



UNIVERSITE CATHOLIQUE
DE LOUVAIN
ECOLE POLYTECHNIQUE DE LOUVAIN
INSTITUTE OF MECHANICS, MATERIALS AND
CIVIL ENGINEERING

Experimental and modeling investigations on droplet entrainment in a PWR hot leg under stratified flow conditions

FRANCOIS HENRY

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Jury Members:

Prof. Y. Bartosiewicz (Supervisor)	Université catholique de Louvain
Ir. M. Valette (Supervisor)	CEA
Dr. L. Rossi	CEA
Prof. M. Giot	Université catholique de Louvain
Prof. H. Anglart	KTH
Prof. H. Jeanmart (Chair)	Université catholique de Louvain

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Extended summary

Today, the nuclear energy has to meet the challenge of the nuclear safety. In this context, the ability to predict with precision the behaviour of a PWR nuclear reactor undergoing a severe accident is a key parameter. However, the flows in the primary circuit during a LOCA are so complex and transient that the complete simulation of the flow behaviour during an accident is currently unachievable. The loss-of-coolant accident in which a breach appears in the primary circuit leading to the depressurization of the nuclear reactor core, particularly emphasizes this flow complexity. Indeed, as a part of the coolant water leaves the primary circuit, the pressure decreases so that the liquid coolant begins to vaporize at the hottest locations of the nuclear reactor core. The flow to simulate to completely predict the nuclear reactor behaviour is therefore multiphase in a very complex geometry.

As a complete simulation of the multiphase flows is unachievable, the behaviour of the primary circuit during a severe accident is currently approached by under-resolved simulations. Thereby, some system codes, like CATHARE or RELAP, aim at modeling the two-phase flow behaviour at a system scale by 1D simulations using the two-fluid governing equations. However, the continuous increase in computational capacity of the computers allows today to enhance the simulation predictions. The CATHARE system code, which simulates the transient two-phase flows occurring during a nuclear accident, is thus currently improved to enable the simulation of a third dispersed phase, either droplets or bubbles. Indeed, the behaviour of these dispersed bubbles and droplets interferes in many transient phenomena occurring during a nuclear accident, so that modeling their dispersions greatly improves the code accuracy. Consequently, increasing the number of governing equations involves the modeling of some additional closure laws, representing the exchange of mass, momentum and energy with the new phase.

Within this framework, this work presents the development of the mass ex-

change term of between the two liquid phases, the liquid film and the droplets, in the particular case of the horizontal stratified flow that may occur in the PWR hot leg. However, a bibliographic review has shown that the entrainment phenomenon remains an understudied field in horizontal pipes. Indeed, most studies focused on vertical pipes of small diameters, so that actual models for the droplet entrainment are not relevant for pipes of the hot leg scale and geometry. Consequently, a new model for the droplet entrainment from a wavy stratified flow in horizontal pipe has been developed in this work in order to predict from a mechanistic point of view the quantity of droplets ripped-off from the liquid film. Otherwise, the literature review also revealed that the existing experimental database do not suit to the assessment of any entrainment model at the PWR hot leg scale. A new experimental facility has therefore been designed and operated in this work to provide the required database of the evolution of the entrainment rate in wavy stratified flow conditions.

The new experimental facility highly differs from the existing facilities by its unusual size ; the pipe diameter is thrice greater than any other experimental facilities while the pipe length is rather small in order to accurately represent a hot leg geometry at a 1/3 scale. This geometry enables the study of the flow axial development and thus, the measurement of the droplet entrainment rate, required for the model assessment. The model approach is to assume the droplet entrainment occurring due to the development and growth of interfacial waves until their break-up. The model relies thus on a mechanistic description of the wave development and fragmentation. Consequently, the evolution of the liquid film interface is a key parameter to investigate within the new experimental facility. Additionally to the evolution of the droplet concentration, the wave characteristics have thus been extensively studied in this work. The key parameters (either the flow conditions, the two-fluid physical properties or the pipe geometry) affecting each step of the phenomenological sequence leading to the wave atomization into droplets, have been highlighted during the experimental campaign.

The proposed model considers thus the droplet entrainment as occurring from the fragmentation of the waves generated at the interface of the stratified flow. The model depends then on a accurate prediction of the interfacial wave geometry (wavelength, amplitude) and of the wave crest fragmentation. The instability wavelength is determined throughout a linear stability analysis of the two-fluid stratified flow while the wave amplitude and the amount of liquid wrested from the wave crest are determined using two force balances on the wave crests.

The linear stability analysis is thus performed on the two-fluid equations and the shear stress closure laws described by Tzotzi and Andritsos [154] and compared to the closure laws of Barnea and Taitel [20], both considered as the most relevant for the modeling of a stratified flow. A dispersion equation is thus achieved, representing the amplification spectrum of the instabilities developing at the interface. The model relies on the assumption that only the most amplified instability develops and grows over the interface, so that the amplification spectrum can be reduced to an unique singularity.

The amount of liquid wrested-off from the liquid film and entrained as droplets is thus determined through two force balances: a first force balance on the wave crest involves the gravity, a lift force and the surface tension. The wave fragmentation is assumed to occur when the force resultant passes from a stabilizing effect to a destabilizing effect. Applying this force balance provides a relation between the wave amplitude and the amount of liquid that is entrained at this wave amplitude. The second force balance is applied to this amount of entrained liquid and involves the gravity and a drag force resulting from the turbulent effects in the gas core. This force balance explicit the capacity of the gas flow to entrain the amount liquid ripped-off from the wave crest. Finally, the model assumes that the wave fragment at the amplitude where the two force balances pass simultaneously to a destabilizing effect ; so, at the amplitude at which the gas core is able to entrain the amount of liquid ripped-off from the wave crest. The characteristic time of the entrainment corresponds to the necessary time for the growing instability to reach the unstable amplitude defined by the two force balances. This parameter has been experimentally derived from the facility data.

The entrainment rate model appears to fit well the experimental data, so that the proposed model is relevant for the pipe of large diameters. The sensitivity study of every parameters conducted on the entrainment parameters, expands the validity of the final proposed correlation on a large range of flow conditions, pipe geometry and two-fluid physical properties.

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List of symbols

Entrainment parameters		
E	Entrained Fraction	—
Γ_E	Entrainment rate	$kg\ m^{-2}\ s^{-1}$
Greeks		
α	Volume Fraction	—
β	Deformation factor (Eq. 1.68)	—
β	Correction factor (Eq. 3.8)	—
γ	Angle	rad
Γ_C	Critical flow in the wall film (Eq. 1.60)	$kg\ m^{-1}\ s^{-1}$
δ	Film thickness	m
λ	Wavelength	m
μ	Viscosity	$kg\ m^{-1}\ s^{-1}$
ρ	Density	$kg\ m^{-3}$
σ	Surface tension	$kg\ s^{-2}$
τ	Shear stress	$kg\ m^{-1}\ s^{-2}$
τ_C	Characteristic time	s
ϕ	Relative humidity	—
ϕ	Angular phase shift	rad
Φ	Source Term	/
Ψ	Conserved quantity	/
Latins		
A	Cross-section	m^2
C	Droplet concentration	$kg\ m^{-2}\ s^{-1}$
C_D	Drag Coefficient	—
d	Droplet Diameter ($= d_{10}$)	μm
d_{05}	Volume Median Diameter	μm
d_{32}	Droplet Sauter Diameter	μm

Latins		
D	Pipe Diameter	m
E	Instability amplitude (Eq. 4.41)	—
f	Friction factor	—
$\mathbf{g}$	Gravity	$m\,s^{-2}$
h	Liquid film height	m
h_0	Height	m
j	Superficial velocity	$m\,s^{-1}$
J	Stress Tensor	/
k	Coefficient	—
L	Length	m
L_A	Characteristic Length (Eq. 1.57)	m
$\dot{M}$	Mass flow-rate	$kg\,s^{-1}$
$\dot{m}$	Surface mass flow-rate	$kg\,s^{-1}\,m^{-2}$
m	Correlating parameters (Eq. 1.61)	—
n	Correlating parameters (Eq. 1.61)	—
$N_{t,d}$	Total number of droplets	—
P	Pressure	bar
P_i	Interfacial Pressure	bar
p	Pressure	bar
p'_v	Saturation pressure	bar
S	Surface	m^2
t	Time	s
T	Stress Tensor	—
U	Instability amplitude (Eq. 4.42)	—
v	Velocity	$m\,s^{-1}$
$\dot{V}$	Volume flow rate	$m^3\,s^{-1}$
$\dot{V}_N$	Normal Volume flow rate	$m_N^3\,h^{-1}$
V_E	Entrained volume	m^3
x	Absolute humidity	$kg_{\text{water}}/kg_{\text{dry air}}$
x_0	Inlet Absolute humidity	$kg_{\text{water}}/kg_{\text{dry air}}$
x	x-axis	—
y	y-axis	—
z	z-axis	—
Z	Impedance	Ω^{-1}
Non-dimensional numbers		
Fr	Froude Number	—
L/D	Not-dimensioned length	—
Re	Reynolds Number	—
We	Weber Number	—

Superscript		
L^*	Saturation Length	m
Subscripts		
c	Critical	
C	Condensation	
d	Liquid droplet	
D	Deposition	
E	Entrainment	
f	Liquid film	
fr	Friction	
g	Gas	
g	Gravity center	
$gr.$	Growth	
i	Interfacial	
I	Imaginary	
k	Phase	
l	Liquid	
p	Peak of distribution	
p	Perturbation	
R	Real	
t	total sum over the pipe cross section	
T	Total	
V	Vaporization	
w	Wall (shear stresses)	
w	Wave (velocity)	
30	Volume average	
∞	Fully developed and permanent	
Mathematics		
$\overline{var}$	Time averaged operator	
$< var >$	Space averaged operator	
$\int var dn$	Space integral operator	

Chapter 1

Introduction

On 11 March 2011, an earthquake off the coast of Japan caused a tsunami that occasioned the greatest nuclear disaster post-Chernobyl over the world: the Fukushima Daiichi nuclear accident. The tsunami wave overwhelmed the protective walls and damaged several nuclear reactors. The backup systems were destroyed and the cooling of the fuel pools stopped. In order to prevent a significant slackening of radioactive particles in the air, astronomical amounts of water were used to cool down the reactors and the fuel pools. As a consequence the evacuation of this water caused a large marine pollution.

In a world where climatic and environmental issues are questioning the energy policies every day, this disaster highlighted the main challenges that the nuclear industry has to overcome to keep nuclear energy as one of the energies of the future.

1.1 General context

1.1.1 The nuclear energy

At the end of 2013, 434 commercial nuclear power reactors¹ were operating all over the world providing 12.98 % of the world's production of electric energy [7]. Factually, nuclear power is the fourth resource used to produce electricity after coal (41 %), natural gas (21.3 %) and hydroelectric power (15.9 %). According to the International Energy Agency (IEA) and only taking into account policies already formally adopted and/or implemented before the accident of Fukushima, the proportion of nuclear energy in the total electricity production

¹Must be added to the 434 reactors in operation around the world for the production of electrical energy, all the nuclear reactors used for medical, military or research applications.

was expected to decrease to 11 % in 2030 [3, 2, 5].

But this world power distribution is not at all representative of the European situation. Those 440 nuclear reactors are indeed mainly dispersed in three regions of the world: North-America, Europe and Japan. In Western Europe, 131 reactor units provide 37.07 % of the production of electrical energy and nuclear energy is therefore the main source of electricity. Throughout the world, only three countries depend on the nuclear energy for more than 50 % of their energy production: **France** (74.12 %), Slovakia (51.80 %) and **Belgium** (51.16 %).

Even if the prospects of construction of new nuclear plants in Europe are far away from the prospects in China, India or Russia, this emphasizes the significant importance of nuclear power over the European energy market. However the energy production by nuclear power facilities is questioned almost everywhere in Europe. Indeed, this technology has to face two major challenges inherent to the operation of current nuclear reactors: the waste management and the reduction of the risk of accidents.

1.1.2 Facing the challenges of the nuclear power production.

The use of a nuclear fuel involves the production of radioactive fission products. Due to the slow decay of these fission products and their volatility, the nuclear energy produced today involves serious risks of contamination or irradiation for tomorrow. Whether a short-term nuclear accident or a leakage of radioactive waste in decades is considered, the use of nuclear energy involves important hazards to human health and environment.

Minimizing the risk of accident is a significant short-term problem as shown by the repeated severe accidents occurred in NPP's for the last forty years [6] : Sellafield (1957), Three Miles Island (1979), Tchernobyl (1989) and Fukushima (2011).

These four severe accidents have involved seven nuclear reactors. To characterize the consequences of the incidents/accidents, the International Nuclear Event Scale (INES) was introduced in 1990 to enable prompt communication of safety-significant information in case of nuclear accidents [5]. As the scale is intended to be logarithmic, each increasing level (from 0 to 7) represents an accident approximately ten times more severe than the previous level as depicted on Figure 1.1.

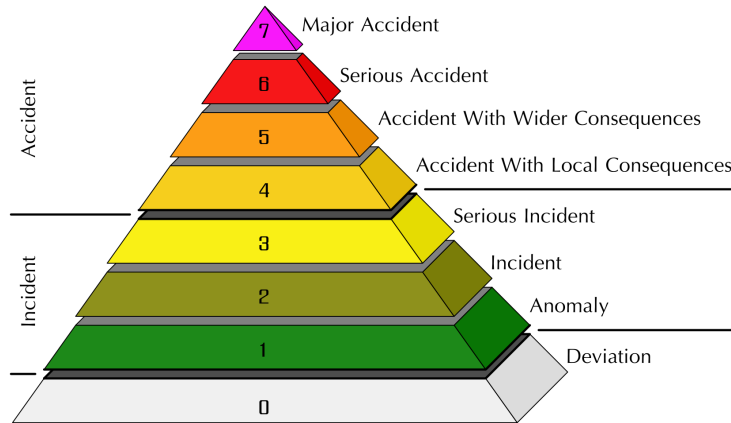


Figure 1.1: International Nuclear Event Scale.

Among these four accidents, the International Atomic Energy Agency (IAEA) has considered two of them as major accidents, ranked on Level 7, the highest level, on the International Nuclear Event Scale: Tchernobyl and Fukushima. Reaching this level implies major release of radioactive material with widespread health and environmental effects. The two others (Sellafield and TMI) are ranked Level 5 as accidents with wide consequences, corresponding to a severe damage to the reactor core but with a limited release of radioactive materials.

Since the start-up of the first nuclear reactor in 1942, 4 more nuclear accidents on non commercial nuclear reactors were worldwide reported, affecting one reprocessing plant for nuclear fuel (Level 6 at Kychtym in 1957), one nuclear laboratory in Canada (Level 5 at Chalk River in 1952), another nuclear laboratory in Switzerland (Level 5 at Lucens in 1969) and one medical installation (Level 5 in Goiânia in 1987). Aside those 8 accidents and not including military data, every other problems have been ranked as accidents Level 4 or incidents involving no release of radioactive materials.

In 2008, while 440 reactors were operating in the world, 125 had already been definitely shut down. The involvement of seven reactors in the four major accidents implies that 1.25% of the worldwide reactors have been implicated, damaged and destroyed by an accident while 0.8 % have been involved in a major accident leading to a dissemination of radioactive particles and endangering the population. In 2008, the industrial nuclear reactors in the world had worked for 14203 cumulative years. A reactor had then been subjected to a severe accident every 3500 years of operation, far away from the expected annual

frequency of 10^{-6} severe accidents per year and per reactor. The importance of accident risks requires their detailed studies in order to improve the safety of nuclear power plants and prevent further accidents.

A major accident occurs as a result of a cascade of reactor component malfunctions or damages with in addition the failure of safety systems to operate as expected. Damages propagate to finally cause the nuclear core melting. In this context, the ability to predict the reactor behavior, firstly in standard operating conditions and secondly after any malfunction occurs is of primary importance. However, this issue requires the knowledge of every physical phenomenon involved in any accident scenario but also the possible responses or malfunctions of any nuclear reactor components (safety systems, nuclear core, cooling systems...). Similarly to any extremely complex installation, the physical phenomena involved in any nuclear accident scenario are so numerous, complicated and non linear that the analytical prediction of the reactor response to an accident is not achievable. The reactor response must then be approached either experimentally or numerically.

1.1.3 Pressurized Water Reactor technology

In Europe, most of nuclear power plants are of PWR type. In order to produce electricity with a steam turbine, the PWR nuclear power plant is divided in two circuits: a primary circuit in which energy is converted from nuclear fission to heat and a secondary circuit in which this energy is converted from heat in electricity, as described in Figure 1.2. A third circuit permits to release the energy not transformable to electricity in the nature. Unlike BWR (Boiling water reactor), the fluid of the primary circuit continuously remains liquid.

The primary circuit is composed of the nuclear core connected to 2 to 4 loops. Each loop consists in a steam generator (i), a pump (j) and two connections to the core named hot (outlet) (h) and cold (inlet) (n) legs (depending on the local temperature) as depicted on Figure 1.3.

The production of energy results from a fission reaction of Uranium particles contained in a set of fuel rods in the nuclear reactor core. The division of a particle by fission generates two or three fission nuclei and a great amount of energy. In order to prevent the rods from melting, this generated heat has to be continuously removed from the core. Then a flow of liquid coolant goes along every fuel rod to maintain the temperature. This nuclear reactor coolant is pressurized to avoid any vaporization in the primary circuit.

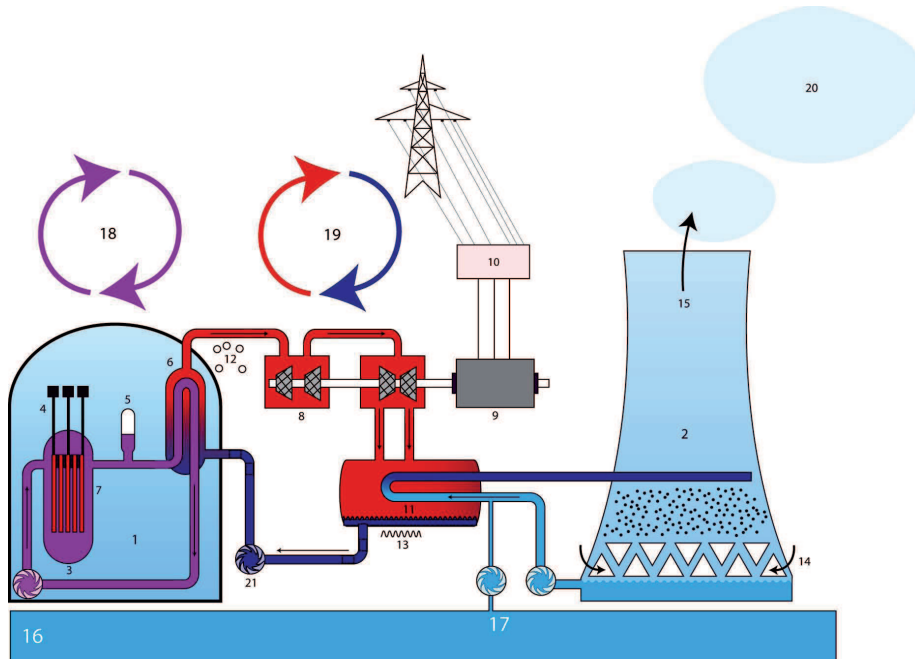


Figure 1.2: General diagram of a nuclear power plant where the color scale stands for the temperature and state of the water: secondary liquid water (dark blue), secondary steam water (red), primary pressurized liquid water (violet). Most component of the nuclear power plant are represented: Nuclear reactor block (1), cooling tower (2), Reactor (3), Control rods (4), Pressurizer (5), Steam generator (6), Nuclear vessel (7), Steam Turbine (8), Generator (9), Transformer (10), Condenser (11), Steam formation (12), Liquid condensation (13), air (14), humid air (15), River (16), Cooling water (17), Primary circuit (18), Secondary circuit (19), Saturated air cloud (20), Secondary pump (21).

The coolant flow goes down through the downcomer and enters the core by the bottom in the lower plenum (c) and rises to the upper plenum (e) as shown in Figure 1.3, evacuating the heat generated in the fuel rods. The flow separates then in hot legs (h). The pressurized water coolant at high temperature flows through the hot legs to reach the steam generator (i). In this element, the fluid of the primary circuit transfers its energy to the secondary circuit. This heat exchange reduces the temperature of the primary coolant by vaporizing the secondary fluid. The coolant returns then to the nuclear core through the cold leg (n) to loop the primary circuit. In order to maintain the pressurization and the flow circulation by compensating pressure losses, a pump (j) is placed between the steam generator and the cold leg. Table 1.1 summarizes the core characteristics.

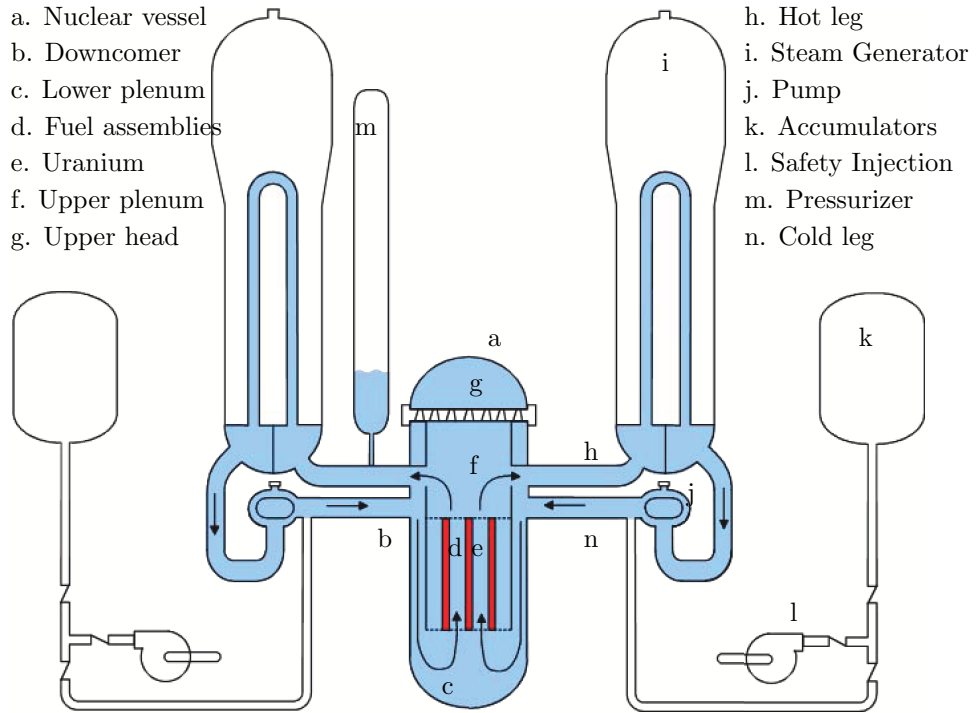


Figure 1.3: General diagram of the primary circuit of a nuclear reactor.

In the secondary circuit, the steam flows to a steam turbine (8) generator (9) which converts thermal energy into mechanical and then electrical energy. The saturated vapor exiting the turbine is then condensed (11) and pumped to the steam generator (21), closing the secondary circuit.

PWR characteristics	
Core pressure	155 bar
Core inlet temperature	286 °C
Core outlet temperature	323 °C
Mass flow	$\pm 60\,000\text{ m}^3/\text{h}$
Electric power	900 MW
Fuel	Enriched Uranium oxide

Table 1.1: General characteristics of a PWR.

A pressurizer is installed in one hot leg in order to compensate the pressure variations that can occur in the primary circuit during normal operation. This vessel contains a large amount of water and serves also as a first emergency

system. The accumulators and powered safety injections consist in the second emergency system for injecting water in the primary circuit. These components contain a large amount of water that can be discharged in order to cool down the reactor.

These safety systems have been sized to respond to the most dangerous accident that serves as reference scenario: the Large Break Loss of Coolant Accident (LBLOCA) in which a double ended break (guillotine) appears in the cold leg of the primary circuit leading to the loss of the primary circuit fluid. Indeed the consequences of this accident count among the worst that can happen to a nuclear reactor as they lead to its complete depressurization, and in case of security failure, to the melting of the reactor core. In the worst case, this accident can finally lead to the reactor vessel destruction and in case of a containment failure to the release of radioactive particles into the atmosphere.

1.1.4 General response to a LBLOCA accident

The only reported occurrence of a Large Break LOCA is the partial core meltdown of an experimental test reactor in Lucens in 1969 (Switzerland) installed in a cavern. This accident has been ranked level 5 on INES scale. During a reactor startup, the occurrence of a leak led to a LOCA. The core meltdown massively contaminated the cavern, which has been sealed since then.

Safety systems have been dimensioned to meet the acceptance criteria of the U.S. NRC, set in 1973 and so avoid the melting of the reactor core in accident situation. Five criteria guarantee that the oxidation rate of fuel cladding remains under 17% so that the reactor core remains coolable. But the most important criterion is that the clad temperature must always remain below 1204°C in order to avoid exothermic oxidation reaction between the water and the zirconium fuel cladding. Indeed the melting and the oxidation of the zirconium lead to a generation of hydrogen, a highly explosive gas. But also, as this reaction is exothermic, this step significantly increases the cladding temperature and amplifies the accident. The safety systems have to cool down the nuclear reactor to prevent the temperature to reach this limit of 1204°C .

A LBLOCA appears as a result of a double ended break on one leg connecting the reactor core to a steam generator, inducing a large leak of the primary coolant water. As soon as the reactor control parameters (temperatures, flow rates, neutron fluxes) start to deviate from the design values, the nuclear reactor emergency shutdown system stops the fission chain reaction by dropping the control rods. However, due to the slow radioactive decay of the fission products,

the nuclear fuel continues to generate a significant amount of heat. A reactor core shutdown from full power still produces about 5 to 6% of the initial heat which then decreases more or less exponentially as explained below. If the independent cooling trains of the Emergency core cooling system (ECCS) fail to operate as designed, this heat may increase the fuel temperature to a damaging point.

The LOCA on cold leg scenario is divided in three phases:

1. The blowdown phase.
2. The refill phase.
3. The core reflooding phase.

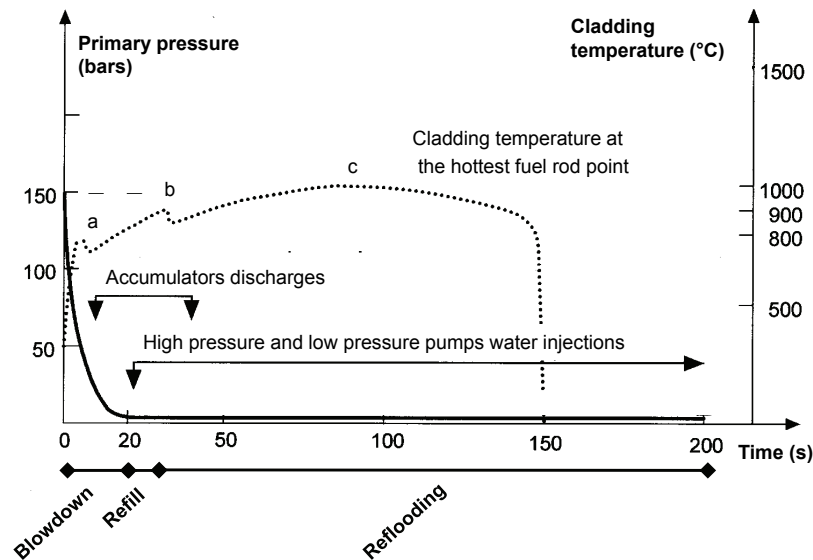


Figure 1.4: Time evolution of the clad temperature and the primary pressure during a LOCA accident.

Blowdown Phase

A break in a cold leg induces a leakage of the primary coolant. This loss of primary fluid causes the depressurization of the primary circuit. The pressure falls to reach the saturation pressure at the hotter locations of the circuit. The coolant begins to vaporize. In less than 1 second, the core power falls to 6%

of the nominal value. By vacuum effect in the reactor core due to the local coolant vaporization, the fuel cladding temperature rapidly increases to reach a first maximum at 800°C after 5 seconds (Point *a* on Figure 1.4). Indeed as the stagnation point is located in the core, the core cooling is very poor. After this 5 seconds, when the core pressure has decreased below the steam generator pressure, the stagnation point suddenly moves to the steam generator. Consequently the flow reverses in the hot legs (see Figure 1.5), the primary liquid falling from the upper plenum and the hot leg through the core. This effect cools down the fuel rod cladding for 5 to 6 seconds. As the depressurization continues, the opening pressure of the first safety system, the discharge of accumulators, is reached at about 11 seconds. At this moment, the mass of water still present in the primary circuit is already only 10% of the initial mass. In about 25 seconds, the pressure in the primary circuit will have totally fallen to reach an equilibrium with the containment pressure around 3 bar. As the stagnation point moves to the steam generators, the temperature drop reverses the heat transfer in the steam generators, enhancing flashing and boiling.

Refill Phase

As the water of the accumulators flows down from the cold legs into the downcomer ring, the refill phase begins 11 seconds after the break. The accumulators dump a huge amount of liquid (up to 1.4 t/s) in the primary circuit. As this liquid reaches the downcomer annulus, the greatest part directly by-passes the core region to reach the break. After a few seconds, the liquid progressively penetrates to the lower plenum facing a countercurrent flow of steam (due to the location of the stagnation point in the steam generators). The bottom of the reactor vessel is gradually filled such as the water level reaches the bottom of the fuel rods 30 to 35 seconds after the accident. At this moment, the accumulators are almost empty and the liquid injections in the primary circuit must then be ensured by the high pressure and low pressure pumps water injection. During this period, the temperature of the fuel clads rises nearly adiabatically until reaching 900°C (Point *b* on Figure 1.4).

Core reflooding Phase

At this time, the high pressure and low pressure pumps go into action and begin to continuously inject liquid water in the primary circuit. Once the liquid level has reached the bottom of the fuel clads in the downcomer, this liquid begins to vaporize due to the high fuel rod temperature. This vaporization cools down the coldest parts of the core which gradually re-wets. Consequently the rise of temperature in the clads stops (after 70 to 80 seconds, see Figure 1.4). The

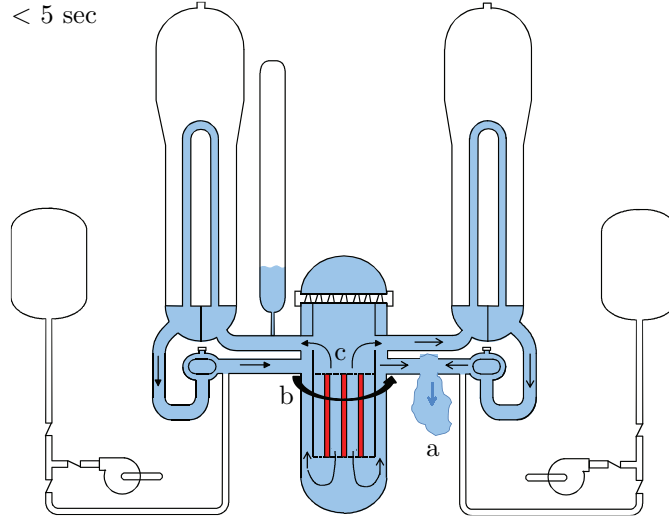
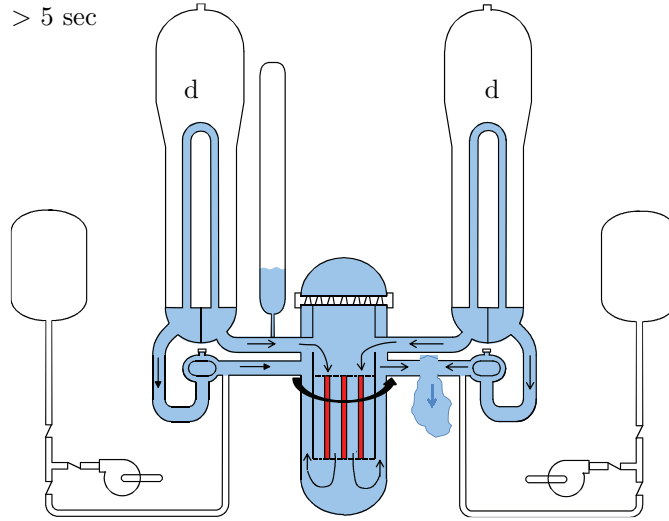
1) < 5 sec2) > 5 sec

Figure 1.5: Evolution of the stagnation position from the core (c) to the steam generators (d) during the bowndown phase following the double ended break (a). Highlights on the by-pass to the break (b) from the non-accidented loops.

vaporization induces a water flow to the upper plenum and the hot legs as depicted on Figure 1.6. As the core reflooding phase begins, the water level in the core undergoes some gravitary oscillations which act on the quantity of liquid still leaking through the breach. Indeed the vaporization blows droplets away from the core liquid-steam interface into the core, the upper plenum and the hot

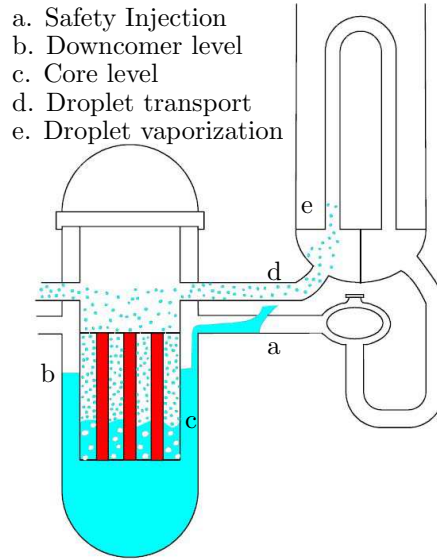


Figure 1.6: Diagram of one loop of a PWR during the flooding process.

legs. Entrained to these hotter regions, those droplets vaporize creating local over pressure. This local vaporization induces then a local flow reversal from core to downcomer and expels a part of the liquid to the upper plenum. As the flow reverses in the core, the liquid level in the downcomer annulus rises back and a part of the cooling liquid can be lost at the break. This local vaporization of droplets initiates the gravitary oscillations of the liquid level in the core. At each oscillation, droplets are expelled from the core to be entrained and vaporized in the core, the upper plenum, the hot legs and the steam generators. This raises the pressure level downstream of the core and maintains the oscillations. When these oscillations are progressively damped and finally shut down, the reflooding occurs with a progressive rise of the liquid quench front which lasts several minutes. The temperature of the fuel clads at the hotter locations rise continuously to a third maximum around 1000°C (Point C on Figure 1.4) until the approach of the quench front which decreases the local temperature before a sudden drop. The correct operation of the safety systems should completely cool down the nuclear core after 150 seconds in case of a LOCA occurrence and enable to meet all accident acceptance criteria.

Depending on the conditions prevailing in the primary circuit, the entrained droplets may vaporize, deposit on walls, form liquid films or be driven by the steam flow. Additional droplets are also entrained from the liquid film in the hot leg. A fraction of these droplets arrives then to the steam generators. The droplets vaporize at the contact of the hotter steam generator walls. These

local phase changes induce pressure losses in the steam generator. These pressure variations act directly on the pressure balance over the nuclear reactor. Consequently the local droplet vaporization acts on the liquid level balance, particularly on the gravitary oscillations in the core. Therefore the quantification of the droplets reaching the steam generators is necessary for improving the reflooding time prediction.

In this context, this work aims at presenting a new correlation for the prediction of the entrainment rate of droplets. In the particular case of the horizontal stratified flows, a mechanistic model for the droplet entrainment is proposed and validated on a new experimental facility.

1.2 A simulation tool for safety analysis [39]

The effectiveness of safety systems and the reactor response to an accident must be verified and validated. Since deliberately causing an accident in a 1/1 scale experimental facility is not conceivable, the complete knowledge of nuclear reactor accidents is experimentally inaccessible and simulations have to be carried out by using computer numerical codes, by applying analytical experiments or partially by using experimental facilities at reduced scales. In addition the complete simulation of nuclear reactor accidents requires to model and couple thermal-hydraulics and fuel neutronics.

Several solutions are available to model the heat transfer and fluid dynamics in a nuclear reactor. Fully simulate the fluid behaviour through each reactor part with Direct Numerical Simulation techniques (DNS) appears to be too complex and expensive. Simulations of lower turbulence resolution like LES are even too expensive to fully simulate the unsteady dynamics of a nuclear accident. Thus, the complete simulation of the primary fluid dynamics during an accident is still achieved with system scale codes (1D and/or partially 3D). A system code solves the conservation equations of mass, momentum and energy (or similar equations) at a system scale using predefined models for most underlying phenomena as interfacial friction terms, mass exchange or interfacial area.

1.2.1 A system code: CATHARE

Several system codes exist as RELAP or CATHARE. These codes simulate the behaviour of a 1D reactor model in near real time depending on the complexity of the computed meshes. In this work, we use the CATHARE code, acronym for Code for Analysis of THERmalhydraulics during an Accident of Reactor and safety Evaluation, jointly developed by CEA, IRSN, AREVA and EDF since 1979.

The CATHARE code was developed as a 1D two-fluid model to simulate the behaviour of two-phase flows. Numerous models have been implemented to model the exchange terms of mass, momentum or energy between the two phases or many other phenomena. Indeed every codes and models respect the fundamental law of conservation of mass of Lavoisier. This principle states that for any system closed to all transfers of matter and energy (both of which have mass), the mass of the system must remain constant over time. Applied to the continuum system that are the fluid mechanics, this principle implies that the rate at which mass enters a system is equal to the rate at which mass

leaves the system. This is formalized in Equations 1.1 that presents the two equations of mass present in the CATHARE code: one equation per state of the water. Left side of the equation describes acceleration, and is composed of time-dependent acceleration and time-independent convective effects. The right side of the equation is in effect a summation of body force, that are the exchange of mass between the two systems (vaporization of water or condensation of vapor). A significant feature of Cauchy equation and consequently all other continuum equations is the presence of convective acceleration: the effect of time-independent acceleration of a flow with respect to space. While individual fluid particles indeed experience time dependent-acceleration, the convective acceleration of the flow field is a spatial effect, one example being fluid speeding up in a nozzle [163]. In these equations, α is the volumic fraction, ρ the density, v the velocity and Γ a mass exchange rate per unit of interfacial area, since the subscripts l and g denotes the liquid and the gas and the subscripts C and V the condensation and the vaporization. A_i denotes the area of the interface between the continuous phases and V the volume of the control volume:

$$\begin{aligned} \frac{\partial}{\partial t} (\alpha_l \rho_l) + \frac{\partial}{\partial z} (\alpha_l \rho_l v_l) &= \left(\frac{A_i}{V} \right) (\Gamma_C - \Gamma_V) \\ \frac{\partial}{\partial t} (\alpha_g \rho_g) + \frac{\partial}{\partial z} (\alpha_g \rho_g v_g) &= \left(\frac{A_i}{V} \right) (\Gamma_V - \Gamma_C) \end{aligned} \quad (1.1)$$

The increase in computers performance on the one side and the need to increase the CATHARE code accuracy on the other side led to the development of a new multi-field version of the CATHARE code in recent years: CATHARE 3.

1.2.2 A new system code: CATHARE 3 [49]

Beside the computational improvement, the main attraction of the CATHARE 3 code is the transition from 2 to 3 fields: gas phase, continuous liquid phase and dispersed phase (liquid or gaseous). More flow interactions between continuous and dispersed phases, like droplets or bubbles, can therefore be considered. The accuracy of the code predictions is then greatly increased. However, increasing the number of fields in the conservation equations requires additional closure relationships. Indeed the exchange terms between the fields increase from 2 to 4 for each conservation equation (see Equations 1.2). The objective of this work is to provide a new mechanistic model for one of these new closure laws: the mass transfer of liquid from the liquid film phase to the droplet phase in an horizontal stratified flow. Indeed as a stratified flow with droplets may appear in the hot leg of a PWR during a LBLOCA, the modeling of the core response requires to quantify the repartition of the liquid phase between the droplets and the liquid film in order to improve the predictions of gravitary oscillations.

The hot leg droplets can indeed have several origins: pulled off from the core liquid interface, entrained from the internal liquid films of the upper plenum, entrained from the hot leg liquid film or generated by a droplet deposition impact on the hot leg liquid film. The modeling of the deposition phenomenon is developed in a parallel work by Neiss [108].

The meaning of the various terms is similar to the 2-phases formulation. However two additional terms of mass exchange appears with each new phase considered. The entrainment term appears in the mass conservation equations (Equations 1.2) as a droplet entrainment rate, which is defined as the amount of water pulled and driven by steam per units of time and surface: Γ_E . The deposition rate appears as Γ_D while the subscripts d and f respectively denotes the droplets and the liquid film.

$$\begin{aligned} \frac{\partial}{\partial t} (\alpha_f \rho_l) + \frac{\partial}{\partial z} (\alpha_f \rho_l v_f) &= \left(\frac{A_i}{V} \right) (\Gamma_{C,f} - \Gamma_{V,f} - \Gamma_E + \Gamma_D) \\ \frac{\partial}{\partial t} (\alpha_d \rho_l) + \frac{\partial}{\partial z} (\alpha_d \rho_l v_d) &= \left(\frac{A_i}{V} \right) (\Gamma_{C,d} - \Gamma_{V,d} + \Gamma_E - \Gamma_D) \\ \frac{\partial}{\partial t} (\alpha_g \rho_g) + \frac{\partial}{\partial z} (\alpha_g \rho_g v_g) &= \left(\frac{A_i}{V} \right) (\Gamma_{V,d} + \Gamma_{V,f} - \Gamma_{C,d} - \Gamma_{C,f}) \end{aligned} \quad (1.2)$$

The objective of this work is to propose a model to quantify the amount of droplets torn off from the surface of a liquid film in a stratified flow for a potential further implementation in the CATHARE 3 code. This information is indeed useful to improve the closure equations of the CATHARE 3 system code. In this context, a literature review was conducted to analyze the strengths and weaknesses of existing entrainment models.

1.3 Literature review

This section presents a review of the studies in the open literature on the droplet entrainment and the stratified flows. As a first point, the droplet entrainment in general in the open literature is briefly described. This review indicates that the droplet entrainment may be quantified by two parameters: the entrained fraction E and the entrainment rate Γ_E . The various models referring to these two parameters in the literature are extensively studied for stratified flows. Finally, a review of the experimental facilities dedicated to stratified flows is proposed.

Figure 1.7 depicts most existing fluid-flows in horizontal or near-horizontal pipes: plug flows, slug flows, annular flows,... Stratified flows, which are ubiquitous in the environment, are fluid flows influenced by the combination of gravity and a density gradient. Stratified and wavy stratified flows implies at least two different fluids (i.e. air-water, air-oil, ...) or a fluid in two different phases (i.e. steam-water).

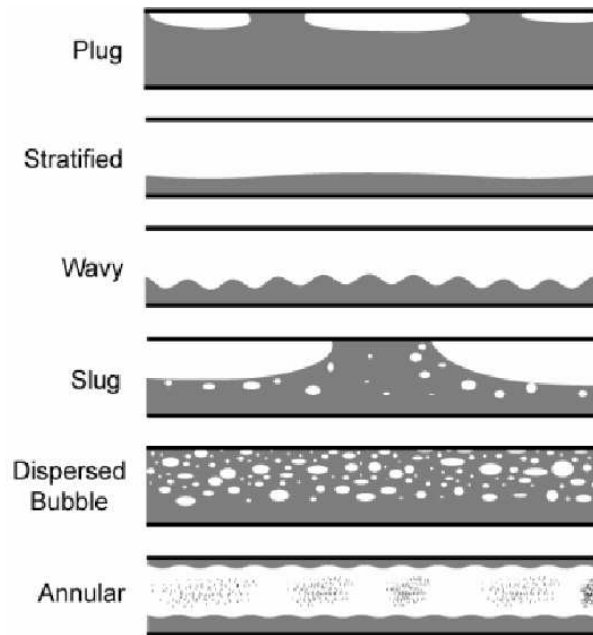


Figure 1.7: Flow patterns in horizontal and slightly upward inclined flow in a tube. Figure from Hewitt et al. [69]

Actually, a stratified flow is a fluid flow in which the gravity overcomes the inertia effect. As a direct consequence, the heaviest phase is maintained in the bottom of the pipe while the lightest phases occupy the rest of the pipe with a

density gradient. The geometry of the stratified flow is mainly characterized by the pipe diameter D , the developing length L , the width of the interface (l_i), the correction factor to an horizontal interface in the cross-section γ and the liquid level in the pipe cross-section $h_l(l_i, \gamma)$ as shown in Figure 1.8. The developing length L is the distance from the inception of the stratified flow: the point of contact between the two fluids. According to the fluid properties (densities, viscosities) but also on the flow kinematics and the geometric parameters, the interface can be convex or concave. The liquid level $h_l(l_i, \gamma)$ is the height of the interface between the two phases in the pipe cross-section. This liquid level evolves from zero at the pipe walls to a maximum value at the center of the pipe cross-section, either in convex or concave interface.

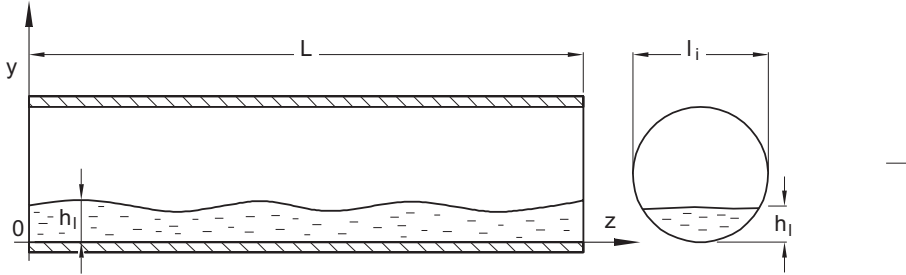


Figure 1.8: Flow description in the control volume CV .

This section aims at describing existing models for droplet entrainment from stratified flows. Engineering models, either theoretical, experimental or numerical models, are commonly used to study complex fluid dynamics problems in the literature. Nevertheless, models need to meet some criterias to correctly represent the studied phenomenon. A model is said to have similitude with the real application when the geometric similarity, the kinematic similarity and the dynamic similarity of the flows are achieved:

- o The geometric similarity is achieved when the model has the same shape than the application.
- o The kinematic similarity is achieved when fluid flow of both the model and real application undergo similar time rates of change motions.
- o The dynamic similarity is achieved when ratios of all forces acting on corresponding fluid and boundary surfaces in the two systems are similar.

To satisfy the above conditions, all parameters required to describe the system must be identified using principles from continuum mechanics. Then, a

dimensional analysis must be used to express the system with as few independent variables and as many dimensionless parameters as possible. The values of the dimensionless parameters are held to be the same for both the scale model and application. This can be done because they are dimensionless and will ensure dynamic similitude between the model and the application.

This similarity is extremely difficult to achieve in stratified flows as the cross-sectional geometric parameters (D , $h_l(l_i, \gamma)$, l_i and γ) present large discrepancies in their dependence with the pipe diameter. As shown in Appendix B.4.1, the liquid level is directly proportional to the liquid fraction α_l which depends of the equilibrium between the interfacial and the parietal shear stresses. The geometric, kinematic and dynamic similarities are cross-correlated such as their achievement is difficult.

1.3.1 Models for droplet entrainment

The droplet entrainment from a liquid film occurs in most multiphase phenomena including an interface between a continuous liquid and a gaseous continuous phase: waves on the surface of an ocean, the oil flow in a pipeline or the rise of the quenching front in a nuclear reactor core offer three typical examples. The droplet entrainment is of importance in these phenomena as mass, momentum and energy transfer processes may be strongly affected by the amount of droplets in the gas phase.

A typical example of the influence of droplets on the energy transfer is the operating of a cooling tower present in most electrical plants, including nuclear power plants. Some water is usually pumped in a river, a lake or an ocean and traveling along the condenser cools down the fluid of the thermal cycle. This secondary water is flowed to the cooling tower and pulverized in a air flow ensured by natural convection. The air humidification by vaporization of the pulverized droplets reduces the amount of energy in the raining water that can then be rejected in the river while the air saturated in humidity leaves at the top of the cooling tower.

The scientific literature on the entrainment phenomena is consequently abundant but there remain many unresolved issues. Indeed, if the literature is quite extensive on flows in open spaces or vertical pipes, the literature on multi-phase flows in horizontal pipes is limited and mainly devoted to annular flows. Experimental studies and modeling are easier in vertical pipe and of bigger importance for the core nuclear engineering (modeling of the core reflooding and/or the heat exchange around the fuel rods). In fact, the two-phase flows in horizontal pipes

have been essentially studied for oil pipeline industrial applications [63, 41, 72]. However the oil flow differs in many aspects from the hot leg stratified flow: fluid properties, pipe dimensions or velocity ranges. Most literature models are thus not directly applicable to the modeling of a hot leg stratified flow.

This section aims at presenting a selected set of models of droplet entrainment from the open literature. All these models are expected to be applicable to evaluate the droplet rate of entrainment or the entrained droplet fraction in horizontal pipes although some are only variations of models designed for vertical or annular flows.

The entrainment phenomenon can be described by two parameters: the entrained mass fraction of droplets E or the entrainment rate Γ_E . The entrained mass fraction of droplets E carried out by the gas stream is defined as the ratio of the droplet mass flow rate $\dot{M}_d$ on the liquid mass flow rate $\dot{M}_l$:

$$E \triangleq \frac{\dot{M}_d}{\dot{M}_l} = \frac{\dot{M}_d}{\dot{M}_d + \dot{M}_f} \quad [-] \quad (1.3)$$

where $\dot{M}_f$ is the liquid film flow rate. The second parameter, the rate of entrainment Γ_E (expressed as a mass per unit of interfacial area and time), represents the quantity of droplets extracted from the continuous liquid phase to be added to the dispersed liquid phase. The entrained droplet fraction E represents the integral effect of the entrainment and deposition phenomena. The two parameters may thus be linked through a mass balance equation of the dispersed liquid phase at the interface under steady state and without phase change conditions, as expressed for example by Kataoka et al. [81] for an annular two-phase flow in Equations 1.4 and 1.5:

$$(\Gamma_E - \Gamma_D) = \Gamma_T = \text{Net mass of liquid taken-off by seconds} \quad (1.4)$$

$$\begin{aligned} \rightarrow \quad \Gamma_E - \Gamma_D &= \frac{V}{A} \frac{\dot{M}_l}{A_i} \left(\frac{\partial E}{\partial z} \right) \\ &= \frac{D \rho_l j_l}{4} \left(\frac{\partial E}{\partial z} \right) \quad [kg \, m^{-2} \, s^{-1}] \end{aligned} \quad (1.5)$$

where Γ_T denotes the total droplet transport rate, D the pipe diameter, ρ_l the liquid density, j_l the liquid superficial velocity and z the pipe axis.

1.3.2 Entrained fraction models

The fraction of entrained droplets E is a more stable parameter to measure and correlate but does not distinct the entrainment from the deposition as it is the result of the balance between these two phenomena. At a critical developing length L^* , the entrainment and the deposition rates are balanced such that the entrainment fraction E does not depend any more on the developing length and becomes constant ($E \neq f\left(\frac{L}{D}\right)$ pour $L \geq L^*$ - Dallman [41] estimated this ratio $\frac{L}{D}$ around 250 for an annular gas-liquid flow). Therefore every correlation that does not include a length z dependence may induce considerable errors under transient or developing flow conditions, in which entrainment and deposition are not balanced.

The influence of the pipe diameter is a crucial parameter. In mono-phasic flow, the representation of a flow in a pipe of high diameter can be simulated in an experiment of smaller dimensions, on the basis of the similitude principles, assuming that every implied phenomena has a similar dependence on the pipe diameter. In multiphasic flows, this strategy meets the problem that the characteristic parameters of the flow present large discrepancies in their diameter dependence as highlighted in the details mathematics in Appendix B.4.1.

Based on experimental data from Cousins et al. [37], Cousins and Hewitt [36], Steen and Wallis [145], Paras and Karabelas [122], Dallman et al. [41], Laurinat et al. [91], Williams et al. [165], Mantilla [101] and Pan and Hanratty [119] both in vertical and horizontal flows, the entrained fraction appears to at least depend on a set of parameters of 10 variables: the superficial velocities j_g and j_l , the densities ρ_g and ρ_l , the viscosities μ_g and μ_l , the surface tension σ , the gravity $\mathbf{g}$, the droplet diameter d and the pipe diameter D (in addition to a z dependence as mentioned previously).

$$E(j_g, j_l, \rho_g, \rho_l, \mu_g, \mu_l, \sigma, \mathbf{g}, d, D, z) = 0 \quad (1.6)$$

According to the Vaschy-Buckingham theorem, as the system depends on three fundamental units (m , kg and s), a equivalent equation between $10 - 3 = 7$ non-dimensional numbers including every parameter may be achieved. Thus the entrained fraction can be expressed as a function of 7 non-dimensional number like, as an example, a Weber of the liquid film, a Weber on the droplet diameter a Froude, a Reynolds for every phase, a density ratio and so on:

$$E\left(W_{ef}, W_{ed}, Fr, Re_g, Re_l, \left(\frac{j_g}{j_l}\right), \left(\frac{\rho_g}{\rho_l}\right)\right) = 0 \quad (1.7)$$

with:

$$We_f \triangleq \frac{\rho_g j_g^2 D}{\sigma} \quad (1.8)$$

$$We_d \triangleq \frac{\rho_g j_g^2 d}{\sigma} \quad (1.9)$$

$$Re_g \triangleq \frac{\rho_g j_g D}{\mu_g} \quad (1.10)$$

$$Re_l \triangleq \frac{\rho_l j_l h_l}{\mu_l} \quad (1.11)$$

$$Fr \triangleq \frac{j_g^2}{g D} \quad (1.12)$$

However many models of entrainment rate Γ_E or entrained fraction E have a simple mechanistic basis: the formation of droplets and their entrainment were assumed to be due to the existence of a shear stress at the interface opposed to the surface tension. Accordingly, the proposed entrainment parameter expressed this antagonism only with a Weber and a Reynolds numbers. Numerous correlations designed for vertical geometries are still based on this two numbers, as neglecting the gravity is less important in annular flows [97, 133].

The free parameters of the entrainment correlations are fitted on experimental data from vertical and/or horizontal flows. The validity of these models is questionable outside the parameter ranges on which the equations were correlated.

Model of Wicks and Dukler [162]

One of the earlier efforts on entrainment models was the study of Wicks and Dukler in 1960 [162]. They reported a correlation that relates an entrainment parameter R to the Lockhart-Martinelli parameter X defined as the ratio between the pressure drops which would exist if each phases flowed alone:

$$X = \left(\frac{(\frac{dP}{dz})_l}{(\frac{dP}{dz})_g} \right)^{1/2} \quad (1.13)$$

This correlation assumes that similarities in the transport of mass and momentum exists between single and two phase flow. The ratio of mass transfer per unit area was correlated with the Lockhart-Martinelli parameter X as the ratio of momentum transfer to explicit the entrainment parameter R :

$$R = \frac{We_c \left(\frac{j_l}{j_g} \right)}{\left(\frac{dP}{dz} \right)_g} \dot{M}_{ld} \quad (1.14)$$

$$X = 0.069 R^{0.39} \quad (1.15)$$

Where We_c is a critical Weber number defined in terms of the spaced-average liquid film thickness $\langle h_l \rangle$: $We_c = \frac{\rho_g v_g^2 \langle h_l \rangle}{\sigma}$. Wicks and Dukler presented their correlation by plotting the entrainment parameter against the Martinelli parameter. Although a reasonable agreement was found, this model does not provide suitable results as their correlation does not depend on many controlling parameters, hidden by the Martinelli factor. Moreover this correlation is dimensional which limits the range of applicability. Finally as this correlation came very early, the range of validity is quite limited by the poor amount of correlated data in vertical flows.

Model of Minh and Huyghe [103]

Minh and Huyghe [103] introduced the entrained fraction E as the entrainment parameter in vertical flows in 1965. They assumed that the entrained fraction of liquid is related to the gas core momentum defined as the homogeneous density of the gas core and the square of the superficial gas velocity: $\langle \bar{\rho}_g \rangle > j_g^2$ where the spaced-average gas density $\langle \bar{\rho}_g \rangle$ takes into account the droplet concentration in the gas core:

$$\langle \bar{\rho}_g \rangle = \rho_g \left[1 + \left(\frac{\rho_l j_l}{\rho_g v_g} \right) \right] \quad (1.16)$$

$$E = f(\langle \bar{\rho}_g \rangle > j_g^2) \quad (1.17)$$

Minh and Huyghe [103] presented their correlation by plotting E against the core momentum. However the use of a dimensional parameter results in uncertainties regarding the dependence of the correlation on the fluid physical properties. Moreover, the range of applicability is unknown as the effects of many factors are not included, especially the Reynolds numbers.

Model of Paleev and Filippovich [118]

In 1966, Paleev and Filippovich [118] improved the model by introducing a dimensionless gas flux for correlating the entrained fraction for vertical flows. Originally defined as a density ratio $\frac{\langle \bar{\rho}_g \rangle}{\rho_l}$ and the product of the liquid viscosity and the superficial gas velocity divided by the surface tension $\frac{\mu_l v_g}{\sigma}$, the dimensionless groups finally correspond to a Weber number combined to a liquid Reynolds number assuming that the velocities are equal at the interface:

$$\frac{We_g}{Re_{lD}} = \frac{\frac{\langle \bar{\rho}_g \rangle v_g^2 D}{\sigma}}{\frac{\rho_l v_l D}{\mu_l}} = \frac{\langle \bar{\rho}_g \rangle}{\rho_l} \left(\frac{\mu_l v_g}{\sigma} \right)^2 \quad \text{if } v_g = v_l \quad (1.18)$$

$$E = f(We_g, Re_{lD}, \frac{\langle \bar{\rho}_g \rangle}{\rho_l}) \quad (1.19)$$

Paleev and Filippovich's correlation fitted few experimental data, including their own (1.6 cm pipe diameter). However, as the effect of the pipe diameter or the liquid Reynolds number were not included, the range of applicability is not well defined.

$$E = 0.015 + 0.44 \log \left(\frac{\bar{\rho}_g}{\rho_l} \left(\frac{\mu_l v_g}{\sigma} \right) 10^4 \right) \quad (1.20)$$

Already in 1970, Hewitt and Hall Taylor [69] have estimated that these correlations of Minh and Huyghe [103] and Paleev and Filipovich [118] have a range of applicability severely limited. Indeed, if these two correlations fit their own data, their crossed predictions are totally incorrect as neither of them account for the effect of diameter.

Model of Wallis [158]

In 1969, Wallis [158] modified the method used by Paleev and Filippovich by substituting the liquid viscosity μ_l by the gas viscosity μ_g , which corresponds to a ratio of a Weber and a gas Reynolds number:

$$\frac{We_g}{Re_{gD}} = \frac{\frac{\bar{\rho}_g v_g^2 D}{\sigma}}{\frac{\rho_g v_g D}{\mu_g}} = \frac{\mu_g v_g}{\sigma} \quad (1.21)$$

$$E = f(We_g, Re_{gD}, \frac{\rho_g}{\rho_l}) \quad (1.22)$$

$$E = 1 - \exp \left[-0.125 \times \left(10^4 \frac{v_g \mu_g}{\sigma} \sqrt{\frac{\rho_g}{\rho_l}} - 1.5 \right) \right] \quad (1.23)$$

Like the Paleev and Filippovich's model, the applicability of this correlation is not well defined as the dimensionless gas flux used does not enough account for the effect of fluid properties. Indeed Kutateladze [88] and Ishii and Grolmes [75] in their works on the onset of entrainment have indicated that the viscosity dependence is far more complicated than those predicted by Paleev and Filipovich [118] or Wallis [158] .

Model of Ishii and Mishima [76, 77, 104]

From 1981 to 1989, Ishii and Mishima developed an advanced mechanistic strategy for the entrainment fraction E based on the mechanistic model of shearing-off of roll wave crests by a gas stream in annular flow in vertical pipe. Droplet entrainment would result whenever the retaining force of surface tension is exceeded by the interfacial shear force exerted by the streaming gas flow. In consequence E is correlated in terms of a Weber number of the entrainment, a density ratio and a liquid film Reynolds number:

$$We = \frac{\rho_g j_g^2 D}{\sigma} \left(\frac{\Delta \rho}{\rho_g} \right)^{\frac{1}{3}} \quad Re_l = \frac{\rho_l j_l D}{\mu_l} \quad (1.24)$$

$$E = f \left(We, Re_l, \frac{\Delta \rho}{\rho_g} \right) \quad (1.25)$$

For a distance far from the tube entrance, their correlation reached a quasi-equilibrium value E_∞ (Equation 1.27):

$$E_\infty = \tanh \left(7.25 \times 10^{-7} \times \left(We \left(\frac{\Delta \rho}{\rho_g} \right)^{\frac{1}{3}} \right)^{1.25} \times Re_l^{0.25} \right) \quad (1.26)$$

$$\text{for } z > 440 D \left(\frac{We \left(\frac{\Delta \rho}{\rho_g} \right)^{\frac{1}{3}}}{Re_l} \right)^{0.25} \quad (1.27)$$

Unlike previous models, their model depends directly on gas and liquid velocities and densities (ρ_g , ρ_l , μ_l and σ). The scale effect is included by the model dependence on diameter. Although this model is quite complete, its validity requires a very long developing length which makes the correlation unsuitable for not fully developed flow. Similarly to the two first models, the available data to fit this correlation were rather limited (data of Cousins et al. [37], Cousins and Hewitt [36], Gill et al. [59] and Steen-Wallis [145]) such as the validity range is quite restricted, especially in terms of pipe diameter (see Table 1.2). Ishii and Mishima's correlation fits their data with a precision of only 40% [76].

	Cousins et al.	Cousins & Hewitt	Steen & Wallis	Gill et al.
Pipe	Vertical	Vertical	Vertical	Vertical
Fluids	Air - water	Air - water	Air - water	Air - water
D [cm]	0.95	3.18	1.07 - 1.56	3.2
$Re_l \times 10^3$	0.37 - 4.2	0.3 - 4.3	0.32 - 6.35	5
P [bar]	2.76	2 - 2.1	1	NC

Table 1.2: Range of validity for Ishii and Mishima's correlation

Model of Oliemans et al. [115]

In 1986, Oliemans et al. [115] proposed a correlation based on a regression analysis of a large set of experimental data including many fluids, pipe diameters (but still restricted below 3.2 cm), Froude or Reynolds Number in annular vertical pipes, from Whalley et al. [161], Gill et al. [58], Cousins et al. [37], Minh and Huyghe [103], Cousins and Hewitt [36], Wurtz [168], Hewitt and Pulling [68], Singh et al. [142], Nigmatulin et al. [110], Keeys et al [82] and their own. Their correlation had a functional but dimensional form:

$$E = \frac{10^{-2.52} \rho_l^{1.08} \rho_g^{0.18} \mu_l^{0.27} \mu_g^{0.28} \sigma^{-1.8} D^{1.72} j_l^{0.7} j_g^{1.44} g^{0.46}}{1 + 10^{-2.52} \rho_l^{1.08} \rho_g^{0.18} \mu_l^{0.27} \mu_g^{0.28} \sigma^{-1.8} D^{1.72} j_l^{0.7} j_g^{1.44} g^{0.46}} \quad (1.28)$$

$$= [kg^{0.01} s^{1.83} m^{-0.93}] \quad (1.29)$$

This correlation depends on every flow parameters and flow conditions but appears to be dimensional.

	Cousins	Cousins/Hewitt	Hewitt/Pulling	Minh/Huyghe
Fluids	Air - water	Air - water	Steam - water	Air - water
D [cm]	0.95	3.18	0.93	0.6 - 1.2
P [bar]	1.2 - 2.7	2 - 2.1	2.4 - 4.5	2.3 - 8.5
L/D [-]	480	230-350	390	NC
	Wurtz	Keays	Nigmatulin	Whalley
Fluids	Steam - water	Steam - water	Steam - water	Air - water
D [cm]	1 - 2	1.27	1.33	3.18
P [bar]	30 - 91	9.1 - 100	1.2 - 3.5	69
L/D [-]	450 - 900	290	300	590

Table 1.3: Range of validity for Oliemans correlation

Model of Hanratty et al. [41, 91, 40, 119]

Hanratty and coworkers such as Dallman (1978) [41], Williams (1981) [165] or Pan (2002) [119] have developed for years a model for the entrained fraction, initially for annular flows in vertical pipes, then for stratified flows in horizontal pipes. The entrained fraction E results from a balance between the liquid film entrainment rate and the droplet deposition rate:

$$\Gamma_D = \Gamma_E \quad (1.30)$$

Modeling the entrainment rate is then based on this equilibrium assumption, such as the entrained fraction E is expressed as a function of the deposition rate Γ_D (see Equation 1.33). This deposition rate Γ_D is assumed to be proportional to the droplet mass concentration C_d by a factor k_d :

$$\Gamma_D = k_d C_d \quad (1.31)$$

Assuming that the droplets are traveling at the same velocity as the gas, the droplet concentration C_d depends on the mass flow rate of droplets $\dot{M}_d$ in the gas

stream. As a consequence this droplet mass flow rate may be directly express as a function of the droplet concentration C_d :

$$C_d = \frac{\dot{M}_d}{\dot{V}_g} = \rho_g \frac{\dot{M}_d}{\dot{M}_g} \quad \leftrightarrow \quad \dot{M}_d = \frac{C_d \dot{M}_g}{\rho_g} \quad (1.32)$$

The entrained fraction may then be directly expressed as a function of the deposition rate Γ_D :

$$E \triangleq \frac{\dot{M}_d}{\dot{M}_l} = \frac{C_d \dot{M}_g}{\rho_g \dot{M}_l} = \frac{\Gamma_D \dot{M}_g}{k_d \rho_g \dot{M}_l} \quad (1.33)$$

Dallman [41] improved the expression of the droplet concentration C_d by relaxing the assumption of equality between the gas and droplet velocities. The parameter s denotes for the velocity ratio between gas and droplets: $s = \frac{v_d}{v_g}$

$$C_d = \frac{\frac{\dot{M}_{ld}}{v_d}}{\frac{\dot{V}_g}{v_g}} = \rho_g \frac{\dot{M}_{ld} s}{\dot{M}_g} \quad \leftrightarrow \quad \dot{M}_{ld} = \frac{C_d \dot{M}_{ld} s}{\rho_g} \quad (1.34)$$

Now, as Hanratty and coworkers assumed a fully developed flow, the equilibrium between entrainment and deposition implies that the entrained fraction E is related to the modeling either of the entrainment rate Γ_E or the deposition rate Γ_D . Finally, after many further works achieved to improve the entrained fraction model, the correlation of Pan and Hanratty [119] includes the effect of gravity and depends on the droplet size. All flow parameters are included in this correlation but the liquid viscosity.

Many studies [118, 103, 115, 77, 158] did not take into account the effects of the liquid velocity on the entrained fraction E . However, several authors [41, 67, 119] have highlighted the existence of a critical limit in the liquid flow rate below which no droplets are entrained: the critical mass flow of the liquid film $\dot{M}_{flc}$, which leads to a similar limit for the entrained fraction: E_m which is included in Pan and Hanratty's correlation [119] for annular flows in horizontal pipes:

$$\frac{(E/E_m)}{(1 - (E/E_m))} = 9 \times 10^{-8} \left(\frac{D v_g^3}{\sigma} (\rho_g \rho_l)^{0.5} \right) \left(\frac{\rho_g^{1-m} \mu_g^m}{d_{32}^{1+m} g \rho_l} \right)^{\left(\frac{1}{(2-m)}\right)} \quad (1.35)$$

where E_m is very difficult to model as in the horizontal model this parameter depends on the circumferential distribution of the liquid film:

$$E_m = 1 - \frac{\pi D \Gamma_c^*}{\dot{M}_l} \quad (1.36)$$

with Γ_c^* the integrated value of the variation along the pipe circumference of the critical level of liquid $\Gamma_c^* =$. This correlation takes into account a droplet

diameter effect together with a gravity effect. However as these correlation dimensions depend on the m exponent, the validity raises theoretical issues. In fact the m coefficient takes a value of 0, 0.6 or 1 depending on the drag coefficient law: $C_D = \left(\frac{v_d d \rho_g}{\mu_g}\right)^{-m}$ with a Newton's law, $m = 0$ and $m = 1$ with a Stokes law. The droplet Sauter diameter d_{32} is calculated based either on the theory of Tatterson et al. [149] or on the theory of Azzopardi [11]. The correlation may then be dimensional according to the m coefficient. However the predictions well matched the experimental results of Dallman [42], Laurinat [90], Williams [164] and Paras and Karabelas [120, 123] essentially for pipes of small diameters (2.31 cm) but diverged for pipes of higher diameters (5.08 cm and 9.53 cm).

	Williams	Laurinat	Dallman	Paras and Karabelas
Geometry	Horizontal	Annular	Horizontal	Horizontal
Fluids	Air - water	Air - water	Air - water	Air - water
D [cm]	9.53	5.08	2.31	5.08
v_g [m/s]	26 - 88	11 - 131	15 - 88	31 - 67
$\dot{M}_l$ [kg/s]	0.12 - 0.86	0.033 - 0.97	0.003 - 0.25	0.04 - 0.39
ρ_g [kg/m ³]	1.3 - 1.85	2.05	1.26 - 2.75	1.3 - 2.3

Table 1.4: Experimental data used by Pan an Hanratty to fit their correlation.

Comparison

Figure 1.9 presents various entrained fraction correlations as a function of the gas superficial velocity for two pipe diameters. Minh and Huyghes correlation [103] and Wicks and Dukler's correlation [162] are not presented as their validity range is quite small and questionable. The correlation of Paleev and Filipovich [118] and the correlation of Wallis [158] do not depend of the pipe diameter. As a consequence, whatever the pipe diameter considered, their correlations predict an identical entrained fraction. The correlations of Ishii and Mishima and of Oliemans highly deviate at the higher diameter of 24 cm: indeed the predicted entrainment inception already appears around $j_g = 5$ m/s and the whole liquid film is supposed to have been entrained ($E = 1$) at $j_g = 20$ m/s. At the higher diameter ($D = 24$ cm), Pan et Hanratty's correlation [119] appears to be the single correlation predicting a possible entrained fraction. However, if the more developed correlations consider a critical mass flow rate of the liquid film, one may question the absence of a critical gas mass flow rate. Indeed, whatever the gas velocity, few droplets appear to be entrained from the liquid film in every presented correlation.

Table 1.5 highlights the completion of those different correlations. Few pa-

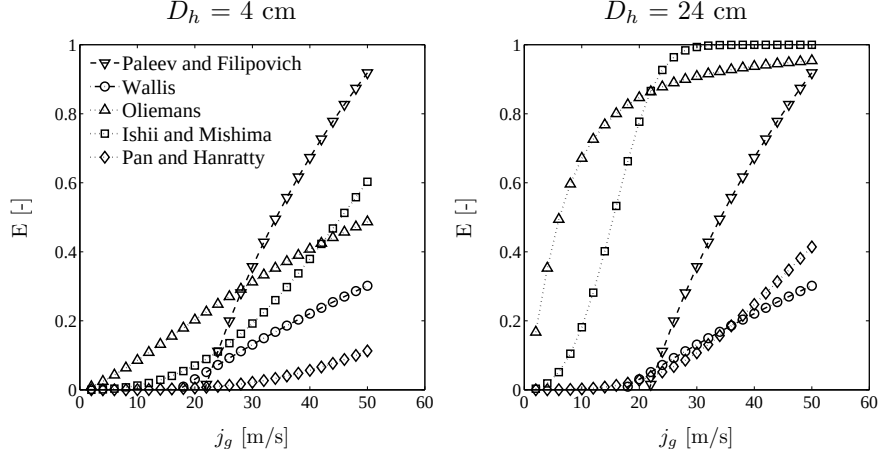


Figure 1.9: Variation of the entrained fraction as a function of the air superficial velocities j_g at $j_l = 0.02$ m/s.

rameters are taken into account in every correlation: the gas velocity j_g and the two densities or the surface tension. Indeed every correlation is based on a similar assumption on the droplet formation, expressed as an antagonism between the stabilizing surface tension and the gas inertia via a Weber number. This approach seems coherent with the modeling of annular flows. However, in stratified flows the stabilizing effect of the gravity adds to the stabilizing effect of the surface tension. Consequently, a Froude number should also be taken this additional effect into account (which is not the case in any correlation (except Oliemans and Hanratty's correlations)).

In every correlation, one Reynolds number takes into account the viscous forces either in the liquid phase or the gaseous phase. Finally one may wonder about the inclusion of a droplet Weber number in a correlation. Indeed the droplet characteristics (size, and velocities) may depend more on its post-entrainment dynamics than on the entrainment process itself.

	Designed for annular flows in vertical pipes	Designed for stratified flows in horizontal pipes	Hydrodynamic equilibrium assumption	Including the critical mass flow of liquid	Including the critical mass flow of gas	Pipe diameter : D	Liquid velocity : j_l	Gas velocity : j_g	Liquid Density : ρ_l	Gas Density : ρ_g	Liquid Viscosity : μ_l	Gas Viscosity : μ_g	Surface tension : σ	Gravity : g
PF	x							x	x	x	x		x	x
W	x		x					x	x	x			x	
O	x		x			x	x	x	x	x	x	x	x	x
IM	x		x			x	x	x	x	x	x		x	
PH	x		x	x		x	x	x	x	x			x	x
Model	x		x	x	x	x	x	x	x	x	x	x	x	x

Table 1.5: Comparison of entrained fraction E models: Paleev and Filipovich (PF), Wallis (W), Oliemans (O), Ishii and Mishima (IM) and Pan and Hanratty (PH) compared to the characteristics that should present an ideal model.

1.3.3 Entrainment rate models

Several models for the entrainment rate simply assume the balance at equilibrium between the entrainment and deposition phenomenon to determine: $\Gamma_E = \Gamma_D$. The developing length is then assumed long enough so that E reaches the saturation and Γ_E directly depends on the droplet deposition phenomenon. These correlations are only applicable to steady and established flows.

Most models derive from models originally developed for vertical pipes and have been more or less carefully adapted to horizontal flows. Thus their conception itself is not adapted to stratified flow and does not account for the specificities of horizontal flows leading to the droplet entrainment.

Then, most models apply to the entrainment phenomenon without explaining the physics underlying the droplet emergence. This is not the case of Holowach et al. who proposed a mechanistic model taking into account the interface dynamics and force balances applied on the generated waves to determine the entrainment rate in a vertical pipe.

Otherwise, it has been argued that the rate of entrainment mainly depends on the heat flux (Milashenko et al. [102]), but most models have been constructed with no such influence at all. Yet other models have tried to gather these two conceptions, see e.g. Hoyer [73]. The literature review presented here is limited to correlations which do not include any heat flux influence since the experiments and the flow to model are assumed to be adiabatic.

Models of Hutchinson and Whalley [161, 159]

The model developed by Hutchinson and Whalley in 1974 [161] and Hutchinson et al. in 1978 [159] considered the rate of entrainment in annular vertical flows as a fraction k [–] of the droplet mass concentration in the gas core in hydrodynamic equilibrium C_∞ [$kg\ m^{-1}\ s^{-2}$].

$$\Gamma_E = k C_\infty \quad (1.37)$$

The droplet formation and entrainment are expected to be due to the existence of a shear stress at the interface τ_i opposed to the surface tension. This antagonism is expressed by assuming the droplet concentration proportional to the Weber number of the liquid film as :

$$C_\infty \propto We_f \quad \text{with} \quad We_f = \frac{\tau_i h_l}{\sigma} \quad (1.38)$$

where h_l denotes the liquid film thickness and τ_i , the interfacial shear stress detailed by Wallis [158]:

$$\tau_i = f_i \rho_g (v_g - v_l)^2 \quad \text{with} \quad f_i = 0.005 \left(1 + 300 \frac{h_l}{D} \right) \quad (1.39)$$

Hutchinson et Whalley fitted the constant parameters to experimental data obtained in vertical pipes and assumed their relationship to remain valid in both horizontal and vertical flows:

$$\Gamma_E = 0.15 \exp(1 + 4.62 \min(2, We_f)) \quad We_f < 3 \quad (1.40)$$

$$\Gamma_E = 0.15 \exp \left(5.76 - \left(\frac{2.303}{We_f} \right)^{\frac{1}{3}} \right) \quad \forall We_f \quad (1.41)$$

This correlation raises two issues: firstly as the film thickness and the interfacial shear stresses (C_∞ , h_l , ...) are not measured in most experimental facilities, some modeling is required to obtain the correlation. Secondly the comparison of this correlation to experimental data has shown a considerable scattering which suggests that a single dimensionless group for correlating C_∞ is not sufficient. Indeed many flow parameters (viscosities, ρ_l , pipe diameter...) are not taken into account in this relationship.

Model of Ueda [155]

Based on a similar rationale, Ueda [155] proposed a correlation in which the entrainment rate is proportional to a slightly different but dimensional number:

$$\Gamma_E = 3.54 \cdot 10^{-3} \left(\frac{\tau_i}{\sigma} \left(\frac{j_l}{\sigma} \right)^{0.6} \right)^{0.57} \quad (1.42)$$

With τ_i expressed as the (complicated) Ueda's model (1981) [156] or the Wallis model.

Although this correlation was fitted on experimental data of Ueda [155] and Tso and Sugawara [153], as this relation is dimensional, the parameter $\frac{j_l}{\sigma}$ is totally questionable. Indeed, apart of a pipe of 1 cm with air-water or air-alcohol fluids, this dimensional correlation is not applicable. Moreover this correlation, as the correlation of Hutchinson and Whalley, is mainly function of the modeling of the interfacial shear stress τ_i .

Model of Kataoka et al. [81]

In 2000, Kataoka, Ishii and Nakayama [81] proposed a correlation for the entrainment rate in annular flows (in vertical pipes) based on Ishii and Mishima's

correlation [104] for the entrained fraction E (developed in Section 1.3.2) and the mass balance proposed in Equation 1.5. Then the proposed entrainment rate Γ_E directly depends on the modeling of the deposition rate Γ_D .

Assuming that the droplet deposition is controlled by a diffusion process, the deposition rate Γ_D is proportional to the concentration C_d of droplets in the core and to a mass transfer coefficient k_d (similarly to Equation 1.31 of Hanratty and coworkers [41, 91, 40, 119]):

$$\Gamma_D \propto k_d C_d \quad (1.43)$$

Many studies, both experimental and theoretical, have been carried out to model this coefficient k_d , leading to several correlations. According to Kataoka et al. [81], although the fullest correlation for the droplet deposition comes from Namie and Ueda [107], Paleev and Filipovich's correlation for deposition [118] which provides similar results in a simpler formulation is preferred.

The entrainment rate Γ_E is then related to the entrainment fraction E and correlated on experimental data from Cousins et al. [36] and Gill et al. [59] summarized in Table 1.6:

$$\Gamma_E = \frac{D \rho_l j_l}{4} \left(\frac{\partial E}{\partial z} \right) + 0.022 \rho_l j_l Re_l^{-0.26} \left(\frac{\mu_g}{\mu_l} \right)^{0.26} E^{0.74} \quad (1.44)$$

This correlation, based on many previous studies by Ishii and coworkers [75, 77, 99], has been slightly improved by modifying few internal parameters by e.g. Sawant et al. in 2009 [134]. This model directly depends on the modeling of the entrained fraction E of Ishii and Mishima [76] (Paragraph 1.3.2). Consequently, as the correlation for E is incorrect at high pipe diameters, the correlation for Γ_E raises theoretical issues.

	Cousins and Hewitt	Gill et al.
Pipe	Vertical	Vertical
Fluids	Air - water	Air - water
D [cm]	3.18	3.2
$Re_l \times 10^3$	0.3 - 4.3	5
P [bar]	2 - 2.1	NC

Table 1.6: Range of validity for Kataoka's correlation

Model of Hewitt and Govan [67]

Hewitt and coworkers [67] conducted a research on the annular flow in vertical pipes resulting in several correlations for the entrainment rate. In 1989, He-

witt and Govan [67] proposed a correlation introducing the critical mass flow rate of the liquid film $\dot{M}_{flc}$ as a parameter. They validated it on a large spectrum of experimental data in terms of fluids, pressures and phase velocities. In its entrainment rate comparative study, Nigmatullin [111] in 1996 particularly emphasized the quality of their correlation.

$$\Gamma_E = \dot{m}_g 5.75 \cdot 10^{-5} \left[(\dot{m}_{fl} - \dot{m}_{fl,c}) \frac{D \rho_l}{\sigma \rho_g^2} \right]^{0.316} \quad (1.45)$$

$$\text{with } \dot{m}_{fl,c} = \frac{\mu_l}{D} Re_{fl,c} = \frac{\eta}{D} \exp \left(5.8504 + 0.4249 \frac{\mu_g}{\mu_l} \sqrt{\frac{\rho_l}{\rho_g}} \right) \quad (1.46)$$

where $\dot{m}$ denotes for the surface mass flow rates in $kg/m^2 s$. This correlation includes dependencies on every flow condition and parameter, including the pipe diameter effect. However, this model has been developed for vertical pipes and does not incorporate the specificities of horizontal flows, highly driven by gravity such as the stratified flows. Therefore, the application of this model to an horizontal pipe better suits to annular flows than to stratified flows. The experimental data references used to fit the correlation are not specified by Hewitt and Govan [67]. Since 2008, this correlation is encoded in the CATHARE-3 system code for calculating the entrainment rate, although it is not specifically adapted to horizontal flow.

Models of Hanratty et al. [41, 91, 40, 119, 135, 136]

Assuming a fully developed flow in annular flow (essentially in vertical pipes), Hanratty and coworkers [41, 91, 40, 119, 135, 136] equated the entrainment rate Γ_E to the deposition rate Γ_D . The entrained fraction E is related either to the entrainment rate or the deposition rate by:

$$E = \frac{\Gamma_E \dot{M}_g}{k_D \rho_g \dot{M}_f} = \frac{\Gamma_D \dot{M}_g}{k_D \rho_g \dot{M}_f} \quad (1.47)$$

Hanratty and coworkers have detailed the entrainment rate Γ_E , according to the earlier work of Taylor [151] in 1940, as proportional to the gas velocity v_g , the two fluid densities (ρ_g , ρ_l) and a dimensionless atomization parameter R_A^* :

$$\Gamma_E = v_g (\rho_g \rho_l)^{0.5} R_A^* \quad (1.48)$$

In 1975, Tatterson [148, 137] proposed a model for R_A^* based on the experimental investigations of Woodmansee and Hanratty [166]. After a large number of simplifying approximations of the wave structure or the pressure fluctuations on small amplitude waves, the parameter R_A^* was expressed as a Weber number based on the average liquid film level around the wall $\langle \bar{h}_l \rangle$:

$$R_A^* = We^* = \frac{\rho_g v_g^2 \langle \bar{h}_l \rangle}{\sigma} \quad (1.49)$$

Such as:

$$\Gamma_E = v_g (\rho_g \rho_l)^{0.5} \frac{\rho_g v_g^2 \bar{h}_l}{\sigma} \quad (1.50)$$

The Tatterson's correlation [148] (Equation 1.50) is then directly connected to a Weber number. However, Dallman et al. [42] showed that this relationship does not satisfy the strong dependence of the entrained fraction E firstly on the liquid film flow rate $\dot{M}_{lf}$ and then on the pipe diameter D . Therefore, the model was improved by Dallman et al. [42], Lopez de Bertodano and Assad [98], Lopez de Bertodano et al. [100], Schadel et al. [136] and by Assad et al. [19] to get a new expression for R_A^* . The parameter R_A^* was then assumed to evolve linearly with the liquid film mass flow rate:

$$R_A^* = \frac{k'_A}{\sigma} \frac{\dot{M}_f}{\pi D} \quad (1.51)$$

The insertion of Equation 1.51 in Equation 1.33 gives:

$$\Gamma_E = \frac{k'_A v_g (\rho_g \rho_l)^{0.5}}{\sigma} \frac{\dot{M}_f}{\pi D} \quad (1.52)$$

The entrainment rate is then linearly dependent on the liquid flow. However, the experimental measurements have shown that the atomization does not occur below the critical rate of the liquid film $\dot{M}_{lfc}$. Dallman [42] then suggested to rewrite R_A^* to include this phenomenon via the critical liquid flow rate $\dot{M}_{lfc}$:

$$R_A^* = \frac{k'_A}{\sigma} \frac{(\dot{M}_{lf} - \dot{M}_{lfc})}{\pi D} \quad (1.53)$$

Moreover, Dallman expressed the factor k'_A as a function of the gas velocity v_g^n in Equations 1.54. Based on experimental results, Schadel et al. [137, 135] considered this dependence to remain valid only for pipe of small diameters. Consequently, the factor n is modified to be equal to 1 for diameter lower than 2.54 cm and zero elsewhere.

$$\begin{aligned} k'_A &= k_A v_g^1 & \text{if } D < 2.54 \text{ cm} \\ k'_A &= k_A & \text{if } D > 2.54 \text{ cm} \end{aligned} \quad (1.54)$$

The presence of a critical flow rate of the liquid film $\dot{M}_{lfc}$ leads to the introduction of the parameter E_m defined as:

$$E_m = 1 - \frac{\dot{M}_{lfc}}{\dot{M}_l} \quad (1.55)$$

For vertical flows, the following relation was derived for entrainment under the fully developed assumption, for a liquid viscosity close to water:

$$\frac{(E/E_m)}{(1 - (E/E_m))} = f \left(\frac{k'_A v_g^3 S D (\rho_g \rho_l)^{0.5}}{4 k_d} \right) \quad (1.56)$$

An alternate formulation of the entrainment rate has been used for which the gas velocity v_g is replaced by $(v_g - v_{gc})$, where v_{gc} denotes the critical gas velocity below which no entrainment is observed. However, Pan and Hanratty [119] argued that there is no physical reason to implement this in horizontal flows where v_{gc} would be dependent on gravitational settlings. Consequently, the practical application of their correlation implies that a droplet entrainment occurs from a liquid film whatever the air velocity. Even for the stable flows, this correlation predicts a small entrainment rate. In consequence, their correlation does not suit for codes in which the transition from the wavy stratified flows to wavy stratified flows with entrainment does not imply a change in the closure laws.

The entrained fraction E in Equation 1.56 is dependent on a k_d parameter. Andreussi in 1983 [11], Govan al. [60] in 1988 and Schadel al. in 1990 [136] have shown that the deposition factor k_d increases with the droplet concentration at larger liquid flows. This could be compensated by the concomitant decrease of k_A but the Schadel's experimental results [136] showed that the growth of k_D is more important than the decrease of k_A .

In 1999, Schadel [136] proposed an adaptation of Equation 1.53 by replacing the pipe perimeter by a characteristic length L_A corresponding to the wetted perimeter.

$$\Gamma_E = k'_A \left[\frac{\dot{m}_{fl} - \dot{m}_{flc}}{L_A} \right] \frac{(\rho_g \rho_l)^{0.5}}{\sigma} v_g^2 \quad (1.57)$$

This characteristic length leads Williams [164] to adapt the work of Hanratty and coworkers for the flows in horizontal pipes, either stratified or annular. The characteristic length no longer corresponds to the wetted perimeter πD , but to the ratio of the wetted sectional area and the height of the liquid film at the bottom of the pipe: $\frac{A_l}{h_0}$ as depicted Figure 1.10, such that:

$$\Gamma_E = k'_A \left[\frac{\dot{M}_{fl} - \dot{M}_{flc}}{A_L/h_0} \right] \frac{(\rho_g \rho_l)^{0.5}}{\sigma} v_g^2 \quad (1.58)$$

The calculation of the entrainment rate therefore requires the sub-modeling of the geometric ratio A_L/h_0 but also of the parameter k_A and the critical flow of the film $\dot{M}_{flc}$. The latter depends on the concentration of the drops into the flow in an horizontal pipe. In annular flow, the modeling of the geometric ratio is immediate. In stratified flows, with a asymmetric liquid film, Williams [164] made the assumption that the interface is flat such that:

$$\frac{A_l}{h_0} \cong \frac{2}{3} D \sin\left(\frac{\gamma}{2}\right) \quad (1.59)$$

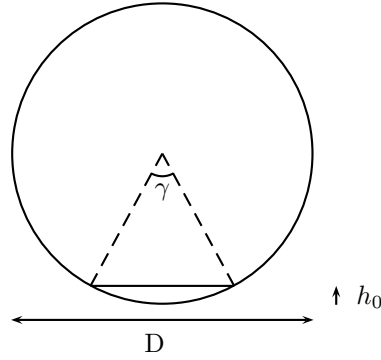


Figure 1.10: Geometry assumed by Williams [164]

where γ denotes the angle of the interface.

Dallman [42], Williams [164] and Pan [119] correlated the k_A and the k_d coefficients with experimental data to obtain a function of several parameters. Unfortunately this one must be correlated in each case experimentally which makes the entrainment correlation hazardous. In horizontal pipe the coefficient k'_A is set to 3×10^{-6} in the final equation of Pan and Hanratty:

$$\Gamma_E = k'_A \left[\frac{\dot{M}_{fl}}{\pi D} - \Gamma_c \right] \frac{(\rho_g \rho_l)^{0.5}}{\sigma} v_g^2 \quad (1.60)$$

with Γ_c defined as the critical flow in the wall film per unit length.

Model of Okawa et al. [113]

In 2003 Okawa et al. proposed also a model in which the entrainment rate is proportional to a Weber number of the liquid film. However, the film thickness is extensively studied in order to represent a correct behaviour of vertical annular flows. This model is function of 3 parameters k_E , n and m which have been slightly adapted by Okawa et al. (2004) [114], Okawa and Kataoka [112], Adamsson and Le Corre (2008) [92] and Anglart and Adamsson (2010) [4].

$$\Gamma_E = k_E \rho_l \pi_E^n \left(\frac{\rho_l}{\rho_g} \right)^m \quad (1.61)$$

$$\pi_E = \frac{f_i \rho_g j_g^2}{\frac{\sigma}{\delta}} \cong We_{lf} \quad (1.62)$$

in which δ is the film thickness calculated from a force balance between the interfacial shear force and the wall friction force acting on liquid film with the liquid film velocity approximated by: $v_f = \frac{D}{4\delta} j_f$ such that δ is equal to:

$$\delta = \sqrt{\frac{f_w \rho_l \dot{M}_f D_h}{f_i \rho_g \dot{M}_g 4}} \quad (1.63)$$

and f_i and f_w the interfacial and wall friction factors (Wallis [158]):

$$f_i = 0.005 \left(1 + 300 \frac{\delta}{D} \right) \quad f_w = \max \left(\frac{16}{Re_f}; 0.005 \right) \quad (1.64)$$

Where Re_f is the liquid film Reynolds number defined by:

$$Re_f = \frac{\rho_l j_f D}{\mu_l} \quad (1.65)$$

The last parameter k_E is dimensional and defined so that the correlation fits the experimental data of Cousins et al. [37], Cousins and Hewitt [36], Hewitt and Pulling [68], Yanai [169], Wurtz [168], Keeys et al. [82], Nigmatullin [110], Whalley [161], Singh et al. [142], Bennet et al. [24], Owen et al. [117] and Asali [18]. This correlation depends on every flow parameter but μ_g the gas viscosity. However, as the correlation is mainly based on the film thickness δ of the annular film, its application to horizontal stratified flows where the film thickness varies circumferentially is questionable.

Range of π_E	k_E [m/s]	n	m
Okawa et al. (2003)			
all	4.79×10^{-4}	1.0	0.111
Okawa et al. (2004)			
$\pi_E < 0.0675$	3.1×10^{-2}	2.3	0
$0.0675 < \pi_E < 0.295$	1.6×10^{-3}	1.2	0
$0.295 < \pi_E$	3.8×10^{-4}	0.5	0
Adamsson and Le Corre (2008)			
$\pi_E < 0.0675$	3.1×10^{-2}	2.3	0
$0.0675 < \pi_E$	1.6×10^{-3}	1.2	0

Table 1.7: Parameters for the Okawa's correlation

Moreover, the use of the parameter k_E , which has the dimension of a velocity, depends on the Weber Number and has no physical meaning (see Table 1.7), rises theoretical doubts since - by similarity analysis - it would have to depend on some more fundamental velocity scale [92]. Finally, the experimental data used to correlate Okawa's relation, defined on Table 1.8, indicate that this relation is not really valid for the modeling of a PWR hot leg. Indeed, this relation has been correlated for the vertical sub-channels in a nuclear fuel assembly with experimental data of little pipe diameters and high pressure.

	Cousins et al.	Cousins & Hewitt	Hewitt & Pulling	Yanai
Fluids	Air/water	Air/water	Steam/water	Steam/water
D [cm]	0.95	3.18	0.93	1.2
P [bar]	1.2 - 2.7	2 - 2.1	2.4 - 4.5	3.4
α_g [-]	0.15 - 0.93	0.46 - 0.81	0.14 - 0.76	0.11 - 0.80
	Wurtz	Keey's et al.	Nigmatulin	Whalley et al.
Fluids	Steam/water	Steam/water	Steam/water	Air/water
D [cm]	1 - 2	1.27	1.33	3.18
P [bar]	30 - 91	9.1 - 100	1.2 - 3.5	69
α_g [-]	0.08 - 0.60	0.15 - 0.69	0.10 - 0.75	0.10 - 0.91
	Singh et al.	Bennet et al.	Owen et al.	Asali
Fluids	Steam/water	Steam/water	Air/water	Air/water
D [cm]	1.25	0.93	3.18	2.29 - 4.2
P [bar]	69	3.8	2.4	1
α_g	0.28 - 0.93	0.20 - 0.80	0.17 - 0.82	0.25 - 0.93

Table 1.8: Experimental data used by Okawa et al. to fit their correlation.

Model of Holowach [70, 71]

Unlike the foregoing models, Holowach [70, 71] presented a mechanistic model for the entrainment rate in vertical annular flows. Based on a quite different approach, the model attempts to take into account the complete phenomenon sequence that leads to the destabilization of the liquid film and the droplet entrainment. Holowach considered the visual observations of Woodmansee and Hanratty [166] as the best scenario to identify the physical phenomena to be included into his model.

According to these observations, the droplet entrainment from a liquid film occurs at the crest of small waves, themselves moving at the surface of waves of larger wavelengths. Holowach's model is based on the volume entrained at each fragmented wave V_E multiplied by the number of these waves N_{CV} in a control volume:

$$\Gamma_E = \rho_l \frac{V_E N_{CV}}{A_{CV} \tau_C} \quad (1.66)$$

To reach an entrainment rate dimension ($kg\ m^{-2}\ s^{-1}$), this result is divided by the characteristic time of the phenomenon τ_C and the interfacial area A_{CV} . So the calculation of this entrainment rate depends on the sub-modeling of these parameters. The main hypothesis is that the short ripple waves are supposed three-dimensional. The control volume has a length equal to the wavelength.

Consequently, as the model is designed for annular flows the number of waves in the control volume is equal to the wetted perimeter P_w divided by this length:

$$N_{CV} = \frac{P_w}{\lambda} \quad (1.67)$$

The interfacial area is then expressed as the product of this characteristic length by the wetted perimeter (corrected by a deformation factor β_c , assumed to be proportional to the void fraction: $\beta_c = \alpha_g^{0.5}$):

$$A_{CV} = P_w \lambda \beta_c \quad (1.68)$$

The characteristic time is defined as the time between two waves fragmentation. So as the necessary time for a wave to travel through the control volume:

$$\tau_C = \frac{\lambda}{v_{gc} - v_{lc}} \quad (1.69)$$

The entrainment model simplifies in Equation 1.70 where only two unknowns remain: the wavelength λ and the entrained volume V_E .

$$\Gamma_e = \rho_l \frac{V_E (v_{gc} - v_{lc})}{\lambda^3 \beta_c} \quad (1.70)$$

The wavelength λ is calculated based on a Hewitt and Hall-Taylor development while the entrained volume is calculated based on a balance of the strength acting on the wave crest. This entrainment correlation potentially includes then the critical mass flow of liquid and gas below which the force balance predicting the entrained volume $V_{e,w}$ ensures a stabilizing effect without entrainment. The final correlation had nevertheless to be fitted on experimental data of Hewitt and Pulling [68], Keeys et al. [82], Singh [142] and Wurtz [168] to ensure a correct correlation. This range of experimental data is extremely particular as the diameter vary around 1 cm and the pressure overstep 3 bars.

$$\Gamma_e = 0.0311 Re_{lf}^{1.67} \left(\frac{\rho_l}{\rho_g} \right)^{7/8} N_\mu^3 \rho_l \frac{V_{e,w} (v_{gc} - v_l)}{\lambda^3 \beta_c} \quad (1.71)$$

with N_μ a viscosity number defined as:

$$N_\mu = \frac{\mu_l}{\rho_l \sigma \left(\frac{\sigma}{g \Delta \rho} \right)^{0.5}} \quad (1.72)$$

The correction fitting factor provided by Holowach et al. is then non negligible. In 2008, Mantilla [101] proposed a short adaptation of Holowach's model [70, 71] to flow in horizontal and inclined pipes and correlated his correlation to his own experimental results. In 2009, Al-Wahaibi and Angeli [9] have adapted the Holowach's concept for horizontal oil-water (liquid-liquid) flows but did not account for the gravity stabilizing effect in their force balance.

	Hewitt and Pulling	Keeys et al.	Singh	Wurtz
D [cm]	0.93	1.26	1.25	1
$Re_g \times 10^3$	45 - 150	240 - 620	110 - 450	84 - 660
$Re_l \times 10^3$	1.6 - 6.8	12 - 48	3.8 - 42	11 - 57
P [bar]	2.39 - 4.46	34.5 - 69	69 - 82.8	30.4 - 91.3

Table 1.9: Experimental data of Holowach et al. for annular flows.

Comparison of the entrainment rate models

Figure 1.11 shows the evolution of the entrainment rates Γ_E predicted by several correlations as a function of the air superficial velocity for two pipe diameters. As these graphs have a logarithmic scale, the difference of amplitude prediction appears to be very important between entrainment rate correlations. One may note that Ueda's correlation predicts questionable results as the relationship is dimensional. In order to provide these evolutions, the interface level is assumed to be constant when increasing the air velocity and proportional to the pipe diameter such that the entrainment rates predicted by Hutchinson appear to be constant. The entrainment correlation of Kataoka et al. predicts a very small entrainment rate. However for fully developed flows, this entrainment correlation is a deposition correlation not developed for horizontal flows greater than 1 cm.

In 2009, Secondi et al. [138] have proposed an evaluation of several entrainment correlations on small vertical pipes against experimental data. Authors concluded that the most reliable correlations are those of Hewitt and Govan [67] and Okawa et al. [113] for pipe of small diameters.

All entrainment correlations presented above have been validated on the experimental results of their authors. Consequently, their expected ranges of validity depend mainly on these databases. The oldest correlations are then accurate only in very specific cases while the newest reach bigger ranges of validity. None of these models have been originally developed specifically for modeling the entrainment phenomenon in stratified flow in horizontal pipe. All were basically developed for annular flows and gradually adapted to stratified flows. Some models are not even modified and do not consider any differences between an annular and a stratified flow. However, most actual versions of these models take into account specific characteristic of a stratified flow: gravity, liquid film asymmetry, etc. The specific characteristics of these models are summarized on Table 1.10.

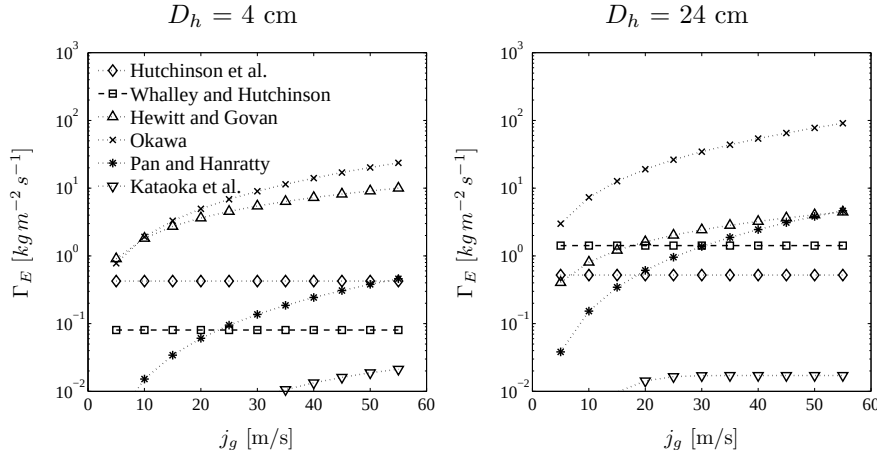


Figure 1.11: Logarithmic variation of the entrained rate as a function of the air superficial velocities j_g at $j_l = 0.02$ m/s

Most correlations do not consider the flow dynamics leading to the destabilization of the interfacial waves and hence the droplet entrainment. Most models do not represent the physics of the problem but interconnect the governing parameters of the flow. In this context, the model of Holowach [70] is particularly interesting as it highlights the mechanics underlying the entrainment in annular flows in vertical pipes. Unfortunately, and precisely because of this feature, this model is not applicable in horizontal pipe for flows governed by gravity. Indeed the direction of acting forces profoundly modifies the physics and the dynamics of the phenomenon.

In conclusion, the existing correlations for the entrainment are not suitable to the PWR geometry and the specific characteristic of a stratified flow. These findings force to reject these models. Factually none of them have been fitted on experimental data approaching the PWR geometry.

With the exception of the correlation of Pan and Hanratty [119], most entrainment relationships do not convince as they are not really designed for the type of flow to simulate. Unfortunately the correlation of Pan and Hanratty [119] deviates from experimental data of greater than 2 cm pipe diameter. A new correlation has then been developed based on a mechanistic approach of the entrainment phenomenon in stratified flow, similarly to the approach of Holowach [70] in vertical pipes.

	Annular flows in vertical pipes Stratified flows in horizontal pipes		Pipe diameter D	Liquid level h_l	Critical mass flow of liquid	Critical mass flow of gas	Stabilizing effect of gravity (Fr)	Weber based concept	Complete mechanistic approach
Hutchinson and Whalley	x			x				x	
Hutchinson et al.	x			x				x	
Ueda et al.	x			x				x	
Kataoka et al.	x		x						
Hewitt and Govan	x		x		x				
Tatterson and Hanratty	x			x				x	
Dallman and Hanratty	x		x		x				
Williams and Hanratty	x	x	x		x				
Pan and Hanratty	x	x	x		x				
Holowach	x			x	(x)	(x)			x
Ideal model		x	x	x	x	x	x	(x)	x

Table 1.10: Comparison of the various model characteristics

Table 1.11 reveals that most entrainment correlations are based on data in vertical pipes, of small diameter. Indeed very few experimental data on horizontal flows in pipe greater than 2 cm have been presented in the open literature. The following section aims at presenting these few experiments on horizontal pipe to estimate their validity to correlate an entrainment rate in a hot leg.

	Pipe diamter [<i>cm</i>]	Entrained Fraction			Entrainement rate					
		Ishii and Mishima	Oliemans	Pan and Hanratty	Kataoka	Hewitt and Govan	Schadel and Hanratty	Pan and Hanratty	Okawa	Holowach
Vertical										
Cousins et al.	0.95	x	x				x		x	
Cousins & Hewitt	3.18	x	x		x				x	
Hewitt & Pulling	0.93		x						x	x
Minh & Huyghe	0.6		x				x			x
Steen & Wallis	1.07	x								
Gill et al.	3.2	x			x					
Yanai et al.	1.2								x	
Wurtz et al.	1		x						x	x
Keey's et al.	1.27		x						x	x
Nigmatulin et al.	1.33		x						x	
Whalley et al.	3.18		x				x		x	
Singh et al.	1.25								x	x
Bennet et al.	0.93								x	
Owen et al.	3.18								x	
Asali et al.	2.29						x		x	
Andreussi et al. [13]	2.29						x			
Leman et al. [93]	4.2						x			
Horizontal										
Dallman	2.31			x					x	
Laurinat	5.08			x					x	
Paras & Karabelas	5.08			x					x	
Williams	9.53			x					x	

Table 1.11: Experiments used to correlate the various relationships for entrainment.

1.3.4 Experimental facilities

The design and construction of experimental facilities dedicated to the study of two-phase flows began in the 1950's. Most experimental facilities have been dedicated to vertical flows, heat transfer simulations or specific geometries. Facilities dedicated to horizontal stratified are not numerous in the literature. However, some issues have been extensively studied as the mechanisms for the transition from stratified to annular flow or the transition to slug flows. Droplet formation, transport and extinction in horizontal flows has been little analyzed in the open literature.

In vertical flows, the measurement of the droplet entrainment is easily achievable using a liquid film extraction method, like Bertodano et al. [99] or Andreussi et al. [11]. Indeed, the action of gravity on annular flow in vertical pipe has no impact on the flow distribution so that the liquid film is homogeneously distributed on the contour. The entrained fraction of liquid (droplets) is deduced from the difference between entering and extracted liquid mass flow rates. Measurements for air and water flows in a vertical pipes have been obtained by e.g. Whalley et al. [160], Gill et al. [58], Collier and Hewitt [35]... (see experimental data used to correlate entrainment models in the previous section).

The gravity action in horizontal pipes involves the flow stratification and hence the liquid film asymmetry around the pipe walls. Achieving an entrainment measurement (E or Γ_E) using a liquid film extraction method is therefore very complicated in horizontal flows. Indeed, most experimental facilities dedicated to horizontal flows are designed to study either the mechanisms of the liquid film formation either the flow regime transitions. The liquid film properties have been extensively studied by Pletcher and McManus [127], Hanratty and coworkers: Dallman [41], Laurinat [91, 89, 90], Williams [164] or by Butterworth and Pulling [32], Fisher and Pearce [50] and Lin et al. [95] although only few studies present statistical analyses of the liquid film: Sekoguchi et al. [139], Fukano et al. [52], Paras and Karabelas [120], Jayanti and Hewitt [79].

Droplet mass balance is still quite unresolved in stratified flows. Although few authors have studied the droplet deposition (Anderson and Russel [10], Butterworth [74] or James et al. [78]), they obtained somewhat diverging results. Yet the entrainment phenomenon is still quite unresolved even if Paras and Karabelas [120], Williams [165], Tayebi et al. [150], Fukano et al. (1989) [51] or Mantilla et al. [101] proposed studies including droplet flux measurements. These few experimental facilities will be evaluated in order to determine if their experimental data are representative of a stratified flow occurring in a hot leg.

This evaluation is based on the flow conditions and two geometrical parameters describing the facility scale effects: the diameter of the horizontal test section D and the analyzed $\frac{L}{D}$ ratios corresponding to the experimental device locations.

In 1991, Paras and Karabelas [120] published the experimental results of a study of the mechanisms leading to the liquid film formation at the top of an annular flow in horizontal conditions. Their work included experimental studies of the liquid film properties along the pipe circumference and experimental data of the droplet spatial distribution. These studies carried out very interesting conclusions like the importance of the gravity effect on the film characteristics or the droplet distribution. However the application of these data to a hot leg stratified flow modeling is problematic because of the large scale difference. Indeed as their facility had not been designed for simulating non fully developed flows, the diameter and the $\frac{L}{D}$ ratio at the test section were respectively equal to 0.0508 m and 300. Authors studied an air-water annular flow in a range of velocity from $j_g = 31$ m/s to 66 m/s and from $j_l = 0.019$ m/s to 0.2 m/s, depicted on the map flow on Figure 1.12 ().

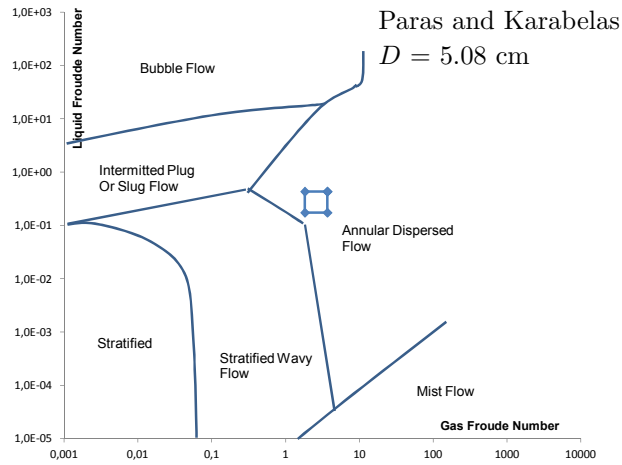


Figure 1.12: Taitel and Dukler's flow map [146] investigated by Paras and Karabelas in horizontal flows.

Hanratty and coworkers developed several experimental setups of increasingly large dimensions. Indeed Dallman et al. [41] in 1984 studied an air-water annular two-phase flow in a test section of only 2.54 cm in diameter (see Figure 1.13) whereas Laurinat et al. [90] investigated a flow in a pipe section of 5.08

cm in 1985. Finally in 1990, Williams [165] proposed a study of the effect of the pipe diameter on horizontal annular two-phase flows. The gravity effect on the liquid film characteristics and on the droplet concentration was extensively studied. The range of velocities extended from $j_l = 0.02$ m/s to 0.08 m/s and from $j_g = 26$ m/s to 88 m/s, as their study object was essentially annular flows (see Figure 1.14). Williams positioned the test section at a $\frac{L}{D}$ ratio of 260 to achieve an established flow. To determine this location, Williams [165] referred to the study of Dallman [41] in which the entrained fraction E did not evolve with the z location between $\frac{L}{D}$ ratio of 230 and 500 for a 2.52 cm pipe diameter.

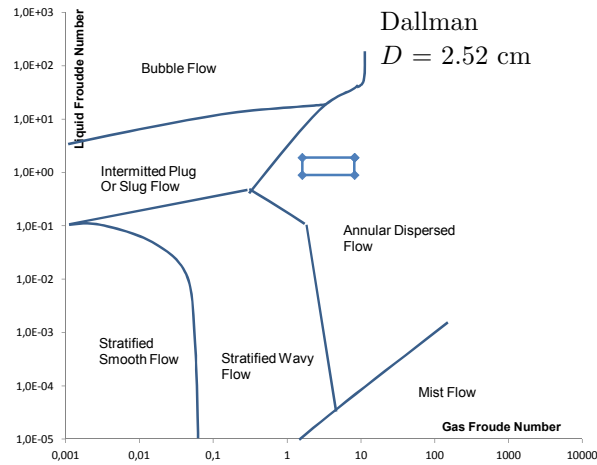


Figure 1.13: Map flow investigated by Dallman in horizontal flows.

Tayebi et al. [150] proposed a study of a two-phase stratified flow of water and hexa-fluorid sulfur at a pressure of 8 bar. This couple of fluids presents a density ratio quite similar as a water-natural gas stratified flow at 60 bar as this facility is designed for a completely different purpose. Their experimental setup had a length of 20 m and a diameter of 10 cm.

Fukano et al. [51] published a droplet flux study in horizontal flows. The experimental facility is a 7 m long channel of 4 cm square section. The studied range of velocities extended from $j_g = 10$ m/s to 40 m/s and from $j_l = 0.001$ m/s to 0.1 m/s. The water is injected through a slot in the pipe, creating waves and enhancing their atomization in order to control the atomization and entrainment initialization. The slot is 0.006m long on the whole width of the

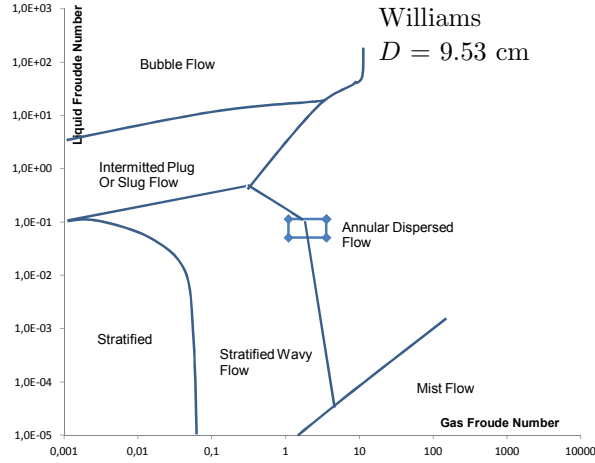


Figure 1.14: Map flow investigated by Williams in horizontal flows.

square section. The test section is in Plexiglas for the flow visualization.

More recently, Mantilla [101] extensively studied an air-water two-phase flow in an horizontal test section of 4.86 cm pipe diameter and a $\frac{L}{D}$ ratio of 253 and in a inclined 15.3 cm pipe diameter but limited to a $\frac{L}{D}$ ratio of 53.

In the 2000s, the CEA has presented the MHYRESA test facility results [54, 57, 25], studying the entrainment of liquid from the upper plenum to the steam generator inlet chamber. In a very large pipe of 0.351 m pipe diameter, the droplet entrainment has been measured in ambient air-water conditions on a large set of air velocity range.

Conclusions

Table 1.12 summarizes the experimental setup characteristics. Some parameters are not specified in every study but the table highlights that every experimental facility has a diameter between 1 and 10 cm and is designed primarily for fully developed flow as the test section location always overcomes 200.

A PWR hot leg has an internal diameter of about 0.72 m and a length of 6.4 m straight pipe and 2.5 m of bend. Figure 1.15 compares the geometrical parameters of the various experimental setup of the literature to the PWR

	Fluids	D cm	L/D -	P bar	$j_l \times 10^2$ m/s	j_g m/s
Dallman	air-water	2.52	550	NS	2 - 50	20 - 100
Laurinat	air-water	5.08	300	NS	1 - 12	20 - 100
Paras-Karabelas	air-water	5.08	300	7	1.9 - 20	31 - 66
Williams	air-water	9.53	260	1	2 - 8	26 - 88
Fukano et al.	air-water	4	NS	NS	0.1 - 10	10 - 40
Tayebi et al.	SF6-water	10	NS	8	25	3.5 - 7
Mantilla et al.	air-water	4.86	253	2	0.35 - 1.8	15 - 80

Table 1.12: Experimental facility characteristics (NS: not specified).

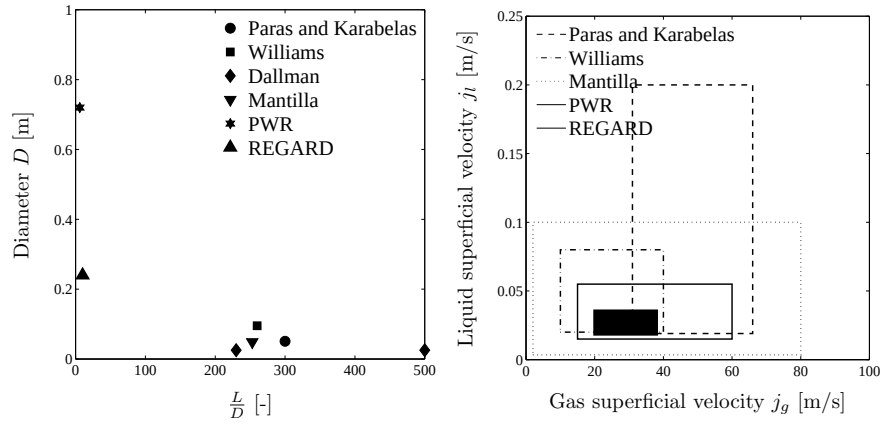


Figure 1.15: Geometrical parameters (left) and analysed range of velocities (right) of many experimental facilities from literature compared to the geometry of a new facility REGARD (solid black) and of a PWR (filled black).

hot leg geometry (and to the REGARD experimental facility that is described in details in the following). Every experimental setup has characteristics very different from those of PWR, as their L/D are very large in order to ensure a fully developed flow and their diameters are far smaller than the hot leg diameter. Indeed, the scale of the greater setup does not overcome $1/8$ in diameter.

Figure 1.15 presents a comparison of the studied range of superficial velocities. These ranges appear to be highly extended and overlapping. However, the superficial velocities are representatives of the mass flow rate ratios and not of the velocities. As highlighted by Williams [165], most phenomena acting on the flow regimes depend differently of the pipe diameter. Thus the pipe diameter has a significant effect on the liquid circumferential distribution and then on the flow regime so that a similar couple of superficial velocities could be

symptomatic of two different flow regimes according to the pipe diameter. The conditions leading to an annular flow on the experimental setup of Dallman [41] may generate a stratified flow in a PWR hot leg. In this way the comparison between Figures 1.12, 1.13 and 1.14 highlights that most authors have essentially studied annular flows.

This literature review of the experimental facilities dedicated to the droplet presence in horizontal flows indicates that the available data are not suited for the validation of a model for the entrainment rate aiming at a hot leg entrainment simulation. Indeed, their dimension scales are too far from a PWR geometry and data on entrained fraction E obtained in a fully developed flow are not relevant for the modeling an entrainment rate Γ_E .

1.4 Methodology

The objective of this work is to provide a model of the entrainment phenomenon suitable to the modeling of a stratified flow in a hot leg of a PWR. As shown in the literature review, current models and current experimental facilities are not suited to represent this phenomenon. The second chapter of this work describes the new experimental facility designed to simulate this type of flow. The experimental data are analyzed in the third chapter. The fourth chapter proposes a new model of the entrainment rate.

1.4.1 A new experimental facility: REGARD

A new experimental facility named REGARD (Recherche Experimentale sur les Gouttes, leur ARrachment et leur Depôt) has then been designed. This installation is dedicated to the validation of the 1D numerical simulation codes for the two-phase horizontal flows which may appear during a LOCA accident in a PWR.

The REGARD facility is primarily designed for investigating the droplet mass transfer phenomena: entrainment and/or deposition. Indeed the droplet entrainment and deposition phenomena are closely related, but the facility is designed to study them separately. The complete test campaign plans to conduct these studies in several geometrical configurations goes from an horizontal pipe to a complete hot leg connected to the water box of the steam generator. However, only experimental campaigns in horizontal pipe have already been achieved and are considered within the framework of this thesis. So the REGARD facility reproduces a stratified flow in a horizontal pipe similar to the type of flow appearing in the hot leg of a PWR during the core reflooding process, without taking account of the upper plenum, the bend and the steam generator.

REGARD works with a main circuit of air, which is coupled to a circuit of water to simulate a stratified flow in the test section. The REGARD facility is designed to match the behaviour of a non-fully developed flow. Consequently this test section consists of an horizontal straight pipe of only 4 m long for an internal diameter of 0.24 m. This diameter is particularly important, almost three times greater than other existing experimental facilities [41, 101, 165] but still three times smaller than a hot leg diameter. The geometrical characteristics of REGARD are compared to standard parameters of a PWR in table 1.13. Among other experimental setups, REGARD differs essentially by these geometric parameters: the diameter D and the length to diameter ratio $\frac{L}{D}$.

	L	D	P	T	Fluids
	[m]	[m]	[bar]	[°C]	
PWR	6.4 + 2.5 (bend)	0.75	3	130	steam - water
REGARD	4	0.24	1.1	30	air - water

Table 1.13: Comparison of the geometric characteristics between REGARD and a PWR hot leg.

		PWR hot leg steam - water	REGARD air - water	
Temperature	T	133.5	30	$^{\circ}C$
Pressure	p	3	1.1	bar
Gas density	ρ_g	1.65	1.26	kg/m^3
Liquid density	ρ_l	932	996	kg/m^3
Gas viscosity	μ_g	$13 \cdot 10^{-6}$	$19 \cdot 10^{-6}$	$Pa \cdot s$
Liquid viscosity	μ_l	$207 \cdot 10^{-6}$	$798 \cdot 10^{-6}$	$Pa \cdot s$
Surface tension	σ	0.052	0.07	kg/s^2

Table 1.14: Comparison of the fluid physical properties between REGARD and a PWR hot leg. The velocity values are varying so that the dimensionless numbers are not presented in this table.

This experimental facility produces an air-water two-phase flow. Introduced at operating pressure and temperature conditions close to the atmosphere, this pair of fluids presents some similar characteristics to the steam flow occurring in the PWR accident to simulate. Indeed, from a physical point of view, the temperature difference between the vapor and the liquid is low in the hot leg during the core reflooding process, especially when the droplet entrainment is large enough. Working with a two-phase flow of constant temperature, without heat transfer, does not significantly change the physics of the problem. Even if the temperature which is kept constant at $30^{\circ}C$ is different from the temperature in the hot leg during the accident. Indeed, from a mechanical point of view, the air-water stratified two-phase flow at low temperature and nearly atmospheric pressure presents a density ratio and a surface tension close to the steam-water flow at higher temperature and pressure (see Table 1.14). The REGARD flow characteristics are then very similar in many ways to the physical phenomenon to reproduce. Furthermore, the use of such a two-phase flow greatly facilitates the system design as this does not require to maintain the

entire system under pressure and at high temperature. Similarly, the relaxation of the pressurization constraint greatly simplifies an already complicated design. Moreover the air-water two-phase flow facilitates the visualization of the flow. Therefore the pipe test section is made of Plexiglas allowing the use of optical non-intrusive measurements which are a significant support for the validation of measuring devices observations. However the air-water two-phase flow implies several differences in fluid properties as depicted in Table 1.14.

1.4.2 A new mechanistic model of the entrainment

The literature review concluded that existing models of the entrainment rate are not suited for the droplet behaviour modeling during the core reflooding process of a LOCA. A new droplet entrainment rate model from a wavy gas-liquid interface has then been developed. The global approach is based on the modeling of the phenomenon sequence leading to the entrainment situation. The formulation of the entrainment rate Γ_E requires to model different steps of the entrainment process: the wave formation, the wave fragmentation, the droplet formation... These models are developed on a mechanistic basis in order to represent the physics of the phenomenon and especially the hydrodynamics. This mechanistic approach suits more accurately to the modeling of transient phenomena since the inception of the phenomenon is taken into account in the sub-models.

The proposed model is based on the observations of wave formation from Woodmansee and Hanratty [166] and confirmed by REGARD high-speed camera visualizations (see Chapter 3.2). The droplet formation mechanism occurs from an instability development which grows in the pipe. This instability reaches an unstable amplitude resulting in the wave fragmentation such that a part of the wave crest is sheared off and blown away by the gas, as shown Figure 1.16. This liquid part then breaks down in several droplet of smaller dimensions.

As the droplet entrainment primarily occurs from a wave fragmentation, the wave characteristic prediction is the subject of the first sub-model of the entrainment modeling. The wave fragmentation is then modeled to quantify the amount of droplets formed from the atomization. In consequence, the proposed model is not primarily a correlation between various dimensionless numbers (Reynolds, Froude, Weber, ...) but the creation of a new dimensionless number representative of the entrainment as proposed in Equation 1.73:

$$\frac{\Gamma_e}{X} = 1 \quad (1.73)$$

The proposed model methodology is inspired from the entrainment model of

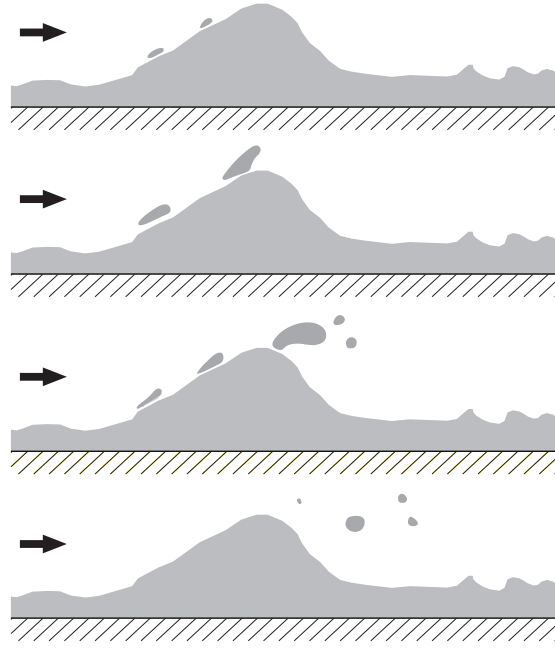


Figure 1.16: Mechanistic description of the droplet entrainment phenomenon described by Woodmansee and Hanratty

Holowach et al. [70, 71] for annular flows in vertical pipe. However the adaptation of this methodology to horizontal pipes highly differentiates every sub-model as the gravity force acts completely differently. Indeed in a vertical configuration, the gravity applies in the flow direction while in horizontal pipes, its influence acts perpendicularly.

The droplet quantity entrained by the gas corresponds to the amount of liquid sheared-off from the waves traveling at the liquid interface. The modeling of the wave development (from growth to fragmentation) and the wave presence over a pipe section lead to an estimation of the entrainment rate as depicted in Equation 1.74. The entrainment rate corresponds to the amount of liquid torn from each wave crests ($N_w \rho_l V_E$) per units of area (A_{CV}) and time (τ_C).

$$\Gamma_e = \rho_l \frac{V_E N_w}{A_{CV} \tau_C} \quad (1.74)$$

This global model requires to model for a defined control volume the characteristic time of the phenomenon (τ_C), the entrained volume at each wave crests (V_E), the number of waves (N_w) and the interfacial area (A_{CV}).

As all these elements depend on the wave geometry, the first developed sub-model consists in a linear stability analysis of the two-phase flow in order to

predict the most amplified wavelength λ_c . The general model is based on the hypothesis that the whole spectrum of amplified wavelengths can be reduced to this single critical wavelength λ_c . The REGARD data on the interfacial frequency spectrum provide a first assessment of this model. The entrained volume (V_E) is then modeled on the basis of a force balance on the wave crest in order to predict the amount of liquid entrained by the flow. The interfacial area is directly obtained from the pipe geometry, few assumptions and REGARD informations on the interface shape. Finally the characteristic time of the entrainment (τ_C) is based on the wave time necessary for the growth of the wave to the critical height. The wave atomization frequency depends then on a ratio between this characteristic time and the time necessary to travel the control volume. This last element is directly based on REGARD experimental data which provide the wave velocity and the wave growth rate.

Chapter 2

Experimental setup and measurement techniques

The previous chapter presented a detailed review of the droplet entrainment predictions which are necessary to correctly simulate the hot leg behaviour. This literature review led to the conclusion that a new model for the entrainment rate is necessary. To perform the validation of this model, several experimental data from a facility with similar characteristics as those of a PWR are required to ensure that the model predictions match the physical reality. Indeed, the characteristics and the published results of existing facilities do not generally account for the stratified flow occurring in the hot leg of a PWR. Therefore a new experimental setup has been designed: the REGARD facility. This setup makes it possible to create locally stratified flow conditions quite similar to those expected in the hot leg.

2.1 Piping description

The overall circuit of REGARD is composed of several elements to generate an air-water two-phase flow in the test section. REGARD works with a main air circuit coupled to a water circuit at the inlet of the test section as depicted in Figure 2.1 and uncoupled by the outlet separator. This test section consists of an horizontal straight pipe of only 4 m long for an internal diameter of 0.24 m to match the behaviour of a non-fully developed flow.

2.1.1 Overall circuit: air and water loops

Figure 2.1 highlights the open circuit of air (in green) and the close circuit of water (blue) joined at the test section (light blue) where air and water are sep-

arately injected. The flow crosses the test section and then enters a separation chamber where the liquid film falls to a tank and most droplets deposit over the wall. The air reaches an upward tank where the remaining droplets are collected before the air is rejected to atmosphere.

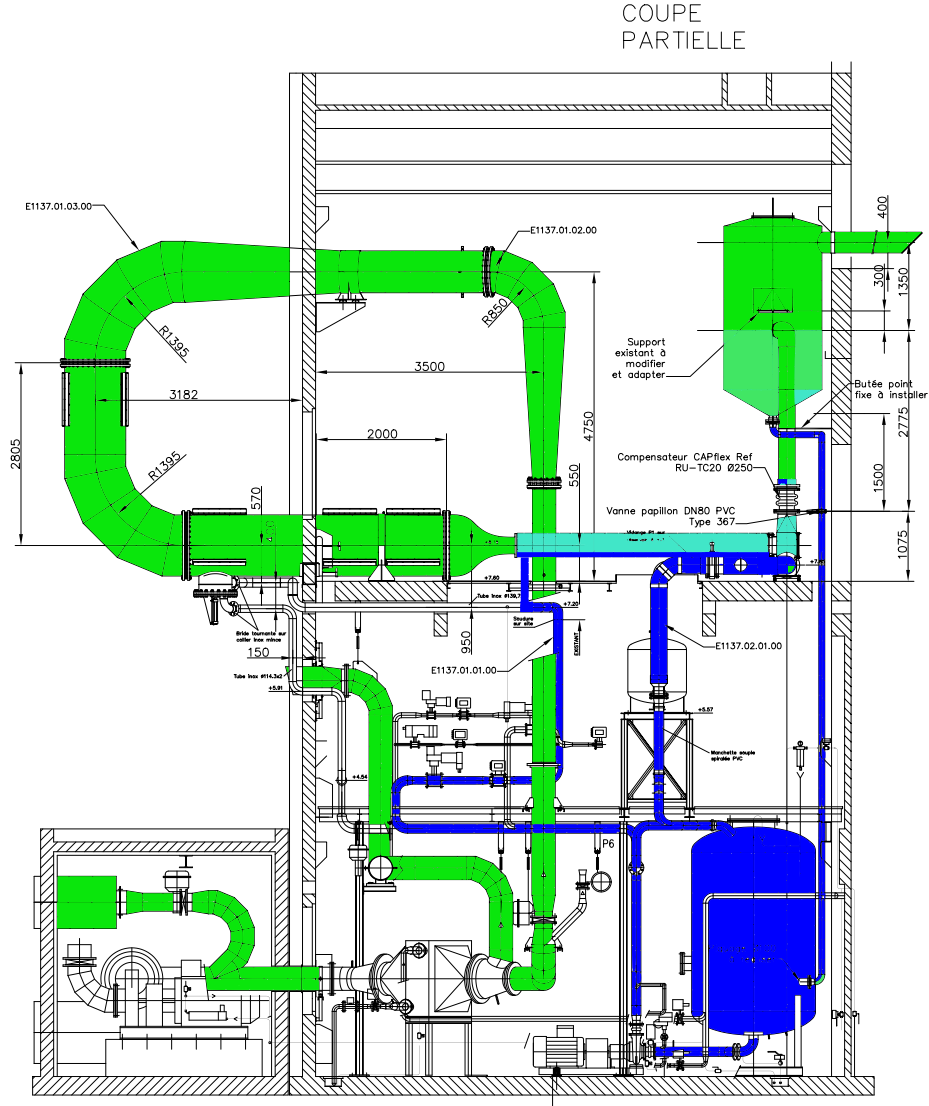


Figure 2.1: Schema of the REGARD facility with highlights of the air open loop (green), the water closed loop (blue) and the test section (light blue).

The air circuit is an open loop in which the air is continuously sucked from outdoor and rejected back outside beyond the separator (depicted in Figure 2.1). A fan draws air at a pressure slightly under 1.5 bar at a mass flow rate $\dot{V}_{N,air}$ between $3000 \text{ Nm}^3/\text{h}$ and $6000 \text{ Nm}^3/\text{h}$. This design presents the disadvantage

that the air characteristics (pressure, temperature and moisture) at the intake change over time (during an experiment and between two experiments) but ensures the absence of pre-existent droplets in the air stream. As the air intake conditions vary over time, a heat exchanger has been placed in the circuit to regulate the temperature of the air flow at 30°C . The air characteristics (pressure and temperature) are then controlled, contrarily to the air humidity which continuously depends on atmospheric conditions.

A flow homogenization system has been installed to ensure a flow as laminar as possible at the test section inlet. The description of the different elements used in the elbows of the circuit to get this homogenization can be found in the technical CEA report [125].

Unlike the air loop, the water loop is a closed circuit. This design permits to limit the water consumption. The water loop is equipped with pumps and also a heat exchanger to maintain the flow at a temperature near 30°C and a flow rate $\dot{V}_{water}$ between $3 \text{ m}^3/\text{h}$ and $6 \text{ m}^3/\text{h}$. The water loop injects the water in the injection segment of the test section and recovers the liquid downstream the separator. The flow-rate ratios $\frac{\dot{V}_{N,air}}{\dot{V}_{water}}$ vary then between 500 and 2000 $\left[\frac{\text{Nm}^3_{air}}{\text{m}^3_{water}}\right]$.

Instrumentation

Flow meters, pressure and temperature sensors are placed in many locations of the different circuits in order to control the flow conditions. The REGARD facility is fitted with many temperature sensors (T_x) and pressure sensors (p_x) in the water and the air loops as shown in Figure 2.2: upstream and downstream the heat exchangers of the air loop and the water loop. The test section (SE) is not equipped with this type of sensors as the pressure drops and temperature fluctuations in the loops and the test section are assessed to be negligible. A system of valves and flow meters controls the mass flow rates in the circuits.

The major loop instruments are then the flowmeters. The flowmeter accuracy is a crucial parameter that depends on the range of flow rates and directly involves the measurement quality in the test section (see calibration curves in [125]). The liquid flowmeters are calibrated such that their uncertainties remain under 5% of the measured values. The air flowmeters are calibrated such that the uncertainty remains under $\pm 10 \text{ Nm}^3/\text{h}$. As the air intake is regulated by a complex system of valves, an auto-regulation system continuously balances the air flow. The set-point of air and liquid flow is set manually by the operator so that the main uncertainties come from his visual appreciation. Therefore, the uncertainty on the air flow is extended up to $\pm 50 \text{ Nm}^3/\text{h}$ (less than $\pm 2\%$).

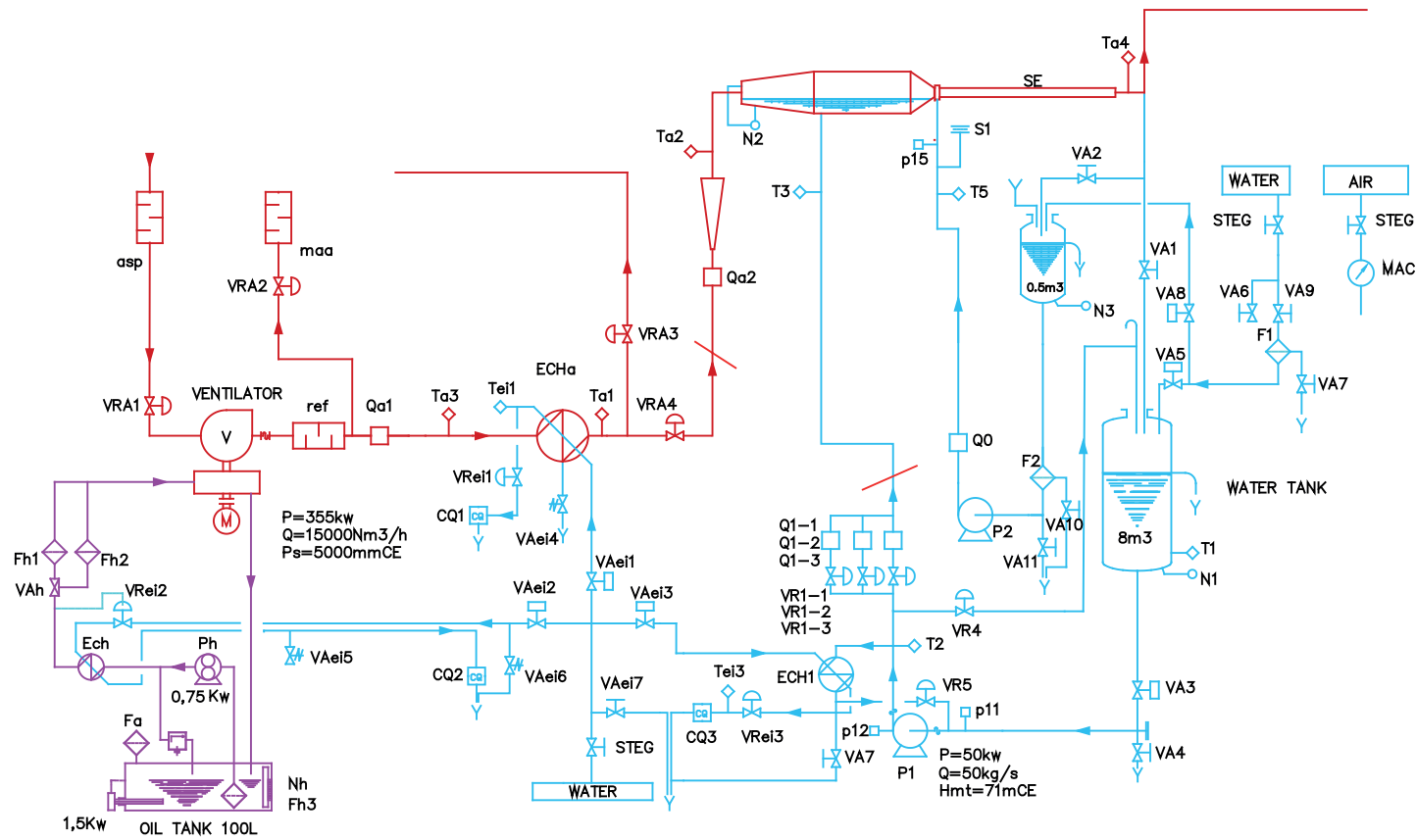


Figure 2.2: Schematic representation of the REGARD facility

2.1.2 The test section

REGARD is designed for the simulation of a non-fully developed wavy stratified flow in an air-water two-phase flow. As the flow conditions are controlled in the overall circuit, the test section is dedicated to the measurement of the flow characteristics and to the flow visualizations. Therefore, the test section consists in a Plexiglas pipe of 4 m long, composed of several interchangeable parts on which are mounted the various measurement devices. The internal pipe diameter is 0.24 m. The total length is achieved by combining two pipe element sizes: Plexiglas pipes of respectively $3.9 \frac{L}{D}$ and $1.95 \frac{L}{D}$ long ; and several connecting flanges.

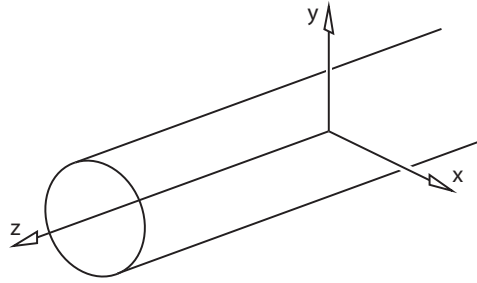


Figure 2.3: Denomination of the three axis.

Among the several flanges and pipe sections, three elements are of particular interest:

- o A large segment of $3.9 \frac{L}{D}$ long has been fitted with the liquid injection so that its location should remain at the test section inlet.
- o An interchangeable large segment has been fitted with an isokinetic probe and a laser system (described in Section 2.2). As this instrumented segment is designed to ensure a rotation around the z-axis in order to reach every location over a cross section, this segment is equipped with two irremovable flanges.
- o Finally an instrumented flange is mountable in every flange position.

Unfortunately this design does not enable the measures of the various experimental devices to be achieved at the same location at the same time. As every segment, but the injection, is interchangeable, the two instrumented elements are mountable at various axial locations along the test section, in order to get data at several different $\frac{L}{D}$ ratios. Figure 2.4 presents a particular example of these arrangements. At the outlet section, the air and water circuits are decoupled as the two-phase flow enters the separator. The centrifugal force separates

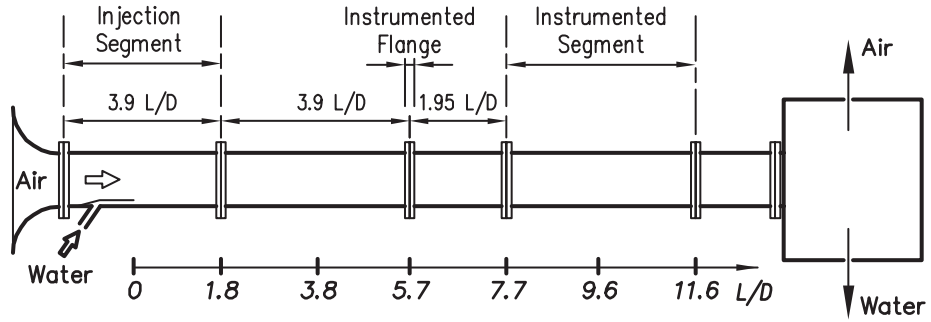


Figure 2.4: Diagram of the Plexiglas test Section with the injection segment, the instrumented segment and the instrumented flange.

most of the droplets from the air flow. The air returns to the ambient while the liquid is collected in a tank to be injected back into the channel inlet.

The liquid injection segment

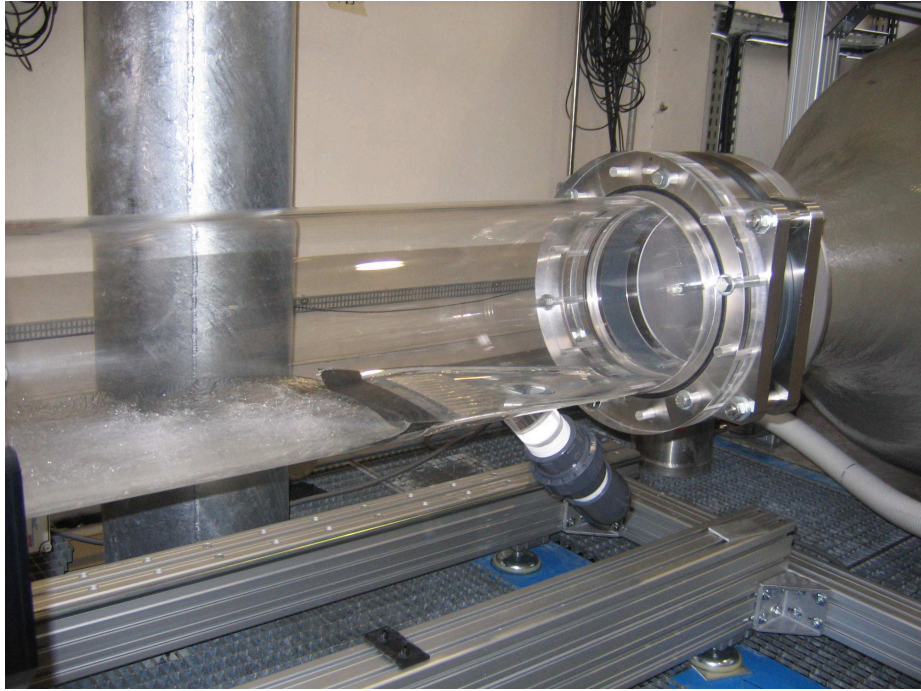


Figure 2.5: Visualization of the water inlet in the test section.

Air and water join in the injection segment as shown on the photography of Figure 2.5. The water inlet nozzle is connected to the bottom of the first segment, while the air loop carries the air flow at the upstream section of this

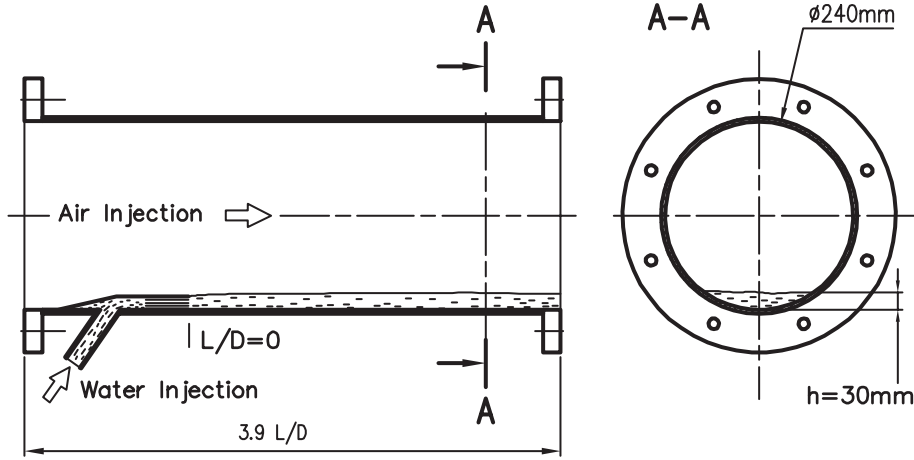


Figure 2.6: Front and lateral view of the liquid injection segment.

segment. In order to ensure an horizontal injection of the liquid, the water inlet is followed by an horizontal plate located 3 cm over the pipe bottom for stratifying the liquid film as depicted in Figure 2.6. By this mean, the water inlet device introduces the liquid flow parallel to the air flow, that is to say horizontally. The water is channeled through some honeycombs (see Figure 2.6) to force a uniform velocity distribution. The contact between the two fluids is considered as the starting point of the two-phase flow and assigned the zero value to the $\frac{L}{D}$ ratio as shown in Figure 2.4.

Instrumented segment and flange

The instrumented segment is a large pipe section fitted with most of the instrumentation and movable in five different axial locations of the test section. The instruments are positioned at the middle of the segment such that the analyzable lengths are reduced to $\frac{L}{D}$ ratio of 3.8 to 11.6 (by steps of 1.95) from the water injection as depicted in Figure 2.4: $\frac{L}{D} = 3.8, 5.7, 7.7, 9.6$ and 11.6 . As the main objective of REGARD is to assess the droplet characteristics, most experimental devices are intrusive in the test section. In order to reach every location in a cross section, the instrumented segment has been designed for rotating around the longitudinal axis z . As this pipe section has been truncated along the instrumented segment to avoid optical deviations, all the experimental devices also rotate with the segment.

The instrumented flange contains the BANJO device dedicated to the liquid film investigations. This element can be mounted at various locations from $\frac{L}{D} = 3.7$ to 11.5 ($0.1 \frac{L}{D}$ upstream from the instrumented segment).

2.2 Instrumentation

The previous paragraphs describe the characteristics of the REGARD facility in terms of operating conditions and geometrical design. This section aims at presenting the complete and innovative instrumentation of REGARD which is fitted with 4 accurate and useful measurement devices:

- o The BANJO. Placed on the instrumented flange, the BANJO is a resistive wire network probe used to quantify the characteristics of the liquid film: liquid film height, liquid film curvature, interfacial waves frequency, wave velocity, etc.
- o A laser particle analyzer is placed on the instrumented segment. This laser instrument is a three laser beam system designed to quantify the main droplet characteristics: three velocity components (Laser droplet velocimetry, LDV) and diameter (Laser granulometry).
- o An isokinetic sampling probe is also placed on the instrumented segment just downstream of the Laser measurement cross section. This intrusive device is designed to measure the local mass flow rate of droplets anywhere over a cross section.
- o A high speed camera can be placed on the instrumented segment instead of the isokinetic probe in order to make inside flow visualizations of the liquid film interface. This apparatus aims at confirming entrainment theories from the literature and thus validate the phenomenon sequence describing the entrainment.

2.2.1 The BANJO flange

The BANJO is a resistive wire probe system placed on the instrumented flange (Figure 2.8), movable at various axial locations of the test section. This instrument is used to continuously measure 9 levels of liquid film using an electrical impedance measurement. The BANJO flange is fitted with nine pairs of resistive wires positioned to make a 3x3 mesh, over three consecutive cross sections, spaced 0.01 m and 0.02 m as depicted on Figure 2.7. At each pipe section, the three pairs of wires are equally distributed, spaced by a quarter diameter. All wires are vertical and not positioned along tilted diameter of the pipe like Paras and Karabelas [120, 165].

A measurement of the impedance of a pair of parallel wires, stretched in the flow, is used to find the height wetted by the liquid, and hence the relative height of the liquid film. Indeed, the part of the wire flooded in the liquid has a lower

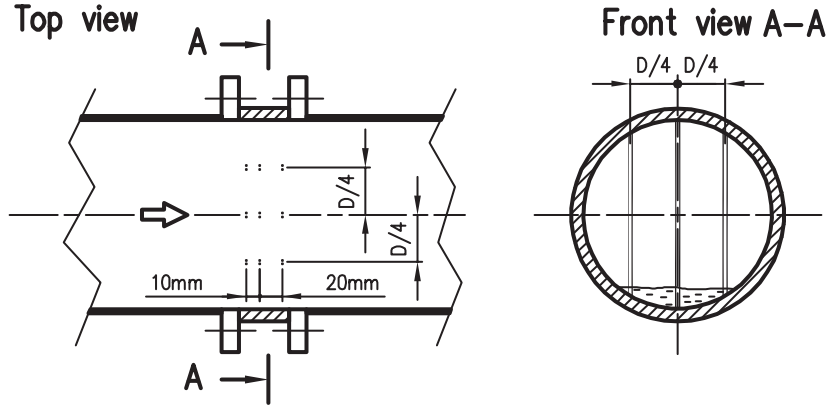


Figure 2.7: Top view and cross sectional view of the 9 pairs of wires.

electrical resistance than the dry part in the air. So by exciting one of the two wires of the pair with a sinusoidal signal and measuring the electric response on the second wire, the flooded length can be estimated. The methodology requires an initial calibration, based on two impedance measurements. This type of instrument is generally based on a capacitive impedance measurement. But for this facility which operates with tap water and not with de-mineralized water, water conductivity permits resistive impedance measurement, much simpler to fabricate than capacitive impedance measurement.:

- o A first measurement in the air only (without water flow) provides the initial (and so maximum because the wires will be totally dry) impedance of the pair of wires: $Z_{max.}$.
- o A second measurement in the water only (without air flow). Without air flow, the liquid interface is plane and the measurement of the impedance of the liquid film corresponds to a 0.03 m height at the center of the pipe: $Z_{3cm.} = Z_{min.}$

The measurement made by each wire pair is an electrical information which must be converted into a liquid film height. This calibration has been found by Pellisson [125] to evolve linearly between the minimum and maximum values previously defined so that the level is calculated based on:

$$h = 0.03 \times \frac{Z_{measured} - Z_{max.}}{Z_{min.} - Z_{max.}} \quad (2.1)$$

The accuracy of a measurement is then estimated between 2% and 5% including

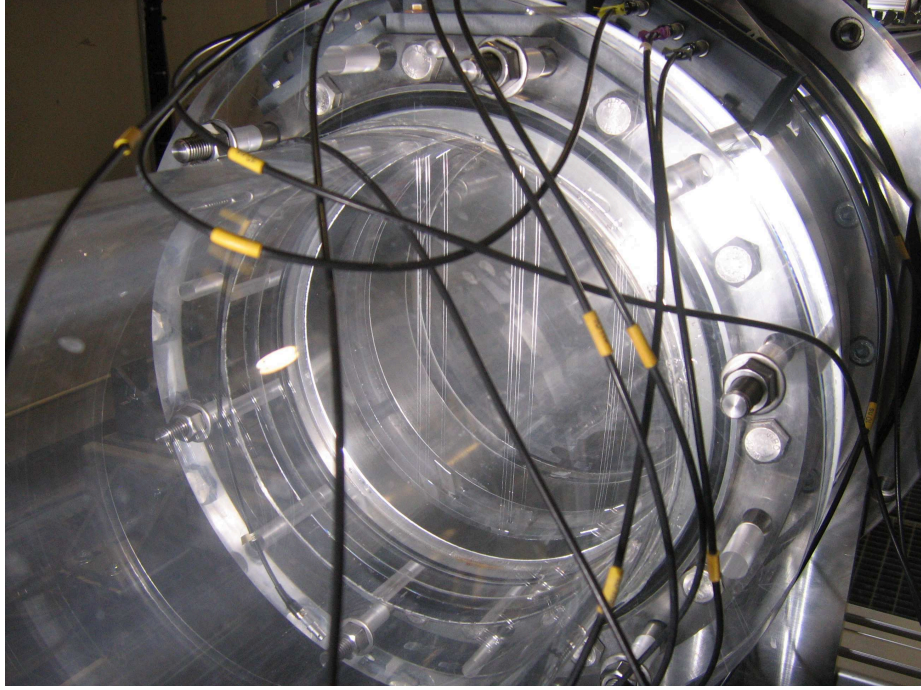


Figure 2.8: Photography of the instrumented flange and the BANJO

the measuring chain uncertainties based on:

$$\frac{\epsilon_h}{h} = \sqrt{\left(\frac{\sqrt{\epsilon_{Z_{meas.}}^2 + \epsilon_{Z_{max.}}^2}}{Z_{meas.} - Z_{max.}}\right)^2 + \left(\frac{\sqrt{\epsilon_{Z_{max.}}^2 + \epsilon_{Z_{min.}}^2}}{Z_{min.} - Z_{max.}}\right)^2} \quad (2.2)$$

For the off-centered six pairs of wires, a simple trigonometric calculation provides the initial liquid height (1.66 cm) at their location when a liquid flow is forced with a 3 cm height at the pipe center without initiating air loop.

In conclusion, the BANJO flange provides nine simultaneous measurements of the liquid level at 9 different but close locations. The interpretation of these data contributes to the validation of the flow model. Indeed three distant liquid levels in a same cross section provide an indication on the average interface curvature. An estimation of the liquid film fraction α_{lf} can then be achieved. Moreover three adjacent signals in the axial direction provide an information on the axial evolution of the liquid interface level. A cross correlation of these signals provides an estimation of the axial and the growth velocities of the waves. Furthermore the separate analysis of each signal provides an indication of the frequency spectrum of the interfacial waves and then of the most amplified wavelength of the liquid film interface.

2.2.2 The LASER granulometer and velocimeter

The particle analyzer is a three component laser equipment used to measure locally the main droplet characteristics: three velocity components and the droplet size. Placed on the instrumented segment, this instrument can be moved in various axial locations.

The particle analyzer is composed of three laser probes. Two of them consist in laser emitting probes, the third is dedicated to the diameter estimation by Doppler phase measurement. The first emitting probe emits two blue beams and two green beams, the second probe emits two purple beams (see Figures 2.12 and 2.9).

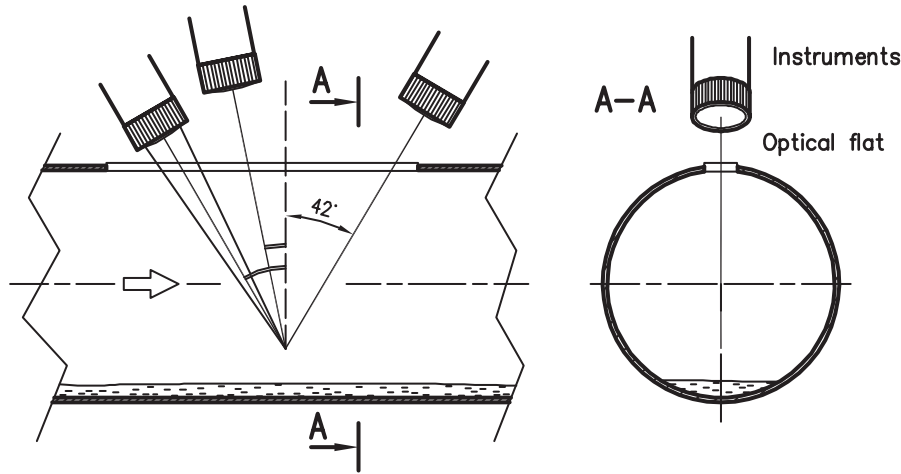


Figure 2.9: Visualization of the particle analyzer and the high-speed camera: granulometer (left probe), emitting beams (center and right probes).

Two laser beams of the same color are designed to converge at a predefined focal surface. The reflection of each beam on a solid or liquid particle crossing this focal surface will create interference fringes at the associated sensor. A simple calculation based on the characteristics of these fringes provides an estimation of the particle velocity in the direction perpendicular to the focal point-sensor axis. The complete methodology and the laser beam characteristics are provided in a CEA technical report [125] (not public). The two laser probes emitting three color beams as shown on Figure 2.9, give access to three velocity components of the particle.

As the pipe segments are cylindrical to get better simulation of a hot leg, the

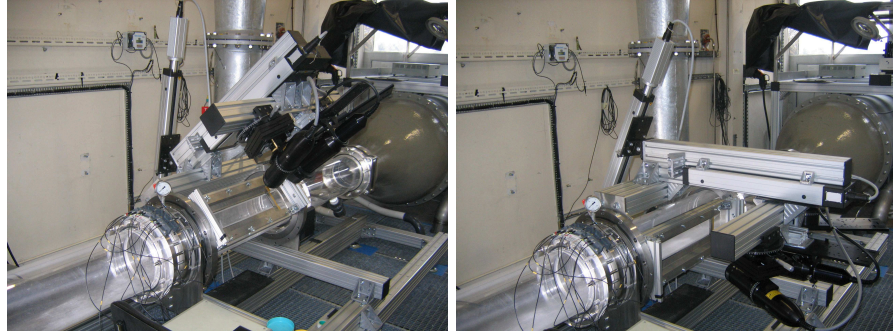


Figure 2.10: Visualization of the particle analyzer rotation at 30° (left) and 90° (right).

upper part of the instrumented segment has been truncated to get a flat surface of 5 cm width at the top of the segment as shown in Figure 2.9. Consequently each optical instrument suffers from less distortion and diffraction.

The third probe is a receptor for one color beam, used to estimate the diameter of the droplets crossing the focal volume. The receptor collects through three parallel slots the reflected beams from the particle surface. The phase shift between the three slots provides an information about the diameter of the particle. The complete methodology and the post-processing are described in [116]. The angle between the emitting probe and the receptor has been selected to be greater than 42° in order to avoid droplet internal reflection [125].

While the BANJO is placed on the instrumented flange, the particle analyzer is located at the center of the instrumented segment. The three probes are mounted on a moving and rotating structure to locate the focal point at any position in the pipe cross section. The whole pipe section rotates together with the instrumentation in order to always have the beams crossing the material through the flat surface. The translational stroke is 0.24 m, equal to one diameter, while the rotational stroke is 90° . Figures 2.10 of the instrumented segment illustrate this rotation.

This design provides access to the detailed study of two of the four quadrants of the pipe section, depicted in Figure 2.11. Indeed a vertical symmetry of the flow is assumed so that studying two quadrants is enough. The 86 point mesh of the data acquisition system has a 2 cm step in both directions with local refinement close to the wall.

Initially, the laser particle analyzer was installed to provide both measure-

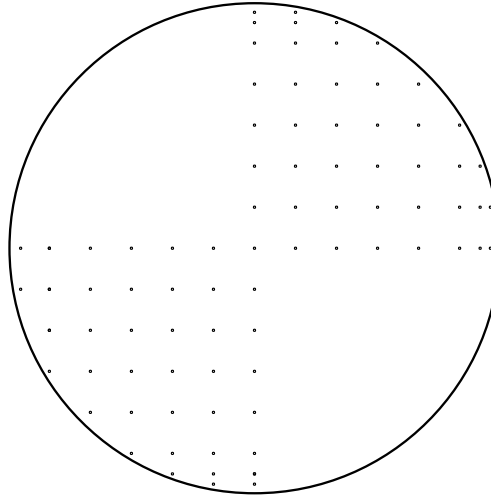


Figure 2.11: 86 measurement positions of the laser particle analyser.

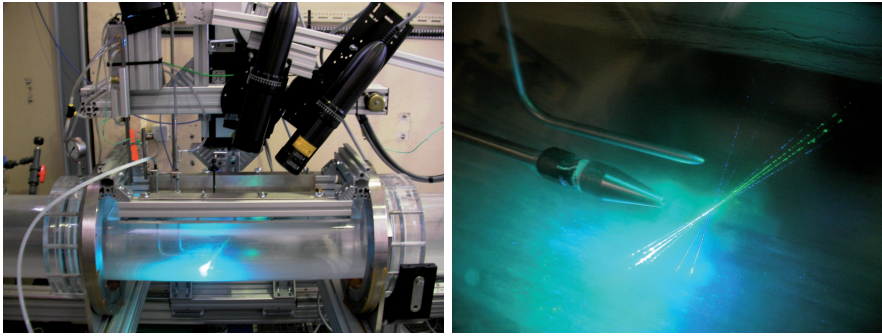


Figure 2.12: Particle sizer in action: Overview (left) and details (right). The right picture shows the focal point of the three double laser beams but also the isokinetic probe and the Pitot tube.

ments of droplet characteristics and mass flow rates at the same axial location. The mass flow rate is unfortunately not available for several technical reasons: firstly the focal volume over which a droplet is simultaneously visible by the three laser beams cannot be accurately assessed. However the number of drops separately collected by each beam is quite different from the quantity of drops simultaneously collected by the three beams and depends on the laser settings. Indeed, as the total power is non uniformly distributed over the three color beams, a modification of the power repartition alters the counting at the three receptors. Moreover the implemented laser probe cannot measure droplets larger than $700 \mu m$ which means that the largest droplets carrying the greatest part of the mass flow rate can not be counted by this experimental apparatus. For

these reasons, both the assessment of the volume measurement and the number of drops are subject to substantial doubts. This ultimately prevents a correct estimation of the total mass flow rate of droplets over the pipe cross section.

However complementary studies [65] have shown that those considerations have very little influence on the qualitative local droplets characteristics (velocity components and diameter). The estimation of the uncertainties on the velocity and diameter measurements are described in details in Appendix A.4.

In conclusion, the LASER system provides several indications on the flow behaviour. Every measured droplet is then characterized by up to three components of velocities and a diameter. The statistical analysis of these characteristics is used to describe the droplet flow. Moreover, associated to the measurements of the mass flow of droplets, these data provide an estimation of the droplet fraction in the pipe.

2.2.3 Isokinetic sampling probe

The mass flow of drops carried by the air flow is the main purpose of the entrainment campaign of the REGARD facility. For the technical reasons previously described, the laser particle analyzer is not able to achieve this measurement. So a isokinetic sampling probe is implemented on the instrumented structure to provide this measurement.

Description

The isokinetic sampling probe is a tube axially placed in the flow and locally sucking a sample of the air flow. This probe is fixed on the structure so as to accurately and continuously follow the motion of the focal point of laser beams but few centimeters downstream (see Figure 2.9 and 2.13). So the mesh used for the isokinetic probe measurements is identical to the particle analyzer mesh excepted close to the wall (due to the size of the probe, this instrument cannot reach the nearest to the wall mesh positions of the laser device), as depicted on Figure 2.14.

The isokinetic system is composed of a vacuum pump, a Pitot tube to measure the velocity of the air and droplet flow and a thermocouple. The isokinetic probe is controlled by the vacuum pump which aims at sucking the air and droplet flow at a velocity equal to 105% of the flow velocity simultaneously measured at the local analysis point. The sucking velocity continuously varies between 97% to 110% during an experiment, as an answer to the fluctuations of

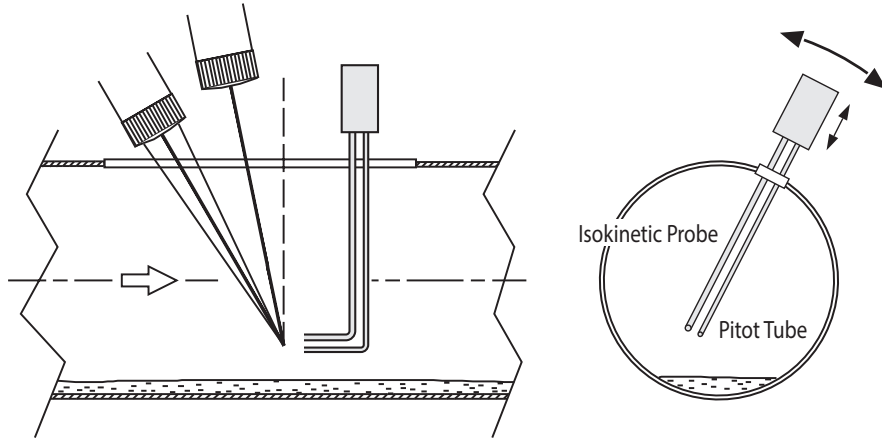


Figure 2.13: Isokinetic probe description.

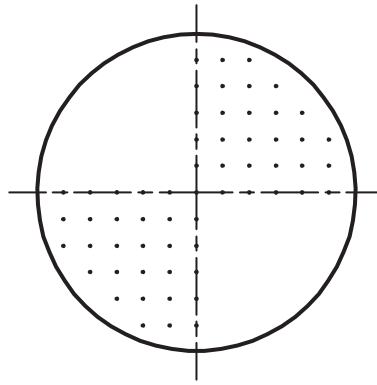


Figure 2.14: All 60 measurement points of the isokinetic sampling probe

the measured air velocity. However the exact isokinetic conditions (100%) are not necessary for an isokinetic probe as pointed out by Romano et al. [132]. The mixture of air and droplets sucked by the isokinetic sampling probe is entrained through the probe into a centrifugation chamber in order to extract the water from the air.

The liquid mass is separated from the air and is continuously weighted. The final mass gives an estimate of the local mass flow rate of droplets sucked from the air flow rate. The inlet diameter of the probe and all the local results are used to integrate the local droplet mass flow rate over the whole cross section.

The centrifuged air flow is sent to a moisture measuring system composed

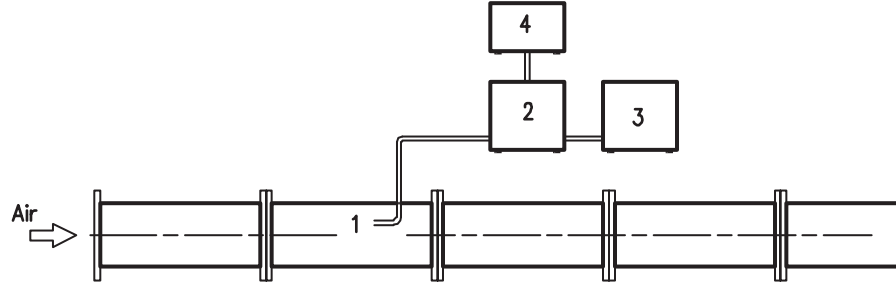


Figure 2.15: Diagram of the isokinetic probe measuring chain.

of a humidity sensor and a temperature sensor (3), downstream of the weighing chamber (2) as shown on Figure 2.15 ((4) represents the acquisition media). However this measurement location may induce additional uncertainties on the measured humidity. Indeed the isokinetic probe suction pipe is rather small so that a part of droplets tend to deposit along this suction pipe forming a local liquid film. The air going through this pipe continues to rise in humidity so that the humidity measured locally becomes slightly greater than the humidity in the test section. The forthcoming centrifugation technique for separating droplets from air may also induce an increase of the relative humidity. Thus the relative humidity measurement is probably slightly overestimated by the measuring system. This effect does not affect the droplet flow measurement because the liquid film developed in the suction pipe early reaches an equilibrium between droplet deposition and entrainment. So the measured droplet flow in the weighing chamber is actually representative of the aspirated flow by the isokinetic probe, provided one would wait for a few minutes for initializing the equilibrium in the suction pipe at the beginning of the experiment.

As the air loop is an open loop, the air flow continuously comes from outside (outdoor ambient air). The external moisture is measured at the beginning of an experimental test but not permanently. Consequently, changes in external atmospheric conditions are unfortunately not reflected in the data.

The measurement of the droplet flow is simply a mass measurement of the liquid sucked in the test section. The accuracy of the balance is 10 mg but the main contribution to droplet mass flow uncertainties comes from other sources. Indeed, as the flow is sucked up during 3 minutes, the quantity of collected liquid in the measuring cylinder is far greater than the accuracy of the balance.

Actually, to access this measurement, the sucked flow must be separated in order to drive the droplets into the weighting system. As this deposition in the cylinder is non continuous, the droplet mass flow evaluation is based on a mass fluctuation minimization technique. As this final step is achieved manually by operators, this induces an uncertainty of 0.003 g/s.

2.2.4 High Speed Camera

The high speed camera helps to visualize the dynamics of droplet entrainment and provides images at a rate up to 675000 frames/seconds on few pixels. The camera is fixed on the rotating and translating structure and equipped with an endoscope in order to follow the motions of the other instrumental devices. This tool is essentially dedicated to the qualitative analysis and visualization in order to confirm theories found in literature more than providing quantitative results.

Technical considerations

Performing flow visualizations from outside of the pipe does not provide accurate enough data to describe the flow behaviour. Indeed, so many effects affect the clearness of the videos such as the circularity of the test section and the associated reflection/refraction effects but also and mainly the presence of droplets depositing on the wall. Therefore, the camera is equipped with an endoscopic system in order to obtain videos close to the liquid film and with many lighting systems. This endoscope is introduced through the port used for the isokinetic probe. Therefore it is impossible to simultaneously perform mass-flow measurement and visualization.

2.3 Measurement campaign

The control parameters of REGARD are the flow rates of air and water. The entrainment measurement campaign is based on a 4 x 2 matrix for air (rows) and water (columns) flow rates. As REGARD is dedicated to the study of non-fully developed flows, this matrix is analyzed at five axial locations of the test section. Consequently, the entrainment campaign can be summarized in a 4 x 2 x 5 matrix or 40 experiments, requiring 10 weeks of experiment.

Four mass flow rates of air were analyzed during the entrainment campaign: 3100 Nm³/h, 3900 Nm³/h, 5400 Nm³/h and 6000 Nm³/h, equivalent ¹ to air

¹With a diameter of 0.24 m and a given temperature, the volumetric flow rate is easily transformed into superficial velocity by the formula: $j_a = \frac{\rho_N}{\rho_a} \frac{\dot{V}_a/3600}{A}$

superficial velocities j_g of 19.8 m/s, 24.9 m/s, 34.4 m/s and 38.2 m/s. The first value 19.8 m/s is a little higher than the initiation of the entrainment phenomenon while the upper one is a little lower than the maximal velocity expected in a hot leg during a LOCA accident (50-60 m/s).

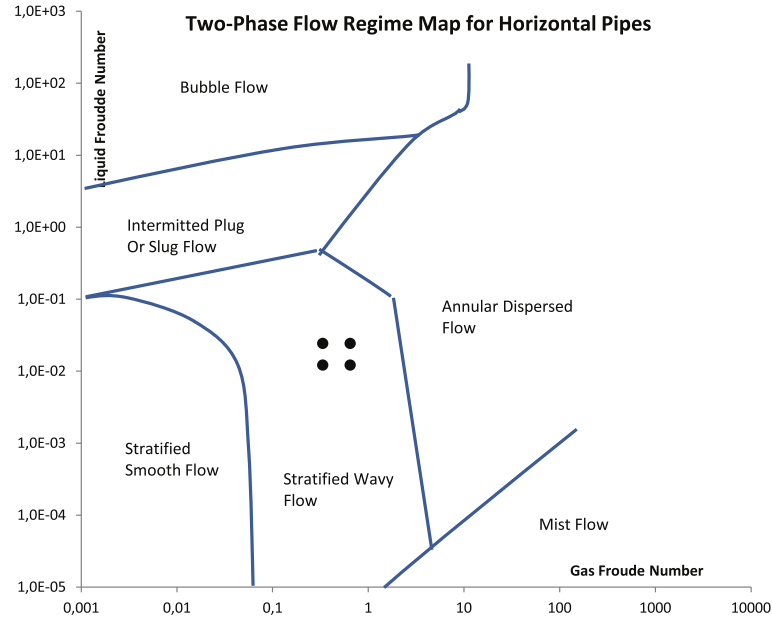


Figure 2.16: Expected flow regime map [147, 83]

Two mass flow rates of liquid water were analyzed: $3 \text{ m}^3/\text{h}$ and $6 \text{ m}^3/\text{h}$, equivalent to liquid superficial velocities j_l of 0.018 m/s and 0.036 m/s. The studied flow regimes are represented (see Figure 2.16) in a flow regime map suited to the characteristics of REGARD (pipe diameter, fluids characteristics) based on the Taitel and Dukler's regime map [147].

The flow behaviour is analyzed by the isokinetic probe and the particle analyzer at 5 axial locations equivalent to $\frac{L}{D}$ ratio of 3.8, 5.7, 7.7, 9.6 and 11.6 while the BANJO measures the flow at 3 slightly different axial locations, equivalent to $\frac{L}{D}$ ratio of: 3.7, 5.6 and 9.5.

2.4 Measurement methodology

The various measurement devices provide a lot of indications on the flow characteristics in the pipe:

1. The Banjo provides:
 - o h_l : The local liquid level measurements
 - o $\bar{h}_l$: The time-averaged local liquid level is calculated from a time-averaging operation of the local liquid level measurements.
 - o α_{lf} : The continuous liquid volume fraction is calculated from the time-averaged local liquid level.
 - o A_{lf} : The continuous liquid cross-section is calculated as the product of the total section by the continuous liquid level volume fraction
 - o $A_g + A_{ld} + A_{lf} = 1$
 - o $\bar{v}_w$: The interfacial wave velocity
 - o $\bar{f}_w$: The interfacial wave frequency
2. The Isokinetic Probe provides:
 - o $\dot{M}_{d,loc}$: The local mass-flow rates of droplets, measurable all over 60 positions of the pipe cross section at the focal point of each of the 5 axial locations L/D , for every of the 8 couples of liquid and gas velocities: Up to 2400 local mass-flow rates of droplets.
 - o $\dot{M}_{v,loc}$: The local mass-flow rates of vapor in the same conditions: Up to 2400 local mass-flow rates of vapor.
3. The Laser System provides:
 - o N_d : The local number of droplets travelling throughout the focal surface at each position over the pipe cross section at the focal point of each axial location, for every couple of liquid and gas velocities: Up to 2400 local positions analyzable
 - o v_d : The velocities of each of these droplets
 - o d : The diameters of each of these droplets

However the entrainment rate Γ_E , which is the main parameter for the validation of the entrainment model, cannot be measured directly. Consequently, Figure 2.17 presents the strategy that will be followed in the following chapter 3 to determine the entrainment rate Γ_E on the basis of the measured parameters.

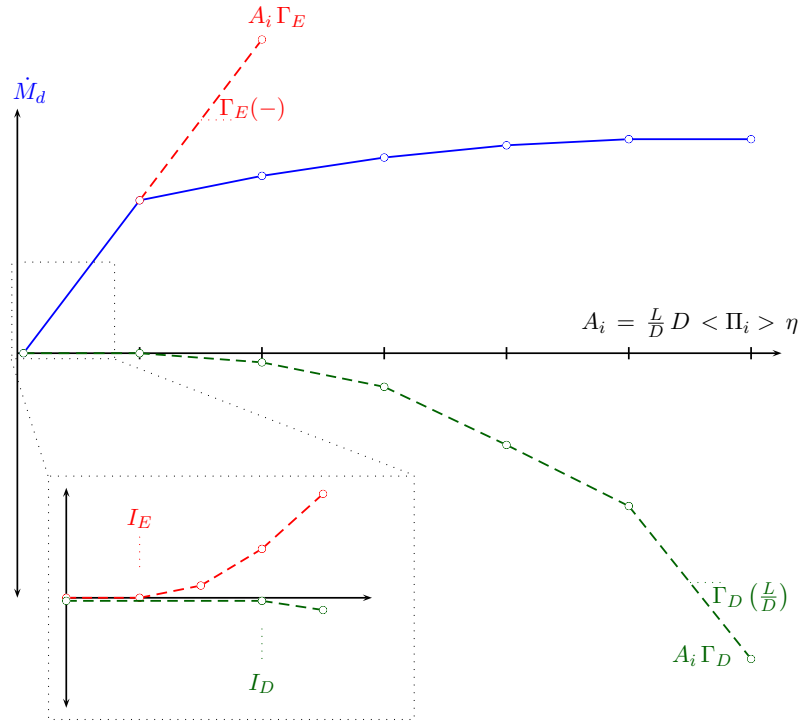


Figure 2.17: Assumed evolution of the entrainment and deposition rates together with the integrated mass flow-rates of droplets along the axial location. I_E and I_D denotes the inception of the entrainment and deposition, while η is the shape factor of the interface and $<\Pi>$ the space averaged curvature of the interface.

The entrainment rate represents the mass of droplets sheared off from the interfacial and entrained in the gas stream, per units of time and interfacial area. Now, the measured flow parameter concerning the droplet mass flux is the time averaged local mass flow rate of droplets: $\dot{M}_{d,loc}$ that may be integrated over the cross-section occupied by the air and the droplets ($A_g + A_{ld}$) to get the total mass flow rate of droplets $\dot{M}_d$ in [kg/s], where $\beta(i)$ is a proportion factor to denote the area on which the mass flow is extended ($\beta(i) = 1$ quite everywhere but near the walls and the interface as described in Appendix A.2):

$$\dot{M}_d = \int_{(A_g + A_{ld})} \dot{M}_{d,loc} dS = \sum_{i=1}^N \beta(i) \dot{M}_{d,i} \quad (2.3)$$

As shown by the blue curve on Figure 2.17, the evolution of this mass flow rate $\dot{M}_d$ with the interfacial area A_i can be consider as corresponding to the variation of droplet mass along the axial location (as D is constant and the wetted perimeter is assumed to evolve very little along the z-axis) due to the combined effects of entrainment and deposition. The slope of this evolution can be written as the total rate of droplets $\Gamma_T(A_i)$ $\left[\frac{kg}{m^2s}\right]$, combination of the entrainment rate Γ_E and the deposition rate Γ_D .

$$\Gamma_E(n) - \Gamma_D(n) = \Gamma_T(n) \quad \text{at the } n \text{ axial location.} \quad (2.4)$$

These three droplet rates $\Gamma_T(z)$, $\Gamma_E(z)$ and $\Gamma_D(z)$ varies along the A_i axis, reduced to a z axis. A general mass balance of the droplet flux along the gas-liquid interface may be written as:

$$\Gamma_E(n) - \Gamma_D(n) = \Gamma_t(n) = \frac{\dot{M}_d(n) - \dot{M}_d(n-1)}{A_i((n-1) \rightarrow n)} \quad (2.5)$$

Under fully-developed conditions, the equilibrium between entrainment and deposition will imply that the mass flow rate of droplets will not evolve any more when moving along the axial location. At a local axial location, the source term of droplet mass (entrainment) will be totally compensated by the loss term of droplet term (deposition): $\Gamma_t = 0$ at equilibrium.

However REGARD is designed to tackle not fully developed flows, so that the mass flow rates of droplets will grow along the z-axis: $\Gamma_T = f(L/D)$. In consequence, the entrainment and deposition rates should be function of the axial location: $(\Gamma_D, \Gamma_E) = f(L/D)$.

Nevertheless a major assumption is made, postulating that the $\frac{L}{D}$ dependence of the entrainment $\Gamma_E(L/D)$ is very short compared to the $\frac{L}{D}$ dependence of the deposition rate $\Gamma_D(L/D)$. With L_E^* the distance at which the entrainment rate becomes constant: $\Gamma_E(L > L^*) = \text{constant}$ and L_D^* the distance at

which the deposition rate becomes constant:

$$L_E \ll L_D \quad (2.6)$$

Indeed the entrainment is assumed to mainly come from the atomization of several wave crests at the gas-liquid interface, neglecting all other entrainment sources (droplet bouncing on the film, bubble fragmentation,...). The dependence of Γ_E on $\frac{L}{D}$ is therefore limited to the short distance necessary for a wave to fragment. The assumption is that, once the first waves will have grown enough to be atomized, the entrainment phenomenon will reach its regime mode and will not depend any more on the axial location. The red curve on Figure 2.17 show that the entrainment rate is constant since a very short axial location.

On the contrary the deposition rate is assumed to mainly depend on the droplet concentration in the pipe, following the main assumptions from the bibliographic review. Consequently the inception length of the droplet deposition is greater than the inception length of the entrainment deposition but mainly the $\frac{L}{D}$ dependence of Γ_D is assumed to be much greater than for entrainment. As shown by the green curve on Figure 2.17, the deposition rate is assumed to slightly increase, following the droplet concentration, until balancing the entrainment rate: $\Gamma_E(L/D) = \Gamma_D(L/D)$. Dallman [166] estimated this value to be greater than a $\frac{L}{D}$ ratio of 250 (60 m applied to REGARD).

To determine the entrainment rate in this work, the deposition is assumed to be negligible between the initiation of the two-phase flow ($\frac{L}{D} = 0$) and the first axial location analyzed ($\frac{L}{D} = 3.8$) such as the slope of the droplet mass flow rates, between 0 and L/D of 3.8 is representative of the entrainment rate Γ_E :

$$\frac{L}{D} < 3.8 \rightarrow \Gamma_D = 0 \quad \text{and} \quad \Gamma_E = \Gamma_T = \frac{\dot{M}_d(3.8) - \dot{M}_d(0)}{A_i(0 \rightarrow 3.8)} \quad (2.7)$$

As a direct consequence, this assumption obviously underestimates the entrainment rate at the saturation Γ_E . The parameters required to determine the entrainment rate are the followings:

$$\Gamma_E = f(\dot{M}_d(3.8), \dot{M}_d(0), A_i) = f(\dot{M}_{d,loc}, A_{lf}, A_i) \quad (2.8)$$

The BANJO provides an estimation of the liquid film fraction α_{lf} , necessary for the integration procedure of the local mass flow of droplets over the pipe section, and also an estimation of the interfacial area A_i . Indeed the interfacial area A_i (described in Appendix A.1) can be calculated on the basis of the liquid level measurements and the interfacial wavelengths of the instabilities (see Equation 3.6 and Figure 3.20) .

The other experimental measured parameters (droplet number, droplet characteristics ...) are used either for the model validation or for the sub-modeling of the mechanistic model. Indeed the wave velocities (axial velocity and growth velocity) from the BANJO are used to model the characteristic time of the entrainment phenomenon τ_e . In contrast, the wave frequency facilitates the validation of the sub-model for the wavelength prediction. The LASER data confirms the entrainment description of Woodmansee and Hanratty [166] but mainly provides the droplet fraction α_{ld} necessary for the validation of the simulations:

$$\alpha_{lf} + \alpha_{ld} + \alpha_g = 1 \quad (2.9)$$

with a droplet volume fraction, equal to the droplet surface fraction (pipe diameter D is constant):

$$\alpha_{ld} = \frac{V_{ld}}{V} = \frac{A_{ld} \times L}{A \times L} = \frac{A_{ld}}{A} = \frac{\sum_{i=1}^{N_{t,d}} A_d(i)}{A} \quad (2.10)$$

where $N_{t,d}$ denotes the total number of droplets over the pipe cross-section, proportional to the integrated mass flow-rates of droplets over the section:

$$N_{t,d} = \frac{\dot{M}_d}{\rho_l \langle \bar{v}_d \rangle \langle \bar{A}_{ld} \rangle} \quad (2.11)$$

where $\langle \bar{v}_d \rangle$ and $\langle \bar{A}_{ld} \rangle$ are the time and cross-sectional space averaged of the droplet velocity and the droplet fraction. Finally these droplet characteristics are of interest for the modeling of the reverse phenomenon of droplet deposition, studied on REGARD by Neiss [108]. Indeed most deposition models require the droplet concentrations, directions and dimensions.

Chapter 3

Experimental results

This chapter is devoted to the presentation of the experimental results collected on REGARD. As the ultimate goal of the experimental campaign is to extensively describe the droplet entrainment phenomenon, several flow characteristics have been studied in details. Indeed, the droplet entrainment is expected to mainly arise from the atomization of interfacial waves of the liquid film. Consequently the liquid film characteristics and the interfacial waves are primarily described in this chapter. The first section addresses the liquid film height variations in order to determine, following a time and space averaging operation, the liquid film fraction α_{lf} . On the first hand, this liquid film fraction is a key parameter of the flow description while on the second hand, this parameter is required for calculating the air and droplet volume fractions, necessary for averaging the droplet measurements: mass flow-rates, diameters and velocities.

The wave characteristics are moreover analyzed in order to validate the sub-models developed for the entrainment rate model. The wave velocity and the wave frequencies are indeed necessary either for the estimation of the most amplified ripple wavelength of the flow λ_c or for the modeling of the entrainment characteristic time τ_C .

The entrained fraction is then analyzed based on the outputs of the isokinetic probe in a second section. The time averaged local mass flow-rates of droplets have been measured all over the pipe cross-section and integrated to get the time averaged and space integrated mass flow-rates of droplets required for the validation of CATHARE. The axial evolution of the entrained fraction is highlighted in order to determine the initial slope of the mass flow-rates of droplets between $\frac{L}{D}$ of 0 and 3.8, expected to be representative of the entrainment rate Γ_E .

In a final section, the droplet characteristics (size and velocities) provided by the LASER setup are presented. This provides complementary information on the droplet dynamic helpful both for the physical understanding of the flow and for a validation of CATHARE pre-existing models. Indeed several models are implemented in CATHARE for the droplet coalescence, transport or interfacial area. Moreover, joining together the mass flow-rates of droplets and the droplet size characteristics (diameters and cross-sections) provide an estimation of the liquid droplet fraction α_{ld} and then on the air fraction α_g (see Equations 2.9 to 2.11).

3.1 Liquid film and interfacial waves

The liquid film investigations are conducted with the BANJO instrument fixed on the instrumented flange. The BANJO, which works at a frequency of 1000 Hz; during 1 to 10 s, measures 9 local heights of the liquid film. Typical evolutions of these levels over time are shown in Figure 3.1.

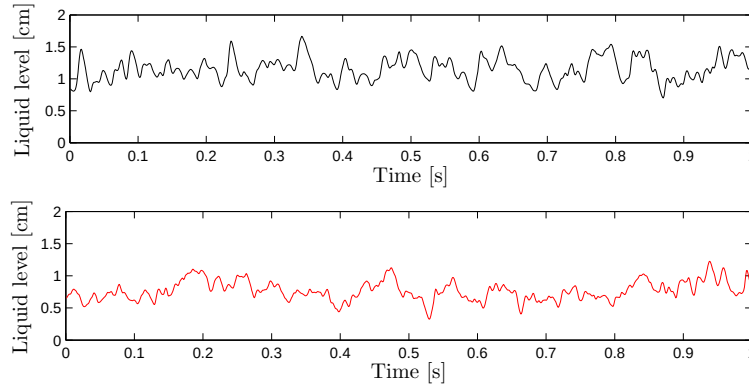


Figure 3.1: Time evolutions of the liquid level measured by the BANJO at $\frac{L}{D} = 5.6$ and $j_g = 19.8$ m/s for the two liquid superficial velocities: $j_l = 0.018$ m/s (up) and $j_l = 0.036$ m/s (bottom).

Depending on the axial location, the measurement duration per run varies between 1 and 10 s, which provides up to 90000 ($9 \times 1000(0)$) data of the local liquid level h_l . The analysis of these measurements provides an information on the behavior of the liquid film. Indeed the time averaged liquid film heights provide an indication on the time averaged curvature of the liquid film and so on the liquid film fraction α_{lf} (as demonstrated in Appendix A.1). This parameter is used to calculate the time and space average section occupied by the gaseous

and the dispersed phases: $\alpha_g + \alpha_{ld} = 1 - \alpha_{lf}$, required in the space averaging operation of the local droplet characteristics (mass flow-rates, size or velocity).

Furthermore the cross-correlation of signals from two axially spaced pairs of wires of the BANJO provides an estimation of the wave celerity by determining the time averaged phase shift between the two signals; while, combined to an autocorrelation, an estimation of the growth velocity of the waves can be achieved. Indeed two axially adjacent wires are close enough to get the wave displacements of the ripple waves (Figure 3.2) as the cross-correlation will capture the evolution of variations of waves of high frequencies: 25 - 100 Hz as an order of magnitude.

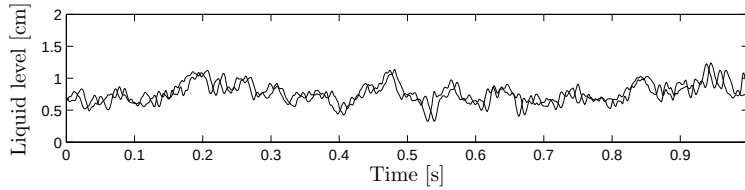


Figure 3.2: Evolution of the liquid levels measured by two axially spaced pairs of wires of the BANJO.

Finally performing a Fast Fourier Transform of a signal provides the frequency spectrum of the wavy interface. This provides insights on the wavy characteristics of the flow, but also, combined to a velocity measurement, an insight on the wavelength spectrum of the ripple waves.

3.1.1 Time average level of the liquid film

The BANJO measures the fluctuations of the liquid film level during 1 s to 10 s depending on the run. This provides 1000 to 10000 values of the liquid level h_l for each of the 9 pair of wires, which may be time averaged to compute a local time averaged liquid film level $\bar{h}_l$. Indeed the flow characterization requires to simplify the interface oscillations to get a time and space averaged over the interface value $< \bar{h}_l >_i$. Moreover the 1D CATHARE 3 code is based on a space and time averaged calculation that required time and space averaged inputs for its validation.

Figures 3.4 and 3.3 illustrate the evolutions of this time averaged level $\bar{h}_l$ at the center of the pipe cross-section:

- o By keeping the superficial liquid velocity j_l constant, the liquid level continuously decreases while the air superficial velocity j_g increases.

- o As expected, increasing the liquid mass flow-rate, while keeping constant the superficial air velocity j_g , involves an augmentation of the liquid level.
- o Finally the liquid level slightly decreases when moving the axial location $\frac{L}{D}$ along the z-axis. This indicates that the equilibrium is not achieved as the liquid level is not stabilized along the z-axis. This imply (as $\dot{m}_l = v_l \alpha_l \rho_l A$) that the velocity v_l is increasing along the z-axis while the liquid fraction α_l is decreasing.

As the liquid is injected with a fixed level of 3 cm at the bottom of the pipe, equal to a $\frac{\bar{h}_l}{D}$ ratio of 0.125 at the center of the pipe cross-section for $\frac{L}{D} = 0$, the interface level has gone down very rapidly along the z-axis to reach the local time averaged levels $\bar{h}_l$. This significant decrease along the axial location indicates that the design of the liquid injection system has been overestimated as the imposed level at $\frac{L}{D} = 0$ is too high for the flow phase velocities.

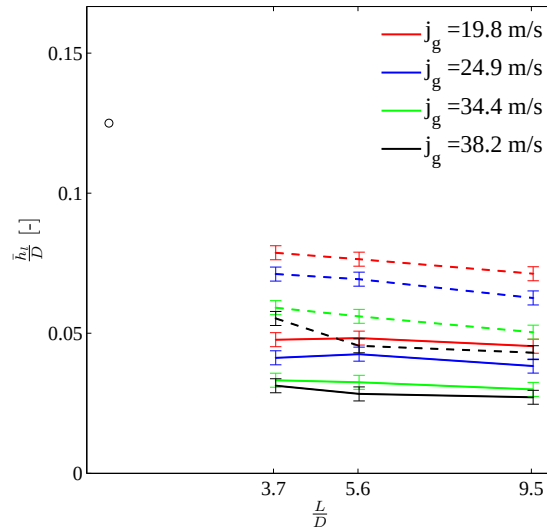


Figure 3.3: Evolution of the time averaged level of liquid at the center of the pipe cross-section at $j_l = 0.018$ m/s (solid lines) and $j_l = 0.036$ m/s (dashed lines).

The measurement uncertainties depend on various parameters such as the setup instrumentation or the post-treatment method. Several experiments conducted with equivalent parameters at different times provide up to 5% difference on the averaged liquid level. Combined to the BANJO uncertainty, the total uncertainties on the liquid level reaches up to $\pm 7.5\%$.

These observations are consistent with those of Paras and Karabelas [123] who specially emphasized the effect of the shear stress at the interface that leads to an acceleration of the liquid film and thus to the reduction of the water level when the air superficial velocity j_g increases.

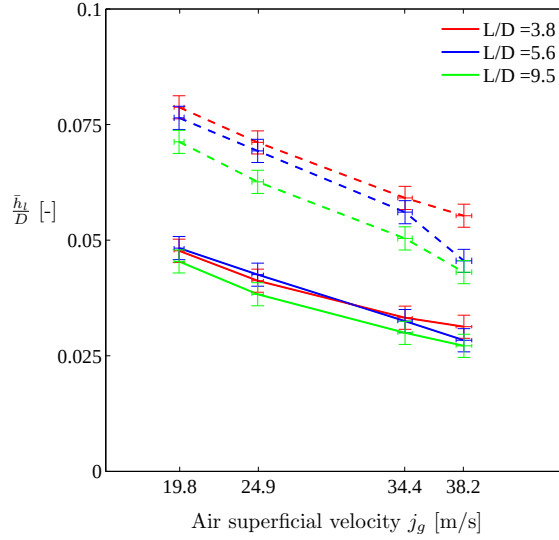


Figure 3.4: Evolution of the time averaged liquid level at the center of the pipe cross-section at $j_l = 0.018$ m/s (solid lines) and $j_l = 0.036$ m/s (dashed lines).

3.1.2 Circumferential distribution of the liquid film

Many authors, such as Williams [165, 164] and Paras and Karabelas [120, 121, 123], have studied the circumferential distribution of the liquid film. Each one noticed a similar behavior of the liquid film as a function of the operating conditions: below an air superficial velocity j_g of 67 m/s for Williams ($D = 9.53$ cm) and 40 m/s for Paras and Karabelas ($D = 5.08$ cm), the liquid film is non uniformly distributed around the walls, essentially due to the gravity which prevails on the turbulent effects [120]. The flow is in transition from annular flow to wavy stratified flow. This liquid film asymmetry is more and more marked as the pipe diameter is large and the transition from stratified to annular flows moves to higher air velocities [165, 123]. This transition depends then on the pipe diameter and on the air superficial velocity but also on the liquid superficial velocity. Indeed increasing the liquid flow-rate makes the circumferential distribution of the liquid film increasingly asymmetric.

To characterize the asymmetry of the liquid film, Williams [165] introduced

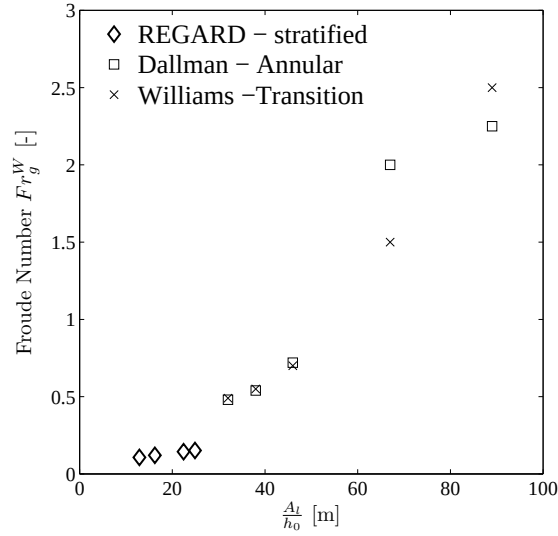


Figure 3.5: $\frac{A_l}{h_0}$ parameter as a function of the Froude Number applied to Dallman [42], Williams [164] and REGARD.

a parameter $\frac{A_l}{h_0}$ which represents the ratio between the flooded area A_l and the height of the liquid film taken from the bottom of the pipe h_0 . The evolution of the transition from a dissymmetric stratified film to a symmetric annular flow depends on a Froude number Fr_g^W defined in Equation 3.1 by Williams [165] (not equal to the Froude number of the surface flow). Indeed a large $\frac{A_l}{h_0}$ and a large Froude number Fr_g^W are symptomatic of an annular flow while a small $\frac{A_l}{h_0}$ and a small Froude number Fr_g^W are representative of an asymmetric flow. Figure 3.5 compares then the Froude number of REGARD Fr_g^W to experimental data of Dallman [42] and Williams [164]. Contrarily to Dallman's experiment [42] ($D = 2.54$ cm) or Williams experiment [164] ($D = 9.53$ cm), both the Froude number Fr_g^W and the $\frac{A_l}{h_0}$ are very small on REGARD. This indicates according to Williams [164] that the circumferential distribution of the film is very asymmetric. These flow characteristics confirm the flow visualizations with a wavy stratified flow meanly circumferentially developed.

$$Fr_g^W = \frac{\dot{j}_g}{\sqrt{g D_h}} \quad (\text{Williams' definition of the Froude Number}) \quad (3.1)$$

The asymmetry of the circumferential distribution of the liquid film is much greater on REGARD, highlighting as well the effect of the diameter on the flow regime. Indeed, Williams [165] ($D = 9.53$ cm) or Paras and Karabelas [123] ($D = 5.08$ cm) would obtain annular flows in the range of velocities that would provide a stratified flow in REGARD. In consequence, as the flow regime and the liquid film characteristics have a significant impact on the entrainment phe-

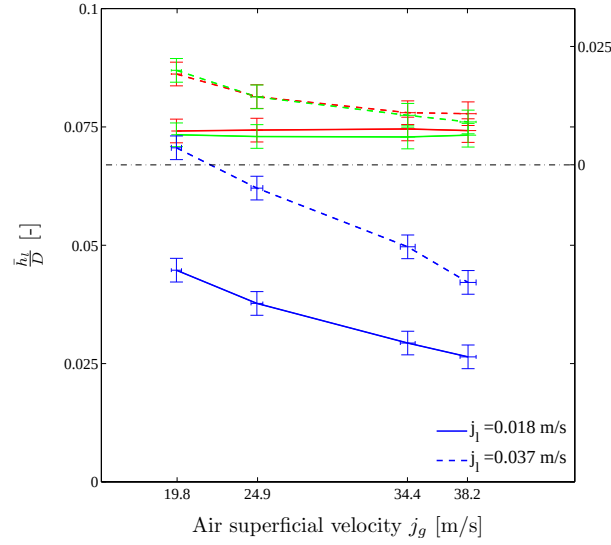


Figure 3.6: Comparison between three liquid level signals at the same cross-section ($\frac{L}{D} = 9.5$) as a function of the air superficial velocity: center (blue) and laterals (green and red), with the wall position at the lateral measurements (black dotted).

nomenon, downsizing the pipe diameter induces significant differences on the flow behavior. This confirms that the usual experimental facilities do not suit the modeling of the hot leg behavior.

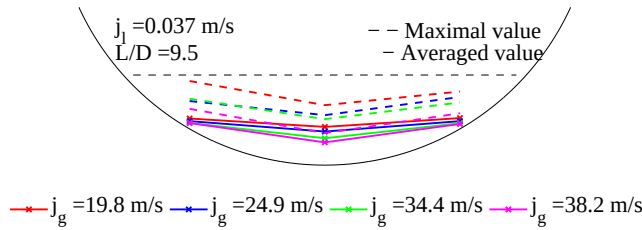


Figure 3.7: Comparison between three liquid level signals at the same cross-section ($\frac{L}{D} = 9.5$) as a function of the air superficial velocity: center (blue) and laterals (green and red), with the wall position at the lateral measurements (black dotted).

In REGARD, the liquid film is thus asymmetric. The interface is given in Figure 3.6 which shows a comparison of the time averaged liquid level $\bar{h}_l$ between three signals over a same cross-section (referenced to zero at the bottom of the

pipe). Whatever the liquid superficial velocity j_l (solid or dashed), the liquid level at the center of the cross-section (blue) decreases with increasing the gas superficial velocity j_g . The liquid level is always lower at the center of the pipe (blue) than on the lateral measurements (red and green). Figure 3.7 highlights the fact that the time averaged liquid levels at the lateral measurements follow the tendencies at the center of the pipe, except that the liquid levels evolve towards a minimal local thickness.

3.1.3 Liquid film volume fraction

Based on the assumption that the shape function is an arc of a circle, a calculation of the average portion of the cross-section area of the liquid film is achieved¹:

$$\alpha_{lf} = \frac{A_{lf}}{A} = \frac{A_{lf}}{\pi \left(\frac{D^2}{4} \right)} \quad (3.2)$$

This calculation provides the time averaged liquid film fraction in the pipe (see Figure 3.8). The liquid film fraction slightly decreases when the air superficial velocity increases at small liquid flow-rates ($j_l = 0.018$ m/s) and decreases somewhat more strongly at the higher liquid flow-rates ($j_l = 0.036$ m/s). One assumes that the shear stress at the interface accelerates the liquid which results in tamping the interface level. Along the axial location, the liquid film fraction also seems to move towards an equilibrium value, that would indicate that the liquid film has ceased to accelerate. An uncertainty on the deduced liquid film fraction is estimated as the difference in fraction between the surface obtained with the measured signals and the surfaces obtained by reducing or increasing the signals by their maximal uncertainty.

This parameter α_{lf} is required both for the post-processing of further experimental results and for the model validation. Indeed, the liquid film fraction enables the calculation of the air and droplet surface fraction and then the cross-section space averaging operation of the droplet characteristics (mass flow-rates, diameter or velocities). Furthermore, the liquid film fraction enables the validation of the interfacial and parietal shear stress laws used in several sub-models of the entrainment model.

¹The method is not really this simple to tackle edge effects. The complete description of the methodology performed for the calculation of the liquid film fraction is provided in Appendix A.1.

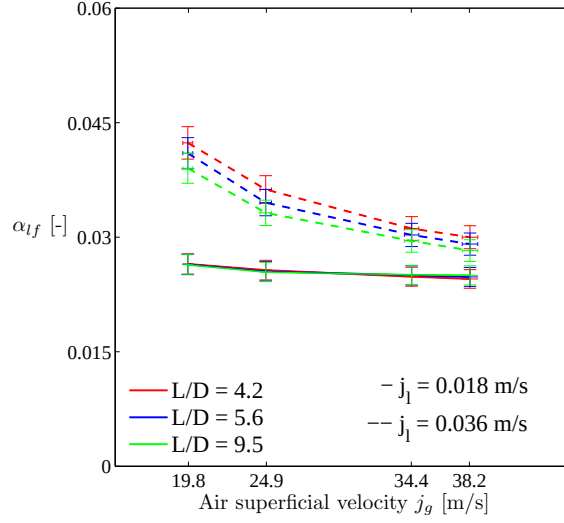


Figure 3.8: Evolution of the liquid film fraction α_{lf} at $j_l = 0.018$ m/s (solid) and $j_l = 0.036$ m/s (dashed).

3.1.4 Interfacial waves

Figure 3.1 has highlighted the oscillations of the interface liquid level. The characterization of these interfacial waves is of interest for the understanding of two-phase stratified flow, particularly those implying a mass transfer due to a wave atomization. Interesting studies have been conducted by Andritsos and Hanratty [14], Paras and Karabelas [123], Shi and Kocamustafaogullari [140] and Ng et al. [109] on interfacial waves.

Andritsos and Hanratty in 1987 [14] studied the time variation of the liquid level in pipes of small (2.53 cm) and intermediate (9.53 cm) diameter to describe the development of the waves. At a fixed mass flow-rate of liquid, the first waves appearing when the air velocity increases are wavelets of small amplitudes, called small-amplitude 2-D waves. Their wavelengths lie between 2 cm and 4 cm. Ng et al. [109]. notice similar wavelets, called 2-D ripples. Finally Shi and Kocamustafaogullari [140] indicated the presence of similar waves of regular structure for low air velocities.

When increasing the air superficial velocity, the wave amplitudes grow while the wavelengths decrease. Beyond a certain air velocity, these small wavelets coalesce to form higher amplitude waves of higher wavelengths called roll-waves. This behavior is similarly highlighted by every authors.

Finally, the structured waves give way to totally 3-D chaotic or totally irregular waves of sharp amplitudes when still increasing the air velocity.

As the entrainment phenomenon results from the atomization of interfacial instabilities, the interfacial wave characteristics are required for the development and the validation of many sub-models of the entrainment phenomenon.

3.1.5 Wave velocity

The time shift Δt between two axially adjacent signals (duration of 1 or 10 seconds, meaning 1000 or 10000 points) enables to determine the local time averaged wave axial velocity $\bar{v}_w = \frac{\Delta z}{\Delta t}$, with the wire space Δz . Since the three pairs of wires are axially spaced, and since the three deduced velocities are almost identical (below the uncertainties), any of the three measurements provides the required velocity. As the accuracy on the wave velocity depends on the accuracy of the time shift between two signals, the most reliable cross-correlation is kept for the description of the wave behavior. Due to the number of waves recorded by samples, the wave velocity is not sensitive to the sample duration.

This wave velocity evolves in a similar manner as the data of Paras and Karabelas [123] and Fukano et al. [51]. As shown in Figure 3.9, increasing the air superficial velocity j_g increases the wave interfacial velocity ; moving along the axial location slightly increases also this velocity.

These results show consistency with the results of Fukano et al. [51] and Paras and Karabelas [123] as shown in Figure 3.10. For a quite similar liquid superficial velocity (0.02 m/s for the two studies compared to 0.018 m/s on REGARD), the wave velocities are quite similar although the pipe diameters are quite different (respectively 2.06 cm and 5.08 cm). Increasing the liquid superficial velocity would increase the wave velocity according to [123, 51]. This effect seems to be embedded by the influence of the decrease of the liquid film level. Indeed as the liquid level is going down along the axial location, the liquid film is somewhat accelerated which results in an acceleration of the interfacial waves.

Figure 3.11 presents the evolution of the time averaged wave velocity $\bar{v}_w$ measured at a lateral position of the pipe cross-section ($x = -6$ cm). The wave velocity continuously increases with the air superficial velocity but depends more strongly on the axial location and the liquid superficial velocity than the wave velocity measured in the center of the pipe cross-section. Moreover the wave velocity is smaller at the lateral positions than in the center of the pipe cross-section.

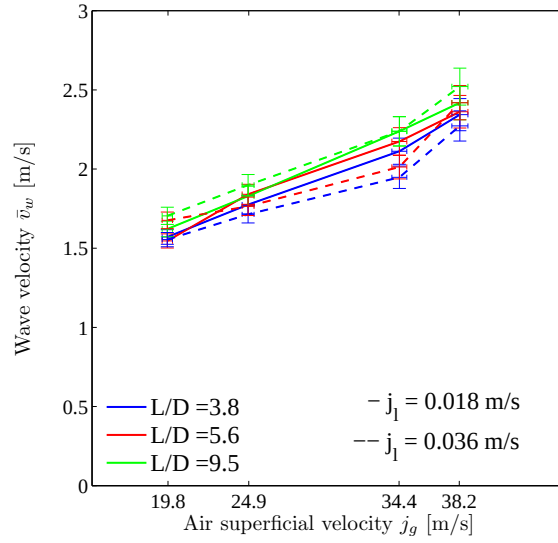


Figure 3.9: Evolution of the axial velocity as a function of the control parameters of the flow: the water and the air superficial velocities (j_l and j_g).

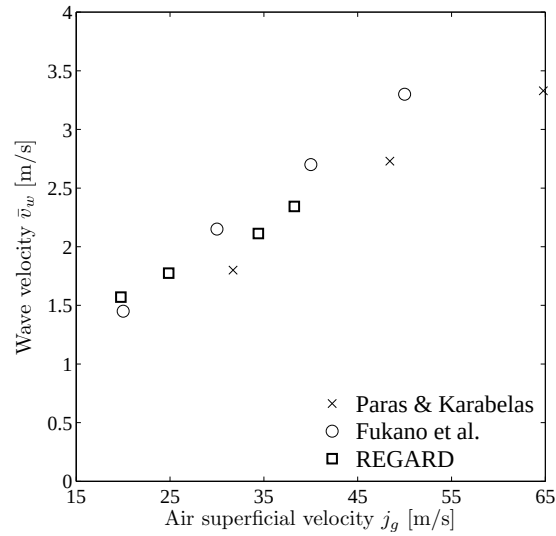


Figure 3.10: Evolution of some axial velocities from the literature [51, 123] compared to REGARD data at $j_l = 0.018$ m/s.

Wave celerity

The wave celerity, defined by Equation 3.3, requires the volume fraction of the liquid film: α_{lf} . The celerity is defined as the difference between the time averaged wave velocity v_w (at the center of the pipe cross section) and the average liquid film velocity. Figure 3.12 presents the variations of this celerity

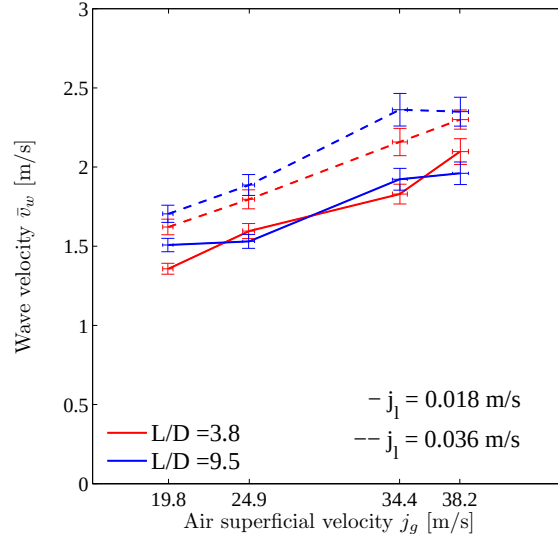


Figure 3.11: Evolution of the axial velocity as a function of the control parameters of the flow: the water and the air superficial velocities (j_l and j_g) at $x = -6$ cm (lateral position).

with operating conditions. Increasing the air superficial velocity j_g results in increasing the wave axial velocity, but as j_l is kept constant, the evolution of the liquid film fraction strongly acts on the wave celerity. The liquid film fraction evolves quite differently depending on j_l as the fraction tends to move towards an equilibrium value. Consequently the wave celerity is larger at the lowest liquid superficial velocity j_l .

$$\bar{c}_w = \bar{v}_w - v_l = \bar{v}_w - \frac{j_l}{\alpha_{lf}} \quad (3.3)$$

$$Fr_w = \frac{c_w}{\sqrt{g h_l}} \quad (3.4)$$

Calculation denoted in Equation 3.3 and the evolutions of the wave celerity must be cautiously considered as the liquid superficial velocity j_l and the liquid film fraction α_{lf} are time and space averaged film characteristics while the wave velocity $\bar{v}_w$ is a local time averaged characteristic. Indeed Figure 3.11 have shown that the wave velocity varies along the pipe circumferential distribution of the liquid film. As a consequence of the circular shape of the pipe, the wave celerity should be calculated all over the pipe cross-section by estimating the circumferential variations of the liquid velocity v_l .

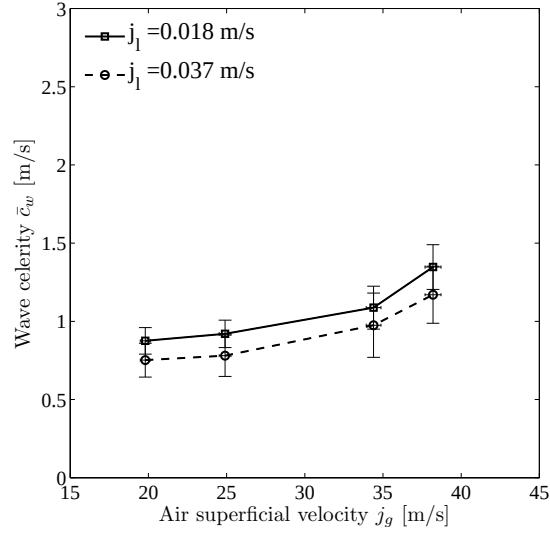


Figure 3.12: Evolution of the time averaged wave celerity c_w depending on operating conditions at the smallest $\frac{L}{D}$.

Wave growth velocity

Following the cross-correlation between two signals from two consecutive pairs of wire, an auto-correlation (detailed in Appendix A.1) provides an estimation of the time averaged wave growth velocity $\bar{v}_{gr}$. The results of such a method are shown in Figure 3.13. The wave growth velocity appears to be very small compared to the waves propagation velocity (15% as an order). The wave growth velocity follows the amplitude fluctuation and the liquid level evolutions, as the wave growth velocity is higher at high liquid level, meaning at high liquid flow.

3.1.6 Wave frequency

The wave frequency conjugated to the wave celerity enables the calculation of the wave wavelength, that are necessary for the validation of sub-models:

$$\lambda = \frac{c}{f} \quad (3.5)$$

The BANJO provides nine time signals of the liquid level in the pipe section. Performing a Fast Fourier Transform (FFT) and/or a Power Spectral Density (PSD) on those signals provide several information on the wave behavior. The frequency spectrum of a signal may be obtained as shown on the x-logarithmic figures (see Figures 3.14 to 3.16). According to Bendat and Pirsol [23] the resulting standard error on the spectrum values is 11%.

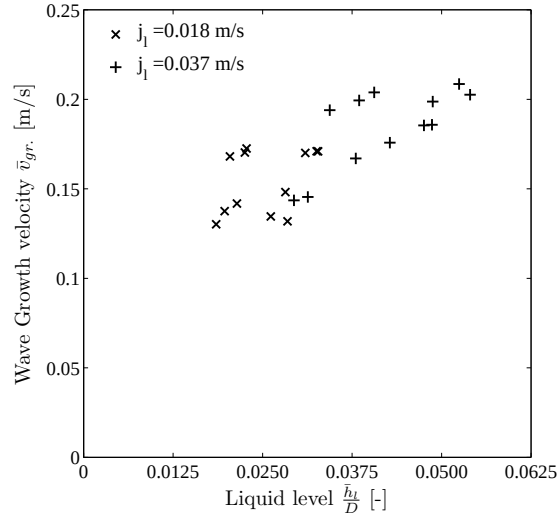


Figure 3.13: Wave growth velocities for several operating conditions: $j_l = 0.018$ m/s (x) and $j_l = 0.036$ m/s (+)

Applying a FFT to various signals provides a large spectrum of frequencies (from 0 to 100 hz/1000hz depending on the length of the signal), containing two major informations. A first large bump, characterized by a series of huge local maxima, from 0.1 to 10 Hz, denotes the characterization of the liquid film with in general a first maximum around 1Hz and sometimes a second peak around 6-12 Hz, relatively pronounced. These results are consistent with the experimental data of Paras and Karabelas [123] which predict a dominant frequency of the spectrum of the liquid film around 10 Hz. A second bump of frequencies, much less pronounced, around 45 to 90 Hz represent the characteristics of the small ripple waves. The power spectral density analysis shows that the frequencies higher than 180 Hz must be considered as noise.

The analyzis of the complete spectrum of frequencies on Figure 3.15 highlights that the dominant frequency of the complete spectrum depends both on the air superficial velocity and the axial location. Moreover the energy of the spectrum seems more distributed on higher frequency when increasing the liquid superficial velocity.

The effect of the circumferential location of the signal is also consistent with the measurements of Paras and Karabelas [121] as shown on Figure 3.16. Indeed the amplitude of the spectrum decreases between the pipe bottom and the location $\theta = 45^\circ$. The amplitude difference is equivalent whatever the frequency. However this amplitude difference is smaller than the 'several orders of magni-

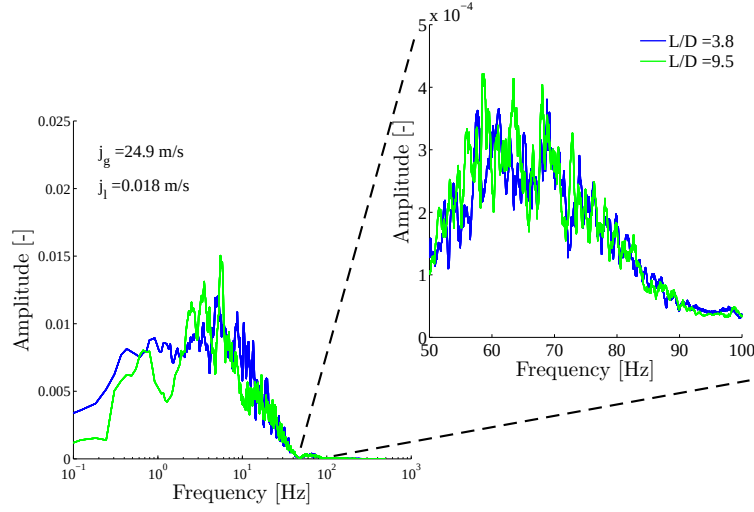


Figure 3.14: Comparison of the FFT of signals at $\frac{L}{D} = 3.7$ (blue) and 9.5 (green) for several operating conditions.

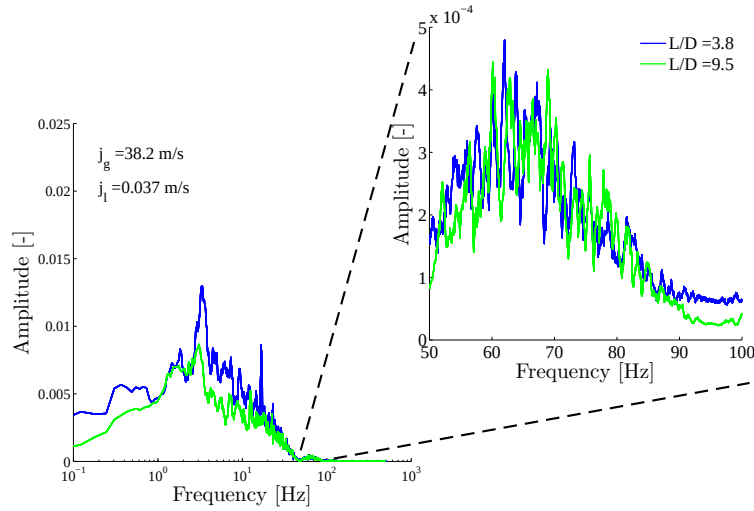


Figure 3.15: Comparison of the FFT of signals at $\frac{L}{D} = 3.7$ (blue) and 9.5 (red) for several operating conditions.

tude' mentioned by Paras and Karabelas [121] as the ratio does not exceed 10.

Finally Figures 3.17 and 3.18 present the evolutions of the critical frequencies of the ripple waves f_c at the interface when increasing the air superficial velocity j_g . The term 'critical' refers to the most amplified ripples of the ripple frequency

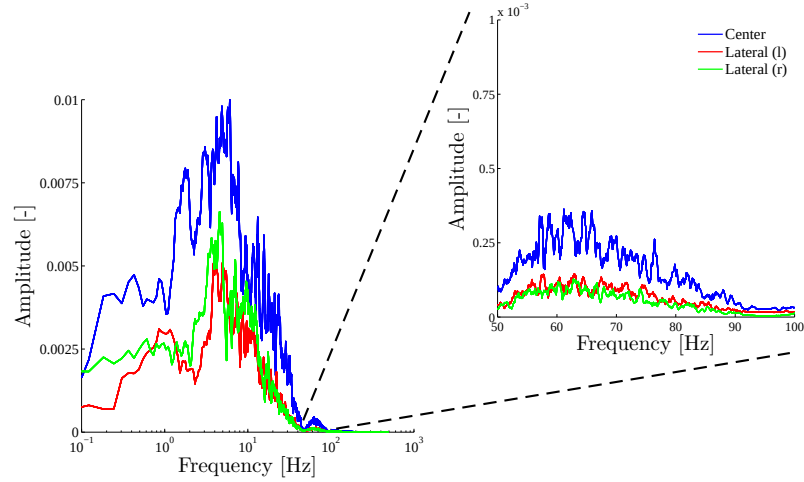


Figure 3.16: Influence of the circumferential location of the signal at fixed conditions : center of the pipe cross-section (blue), lateral positions (red and green).

spectrum: i.e. the ripple frequency corresponding to the amplitude peak. The ripple critical frequency slightly increases with the air superficial velocity. This would signify that the number of fragmentable waves per unit of interfacial area increases with the air superficial velocity. The variations along the z -axis are too small to be interpreted as certainly beyond the measurement uncertainty. These two trends appear to be consistent with the proposed model of wave formation that should not depend on the $\frac{L}{D}$ ratio. The effect of the liquid superficial velocity is hardly analyzable, as the evolution seems inversely proportional to the liquid fraction. The trends are consistent with literature data of Paras and Karabelas [122, 123] where the main ripple frequencies increase while increasing the superficial air superficial velocity j_g .

3.1.7 Synthesis on the liquid film investigation

The BANJO provides many insights on the liquid film behavior. However REGARD has not been especially designed for the analysis of the circumferential liquid film formation around the pipe section contrary to most experimental facilities [51, 121, 122, 123, 165]. REGARD was primarily designed for the analysis of the dispersed flows. The BANJO has then not been used in an optimal operation during the measurements campaigns as most of the recorder data have been measured in a complementary spirit to the entrainment and deposition

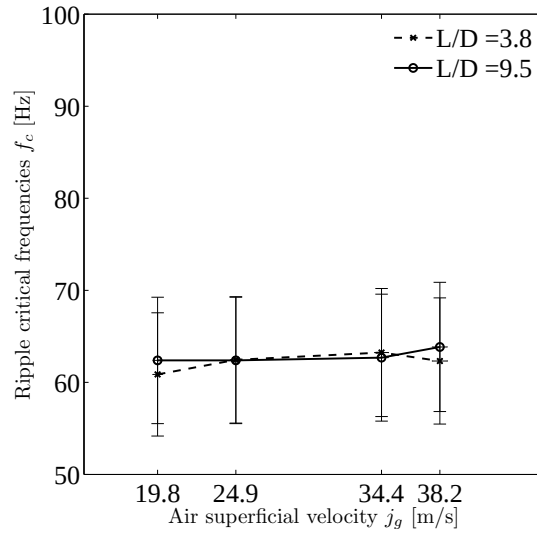


Figure 3.17: Evolution of the ripple wave frequencies as a function of the air superficial velocity at $j_l = 0.018$ m/s.

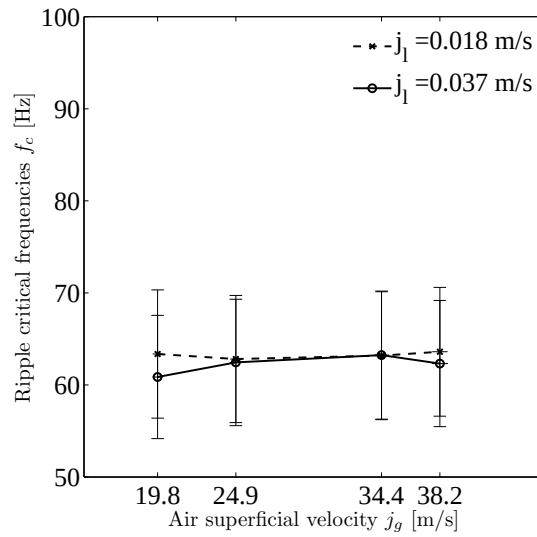


Figure 3.18: Evolution of the ripple wave frequencies as a function of the air superficial velocity at $\frac{L}{D} = 5.7$.

measurements of droplet mass flow-rates. The added-value of this instrument has only been highlighted during the post-treatment analysis of the rough data. At that moment, it appears that a complete and specific measurement campaign of liquid film analysis would have provided many more complementary information and released several assumption constraints.

The BANJO provides data on many liquid film flow parameters: the time average liquid film distribution $\bar{h}_l$ in three positions per cross-section, the liquid film fraction α_{lf} or the wave characteristics (frequency and velocity). The analysis of those data will significantly contribute to the modeling of the entrainment process.

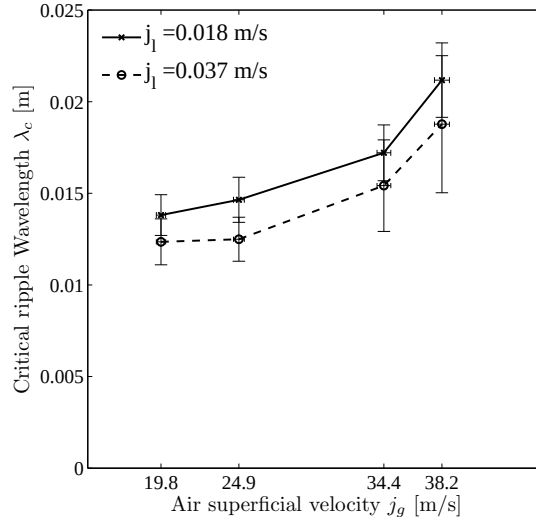


Figure 3.19: Evolution of the time and space averaged critical ripple wavelength depending on operating conditions.

Combined to the determined time averaged velocity of the ripple waves $\bar{v}_w$, the critical ripple frequency allows a determination of the critical ripple wavelength, presented in Figure 3.19. The critical ripple wavelength increases with the air superficial velocity j_g and the liquid flow j_l . This latter effect comes essentially from the evolution of the liquid film fraction depending on j_l . As increasing j_l does not influence the time averaged wave velocity v_w but acts directly on the liquid film fraction α_{lf} , the wave celerity c_w is significantly function of the liquid flow j_l . As a consequence, despite that the influence on the frequency f_c is too small to be highlighted, the critical ripple wavelength λ_c increases with the liquid flow j_l .

The critical ripple wavelength λ_c does not significantly evolve along the pipe z-axis. This is consistent with the proposed general model for the entrainment rate which is expected to increase with the air flow j_g , increase with the liquid flow j_l and to be already constant along the axial location at the analyzed $\frac{L}{D}$ locations.

However, as the estimation of the wave celerity must be cautiously considered due to the circumferential variation of the characteristics used for its determination, the amplitude of the critical ripple wavelength must also be cautiously considered.

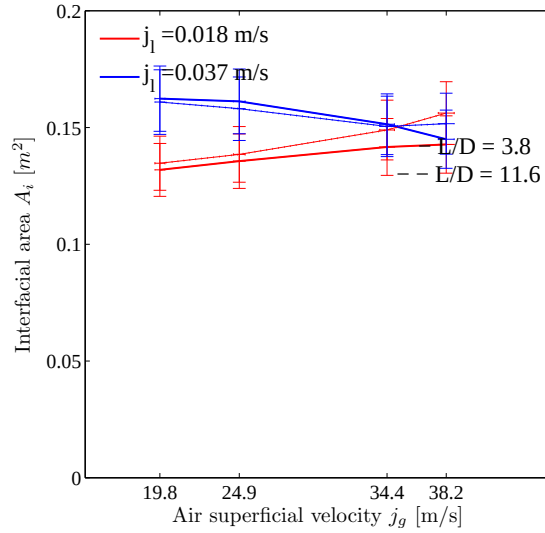


Figure 3.20: Evolution of the interfacial area on a pipe length of $3.75 \frac{L}{D}$ with the air superficial velocity at two axial locations and for two liquid flow-rates.

Finally, Figure 3.20 presents the evolution of the interfacial area A_i , calculated based on the developed length of the liquid film (taken into account the wave oscillations) in Equation 3.6 where $\langle |\Delta \bar{h}_l| \rangle$ is the space and time averaged fluctuations of the liquid film.

$$A_i = l_i \times \int_0^L \left(\sqrt{1 + \left(\langle \Delta \bar{h}_l \rangle \sin \left(\frac{2\pi z}{\lambda_c} \right) \right)^2} \right) dz \quad (3.6)$$

This interfacial area will be used to calculate from the global mass flow-rates of droplets, the droplet rate between two axial locations, and hence to deduce the entrainment rate. This interfacial area highly varies with the two air and liquid superficial velocities and along the axial location. Obviously, the effects are not direct but come from the variations of the local time averaged thickness of the liquid film but also from the ripple wave amplitudes and the interfacial wave velocity.

Data analysis

The liquid level follows expected trends: increasing with the liquid flow j_l and decreasing with the air flow j_g . The effect of the shear stress at the interface, which depends on both velocities, may then be highlighted. Increasing the air superficial velocity leads then to an acceleration of the liquid film velocity such that the liquid film level decreases. Probably due to the injection system not adequately designed for the studied range of phase velocities and to the current development of a non yet fully developed flow, moving along the axial location conducts to a decrease of the height of the liquid film.

The liquid film fraction based on an arc of circle assumption for the interface curvature presents a similar evolution. However the liquid film fraction seems to evolve towards an equilibrium value when increasing the air superficial velocity. Indeed at low liquid flow, such an increase does not influence the liquid film fraction while at the higher liquid flow, the decreasing evolution seems to be tamped to move towards a similar value.

The analysis of time variations of the interface highlights the wavy character of the flow. Indeed, local variations of the liquid height are important. Due to the shallow water effect, these variations are not symmetrical around the time average value of the interface so that the waves depth is lower and less steep than the wave crests. A Fast Fourier Transform and a Power Spectral Density highlights the spectrum of frequencies of the interface. The estimated critical ripple frequency (the frequency corresponding to the maximum of the ripple spectrum) appears to increase with the air superficial velocity and to be not very influenced by the liquid flow or the axial location.

Further improvements

The analysis of the BANJO data provides many insights on the improvement to bring to the experimental setup, either on the setup or on the experiment methodology. Indeed, to achieve a clearer campaign on the liquid film characteristics, the liquid injection setup should be improved in order to allow a control on the inlet liquid level. Ideally, this parameter should be variable along the test campaign. However to more accurately represent the inlet of a hot leg in a PWR, the injection system may be completely reconsidered.

The BANJO instrument could be modified to reconsider the relative positioning of the 9 couples of wires. Indeed in the current configuration, many data are redundant and only usable in a verification process of the other signals.

Positioning the wires in a less regular but more complete mesh would increase the modeling accuracy of the liquid film. Finally, a complementary system of liquid level measurement using infrasonic vibration would considerably enhance the confidence on BANJO data.

However, the measurement methodology itself could be improved to enhance the BANJO measurements. Indeed, a systematic use of the BANJO instrument would have provided many more measurements of the liquid level, as $90 \times 40 = 3600$ data could have been measured compared to approximately 50 measurements really achieved during the test campaign. Moreover, increasing the buffer of the BANJO acquisition, limited to 9×10000 measurements, meaning 10 seconds of measurements would increase the data precision. Indeed the isokinetic probe and the laser systems stay 180 seconds at each mesh positions, while the BANJO is limited to 10 seconds, at a 1000 Hz frequency. Increasing the number of BANJO runs would provide a basis for a statistical analysis more suitable for the averaging operation and the result confidence.

j_g	j_l	$\frac{L}{D}$	v_w	$v_{gr.}$	f_c	λ_c	α_f
m/s	m/s	—	m/s	m/s	Hz	m	—
19.8	0.018	3.7	1.36	0.055	62.1	0.0107	0.027
24.9	0.018	3.7	1.60	0.061	62.3	0.0141	0.026
34.4	0.018	3.7	1.83	0.050	61.6	0.0190	0.025
38.2	0.018	3.7	2.10	0.053	71.8	0.0218	0.024
19.8	0.036	3.7	1.62	0.234	59.8	0.0126	0.042
24.9	0.036	3.7	1.80	0.184	61.0	0.0128	0.036
34.4	0.036	3.7	2.16	0.122	63.1	0.0155	0.031
38.2	0.036	3.7	1.81	0.046	63.2	NC	0.027
19.8	0.018	5.6	NC.	0.083	58.7	NC	0.031
24.9	0.018	5.6	NC.	0.062	57.8	NC	0.028
34.4	0.018	5.6	NC.	0.049	63.6	NC	0.026
38.2	0.018	5.6	NC.	0.071	58.7	NC	0.028
19.8	0.036	5.6	1.68	0.258	58.7	NC	0.045
24.9	0.036	5.6	1.74	0.166	56.8	NC	0.038
34.4	0.036	5.6	2.42	0.106	52.8	NC	0.031
38.2	0.036	5.6	2.83	0.086	59.7	NC	0.034
19.8	0.018	9.5	1.51	0.061	58.0	0.0140	0.026
24.9	0.018	9.5	1.53	0.057	56.4	0.0143	0.025
34.4	0.018	9.5	1.92	0.060	59.7	0.0199	0.025
38.2	0.018	9.5	1.96	0.063	59.2	0.0212	0.026
19.8	0.036	9.5	1.70	0.215	60.7	0.0125	0.039
24.9	0.036	9.5	1.89	0.155	61.5	0.0126	0.033
34.4	0.036	9.5	2.36	0.103	63.0	0.0177	0.030
38.2	0.036	9.5	2.22	0.086	64.0	NC	0.028

Table 3.1: Summary of Banjo data (NC: Not Calculable, essentially due to the too high uncertainty.).

3.2 High-speed camera investigations

The literature review underlines that most experimental facilities are totally or partially designed for flow visualizations. However recorded experimental investigations are relatively rare mainly due to the timeliness and flow pattern. Nevertheless data and observations have led to a description of the interfacial flow behavior. An interfacial model of the wave development in a stratified flow have been early proposed following their flow visualizations by Hewitt and Hall-Taylor [69] and Woodmansee and Hanratty [166]. In their experimentations of liquid film atomization, at a relatively important air velocity (10 m/s), they primarily depict the apparition and development of some waves of - great - dimensions, whether considering their wavelength, their width or their height. These 'roll waves' present a 2-D profile progressing over the whole width of the square pipe. According to Woodmansee and Hanratty [166], the gradient of velocity along the wave crest generates the development of instabilities of small amplitude and small wavelength: the ripple-waves. Initiated at the basis of the roll-waves, these ripples accelerate, develop and grow along the roll-wave crest. Depending on flow conditions, these ripple-waves may break during the amplification and destabilization phase. This interface behaviour has been similarly detailed by several authors [164, 70, 101, 47, 126, 144, 128, 131] in stratified flow for this range of air velocities. At higher air velocity (≥ 20 m/s), the roll-waves cease to appear/exist on the flow visualizations as the interface becomes increasingly chaotic with a massive development of ripple-waves which develop, destabilize and finally break [164]. If roll-waves still appear, their existence is totally hidden in the flow visualizations by the faster development of 3-D ripple-waves.

The wave destabilization leading to the atomization of the ripple-wave crests as droplets may mainly take two forms:

- o formation of filaments transverse to the flow [166, 69, 47, 128],
- o formation of filaments longitudinal to the flow [126, 144, 131, 128].

In 1969, the ripple-wave break-up by formation of filaments transverse to the flow leading to the wave crest atomization was not fully developed in the flow description of Woodmansee and Hanratty [166] but has since been completed by several authors [128, 126]. The small ripple-waves are faster than the roll-waves, as the instability theory assumes the wave celerity to be proportional to $k^{1/2}$, (k is the wave number). Therefore the ripple-wave travels along the crest of the roll-wave. Due to the action of the faster gas on the shape of the gas-liquid interface, the instabilities (Figure 3.21, step a) grow and stretch (Figure 3.21,

step b). The perturbation becomes then bi-dimensional. At the extremities, the curvature effects leads to a local overpressure due to Laplace's forces. This overpressure conducts to the formation of a cylindrical filament transverse to the flow [126] (Figure 3.21, step c). This cylindrical filament separates from the liquid sheet and destabilizes then as droplets following the mechanism of capillary break from Rayleigh.

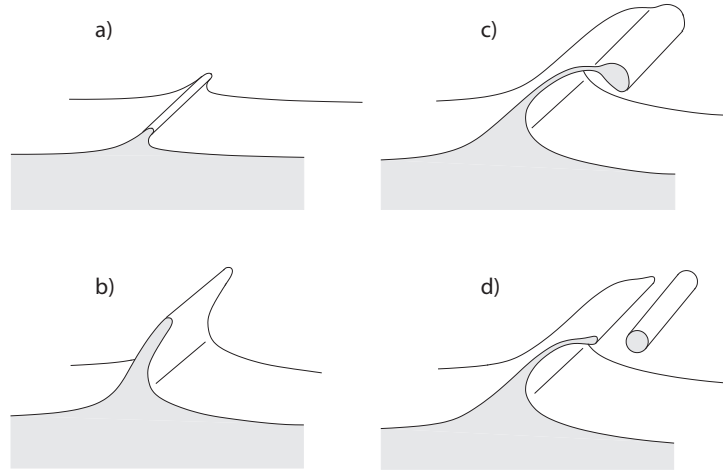


Figure 3.21: Wave atomization following the formation of filaments transverse to the flow.

A second mechanism of wave atomization has been proposed [126, 144, 131, 128] assuming that the droplet formation follows the formation of liquid filaments at the wave crest, longitudinal to the flow. The mechanism of formation of the liquid filaments is similar to the formation of filaments transverse to the flow. However, the transverse filament destabilizes before being separated from the wave crest due to the action of the surface tension. Consequently, the transverse filament deforms in a succession of swellings and contractions (Figure 3.22, step a). Due to the shear of the gas flow, the swellings deform and stretch out, taking the form of longitudinal fingers [126] which destabilize as droplets according to the procedure of Rayleigh (Figure 3.22, step b).

In order to confirm the reported flow visualizations and to validate the general philosophy of the entrainment model, REGARD is partially designed to achieve several internal or external visualizations of the flow pattern. The visualizations have been realized at 5400 fps on 1024 x 1024 pixels with a Photron FASTCAM SA 1.1 to achieve a good agreement between the frequency and the resolution of the video capture.

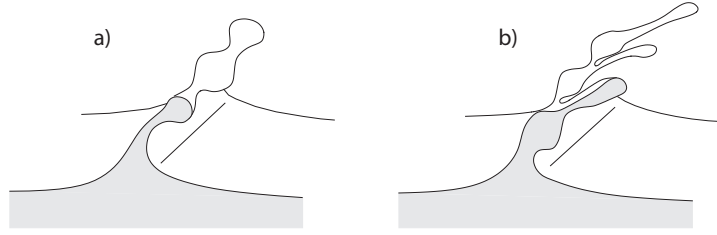


Figure 3.22: Wave atomization following the formation of filaments longitudinal to the flow.

However, as a first conclusions of the flow visualizations on REGARD, the wave deformation appears to be not as 2-dimensional as previously mentioned in the literature. Indeed as the waves are mainly 3-dimensional, the action of the air flow is not homogeneous at the wave crest. Consequently the center of the wave crest is faster than the extremities, deforming the wave crest in a arc of circle form. The transverse filaments are therefore not really 2-dimensional as depicted in Figures 3.21 and 3.22 but takes a form of half a torus.

3.2.1 Wave atomization description

Figure 3.23 presents four consecutive ($\Delta t = 0.37ms$) lateral (plane z-y) photographs (left) of the air-liquid interface on REGARD and highlights on several wave breakups (right). As the liquid film is very wavy, several instabilities grow over the interface. Many waves break leading to the emergence of several droplets.

Figure 3.23 shows mainly the evolution of two wavelets over the air-liquid interface. The wavelets on the left is in a growth and destabilizing process while the wavelet on the right is already destabilized and begins to break. The left wavelet present a half a circle form where the center of the wave crest is faster than the edges. This instability develops and stretches out until beginning to form two small fingers. The right wavelet is already destabilized and fingers are present in the first graph. These fingers are stretched out when the wave moves along the z-axis until they fragment in two main parts. This set of photographs highlights that the wave atomization appearing on REGARD for the analyzed set of fluid conditions is a wave fragmentation following the development of a longitudinal filament (finger) of liquid rather than a transverse filament. The study of the flow visualizations shows that the descriptions of Rizk et al. [131] or Stapper et al. [144] properly depicts the interfacial flow on REGARD.

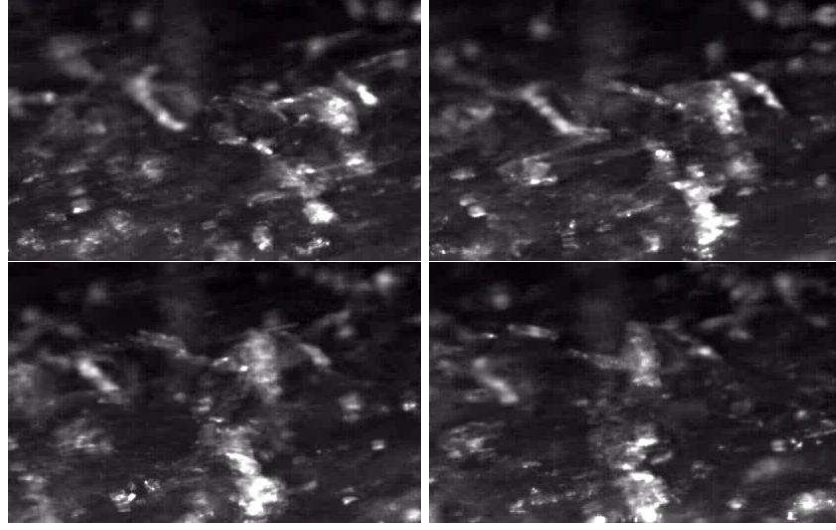


Figure 3.23: Lateral view photographs of the entrainment inception showing wave fragmentation and droplet formation. (Flow is going from right to left.)

Photography 3.24 presents a top view of a wave fragmentation process. The circular form of the wave is particularly clear on the photograph such as the presence of several fingers forming at the center of the wavelet crest. This picture confirms that the wave atomization follows the mechanism described by Rizk et al. [131].

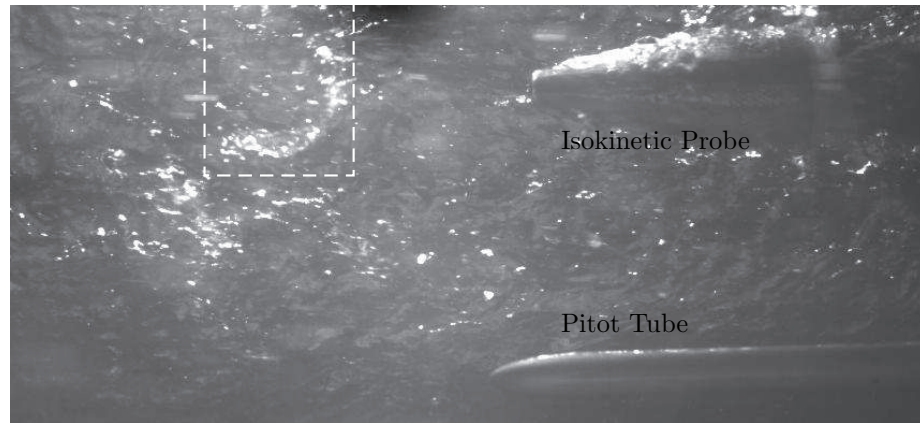


Figure 3.24: Top view photograph of the stratified flow at $j_l = 0.018$ m/s and $j_g = 24.9$ m/s. Visualization of the flow, the isokinetic probe (up) and the tube Pitot (down). (Flow is going from left to right.)

Two phenomena differs from the expected behavior of the flow. Indeed if

the main contributor to the droplet creation is the break of unstable waves, two other contributors are visualized. The impact of droplets depositing on the liquid film provokes the ejection of new droplets. More rare but present, the fragmentation of bubbles emerging at the surface of the liquid film seems to provoke also the ejection of droplets toward the streaming gas.

3.2.2 Droplet visualization

Photography 3.25 show a top view (plane x-y) of the flow at $\frac{L}{D} = 9.6$ and high air flow ($j_g = 38.2$ m/s). The many very small lines are droplets traveling near 40 m/s (top) and 30 m/s (bottom) (Droplets crossed the analyzed section by camera in 5 ms, 30 frames) while spherical and luminous objects are mainly bubbles trapped in the liquid film or big drops at the crest of the waves. Differentiation is made according to the reflection of light and depending on their velocities.

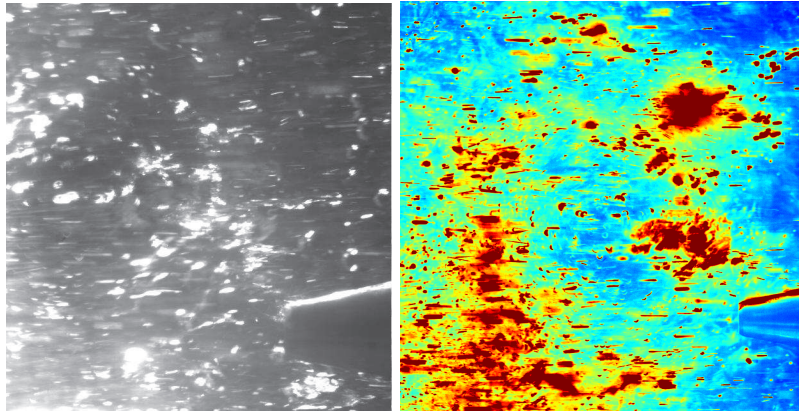


Figure 3.25: Top view photography of the stratified flow at $j_l = 0.018$ m/s and $j_g = 38.2$ m/s (left) and $j_g = 24.9$ m/s (right).

Figure 3.25 shows that the droplets are traveling quite parallel to the flow direction, so that the droplet velocity component in the direction of the flow v_z appears to be far greater than the transverse velocity component v_x . As the isokinetic probe appears in the figure, the droplet velocity may be estimated by calculating the number of pixels travel during the impressing time of the camera. As these results completely confirm the more precise laser measurements presented here after, this subject is no more treated in this section.

3.3 Droplet mass flux

The previous sections aim to describe the interface behaviour and the wave characteristics. The atomization of interfacial waves conducts to droplet entrainment. This third section aims to present measurements of the mass flow-rates of droplets. Due to the measurement technique, only time averaged data are available. In a first time, local (and time averaged) mass flow-rates of droplets $\dot{M}_{d,i}$ [kg/s] are studied at the i positions in the cross-section. In a second time, applying the air cross-sectional space averaging operation leads to a measurement of the integrated mass flow-rates of droplets over a pipe cross-section $\dot{M}_d$ [kg/s]. The entrained fraction E [-] may then be evaluated from Equation 1.3 as well as the entrainment rate Γ_E [kg/m² s].

The droplet mass flow-rates have rarely been directly studied in the literature. Most authors (Dallman [41], Laurinat [90] or Mantilla [101]) deduced the entrained fraction of liquid E from a liquid film extraction technique: the space averaged mass flux of droplets $\dot{M}_d$ is assumed to be equal to the difference between two distant measurements of the liquid film mass flow-rates, the liquid flow-rate introduced in the test section $\dot{M}_{lf,0}$ and the liquid flow-rate extracted at the measurement location $\dot{M}_{lf,1}$. The liquid film is extracted over a small proportion of the circumference (θ) and an integration over the pipe circumference provides a global measurement of the total liquid film mass flow-rate.

$$E_{0 \rightarrow 1} = \frac{\dot{M}_d}{\dot{M}_{lf,1}} = \frac{\dot{M}_{lf,0} - \int_0^{2\pi} \dot{M}_{lf,1,\theta} d\theta}{\dot{M}_{lf,1}} \quad (3.7)$$

This indirect technique is easily achievable in annular flows in which the liquid film is uniform over the pipe circumference but very complex to achieve in stratified flows. In case of horizontal flow, indirect techniques are extremely difficult to set up. Indeed gravity imposes an asymmetric distribution of the liquid film over the wall circumference (as shown in section 3.1.2) but also of the entrained droplet concentration. The circumferential distribution of droplets is expected to be generally non-uniform as shown by Williams [164] or Paras and Karabelas [120, 121]. To evidence this heterogeneity, the mass flow of droplets must be directly measured at many local positions over the pipe cross-section. In the open literature, local direct measurements of the mass flow-rates of droplets in horizontal flows have been achieved by Paras and Karabelas [120, 121] and Williams [164].

Studying annular flows, Paras and Karabelas [120, 121] have used an isokinetic probe in order to collect the mass flux of droplets along different diameters in the cross-section: the horizontal, the vertical and the 45° axis passing through

the center of the pipe. They concluded on the strong influence of the gravity on the vertical profiles which presented a strong stratification. Indeed they noticed variations by a factor 10 between the mass fluxes measured at the top and the bottom of the pipe. As the stratification is essentially vertical, the horizontal profiles were quite uniform.

In their study of transitional flows from stratified to annular, Williams et al. [164] also used an isokinetic probe to collect local mass fluxes of droplets all over the pipe cross-section. They also highlighted a large variation of the droplet flux along the vertical axis. They showed that the droplet flux is low and quite constant near the top of the pipe. Similarly to the data of Dallman [41], Williams [164] and Paras and Karabelas [120] observed a maximum of the droplet mass flow distribution under high air velocity conditions in the upper quadrant². As the position of this maximum has no intuitive physical meaning, whatever the flow is annular or stratified, they suggested that this is a consequence of a secondary flow. In contrast to Paras and Karabelas [120], Williams [164] presented some maps of the droplet flux over a cross-section which reveal that the horizontal profile of droplet flux ceases to be uniform at high air velocities (over 30 m/s, but probably dependent on the pipe diameter D).

In the REGARD experiment, an isokinetic probe provides up to 75 local measurements $\dot{M}_{d,i}$ over a cross-section, time averaged on the 180 seconds of the run duration. The total mass flow-rate of droplets $\dot{M}_d$ at a fixed axial location for a couple of phase velocities must be estimated by an integration of these local data $\dot{M}_{d,i}$ over the pipe cross-section:

$$\dot{M}_d = \sum_{i=1}^{75} \frac{\dot{M}_{d,i}}{t} \beta_i \frac{S_{mesh}}{S_{probe}} \quad (3.8)$$

where $\dot{M}_{d,i}$ denotes the measured mass of liquid, t equals 180 seconds, β_i denotes the correcting factor of the integration (detailed in Appendix A.2) and S_{mesh} and S_{probe} denotes the surface of a mesh cells and the aspirating surface of the isokinetic probe. The injected mass flow-rate of liquid ($\dot{M}_l = \rho_l j_l A$) is controlled in REGARD. Consequently, the isokinetic probe is aimed to quantify the entrained fraction of droplets ($E = \frac{\dot{M}_d}{\dot{M}_l} [-]$) within the gas stream. The integrated final data $\dot{M}_d$ is representative of the total mass exchange of liquid between the liquid film and the droplets. To account for additional effects of mass transfer by evaporation, the isokinetic probe is moreover equipped with a

²This maximum appears at velocities far greater than the velocities studied in this work. For that reason and as the transition to annular flows is moved to higher air flows with the large diameter of REGARD, it is not possible to assess this phenomenon.

quantification system of the air humidity. The isokinetic probe results include both measurement of the local mass flow and the local humidity of the air. The post-processing method analyzes these results to provide an integrated measure of the total mass flow of droplets, with or without taking into account the evaporation contribution.

The isokinetic probe has to suck the mixed flow of air and droplets at a velocity close to the local velocity. A Pitot tube is mounted on the instrument to locally measure the velocity of the gas stream.

3.3.1 Local mass flow-rates of droplets $\dot{M}_{d,i}$

Map of droplet mass flow-rates

The experimental outputs of the isokinetic probe for an experiment consist in 75 local (and time averaged) mass flow-rates of droplets, distributed over the pipe cross-section. The local measurements are actually representative of the droplet mass flows over the suction section of the isokinetic probe. To complete the pipe section occupied by the dispersed phase, these local measurements are extrapolated over a mesh element centered on the suction section. A map of these measurements $\dot{M}_{d,i}$, space averaged over a mesh cell to get the surface mass flow-rates $\dot{m}_d$ in $[kg\,m^{-2}\,s^{-1}]$, is presented on Figure 3.26. At the positions where the local mass flow-rates are too low to be measured or below the measurement uncertainty, no value was plotted on the maps (white space).

The expected heterogeneity in the droplet concentration due to the action of the gravity is particularly highlighted. Indeed the mass flux densities are especially large at the bottom of the pipe and negligible in the upper part. This stratification marks substantially along a vertical diameter as illustrated in Figure 3.27 in which the left and right maps corresponds to two axial locations for several air velocities at the low liquid flow: $j_l = 0.018$ m/s. The maps emphasize a particular accumulation of drops in the near-wall region or close to the liquid film. Whatever the flow conditions, the droplet dispersion over the pipe cross-section is very uneven so that in 85% of experiments, no droplet mass flow can be measured at upper positions of the grid. The accumulation of liquid in the weighing system is simply zero or does not exceed the weighing accuracy during the 3 minutes of measurement duration at a local position. To actually gain access to droplet mass flow information in an upper quadrant, it would have been required to significantly increase the duration of measurements at each mesh. But such an increase would have been problematic because of the consequent increase in the run duration (already one day per run of 75 local

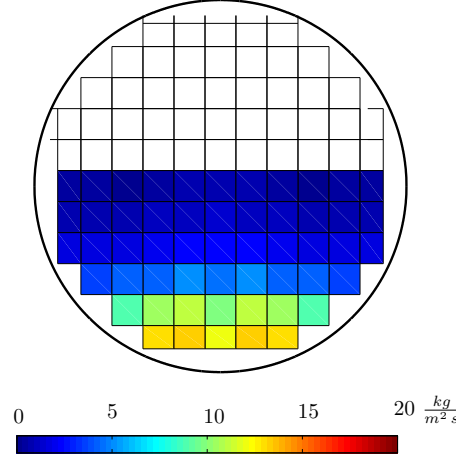


Figure 3.26: Map of local mass flux densities of droplets $\dot{m}_{d,i}$, at $j_g = 19.8$ m/s, $j_l = 0.036$ m/s and $\frac{L}{D} = 11.6$.

positions, taking into account the time required to calibrate the setup, initiate the flow and the rotational and translational Stokes of the isokinetic probe).

Locally the time averaged local mass flow-rates $\dot{m}_d$ vary by a factor up to 20 when increasing the air superficial velocity j_g . The effect of the air velocity is very important on the droplet flux. The slope of this increase is also stratified as the variations are greater at the lower mesh cells than in the upper cells, so that at the highest positions, the mass flux could even be considered as negligible but practically independent of the air flow at those axial locations. A detailed study will be performed in the following paragraphs to assess these effects.

Figure 3.27 also emphasizes the evolution of the maps of mass flux depending on the axial location $\frac{L}{D}$. The stratification of the droplet flow-rates varies along the z-axis so slightly that a detailed study is also required to precisely analyze the effect. Anyway, the number of mesh cells in which the mass flow-rates exceed the measurement uncertainties increases along the axial location. This local increase in the number of droplet is symptomatic of the diffusion of droplets ejected from the interface and entrained within the gas stream.

The effect of the liquid flow on the droplet mass flow appears mainly on Figure Figures 3.28. The dispersion of the mass fluxes of droplet in the pipe cross-section significantly increases, so that the local mass flow-rates are greater

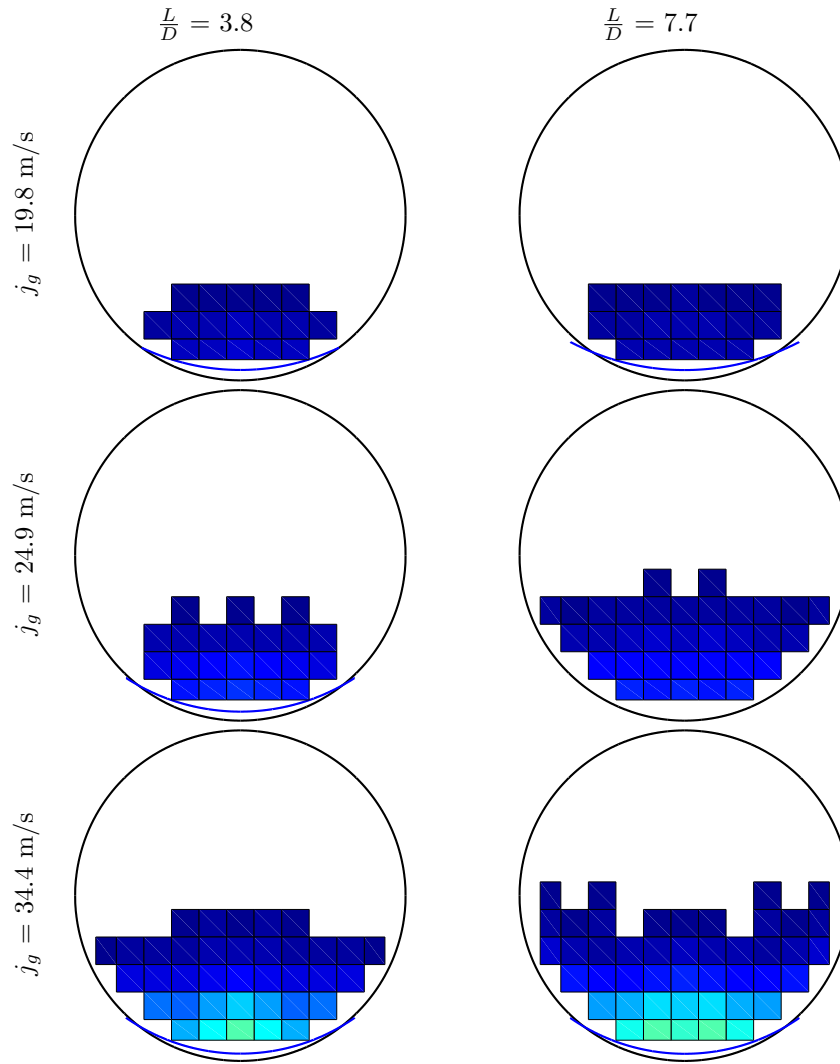


Figure 3.27: Maps of the droplet mass mass flux densities depending on flow conditions at $j_l = 0.018$ m/s

everywhere. This effect especially marks over the lower mesh cells. This local increase is probably due to the conjunction of two effects; the rise of interface with the liquid flow and the extension of the interfacial area. The first effect gets the mesh cells closer to the interface level and then to the local wave fragmentation and droplet generation while the second effect increases the number of interfacial waves and then the number of droplet sources.

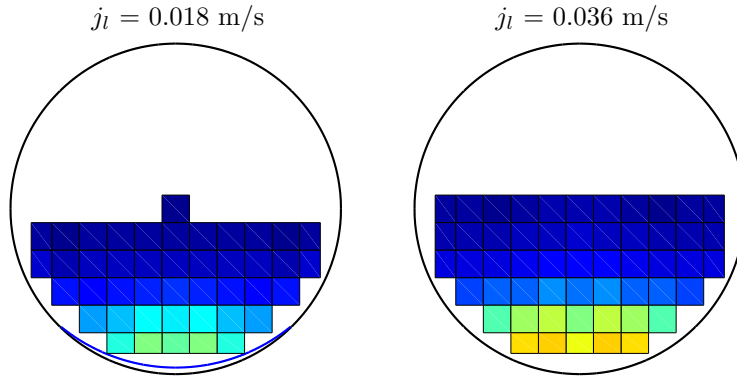


Figure 3.28: Maps of the droplet mass flux densities at $j_g = 34.4$ m/s and at $\frac{L}{D} = 5.7$.

Local analysis

Figures 3.29 and 3.30 represent the evolution of the droplet mass flux locally-measured along six vertically aligned points. The x-axis represents the position of the points on this vertical line with respect to a reference at the center of the pipe, while the y-axis indicates the measured mass flow $\dot{M}_d$ in [g/s]. These curves show that the flow goes to zero when the probe moves up along the vertical axis as the probe does not measure any droplet flow in most of the upper portions of the pipe. The uncertainties are not represented on this graph in order to not overloaded it but they have a constant value of 0.003 g/s. Therefore the uncertainty is large compared to the little flux of droplets and small compared to the bigger fluxes (from 150 % to 1 % according to flow conditions and mesh positions).

The droplet flux is maximum at the interface with the liquid film (at the bottom) and decreases towards the pipe centerline. As expected, the droplet mass flux increases significantly with the gas flow, slightly with the liquid flow and slightly along the axial location. This latter is symptomatic of a flow where droplet entrainment and deposition are not yet balanced. Indeed the local increase of droplet flux along the z-axis shows that the droplet diffusion is not yet stabilized so that the droplets are still dispersing over the pipe cross-section. The influence of the air velocity is very important as doubling the air velocity may increase by a factor 10 the local mass flows of droplets. The number of mesh cells in which a droplet flux measurement could be achieved also increases with the air velocity as the vertical droplet diffusion is accelerated by the increase of air velocity. So the air velocity, in a first effect, increases the droplet generation at the interface and, in a second effect, favors their dispersion over

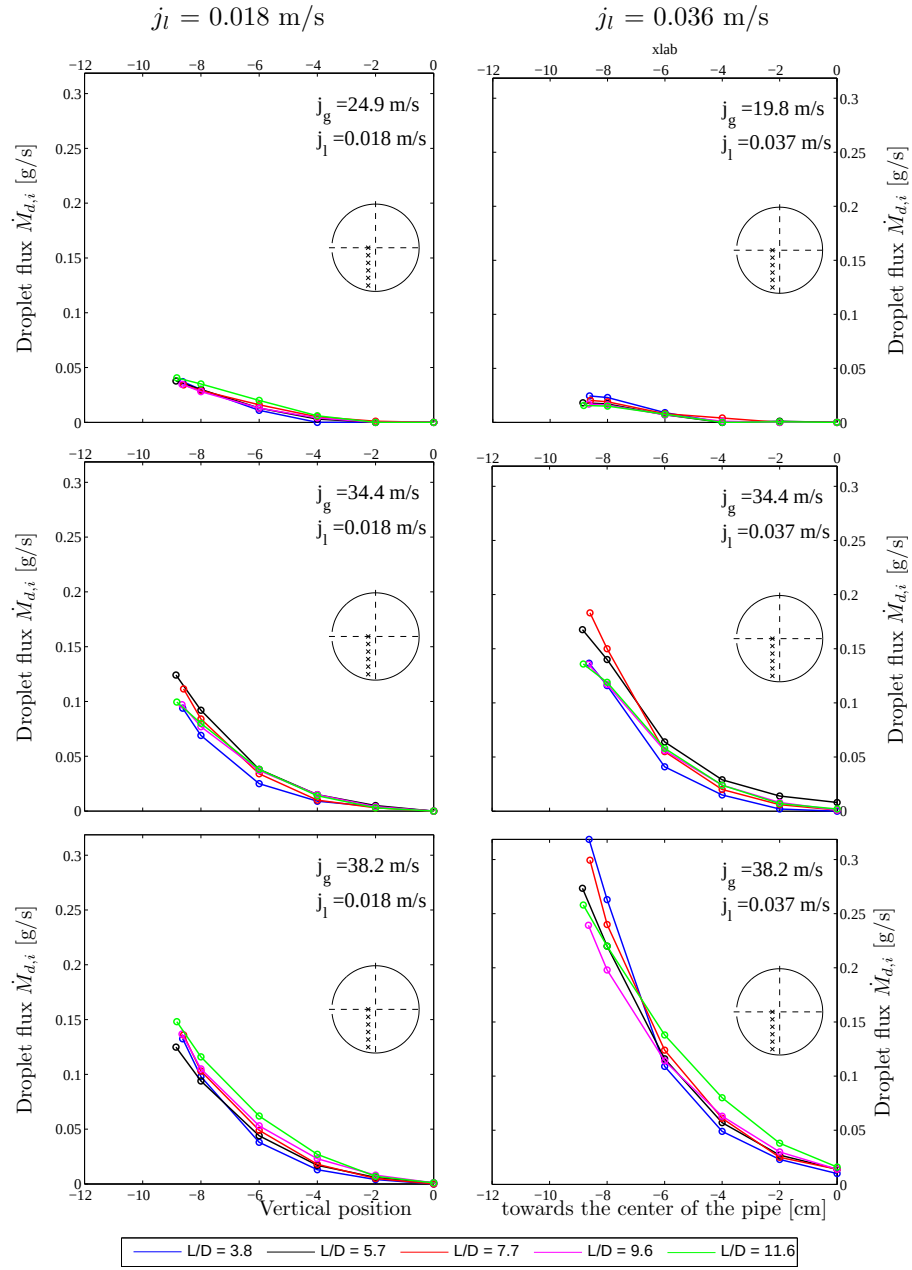


Figure 3.29: Evolution of the local mass flow-rates of droplets along a vertical axis in the pipe section (2 cm spacing between each local position) distant of 2 cm from the vertical diameter.

the pipe cross-section. The effect of liquid velocity is dual: obviously, the rise of the liquid film height $\bar{h}_l$ moves forward the curves by some 1 cm along the

vertical y-axis, but the increase of the liquid velocity and the interfacial area also generates an higher number of ripple waves and then of droplet sources.

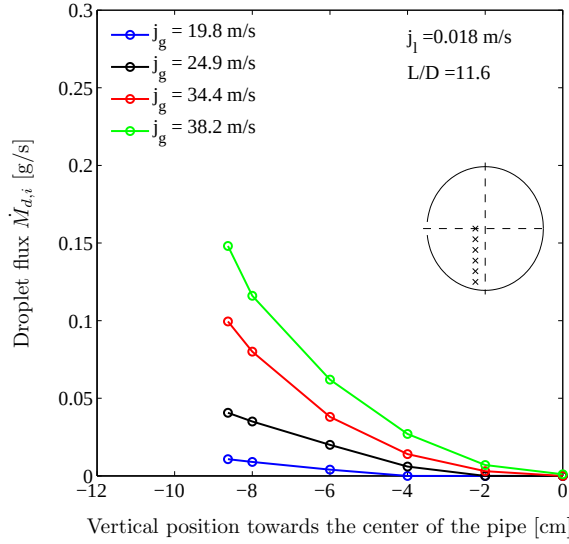


Figure 3.30: Evolution of the local mass flow-rates of droplets along a vertical axis in the pipe section (-2 cm parallel to the vertical diameter).

Figure 3.31 highlights the horizontal dispersion of the mass flux along an horizontal chord as a function of the air velocity j_g . In contrast to Paras and Karabelas [120] but similarly to Williams [164], at high air flows (30 m/s), the droplet flow-rates are not totally uniform. Indeed the mass flows may be higher near the walls than at the center of the pipe due either to the re-entrainment of droplet deposited on the upper part of the wall or to the circumferential development of the liquid film. The horizontal dispersion of the droplet mass flux does not follow the air velocity profile as expected but differs close to the walls.

Figure 3.32 presents the local evolution of the time averaged mass flow-rates of droplet $\dot{M}_{d,i}$ in 4 mesh cells of the pipe cross-section. As a first effect, the droplet mass flow-rates always significantly increase with the flow of air. And in a second effect, the slope of these augmentations appears to be quite stratified as the mass flux increases much greater in the lower mesh cells than in the upper cells. This means that the diffusion is quite controlled by the gravity as the number of droplets able to reach the upper positions is quite smaller than the number of droplets reaching the lower positions.

The droplet flow slightly but continuously increases when moving along the axial location $\frac{L}{D}$. The absolute difference between the axial location does not

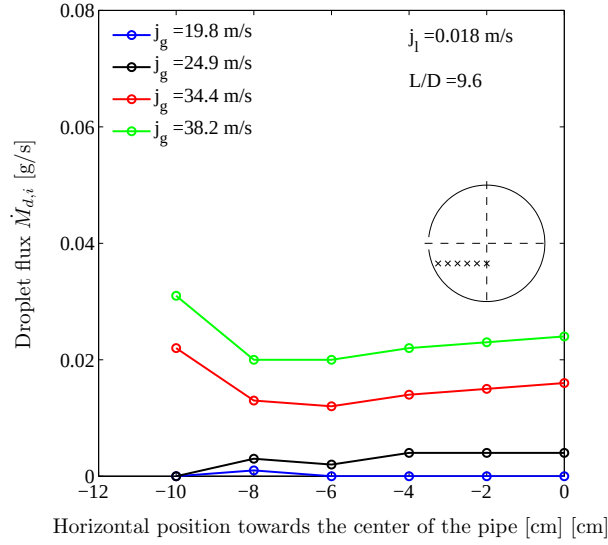


Figure 3.31: Evolution of the local mass flow-rates of droplets along an horizontal axis at $h = 8\text{ cm}$ (-4 cm parallel to the horizontal diameter).

seem to be affected by the vertical position.

3.3.2 Droplet mass flux integrated over the pipe cross-section $\dot{M}_d$

The distribution of the droplet flow over the cross-section is averaged using an integral approach to get the total droplet mass flow-rate over the air and droplet cross-section: $(1 - \alpha_l) A$. Over a pipe section of 24 cm in diameter, 75 time averaged mass flow-rates distributed nearly uniformly in the pipe are measured per run. Since the probe has an inlet diameter of 4 mm, the proportion of the area covered by the samples corresponds to approximately 2% of the pipe section. The dispersion of the droplets must then be cautiously integrated in order to minimize the integration uncertainties (see Appendix A.2).

Figures 3.33 and 3.34 show the evolutions of the integrated mass flow-rate of droplets depending on phase velocities or moving downwards. As expected in the measurement methodology section (see Figure 2.17), as the flow is not yet fully developed, entrainment and deposition are not yet balanced so that the integrated mass flow-rates of droplets increase when moving along the axial location $\frac{L}{D}$. The profiles reveal that the increase of the integrated mass flow-rate of droplets follow the expected tendencies, moving towards a saturation. The inception of the deposition progressively reduces the increase of droplet concentration in the airflow until reaching the equilibrium balance.

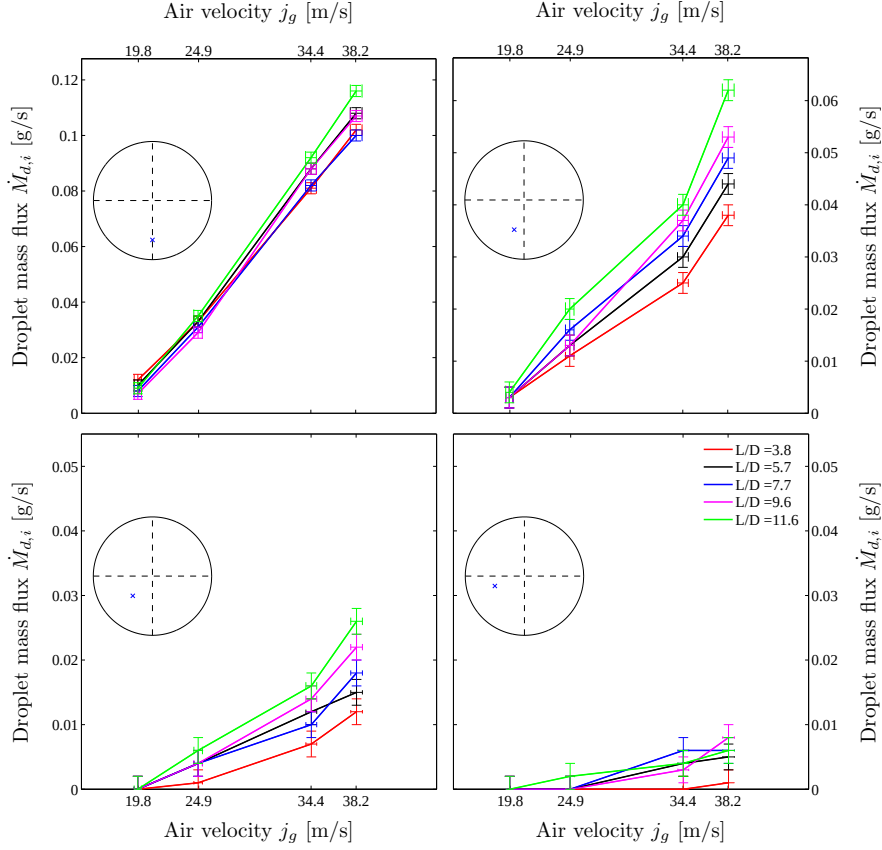


Figure 3.32: Evolution of the local mass flow of droplet for several positions over the cross-section at $j_l = 0.018$ m/s.

Nevertheless the initial important gradient of droplet glow generated may partially be due to the presence of a surface instability at the junction of the two continuous (air and water) phases at $\frac{L}{D} = 0$, due to the liquid injection technique. This instability may destabilize the interface leading to an unexpected local wave fragmentation which could slightly contribute to the droplet flow.

Otherwise the droplet mass flow-rates significantly increase with the air velocity but also with the liquid velocity as depicted on Figure 3.34. The direct implication of the liquid mass flow effect is however questionable as the effect may mainly come from the interface behavior, highly influenced by the liquid velocity (see Section 3.1.2 for details).

Operating an extrapolation from Figure 3.34 determines the initiation velocity of the entrainment phenomenon. Whatever the distance or the water mass flow-rate, this initiation velocity would appear between 15 m/s to 18 m/s, which

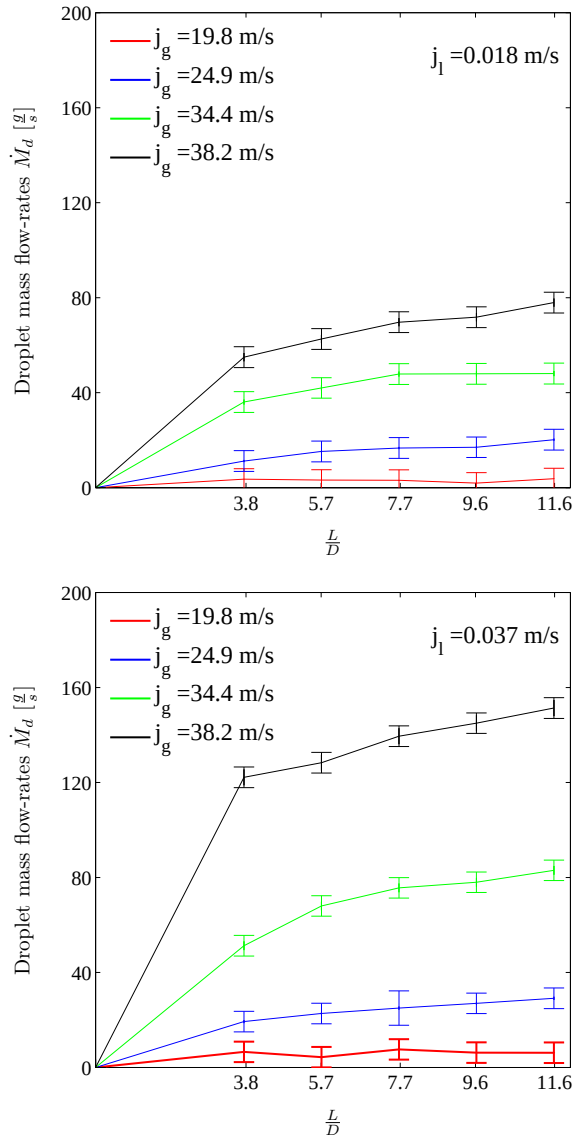


Figure 3.33: Evolution of the integrated droplet mass flow-rates as a function of the axial location $\frac{L}{D}$.

is a little higher than the data of Mantilla [101] who found an onset of entrainment by a pressure drop method at around 10 m/s but is similar to the data of Paras and Karabelas [121] and Williams [164]. However the uncertainty on such an extrapolation from only 4 data is so important that this information must be cautiously taken into account. In order to determine the limits of entrainment inception, an additional study on a large range of velocity should have been conducted in a position of the isokinetic probe very close to the interface, while extending the measuring time of the mass measurement.

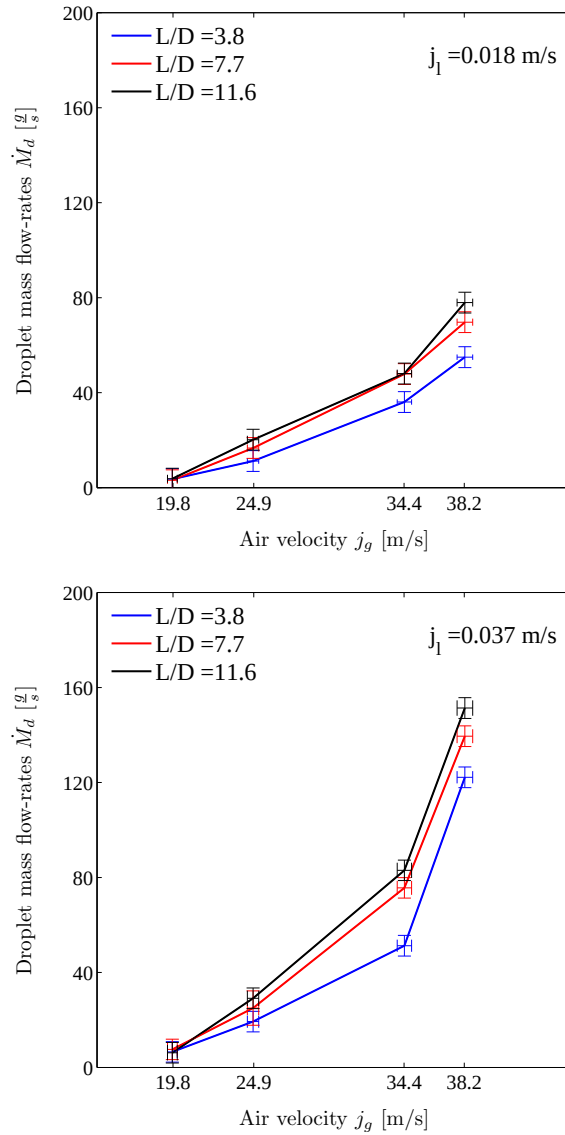


Figure 3.34: Evolution of the integrated droplet mass flow-rates as a function of the air velocity j_g .

3.3.3 Local and integrated measurements of humidity

As the circuit of air is an open loop, the inlet air is continuously sucked from outside to be injected in the test section. Some air properties, like air pressure and air temperature, are regulated by several compressors and heat exchangers. Others, like air humidity, depend on the outlet atmospheric conditions. At the inlet the saturation in water of the air is neither imposed nor controlled. Consequently the air humidity increases when brought into contact with the liquid

flow, by local vaporization either of the liquid film or of several smaller droplets. The following paragraphs aim at quantifying this vaporization.

The relative humidity ϕ of the airflow is measured downstream of the weighing system of the isokinetic probe. Combining this local humidity measurement with the initial measurements of relative humidity and initial temperature outside at the beginning of the experimental run and with the temperature of the air regulated in the air loop, the variations of the mass flow-rates of steam in the airflow may be estimated. This provides an important input on the mass exchanges by vaporization.

Maps of relative humidity ϕ

Similarly to the maps of droplet mass flow-rates (Figure 3.27), the maps of relative humidity ϕ appears to be significantly stratified as depicted in Figure 3.35. The relative humidity is higher at the bottom of the pipe cross-section and increases with the local mass flow of droplets. The evolutions in relative humidity ϕ are very similar to the evolutions in droplet mass flow-rates, except for few characteristics. The increase of air humidity with the flow conditions is limited by the 100% saturation. The inlet air is already humid, so that the relative humidity over the pipe cross-section is uniform at $\frac{L}{D} = 0$. Finally the diffusion of the humidity appears to be a phenomenon far faster than the diffusion of the droplet flux. Inter alia, the droplet flux in the upper locations, which is too small to be considered, contributes anyway to the local droplet vaporization. Moreover, the smallest droplets which are the most (susceptible) to vaporize, diffuse faster as turbulence overcomes gravity.

As the air velocity increases, the relative humidity increases throughout the lower quadrant until a quasi-homogeneous distribution of important values (80% - 100%). In the lower mesh cells, these values may reach quite the saturation (95%-100%). When the air flow has reached the saturation in steam, the relative humidity ϕ measurement is no more dependent on the flow parameters. In the upper mesh cells where the relative humidity is relatively low, the humidity measurement appears to be quite stratified, which means that little droplets reaching these positions still contribute to vaporization. Finally, this phenomenon slows down the diffusion of the droplets.

Humidity profiles in the pipe

The relative humidity measurement is qualitatively interesting but for a quantitative analysis, this measure is not readily usable. The relative humidity ϕ is

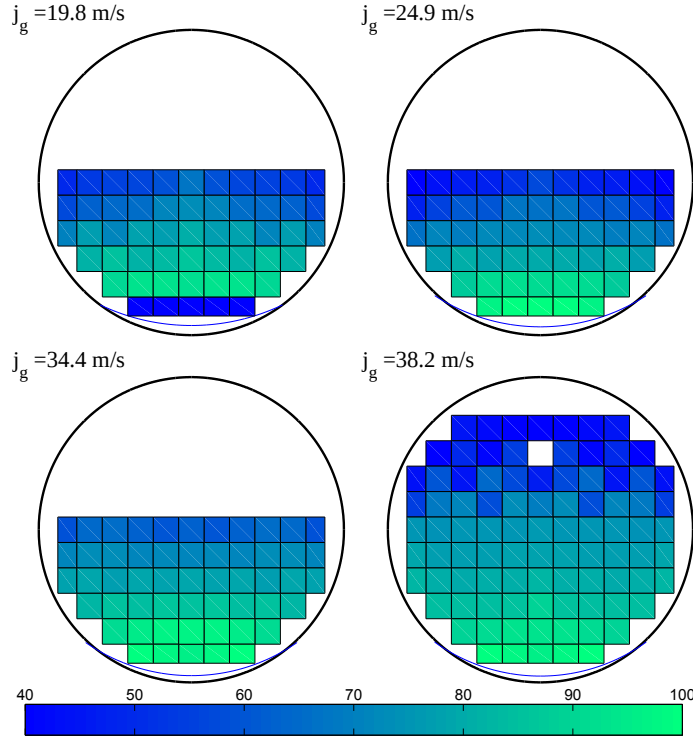


Figure 3.35: Mapping of the relative humidity for different air velocities at $\frac{L}{D}$ of 3.8 and higher liquid velocity: $j_l = 0.036$ m/s.

then converted at each mesh point into an absolute humidity x :

$$x \equiv \frac{\dot{M}_{\text{steam}}}{\dot{M}_{\text{dry air}}} = 0.622 \frac{\phi p'_v}{p - \phi p'_v} \quad \left[\frac{kg_{\text{water}}}{kg_{\text{dry air}}} \right] \quad (3.9)$$

where p'_v denotes the saturation pressure at the temperature T : $p'_v = p'_v(T)$. The associated uncertainties depend then on the uncertainties on the pressure, on the saturation pressure (and hence the temperature in the test section) and on the relative humidity:

$$\Delta x = 0.622 \sqrt{\left(\frac{p'_v p}{(p - \phi p'_v)^2} \Delta \phi \right)^2 + \left(\frac{\phi p}{(p - \phi p'_v)^2} \Delta p'_v \right)^2 + \left(\frac{-\phi p'_v}{(p - \phi p'_v)^2} \Delta p \right)^2} \quad (3.10)$$

with $\Delta p'_v$, Δp and $\Delta \phi$ the uncertainties on the saturation pressure, the pressure and the relative humidity respectively. The uncertainty on the saturation pressure is estimated by taking into account an uncertainty of 3°C on the temperature ; the pressure uncertainty corresponds to 5% of the pressure and the uncertainty of the relative humidity is 5% of the measured relative humidity.

The mass flow-rate of steam $\dot{M}_s$ is deduced from the difference in humidity between the inlet absolute humidity (x_0) assumed to be representative of the absolute humidity of the air continuously sucked from outside, and the calculated absolute humidity:

$$\Delta \dot{M}_{s,local} = \left(\frac{x - x_0}{1 + x_0} \right) \dot{M}_{g,local} \left[\frac{kg_{water}}{s} \right] \quad (3.11)$$

As:

$$\dot{M}_{g,inlet} = (1 + x_0) \dot{M}_{dry\ air,inlet} \quad (3.12)$$

The final uncertainty on the steam mass flow-rates (see Equation 3.13) is then dependent on the variations of the outside atmospheric conditions (x depends on pressure, temperature and relative humidity) but also on the regulated temperature in the test section, the pressure and the humidity:

$$\Delta \dot{M}_{s,local} = \dot{M}_{s,local} \sqrt{\left(\frac{\sqrt{\Delta^2 x + \Delta^2 x_0}}{x - x_0} \right)^2 + \left(\frac{\sqrt{\Delta^2 x_0}}{1 + x_0} \right)^2 + \left(\frac{\Delta \dot{m}_{g,local}}{\dot{m}_{g,local}} \right)^2} \quad (3.13)$$

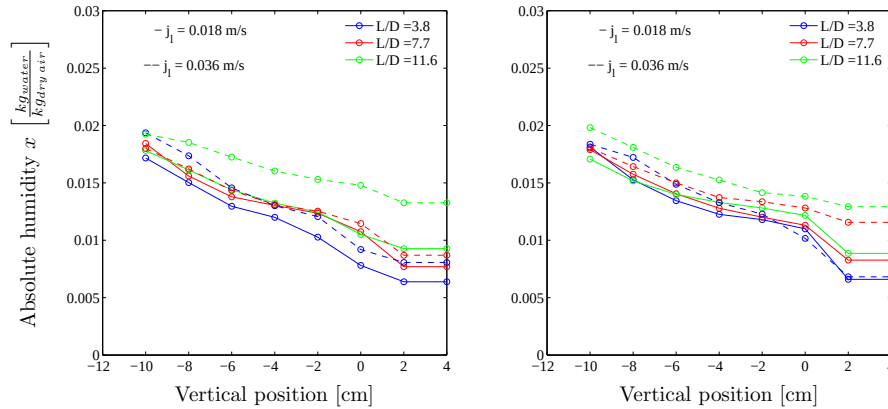


Figure 3.36: Evolution of the absolute humidity along the vertical diameter of the pipe cross-section.

Similarly to the evolutions of the droplet mass flow-rates in Figures 3.29 and 3.30, the evolution of the absolute humidity along a vertical line is proposed in Figures 3.36 and 3.37 which show large variations for the absolute humidity over the pipe cross-section.

At the bottom of the pipe, the steam flow-rate has already reached the saturation in humidity. However between the middle and the bottom of the pipe the steam flow may still increase by a factor of 2 to 3.

Two experiments (at locations of 5.7 and 9.6) have been conducted in May and June 2011. The measured steam mass flow-rates are always greater than in the three other axial locations studied in winter (October to December 2010). The difference may reach 20 to 50% depending on the flow conditions and the probe position over the cross-section. The only variation on REGARD between these two periods is the atmospheric conditions. Consequently this difference highlights that the flow conditions have not could be maintain completely similar over time.

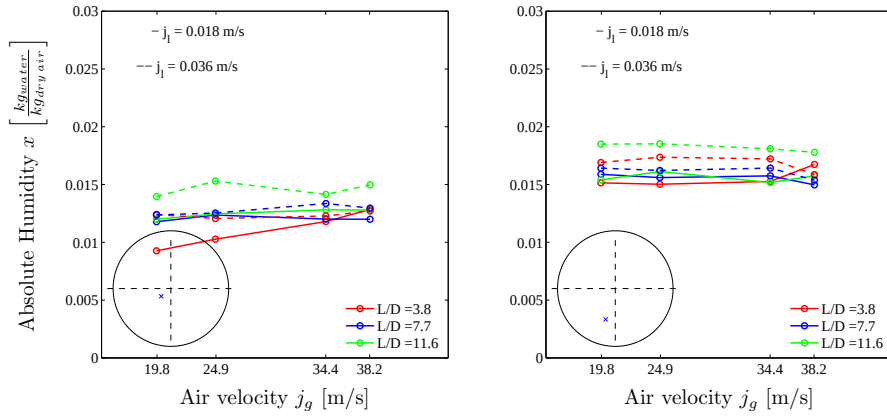


Figure 3.37: Local evolution of the absolute humidity for 2 mesh positions along a vertical axis when increasing the superficial air velocity j_g .

Mass flow-rates of steam $\dot{M}_s$ integrated over the cross-section

The total mass flow-rates of steam may be determined by performing an integration operation on the local time averaged mass flow-rates of steam over a pipe cross-section. The difference between the mass flow-rates of steam estimated in the test section and the mass flow-rates initially present at the intake air and measured at the beginning of the run, corresponds to the mass transfer by vaporization either of the liquid film or of the droplet flow.

Figure 3.38 presents the importance of the amount of steam in the air. The mass flow-rates of steam depend very little on the axial location $\frac{L}{D}$. The mass flow-rates of steam are not at all negligible compared to the mass flow-rates of droplets (see Figures 3.33 and 3.34 for comparison). Indeed the ratio between the vapor flow and the droplet flow varies between 0.15 and 2 depending on flow conditions. As expected due to the definition of the mass flow-rates of steam (Equation 3.11), the flux of steam directly follows the evolution of the air flow-rate j_g .

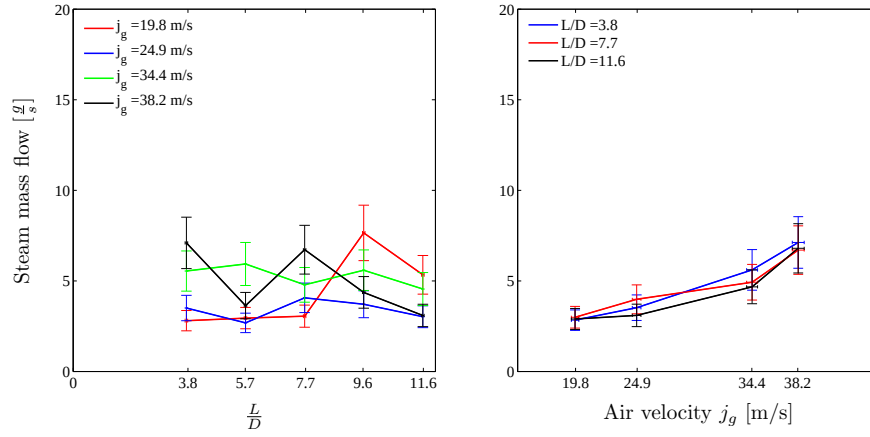


Figure 3.38: Evolution of the integrated steam mass flow-rates at: $j_l = 0.018$ m/s depending on the axial location (left) and the air velocity (right).

The measurement of relative humidity appears to be important for a complete analysis of the amount of liquid lost by the liquid film during an experiment on air-water stratified flow. Indeed in a vapor-water stratified flow, the humidification phenomenon has not to be considered. Surprisingly, while most reported experiments on horizontal stratified flow deals with liquid-air flows, none of them presents data on humidity measurements. Obviously every other studies concern a fully developed flow. One can assume that the air humidity has already reached the saturation all over the pipe cross-section at the measuring location. This effect could then no more be taken into account as no additional droplet vaporization occurs.

3.3.4 Synthesis on the droplet mass flux

Entrained fraction E and Entrainment rate Γ_E

The steam mass flow-rate is assumed to predominately come from the vaporization of the droplets rather than the liquid film. Consequently, and reminding that this droplet vaporization will not occur in an adiabatic steam flow (like the flow that will occur in a hot leg of a PWR during a LOCA), the mass flow-rates of droplets integrated over the air cross-section $\dot{M}_d$ is combined to the mass flow-rates of steam, integrated over the same air cross-section $\dot{M}_s$ to finally characterize the total mass flow-rate of the dispersed phase over the pipe cross-section in the entrained fraction E :

$$E = \frac{\dot{M}_d + \dot{M}_s}{j_l \times \pi \frac{D^2}{4}} \quad (3.14)$$

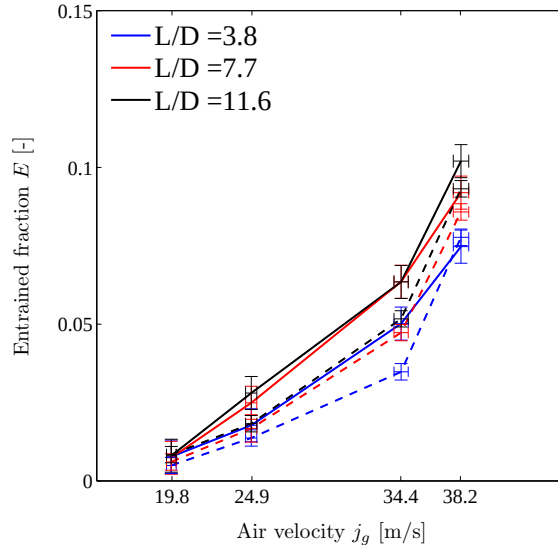


Figure 3.39: Evolution of the entrained fraction E as a function of the air velocity: $j_l = 0.018$ m/s (solid) and 0.036 m/s (dashed).

The entrained fraction E evolves as expected in the theoretical methodology (see Section 2.4) with the control parameters as presented in Figures 3.39 and 3.40. Increasing the air velocity induces a sharp augmentation of the mass flow-rates of droplets. Indeed both mass flow-rates of steam and droplets increase with the air superficial velocity j_g . Furthermore, the contribution of steam to the total mass flow-rates of droplets may be important so as surpassing the droplet contribution at the smallest air velocity: $j_g = 19.8$ m/s. This mean that at this velocity most of droplets extracted from the liquid film are vaporized in the dry air.

The influence of the liquid flow on the entrained fraction E is more questionable. Indeed Mantilla [101] has observed an increase of the droplet mass flow-rates while several other experimental data [165] and theories of the literature (Hanratty et al. [136, 40]) did not meet such effect and, in consequence, did not take j_l into account in their correlation for E . In fact, the entrained fraction E appears to be much more dependent of the interfacial area A_i (see Equation 3.6 for details) than directly on the liquid flow $\dot{M}_l$. To highlight this dependence, Figure 3.41 presents a comparison of the entrained fractions, per unit of the interfacial area, E^* at $j_l = 0.018$ m/s and $j_l = 0.036$ m/s:

$$E^* = \frac{E}{A_i} \quad (3.15)$$

where A_I represent the developed surface of the interface, taking into ac-

count the average height variations along the x-axis and z-axis (determined from Figure 3.20). The quantity of droplets entrained in the air depends mainly on the wave characteristics (average height variations, frequencies) and the interfacial surface curvature in the pipe cross-section.

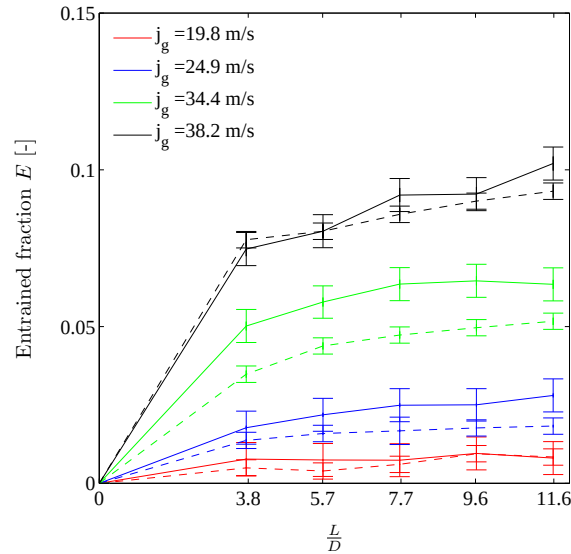


Figure 3.40: Evolution of the entrained fraction E depending on the axial location: $j_l = 0.018$ m/s (solid) and 0.036 m/s (dashed).

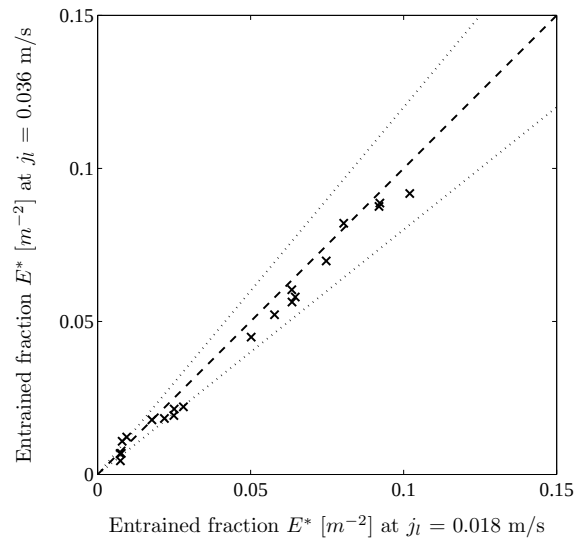


Figure 3.41: Ratio between the entrained fraction per unit of interfacial area at $j_l = 0.018$ m/s and $j_l = 0.036$ m/s

For a same range of liquid and gas velocities, the entrained fractions E measured by Mantilla [101], Paras and Karabelas [120], Williams [164] and REGARD are very different (see Figure 3.42). As the ranges of velocities overlap, the difference in entrainment fraction measurement must come essentially from the influence of the diameter. However as entrained fraction may differ in literature for pipes of same diameters and flows of same velocities, an effect of the gas pressure or the measurement system should perhaps be considered.

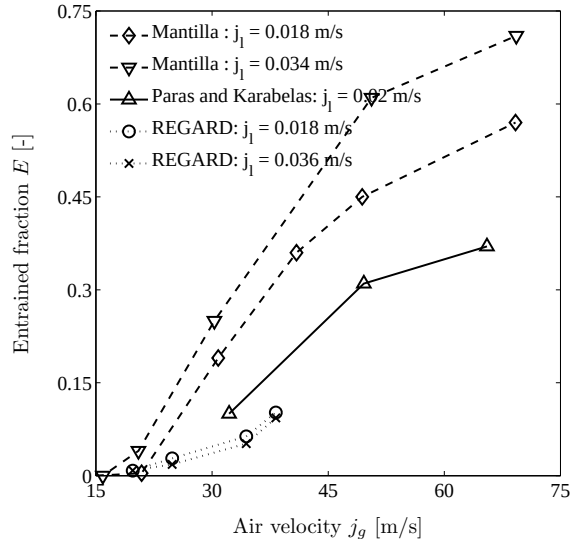


Figure 3.42: Entrained fraction from REGARD, Mantilla on a 4.86 cm pipe diameter and Paras and Karabelas on a pipe diameter of 5.08 cm.

Coming from the evolution of the entrained fraction with the axial location, the entrainment rate Γ_E , presented in Figure 3.43, appears to primarily increase with the air superficial velocity j_g and slightly with the liquid superficial velocity j_l .

Further improvements

Many ways may be considered to improve the measurements of the mass flow-rates of droplets with the isokinetic probe system. Most of them imply to increase the run duration and to realize more additional studies:

- o Increasing the measuring duration beyond 180 seconds at each mesh point would increase both the number of mesh positions analyzable (in which a mass-flow rate exceeds the precision of the weighting system) and the precision on these measurements. At minimum, increasing the measuring duration beyond 180 seconds for the weakest values so as to reach the minimum value for the measuring accuracy.

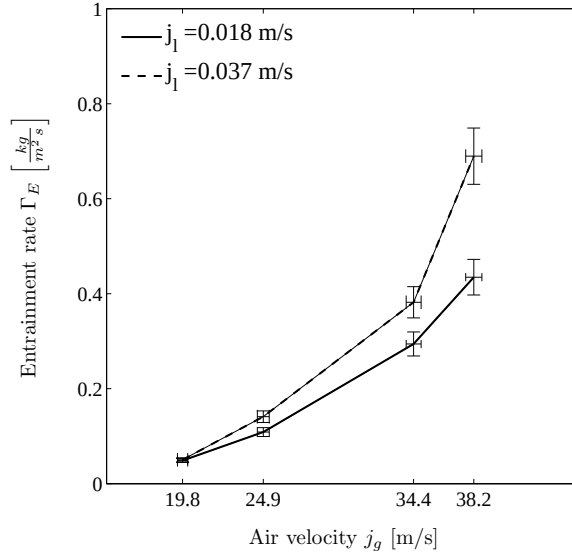


Figure 3.43: Evolution of the entrainment rate Γ_E with the air superficial velocity j_g

- o Extending the experiment campaign would increase the understanding on the evolution of the mass flow-rates of droplets depending on flow parameters.
- o A modification of the injection segment in order to release the constraints at $\frac{L}{D} = 3.8$. Increasing the range of analyzed axial locations to provide data at $\frac{L}{D}$ of 1, 2 or 3 would improve the precision on the deduced entrainment rate Γ_E .
- o Finally, the lower mesh positions should have been extensively studied in order to determine the phase velocities of the entrainment inception.

However, few modifications should also be implemented on the measuring system of the isokinetic probe in order to reduce the uncertainties and enhance the precision:

- o The final measure from the weighing system of the mass of droplets sucked by the isokinetic probe should no more depend on an human data manipulation (cfr Appendix A.2 for details). This could be achieved by a slight improvement of the post-treatment code.
- o The outlet atmospheric conditions should be continuously measured to reduce the uncertainties on the inlet air conditions.
- o An air saturation in humidity system should be implemented on REGARD in order to eliminate the humidity considerations.

3.4 Droplet characteristics

The previous sections devoted to the analysis of the data obtained with the BANJO and the isokinetic probe highlight the double stratification of the flow: both the continuous air-water phases and the dispersed phase of droplets are stratified over the pipe cross-section. However the data on the droplet mass flow-rates are not sufficient to completely describe the flow, and especially the droplet behaviour. Indeed, the isokinetic probe provides indications on the dispersion of the droplet mass flow-rates but does not characterize these droplets in terms of diameters, volumes or velocities. To enhance the description of the flow, a laser measurement system of both droplet size and velocities (granulometry and velocimetry) has been implanted on REGARD.

The use of laser technology to assess droplet characteristics is almost unique in the literature. Indeed this technique is quite innovative. This non-intrusive instrument requires a transparent flat space, unobstructed either by the liquid film or any the other instrument devices, crossed by the laser beams. This measurement is then very difficult to achieve in annular flow. Moreover the interest of this technique to analyze separately several different positions over a pipe cross-section is essentially useful for test sections of large diameters in which the droplet dispersion is important. In consequence, this technique has not yet been implemented on many experimental facilities. However, on horizontal flows, Simmons and Hanratty [141] and Al-Sarkhi and Hanratty [8] have used a laser granulometer system to characterize the droplet size in a air-water flow in the pipe of 9.53 cm diameter (facility used by Williams et al. [165]: from stratified to annular flows). These droplet size measurements constitute the only sets of data, usable for the comparison of droplet characteristics.

More recently, Patruno et al. [124] presented an interesting study of the droplet size distribution obtained with a high-speed camera technique. However, the fluids (Exxsol $D60^{TM}$ and nitrogen) and the flow characteristics do not suit for a data comparison with REGARD.

The granulometer and velocimeter laser systems are used to associate the three components of the velocity and a diameter to every droplet passing through the surface generated by the intersection of the laser beams. The description of these two techniques is detailed in Appendix A.3. In quantitative terms, the measurements of the numbers of droplets crossing this surface are questionable for several technical reasons. Indeed these numbers of droplets strongly depend both on the angle of intersection of the laser beams, the size of the intersection

area and on the settings of the laser post-processing data. The uncertainties on these parameters are very important. In consequence, laser systems are still not adequate to measure the droplet mass flow-rates and to confirm the data obtained with the isokinetic probe.

However, in qualitative terms, the LASER system provides some interesting data on the droplet size and velocities, as the laser settings appear to not strongly affect the droplet characteristics. The description of the post-processing of the granulometer data and the effect of analysis modes (beam coincidence or no-coincidence) are detailed in Appendix A.3.3. Indeed select the three laser beams simultaneously (coincidence mode) or separately (no-coincidence mode) strongly influences the spectral information.

In a first part, the analysis of the droplet sizes provides many insights: time and space averaged diameter $\langle \bar{d} \rangle$, time and space averaged Sauter diameter $\langle \bar{d}_{32} \rangle$, time and space averaged volume and time and space averaged surface. Moreover, similarly to the analysis of the droplet mass flow-rates, the droplet size characteristics are initially considered locally at the local scale of the mesh cells.

The second part is devoted to the velocity analysis, similarly both locally and at the pipe cross-section scale. Finally, these two features are joined to the isokinetic probe data on droplet mass flow-rates to conduct a global analysis in order to determine an estimation of the droplet volume fraction α_d .

3.4.1 Droplet diameter

Similarly to the isokinetic probe studies, measurements could be performed only in a lower quadrant of the pipe cross-section in most experimental runs. Indeed, in the upper quadrants, the number of droplet measurements was often zero or not sufficient to validate a statistical study.

A preliminary analysis on the distribution function of the droplet characteristics (see Appendix A.3) have shown that the validity of droplet data at certain local mesh cells of the pipe cross-section was controversial, in particular close to the liquid film. Indeed the laser system seems to confuse sometimes the liquid film and droplet characteristics, providing extremely large droplets. Consequently the laser analysis requires to adequately select the measurement cells. Finally the real number of analyzable positions is far lower than the 86 theoretical positions (varying between 26 and 78 depending on operating conditions).

The uncertainty on a measurement of the droplet diameter is difficult to estimate. Indeed the granulometer provides diameter data only for droplets considered as spherical but the laser system manufacturer does not provide any information on its granulometer precision. All non spherical droplets are nevertheless expected to be rejected by the instrument. But this effect should tend to more reject more bigger deformable droplets than smaller droplets (remaining almost spherical due to the effect of surface tension).

Moreover a modification in laser settings changes the relative counting between the 3 probes and then the relative number of droplets characterized by a diameter (as the diameter measurement is associated to only one laser beam). The calculation of the uncertainty on a diameter measurement could then not be achieved. For these reasons, we have not been able to obtain the uncertainty level.

Droplet distribution according to their diameters.

The paragraph deals with the distributions of droplet number and volume, both at a the local and global scales. As an introductory result, Figures 3.44, 3.45 and 3.46 present the probability density function (pdf) of the droplet distribution at the global scale of the complete cross-section, both quantitatively and qualitatively (adimensionalized by the maximum value). These graphs emphasize a marked maximum, generally located around $60 \mu m$. These distributions are wide as their spectra extends from $10 \mu m$ to $670 \mu m$ (which corresponds to the higher perception set for the laser granulometer: $700 \mu m$) and are then not symmetric around the maximum. The asymmetry of the distribution implies that the average diameters $\langle \bar{d} \rangle$ are always greater than the peak diameter $\langle \bar{d}_p \rangle$.

Figures 3.44 and 3.45 present the evolution of the profile of the droplet distribution with the flow conditions (j_g , j_l) and along the axial axis. Increasing the air velocity acts directly on the distribution: in quantitative terms, the number of characterized droplets increases significantly by a 10^3 factor for the smaller droplets (of the peak size) and by 10^2 to 10^1 factors for the higher range of diameters. The number of small droplets increase more than the number of large droplets. Indeed, if in qualitative terms the peak location is little affected by the air velocity, increasing j_g commensurately reduces the proportion of large droplets, as the distributions become increasingly thin. As an example, at the axial location $\frac{L}{D}$ of 7.7, the peak diameter range, extended from 50 to $120 \mu m$ at the smallest air velocity ($j_g = 19.8$ m/s), shrinks to a sharp range of 50-60 μm at the highest velocity ($j_g = 38.2$ m/s).

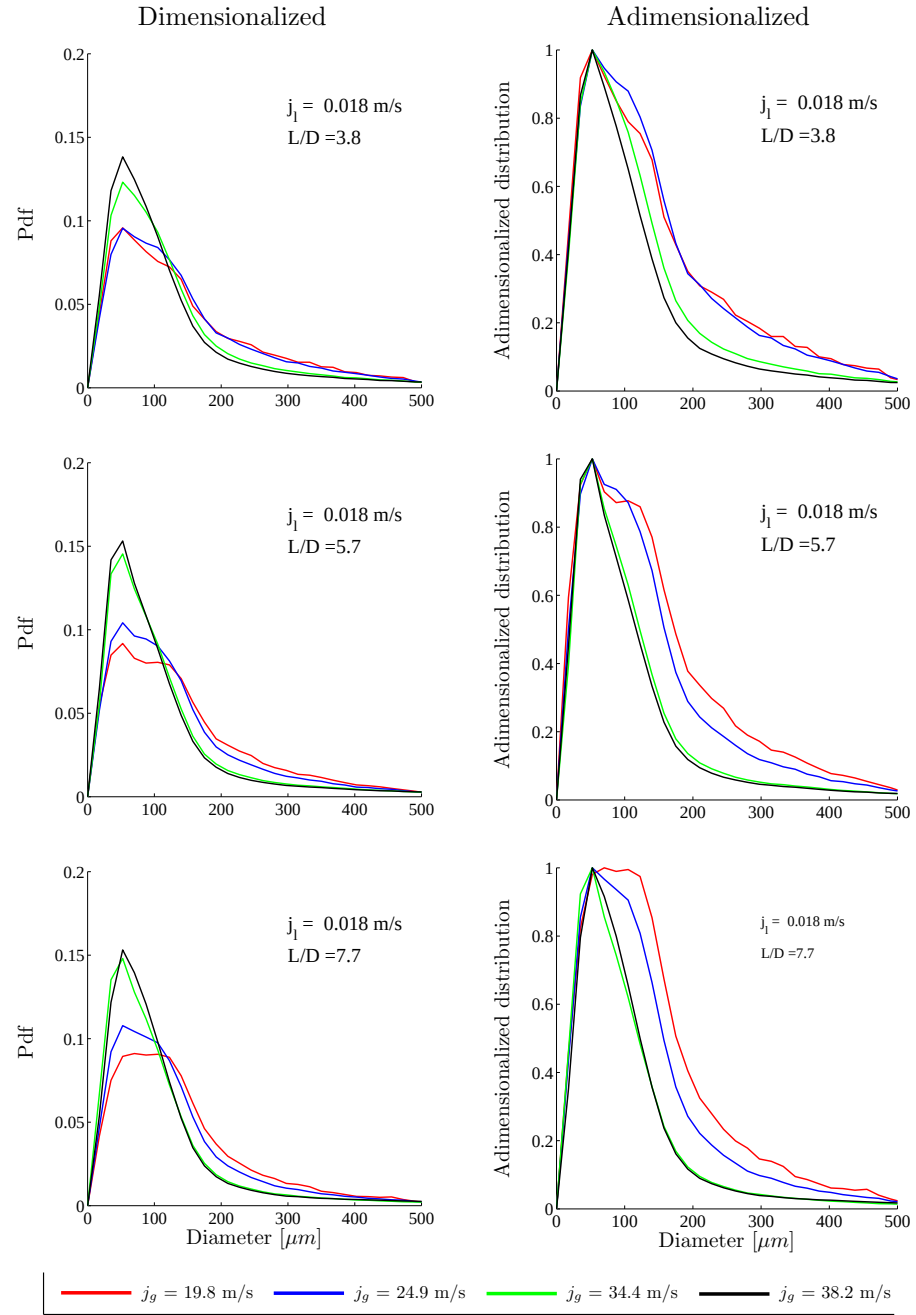


Figure 3.44: Droplet probability density functions (step: $17.5 \mu m$) over the pipe cross-section when increasing the air superficial velocity j_g .

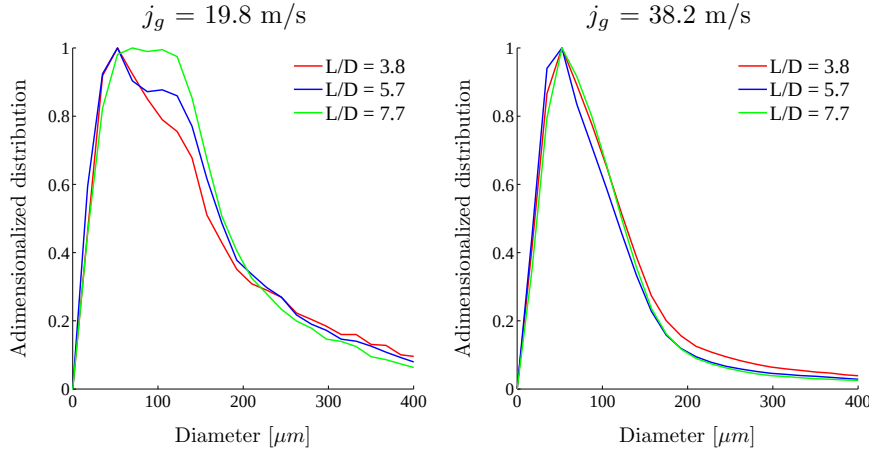


Figure 3.45: Adimensionalized droplet distributions over the pipe cross-section as a function of the axial location at $j_l = 0.018$ m/s.

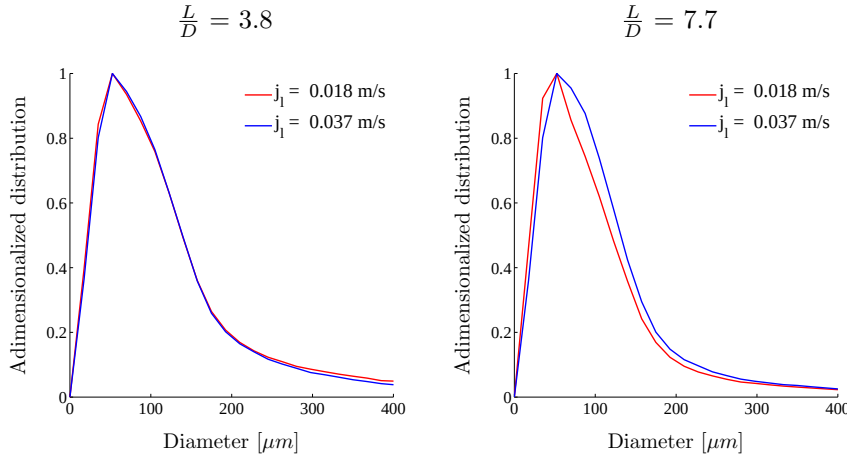


Figure 3.46: Adimensionalized droplet distributions over the pipe cross-section as a function of the liquid velocity j_l at $j_g = 34.4$ m/s.

Moving along the z -axis (see Figure 3.45) or increasing the liquid superficial velocity j_l (see Figure 3.46) have a smaller effect on the distribution profile. Finally the evolution of droplet distribution follows quite similarly the evolutions of mass flow-rates measured by the isokinetic probe. Moving along the z -axis makes the droplet distribution increasingly thin as the proportion of large droplets decreases.

Figure 3.46 shows that the liquid superficial velocity has a very little influence on the droplet distribution in terms both of distribution profile and measured number of droplets.

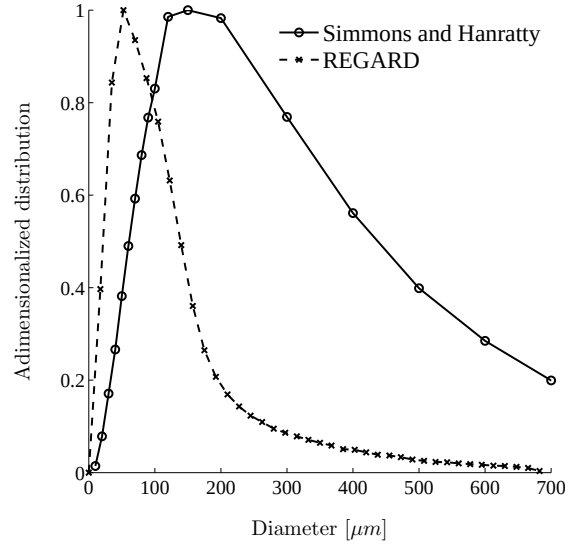


Figure 3.47: Comparison with data from Simmons and Hanratty [141] ($j_l = 0.016$ m/s and $j_g = 36$ m/s) on a pipe of 9.53 cm diameter.

Figure 3.47 presents a comparison of the probability density functions presented by Simmons and Hanratty [141] on a facility of 9.53 cm diameter and on REGARD for a similar range of superficial velocities ($j_l = 0.018$ m/s on REGARD and 0.016 m/s for Simmons and Hanratty [141] and $j_g = 34.4$ m/s on REGARD and 36 m/s for Simmons and Hanratty [141]).

Local investigations

At the global scale, the droplet distributions keep a similar profile whatever the flow conditions. In order to study locally the droplet behaviour, Figures 3.48 and 3.49 present the local investigations on the droplet distributions. These graphs show the evolutions of the local droplet distributions along respectively a vertical and a horizontal axis, similarly to the mass flow-rate investigation.

In Figure 3.48, along a vertical axis, the maximum number of droplets appear at the bottom of the pipe (red or blue curves) near the liquid film. Almost no droplet has been counted in the highest positions (fushia curves). However the few droplets counted in these higher positions have a large diameter far greater than the average peak diameter of the overall cross-section. Consequently the local droplet averaged diameters $\bar{d}$ are greater (200 - 250 μm) at the highest positions. At the lowest positions (red curves), the spectrum of the distribu-

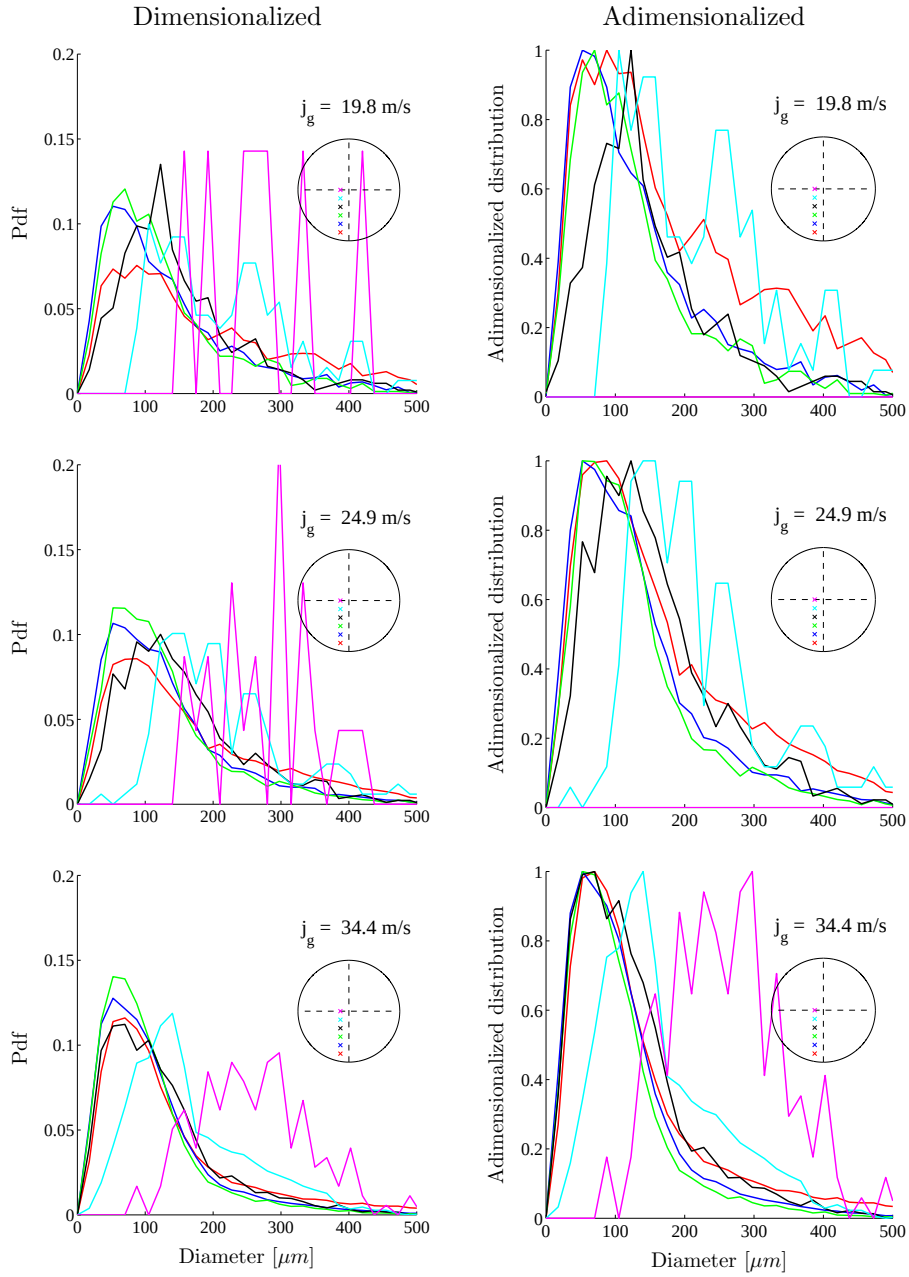


Figure 3.48: Evolution of the local distributions of the number of droplets along a vertical axis at $\frac{L}{D} = 3.8$ for $j_l = 0.018$ m/s.

tions are wider which results in a peak at a slightly larger diameter than the average value and in a proportionally greater proportion of droplets of large diameter. Therefore the local average droplet volume and the local average mass flow-rates are greater in the lower and upper positions than at the center

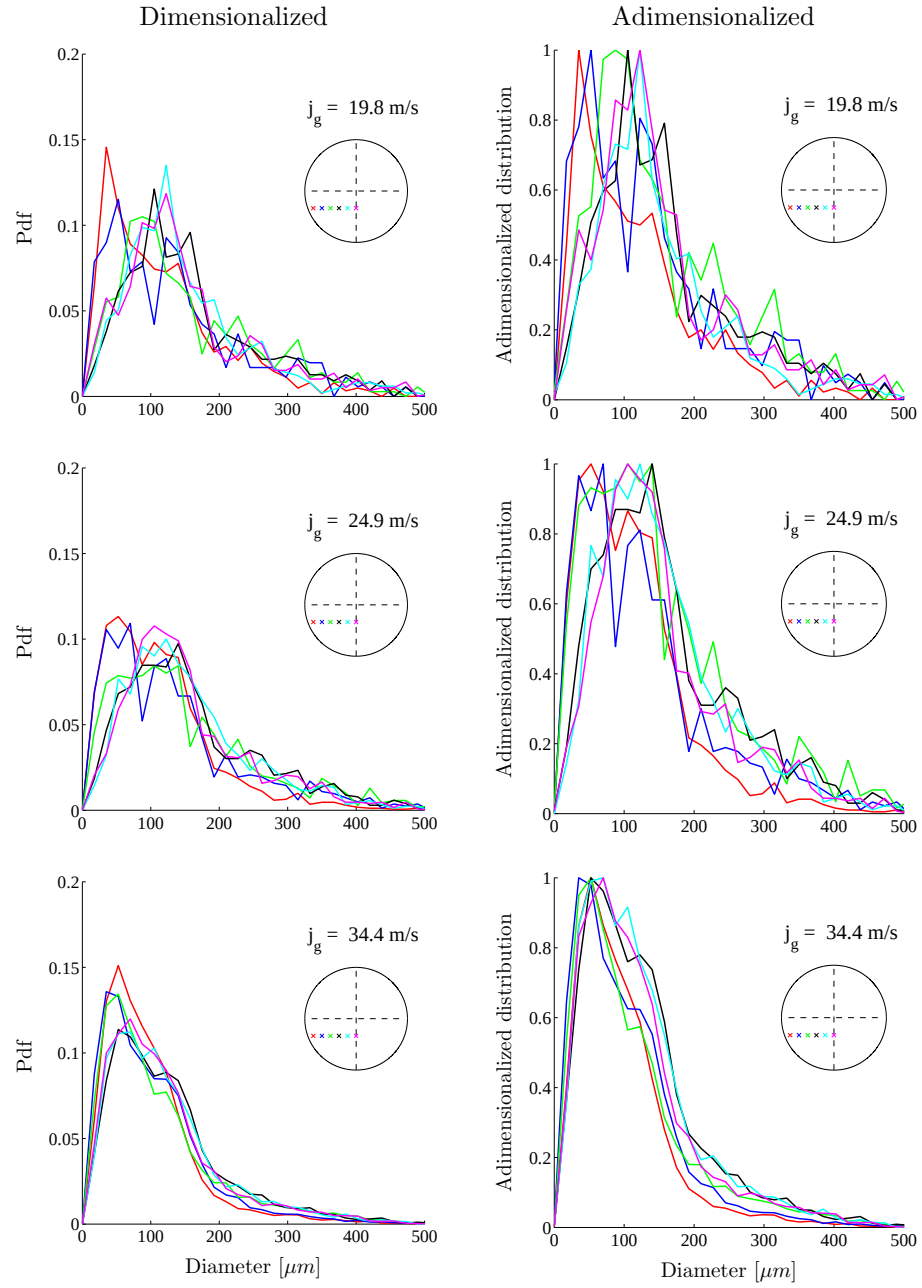


Figure 3.49: Horizontal evolution of the local of the droplet number distributions.

of the lower quadrant. The profile of the dimensionless distributions becomes increasingly smoother when the airflow increases, which is symptomatic of an increase of the proportion of the small droplets.

In Figure 3.49, along an horizontal axis, the evolutions of these distributions show that the droplets are systematically more numerous near the wall region (red curves). At $j_g = 19.8$ m/s, the droplet number decreases when the probe moves axially toward the center of the pipe and then increases again when approaching the center of the pipe. However this phenomenon disappears when increasing the air velocity. At the nearest position from the wall, the number of droplets is quite similar all over the x-axis, following the expected profile for the air velocity. At low air flow-rates, as the droplets are not numerous, the peak diameters of the distribution highly depend on the x-axis position. However the number of analyzed droplets is so reduced that the quality of the statistical analysis is questionable. Increasing the air flow-rate induces an increase of the droplet number and leads to a peak diameter independent of the x-axis value.

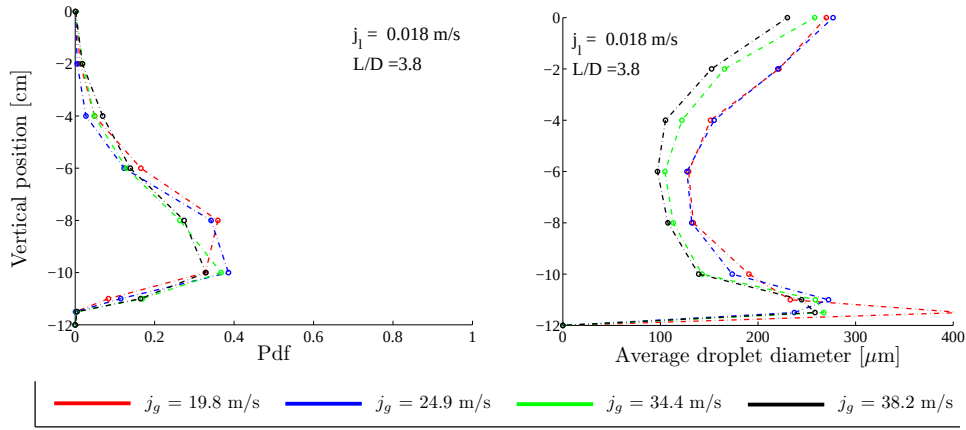


Figure 3.50: Evolution of droplet characteristics along a vertical axis.

Data analysis

In summary the droplet spectra depend on the flow conditions (liquid and air velocities) and on the axial location. The dispersion of the droplets all over the pipe cross-section mainly depends on their initial energy, velocity and direction (and also on the collision risks). The conjunction of these two dependences is highlighted in Figure 3.50 which represents the evolution, along the central vertical axis, of the droplet dispersion (left) and of the time averaged droplet diameter $\bar{d}$ (right). The droplet dispersed phase is stratified such that the maximal number of droplets appears between -10 cm and -8 cm (referenced at the center of the pipe), just above the interface while the minimum appears on the highest mesh cells. As the number of droplets decreases significantly when rising in the pipe along the y-axis, the average diameters $\bar{d}$ goes through a minimum (around -6 cm). Indeed the average diameter is important ($300 \mu m$) above the

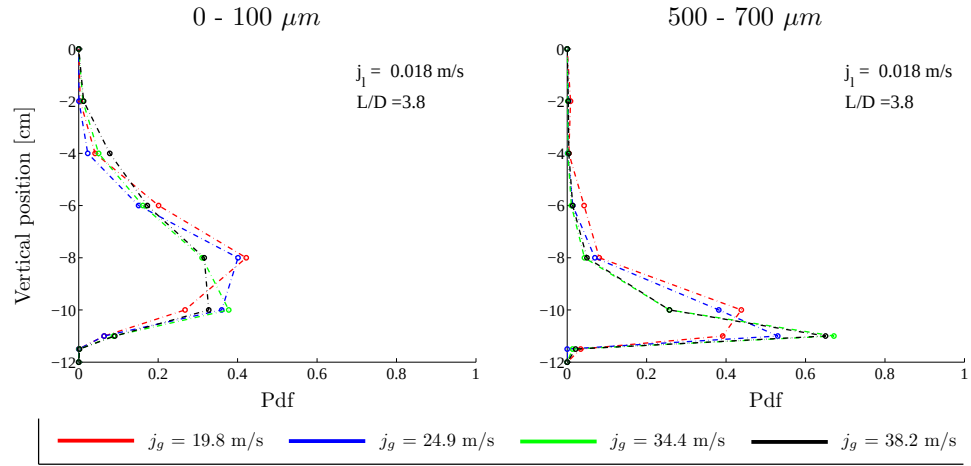


Figure 3.51: Comparison between two vertical droplet number evolutions.

interface, then decreases significantly to 100 - 150 μm when slightly rising upward in the pipe and finally rises again to 220 - 300 μm when the probe reaches the highest positions.

Figure 3.51 shows that the droplet dispersions along a vertical axis for two ranges of droplet diameters are differently distributed. Indeed these graphs show that, whatever the air velocity, around 80% of the droplets of large diameter are located just above the interface (at -10 cm) while the small droplets are more uniformly dispersed from -10 cm to -6 cm. The large droplets, created over the interface by the wave fragmentation, seem to hardly overcome their creation position. Several scenarios may explain their absence at the upper mesh positions: first, these bigger droplets could be too big to be stable and would fragment in several smaller droplets. Second, due to their large size and their slower velocity, the risks of collisions may be more important, also leading to their fragmentation. Finally, due to their highest weight, these droplets would overcome the turbulent effect and would deposit right after their creation.

Maps of droplet time averaged diameter

Figure 3.52 presents the maps of the time averaged droplet diameters $\bar{d}$ (right) and the associated maps of droplet dispersion (left). Similarly to the maps of droplet mass flux, the stratification of the droplet phase over the pipe cross-section is important. While the dispersion of the average diameter appears to be very heterogeneous but not stratified. Indeed the droplets present an average diameter greater in areas near the liquid film, near the walls but also at the highest positions of the pipe cross-section.

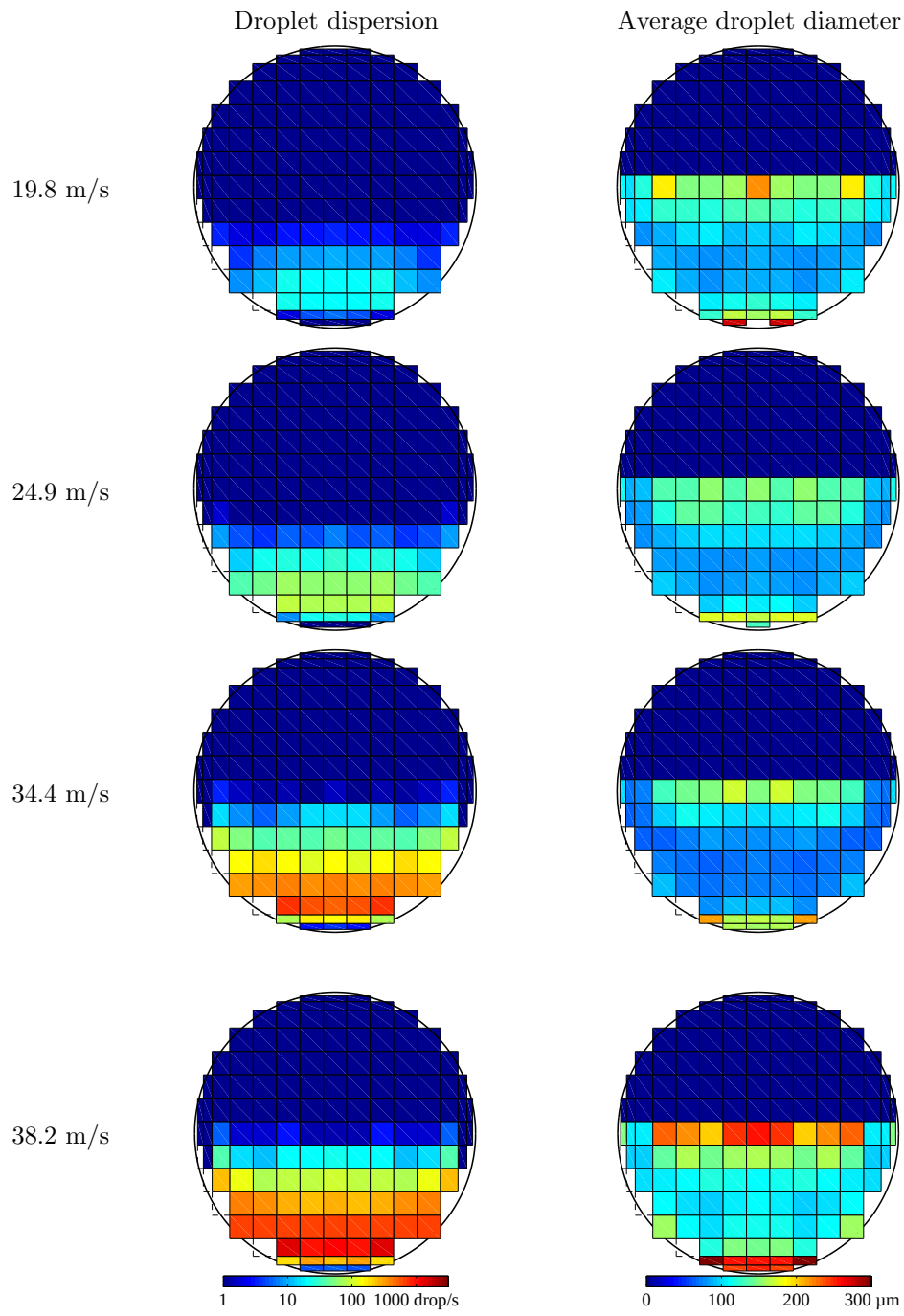


Figure 3.52: Map comparison between the droplet dispersion (left) and their average diameter $\bar{d}$ (right)

3.4.2 Droplet volume

The previous paragraph highlights the stratification of the droplet dispersion and the heterogeneity of the local time averaged droplet diameter $\bar{d}$ all over the pipe cross-section. The evolution of the total local volume of the $N_d(n)$ droplets in cell n $V_d(n)$ (as measured droplets are spherical) is therefore hard to predict.

$$V_d(n) = \sum_{i=1}^{N_d(n)} \frac{4}{3} \pi \left(\frac{d(i,n)}{2} \right)^3 \quad (3.16)$$

The maps of the total volume at each cell are presented in Figure 3.53 to tackle this question. The local droplet volume appears extremely stratified. Indeed most of the global volume of droplets are clearly located above the interface. The majority of the volume torn off from the liquid film appears in the lower part of the streaming gas flow. However, the contribution to upper zones increases slightly with the axial location, as droplets are rising in the pipe cross-section.

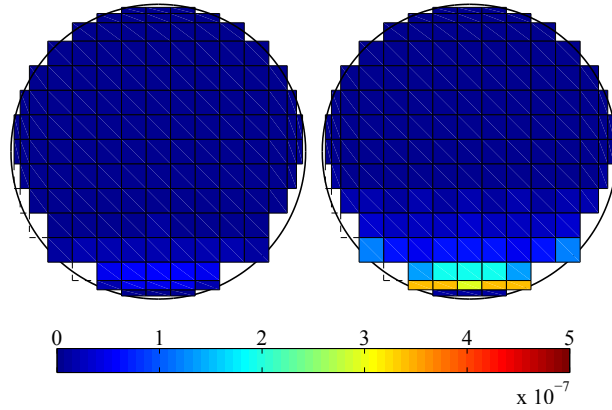


Figure 3.53: Maps of total local droplet volume V_d at $\frac{L}{D} = 3.8$ and $j_g = 24.9$ m/s (left) and 38.2 m/s (right).

The stratification of the local number of droplets $N_d(n)$ totally overcomes the heterogeneity of the dispersion of the droplet diameters. To quantify this effect, the probability density functions of the droplet volume V_{step}/V_t on a global scale (with V_{step} the total volume of droplet per step of $17.5 \mu\text{m}$ of droplet diameter and V_t the total volume of droplet over the pipe cross-section $V_t = \int V_d(n)dn$) are compared to the probability density functions of the number of droplets in Figure 3.54. The two profiles appear totally different. Indeed, as the diameter plays at the power three on the volume, a droplet of $600 \mu\text{m}$ has an equivalent weight to 1000 droplets of $60 \mu\text{m}$. Few very large drops are enough to

completely deform the distribution and move the position of the peak of the volume towards higher diameters. While the peaks of the number of droplets was around 50 to 60 μm and did not depend on the flow characteristics, the peaks of the volume distribution appear highly extended around 300 - 450 μm at low air velocities and outright form a large plateau for higher air flow-rates.

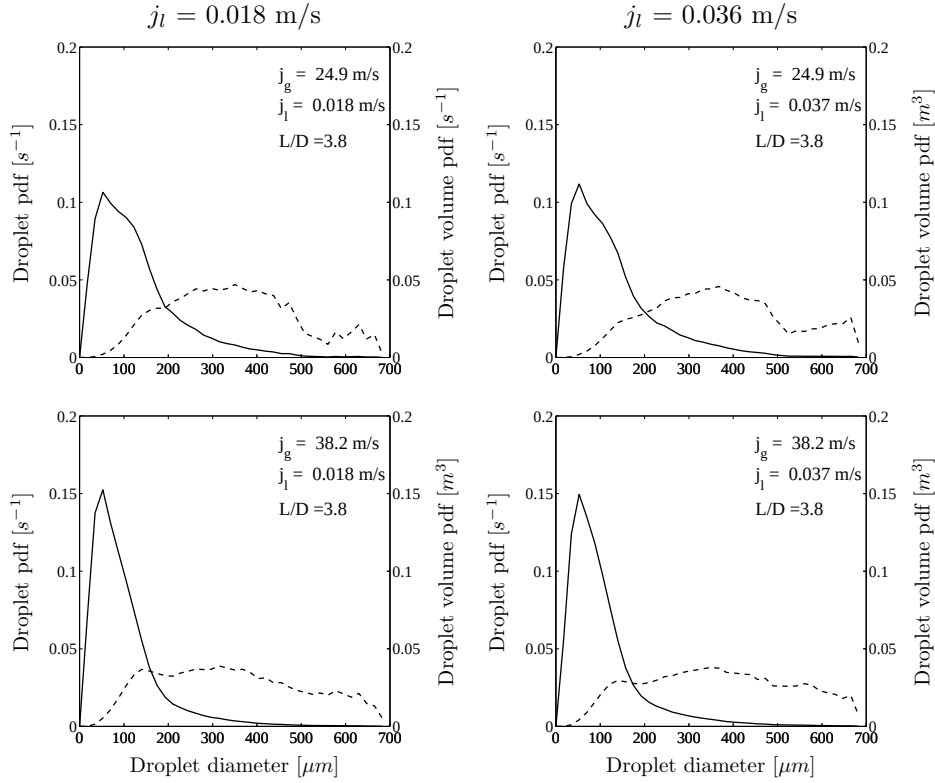


Figure 3.54: Evolution of the droplet number (continuous lines) and volume probability density function (dashed lines) (with a x-step of 17.5 μm) at $j_g = 24.9$ m/s (top) and 38.2 m/s (bottom) at $\frac{L}{D} = 3.8$.

The y-scales in Figure 3.54 does not indicate that the number of measured droplet have been multiplied by 20 when increasing the air superficial velocity j_g from 24.9 m/s to 38.2 m/s while the entrained volume of droplet has been multiplied by 10. The air velocity therefore strongly affects both the droplet volume and the droplet number observed over the pipe cross-section.

Figure 3.55 shows the effect of the liquid superficial velocity j_l on the volume distribution. The study of the isokinetic probe data has revealed that this liquid superficial velocity j_l directly influences the mass of droplets over the pipe cross-

section. In a similar way, the graphs show that doubling the liquid flow results in approximatively doubling the volume of droplets. This increase is equitably spread over the overall range of diameters. Therefore the proportion of droplets of every size remains quite constant along j_l . This could be explained by an augmentation of the interfacial area more than by an increase of the volume wrest-off at the crest of each ripple-waves by the air.

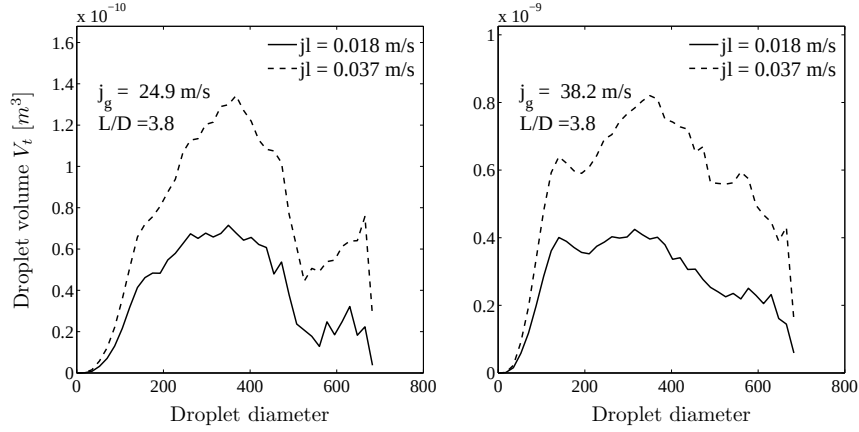


Figure 3.55: Comparison between the volume distributions for the two liquid flow-rates.

3.4.3 Synthesis on droplet characteristics

In order to summarize the droplet size characteristics, Figures 3.57 to 3.61 present the evolution of the droplet time and cross-section averaged diameters $\langle \bar{d} \rangle$ as a function of the air superficial velocity j_g (Figures 3.57 to 3.59) or along the axial location $\frac{L}{D}$ (Figure 3.60). The average diameter $\langle \bar{d} \rangle$ appears to decrease quasi linearly while increasing the air superficial velocity or moving along the axial location. This is mainly caused by the proportionally greater augmentation of the number of droplets of small size compared to bigger ones.

However the data of REGARD significantly differ from the predictions of Tatterson et al. [149] in 1977 or Kataoka and al. [80] which proposed in 1983 a correlation for the volume median diameter $\bar{d}_{0.5}$ in annular flow (Equation 3.18) based on a mechanistic model (in a similar way to Ishii and Grolmes [75]), correlated on a large set of experimental datas [162, 36, 96]. Their correlation relates however the average droplet diameter $\bar{d}$ to the volume median diameter $d_{0.5}$ such that the predicted average diameter $\bar{d}$ evolves from $211 \mu\text{m}$ to $571 \mu\text{m}$ according to flow conditions:

Tatterson's correlation

$$d_{0.5} = 0.106 \frac{\sigma}{\rho_g j_g^2} Re_g^{1.1} \left(\frac{\mu_g^2}{D \sigma \rho_g} \right)^{0.5} \quad (3.17)$$

Kataoka's correlation

$$d = 0.313 \times d_{0.5}$$

$$d_{0.5} = 0.028 \frac{\sigma}{\rho_g j_g^2} Re_{lf}^{-1/6} Re_g^{2/3} \left(\frac{\rho_g}{\rho_l} \right)^{-1/3} \left(\frac{\mu_g}{\mu_l} \right)^{2/3} \quad (3.18)$$

Moreover the comparison with the data of Simmons and Hanratty [141] (Figure 3.47) or Al-Sharki and Hanratty [8], also show a considerable difference in terms of droplet probability density function (pdf). However, the two studies analyze the flux of droplets only at the center line of two smaller experimental setup (pipe diameters of 2.54 cm and 9.53 cm). Figure 3.56 presents the evolution of the volume median diameter d_{50} (dotted lines) reported in three studies (Simmons and Hanratty, Al-Sharki and Hanratty and REGARD) compared to the prediction of Tatterson (solid lines) and Kataoka (dashed lines). The graphs show a significant difference between the experimental results and the predictions. However, these graphs show also that the trends observed in the volume median diameter considerably differ between REGARD and the two experimental studies. This may be due to the fact that in REGARD a considerable number of small droplets disappeared by vaporization in contact with the air. As the number of droplet vaporized is less sensitive to the air velocity than the number of created droplet, the volume median diameter is more affected by the loss of smaller droplets at low air velocity than at high air velocity.

Figure 3.59 show that the Sauter diameter measured on REGARD follows the evolutions presented by Al-Sharki and Hanratty [8]. The Sauter diameter increases with the pipe diameter and decreases with the air superficial velocity. The effect of the air velocity also follows the evolution highlighted by Simmons and Hanratty[141], Al-Sharki and Hanratty [8] and Patruno et al. [124].

Maps have shown that the droplet dispersion is very heterogeneous with a massive presence of droplets close to the liquid film. The stratification is so important that in most experiments, no droplets have managed to reach the upper positions of the pipe cross-section. Even if the stratification is symptomatic of a horizontal flow highly driven by gravity, the importance of the droplet dispersions mainly depends on the flow development, directed by the operating air velocity j_g along the axial location. Finally the liquid velocity j_l has a quite little effect on the droplet dispersion apart from the augmentation of the interfacial area and thus on the droplet generation instabilities. The effect

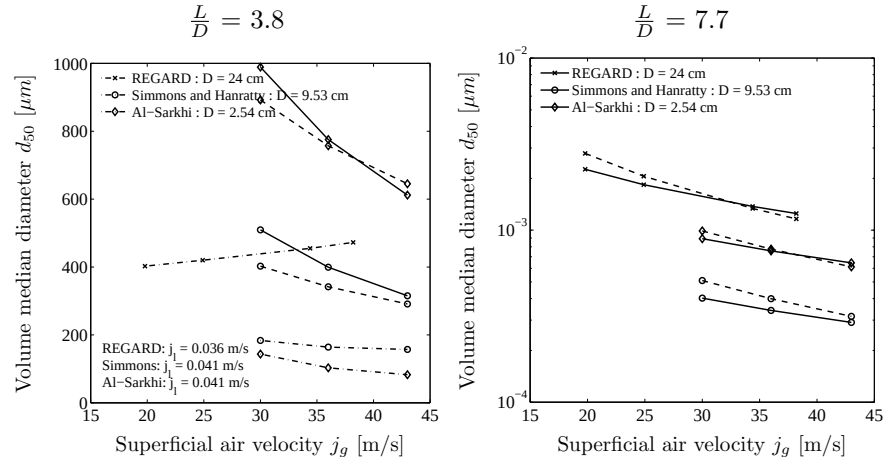


Figure 3.56: Evolution of the volume median diameter d_{50} over the total section as a function of the air flow.

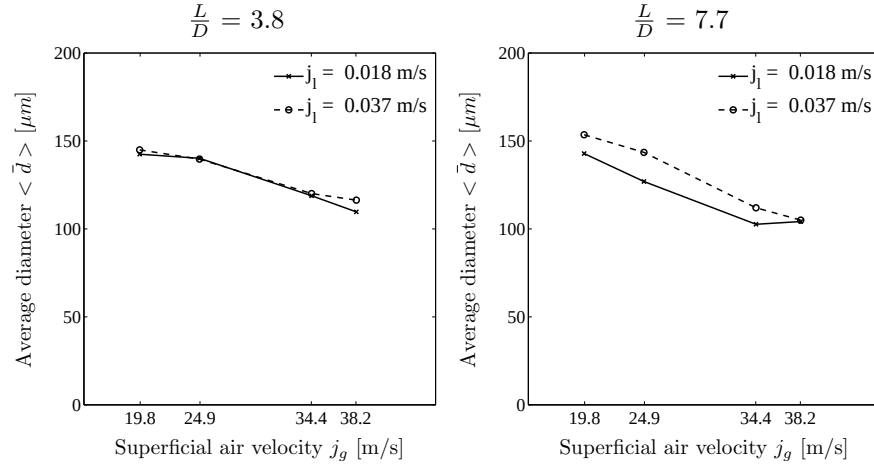


Figure 3.57: Evolution of the average diameter over the total section as a function of the air flow.

of the flow development appears directly at the first axial locations (close to the inception of the droplet entrainment) where all droplets are located in a small layer over the liquid film while when the probes move along the axial location, the droplets disperse upwards and along the wall.

However the behaviour of a droplet is highly dependent on its diameter. The spectrum of the distributions have shown that different categories of droplets can be detected depending on their diameter. The vast majority of droplets have a small diameter: $20 - 120 \mu\text{m}$. According to the operating conditions, from 67% to 83% of the droplets belong to this category, while 84 to 92% of

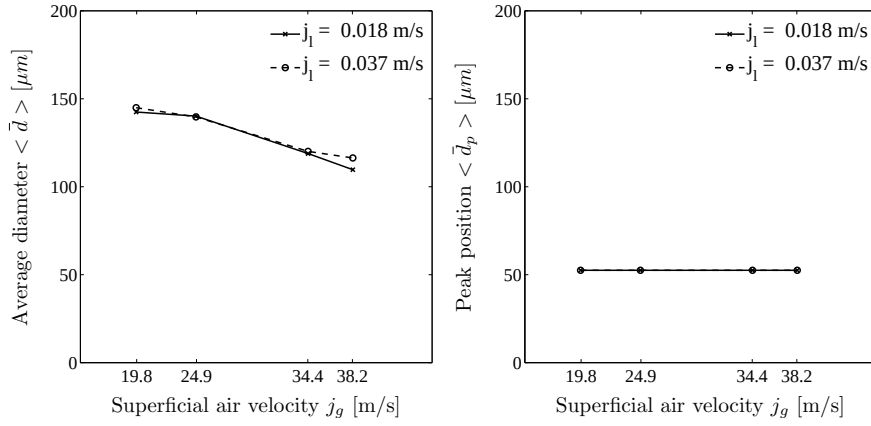


Figure 3.58: Evolution of the average diameter (left) and the peak position (right) as a function of the air velocity.

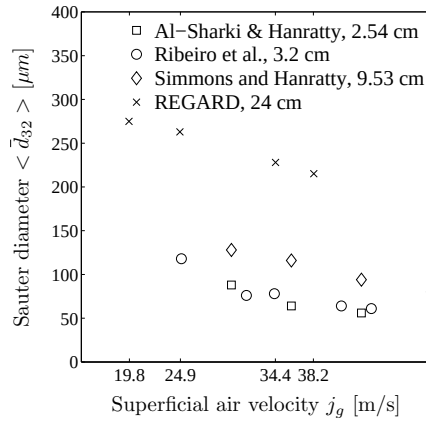


Figure 3.59: Evolution of the Sauter diameter as a function of the pipe diameter between several experimental data: Al-Sharki and Hanratty [8], Ribeiro et al. [129] and Simmons and Hanratty [141] for a liquid superficial velocity around 0.4 m/s.

droplets have a diameter lower than 250 μm . Then, another category of droplets contains some 10% of the droplets in the diameter range of 300 μm to 700 μm and only 1.2 to 2.8% of droplets belong to the larger than 500 μm category.

The droplets of largest diameters (bigger than 500 μm) are very little but each of these droplets counts in terms of volume for 1000 droplets of the peak size. This means that this category highly contributes to the entrainment fraction. Almost no droplets of this size have been counted in upper positions which means that most of them quickly fragment after their creation. A collision with a smaller droplet is almost inevitable taking into account the density of droplets.

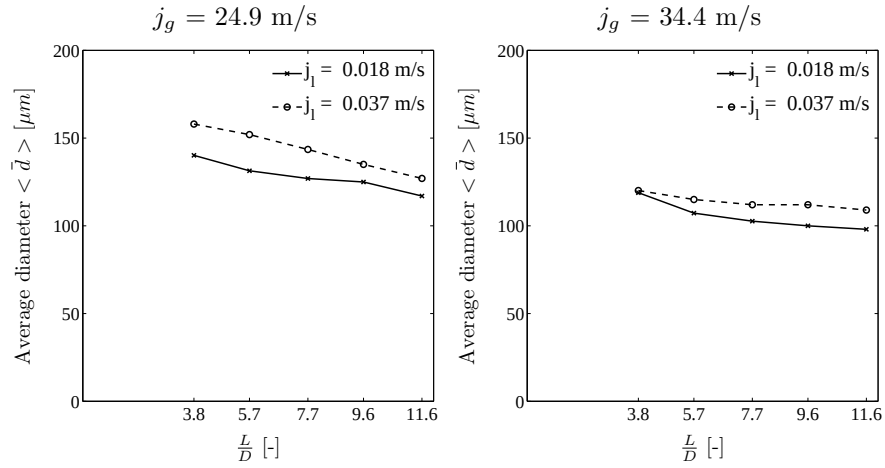


Figure 3.60: Evolution of the average diameter as a function of the axial location

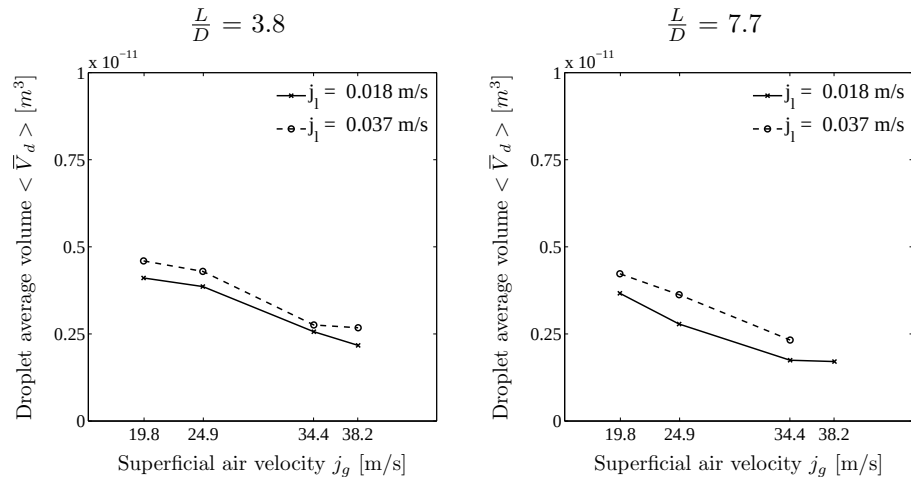


Figure 3.61: Evolution of the average volume on the total section as a function of the air flow

The smallest droplets (10-150 μm , quite mono-dispersed around a peak at 60 μm) disperse over the pipe section with a huge stratification. A vertical gradient of number of droplet appears then due to the action of gravity with a maximal number of droplets near the liquid film. The number of droplets along this vertical axis decreases with a slope depending on the operating conditions.

Larger droplets correspond mainly to first droplets generated by a wave atomization. This wave fragmentation leads to the appearance of large but

unstable droplets that are torn off the liquid film and expelled in the gas. These bigger droplets fragment or collide such as none of them survives far away from the liquid film. Thus a generation of small and middle size droplets appears in the pipe. The ejection conducts these middle size droplets over the pipe section even if the majority rapidly breaks into smaller droplets. This break up may come from their destabilization but more probably from a collision with other droplets. The large droplets are essentially located very close to the liquid film so that only the REGARD experiment measures them. Indeed, as Simmons and Hanratty[141], Al-Sharki and Hanratty [8] and Patruno et al. [124] measure the droplet close to the centerline of the pipe cross-section, these large droplets cannot be analyzed. As a consequence, the evolution of the $d_{v=50\%}$, the droplet diameter beyond which is located half of the total volume of droplets is completely different between the data of REGARD and their reported data.

3.4.4 Droplet velocity components

This part is devoted to the analysis of velocity components provided by the laser particle sizer. The three velocity components, provided in the three directions perpendicular to the three laser beams are reconstructed as axial (v_z), lateral (v_x) and vertical components (v_y) (see Appendix A.4). Unfortunately, the positioning of the laser probes which reduces the uncertainty on the axial component and the high difference in velocity amplitude in the three directions, strongly affects the uncertainties on the lateral and vertical velocity components, which are therefore useless for the flow description (see Appendix A.4.3). However, the analysis of the axial component of the velocity provides an interesting insight on the flow.

Local air velocity v_g

In order to suck the air-droplet flow at an adequate speed, the gas velocity is continuously measured at the analyzed mesh cell by the Pitot tube. Figure 3.62 shows the evolution of the local time averaged air velocity $\bar{v}_g$ at the center of the pipe cross-section for several axial locations. The gas velocity $\bar{v}_g$ is not really affected by the liquid superficial velocity j_l and appears constant along the z-axis. Obviously the air velocity $\bar{v}_g$ increases quasi linearly with the air superficial velocity j_g . Quantitatively, the air velocity $\bar{v}_g$ at the center of the pipe approaches nearly 150% of the air superficial velocity over the air cross-section.

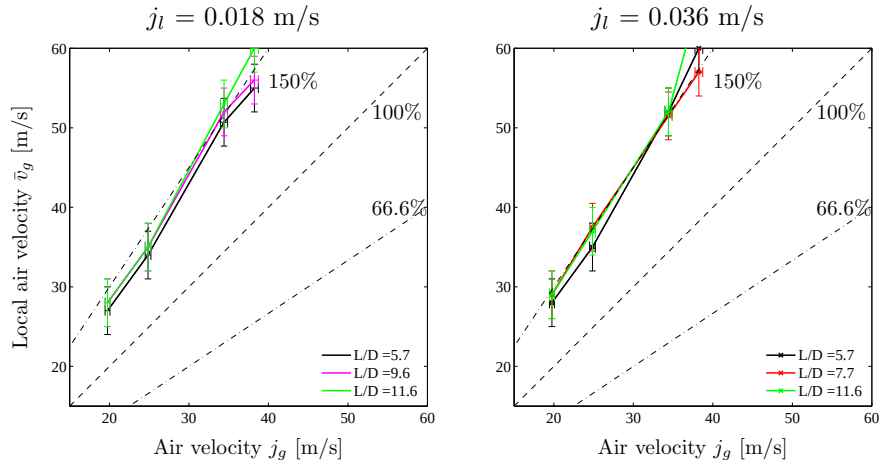


Figure 3.62: Evolution of the air velocity at the center of the pipe measured by the Pitot tube.

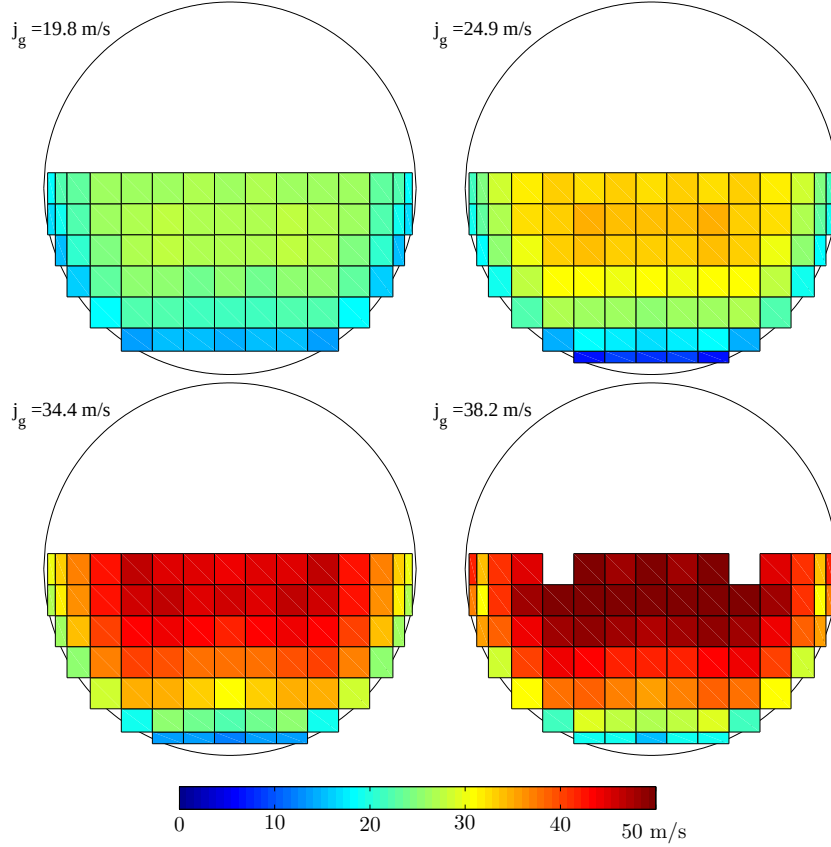
Axial velocity component v_z 

Figure 3.63: Comparison of the maps of local average axial velocity component $\bar{v}_z$ as a function of the air superficial velocity j_g at $j_l = 0.018$ m/s and $\frac{L}{D} = 7.7$

To characterize the local droplet behaviour, it is necessary to rely on an averaging operation to bring around 10000 local components in one direction to a single information:

- o The standard average droplet velocity: $v_{d,10}$ defined as the sum of the droplet velocities divided by the number of droplet:

$$\bar{v}_z \equiv \bar{v}_{(d,10)_z} = \frac{\sum_{i=1}^{N_d} v_{d,z}(i)}{N_d} \quad (3.19)$$

where N_d is the local number of droplets characterized by three velocity components, $v_{d,z}(i)$ the axial velocity component of the i^{th} measured droplet.

- o The cross-section averaged standard average droplet velocity:

$$\langle \bar{v}_z \rangle = \frac{\sum_{j=1}^{88} \bar{v}_z(j)}{88} \quad (3.20)$$

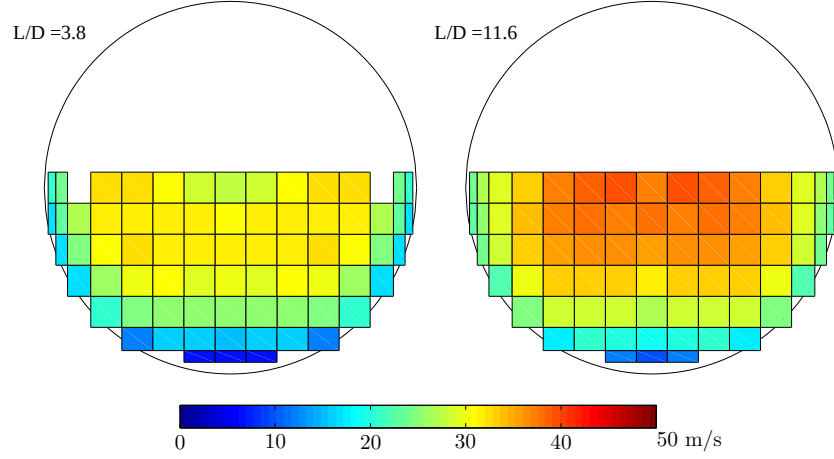


Figure 3.64: Comparison of the maps of local average axial velocity component: $\bar{v}_z$ depending on the axial location $\frac{L}{D}$ at $j_l = 0.018$ m/s and $j_g = 24.9$ m/s.

Figures 3.63 and 3.64 present the evolution of the maps of the local averaged axial velocities $\bar{v}_z$ respectively when the air superficial velocity j_g is increased and along the axial location. The droplet axial velocity increases significantly with the air velocity, substantially at the center of the pipe cross-section. Indeed the droplet axial velocity components are minimal near the walls and the liquid film and increase when moving away toward the center of the flow.

Figure 3.65 presents the evolution of the axial velocity averaged over the pipe cross-section $\langle \bar{v}_z \rangle$ depending on the flow variables. This figure shows that the average droplet velocity increases along the pipe location, as the number of smaller but faster droplet increases.

Indeed Figure 3.66 shows that the average axial velocity is inversely proportional to the droplet diameter, especially at the highest air superficial velocity j_g and close to the entrainment inception ($\frac{L}{D} = 3.8$). Close to the pipe outlet, the variation of average velocity $\langle \bar{v}_z \rangle$ for the lowest air superficial velocity ($j_g = 19.8$ m/s and 24.9 m/s) is decreasing. This could signify that at these velocities, the length of the pipe seems sufficient so that every droplet, even the largest, has accelerated to a regime velocity. Obviously, it's impossible to refute a plausible role of the droplet deposition on these evolutions.

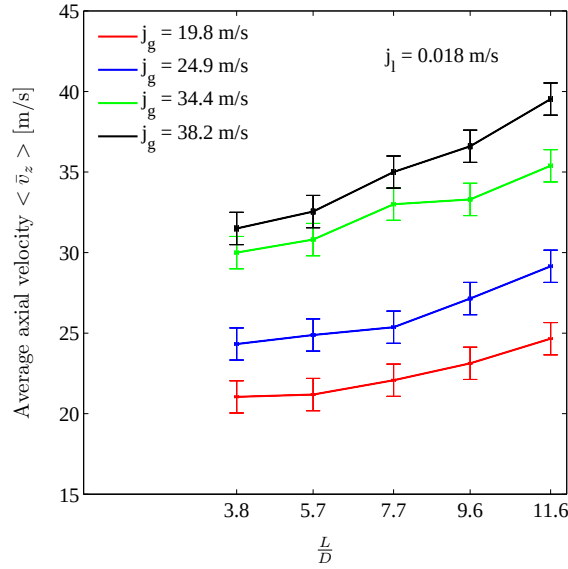


Figure 3.65: Evolution of the average axial velocity component $\langle \bar{v}_z \rangle$ depending on the axial location $\frac{L}{D}$.

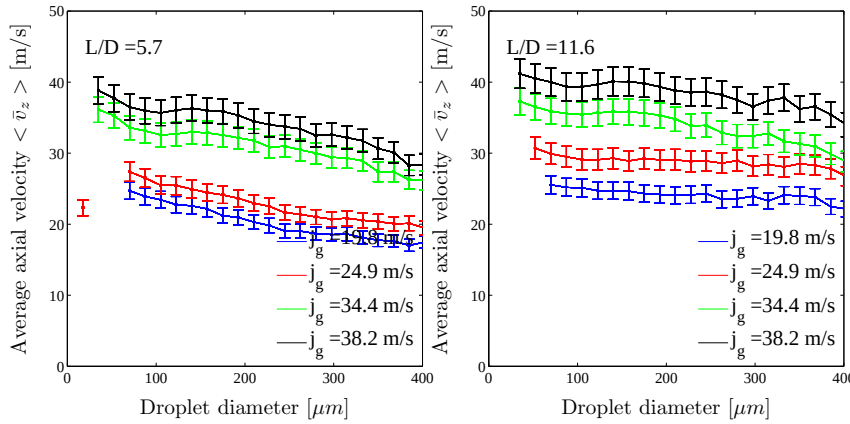


Figure 3.66: Average axial velocity as a function of the droplets diameter at $\frac{L}{D} = 3.8$ (left) and 11.6 (right).

Profile of the axial velocity $\bar{v}_z$ over the pipe cross-section

Figures 3.67 to 3.70 presents the local evolution of the axial velocity $\bar{v}_z$ with the flow parameters. The average droplet local axial velocity $\bar{v}_z$ highly depends on the local position over the cross-section as shown in Figure 3.67. The average droplet axial velocity $\bar{v}_z$ increases nearly linearly with the air superficial velocity j_g . However the slope of the increase depends on the position over the pipe cross-section, as proportional to the air velocity. Along the axial location, the droplet axial velocity slightly increases at the highest mesh position, while seems

constant at the two intermediate positions. The droplet seems to have reached a saturation regime in these two positions, which has not yet been reached at the highest position. At the lowest position, the difference between the three axial location is probably due to the increase of the distance relative to the liquid level. As expected, at the center of the pipe cross-section, the droplet averaged axial velocity appears to be lower than the air velocity measured by the Pitot.

Figures 3.68 and 3.69 present the evolution of the axial velocity $\bar{v}_z$ respectively along a vertical chord and along an horizontal chord, when moving along the axial location and increasing the air superficial velocity j_g .

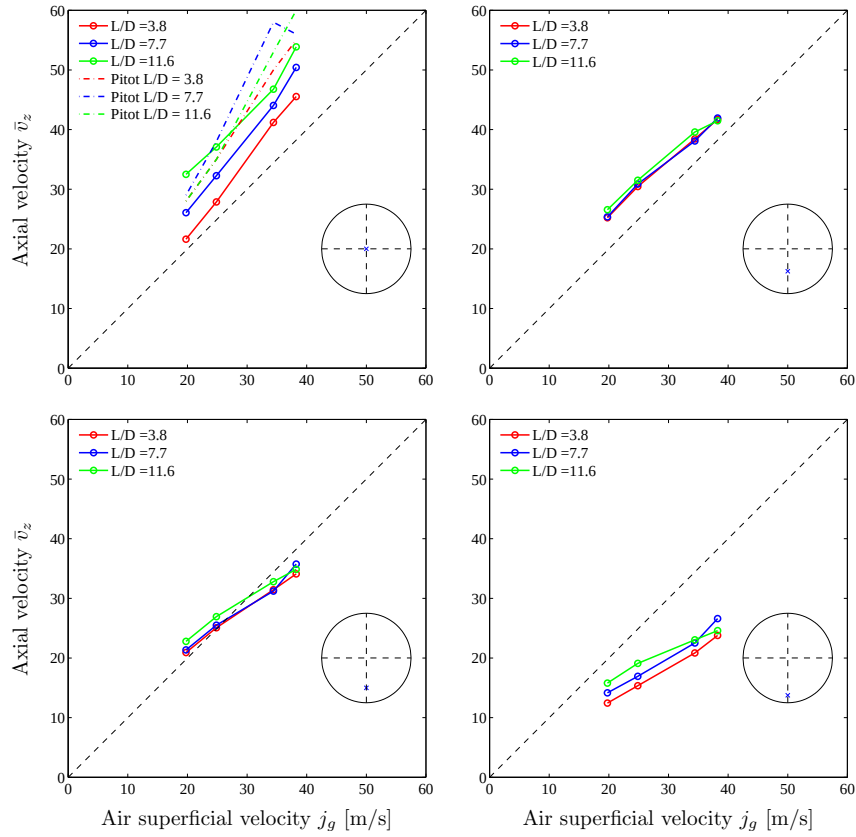


Figure 3.67: Evolution of the mean axial component of velocity $\bar{v}_z$ at different positions of the pipe cross-section while the air velocity increases for three axial locations and a fixed velocity of water ($j_l = 0.018$ m/s).

The vertical position of the maximum axial velocity along the vertical chord increases when moving along the axial location $\frac{L}{D}$ (passing from -4 cm, referenced to the centerline to 0 cm or higher). Increasing the air flow j_g results

in increasing the local droplet velocities all over the cross-section but also in moving upward the local position of the maximum droplet velocity. The liquid superficial velocity appears to have a too small effect to be analyzed. In fact, unlike the air velocity v_g , the axial droplet velocity is not maximum at the center of the vertical chord. In addition, the vertical profile is more stiff than the horizontal profile. Indeed the horizontal profile is little different as the velocity is quite constant from -6 to 0 cm (referenced at the center of the horizontal chord) and already important at the close positions to the wall, resulting in a smoother horizontal profile than the vertical evolution.

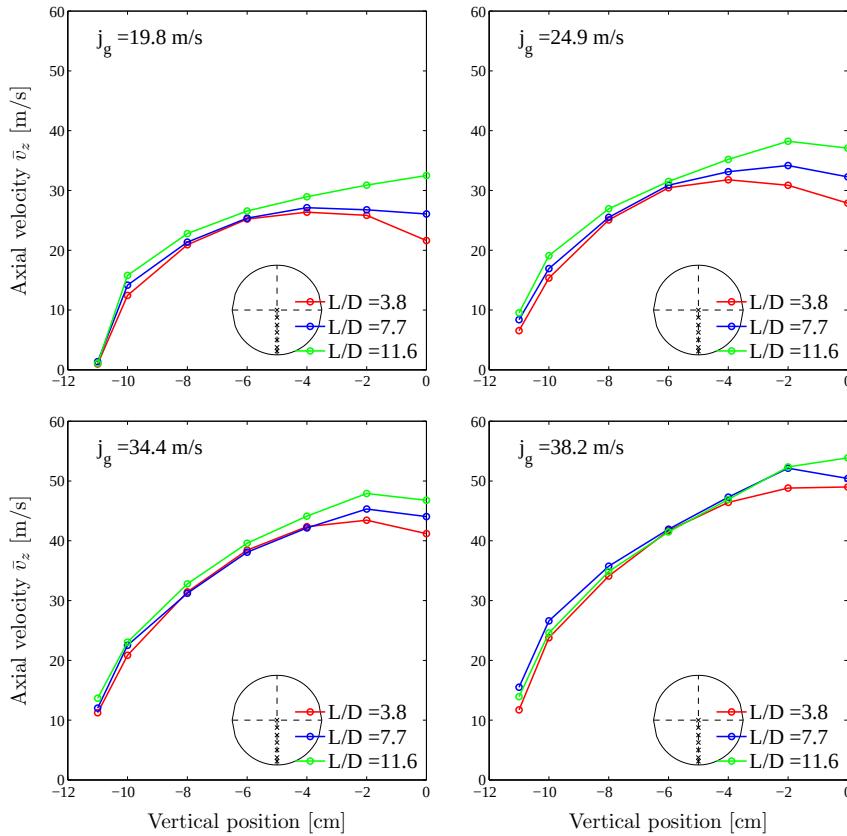


Figure 3.68: Evolution of the droplets mean axial velocity component v_{10} along a vertical axis located in the center of the pipe for different locations in the instrumented pipe.

Figure 3.70 presents the evolution of the axial velocity $\bar{v}_z$ for different local positions along a vertical axis compared to the air velocity measured by the Pitot in the center of the pipe cross-section³. The liquid film velocity has no

³As a reminder, the gas velocity measured by the Pitot has not been recorded elsewhere

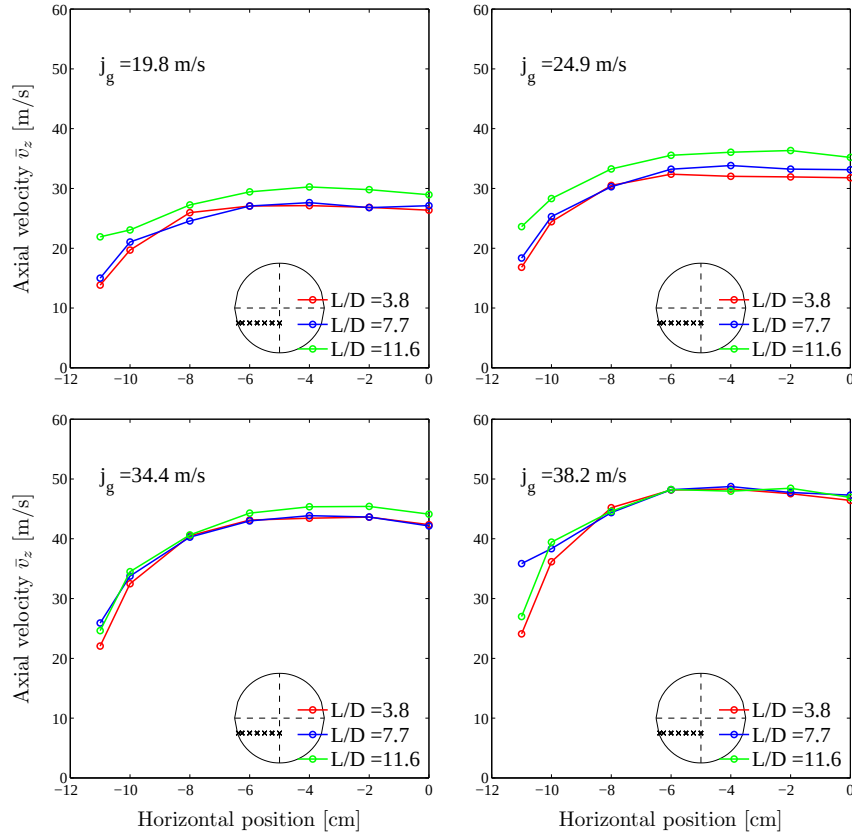


Figure 3.69: Evolution of the droplets mean axial velocity component v_{10} along an horizontal axis located 4 cm below the center of the pipe for different locations in the instrumented pipe.

influence on the droplet axial velocity for the upper positions but significantly acts on the droplet velocity in the lower positions. Indeed as the liquid film is 50% higher at $j_l = 0.036$ m/s than at $j_l = 0.018$ m/s (see Figure 3.4), while the wave velocity is slightly lower (see Figure 3.9), the droplets are sheared off from the liquid film at a slightly upper position at $j_l = 0.036$ m/s than at $j_l = 0.018$ m/s. A comparison of the droplet axial velocity at a fixed position indicates that the droplets have already been more accelerated for a liquid flow of $j_l = 0.018$ m/s than of $j_l = 0.036$ m/s. Conversely in the upper positions, the droplet have already been accelerated to their higher velocities so that the liquid flow has no more influence on their velocity. In summary, the influence area of the liquid film on the droplet velocity increases with the liquid velocity j_l .

than in the center of the pipe cross-section

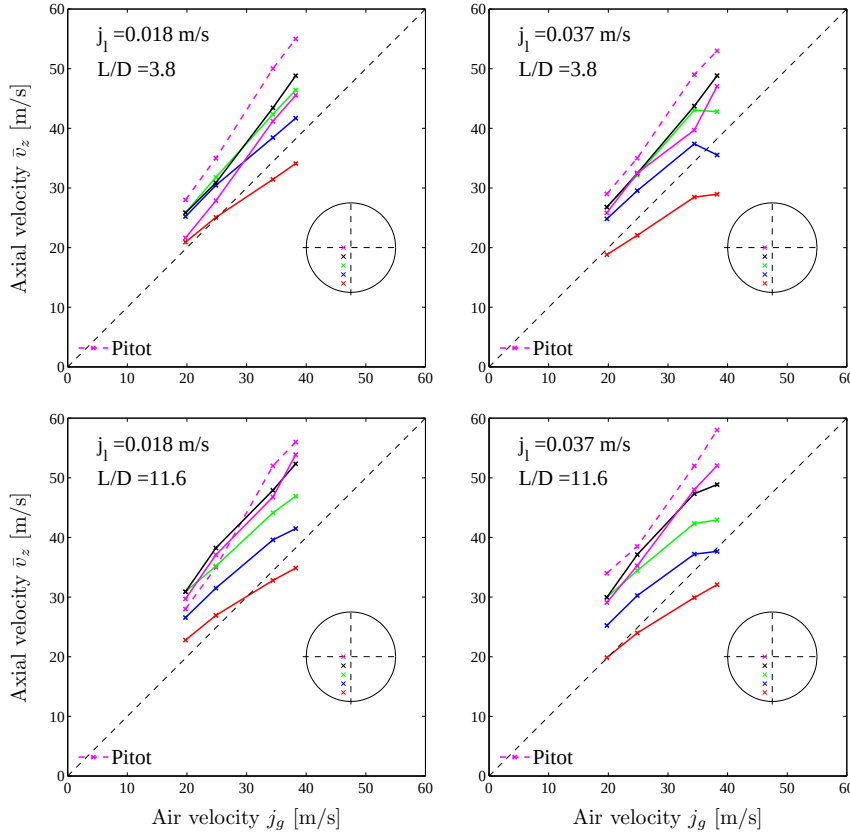


Figure 3.70: Evolution according to the air velocity, of the Pitot velocity at the center of the pipe and of the droplets local average axial velocity component $\bar{v}_z$ at different mesh cells of the pipe cross-section.

3.4.5 Lateral and vertical velocity components

The particle sizer provides three velocity measurements in three particular directions. A reconstruction of the velocities in the axial, vertical and lateral directions is easily achievable. However, the uncertainties on the reconstructed lateral and vertical component are so important (200%) that these data are not usable.

3.4.6 Synthesis on droplet velocity components

Figures 3.72 and 3.73 present the evolution of the droplet axial velocity component $\bar{v}_z$ along the vertical diameter of the pipe, combined to the velocity measurements from the Pitot tube and the BANJO. Figure 3.72 presents the evolution when increasing the air superficial velocity j_g while Figure 3.73 presents the influence of the liquid superficial velocity j_l .

Droplet volume fraction

The last flow parameters to be determined are the droplet volume fraction α_{ld} and the gas volume fraction α_g . As detailed in Equations 2.10 and 2.11, the determination of the droplet volume fraction α_{ld} is achieved from the total number of droplets, the average droplet axial velocity and the droplet mass flow-rate. Figures 3.71 present then the evolution of the droplet volume fraction along the action location and with the air superficial velocity. The last parameter, the gas volume fraction is therefore determined from the mass conservation equation summarized in Equation 2.9.

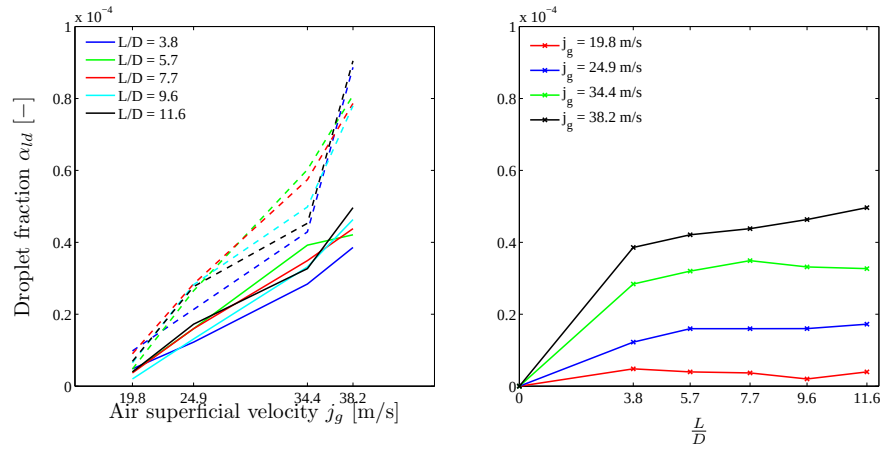


Figure 3.71: Evolution of droplet fraction (at $j_l = 0.018$ m/s (solid) and $j_l = 0.036$ m/s (dashed)).

3.4.7 Technical improvements

The laser granulometry and velocimetry techniques are very promising methods for the droplet characterization. However, the laser technology is still too little experienced and is therefore perfectible for improving the measurement quality and reducing the measurement uncertainties. As a first but main advice, the three laser probes should be gather together on a single probe in order to eliminate the uncertainties on the relative positioning between the three laser beams.

Moreover, the orientation of the three laser beams should also be reconsidered in order to minimize the uncertainties over the three reconstructed velocity component, not only the axial component. Indeed, in the current configuration, the only usable information on the vertical (v_y) and lateral (v_x) components is their signs as their amplitudes are subject to caution.

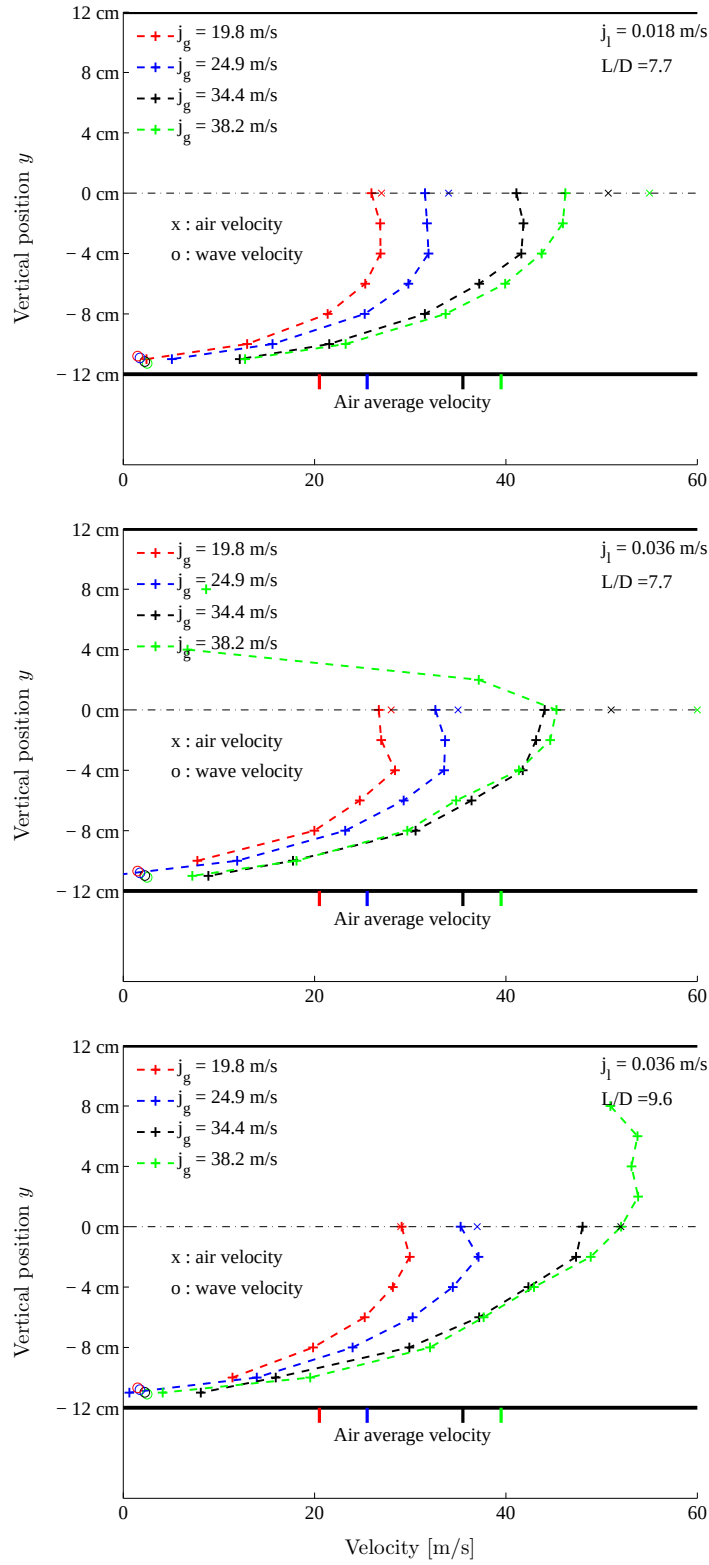


Figure 3.72: Evolution of the profile of the axial velocity $\bar{v}_z$ in the pipe when increasing the air superficial velocity j_g , compared to the measurement of the wave velocity (cross) and the air velocity by the Pitot (circle).

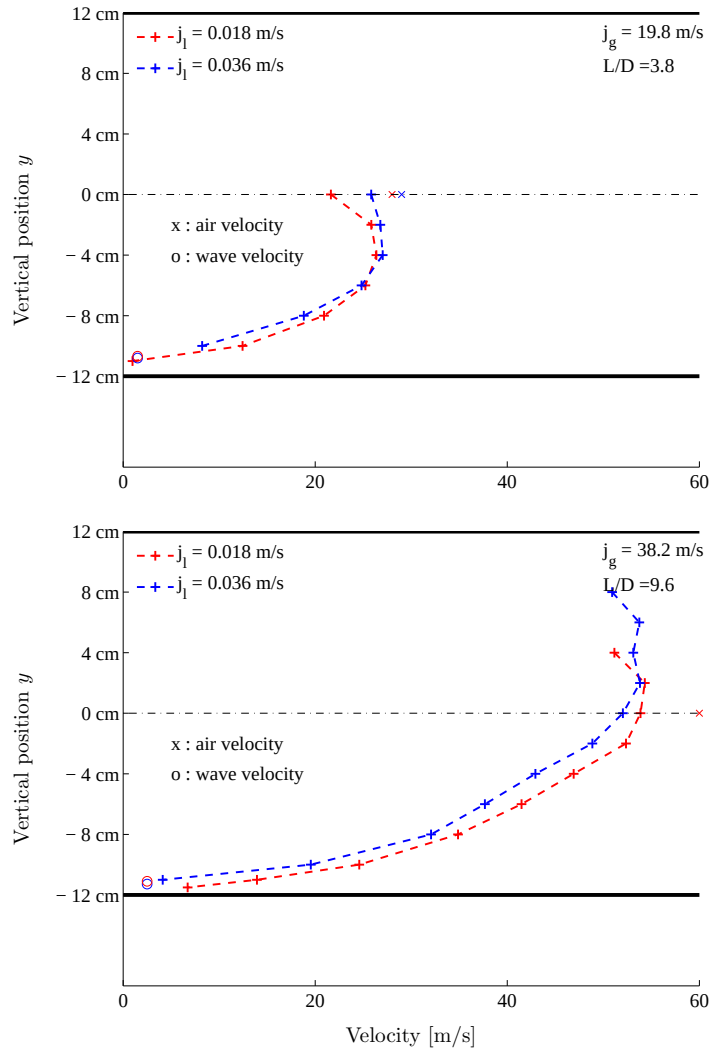


Figure 3.73: Evolution of the profile of the axial velocity $\bar{v}_z$ in the pipe when increasing the liquid superficial velocity j_l , compared to the measurement of the wave velocity (cross) and the air velocity by the Pitot (circle).

3.5 Synthesis on the experimental data

The REGARD experimental facility has been developed in order to provide several various insights on the behavior of an air-liquid wavy stratified flow occurring with droplet entrainment in an horizontal circular pipe of large pipe diameter ($D = 24$ cm). Therefore several measuring devices (BANJO, isokinetic probe, laser system) provide a database on the flow behavior, both in terms of gas-liquid interface behavior or in terms of droplet characteristics.

The wavy stratified flow with entrainment has been studied at five axial locations along the pipe z-axis, for eight flow conditions of gas and liquid superficial velocities. A test matrix of 40 experiments have then be conducted: four air superficial velocities (19.8 m/s, 24.9 m/s, 34.4 m/s and 38.2 m/s) and two liquid superficial velocities (0.018 m/s and 0.036 m/s) for five axial locations ($L/D = 3.8$ 5.7 7.7 9.6 and 11.6). Taking into account the values of the liquid film fraction, these superficial velocities (j_g and j_l) correspond to a liquid velocity (v_l) varying between 0.5 m/s and 1.5 m/s and to an air velocity (v_g) varying 20 m/s and 40 m/s, which completely correspond to a wavy stratified flow (as illustrated in Figure 2.16).

The five studied axial locations ensure the flow to be non-fully developed and established. Therefore the development of the stratified flow has could be extensively studied.

3.5.1 Flow behaviour description

Every instrumental device contributes to the description of the flow behaviour. A review on the flow descriptions has shown that in stratified flow, the droplet formation comes from the destabilization of the liquid film interface. However the several steps of the wave formation depend on the pipe geometry and the flow conditions (air and liquid velocities and interface level). Indeed from a stable stratified flow (without interfacial waves), increasing the air velocity leads to the following succession of interfacial disturbances (Section 3.1.4, page 91):

- o At very low air velocities, some 2-D wavelets of small amplitudes appear over the interface. Due to the uneven level of liquid over the pipe cross-section, these regular 2-D structures deform in a structured diamond wave network.
- o An increase of the air velocity leads to the amplitude growth while the wavelengths decrease. But beyond a certain air velocity, these small

wavelets coalesce to form higher amplitude waves of higher wavelengths called 2-D roll-waves.

- o Small faster 3-D wavelets begin to appear over the 2-D roll-waves traveling from the wave bottom to the wave crest. Depending on the flow conditions, these ripple-waves may deform and break at the roll-wave crests, starting the droplet formation.
- o Still increasing the air velocity, the structured 2-D waves give way to a 3-D network of ripple-waves. This transition highly depends on the wave geometry as it happens at smaller air velocities in a cylindrical pipe than in a square pipe (due to the destabilizing effect of the not-transversely uniform wave velocity).

The study of the flow visualizations leads to the conclusion that for the range of phase velocities analyzed in REGARD, the wavelets (ripple-waves) traveling over the gas-liquid interface rank in the fourth category. Indeed for the range of air velocities of 20 m/s to 40 m/s, the presence of roll-waves appears to be hidden by the dynamics of the ripple-waves. The wavelets fragment, grow, stretch-out so rapidly that the interface becomes too chaotic to notice the presence of an underlying roll-wave. Moreover the flow visualizations indicate that the wavelets follow the destabilizing mechanism highlighted by Rizk et al. [131]. Due to the action of the faster gas on the curvature of the gas-liquid interface, several instabilities develop, grow and stretch over the interface. These ripple-waves of small amplitudes and wavelengths, but high frequency, destabilize leading to the formation of a cylindrical filament transverse to the flow. This transverse filament also destabilizes before being separated from the ripple-wave crest due to the action of the surface tension. Consequently, the transverse filament deforms in a succession of swellings and contractions. Due to the shear of the gas flow, the swellings deform and stretch out, taking the form of longitudinal fingers which destabilize as droplets according to the procedure of Rayleigh. The wavelets appear then quite uniformly over the interface as some 3-dimensional curved structures which destabilize primarily as some transverse filaments which degenerate in longitudinal filaments of liquid, similar to little fingers. The analysis of BANJO data show that the characteristics of these ripple-waves depend on the flow conditions. The wave velocity mainly depends on the air superficial velocity j_g while the frequency mainly depends on the liquid level. The number of interfacial 3-D ripple-waves traveling over the interface in a control volume depends then primarily on every flow conditions and indirectly on the pipe geometry.

The fragmentation of these ripple waves as longitudinal filaments leads to the atomization of these liquid fingers as droplets. The behaviour of the gaseous phase with dispersed droplets of the air-liquid stratified flow has been extensively analyzed. Indeed the various experimental data have mainly highlighted the influence of two phenomena on the two-phase air-liquid flow occurring in REGARD:

- o The gravity force which acts perpendicularly to the flow and significantly overcome the turbulent and gas effects, ensuring therefore the stratification of the flow. Contrary to an annular flow, the gravity maintains then the liquid film at the bottom of the pipe cross-section. The flow is elsewhere stratified at two levels: On the one side, the two continuous phases (air and liquid) of very different densities are stratified such that the liquid continuous film (heavy phase) occupies the bottom of the pipe and the gas (light phase) occupies the top of the pipe. On the other side, the dispersed phase is also stratified as the droplets are heterogeneously dispersed over the pipe cross-section with a large presence in the bottom of the gas cross-section and a small density of droplets in the top of the pipe cross-section.
- o The development of the flow which significantly acts on the flow characteristics. Indeed as the measuring devices have been positioned close to the junction of the two phases (air and liquid), the wavy stratified flow has been studied early in its development. Consequently the evolution of every flow parameter along the z-axis direction contributes to the description of the inception of droplet entrainment and the development of the dispersed phase.

The flow analyzed in REGARD is then primarily stratified and in development. None of the three phases (liquid film, dispersed droplet and air) reaches an equilibrium state in the REGARD facility. Indeed the liquid level h_l is decreasing along the z-axis, as the thickness of the liquid injection layer has been overestimated in the facility design while the air velocity is slightly accelerating. The dispersed phase is initiating its development so that the three phase volume fractions (α_g , α_{lf} and α_{ld}) continuously vary when moving along the axial location $\frac{L}{D}$.

The visual observations confirm the literature data as the droplets primarily come from the atomization of the liquid interface. As the air-liquid flow is stratified at the studied air and water mass flow-rates, most droplets are formed in the bottom of the pipe cross-section. These droplets, observed in the lower

layers of the pipe cross-section, directly above the liquid level at the closest location from the liquid injection, continuously scatter and spread all over the gas cross-section along the z-axis. The mass flow of droplets increases then locally all over the pipe cross-section.

As the droplets primarily come from the atomization of the liquid filaments torn from the ripple-waves, the droplets initially formed are relatively large (300 - 700 μm) but are likely to fragment or deposit. Indeed these large drops, present in the lowest layer of the dispersed phase, are either unstable or at risk of collision and consequently break in several smaller droplets. Or as the drop velocities are conversely proportional to their size, their initial velocity is too small to avoid the almost immediate deposition. The stratification of the droplet phase is consequently more pronounced for the larger droplets than for the smaller ones, as most of the larger droplets are located just above the liquid interface.

The smaller droplets (50 - 100 μm) come therefore from the atomization either of the liquid filaments or the first larger drops. With a size ensuring their stability, these droplets disappear from the flow only due to their deposition. As a consequence, the number of these small droplets is much more important than the number of the large droplets, up to a proportion of 85% of the droplets observed over the complete pipe cross-section and up to 100% in the middle layers of the flow. Indeed as their velocities are higher than the larger droplet velocities, the trajectory of these droplets is less affected by the gravity than the larger droplets so that they disperse more easily over the pipe cross-section. However these smaller droplets are heterogeneously dispersed, denoting some stratification, both because of the developing length and the action of the gravity. Indeed, the gravity causes the drop down and deposit on the interface so that only droplets expelled with a sufficient energy and an adequate initial direction, can reach the highest layers of the pipe cross-section before the gravity overcomes their vertical component of inertia. Moreover these droplets have an axial velocity much greater than their vertical velocity so that, even with the initial conditions to reach the upper layer of the pipe cross-section, the length of the pipe is too short for them to access it in the REGARD facility. Locally, the droplet flux therefore comprises both ascending and falling droplets. The local balance is thus achieved when the two components become equal. At the global scale of the pipe cross-section, the balance between droplet deposition and entrainment is therefore achieved when local balance is achieved.

The droplet dispersed phase is then highly and doubly stratified: both in terms of droplet number and in terms of droplet size. The lowest layers present a large number of drops and a large proportion of large droplets ; these two parameters decrease along the vertical diameter as the droplets become fewer and fewer while the average diameter decreases. However, the proportion of larger droplets rises again in the middle layer of the pipe cross-section as the number of droplets become relatively small.

Finally this double stratification of the droplet dispersed phase has a strong implication on the observed volume of droplets. Indeed as the droplets are both more numerous and greater in the lowest layers of the cross-section, the local volume of droplets just above the interface is far greater than anywhere else. The local mass flow-rates of droplets are then also stratified as the droplet fluxes decrease approximately hyperbolically when rising along the vertical diameter.

The effect of the liquid film flow-rate essentially appears in the variation of the averaged liquid level. Indeed, most wave characteristics appear to be quasi unchanged by the increase of the liquid superficial velocity. Both the averaged velocity and the averaged frequency of the waves remain unchanged. However, the wave amplitude is little affected. The main implication of the liquid superficial velocity is then the increase of the liquid level and the subsequent increase of the wetted perimeter and the interfacial area. As a main consequence, the liquid superficial velocity affects more the number of waves than the wave characteristics.

At the little liquid mass-flow rates studied in this work, increasing the liquid superficial velocity has no influence on the gaseous phase. However, the increase of the liquid level has two indirect effects on the dispersed phase: first, the augmentation of the number of waves imply the augmentation of the total amount of liquid wrested at the ripple-wave crests ; second, as the vertical position of the waves is slightly higher, the droplets have a slightly higher probability to reach the upper layers of the pipe cross-section. Thus a mass flow-rate of droplets has been observed in the upper layers at the highest liquid superficial velocities. The conjunction of these two effects makes the mass flow-rate of droplets observable in the upper layers at the highest liquid superficial velocities.

Finally, the effect of the air superficial velocity on the dispersed flow is essential. Indeed, modifying the air velocity acts at every scale of the air-liquid two-phase flow. The wave velocity follows the air velocity and increases quasi linearly when the air flow-rate is increased. As a consequence, the wave devel-

opment increases faster so that the number of waves present in a control volume increases. The quantity of droplet generated increases therefore with the air superficial velocity. Moreover as their velocity increases as well, the number of droplets traveling through the control volume increases too. The droplet mass flow-rates increase thus everywhere over the pipe cross-section. The entrained fraction is extremely sensitive to this parameter.

The air velocity acts also on the droplet spectrum as the smaller droplets become proportionally more numerous when the air superficial velocity increases. Indeed the smaller droplets are proportionally more accelerated than the larger droplets so that the number of smaller droplets observed traveling the control volume, increase more significantly than the number of large droplets.

Chapter 4

A mechanistic model for the droplet entrainment rate Γ_E

The bibliographic review detailed in Chapter 1.3 concluded that usual entrainment rate models are not fully adapted to the modeling of a stratified flow in the hot leg of a PWR. Indeed, these entrainment models have been primarily developed for fully established flows and do not take deeply into account the fluid mechanics leading to the wave fragmentation. The parameters of these models have been predominately correlated on experimental facilities of small diameter dimensions, mainly in annular flows and vertical pipes. Therefore the effect of the pipe size does not appropriately appear in most of these usual models.

In Chapter 2, in order to provide more suitable experimental data for a model validation, a new experimental facility of greater scale has been designed and operated, and its data analyzed in Chapter 3. Data on entrainment rate Γ_E have been obtained at a 1/3 scale of a hot leg, both in length and diameter, three to ten times greater than usual experimental data. The data of REGARD provide an acceptable database for the validation of the entrainment rate model at a 0.24 m pipe diameter.

Chapter 4 aims at presenting the entrainment rate model developed in a mechanistic approach of the fluid mechanics in a stratified flow. The general approach is primarily detailed based on the flow conception depicted by Woodmansee and Hanratty [166]. The global model requires some additional modeling of various parameters that are completely described in the following sections.

As the modeling of these parameters may require a subsequent calculation time, a correlated version of the model, less expensive in computation time, is finally developed for the implementation in CATHARE 3.

4.1 General approach

In 1969, Woodmansee and Hanratty [166] made some flow observations on a stratified flow with droplet entrainment. Their analysis of the entrainment process has been validated by several further studies (e.g. [51, 119, 101, 70, 9]) and confirmed by the flow visualizations on REGARD (see Section 3.2). The droplet entrainment phenomenon from the liquid film mainly occurs in these experimental conditions from a wave atomization, depicted in Figure 4.1.

One may point out that various other entrainment processes have been proposed in the literature [51, 144, 47, 1, 130]. Indeed, Hewitt and Hall-Taylor [69] and Kuo et al. [87], in 1995, described the following other droplet entrainment processes:

- o The wave undercutting occurring in counter-current stratified or annular flow.
- o The wave rolling occurring in co-current stratified or annular flow, described as the major entrainment process in this work.
- o The bubble bursting at liquid interfaces, which induces a droplet emission.
- o The droplet deposition, which induces a mechanical shock at the interface which generates droplets.

However, under the flow conditions predicted in the hot legs of a PWR, measured and visualized on REGARD, the stratification characteristics of the flow ensures that the wave atomization primarily occurs from the wave atomization phenomenon and, secondarily, from the droplet deposition. Indeed, once droplets begin to deposit, a re-entrainment process of droplets may be considered due to their impact on the liquid film. Nevertheless this process should not be taken into account in the entrainment but in the deposition modeling.

The droplet entrainment is assumed to result from the development of interfacial disturbances that grow over the interface until reaching the unstable amplitude of interfacial Kelvin-Helmholtz instabilities and then fragmenting. Indeed a consequent shear stress appears due to the difference of velocities between the two phases at the interface, initializing Kelvin-Helmholtz instabilities.

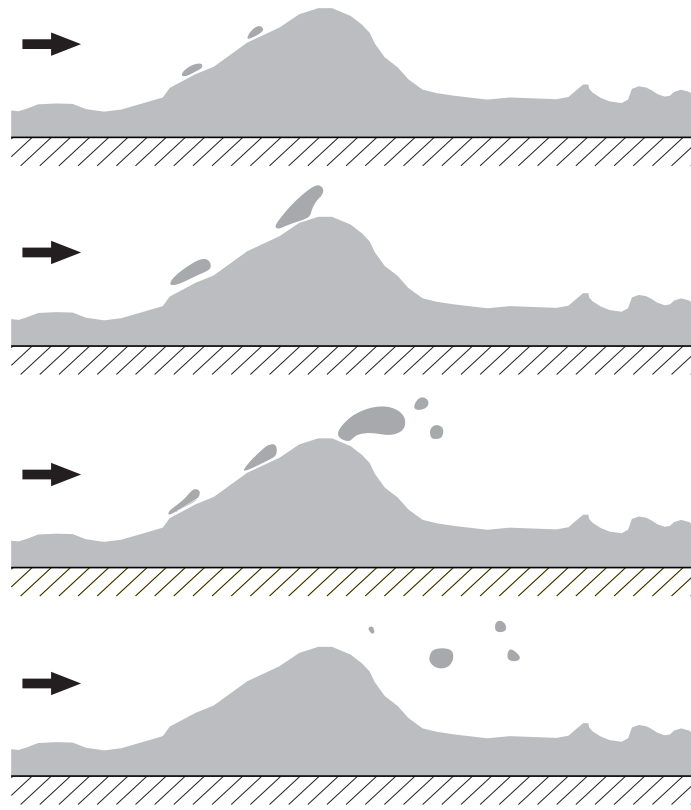


Figure 4.1: Mechanistic description of the droplet entrainment phenomenon described by Woodmansee and Hanratty [166].

According to the flow conditions (phase velocities, phase densities, surface tension,...) this instability may amplify and then grow both in time and space or be tamped. The stratified flow may then be wavy or smooth. When the instability begins to grow in the wavy stratified flow, several small-amplitude waves appear at the interface and begin to grow. The distribution of the phase velocities around the wave profiles locally modifies the shear-stresses, resulting in a deformation of the shape of the waves. Depending on the flow conditions, the wave may grow to a critical amplitude beyond which the destabilizing forces acting on the wave overcome the gravity and the surface tension forces, leading to the wrenching of the top of the wave. The flow turns then from a wavy stratified flow into a droplet stratified flow. The transition between these three stratified regimes essentially depends on the two phase velocities. On the other hand, acting on the liquid to water fraction at fixed velocity conditions may turn the stratified flow to a slug or a churn flow. However a last transition may come at very high air velocities: the transition from a stratified flow to an annular flow. The following modelings do not suit however to these three flow

regimes (churn, slug or annular). Indeed the force balance acting on the wave is especially formulated for stratified flows. According to usual experimental visualizations [166, 69, 47, 126, 144, 131, 128] (described in Section 3.2), the break-up of the wave which finally results in the creation of several droplets may go through by several different stages. Indeed the fragmentation process may take many forms as the wave crests deform in unstable structures: rings, fingers, ... before finally be atomized into droplets. Anyway the wave crest is taken away from the interface and entrained as droplets in the air stream.

Geometrical assumptions on the interface shape

The present model assumes several hypotheses on the shape of the interface:

- o The model is developed on a control volume: CV , defined by an interfacial area A_{CV} of length L and width l_i as depicted in Figure 4.2. The liquid level in the pipe cross-section is characterized by the parameter h_l symbolizing the liquid height at the deepest location of the pipe cross section and the parameter $\bar{h}_l$ symbolizing the cross-sectional averaged liquid height.

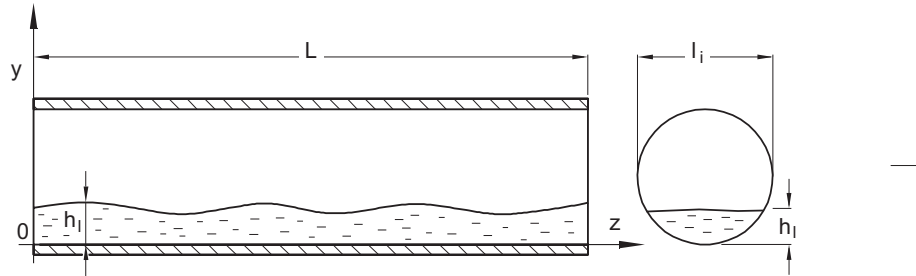


Figure 4.2: Flow description in the control volume CV .

- o The interfacial area is assumed to be flat so that the intersection between the pipe cross section and the interface is an horizontal chord, of width l_i .
- o Each interfacial wave has the same three-dimensional sine wave geometry. The wavelengths in every direction are identical so that A_{CV} is proportional to the square of the wavelength of the instabilities (λ).

Model

The dispersion of the waves over the interface is assumed to be homogeneous and all interfacial waves have similar characteristics (geometry and velocity). The space variation of the liquid height in the pipe cross-section is indeed assumed

to have no influence on the wave characteristics. Consequently, the amounts of water torn out at the crest of each wave are assumed to be identical. The total mass of liquid wrest off from the interface is thus assumed to be proportional to the number of waves that develops on the liquid film multiplied by the entrained amount of liquid per wave:

$$M_{\text{entrained,CV}} = N_{\text{waves}} \times M_{\text{entrained per waves}} \quad (4.1)$$

To express an entrainment rate as a flux density $\Gamma_{\text{E,CV}}$ in $[kg\,m^{-2}\,s^{-1}]$ in the control volume CV , this amount of liquid entrained in a control volume ($M_{\text{entrained,CV}}$) is divided by the interfacial area (A_{CV}) and the characteristic time of the entrained phenomenon (τ_C). The amount of liquid entrained per wave is obviously proportional to the entrained volume of liquid ($V_{\text{entrained}}$) and the liquid density (ρ_l). Thus:

$$\Gamma_{\text{E,CV}} = \frac{N_w \times \rho_l \times V_E}{A_{CV} \times \tau_{C,E}} \quad [kg\,m^{-2}\,s^{-1}] \quad (4.2)$$

The number of waves ($N_w = N_w(A_{CV})$) in the control volume depends on the size of the interface area and the wavelength of the instabilities. The entrained volume (V_E) depends on the wave geometry (amplitude, wavelength) at the fragmentation. The interfacial area (A_{CV}) depends finally on the control volume and the interfacial shape while the characteristic time is the time necessary for a wave to travel the control volume and depends then also on the wave geometry and on the wave velocities.

The interface area of the stratified flow is expressed as the product of the interfacial width l_i by the length of the control volume L :

$$A_{CV} = l_i \times L \quad (4.3)$$

The number of fragmented waves $N_w(A_{CV})$ in the control volume is proportional to the total number of waves $N_t(A_{CV})$ in the control volume:

$$N_w = \left(\frac{N_w}{N_t} \right) N_t = \kappa N_t \quad (4.4)$$

where κ denotes the proportion of waves which fragment. The total number of waves in the control volume is directly proportional to the area of the control volume:

$$N_t = \left(\frac{L}{\lambda_c} \right) \times \left(\frac{l_i}{\lambda_c} \right) \quad (4.5)$$

Finally, the entrainment rate is expressed as:

$$\begin{aligned} \Gamma_{\text{E,CV}} &= \frac{\kappa \left(\frac{L}{\lambda_c} \right) \left(\frac{l_i}{\lambda_c} \right) \rho_l V_E}{l_i L \tau_C} \\ &= \frac{\kappa \rho_l V_E(\lambda)}{\lambda_c^2 \tau_C(\lambda)} \end{aligned} \quad (4.6)$$

Consequently this model requires mainly the determination of the instability wavelength λ . A first sub-model, based on a linear stability analysis of the stratified flow, has then been developed to estimate the most amplified wavelength. This critical wavelength is assumed to be the only one developing at the interface.

$$\lambda_c = \lambda(\text{linear stability analysis}) \quad (4.7)$$

As the wave fragmentation occurs when the destabilizing effects of the air velocity (pressure variation) on the wave overcome the retaining gravity and surface tension, a second sub-model implements then a force balance on the wave crests to determine the entrained volume.

$$V_E = V(\text{force balances}) \quad (4.8)$$

Finally a last sub-model has been developed to express the characteristic time of the entrained phenomenon τ_C . The proportion of waves which fragment κ is arbitrarily imposed in the model to 1, assuming that every wave in the control volume breaks up. The shape factor γ is geometrically determined from the characterization of the interface assumed in the linear stability analysis.

4.2 Sub-model of the critical wavelength

As depicted in the previous section, modeling the wavelength of the critical interfacial instabilities λ_c is an essential part of the mechanistic model of entrainment. This modeling is achieved throughout an analysis of the stability of the stratified flow towards an infinitesimal perturbation. The most amplified wavelength of the perturbation spectrum will be assumed to represent the waves that will grow over the interface of the stratified flow.

Linear stability analyses of an horizontal flow have been extensively performed in the literature, mainly devoted to stable smooth flows and stable wavy flows. The transient formulation based on the two-fluid model results in a Kelvin-Helmholtz type of instability. Barnea and Taitel [21] indicate that this approach is not suited for analyzing the steady-state stability of annular flows, since steady-state annular flow is inherently unstable with respect to the interfacial structure.

In horizontal flows, the purpose of the linear stability analysis was most of the time the determination of the transition from a stable smooth stratified flow to a wavy unstable stratified flow or annular flow. The first studies were obviously simple and applied to inviscid fluids, as the works of Kordyban and Ranov (1970) [85], Taitel and Dukler (1976) [146], Kordyban (1977) [84] or Mishima and Ishii in 1980 [104]. These analyses neglected the interfacial and parietal shear stresses and, even sometimes the surface tension. More recently, Lin and Hanratty (1986) [94], Wu and al. (1987) [167], Andritsos and al. (1989) [16], Barnea (1991) [20], Crowley and al. (1992) [38], Barnea and Taitel (1993) [21], Funada and Joseph [53] or Guo and al. in 2002 and 2007 [62, 61] developed viscous analyses which use the two-fluid model and simulates the interfacial and parietal shear stresses. The major outcome of these works is certainly the flow map proposed by Taitel and Dukler in 1976 [146].

The recent works separately attempt to increase the precision of the transition criteria by introducing various effects in the two-fluid model. Brauner and Maron in 1993 [28] introduced an interfacial shear stress depending on the local interfacial slope while in 1998 [29] they introduced general modifications to take into account the curved interface in the two-fluid model. Guo et al. in 2002 [62] or Ansari and Sani [17] in 2006 have taken into account non-linear conservation equations with (Ansari) or without (Guo) the effect of surface tension.

In 1993 Barnea and Taitel [21] discussed the use of the viscous and non-

viscous approaches of the Kelvin-Helmholtz stability criteria for stratified flows. The neutral stability lines obtained from the viscous analysis and the inviscid analysis were different for low liquid viscosities (1 cP: water at atmospheric pressure and 20°C) but similar for higher viscosities (5000 cP). The position and maxima of the amplification factor based either on a inviscid or viscous Kelvin-Helmholtz analysis are quite identical. However, in the range where no-amplification is predicted by the inviscid Kelvin-Helmholtz analysis, the amplification factor predicted by the viscous analysis may be positive (unstable) or negative (stable) so that the neutral stability lines are quite different according to the analysis lines (even if the maxima of the amplification spectrum are identical). Their interpretation which supports the previous work of Barnea (1991) [20], indicates that the inviscid Kelvin-Helmholtz analysis suits better to determine the transition from smooth stratified to wavy stratified flow while the viscous Kelvin-Helmholtz analysis predicts the transition either to slug or annular (depending on the void fraction). However their personal interpretation highly differs from i.e. Andritsos and Hanratty [16] who also relate the wave type to the amplification factor.

4.2.1 Concept of linear stability analysis

A linear stability analysis studies the stability of the flow towards a perturbation h' of very small amplitude and any frequency f or wavenumber k from a fully developed and permanent flow (depicted in Table 4.1). The perturbation may grow whether in space (along the z -axis) when $\Im[k] < 0$ or in time when $\Im[\omega] > 0$. According to de Crécy [44], the model is spatially stable if:

$$\forall \omega \in \Re \rightarrow \frac{\omega}{\Re(k)} \Im(k) \leq 0 \quad (4.9)$$

for every root of the dispersion equation. As the parameter $\frac{\omega}{\Re(k)}$ corresponds to the propagation velocity of the perturbations, this criterion expresses the condition for non-amplification of the perturbations. However Micaelli (1981)[44] had indicated that this criterion is not always justified but as the perturbations analyzed in this work may be assimilated to waves propagating from the excitation point, this criterion remains valid. Moreover, according to Gaster [55, 56] and Micaelli [44], the spatial stability of the flow is sufficient to also ensure the time stability.

The following paragraphs aim at describing the essential steps of the linear stability analysis, beginning with a short description of the conservation and closure laws used in the model and continuing with the details of the analysis method. The complete linear stability analysis, including every step as the

Table 4.1: Synthesis on the type of perturbations studied by an hyperbolicity or a linear stability analysis.

	Amplitude	Frequency ω	Wave Number k
Hyperbolicity study	$\rightarrow 0$	$\rightarrow \infty$	$\rightarrow \infty$
Linear stability study	$\rightarrow 0$	Any	Any

hyperbolicity limit or the derivations is detailed in Appendix B, Sections B.2 and B.3.

4.2.2 Conservation laws

The linear stability analysis is conducted on a stratified two-phase flow of air and water assuming several hypotheses:

- o The two fluids are assumed to be incompressible and non-expandable. The governing conservation equations of energy can then be uncoupled so that the two-fluid model can be reduced to the conservation laws of mass and momentum.
- o The instability wavelength, frequency and velocity are assumed to be stable and constant in time and space once the instability begins to grow, so that only the instability amplitude evolves during the wave development.
- o The interfacial instabilities are assumed to have a wavelength level of a similar size as the interfacial height so that the shallow water equations may be assumed.
- o The initial system is assumed to be stable in time and space (no perturbations). The perturbation initiates the growth of the instabilities but but does not apply to the wave development since wave fragmentation and atomization are not considered, so as paradoxally neither wave fragmentation nor droplet entrainment are considered in the wave equation development. Consequently no mass transfer between the liquid phases are considered for the linear stability analysis.

Starting from a general form of a local instantaneous balance equations (derived from Delhayé [46] and brilliantly taught by Bartosiewicz [22]):

$$\frac{\partial}{\partial t} (\rho_k \Psi_k) + \nabla \cdot (\rho_k \Psi_k \vec{v}_k) + \nabla \cdot \vec{J}_k - \rho_k \Phi_k = 0 \quad (4.10)$$

Assuming for the conservation equation of mass that there is no spontaneous generation of mass ($\Phi_k = 0$), that there is no mass transfer at the interfaces,

Table 4.2: Flux and term sources for the mass and momentum balance equations.

	Ψ_k	J_k	Φ_k
Mass	1	0	0
Momentum	w_k	$-\left(\vec{\tau}_k - p_k \vec{1}\right)$	$\vec{g}$

either at walls or at the interface ($J_k = 0$), and as the mass is already represented by the local density ρ_k , the general balance equation may be written as Equation 4.11 for the mass. On the other hand, the momentum conservation equation becomes Equation 4.12, assuming that the only external force acting on the flow is the gravity ($\Phi_k = \vec{g}$) and that the momentum transfer is described by the usual stress tensor $\vec{T}_k = -\left(\vec{\tau}_k - p_k \vec{1}\right)$.

Table 4.2 presents the three flux and term sources in the mass and momentum balance equations:

$$\frac{\partial}{\partial t}(\rho_k) + \nabla \cdot (\rho_k \vec{v}_k) = 0 \quad (4.11)$$

$$\frac{\partial}{\partial t}(\rho_k w_k) + \nabla \cdot (\rho_k w_k \vec{v}_k) + \nabla \cdot (-\vec{T}_k) - \rho_k \vec{g} = 0 \quad (4.12)$$

Let us integrate Equations 4.11 and 4.12 over the phase cross section A_k , apply the Leibniz and Gauss theorems and the averaging operators, and let us make the following assumptions¹, Section B.2:

- o The geometry is 1D.
- o The phase densities are constant over the cross section A_k of the phase k :

$$\langle \rho_k \rangle_k = \rho_k, \quad (4.13)$$

- o In a 1D model, the average phase velocities w_k can be expressed so that:

$$\langle w_k \rangle_k = v_k, \quad (4.14)$$

- o The pipe is horizontal, linear and not deformable. The pipe cross section A is then constant ; thus:

$$\frac{\partial A}{\partial t} = 0 \quad \text{and} \quad \frac{\partial A}{\partial z} = 0, \quad (4.15)$$

- o The axial momentum diffusion is negligible.

¹Every step is detailed in Appendix B

- o No mass transfer between the gazeous and the liquid phase.
- o No mass transfer of liquid between the two liquid phases:

$$\Gamma_E = 0 \quad \text{and} \quad \Gamma_D = 0, \quad (4.16)$$

- o The pipe is horizontal; thus the action of the gravity along the z-axis:

$$A_k \rho_k g_z = 0, \quad (4.17)$$

- o Momentum transfer due to the mass transfer is null as no mass transfer is assumed.

$$A (M v_{k,i}) = 0. \quad (4.18)$$

The mass and momentum balance equations are finally derived:

$$\begin{aligned} \frac{\partial}{\partial t} (\alpha_k \rho_k) + \frac{\partial}{\partial z} (\alpha_k \rho_k v_k) &= 0 \\ \frac{\partial}{\partial t} (A_k \rho_k v_k) + \frac{\partial}{\partial z} (A_k \rho_k v_k^2) + A_k \frac{\partial}{\partial z} \langle p_k \rangle &= A F_{fr,k} + A F_{fr,i,k} \end{aligned} \quad (4.19)$$

where F_{fr} denotes the acting friction forces, at the wall on phase k : $F_{fr,k}$ and at the interface on phase k : $F_{fr,i,k}$. Obviously, the force balance at the interface requires: $F_{fr,i,g} = -F_{fr,i,l}$

4.2.3 Closure laws

The final balance equations for mass and momentum (Equations 4.19) require additional modeling for the closure relations of the friction forces $F_{fr,k}$, $F_{fr,i,k}$ the average pressure $\langle p_k \rangle$ and the interfacial pressure.

Interfacial pressure

Indeed, the vertical repartition of pressure is assumed to be hydrostatic over the cross section:

$$\langle p_k \rangle = P_{i,k} + \rho_k g_{\perp} (h_i - h_{g,k}) \quad (4.20)$$

where $P_{i,k}$ denotes the interfacial pressure of the phase k , $g_{\perp}$ the component of gravity perpendicular to the flow direction and $h_{g,k}$ the position of the gravity center of the phase k .

Two non-miscible fluids separated by a curved interface present a pressure difference between the two phases at the interface. As this difference comes essentially from the surface tension, the interfacial jump of pressure is expressed via a Laplace's law:

$$P_{i,g} - P_{i,l} = \frac{\sigma}{R_{Laplace}} = \sigma \left(\frac{\delta^2 h_l}{\delta z^2} \right) \quad (4.21)$$

where $R_{Laplace}$ is defined as the Laplace's radius, equal to the curvature of the interface along the z-axis (as the geometry is assumed to be 2D).

Friction forces

The friction forces $F_{fr,k}$ are defined as:

$$F_{fr,k} = \tau_{w,k} \frac{S_k}{A} \quad F_{fr,k,i} = \tau_{i,k} \frac{S_i}{A} \quad (4.22)$$

where $\tau_{w,k}$ and $\tau_{i,k}$ respectively denotes the parietal and interfacial shear stresses on phase k , while S_i and S_k respectively denotes the phase areas.

Parietal shear stresses

The modeling of the parietal shear stresses follows then a standard formulation:

$$\tau_{w,k} = \frac{1}{2} f_k \rho_k u_k |u_k| = \tau_k \quad (4.23)$$

where f_k denotes the friction factor of the phase k . Over time, several models of the parietal friction factors f_k have been presented in the literature. The model of Wallis [158] has been extensively used for years but currently the model of Taitel and Dukler [146] (Equations 4.24) is frequently preferred since the predicted variations fit well various experimental data for a smooth pipe:

$$\begin{aligned} f_k &= C_k Re_k^{-n_k} \\ \text{if } Re_k > 2000 &\Rightarrow C_k = 0.046 \quad n_k = 0.2 \\ \text{else } Re_k < 2000 &\Rightarrow C_k = 16 \quad n_k = 1 \end{aligned} \quad (4.24)$$

The CATHARE code uses a slightly different friction factor defined as:

$$\begin{aligned} f_k &= \max(16 Re_k^{-1}, 0.079 Re_k^{-0.025}, 0.003) \\ \text{if } 16 Re_k^{-1} < 0.079 Re_k^{-0.025} &\Rightarrow C_k = 0.079 \quad n_k = 0.025 \\ \text{if } 16 Re_k^{-1} > 0.079 Re_k^{-0.025} &\Rightarrow C_k = 16 \quad n_k = 1 \end{aligned} \quad (4.25)$$

Interfacial shear stress

The way of modeling the interfacial shear between the phases at the free interface has been always recognized as critical in determining the resulting stability characteristics. Its evaluation is complex since the variation of the stresses along the interface critically depends on the unknown structure and characteristics of the mobile wavy interface [27]. The conventional way to model the interfacial shear stress is by assuming a quasi-steady relation in terms of the local layer

depth and the instantaneous phase velocities: $\tau_i = \tau_i(h_l, j_g, j_l)$.

The interfacial shear stress is then expressed in a similar way as the parietal shear stresses:

$$\tau_{i,k} = \frac{1}{2} f_i \rho_g (u_g - u_l) |u_g - u_l| = \tau_i \quad (4.26)$$

The interfacial friction factor has been extensively modeled in the literature. One of the first contributions has come from Cohen and Hanratty in 1965 [34], assuming the friction factor to be constant whatever the flow conditions:

$$f_i = 0.0142 \quad (4.27)$$

Currently many authors based their work on the friction factor proposed by Taitel and Dukler [146], assuming the interfacial friction factor equal to the parietal friction factor of the gaseous phase. In this approach, the liquid phase is assumed to have a behavior similar to a static wall:

$$f_i = f_g = C_g Re_g^{-n_g} \quad (4.28)$$

$$\begin{array}{ll} \text{if } Re_g > 2000 & \Rightarrow C_g = 0.046 \quad n_g = 0.2 \\ \text{else } Re_g < 2000 & \Rightarrow C_g = 16 \quad n_g = 1 \end{array}$$

However the friction factor may be quite small for different flow conditions. To ensure a minimal value to the friction factor, Barnea and Taitel [20] have proposed a slight improvement on the friction factor, combining the approaches of Cohen and Hanratty [34] and Taitel and Dukler [146]:

$$\begin{array}{ll} \text{if } f_g > 0.0142 & f_i = f_g \\ \text{else} & f_i = 0.0142 \end{array} \quad (4.29)$$

Finally, a similar approach is used in the CATHARE code, where the friction factor is a mixed approach between a constant term and the parietal friction factor. However the friction factor contains an additional term defined as:

$$f_i = \max \left(0.003; \frac{0.079}{Re_g^{0.25}} \right) \times \min \left(100; 1 + 0.038 \frac{Re_g^{0.97}}{Fr_g^{1.3}} \frac{\alpha_l^{0.5}}{\alpha_g^{0.44}} \right) \quad (4.30)$$

However, according to Tzotzi and Andritsos [154], "the Taitel-Dukler model fails in presence of interfacial waves, mainly due to the assumption that $f_i = f_g$. In addition the liquid-to-wall shear stress also contributes to the deviation of experimental results with predictions by the Taitel-Dukler model". Indeed Kowalski [86] found that this model of liquid-to-wall shear stress significantly underestimates the liquid wall friction factor. Similar observations have been found by Andreussi and Persen [12], Spedding and Hand [143] or Andritsos and Hanratty

[15]. Andritsos and Hanratty [15] proposed then a semi-theoretical correlation that accounts for the influence of waves on both the interfacial and the wall-to-liquid friction factors.

The relation between f_i and f_g in this model takes into account the significant increase of the ratio $\frac{f_i}{f_g}$ in the large-amplitude wave region:

$$\begin{aligned} \frac{f_i}{f_g} &= 1 & \text{for } j_g < j_{g,trans.} \\ \frac{f_i}{f_g} &= 1 + 15 \frac{h}{D} \left(\frac{j_g}{j_{g,trans.}} - 1 \right) & \text{for } j_g > j_{g,trans.} \end{aligned} \quad (4.31)$$

where $j_{g,trans.}$ is the gas superficial velocity corresponding to the estimated transition to large-amplitude waves ($j_{g,trans.} = 5 \text{ m/s}$ for an air-liquid two-phase flow).

4.2.4 Void fraction calculation

In the following developments, the void fraction α_g is determined assuming an hydrostatic equilibrium between the two phases. The liquid fraction α_l is then immediatly deduced: $\alpha_l = 1 - \alpha_g$. The complete development is detailed in Section 5.2.1.

4.2.5 Linear stability analysis

Based on the final Equations 4.19 of mass and momentum, the basic set of equations for the linear stability analysis is:

$$\begin{aligned} \frac{\partial}{\partial t} (\alpha_g \rho_g) + \frac{\partial}{\partial z} (\alpha_g \rho_g v_g) &= 0 \\ \frac{\partial}{\partial t} (\alpha_l \rho_l) + \frac{\partial}{\partial z} (\alpha_l \rho_l v_l) &= 0 \end{aligned} \quad (4.32)$$

$$\begin{aligned} \frac{\partial}{\partial t} (\alpha_g \rho_g u_g) + \frac{\partial}{\partial z} (\alpha_g \rho_g u_g^2) + \alpha_g \frac{\partial p_g}{\partial z} &= -\tau_g \frac{S_g}{A} - \tau_i \frac{S_i}{A} \\ \frac{\partial}{\partial t} (\alpha_l \rho_l u_l) + \frac{\partial}{\partial z} (\alpha_l \rho_l u_l^2) + \alpha_l \frac{\partial p_l}{\partial z} &= -\tau_l \frac{S_l}{A} + \tau_i \frac{S_i}{A} \end{aligned}$$

Passing to a non-conservative formulation and eliminating the derivative on α_k in the momentum equations by using a derivative on the velocity v_k obtained from the mass equations, the momentum equations becomes:

$$\begin{aligned} \alpha_g \rho_g \left[\frac{\partial u_g}{\partial t} + u_g \frac{\partial u_g}{\partial z} \right] + \alpha_g \frac{\partial p_g}{\partial z} &= -\tau_g \frac{S_g}{A} - \tau_i \frac{S_i}{A} \\ \alpha_l \rho_l \left[\frac{\partial u_l}{\partial t} + u_l \frac{\partial u_l}{\partial z} \right] + \alpha_l \frac{\partial p_l}{\partial z} &= -\tau_l \frac{S_l}{A} + \tau_i \frac{S_i}{A} \end{aligned} \quad (4.33)$$

Introducing Laplace's law on the interfacial gap of pressure at the interface (Closure Law 4.21) and assuming the pressure distribution to be hydrostatic (Closure Law 4.20), the two momentum equations are put together:

$$\begin{aligned}
& \rho_l \left[\frac{\partial u_l}{\partial t} + u_l \frac{\partial u_l}{\partial z} \right] - \rho_g \left[\frac{\partial u_g}{\partial t} + u_g \frac{\partial u_g}{\partial z} \right] \\
& - \sigma \frac{\partial^3 h_i}{\partial z^3} + (\rho_l - \rho_g) g \frac{\partial h_i}{\partial z} \\
& - \left(\rho_l \frac{\partial h_{g,l}}{\partial h_i} - \rho_g \frac{\partial h_{g,g}}{\partial h_i} \right) g \frac{\partial h_i}{\partial z} \\
& = -\tau_l \frac{1}{\alpha_l} \frac{S_l}{A} + \tau_g \frac{1}{\alpha_g} \frac{S_g}{A} + \tau_i \frac{S_i}{A} \left(\frac{1}{\alpha_g} - \frac{1}{\alpha_l} \right)
\end{aligned} \tag{4.34}$$

A small perturbation is then applied to the different variables: the velocities u_k , the volume fractions α_k and thus the levels of the phases and interface in the pipe h_k , where the symbol $\bar{}$ denotes the steady-state value and the $'$ stands for the perturbation:

$$\begin{aligned}
u_g &= \bar{u}_g + u'_g \\
u_l &= \bar{u}_l + u'_l \\
h_i &= \bar{h}_i + h'_i \\
h_{g,l} &= \bar{h}_{g,l} + h'_{g,l} \\
h_{g,g} &= \bar{h}_{g,g} + h'_{g,g} \\
\alpha_g &= \bar{\alpha}_g + \alpha'_g \\
\alpha_l &= \bar{\alpha}_l + \alpha'_l
\end{aligned} \tag{4.35}$$

The perturbed equations become then:

$$\begin{aligned}
& (\bar{\alpha}_l + \alpha'_l) (\bar{\alpha}_g + \alpha'_g) \rho_l \left[\frac{\partial (\bar{u}_l + u'_l)}{\partial t} + (\bar{u}_l + u'_l) \frac{\partial (\bar{u}_l + u'_l)}{\partial z} \right] \\
& - (\bar{\alpha}_l + \alpha'_l) (\bar{\alpha}_g + \alpha'_g) \rho_g \left[\frac{\partial (\bar{u}_g + u'_g)}{\partial t} + (\bar{u}_g + u'_g) \frac{\partial (\bar{u}_g + u'_g)}{\partial z} \right] \\
& - (\bar{\alpha}_g + \alpha'_g) (\bar{\alpha}_l + \alpha'_l) \sigma \frac{\partial^3 (\bar{h}_i + h'_i)}{\partial z^3} \\
& + (\bar{\alpha}_g + \alpha'_g) (\bar{\alpha}_l + \alpha'_l) (\rho_l - \rho_g) g \frac{\partial (\bar{h}_i + h'_i)}{\partial z} \\
& - (\bar{\alpha}_g + \alpha'_g) (\bar{\alpha}_l + \alpha'_l) \left(\rho_l \frac{\partial (\bar{h}_{g,l} + h'_{g,l})}{\partial h_i} - \rho_g \frac{\partial (\bar{h}_{g,g} + h'_{g,g})}{\partial h_i} \right) g \frac{\partial (\bar{h}_i + h'_i)}{\partial z} \\
& = -(\bar{\alpha}_g + \alpha'_g) (\bar{\tau}_l + \tau'_l) \frac{S_l}{A} + (\bar{\alpha}_g + \alpha'_g) (\bar{\tau}_i + \tau'_i) \frac{S_i}{A} \\
& + (\bar{\alpha}_l + \alpha'_l) (\bar{\tau}_g + \tau'_g) \frac{S_g}{A} + (\bar{\alpha}_l + \alpha'_l) (\bar{\tau}_i + \tau'_i) \frac{S_i}{A}
\end{aligned} \tag{4.36}$$

This equation is simplified in several steps: first, as the linear stability analysis focuses on the perturbations, the stable equation before perturbation (Equation 4.34) is subtracted from the perturbed equation. Then, as the flow is supposed to be fully developed, every differentiation on an average variable is assumed to be zero. The second order terms are finally neglected to get:

$$\begin{aligned}
& \rho_l \frac{\partial u'_l}{\partial t} + \rho_l \bar{u}_l \frac{\partial u'_l}{\partial z} - \rho_g \frac{\partial u'_g}{\partial t} - \rho_g \bar{u}_g \frac{\partial u'_g}{\partial z} \\
& - \sigma \frac{\partial^3 h'_i}{\partial z^3} + (\rho_l - \rho_g) g \frac{\partial h'_i}{\partial z} - \left(\rho_l \frac{\partial \bar{h}_{g,l}}{\partial h_i} - \rho_g \frac{\partial \bar{h}_{g,g}}{\partial h_i} \right) g \frac{\partial h'_i}{\partial z} \\
& = -\frac{\tau'_l}{\alpha_l} \frac{S_l}{A} + \frac{\tau'_i}{\alpha_l} \frac{S_i}{A} + \frac{\tau'_g}{\alpha_g} \frac{S_g}{A} + \frac{\tau'_i}{\alpha_g} \frac{S_i}{A}
\end{aligned} \tag{4.37}$$

To study the stability of the system, this equation is derived with respect to the spatial variable z to become:

$$\begin{aligned}
& \rho_l \left[\frac{\partial^2 u'_l}{\partial t \partial z} + \frac{\partial \bar{u}_l}{\partial z} \frac{\partial u'_l}{\partial z} + \bar{u}_l \frac{\partial^2 u'_l}{\partial z^2} \right] \\
& - \rho_g \left[\frac{\partial^2 u'_g}{\partial t \partial z} + \frac{\partial \bar{u}_g}{\partial z} \frac{\partial u'_g}{\partial z} + \bar{u}_g \frac{\partial^2 u'_g}{\partial z^2} \right] \\
& - \sigma \frac{\partial^4 h'_i}{\partial z^4} + (\rho_l - \rho_g) g \frac{\partial^2 h'_i}{\partial z^2} - \left(\rho_l \frac{\partial \bar{h}_{g,l}}{\partial h_i} - \rho_g \frac{\partial \bar{h}_{g,g}}{\partial h_i} \right) g \frac{\partial^2 h'_i}{\partial z^2} \\
& = \frac{\partial}{\partial z} \left(\tau'_g \bar{\alpha}_l \frac{S_g}{A} + \tau'_i \bar{\alpha}_l \frac{S_i}{A} - \tau'_l \bar{\alpha}_g \frac{S_l}{A} + \tau'_i \bar{\alpha}_g \frac{S_i}{A} \right) \\
& = \frac{\partial F'(\alpha'_l, u'_l, u'_g)}{\partial z}
\end{aligned} \tag{4.38}$$

where F' is equal to the perturbed part of F which is a writing simplification of the shear stress contributions:

$$\begin{aligned}
F &= -(\bar{\alpha}_g + \alpha'_g) (\bar{\tau}_l + \tau'_l) \frac{S_l}{A} + (\bar{\alpha}_g + \alpha'_g) (\bar{\tau}_i + \tau'_i) \frac{S_i}{A} \\
&+ (\bar{\alpha}_l + \alpha'_l) (\bar{\tau}_g + \tau'_g) \frac{S_g}{A} + (\bar{\alpha}_l + \alpha'_l) (\bar{\tau}_i + \tau'_i) \frac{S_i}{A} \\
F &= -\frac{\tau'_l}{\alpha_l} \frac{S_l}{A} + \frac{\tau'_i}{\alpha_l} \frac{S_i}{A} + \frac{\tau'_g}{\alpha_g} \frac{S_g}{A} + \frac{\tau'_i}{\alpha_g} \frac{S_i}{A}
\end{aligned} \tag{4.39}$$

The derivatives of the velocities v'_k are then expressed as derivatives on the interface level h'_i . Eliminating the fully developed terms present in the formulation

and simplifying the formulation, the equation becomes:

$$\begin{aligned}
& \left[\frac{\partial^2 h'_i}{\partial t^2} \right] \left[\frac{\rho_l}{\alpha_l} + \frac{\rho_g}{\alpha_g} \right] \\
& + \left[\frac{\partial^2 h'_i}{\partial z \partial t} \right] \left[2 \frac{\rho_l}{\alpha_l} \bar{u}_l + 2 \frac{\rho_g}{\alpha_g} \bar{u}_g \right] \\
& + \left[\frac{\partial^2 h'_i}{\partial z^2} \right] \left[\frac{\rho_l}{\alpha_l} \bar{u}_l^2 + \frac{\rho_g}{\alpha_g} \bar{u}_g^2 - (\rho_l - \rho_g) g \frac{dh_i}{d\alpha_l} - \left(\rho_l \frac{\partial \bar{h}_{g,l}}{\partial \alpha_l} - \rho_g \frac{\partial \bar{h}_{g,g}}{\partial \alpha_l} \right) g \right] \\
& + \left[\frac{\partial^4 h'_i}{\partial z^4} \right] \left[\frac{dh_i}{d\alpha_l} \sigma \right] \\
& - \left[\frac{\partial h'_i}{\partial t} \right] \left[\frac{\partial F}{\partial u_g} \frac{1}{\alpha_g} - \frac{\partial F}{\partial u_l} \frac{1}{\alpha_l} \right] \\
& - \left[\frac{\partial h'_i}{\partial z} \right] \left[\frac{\partial F}{\partial u_g} \frac{\bar{u}_g}{\alpha_g} - \frac{\partial F}{\partial u_l} \frac{\bar{u}_l}{\alpha_l} + \frac{\partial F}{\partial \alpha_l} \right] \\
& = 0
\end{aligned} \tag{4.40}$$

The complete set of perturbed equations has then been reduced to an equation depending on several derivatives of the interface level h'_i . A general form of this perturbation may then be assumed, as the general form of a wave equation (Equation 4.41) is:

$$\Delta \vec{E} = \frac{1}{v_p^2} \frac{\partial^2 \vec{E}}{\partial t^2} \tag{4.41}$$

where $\vec{E}$ denotes the amplitude and the polarization of the wave, v_p denotes the wave propagation velocity associated to the wave number k and the pulsation ω via $(v_p = \frac{\omega}{k})$. This becomes Equation 4.42 in a homogeneous, linear and isotropic one-dimensional field:

$$\frac{\partial^2 U}{\partial z^2} = \frac{1}{v_p^2} \frac{\partial^2 U}{\partial t^2} \tag{4.42}$$

Along the real z direction, the general solution is:

$$U(z, t) = f(kz - \omega t) + g(kz + \omega t) \tag{4.43}$$

The first term is a wave propagating along the positive z while the second propagates along the negative z . As the general formulation (D'Alembert's equation) 4.42 is linear, the solutions may be assumed to be sinusoidal for every position of the 1-D space in the propagation direction:

$$U(z, t) = A \cos(\phi - kz + \omega t) \tag{4.44}$$

$$U(z, t) = A_0 \exp^{i\phi} \exp^{i(\omega t - kz)} \tag{4.45}$$

This justifies the periodic form assumed for the perturbation of the interface level h'_i (4.46):

$$h'_i = \epsilon \exp^{i(\omega t - kz)} \tag{4.46}$$

Implemented in Equation 4.40, a dispersion equation is obtained:

$$\omega^2 - 2(\mathbf{a}k - \mathbf{b}i)\omega + \mathbf{c}k^2 - \mathbf{d}k^4 - \mathbf{e}ki \quad (4.47)$$

with the following coefficients:

$$\begin{aligned} \rho_{tot} &= \left[\frac{\rho_l}{\alpha_l} + \frac{\rho_g}{\alpha_g} \right] \\ \mathbf{a} &= \frac{1}{\rho_{tot}} \left[\frac{\rho_l}{\alpha_l} \bar{u}_l + \frac{\rho_g}{\alpha_g} \bar{u}_g \right] \\ \mathbf{b} &= \frac{1}{\rho_{tot}} \frac{1}{2} \left[\frac{1}{\alpha_l} \left(\frac{\partial F}{\partial u_l} \right)_{u_g, \alpha_l} - \frac{1}{\alpha_g} \left(\frac{\partial F}{\partial u_g} \right)_{u_l, \alpha_l} \right] \\ \mathbf{c} &= \frac{1}{\rho_{tot}} \left[\frac{\rho_l}{\alpha_l} \bar{u}_l^2 + \frac{\rho_g}{\alpha_g} \bar{u}_g^2 - (\rho_l - \rho_g) g \frac{dh_i}{d\alpha_l} - \left(\rho_l \frac{\partial \bar{h}_{g,l}}{\partial \alpha_l} - \rho_g \frac{\partial \bar{h}_{g,g}}{\partial \alpha_l} \right) g \right] \\ \mathbf{d} &= \frac{1}{\rho_{tot}} \left[\frac{dh_i}{d\alpha_l} \sigma \right] \\ \mathbf{e} &= -\frac{1}{\rho_{tot}} \left[\frac{\bar{u}_g}{\alpha_g} \left(\frac{\partial F}{\partial u_g} \right)_{u_l, \alpha_l} - \frac{\bar{u}_l}{\alpha_l} \left(\frac{\partial F}{\partial u_l} \right)_{u_g, \alpha_l} + \left(\frac{\partial F}{\partial \alpha_l} \right)_{u_g, u_l} \right] \end{aligned} \quad (4.48)$$

With the discriminant:

$$\Delta_i = 4(\mathbf{a}k - \mathbf{b}i)^2 - 4(\mathbf{c}k^2 - \mathbf{d}k^4 - \mathbf{e}ki) \quad (4.49)$$

The solutions are

$$\omega = (\mathbf{a}k - \mathbf{b}i) \pm \sqrt{(\mathbf{a}k - \mathbf{b}i)^2 - (\mathbf{c}k^2 - \mathbf{d}k^4 - \mathbf{e}ki)} \quad (4.50)$$

or, in polar formulation:

$$\begin{aligned} \omega_1 &= (\mathbf{a}k - \mathbf{b}i) + \sqrt[4]{\beta^2 + \gamma^2} \exp^{i\frac{1}{2} \arctan\left(\frac{\gamma}{\beta}\right)} \\ \omega_2 &= (\mathbf{a}k - \mathbf{b}i) + \sqrt[4]{\beta^2 + \gamma^2} \exp^{i\frac{1}{2} [\arctan\left(\frac{\gamma}{\beta}\right) + 2\pi]} \end{aligned} \quad (4.51)$$

with the coefficients β and γ :

$$\begin{aligned} \beta &= (\mathbf{a}^2 - \mathbf{c})k^2 + \mathbf{d}k^4 - \mathbf{b}^2 \\ \gamma &= \mathbf{e}k - 2\mathbf{a}b k \end{aligned} \quad (4.52)$$

The linear stability analysis provides then a relationship (Equation 4.51 or 4.50) between the angular frequency w and the wave number k of the perturbation.

Decoupling the angular frequency into a real component and a imaginary component such that: $\omega = \omega_R + i\omega_I$, the interface level perturbation h'_i gets the following formulation:

$$\begin{aligned} h'_i &= \epsilon \exp^{i((\omega_R + i\omega_I)t - kz)} \\ &= \epsilon \exp^{i(\omega_R t - kz) - \omega_I t} \\ &= \epsilon \exp^{i(\omega_R t - kz)} \exp^{-\omega_I t} \end{aligned} \quad (4.53)$$

The first term corresponds to the solution of the wave equation while the second term denotes the amplification of this solution. Indeed, for every ω_I which makes this term bigger than 1, so for every ω_I smaller than zero, the wave will be amplified with time (and then in space as demonstrated following Micaelli, in De Crecy [44]).

$$\forall \omega_I < 0 \Rightarrow \text{Unstable solution} \quad (4.54)$$

However, the amplitude of the amplification factor $\exp^{-\omega_I t}$ does not enable to determine the growth velocity of the instability. Indeed, first this would depend on the arbitrary initial amplitude of the perturbation ϵ as the growth velocity is proportional to the product of the initial perturbation amplitude by the amplification factor:

$$v_{\text{growth}} \propto \epsilon (-\omega_I) \quad (4.55)$$

Second, once the instability is initiated, the linear stability analysis no longer applies as non-linear effects become increasingly important so that the amplification factor is no longer representative of the growth velocity.

The wave amplification factor is then equal to $-\omega_I$. Figure 4.3 presents a typical example of solution of this linear stability analysis, for a given set of flow conditions, calculated with the proposed coefficients achieved from this proposed linear stability analysis. The figure shows that for the wave propagating in the flow directions ($\omega_R \geq 0$), every $\omega_R \geq 20$ has an unstable ω_I so that the corresponding wavelength is going to grow in time and space.

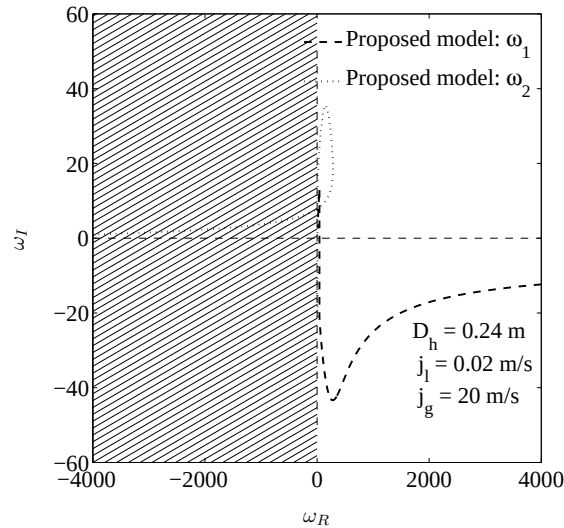


Figure 4.3: Example of solutions of the linear stability analysis.

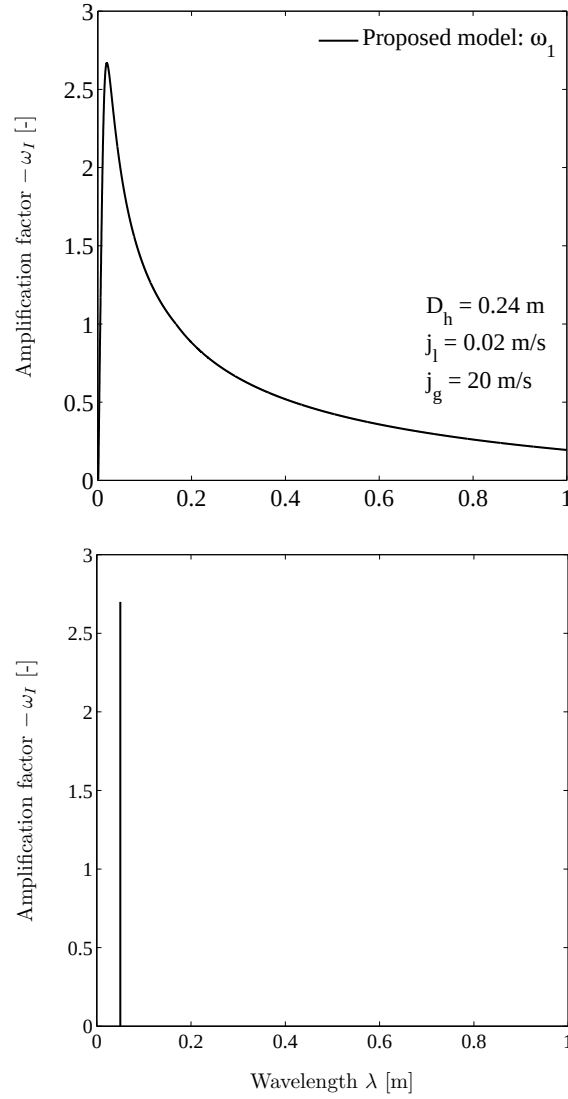


Figure 4.4: Associated solutions, of Figure 4.3, presenting the spectrum of amplification factors for a large range of wavelengths (top) and the assumed simplified amplification factor spectrum used in the wavelength model (bottom).

Figures 4.3 and 4.4 highlight that a large range of the perturbation spectrum presents a positive amplification factor ($\omega_I < 0$). Thus waves in a large range of wavelengths, are amplified at the interface. The major assumption made is that this spectrum of amplified wavelengths can be reduced to the only wavelength corresponding to the greatest amplification factor. The other wavelengths, that would grow slower, are assumed to be overcome by the most amplified wavelength. Every wavelet developing over the interface is then assumed to have the

same critical wavelength λ_c^2 .

This result is consistent with the usual linear stability analysis of the literature [21, 20, 16, 38, 53, 62, 61]. However, in a first hand, as the closure laws differ between this model and the referred literature studies, and in a second hand, as usual models apply more severe assumptions to the proposed equation resolution, the final coefficients in Equation 4.48 may differ from the literature coefficients, either considerably (compared to Guo et al. [62, 61]) or moderately (compared to Barnea and Taitel [21], see Appendix B.3.4, Equations B.85).

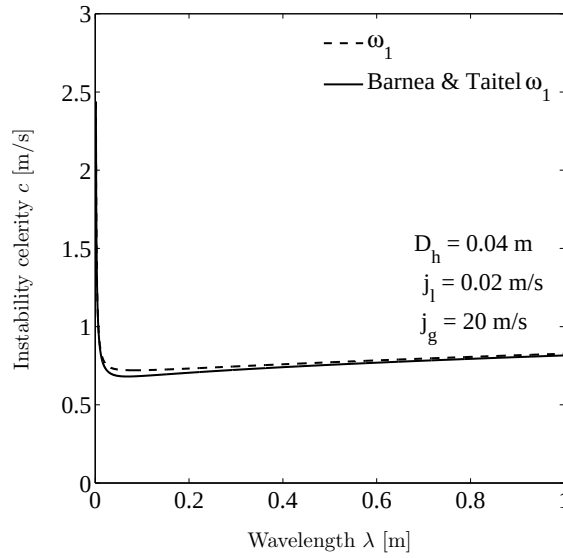


Figure 4.5: Evolution of the instability celerity as a function of the instability wavelength.

Figure 4.5 presents the evolution of the wave celerity c associated to the instability as a function of its wavelength λ . This graph shows that for every wavelength which will be amplified the wave celerity is quite uniform and a little smaller than 1 m/s whatever the proposed linear stability analysis formulation or the Barnea and Taitel formulation.

Additional modeling

Several additional models from the literature have been tested on the linear stability analysis. For example, Brauner and Moaron [27] have added a modi-

²This critical wavelength corresponds to the wave number k which satisfies the equation $\frac{\partial(-\omega_I)}{\partial k} = 0$. However there are no analytic solution to this equation so that the maximum of the amplification factor must be numerically approached.

fication on the interfacial shear stress to account for the memory effect of the wave shape by increasing the shear stress at the windward side and relax it at the downward side. In the same time, they considered the use of shape factors in the governing equations of the linear stability analysis [26] followed in that way by Guo et al. [62, 61]. However, if these additional modelings are presented as improving the determination of the boundary between the various flow regimes, their effect in the implementation in the present linear stability analysis cannot be validated. Indeed, the experimental assessment is not accurate enough to determine if their effect is positive or negative.

4.3 Sub-model of the entrained volume at the wave crest

The previous section aims at describing the linear stability analysis providing the most amplified wavelength of the interfacial wave spectrum: λ_c . This section provides thus one of the three major parameters of the entrainment rate model. This third section presents the methodology followed to calculate the entrained volume of liquid at the crest of an interfacial wave of wavelength λ_c .

4.3.1 Force balance at the wave crest

The underlying physical behaviour for the droplet entrainment is believed to be the development of Kelvin-Helmholtz instabilities as observed by Woodmansee and Hanratty [166]. In general, this class of instabilities involves the interplay between various types of forces as described by Brennen [30]:

- o A buoyancy force due to gravity and proportional to the difference in densities of the two fluids. In horizontal stratified flow, this force is stabilizing.
- o A surface tension force that is always stabilizing regardless of the type of flow.
- o A Bernoulli effect that implies a change in the pressure acting on the interface caused by a change in velocity resulting from the wave growth. To illustrate this effect, Figure 4.6 shows the streamlines of the two-phase flow around the interface. Obviously, the amount of distortion of the streamlines decreases with increasing the distance from the interface.
 ‘As the gas flows around the curves of the streamlines, centrifugal forces are set up and these must be balanced by a pressure gradient in the direction normal to the streamline’ ; according to Hewitt and Hall-Taylor [69]. At the wave crests (surface of positive curvature), the pressure at the interface will be lower than the pressure at infinity. Thus the gas flowing past the wavy interface generates a suction pressure over the crest tending to cause the wave distortion. This effect is always destabilizing. The Bernoulli effect is particularly important in the destabilization of the stratified flow leading to the transition to slug flows [94, 106].
- o A viscous effect of skin friction at the interface between the two fluids. ‘Because of the viscous effects in the boundary layer near the interface, inertia is insufficient to overcome the horizontal gradient of pressure generated by the succession of crests and troughs and separation takes place on the leeward side of the wave. This separation causes the streamlines

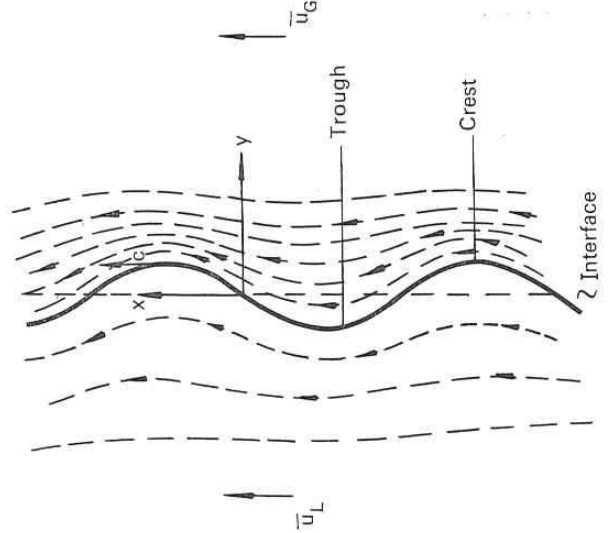


Figure 4.6: Streamlines in a two-phase flow with wavy interface from Hewitt and Hall-Taylor [69].

further from the interface to be displaced. As a consequence, a normal stress in phase with the wave slope is set up, there being an increased pressure on the windward side of the crest and a reduced pressure on the leeward side' [69].

The modeling of the entrained volume at the crest of a wave is developed on the basis of a force balance over the wave, similarly to Holowach et al. in vertical annular flow [71]. The modeling of a force balance acting on a wave crest in order to characterize a pipe flow has been primarily presented in 1975 by Ishii and Grolmes [75] in vertical flow in order to determine the inception of the entrainment phenomenon. Their model takes into account a drag force and a surface tension force in a vertical force balance parallel to the flow direction. The strategy has been developed later by Holowach [71] in 2002 in annular vertical flows. Their modeling incorporates the gravity in a vertical force balance in the flow direction and greatly improves the sub-modeling of the wave geometry and the acting forces. However various basic assumptions from Ishii and Grolmes [75] are conserved in the model such as the quantification of the wave amplitude. A recent attempt to adapt this strategy to horizontal and inclined pipe has been carried out by Mantilla in 2008 [101]. Following the previous studies, the entrainment inception and the entrained mass of liquid are modeled

based on a force balance in the flow direction. Only the drag force and the surface tension are considered in horizontal pipes, as the gravity is normal to the force balance. Neglecting the stabilizing effect of the gravity on the wave fragmentation seems questionable in horizontal flows.

In the proposed model, the wave fragmentation is assumed to occur when the vertical resultant of the forces acting on a ripple wave crest changes from vertical downward to upward direction, e.g. passing through the equilibrium situation. The forces included in the model are the gravity $F_{g,y}$, the surface tension $F_{\sigma,y}$, the drag $F_{d,y}$ and the lift forces $F_{l,y}$. Compared to reference [71], a lift and a gravity forces are thus added, and the components of these forces have different orientations.

As a consequence of the assumed orientation of the forces, the proposed model is only valid for stratified flows in horizontal pipes where the liquid film is maintained in the bottom of the pipe and does not suit to annular or transitional flows in which the circumferential development of the liquid film could be important. The force balance in the y direction when the conditions are fulfilled to pull-up a certain part of the wave into droplets is:

$$F_{D,y} + F_{L,y} + F_{g,y} + F_{\sigma,y} = 0 \quad (4.56)$$

where these different forces are detailed below in Equations 4.61 ($F_{D,y}$), 4.62 ($F_{L,y}$), 4.73 ($F_{g,y}$) and 4.77 ($F_{\sigma,y}$) and depicted in Figure 4.7.

This approach completely differentiates the present model from the reference models of Ishii and Grolmes [75], Holowach [71] and Mantilla [101] which consider the force balance in the flow direction z , following Equation 4.57:

$$F_{D,z} + F_{L,z} + F_{g,z} + F_{\sigma,z} = 0 \quad (4.57)$$

In vertical flows, for Holowach et al. [71], the lift force component is zero so that:

$$F_{D,z} + F_{g,z} + F_{\sigma,z} = 0 \quad (4.58)$$

However, in horizontal flow, for Mantilla [101], both the gravity and the lift forces are normal to the z direction so that the force balance reduces to:

$$F_{D,z} + F_{\sigma,z} = 0 \quad (4.59)$$

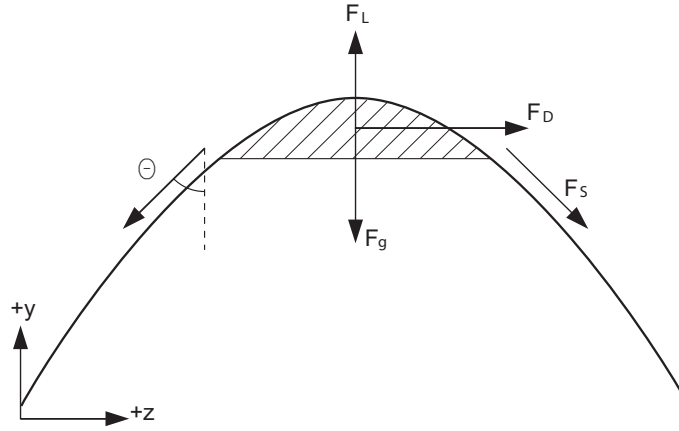


Figure 4.7: Force balance at the crest of a three-dimensional sine ripple wave geometry (Figure from Hewitt and Hall-Taylor [69]).

Wave geometry

A wave geometry must be characterized for a force balance on the wave crest. At first, the wave is assumed to take a sinusoidal shape, being a three-dimensional sine body as depicted in Figure 4.8 (top). The wave equation is therefore:

$$y(x, z) = h \left(\sin \left(\frac{2\pi x}{\lambda_c} \right) + \sin \left(\frac{2\pi z}{\lambda_c} \right) \right) \quad (4.60)$$

This geometry highly simplifies the quantification of the geometry. Every parametric geometry is more easily depicted based on a three-dimensional sine wave geometry. However the three-dimensional sine wave geometry seems to be a restrictive assumption both in terms of force formulation than in terms of force directions. The wave is continuously deformed to be not symmetric around the wave crest top. The slope is quasi vertical downstream to the wave top, and consequently greater than upstream. The deformed wave geometry takes a wave equation similar to Equation 4.67. Finally, the three-dimensional wave geometry deforms until becomes a quarter of an ellipsoid of characteristic parameters equal to h , $\lambda/4$ and $\lambda/2$. The wave deformation is depicted in Figure 4.8 (from top to bottom).

The parametrization of the three-dimensional sine wave geometry depends on three geometrical dimensions: the wavelength λ , the amplitude h and the location of the wave fragmentation characterized by a correlating dimension b , as depicted in Figure 4.10 (in the plane $y-x$ (top) and $y-z$ (bottom)). The assumed wave Equation 4.60 implies that the geometry in the plane $y-x$ and $y-z$ are identical.

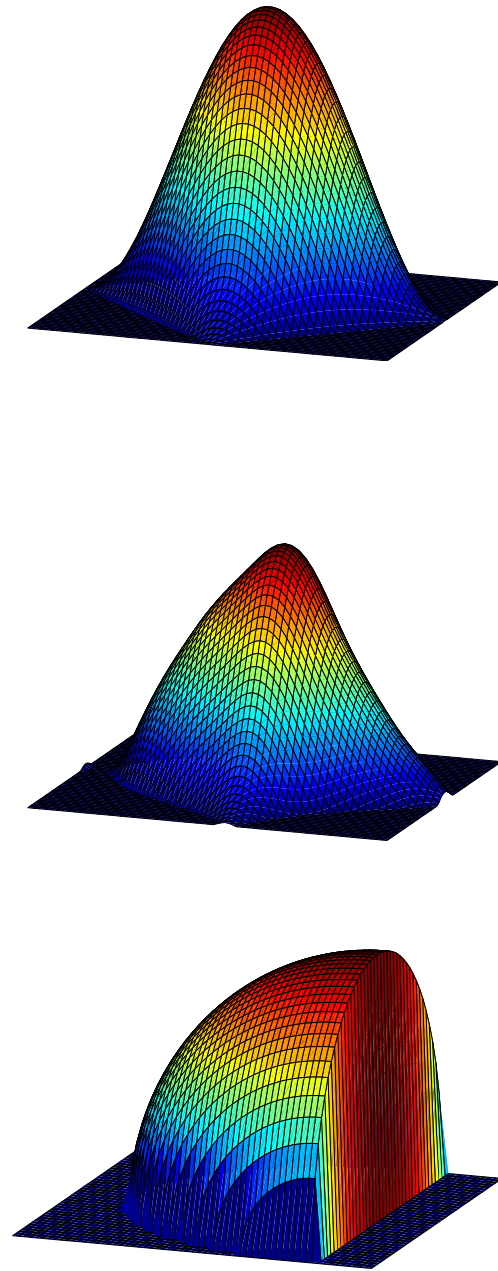


Figure 4.8: Three-dimensional sine wave geometry (top), deformed three-dimensional sine wave geometry (center) and quarter-ellipsoid wave geometry (bottom).

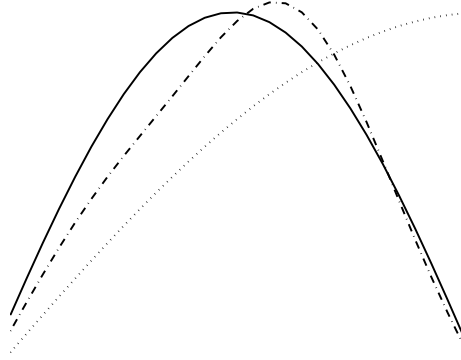


Figure 4.9: Sine wave geometry in the $y - z$ plane and assumed deformed geometry.

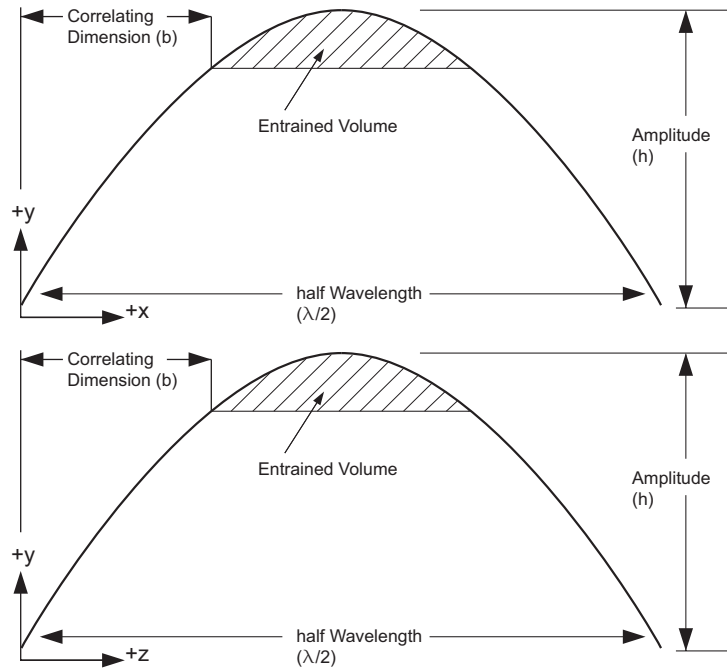


Figure 4.10: Main geometric parameters describing a three-dimensional sine ripple wave geometry: λ , h and b in the planes $y - x$ (top) and $y - z$ (bottom).

The location of the fragmentation is characterized by the correlating dimension b which corresponds to the distance from the wave crest basis ($y = 0$, $z = 0$) to the wave crest fragmentation. This parameter characterizes the proportion of the wave which is entrained by the air flow.

Wave crest lift/drag force calculation

The lift force denotes the Bernoulli effect on the wave geometry while the drag force represents the force opposed to the motion of the wave in the gas. The lift/drag forces (Equation 4.61 and 4.62 in horizontal flows) are usually calculated by giving a lift/drag coefficient C_L/C_D , using the relative velocity between the ripple wave crest and the gas core ($v_g - v_w$), and by estimating the effective area A_D and A_L , on which the drag force and the lift force act by a surface integration. In horizontal stratified flows, the vertical drag component is zero while the horizontal lift component is zero:

$$\begin{aligned} F_{D,y} &= 0 \\ F_{D,z} &= 0.5 \rho_g C_D A_D (v_g - v_w)^2 \end{aligned} \quad (4.61)$$

$$\begin{aligned} F_{L,y} &= 0.5 \rho_g C_L A_L (v_g - v_w)^2 \\ F_{L,z} &= 0 \end{aligned} \quad (4.62)$$

Many correlations could be used for evaluating the drag coefficient C_D . For instance, Holowach et al. [71] and Mantilla [101] used a drag coefficient (Equation 4.63) based on the interfacial area analysis of Kataoka et al. [81]. In their analysis, Kataoka et al. [81] related the drag coefficient on the wave crest to the Weber number based on a relation between the interfacial shear stress τ_i and the drag force and correlated on several experimental data [162, 36, 96]:

$$C_D = 286 Re_{lf}^{1/6} Re_g^{-2/3} \left(\frac{\rho_g}{\rho_{lf}} \right)^{1/3} \left(\frac{\mu_g}{\mu_l} \right)^{-2/3} \quad (4.63)$$

In a first step, following the method of Holowach et al. [71] and Mantilla [101], the area normal to the flow on which the drag force acts is estimated equivalent to the projection of the three-dimensional sine wave on the $y-x$ plane passing through the wave top. As the wave is assumed to have a three-dimensional sine topology, the equation of the sine wave in this $y-x$ plane is given in terms of the amplitude h and the wavelength λ :

$$y = h \sin \left(\frac{2\pi x}{\lambda_c} \right) \quad (4.64)$$

The surface integration of Equation 4.64 gives the drag area $A_E = A_E(h, b, \lambda)$ used by Holowach et al. [71] and Mantilla [101] as a function of the three geometric parameters:

$$\begin{aligned} A_D(h, b, \lambda) &= \int_b^{\frac{\lambda}{2}-b} h \sin \left(\frac{2\pi z}{\lambda} \right) dz - h \sin \left(\frac{2\pi b}{\lambda} \right) \left(\frac{\lambda}{2} - b \right) \\ &= \frac{h\lambda}{\pi} \cos \left(\frac{2\pi b}{\lambda} \right) - \left(\frac{h\lambda}{2} - 2hb \right) \sin \left(\frac{2\pi b}{\lambda} \right) \end{aligned} \quad (4.65)$$

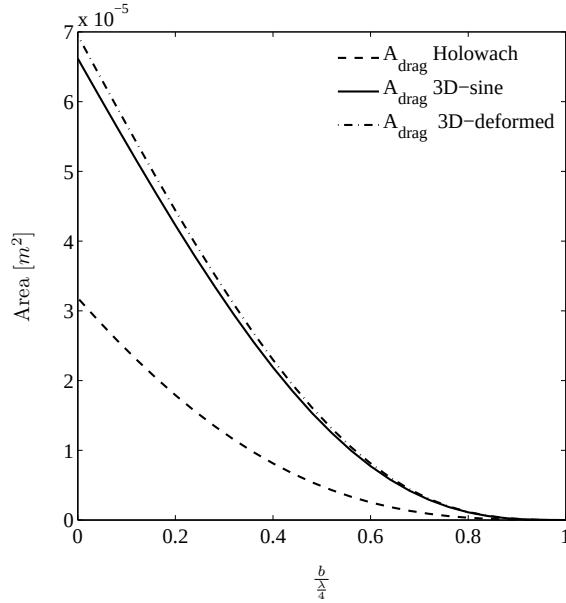


Figure 4.11: Evolution of the drag areas as a function of the correlating dimension b for the present model with the three-dimensional sine wave geometry (Equations 4.64 in dashed line and 4.71 in solid line) and a deformed wave geometry (Equation 4.67 in dotted and dashed line), Holowach's model [71] (Equation 4.65) similar to Mantilla's model [101] at a fixed wave geometry (h, λ).

In a second more precise step, the areas on which the drag and lift forces act are estimated by a surface area of revolution of the cosine function around the wave top from $x = b$ to $x = \frac{\lambda}{4}$ (completely described in Appendix B.5):

$$\begin{aligned}
 A_D(h, b, \lambda) &= 0.5 \int_b^{\lambda/4} 2\pi y(x) \sqrt{1 + y'(x)^2} dx \\
 &= -0.5 \frac{h\lambda}{2} \left[\cos\left(\frac{2\pi x}{\lambda}\right) \sqrt{1 + \left(\frac{2\pi h}{\lambda} \cos\left(\frac{2\pi x}{\lambda}\right)\right)^2} \right]_b^{\lambda/4} \\
 &\quad + \left[\frac{\operatorname{arcsinh}\left(\frac{2\pi h}{\lambda} \cos\left(\frac{2\pi x}{\lambda}\right)\right)}{\frac{2\pi h}{\lambda}} \right]_b^{\lambda/4} - 2\pi \left(\frac{\lambda}{4} - b\right) h \sin\left(\frac{2\pi b}{\lambda}\right)
 \end{aligned} \tag{4.66}$$

Finally the surface area of revolution is applied to the functions characterizing the deformation of the sine-wave geometry, defined with a form similar to Equation 4.67:

$$y = \frac{\pi}{4} + 0.32 \sin\left(-\frac{\pi}{6} + \frac{2\pi Z}{\lambda}\right)^4 \times \sin\left(\frac{2\pi z}{\lambda}\right) \tag{4.67}$$

The simplified (Holowach et al. [71] and Mantilla [101]) and the precise sur-

face (Present Model) integrations are compared in Figure 4.11. This figure shows large discrepancies between the two approaches. However, the difference between the three-dimensional sine wave geometry and the three-dimensional deformed wave geometry are tenuous as the wave deformation little affects the area normal to the flow.

Lift force calculation

The lift coefficient C_L is more complicated to determine, as the modeling of the Bernoulli effect is rarely done in the literature. The Bernoulli effect is assumed based on the calculation of the pressure gradient at the interface:

$$-\frac{dp}{dn} = \rho_g K_{crest} \cos(\theta) (v_g - v_w)^2 \quad (4.68)$$

where K_{crest} denotes the local curvature of the streamlines and θ the local angle of the tangent at the wave function. An integration from the wave interface to infinity gives:

$$F_{L,y} = \rho_g A_L K_{crest} \cos(\theta) (v_g - v_w)^2 \quad (4.69)$$

The lift force is finally written in a similar form than the drag force:

$$F_{L,y} = 0.5 \rho_g C_L A_L (v_g - v_w)^2 \quad (4.70)$$

with the lift area A_L defined similarly to the drag area A_D :

$$\begin{aligned} A_L(h, b, \lambda) &= \int_b^{\lambda/4} 2\pi y(x) \sqrt{1 + y'(x)^2} dx \\ &= -\frac{h\lambda}{2} \left[\cos\left(\frac{2\pi x}{\lambda}\right) \sqrt{1 + \left(\frac{2\pi h}{\lambda} \cos\left(\frac{2\pi x}{\lambda}\right)\right)^2} \right]_b^{\lambda/4} \\ &\quad + \left[\frac{\operatorname{arcsinh}\left(\frac{2\pi h}{\lambda} \cos\left(\frac{2\pi x}{\lambda}\right)\right)}{\frac{2\pi h}{\lambda}} \right]_b^{\lambda/4} - 2\pi \left(\frac{\lambda}{4} - b\right) h \sin\left(\frac{2\pi b}{\lambda}\right) \end{aligned} \quad (4.71)$$

The drag/lift forces are finally functions of the gas density ρ_g , the flow conditions (phase and wave velocities) but mainly on the wave geometrical parameters.

Figure 4.15 presents the evolution of the lift area with the correlating dimension b . For the three-dimensional sine wave geometry, the lift area is exactly twice the drag area. The ratio slightly differs when the wave is deformed even is the difference in terms of area during the deformation remains tenuous.

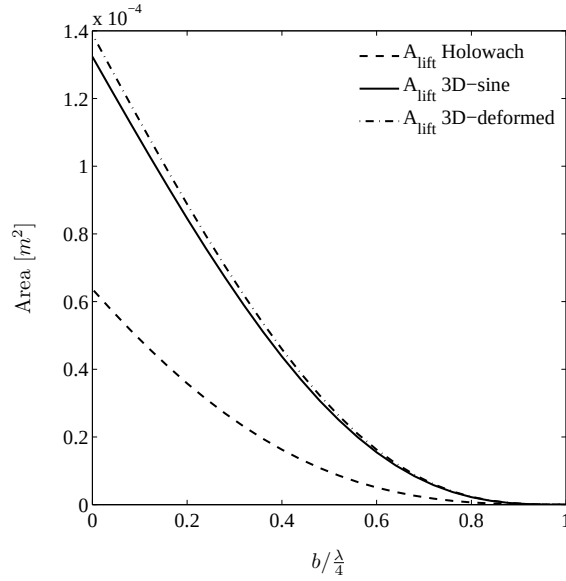


Figure 4.12: Evolution of the lift areas as a function of the correlating dimension b for the present model, Holowach's model and Mantilla's model at a fixed wave geometry (h, λ) applying the same equations than in Figure 4.11.

Wave crest gravitational force calculation

The gravitational force acting on the wave crest is expressed as:

$$F_{g,y} = - \int \int \int \rho_w(x, y, z) g F(x, y, z) dx dy dz \quad (4.72)$$

where $F(x, y, z) = 0$ denotes the shape function of the wave crest and ρ_w the local density of the wave crest. Equation 4.72 simplifies into Equation 4.73 by assuming that the wave crest density is homogeneous and corresponding to the liquid density:

$$F_{g,y} = -\rho_l g V_E \quad (4.73)$$

where the entrained volume $V_E = V_E(h, b, \lambda)$ on which the gravitational force acts depending on the wave geometry. Assuming in a first step the three-dimensional sine wave geometry, the entrained volume V_E is defined by using the shell integration method³ depicted in Appendix B.5, assuming the cosine form of the wave equation:

$$y(z) = h \cos\left(\frac{2\pi z}{\lambda}\right) \quad (4.74)$$

³This shell integration method on a cosine function provides a more simple equation of the entrained volume than the disc integration method on an arc-cosine function as used by Holowach et al. [71] and Mantilla [101].

$$\begin{aligned}
V_E(h, b, \lambda) &= \int_0^{\frac{\lambda}{4}-b} 2\pi z y(z) dz \\
&= \frac{\lambda^2 h}{2\pi} \left(\frac{2\pi(\frac{\lambda}{4}-b)}{\lambda} \sin\left(\frac{2\pi(\frac{\lambda}{4}-b)}{\lambda}\right) + \cos\left(\frac{2\pi(\frac{\lambda}{4}-b)}{\lambda}\right) - 1 \right) \\
&\quad - \left(h \sin\left(\frac{2\pi b}{\lambda}\right) \pi \left(\frac{\lambda}{4} - b\right)^2 \right)
\end{aligned} \tag{4.75}$$

Figure 4.13 presents the evolution of the entrained volume at the wave top. As first sight, the shell integration method leads to a more realistic result than the disc integration method used by Holowach et al. [71] and Mantilla [101]. Nevertheless, the deformation of the three-dimensional geometry to a quarter of ellipsoid has a little influence on the entrained volume. The entrained volume slightly increases with the deformation until reaching the volume entrained at the wave crest of a three-dimensional half-ellipsoid (a maximum difference of 37%).

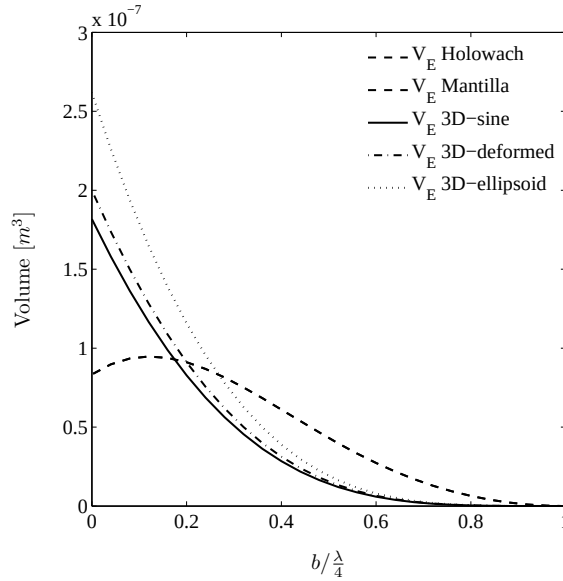


Figure 4.13: Evolution of the entrained volume at the wave crest as a function of the correlating dimension b for the present model and Holowach's model at a fixed wave geometry (h, λ) .

Wave crest surface tension force calculation

The surface tension acting on the wave crest is given by:

$$F_\sigma = - \int_{h(b)}^{h_{max}} \int_0^{2\pi} \sigma(h, \psi) K_{crest}(h, \psi) R(h, \psi) d\psi dh \tag{4.76}$$

In the force balance along the vertical axis, the vertical component of the surface tension is:

$$F_{\sigma,y} = -F_{\sigma} \sin(\theta) \quad (4.77)$$

where θ is the tangent to the slope at the fragmentation. In a first step, following the approach of Holowach et al. [71] and Mantilla [101] in order to calculate the surface tension that acts to maintain the wave crest geometry and resists the entrainment, several assumptions are made to enhance the calculation of the wave crest curvature and the effective area on which applies the surface tension. In this first step, the surface tension force is expressed as:

$$F_{\sigma,y} = -\sigma A_{\sigma} K_{\text{crest}} \sin(\theta) \quad (4.78)$$

where σ the surface tension is considered constant all over the wave crest, K_{crest} is the average curvature at the crest area and A_{σ} is the effective area on which the surface tension acts. To evaluate these two geometrical parameters necessary for the surface tension modeling, several assumptions are made:

- o For Holowach et al. [71] and Mantilla [101], the effective area on which the surface tension acts is equivalent to the top projection of the wave crest, a disc of radius $(\frac{\lambda}{4} - b)$. Nevertheless as the wave crest is stretched during the wave deformation, the area in which the entrained wave crest maintains contact with the rest of the wave may decrease by an arbitrary factor k_{σ} so that:

$$A_{\sigma}(b, \lambda) = \frac{\pi \left(\frac{\lambda}{4} - b\right)^2}{k_{\sigma}} \quad (4.79)$$

In its mechanistic model, Mantilla [101] considers a factor k_{σ} equal to 1 so that no deformation is assumed, while Holowach et al. [71] considers a factor k_{σ} equal to 4.

- o For Holowach et al. [71], the wave curvature is obtained by assuming the sine quantification of the wave crest so that:

$$\begin{aligned} K_{\text{crest}}(h) &= \frac{y''(z)}{(1 + (y'(z))^2)^{3/2}} \\ &= \frac{\left| -\frac{4\pi^2 h}{\lambda^2} \sin\left(\frac{2\pi z}{\lambda}\right) \right|}{\left(1 + \left(\frac{2\pi}{\lambda} \cos\left(\frac{2\pi z}{\lambda}\right)\right)^2\right)^{3/2}} \\ K_{\text{crest}} &= \frac{4\pi^2 h}{\lambda^2} \end{aligned} \quad (4.80)$$

The variation of the curvature along the wave height is not considered and the curvature is averaged to its local value at the wave top.

In a different approach, Mantilla [101] calculates the wave curvature based on its experimental correlations of wave amplitude and base wave width, applied to its force balance. However the application of these Mantilla's correlations does not fit the experimental data from literature and have not been considered in the present work.

- o The vertical projection of the surface tension is obtained by calculating the angle θ . The tangent of the angle θ is equal to the slope of the sine wave, which is determined by differentiating Equation 4.74 denoting the sine shape of the wave so that:

$$\begin{aligned} y &= h \sin\left(\frac{2\pi z}{\lambda}\right) \\ \frac{dy}{dz} &= \frac{2\pi}{\lambda} h \cos\left(\frac{2\pi z}{\lambda}\right) \end{aligned} \quad (4.81)$$

At the fragmentation position of the wave crest ($z = b$), the angle θ equals then:

$$\theta = \arctan\left(\frac{2\pi}{\lambda} h \cos\left(\frac{2\pi b}{\lambda}\right)\right) \quad (4.82)$$

- o In the present model, the vertical resultant of the surface tension at the wave crest fragmentation has been enhanced in two steps. First, assuming the three-dimensional sine wave geometry, the curvature is assumed constant along the circumference: $K_{crest}(h, \psi) = K_{crest}(h)$. Then by integrating $K_{crest}(h)$ from the wave top to the fragmentation the local curvature, the average curvature is determined. In the same time, the area on which the surface tension is exerted is determined following the surface integration of the wave crest. In a second step, assuming the three-dimensional deformed wave geometry, the surface tension force is obtained from the direct integration of Equation 4.83:

$$\begin{aligned} F_{\sigma, y} &= - \int_{h(b)}^{h_{max}} \int_0^{2\pi} \sigma K_{crest}(h) R(h) \sin(\theta) d\psi dh \\ &= - \int_{h^* \sin(\frac{2\pi b}{\lambda})}^{h^*} \sigma \frac{\left| -\frac{4\pi^2 h}{\lambda^2} \sin\left(\frac{2\pi z}{\lambda}\right) \right|}{\left(1 + \left(\frac{2\pi}{\lambda} h \cos\left(\frac{2\pi z}{\lambda}\right)\right)^2\right)^{3/2}} 2\pi \left(\frac{\lambda}{4} - b\right) \times \dots \\ &\quad \sin\left(\arctan\left(\frac{2\pi}{\lambda} h \cos\left(\frac{2\pi b}{\lambda}\right)\right)\right) dh \end{aligned} \quad (4.83)$$

Figure 4.14 presents the discrepancies between the local curvature of the wave crest at the wave top and the effective evolution along the height of the wave. The solid line presents the evolution of the averaged curvature from the wave top to the fragmentation. As a direct consequence of these

discrepancies, the amplitude of the surface tension forces may be highly different between the Mantilla's model, Holowach's model and the present model.

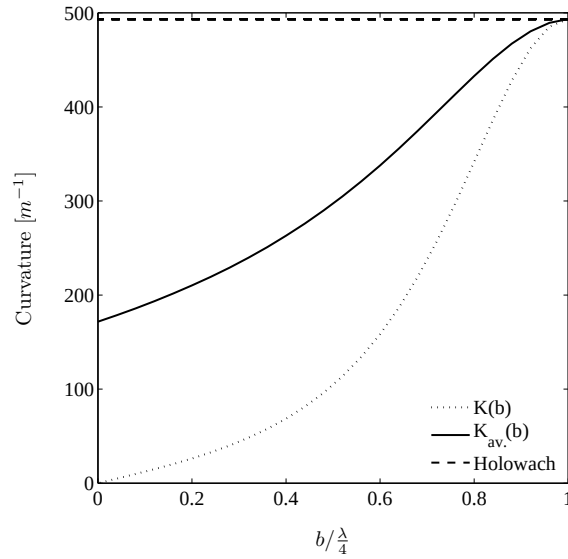


Figure 4.14: Evolution of the curvature as a function of the correlating dimension b for the present model and Holowach's model. At a fixed wave geometry (h, λ) , the local curvature $K(b)$ and the averaged curvature from the wave top to b : K_{av} , are plotted.

Figure 4.15 presents the discrepancies in the area on which the surface tension is exerted between the literature models and the present model. Again, the three models present a considerable difference in terms of area while the two three-dimensional geometries little differ.

- o However, due to the various forces acting on the wave, the wave crest deforms in its developing process leading to its fragmentation. The original sine function evolves thus into a complicated function. The methodology is thus finally applied to an assumed wave geometry to enhance the modeling of the surface tension.

Force balance at the wave crest

The geometrical parameters of these forces are expressed as functions of three parameters: h the amplitude of the wave at the onset of fragmentation, b a geometrical parameter corresponding to the horizontal distance between the base of the wave and the location of the fragmentation, and λ the wavelength of

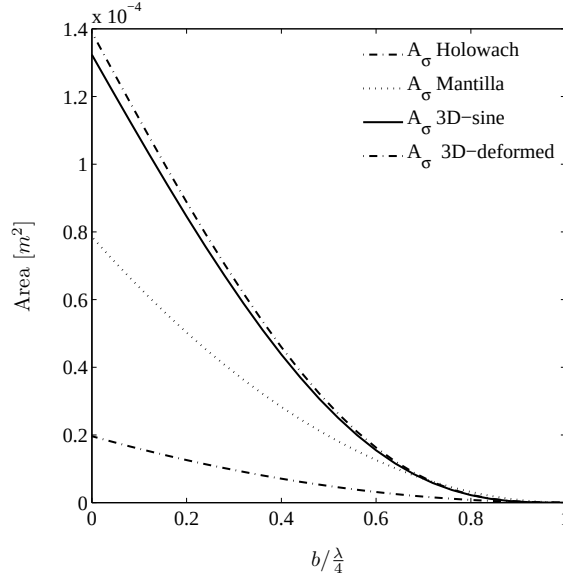


Figure 4.15: Evolution of the surface tension areas as a function of the correlating dimension b for the present model, Holowach's model and Mantilla's model at a fixed wave geometry (h, λ) .

the ripple. The sum of the various forces exerted on the crest of the wave is a function of these three parameters. By knowing the amplitude h at the onset of fragmentation, the entrained volume is found out by simple iterations on the size b until the force balance is checked. This requires to know the evolution of the amplification factor up to the onset of the entrainment phenomenon.

Figure 4.16 presents an example of the force balance evolution depending on the correlating dimension b at a fixed wave geometry (λ, h) . This graph shows that the force balance passes through positive to negative at the critical amplitude $h_c = h \sin\left(\frac{2\pi b_c}{\lambda}\right)$ which characterizes the eventual fragmentation position for this wave geometry.

The force balance in the direction of the flow in an horizontal flow, along the z -axis, is unachievable in the three-dimensional sine wave geometry, although this is performed by Mantilla [101]. Indeed, the assumption of the sine geometry imposes the horizontal component of the surface tension to zero to this direction so that only the drag force acts in the horizontal force balance. The force balance is thus always negative.

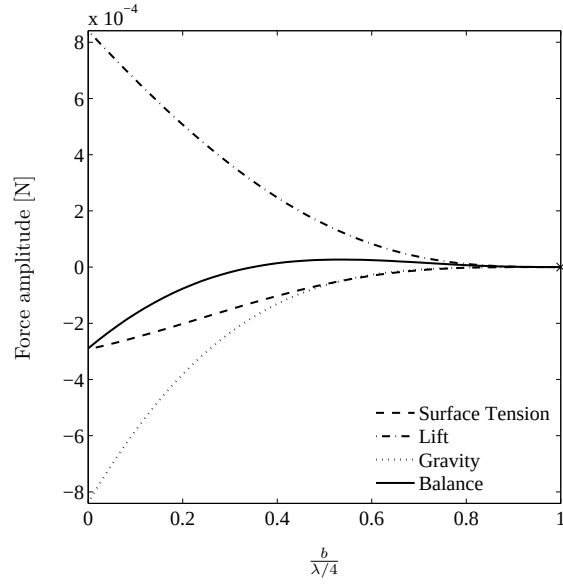


Figure 4.16: Example of the evolution of the various forces exerting in the vertical y -direction on the wave crest as a function of the correlating dimension b at a fixed wave geometry.

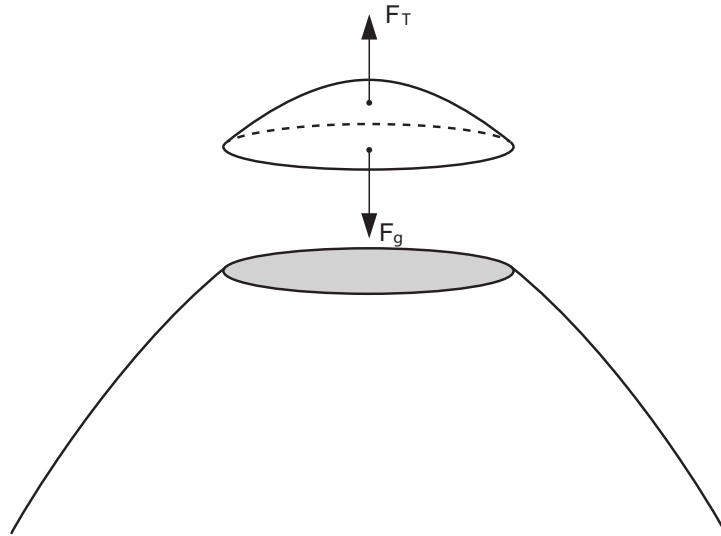


Figure 4.17: Force balance on the entrained volume of liquid.

4.3.2 Force balance on the entrained volume

A second force balance is applied on the entrained amount of liquid after separation to determine whether the turbulent effect overcomes the gravity effect

and enables the entrainment of the entrained volume as several droplets. The force balance implies then a gravity force $F_{g,d}$ and a turbulent force F_T as in Equation 4.84, depicted in Figure 4.17:

$$F_{g,d} + F_T = 0 \quad (4.84)$$

The gravity force is applied on the amount of liquid entrained wrested off from the wave crest:

$$F_{g,d} = -\rho_l g V_T \quad (4.85)$$

The entrained volume of droplets V_T , assumed to be an half-ellipsoid, is then equal to the entrained volume from the wave crest V_E so that the radius of the entrained volume of droplets R_E is equal to:

$$V_T = \frac{2}{3} \pi R_T^2 (h - h_f) = V_E \quad \rightarrow \quad R_T = \left(\frac{3}{2} \frac{V_E}{\pi (h - h_f)} \right)^{1/2} \quad (4.86)$$

where h_f denotes the wave amplitude at which the wave fragments. The turbulent effect applies as a vertical force, in a formulation similar to a drag/lift force so that:

$$F_T = C_{D_T} \rho_g A_T (v_T - v_d)^2 \quad (4.87)$$

where C_{D_T} denotes the drag coefficient of the entrained volume of liquid including the suspension effect and the drag effect, A_T the area of the entrained volume of liquid on which the drag applies, v_d the vertical component of droplet velocity (assumed to be zero at the droplet cration) and v_T the turbulent velocity. In a first step, the drag coefficient C_{D_T} is modeled following Kataoka et al. [80] on the basis of a relation between the critical Weber number from the droplets entrained from wave crests and their drag coefficients. This relation comes from the force balance on the wave crest ligaments:

$$C_{D_T} = \frac{8}{We_c} \quad (4.88)$$

An expression of the drag coefficient as a function of the Reynolds numbers of the gas and the liquid and of the density and viscosity ratios is achieved and correlated on various experimental data [162, 36, 96]:

$$C_{D_T} = 286 Re_{lf}^{1/6} Re_g^{-2/3} \left(\frac{\rho_g}{\rho_{lf}} \right)^{1/3} \left(\frac{\mu_g}{\mu_l} \right)^{-2/3} \quad (4.89)$$

In a second step taking into account the non-sphericity of the entrained volume of liquid, the drag coefficient is assumed equal to the drag coefficient of an immersed hemisphere body presenting its plane face at the flow:

$$C_{D_T} = 1.2 \quad (4.90)$$

The entrained volume velocity along the vertical direction may be expressed based on a modeling of the turbulent kinetic energy of the gas or vapor k_∞ , assuming that the turbulence is isotropic in the air field.

$$v_T = \sqrt{\frac{2}{3} k_\infty} \quad (4.91)$$

The intensity of the turbulent kinetic energy is assumed to be identical to the case of the horizontal pipe for fully developed flow, uniform and unidimensional flows, expressed following Chandesris et al. [33]:

$$k_\infty = 0.0367 v_g^2 Re_g^{-1/6} \quad (4.92)$$

where Re_g denotes the Reynolds number based on the hydraulic diameter and the bulk velocity. Finally the area on which the drag force applies may then be expressed as the surface of an hemisphere of radius R_T

$$A_T = \pi R_T^2 \quad (4.93)$$

Figure 4.18 presents an example of the force balance on the half-sphere of liquid resulting from the fragmentation of the wave crest.

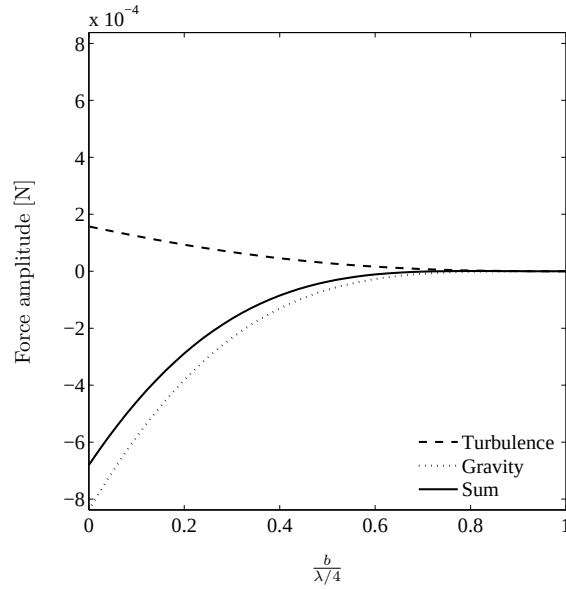


Figure 4.18: Example of the evolution of the forces exerting in the vertical y-direction on the half-sphere of liquid entrained at the wave crest at a fixed wave geometry.

4.3.3 Determination of the wave amplitude at the fragmentation

For every instability wavelength λ_c , each force balance provides a the axial position of the wave fragmentation as a function of the wave amplitude. Indeed at a given wavelength, the force balance solutions show that the volume fraction of the wave that will be sheared-off varies with the wave amplitude. For each of the n force balances, the solution of the amplitude at which the wave fragments ($h_{f,n}$) is a function of the wave parameter $b_{c,n}$:

$$h_{f,n} = h \sin \left(\frac{2\pi b_{c,n}}{\lambda_c} \right) \quad (4.94)$$

Each force balance is therefore characterized by the couple $(b_{c,n}, h_{f,n})$. The two equations 4.95 depending on the wave amplitude h and the position of the wave fragmentation b_c are summarized here after:

$$f_1(\lambda_c, h_{f,1}, b_{c,1}) = 0 \quad f_2(\lambda_c, h_{f,2}, b_{c,2}) = 0 \quad (4.95)$$

So as:

$$b_{c,1} = f(h_f) \quad b_{c,2} = f(h_f) \quad (4.96)$$

Determining the amplitude of the wave at which fragmentation occurs is carried out by determining the wave amplitude h_f at which the two force balances (Equation 4.56 and Equation 4.84 summarized as 4.96) simultaneously pass from a stabilizing effect to a destabilizing effect. This corresponds to the wave amplitude at which the two fragmentation position are equals:

$$b_{c,1}(h_f) = b_{c,2}(h_f) \quad (4.97)$$

where $b_{c,n}(h_f)$ denotes the evolution of the critical correlating dimension b_c at which the n force balance passes by zero as a function of the wave amplitude h . This criterion is illustrated in Figure 4.19 which presents an example of the evolution of the critical correlating dimension $b_{c,n}$ at which the direction of each force is reversed, as a function of the wave amplitude h at a fixed instability wavelength λ .

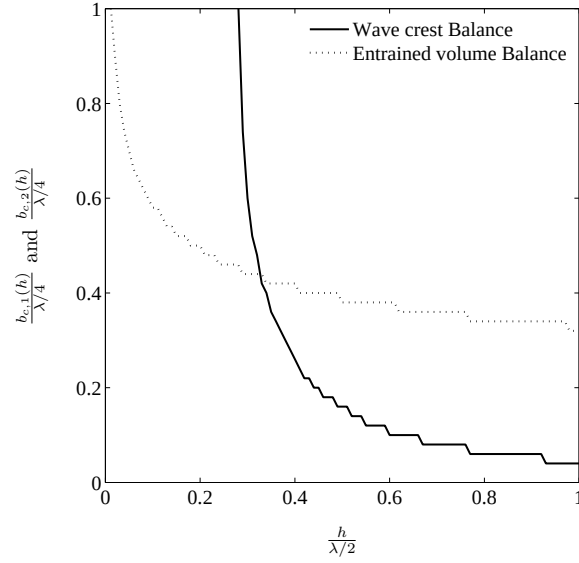


Figure 4.19: Determination of the wave amplitude at which the wave fragments.

4.4 Characteristic time τ_C of the entrainment rate Γ_E

The characteristic time of the entrainment rate τ_C is defined as the time required for a wavelet to develop until reaching an unstable amplitude at which the fragmentation occurs. The characteristic time is therefore related to the ratio between the wave amplitude and the wave growth velocity at this amplitude. Theoretically, at the inception of the instability, the instability has an infinitesimal amplitude ϵ (Equation B.82) and begins to exponentially grow in space and time. The slope of the time growth is characterized by the amplification factor $(-\omega_I)$ given by the linear stability analysis. However, once the wave amplitude has a little increased, the linear stability analysis is no longer valid and the amplification factor changes due to non-linear effects. From the initial exponential slope, the growth velocity decreases due to the stabilizing effects (gravity, surface tension, ...) on the instability. Consequently, the amplification factor given by the linear stability analysis is not suitable for determining the evolution of the wave growth velocity with the wave amplitude.

Furthermore, the literature about the wave growth in annular or stratified flows is very poor. Indeed, if few authors had tried to predict the amplitude of stable roll waves at a small air velocity [64, 31, 105, 13] or to correlate experimental measurements of wave amplitude [75, 101, 70], none of them has

proposed a correlation predicting the ripple-wave growth at a gas-liquid interface. Actually, the wave development is still an understudied field.

In this work, the characteristic time of the entrainment rate τ_C is the time required for a wave to develop until reaching the fragmenting amplitude. This point highly differentiates this work from Holowach's model [70] which estimates the period of the entrainment phenomenon as equal to the time scale that the gas core passes over the liquid film. Mantilla [101] considers the characteristic time of the entrainment rate as equal to the inverse of the wave frequency.

The characteristic time of the entrainment rate is therefore defined as:

$$\tau_C = \frac{h_{max}}{\bar{v}_{gr.}} \quad (4.98)$$

where h_{max} denotes the wave amplitude h at the fragmentation, while $\bar{v}_{gr.}$ denotes the average growth velocity of the instability from the instability inception to the fragmentation. The wave amplitude at the fragmentation is determined by the two force balances (Equations 4.56 and 4.84) while the wave growth velocity $v_{gr.}$ is experimentally estimated from the wave amplitude variations measured on REGARD.

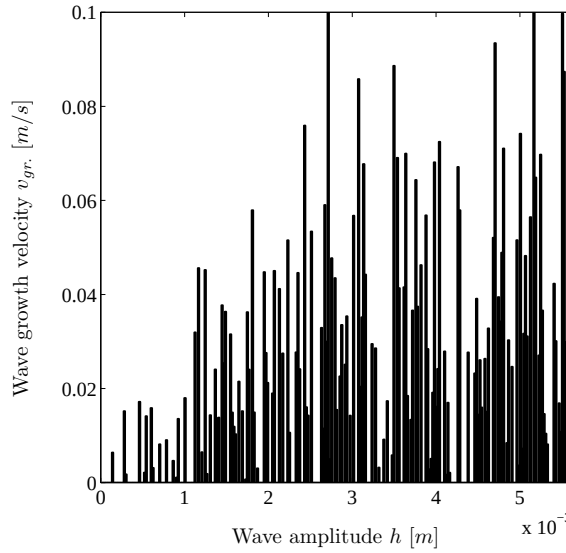


Figure 4.20: Evolution of the growth velocity with the wave amplitude based on REGARD experimental data.

Figure 4.20 presents the evolution of the wave growth velocity as a function of the wave amplitude: $v_{gr.}(h)$. Although the evolution is rather noisy, the

wave growth velocity is assumed to linearly increase with the wave amplitude by a factor $\xi = \xi(j_g, j_l)$. The slope of this augmentation is depicted in Figure 4.21. This second graph highlights that the ξ factor linearly increases with the air superficial velocity j_g but appears constant between the two tested liquid superficial velocity j_l ($j_l = 0.018 \text{ m/s}$ and $j_l = 0.037 \text{ m/s}$) and independent of the axial location of the measuring devices.

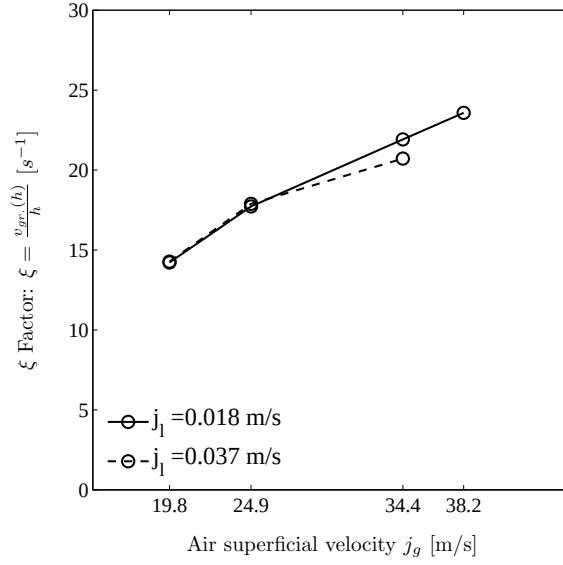


Figure 4.21: Evolution of the factor $\xi(j_g, j_l)$ with the superficial air velocity j_g and the liquid superficial velocity j_l , based on experimental results from REGARD.

As the wave growth velocity is assumed to linearly increase with the wave amplitude, the average wave growth velocity during the instability development until fragmentation is:

$$\bar{v}_{gr.} = \frac{1}{(h_{max} - h_0)} \int_{h_0}^{h_{max}} \xi h dh = \frac{\xi (h_{max} - h_0)}{2} \quad (4.99)$$

Consequently, as the initial wave amplitude h_0 is zero, Equation 4.98 simplifies in Equation 4.100:

$$\tau_C = \frac{h_{max}}{\bar{v}_{gr.}} = \frac{2}{\xi} \quad (4.100)$$

The characteristic time of the entrainment phenomenon appears to slightly decrease with the air superficial velocity as depicted in Figure 4.22. Consequently the effect of the increase of the the air superficial velocity on the characteristic time of the entrainment is to increase the entrainment rate Γ_E (see Equation

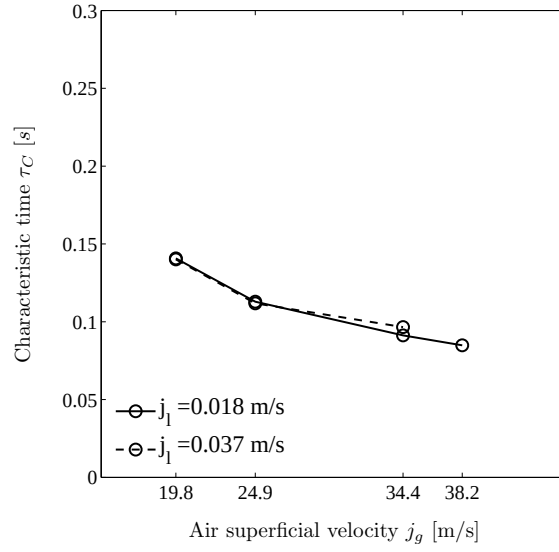


Figure 4.22: Evolution of the characteristic time based on experimental results from REGARD.

4.6). The characteristic time is finally assumed to be inversely proportional to the critical wavelength λ_c .

4.5 Proportion of waves fragmented κ

Every wave is assumed to fragment so that the proportion of waves fragmented κ is arbitrarily assumed to be one.

$$\kappa = 1 \quad (4.101)$$

Chapter 5

Model assessment

The bibliographic review, detailed in the Chapter 1.3, concluded that usual entrainment models are not suited to modeling of an horizontal flow in a pipe of large diameter such as in a hot leg of a PWR. Consequently, a new model has been developed in Chapter 4, based on a mechanistic approach of fluid dynamics leading to wave interface breakup and droplet entrainment. This model, based on the wave formation at the interface and the growth to the fragmentation process, requires different sub-models, detailed in the previous chapter.

In addition, a literature review on the existing experimental facilities has shown that the existing data do not totally suit to the validation of the developed model. Thus, the new experimental facility REGARD of greater size has been designed in the Chapter 2 and many flow measurements have been conducted in Chapter 3 in order to provide a new database more suited to the modeling of a stratified flow in a PWR hot leg.

Chapter 5 is dedicated to the assessment of the entrainment model against the REGARD experimental data. An introductory section is dedicated to the presentation of the model philosophy in which every step leading to the determination of the entrainment rate is summarized. As this architecture is based on the assembly of various sub-models (critical wavelength, entrained volume, ...), the second section is devoted to the evaluation of each sub-model concerning the entrainment rate model. First, as the constitutive equations of fluid mechanics appear in most sub-models, an evaluation of the closure laws of interfacial and parietal shear stresses is performed: the liquid film fraction predicted by the flow balance of the two-fluid model in a fully developed flow is compared to the REGARD experimental data. Afterwards, the sub-model dedicated to the critical wavelength of the instabilities is compared to the REGARD experimental

data over the averaged interfacial instability frequencies.

The calculation of the entrained volume of liquid at the crest of a ripple-wave is not assessable against the experimental data whereas the characteristic time of the entrainment has been experimentally developed based on the REGARD facility data. The last assessment is thus directly the evaluation of the entrainment rate model.

Finally, a correlation for the droplet entrainment rate is derived from a database obtained by applying the entrainment rate model to a large set of flow conditions (flow velocities, flow characteristics, pipe geometry,...).

5.1 Sub-model assembly

Figure 5.1 presents the philosophy of the sub-model assembly leading to the determination of the entrainment rate. The input bases of most sub-models are the flow conditions (j_g, j_l), the pipe diameter (D_h) and the two-fluid physical properties (density, viscosity, surface tension).

The first sub-model calculates the interfacial and parietal shear-stresses based either on the Barnea and Taitel's formulation [20, 147], on the CATHARE formulation [49] or on the Andritsos and Hanratty's formulation [154]. On the basis of the two-fluid equilibrium, the liquid film fraction α_l and the void fraction α_g are determined in a second sub-model. The liquid film fraction, which may be separately imposed, is used in a geometrical sub-model calculating the geometric dimensions, e.g. the liquid level, the interfacial length, the liquid hydraulic diameter and the gas hydraulic diameter. All these flow characteristics constitute the inputs of the linear stability analysis which provides the critical wavelength necessary to the entrainment rate model. The entrained volume is determined on the basis of this critical wavelength, considering a wave lift coefficient and an entrained volume drag coefficient. The final sub-model of the characteristic time is experimentally derived from the REGARD data. The entrainment rate Γ_E is then determined from the inputs provided by the three main sub-models.

5.2 Sub-model assessment

This section is dedicated to the evaluation of the sub-modeling of the closure laws. In a first point, the closure laws of interfacial and parietal shear stresses are evaluated. However, the REGARD experimental data does not enable the

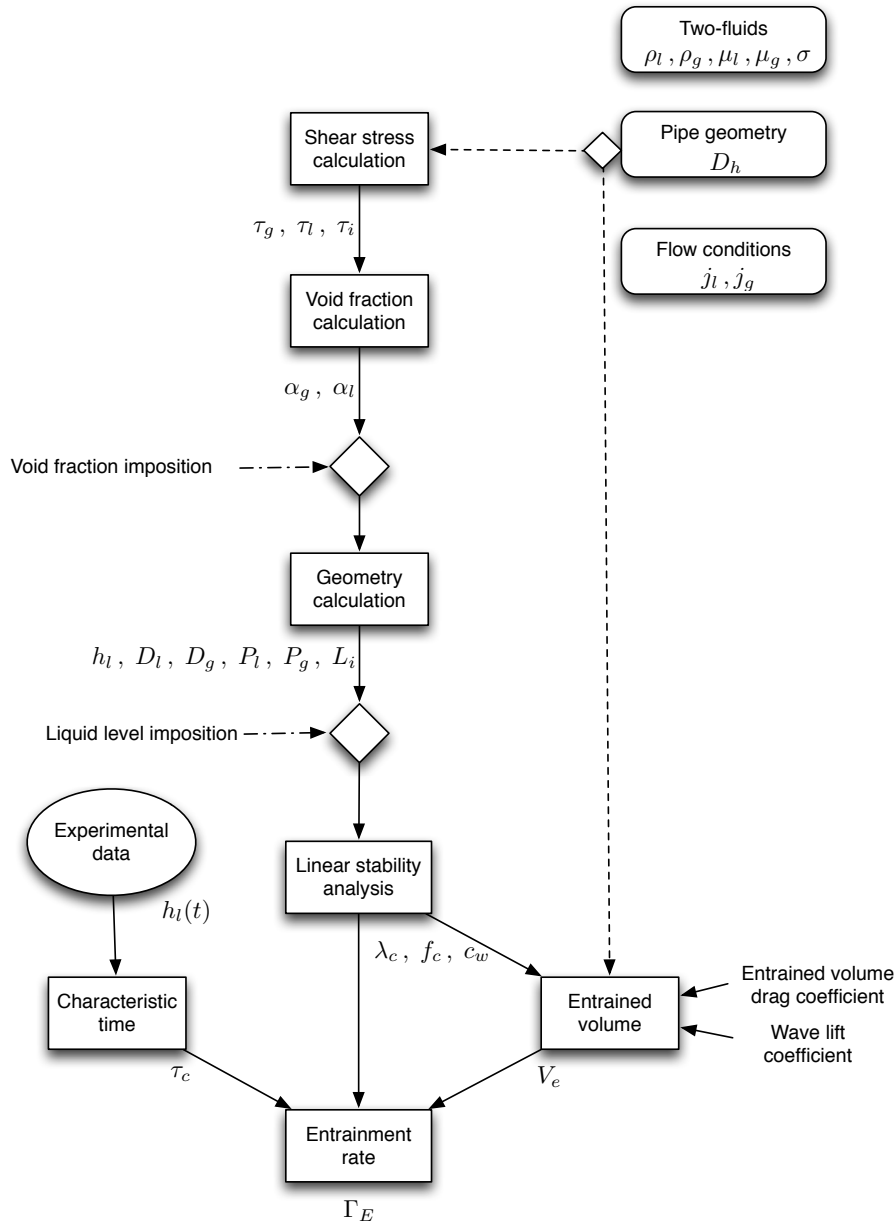


Figure 5.1: Methodology for the entrainment rate calculations.

evaluation of neither the pressure gradient formulation nor the interfacial pressure jump. These two closure laws must be then supposed valid for the stratified flow modeling and cannot be validated.

Afterwards the sub-models required for the entrainment rate modeling are

evaluated. The predictions from the linear stability analysis in terms of critical wavelength and critical frequencies are then evaluated against the experimental data on the interface characteristics from the BANJO device.

5.2.1 Closure laws: shear-stresses

To validate the selection of the shear-stress closure laws, the gas fraction α_g predicted by the two-fluid model with the various closure laws is compared to the experimental data from REGARD. To easily express the gas fraction as a function of the interfacial and parietal shear stresses, a fully-developed flow is assumed. A fully-developed two-fluid flow along the z-axis direction, without mass transfer and considering the two fluids incompressible and not expandable in a pipe of constant cross section (with no axial pressure losses) means that:

$$\frac{\partial \alpha_k}{\partial z} = 0 \quad (k = g, l) \quad (5.1)$$

Mass conservation equations

The general equations of mass conservation are derived from Equations 4.19:

$$\frac{\partial}{\partial z} (\alpha_l u_l) = 0 \quad (5.2)$$

$$\frac{\partial}{\partial z} (\alpha_g u_g) = 0 \quad (5.3)$$

Momentum conservation equations

Identically, the general equations of momentum conservation are then:

$$\rho_l \frac{\partial}{\partial z} (\alpha_l u_l^2) + \alpha_l \frac{\partial \langle p_l \rangle}{\partial z} = \frac{\chi_i \tau_i}{A} - \frac{\chi_l \tau_l}{A} \quad (5.4)$$

$$\rho_g \frac{\partial}{\partial z} (\alpha_g u_g^2) + \alpha_g \frac{\partial \langle p_g \rangle}{\partial z} = -\frac{\chi_i \tau_i}{A} - \frac{\chi_g \tau_g}{A} \quad (5.5)$$

Multiplying Equation 5.4 by the gas fraction α_g and Equation 5.5 by the liquid fraction α_l , and subtracting the two equations, one may write the cross-balance equation:

$$\begin{aligned} & \rho_l \alpha_g \frac{\partial}{\partial z} (\alpha_l u_l^2) - \rho_g \alpha_l \frac{\partial}{\partial z} (\alpha_g u_g^2) + \alpha_l \alpha_g \frac{\partial \langle p_l \rangle}{\partial z} - \alpha_l \alpha_g \frac{\partial \langle p_g \rangle}{\partial z} \\ &= \frac{\chi_i \tau_i}{A} - \alpha_g \frac{\chi_l \tau_l}{A} + \alpha_l \frac{\chi_g \tau_g}{A} \end{aligned} \quad (5.6)$$

As every derivative along the z-axis are zero as the flow is fully-developed, a single expression of the gas fraction is achieved for a fully-developed stratified flow in an horizontal pipe:

$$\alpha_g = \frac{\chi_i \tau_i + \chi_g \tau_g}{\chi_l \tau_l + \chi_g \tau_g} \quad (5.7)$$

This gas fraction prediction α_g depends then only of the formulation of the interfacial (τ_i) and parietal (τ_l , τ_g) shear stresses and of the interfacial geometry (χ_i , χ_l , χ_g). Figure 5.2 presents the evolution of the predicted gas fraction for the set of shear stress closure laws used by Barnea and Taitel [20], Taitel and Dukler [147], Emonot et al. on CATHARE 3 [49] and Tzotzi and Andritsos [154]. The model of Barnea and Taitel is presented in two different formulations: the standard formulation considers the Reynolds number in Equation 4.24 as:

$$Re_k = \frac{\rho_k D_k v_k}{\mu_k} \quad (5.8)$$

while the second simplified formulation (denoted Barnea & Taitel f(Dh) in the followings) expresses the Reynolds number as:

$$Re_k = \frac{\rho_k D j_k}{\mu_k} \quad (5.9)$$

This second formulation is less accurate but easier to implement. Every formulation of the interfacial and parietal shear forces conducts to a similar evolution of the gas fraction α_g . Increasing the air superficial velocity leads to an increase of the gas fraction. However, the fourth formulations slightly differ in terms of magnitude. The modifications made on the wall-to-liquid friction factor f_l and on the interfacial friction factor f_i by Tzotzi et Andritsos [154] in Equation 4.31 (Section 4.2.3) compared to the formulations of Barnea and Taitel [20] considerably change the predicted values of the gas fraction. Indeed, the gas fraction is very sensitive to the wall-to-liquid friction factor and to the interfacial friction factor.

To evaluate the predictions of the various models, Figure 5.3 shows then the evolutions with the air superficial velocity compared to experimental data from REGARD. The model presented by Tzotzi and Andritsos [154], based on Andritsos and Hanratty [15] shows the best agreement with the experimental data of REGARD. Similarly to Spedding and Hand [143] and Shi and Kocamustafogullari [140], Tzotzi and Andritsos model [154] modifying the ratio f_i/f_g (Equation 4.31) and the liquid-to-wall friction factor f_l is proved to be adequate to predict liquid film height in wavy stratified flows. Consequently Tzotzi and Andritsos model [154] has been preferred to Barnea and Taitel's model or CATHARE's model, and implemented in the two-fluid model for stratified flow and used in the frame of the linear stability analysis.

5.2.2 Linear stability analysis

The linear stability analysis provides the dispersion equation of the flow, a relation between the angular frequency ω and the phase number k of the instability

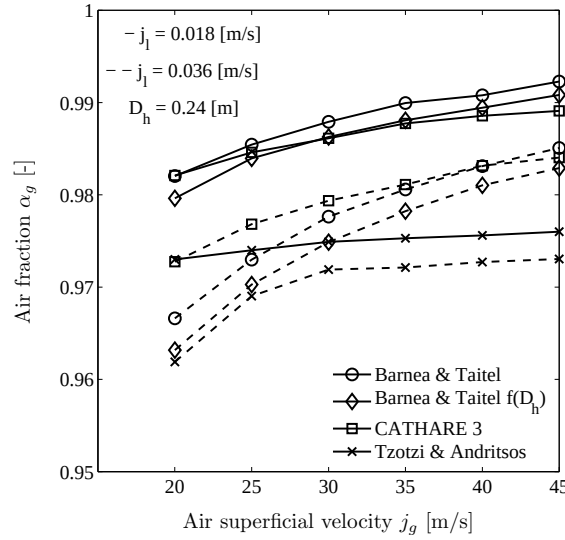


Figure 5.2: Evolution of the air fraction with the air superficial velocity for various set of shear stress closure laws.

spectrum. The ratio between the two parameters corresponds to the instability celerity c_i :

$$c_i = \frac{\omega}{k} \quad (5.10)$$

This provides the spectra of either the wavelengths or the frequencies of the instabilities. This section is firstly devoted to the evaluation of the linear stability results against the experimental data obtained from the BANJO. Afterwards, a sensitivity study is achieved according the following parameters:

- o the flow parameters: j_g , v_g , j_l , v_l , α_l
- o the pipe geometry: D_h
- o the friction factors magnitude: f_i , f_g and f_l
- o the two-fluid physical properties: ρ_l , ρ_g , μ_l , μ_g and σ .

Both the frequency and the wavelength of the instabilities have been experimentally determined from the time evolution of the interfacial waves. However, authors have a greater confidence in the determination of the instability critical frequencies f_c than in the instability wavelengths λ_c , which are indirectly determined. The assessment of the linear stability analysis is then primarily performed within the frequency spectrum.

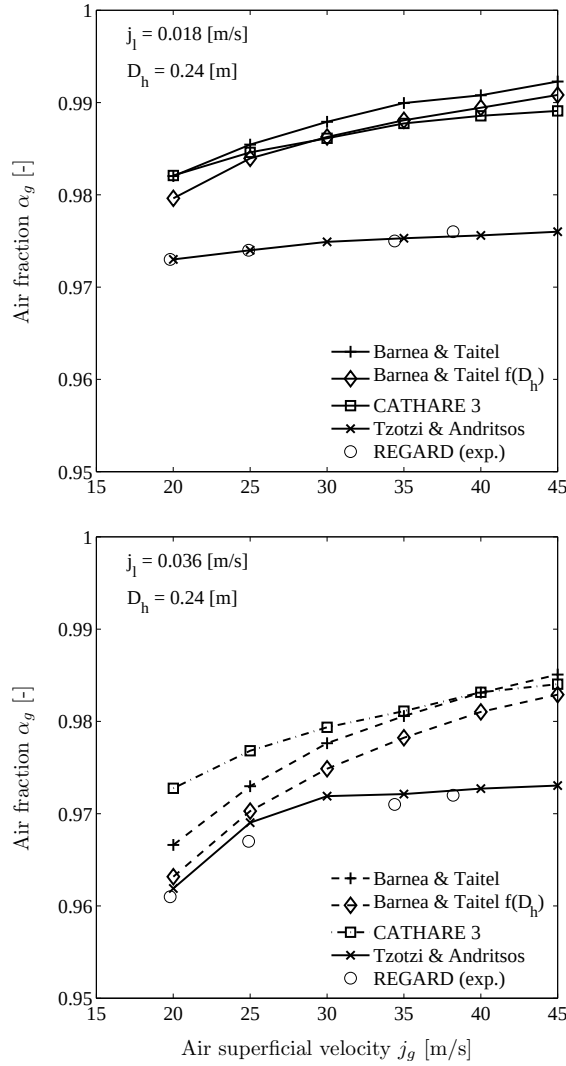


Figure 5.3: Comparaison of the evolution of the air fraction with the air superficial velocity for various sets of shear stress closure laws with experimental data from REGARD.

Critical instability frequency f_c

The linear stability analysis has been primarily implemented to provide a relationship between the angular frequency ω and the wave number k in order to find the amplification spectrum of the instabilities. Indeed, it has been shown that the imaginary part of the angular frequency ω_I is representative of the amplification factor. Consequently, the characteristics of the instability of the greatest amplification factor are determined: k_c and ω_c . The instability critical frequency and critical wavelength are then determined as:

$$\lambda_c = \frac{2\pi}{k_c} \quad , \quad f_c = \frac{\frac{\omega(c)}{k(c)}}{\lambda_c} \quad (5.11)$$

As a second order equation in ω , the dispersion equation presents two solutions to the linear stability analysis for every wave number k_i . An extensive study on the solution spectrum has shown that the common term to the two solutions is always negative (for the stratified set of equations), so that the two solutions are never together positive for a same wave number k . Depending on the flow conditions, the two solutions can be either negative whatever the wave number or partially positive for a range of wave numbers.

Figure 5.4 shows a good agreement between the experimental data and the theoretical results. The critical frequency appears to be quite stable around 60 Hz. In fact, this linear and constant evolution is due to the conjunction of the air velocity effect (which tends to increase the critical frequency at a constant liquid film fraction) and of the liquid film fraction effect (which tends to decrease the critical frequency at a constant air velocity). The two effects compensate each other so that the critical frequency appears constant. The good agreement with the experimental data must be cautiously considered as the accuracy of the REGARD critical frequency is about 10-15%.

Critical instability wavelength λ_c

The linear stability analysis predicts an instability frequency close to the experimental measurements. However, the wave celerity predicted by the linear stability analysis differs from the wave celerity deduced from the wave velocity. As a direct consequence, the evolution of the instability wavelength calculated from the experimental data is not well approached by the linear stability analysis as shown in Figure 5.5.

Figure 5.6 presents the evolution of the critical wavelengths λ_c predicted by the linear stability analysis as a function of the air and liquid superficial velocities at the REGARD two-fluid physical properties and characteristics and pipe geometry.

Figure 5.6 shows that the predicted critical wavelengths λ_c have an order of amplitude similar to the experimental data (0.02 m). However the critical wavelength λ_c decreases with the air superficial velocity while the experimental data show an increase. This is coherent with the entrainment rate model (Equation 4.6) and the experimental data, because decreasing the critical wavelength has a positive effect on the entrainment rate amplitude (without considering the

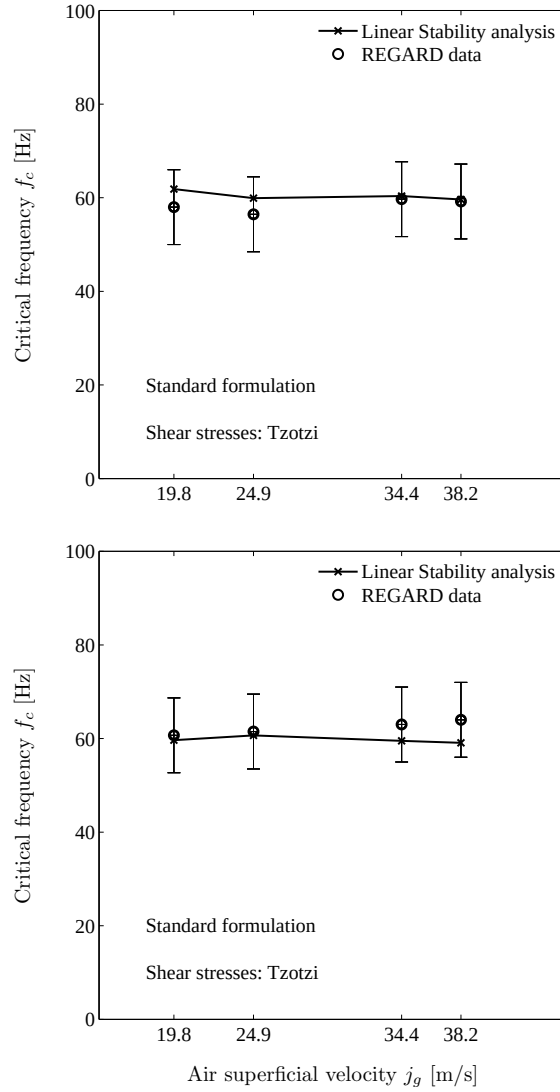


Figure 5.4: Evolution of the predicted ripple-wave critical frequencies from the linear stability analysis against experimental data from the BANJO instrument located at the largest studied $\frac{L}{D}$ ratio on the REGARD experiment for $j_l = 0.018$ m/s (top) and $j_l = 0.036$ m/s (bottom).

effect on the entrained volume). In fact, experimentally, as a consequence of the interfacial cross-sectional shape, the liquid velocity v_l is extremely heterogeneous over the liquid film interface. The space averaged liquid velocity $\langle \bar{v}_l \rangle$, calculated as the ratio between the liquid flow-rate $\dot{M}_l$ to the liquid film fraction α_{lf} , is therefore not representative of the local velocity of the liquid flow at the positions over the pipe cross-section where the interfacial wave velocities and frequencies are measured. Consequently the wave celerity, calculated from a

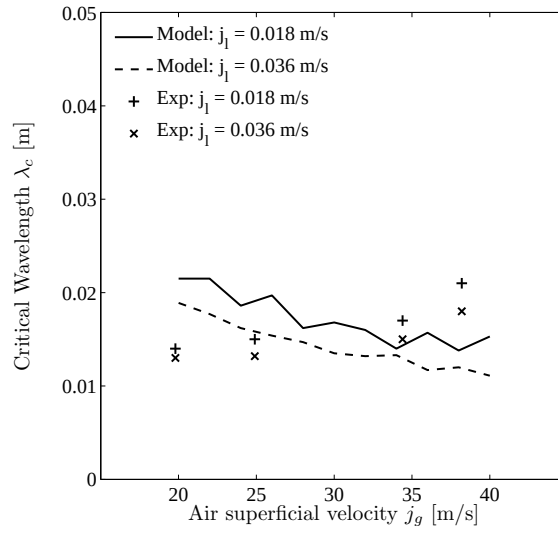


Figure 5.5: Comparison between the predictions of the linear stability analysis and the derived data from REGARD.

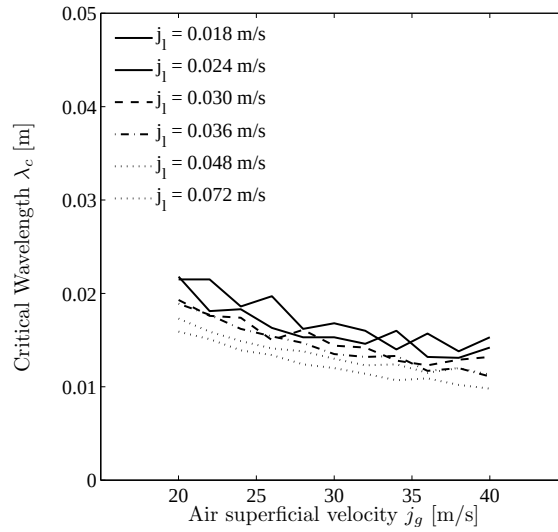


Figure 5.6: Evolution of the predicted critical ripple-wave wavelengths from the linear stability analysis.

space averaged liquid velocity $\langle \bar{v}_l \rangle$, does not correspond to the wave celerity predicted by the linear stability analysis.

Figure 5.6 shows thus a significant influence of the air superficial velocity j_g on the critical wavelength λ_c . The liquid superficial velocity j_l have a smaller but non negligible effect on the critical wavelength λ_c . Increasing the liquid

superficial velocity at a fixed air superficial velocity leads to an decrease of the critical wavelength.

Sensitivity to the liquid film fraction

In Figure 5.5, the critical wavelengths predicted by the linear stability analysis are compared to REGARD experimental data at similar flow conditions while the liquid film fraction α_l is deduced from Equation 5.7. Therefore, to examine the effect of the liquid film fraction α_l , Figure 5.7 presents its influence on the critical wavelength by showing the evolution of the critical wavelengths when the liquid film fraction is kept constant whatever the air superficial velocity. The liquid film fraction α_l appears to be a key parameter in the linear stability analysis. At a constant liquid film fraction, the critical wavelength linearly stays constant or suddenly increases with the air superficial velocity j_g . Increasing the liquid film fraction α_l (decreasing the void fraction α_g), follows the tendencies of the influence of the air and liquid superficial velocities. This indicates that the effect of the void fraction on the critical wavelength prediction can be assumed to be taken into account by the two-fluid velocity effects.

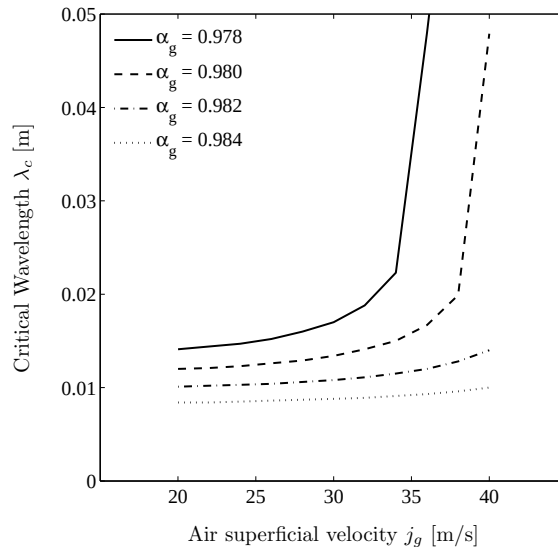


Figure 5.7: Influence of the liquid film fraction on the evolution of the critical wavelength λ_c when increasing the air superficial velocity j_g .

Sensitivity to the pipe geometry

Figure 5.8 presents the effect of the pipe diameter on the critical wavelength λ_c . Similarly to the evolution of the shear stresses with the pipe diameter, the critical wavelength is highly influenced by the pipe diameter. At a smaller pipe

diameter, the critical wavelength is smaller. Consequently the entrainment rate is proportionally higher at a smaller pipe diameter. The influence of the pipe diameter on the critical wavelength predictions is therefore consistent with the literature data (see Chapter 1.3).

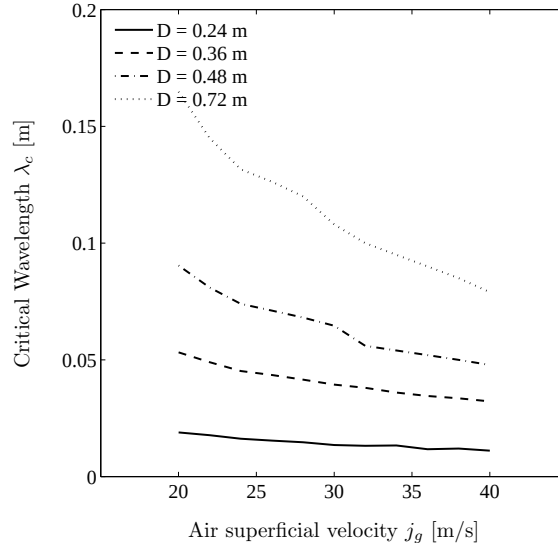


Figure 5.8: Influence of the pipe diameter on the evolution of the critical wavelength λ_c when increasing the air superficial velocity j_g .

Sensitivity to the shear-stress modeling

Figure 5.9 presents the influence on the shear-stress modeling on the prediction of the critical wavelength. The usual model of Barnea and Taitel [21] underpredicts these two friction factors compared to Tzotzi and Andritsos [154] and CATHARE-3 [49]. This graph shows thus the relative importance of the shear stress modeling. To investigate in details this influence, Figures 5.10 to 5.12 present the influence of the friction factors, separately modified, on the evolution of the critical wavelength. Figure 5.13 presents the influence of the surface tension on the critical wavelength.

The linear stability analysis appears to be extremely sensitive to the modeling of the liquid-to-wall friction factor f_l and the interfacial friction factor f_i in the modeling of the shear-stresses as depicted in Figures 5.10 and 5.12. Increasing these friction factors by a factor f_l^* or f_i^* induces a diminution of the critical wavelength. On the other side, the gas-to-wall friction factor f_g has quasi no influence on the critical wavelength as depicted in Figure 5.11.

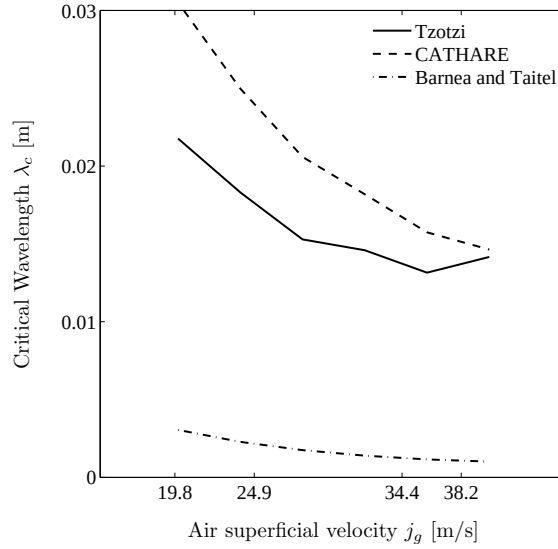


Figure 5.9: Influence of the shear-stress modeling on the evolution of the critical wavelength λ_c when increasing the air superficial velocity j_g .

Figure 5.13 presents the influence of the surface tension on the critical wavelength expressed via a critical Froude number Fr_λ based on the velocity difference.

$$Fr_\lambda = \frac{(v_g - v_l)}{g \lambda_c} \quad (5.12)$$

The amplitude of the surface tension σ is also an influent factor on the wave stability as increasing the surface tension leads to an augmentation of the critical wavelength. The influences of the liquid-to-wall friction factor f_l and of the interfacial friction factor f_i correspond to the influence of the two-fluid physical properties. Indeed Figures 5.14, 5.15, 5.16 and 5.17 which present the influence of the two-fluid characteristics on the critical Froude number, show that the friction factors follows the evolution of the densities and the viscosities at a fixed pipe diameter and two-fluid superficial velocities.

Synthesis on the parameters affecting the linear stability analysis

The linear stability analysis appears to predict a critical wavelength which is extremely sensitive to the air superficial velocity and the pipe diameter. The liquid superficial velocity has a smaller but non negligible influence on the critical wavelength.

The modeling of the closure laws is particularly important as the liquid-to-wall and the interfacial friction factors significantly acts on the critical wavelengths. One notices however that simultaneously increasing both friction fac-

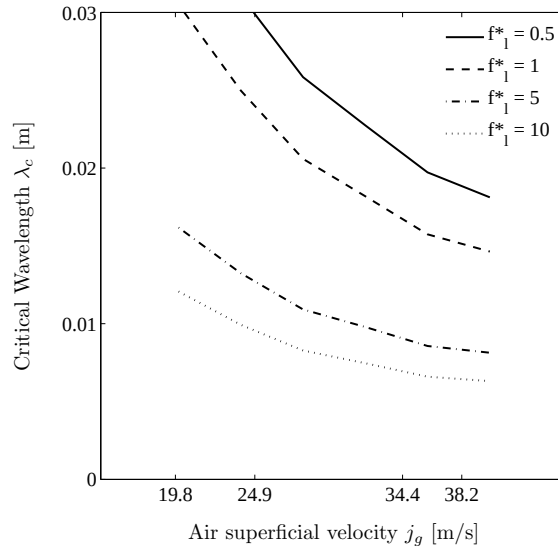


Figure 5.10: Influence of the liquid-to-wall friction factor on the evolution of the critical wavelength λ_c when increasing the air superficial velocity j_g .

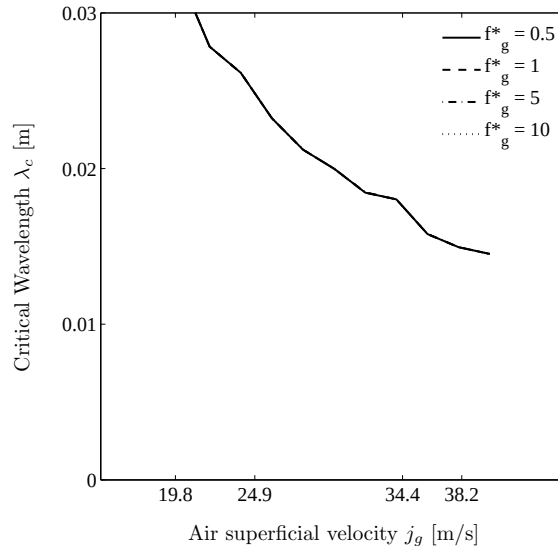


Figure 5.11: Influence of the gas-to-wall friction factor on the evolution of the critical wavelength λ_c when increasing the air superficial velocity.

tors restricts and compensates their separated influence. The gas-to-wall friction factors has no influence on the critical wavelengths. The amplitude of the surface tension has also a non-negligible influence on the critical wavelengths.

Finally the critical wavelength appears to be a function of the two-fluid

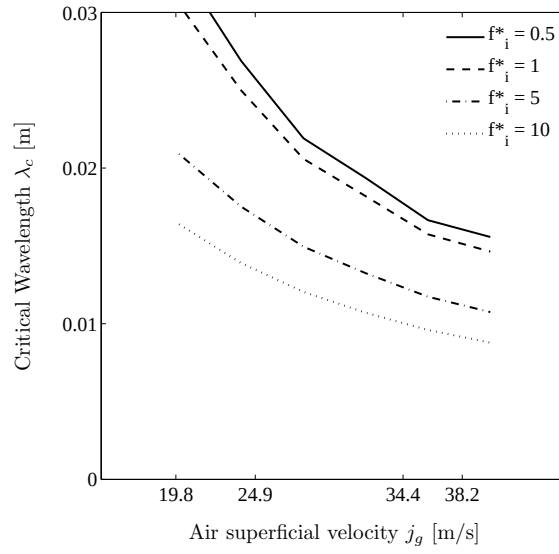


Figure 5.12: Influence of the interfacial friction factor on the evolution of the critical wavelength λ_c when increasing the air superficial velocity.

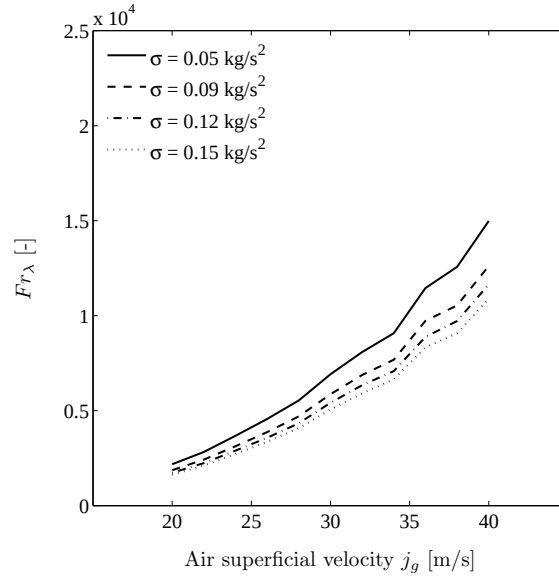


Figure 5.13: Influence of the surface tension on the evolution of the critical wavelength λ_c when increasing the air superficial velocity j_g .

characteristics and the flow conditions so that:

$$\lambda_c = \lambda(\rho_g, \rho_l, \mu_l, \sigma; j_g, j_l, D) \quad (5.13)$$

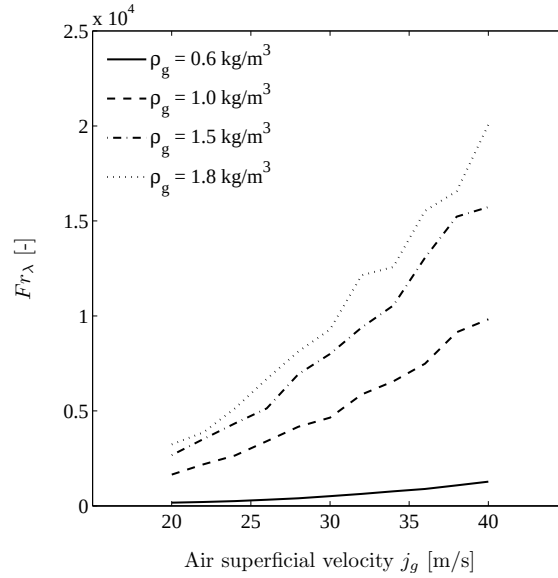


Figure 5.14: Influence of the gas density on the evolution of the ripple-wave critical wavelength λ_c when increasing the air superficial velocity j_g .

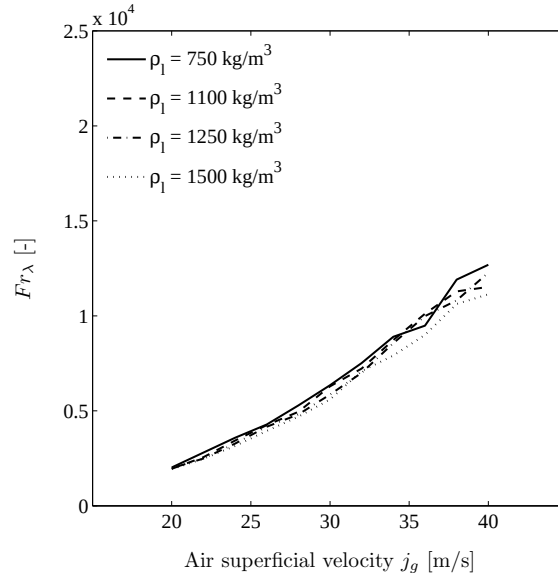


Figure 5.15: Influence of the liquid density on the evolution of the critical wavelength λ_c when increasing the air superficial velocity j_g .

5.2.3 Entrained volume at the wave crest

In an horizontal pipe, the force balance at the wave crest involves three forces: a gravity force, a surface tension force and a lift force. The force balance depends thus on the wave geometry for the determination of the volume and areas on

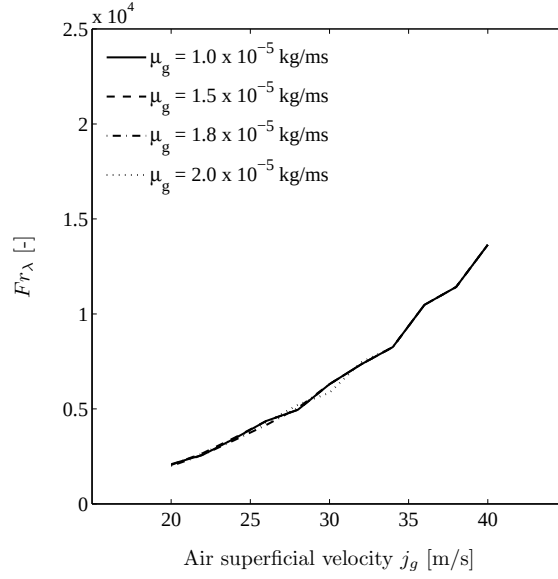


Figure 5.16: Influence of the gas viscosity on the evolution of the critical wavelength λ_c when increasing the air superficial velocity j_g .

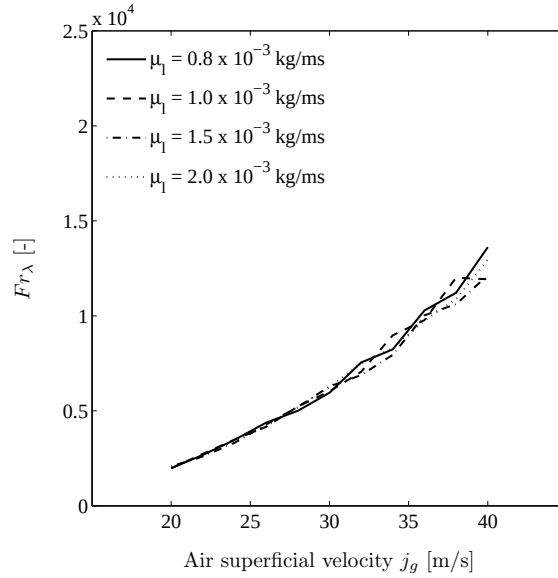


Figure 5.17: Influence of the liquid viscosity on the evolution of the critical wavelength λ_c when increasing the air superficial velocity j_g .

which these forces act but also on the flow parameters used in the force formulation. Besides the three geometrical parameters (b , h and λ) describing the wave geometry, the force balance on the wave crest is therefore function of the liquid density ρ_l (gravity), the gas density ρ_g (lift force), the lift coefficient C_L and the surface tension σ . The different forces are linearly dependent on these

parameters so that the sensitivity of the model to their variations is important. However, if the two-fluid physical properties (ρ_l , ρ_g and σ) are well known, the lift coefficient is arbitrarily modeled. Consequently at a fixed wave geometry, the correlating dimension $b_{c,1}$ at which the force balance passes through zero, highly evolves with the lift coefficient.

$$b_{c,1}(h) = f(\rho_l, \rho_g, \sigma; \lambda; C_L; j_g, j_l) \quad (5.14)$$

Similarly, the second force balance, made on the volume entrained from the wave crest, is dependent on the liquid density ρ_l , the gas density ρ_g and the entrained volume drag coefficient C_{D_T} in addition to the wave geometry (b , h and λ). The correlating dimension $b_{c,2}$ at which the second force balance passes through zero depends thus on the entrained volume drag coefficient C_{D_T} but not on the wave lift coefficient C_L used in the first force balance. As the second force balance is independent of the lift coefficient, modifying this parameter leads to an increase of the wave amplitude h_f (Figure 5.18) at which the two force balance passes simultaneously by zero and does not act on the entrained volume of liquid sheared-off at the wave crest. The entrained volume appears thus to be quasi independent of the lift and the drag coefficients as depicted in Figure 5.18 on which the three curves are merging.

$$b_{c,2}(h) = f(\rho_l, \rho_g; \lambda; C_{D_T} j_g) \quad (5.15)$$

The wave amplitude $h_f = h(b_{c,1}(h) = b_{c,2}(h))$ at which the fragmentation occurs, is thus dependent on the wave lift coefficient C_L and on the entrained volume drag coefficient C_D such as on all the parameters affecting the two force balances:

$$h_f = f(b_{c,1} \cup b_{c,2}) = f(\rho_l, \rho_g, \sigma; \lambda; C_L, C_{D_T}; j_g, j_l) \quad (5.16)$$

The drag coefficient C_{D_T} on an hemisphere is well known and equal to 1.2. At the opposite, the lift coefficient C_L on a sine half-body has rarely been studied. This lift coefficient used in the simplified model addressed to CATHARE has therefore been fitted to predict the wave amplitude measured on REGARD: $C_L \approx 0.3$. However, Figure 5.18 has shown that beyond a lift coefficient of 0.1, the entrained volume V_E does not depend on this parameter.

Equation 5.16 has synthesized the dependency of the wave amplitude on the flow physical properties (ρ_l , ρ_g , σ), on the wave geometry (λ_c) and on the assumed force coefficients (C_L , C_{D_T}) in addition to the flow conditions (j_g , j_l). The entrained volume V_E could depend therefore on all these parameters. Indeed, at a fixed air superficial velocity j_g , both the wave amplitude h and the correlating dimension $b_{c,1}$ increase with the wavelength λ_c so that the wave

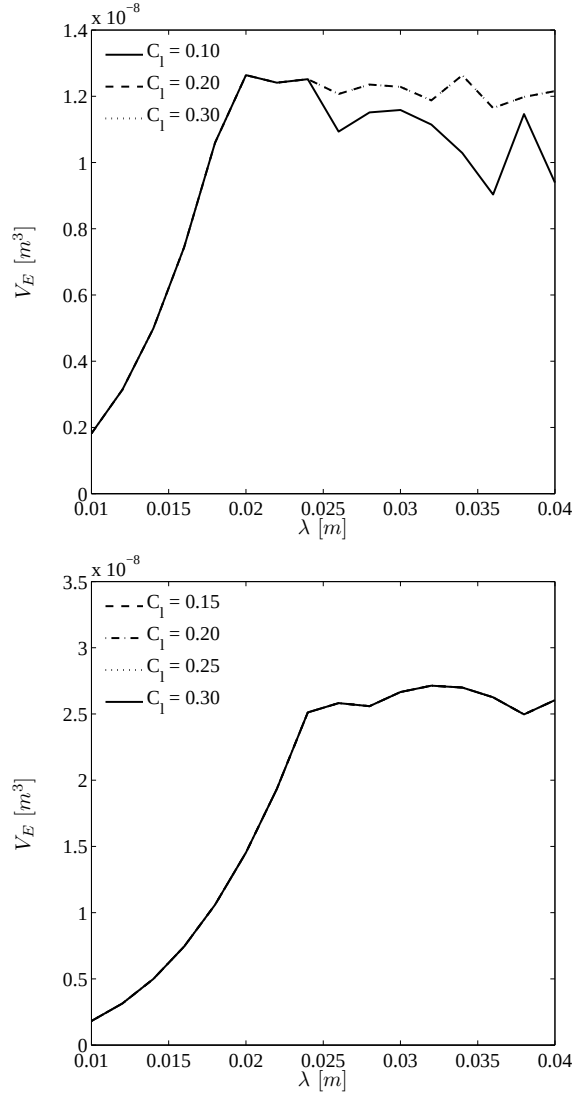


Figure 5.18: Evolution of the entrained volume as a function of the instability wavelength according to the lift coefficient at two different air superficial velocities (30 m/s (top) and 40 m/s (bottom)).

amplitude wrested-off from the top of the wave ($h - h_f$) remains quasi constant as described in Equation 5.17. Consequently, the entrained volume V_E is only slightly dependent on the wavelength as shown in Figure 5.19. This graph illustrates the evolution of the entrained volume as a function of the critical wavelength for various air superficial velocities j_g . The evolution can be separates in two different evolutions:

- o for the shortest wavelengths, the entrained volume appears to be directly

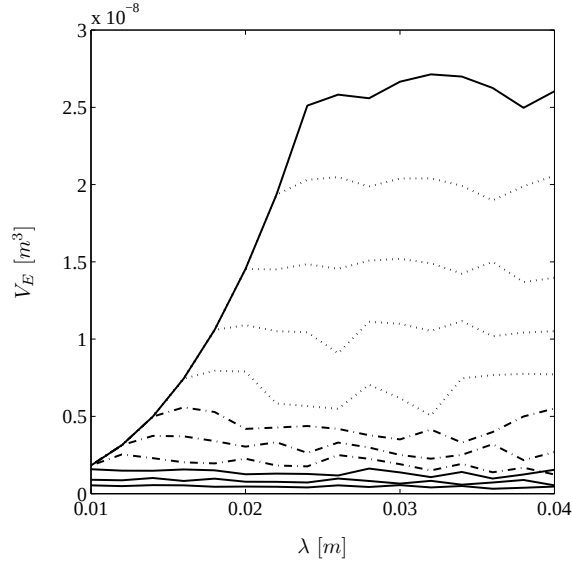


Figure 5.19: Evolution of the entrained volume as a function of the critical wavelength at various air superficial velocities j_g from 20 m/s to 40 m/s by steps of 2 m/s.

proportional to the wavelength independently of the air superficial velocity until reaching a limit wavelength λ_c^* ;

- o beyond which the entrained volume does not depend anymore on the critical wavelength but significantly varies with the air superficial velocity j_g . Indeed contrarily to the lift and drag coefficients which are separately involved in the two force balances, the air velocity acts on the two force equilibrium, directly affecting the wave amplitude at which the fragmentation occurs. The entrained volume at the wave crest V_E and the limit wavelength λ_c^* highly depend thus on the air superficial velocity j_g as shown in Figure 5.19.

$$\begin{aligned}
 \text{at } \lambda_c > \lambda_c^* \quad \text{if } \lambda_c \nearrow &\rightarrow b_{c,1}(h) \nearrow \\
 &\rightarrow b_{c,2}(h) \quad \text{cst} \\
 &\rightarrow h_f(b_{c,1} = b_{c,2}) \nearrow \\
 &\rightarrow (h - h_f) \quad \text{cst} \\
 &\rightarrow V_E \text{ cst}
 \end{aligned} \tag{5.17}$$

Figure 5.20 highlights that the entrained volume V_E highly increases with the air superficial velocity j_g . Otherwise, the liquid superficial velocity j_l and the pipe diameter D have no direct influence on the entrained volume at a fixed wavelength λ_c . Moreover Figure 5.20 presents the transitional air superficial velocity j_g^* at which the entrainment begins to occur. The predicted transitional

air superficial velocity of 15 m/s is comparable to the values extrapolated from the REGARD experimental data.

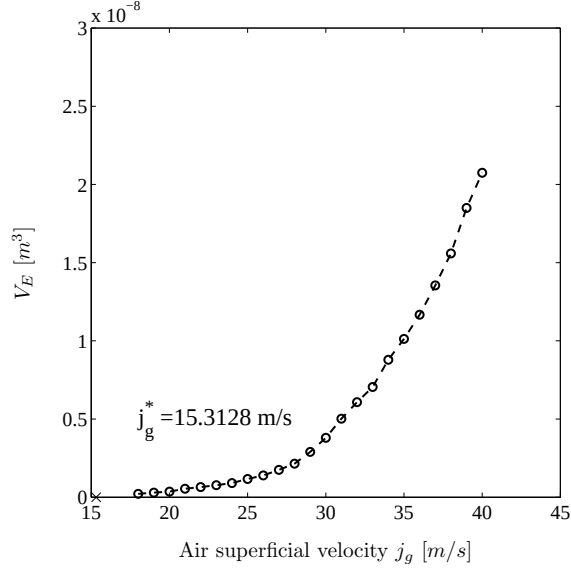


Figure 5.20: Evolution of the entrained volume as a function of the critical wavelength when increasing the air superficial velocity j_g from 20 m/s by steps of 2 m/s.

Figures 5.21, 5.22 and 5.23 highlight the effect of the physical properties (ρ_l, ρ_g, σ) on the entrained volume V_E beyond the limit critical wavelength λ_c^* . Figures 5.21 and 5.22 show that the entrained volume V_E significantly evolves with the gas density and the liquid density while Figure 5.23 shows no significant influence of the surface tension. Finally the entrained volume V_E appears to depend only on the parameters affecting both the two force balances, as synthesized in Equations 5.18.

$$\begin{aligned} V_E &= f(b_{c,1} \cap b_{c,2}) = f(\lambda) && \text{if } \lambda < \lambda_c^* \\ V_E &= f(b_{c,1} \cap b_{c,2}) = f(\rho_l, \rho_g; j_g) && \text{if } \lambda > \lambda_c^* \end{aligned} \quad (5.18)$$

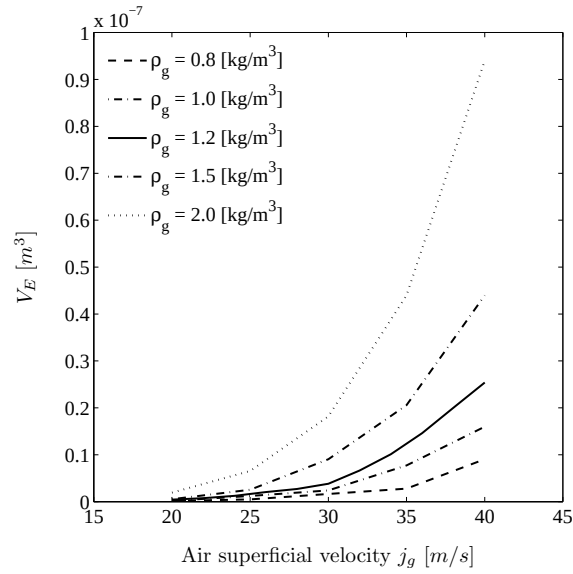


Figure 5.21: Evolution of the entrained volume as a function of the air superficial velocity j_g according to the air density ρ_g .

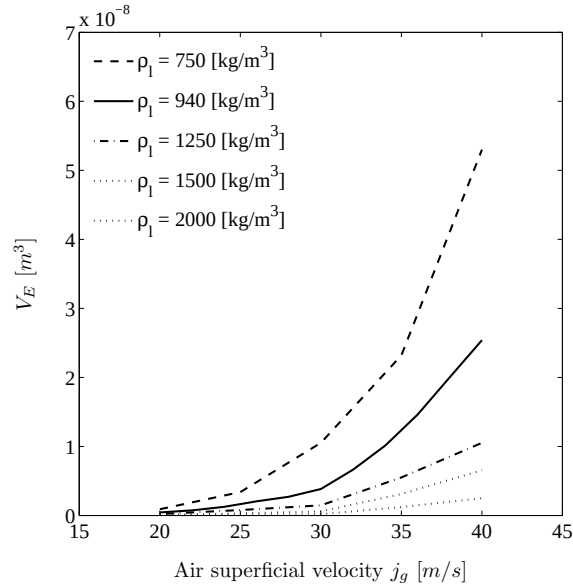


Figure 5.22: Evolution of the entrained volume as a function of the air superficial velocity j_g according to the liquid density ρ_l .

5.3 Entrainment rate model assessment

The assessment of the entrainment rate model is performed against the experimental data from REGARD, obtained in Section 3.3.4. Figure 5.24 presents the comparison between the REGARD experimental data and the model predic-

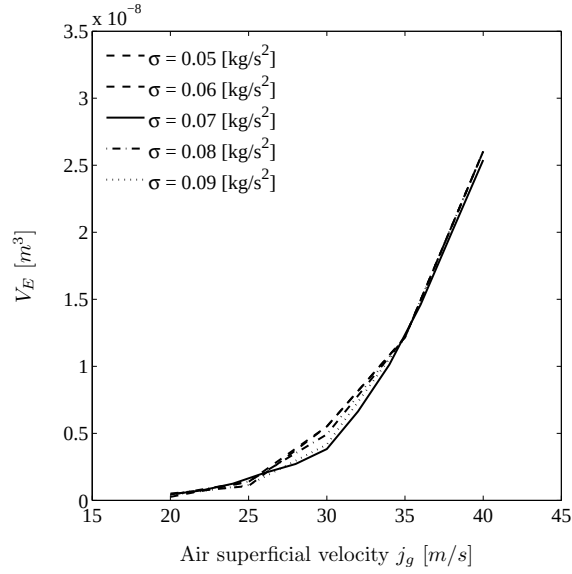


Figure 5.23: Evolution of the entrained volume as a function of the air superficial velocity j_g according to the surface tension σ .

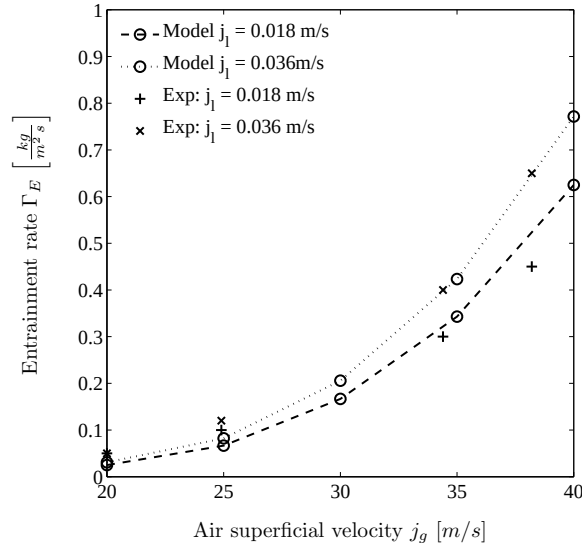


Figure 5.24: Comparison between experimental data and model outputs for the entrainment rate.

tions for the entrainment rate Γ_E . The model fits well the experimental results although the predictions of the critical wavelengths highly differs.

The model predicts a significant increase of the entrainment rate with the air superficial velocity j_g and a slight increase with the liquid superficial ve-

locity j_l . This liquid velocity influence comes from the critical wavelength λ_c dependence on the liquid superficial velocity while the gas velocity influence separately comes from the critical wavelength λ_c , the entrained volume V_E and the characteristic time τ_C dependences. As the critical wavelength predicted by the proposed model is function of the physical properties (densities, viscosities, surface tension), the entrainment rate Γ_E model also depends more or less on all these properties.

The model appears then to be relevant to model the entrainment rate occurring in a air-water two-phase flow in a horizontal pipe of large diameter. Indeed the model predictions fit well the experimental data from REGARD. Nevertheless this model assessment out from the operating conditions experimentally studied with REGARD is hardly achievable. Indeed, this model lays on experimental measurements of a characteristic time which are not available in the literature at others pipe geometries or flow conditions. Consequently, the range of validity of the characteristic time sub-model, and hence of the entrainment rate model, is quite restricted. Moreover, experimental entrainment rate measurements (and/or characteristic time measurements) are not provided in the literature, so that the model is hardly assessable out of the REGARD range.

5.4 Droplet entrainment rate correlation

A mechanistic model for the entrainment rate Γ_E has been developed in Chapter 4. Chapter 5 has been primarily devoted to the assessment of this proposed entrainment rate model which appears to fit well the experimental data from REGARD. However, the complete model requires a lots of sub-calculations: the complete spectrum of the instability wavelengths is calculated, while the force balances are applied on a large set of wave amplitude for each critical wavelength. Consequently, applying the complete model requires too many calculation operations and thus, too many computation time to be directly implemented in a system code. In order to facilitate a further implementation of this model into a system code (like CATHARE 3 or RELAP), a correlation of the entrainment rate as a simple function of the flow conditions and the fluid characteristics has to be derived from the model predictions. This section firstly presents thus the setup of the correlations concerning the three sub-models used in the entrainment rate Γ_E model, before comparing the correlation predictions to the experimental data from REGARD.

5.4.1 Entrainment rate Γ_E^* correlation

The final general model of the entrainment rate Γ_E has been expressed in Equation 4.6. Assuming that the proportion of waves κ which fragments is equal to one so that every wave fragments, the entrainment rate model simplifies in:

$$\Gamma_E = \frac{\rho_l V_E}{\lambda_c^2 \tau_C} \quad (5.19)$$

This entrainment rate correlation requires then three sub-correlations: for the critical wavelength λ_c^* , the entrained volume V_E^* and the characteristic time τ_C^* . The following sub-sections propose these three correlations derived from the various models proposed in Chapter 4 to express the entrainment rate correlation:

$$\Gamma_E^* = \frac{\rho_l V_E^*}{(\lambda_c^*)^2 \tau_C^*} \quad (5.20)$$

5.4.2 Critical wavelength correlation

The sensitivity study of the critical wavelength model has shown that the critical wavelength λ_c depends on the two-fluid physical properties in addition to the flow conditions (see Equation 5.13) and the pipe diameter:

$$\lambda_c = \lambda_c(\rho_g, \rho_l, \mu_l, \mu_g, \sigma; j_g, j_l, \alpha_l, D) \quad (5.21)$$

The individual dependence of every parameter would lead to express the critical wavelength λ_c^* correlation, written as a Froude number (Equation 5.12),

as:

$$Fr_{\lambda_c^*} = \frac{(v_g - v_l)^2}{g \lambda_c^*} \propto \rho_g^{2.64}, \rho_l^{-0.14}, \mu_l^{-0.015}, \mu_g^{0.0024}, \sigma^{-0.28}, f(j_g, j_l, \alpha_l, D) \quad (5.22)$$

For examples, the dependence of the Critical Froude Number Fr_λ on the liquid density ρ_l and the surface tension σ are depicted in Figure 5.25.

The Vaschy-Buckingham theorem indicates that the dimensionless parameter characterizing λ (function of 9 physical variables and 3 independent physical units) may be constructed with $9 - 3 = 6$ dimensionless parameters. In Chapter 1.3, the relevant dimensionless parameters to model the entrainment phenomenon has been depicted (see Equations 1.12), so that:

$$Fr_{\lambda_c^*} = \mathbf{A} We^{\mathbf{a}} Re_g^{\mathbf{b}} Re_l^{\mathbf{c}} Fr_D^{\mathbf{d}} \left(\frac{\rho_g}{\rho_l} \right)^{\mathbf{e}} \left(\frac{\mu_g}{\mu_l} \right)^{\mathbf{f}} \left(\frac{D}{\lambda_c^*} \right)^{\mathbf{g}} \quad (5.23)$$

where $\mathbf{A}$, $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$, $\mathbf{e}$, $\mathbf{f}$ and $\mathbf{g}$ denote rational numbers. Finally in a dimensionless form, the critical wavelength may be written as:

$$Fr_{\lambda_c^*} = 0.027 We^{0.19} Re_g^{-0.77} Re_l^{0.14} Fr_D^{1.55} \left(\frac{\rho_g}{\rho_l} \right)^{1.01} \left(\frac{\mu_g}{\mu_l} \right)^{-0.34} \left(\frac{D}{\lambda_c^*} \right)^{1.3} \quad (5.24)$$

The optimal coefficients have been found by least squares method on a large set

Variable	Units	Min.	Max.
D	$[m]$	0.02	1
j_g	$[m/s]$	v_g^*	50
j_l	$[m/s]$	0.01	0.05
ρ_g	$[kg/m^3]$	0.8	2
ρ_l	$[kg/m^3]$	750	2000
μ_g	$[kg/m\ s]$	1.0×10^{-5}	2.0×10^{-5}
μ_l	$[kg/m\ s]$	0.8×10^{-3}	2.0×10^{-3}
σ	$[kg/s^2]$	0.03	0.15

Table 5.1: Extended range of the flow variables and the flow conditions analyzed to achieve the critical wavelength correlation and the entrainment rate correlation.

of 1000 model predictions achieved by varying every model variables by factors from 0.2 to 5 and resumed in Table 5.1. For the shortest pipe diameters, the closure laws for interfacial and parietal shear stresses have been adapted to coincide with the Barnea and Taitel's laws (Equations 4.29 and 4.24), fully relevant for pipe of small diameters according to literature reviews[20, 12]. However the

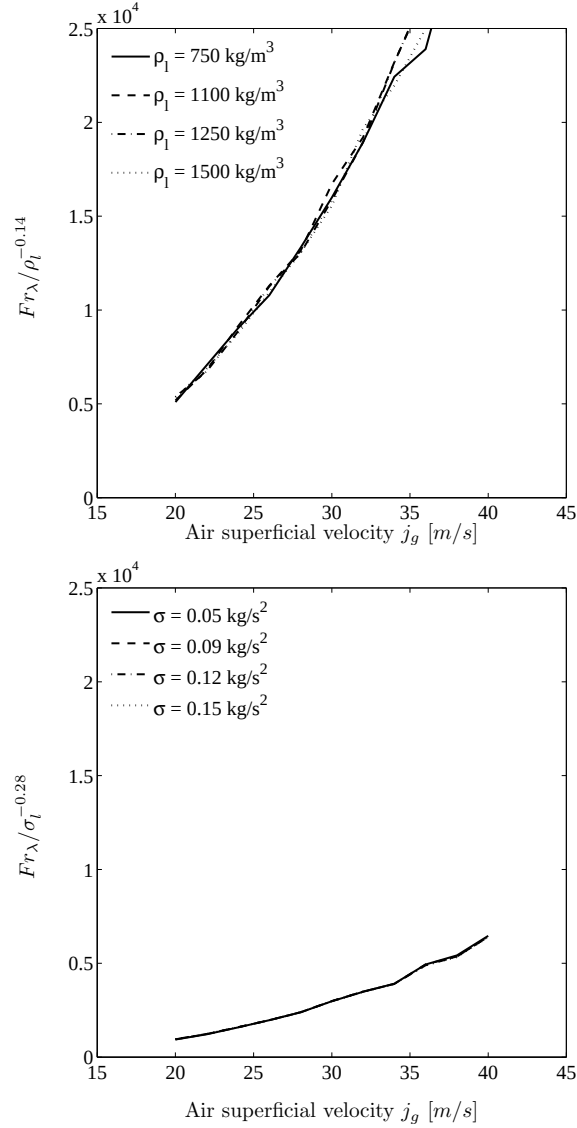


Figure 5.25: Evaluation of the dependence of the correlation on some flow physical properties: the liquid density ρ_l (top) and the surface tension σ (bottom), as a function of the air superficial velocity while every other parameters (j_l , D , ρ_g , μ_l , μ_g) are kept constant.

individual dependencies of every parameters can not be perfectly met by this set of adimensional numbers so that the critical wavelength correlation λ_c^* does not completely fit the critical wavelength model λ_c predictions. Less than 15% of the correlation predictions differ by 10% from the model results while every correlation predictions satisfy the model results with a maximal difference of 35% as shown in Figure 5.26.

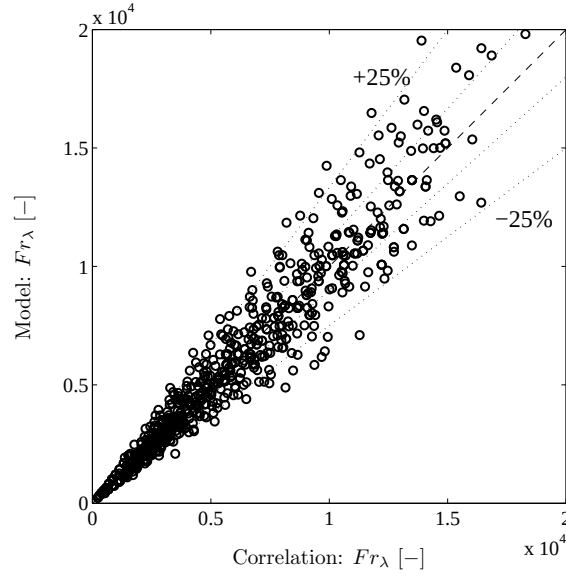


Figure 5.26: Comparison between model data on the critical Froude Number and the correlation predictions with the correlation coefficients in Equation 5.24 determined by least square method.

5.4.3 Entrained volume correlation

Figure 5.19 has shown that the entrained volume mainly depends on two phenomena. Under the limit critical wavelength λ_c^* depending on the flow conditions and the fluid characteristics (see Section 5.2.3), the entrained volume V_E is only dependent on the critical instability wavelength λ_c (see Equation 5.18). Beyond this limit, the entrained volume does not depend any more on the wave geometry. Indeed the wave height h at which the volume is wrested-off from the wave, linearly evolves with the critical wavelength λ_c , so that the height of the entrained volume ($h - h_w$) appears independent of the critical wavelength. The entrained volume V_E is thereby essentially function of the flow conditions and the fluid densities (see Equation 5.18). The entrained volume correlation V_E^* should then express this two tendencies as depicted in Equation 5.25:

$$V_E^* = \min \left(V_{E, \lambda_c < \lambda_c^*}^*, V_{E, \lambda_c > \lambda_c^*}^* \right)$$

$$\text{with } V_{E, \lambda_c > \lambda_c^*}^* = V_{E, \lambda_c > \lambda_c^*}^* (\rho_g, \rho_l, j_g) \quad \text{and} \quad V_{E, \lambda_c < \lambda_c^*}^* = V_{E, \lambda_c < \lambda_c^*}^* (\lambda)$$
(5.25)

Below the limit critical wavelength: $V_{E, \lambda_c < \lambda_c^*}^*$

Below the limit critical wavelength $\lambda_c < \lambda_c^*$, function of the flow characteristics ($\lambda_c^* = \lambda_c^* (\rho_g, \rho_l, j_g)$), the entrained volume V_E (predicted by the proposed

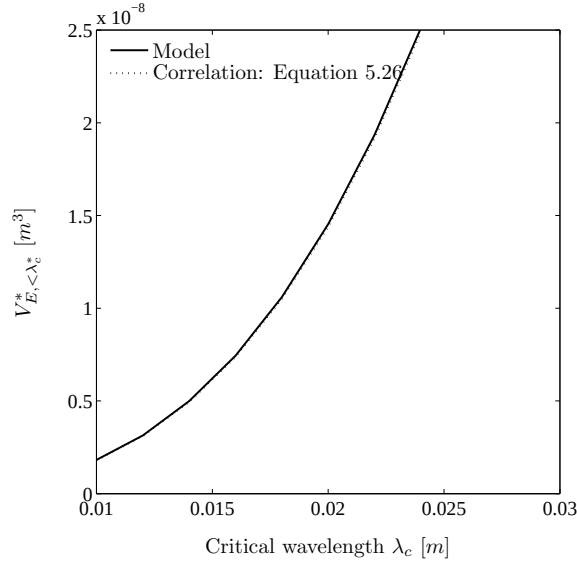


Figure 5.27: Comparison between the model results and the correlation results for the entrained volume $V_{E, \lambda_c < \lambda_c^*}^*$ (Equation 5.26) as a function of the critical wavelength λ_c .

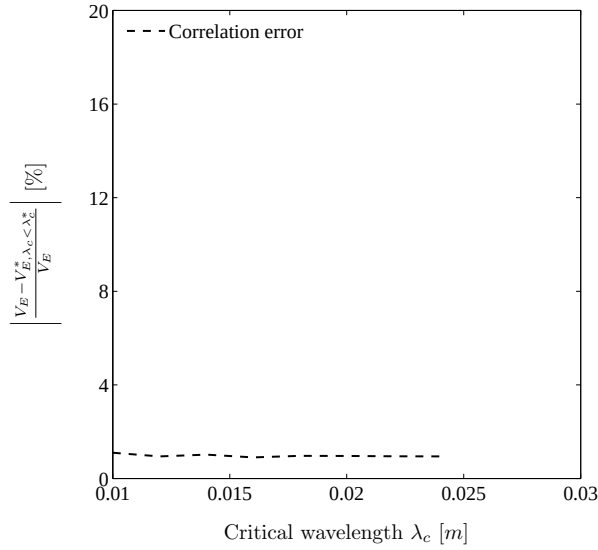


Figure 5.28: Difference between the model results and the correlation results for the entrained volume $V_{E, \lambda_c < \lambda_c^*}^*$ (Equation 5.26) as a function of the critical wavelength λ_c .

model) is only proportional to λ^3 as shown in Figure 5.27. The entrained volume correlation beyond the limit critical wavelength $V_{E, \lambda_c < \lambda_c^*}^*$, has to follow this dependence, as written in Equation 5.26. The correlation predictions are compared to the model predictions in Figure 5.27 which shows a good agreement

between the model and the correlation predictions. Figure 5.28 shows that the difference between the model and the correlation remains around 1%.

$$V_{E,\lambda_c < \lambda_c^*}^* = 0.018 \lambda_c^3 \quad (5.26)$$

Beyond the limit critical wavelength: $V_{E,\lambda_c > \lambda_c^*}^*$

Beyond the transient critical wavelength λ_c^* , the sensitivity study has shown that the entrained volume is proportional to the fluid densities and the air velocity (as shown in Figure 5.19). An extensive study of the entrained volume sensitivity to the air superficial velocity has shown a critical air velocity v_g^* below which the model does not predict any solutions for the entrained volume. Below this threshold, no entrainment is predicted. The sensitivity study leads to the conclusion that the entrained volume may be considered as proportional to the difference between the air velocity and this wave fragmentation limit. Otherwise, the entrained volume may be considered as proportional to the two-fluid density ratio so that the entrained volume may be written as Equation 5.27:

$$V_{E,\lambda_c > \lambda_c^*}^* = \mathbf{a} \left(\frac{\rho_g}{\rho_l} \right)^{\mathbf{b}} (v_g - v_g^*)^3 \quad (5.27)$$

Figure 5.29 compares the model results and the correlation predictions for various density ratios and shows a good agreement. Figure 5.30 presents then the prediction error of the correlation compared to the model data. The correlation better suits for large air superficial velocities j_g where the entrainment is important, than at the small air superficial velocities at which the entrained volume is over-predicted (10% of difference). The coefficients applied in the entrained volume correlation: $\mathbf{a}$ and $\mathbf{b}$, has been fitted to tackle the evolution of the entrained volume with the density ratio as shown in Figure 5.31. Assuming $\mathbf{b} = 2.69$ and $\mathbf{a} = 10^{-4}$ ensures a good agreement between the model data and the correlation predictions.

Correlation: V_E^*

Finally, the entrained volume correlation defined in Equation 5.25 takes the following form:

$$V_E^* = \min \left(10^{-4} \left(\frac{\rho_g}{\rho_l} \right)^{2.69} (v_g - v_g^*)^3 ; 0.018 \lambda_c^3 \right) \quad (5.28)$$

In Equation 5.28, the term 10^{-4} is dimensionalized as expressed in $[s^3]$. The critical air velocity v_g^* at which the entrainment begins, for the range of studied flow conditions and the fluid characteristics, slightly varies around 15.3 $[m/s]$, as shown in Figure 5.20.

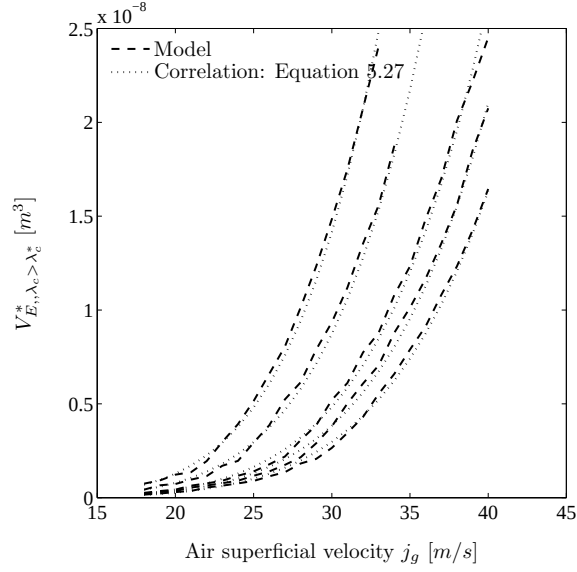


Figure 5.29: Comparison between the model results and the correlation results for the entrained volume $V_{E, \lambda_c > \lambda_c^*}^*$ (Equation 5.27) as a function of the air superficial velocity j_g .

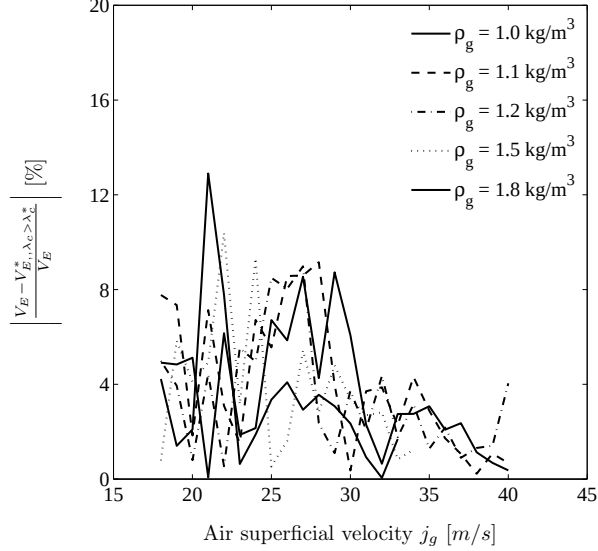


Figure 5.30: Difference between the model results and the correlation results for the entrained volume $V_{E, \lambda_c > \lambda_c^*}^*$ (Equation 5.27) as a function of the air superficial velocity j_g .

5.4.4 Characteristic time correlation τ_C^*

The characteristic time τ_C has been shown to mainly evolve with the critical wavelength λ_c and hence with the air superficial velocity j_g (Equation 4.22).

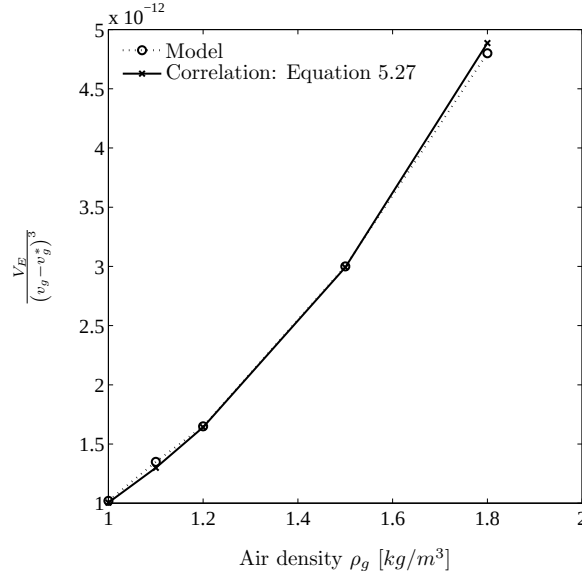


Figure 5.31: Comparison between the model results and the correlation predictions for the fluid density ratio $\mathbf{a} \left(\frac{\rho_g}{\rho_l} \right)^{\mathbf{b}}$ in Equation 5.27.

The influence of the liquid superficial velocity j_l appears to be indeterminable. As the characteristic time has been experimentally obtained, the influence of the two-fluid properties (densities, viscosities, surface tension) or the pipe geometry (diameter) could not have been studied. Consequently, the characteristic time correlation is assumed to be proportional to the critical wavelength and to follow its evolution with the air superficial velocity j_g . Based on the critical wavelength predicted by the proposed model, the characteristic time correlation is:

$$\begin{aligned}
 \tau_C^* &= \tau_C^*(\lambda_c(j_g)) \\
 &= \frac{2 \times 10^{-4}}{\lambda_c^*(j_g)} \\
 &\approx \frac{j_g}{200}
 \end{aligned} \tag{5.29}$$

5.4.5 Assessment of the entrainment rate correlation against the REGARD experimental data

Figures 5.34 and 5.32 present the comparison between the experimental data on the entrainment rate Γ_E obtained with the REGARD facility and the correlation predictions for the entrainment rate Γ_E^* . Figure 5.32 presents the evolution of the entrainment rate performed with Tzotzi and Andritsos friction laws and the linear stability model proposed in this work, compared to the experimental data from REGARD.

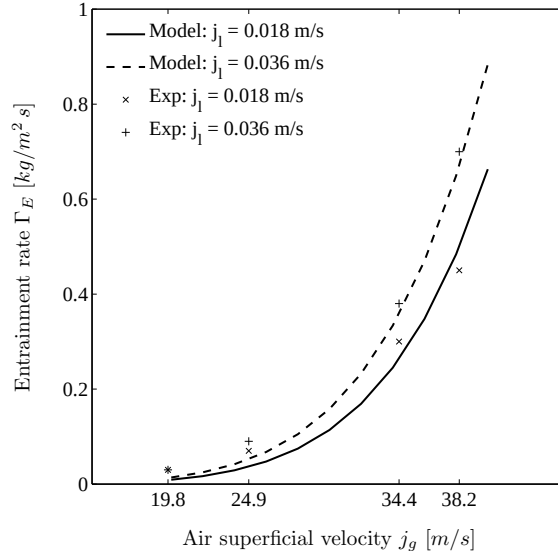


Figure 5.32: Evolution of the entrainment rate correlation, performed with Tzotzi and Andritsos friction laws and proposed linear stability analysis, as a function of the air superficial velocity compared to experimental data from REGARD.

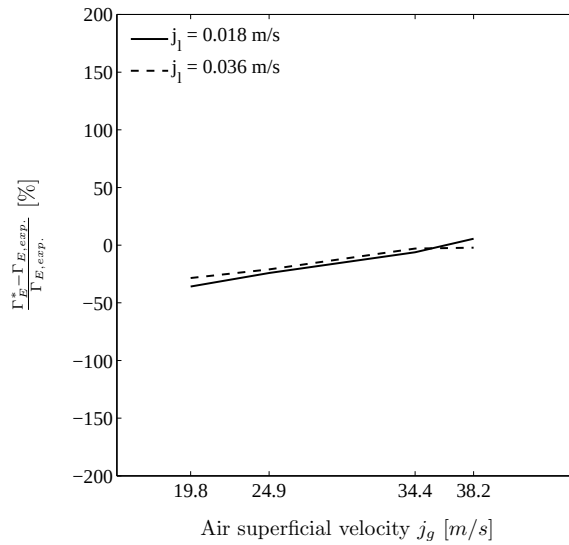


Figure 5.33: Difference between the entrainment rate correlation, performed with Tzotzi and Andritsos friction laws and proposed linear stability analysis, as a function of the air superficial velocity compared to experimental data from REGARD.

The critical wavelength, calculated with Tzotzi and Andritsos friction laws

and the proposed linear stability model, appears to induce a better agreement between the correlation predictions and the experimental data. Indeed Figure 5.33 indicates that the difference between the two curves remain under 40% with an average difference of 14.4% for the eight experimental conditions. The proposed correlation is thus fully relevant to model the entrainment rate occurring in the REGARD experimental facility. However the question remains on the validity of the predictions on a large range of pipe diameters, flow conditions or two-fluid physical properties. The two following sub-sections aim at studying the proposed correlation validity at two different pipe scales.

In order to emphasize the effect of the friction laws and of the linear stability analysis formulation, Figure 5.34 presents the evolution of the entrainment rate performed assuming the Barnea and Taitel's friction laws and linear stability analysis models a function of the air superficial velocity j_g . Figure 5.35 indicates that the model predictions fit in this case the experimental data with an accuracy between 25% and 175%. The order of magnitude is thus correct even if the predictions are not sufficiently precise.

5.4.6 Correlation predictions compared to Hewitt measurements [66]

Figure 5.32 presents the comparison between the present model and the entrainment rate $\Gamma_{E,Hewitt}$, deduced from the z-axis space evolution of the mass flux of droplets measured by Hewitt [66] in a circular pipe of 3.175 pipe diameter at a superficial air velocity j_g of 38 m/s (air-liquid two-phase flow). The entrainment rate appears to be greater at this small pipe diameter than at the 0.24 m pipe diameter of REGARD. This is consistent with the tendencies predicted by the entrainment model.

5.4.7 Correlation predictions at the PWR hot leg scale

As a direct consequence of the influence of the pipe diameter on the critical wavelength (detailed in Figure 5.8), the entrainment rate predictions in a pipe of a PWR hot leg size are smallest than at the REGARD scale. Indeed, the critical wavelength λ_c increases with the pipe diameter. The denominator of the entrainment rate increases thus so that the entrainment rate decreases at the same fluid characteristics and flow conditions. Figure 5.37 presents the entrainment rate predictions, achieved with the entrainment rate correlation in Equation 5.19, performed at the PWR hot leg scale ($D = 0.72\text{ m}$), thrice the REGARD pipe diameter scale ($D = 0.24\text{ m}$). Figures 5.37, 5.32 and 5.36 indicate that the entrainment rate Γ_E^* decreases with the pipe diameter D , increases

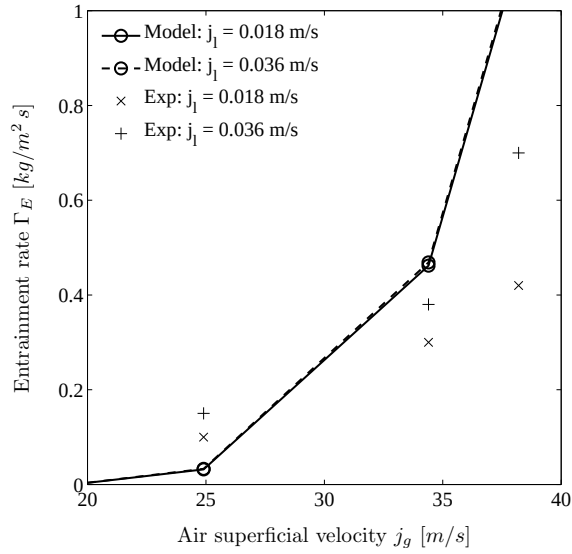


Figure 5.34: Evolution of the entrainment rate correlation, performed with Barnea and Taitel's friction laws and linear stability analysis, as a function of the air superficial velocity compared to experimental datas from REGARD.

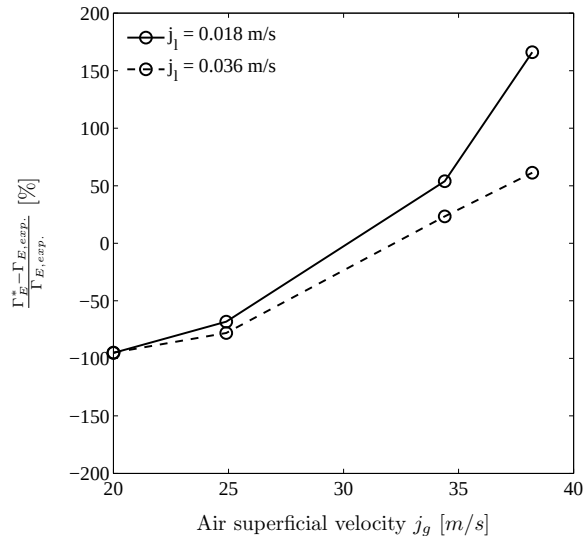


Figure 5.35: Difference between the entrainment rate correlation, performed with Barnea and Taitel's friction laws and linear stability analysis, as a function of the air superficial velocity compared to experimental datas from REGARD.

with the air superficial velocity j_g and increases with the liquid superficial velocity j_l .

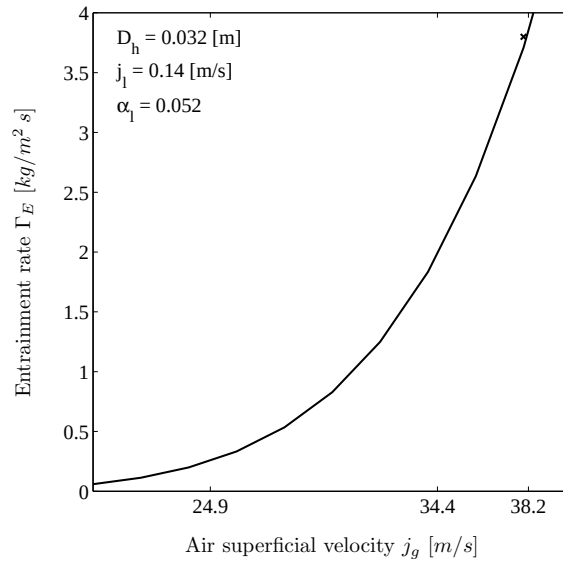


Figure 5.36: Evolution of the entrainment rate correlation, performed with Tzotzi and Andritsos friction laws and proposed linear stability analysis, as a function of the air superficial velocity compared to (x) Hewitt measurement [66] achieved on a air-liquid two-phase flow in a 3.175 cm pipe diameter facility at a air superficial velocity of 38 m/s.

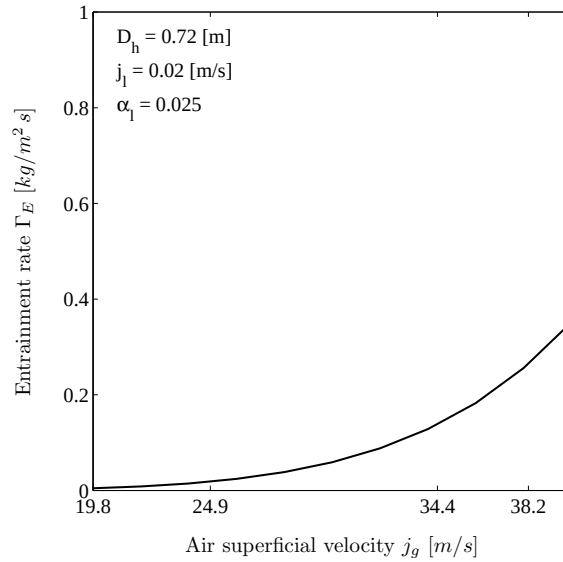


Figure 5.37: Evolution of the entrainment rate correlation, performed with Tzotzi and Andritsos friction laws and proposed linear stability analysis, as a function of the air superficial velocity at the PWR hot leg scale.

5.5 Model conclusions

A new mechanistic model, based on a description of the phenomena leading to the wave fragmentation and atomization, has been developed in this work in order to determine the amount of liquid wrested-off from the liquid film in a stratified flow in a horizontal circular pipe, and entrained as droplets. The phenomenological model considers that the droplet entrainment comes from the development and the growth of interfacial Kelvin-Helmoltz instabilities. The model is finally based on the determination of three sub-models: the instability critical wavelength λ_c , the entrained volume at the crest of an instability V_E and the characteristic time of the entrainment phenomenon τ_C , as proposed in Equation 4.6. The instability wavelengths are determined by performing a linear stability analysis of the two-fluid stratified flow. The amount of liquid entrained at the crest of these instabilities V_E is determined throughout a force balance at the wave crest and a force balance on the entrained volume of liquid. The characteristic time has been derived from experimental data from the REGARD facility.

The proposed model fits well the experimental data from REGARD as shown in Figure 5.24. However, the entrainment rate model requires several sub-calculations and computational time to be directly used in a system-code. Consequently, an entrainment rate correlation Γ_E^* has been derived from the entrainment rate model Γ_E in order to reduce the number of required calculations. To keep a mechanistic approach in the correlation, a correlation of each sub-model has been written in order to conserve the parameter dependence of every sub-model in the entrainment rate correlation. Thereby, the entrainment rate correlation is based on three sub-correlations for the critical wavelength λ_c^* , the entrained volume V_E^* and the characteristic time τ_C^* (see Equation 5.19). Table 5.2 summarizes the parameter dependence of every sub-model and sub-correlations defined in this work. The evolution of the sub-model predictions with the physical properties and the flow conditions has been extensively studied except for the characteristic time which is experimentally determined and therefore only assessed on the experimental two-fluid conditions.

The proposed correlation fits well too the experimental data from REGARD as shown in Figure 5.32. At the REGARD scale, the difference between the entrainment rate model and the experimental data is kept below 30% while the difference between the entrainment rate model and the entrainment rate correlations evolves around 0% and 20%. The model has been slightly adapted at the level of characteristic time to introduce a pipe diameter dependence, unevalu-

able on the single basis of the REGARD data. Consequently, the model also fits the experimental data from Hewitt [66] and predicts an entrainment rate of a similar order of magnitude at the hot leg scale. These entrainment rate predictions highly differentiate the present model from the literature entrainment rate models (see Section 1.3, Figure 1.11). Indeed, compared to existing models, the entrainment rate amplitude remains in a similar order of magnitude for a range of pipe diameter from 0.1 m to 1 m, while most existing correlations predict a significant variation. Conserving a similar order of magnitude is coherent with the proposed entrainment approach, as the interface should keep a similar behaviour for these pipe scales. The proposed model is therefore relevant for the simulation of the droplet entrainment from a stratified liquid film in horizontal pipes (diameter from 0.1 m to 1 m).

The entrainment rate correlation predicts that the entrainment rate increases with the air superficial velocity and the liquid superficial velocity while increasing the pipe diameter induces a decrease of the entrainment rate. The two-fluid densities act on the entrained volume and the critical wavelength (but undetermined on the characteristic time) while the two-fluid viscosities and the surface tension only act on the critical wavelength.

			j_g	j_l	D	ρ_g	ρ_l	μ_g	μ_l	σ
Model	Critical wavelength	λ_c	x	x	x	x	x	x	x	x
	Entrained Volume	V_E	x	-	-	x	x	-	-	-
	Characteristic time	τ_C	x	-	?	?	?	?	?	?
	Entrainment rate model	Γ_E	x	x	x	x	x	x	x	x
Correlation	Critical wavelength	λ_c^*	x	x	x	x	x	x	x	x
	Entrained Volume	V_E^*	x	-	-	x	x	-	-	-
	Characteristic time	τ_C^*	x	-	i	-	-	-	-	-
	Entrainment rate model	Γ_E^*	x	x	x	x	x	x	x	x

Table 5.2: Dependence on the flow conditions, pipe geometry and physical properties of the sub-models and the sub-correlations presented in the present work: (x) taken into account in the model, (i) imposed dependence, (-) not dependent, (?) unknown dependence.

5.6 Model perspectives

The proposed model fits thus well the experimental data from REGARD. However, similarly to every mechanistic model, the description of the phenomenon

sequence leading to the entrainment rate could be deepened and the entrainment rate model could be improved in many ways in further studies. Indeed, the modeling of various parameters is still perfectible and some stratified flow considerations are currently not deeply taken into account:

- First of all, the proposed model for the entrainment rate only considers the amount of liquid entrained from the wave development and fragmentation. Every other entrainment sources are neglected and not considered. Both the importance or the interactions with the wave development of these additional droplet entrainment sources could be additionally modeled.
- The proposed model is based on the wave development and considers various sub-models which can be separately improved:
- The sub-model for the characteristic time is experimentally achieved on the only basis of the REGARD experimental data. The range of validity is therefore extremely limited, compared to the two other sub-models (see Figure 5.38). One major improvement should be thus to extend the range of validity by performing some BANJO measurements of the interfacial film evolutions on several additional couples of superficial velocities at the REGARD scale. Moreover, some measurements of the characteristic time should be obtained from experimental facilities of various scales to extend the diameter range of validity. In the same spirit, studying various two-fluid couples would allow to determine the physical property dependences of the characteristic time and therefore increase the validity range of the correlations.

Finally, a mechanistic model of the characteristic time could be described in order to eliminate the experimental data from the entrainment rate model. This would imply to extensively address the totally under-studied of the wave growth in time and space of small amplitude waves in stratified flow in pipes. Indeed, the study of the wave development to describe the complete progression of the wave amplitude until its fragmentation appears to be very perfectible.

- The sub-model for the interfacial area could be significantly improved in order to take into account the edge effects, the circumferential variation of the liquid film height (as the pipe is circular) and their induced effects on the local critical wavelengths. Indeed, the flow visualizations have shown that the interface presents a diamond structure due to the difference in terms of velocity and wavelength of the interfacial instabilities due to the

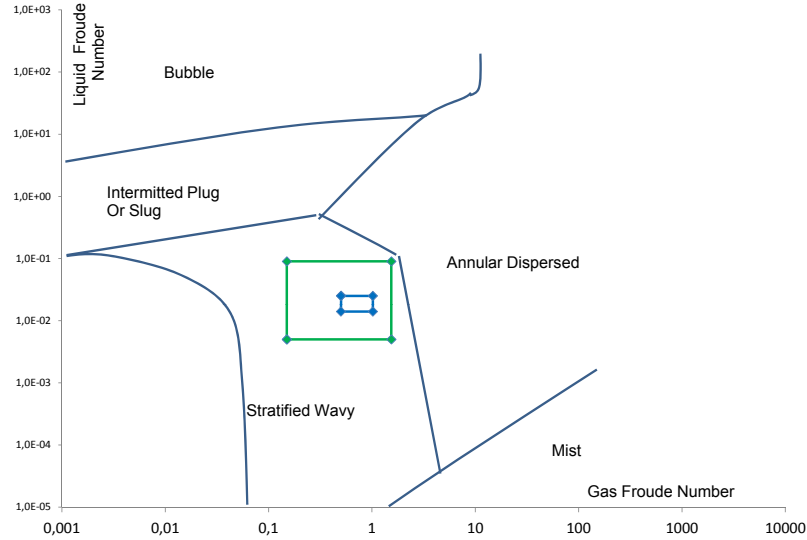


Figure 5.38: Map flow of the studied range of two-fluid Froude Numbers for the characteristic time (blue) and the entrained volume and the critical wavelength (green).

local liquid depth, so that the critical wavelength is not homogeneous all over the liquid film circumference.

- The sub-model characterizing the critical wavelength could be slightly improved in many ways:
 - o Firstly, the critical wavelength is achieved on the basis of a **linear** stability analysis. Performing a non-linear stability analysis could improve the prediction of the amplification spectrum, according to Guo et al. [62, 61]. A non-linear analysis would provide an indication of the evolution of the wave amplification factor and the critical wavelength with the wave amplitude.
 - o The linear stability analysis is based on a simple two-fluid model. The modification of the two-fluid equations to take into account the wave asymmetry [29, 17] or the form factor [20, 21, 62, 61] could improve the critical wavelength model even if some preliminary prospections on these effects have lead to no assessable results (as not significant).
 - o Mainly, the bibliographic review has shown that the adaptation of the closure laws to the two-fluid flows still remains an unresolved

topic. Indeed the closure laws of Barnea and Taitel [20], CATHARE [157] or Tzotzi and Andritsos [154] does not completely correspond to the modeling of a highly stratified two-phase flow. However, the critical wavelength is highly dependent on these closure laws so that a major improvement on this topic could significantly increase the validity of the critical wavelength model. Moreover the range of validity of the various closure laws is not well experimentally defined. Therefore, achieving an extensive study on the closure laws in order to define some laws adapted to the wavy stratified flows in pipes of high diameter would bring a significant enhancement.

- The sub-model characterizing the entrained volume could also be slightly improved, similarly to the characteristic time, with a sub-model characterizing the wave growth from its apparition to its atomization. Characterizing the wave geometry evolution (amplitude, shape, growth velocity) would enhance the entrained volume sub-model. Otherwise the aerodynamic characteristics of a wave could be studied in order to provide the wave lift coefficient used in the force balances.
- Finally, increasing the range of flow conditions and two fluid physical properties on which the various sub-models have been assessed would significantly increase the validity range of the entrainment rate correlation.
- Theoretically, the most interesting improvement would be to generalize the model to every two-phase flow, from the stratified flow in inclined pipes of very high diameters to annular flow in vertical pipes of very small diameters. Indeed, the proposed phenomenological approach of the droplet entrainment should remain valid whatever the flow configuration. This would require to adapt every sub-models to take into account the pipe orientation.

Chapter 6

Conclusion

To enhance the simulation of nuclear accidents, the system code CATHARE is continuously improved. The version, currently in development, adds an additional droplet field to the classical two-fluid (liquid-vapor) 6-equation model. Consequently, several new closure laws are required to simulate the exchange terms between the conservation equations. In this framework, the objective of this work was to develop a closure law for the droplet entrainment from a liquid film in a stratified flow in the hot leg of a PWR.

An extensive literature review has shown that the droplet entrainment phenomenon in horizontal pipe remains an understudied field of two-phase flows. Indeed, the research forces have mainly focused in the droplet entrainment in annular flow in vertical pipes of small size. Consequently, the literature review has led to the conclusion that the existing models do not suit to the modeling of the droplet entrainment in horizontal pipes of high diameter.

Otherwise, the literature review has also highlighted that the existing experimental databases do not enable the assessment of any entrainment model at the PWR hot leg scale. A new experimental facility has therefore been designed and operated in order to provide a droplet entrainment rate database from a wavy stratified flow at a 1/3 scale of a PWR hot leg. This experimental facility has been fitted with various experimental devices in order to continuously measure the two-phase flow characteristics. The REGARD experimental facility provided thus a large database on the droplet concentration, the droplet fluxes and the liquid film height evolutions. Experimental measurements of the droplet entrainment rate from the liquid film has thus been determined, providing therefore the required experimental database in pipes of high diameters for the model assessments.

In addition to the entrainment rate measurements, the operation of the REGARD facility has enabled to achieve an extensive description of the two-phase flow behaviour in an wavy stratified flow with droplet entrainment. The two-phase flow visualizations, with an high-speed camera, have shown that the main source of droplet is the fragmentation of interfacial wave crests and the consecutive atomization of the wrested amount of liquid. Consequently, the behaviour of the liquid film has been extensively studied with a resistive probe. Numerous measurements of the time evolution of the liquid film have provided informations on the evolution of the liquid film fraction, the wave celerity and the wave frequency.

The droplet measurements have shown a huge vertical stratification of the droplet mass-flow rates and a significant influence of the developing length. Indeed, as the droplets have a much greater axial velocity than a vertical velocity, the necessary distance to rise from the wave crest atomization to the upper positions of the pipe cross-section are far greater than the PWR hot leg length. The droplet flow is therefore doubly stratified, firstly due to the inherent effect of the gravity and secondly due to the unfinished development of the flow. The space-averaging operation is thus extremely important to achieve a correct droplet entrainment measurement.

A new mechanistic model has been developed in this work to determine the entrainment rate of droplets. Based on the REGARD experimental visualizations, the droplet entrainment is primarily assumed to occur from the fragmentation of interfacial waves. Due to the velocity difference between the two-phases at the interface, Kelvin-Helmholtz instabilities develop over the interface. These waves grow until reaching an unstable amplitude at which the wave tops fragment and atomize in several droplets. The proposed model considers this phenomenological sequence to determine the droplet entrainment rate. The entrainment rate model is then based on various sub-models characterizing the instability wavelength, the amount of liquid entrained at each wave crest and the necessary time for a wave to grow to its unstable amplitude.

The sub-model characterizing the critical wavelength of the interfacial instabilities has been developed on the basis of a linear stability analysis of the two-phase stratified flow in order to determine the wavelength spectrum of the amplified instabilities. The sub-model assumes that every instabilities which develop over the interface, have a similar wavelength corresponding to the most amplified instability. The linear stability analysis is performed on the basis of

the two-fluid model, using various friction laws. The instability frequency spectrum appears to fit well the experimental data from REGARD.

The sub-model characterizing the entrained volume is developed on the basis of two force balances, one at the wave crest and one on the entrained amount of liquid. The sub-model philosophy is that the wave crest atomizes at the amplitude where the two force balances simultaneously pass from stabilizing to destabilizing. The force balance at the wave crest implies the gravity, a lift force and the surface tension in a horizontal pipe while the second force balance on the entrained amount of liquid implies the gravity and a turbulent drag force.

The sub-model characterizing the characteristic time of the entrainment corresponds to the time necessary for a wave to develop until reaching its unstable amplitude. This parameter is experimentally determined on the basis of the wave growth velocity of the interfacial instabilities in the REGARD facility.

The global model for the entrainment rate fits well the experimental data from the REGARD facility. The amount of liquid wrested-off from the liquid film per units of time and surface are correctly predicted by the proposed model which appears thus to be relevant for the modeling of entrainment in a horizontal stratified flow in a pipe of 0.24 m diameter.

In order to reduce the calculation and thus the computational time, these sub-models have been simplified in several correlations of the two-fluid physical properties, the flow conditions and the pipe geometry. These sub-correlations fit well the predictions of the sub-models and consequently, the predictions of the entrainment rate correlation fits also well the entrainment rate model predictions.

In conclusion, the initial objective has been achieved in this work. A totally new model for the entrainment rate has been designed based on a mechanistic approach of the phenomenological sequence leading to the pulling-off of droplets from the liquid film. The model assessment have been performed against an experimental facility designed at a 1/3 scale of a PWR hot leg and operated at similar flow conditions. The model has appeared relevant for the modeling of the droplet entrainment in stratified flow in horizontal pipes of large diameter. As the model has been operated over a large series of flow parameters, the final correlation appears to remain relevant for wavy stratified flow in pipes from small to large diameters for couples of fluids of physical properties near to those of air-water at 30°C and 1.1 bar.

Appendix A

Experimental Appendix

A.1 Liquid volume fraction α_{lf}

The object of this section is to detail the calculation of the liquid volume fraction α_{lf} from the time averaged three liquid level measurements $\bar{h}_l$ obtained from the BANJO data over a pipe cross-section as depicted on Figure A.1:

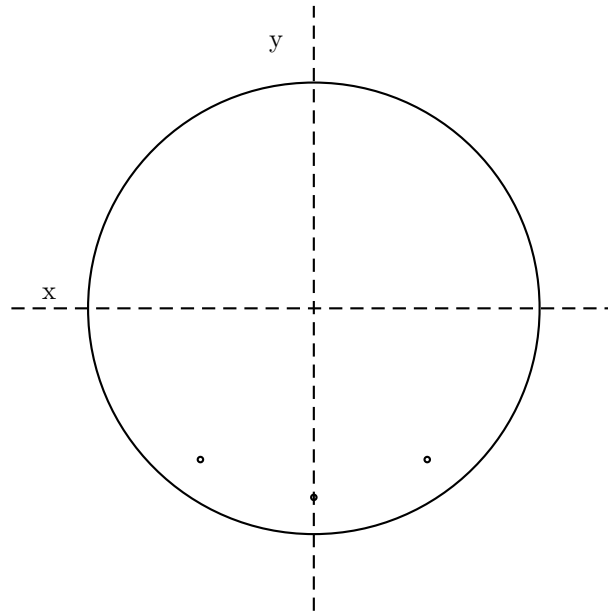


Figure A.1: Three liquid level measurements in a pipe cross-section.

The interface is assumed to be an arc of circle. Consequently the circle passing through these three positions may be determined as shown in Figure A.2:

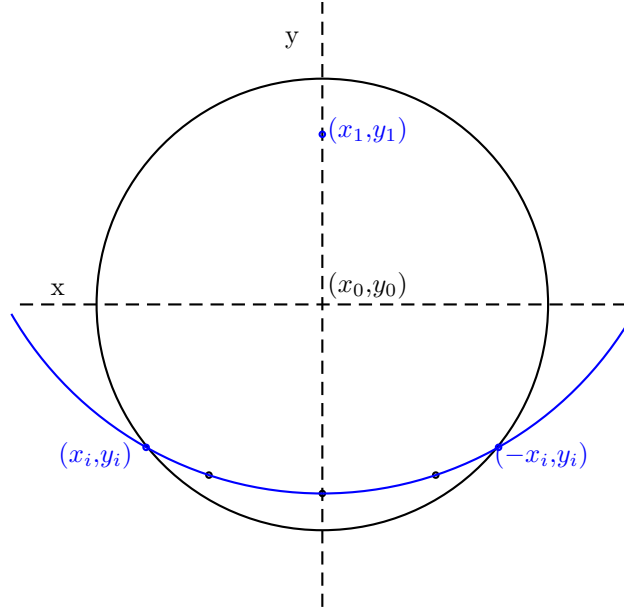


Figure A.2: Determination of the circle passing through the three liquid level measurements.

A trigonometric calculation allows to determine the coordinates of the intersection points (x_i, y_i) between the pipe circle $((x_0, y_0), R_0)$ and the extrapolated interface circle $((x_1, y_1), R_1)$:

$$(x_i - x_0)^2 + (y_i - y_0)^2 = R_0^2 \quad (\text{A.1})$$

$$(x_i - x_1)^2 + (y_i - y_1)^2 = R_1^2 \quad (\text{A.2})$$

The two sections of disc under the line defined by the two intersection points are then calculated on the basis of the two intersection angle δ_0 and δ_1 :

$$S_0 = \delta_0 \pi R_0^2 - 2 \times \frac{(x_i - x_0) \times (y_i - y_0)}{2} \quad (\text{A.3})$$

$$S_1 = \delta_1 \pi R_1^2 - 2 \times \frac{(x_i - x_1) \times (y_i - y_1)}{2} \quad (\text{A.4})$$

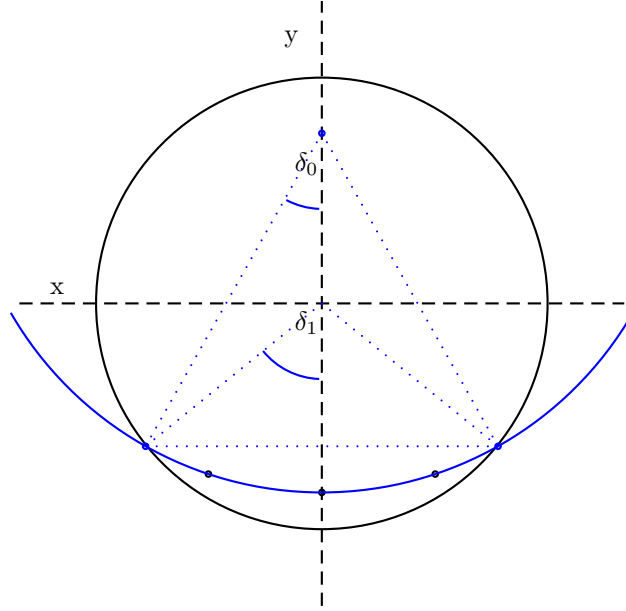


Figure A.3: Determination of the circle passing through the three liquid level measurements.

Finally the area under the interface, considered as representative of the liquid film volume fraction, is determined as the difference between the two sections:

$$\alpha_{lf} = S_0 - S_1 \quad (\text{A.5})$$

The uncertainty on the α_{lf} calculation is obtained by determining the difference in α_{lf} obtained considering the uncertainties on the liquid level measurement $\bar{h}_l$.

A.2 Isokinetic Probe

This section aims at describing the calculation methods used in the post-treatment of the data obtained with the isokinetic probe. The first paragraph deal with the integration procedure of the droplet mass flow-rates measured all over the pipe cross-section.

A.2.1 Droplet flux integration

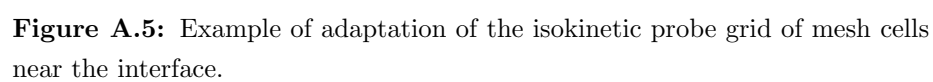
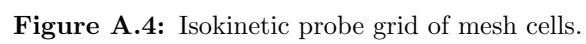
The pipe cross-section is divided as a grid of 75 mesh cells. Most of these cells have a similar size but as the mesh is refined near the walls, few mesh have a smaller size. In consequence, a different weight β_i should be attribute to these cells for the integration procedure of the local mass flow-rates $M_{d,i}$:

$$\dot{M}_d = \sum_{i=1}^{75} \frac{M_{d,i}}{t} \beta_i \frac{S_{mesh}}{S_{probe}} \quad (\text{A.6})$$

In this equation, the time t is by default equal to 180 seconds, except when the aspirated mass flow-rates of droplets was so important that the weighing machine reaches its maximum capacity. S_{mesh} is the default size of a mesh cell: $2 \times 2 \text{ cm}^2$; β_i the local correction factor for the mesh size, equaled to 1 by default; S_{probe} is the aspiration section of the isokinetic probe. Figure A.4 shows the various mesh cells in the pipe section and the local default values for the β_i correction factor.

However, the lower mesh cells may be partially under water during the measurement duration. In consequence, the position of the interface is taken into account in the β_i coefficients (from a to f) in the lower positions of the grid. The interface average position but also the interface maximum amplitude are taken into account to determine wether a mesh cell is partially flooded. The β_i coefficient is then set to zero in case of flooding. However this technique distort the validity of the integration procedure. In this case, the deleted mesh cells are replaced by new mesh cells positioned between the interface level and the lowest not-deleted cell as shown in Figure A.5:

The local mass flow-rates $M_{d,i}$ in these new mesh cells are extrapolated, following a cubic spline, from the local mass flow-rates measured in the vertical upper mesh cells, as shown in Figure A.6.



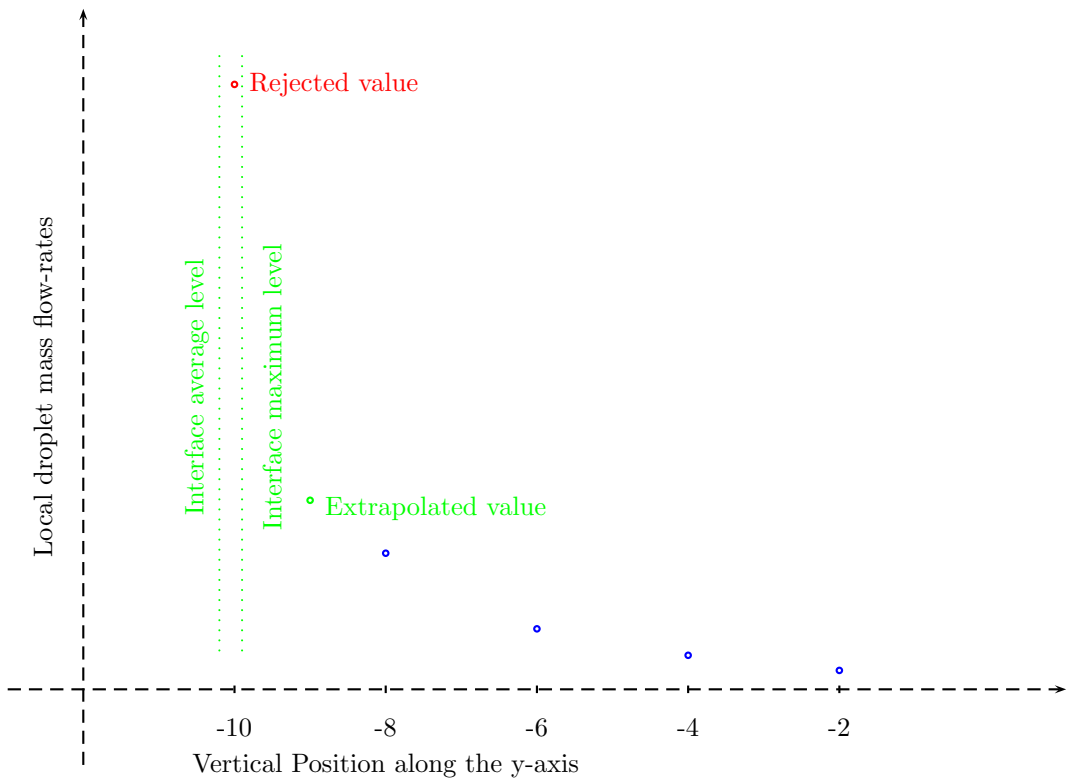


Figure A.6: Isokinetic probe: extrapolation of the local mass flow-rates.

A.3 Laser Preliminary analysis

This section aims at presenting the preliminary considerations on the laser settings. The first paragraphs are dedicated to a short description of the laser granulometry and velocimetry used to determine the droplet characteristics. These descriptions are highly inspired by the product descriptions of Dantec Dynamics [43] and TSI [152] and The laser Doppler techniques by L. Brain [48]. As a second paragraph, the laser measuring mode is analyzed.

A.3.1 Laser Doppler velocimetry (LDV) [48, 43, 152]

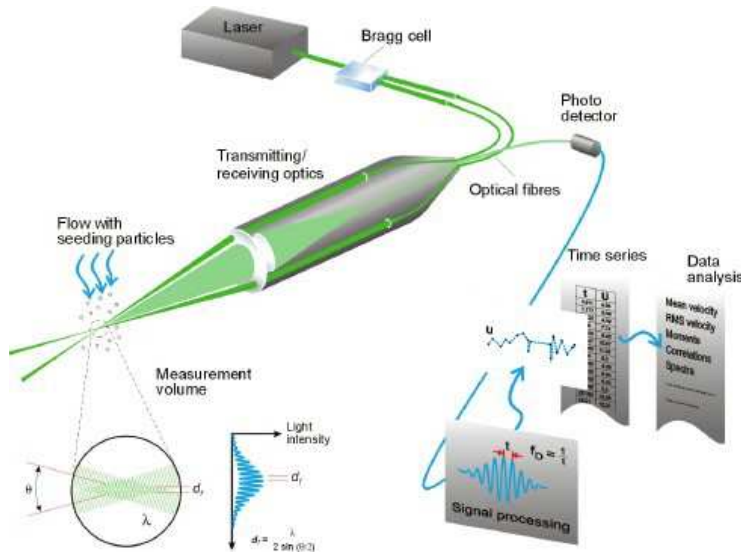


Figure A.7: Laser Doppler velocimetry: principles (Figure from Dantec Dynamics [43]).

Laser Doppler velocimetry (LDV) is the technique of using the Doppler shift in a laser beam to measure the velocity in transparent or semi-transparent fluid flows, described in Figure A.7. LDV crosses two beams of collimated, monochromatic, and coherent laser light in the flow of the fluid being measured. The two beams are usually obtained by splitting a single beam, thus ensuring coherence between the two. A transmitting optics focuses the beams to intersect at their waists (the focal point of a laser beam), where they interfere and generate a set of straight fringes. As particles (either naturally occurring or induced) entrained in the fluid pass through the fringes, they reflect light that is then collected by a receiving optics and focused on a photodetector (typically an avalanche photodiode).

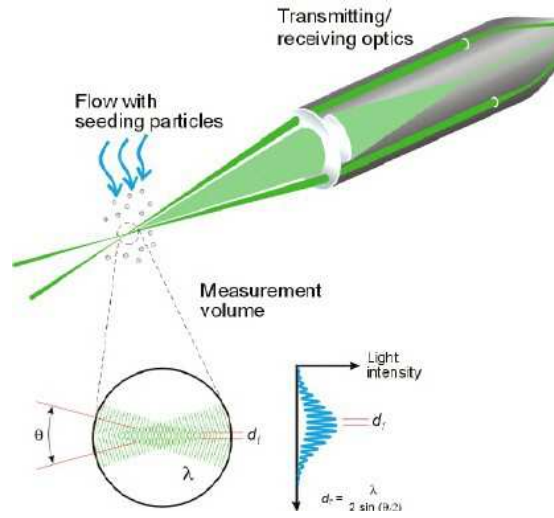


Figure A.8: Laser Doppler velocimetry: velocity measurements (Figure from Dantec Dynamics [43]).

The reflected light fluctuates in intensity, the frequency of which is equivalent to the Doppler shift between the incident and scattered light, and is thus proportional to the component of particle velocity which lies in the plane of two laser beams (see Figure A.8). If the sensor is aligned to the flow such that the fringes are perpendicular to the flow direction, the electrical signal from the photodetector will then be proportional to the full particle velocity. By combining three devices with different wavelengths, all three flow velocity components can be simultaneously measured.

A.3.2 Phase Doppler Particle Analyzer (PDPA) [48, 43, 152]

The PDPA technique is an extension of laser Doppler velocimetry and is based on light-scattering interferometry as illustrated in Figure A.9. The measurement point is defined by the intersection of two focused laser beams and the measurements are performed on single particles as they move through the sample volume. Particles thereby scatter light from both laser beams, generating an optical interference pattern. Both scattered waves interfere in space and create a beat signal with a frequency which is proportional to the velocity of the particle. A receiving optics placed at a well-chosen off-axis location projects a portion of the scattered light onto multiple detectors. Each detector converts the optical signal into a Doppler burst with a frequency linearly proportional to

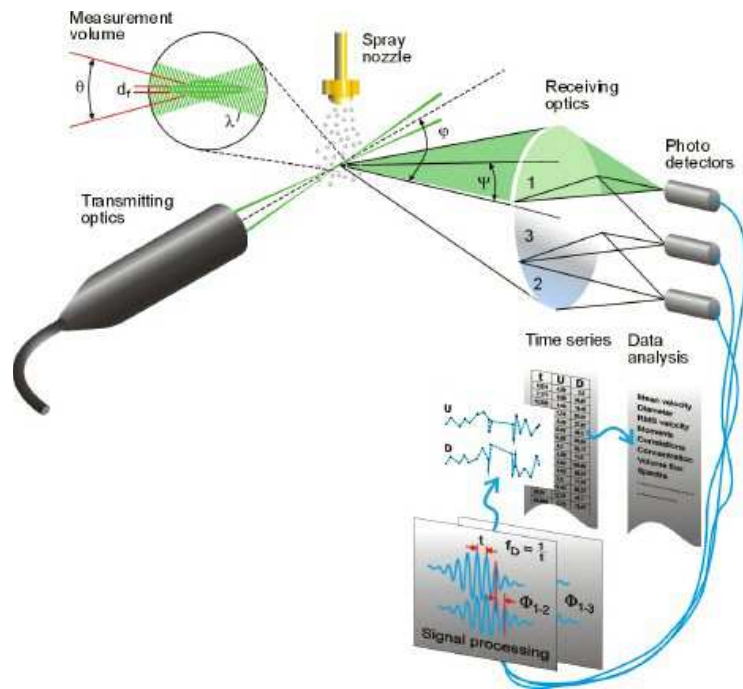


Figure A.9: Phase Doppler Particle Analyzer: principles (Figure from Dantec Dynamics [43]).

the particle velocity. The phase shift between the Doppler signals from different detectors is proportional to the diameter of the particle.

A.3.3 Laser modes

The droplet characteristics may be analyzed in two modes: with or without the beam coincidence. In the first case, a droplet is analyzed only when its movement has been simultaneously seen by the three laser probes. Each measured droplet is then characterized by three components of velocity and in most cases a diameter. In the second mode, each droplet seen by one of the three laser beams are collected. The number of partially analyzed droplets is then far greater but their characterization is not complete.

An important point of the laser analysis is to determine whether the restricted set of fully characterized droplet is representative of the more complete set of droplets not fully representative and to compare the size of the two set of datas (Table A.1). The number of droplet fully characterized is then very small compared to the number of event measured by the three laser beams: 10.75%. Moreover, the difference in number of characterized droplets seen by each mode highly depends on the mesh position as illustrated in Figure A.10

Table A.1: Example of number of droplet counted by each laser beam for one mesh cell of one experiment.

Number	Laser 1	Laser 2	Laser 3	Diameter	Total
With Coincidence	13364	13364	13364	13364	13364
Without Coincidence	52470	77509	121519	48549	124328
	25.47%	17.24%	11%	27.53%	10.75%

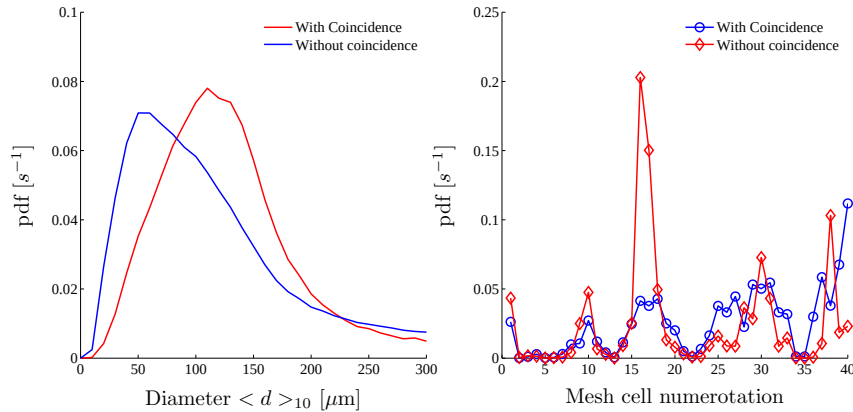


Figure A.10: Local difference of number of characterized droplet according to the laser mode.

Figures A.11 to A.14 analyze the difference in terms of velocities and diameter between the two modes. On the left graphs, the space averaged distribution are compared while on the right graphs, the average characteristics are compared on the various local mesh cells. Figure A.11 show the diameter evolution between the two modes. It appears directly that the diameter probability density function are totally different. Moreover this difference appears on quite every mesh cells. This indicates that in coincidence mode most small droplets are not counted. As the power attributed to the three beams differ, one of them is unable to see the smallest droplet and loose information. In consequence, for the analysis of the size data, the mode with coincidence is not sufficient to analyze the droplet flow.

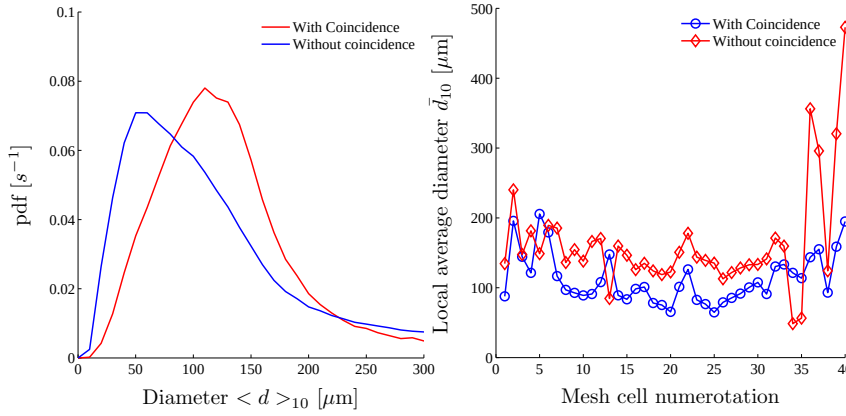


Figure A.11: Comparison of the measured diameters between the two modes.

Figure A.12 shows that the characteristics of the velocity 1 are quite identical whatever the mode selected. Both in terms of distribution or local average value.

On the opposite, Figures A.13 and A.14 show that the distribution differs between the two modes, especially for the velocity 3. But, contrarily to the diameter measurement, this difference is not homogeneously distributed on every mesh cells. Indeed, the difference comes essentially from the measurements in few points (2, 5, 6, 13, ...). An analysis of the position of these mesh cells indicates that these cells are positionned very low in the pipe cross-section, close to the interface or too high in the pipe cross-section for the number of droplet to be statistically acceptable (shown in Figure A.15). Finally, one may assume that these measurements can be considered as noise (as these droplets are seen only by one of the three beams) and neglect these droplet velocities as the local droplet dynamics at these positions is questionable. Finally, for the global study

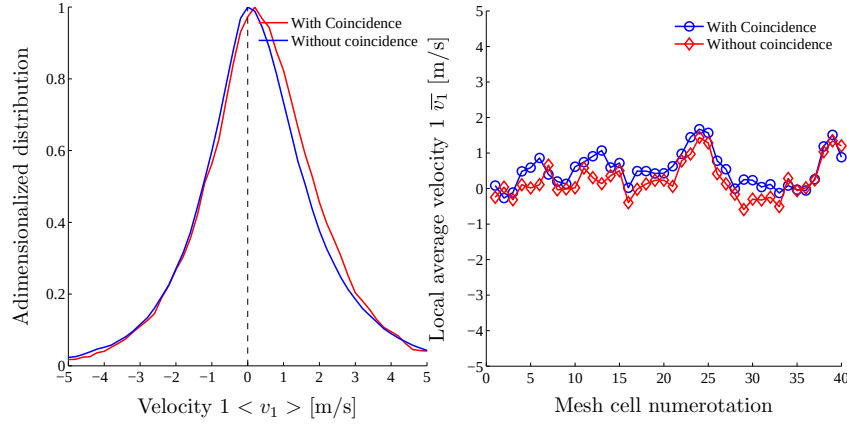


Figure A.12: Comparison of the measured velocity 1 between the two modes.

of the droplet characteristics, the set of analyzed mesh cells have been slightly reduced to avoid these measurements.

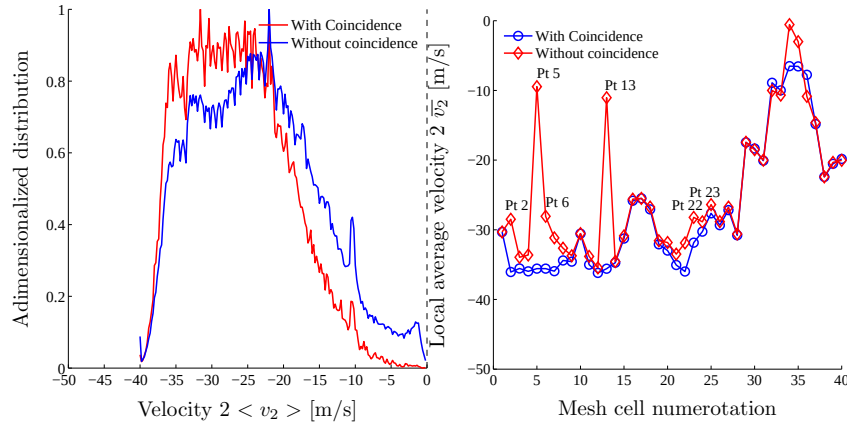


Figure A.13: Comparison of the measured velocity 2 between the two modes.

In conclusion, the coincidence mode can be used for the analyzed of the droplet velocity components while the non-coincidence mode must be used whatever the flow conditions for the analysis of the size data. Finally Figure A.16 shows the difference in terms of axial and vertical components of velocity between the droplets associated to a diameter and the droplets whose the diameter could not been measured. This indicates that droplets rejected by the granulometer as not spherical are slower that the average velocity of droplet associated to a diameter. This means that the rejected droplet may be considered as large droplets. This enforce again the problematic of the laser settings which reject larger droplets, higher contributors to the droplet volume.

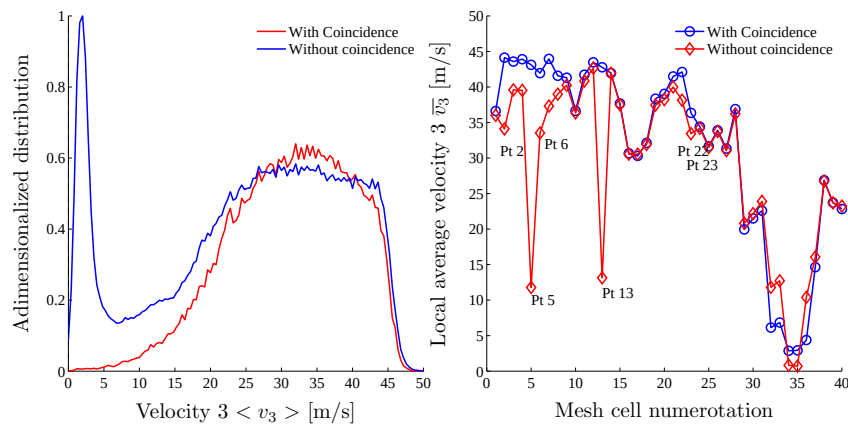


Figure A.14: Comparison of the measured velocity 3 between the two modes.

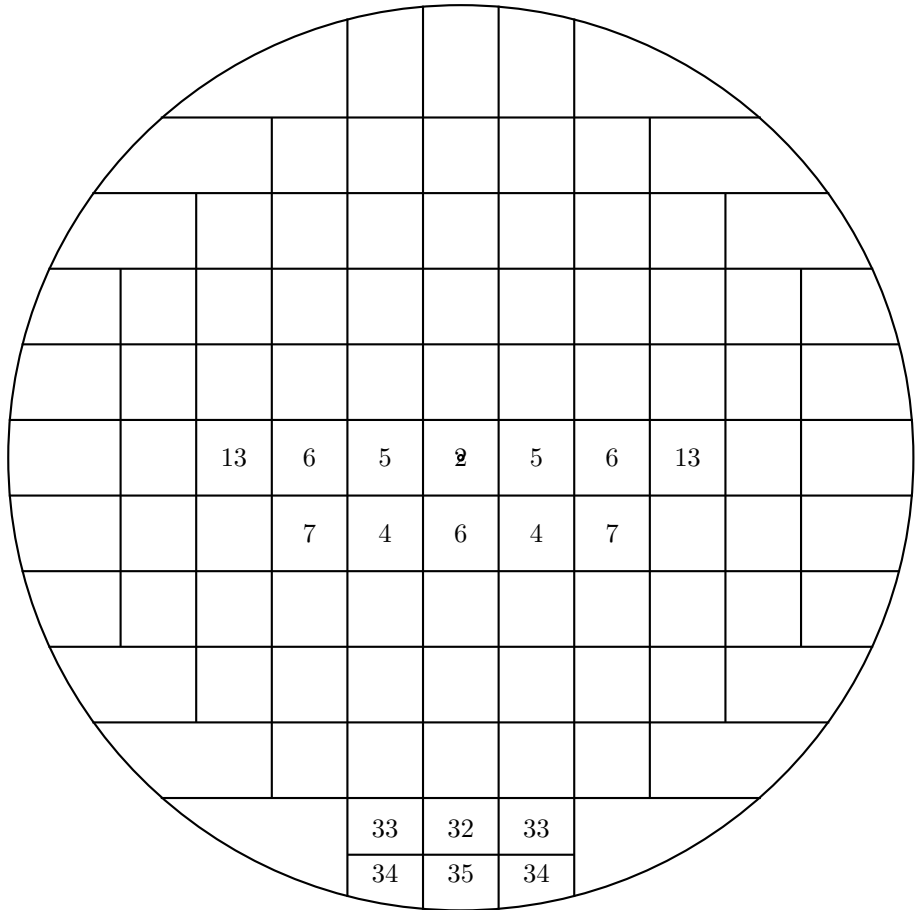


Figure A.15: Position of the mesh cells questionable in Figures A.13 and A.14.

Figure A.15 has shown that the difference in terms of droplet characteristics

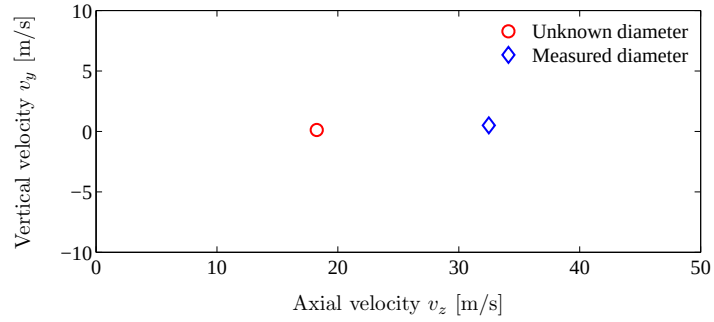


Figure A.16: Difference between droplets associated to a diameter and droplets whose the diameter has not been measured.

are particularly emphasized in few specified mesh cells. Indeed, adding these points to space average the droplet characteristics induce additional effects as shown in Figures A.17. These two graphs show that the pdf of the droplet distribution can include some additional peaks when considering the 'strange' mesh cells. Using the complete laser grid implies additional peaks which can even be the main peaks if the number of droplets is small.

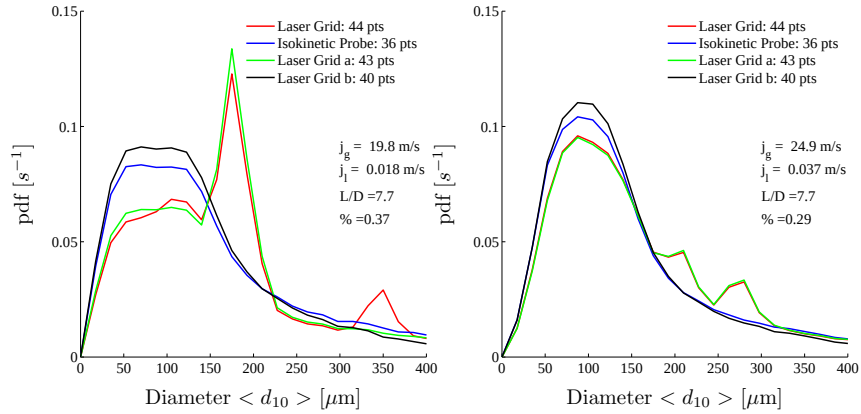


Figure A.17: Evolution of the profiles of the probability density functions as a function of the mesh grid selection (the % number indicates the number of droplets associated to a diameter value at the operating conditions).

A.4 Laser settings

The object of this paragraph is to explicit the calculation of the three orthonormal components of the droplet velocity (axial, lateral and vertical components) from the three velocity measurements made by the laser probes.

The droplet velocity instrument consists in two laser probes. The first one emits two laser beams (one green, in blue) in two perpendicular directions. This probe is fixed on the measurement structure such that the fringe pattern of the first beam (blue) is aligned with the direction of flow. Consequently, the second beam (green) emits in a plane perpendicular to the direction of the flow. The second probe emits the violet laser beam in the same longitudinal plane than the blue one, aligned with the direction of the flow. To obtain useful measurements, the angle of incidence of the two probes is obviously different.

This means that two probes emit laser beams in the same longitudinal plane allowing to deduce the axial component of velocity of the drops in the direction of flow. The combination of the informations of the three laser probes permits in theory to calculate the two others orthonormal components of velocity in a plane perpendicular to the direction of the flow.

So a first part of this section is to describe the transformation equations in a longitudinal plane, while a second part describes the transformation equations in the plane perpendicular to the direction of flow. The following of the section describes the calculation of uncertainties on these measures.

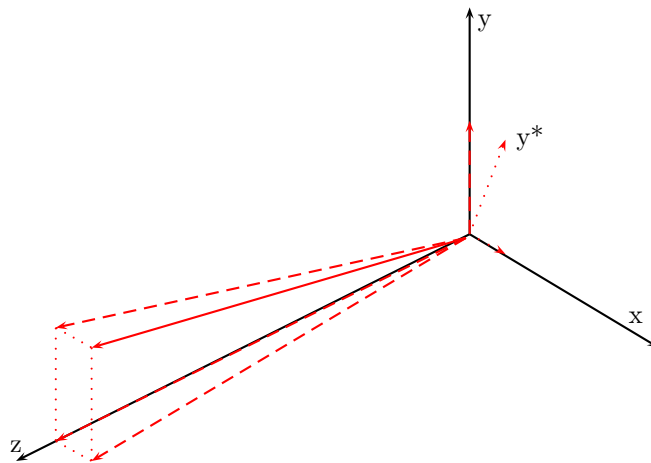


Figure A.18: Direction of a droplet and orthonormal components of velocity

A.4.1 In a longitudinal plane

In the longitudinal plane, two laser beams are emitted by the probes lasers: the blue beam and the violet beam. Tabular A.2 refers the two angles of incidence of the laser beams while Figure A.19 represents the velocity components measured by the two probes in this plane. The blue beam thus measures a negative velocity and the violet a positive one.

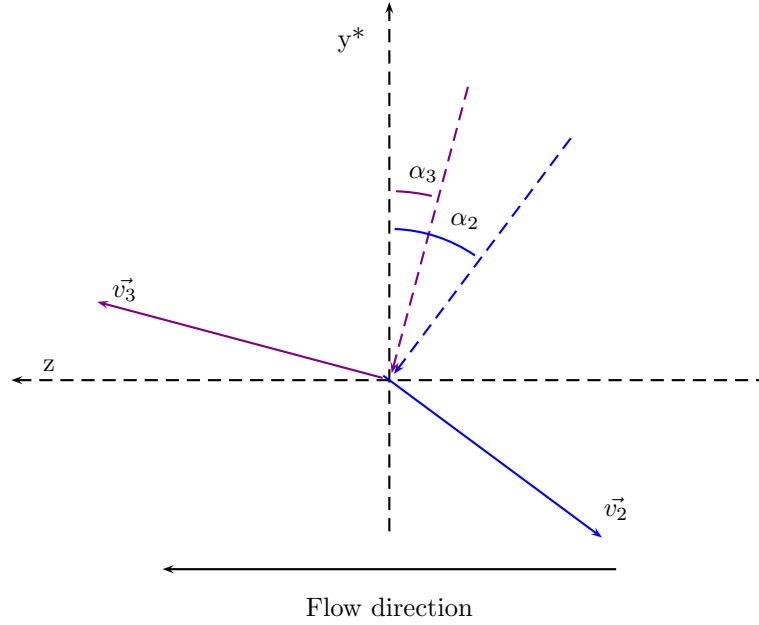


Figure A.19: Velocities measured in the longitudinal plane defined by the z-axis and the y^* -axis

Figure A.20 shows the first step of the reconstruction of the two orthonormal components of velocity in this plane. The two measures of velocity made by the lasers system corresponds to projections of the velocity $\vec{v}$ on the two directions perpendicular to the laser beams. While the velocity $\vec{v}$ is itself a projection of the droplet velocity $\vec{\omega}$ on the longitudinal plane. The projected velocity $\vec{v}$ is characterized by an amplitude v and a angle θ . Equations A.8 to A.13

Table A.2: Positionning angles of the laser probes

α	β
36.5°	15°

aim at describing the calculations required to reconstruct those orthonormal

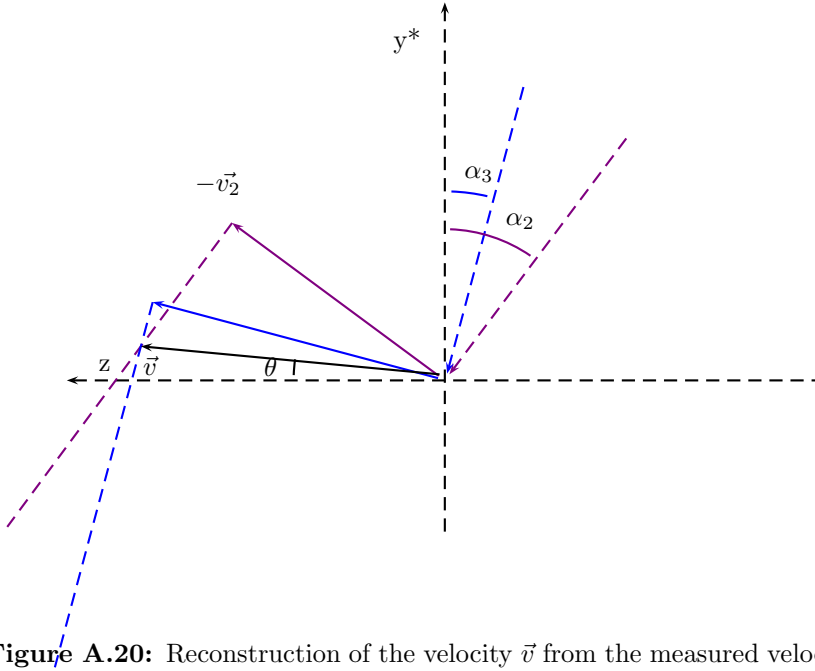


Figure A.20: Reconstruction of the velocity $\vec{v}$ from the measured velocities in the longitudinal plane defined by the z-axis and the y^* -axis

components from the indication of velocity made by the laser probes. The velocities measured by the laser probes correspond to the projection of the droplet velocity on the direction perpendicular to the laser beam. This provide a matrix system (Equation A.8) with two unknowns (the amplitude and the angle of the projection of the droplet velocity in the longitudinal plane v).

$$v_2 = v \cos(\alpha_2 - \theta) \quad (\text{A.7})$$

$$v_3 = v \cos(\alpha_3 - \theta) \quad (\text{A.8})$$

The following equations describe the calculation procedure to determine this two unknowns. The first step consists to expand the cosines difference (Equation A.10) while the second (Equation A.11) permits to eliminate one of the two unknowns (the amplitude v) from the system.

$$v_2 = v (\cos \alpha_2 \cos \theta + \sin \alpha_2 \sin \theta) \quad (\text{A.9})$$

$$v_3 = v (\cos \alpha_3 \cos \theta + \sin \alpha_3 \sin \theta) \quad (\text{A.10})$$

$$\frac{v_2}{v_3} = \frac{\cos \alpha_2 \cos \theta + \sin \alpha_2 \sin \theta}{\cos \alpha_3 \cos \theta + \sin \alpha_3 \sin \theta} \quad (\text{A.11})$$

The last step consists in isolating the unknown θ from:

$$\frac{v_2}{v_3} = \frac{\cos \alpha_2 + \sin \alpha_2 \tan \theta}{\cos \alpha_3 + \sin \alpha_3 \tan \theta} \quad (\text{A.12})$$

$$\tan \theta = \frac{v_3 \cos \alpha_2 - v_2 \cos \alpha_3}{v_2 \sin \alpha_3 - v_3 \sin \alpha_2} \quad (\text{A.13})$$

Based on the calculation of the angle θ , the amplitude of the projected velocity is:

$$v = \frac{v_2}{\cos(\alpha_2 - \theta)} \quad (\text{A.14})$$

This provides the axial component of velocity and a velocity component perpendicular to the flow direction:

$$v_{axial} = v \cos \theta \quad (\text{A.15})$$

$$v_{perp.} = v \sin \theta \quad (\text{A.16})$$

A.4.2 In a perpendicular plane

In the plane perpendicular to the flow direction (plane x/y), two components of velocity are available: the green laser measured component and the reconstructed component of velocity (v) from the previous paragraph. These two components are perpendicular as shown in Figure A.21. The velocities are measured with an angle defined by the position of the mesh cells analyzed (x_γ/y_γ), reached by the rotation of the instrumented segment.

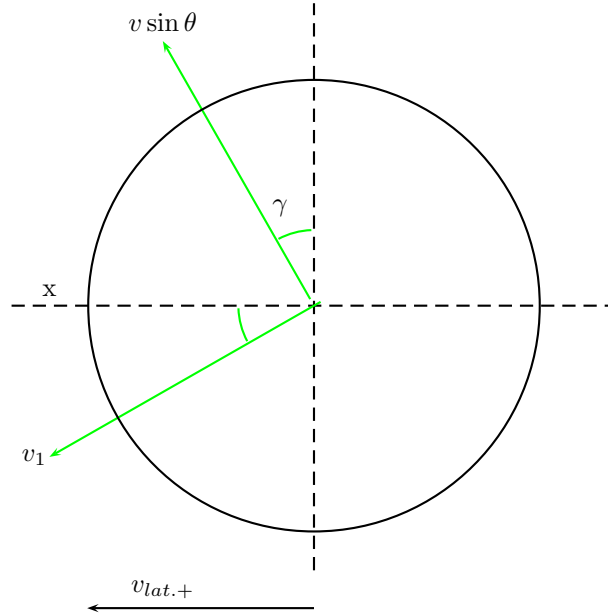


Figure A.21: Velocities measured in the perpendicular plane to the flow

The granulometer rotation is provided by the mesh position of the laser coincidence. With the associated coordinates, the rotation angle of the laser

instruments may be determined by Equation A.17 as illustrated in Figure A.22:

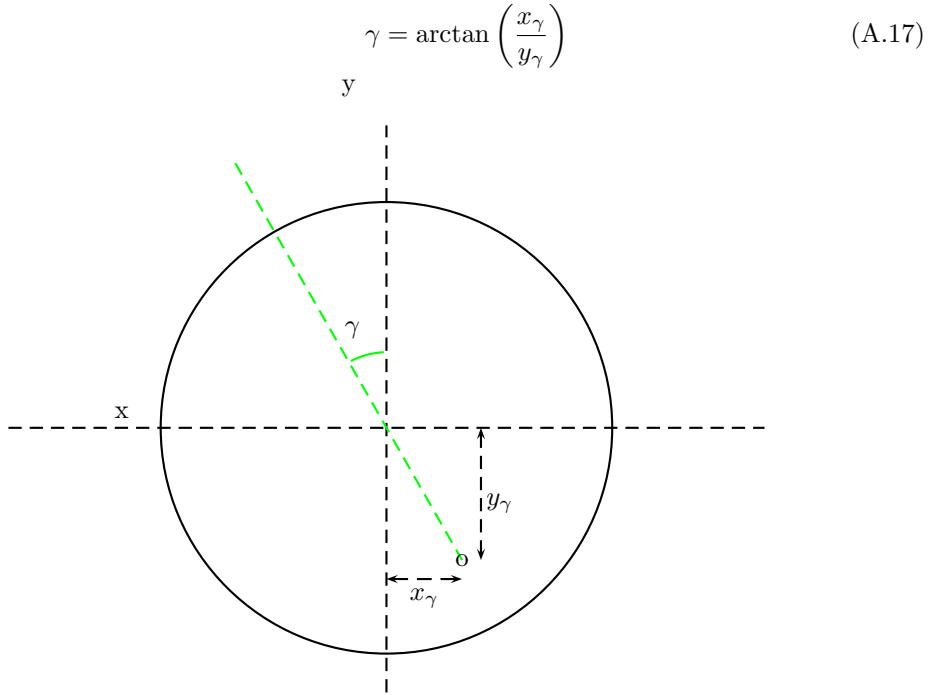


Figure A.22: Velocities measured in the perpendicular plane to the flow

This vector reconstruction is detailed on Figure A.23:

Indeed the reconstructed velocity ω appears in the formulation of the two velocities v_0 (obtained from v) and v_1 :

$$v \sin(\theta) = v_0 = \omega \cos\left(\frac{\pi}{2} - \phi - \gamma\right) \quad (\text{A.18})$$

$$v_1 = \omega \cos(\phi + \gamma) \quad (\text{A.19})$$

Various trigonometric simplifications lead to the velocity decomposition of Equation A.23:

$$v_0 = \omega \sin(\phi + \gamma) \quad (\text{A.20})$$

$$v_1 = \omega \cos(\phi + \gamma) \quad (\text{A.21})$$

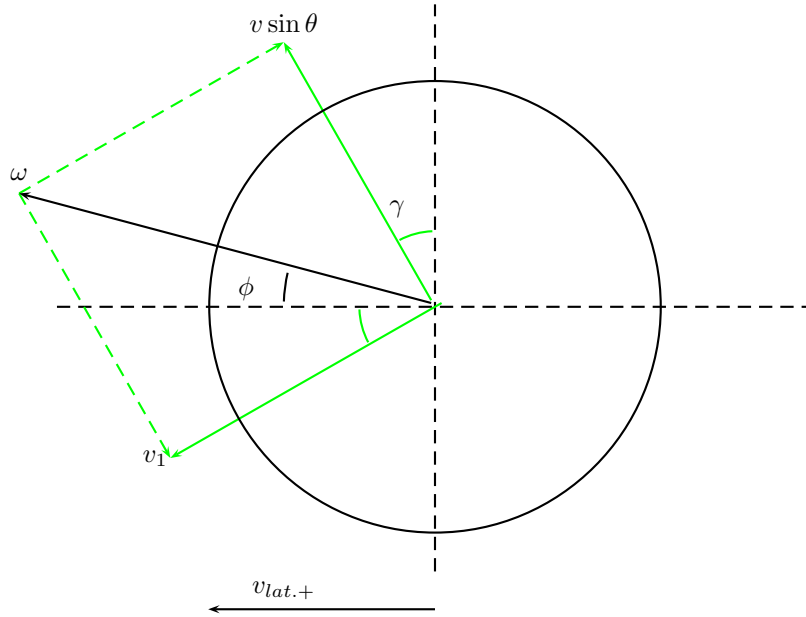


Figure A.23: Velocities measured in the perpendicular plane to the flow

$$v_0 = \omega (\sin \phi \cos \gamma + \sin \gamma \cos \phi) \quad (\text{A.22})$$

$$v_1 = \omega (\cos \phi \cos \gamma - \sin \gamma \sin \phi) \quad (\text{A.23})$$

$$v_0 (\cos \phi \cos \gamma - \sin \gamma \sin \phi) = v_1 (\sin \phi \cos \gamma + \sin \gamma \cos \phi) \quad (\text{A.24})$$

$$v_0 (\cos \gamma - \sin \gamma \tan \phi) = v_1 (\tan \phi \cos \gamma + \sin \gamma) \quad (\text{A.25})$$

The angle ϕ may then be expressed as a function of the measured velocities and the imposed rotational angle γ :

$$\tan \phi = \frac{v_0 \cos \gamma - v_1 \sin \gamma}{v_1 \cos \gamma + v_0 \sin \gamma} \quad (\text{A.26})$$

The velocity ω may then be directly expressed as a function of the measured velocity v_1 , the geometric angle γ and the calculated angle ϕ .

$$\omega = \frac{v_1}{\cos(\phi + \gamma)} \quad (\text{A.27})$$

Finally the lateral x-component and the vertical y-component of velocity are calculated from ω and ϕ :

$$v_{x.} = \omega \cos \phi \quad (\text{A.28})$$

$$v_{y.} = \omega \sin \phi \quad (\text{A.29})$$

$$= v \sin \theta \sin \phi \quad (\text{A.30})$$

A.4.3 Uncertainty calculation

Based on the standard deviation approach of the uncertainty propagation, the uncertainty on the calculation of the tangent of the angle θ is:

$$\begin{aligned} \epsilon_{\tan \theta} = \tan \theta \times & \left[\frac{\sqrt{(\cos \alpha_2 \epsilon_{v_3})^2 + (-v_3 \sin \alpha_2 \epsilon_{\alpha_2})^2 + (\cos \alpha_3 \epsilon_{v_2})^2 + (-v_2 \sin \alpha_3 \epsilon_{\alpha_3})^2}}{v_3 \cos \alpha_2 - v_2 \cos \alpha_3} \right] \\ & + \tan \theta \times \left[\frac{\sqrt{(\sin \alpha_3 \epsilon_{v_2})^2 + (v_2 \cos \alpha_3 \epsilon_{\alpha_3})^2 + (\sin \alpha_2 \epsilon_{v_3})^2 + (v_3 \sin \alpha_2 \epsilon_{\alpha_2})^2}}{v_2 \sin \alpha_3 - v_3 \sin \alpha_2} \right] \end{aligned} \quad (\text{A.31})$$

The uncertainty on the angle θ may then be expressed from the uncertainty on the tangent $\epsilon_{\tan \theta}$ such that:

$$\epsilon_{\theta} = \sqrt{\left(\frac{1}{\left(\frac{v_3 \cos \alpha_2 - v_2 \cos \alpha_3}{v_2 \sin \alpha_3 - v_3 \sin \alpha_2} \right)^2 + 1} \right)^2 \epsilon_{\tan \theta}^2} \quad (\text{A.32})$$

The uncertainty on the reconstructed velocity v in the longitudinal plane (z/y^*) is thus:

$$\epsilon_v = \sqrt{(\epsilon_2 \cos(36.5 - \theta))^2 + \left(-v_2 \sin(36.5 - \theta) \sqrt{\epsilon_{\alpha_2}^2 + \epsilon_{\theta}^2} \right)^2} \quad (\text{A.33})$$

Such as the uncertainties on the axial component v_z and the laser oriented component v_{y*} may be calculated:

$$\epsilon_{v_z} = \sqrt{(\epsilon_v \cos \theta)^2 + (-v \sin \theta \epsilon_{\theta})^2} \quad (\text{A.34})$$

$$\epsilon_{v_{y*}} = \sqrt{(\epsilon_v \sin \theta)^2 + (v \cos \theta \epsilon_{\theta})^2} \quad (\text{A.35})$$

The uncertainties on the vertical z -component and the lateral x -component of velocity may then be estimated from the uncertainties on the angle γ (dependent on the precision of the mesh positioning), on the measured velocity v_1 , on the calculated velocity v_0 and then on the relative angle ϕ :

$$\epsilon_{\gamma} = \sqrt{\left(\frac{y_{\gamma}}{x_{\gamma}^2 + y_{\gamma}^2} \epsilon_{y_{\gamma}}^2 \right)^2 + \left(\frac{-x_{\gamma}}{x_{\gamma}^2 + y_{\gamma}^2} \epsilon_{x_{\gamma}}^2 \right)^2} \quad (\text{A.36})$$

$$\begin{aligned} \epsilon_{\tan \phi} = & \tan \phi \left[\frac{\sqrt{(\cos \gamma \epsilon_{v_0})^2 + (-\sin \gamma v_0 \epsilon_\gamma)^2 + (\sin \gamma \epsilon_{v_1})^2 + (\cos \gamma v_1 \epsilon_\gamma)^2}}{v_0 \cos \gamma - v_1 \sin \gamma} \right] \\ & + \tan \phi \left[\frac{\sqrt{(\cos \gamma \epsilon_{v_1})^2 + (\cos \gamma v_0 \epsilon_\gamma)^2 + (\sin \gamma \epsilon_{v_0})^2 + (-\sin \gamma v_1 \epsilon_\gamma)^2}}{v_1 \cos \gamma + v_0 \sin \gamma} \right] \end{aligned} \quad (\text{A.37})$$

$$\epsilon_\phi = \sqrt{\left(\frac{1}{\left(\frac{v_0 \cos \gamma - v_1 \sin \gamma}{v_1 \cos \gamma + v_0 \sin \gamma} \right)^2 + 1} \right)^2} \epsilon_{\tan \phi}^2 \quad (\text{A.38})$$

$$\epsilon_\omega = \omega \sqrt{\left(\frac{\epsilon_{v_1}}{v_1} \right)^2 + \left(\frac{-\sin(\phi + \gamma) (\epsilon_\phi + \epsilon_\gamma)}{\cos(\phi + \gamma)} \right)^2} \quad (\text{A.39})$$

Finally the uncertainties on the lateral and vertical component of the droplet velocity are:

$$\epsilon_{v_x} = \sqrt{(\epsilon_\omega \cos \phi)^2 + (\omega \sin \phi \epsilon_\phi)^2} \quad (\text{A.40})$$

$$\epsilon_{v_y} = \sqrt{(\epsilon_\omega \sin \phi)^2 + (\omega \cos \phi \epsilon_\phi)^2} \quad (\text{A.41})$$

Table A.3: Order of magnitude of the uncertainties on the velocity components

	ϵ_{v_x}	ϵ_{v_y}	ϵ_{v_z}
	5-10%	150%	250%

A.5 Detailed results

$\frac{L}{D}$	air			Liquid film			Liquid droplets			Diameter			E
	$\dot{V}_g$ m_N^3/h	$\langle \bar{v} \rangle_g$ m/s	α_g —	$\dot{V}_f$ m^3/h	v_f m/s	α_f —	$\dot{V}_d$ m^3/h	v_d m/s	α_d —	$\langle \bar{d} \rangle$ μm	d_{32} μm	$\dot{V}_{hum.}$ m^3/h	
3.8	3100	18.94	0.9684	3	0.5856	0.0315	0.0221	17.4	0.078 e-4	199.8	392.2	0.0104	
5.7	3100	18.93	0.9687	3	0.5902	0.0313	0.0214	18.9	0.070 e-4	218.5	316.6	0.0111	
7.7	3100	18.93	0.9686	3	0.5891	0.0312	0.0213	20.9	0.063 e-4	185.2	298.4	0.0111	
9.6	3100	18.93	0.9685	3	0.5886	0.0312	0.0284	23.6	0.074 e-4	187.1	305.7	0.0221	
11.6	3100	18.93	0.9686	3	0.5904	0.0311	0.0288	23.5	0.075 e-4	161.1	337.6	0.0161	
3.8	3900	23.81	0.9691	3	0.6017	0.0306	0.0536	19.4	0.170 e-4	195.1	385.0	0.0132	
5.7	3900	23.81	0.9691	3	0.6031	0.0306	0.0632	21.9	0.177 e-4	187.1	321.1	0.0111	
7.7	3900	23.80	0.9694	3	0.6115	0.0305	0.0724	22.9	0.194 e-4	165.3	301.2	0.0149	
9.6	3900	23.81	0.9689	3	0.6015	0.0304	0.0830	24.7	0.207 e-4	179.3	300.2	0.0144	
11.6	3900	23.81	0.9691	3	0.6064	0.0303	0.0826	26.8	0.189 e-4	163.3	437.0	0.0118	
3.8	5400	32.95	0.9696	3	0.6248	0.0295	0.1508	24.3	0.382 e-4	167.4	397.5	0.0219	
5.7	5400	32.95	0.9697	3	0.6299	0.0295	0.1754	25.2	0.428 e-4	166.8	475.4	0.0238	
7.7	5400	32.97	0.9688	3	0.6138	0.0294	0.1904	27.2	0.430 e-4	141.0	380.0	0.0195	
9.6	5400	32.95	0.9696	3	0.6301	0.0295	0.1969	26.1	0.463 e-4	162.5	373.7	0.0227	
11.6	5400	32.96	0.9693	3	0.6244	0.0296	0.1962	26.1	0.462 e-4	142.9	417.1	0.0188	
3.8	6000	36.61	0.9695	3	0.6335	0.0291	0.2313	26.1	0.544 e-4			0.0279	
5.7	6000	36.62	0.9694	3	0.6324	0.0291	0.2414	26.4	0.562 e-4	143.9	440.5	0.0177	
7.7	6000	36.62	0.9693	3	0.6350	0.0291	0.2810	31.2	0.552 e-4			0.0262	
9.6	6000	36.62	0.9692	3	0.6325	0.0291	0.2767	28.7	0.592 e-4	169.1	391.2	0.0203	
11.6	6000	36.63	0.9691	3	0.6333	0.0291	0.2928	31.2	0.565 e-4	133.1	424.3	0.0153	

Table A.4: Summary of measured and calculated flow parameters at the liquid superficial velocity $j_l = 0.018$ m/s for every axial locations and variations of the air superficial velocity.

Appendix B

Model Appendix

B.1 Linear stability analysis: details

The studied problem is the modeling of a 1D stratified two-phase flow in an horizontal pipe in order to predict the frequency spectrum of the instabilities developing at the interface. A linear stability analysis is performed on the conservation equations governing the two-fluid approach.

B.2 Conservation equation

Obtaining the basis forms of the governing equations is extensively described in this annex. The basic theorems of Leibniz and the divergence are firstly remind before describing the governing conservation equations of mass and momentum. As the two fluids are assumed to be not-expandable, the conservation equation of energy is decouple and no analyze in this work.

Leibniz Theorem

$$\frac{\partial}{\partial t} \int_A f dA = \int_A \frac{\partial f}{\partial t} dA + \int_C f \vec{v}_{A_w} \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \quad (\text{B.1})$$

$$\Rightarrow \int_A \frac{\partial f}{\partial t} dA = \frac{\partial}{\partial t} \int_A f dA - \int_C f \vec{v}_{A_w} \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \quad (\text{B.2})$$

Divergence Theorem

$$\int_A \nabla \cdot \vec{f} dA = \frac{\partial}{\partial z} \int_A f_z dA + \int_C \vec{f} \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \quad (\text{B.3})$$

B.2.1 Mass conservation equations

From the general instantaneous formulation of a conservation equation:

$$\frac{\partial}{\partial t} (\rho_k \Psi_k) + \nabla \cdot (\rho_k \Psi_k \vec{v}_k) + \nabla \cdot \vec{J}_k - \rho_k \Phi_k = 0 \quad (\text{B.4})$$

with:

$$\Psi_k = 1 \quad J_k = 0 \quad \Phi_k = 0 \quad (\text{B.5})$$

$$\frac{\partial}{\partial t} \rho_k + \nabla \cdot (\rho_k \vec{v}_k) = 0 \quad (\text{B.6})$$

The integration over the cross-section of the phase k (A_k) leads to:

$$\int_{A_k} \frac{\partial}{\partial t} \rho_k dA + \int_{A_k} \nabla \cdot (\rho_k \vec{v}_k) dA = 0 \quad (\text{B.7})$$

Various manipulations on Equation B.7 simplify the conservation equation of mass such as:

Applying the theorem of Leibniz (Equation B.2) and the theorem of the Divergence (Equation B.3) ¹, the conservation Equation (B.7) becomes:

$$\begin{aligned} & \frac{\partial}{\partial t} \int_{A_k} \rho_k dA - \int_C \rho_k \vec{v}_{A_w} \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,\tau}} \\ & + \frac{\partial}{\partial z} \int_A (\rho_k \vec{v}_k)_z dA + \int_C (\rho_k \vec{v}_k) \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,\tau}} = 0 \end{aligned} \quad (\text{B.8})$$

Applying various average operators assuming that the fluid density is homogeneous over the phase or that the velocity profile of $\vec{v}_k$ can be averaged by a component ω_k such as:

$$\frac{\partial}{\partial t} \int_{A_k} \rho_k dA = \frac{\partial}{\partial t} (A_k < \rho_k >_k) \quad (\text{B.9})$$

$$\frac{\partial}{\partial z} \int_A (\rho_k \vec{v}_k)_z dA = \frac{\partial}{\partial z} (A_k < \rho_k \omega_k >_k) \quad (\text{B.10})$$

1

$$\begin{aligned} & \frac{\partial}{\partial t} \int_A f dA = \int_A \frac{\partial f}{\partial t} dA + \int_C f \vec{v}_{A_w} \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\ \Rightarrow & \int_A \frac{\partial f}{\partial t} dA = \frac{\partial}{\partial t} \int_A f dA - \int_C f \vec{v}_{A_w} \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\ & \int_A \nabla \cdot \vec{f} dA = \frac{\partial}{\partial z} \int_A f_z dA + \int_C \vec{f} \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \end{aligned}$$

Modifying the transfer terms at the walls so that:

$$\begin{aligned}
& - \int_C \rho_k \vec{v}_{Aw} \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} + \int_C (\rho_k \vec{v}_k) \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
& = \int_C (\rho_k (\vec{v}_k - \vec{v}_{Aw})) \cdot \vec{n} \\
& = \int_{\xi(z,t)} (\rho_k (\vec{v}_k - \vec{v}_{Aw})) \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
& + \int_{\xi_k(z,t)} (\rho_k (\vec{v}_k - \vec{v}_{Aw})) \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}}
\end{aligned} \tag{B.11}$$

where $\xi_k(z, t)$ denotes the interface between the phase k and the pipe wall and $\xi(z, t)$ the interface between the two phases. The first term disappears by considering the walls to be not porous. The mass transfer between phase may also be neglected for the linear stability analysis.

Assuming the pipe section A to be a constant.

Finally, Equation (B.8) becomes:

$$\boxed{\frac{\partial}{\partial t} (\alpha_k < \rho_k >) + \frac{\partial}{\partial z} (\alpha_k < \rho_k \omega_k >) = 0} \tag{B.12}$$

B.2.2 Momentum equations

From the general instantaneous formulation of a conservation equation:

$$\frac{\partial}{\partial t} (\rho_k \Psi_k) + \nabla \cdot (\rho_k \Psi_k \vec{v}_k) + \nabla \cdot \vec{J}_k - \rho_k \Phi_k = 0 \tag{B.13}$$

With the particular variables associated to the momentum equations:

$$\Psi_k = w_k \quad J_k = -\vec{T}_k = -\left(\vec{\tau}_k - p_k \vec{1}\right) \quad \Phi_k = \vec{g} \tag{B.14}$$

Equation B.13 becomes:

$$\frac{\partial}{\partial t} (\rho_k w_k) + \nabla \cdot (\rho_k w_k \vec{v}_k) + \nabla \cdot (-\vec{T}_k) - \rho_k \vec{g} = 0 \tag{B.15}$$

By integrating over the phase cross-section A_k :

$$\int_{A_k} \frac{\partial}{\partial t} (\rho_k w_k) dA + \int_{A_k} \nabla \cdot (\rho_k w_k \vec{v}_k) dA = \int_{A_k} \nabla \cdot \vec{T}_k dA + \int_{A_k} \rho_k \vec{g}_z dA = 0 \tag{B.16}$$

and applying Leibniz (Equation B.1) and the Divergence theorem over a section (Equation B.3)², each term of Equation B.16 becomes:

$$\begin{aligned}
 \int_{A_k} \frac{\partial}{\partial t} (\rho_k w_k) dA &= \frac{\partial}{\partial t} \int_{A_k} (\rho_k w_k) dA - \int_C \rho w_k \vec{v}_{kAw} \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
 \int_{A_k} \nabla \cdot (\rho_k w_k \vec{v}_k) dA &= \frac{\partial}{\partial z} \int_{A_k} (\rho_k w_k \vec{v}_k)_z dA + \int_C (\rho_k w_k \vec{v}_k) \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
 \int_{A_k} \nabla \cdot \vec{T}_k dA &= \frac{\partial}{\partial z} \int_{A_k} (\vec{T}_k)_z dA + \int_C \vec{T}_k \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
 \int_{A_k} \rho_k \vec{g}_z dA &= \int_{A_k} \rho_k \vec{g}_z dA
 \end{aligned} \tag{B.17}$$

The conservation equation of momentum is therefore:

$$\begin{aligned}
 &\frac{\partial}{\partial t} \int_{A_k} (\rho_k w_k) dA - \int_C \rho w_k \vec{v}_{kAw} \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
 &+ \frac{\partial}{\partial z} \int_{A_k} (\rho_k w_k \vec{v}_k)_z dA + \int_C (\rho_k w_k \vec{v}_k) \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
 &+ \frac{\partial}{\partial z} \int_{A_k} (\vec{T}_k)_z dA + \int_C \vec{T}_k \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
 &+ \int_{A_k} \rho_k \vec{g}_z dA = 0
 \end{aligned} \tag{B.18}$$

This formulation can be simplified by assuming several simplifications such as:

Applying the average operators:

$$\begin{aligned}
 \frac{\partial}{\partial t} \left[\int_{A_k} (\rho_k w_k) dA \right] &= \frac{\partial}{\partial t} \left[\langle \rho_k w_k \rangle_k \int_{A_k} dA \right] = \frac{\partial}{\partial t} (A_k \langle \rho_k w_k \rangle_k) \\
 \frac{\partial}{\partial z} \left[\int_{A_k} (\rho_k w_k \vec{v}_k)_z dA \right] &= \frac{\partial}{\partial z} \left[\langle \rho_k w_k w_k \rangle_k \int_{A_k} dA \right] \\
 &= \frac{\partial}{\partial z} (A_k \langle \rho_k w_k^2 \rangle_k) \\
 \int_{A_k} \rho_k \vec{g}_z dA &= \langle \rho_k \vec{g} \rangle_k \int_{A_k} dA = A_k \langle \rho_k g_z \rangle_k
 \end{aligned} \tag{B.19}$$

²

$$\begin{aligned}
 \frac{\partial}{\partial t} \int_A f dA &= \int_A \frac{\partial f}{\partial t} dA + \int_C f \vec{v}_{Aw} \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
 \Rightarrow \int_A \frac{\partial f}{\partial t} dA &= \frac{\partial}{\partial t} \int_A f dA - \int_C f \vec{v}_{Aw} \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
 \int_A \nabla \cdot \vec{f} dA &= \frac{\partial}{\partial z} \int_A f_z dA + \int_C \vec{f} \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}}
 \end{aligned}$$

Equation B.18 evolves to:

$$\begin{aligned}
& \frac{\partial}{\partial t} (A_k < \rho_k w_k >_k) + \frac{\partial}{\partial z} (A_k < \rho_k w_k^2 >_k) - A_k < \rho_k g_z >_k \\
& - \int_C \rho w_k \vec{v}_{kA_w} \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} + \int_C (\rho_k w_k \vec{v}_k) \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
& + \frac{\partial}{\partial z} \int_{A_k} \left(\vec{T}_k \right)_z dA + \int_C \vec{T}_k \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} = 0
\end{aligned} \tag{B.20}$$

Simplifying the equation by splitting the terms containing $\vec{T}_k$:

$$\begin{aligned}
\frac{\partial}{\partial z} \int_{A_k} \left(\vec{T}_k \right)_z dA &= \frac{\partial}{\partial z} \int_{A_k} \left(- \left(\vec{\tau}_k - p_k \vec{1} \right) \right)_z dA \\
&= \frac{\partial}{\partial z} \int_{A_k} \left(p_k \vec{1} \right)_z dA - \frac{\partial}{\partial z} \int_{A_k} \left(\vec{\tau}_k \right)_z dA \\
&= \frac{\partial}{\partial z} < p_k >_k \int_{A_k} dA - \frac{\partial}{\partial z} < (\vec{n}_k \cdot \tau_k) \cdot \vec{n}_z >_k \int_{A_k} dA \\
&= \frac{\partial}{\partial z} (A_k < p_k >_k) - \frac{\partial}{\partial z} (A_k < (\vec{n}_k \cdot \tau_k) \cdot \vec{n}_z >_k)
\end{aligned} \tag{B.21}$$

$$\begin{aligned}
\int_C \vec{T}_k \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} &= \int_{\xi(z,t)} \vec{T}_k \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,\tau}} + \int_{\xi_k(z,t)} \vec{T}_k \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
&= \int_{\xi(z,t)} \vec{n}_z \cdot (\vec{n}_k \cdot \vec{T}_k) \frac{d\xi}{\vec{n}_k \cdot \vec{n}_{k,C}} + \int_{\xi_k(z,t)} \vec{n}_z \cdot (\vec{n}_k \cdot \vec{T}_k) \frac{d\xi}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
&= \int_{\xi(z,t)} \vec{n}_z \cdot (\vec{n}_k \cdot \left(\vec{\tau}_k - p_k \vec{1} \right)) \frac{d\xi}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
&+ \int_{\xi_k(z,t)} \vec{n}_z \cdot (\vec{n}_k \cdot \left(\vec{\tau}_k - p_k \vec{1} \right)) \frac{d\xi}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
&= \int_{\xi(z,t)} \vec{n}_z \cdot (\vec{n}_k \cdot \vec{\tau}_k) \frac{d\xi}{\vec{n}_k \cdot \vec{n}_{k,C}} + \int_{\xi_k(z,t)} \vec{n}_z \cdot (\vec{n}_k \cdot \vec{\tau}_k) \frac{d\xi}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
&- \int_{\xi(z,t)} \vec{n}_z \cdot (\vec{n}_k \cdot p_k \vec{1}) \frac{d\xi}{\vec{n}_k \cdot \vec{n}_{k,C}} - \int_{\xi_k(z,t)} \vec{n}_z \cdot (\vec{n}_k \cdot p_k \vec{1}) \frac{d\xi}{\vec{n}_k \cdot \vec{n}_{k,C}}
\end{aligned} \tag{B.22}$$

$$\begin{aligned}
& - \int_C \rho w_k \vec{v}_{kA_w} \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} + \int_C (\rho_k w_k \vec{v}_k) \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
&= - \int_{\xi(z,t)} \vec{n}_z \cdot ((\vec{v}_k - \vec{v}_{kA_w}) \omega_k) \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
&= - \int_{\xi(z,t)} \vec{n}_z \cdot (\dot{m} v_k) \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}}
\end{aligned} \tag{B.23}$$

where $(\dot{m} v_k)$ denotes the transfer of momentum at the interface between the two fluids

Equation B.18 simplifies in:

$$\begin{aligned}
 & \frac{\partial}{\partial t} (A_k < \rho_k w_k >_k) + \frac{\partial}{\partial z} (A_k < \rho_k w_k^2 >_k) - A_k < \rho_k g_z >_k \\
 & + \frac{\partial}{\partial z} (A_k < p_k >_k) - \frac{\partial}{\partial z} (A_k < (\vec{n}_k \cdot \tau_k) \cdot \vec{n}_z >_k) \\
 & = - \int_{\xi(z,t)} \vec{n}_z \cdot (\dot{m} v_k) \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
 & + \int_{\xi(z,t)} \vec{n}_z \cdot (\vec{n}_k \cdot \vec{\tau}_k) \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} + \int_{\xi_k(z,t)} \vec{n}_z \cdot (\vec{n}_k \cdot \vec{\tau}_k) \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\
 & - \int_{\xi(z,t)} \vec{n}_z \cdot (\vec{n}_k \cdot p_k \vec{1}) \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} - \int_{\xi_k(z,t)} \vec{n}_z \cdot (\vec{n}_k \cdot p_k \vec{1}) \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}}
 \end{aligned} \tag{B.24}$$

One may identify the following terms in this equation:

o **Axial diffusion of momentum**

$$\frac{\partial}{\partial z} (A_k < (\vec{n}_k \cdot \tau_k) \cdot \vec{n}_z >_k) \tag{B.25}$$

o **Interfacial shear stress**

$$\int_{\xi(z,t)} \vec{n}_z \cdot (\vec{n}_k \cdot \vec{\tau}_k) \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} = A F_{fr,i,k} \tag{B.26}$$

o **Parietal shear stress**

$$\int_{\xi_k(z,t)} \vec{n}_z \cdot (\vec{n}_k \cdot \vec{\tau}_k) \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} = A F_{fr,w,k} \tag{B.27}$$

o **Momentum transfer per mass transfer**

$$\int_{\xi(z,t)} \vec{n}_z \cdot (\dot{m} v_k) \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} = -A (M v_{ki}) \tag{B.28}$$

o **Interfacial pressure**

$$\int_{\xi(z,t) \cup \xi_k(z,t)} \vec{n}_z \cdot (\vec{n}_k \cdot p_k \vec{1}) \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} = \int_{\xi(z,t) \cup \xi_k(z,t)} p_k \vec{n}_k \cdot \vec{n}_z \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \tag{B.29}$$

This is equivalent to the equations from Delhayé [45]. However to simplify the equation, the following assumptions are additionally made:

o $< \rho_k >_k = \rho_k$

o $< \omega_k >_k = v_k$

o The axial diffusion of momentum is negligible.

Consequently the governing equation of momentum becomes:

$$\left[\begin{aligned} & \frac{\partial}{\partial t} (A_k \rho_k v_k) + \frac{\partial}{\partial z} (A_k \rho_k v_k^2) - A_k \rho_k g_z + \frac{\partial}{\partial z} (A_k < p_k >) \\ & + \int_{\xi(z,t) \cup \xi_k(z,t)} p_k \vec{n}_k \cdot \vec{n}_z \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} = A F_{fr,k} + A F_{fr,i,k} + A (M v_{ki}) \end{aligned} \right] \quad (\text{B.30})$$

This formulation can be finally simplified by assuming various mathematical operations:

- o Introducing the average pressure of the phase k as a relation between the interfacial pressure and the phase cross-section dimensions:

$$< p_k > = P_i + \rho_k g_\perp (h_i - h_{g,k}) \quad (\text{B.31})$$

and then introducing a gap of pressure at the interface via a Laplace's law:

$$< p_k > = P_{i,k} + \rho_k g_\perp (h_i - h_{g,k}) \quad (\text{B.32})$$

So that:

$$\int_{\xi(z,t) \cup \xi_k(z,t)} p_k \vec{n}_k \cdot \vec{n}_z \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} = \int_C p_k \vec{n}_k \cdot \vec{n}_z \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \quad (\text{B.33})$$

- o Applying the Divergence theorem in the particular case where $\vec{f} = \vec{n}_z$, this term may simplify into:

$$\begin{aligned} \int_A \nabla \cdot \vec{f} dA &= \frac{\partial}{\partial z} \int_A f_z dA + \int_C \vec{f} \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\ \int_{A_k} \nabla \cdot \vec{n}_z dA &= \frac{\partial}{\partial z} \int_{A_k} (\vec{n}_z)_z dA + \int_C \vec{n}_z \cdot \vec{n} \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \end{aligned} \quad (\text{B.34})$$

$$\begin{aligned} 0 &= \frac{\partial A_k}{\partial z} + \int_C \vec{n}_k \cdot \vec{n}_z \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} \\ \int_{\xi(z,t) \cup \xi_k(z,t)} p_k \vec{n}_k \cdot \vec{n}_z \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} &= < p_k > \int_C \vec{n}_k \cdot \vec{n}_z \frac{dC}{\vec{n}_k \cdot \vec{n}_{k,C}} = - < p_k > \frac{\partial A_k}{\partial z} \end{aligned} \quad (\text{B.35})$$

Therefore Equation B.30 comes to:

$$\begin{aligned} & \frac{\partial}{\partial t} (A_k \rho_k v_k) + \frac{\partial}{\partial z} (A_k \rho_k v_k^2) - A_k \rho_k g_z + \frac{\partial}{\partial z} (A_k < p_k >) \\ & - < p_k > \frac{\partial A_k}{\partial z} = A F_{fr,k} + A F_{fr,i,k} + A (M v_{ki}) \end{aligned} \quad (\text{B.36})$$

And finally:

$$\begin{aligned} & \frac{\partial}{\partial t} (A_k \rho_k v_k) + \frac{\partial}{\partial z} (A_k \rho_k v_k^2) - A_k \rho_k g_z + A_k \frac{\partial}{\partial z} < p_k > \\ & = A F_{fr,k} + A F_{fr,i,k} + A (M v_{ki}) \end{aligned} \quad (\text{B.37})$$

Rewriting the gradient of pressure:

$$\begin{aligned}
 \frac{\partial}{\partial z} \langle p_k \rangle &= \frac{\partial}{\partial z} P_{i,k} + \rho_k g \frac{\partial}{\partial z} (h_i - h_{g,k}) \\
 &= \frac{\partial}{\partial z} P_{i,k} + \rho_k g \frac{\partial h_i}{\partial z} - \rho_k g \frac{\partial h_{g,k}}{\partial z} \\
 &= \frac{\partial}{\partial z} P_{i,k} + \rho_k g \frac{\partial h_i}{\partial z} - \rho_k g \left(\frac{\partial h_{g,k}}{\partial h_i} \right) \frac{\partial h_i}{\partial z}
 \end{aligned} \tag{B.38}$$

With no mass transfer at the interface, the final equations are thus:

$$\begin{aligned}
 &\frac{\partial}{\partial t} (A_k \rho_k v_k) + \frac{\partial}{\partial z} (A_k \rho_k v_k^2) - A_k \rho_k g_z \\
 &+ A_k \frac{\partial P_{i,k}}{\partial z} + \rho_k A_k g_y \frac{\partial h_i}{\partial z} - \rho_k A_k g_y \left(\frac{\partial h_{g,k}}{\partial h_i} \right) \frac{\partial h_i}{\partial z} \\
 &= A F_{fr,k} + A F_{fr,i,k}
 \end{aligned} \tag{B.39}$$

B.3 Details of linear stability analysis

B.3.1 Mass equations of conservation

From Equation B.12,

$$\frac{\partial}{\partial t} (\alpha_k \rho_k) + \frac{\partial}{\partial z} (\alpha_k \rho_k v_k) = 0 \tag{B.40}$$

one may relate the variations of α_k to variations of h_k :

$$\rho_k \frac{\partial h_k}{\partial t} \frac{\partial \alpha_k}{\partial h_k} + \rho_k \alpha_k \frac{\partial v_k}{\partial z} + \rho_k v_k \frac{\partial h_k}{\partial z} \frac{\partial \alpha_k}{\partial h_k} = 0 \tag{B.41}$$

so that:

$$\Rightarrow \frac{\partial h_k}{\partial t} + \frac{\partial h_k}{\partial \alpha_k} \alpha_k \frac{\partial v_k}{\partial z} + v_k \frac{\partial h_k}{\partial z} = 0 \tag{B.42}$$

Applying a perturbation to the equation variables: phase fraction (α_k), phase velocity (v_k) and level of the phase in the pipe cross-section (h_k):

$$\bar{\alpha}_k + \alpha'_k \quad \bar{v}_k + v'_k \quad \text{et} \quad h_k = \bar{h}_k + h'_k \tag{B.43}$$

The perturbations of the variables lead to:

$$\begin{aligned}
 &\frac{\partial \bar{h}_k}{\partial t} + \frac{\partial h'_k}{\partial t} \\
 &+ \frac{\partial \bar{h}_k}{\partial \alpha_k} \bar{\alpha}_k \frac{\partial \bar{v}_k}{\partial z} + \frac{\partial \bar{h}_k}{\partial \alpha_k} \bar{\alpha}_k \frac{\partial v'_k}{\partial z} + \frac{\partial h'_k}{\partial \alpha_k} \bar{\alpha}_k \frac{\partial \bar{v}_k}{\partial z} + \frac{\partial h'_k}{\partial \alpha_k} \bar{\alpha}_k \frac{\partial v'_k}{\partial z} \\
 &+ \frac{\partial \bar{h}_k}{\partial \alpha_k} \alpha'_k \frac{\partial \bar{v}_k}{\partial z} + \frac{\partial \bar{h}_k}{\partial \alpha_k} \alpha'_k \frac{\partial v'_k}{\partial z} + \frac{\partial h'_k}{\partial \alpha_k} \alpha'_k \frac{\partial \bar{v}_k}{\partial z} + \frac{\partial h'_k}{\partial \alpha_k} \alpha'_k \frac{\partial v'_k}{\partial z} \\
 &+ \bar{v}_k \frac{\partial h'_k}{\partial z} + \bar{v}_k \frac{\partial \bar{h}_k}{\partial z} + v'_k \frac{\partial h'_k}{\partial z} + v'_k \frac{\partial \bar{h}_k}{\partial z} = 0
 \end{aligned} \tag{B.44}$$

Neglecting second orders terms, substracting the non-perturbed equations and the developed flow terms, the equations become:

$$\frac{\partial h'_k}{\partial t} + \frac{\partial \bar{h}_k}{\partial \alpha_k} \bar{\alpha}_k \frac{\partial u'_k}{\partial z} + \bar{u}_k \frac{\partial h'_k}{\partial z} = 0 \quad (\text{B.45})$$

For the gaseous and liquid phases:

$$\begin{aligned} \frac{\partial u'_g}{\partial z} &= - \left[\frac{\partial h'_g}{\partial t} + \bar{u}_g \frac{\partial h'_g}{\partial z} \right] \frac{1}{\bar{\alpha}_g} \frac{\partial \alpha_g}{\partial h_g} \\ \frac{\partial u'_l}{\partial z} &= - \left[\frac{\partial h'_l}{\partial t} + \bar{u}_l \frac{\partial h'_l}{\partial z} \right] \frac{1}{\bar{\alpha}_l} \frac{\partial \alpha_l}{\partial h_l} \end{aligned} \quad (\text{B.46})$$

And as two functions of the same variable of level: the liquid level (h_l):

$$\begin{aligned} \frac{\partial u'_g}{\partial z} &= \left[\frac{\partial h'_l}{\partial t} + \bar{u}_g \frac{\partial h'_l}{\partial z} \right] \frac{1}{\bar{\alpha}_g} \frac{\partial \alpha_l}{\partial h_l} \\ \frac{\partial u'_l}{\partial z} &= - \left[\frac{\partial h'_l}{\partial t} + \bar{u}_l \frac{\partial h'_l}{\partial z} \right] \frac{1}{\bar{\alpha}_l} \frac{\partial \alpha_l}{\partial h_l} \end{aligned} \quad (\text{B.47})$$

B.3.2 Momentum conservation equations

From the Equation B.39 in which the term $A_k \rho_k g_z$ is set to zero by assuming the pipe to be horizontal:

$$\frac{\partial}{\partial t} (A_k \rho_k v_k) + \frac{\partial}{\partial z} (A_k \rho_k v_k^2) + A_k \frac{\partial}{\partial z} \langle p_k \rangle = A F_{fr,k} + A F_{fr,i,k} \quad (\text{B.48})$$

and applied to the liquid and gaseous continuous phases:

$$\frac{\partial}{\partial t} (\alpha_g \rho_g u_g) + \frac{\partial}{\partial z} (\alpha_g \rho_g u_g^2) + \alpha_g \frac{\partial p_g}{\partial z} = -\tau_g \frac{S_g}{A} - \tau_i \frac{S_i}{A} \quad (\text{B.49})$$

$$\frac{\partial}{\partial t} (\alpha_l \rho_l u_l) + \frac{\partial}{\partial z} (\alpha_l \rho_l u_l^2) + \alpha_l \frac{\partial p_l}{\partial z} = -\tau_l \frac{S_l}{A} + \tau_i \frac{S_i}{A} \quad (\text{B.50})$$

Obtaining a non-conservative formulation of the gaseous equation:

$$\alpha_g \rho_g \frac{\partial u_g}{\partial t} + \underline{\rho_g u_g \frac{\partial \alpha_g}{\partial t}} + \alpha_g \rho_g \frac{\partial u_g^2}{\partial z} + \underline{\rho_g u_g^2 \frac{\partial \alpha_g}{\partial z}} + \alpha_g \frac{\partial p_g}{\partial z} = -\tau_g \frac{S_g}{A} - \tau_i \frac{S_i}{A} \quad (\text{B.51})$$

The second and the fourth term are transformed via Equation B.40 obtained from the mass conservation equation development:

$$\alpha_g \rho_g \frac{\partial u_g}{\partial t} - \rho_g \alpha_g u_g \frac{\partial u_g}{\partial z} + \alpha_g \rho_g \frac{\partial u_g^2}{\partial z} + \alpha_g \frac{\partial p_g}{\partial z} = -\tau_g \frac{S_g}{A} - \tau_i \frac{S_i}{A} \quad (\text{B.52})$$

This equation is simplified via Equation B.53) on the derivation of the phase velocity:

$$\frac{\partial u_g^2}{\partial z} = u_g \frac{\partial u_g}{\partial z} + u_g \frac{\partial u_g}{\partial z} \quad (\text{B.53})$$

such that:

$$\alpha_g \rho_g \left[\frac{\partial u_g}{\partial t} + u_g \frac{\partial u_g}{\partial z} \right] + \alpha_g \frac{\partial p_g}{\partial z} = -\tau_g \frac{S_g}{A} - \tau_i \frac{S_i}{A} \quad (\text{B.54})$$

And developing the gradient of pressure:

$$\alpha_g \rho_g \left[\frac{\partial u_g}{\partial t} + u_g \frac{\partial u_g}{\partial z} \right] + \alpha_g \frac{\partial P_{i,g}}{\partial z} + \alpha_g \rho_g g \frac{\partial (h_i - h_{g,g})}{\partial z} = -\tau_g \frac{S_g}{A} - \tau_i \frac{S_i}{A} \quad (\text{B.55})$$

A similar formulation is obtained for the liquid phase:

$$\alpha_l \rho_l \left[\frac{\partial u_l}{\partial t} + u_l \frac{\partial u_l}{\partial z} \right] + \alpha_l \frac{\partial P_{i,l}}{\partial z} + \alpha_l \rho_l g \frac{\partial (h_i - h_{g,l})}{\partial z} = -\tau_l \frac{S_l}{A} + \tau_i \frac{S_i}{A} \quad (\text{B.56})$$

These two equations are put together via the Laplace's law on the gap of pressure at the interface:

$$\begin{aligned} P_{i,g} - P_{i,l} &= \sigma \frac{\partial^2 h_i}{\partial z^2} \\ \frac{\partial P_{i,g}}{\partial z} - \frac{\partial P_{i,l}}{\partial z} &= \sigma \frac{\partial^3 h_i}{\partial z^3} \end{aligned} \quad (\text{B.57})$$

such that:

$$\alpha_l \rho_l \left[\frac{\partial u_l}{\partial t} + u_l \frac{\partial u_l}{\partial z} \right] + \alpha_l \left[\frac{\partial P_{i,g}}{\partial z} - \sigma \frac{\partial^3 h_i}{\partial z^3} \right] + \alpha_l \rho_l g \frac{\partial (h_i - h_{g,l})}{\partial z} = -\tau_l \frac{S_l}{A} + \tau_i \frac{S_i}{A} \quad (\text{B.58})$$

Isolating the gradient of the interfacial pressure of the gaseous phase:

$$\frac{\partial P_{i,g}}{\partial z} = \frac{1}{\alpha_g} \left[-\tau_g \frac{S_g}{A} - \tau_i \frac{S_i}{A} - \alpha_g \rho_g \left[\frac{\partial u_g}{\partial t} + u_g \frac{\partial u_g}{\partial z} \right] - \alpha_g \rho_g g \frac{\partial (h_i - h_{g,g})}{\partial z} \right] \quad (\text{B.59})$$

and introducing it in the liquid phase equation, this provides an unique equation for the two continuous phases:

$$\begin{aligned} & \alpha_l \rho_l \left[\frac{\partial u_l}{\partial t} + u_l \frac{\partial u_l}{\partial z} \right] - \alpha_l \sigma \frac{\partial^3 h_i}{\partial z^3} + \alpha_l \rho_l g \frac{\partial (h_i - h_{g,l})}{\partial z} - \alpha_l \sigma \frac{\partial^3 h_i}{\partial z^3} \\ & + \frac{\alpha_l}{\alpha_g} \left[-\tau_g \frac{S_g}{A} - \tau_i \frac{S_i}{A} - \alpha_g \rho_g \left[\frac{\partial u_g}{\partial t} + u_g \frac{\partial u_g}{\partial z} \right] - \alpha_g \rho_g g \frac{\partial (h_i - h_{g,g})}{\partial z} \right] \quad (\text{B.60}) \\ & = -\tau_l \frac{S_l}{A} + \tau_i \frac{S_i}{A} \end{aligned}$$

As $\frac{\partial \alpha_g}{\partial z} = \frac{\partial 1 - \alpha_l}{\partial z} = (-1) \frac{\partial \alpha_l}{\partial z}$, this equation becomes:

$$\begin{aligned} & \alpha_l \alpha_g \rho_l \left[\frac{\partial u_l}{\partial t} + u_l \frac{\partial u_l}{\partial z} \right] + \alpha_g \alpha_l \rho_l g \frac{\partial (h_i - h_{g,l})}{\partial z} \\ & - \alpha_l \alpha_g \rho_g \left[\frac{\partial u_g}{\partial t} + u_g \frac{\partial u_g}{\partial z} \right] - \alpha_l \alpha_g \rho_g g \frac{\partial (h_i - h_{g,g})}{\partial z} \quad (\text{B.61}) \\ & - \alpha_g \alpha_l \sigma \frac{\partial^3 h_i}{\partial z^3} = -\alpha_g \tau_l \frac{S_l}{A} + \alpha_g \tau_i \frac{S_i}{A} + \alpha_l \tau_g \frac{S_g}{A} + \alpha_l \tau_i \frac{S_i}{A} \end{aligned}$$

Finally the referenced equation is reached:

$$\begin{aligned}
& \alpha_l \alpha_g \rho_l \left[\frac{\partial u_l}{\partial t} + u_l \frac{\partial u_l}{\partial z} \right] - \alpha_l \alpha_g \rho_g \left[\frac{\partial u_g}{\partial t} + u_g \frac{\partial u_g}{\partial z} \right] \\
& - \alpha_g \alpha_l \sigma \frac{\partial^3 h_i}{\partial z^3} + \alpha_g \alpha_l (\rho_l - \rho_g) g \frac{\partial h_i}{\partial z} - \alpha_g \alpha_l \left(\rho_l \frac{\partial h_{g,l}}{\partial h_i} - \rho_g \frac{\partial h_{g,g}}{\partial h_i} \right) g \frac{\partial h_i}{\partial z} \\
& = -\alpha_g \tau_l \frac{S_l}{A} + \alpha_g \tau_i \frac{S_i}{A} + \alpha_l \tau_g \frac{S_g}{A} + \alpha_l \tau_i \frac{S_i}{A}
\end{aligned} \tag{B.62}$$

And in a simplified formulation:

$$\begin{aligned}
& \rho_l \left[\frac{\partial u_l}{\partial t} + u_l \frac{\partial u_l}{\partial z} \right] - \rho_g \left[\frac{\partial u_g}{\partial t} + u_g \frac{\partial u_g}{\partial z} \right] - \sigma \frac{\partial^3 h_i}{\partial z^3} \\
& + (\rho_l - \rho_g) g \frac{\partial h_i}{\partial z} - \left(\rho_l \frac{\partial h_{g,l}}{\partial h_i} - \rho_g \frac{\partial h_{g,g}}{\partial h_i} \right) g \frac{\partial h_i}{\partial z} = -\tau_l \frac{1}{\alpha_l} \frac{S_l}{A} + \tau_g \frac{1}{\alpha_g} \frac{S_g}{A} + \tau_i \frac{S_i}{A} \left(\frac{1}{\alpha_g} - \frac{1}{\alpha_l} \right)
\end{aligned} \tag{B.63}$$

Similarly to the mass conservation equation, the stability of the model requires to perturbate the equation. Every variable are then decomposed in a stable term and in a perturbed term, oscillating around the stable value:

$$\begin{aligned}
u_g &= \bar{u}_g + u'_g \\
u_l &= \bar{u}_l + u'_l \\
h_i &= \bar{h}_i + h'_i \\
h_{g,l} &= \bar{h}_{g,l} + h'_{g,l} \\
h_{g,g} &= \bar{h}_{g,g} + h'_{g,g} \\
\alpha_g &= \bar{\alpha}_g + \alpha'_g \\
\alpha_l &= \bar{\alpha}_l + \alpha'_l
\end{aligned} \tag{B.64}$$

As $\alpha_g + \alpha_l = 1$ and $\bar{\alpha}_g + \bar{\alpha}_l = 1$, $\alpha'_g = -\alpha'_l$.

Applying these perturbations to the referenced Equation (Equation B.63):

$$\begin{aligned}
& (\bar{\alpha}_l + \alpha'_l) (\bar{\alpha}_g + \alpha'_g) \rho_l \left[\frac{\partial (\bar{u}_l + u'_l)}{\partial t} + (\bar{u}_l + u'_l) \frac{\partial (\bar{u}_l + u'_l)}{\partial z} \right] \\
& - (\bar{\alpha}_l + \alpha'_l) (\bar{\alpha}_g + \alpha'_g) \rho_g \left[\frac{\partial (\bar{u}_g + u'_g)}{\partial t} + (\bar{u}_g + u'_g) \frac{\partial (\bar{u}_g + u'_g)}{\partial z} \right] \\
& - (\bar{\alpha}_g + \alpha'_g) (\bar{\alpha}_l + \alpha'_l) \sigma \frac{\partial^3 (\bar{h}_i + h'_i)}{\partial z^3} \\
& + (\bar{\alpha}_g + \alpha'_g) (\bar{\alpha}_l + \alpha'_l) (\rho_l - \rho_g) g \frac{\partial (\bar{h}_i + h'_i)}{\partial z} \\
& - (\bar{\alpha}_g + \alpha'_g) (\bar{\alpha}_l + \alpha'_l) \left(\rho_l \frac{\partial (\bar{h}_{g,l} + h'_{g,l})}{\partial h_i} - \rho_g \frac{\partial (\bar{h}_{g,g} + h'_{g,g})}{\partial h_i} \right) g \frac{\partial (\bar{h}_i + h'_i)}{\partial z} \\
& = - (\bar{\alpha}_g + \alpha'_g) (\bar{\tau}_l + \tau'_l) \frac{S_l}{A} + (\bar{\alpha}_g + \alpha'_g) (\bar{\tau}_i + \tau'_i) \frac{S_i}{A} \\
& + (\bar{\alpha}_l + \alpha'_l) (\bar{\tau}_g + \tau'_g) \frac{S_g}{A} + (\bar{\alpha}_l + \alpha'_l) (\bar{\tau}_i + \tau'_i) \frac{S_i}{A}
\end{aligned} \tag{B.65}$$

And simplifying this equation by eliminating second order terms, the non-perturbed system, the terms: $\frac{\partial \bar{u}_g}{\partial z}$, $\frac{\partial \bar{p}_g}{\partial z}$ and $\frac{\partial \bar{\alpha}_g}{\partial z}$ representative of a fully developed flow such as $\frac{\partial \bar{u}_g}{\partial t}$ and $\frac{\partial \bar{\alpha}_g}{\partial z}$ representative of a permanent flow:

$$\begin{aligned}
& \bar{\alpha}_l \bar{\alpha}_g \rho_l \frac{\partial u'_l}{\partial t} \\
& + \bar{\alpha}_l \bar{\alpha}_g \rho_l \bar{u}_l \frac{\partial u'_l}{\partial z} \\
& - \bar{\alpha}_l \bar{\alpha}_g \rho_g \frac{\partial u'_g}{\partial t} \\
& - \bar{\alpha}_l \bar{\alpha}_g \rho_g \bar{u}_g \frac{\partial u'_g}{\partial z} \\
& - \bar{\alpha}_g \bar{\alpha}_l \sigma \frac{\partial^3 h'_i}{\partial z^3} \\
& + \bar{\alpha}_g \bar{\alpha}_l (\rho_l - \rho_g) g \frac{\partial h'_i}{\partial z} \\
& - \bar{\alpha}_g \bar{\alpha}_l \left(\rho_l \frac{\partial \bar{h}_{g,l}}{\partial h_i} - \rho_g \frac{\partial \bar{h}_{g,g}}{\partial h_i} \right) g \frac{\partial h'_i}{\partial z} \\
& = - \bar{\alpha}_g \tau'_l \frac{S_l}{A} + \bar{\alpha}_g \tau'_i \frac{S_i}{A} + \bar{\alpha}_l \tau'_g \frac{S_g}{A} + \bar{\alpha}_l \tau'_i \frac{S_i}{A}
\end{aligned} \tag{B.66}$$

Simplifying the notations by dividing every terms by the product of the two

volume fractions ($\alpha_g \alpha_l$) :

$$\begin{aligned}
 & \rho_l \frac{\partial u'_l}{\partial t} + \rho_l \bar{u}_l \frac{\partial u'_l}{\partial z} - \rho_g \frac{\partial u'_g}{\partial t} - \rho_g \bar{u}_g \frac{\partial u'_g}{\partial z} \\
 & - \sigma \frac{\partial^3 h'_i}{\partial z^3} + (\rho_l - \rho_g) g \frac{\partial h'_i}{\partial z} - \left(\rho_l \frac{\partial h_{g,l}^-}{\partial h_i} - \rho_g \frac{\partial h_{g,g}^-}{\partial h_i} \right) g \frac{\partial h'_i}{\partial z} \\
 & = -\frac{\tau'_l}{\alpha_l} \frac{S_l}{A} + \frac{\tau'_i}{\alpha_l} \frac{S_i}{A} + \frac{\tau'_g}{\alpha_g} \frac{S_g}{A} + \frac{\tau'_i}{\alpha_g} \frac{S_i}{A}
 \end{aligned} \tag{B.67}$$

B.3.3 Linear stability analysis

In Equation B.67, two types of differential terms appear: differential of velocity $\frac{\partial u'_g}{\partial z}, \frac{\partial u'_l}{\partial z}$ and differential of liquid level $\frac{\partial h'_i}{\partial z}$. The equation is simplified by a differentiation along the z-axis of the pipe z : ($\frac{\partial F}{\partial z}$):

$$\begin{aligned}
 & \rho_l \left[\frac{\partial^2 u'_l}{\partial t \partial z} + \frac{\partial \bar{u}_l}{\partial z} \frac{\partial u'_l}{\partial z} + \bar{u}_l \frac{\partial^2 u'_l}{\partial z^2} \right] - \rho_g \left[\frac{\partial^2 u'_g}{\partial t \partial z} + \frac{\partial \bar{u}_g}{\partial z} \frac{\partial u'_g}{\partial z} + \bar{u}_g \frac{\partial^2 u'_g}{\partial z^2} \right] \\
 & - \sigma \frac{\partial^4 h'_i}{\partial z^4} + (\rho_l - \rho_g) g \frac{\partial^2 h'_i}{\partial z^2} - \left(\rho_l \frac{\partial h_{g,l}^-}{\partial h_i} - \rho_g \frac{\partial h_{g,g}^-}{\partial h_i} \right) g \frac{\partial^2 h'_i}{\partial z^2} \\
 & = \frac{\partial}{\partial z} \left(+\tau'_g \bar{\alpha}_l \frac{S_g}{A} + \tau'_i \bar{\alpha}_l \frac{S_i}{A} - \tau'_l \bar{\alpha}_g \frac{S_l}{A} + \tau'_i \bar{\alpha}_g \frac{S_i}{A} \right) = \frac{\partial F'(\alpha'_l, u'_l, u'_g)}{\partial z}
 \end{aligned} \tag{B.68}$$

Via relations (Equations B.69 and B.70) from the mass conservation equation, the two types of differential simplified:

$$\begin{aligned}
 \frac{\partial^2 u'_l}{\partial t \partial z} &= \frac{-1}{\bar{\alpha}_l} \frac{\partial^2 \alpha'_l}{\partial t^2} + \frac{-\bar{u}_l}{\bar{\alpha}_l} \frac{\partial^2 \alpha'_l}{\partial z \partial t} \\
 \frac{\partial^2 u'_l}{\partial z^2} &= \frac{-1}{\bar{\alpha}_l} \frac{\partial^2 \alpha'_l}{\partial t \partial z} + \frac{-\bar{u}_l}{\bar{\alpha}_l} \frac{\partial^2 \alpha'_l}{\partial z^2} \\
 \frac{\partial^2 u'_g}{\partial t \partial z} &= \frac{1}{\bar{\alpha}_g} \frac{\partial^2 \alpha'_l}{\partial t^2} + \frac{\bar{u}_g}{\bar{\alpha}_g} \frac{\partial^2 \alpha'_l}{\partial z \partial t} \\
 \frac{\partial^2 u'_g}{\partial z^2} &= \frac{1}{\bar{\alpha}_g} \frac{\partial^2 \alpha'_l}{\partial t \partial z} + \frac{\bar{u}_g}{\bar{\alpha}_g} \frac{\partial^2 \alpha'_l}{\partial z^2}
 \end{aligned} \tag{B.69}$$

$$\begin{aligned}
 \frac{\partial^2 u'_l}{\partial t \partial z} &= \frac{-1}{\bar{\alpha}_l} \frac{d\alpha_l}{dh_i} \left[\frac{\partial^2 h'_i}{\partial t^2} + \bar{u}_l \frac{\partial^2 h'_i}{\partial z \partial t} + \frac{\partial \bar{u}_l}{\partial t} \frac{\partial h'_i}{\partial z} \right] \\
 \frac{\partial^2 u'_l}{\partial z^2} &= \frac{-1}{\bar{\alpha}_l} \frac{d\alpha_l}{dh_i} \left[\frac{\partial^2 h'_i}{\partial t \partial z} + \bar{u}_l \frac{\partial^2 h'_i}{\partial z^2} + \frac{\partial \bar{u}_l}{\partial z} \frac{\partial h'_i}{\partial z} \right] \\
 \frac{\partial^2 u'_g}{\partial t \partial z} &= \frac{1}{\bar{\alpha}_g} \frac{d\alpha_l}{dh_i} \left[\frac{\partial^2 h'_i}{\partial t^2} + \bar{u}_g \frac{\partial^2 h'_i}{\partial z \partial t} + \frac{\partial \bar{u}_g}{\partial t} \frac{\partial h'_i}{\partial z} \right] \\
 \frac{\partial^2 u'_g}{\partial z^2} &= \frac{1}{\bar{\alpha}_g} \frac{d\alpha_l}{dh_i} \left[\frac{\partial^2 h'_i}{\partial t \partial z} + \bar{u}_g \frac{\partial^2 h'_i}{\partial z^2} + \frac{\partial \bar{u}_g}{\partial z} \frac{\partial h'_i}{\partial z} \right]
 \end{aligned} \tag{B.70}$$

Eliminating the terms of established flow:

$$\begin{aligned}
& -\frac{\rho_l}{\bar{\alpha}_l} \frac{d\alpha_l}{dh_i} \left[\left[\frac{\partial^2 h'_i}{\partial t^2} + \bar{u}_l \frac{\partial^2 h'_i}{\partial z \partial t} \right] + \bar{u}_l \left[\frac{\partial^2 h'_i}{\partial t \partial z} + \bar{u}_l \frac{\partial^2 h'_i}{\partial z^2} \right] \right] \\
& -\frac{\rho_g}{\bar{\alpha}_g} \frac{d\alpha_l}{dh_i} \left[\left[\frac{\partial^2 h'_i}{\partial t^2} + \bar{u}_g \frac{\partial^2 h'_i}{\partial z \partial t} \right] + \bar{u}_g \left[\frac{\partial^2 h'_i}{\partial t \partial z} + \bar{u}_g \frac{\partial^2 h'_i}{\partial z^2} \right] \right] \\
& -\sigma \frac{\partial^4 h'_i}{\partial z^4} + (\rho_l - \rho_g) g \frac{\partial^2 h'_i}{\partial z^2} - \left(\rho_l \frac{\partial h_{g,l}^-}{\partial h_i} - \rho_g \frac{\partial h_{g,g}^-}{\partial h_i} \right) g \frac{\partial^2 h'_i}{\partial z^2} \\
& = \left(\frac{\partial F}{\partial u_l} \frac{\partial u'_l}{\partial z} \right) + \left(\frac{\partial F}{\partial u_g} \frac{\partial u'_g}{\partial z} \right) + \left(\frac{\partial F}{\partial \alpha_l} \frac{\partial \alpha_l}{\partial z} \right)
\end{aligned} \tag{B.71}$$

or in a more clearer formulation:

$$\begin{aligned}
& \left[\frac{\partial^2 h'_i}{\partial t^2} \right] \left[-\frac{\rho_l}{\bar{\alpha}_l} \frac{d\alpha_l}{dh_i} - \frac{\rho_g}{\bar{\alpha}_g} \frac{d\alpha_l}{dh_i} \right] \\
& + \left[\frac{\partial^2 h'_i}{\partial z \partial t} \right] \left[-\frac{\rho_l}{\bar{\alpha}_l} \frac{d\alpha_l}{dh_i} \bar{u}_l - \frac{\rho_l}{\bar{\alpha}_l} \frac{d\alpha_l}{dh_i} \bar{u}_l - \frac{\rho_g}{\bar{\alpha}_g} \frac{d\alpha_l}{dh_i} \bar{u}_g - \frac{\rho_g}{\bar{\alpha}_g} \frac{d\alpha_l}{dh_i} \bar{u}_g \right] \\
& + \left[\frac{\partial^2 h'_i}{\partial z^2} \right] \left[-\frac{\rho_l}{\bar{\alpha}_l} \frac{d\alpha_l}{dh_i} \bar{u}_l^2 - \frac{\rho_g}{\bar{\alpha}_g} \frac{d\alpha_l}{dh_i} \bar{u}_g^2 + (\rho_l - \rho_g) g - \left(\rho_l \frac{\partial h_{g,l}^-}{\partial h_i} - \rho_g \frac{\partial h_{g,g}^-}{\partial h_i} \right) g \right] \\
& + \left[\frac{\partial^4 h'_i}{\partial z^4} \right] (-\sigma) \\
& = \left[\frac{\partial F}{\partial u_l} \frac{1}{\alpha_l} \frac{\partial \alpha_l}{\partial h_i} \left(-\frac{\partial h'_i}{\partial t} - \bar{u}_l \frac{\partial h'_i}{\partial z} \right) + \frac{\partial F}{\partial u_g} \frac{1}{\alpha_g} \frac{\partial \alpha_l}{\partial h_i} \left(\frac{\partial h'_i}{\partial t} + \bar{u}_g \frac{\partial h'_i}{\partial z} \right) + \frac{\partial F}{\partial \alpha_l} \frac{\partial \alpha_l}{\partial h_i} \frac{\partial h_i}{\partial z} \right]
\end{aligned} \tag{B.72}$$

so that:

$$\begin{aligned}
& \left[\frac{\partial^2 h'_i}{\partial t^2} \right] \left[\frac{\rho_l}{\bar{\alpha}_l} + \frac{\rho_g}{\bar{\alpha}_g} \right] \\
& + \left[\frac{\partial^2 h'_i}{\partial z \partial t} \right] \left[2 \frac{\rho_l}{\bar{\alpha}_l} \bar{u}_l + 2 \frac{\rho_g}{\bar{\alpha}_g} \bar{u}_g \right] \\
& + \left[\frac{\partial^2 h'_i}{\partial z^2} \right] \left[\frac{\rho_l}{\bar{\alpha}_l} \bar{u}_l^2 + \frac{\rho_g}{\bar{\alpha}_g} \bar{u}_g^2 - (\rho_l - \rho_g) g \frac{dh_i}{d\alpha_l} - \left(\rho_l \frac{\partial h_{g,l}^-}{\partial \alpha_l} - \rho_g \frac{\partial h_{g,g}^-}{\partial \alpha_l} \right) g \right] \\
& + \left[\frac{\partial^4 h'_i}{\partial z^4} \right] \left[\frac{dh_i}{d\alpha_l} \sigma \right] \\
& - \left[\frac{\partial h'_i}{\partial t} \right] \left[\frac{\partial F}{\partial u_g} \frac{1}{\alpha_g} - \frac{\partial F}{\partial u_l} \frac{1}{\alpha_l} \right] \\
& - \left[\frac{\partial h'_i}{\partial z} \right] \left[\frac{\partial F}{\partial u_g} \frac{\bar{u}_g}{\alpha_g} - \frac{\partial F}{\partial u_l} \frac{\bar{u}_l}{\alpha_l} + \frac{\partial F}{\partial \alpha_l} \right] \\
& = 0
\end{aligned} \tag{B.73}$$

The perturbation is assumed to have the following formulation:

$$h'_i = \epsilon \exp^i(\omega t - k z) \tag{B.74}$$

Such that the derivation terms become:

$$\begin{aligned}
\left[\frac{\partial h'_i}{\partial t} \right] &= \epsilon i \omega \exp^{i(\omega t - k z)} \\
\left[\frac{\partial^2 h'_i}{\partial t^2} \right] &= -\epsilon \omega^2 \exp^{i(\omega t - k z)} \\
\left[\frac{\partial h'_i}{\partial z} \right] &= -\epsilon i k \exp^{i(\omega t - k z)} \\
\left[\frac{\partial^2 h'_i}{\partial z^2} \right] &= -\epsilon k^2 \exp^{i(\omega t - k z)} \\
\left[\frac{\partial^2 h'_i}{\partial z \partial t} \right] &= \epsilon \omega k \exp^{i(\omega t - k z)} \\
\left[\frac{\partial^4 h'_i}{\partial z^4} \right] &= \epsilon k^4 \exp^{i(\omega t - k z)}
\end{aligned} \tag{B.75}$$

Finally the dispersion equation is obtained

$$\omega^2 - 2(\mathbf{a}k - \mathbf{b}i)\omega + \mathbf{c}k^2 - \mathbf{d}k^4 - \mathbf{e}k i \tag{B.76}$$

with the following coefficients:

$$\begin{aligned}
\rho_{tot} &= \left[\frac{\rho_l}{\bar{\alpha}_l} + \frac{\rho_g}{\bar{\alpha}_g} \right] \\
\mathbf{a} &= \frac{1}{\rho_{tot}} \left[\frac{\rho_l}{\bar{\alpha}_l} \bar{u}_l + \frac{\rho_g}{\bar{\alpha}_g} \bar{u}_g \right] \\
\mathbf{b} &= \frac{1}{\rho_{tot}} \frac{1}{2} \left[\frac{1}{\alpha_l} \left(\frac{\partial F}{\partial u_l} \right)_{u_g, \alpha_l} - \frac{1}{\alpha_g} \left(\frac{\partial F}{\partial u_g} \right)_{u_l, \alpha_l} \right] \\
\mathbf{c} &= \frac{1}{\rho_{tot}} \left[\frac{\rho_l}{\bar{\alpha}_l} \bar{u}_l^2 + \frac{\rho_g}{\bar{\alpha}_g} \bar{u}_g^2 - (\rho_l - \rho_g) g \frac{dh_i}{d\alpha_l} - \left(\rho_l \frac{\partial h_{g,l}}{\partial \alpha_l} - \rho_g \frac{\partial h_{g,g}}{\partial \alpha_l} \right) g \right] \\
\mathbf{d} &= \frac{1}{\rho_{tot}} \left[\frac{dh_i}{d\alpha_l} \sigma \right] \\
\mathbf{e} &= -\frac{1}{\rho_{tot}} \left[\frac{\bar{u}_g}{\alpha_g} \left(\frac{\partial F}{\partial u_g} \right)_{u_l, \alpha_l} - \frac{\bar{u}_l}{\alpha_l} \left(\frac{\partial F}{\partial u_l} \right)_{u_g, \alpha_l} + \left(\frac{\partial F}{\partial \alpha_l} \right)_{u_g, u_l} \right]
\end{aligned} \tag{B.77}$$

So, by calculating the discriminant of the dispersion equation:

$$\Delta_i = (\mathbf{a}k - \mathbf{b}i) - (\mathbf{c}k^2 - \mathbf{d}k^4 - \mathbf{e}k i) \tag{B.78}$$

the solutions o

$$\omega = (\mathbf{a}k - \mathbf{b}i) \pm \sqrt{(\mathbf{a}k - \mathbf{b}i) - (\mathbf{c}k^2 - \mathbf{d}k^4 - \mathbf{e}k i)} \tag{B.79}$$

Or, in a polar form:

$$\begin{aligned}
\omega_1 &= (\mathbf{a}k - \mathbf{b}i) + \sqrt[4]{\beta^2 + \gamma^2} \exp i \frac{1}{2} \arctan \left(\frac{\gamma}{\beta} \right) \\
\omega_2 &= (\mathbf{a}k - \mathbf{b}i) + \sqrt[4]{\beta^2 + \gamma^2} \exp i \frac{1}{2} \left[\arctan \left(\frac{\gamma}{\beta} \right) + 2\pi \right]
\end{aligned} \tag{B.80}$$

with the coefficients β and γ :

$$\begin{aligned}\beta &= (\mathbf{a}^2 - \mathbf{c}) k^2 + \mathbf{d} k^4 - \mathbf{b}^2 \\ \gamma &= \mathbf{e} k - 2 \mathbf{a} \mathbf{b} k\end{aligned}\tag{B.81}$$

Decoupling the angular frequency in a real and a imaginary part: $\omega = \omega_R + i \omega_I$, the interfacial level perturbation h'_i takes the following formulation:

$$\begin{aligned}h'_i &= \epsilon \exp^{i((\omega_R + i \omega_I) t - k z)} = \epsilon \exp^{i(\omega_R t - k z) - \omega_I t} \\ &= \epsilon \exp^{i(\omega_R t - k z)} \exp^{-\omega_I t}\end{aligned}\tag{B.82}$$

In a second part, considering the linear stability analysis applied not to the interfacial level in the center of the pipe but to the cross-section averaged level $\langle h_i \rangle$ such as:

$$\frac{\partial \bar{h}_i}{\partial x} = \frac{\partial \bar{h}_i}{\partial h_i} \frac{\partial h_i}{\partial x} \quad (x = z, t, \dots)\tag{B.83}$$

The coefficients of the dispersion equation becomes then:

$$\begin{aligned}\rho_{tot} &= F^2 \times \left[\frac{\rho_l}{\alpha_l} + \frac{\rho_g}{\alpha_g} \right] \\ \mathbf{a} &= F^2 \times \frac{1}{\rho_{tot}} \left[\frac{\rho_l}{\alpha_l} \bar{u}_l + \frac{\rho_g}{\alpha_g} \bar{u}_g \right] \\ \mathbf{b} &= F \times \frac{1}{\rho_{tot}} \frac{1}{2} \left[\frac{1}{\alpha_l} \left(\frac{\partial F}{\partial u_l} \right)_{u_g, \alpha_l} - \frac{1}{\alpha_g} \left(\frac{\partial F}{\partial u_g} \right)_{u_l, \alpha_l} \right] \\ \mathbf{c} &= F^2 \times \frac{1}{\rho_{tot}} \left[\frac{\rho_l}{\alpha_l} \bar{u}_l^2 + \frac{\rho_g}{\alpha_g} \bar{u}_g^2 - (\rho_l - \rho_g) g \frac{dh_i}{d\alpha_l} - \left(\rho_l \frac{\partial \bar{h}_{g,l}}{\partial \alpha_l} - \rho_g \frac{\partial \bar{h}_{g,g}}{\partial \alpha_l} \right) g \right] \\ \mathbf{d} &= F^4 \times \frac{1}{\rho_{tot}} \left[\frac{dh_i}{d\alpha_l} \sigma \right] \\ \mathbf{e} &= F \times -\frac{1}{\rho_{tot}} \left[\frac{\bar{u}_g}{\alpha_g} \left(\frac{\partial F}{\partial u_g} \right)_{u_l, \alpha_l} - \frac{\bar{u}_l}{\alpha_l} \left(\frac{\partial F}{\partial u_l} \right)_{u_g, \alpha_l} + \left(\frac{\partial F}{\partial \alpha_l} \right)_{u_g, u_l} \right]\end{aligned}\tag{B.84}$$

B.3.4 Comparison with Barnea and Taitel [21]

The dispersion equation achieved by Barnea and Taitel [21] has a similar form but slightly differs from the presented linear stability analysis. In fact, only

three coefficients are no similarly formulated:

$$\begin{aligned}
 \text{avec } \rho_{tot} &= \left[\frac{\rho_l}{\bar{\alpha}_l} + \frac{\rho_g}{\bar{\alpha}_g} \right] \\
 \mathbf{a} &= \frac{1}{\rho_{tot}} \left[\frac{\rho_l}{\bar{\alpha}_l} \bar{u}_l + \frac{\rho_g}{\bar{\alpha}_g} \bar{u}_g \right] \\
 \mathbf{b} &= \frac{1}{\rho_{tot}} \frac{1}{2} \left[\left(\frac{\partial F}{\partial j_g} \right)_{j_l, \alpha_l} - \left(\frac{\partial F}{\partial j_l} \right)_{j_g, \alpha_l} \right] \\
 \mathbf{c} &= \frac{1}{\rho_{tot}} \left[\frac{\rho_l}{\bar{\alpha}_l} \bar{u}_l^2 + \frac{\rho_g}{\bar{\alpha}_g} \bar{u}_g^2 - (\rho_l - \rho_g) g \frac{dh_i}{d\alpha_l} \right] \\
 \mathbf{d} &= \frac{1}{\rho_{tot}} \left[\frac{dh_i}{d\alpha_l} \sigma \right] \\
 \mathbf{e} &= -\frac{1}{\rho_{tot}} \left[\left(\frac{\partial F}{\partial \alpha_l} \right)_{j_g, j_l} \right]
 \end{aligned} \tag{B.85}$$

Coefficients $\mathbf{b}$, $\mathbf{c}$ et $\mathbf{e}$ differ between the two modelings.

B.4 Mathematics Appendix

B.4.1 Geometrical relations

The phase and pipe cross-sections may be expressed as functions of two variables: the pipe radius R and the phase surface fraction α_k :

$$A = \frac{\pi D_h^2}{4} \tag{B.86}$$

$$A_l = \frac{\alpha_l \pi D_h^2}{4} = \alpha_l A \tag{B.87}$$

$$A_g = \frac{\alpha_g \pi D_h^2}{4} = \alpha_g A \tag{B.88}$$

The perimeters of the phase cross-sections are expressed as a function of the liquid level in the center of the pipe cross section $h_l(R, \alpha_l)$. The cross-sections

are therefore expressed as functions of this liquid level $A_l(R, h_l)$:

$$\begin{aligned}
 A_l(\alpha_l, R) &= A_{\text{pipe section portion}} - A_{\text{triangle subtended by the chord}} = A_l(h_l, R) \\
 \alpha_l A(R) &= \frac{\phi}{2\pi} \pi R^2 - 2 \left[\frac{(R - h_l) \times [R \sin(\arccos(\frac{R-h_l}{R}))]}{2} \right] \\
 &= \frac{\phi}{2} \frac{1}{\pi} A - \frac{\pi}{\pi} \left[R \left(1 - \frac{h_l}{R} \right) \right] \times R \times \sin \left(\arccos \left(\frac{R - h_l}{R} \right) \right) \\
 \alpha_l &= \frac{\arccos(1 - \frac{h_l}{R})}{\pi} - \frac{(1 - \frac{h_l}{R}) \times [\sin(\arccos(1 - \frac{h_l}{R}))]}{\pi} \\
 \alpha_l &= \alpha_l(h_l)
 \end{aligned} \tag{B.89}$$

The various interfacial lengths may then be derived:

$$L_l = D_h \arccos \left(\frac{R - h_l}{R} \right) \tag{B.90}$$

$$L_g = \pi D_h - D_h \arccos \left(\frac{R - h_l}{R} \right) \tag{B.91}$$

$$L_i = D_h \sin \left(\arccos \left(\frac{R - h_l}{R} \right) \right) \tag{B.92}$$

Such as the phase perimeters:

$$P_l = L_l + L_i \tag{B.93}$$

$$P_g = L_g + L_i \tag{B.94}$$

Such as the hydraulic diameters of the phase cross-sections:

$$D_{h,l} = \frac{4 A_l}{P_l} \tag{B.95}$$

$$D_{h,g} = \frac{4 A_g}{P_g} \tag{B.96}$$

The position of the gravity center follows:

$$\begin{aligned}
 h_{g,l} &= R \frac{-2 \sin^3 \left(\arccos \left(\frac{R-h_l}{R} \right) \right) + 3 \left[\arccos \left(\frac{R-h_l}{R} \right) - \frac{1}{2} \sin \left(\arccos \left(\frac{R-h_l}{R} \right) \right) \right]}{3 \left[\arccos \left(\frac{R-h_l}{R} \right) - \frac{1}{2} \sin \left(2 \arccos \left(\frac{R-h_l}{R} \right) \right) \right]} \\
 &= \frac{R}{3} \left(\frac{-2 \sin^3 \left(\arccos \left(\frac{R-h_l}{R} \right) \right)}{\arccos \left(\frac{R-h_l}{R} \right) - \frac{1}{2} \sin \left(2 \arccos \left(\frac{R-h_l}{R} \right) \right)} + 1 \right)
 \end{aligned} \tag{B.97}$$

$$\begin{aligned}
h_{g,g} &= R \frac{2 \sin^3 \left(\arccos \left(\frac{R-hl}{R} \right) \right) + 3 \left[\pi - \arccos \left(\frac{R-hl}{R} \right) + \frac{1}{2} \sin \left(2 \arccos \left(\frac{R-hl}{R} \right) \right) \right]}{3 \left[\pi - \arccos \left(\frac{R-hl}{R} \right) + \frac{1}{2} \sin \left(2 \arccos \left(\frac{R-hl}{R} \right) \right) \right]} \\
&= \frac{R}{3} \left(\frac{-2 \sin^3 \left(\arccos \left(\frac{R-hl}{R} \right) \right)}{-\pi + \arccos \left(\frac{R-hl}{R} \right) - \frac{1}{2} \sin \left(2 \arccos \left(\frac{R-hl}{R} \right) \right)} + 1 \right)
\end{aligned} \tag{B.98}$$

Derivation along h_l

Assuming $\left(1 - \frac{h_l}{R}\right) = \kappa$, the liquid film fraction may be expressed as:

$$\alpha_l = \frac{\arccos \kappa}{\pi} - \frac{\kappa \times [\sin(\arccos \kappa)]}{\pi} \tag{B.99}$$

The derivations of the gravity center positions become then:

$$\begin{aligned}
\frac{\partial h_{g,l}}{\partial h_l} &= \frac{-2R}{3} \frac{\left[\arccos \left(\frac{R-hl}{R} \right) - \frac{1}{2} \sin \left(2 \arccos \left(\frac{R-hl}{R} \right) \right) \right] \frac{\partial [\sin^3(\arccos(\frac{R-hl}{R}))]}{\partial h_l}}{\left[\arccos \left(\frac{R-hl}{R} \right) - \frac{1}{2} \sin \left(2 \arccos \left(\frac{R-hl}{R} \right) \right) \right]^2} \\
&\quad - \frac{-2R \sin^3 \left(\arccos \left(\frac{R-hl}{R} \right) \right) \frac{\partial [\arccos(\frac{R-hl}{R}) - \frac{1}{2} \sin(2 \arccos(\frac{R-hl}{R}))]}{\partial h_l}}{3 \left[\arccos \left(\frac{R-hl}{R} \right) - \frac{1}{2} \sin \left(2 \arccos \left(\frac{R-hl}{R} \right) \right) \right]^2} \\
\frac{\partial h_{g,l}}{\partial h_l} &= \frac{-2R}{3} \frac{\left[\arccos \kappa - \frac{1}{2} \sin(2 \arccos \kappa) \right] \frac{-1}{R} \left[3 \sin^2(\arccos \kappa) \kappa \frac{-1}{\sqrt{1-\kappa^2}} \right]}{\left[\arccos \kappa - \frac{1}{2} \sin(2 \arccos \kappa) \right]^2} \\
&\quad + \frac{2R}{3} \frac{\sin^3(\arccos \kappa) \frac{-1}{R} \frac{-1}{\sqrt{1-\kappa^2}}}{\left[\arccos \kappa - \frac{1}{2} \sin(2 \arccos \kappa) \right]^2} \\
&\quad - \frac{2R \sin^3(\arccos \kappa) \cos(2 \arccos(\kappa)) \frac{-1}{\sqrt{1-\kappa^2}} \frac{-1}{R}}{3 \left[\arccos \kappa - \frac{1}{2} \sin(2 \arccos \kappa) \right]^2} \\
\frac{\partial h_{g,l}}{\partial h_l} &= \frac{2}{3} \frac{\left[\arccos \kappa - \frac{1}{2} \sin(2 \arccos \kappa) \right] \left[3 \sin^2(\arccos \kappa) \kappa \right]}{\left[\arccos \kappa - \frac{1}{2} \sin(2 \arccos \kappa) \right]^2} \\
&\quad + \frac{2}{3} \frac{\sin^3(\arccos \kappa) (\cos(2 \arccos(\kappa)) - 1)}{\left[\arccos \kappa - \frac{1}{2} \sin(2 \arccos \kappa) \right]^2}
\end{aligned} \tag{B.100}$$

$$\begin{aligned}
\frac{\partial h_{g,g}}{\partial h_l} &= \frac{-2R}{3} \frac{\left[\arccos\left(\frac{R-h_l}{R}\right) - \frac{1}{2} \sin\left(2 \arccos\left(\frac{R-h_l}{R}\right)\right) - \pi \right] \frac{\partial \left[\sin^3\left(\arccos\left(\frac{R-h_l}{R}\right)\right) \right]}{\partial h_l}}{\left[\arccos\left(\frac{R-h_l}{R}\right) - \frac{1}{2} \sin\left(2 \arccos\left(\frac{R-h_l}{R}\right)\right) - \pi \right]^2} \\
&\quad - \frac{-2R \sin^3\left(\arccos\left(\frac{R-h_l}{R}\right)\right) \frac{\partial \left[\arccos\left(\frac{R-h_l}{R}\right) - \frac{1}{2} \sin\left(2 \arccos\left(\frac{R-h_l}{R}\right)\right) - \pi \right]}{\partial h_l}}{3 \left[\arccos\left(\frac{R-h_l}{R}\right) - \frac{1}{2} \sin\left(2 \arccos\left(\frac{R-h_l}{R}\right)\right) - \pi \right]^2} \\
\frac{\partial h_{g,g}}{\partial h_l} &= \frac{-2R}{3} \frac{\left[\arccos \kappa - \frac{1}{2} \sin\left(2 \arccos \kappa\right) - \pi \right] \frac{-1}{R} \left[3 \sin^2\left(\arccos \kappa\right) \kappa \frac{-1}{\sqrt{1-\kappa^2}} \right]}{\left[\arccos \kappa - \frac{1}{2} \sin\left(2 \arccos \kappa\right) - \pi \right]^2} \\
&\quad + \frac{2R}{3} \frac{\sin^3\left(\arccos \kappa\right) \frac{-1}{R} \frac{-1}{\sqrt{1-\kappa^2}}}{\left[\arccos \kappa - \frac{1}{2} \sin\left(2 \arccos \kappa\right) - \pi \right]^2} \\
&\quad - \frac{2R \sin^3\left(\arccos \kappa\right) \cos\left(2 \arccos(\kappa)\right) \frac{-1}{\sqrt{1-\kappa^2}} \frac{-1}{R}}{3 \left[\arccos \kappa - \frac{1}{2} \sin\left(2 \arccos \kappa\right) - \pi \right]^2} \\
\frac{\partial h_{g,g}}{\partial h_l} &= \frac{2}{3} \frac{\left[\arccos \kappa - \frac{1}{2} \sin\left(2 \arccos \kappa\right) - \pi \right] \left[3 \sin^2\left(\arccos \kappa\right) \kappa \right]}{\left[\arccos \kappa - \frac{1}{2} \sin\left(2 \arccos \kappa\right) - \pi \right]^2} \\
&\quad + \frac{2}{3} \frac{\sin^3\left(\arccos \kappa\right) \left(\cos\left(2 \arccos(\kappa)\right) - 1 \right)}{\left[\arccos \kappa - \frac{1}{2} \sin\left(2 \arccos \kappa\right) - \pi \right]^2}
\end{aligned} \tag{B.101}$$

The average liquid height over the liquid phase section A_l :

$$\begin{aligned}
\bar{h}_l &= \frac{R \arccos\left(\frac{R-h_l}{R}\right)}{2 \sin\left(\arccos\left(\frac{R-h_l}{R}\right)\right)} - \frac{R}{2} \left(\frac{R-h_l}{R} \right) \\
&= \frac{R \arccos \kappa}{2 \sin\left(\arccos \kappa\right)} - \frac{R}{2} \kappa \\
&= \left(\frac{R}{2} \right) \left(\frac{\arccos \kappa}{\sin\left(\arccos \kappa\right)} - \kappa \right)
\end{aligned} \tag{B.102}$$

So that the derivation along the liquid level at the center of the pipe cross-section becomes:

$$\begin{aligned}
\frac{\partial \bar{h}_l}{\partial h_l} &= \left(\frac{\partial \bar{h}_l}{\partial \kappa} \right) \left(\frac{\partial \kappa}{\partial h_l} \right) \\
&= \left(\frac{\partial \bar{h}_l}{\partial \kappa} \right) \left(\frac{-1}{R} \right)
\end{aligned} \tag{B.103}$$

The two terms are calculated as:

$$\begin{aligned}\frac{\partial \bar{h}_l}{\partial \kappa} &= \left(\frac{R}{2}\right) \left(\frac{\left[\frac{\partial}{\partial \kappa} (\arccos \kappa) \right] \sin (\arccos \kappa) - \arccos \kappa \left[\frac{\partial}{\partial \kappa} (\sin (\arccos \kappa)) \right]}{\sin^2 (\arccos \kappa)} - 1 \right) \\ &= \left(\frac{R}{2}\right) \left(\frac{-1 + \kappa \left(\frac{1}{\sqrt{1-\kappa^2}} \right) \arccos \kappa}{1 - \kappa^2} - 1 \right)\end{aligned}$$

$$\begin{aligned}\frac{\partial \alpha_l}{\partial \kappa} &= \frac{1}{\pi} \times \left[\frac{\partial \arccos \kappa}{\partial \kappa} - \sin (\arccos \kappa) - \kappa \times \frac{\partial [\sin (\arccos \kappa)]}{\partial \kappa} \right] \\ &= \frac{1}{\pi} \times \left[\frac{\partial \arccos \kappa}{\partial \kappa} - \sin (\arccos \kappa) - \kappa \times \frac{\partial \arccos \kappa}{\partial \kappa} [\cos (\arccos \kappa)] \right] \\ &= \frac{-1}{\pi} \times \left[\sqrt{1 - \kappa^2} + \sin (\arccos \kappa) \right] \\ &= \frac{-2}{\pi} \sqrt{1 - \kappa^2}\end{aligned}$$

$$\begin{aligned}\frac{\partial \alpha_l}{\partial h_l} &= \frac{\partial \alpha_l}{\partial \kappa} \frac{\partial \kappa}{\partial h_l} \\ &= \frac{-1}{R} \times \frac{\partial \alpha_l}{\partial \kappa} \\ &= \frac{2}{R \pi} \sqrt{1 - \kappa^2}\end{aligned}$$

(B.104)

Derivation of the shear stresses

Equations B.105 explicit the expression of the term F denoting the shear stresses acting in the two-fluid model and the expressions of the derivatives along the liquid film fraction and the phase velocities:

$$\begin{aligned}
 F &= -\tau_l \frac{L_l}{A_l} + \tau_g \frac{L_g}{A_g} + \tau_i L_i \left(\frac{1}{A_l} + \frac{1}{A_g} \right) \\
 \frac{\partial F}{\partial \alpha_l} &= -\frac{\partial \left(\tau_l \frac{L_l}{A_l} \right)}{\partial \alpha_l} + \frac{\partial \left(\tau_g \frac{L_g}{A_g} \right)}{\partial \alpha_l} + \frac{\partial \left(\tau_i L_i \left(\frac{1}{A_l} + \frac{1}{A_g} \right) \right)}{\partial \alpha_l} \\
 &= -\left(\frac{L_l}{A_l} \right) \frac{\partial \tau_l}{\partial \alpha_l} - \left(\frac{\tau_l}{A_l} \right) \frac{\partial L_l}{\partial \alpha_l} + \left(\frac{\tau_l L_l}{A_l^2} \right) \frac{\partial A_l}{\partial \alpha_l} + \left(\frac{L_g}{A_g} \right) \frac{\partial \tau_g}{\partial \alpha_l} + \left(\frac{\tau_g}{A_g} \right) \frac{\partial L_g}{\partial \alpha_l} - \left(\frac{\tau_g L_g}{A_g^2} \right) \frac{\partial A_g}{\partial \alpha_l} \\
 &\quad + L_i \left(\frac{1}{A_l} + \frac{1}{A_g} \right) \frac{\partial \tau_i}{\partial \alpha_l} + \tau_i \left(\frac{1}{A_l} + \frac{1}{A_g} \right) \frac{\partial L_i}{\partial \alpha_l} + \tau_i L_i \frac{\partial \left(\frac{1}{A_l} + \frac{1}{A_g} \right)}{\partial \alpha_l} \\
 \frac{\partial F}{\partial u_l} &= -\frac{\partial \left(\tau_l \frac{L_l}{A_l} \right)}{\partial u_l} + \tau_g \frac{\partial \left(\frac{L_g}{A_g} \right)}{\partial u_l} + \frac{\partial \left(\tau_i L_i \left(\frac{1}{A_l} + \frac{1}{A_g} \right) \right)}{\partial u_l} \\
 &= -\left(\frac{L_l}{A_l} \right) \frac{\partial \tau_l}{\partial u_l} + \left(\frac{L_g}{A_g} \right) \frac{\partial \tau_g}{\partial u_l} + L_i \left(\frac{1}{A_l} + \frac{1}{A_g} \right) \frac{\partial \tau_i}{\partial u_l} \\
 \frac{\partial F}{\partial u_g} &= -\frac{\partial \left(\tau_l \frac{L_l}{A_l} \right)}{\partial u_g} + \frac{\partial \tau_g \left(\frac{L_g}{A_g} \right)}{\partial u_g} + \frac{\partial \left(\tau_i L_i \left(\frac{1}{A_l} + \frac{1}{A_g} \right) \right)}{\partial u_g} \\
 &= -\left(\frac{L_l}{A_l} \right) \frac{\partial \tau_l}{\partial u_g} + \left(\frac{L_g}{A_g} \right) \frac{\partial \tau_g}{\partial u_g} + L_i \left(\frac{1}{A_l} + \frac{1}{A_g} \right) \frac{\partial \tau_i}{\partial u_g}
 \end{aligned} \tag{B.105}$$

Equations B.105 involve derivatives of the shear stresses τ along the liquid film fraction and the phase velocities but also derivatives of the geometry parameters along the liquid film fraction. Equations B.106 explicit then the derivatives of the phase cross-section along the liquid film fraction while Equations B.107 explicits the derivatives of the phase chords and interfacial length along the liquid film fraction.

$$\begin{aligned}
 \frac{\partial A_l}{\partial \alpha_l} &= A \frac{\partial \alpha_l}{\partial \alpha_l} = A \\
 \frac{\partial A_g}{\partial \alpha_l} &= A \frac{\partial (1 - \alpha_l)}{\partial \alpha_l} = -A \\
 \frac{\partial \left(\frac{1}{A_g} + \frac{1}{A_l} \right)}{\partial \alpha_l} &= \frac{1}{A} \left(\frac{1}{\alpha_g^2} - \frac{1}{\alpha_l^2} \right)
 \end{aligned} \tag{B.106}$$

$$\begin{aligned}
\frac{\partial L_l}{\partial \alpha_l} &= \frac{\partial L_l}{\partial \kappa} \frac{\partial \kappa}{\partial \alpha_l} \\
\frac{\partial L_g}{\partial \alpha_l} &= \frac{\partial L_g}{\partial \kappa} \frac{\partial \kappa}{\partial \alpha_l} \\
\frac{\partial L_i}{\partial \alpha_l} &= \frac{\partial L_i}{\partial \kappa} \frac{\partial \kappa}{\partial \alpha_l} \\
\frac{\partial L_l}{\partial \kappa} &= -D_h \frac{1}{\sqrt{1-\kappa^2}} \\
\frac{\partial L_g}{\partial \kappa} &= D_h \frac{1}{\sqrt{1-\kappa^2}} \\
\frac{\partial L_i}{\partial \kappa} &= -D_h \frac{\kappa}{\sqrt{1-\kappa^2}} \\
\frac{\partial \kappa}{\partial \alpha_l} &= \left(\frac{\partial \alpha_l}{\partial \kappa} \right)^{-1}
\end{aligned} \tag{B.107}$$

Equations B.107 involve the derivative of the interfacial length and the phase chords along the geometric parameter κ . These derivatives are thus also presented in Equations B.107. Equations B.108 explicit the derivatives of these lengths, but also of the induced perimeters, along the liquid film fraction.

$$\begin{aligned}
\frac{\partial L_l}{\partial \alpha_l} &= \frac{\partial L_l}{\partial \kappa} \left(\frac{\partial \alpha_l}{\partial \kappa} \right)^{-1} \\
&= D_h \frac{-1}{\sqrt{1-\kappa^2}} \times \frac{-\pi}{2\sqrt{1-\kappa^2}} \\
&= + \frac{\pi}{2} D_h \frac{1}{1-\kappa^2} \\
\frac{\partial L_g}{\partial \alpha_l} &= \frac{\partial L_g}{\partial \kappa} \left(\frac{\partial \alpha_l}{\partial \kappa} \right)^{-1} \\
&= -D_h \frac{-1}{\sqrt{1-\kappa^2}} \times \frac{-\pi}{2\sqrt{1-\kappa^2}} \\
&= - \frac{\pi}{2} D_h \frac{1}{1-\kappa^2} \\
\frac{\partial L_i}{\partial \alpha_l} &= \frac{\partial L_i}{\partial \kappa} \left(\frac{\partial \alpha_l}{\partial \kappa} \right)^{-1} \\
&= -D_h \frac{\kappa}{\sqrt{1-\kappa^2}} \times \frac{-\pi}{2\sqrt{1-\kappa^2}} \\
&= \frac{\pi}{2} D_h \frac{\kappa}{1-\kappa^2} \\
\frac{\partial P_l}{\partial \alpha_l} &= \frac{\partial (L_l + L_i)}{\partial \kappa} \left(\frac{\partial \alpha_l}{\partial \kappa} \right)^{-1} \\
&= + \frac{\pi}{2} D_h \frac{1+\kappa}{1-\kappa^2} \\
&= \frac{\pi}{2} D_h \frac{\kappa}{1+\kappa}
\end{aligned} \tag{B.108}$$

$$\begin{aligned}
\frac{\partial P_g}{\partial \alpha_l} &= \frac{\partial (L_g + L_i)}{\partial \kappa} \left(\frac{\partial \alpha_l}{\partial \kappa} \right)^{-1} \\
&= + \frac{\pi}{2} D_h \frac{\kappa - 1}{1 - \kappa^2} \\
&= - \frac{\pi}{2} D_h \frac{\kappa}{1 - \kappa}
\end{aligned} \tag{B.109}$$

Equation B.110 presents the derivatives along the liquid film fraction of the hydraulic diameter of the two phases.

$$\begin{aligned}
\frac{\partial D_l}{\partial \alpha_l} &= \frac{\partial (4 A_l P_l^{-1})}{\partial \alpha_l} \\
&= 4 A_l \frac{\partial P_l^{-1}}{\partial \alpha_l} + 4 P_l^{-1} \frac{\partial A_l}{\partial \alpha_l} \\
&= 4 A_l (-1) P_l^{-2} \frac{\partial P_l}{\partial \alpha_l} + 4 P_l^{-1} A
\end{aligned} \tag{B.110}$$

$$\begin{aligned}
\frac{\partial D_g}{\partial \alpha_l} &= \frac{\partial (4 A_g P_g^{-1})}{\partial \alpha_l} \\
&= 4 A_g \frac{\partial P_g^{-1}}{\partial \alpha_l} + 4 P_g^{-1} \frac{\partial A_g}{\partial \alpha_l} \\
&= 4 A_g (-1) P_g^{-2} \frac{\partial P_g}{\partial \alpha_l} + 4 P_g^{-1} - A
\end{aligned}$$

Derivation of the parietal shear stress

Equation B.111 presents each derivatives of the parietal shear stress τ_l .

$$\begin{aligned}
 \tau_l &= \frac{1}{2} \rho_l f_l v_l^2 \\
 &= \frac{1}{2} \rho_l C_l Re^{-n_l} v_l^2 \\
 &= \frac{1}{2} C_l \rho_l^{1-n_l} \mu_l^{n_l} D_l^{-n_l} v_l^{2-n_l} \\
 &= K_l D_l^{-n_l} v_l^{2-n_l}
 \end{aligned}$$

$$\begin{aligned}
 \left(\frac{\partial \tau_l}{\partial v_l} \right)_{v_g, \alpha_l} &= K_l D_l^{-n_l} \frac{\partial v_l^{(2-n_l)}}{\partial v_l} \\
 &= K_l D_l^{-n_l} (2 - n_l) v_l^{(1-n_l)}
 \end{aligned}$$

$$\begin{aligned}
 \left(\frac{\partial \tau_l}{\partial j_l} \right)_{j_g, \alpha_l} &= K_l \frac{D_l^{-n_l}}{\alpha_l^{(2-n_l)}} \frac{\partial j_l^{(2-n_l)}}{\partial j_l} \\
 &= K_l \frac{D_l^{-n_l}}{\alpha_l^{(2-n_l)}} (2 - n_l) j_l^{(1-n_l)}
 \end{aligned}$$

$$\begin{aligned}
 \left(\frac{\partial \tau_l}{\partial \alpha_l} \right)_{v_g, v_l} &= K_l v_l^{(2-n_l)} \left[\frac{\partial D_l^{-n_l}}{\partial \alpha_l} \right] \\
 &= K_l v_l^{(2-n_l)} \left[(-n_l) D_l^{(-n_l-1)} \frac{\partial D_l}{\partial \alpha_l} \right]
 \end{aligned}$$

$$\begin{aligned}
 \left(\frac{\partial \tau_l}{\partial \alpha_l} \right)_{j_g, j_l} &= K_l \left[\frac{\partial \left(\frac{j_l^{(2-n_l)}}{\alpha_l^{(2-n_l)}} D_l^{-n_l} \right)}{\partial \alpha_l} \right] \\
 &= K_l \left[D_l^{-n_l} \frac{\partial \left(j_l^{(2-n_l)} \alpha_l^{(n_l-2)} \right)}{\partial \alpha_l} + j_l^{(2-n_l)} \alpha_l^{(n_l-2)} \frac{\partial (D_l^{-n_l})}{\partial \alpha_l} \right] \\
 &= K_l j_l^{(2-n_l)} \left[D_l^{-n_l} (n_l - 2) \alpha_l^{(n_l-2-1)} + \alpha_l^{(n_l-2)} \frac{\partial (D_l^{-n_l})}{\partial \alpha_l} \right] \\
 &= K_l j_l^{(2-n_l)} \left[D_l^{-n_l} (n_l - 2) \alpha_l^{(n_l-2-1)} + \alpha_l^{(n_l-2)} \left((-n_l) D_l^{(-n_l-1)} \frac{\partial D_l}{\partial \alpha_l} \right) \right]
 \end{aligned}$$

(B.111)

Derivation of the interfacial shear stress

Equation B.112 presents each derivatives of the interfacial shear stress τ_i when considering a constant friction factor.

$$\begin{aligned}
\tau_i &= \frac{1}{2} 0.014 \rho_g (v_g - v_l) |v_g - v_l| \\
\left(\frac{\partial \tau_i}{\partial v_l} \right)_{v_g, \alpha_l} &= \frac{1}{2} 0.014 \rho_g \frac{\partial (v_g^2 + v_l^2 - 2 v_g v_l)}{\partial v_l} \\
&= \frac{1}{2} 0.014 \rho_g (2 v_l - 2 v_g) \\
\left(\frac{\partial \tau_i}{\partial j_l} \right)_{j_g, \alpha_l} &= \frac{1}{2} 0.014 \rho_g \frac{\partial \left(\left(\frac{j_g^2}{\alpha_g^2} \right) + \left(\frac{j_l^2}{\alpha_l^2} \right) - 2 \frac{j_g}{\alpha_g} \frac{j_l}{\alpha_l} \right)}{\partial v_l} \\
&= \frac{1}{2} 0.014 \rho_g \left(2 \frac{j_l}{\alpha_l^2} - 2 \frac{j_g}{\alpha_l \alpha_g} \right) \\
&= \frac{1}{2} 0.014 \rho_g (2 v_l - 2 v_g) \frac{1}{\alpha_l} \\
\left(\frac{\partial \tau_i}{\partial v_g} \right)_{v_l, \alpha_l} &= \frac{1}{2} 0.014 \rho_g (2 v_g - 2 v_l) \\
\left(\frac{\partial \tau_i}{\partial j_g} \right)_{j_l, \alpha_l} &= \frac{1}{2} 0.014 \rho_g (2 v_g - 2 v_g) \frac{1}{\alpha_g} \\
\left(\frac{\partial \tau_i}{\partial \alpha_l} \right)_{v_l, v_g} &= 0 \\
\left(\frac{\partial \tau_i}{\partial \alpha_l} \right)_{j_l, j_g} &= \frac{1}{2} 0.014 \rho_g \frac{\partial \left(\left(\frac{j_g^2}{\alpha_g^2} \right) + \left(\frac{j_l^2}{\alpha_l^2} \right) - 2 \frac{j_g}{\alpha_g} \frac{j_l}{\alpha_l} \right)}{\partial \alpha_l} \\
&= \frac{1}{2} 0.014 \rho_g \frac{\partial (j_g^2 \alpha_g^{-2} + j_l^2 \alpha_l^{-2} - 2 j_g j_l \alpha_l^{-1} \alpha_g^{-1})}{\partial \alpha_l} \\
&= \frac{1}{2} 0.014 \rho_g \left[j_g^2 \frac{\partial (\alpha_g^{-2})}{\partial \alpha_l} + j_l^2 \frac{\partial (\alpha_l^{-2})}{\partial \alpha_l} - 2 j_g j_l \frac{\partial (\alpha_l^{-1} \alpha_g^{-1})}{\partial \alpha_l} \right] \\
&= \frac{1}{2} 0.014 \rho_g \left[j_g^2 (-2) \alpha_g^{-3} \frac{\partial \alpha_g}{\partial \alpha_l} + (-2) j_l^2 \alpha_l^{-3} - 2 j_g j_l \left(\alpha_l^{-1} \frac{\partial \alpha_g^{-1}}{\partial \alpha_l} + \alpha_g^{-1} \frac{\partial \alpha_l^{-1}}{\partial \alpha_l} \right) \right] \\
&= \frac{1}{2} 0.014 \rho_g [j_g^2 (-2) \alpha_g^{-3} (-1) + (-2) j_l^2 \alpha_l^{-3} - 2 j_g j_l (\alpha_l^{-1} (-1) \alpha_g^{-2} (-1) + \alpha_g^{-1} (-1) \alpha_l^{-2})] \\
&= \frac{1}{2} 0.014 \rho_g \left[\frac{2 v_g^2}{\alpha_g} - \frac{2 v_l^2}{\alpha_l} - 2 v_g v_l \left(\frac{1}{\alpha_l} - \frac{1}{\alpha_g} \right) \right]
\end{aligned} \tag{B.112}$$

Derivation of the interfacial shear stress

Equation B.113 presents each derivatives of the interfacial shear stress τ_i when considering a friction factor equivalent to the gas friction factor.

$$\begin{aligned}
\tau_i &= \frac{1}{2} f_g \rho_g (v_g - v_l) |v_g - v_l| \\
&= \frac{1}{2} C_g \rho_g^{(1-n_g)} \mu_g^{n_g} D_g^{-n_g} v_g^{-n_g} (v_g^2 + v_l^2 - 2 v_g v_l) \\
&= K_g D_g^{-n_g} v_g^{-n_g} (v_g^2 + v_l^2 - 2 v_g v_l) \\
\\
\left(\frac{\partial \tau_i}{\partial v_l} \right)_{v_g, \alpha_l} &= K_g D_g^{-n_g} v_g^{-n_g} (2 v_l - 2 v_g) \\
\left(\frac{\partial \tau_i}{\partial j_l} \right)_{j_g, \alpha_l} &= K_g D_g^{-n_g} v_g^{-n_g} (2 v_l - 2 v_g) \frac{1}{\alpha_l} \\
\left(\frac{\partial \tau_i}{\partial v_g} \right)_{v_l, \alpha_l} &= K_g D_g^{-n_g} \left((2 - n_g) v_g^{(1-n_g)} - n_g v_l^2 v_g^{(-n_g-1)} - 2 v_l (1 - n_g) v_g^{-n_g} \right) \\
\left(\frac{\partial \tau_i}{\partial j_g} \right)_{j_l, \alpha_l} &= K_g D_g^{-n_g} \left(\frac{(2 - n_g) j_g^{(1-n_g)}}{\alpha_g^{(2-n_g)}} - \frac{n_g v_l^2 j_g^{(-n_g-1)}}{\alpha_g^{(-n_g)}} - \frac{2 v_l (1 - n_g) j_g^{-n_g}}{\alpha_g^{(1-n_g)}} \right) \\
\\
\left(\frac{\partial \tau_i}{\partial \alpha_l} \right)_{v_l, v_g} &= K_g v_g^{-n_g} (v_g^2 + v_l^2 - 2 v_g v_l) \frac{\partial D_g^{-n_g}}{\partial \alpha_l} \\
&= K_g v_g^{-n_g} (v_g^2 + v_l^2 - 2 v_g v_l) (-n_g) D_g^{-n_g-1} \frac{\partial D_g}{\partial \alpha_l} \\
\left(\frac{\partial \tau_i}{\partial \alpha_l} \right)_{j_l, j_g} &= K_g \frac{\partial \left[\left(\frac{j_g}{\alpha_g} \right)^{-n_g} \left(\left(\frac{j_g}{\alpha_g} \right)^2 + \left(\frac{j_l}{\alpha_l} \right)^2 - 2 \left(\frac{j_g}{\alpha_g} \right) \left(\frac{j_l}{\alpha_l} \right) \right) D_g^{-n_g} \right]}{\partial \alpha_l} \\
&= K_g \frac{\partial \left(j_g^{(2-n_g)} \alpha_g^{(n_g-2)} + j_l^2 j_g^{(n_g)} \alpha_g^{(n_g)} \alpha_l^{-2} - 2 j_l j_g^{(1-n_g)} \alpha_g^{(n_g-1)} \alpha_l \right) D_g^{-n_g}}{\partial \alpha_l} \\
&= K_g D_g^{-n_g} \frac{\partial \left(j_g^{(2-n_g)} \alpha_g^{(n_g-2)} + j_l^2 j_g^{(-n_g)} \alpha_g^{n_g} \alpha_l^{-2} - 2 j_l j_g^{(1-n_g)} \alpha_g^{(n_g-1)} \alpha_l \right)}{\partial \alpha_l} \\
&\quad + \frac{1}{2} C_g \rho_g^{(1-n_g)} \mu_g^{n_g} \left(j_g^{(2-n_g)} \alpha_g^{(n_g-2)} + j_l^2 j_g^{(-n_g)} \alpha_g^{n_g} \alpha_l^{-2} - 2 j_l j_g^{(1-n_g)} \alpha_g^{(n_g-1)} \alpha_l \right) \frac{\partial D_g^{-n_g}}{\partial \alpha_l} \\
&= K_g D_g^{-n_g} \left[-(n_g - 2) j_g^{(2-n_g)} \alpha_g^{(n_g-3)} + j_l^2 j_g^{(-n_g)} \left(-2 \alpha_l^{-3} \alpha_g^{n_g} - n_g \alpha_l^{-2} \alpha_g^{(n_g-1)} \right) - 2 j_l j_g^{(1-n_g)} \left(\alpha_g^{(n_g-1)} \alpha_l \right) \right. \\
&\quad \left. + \frac{1}{2} C_g \rho_g^{(1-n_g)} \mu_g^{n_g} \left(j_g^{(2-n_g)} \alpha_g^{(n_g-2)} + j_l^2 j_g^{(-n_g)} \alpha_g^{n_g} \alpha_l^{-2} - 2 j_l j_g^{(1-n_g)} \alpha_g^{(n_g-1)} \alpha_l \right) \right. \\
&\quad \left. \times (-n_g) D_g^{(-n_g-1)} \frac{\partial D_g}{\partial \alpha_l} \right]
\end{aligned}$$

(B.113)

B.4.2 Interfacial gap of pressure

Figure B.1 presents the evolution of the pressure along the vertical axis y taking into account both the interfacial gap of pressure at the interface, characterized by the Laplace's law, and the hydrostatic repartition of pressure in the two separated phases.

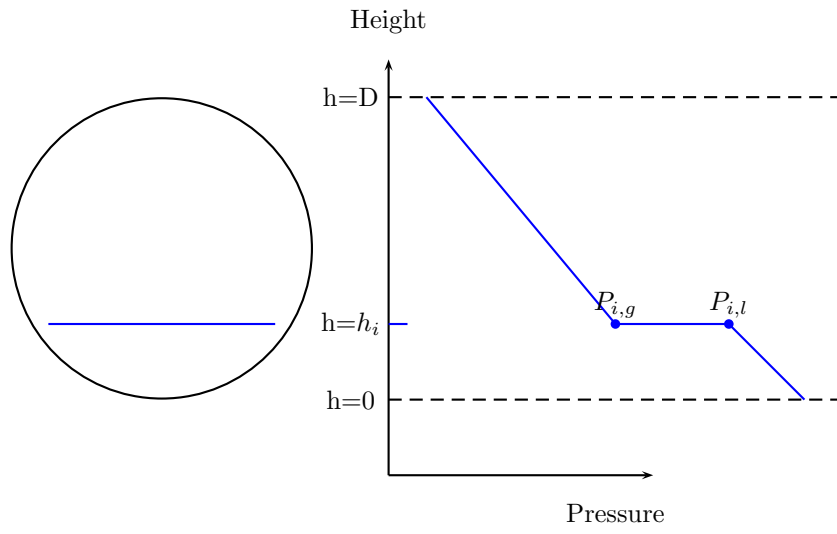


Figure B.1: Evolution of the pressure along the vertical axis y .

B.5 Force balance: geometric integration

Calculation of the drag area by surface area of revolution method

$$\begin{aligned}
 A_L(h, b, \lambda) &= \int_b^{\lambda/4} 2\pi y(x) \sqrt{1 + y'(x)^2} dx \\
 &= -\frac{h\lambda}{2} \left[\cos\left(\frac{2\pi x}{\lambda}\right) \sqrt{1 + \left(\frac{2\pi h}{\lambda} \cos\left(\frac{2\pi x}{\lambda}\right)\right)^2} \right]_b^{\lambda/4} \\
 &\quad + \left[\frac{\operatorname{arcsinh}\left(\frac{2\pi h}{\lambda} \cos\left(\frac{2\pi x}{\lambda}\right)\right)}{\frac{2\pi h}{\lambda}} \right]_b^{\lambda/4} - 2\pi \left(\frac{\lambda}{4} - b\right) h \sin\left(\frac{2\pi b}{\lambda}\right)
 \end{aligned} \tag{B.114}$$

Calculation of the entrained volume by shell integration method

$$V = 2\pi \int_a^b x f(x) dx \tag{B.115}$$

$$f(x) = h \cos\left(\frac{2\pi x}{\lambda}\right) \tag{B.116}$$

$$V = 2\pi \int_0^t x h \cos\left(\frac{2\pi x}{\lambda}\right) dx \tag{B.117}$$

$$V = \lambda h \int_0^t \frac{2\pi x}{\lambda} \cos\left(\frac{2\pi x}{\lambda}\right) dx \tag{B.118}$$

$$s = \frac{2\pi x}{\lambda} \quad ds = \frac{2\pi}{\lambda} dx \tag{B.119}$$

$$\text{en } x = t \quad \Rightarrow \quad s = \frac{2\pi t}{\lambda} \tag{B.120}$$

$$V = \lambda h \int_0^{\frac{2\pi t}{\lambda}} s \cos(s) \left(\frac{\lambda}{2\pi} ds\right) \tag{B.121}$$

$$V = \frac{\lambda^2 h}{2\pi} \int_0^{\frac{2\pi t}{\lambda}} s \cos(s) ds \tag{B.122}$$

$$V = \frac{\lambda^2 h}{2\pi} [s \sin(s) + \cos(s)]_0^{\frac{2\pi t}{\lambda}} \tag{B.123}$$

$$V = \frac{\lambda^2 h}{2\pi} \left[\left(\frac{2\pi t}{\lambda}\right) \sin\left(\frac{2\pi t}{\lambda}\right) + \cos\left(\frac{2\pi t}{\lambda}\right) - 1 \right] \tag{B.124}$$

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