_n, Y \in R_n$. Such a R_n exists for all $n \in \mathbb{N}$ because $K_n \subseteq L_w^Y$ and $L_w^Y \cap U(Y) = \emptyset$. Let the sequence $\{z_n \in \mathcal{X}, n \in \mathbb{N}\}$ be defined by $z_n = \alpha_n z$ for some $\alpha_n \geq 0$ and $z_n \in \partial K_n$. Clearly, $\lim_n z_n = z$. By (A12), $W(z_n, R_n, z) \leq w$ for all $n \in \mathbb{N}$, violating Cb.

Case 2: $\bar{L}_w^Y \cap U(Y) = \emptyset$. Let $X_w^Y = \partial \bar{L}_w^Y$. The proof that $W(X_w^Y, Y) = w$ can follow some existing proof, for instance the one developed in Fleurbaey and Maniquet (2017a), from which we derive that $W(X, Y) \leq w$ for all $X \subset L_w^Y$ by Mx and Cont. $\square$

Proof of (C) The proof is similar to the proof of (A) above and is therefore omitted. $\square$

Proof of (B) and (D) They follow from the proofs of (A) and (C) above in the same way as claims (B) and (D) follow from (A) and (C) in Lemma 5. $\square$

Theorem 5

Proof Statement 1) is proven in the proofs of statements 1) of Theorems 3 and 4 for the $\mathcal{R}^c$ domain. The simple proof that it is valid over the entire domain $\mathcal{R}$ is omitted. It mimicks proofs of similar claims in, for instance, Fleurbaey and Maniquet (2017b).

Statement 2) follows from the proofs of statement 2) of Theorems 3 and 4 when Lemma 5 is replaced with Lemma 8. $\square$

Appendix B The necessities-equivalent well-being measure

Let us assume that one good, say good 1, gathers all the necessities so that no bundle not containing good 1 can be better than a bundle with a positive amount of necessities. With a slight generalization of this idea, let us define the domain of preferences having the property that all indifference contours have a non-empty intersection with the axis of good 1: we say that $R \in \mathcal{R}^1$ if and only if for all $x \in \mathcal{X}$, there exists $z^1 \in \mathcal{X}$ such that $I(x, R) \cap A^1 = \{z^1\}$.¹⁹ Observe that $\mathcal{R}^c \subset \mathcal{R}^1 \subset \mathcal{R}$.

Definition 5. *The necessities-equivalent well-being measure is the function $W_{nec} : \mathcal{X} \times \mathcal{R}^1 \times \mathcal{X} \rightarrow \mathbb{R}_+$ such that, for all $(x, R, y) \in \mathcal{X} \times \mathcal{R}^1 \times \mathcal{X}$,*

$$W_{nec}(x, R, y) = \frac{a^1(I(x, R))}{a^1(I(y, R))}$$

Following the axiomatic analysis in Section 5, we obtain the characterization of W_{nec} - referred to as W_1 in the main text - as a corollary of the axiomatic characterization of W_k .

Corollary 1 (of Theorem 2). *1) Well-being measure W_{nec} satisfies Supremum Contour at Consumption (Sup-x), Supremum Contour at Reference (Sup-y), Infimum Contour at Consumption (Inf-x) and Infimum Contour at Reference (Inf-y) over the domain $\mathcal{X} \times \mathcal{R}^1 \times \mathcal{X}$.*

2) Conversely, if a well-being measure $W : \mathcal{X} \times \mathcal{R}^1 \times \mathcal{X} \rightarrow \mathbb{R}$ satisfies any three axioms among Supremum Contour at Consumption (Sup-x), Supremum Contour at Reference (Sup-y), Infimum Contour at Consumption (Inf-x) and Infimum Contour at Reference (Inf-y), then $W = W_{nec}$.

Proof It is routine to check that W_{nec} satisfies the axioms over $\mathcal{R}^1$ for $L_w^Y, L_w^X, U_w^Y, U_w^X$ as defined in the proof of Theorem 2 for $k = 1$.

Statement 2) immediately follows from Theorem 2, which proves that $W = W_k$ for some k , and the fact that $k = 1$ is the only value of k that gives a well-defined measure. $\square$

Given that $\mathcal{R}^1$ seems to us a more natural domain than $\mathcal{R}^c$, and given that W_{nec} is the only well-being measure satisfying the axioms over that domain – whereas there are K W_k measures – we consider that W_{nec} is more important than W_k , especially in view of empirical applications.

¹⁹Following the notation introduced in Section 3, A^1 denotes the good 1 axis. Formally, $x \in A^1 \Leftrightarrow x_\ell = 0, \forall \ell \in \{2, \dots, K\}$.

Appendix C Extensions

In our analysis, a special role is played by homothetic preferences. The postulate is that when preferences are homothetic, we *know* how to measure well-being in some situations: if the reference bundle is a multiple of the consumption bundle, then well-being should only depend on this multiple. This is strongly related to the situations to which this theory is supposed to be applied: all goods should be considered equally important, no distinction can be made across goods.

The theory, however, easily generalizes to other situations and models. The most obvious alternative is the one in which one good plays a particular role so that the most natural preferences are quasi-linear preferences. Let us assume that good 1 is this special good, the consumption of which can be unboundedly negative or positive (and we redefine $\mathcal{X}$ accordingly). The domain of quasi-linear preferences is denoted $\mathcal{R}^{ql}$, and for $R \in \mathcal{R}^{ql}$, we use utility function u to represent R provided there exists some $v : \mathbb{R}_+^{K \setminus \{1\}} \rightarrow \mathbb{R}$ such that $u(x) = v(x_{-1}) + x_1$, where x_{-1} is obtained from x after removing its first component. In this new model, we need to replace our axiom of Homotheticity with the following axiom of Quasi Linearity: when we compare two situations in which the reference bundle is equal to the consumption bundle except for the quantity of good 1, then well-being should be the same as soon as preferences are quasi linear:

Quasi Linearity (QL) - For all $x = (x_1, x_{-1})$ and $\lambda \in \mathbb{R}$, if $R, R' \in \mathcal{R}^{ql}$, then

$$W(x, R, (x_1 + \lambda, x_{-1})) = W(x, R', (x_1 + \lambda, x_{-1})).$$

With this new axiom, all the reasonings that are developed in the paper remain valid provided homothetic transforms are replaced with translation along the good 1-axis. For instance, the analogue of Deaton's measure is defined as follows: the well-being of an agent is measured by the distance by which reference bundle y has to be translated to become indifferent to x :

$$W_y(x, R, y) = \omega \Leftrightarrow (y_1 - \omega, y_{-1}) \in I(x, R).$$

The other measures require to define the quasi-linear preferences obtained by translation of a given indifference contour. By analogy with what we write in the main text, let $R_{I(x, R)} \in \mathcal{R}^{ql}$ denote the quasi-linear preferences obtained by translation of the indifference contour at x for preferences R . Then, we can rewrite the variant of $W_{\max}$, for instance, as

$$W_{\max}(x, R, y) = u(x) - \min_{y' \in U(y, R)} u(y'),$$

with u representing quasi-linear preferences $R_{I(x, R)}$.

All our results can be adjusted to this modified model provided definitions and proofs are adjusted accordingly.

Finally, let us come back to our assumption that well-being is decreasing in reference and see how this assumption can be relaxed. All models consistent with observed choices in experiments have the property that well-being is *increasing* in reference

consumption in situations in which consumption is preferred over reference. This violates our axiom of Reference Preference, which would then need to be reformulated so as to allow for opposite effect of changes in reference consumption as a function of the relative preference towards consumption and reference:

Reference Preference' (P'y): for all $x, y, y' \in \mathcal{X}$ and $R \in \mathcal{R}^c$, if yIy' then $W(x, R, y) = W(x, R, y')$, if $yPy'Px$ then $W(x, R, y) < W(x, R, y')$, and if $xPyPy'$ then $W(x, R, y) > W(x, R, y')$.

This would then require to modify the definition of our well-being bound axioms when they apply to reference consumption, by distinguishing between how well-being measures should behave above and below the reference consumption. For instance, Infimum Contour at Reference can be redefined as follows.

Infimum Contour at Reference' (Inf'-y) - For all $x, y, y' \in \mathcal{X}$ and $R, R', R'' \in \mathcal{R}^c$, if $U(y, R) = \text{ch } [U(y', R') \cup U(y', R'')]$ and $I(x, R) = I(x, R') = I(x, R'')$, then

- if yRx : $W(x, R, y) = \max \{W(x, R', y'), W(x, R'', y')\}$,
- if xRy : $W(x, R, y) = \min \{W(x, R', y'), W(x, R'', y')\}$.

Many new measures satisfy the axioms. For instance, the following measure satisfies Supremum Contour at Consumption (Sup-x) and Infimum Contour at Reference' (Inf'-y). It coincides with $W_{\max}$ when yRx , and it works as follows when xPy : there is a strictly positive bundle, $r \in \mathbb{R}_{++}^K$, and a strictly positive price vector, $p \in \mathbb{R}_{++}^K$, bundle x is evaluated by computing the multiple of r that is indifferent to x , y is evaluated by the equivalent income, at price vector p , expressed as a multiple of r , and well-being is the ratio of the evaluation of x over that of y .

Definition 6. *The r, p well-being measure is the function $W_{r,p} : \mathcal{X} \times \mathcal{R}^c \times \mathcal{X} \rightarrow \mathbb{R}_+$ such that, for all $(x, R, y) \in \mathcal{X} \times \mathcal{R}^c \times \mathcal{X}$,*

- if yRx : $W_{r,p} = W_{\max}$,
- if xPy : $W_{r,p}(x, R, y) = \frac{e(x)}{e(y)}$, where $e(x) = e \Leftrightarrow xIer$ and $e(y) = e \Leftrightarrow I(y, R) \succ^A \{y' \in \mathcal{X} | py' = er\}$.

Somehow surprisingly, the basic axiom that is violated by this measure and that is the most difficult to combine with the new axioms is Normalization, the requirement that $W(y, R, y) = 1$. Indeed, to be consistent with the experimental literature on other-regarding preferences, one would need the property that as soon as consumption is preferred over reference, well-being is increasing in consumption independently of the reference: $W(x, R, y) \geq W(x', R, y')$ if and only if xRx' . If one believes that normative analysis should be consistent with this experimental evidence, then Normalization should be dropped, and our analysis should completely change.

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