

Aircraft Wake Sensing and Tracking Strategies towards Operational Formation Flight

Assessment of data assimilation, reinforcement learning and
control approaches within large eddy simulations

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Introduction

Twice a year, the sky comes to life with migratory birds flying to and from their wintering or breeding grounds. It seems instinctive for them to align in the sky and to draw these impressive arrows, which one qualifies as "V-shapes", see Picture 1. It is now about a century since mankind started to colonize the skies, and yet it cannot boast of having reproduced flight patterns as impressive as those of migratory birds. This attraction for formation flight among birds is still debated, but the leading explanation lies in the existence of a wake in a region behind each individual, and that each can perceive. How does that wake perception translates for airplanes?

Context and motivations

In fluid mechanics, the wake of any flow device is the signature of its producing forces, extracting energy, or simply moving through the fluid medium. Through the wake, one can thus observe the reaction of the flow to the action of the device. Just like birds, airplanes produce lifting forces that allow them to overcome the force of gravity and therefore remain in a steady flight. Unlike them, however, airplanes do not compensate drag by flapping their wings, but use a propulsion system, i.e., a propeller or a jet engine. This steady configuration leads to the emission of a wake consisting of two parallel vortices and made visible by the condensation trails of the engines, as shown on Picture 2. These two vortices modify their environment by inducing swirling movements of the surrounding air. A following aircraft that would enter such a perturbed environment could experience a strong rolling moment or a strong downwash velocity that could be hazardous.

In order to limit the risk of incidents, the International Civil Aviation Organization (ICAO) thus imposes minimum separation distances between aircraft, which explains the existence of the well-known air lanes and time intervals between successive take-offs or landings [121]. The increase in air traffic over the last decades and projected for the years to come [53] has nonetheless increased the risk related to wake encounter, especially near airports where the concentration of the air traffic is very high. In the busiest airports, we observe a saturation of the air traffic. The reduction of the separation between aircraft is therefore of major economic interest, which has led and continues to lead to major research and operational efforts to improve airport traffic while guaranteeing safety. Large scale projects, among others in Europe, have been dedicated to the problem, e.g., the European Wake Vortex Recategorisation (RECAT-EU).

The knowledge of the dynamics of the wakes produced by aircraft in cruise condition is also of major interest in order to avoid wake encounters. Aircraft wakes are indeed powerful long-lasting structures whose behavior depends on the external environment such as turbulence, stratification, wind, etc [126, 76, 45]. The understanding of the mechanisms governing their propagation, decay and demise is therefore essential to ensure the safety of aircraft flying in the same air lanes. All these studies have helped define the rules to ensure the safety of airplanes.

Formation flight strategies observed among migratory birds are in a dramatic contrast with the very cautious policy of the Air Traffic Management regulations. Birds indeed seems to have mastered the risks associated with wake vortex interactions in order to harvest energy. Theoretical studies have illustrated that a group of 25 birds flying in a V-shaped formation could experience 71% range increase comparing with a bird flying alone [94]. Also, experimental results indicate that pelicans in formation beat their wings less frequently and glide for longer periods, which results in 11% to 14% reductions in heart rate as compared to lone flyers [146].

The study of formation flight for aircraft is not new and has already been studied in the past, in particular for military applications. Indeed, where birds fly in formation with an energy reduction objective, military aircraft historically fly in formation for mutual support and strategic reasons. Even more spectacularly, fighters can fly just behind a cargo aircraft for air-to-air refueling tasks in order to extend the range of the mission. The interactions with the strong wake of the tanker makes the operation more difficult for the receiver to maintain its position and to detect the refueling probe [50]. Such flight strategies therefore require among other things a strong knowledge of the wake vortices behavior, and complex control and estimation strategies.

More recently, formation flight for civilian applications has been contemplated. Similarly to birds, a following aircraft can consider taking advantage of the vertical upwash winds induced by the swirling movement of the wake vortices produced by a preceding aircraft. Early investigations conducted on a pair of aircraft (leader + follower) indeed showed that substantial fuel reduction (i.e., up to 15%) can be achieved by the follower, both theoretically [77, 101, 10] and experimentally [9, 140, 71]. The knowledge of wake dynamics and behavior accumulated over the past decades also led the aircraft manufacturers to focus on the formation flight concept. After Boeing's ecoDemonstrator platform in 2018 [57], Airbus has recently tackled the problem and was able to demonstrate the operational feasibility of formation flight in November 2021 [2], as shown in Picture 3. The transatlantic crossing has confirmed the potential for more than a 5% fuel saving for the follower. Those studies and demonstrations open the way to new research perspectives in the field of aircraft wake dynamics. Where most of the past work has focused on wake encounter and its consequences [95, 37, 11], the wake of aircraft formations shows different behaviors than the wake of an isolated aircraft [139]. It will become essential to study those behaviors before considering implementing formation flight strategies for civilian aviation.

With the current and growing need for new technologies to lower fuel costs, the design of strategies to meet the challenges of formation flying and the analysis of the impact of such flight strategies on the air traffic management is therefore coherent. Despite rather extensive experimental and numerical studies of the concept, further research is still needed before it becomes a common operational practice. In particular, the follower aircraft needs to locate the wake vortices or estimate its characteristics.

Data assimilation and control in flow-mediated environment

The understanding of wakes and their dynamics facilitates the perspective of setting up formation flight. In particular, it provides us with information about the ideal position in those wakes in order to achieve the energy saving objective. To reach that position, it becomes first necessary to localize the wake. This starts with the observation of the environment to detect the wake vortices, before setting up an approach strategy. For that purpose, visual information about the leader's position takes all its importance when dealing with fighters or migratory birds in close formation flight, as the wake directly follows the position the leader [125, 134]. Nevertheless, civilian aviation rather targets the concept of extended formation flight [100] that better addresses safety concerns. In that case, the follower flies at several tens of wingspans downstream of the leader. Because of the increased distance between the leader and the follower, instabilities such



Picture 1: V-shape formation of migratory birds flying to their wintering grounds, observed in Olloy-sur-Viroin. Copyright Ignace Ransquin.



Picture 2: Aircraft wake vortices, made visible through contrail made of condensed water produced by the engines. Copyright Johan Kellerman.



Picture 3: Formation flight strategy performed by two Airbus A350. Copyright Sylvain Ramadier, Airbus.



Picture 4: Gliders exploiting the same thermal updraft, nearby Saint-Hubert. Copyright Philippe Chatelain.

as meandering [52], the interactions of wakes produced by different aircraft in a formation [139], and environmental factors [75] may affect the motion of the vortices. Therefore, the localization of the leader's wake position based on visual information about the leader's position solely is not sufficient. Extra sensors that gather additional information are required to define the appropriate and safe actions when dealing with aircraft flying in extended formation flight [149].

In this context, biomimetic inspiration can once again provide a path to enhance the adaptivity and thence the efficiency of flow sensing. Birds do it naturally, as proven experimentally by recent research on free-flying ibises in "V-shape" formation [112]. Portugal et al. have indeed hypothesized that they might possess the capability to sense the upwash wake generated by leading birds and then minimize their energy expenditure by flying in the aerodynamically optimal position. Yet, which sensorimotor cues of the flow are available to birds and how they are exploited remains an open problem. Following a similar principle, numerous birds of prey manage to maintain themselves in the sky for hours without flapping their wings but rather using thermal updrafts for soaring over hundreds of kilometers. Man-made devices have already adopted the same strategy in the context of soaring flight and the design of efficient gliders, see Picture 4. Going one step further, autonomous unmanned gliders are now able to exploit their environment alone or in collaboration with other autonomous gliders thanks to a perception of the surrounding flow [92]. In addition, those brilliant examples of flow perception are not limited to the aerial kingdom: in water, the synchronized swimming of fish schools, their ability to efficiently hunt, even in dark environment, or to communicate are partly due to their lateral line which is the system of sensory organs that enables them to perceive the surrounding flow environment [25, 144, 157].

These examples are characterized by the use of indirect measurements of the flow in order to be aware of the environment or to take an action. The flow disturbances are either deduced from local velocity or pressure sensors [144] or from measurements of the dynamics of the objects in the flow itself [91, 81]. This information has therefore first to be assimilated or learned in order to take the appropriate action towards a specific objective.

The use of filtering and Bayesian assimilation tools has already been considered in the context of formation flight in order to determine the characteristics of a wake over time. These same tools have also been used on autonomous gliders to guide them towards estimated thermal positions. Yet, those techniques still require specific instrumentation to be installed aboard the aircraft, which comes with a certain cost and design constraints. The research efforts conducted by Hanson [69], Chichka et al. [28] or Caprace et al. [21] suggest that the use of aircraft dynamics measurements are promis-

ing towards determining the optimal position of an aircraft in the wake of a leader, just as birds seem to perform during their flight. In addition to data assimilation techniques, machine learning tools have been developed to understand how devices could learn to optimize certain performances in flow-mediated environments [16]. Focusing on the determination of the optimal actions that a device should take to achieve a given goal in a simulated environment, recent efforts have shown how to teach a swimmer to exploit the wakes of preceding fish [143], or a glider to fly in unknown thermals [119], using the Reinforcement Learning framework [136].

Prospects are therefore to be explored in the context of formation flight. Dedicated techniques are investigated in this work with the goal of determining under which conditions reliable estimation and learning can be performed. We are specifically interested in solutions that would not require a communication between the two aircraft, but rather exploit dynamic measurements, thereby following inspiration by our natural environment's design.

Numerical simulations for the multidisciplinary formation flight problem

Research efforts in fluid mechanics and in particular in the study of wake behavior and propagation have increased considerably with the emergence of numerical simulation techniques. This is even more striking with the rapid increase in the performance of modern computers. Aircraft applications have been the beneficiaries and also the precursors of this numerical revolution, with the development of a wide variety of simulation methods epitomized by the European Fundamental Research on Aircraft Wake Phenomena (FAR-Wake, 2005-2008) [42], later reviewed by Hallock and Holzäpfel [67].

Among these methods, the ones based on the vorticity-velocity formulation of the Navier-Stokes equations are particularly efficient [98]. The research group of Winckelmans et al. [151] and Chatelain et al. [26] have been involved in the development of such methods, with a particular interest for Large Eddy Simulations (LES) of wind turbine and aircraft wakes. The study of formation flight, and in particular of the resulting wake and its impact on the aircraft of the formation, is in the continuity of these latter works. The state-of-the-art Vortex-Particle-Mesh (VPM) method seems therefore appropriate for the design of a high-fidelity flight environment in which to simulate formation flight. This LES method was improved over the years with key ingredients that guarantee both numerical and computational efficiency.

Immersed lifting lines were for instance introduced, thus replacing the effect of slender lifting devices on the flow by an equivalent bound vortex. This reduces the computational costs and thus allows to simulate the wake of the devices over long distances, using space developing simulations. The latter makes capturing the full phenomenological chain of wake propagation possible [22, 23] whereas time developing simulations approximate the spatial evolution by the temporal evolution of a finite segment of wake. The latter technique exploits the fact that aircraft vortices exhibit slow variations along its axial direction. Still, a calibrated combination of space developing and time developing VPM simulations can be a suitable approach when the understanding of the problem requires to model both short and long-term scales phenomena such as in formation flight context.

An isolated aircraft is easily modeled by a fixed wing in a numerical environment, but formation flight exhibits more complexity as it involves quite directly the aircraft dynamics. A follower indeed undergoes aerodynamic interferences due to the leader wake proximity. The coupling of VPM to the multi-body integrator ROBOTRAN [49] allows to take these effects into account, provided that the appropriate control surfaces of the aircraft and efficient control laws are implemented. This leads to a high-fidelity simulation tool involving both unsteady wake propagation, aircraft dynamics, and control. By combining such a tool with a simplified model that captures those physics at a sufficient accuracy, it becomes possible to consider and develop formation flight strategies involving data assimilation or learning, by comparing measurements obtained in these two tools.

Objectives

The previous sections have introduced the general topics and the context of this thesis. The discussions therein have moreover identified important research questions concerning formation flight for civil aviation applications. They can be summarized as follows:

- What are the physics that need to be modeled in order to numerically simulate formation flight? In particular, is it enough to combine wake flow simulation to aircraft dynamics computation in order to model the large-scale phenomena observed in formation flight? To what extent models with different accuracy levels can be used to represent formation flight?
- How can we use the dynamics of an aircraft as measurements of the flow that surrounds it, and in particular of a wake that impacts it? How can we associate

an aircraft dynamics model with these measurements in order to predict the characteristics of that wake? Is that aircraft dynamics model absolutely necessary or are model-free-based approaches also efficient?

- How can one quantify the adequacy and efficiency of measurements, and in particular measurements of the dynamics of an aircraft, towards the prediction of wake characteristics ? Is that wake measurement ability homogeneous in space behind that wake? How can one benefit from potential differences in the ability to measure a wake in order to design more efficient control strategies?
- How does the wake of an aircraft formation behave compared to the wake of an isolated aircraft? What is its long-term impact, in particular on the Air Traffic Management?

Outline

We open this work in Chapter 1 with the presentation of the simulation tools developed for the evaluation of formation flight. We describe the building blocks required to reproduce the aerodynamic effects of formation flight on an aircraft, and the consequences on the aircraft dynamics. We describe in this first chapter two different formation flight models characterized by two different levels of fidelity and accuracy, in particular for the modeling of the wake propagation. The first one uses a quasi-static wake model based on the Prandtl Lifting Line theory while the second one uses the VPM simulation tool to propagate the wake in time and space. We subsequently describe the dynamic equation of motion of a six degrees-of-freedom aircraft with states and conventions that are related. The achievement of this chapter consists in the control of the complete dynamics of an aircraft within Large Eddy Simulations and the comparison with a simplified model.

In Chapter 2, we use the simulation tools developed in Chapter 1 in order to design a wake tracking strategy that enables a follower aircraft to detect the position of the wake produced by a leader aircraft, without the need for information coming from that leader. The follower uses measurements of its dynamics to feed an ensemble Kalman filter-based estimator that makes use of the simplified aircraft model. We highlight in this chapter the need for appropriate measurement and appropriate accuracy in order to be able to deduce the position of a wake over time in the objective of energy consumption reduction.

Drawing inspiration from a bird, Chapter 3 focuses on the tracking of the optimal position in the wake of a leader through the use of a Reinforcement Learning algorithm

within an adapted simulation framework. The follower uses appropriate acceleration measurements to determine the velocity required to converge towards the sweet spot where it would benefit from the largest energy savings.

These two chapters use dynamics measurements to predict and converge to an optimal position in the wake of a leader. In Chapter 4, we shift our focus to the observability of the characteristics of the leader's wake when dealing with these measurements. In particular, we show that, depending on the position of the follower in the wake, measurements do not provide similar guarantees in terms of wake estimation. We finally introduce a control strategy that maximizes the observability of the wake when the follower tries to reach a desired position.

In Chapter 5, we evaluate the impact of formation flight on the wake propagation. This study requires to propagate the wake of formations over long distances, while taking the initial wake emission into account. We therefore introduce an appropriate simulation methodology that combines both Space Developing and Time Developing simulations in order to account for all the phenomenological chain behind wake propagation. We then perform simulations of various formation flight configurations, where the relative position of the follower with respect to the leader is assumed uncertain. This enables the study of the impact of this uncertainty on both the wake vortices characteristics and the Air Traffic Management.

Finally, we draw our conclusions and offer some perspectives for future works in terms of the uses of the developed methodologies and their potential extensions.

Aircraft and Wake Models

1.1 Introduction

Formation flying, as a bio-inspired technique, has long been known for its aerodynamic benefits [150, 128]. Large fuel savings are at stake when it comes to the integration of that technique into commercial aviation. Therefore, the topic is intensively researched today (e.g., [71, 57]), even in an industrial context as shown by the recent transatlantic crossing performed by two airbus A350 flying one behind the other in November 2021 [2]. Such an experiment shows the relevance of the formation flight strategy for long-haul flights (i.e., more than a 5% fuel saving), but also to highlight the various challenges that need to be tackled before imagining this flight technique on the long term.

In the context of this work, we do not have access to actual aircraft or actual flight data. In order to understand the physics that govern flight and wake dynamics, we rather rely on numerical models. The latter can be used to represent the physical phenomena involved in formation flight and in particular the effect of that flight technique on the dynamics of the aircraft and their wakes. In addition, they make it possible to use tools from other engineering disciplines in order to exploit their power and efficiency in the context of formation flight, e.g., data assimilation, control, machine learning tools. Indeed, filtering tools, such as the Kalman filter, involve prediction steps that require a representation of the physical phenomena, i.e., a model of the reality.

In this work, we exploit several models that differ from each other by their level of fidelity and accuracy. A high-fidelity model that we call the *simulated flight environment*, is used to simulate the unsteady physics of formation flight as accurately as possible, including aircraft aerodynamics and dynamics, and wake propagation. A Vortex

Particle-Mesh (VPM) method [26] is indeed used to perform Large Eddy Simulations of the wake flow behind aircraft whose lifting surfaces are modeled as Immersed Lifting and Dragging Lines [23]. This simulated flight environment represents reality and is responsible to produce reference measurements, i.e., *simulated flight data*, for the tools developed in the next chapters.

We also define a second model that we call the *simplified* or *surrogate model*. It differs from the simulated flight environment by a less accurate representation of the physics involved in formation flight. The Prandtl Lifting Line theory is implemented as a quasi-static simplified aerodynamic model without any representation of the wake dynamics. However, it enables the computation of the flight mechanics of the aircraft and to control it over time. Its main advantage lies in its low computational cost which permits its use in real time applications within more elaborate control tools. The surrogate model also allows the prediction of some behaviors of the simulated flight environment, e.g., the aircraft dynamics, in order to develop the appropriate autopilot to control these dynamics.

The dynamics and aerodynamics of an aircraft in formation flight are more complex compared to the dynamics observed when it is not subjected to the wake of a leader. This is caused by the aerodynamic interferences that arise from the wake generated by the leader, i.e., wake induced forces and moments, which means new terms in the equations of motion of the aircraft. Because those formation flight interactions significantly alter the follower's dynamics, their effects have to be sufficiently captured when dealing with control law modeling in order to ensure reliable performance of the control system in an actual operating environment.

Throughout this work, we will juggle with both the surrogate model and the simulated flight environment to describe and predict the physical phenomena that explain formation flight and its efficiency. This explains the detailed description of both models in this chapter. First, we qualitatively describe in Section 1.2 the aerodynamic phenomena that lead to the benefit of formation flight. We then describe in Section 1.3 how we design the simplified aircraft model that allows us to quantify the impact of formation flight on the aircraft, in terms of induced forces and torques. We first present the Prandtl Lifting Line theory that predicts the aerodynamic efforts on a lifting surface, before computing those efforts on an entire aircraft. The later enable the computation of the accelerations undergone by the aircraft and to deduce the equations of motion that are presented in Section 1.4. We finally describe the autopilot designed to control these dynamics. We then present in Section 1.5 the virtual flight environment which is considered as the reference environment to validate the tools and theory developed in the rest of this work. This environment is based on Large Eddy Simulations and allows

the computation of the complete dynamics of the wake of an isolated aircraft or a formation. We have integrated the virtual flight environment the complete dynamics of an aircraft as well as the aircraft controller schemes, similarly to the surrogate model. We compare the dynamics computed by both the surrogate model and the virtual environment in order to assess their agreement and validate their combined use. We close this chapter with some conclusions in Section 1.6.

1.2 Formation flight aerodynamics

1.2.1 An aircraft wake as a two-vortex system

In this section, we first present concepts related to aircraft wake in order to introduce the origin of the benefit of formation flight. This starts with the definition of the main characteristics of aircraft wake vortices that are deduced from the properties of the aircraft that generates them. The wake of a flying aircraft first consists in a vortex sheet that emanates from the wing and rolls up to form far downstream a two-vortex system (2VS). The two counter-rotating vortices of the 2VS are defined by their circulation Γ_0 and $-\Gamma_0$ and a separations b_0 as described in Fig. 1.1.

We first define the root circulation of the vortex system

$$\Gamma_0 = \int_0^{b/2} -\frac{d\Gamma(y)}{dy} dy = \Gamma(0), \quad (1.1)$$

with $\Gamma(y)$ the circulation distribution of the vortex sheet emitted by the aircraft main wing. By neglecting the vortex sheets associated to the vertical and horizontal tail planes of the aircraft, the conservation of momentum directly relates the lift force on the aircraft to the circulation distribution of the wing using

$$L = \rho U_\infty \int_{-b/2}^{b/2} \Gamma(y) dy \quad (1.2)$$

$$= -\rho U_\infty \int_{-b/2}^{b/2} y \frac{d\Gamma(y)}{dy} dy, \quad (1.3)$$

with ρ the ambient air density and U_∞ the cruise speed of the aircraft. We then define the total impulse ($\Gamma_0 b_0$) of the wing, i.e.,

$$\Gamma_0 b_0 = - \int_{-b/2}^{b/2} y \frac{d\Gamma(y)}{dy} dy. \quad (1.4)$$

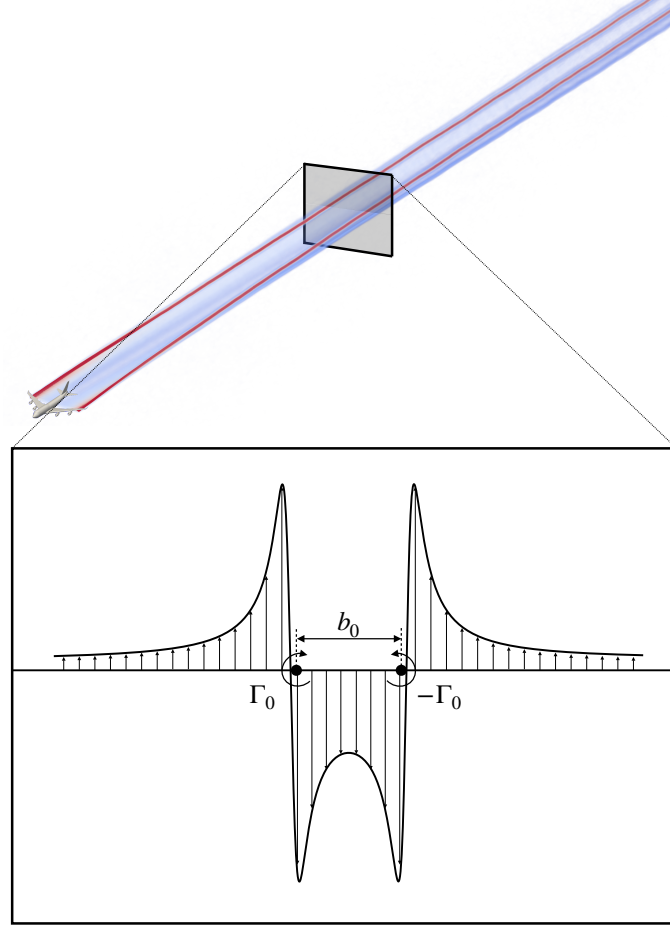


Figure 1.1: Visualization of the vortex sheet roll-up behind an aircraft into a two vortex system (top). This vortex system induces upwash or downwash velocities on its environment, such as represented for a line passing through the two vortices (bottom).

From Eqs. 1.3 and 1.4, we deduce

$$L = \rho U_\infty \Gamma_0 b_0, \quad (1.5)$$

$$b_0 = \frac{-1}{\Gamma_0} \int_{-b/2}^{b/2} y \frac{d\Gamma(y)}{dy} dy. \quad (1.6)$$

In the case of an elliptical circulation distribution, one deduces $b_0 = \frac{\pi b}{4}$, with b the span of the aircraft wing. The aircraft lift is also related to its weight. For an aircraft

in steady level flight, we have

$$L = mg, \quad (1.7)$$

where g is the gravity and m the mass of the aircraft. From Eqs. 1.5 and 1.7 one deduces the main characteristics of the 2VS from the aircraft geometry and the flight conditions.

As observed in Fig. 1.1 each one of the two rolled-up vortices induces velocity on their surrounding environment. This velocity is characterized by the tangential velocity profile $u_\theta(r)$ computed for radial positions r from the center of each vortex. This profile is computed as

$$u_\theta(r) = \frac{\Gamma(r)}{2\pi r}, \quad (1.8)$$

where

$$\Gamma(r) = \int_0^{2\pi} \mathbf{u}(r, \theta) \cdot \hat{\mathbf{e}}_\theta r d\theta = \int_0^{2\pi} \int_0^r \omega(r, \theta) \cdot \hat{\mathbf{e}}_\theta r' dr' d\theta \quad (1.9)$$

is the circulation distribution of the vortex, measured from the velocity field $\mathbf{u}$. From the tangential velocity profile $u_\theta(r)$, we introduce the definition of vortex core radius, r_c , defined as the position where the azimuthal velocity $u_\theta(r)$ is maximum, i.e., $r_c = \arg \max u_\theta(r)$. The exact value of that radius core size is still somewhat undetermined as it is indeed very difficult to capture the vortex core details of real aircraft wakes both experimentally and numerically. When considering the energy of the 2VS and fitting each vortex with a simplified model, one obtains that the core radius r_c ranges from 3% to 5% of the aircraft wingspan b [60, 14].

Rather than simulating the complete vortex sheet roll-up from its emission to the formation of the 2VS, different models have been developed to represent wake vortex circulation and tangential velocity distributions of the rolled-up vortices of the 2VS, with a reduced amount of parameters. This work does not intend to compare or to develop any new wake models. Rather, in the following chapters, we use the *Low Order Algebraic* (LOA) (or *Moore-Rosenhead*, or *Burnham-Hallock*, see Appendix A) model whenever we have to represent the influence of wake vortices on their environment, i.e., in the surrogate model of an aircraft formation. The LOA model is characterized by one single inner length scale which plays the role of tuning parameter for the model, i.e.,

$$\frac{\Gamma(r)}{\Gamma_0} = \frac{r^2}{r^2 + r_c^2}, \quad (1.10)$$

from which the tangential velocity is deduced using Eq. (1.8). From a vortex separation b_0 and a core radius r_c , one computes the velocity field induced by a vortex pair on its environment, as represented in Fig. 1.1. The interested reader is invited to read

the Appendix A for an enumeration of other aircraft wake models described in the literature.

From all the aforementioned definitions, we deduce an estimation of the sinking velocity of the vortex pair that results from the tangential velocity that each vortex induces on the other

$$V_0 = \frac{\Gamma_0}{2\pi b_0}, \quad (1.11)$$

and a corresponding time scale

$$t_0 = \frac{b_0}{V_0}. \quad (1.12)$$

All the global properties $U_\infty, b, \Gamma_0, b_0, V_0$ and t_0 that we have just described are the governing parameters of the physics behind formation flight. They are therefore used recurrently throughout the following work.

1.2.2 Aerodynamic impacts of formation flight

Fig. 1.1 shows the main consequences of the aircraft wake vortices on its environment, namely downwash areas between the vortices and upwash areas outside. In a very simplified way, these upwash regions carry and push upwards any object that would be flying in those areas, e.g., a follower aircraft that would want to reduce its energy consumption. Qualitatively, the benefit of formation flight is attributed to the rotation of the resultant forces on the follower aircraft. As described in Fig. 1.2a the upwash modifies the air velocity vector perceived by the follower, i.e., $\mathbf{U}_\infty \rightarrow \mathbf{U}'$. The lift (resp. drag) forces being perpendicular (resp. parallel) to the velocity wind vector, it is thus modified, which translates into a difference $\Delta \mathbf{D}$ (resp. $\Delta \mathbf{L}$) of the resulting horizontal (resp. vertical) force in an inertial frame. As a main consequence, the thrust required to balance the forces is reduced as is the energy consumption of the follower.

These physical phenomena are at the origin of the V-shapes observed during the migration of birds. The following birds need to position on either side of a leading bird wake in order to benefit from the upwash wind and the resulting required thrust reduction. On the other hand, the position just behind the leader is detrimental because of a downwash wind as observed in Fig. 1.2b.

The impact of formation flight on a follower is not limited to the 2D aerodynamic effects observed in Fig. 1.2a. As observed in Fig. 1.2b the velocity profile u_θ induced by the vortex pair of the leader on the follower is not constant along the wing span. The increase in lift induced on the left wing is greater than the increase in lift induced on the right wing which therefore results in an induced rolling moment that needs to be compensated by the follower in order to maintain the position. Similarly, the induced

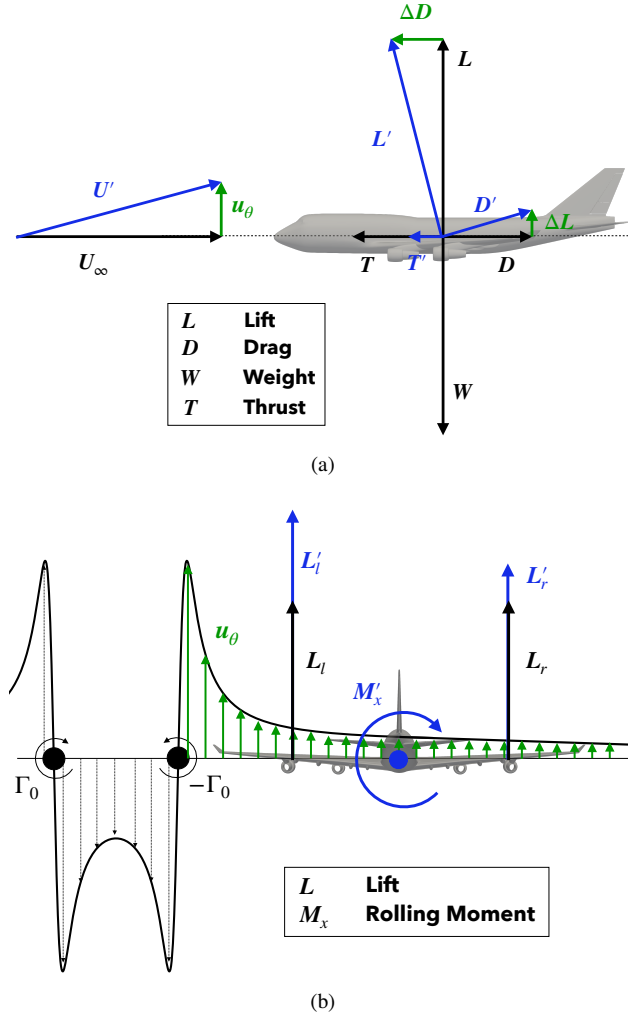


Figure 1.2: Main resultant efforts on the follower aircraft due to vertical upwash wind generated by the leader's wake vortices. The baseline forces are drawn in black and formation resulting forces in blue.

drag forces on the left and right wings also differ and lead to a yawing moment. All the aerodynamic loads on the follower are thus impacted by the presence of the vortices in the environment. Due to the non-linearities caused by the wake vortices on the induced velocities and the great variance depending on the position of the follower in the wake

of the leader, one needs an appropriate model to quantitatively compute the variation of the loads on the follower.

1.3 Simplified aerodynamic model for aircraft in formation flight

Contrary to the qualitative approach described in Section 1.2 the quantification of the impact of formation flight on a follower requires the computation of the aerodynamic efforts on that aircraft. In particular, those aerodynamic efforts need to take into account the effect of the leader's wake vortices. The aerodynamic model to be designed also have to consider all the components of an aircraft essential to its flight, i.e, its wings, Vertical and Horizontal Tail Planes (VTP and HTP), usually called the lifting surfaces of the aircraft.

As described in Section 1.2, the influence of leader's wake is accounted through the LOA wake model which enables the computation of the velocities induced by the vortices from geometrical parameters of the leader. In this section, we first present the Prandtl Lifting Line theory which is used as the basic building block for the design of the aerodynamic model of the follower. Indeed, from that theory, one can compute the aerodynamic efforts on a lifting surface from geometrical parameters and velocities perceived along that surface, e.g., wake induced velocities. Applying the Prandtl Lifting Line theory to the wing, VTP and HTP of the follower, a complete quantitative aerodynamic model can be build for that aircraft. This is further described in this section. We finish with results computed by the entire model.

1.3.1 Classical Prandtl Lifting Line

The Lifting Line theory is a powerful mathematical model that predicts the circulation distribution $\Gamma(y)$ and the loads of 3D slender devices as it is required to evaluate the impact of the formation flight on a follower. It was initially formulated more than one hundred years ago by Prandtl [113]. Based on geometrical parameters of a wing, the theory is able to evaluate at a low computational cost and with a fairly good accuracy, the main efforts that a wing undergoes when moving in its environment. This explains why this theory is always used for preliminary designs and for aerodynamic predictions. Yet, the Prandtl Lifting Line (PLL) has its limitations: the wing must have a sufficiently high aspect ratio $AR > 7 \dots 8$ [83] and is limited to a succession of airfoils that lie on a straight line (i.e., no sweep nor dihedral). The PLL also assumes steadiness and is limited to small angles of attack. Improvements have therefore been brought to

the initial theory to extend its use while continuing to benefit from its simplicity (see [64, 110, 120, 111, 129]).

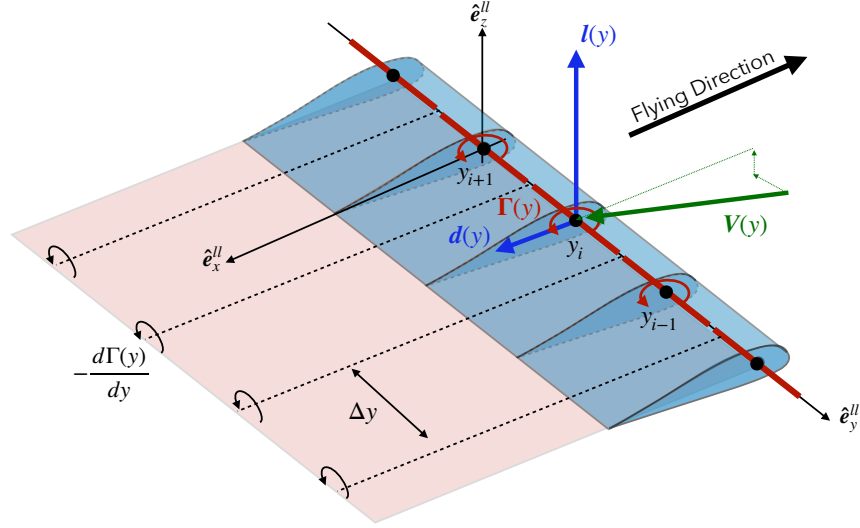


Figure 1.3: Aerodynamic loads computed on the PLL based on the circulation distribution.

We choose the PLL theory as a background model for its simplicity and computational affordability. It consists in replacing the lifting surface, e.g., a wing, with a finite number of segments on which is associated an horseshoe vortex, as described in Fig. 1.3. The latter is centered around the lifting surface segment centre-line, and bounded to the quarter-chord line. For the sake of simplicity, we define the lifting line frame $(ox^{ll}y^{ll}z^{ll})$ whose origin is at the center of the lifting line, axis $\hat{e}_x^{ll}$ is opposite to the flying direction, axis $\hat{e}_y^{ll}$ is aligned with the lifting surface and axis $\hat{e}_z^{ll}$ is pointing upwards, the three axes being perpendicular. The circulation $\Gamma(y)$ associated to each horse-shoe vortex and aligned with $\hat{e}_y^{ll}$ is computed assuming that the aerodynamics of the wing segment can be described using 2D relations. It neglects the transverse flow component. On the one hand, the Kutta-Joukowski theorem [87, 79] gives the sectional lift density

$$l(y) = \rho U(y) \Gamma(y), \quad (1.13)$$

where U is the streamwise component of the air speed vector $V(y)$ at the segment, i.e., $V(y) = [U, V, W]^T$. On the other hand, the definition of the sectional lift coefficient

$$C_l(y) = \frac{l(y)}{\frac{1}{2}\rho U(y)^2 c(y)} = f(\alpha_e(y)) \simeq a_0 \alpha_e(y), \quad (1.14)$$

where f is a function retrieved from tabulated data that related the lift coefficient to the effective angle of attack $\alpha_e(y)$ measured at the segment. We assume a linear relation in this work, with a slope a_0 . The effective angle attack is written as

$$\alpha_e(y) = \alpha_g(y) + \tau(y)\delta_{CS}(\tau) + \frac{W(y)}{U(y)} - \epsilon(y). \quad (1.15)$$

In this equation, $\alpha_g(y)$ is the geometrical angle of attack of the segment that can vary along the span if the wing is twisted. δ_{CS} is the deflection of a control surface associated to the lifting surface (e.g., an aileron), and whose contribution to the 2D lift is modeled as a local modification of the angle of attack through the factor $\tau(y)$. This factor is non-zero at the position of the control surface and zero elsewhere (e.g., outboards ailerons only act near the wing tips). The term $\frac{W}{U}$ is the equivalent contribution to the angle of attack of the vertical airspeed component measured at the segment. Through this term, the PLL theory shows its ability to take into account both the contribution of the upwash or downwash induced by the environment (particularly, the leader's wake induced velocity), and the contribution of the movement of the wing. Indeed, the airspeed perceived in $\mathbf{x} = [x, y, z]$ is computed using

$$\mathbf{V} = \mathbf{V}_{env}(\mathbf{x}) - (\mathbf{V}_K + \boldsymbol{\Omega} \times \mathbf{x}), \quad (1.16)$$

where $\mathbf{V}_{env}$ is the velocity of measured in the environment (e.g., the leader wake-induced velocity or any flow turbulence), $\mathbf{V}_K$ is the flight path velocity of the wing, $\boldsymbol{\Omega}$ is the angular velocity of the wing, and $\times$ the cross product. Finally, $\epsilon(y)$ is the downwash angle which is deduced from the Biot-Savart law. It is due to the velocity $\mathbf{V}_{ind}$ induced by the lifting surface wake on itself and the environment. In this model, we assume that $V \ll U$ and $W \ll U$, i.e., the perturbations of the environment and the dynamics of the aircraft are much smaller than the flying velocity that is aligned with $\hat{e}_x^{ll}$. We thus assume that the vortex sheet produced by the wing is also aligned with $\hat{e}_x^{ll}$, as it is drawn in Fig. 1.3. We use the mathematical notations used by Caprace et al. [20] to deduce the induced velocity V_{ind} from the singular wing vortex tube and the singular wake vortex sheet that are used in the original lifting line theory. The resulting vorticity field is thus also singular, i.e.,

$$\boldsymbol{\omega}(\mathbf{x}) = \delta(x)\Gamma(y)\delta(z)\hat{e}_y^{ll} + H(x)\left(-\frac{d\Gamma(y)}{dy}\right)\delta(z)\hat{e}_x^{ll}, \quad (1.17)$$

where δ is the Dirac delta and $H(x)$ the Heavyside function. The first term of this equation is the bound vorticity of the lifting line and the second one is the shed vorticity through the vortex sheet. In this form, the equation assumes the lifting line is perpendicular to the airspeed vector $\mathbf{V}$ and the vortex sheet is propagated in the $\hat{\mathbf{e}}_x^{ll}$ direction. In those conditions, the resulting self-induced velocity is

$$\begin{aligned} \mathbf{V}_{ind}(\mathbf{x}) &= (\mathbf{K}(\mathbf{x}) \times) * \boldsymbol{\omega}(\mathbf{x}) \\ &= \frac{1}{4\pi} \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{1}{y-y'} \left(-\frac{d\Gamma(y')}{dy'} \right) dy' \hat{\mathbf{e}}_z, \end{aligned} \quad (1.18)$$

with $(\mathbf{K}(\mathbf{x}) \times) = -(1/4\pi\|\mathbf{x}\|^3) \mathbf{x} \times$ the 3D Biot-Savart kernel, $\times$ the scalar product and $*$ the 3D convolution operator. The downwash angle measured on the lifting line is finally

$$\epsilon(y) = -\frac{\mathbf{V}_{ind}(y) \cdot \hat{\mathbf{e}}_z}{U(y) + \mathbf{V}_{ind}(y) \cdot \hat{\mathbf{e}}_x} = -\frac{1}{4\pi U(y)} \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{1}{y-y'} \left(-\frac{d\Gamma}{dy'} \right) dy'. \quad (1.19)$$

By combining Eqs. 1.13, 1.14, 1.15, and 1.19, we deduce the Prandtl's integrodifferential equation for the circulation distribution,

$$\begin{aligned} \rho U(y) \Gamma(y) &= \frac{1}{2} \rho U(y)^2 c(y) a_0 \left(\alpha_g(y) + \tau(y) \delta_{CS} + \frac{W(y)}{U(y)} - \epsilon(y) \right) \\ \Gamma(y) &= \frac{1}{2} a_0 c(y) U(y) \left(\alpha_g(y) + \tau(y) \delta_{CS} + \frac{W(y)}{U(y)} \right. \\ &\quad \left. - \frac{1}{4\pi U(y)} \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{1}{y-y'} \left(\frac{d\Gamma}{dy'} \right) dy' \right). \end{aligned} \quad (1.20)$$

The aerodynamic loads are readily deduced from the circulation distribution. Applying the vortex lifting law to a differential segment $d\mathbf{l}$ of the lifting line, we have

$$\begin{cases} d\mathbf{F} &= \rho \Gamma \mathbf{V} \times d\mathbf{l} \\ d\mathbf{M} &= \rho \Gamma \mathbf{x} \times (\mathbf{V} \times d\mathbf{l}) \end{cases}. \quad (1.21)$$

As for the sectional lift coefficient from Eq. (1.14), we also use polar functions in order to take into account the lifting surface parasitic drag and pitching moment. We then compute the vector components of the force $\mathbf{F} = [F_x^{ll}, F_y^{ll}, F_z^{ll}]_{ll}^T$ and torque $\mathbf{M} =$

$\left[M_x^{ll}, M_y^{ll}, M_z^{ll} \right]_{ll}^T$ of the lifting surface that is aligned with $\hat{e}_y^{ll}$ in Fig. 1.3, i.e.,

$$\begin{cases} F_x^{ll} &= \rho \int_{-\frac{b}{2}}^{\frac{b}{2}} \left[\Gamma(y) U(y) \left(\epsilon(y) - \frac{W(y)}{U(y)} \right) + \frac{1}{2} c(y) U(y)^2 C_{d,0} \right] dy \\ F_y^{ll} &= 0 \\ F_z^{ll} &= \rho \int_{-\frac{b}{2}}^{\frac{b}{2}} \Gamma(y) U(y) dy \\ M_x^{ll} &= \rho \int_{-\frac{b}{2}}^{\frac{b}{2}} \Gamma(y) U(y) y dy \\ M_y^{ll} &= \rho \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{1}{2} c(y)^2 U(y)^2 C_{m,0} dy \\ M_z^{ll} &= -\rho \int_{-\frac{b}{2}}^{\frac{b}{2}} \left[\Gamma(y) U(y) \left(\epsilon(y) - \frac{W(y)}{U(y)} \right) + \frac{1}{2} c(y) U(y)^2 C_{d,0} \right] y dy \end{cases}, \quad (1.22)$$

where $C_{d,0}$ and $C_{m,0}$ are the parasitic drag and pitching moment coefficients that are retrieved from tabulated data. As the theory neglects the transverse flow component, there is no force component computed along the span direction, i.e., $\hat{e}_y^{ll}$.

The evaluation of the aerodynamic efforts thus requires to solve the Prandtl's integro-differential Eq. (1.20). An efficient resolution technique consists in projecting the solution on an orthogonal basis of functions. The segmentation of the lifting line into M pieces is associated to the use of N sinusoidal basis functions and N basis coefficient B_n such that $N < M$. We define

$$y(\theta) = -\frac{b}{2} \cos(\theta) \text{ with } \theta \in [0, \pi], \quad (1.23)$$

$$\Gamma(y) = \Gamma(\theta) = U_\infty b \sum_{n=1}^N B_n \sin(n\theta), \quad (1.24)$$

where U_∞ is the cruise speed of the aircraft. By replacing the two previous equations in Eq. (1.20), we have

$$\begin{aligned} \sum_{n=1}^N B_n \sin(n\theta) &= \frac{c(\theta)}{2b} a_0 \left[(\alpha_g(\theta) + \tau(\theta) \delta_{CS}) \frac{U(\theta)}{U_\infty} + \frac{W(\theta)}{U_\infty} \right. \\ &\quad \left. - \frac{1}{4\pi U_\infty} \int_0^\pi \frac{U_\infty b \sum_{n=1}^N n B_n \cos(n\theta') d\theta'}{-\frac{b}{2} (\cos(\theta) - \cos(\theta'))} \right]. \end{aligned} \quad (1.25)$$

Using the *Glauert integral* [62] that specifies $\int_0^\pi \frac{\cos(n\theta')}{(\cos(\theta') - \cos(\theta))} d\theta' = \frac{\pi \sin(n\theta)}{\sin(\theta)}$, we get

$$\sum_{n=1}^N B_n \left[\sin(n\theta) + \frac{1}{4} \frac{c(\theta)}{b} a_0 \frac{n \sin(n\theta)}{\sin(\theta)} \right] = \frac{c(\theta)}{2b} a_0 \left[(\alpha_g(\theta) + \tau(\theta) \delta_{CS}) \frac{U(\theta)}{U_\infty} + \frac{W(\theta)}{U_\infty} \right]. \quad (1.26)$$

As $N < M$, the previous equation is solved using a least-square method to find the value of the N coefficients B_n that minimizes the sum of the square of the residuals. While such a methodology is not mandatory for the resolution, it provides a continuous and regularized solution for Eq. (1.20).

1.3.2 The PLL theory for an aircraft with three lifting surfaces

The aerodynamics of an aircraft are mostly governed by three major lifting surfaces already mentioned, i.e., the wing, the VTP and the HTP. The wing generates the most important part of the lift of the aircraft and is equipped with ailerons used for the control of the lateral dynamics of the aircraft. The main role of the VTP is to provide control, stability and trim in yaw with the help of the rudder. Finally, the function of HTP is to provide stability and control in pitch. In particular, the HTP helps adjusting for changes in position of the centre of pressure or centre of gravity caused by changes in speed and altitude. As depicted in Fig. 1.4 we model an aircraft by its simplest, considering only the wing, the HTP and the VTP for the computation of its aerodynamics, using the PLL theory. We assume the VTP and HTP are perpendicular to each other forming together an inverted "T", as described in Fig. 1.4. This tail is located at the distance d_t downstream the main wing.

The evaluation of the forces on the wing follows the exact same methodology as the one described throughout Section 1.3.1. Indeed, we assume that the tail does not influence the wing vorticity generation. While the same approach could also be used for the VTP and HTP, it would not take their mutual influence into account. Indeed, the vortex sheet and the bound vortex of the HTP directly impact the vortex sheet and the bound vortex of the VTP, and vice versa. Joint integro-differential equations have thus to be deduced and solved in order to obtain their circulation distribution $\Gamma_{HTP}(y)$ and $\Gamma_{VTP}(z)$. By adapting Eq. (1.17), the singular vorticity field associated to the VTP and HTP lifting lines is written

$$\begin{aligned} \boldsymbol{\omega}(\mathbf{x}) = & \delta(x) \Gamma_{HTP}(y) \delta(z) \hat{\mathbf{e}}_y + H(x) \left(-\frac{d\Gamma_{HTP}(y)}{dy} \right) \delta(z) \hat{\mathbf{e}}_x \\ & + \delta(x) \Gamma_{VTP}(z) \delta(y) \hat{\mathbf{e}}_z + H(x) \left(-\frac{d\Gamma_{VTP}(z)}{dz} \right) \delta(y) \hat{\mathbf{e}}_x. \end{aligned} \quad (1.27)$$

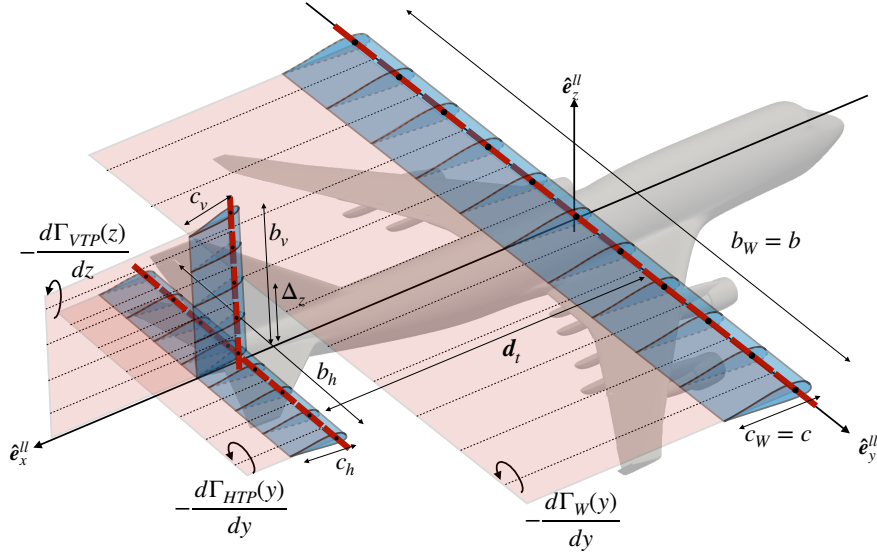


Figure 1.4: Aircraft represented as three PLLs, i.e., the wing, the HTP and the VTP, each one emitting its own vortex sheet.

Using Eq. (1.18), we compute the velocity induced by the four contributions of the previous vorticity field on the HTP and VTP, i.e.,

$$\begin{aligned}
 \mathbf{V}_{\text{ind}}^{HTP}(y) = & \mathbf{0} \\
 & + \frac{1}{4\pi} \int_{-\frac{b_h}{2}}^{\frac{b_h}{2}} \frac{1}{y-y'} \left(-\frac{d\Gamma_{HTP}(y')}{dy'} \right) dy' \hat{e}_z^{ll} \\
 & + \frac{-1}{4\pi} \int_{-\frac{b_{lv}}{2}+\Delta_z}^{\frac{b_{lv}}{2}+\Delta_z} \frac{y}{\sqrt{y^2+z'^2}} {}^3\Gamma_{VTP}(z') dz' \hat{e}_x^{ll} \\
 & + \frac{1}{4\pi} \int_{-\frac{b_{lv}}{2}+\Delta_z}^{\frac{b_{lv}}{2}+\Delta_z} \frac{y}{y^2+z'^2} \left(-\frac{d\Gamma_{VTP}(z')}{dz'} \right) dz' \hat{e}_z^{ll} \\
 & + \frac{1}{4\pi} \int_{-\frac{b_{lv}}{2}+\Delta_z}^{\frac{b_{lv}}{2}+\Delta_z} \frac{z'}{y^2+z'^2} \left(-\frac{d\Gamma_{VTP}(z')}{dz'} \right) dz' \hat{e}_y^{ll}, \quad (1.28)
 \end{aligned}$$

$$\begin{aligned}
\mathbf{V}_{\text{ind}}^{VTP}(z) = & \frac{1}{4\pi} \int_{-\frac{b_h}{2}}^{\frac{b_h}{2}} \frac{z}{\sqrt{y'^2 + z^2}}^3 \Gamma_{HTP}(y') dy' \hat{e}_x^{ll} \\
& + \frac{-1}{4\pi} \int_{-\frac{b_h}{2}}^{\frac{b_h}{2}} \frac{y'}{y'^2 + z^2} \left(-\frac{d\Gamma_{HTP}(y')}{dy'} \right) dy' \hat{e}_z^{ll} \\
& + \frac{-1}{4\pi} \int_{-\frac{b_h}{2}}^{\frac{b_h}{2}} \frac{z}{y'^2 + z^2} \left(-\frac{d\Gamma_{HTP}(y')}{dy'} \right) dy' \hat{e}_y^{ll} \\
& + 0 \\
& + \frac{-1}{4\pi} \int_{-\frac{b_v}{2} + \Delta_z}^{\frac{b_v}{2} + \Delta_z} \frac{1}{z - z'} \left(-\frac{d\Gamma_{VTP}(z')}{dz'} \right) dz' \hat{e}_y^{ll}, \quad (1.29)
\end{aligned}$$

where $y \in [-b_h/2, +b_h/2]$ on the HTP and $z \in [-b_v/2 + \Delta_z, +b_v/2 + \Delta_z]$ on the VTP, and where b_v (resp. b_h) is the span of the VTP (resp. HTP) and Δ_z is the vertical offset of the VTP in the $\hat{e}_z$ direction, see Fig. 1.4 for a visual representation. Each line of Eqs. 1.28 and 1.29 corresponds to the influence of each term of Eq. (1.27) to the induced velocities.

The joint integro-differential equations for the HTP and VTP are deduced similarly as Eq. (1.20), and we obtain

$$\begin{aligned}
\Gamma_{HTP}(y) = & \frac{1}{2} a_0 c_h(y) U(y) \left((\alpha_{g,h}(y) + \tau(y) \delta_{HT}) + \frac{W(y)}{U(y)} + \frac{\mathbf{V}_{\text{ind}}^{HT}(y) \cdot \hat{e}_z^{ll}}{U(y)} \right) \\
= & \frac{1}{2} a_0 c_h(y) U(y) \left((\alpha_{g,h}(y) + \tau(y) \delta_{CS}) + \frac{W(y)}{U(y)} \right. \\
& - \frac{1}{4\pi U(y)} \int_{-\frac{b_h}{2}}^{\frac{b_h}{2}} \frac{1}{y - y'} \left(\frac{d\Gamma_{HTP}(y')}{dy'} \right) dy' \\
& \left. - \frac{1}{4\pi U(y)} \int_{-\frac{b_v}{2} + \Delta_z}^{\frac{b_v}{2} + \Delta_z} \frac{y}{y^2 + z'^2} \left(\frac{d\Gamma_{VTP}(z')}{dz'} \right) dz' \right), \quad (1.30)
\end{aligned}$$

$$\begin{aligned}
\Gamma_{VTP}(z) &= \frac{1}{2} a_0 c_v(z) U(z) \left((\alpha_{g,v}(z) + \tau(z) \delta_{VT}) - \frac{V(z)}{U(z)} - \frac{V_{ind}^{VTP}(z) \cdot \hat{e}_y''}{U(z)} \right) \\
&= \frac{1}{2} a_0 c_v(z) U(z) \left((\alpha_{g,v}(z) + \tau(z) \delta_{VT}) - \frac{V(z)}{U(z)} \right. \\
&\quad \left. - \frac{1}{4\pi U(z)} \int_{\frac{-b_h}{2}}^{\frac{b_h}{2}} \frac{z}{y'^2 + z^2} \left(\frac{d\Gamma_{HTP}(y')}{dy'} \right) dy' \right. \\
&\quad \left. - \frac{1}{4\pi U(z)} \int_{\frac{-b_v}{2} + \Delta_z}^{\frac{b_v}{2} + \Delta_z} \frac{1}{z - z'} \left(\frac{d\Gamma_{VTP}(z')}{dz'} \right) dz' \right), \tag{1.31}
\end{aligned}$$

where $c_v(z)$ (resp. $c_h(y)$) is the VTP (resp. HTP) chord distribution, $\alpha_{g,v}(z)$ (resp. $\alpha_{g,h}(y)$) is the geometrical angle of attack and δ_{VT} (resp. δ_{VT}) is the control surface associated to the VTP (resp. HTP), i.e., the rudder (resp. the elevator). In the previous equations, the influence of the wing wake vortex sheet on the tail is accounted through the components $W(y)$ and $V(z)$ of the airspeed. Indeed, we first compute the wing circulation distribution $\Gamma_W(y)$ and the resulting vortex sheet $-\frac{\Gamma_W(y)}{dy}$ assuming it is independent from the tail wake. The wing vorticity thus modifies the airspeed vector $\mathbf{V}$ through the environment velocity term $\mathbf{V}_{env}$ from Eq. (1.16). That velocity can be computed using Eq. (1.17) and the circulation distribution $\Gamma_W(y)$ computed on the wing. The resolution of Eqs. 1.30 and 1.31 is performed following the same methodology as the one described through Eqs. 1.23, 1.24, 1.25 and 1.26 with the introduction of two variable changes this time, one for $\Gamma_{VTP}(z)$ and the other for $\Gamma_{HTP}(y)$.

The forces and torques on the HTP and VTP are computed with the Eqs. 1.21 and are added to those computed for the wing. The aerodynamic effort of the three lifting surfaces are then reprojected on the center of mass of the aircraft. This allows the completion of the equation of movement of the aircraft and to integrates its kinematics, see Section 1.4.

1.3.3 Drawbacks of the model

The model described in the previous sections allows the evaluation of the main aerodynamic efforts on an aircraft while considering the influence of a disturbed external environment (e.g., wake vortices produced by a leading aircraft) through the term $\mathbf{V}_{env}$ in Eq. (1.16). The kinematics of the aircraft is also taken into account through the velocity of each of the lifting surfaces, i.e., $\mathbf{V}_K + \boldsymbol{\Omega} \times \mathbf{x}$ in the same equation. Finally, control commands are taken into account through the deflection angle δ_{CS} which lo-

cally modifies the aerodynamics (i.e., the polar) of the lifting surfaces in Eq. (1.15). However, this model does not account for the intrinsic unsteadiness of the complete dynamics of an aircraft and its wake. The model is rather considered as quasi-static. Indeed, the wakes produced by the lifting lines are modeled as straight infinite vortex sheets aligned with $\hat{e}_x^{II}$. In the classical PLL theory, the impact of any change in the velocity of the aircraft, airspeed or control command on the sheets is considered as instantaneous on the vortex sheet. Therefore, the transient until the new steady state is not captured. Several efforts have led to generalized methodologies allowing to take these unsteady aspects into account (,e.g., [129, 1]).

Also, this quasi-static hypothesis does not enable the modeling of the roll-up of the vortex sheets emitted by the lifting surfaces. The dynamics of the wake generated by an aircraft can therefore not be studied by such an approach. In particular, this highlights the difficulty of modeling the influence of the main wing of the aircraft on its tail. The wing vortex sheet has already started to roll up when impacting the tail, but it cannot be considered as two fully rolled-up vortices yet. Vortex models described in Appendix A can therefore not be used as straightforwardly.

Moreover, the lifting surfaces modeled in this section are assumed to be rigid and they do not deform due to the aerodynamic efforts. The control authorities are also assumed to be perfectly known in the form of a change in the polar data as a function of the deflection angle. Therefore, they do not take the influence of the flow disturbance that they involve into account. In an operational context, the use of a more refined aero-elastic model would entail the accounting for uncertainties of the aircraft elastic response, and the uncertainty on the control authorities due to the structure deformation and the flow adaptation [51].

Finally, our model cannot take into account more realistic geometries (,i.e., dihedral, deflection effects, etc) of the lifting surfaces, nor can it model any wingtip devices compared to adaptations of the method [64, 110, 120]. Compressibility effects are also not modeled and would require additionnal corrections. The parasitic drag of the lifting surfaces that results from the boundary layers deploying along the surfaces and that leads among other things to a deficit of the axial flow velocity is not taken into account either. Such simplifications lead to neglecting some crucial phenomena. For instance, in the case of the modeling of the tail, the wall effect (i.e., velocity deficit) perceived at the foot of the VTP that results from the boundary layer induced by the HTP is not reproduced.

For the sake of simplicity, all the aforementioned effects are not taken into account in this work. They could be all combined if one wanted to compute more accurate aerodynamics. Also, one could consider using other modeling theory that innately

take these effects into account (, i.e., vortex lattice method) or even build a model using actual flight data. The later would enable the computation of actual aerodynamic derivatives or corrections that would be implemented on a chosen modeling method. Nevertheless, our goal is not to perfectly compute the aerodynamics of an existing aircraft. The objective of the model described in this chapter is rather to compute simple aerodynamics that can be used to test innovative control tools that will be described in the rest of this work.

1.3.4 Circulations and efforts along the lifting surfaces

In this section, we present the aerodynamics as computed by the model designed in Sections 1.3.2 and 1.3.2. This allows us to introduce the geometric characteristics of the two aircraft used throughout this work. The first simulated aircraft is based on the Airbus A350 flying at 250 m/s at FL330. The second one is based on a much smaller aircraft, the Cessna C172 flying at 50 m/s at mean sea level. The main characteristics are listed in Table 1.1 In the following figures, we use the A350 aircraft configuration to illustrate the performances of the model.

Aerodynamics of the wing

We first compute in Fig. 1.5 the circulation distribution along the wing for three different configurations. The first one consists in the aircraft wing flying in steady flight. The second one consists in the same wing the ailerons of which are deployed in order to initiate a rolling maneuver. The last configuration consists in the aircraft wing during the rolling maneuver, i.e., ailerons in neutral position and the wing with a non-zero rolling rate.

The A350-like aircraft modeled in this work has a non-twisted elliptically-shaped wing, i.e., $\alpha_{g,w}(y) = \alpha_{0,w}$ and $c_w(y) = \frac{4b}{\pi R} \sqrt{1 - \left(\frac{2y}{b}\right)^2}$. In steady flight, the resulting circulation wing distribution is also elliptic, i.e., $\Gamma_w(y) = \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$, where Γ_0 depends on the flying velocity U_∞ , the angle of attack $\alpha_{0,w}$ and the aspect ratio $\mathcal{R}$. This circulation is the reference in Fig. 1.5. Let's note that such an elliptic distribution is not targeted by large commercial airliners and is therefore not representative of them. However, in the context of this work, we do not aim to accurately reproduce these aircraft. Such chord and circulation distributions are here used for numerical reasons, in order to avoid strong gradients or discontinuities that implies constraining numerical

<u>A350</u>			
	<u>Wing</u>	<u>HTP</u>	<u>VTP</u>
Span	$b = 64.5m$	$b_h/b = 0.35$	$b_v/b = 0.19$
Aspect ratio	$\mathcal{R} = 6.8$	$\mathcal{R}_h = 4$	$\mathcal{R}_v = 4.4$
Shape	Ellipse	Ellipse	Ellipse
Incidence	$\alpha_{0,w} = 4.5^\circ$	$\alpha_{0,h} = 1.3^\circ$	$\alpha_{0,v} = 0^\circ$
Sectional lift slope	$a_0 = 2\pi$	$a_0 = 2\pi$	$a_0 = 2\pi$
Cruise speed	$U_\infty = 250m/s$		
Mass	$m = 300000kg$		
Air density	$\rho = 0.413kg/m^3$		
Zero-lift drag coefficient	$C_{d,0} = 0.01$		
<u>C172</u>			
	<u>Wing</u>	<u>HTP</u>	<u>VTP</u>
Span	$b = 11m$	$b_h/b = 0.26$	$b_v/b = 0.22$
Aspect ratio	$\mathcal{R} = 7.5$	$\mathcal{R}_h = 4$	$\mathcal{R}_v = 4$
Shape	Ellipse	Rectangle	Rectangle
Incidence	$\alpha_{0,w} = 4^\circ$	$\alpha_{0,h} = 1.42^\circ$	$\alpha_{0,v} = 0^\circ$
Sectional lift slope	$a_0 = 2\pi$	$a_0 = 2\pi$	$a_0 = 2\pi$
Cruise speed	$U_\infty = 50m/s$		
Mass	$m = 700kg$		
Air density	$\rho = 1.225kg/m^3$		
Zero-lift drag coefficient	$C_{d,0} = 0.01$		

Table 1.1: Main characteristics of the A350-like and C172-like aircraft.

Configuration	C_X	C_Y	C_Z	C_{M_l}	C_{M_m}	C_{M_n}
Reference (—)	0.0168	0	0.381	0	0	0
$\delta_{CS} = 10^\circ$ (--)	0.0169	0	0.381	0.0057	0	-0.0003
Rolling Rate = 5° (---)	0.0167	0	0.381	-0.0056	0	-0.0002

Table 1.2: Wing contribution to the aerodynamic efforts of the A350 aircraft model. Comparison of the three wing configurations from Fig. 1.5. Dimensionless coefficients are used, i.e., $C_i = \frac{F_i}{1/2\rho U_\infty^2 S}$ and $C_{M_i} = \frac{M_i}{1/2\rho U_\infty^2 S b}$.

discretizations. This is all the more important for the large eddy simulations that are performed later.

We then observe the impact of the control surface on the circulation. The asymmetric deflection of the ailerons leads to a local decrease of the circulation on the left-hand side of the wing where the effect of the ailerons is modeled as a local decrease of the geometric attack angle. On the right-hand side, on the contrary, it leads to a local increase of the circulation. The overall and desired effect is the positive rolling moment, i.e.,

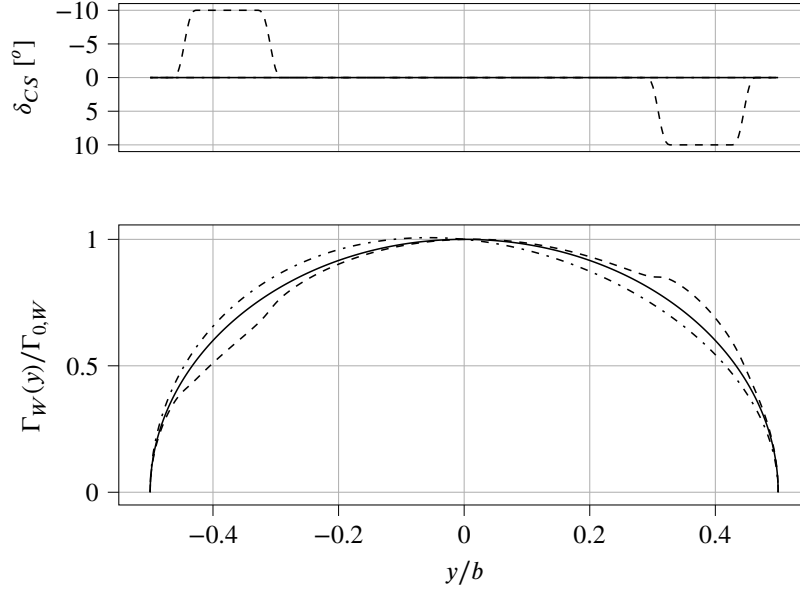


Figure 1.5: Wing circulation distribution for three configurations. Reference wing: (—); wing with 10° ailerons deflection: (---); wing during 5°/s rolling rate maneuver: (-.-).

rotation around the $\hat{e}_x^{ll}$ induced by the asymmetry of the lifting force on both sides of the wing, as presented in Table 1.2. We also measure a negative yawing moment, i.e., rotation around the $\hat{e}_z^{ll}$, called the adverse yaw that exists due to asymmetric induced drag perceived between both sides of the wing.

We finally assess the impact of a positive rolling rate, i.e., the left wing moving downwards, the right wing moving upwards, on the distribution circulation. We observe a more global modification of the circulation, as the entire wing undergoes the influence of the rolling movement. The lift force perceived on the left wing is more important than the right wing. This induces a negative rolling moment in opposition to the rolling rate, see Table 1.2. That is the so-called roll damping. A yawing moment also appears due to the drag asymmetry.

Aerodynamics of the tail

We now briefly investigate the impact of taking the mutual influence of the HTP and VTP on the aerodynamics of the tail into account as well as the influence of the wing

wake. In the following configurations, we compute the circulation along the HTP and VTP. The former is characterized by a geometrical angle of attack $\alpha_{g,h,0} > 0$ (see Table 1.1), and the latter is characterized by a non-zero control surface deflection $\delta_{VT} = 10^\circ$ (i.e., non-zero rudder deflection). The first configuration computes the circulation of the HTP and VTP taking their mutual interactions into account, i.e., using the methodology described in Section 1.3.2. The second one, however, assumes that VTP and HTP are two independent lifting surfaces, i.e., their circulation being computed in a similar way as the main wing (see Section 1.3.1). Finally, the last configuration assumes that the HTP and HTP exert their mutual influence. However, we do not take the influence of the wing wake on the tail into account. Results are presented in Fig. 1.6.

On the first configuration, we observe the negative circulation of the HTP while its geometrical angle of attack $\alpha_{g,h,0}$ is greater than zero. This should generate a positive circulation as for the wing in Fig. 1.5. This negative circulation is due to the downwash induced by the wing on the tail that modifies the effective angle of attack perceived by the HTP to the point of making it negative. The HTP thus generates a negative lift, see Table 1.3. We also observed that the circulation and vortex sheet generated by the VTP due to the deflection of the rudder induce an asymmetry on the HTP circulation distribution and an increase of the circulation on the left-hand side of the HTP. In return, this asymmetry generates an asymmetric HTP vortex sheet that induces a non-zero net velocity component in the $\hat{e}_y^{ll}$ direction. This component induces a local modification of the angle of attack on the VTP which modifies its circulation, in particular for the VTP section close to the HTP, i.e., $z \simeq 0$ in Fig. 1.6b. Those interactions are not modeled in the second configuration, where we observe an elliptic circulation distribution for the VTP, because of its elliptic geometry. Even without the mutual influence of the HTP and VTP, the circulation distribution of the HTP is not elliptic. This is once again explained by the velocity induced by the wing wake on the HTP which is not uniform along the HTP. This tends to flatten its distribution. Finally, we compute the circulation along the HTP and VTP without the influence of the wing wake. We observe a positive circulation on the HTP as there is no downwash to cancel its positive geometric angle of attack. We also observe that the circulation along the VTP is not impacted by the presence of the wing vortex sheet, as the latter is symmetric with respect to the $\hat{e}_x^{ll} - \hat{e}_z^{ll}$ plane.

From a global point of view, we observe that the major impact of the modeling of the mutual influence of the HTP and VTP is the 10% decrease of the side force C_Y , compared to the configurations with no influence. This side force is exerted on the VTP which generates the yawing moment. Therefore, the yawing moment is also 10%

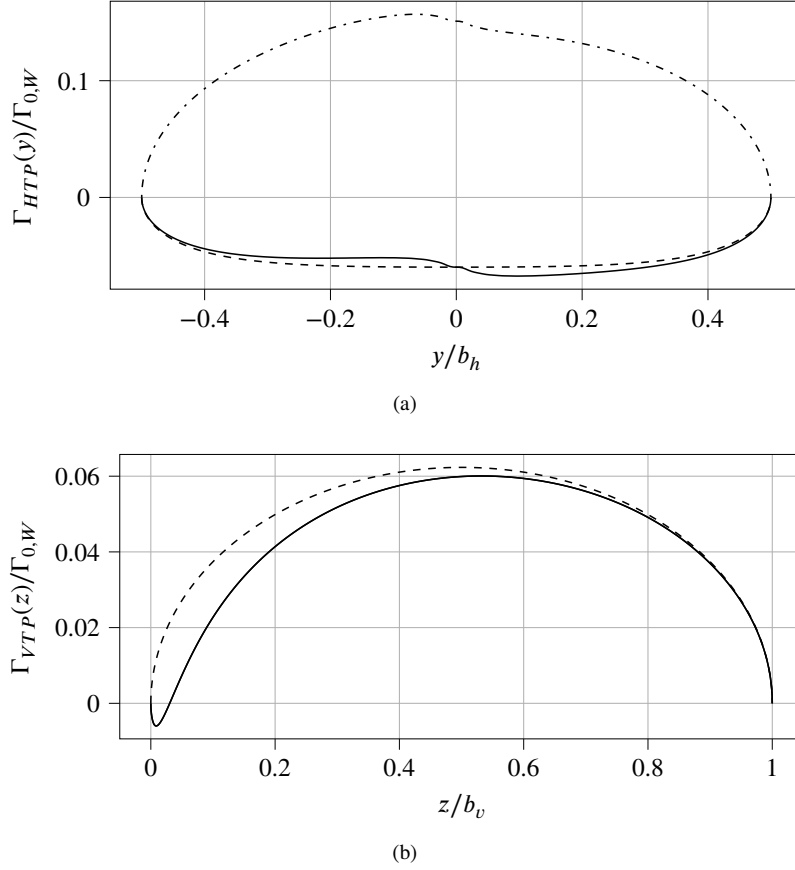


Figure 1.6: Circulation distribution along the HTP (a) and VTP (b) for three tail models. Mutual interactions between the HTP and VTP, with influence of the wing vortex sheet (—); No mutual interactions, with influence of the wing vortex sheet (---); Mutual interactions between the HTP and VTP, with no influence of the wing vortex sheet (-.-).

reduced. The impact of the wing sheet on the tail mainly consists in a modification of the vertical force C_Z exerted on the tail. In cases 1 and 2, when the influence of the wing is taken into account, the vertical force induced by the HTP is positive, while in case 3, with no influence of the wing, the latter is negative. The pitching moment it induces is therefore also impacted which has a strong consequence on the aircraft stability.

Configuration	C_X	C_Y	C_Z	C_{M_l}	C_{M_m}	C_{M_n}
Case 1 (—)	0.0025	-0.0041	-0.0096	0.0004	0.0061	-0.0025
Case 2 (- -)	0.0025	-0.0045	-0.0096	0.0005	0.0061	-0.0028
Case 3 (- · -)	0.0030	-0.0041	0.0218	0.0004	-0.0134	-0.0028

Table 1.3: Tail contribution to the aerodynamic efforts of the A350 aircraft model for the 3 configurations presented in Fig. 1.6. Efforts are addimensionalized, i.e., $C_i = \frac{F_i}{1/2\rho U_\infty^2 S}$ and $C_{M_i} = \frac{M_i}{1/2\rho U_\infty^2 S b}$.

Qualitative impact of formation flight

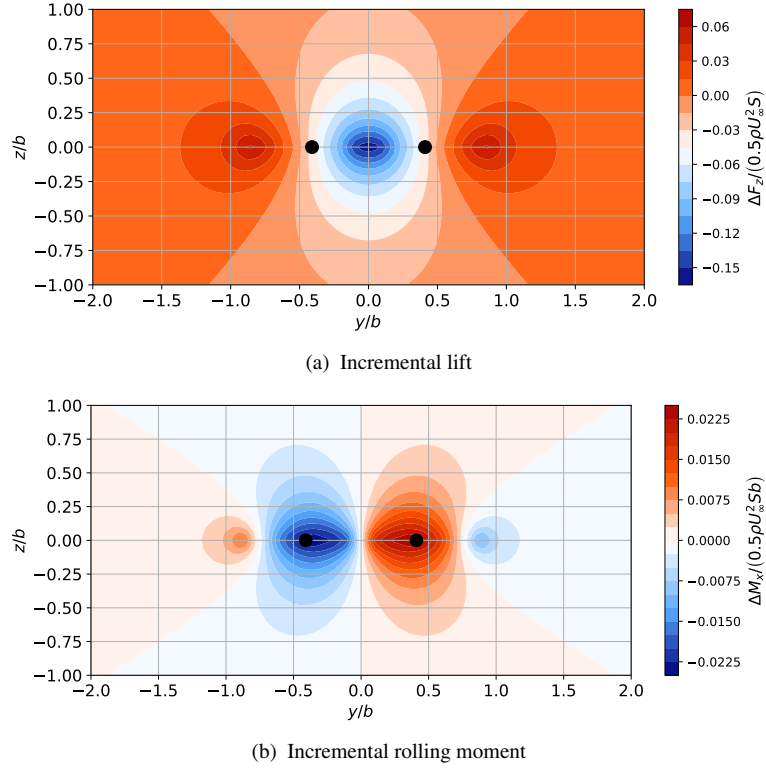


Figure 1.7: Contour plots of the incremental efforts experienced by a A350-like aircraft flying in the wake of a similar aircraft. The two black circles (●) indicate the position of the wake vortices.

Finally, we can quantify the aerodynamic impact of formation flight on a follower that flies in the wake of a leader, as already highlighted qualitatively in Section 1.2.2. We analyse a scenario similar to the one depicted in Fig. 1.2. The wake of a leader is modeled as two LOA vortices with intensity Γ_0 , see Eq. (1.10), and separated by a distance b_0 . We compute the incremental forces and torques induced by the upwash of the wake on the follower, i.e.,

$$\Delta F_i = F_i^{FF} - F_i^{isol} \quad (1.32)$$

$$\Delta M_i = M_i^{FF} - M_i^{isol} \quad (1.33)$$

where F_i^{isol} and M_i^{isol} are the efforts computed on an isolated aircraft and F_i^{FF} and M_i^{FF} are the efforts computed on the follower flying in the wake of the leader. The subscript $\cdot_i$ refers to one of the three components x , y or z .

We observe in Fig. 1.7a that on either side of the leader's wake, the follower experiences a positive vertical force increment ΔF_z due to the upwash wind induced by the wake. On the contrary, this increment is negative in between the wake vortices, due to a downwash wind in that area. We observe that the maximum increment is located at $(y, z) \simeq (0.9b, 0.0b)$. That position would be the optimal one for the follower in the wake of the leader. The rolling moment increment ΔM_x is depicted in Fig. 1.7b. As explained in Section 1.2.2, the follower undergoes a rolling moment that tends to move it away from the wake of the leader when it is located either side of the latter. The rolling moment tends to bring the follower into the wake when it is in the inside. Let's note that at the optimal positions, we compute a rolling moment increment $\frac{|\Delta M_x|}{1/2\rho U_\infty^2 Sb} \simeq 0.01$ that corresponds to values found in the literature [68].

1.4 Aircraft flight dynamics and control

A flight mechanics model uses the forces and torques applied on the aircraft and integrates the aircraft 6 degrees-of-freedom (6DOF) equations of motion over time. Because of the 6DOF, the description of the motion of the aircraft can be relatively complex. Therefore, motion is usually described by a number of variables which are related to a suitably chosen system of axes presented in Figs. 1.8a and 1.8b.

In the later, $(ox^i y^i z^i)$, $(ox^b y^b z^b)$ and $(ox^w y^w z^w)$ are the inertial, body and aerodynamic frames respectively. The three angles ϕ , θ and ψ characterize the orientation of an aircraft with respect to the inertial frame and are referred to as the attitude of the aircraft. The angle of attack α and sideslip angle β characterize the orientation of an aircraft with respect to the aerodynamic frame. In this section, we present the dynamic states that govern the motion of the aircraft as used in our model and write the related

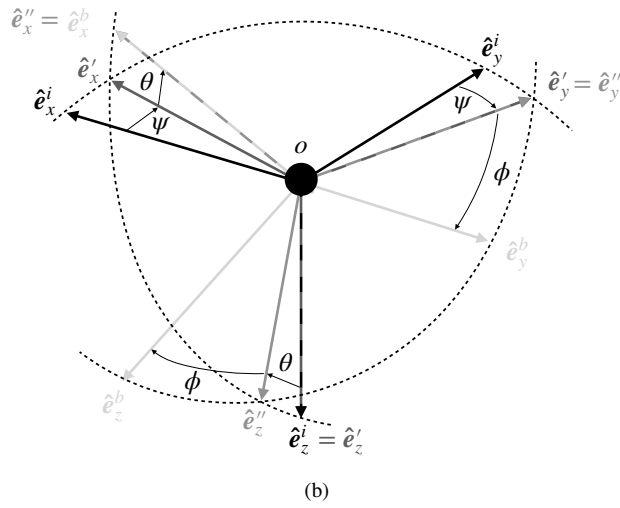
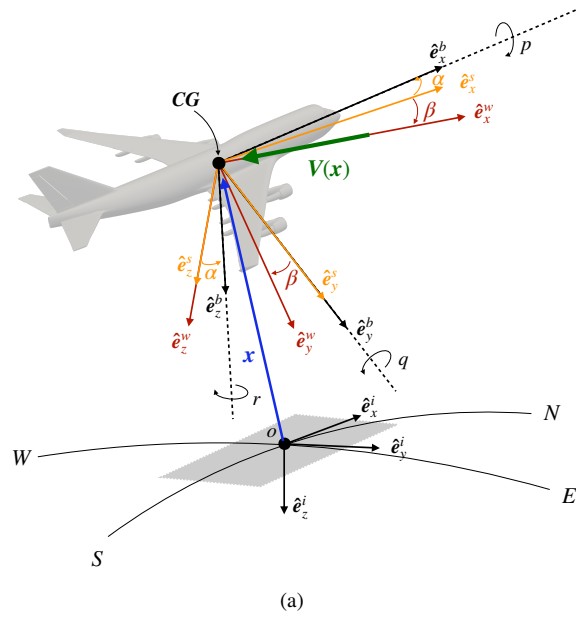


Figure 1.8: Frames used for the definition of aircraft dynamics (a). Euler angles from the inertial frame to the aircraft frame (b).

equation of motions. Then, we describe the controller that is used to pilot the aircraft along defined trajectories. This section aims to present the assumptions used for the design of our application. A more comprehensive coverage of the fundamentals behind aircraft dynamics can be found in [34].

1.4.1 Aircraft dynamic states and equations of motion

The first formal derivation of the equations of motion for a rigid symmetric aircraft is usually attributed to Bryan [17]. Their objective is to realize Newton's second law of motion for each of the six degrees of freedom which simply states that

$$\mathbf{F} = m\mathbf{a}. \quad (1.34)$$

For the rotary degrees of freedom, the mass and acceleration become moment of inertia and angular acceleration respectively whilst the force becomes the torque. The derivation of the equations of motion requires that Eq. (1.34) is expressed in terms of its dynamic states.

We first define the position of the aircraft in the inertial frame $\mathbf{x} = [x, y, z]^T$. The evolution of the inertial position in time depends on the flight path velocity $\mathbf{V}_K = [U_K, V_K, W_K]^T$ which is computed from the resultant total force vector on the aircraft, i.e.,

$$\mathbf{F} = \mathbf{F}^{\text{aero}} + \mathbf{F}^g + \mathbf{F}^{Th} = \left[F_x^i, F_y^i, F_z^i \right]^T, \quad (1.35)$$

where $\mathbf{F}^{\text{aero}}$ is the resultant aerodynamics force vector computed from Eq. (1.22), $\mathbf{F}^g = [0, 0, mg]^T$ is the gravity force and $\mathbf{F}^{Th} = [\delta_{Th} F_{Th,0}, 0, 0]^T$ is the thrust vector generated by the engines. Using the appropriate transformation matrices between the different frames, one deduces the components F_x^i, F_y^i and F_z^i of the resulting force vector $\mathbf{F}$ in the inertial frame.

The angular position consists of three rotations with ϕ the bank, θ the pitch and ψ the yaw. As depicted in Fig. 1.8b, they describe the transformation from the inertial frame to the aircraft frame and are integrated over time using the angular body rates $[p, q, r]^T$. Those body rates evolve over time depending on the total torque on the aircraft, i.e.,

$$\mathbf{M} = \mathbf{M}^{\text{aero}} + \mathbf{M}^{Th} = \left[M_x, M_y, M_z \right]^T, \quad (1.36)$$

where $\mathbf{M}^{\text{aero}}$ is the resultant aerodynamic torque vector computed from Eq. (1.22) and $\mathbf{M}^{Th}$ is the torque due to the engine thrust. We deduce the components M_x^b, M_y^b and M_z^b of the torque vector in the aircraft body frame this time, for an efficient integration of the body rates. Due to the symmetry of the aircraft with respect to the plane defined

by $\hat{e}_x^b$ and $\hat{e}_z^b$, the products of inertia $I_{xy} = I_{yz} = 0$. We also assume that the aircraft body axes are aligned to be principal inertia axes. In this special case, the remaining product of inertia I_{xz} is also zero (see [34]). The equations of motion for the three translations and the three rotations are thus

$$\begin{aligned} m\dot{\mathbf{V}}_K &= \mathbf{F}, & (1.37) \\ \dot{\mathbf{x}}_K &= \mathbf{V}_K, & (1.38) \end{aligned} \quad \begin{cases} I_{xx}\dot{p} - (I_{yy} - I_{zz})qr = M_x \\ I_{xx}\dot{q} - (I_{xx} - I_{zz})pr = M_y \\ I_{xx}\dot{r} - (I_{xx} - I_{yy})pq = M_z \end{cases}, \quad (1.39)$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi \tan\theta & \cos\phi \tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi \sec\theta & \cos\phi \sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (1.40)$$

where m is the mass of the aircraft and $[I_{xx}, I_{yy}, I_{zz}]$ are the moment of inertia about the principal axes.

For an aircraft flying in the absence of external disturbances, its dynamics involve twelve states, i.e.,

$$\mathbf{q}_{dyn} = [x, y, z, U_K, V_K, W_K, \phi, \theta, \psi, p, q, r]. \quad (1.41)$$

In addition to these states, four main commands are classically attributed to aircraft dynamics for its control,

$$\mathbf{u}_c = [\bar{\delta}_{Ail}, \bar{\delta}_{VT}, \bar{\delta}_{HT}, \bar{\delta}_{Th}], \quad (1.42)$$

where $\bar{\delta}_{Ail}$ is the command of the wing ailerons deflection, $\bar{\delta}_{VT}$ is the command of the rudder deflection, $\bar{\delta}_{HT}$ is the command of the elevator deflection and $\bar{\delta}_{Th}$ is the thrust command applied to the engine. The latter is expressed as a percentage of the reference axial thrust force $F_{Th,0}$ which is the force required to fly in a steady environment at the cruise speed U_∞ . In this work, we consider two different dynamics for these control commands. The first ones are associated to the C172-like aircraft defined in Table 1.1 and are characterized by an instantaneous response time from the command $\mathbf{u}_c$ to the effective control surface deflections or the effective thrust, i.e., $\mathbf{q}_{cont} = [\delta_{Ail}, \delta_{VT}, \delta_{HT}, \delta_{Th}]$. In other words, we have

$$\mathbf{q}_{cont} = \mathbf{u}_c. \quad (1.43)$$

The second ones are associated to the A350-like aircraft defined in Table 1.1 and are characterized by a first-order actuation lag for all the commands. This means that the

effective surface deflections and thrust follow the dynamics

$$\begin{cases} \dot{\delta}_{Ail} = -\frac{1}{\tau_{Ail}} (\delta_{Ail} - \bar{\delta}_{Ail}) \\ \dot{\delta}_{VT} = -\frac{1}{\tau_{VT}} (\delta_{VT} - \bar{\delta}_{VT}) \\ \dot{\delta}_{HT} = -\frac{1}{\tau_{HT}} (\delta_{HT} - \bar{\delta}_{HT}) \\ \dot{\delta}_{Th} = -\frac{1}{\tau_{Th}} (\delta_{Th} - \bar{\delta}_{Th}) \end{cases}, \quad (1.44)$$

where $\tau_{Ail}, \tau_{HT}, \tau_{VT}, \tau_{Th}$ are the time constants of the dynamics, and the overbar $\bar{\cdot}$ is the command $\mathbf{u}_c$ desired.

We can finally write the complete dynamics of the aircraft as

$$\dot{\mathbf{q}}(t) = \mathbf{f}(\mathbf{q}, \mathbf{u}_c, t), \quad (1.45)$$

where $\mathbf{q} = \mathbf{q}_{dyn} \cup \mathbf{q}_{cont}$ gathers the dynamic states of the aircraft and of its actuation, $\mathbf{f}$ gathers all the dynamic equations, i.e., Eqs. 1.37, 1.38, 1.39, 1.40 and Eq. (1.43) or Eq. (1.44), as well as all the aerodynamic forces described in Section 1.3. Indeed, the aerodynamic forces and torques are directly dependent on the states $\mathbf{q}$ through Eq. (1.16) and the appropriate frame changes, i.e., presented in Figs. 1.8a and 1.8b.

1.4.2 Aircraft controller design

It is observed in Eq. (1.45) that the aircraft dynamics are strongly non-linear. Such a representation is not appropriate to build simple control laws for the 6DOF dynamics. The design of an autopilot and most of the computation of the feedback gains that it implies are far more easily performed based on a linear representation of the aircraft dynamics. Indeed, the evaluation of stability derivatives computed around a trimmed level flight enables to determine the linear dynamics of the aircraft around that level flight. This leads to a representation of the form

$$\Delta \dot{\mathbf{q}}(t) = \mathbf{A} \Delta \mathbf{q}(t) + \mathbf{B} \Delta \mathbf{u}_c(t), \quad (1.46)$$

where $\mathbf{A}$ and $\mathbf{B}$ are the linearized matrices of the state space representation and are computed from the aerodynamic model using finite differences. $\Delta \mathbf{q}$ (resp. $\Delta \mathbf{u}_c$) gathers the perturbation of the dynamic states (resp. the command) around the steady configuration $\mathbf{q}_0$ (resp. the command $\mathbf{u}_{c,0}$ required to maintain the steady state), i.e.,

$$\mathbf{q}(t) = \mathbf{q}_0 + \Delta \mathbf{q}(t), \quad (1.47)$$

such that $\dot{\mathbf{q}}_0 = \mathbf{0}$. In the rest of this work, for the sake of simpler notation, we use $\mathbf{q}$ (resp. $\mathbf{u}_c$) instead of $\Delta \mathbf{q}(t)$ (resp. $\Delta \mathbf{u}_c$) since we consider the reference $\mathbf{q}_0$ to be a level flight where all states are zero, except for the axial velocity, i.e., $U_{K,0} = U_\infty$.

In this linearized representation, the behavior of simple fixed-wing aircraft can be separated into longitudinal and lateral dynamics which are linearly uncoupled [34], simplifying the control problem. The longitudinal dynamics involve the positions x and z and the associated flight path velocity components U_K and W_K as well as the pitch angle θ and pitch rate q . The lateral dynamics involve the lateral position y and lateral velocity component V_K as well as the roll ϕ and yaw ψ angles and rates p and r . The ailerons and rudder deflections δ_{Ail} and δ_{VT} are attributed to the lateral dynamics while the elevator deflection δ_{HT} and thrust command δ_{Th} are attributed to the longitudinal dynamics. The linearized matrices and the separation in lateral and longitudinal dynamics are presented in Appendix B for both the C172-like and A350-like aircraft.

The designed autopilot pursues two different objectives. The first mode, hereafter referred to as *position hold*, enables an aircraft to maintain a fixed desired position $\mathbf{x}^{ref}$, no matter the influence of external interferences, i.e., gust or leader aircraft wake. In the second mode, the autopilot actively targets a trajectory $\mathbf{x}^{target}$; this mode is called *tracking*. Acting on the complete 6DOF dynamics of the aircraft, the autopilot commands the control inputs $\mathbf{u}_c$ of the aircraft in order to maintain a fixed position or to follow a trajectory (here related to a moving target in formation flight, see later).

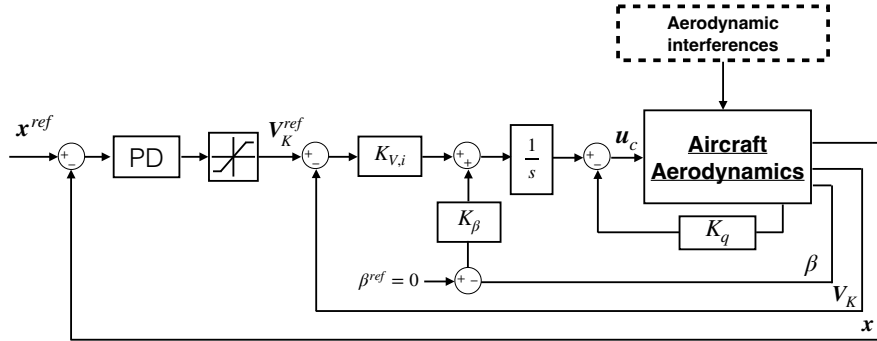


Figure 1.9: Complete structure of the control scheme.

The structure of the autopilot is shown in Fig. 1.9; it is similar to the one designed in [12]. It is designed to be robust under aerodynamic interferences and simple enough to be integrated into existing simulation tools. It uses a full state measurement. The velocity tracking control is implemented using three internal control loops, see Fig. 1.9. One is proportional to the dynamic states of the aircraft (K_q), the second is proportional to the integral of the error on the side slip angle (K_{β}), and the third one is proportional to the integral of the error on the inertial speeds ($K_{V,i}$). The gains of those

three first loops are designed using a linear quadratic regulator (LQR). The autopilot is designed to make the aircraft fly at 0° sideslip angle i.e., $\beta^{\text{ref}} = 0^\circ$. As an additional layer, the reference flight path velocity $\mathbf{V}_K^{\text{ref}}$ is computed over time using three inertial position tracking loops. Each one of them implies a proportional-derivative (PD) controller that imposes the velocity target for the internal loops. Those velocities are limited through rate limiters in order to maintain reasonable velocity targets.

1.5 A simulated high-fidelity flight environment as a reality

This section presents the Vortex Particle-Mesh (VPM) method used to realize Large Eddy Simulations (LES) of the wake flow behind aircraft whose lifting surfaces are modeled as Immersed Lifting and Dragging Lines (ILDL). In the context of this work, such tools present two main advantages. First, the comprehensive simulation of the complete roll-up process of the vortex sheet that emanates from the lifting surfaces of the aircraft, including the subsequent instabilities and the ensuing turbulence, is within reach thence waving the need for vortex models when considering the evaluation of the influence of aircraft wakes on its environment, e.g., a follower surfing the wake, impact of the wing on the tail. Secondly, instead of considering flight measurements which are kept confidential by aircraft manufacturers, LES of aircraft and their wake can be used as high-fidelity representation of the real flight environment of aircraft. This *simulated flight environment* thus represents the "reality" in the rest of this work and produce reference measurements to be compared to those produced by the quasi-static representation described in section Section 1.3.

The simulation of complex situations, such as aircraft in formation flight, requires however to model if not all, a part of the dynamics of the aircraft. Indeed, as highlighted in Figs. 1.2a and 1.2b induced aerodynamic effects need to be compensated in order to maintain appropriate trajectories. LES of aircraft wakes are mainly performed to study the propagation of the wake of fixed wings. Here, we also implement in the high-fidelity VPM flight environment the complete dynamics of an aircraft, similar to those presented in Fig. 1.4, i.e., three lifting surfaces with their associated control surfaces (ailerons, rudder and elevator), which allows the study of their impact on the wake propagation. We first present in Section 1.5.1 the equations behind VPM method. We then introduce in Section 1.5.2 the ILDL model used to represent the lifting surfaces of an aircraft. We finally describe in Section Section 1.5.3 the way the dynamics and control laws are implemented in the VPM solver. Finally, we present in Section 1.5.4 a comparison of the aerodynamics and flight mechanics computed by the simplified model and the simulated flight environment.

1.5.1 Flow solver

In this work, we use a state-of-the-art Vortex-Particle Mesh (VPM) method to simulate the wake flow behind an aircraft. It builds upon the Vortex-in-Cell [36] technique, which combines both particles and a mesh. This makes it an hybrid approach among the many variants of vortex methods [98]. The method is based on the vorticity-velocity formulation ($\boldsymbol{\omega} - \mathbf{u}$) of the Navier-Stokes equations for incompressible flows ($\nabla \cdot \mathbf{u} = 0$), i.e.,

$$\frac{D\boldsymbol{\omega}}{Dt} = (\nabla \mathbf{u}) \cdot \boldsymbol{\omega} + \nu \nabla^2 \boldsymbol{\omega} + \nabla \cdot \mathbf{T}^M, \quad (1.48)$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla$ represents the Lagrangian derivative, ν is the dynamic viscosity and $\mathbf{T}^M$ is the sub-grid scales (SGS) model, i.e., here a Regularized Variational Multiscale (RVM) type model developed by [78]. In that model, the SGS acts on the "resolved small scales vorticity field" $\boldsymbol{\omega}^s$, i.e.,

$$\nabla \cdot \mathbf{T}^M = -\nabla \times (\nu_{SGS} \nabla \times \boldsymbol{\omega}^s), \quad (1.49)$$

where $\boldsymbol{\omega}^s$ is obtained using efficient iterative filtering. The eddy viscosity is evaluated as $\nu_{SGS} = C_r^{(n)} h^2 (\mathbf{S} : \mathbf{S})^{1/2}$, where $\mathbf{S}$ is the strain rate of the velocity field, h is the mesh size, and $C_r^{(n)}$ is calibrated according to [31]. The velocity field can be decomposed as $\mathbf{u} = U_\infty \hat{\mathbf{e}}_x + \mathbf{u}_\omega$ in which one recovers $\mathbf{u}_\omega$ from the vorticity through the resolution of the Poisson equation

$$\nabla^2 \mathbf{u}_\omega = -\nabla \times \boldsymbol{\omega}. \quad (1.50)$$

Similarly to classical vortex particle methods, the advection term is handled in a Lagrangian way. The flow is discretized using a set of particles characterized by a position $\mathbf{x}_p$, a volume V_p and a strength $\boldsymbol{\alpha}_p = \int_{V_p} \boldsymbol{\omega} d\mathbf{x}$. Particles are advected by the flow field and their strength is modified to account for the vortex stretching, diffusion and subgrid-scale modeling. One can integrate Eq. (1.48) over a particle volume through a mid-point quadrature with the assumption of the particle discretization, see [152, 36] for a detailed analysis. One then finds a set of differential equations for the particles position and strength

$$\frac{d\mathbf{x}_p}{dt} = \mathbf{u}(\mathbf{x}_p) \quad (1.51)$$

$$\frac{d\boldsymbol{\alpha}_p}{dt} = \int_{V_p} ((\boldsymbol{\omega} \cdot \nabla) \mathbf{u} + \nu \nabla^2 \boldsymbol{\omega} + \nabla \cdot \mathbf{T}^M) d\mathbf{x}. \quad (1.52)$$

The previous equations are here solved using a third order Runge-Kutta integration scheme. Since particles are used solely for the advection treatment, they enable the

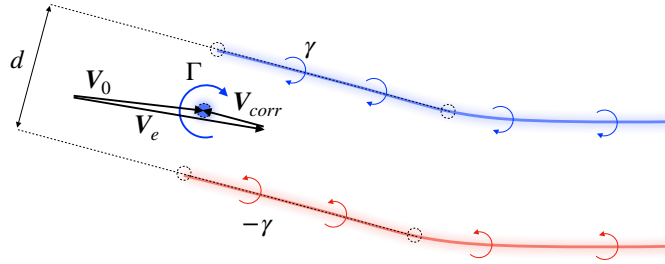
avoidance dispersion errors and waive the classical Courant-Friedrichs-Lewy condition inherent to Eulerian solvers, hence allowing for CFL numbers $\left(\frac{\|u\|\Delta t}{h}\right)$ above unity, with the h mesh size of the uniform grid Δt the integration time step. However, a Lagrangian CFL ($\|\omega\|\Delta t$) and Lagrangian deformation ($\|\nabla u\|\Delta t$) have to remain under a prescribed value.

In the present hybrid VPM method, all the spatial differential operations are computed on a uniform underlying mesh, thus in an Eulerian manner, which offers several advantages. First, the velocity field, Eq. (1.50), is solved using a Fourier-based fast Poisson Solver which allows the combination of unbounded directions and inlet/outlet boundaries. The right-hand side of Eq. (1.52) uses fourth-order finite differences, and the turbulence SGS model ($\nabla \cdot \mathbf{T}^M$) also uses the grid and is implemented as in [30]. Finally, the mesh is exploited to remedy the loss of accuracy due to Lagrangian distortion through remeshing. This consists in the periodic regularization onto a grid of the particle set via high order interpolation schemes to recover information back and forth between the particles and the mesh. Reprojection of the vorticity field is also periodically applied to maintain the solenoidal property of the vorticity field which is otherwise not directly enforced by the solver. It is worth mentioning that the Reynolds number $Re = \frac{U_\infty b}{\nu}$ that characterize the studied flight conditions is high. As a consequence, the dissipation that will be observed in the simulations is essentially due to the SGS term $\nabla \cdot \mathbf{T}^M$ as well as the remeshing and reprojection numerical steps, rather than the fluid viscosity ν . Results are thus dependent on the quality of the numerical models and methods implemented. We refer to the reviews by Cottet and Komoutsakos [36] and Winckelmans [152] for an in-depth description of the application of VPM methods and the theory behind. Additionally, a description of the present implementation on massively parallel architecture has been provided by Chatelain et al [26]. This VPM method is used in various applications, ranging from aircraft wakes [23, 118] and helicopter wakes [24, 19] to vertical axis wind turbines or horizontal axis wind turbine wakes [27, 6, 35].

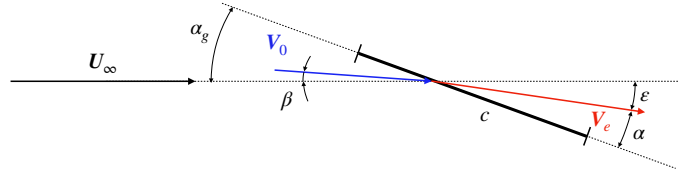
1.5.2 Aircraft model using Immersed Lifting and Dragging Lines

The generation of vorticity along the lifting surfaces of the aircraft (i.e., wing, HTP and VTP) is accounted for through a lifting line model inspired from the PLL theory as described in [113]. The model used in this work however extends the PLL to account for the airfoil parasitic drag using the implementation proposed by [23] called the Immersed Lifting and Dragging Line (ILDL) model. Like the Actuator Line Method [131], the ILDL treats the generation of the vorticity in the flow instead of applying

a force in the volume. As we consider slender wings, the flow around spanwise elements of the wing can be treated as essentially two-dimensional. The elements are thus approximated as a bound vortex of local circulation Γ concentrated along the quarter chord line, to which are associated two trailing vortex sheets of circulation per unit length $-\gamma$ and γ which mimic the wake velocity deficit. The two vortex sheets are separated by a distance d , therefore modeling a momentum sink located on the airfoil, or equivalently a vorticity dipole source of strength $\mu = d\gamma$, see Fig. 1.10a.



(a) ILDL with bound vortex at the quarter chord, and two shed vortex sheets.



(b) The reference direction is given by the upstream flow velocity U_∞ . Then, for each spanwise element, we define the geometric angle of attack α_g with respect to the airfoil chord c ; the downwash angle ϵ and the local effective angle of attack $\alpha = \alpha_g - \epsilon$, using the local effective upstream velocity V_e ; and the angle β between the local measured velocity V_0 and the reference direction.

Figure 1.10: Schematic 2-D views of the lifting and dragging line in steady conditions. Reproduced from Caprace et al. [23].

For each spanwise element, we define the sectional lift and drag as $L_{2D} = L_{2D} \hat{e}_L$ and $D_{2D} = D_{2D} \hat{e}_D$, as well as the bound circulation $\Gamma = \Gamma \hat{e}_\Gamma$. We also define the local velocity V_0 , that is the velocity "as measure on the line element", in the sectional plane of the element, see Fig. 1.10b. We also define the "local effective upstream velocity" V_e , i.e., the velocity used in the usual definition of the airfoil aerodynamic coefficients,

$$L_{2D} = \frac{1}{2} \rho |V_e|^2 c C_l(\alpha, Re) , \quad (1.53)$$

$$D_{2D} = \frac{1}{2} \rho |V_e|^2 c C_d(\alpha, Re) , \quad (1.54)$$

where c is the chord of the element, ρ is the fluid density, C_l and C_d are the lift and drag coefficient retrieved from tabulated polar data, based on the local angle of attack α and the Reynolds number Re . It is clear that $\mathbf{V}_0 \neq \mathbf{V}_e$ when the model accounts for the parasitic drag. Because of the velocity induced by the dipole vortex sheets due to the drag, $\mathbf{V}_e$ differs from $\mathbf{V}_0$ by a correction $\mathbf{V}_{corr}$. Then, the closure of the model necessitates relation between the aerodynamic forces and the circulation Γ and the dipole intensity μ of the local 2-D segment. They are recovered from the Kutta-Joukowski theorem and the relation between the drag and the flux of momentum imposed by the vortex dipole to the flow, respectively, i.e.,

$$\mathbf{L}_{2D}(t) = \rho \mathbf{V}_0(t) \times \Gamma(t) \quad (1.55)$$

$$\mathbf{D}_{2D}(t) = \rho \mathbf{V}_0(t) \mu(t) . \quad (1.56)$$

For a numerical implementation, the bound vortex and shed vortex sheet are discretized along the wing span as line and sheet of vortex particles, thickened through a spatial mollification. Required for numerical purposes, this mollification corresponds to a Gaussian spreading (of mollification factor σ_l and σ_d for the lift and drag respectively) that covers a few mesh sizes h in the direction $\hat{\mathbf{e}}_L$. The vorticity shed in the flow as new particles (mollified as well) is deduced from the temporal and spatial variation of the bound and sheet vorticity. These new particles are then merged with the existing vorticity in the flow. The ILDL method thus preserves the Lagrangian character of the overall numerical method unlike the ALM. We refer to [23] for a detailed description of the entire methodology and its implementation into the VPM method.

1.5.3 Aircraft dynamics and control in VPM

The rigid motion of the aircraft is captured through a coupling of the flow solver with the multi-body integrator ROBOTRAN [49]. It uses symbolic generation to determine the equations of the direct dynamics of a model. This approach enables the resolution of the dynamics of complex multi-body systems. This system reduces to the 6DOF dynamics of an aircraft in this work. In VPM, the aircraft is modeled similarly to the one presented in Fig. 1.4. It consists of three lifting surfaces of chosen dimensions, rigidly associated, and whose flexibility is not taken into account. In VPM, the lifting surfaces are represented using the ILDL model that enables the computation of the forces on the wing, VTP and HTP. Those forces are provided to ROBOTRAN which uses a different time integrator to advance the state of the aircraft over time. The positions and velocities are then updated in the flow simulation and the process is iterated.

Similarly to the simplified model described in Section 1.3, the control surfaces (i.e., ailerons, elevator and rudder) are modeled in VPM through a local modification

of the geometric angle of attack of the lifting surfaces. Their impact is thus directly accounted for through the tabulated data used for the resolution of the ILDL. Indeed, the ILDL model imposes the modification of lift and drag that results from the local change of the angle of attack. At each time step, the same controller as the one designed in Section 1.4.2 exploits measurements of the dynamics computed by ROBOTRAN in order to compute the appropriate control inputs required to maintain a position in the flow or to follow a desired trajectory.

1.5.4 Comparison between simplified model and the simulated environment

In this section, we compare the aerodynamics computed by both the simplified model and the ILDL model in VPM for an A350-like aircraft (see Table 1.1). We then compare the dynamics of the controlled aircraft in both models to observe their differences. In this work, if not explicitly stated, the mollification of the lifting lines covers five mesh sizes h , with the numerical parameters $\frac{\sigma_l}{h} = 1$, $\frac{\sigma_d}{h} = 1$ and $\frac{d}{h} = 4$, i.e., layout II described in [23].

Lifting surfaces circulation distribution

We present in Fig. 1.11 the circulation distributions along the three lifting surfaces of the aircraft, as computed by the ILDL model in VPM and the simplified model described in Section 1.3. The aircraft is flying in the same configuration as in Fig. 1.6 i.e., the HTP with a geometrical angle of attack $\alpha_{g,h,0} > 0$ and the VTP with a non-zero control surface deflection (i.e., non-zero rudder deflection), $\delta_{VT} = 10^\circ$. The reference parameters of the VPM simulation used to represent the simulated flight environment are reported in Table 1.4.

Table 1.4: Numerical parameters used in the VPM simulations.

Domain size $(L_x \times L_y \times L_z)$	$8b \times 4b \times 2b$
Size of the cubic mesh cells	$h/b = 1/64$

We first observe the good agreement between the two circulation distributions of the wing. The main differences between the two models are the modeling of the velocity deficit induced by the drag in the computation of the effective attack angle and the mollification of the circulation distribution for numerical reasons. We thus observe that those influences are not sufficient to induce important changes in the aerodynamic load undergone by the wing. The simplified model appears to be a good approximation

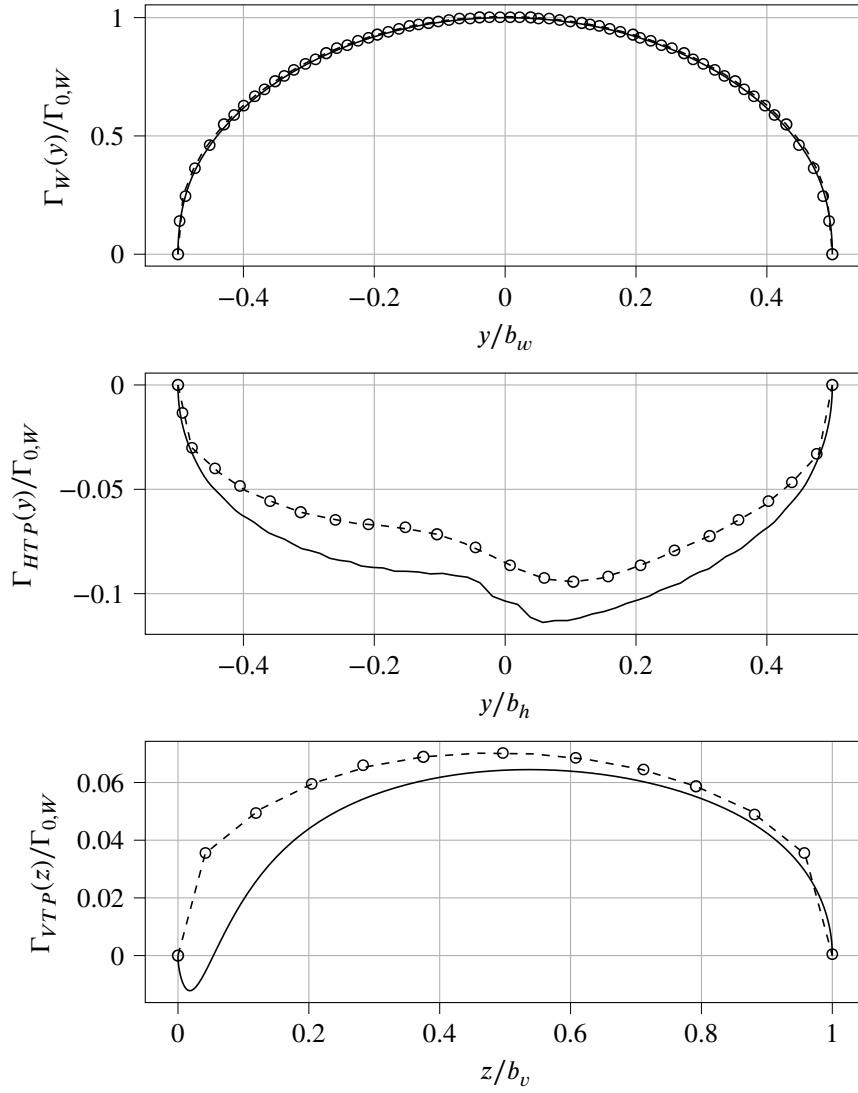


Figure 1.11: Circulation distributions along the wing (top), HTP (middle) and VTP (bottom) computed by the simplified aircraft model (—) and the ILDL implemented in VPM as the simulated flight environment (- -). The circles ($\odot$) represent the numerical discretization of the lifting surfaces.

of the VPM model. This is consistent, as the models are all based on PLL, with ILDL including a bit more physics. Also, we consider an aircraft in steady flight. This is therefore in good agreement with the quasi-static hypothesis of the PLL model used for the simplified aircraft, see Section 1.3.3.

The circulation distributions of the HTP and VTP are not as accurately modeled as the wing circulation distribution. Despite the proper capture of the VTP-induced deformation of the HTP circulation (i.e., left-right asymmetry), the intensity of the circulation is not accurately computed by the simplified aircraft. This is due to the wing wake induced velocity that is computed by an infinite vortex sheet in the simplified model while the roll-up of the sheet is taken into account in the VPM model. The inaccuracy in the computation of that velocity induces different effective angles of attack and different circulation distributions. Regarding the VTP, we observe that the circulation profile is also not accurately reproduced. In particular, the sign change computed by the simplified model for $z/b_v \in [0, 0.05]$ is not observed in the VPM model. This error is explained by two major differences. First, the mollification that is used in the VPM model (and not in the simplified one) induces some thickness of the lifting line that represents the HTP. This mollification smoothes the singularities that are intrinsic to the classical PLL model. Also, the inherent simplifications of the simplified model, such as not taking into account the velocities induced by the bound vortex, also lead to the computation of approximate induced velocities and therefore to circulation errors.

Finally, let us note that the PLL theory does not model all the aerodynamic effects along the lifting surfaces, as highlighted in Section 1.3.3. In particular, the boundary layers are not represented in this model. Therefore, the "wall" effects are not taken into account, in particular at the intersection of the VTP and the HTP. The observations in Fig. 1.11 are therefore to be considered with caution. Models based on alternate theories (, e.g., Vortex Lattice Method) would allow to take some additional behaviors into account and to perform comparative analysis.

Controlled aircraft dynamics

Despite the difference observed in the computation of lifting surface circulation, we validate here the simplified model by comparing the aircraft dynamic responses due to oscillating disturbances with those of an identical aircraft flying in the VPM environment. The geometrical parameters of the aircraft used in this section are reported in Table 1.1. The same controller described in Section 1.4.2 is used in both the VPM environment and the surrogate model. To validate the latter, we analyse a scenario involving a leader and a follower, where the leader tracks an oscillating trajectory and

the follower tries to maintain a constant position in time. The reference parameters of the VPM simulation used to reproduce the simulated flight environment are similar to the one used in Fig. 1.11 and are reported in Table 1.4. In this section, the computational domain includes two aircraft that are separated by a longitudinal distance of $4b$. These conditions are not consistent with the hypothesis of formation flight for civilian application. However, the choice of a small longitudinal separation distance was made to keep a relatively small time-to-solution for the simulation.

In the scenario presented in this section, the leader aircraft is instructed to follow a sinusoidal target trajectory in the $\hat{e}_y^i - \hat{e}_z^i$ plane and to maintain a constant cruise speed U_∞ in the streamwise direction, i.e.,

$$x_L^{ref}(t) = \begin{bmatrix} A_y \left[1 - \cos \left(\frac{2\pi}{T_y} (t - t_{start}) \right) \right] \\ A_z \sin \left(\frac{2\pi}{T_z} (t - t_{start}) \right) \end{bmatrix}_i, \forall t > t_{start}, \text{ otherwise } x_L^{ref}(t) = \begin{bmatrix} U_\infty t \\ 0 \\ 0 \end{bmatrix}_i. \quad (1.57)$$

The oscillation periods $T_y = T_z = 0.4T_{ph}$ are expressed in terms of the phugoid period $T_{ph} \simeq 125s$, as estimated from the linearized dynamics of Section 1.4.2. We chose $A_y = 0.4b$ and $A_z = 0.3b$. Simulations (not shown here) confirm that the leader manages to perform the oscillatory maneuvers in the lateral and in the vertical directions using responsive control commands. However, a constant delay always exists both in the lateral and vertical axes between the target and the position of the aircraft mostly due to the use of speed limiters (shown in the block diagram of Fig. 1.9, which prevents the saturation of control commands).

The follower is instructed to maintain a prescribed position $(y_F^{ref}, z_F^{ref}) = (1.1b, 0)$, in the $\hat{e}_y^i - \hat{e}_z^i$ plane, while evolving in close proximity of the oscillatory wake of the leader and flying at the cruise speed U_∞ along the longitudinal axis $\hat{e}_x^i$. However, through aerodynamic interactions, the dynamics of the second aircraft are subjected to the influence of the wake produced by the first one.

Figs. 1.12 and 1.13 depict the follower control commands and dynamics under aerodynamic loads computed by the simulated flight environment and by the surrogate model. The wake starts oscillating at $t_{start} \simeq 40t^*$, with $t^* = \frac{b}{U_\infty}$, which explains the changes in the dynamic states from that time. The same autopilot and a similar lifting line model are used in both the simulated flight environment and the surrogate model, which explains their very similar behaviors. However, differences are observed either before and after t_{start} (i.e., when the wake starts oscillating), because of the hypotheses underlying the surrogate model. Indeed, as mentioned in Section 1.2, the surrogate

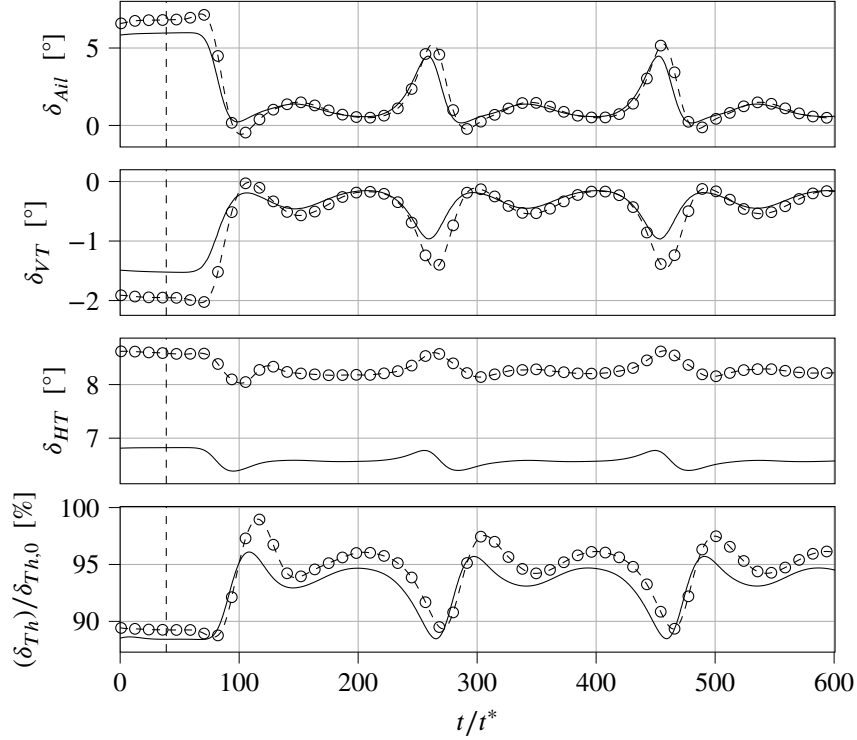


Figure 1.12: Control commands of the follower aircraft, maintaining its position while being impacted by the oscillating wake of the leader: comparison between the simplified aircraft model (—) and the simulated flight environment (-○ -) dynamics.

model assumes the wake of the leader to be made of two rolled-up counterrotating vortex tubes. Conversely, we assume that the vortex sheet shed by the wing has not started to roll when it reaches the tail. Those assumptions are not in perfect agreement with the VPM environment, which precisely captures the rollup of the vortex sheets. This explains the offset observed on the dynamic state and, in particular, on the elevator deflection input (δ_{HT}), the pitch angle (θ), and the additional required thrust (δ_{Th}), as they are interdependent through the longitudinal dynamics. The quasi-static hypothesis of the surrogate model (see Section 1.3.3) makes it also more reactive than the simulated flight environment that captures the wake dynamics transient.

Despite those assumptions, the evolution of all of the 6DOF states agrees fairly well over time, illustrating the validity of the surrogate model versus the simulated flight environment. The turbulence that will be latter added to the flow will of course

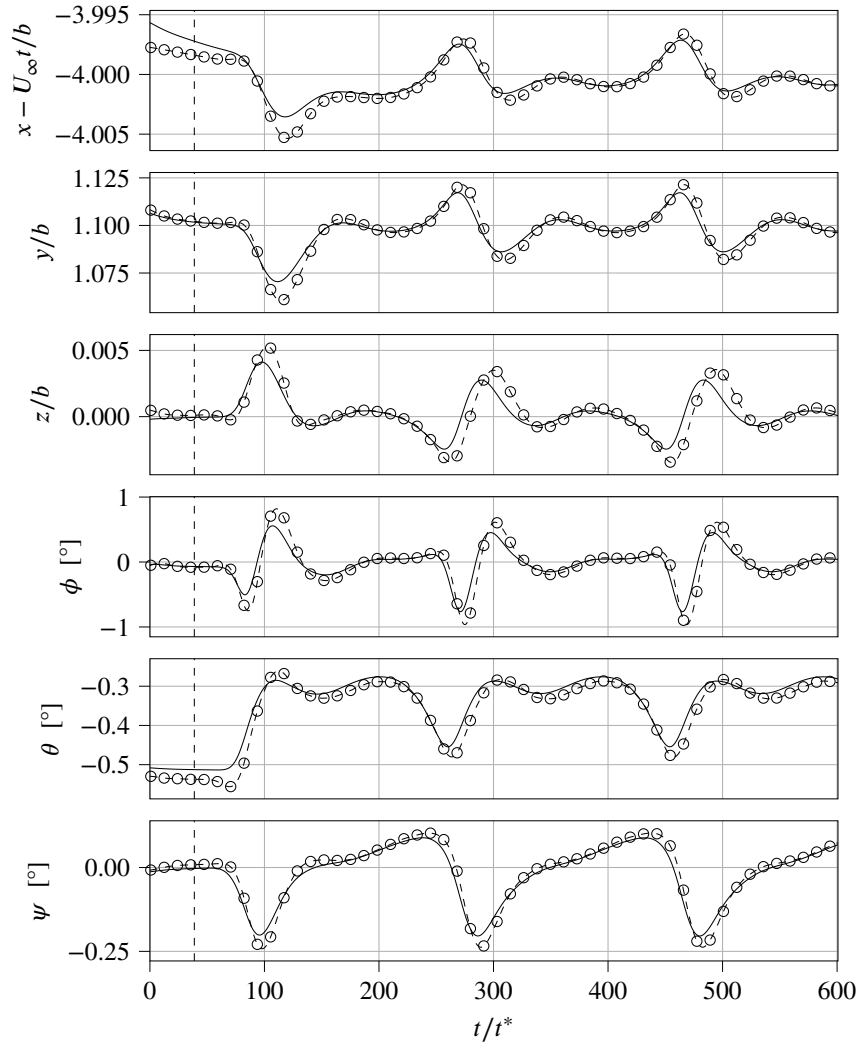


Figure 1.13: Dynamic response of the follower aircraft, maintaining its position while being impacted by the oscillating wake of the leader: comparison between the simplified aircraft model (—) and the simulated flight environment (-○ -) dynamics.

perturb the dynamics of the aircraft in the simulated flight environment. The general behavior of the aircraft will nonetheless follow similar trends as long as the turbulence do not exceed some intensities.

Let us finally highlight several specific behaviors of the controller observed in Fig. 1.12. The non-zero average deflections of the ailerons and the rudder compensate for the rolling moment induced by the impacting wake and maintain 0° of slip. Also, one should observe that the required thrust command δ_{Th} is smaller than required by an aircraft flying outside any wake (i.e., $\delta_{Th} < \delta_{Th,0} = 100\%$). This reduction is to first-order approximation attributed to the rotation of the resulting aerodynamic force vector arising from the presence of the upwash induced by the leader's wake, as described qualitatively in Fig. 1.2a. The rotation of the force vector translates into an apparent drag reduction experienced by the follower and, therefore, to a smaller required thrust command. Let us note that, in this work, the controllers of the leader and the follower are designed to maintain a streamwise velocity, $U_{K,F}^{ref} = U_\infty$. Such a strong constraint can lead to non-realistic engines behaviors (, e.g., in Fig. 1.12) that are clearly not observed on actual aircraft engines that are driven by dynamics with slower time response. The streamwise dynamics of formation flight (e.g., different streamwise velocity, safety distance, approach of the planes, etc.) are not intended to be optimized in the framework of this work, explaining the choice for a constant streamwise velocity.

1.6 Conclusions

In this chapter, we have worked on the development of numerical tools used to model aircraft flight dynamics, from the computation of forces on the lifting surfaces to the derivation of the equations of motion. In particular, we have taken care to be able to represent the impact of the wake of a leader aircraft on the aerodynamics and dynamics of a follower, in order to be able to model at least two aircraft in formation flight.

We have developed two different aircraft models, characterized mainly by the degree of accuracy attributed to the modeling of the wake propagation of the aircraft. The first, called the *surrogate model*, is based on the Prandtl Lifting Line theory in order to evaluate the forces on the lifting surfaces, of which there are three on the considered aircraft, i.e., the wing, the Vertical and the Horizontal Tail Planes. The second model called the *simulated flight environment* is based on a Vortex Particle Mesh method to perform Large Eddy Simulations of the wake flow behind aircraft whose three same lifting surfaces are modeled as Immersed Lifting and Dragging Lines. For both mod-

els, an autopilot is developed in order to control the six-degree-of-freedom dynamics of the aircraft that are integrated over time from the resulting efforts.

In the absence of real aircraft flight data provided by manufacturers, the virtual flight environment allows the generation of *simulated flight data* which can then be compared to similar measurements reproduced by the surrogate model. The latter has the particularity of being computationally cheaper and can therefore be used in onboard control tools operating in real-time, provided that it models the flight data accurately enough. We show in this chapter a good agreement between the dynamic states computed by the two models, which opens the perspectives of using the surrogate model in estimation tools, i.e., Kalman filters, for the predication of phenomena in the simulated flight environment.

That good agreement is to be attributed to the similarity between the mathematical theory used to design the two models. The Immersed Lifting and Dragging Line is indeed a more elaborate model that derives from the Prandtl Lifting Line theory for the lifting part, while including an additional dragging part. Such an approach deviates from a more realistic design of aircraft surrogate models which are obtained from flight test data and are built from aerodynamic derivatives deduced from them. An alternative model could have been obtained by computing aerodynamic derivatives from the virtual flight environment, as we did to design the autopilot from aerodynamic derivatives deduced from the surrogate aircraft model around a trim level flight.

In any case, models remain what they are, namely mathematical representations of the physics that one chooses to simulate. By not including, for example, compressibility phenomena, aero-elastic interactions, interactions between the flow and the nacelle or the boundary layers the aircraft and flow dynamics computed here will not be able to reproduce actual flight data. The wake propagation, however, is accurately computed by the simulated flight environment thanks to the VPM flow solver. This wake being the origin of the formation flight, the models as they have been designed will be able to give deeper insights about the physics behind formation flight and how to benefit from it.

Wake Vortex Detection and Tracking for Aircraft in Formation Flight

The content of this chapter is largely inspired by an article published in the Journal of Guidance Control and Dynamics, entitled "Wake Vortex Detection and Tracking for Aircraft in Formation Flight" [117], by I. Ransquin, D.-G. Caprace, M. Duponcheel and P. Chatelain.

2.1 Introduction

While migratory birds usually fly in close formation, civil aviation safety constraints permit only extended formation flight. In that case, the follower flies at several tens of wingspans downstream of the leader. Because wake vortices are energetic and long-lasting flow structures, the follower still benefits from a positive interaction with the leader's wake vortices [100, 10]. Additionally, because of the longer distance between the leader and the follower, instabilities such as meandering [124], the interactions of wakes produced by different aircraft in a formation [139], and environmental factors [75] may affect the motion of the vortices. It is therefore essential for the follower to continuously locate the wake vortices. This information can then be provided to the pilot, or an autopilot, in order to maximize the fuel savings during the flight.

Several methods allowing the follower to track the vortices have been proposed, yet not many of them were tested during actual formation flight experiments. In a first family of techniques, the position and strength of the vortices are determined from a

direct measurement of the flow field [40, 97, 123], e.g., through a LIDAR installed on the aircraft. Today, even though ground-based measurements provide sufficiently reliable results [67], the use of such technologies still poses significant challenges for onboard wake measurements [135].

Alternative vortex detection techniques rely on indirect measurements and on estimations of the wake vortex parameters. In a first example [72], distributed pressure measurements are used on the follower wing and are processed by an estimator, essentially based on a Extended Kalman Filter (EKF), to deduce the parameters of the wake vortices currently influencing the follower. This technique showed promising results in several flight configurations, especially for deducing the position of the vortices, even though some observability issues were highlighted [47]. This technique still requires specific instrumentation to be installed onboard the aircraft though, which comes with a certain cost and design constraints. The use of conventionally available measurements has therefore been envisioned in the context of formation flight, using for instance the dynamics of the aircraft themselves feeding on peak-seeking algorithms [28, 12]. Those online techniques continuously direct the airplane towards the direction of steepest descent of a function of flight parameters (i.e., gradient descent minimization algorithm), which provides an estimation of the airplane drag or fuel consumption. A drawback to those approaches is the need for linearization of the system leading to the objective function and the absence of real-time estimation of the optimal position; merely the algorithm provides a direction to follow. An alternative to peak-seeking algorithms has been proposed by Hanson [69] using a fuzzy logic technique for determining and maintaining a constant lateral distance between a follower and the wake of a leader. Unlike a peak-seeking algorithms [28, 12, 15, 125], this fuzzy logic signal) in order to determine the wake position.

The present work pursues the effort initiated in [21] on wake vortex detection based on data assimilation tools (i.e., Kalman filters). Two types of data were considered in that study; a first one consisted in circulation measurements along the main wing feeding the estimation of the position of an incoming wake, as suggested in [72]. A second step considered simplified aircraft dynamics. The measurements consisted solely of the dynamics of an aircraft whose motion is restricted to a translation along the vertical ∂_z^l axis, with a fixed attitude. The position, velocity, and acceleration along that axis were submitted to an idealized spring-mass system. The present study is intended to enhance the vortex detection and tracking technique by exploring the use of the full 6DOF aircraft flight dynamics and control actions (i.e., control surface deflections) as measurements to sense the position of a wake. Such an approach is motivated by the direct availability of those measurements on current aircraft without requiring any ad-

ditional hardware devices, as opposed to the above pressure-based or LIDAR-based schemes. The choice for a reduced amount of additional equipment makes the resulting wake sensing strategy deployable within a short- to mid-term horizon compare to other strategies. While the use of simplified dynamics invoked in [21] showed promising results, we want to study the performance and the reliability of the methodology in realistic situations, i.e., involving full unconstrained aircraft. The 6DOF dynamics are here coupled with an autopilot in order to study the interactions between the sensing strategy and the control scheme of the follower. The full framework developed in this work enables the study of efficient strategies for the tracking of the leader's wake, which will allow the follower to maintain a fixed relative position compared to the wake vortices. To this end, and akin to [21], the estimation procedure used in this work exploits a state model of the process, itself based on a Lifting Line model that characterizes the aerodynamics of the follower. It also takes into account the nonlinear dynamics of the follower controlled by an autopilot including the influence of the leader's wake vortices. Here, the estimator itself makes use of an Ensemble Kalman Filter (EnKF), better suited to handle the nonlinearity of the process [141] than the EKF.

We employ LES in order to capture the production of the wake vortices originating from the leader, their roll-up, and the interaction with some ambient turbulence and with the follower and its wake. The aircraft are represented by several immersed lifting lines in the LES and their 6DOF dynamics are simulated in this unsteady flow flight. Every aircraft in the formation is equipped with an autopilot that commands the control surfaces and therefore the dynamics of the aircraft within the LES. Simulated dynamic measurements are extracted from those simulations and serve as a reference for the estimator.

Two main flight situations are established and are thoroughly investigated in this work. The first one explores the ability for a follower flying on a predefined trajectory to use measurements obtained from its dynamics in order to estimate the position of the wake produced by a static or a moving leader, whose geometry (i.e., mass and dimensions) is known. The second one investigates the situation where the follower detects the position of the wake of a moving leader and concurrently follows a target position relative to that estimated wake location as a way to constantly benefit from the wake upwash. In the first case, the trajectory of the follower is independent of the dynamics of the leader, whereas in the second situation, the trajectory of the follower relies on the estimation of the wake location and is therefore highly influenced by the leader's behavior. The performance of the estimator is therefore impacted, as well as the flying performance of the follower (i.e., reduction in thrust command). These simulations

suggest that the use of the follower dynamics is sufficient to determine the position of an impacting wake. We show the performance of the estimator for different flight configurations. At the same time, we also investigate the ability of the estimator to determine the intensity of the wake of a leader, in addition to determining its position. Indeed, while the geometry of a leader does not change during a given flight, its mass decreases slowly, inducing a slow change in its wake intensity. Finally, our results highlight the known observability issues [72, 47] that arise from the multiplicative compensation between the wake intensity and the lateral position when dealing with more challenging configurations (i.e., a moving wake or strong turbulence).

The contributions of this work lie in the application of an EnKF that processes measurements of a follower's dynamics in order to estimate and track the position of a leader's wake and in the validation of the sensing and control scheme in a high-fidelity simulated flight environment based on LES. This chapter is structured as follows. Section 2.2 introduces the complete detection architecture. The latter is built in a simulated flight environment in which we simulate various configurations of formation flight and the estimator itself that propagates a low-fidelity state model of those configurations. Section 2.3 assesses the estimation quality of the wake position for the two considered flight situations. We also evaluate the ability of the estimator to determine the intensity of the wake, in addition to its position. Considerations of the observability of the dynamics based estimation strategy are therefore discussed. Concluding remarks are made in Section 2.5.

2.2 Design of the wake detection and tracking strategy

The simulated flight environment described in Section 1.5 represents our reality and will produce the reference measurements that feed the estimator. The latter makes use of an EnKF in order to process those measurement and to estimate the characteristics of the wake influencing the follower. This is done through the exploitation of a state model representation of the two-aircraft formation, called the surrogate model as opposed to the flight environment. Fig. 2.1 is a schematic representation of the complete architecture. The latter combines the simulated flight environment, the EnKF, the surrogate model, and a filtering tool used to provide a smooth target to the autopilot.

The current section introduces the simulated flight environment in Section 2.2.1. Section 2.2.2 presents the equations of the EnKF, which relies on the surrogate model described in Section 2.2.3. The filtering procedure is described in Section 2.2.4.

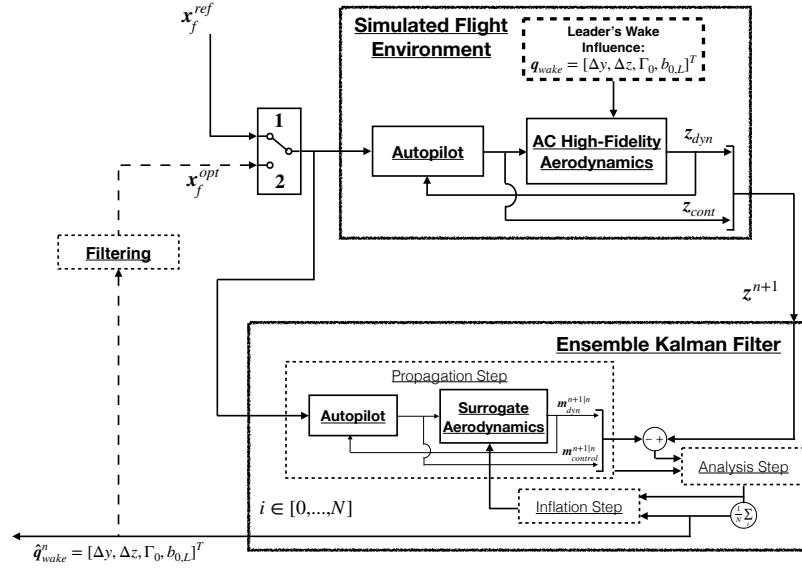


Figure 2.1: Block diagram of the tracking strategy including the simulated flight environment, the estimator, and a filtering tool to provide a smooth target.

2.2.1 Simulated flight environment

A sensing strategy is evaluated by its ability to perform in an environment as close as possible to real flight conditions, which means in this work the nonlinear and unsteady environment produced by the wake of a leader and a follower flying in extended formation. We use the simulated flight environment described in Section 1.5 to simulate the unsteady wake developed by this system of two aircraft. Indeed, the VPM method [26] is used to realize LES of the wake flow behind aircraft, whose lifting surfaces are modeled as ILDL. In the present application, the leader and the follower aircraft are based on the Cessna C172 and thus have identical parameters described in Table 1.1. Following the methodology described in Section 1.5.2, each lifting line uses the local flow velocity and tabulated airfoil data to determine the bound vorticity and the vorticity sheet that must be shed in the wake as a signature of lift and drag.

Within the flow simulation, the motion of each aircraft is computed through the time-integration of their respective 6DOF equations of motion using the multi-body integrator ROBOTRAN as presented in Section 1.5.3. Control surfaces at each lifting surface (i.e., ailerons, rudder and elevator) are also taken into account as a local modification of the polar in order to model the corresponding change in aerodynamic loads.

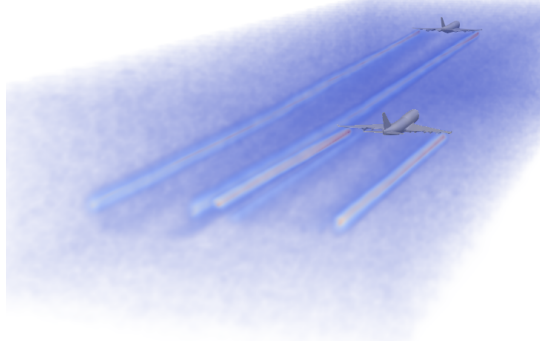


Figure 2.2: Volume rendering of the vorticity in the wake of two-aircraft formation, with ambient turbulence. Immersed Lifting and Dragging Lines are displayed as realistic aircraft.

In this work, ambient turbulence is added to the simulation to reproduce as closely as possible actual flight and wake conditions. From the estimator point of view, this corresponds to a natural source of process noise in the estimation routine, as the turbulence is not modeled in the surrogate model described in Section 2.2.3. Synthetic turbulence, reproducing the spectrum of Homogenous Isotropic Turbulence (HIT), and generated using the Mann algorithm [96], is smoothly injected at the inlet of the computational domain. This affects the wake generation mechanism and may trigger instabilities. Fig. 2.2 is a visual representation of the turbulent environment in which the two-aircraft formation evolves, and in which the wake detection strategy performs.

2.2.2 Estimator and Kalman filtering

This work relies on the discrete-time Kalman filter framework to estimate the characteristics of the leader's wake vortices. With regard to the estimator, those characteristics are represented by a wake state vector $\mathbf{q}_{\text{wake}}$. Associated to the follower's dynamics state vector $\mathbf{q}_{\text{dyn}}$, they form the state vector $\mathbf{q}$ of size M that is propagated by the estimator over time, thanks to the surrogate model. The propagation step produces the predicted measurement vector $\mathbf{m}$, which is compared to the true measurement vector $\mathbf{z}$ in order to produce an updated estimated state during the analysis step. The state vector $\mathbf{q}$, the measurements $\mathbf{z}$ (or $\mathbf{m}$), and the surrogate model are described in detail in Section 2.2.3. We focus on the description of the filtering methodology here.

The nonlinearity of the system under investigation associated to the complex flow environment makes the classical Kalman Filter (KF) for linear systems inadequate. The Extended Kalman Filter (EKF), which requires the linearization of the system about

the current state, was also proven unsuccessful in some configurations when applied to the current problem [72]. Biases on the solution, or even diverging behaviors were observed, and are likely related to the observability structure of the wake, or to the nonlinearity through an uncontrolled growth of the error covariance. To overcome these issues, we here use the Ensemble Kalman Filter (EnKF) [55, 5] with implicit linearized formulation for the measurements [54], schematically described in Fig. 2.3 and in the continuity of the work carried out in [21].

The EnKF uses a Monte Carlo approach to propagate the probability density function (PDF) of the states through the system, meaning that the state vector $\mathbf{q}$ is represented by a discrete ensemble of size N . Then, approximate statistics of the propagated distribution can be computed from the ensemble and the resulting mean and covariance matrices are used as in a classical KF. An analysis step and a covariance inflation step follow the propagation step every time the filtered state is iterated.

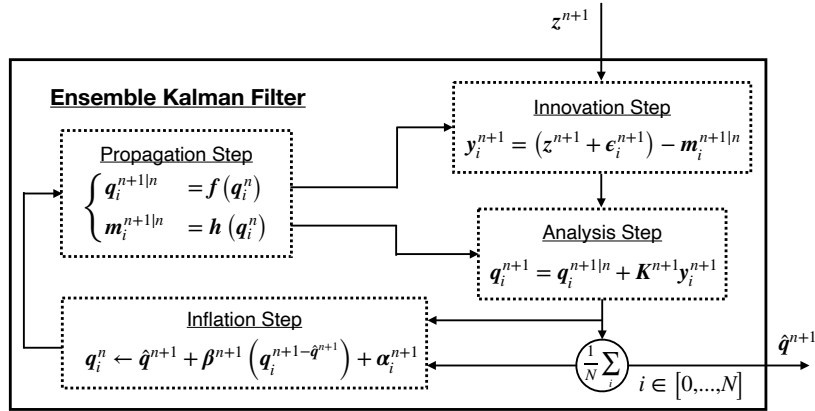


Figure 2.3: Ensemble Kalman Filter scheme.

The propagation step consists in the forward time evolution of each member $\mathbf{q}_i^n$ of the ensemble from iteration n to $n + 1$ through the surrogate model of the flight environment

$$\begin{cases} \mathbf{q}_i^{n+1|n} &= \mathbf{f}(\mathbf{q}_i^n) \\ \mathbf{m}_i^{n+1|n} &= \mathbf{h}(\mathbf{q}_i^n) \end{cases}, \quad (2.1)$$

where $(.)^{n+1|n}$ denotes the propagation of the model without any measurements incorporated in the estimation process, $\mathbf{m}$ is the measurement vector, and $\mathbf{f}$ and $\mathbf{h}$ are non-linear functions describing the dynamics of the state model. Unlike the classic

definition of the KF, note that the state model does not include any process noise usually denoted as $\mathbf{w}$. We can now estimate the a priori state mean and covariance based on the propagated ensemble, i.e.,:

$$\hat{\mathbf{q}}^{n+1|n} = \frac{1}{N} \sum_{i=1}^N \mathbf{q}_i^{n+1|n}, \quad (2.2)$$

$$\mathbf{P}^{n+1|n} = \frac{1}{N-1} \sum_{i=1}^N \left(\mathbf{q}_i^{n+1|n} - \hat{\mathbf{q}}^{n+1|n} \right) \left(\mathbf{q}_i^{n+1|n} - \hat{\mathbf{q}}^{n+1|n} \right)^T. \quad (2.3)$$

The innovation step is performed by comparing the true measurements $\mathbf{z}^{n+1}$, i.e. those obtained from the simulated flight environment at the current assimilation time, and the measurement vector $\mathbf{m}_i^{n+1|n}$ of each element of the ensemble,

$$\mathbf{y}_i^{n+1} = \left(\mathbf{z}^{n+1} + \boldsymbol{\epsilon}_i^{n+1} \right) - \mathbf{m}_i^{n+1|n}, \quad (2.4)$$

where the random vector $\boldsymbol{\epsilon}_i^{n+1} \sim \mathcal{N}(0, \mathbf{R})$ is an additive zero-mean Gaussian measurement noise with covariance $\mathbf{R}$. Unlike the process noise, the measurement noise is thus explicitly introduced within the simulations and then allows the innovation step to compare independent random vectors. Indeed, we cannot use the same measurement $\mathbf{z}^{n+1}$ for all the ensemble members, as Burgers et al. [18] found that doing so in an ensemble no longer reflects the fact that $\mathbf{z}^{n+1}$ is drawn from a random variable, and will introduce spurious correlations in the ensemble covariance [41]. The measurement noise has a very specific physical meaning, as it is directly related to the precision of sensors used in a real application. This has to be kept in mind when tuning the values of the measurement noise covariance matrix $\mathbf{R}$.

Then, the analysis step starts with the use of the sample covariance to compute the Kalman gains $\mathbf{K}^{n+1}$, using the standard definition [55] without any further modification, i.e.,

$$\mathbf{K}^{n+1} = \mathbf{P}^{n+1|n} \left(\mathbf{H}^{n+1|n} \right)^T \left(\mathbf{R}^{n+1} + \mathbf{H}^{n+1|n} \mathbf{P}^{n+1|n} \left(\mathbf{H}^{n+1|n} \right)^T \right)^{-1}, \quad (2.5)$$

where $\mathbf{H}^{n+1|n}$ is the measurement matrix obtained in the frame of an implicit linearized formulation [39]. In practice, it means that the measurement matrix $\mathbf{H}^{n+1|n}$ is not directly computed. Instead, an approximation of the products $\mathbf{P}^{n+1|n} \left(\mathbf{H}^{n+1|n} \right)^T$ and $\mathbf{H}^{n+1|n} \mathbf{P}^{n+1|n} \left(\mathbf{H}^{n+1|n} \right)^T$ can be computed given the prior ensemble [54]. All the members of the ensemble thus share the same Kalman gain, but have their own innovation. Indeed, each member of the ensemble is individually updated using the Kalman gain in its classical form,

$$\mathbf{q}_i^{n+1} = \mathbf{q}_i^{n+1|n} + \mathbf{K}^{n+1} \mathbf{y}_i^{n+1}. \quad (2.6)$$

We deduce the updated *a posteriori* state and covariance matrix of the system by taking the mean $\hat{\mathbf{q}}^{n+1}$ and covariance $\mathbf{P}^{n+1}$ of the updated ensemble members:

$$\hat{\mathbf{q}}^{n+1} = \frac{1}{N} \sum_{i=1}^N \mathbf{q}_i^{n+1}, \quad (2.7)$$

$$\mathbf{P}^{n+1} = \frac{1}{N-1} \sum_{i=1}^N \left(\mathbf{q}_i^{n+1} - \hat{\mathbf{q}}^{n+1} \right) \left(\mathbf{q}_i^{n+1} - \hat{\mathbf{q}}^{n+1} \right)^T. \quad (2.8)$$

Similarly to the classical KF, the EnKF minimizes the covariance of the estimated state distribution $\mathbf{P}^{n+1}$. However, as opposed to a KF, the covariance of the states is not bounded from below by the covariance of the state disturbance. Indeed, the EnKF does not process any process noise when it computes the covariance, as already mentioned. As described in [41], this process noise is implicitly added to the ensemble members in the EnKF, through the "Monte Carlo" formulation of the KF. Therefore, the progressive decrease of the ensemble covariance $\mathbf{P}^{n+1|n}$, despite a non-zero measurement covariance $\mathbf{R}$, would yield negligible Kalman gain $\mathbf{K}^{n+1}$. Hence, the innovation vector would be ignored in Eq. (2.6), rendering the filter totally ineffective. Covariance inflation is thus implemented by altering the state ensemble of each vector. The Relaxation To Prior Spread (RTPS) technique [148], as well as additive inflation, lead to

$$\mathbf{q}_i^n \leftarrow \hat{\mathbf{q}}^{n+1} + \boldsymbol{\beta}^{n+1} \left(\mathbf{q}_i^{n+1} - \hat{\mathbf{q}}^{n+1} \right) + \boldsymbol{\alpha}_i^{n+1}, \quad (2.9)$$

with $\boldsymbol{\beta}^{n+1}$ a diagonal matrix such that $\beta_{jj}^{n+1} = 1 + \theta \left(\frac{\sigma_{jj}^b - \sigma_{jj}^a}{\sigma_{jj}^a} \right)$, where $\boldsymbol{\sigma}^a$ and $\boldsymbol{\sigma}^b$ are diagonal matrices with the standard deviation of the prior $\mathbf{q}_i^{n+1|n}$ and of the posterior $\mathbf{q}_i^{n+1}$ states, respectively, $\theta = 0.7$ and $j \in [0, \dots, M]$. $\boldsymbol{\alpha}_i^{n+1} \sim \mathcal{N}(0, \mathbf{Q}_\alpha)$ is the small random Gaussian additive inflation whose covariance matrix is $\mathbf{Q}_\alpha$. The multiplicative inflation $\boldsymbol{\beta}^{n+1} \left(\mathbf{q}_i^{n+1} - \hat{\mathbf{q}}^{n+1} \right)$ is used to spread the ensemble around the state mean while the additive inflation $\boldsymbol{\alpha}_i^{n+1}$ is used as a representation of the process noise that is deliberately missing in the surrogate model in Eq. (2.1). More details on the theoretical background to the EnKF can be found in [55] and [5]. In addition to its ease of implementation, this approach generally enables the filtering of systems with a very large number of degrees of freedom up to several millions [39]. The EnKF processes measurements provided by those large systems thanks to models involving a reduced number of states (e.g., of the order of a hundred in [39]) and therefore estimates those systems at a more reasonable cost. In this work, the states are the ones involved in the equations of motion of an aircraft subjected to the influence of an external wake, i.e., 12 for the follower's dynamics and 4 for the leader's wake dynamics, see Section 2.2.4.

We also use the EnKF to exploit its ability to handle nonlinear processes, which is required for an accurate estimation of the wake characteristics as highlighted in [72] and [21].

2.2.3 Surrogate model

We adopt a state model representation for the process formally described by $\mathbf{f}$ and $\mathbf{h}$ in Section 2.2.2. This model has to be computationally affordable as it is evaluated at every iteration of the estimation process, and for each member of the ensemble. In order to ensure an accurate estimation of the relative position of the follower with respect to the instantaneous position of the leader's wake, the measurements $\mathbf{m}$ produced by the surrogate model have to be similar to the measurements $\mathbf{z}$ provided by the simulated flight-environment for identical flight configurations. Since the detection strategy relies on the follower dynamics, the surrogate model has to correctly reproduce the aerodynamic loads undergone by the follower in those configurations. These characteristics are found in the simplified model described in Sections 1.3 and 1.4. As shown in Section 1.5.4, the aircraft dynamics produced by that model are similar to the dynamics produced by the simulated flight environment described in Sections 1.5.1, 1.5.2 and 1.5.3. The association of the simplified model to the EnKF in order to predict the simulated flight environment thus seems appropriate.

The states of our two-aircraft formation model consist of the characteristics of the leader's wake $\mathbf{q}_{wake}$ and the dynamics of the follower $\mathbf{q}_{dyn}$. In this section, we describe each one of them, and we derive the discrete-time state equations that characterize the process that is propagated by the EnKF.

Aerodynamics

As described in Section 1.3, the Prandtl Lifting Line theory is implemented for each of the three lifting surface of the follower as the simplified aerodynamic model of the simulated flight environment for the estimator. The simplified model is synthesized in Fig. 2.4 including the leader's wake vortices.

The wake of the leader is modeled as two straight counter-rotating vortex tubes of circulation $\Gamma_{0,L}$ and spacing $b_{0,L}$. Those two parameters are directly related to the mass m and the geometry of the leader from Eqs. 1.5 and 1.7. While Flanzer et al. [57] use the Automatic Dependent Surveillance-Broadcast (ADS-B) for an experimental formation flight application in order to determine the wind speed, we hypothesis a reasonable extension of the ADS-B, namely the episodic broadcast of minimal leader's properties, in particular its mass m . In this work, this hypothesis translates in our as-

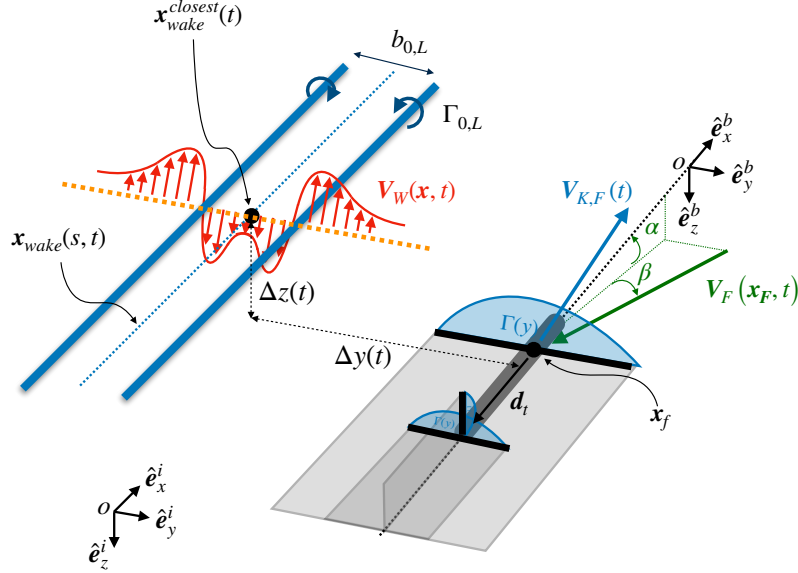


Figure 2.4: Aerodynamic model of the follower with three lifting lines (black), their wake (light grey) and their circulation distributions (light blue). The velocities induced by the leader's two vortices are depicted in red.

sumption that the leader's properties are initially known, and vary little within the time scales considered. The vortices are aligned with the inertial axis $\hat{\mathbf{e}}_x^i$ (which is parallel to the ground), and are regularized using a LOA wake vortex model, with a core radius $r_c/b = 0.05$ [14], see Appendix A for a more detailed description.

We use the quantities $[\Delta y, \Delta z]_i$ to represent the relative distance between the follower and the position of the leader's wake. Let us define $\mathbf{x}_{\text{wake}}(s, t)$ as the function describing the position of the centerline of the leader's wake with respect to time and a parameter s (i.e., parametric representation of the line). We also define the time varying position of the wake $\mathbf{x}_{\text{wake}}^{\text{closest}}(t)$ as the point belonging to the centerline $\mathbf{x}_{\text{wake}}(s, t)$ such that the distance to the follower position $\mathbf{x}_F(t)$ is minimal, i.e.,

$$\mathbf{x}_{\text{wake}}^{\text{closest}}(t) = \min_s (\|\mathbf{x}_{\text{wake}}(s, t) - \mathbf{x}_F(t)\|). \quad (2.10)$$

Δy and Δz are the vertical and lateral components of the vector $\mathbf{x}_{\text{wake}}^{\text{closest}} - \mathbf{x}_F$, where $\mathbf{x}_F$ is the position of the follower. Therefore, they measure the relative distance of the follower to the wake and not to the position of the leader. Unlike this relative distance, which can significantly vary depending on external factors [75] and on the trajectories

of the leader and of the follower, the circulation $\Gamma_{0,L}$ vary slowly (, i.e., with the mass of the leader) and the spacing $b_{0,L}$ remains very constant over time.

The wake-induced velocities $\mathbf{V}_W(\mathbf{x}, t)$ on the follower's wing, VTP and HTP depend on the spanwise location as well as on the follower's angular position relative to the leader's wake. The complete dynamics of the follower, combined with the induced velocities, lead to the airspeed vector $\mathbf{V}_F(\mathbf{x}_F, t)$ of the follower, see Eq. (1.16). When measured at the center of mass of the follower, this relative air velocity vector enables the computation of the related aerodynamic angles α and β , i.e., the angle of attack and the angle of slip, respectively.

In summary, for the aerodynamic part of the model, the states characterizing the wake of the leader are

$$\mathbf{q}_{\text{wake}} = [\Delta y, \Delta z, \Gamma_{0,L}, b_{0,L}]^T. \quad (2.11)$$

Those states are the unknown variables of the complete system, as their dynamics is *a priori* unknown. The goal of the estimator is to determine the value of those states over time. In the simplified model, the dynamics associated to the wake states are considered to be constant over time, meaning that except for any process noise, those states remains constant during the propagation step of the EnKF. In other words, we have the discrete-time propagation equation for each member of the ensemble,

$$\mathbf{q}_{i, \text{wake}}^{n+1|n} = \mathbf{q}_{i, \text{wake}}^n. \quad (2.12)$$

We recall that the process noise is not explicitly represented in the propagation step of the EnKF, as observed in the previous equation. Nevertheless, covariance state inflation is implemented and alters the wake state vector of each ensemble at each estimation step in a similar form to a random walk dynamics of the wake states, see Eq. (2.9). This inflation step enables the capture of the variation of the wake states, in particular Δy and Δz , during a estimation step.

The relative positions $(\Delta y, \Delta z)$ cannot be directly measured over time; the objective of our technique is precisely to determine them using aircraft dynamics measurements. The spacing $b_{0,L}$ is constantly supplied to the EnKF as a measurement in the vectors $\mathbf{z}$ and $\mathbf{m}$. Indeed, aircraft flying in extended formation flight configurations are assumed to know the geometrical properties of each other. Finally, regarding the wake intensity $\Gamma_{0,L}$, we initially assume that it is available in the measurement vectors $\mathbf{z}$ and $\mathbf{m}$ over time. This assumption permits us to evaluate the ability of the detection strategy to determine the position of a wake of a leader. Section 2.3 assesses the ability of the EnKF to deal with the state $\Gamma_{0,L}$ unmeasured and thus *a priori* unknown over time in order to test the robustness of the filter (i.e. the measurement vectors $\mathbf{z}$ and $\mathbf{m}$

will not include $\Gamma_{0,L}$). Therefore, for the aerodynamic part of the model, the list of available measurements is

$$\mathbf{m}_{\text{angle}} = [\alpha, \beta]^T, \quad (2.13) \quad \mathbf{m}_{\text{wake}} = [\Gamma_{0,L}, b_{0,L}]^T, \quad (2.14)$$

with the measurement $\Gamma_{0,L}$ being included in the wake measurement only.

Flight Dynamics and control

The flight mechanic model uses the forces and moments computed by the aerodynamic model and integrates the aircraft 6DOF equations of motion over time in the same way as described in Section 1.4. This allows the prediction of the position, attitude, and velocity of the follower over one time step of the filter. With that flight dynamics model, the state vector can be augmented with

$$\mathbf{q}_{\text{dyn}} = [x_F, y_F, z_F, U_{K,F}, V_{K,F}, W_{K,F}, \phi, \theta, \psi, p, q, r]^T, \quad (2.15)$$

and this turns readily into the additional available measurements

$$\mathbf{m}_{\text{dyn}} = [x_F, y_F, z_F, U_{K,F}, V_{K,F}, W_{K,F}, \phi, \theta, \psi, p, q, r]^T. \quad (2.16)$$

To the dynamics states and measurements, we associated the autopilot described in Section 1.4.2. It is used in *position hold* mode (, i.e., switch 1 in Fig. 2.1) or in *wake tracking* mode (, i.e., switch 2 in Fig. 2.1). With the developed autopilot, the available measurement vector can be augmented with

$$\mathbf{m}_{\text{control}} = \mathbf{u}_c = [\delta_{Ail}, \delta_{VTP}, \delta_{HTP}, \delta_{th}]^T, \quad (2.17)$$

as control commands provide information regarding the follower configuration¹.

The dynamic equations that govern the states $\mathbf{q}_{\text{dyn}}$ as well as the measurements $\mathbf{m}_{\text{dyn}} \cup \mathbf{m}_{\text{control}}$ are all described in Section 1.4 in their continuous-time formulation. The EnKF dealing with discrete-time equations, one can easily reformulate the dynamic equation leading the formulation

$$\begin{cases} \mathbf{q}_{i,\text{dyn}}^{n+1|n} = \mathbf{f}_{\text{dyn}} \left(\mathbf{q}_{i,\text{dyn}}^n, \mathbf{u}_c^n, \mathbf{q}_{i,\text{wake}}^n \right) \\ \mathbf{m}_{i,\text{dyn}}^{n+1|n} = \mathbf{h}_{\text{dyn}} \left(\mathbf{q}_{i,\text{dyn}}^n, \mathbf{u}_c^n, \mathbf{q}_{i,\text{wake}}^n \right) \end{cases}, \quad (2.18)$$

¹In this chapter, we use the C172-like aircraft (see Table 1.1). For that aircraft, the response time from the command $\mathbf{u}_c$ to the effective control surface deflections or the effective thrust is assumed to be instantaneous, see Section 1.4.1. This explains that $\mathbf{u}_c = \mathbf{q}_{\text{cont}}$.

through a numerical integration scheme, i.e., Runge-Kutta 4. $\mathbf{f}_{dyn}$ and $\mathbf{h}_{dyn}$ are non-linear function describing the dynamic states propagation over one EnKF iteration. They are function of the leader's wake state parameters, as the aerodynamic loads on the follower are impacted by the leader's wake.

2.2.4 Summary of the detection strategy

The complete estimator propagates sixteen states using the surrogate model formally described by $\mathbf{f}$ and $\mathbf{h}$ in Eq. (2.1), and more precisely defined by Eqs. 2.11 and 2.15. Four are the states characterizing the wake of the leader $\mathbf{q}_{wake}$ and twelve states come from the dynamics of the follower $\mathbf{q}_{dyn}$. The temporal evolution of the follower's dynamic states $\mathbf{q}_{dyn}$ are measured by the follower to certain precision. Although the dynamics of the states of the leader's wake $\mathbf{q}_{wake}$ are unknown, the EnKF propagates the value of those states in order to produce an a priori estimate of the measurements, i.e., the vector $\mathbf{m}$. This vector is compared to the true measurement vector $\mathbf{z}$ in order to update the estimated states during the analysis step of the estimation. Thanks to classical onboard sensors (i.e., Inertial Measurements Units (IMU), Global Navigation Satellite Systems (GNSS), Inertial Navigation System (INS)), the twelve dynamic states benefit from direct measurements from the follower's perspective, i.e., $\mathbf{z}_{dyn}$ are compared to $\mathbf{m}_{dyn}$. The autopilot also provides additional information thanks to the control commands and the measured aerodynamic angles, i.e., $\mathbf{z}_{control}$ (resp., $\mathbf{z}_{angle}$) are compared to $\mathbf{m}_{control}$ (resp., $\mathbf{m}_{angle}$). While the positional states $\Delta y, \Delta z$ are not directly measured by the follower, we assume the latter benefits from direct informations regarding the geometry and mass of the leader. In practice, this can be achieved using a communication tool that episodically transmits the relevant information from the leader to the follower (e.g. via an ADS-B with extended transmission capabilities). It assumes that the geometrical properties of the leader are initially known, and vary little in the time scales considered. This leads to the additional wake measurements, i.e., $\mathbf{z}_{wake}$ are compared to $\mathbf{m}_{wakes}$. Taking up the notation described in Section 2.2.2, we have

$$\begin{cases} \mathbf{q} = \mathbf{q}_{dyn} \cup \mathbf{q}_{wake} \\ \mathbf{m} = \mathbf{m}_{dyn} \cup \mathbf{m}_{control} \cup \mathbf{m}_{angle} \cup \mathbf{m}_{wake} \\ \mathbf{z} = \mathbf{z}_{dyn} \cup \mathbf{z}_{control} \cup \mathbf{z}_{angle} \cup \mathbf{z}_{wake} \end{cases} \quad (2.19)$$

Section 2.3 assesses the ability of the estimator to determine the intensity of the wake of the leader in addition to determining its position. In practical terms, we will investigate the performance of the estimator when the wake measurement $\mathbf{z}_{wake} = \Gamma_{0,L}$

is not used, leading to three unmeasured wake states, namely the relative position and wake intensity i.e. $[\Delta y, \Delta z, \Gamma_{0,L}]$.

2.3 Results

Various simulations were performed in order to assess the efficiency of the detection and tracking strategies. The aircraft modeled in our framework is much like the Cessna-172 private aircraft. The geometrical parameters of the aircraft are reported in Table 1.1. In the different scenarios, the leader either follows a fix target, or is instructed to follow a sinusoidal target trajectory in the $\hat{e}_y^i - \hat{e}_z^i$ plane, similar to the one described in Eq. (1.57). It is always asked to maintain a constant cruise speed U_∞ along the streamwise direction $\hat{e}_x^i$. The parameters of the oscillations (i.e. amplitudes A_y, A_z , periods T_y, T_z) are reported in Table 2.1. The oscillation periods are expressed in terms of the period of the phugoid $T_{\text{phugoid}} = 26\text{s}$, as estimated from the linearized dynamics described in Section 1.4.1. The follower is either instructed to maintain a prescribed position in the $(oy_i z_i)$ plane, while evolving in close proximity of the wake of the leader, or is instructed to track the wake of the leader while flying at the cruise speed U_∞ along the longitudinal axis x_i .

The reference parameters of the VPM simulation used to produce the simulated flight environment are reported in Table 2.2. We used a longitudinal distance of $5b$ between the two aircraft. These conditions are not consistent with the hypothesis of extended formation flight. In particular, all the behaviors observed in the far wake behind an isolated aircraft (, i.e., wake meandering [124], vortex wandering [52], etc) that could impact the performance of the estimation strategy are not represented within the LES of two aircraft separated by a $5b$ distance. Moreover, a $5b$ distance is not compatible with the Traffic Collision Avoidance System (TCAS) that equips modern aircraft. Such equipment would prevent such a short relative distance between the aircraft for obvious safety reasons. Even in extended formation flight (, i.e., $50 - 100b$ between the aircraft), the TCAS would have to be adapted. However, as already mentioned in Chapter 1, this work is devoted to the study of the tracking of the wake in the $\hat{e}_y^i - \hat{e}_z^i$ explaining the short relative distance and a constant streamwise velocity target $U_K = U_\infty$. Moreover, the choice of a small longitudinal separation distance was made to keep a relatively small time-to-solution for the simulation. Furthermore, the fact that the leader's vortices are not fully rolled-up when they reach the follower can be seen as a source of process noise which the estimator is designed to tolerate. Under those conditions, the simulation was run on 256 CPUs for a maximum of 5 hours. It is worth noticing that with the mesh resolution used, i.e., $h/b = 1/64$, the VTP and

HTP might not be resolved enough to accurately capture the dipole of the ILDL model, which will impact the emission of vorticity related to the drag. The interested reader will find more related information in [23].

	Config. A	Config. B
A_y/b	0.25	0.4
A_z/b	0.5	0.5
T_y/T_{phugoid}	1	1.5
T_z/T_{phugoid}	1	3

Table 2.1: Parameters of the oscillating movement performed by the leader.

Domain size $(L_x \times L_y \times L_z)$	$8b \times 4b \times 2b$
Size of the cubic mesh cells	$h/b = 1/64$
Eddy Dissipation Rate (EDR)	$\epsilon^* = \frac{\epsilon b_0}{V_0^{*3}} \simeq 0.02$

Table 2.2: Numerical parameters used in the VPM simulations .

Unlike the simulation described in Section 1.5.4, synthetic turbulence is injected at the inlet. It is characterized by a dimensionless Eddy Dissipation Rate (EDR) $\epsilon^* = \frac{\epsilon b_0}{V_0^3} \simeq 0.02$, corresponding to strong turbulence [44]. This presents a quite challenging environment to assess the performances of our estimator, as extended formation flight for energy savings is likely to be implemented under low turbulence conditions.

As described in Section 2.2.2, the filter requires the initialization of state additive inflations α_i^{n+1} and measurement noises ϵ_i^{n+1} . Additive state inflation consists of adding a sample from the Gaussian distribution $\mathcal{N}(0, \mathbf{Q}_\alpha)$ to each member of the ensemble before the propagation step. The covariance matrix $\mathbf{Q}_\alpha$ is made up of a series of parameters used to tune the filter, which are provided in Appendix C. Regarding the measurements noises ϵ_i^{n+1} , they are directly related to the precision of the sensors used onboard the aircraft to provide the true measurements $\mathbf{z}$. The measurement noise covariance matrix $\mathbf{R}$ is defined accordingly, i.e, as the confidence of the filter in these measurements. The covariance associated with the measurement of the side slip angle β reflects the poor quality of the measurement provided by commonly used instruments. Two different noise measurement sets are used in this work, in order to compare the influence of coarse or accurate measurements on the performance of the filter. They are also given in Appendix C.

This work considers two configurations of the global filtering scheme, as described in Fig. 2.3. Section 2.3.1 first verifies the ability for a follower to solely use measure-

ments coming from its dynamics in order to detect the unknown parameters of the wake produced by a leader. We also aim to quantify the accuracy of the estimated wake characteristics in this configuration, where the follower maintains a predefined trajectory $\mathbf{x}_F^{ref}$ using the *position hold* autopilot (switch 1 in Fig. 2.3). Then the follower detects the position of the wake of a leader and, at the same time, follows the appropriate position in order to constantly benefit from the largest wake upwash. In order to perform this tracking, the estimation of the position of the leader's wake $\hat{\mathbf{q}}_{wake}^n$ is provided to a wake analysis module (see Fig. 2.3) that filters the estimation. This provides an optimal target $\mathbf{x}_F^{opt}$ to be followed by the autopilot of the follower in the *wake tracking* mode (switch 2 in Fig. 2.3). This is described in Section 2.3.2.

2.3.1 Wake detection

Detection of a stationary wake

This section shows that the envisioned estimation scheme is able to sense the wake and to determine the position of a non-maneuvering leader's wake over time. The leader follows a fixed target $(y_L^{ref}, z_L^{ref}) = (0.0b, 0.0b)$ and adapts its commands according to the perturbations in the ambient turbulence. The follower holds the position $(y_F^{ref}, z_F^{ref}) = (1.1b, 0.0b)$ which is a bit further away from the optimal one in the wake of the leader (, i.e., $(\Delta y^{opt}, \Delta z^{opt}) \simeq (\pm 0.9b, 0.0b)$, see Section 1.3.4). This relative position is used recurrently in this work. It is chosen as a compromise between maximizing the energy savings, and maintaining some distance with the leader's wake for safety reasons (, i.e., to limit wake encounter risks). In a more practical way, the controller described in Section 1.4.2 also did not showed to be able to lead the follower in a steady configuration for positions too close from the leader. Nevertheless, that position remains a relevant and realistic choice as proven by experimental studies [71, 70]. Both the leader and the follower fly at the cruise speed U_∞ along the $\hat{\mathbf{e}}_x^i$ axis. The EnKF uses $N = 100$ ensemble members, as in the rest of this work. The estimation is performed at a rate $\Delta t_{EnKF} = 0.2 \cdot t^*$ with $t^* = \frac{b}{U_\infty}$ the convective time. The levels of noise associated with the sensor measurements are considered to be high in this section; they are provided in Appendix C.

Observe in Fig. 2.5 the performance of the EnKF in estimating the wake states $\mathbf{q}_{wake}$ for two filter configurations. *Filter 1* deals with a full wake measurement vector (i.e., the estimator assumes the follower knows the leader's characteristics $\mathbf{z}_{wake} = [\Gamma_{0,L}, b_{0,L}]$) whereas *Filter 2* uses only the measurement of the spacing between the vortices, i.e., $\mathbf{z}_{wake} = [b_{0,L}]$. The wake intensity $\Gamma_{0,L}$ is thus not directly provided

to *Filter 2*, but along with the position of the wake (Δy and Δz) is considered to be unknown. Table 2.3 synthesizes the differences between the filters.

	<i>Filter 1</i>	<i>Filter 2</i>	<i>Filter 3</i>
$\mathbf{z}_{wake}$	$[\Gamma_{0,L}, b_{0,L}]$	$[b_{0,L}]$	$[\Gamma_{0,L}, b_{0,L}]$
$R_\phi = R_\theta = R_\psi$	1°	1°	0.1°

Table 2.3: Parameters distinguishing the three *Filters* used (other parameters are identical for the three filters, see Appendix C).

First consider *Filter 1*, involving the complete wake measurement vector. The convergence of the filter is obtained after 25 iterations, corresponding to a simulation time $t \simeq 5t^*$. A good estimate is obtained for the four wake states $\mathbf{q}_{wake} = [\Delta y, \Delta z, \Gamma_{0,L}, b_{0,L}]^T$. This result was expected regarding the estimation of the wake intensity $\Gamma_{0,L}$ and the spacing between the vortices $b_{0,L}$, as the estimator benefits from a direct measurement of their value through the vector $\mathbf{z}_{wake} = [\Gamma_{0,L}, b_{0,L}]$. Even though the standard deviation of the associated measurement noise was set to 30% for $\Gamma_{0,L}$ and 20% for $b_{0,L}$, the estimations remain within 5% of their actual value. Also, the confidence intervals associated to those states and computed as the *a posteriori* variance of the estimate (see Eq. (2.8)) remain within 15% of their actual value. However, we are more interested in the performance of the estimator in estimating the position of the wake, as no direct measurements are used by the estimator for those states. Estimates that remain within 10% of the wing span are obtained for both Δy and Δz , which is a satisfactory performance for a formation flight application. The confidence intervals for both states seem constant over the estimation process and remain within 30% of the wing span.

We now analyze the performances of *Filter 2* which uses only the spacing between the vortices as wake measurement, i.e., $\mathbf{z}_{wake} = [b_{0,L}]$, rather than $\mathbf{z}_{wake} = [\Gamma_{0,L}, b_{0,L}]$. The spacing between the vortices $b_{0,L}$ still benefits from a good estimation, because of the use of a direct measurement of that state. Like *Filter 1*, *Filter 2* converges to an accurate estimation of the vertical position Δz after 25 iterations. The main differences between the two filters appear in the estimation of $\Gamma_{0,L}$ and Δy . Indeed, *Filter 2* poorly estimates the wake intensity $\Gamma_{0,L}$; its estimate is twice the actual value with a poor confidence interval. Hemati et al. [72] had already highlighted this inaccuracy. They explained that much of the process uncertainty was lumped into the estimation of $\Gamma_{0,L}$, rather than into the position estimates Δy or Δz , which showed better accuracy in their study. However, we also observe here that the lateral position estimate Δy exhibits a significative offset, close to 30% of the wingspan. There is indeed an intrinsic correlation between $\Gamma_{0,L}$ and Δy . The inaccurate wake inten-

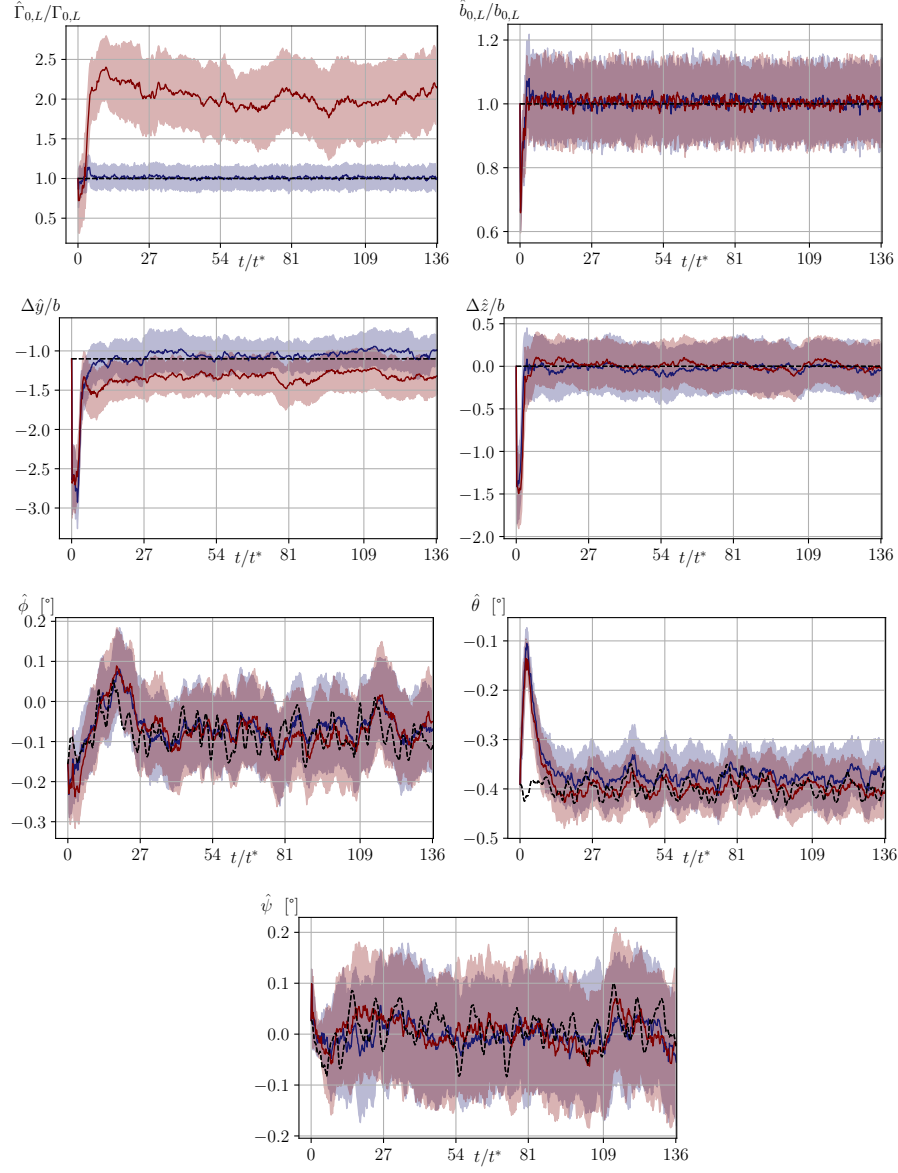


Figure 2.5: Detection of a steady wake. The black dashed line (—) indicates the actual values of the states. The blue lines (—) (resp. red (—)) are the estimates with their *a posteriori* $1 - \sigma$ confidence interval produced by *Filter 1* (resp. *Filter 2*).

sity estimation leads in fact to an over-estimated lateral-distance estimate as, from the follower's perspective, a strong distant wake generates aerodynamic effects which can hardly be distinguished from those of a closer but weaker wake. This is explained by the mechanism behind the influence of the leader's wake on the follower, which is governed by the Bio-Savart law that depends both on the wake intensity Γ_0 and the relative position $(\Delta y, \Delta z)$. An analysis of the sensitivity of the induced wind with respect to the wake intensity Γ_0 and the lateral position Δy is provided in Appendix D. It highlights a linear dependance of the upwash wind with respect to the wake intensity. The dependance with respect to the relative position is more complex and is characterized by a (non-linear) decrease of the upwash wind when the relative position increases. The upwash wind is responsible for the induced aerodynamic effects, which leads to complex sensitivity of the dynamics to the relative position in particular.

This observability issue was also put forward in [21] using other sets of measurements. In the present case, the analysis of the dynamic states further explains this issue. For the sake of concision, we focus only on the analysis of the attitude angles of the follower, also presented in Fig. 2.5. Because they are also measured, the roll, pitch and yaw (as well as the position, translation velocity, and rotation rates) should be accurately estimated by the filter. The two filter configurations show similar behaviors regarding the roll and the yaw. Indeed, except for the oscillations that are attributed to the ambient turbulence, the average angles are correctly reproduced by the two filter configurations. However, the pitch angle is not estimated in the same way by the two filters, as observed in Fig. 2.5. *Filter 2* seems to accurately reproduce the reference pitch value on average, while the first one estimates the pitch angle with an offset that is close to the one observed in Fig. 1.13 where we have compared the dynamics produced by the simulated environment and the surrogate model. In addition to the intrinsic responsible for the offset observed on the wake intensity estimate $\Gamma_{0,L}$. Indeed, in not constraining $\Gamma_{0,L}$ with information provided by an ADS-B $\mathbf{z}_{wake} = [\Gamma_{0,L}]$, we give the filter the opportunity to fit the dynamic states with more accuracy and, in particular, filter the opportunity to fit the dynamic states with more accuracy and, in particular, to fit the pitch angle. While adapted filter techniques exist to deal with those bias errors [58, 156], we keep our filters as simple as possible to identify the origin of the wake detection issues. Since all the states associated with the longitudinal dynamics (i.e., translations along $\hat{\mathbf{e}}_x^i$ and $\hat{\mathbf{e}}_z^i$ axis, and rotation about the $\hat{\mathbf{e}}_y^i$ axis) are intrinsically correlated and, since the pitch angle measurement affects the estimation of the lateral position, we deduce that the longitudinal dynamics of the aircraft have a significant influence on the estimation of the lateral position Δy and the wake intensity $\Gamma_{0,L}$ and to be much affected by errors on the longitudinal dynamics.

It is worth mentioning that a substantial lateral estimation error has to be avoided, as it might lead to a wake encounter (i.e. the follower airplane entering one of the leader's wake vortices, leading to unwanted aircraft response and possibly severe consequences) as soon as the autopilot is switched to the *wake tracking* mode. This justifies the use of a measurement of the wake intensity $\Gamma_{0,L}$, as tested in the first configuration. Let us mention that the use of a measurement of the spacing between the wake vortices $b_{0,L}$ has a similar impact as the use of a measurement of the wake intensity $\Gamma_{0,L}$, i.e., an improved wake position estimation. We will therefore assume that $\Gamma_{0,L}$ and $b_{0,L}$ are measured in the rest of this work.

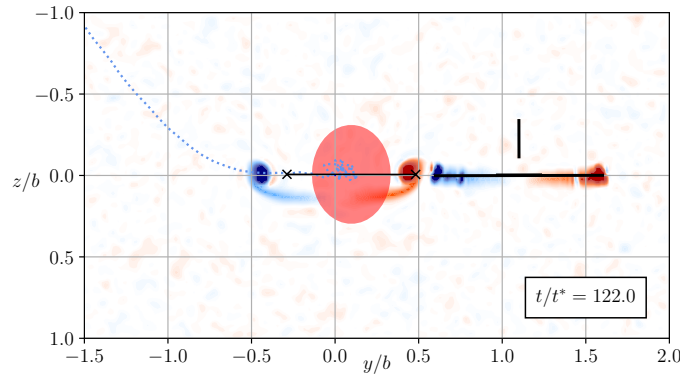


Figure 2.6: Cross section of the streamwise vorticity behind the follower (—). The dotted line (···) is the history of the estimated position of the leader's wake, with the current estimation (× — ×). The shaded area (●) represents $1 - \sigma$ confidence interval of the estimate.

Fig. 2.6 presents a visual representation of the detection methodology that will be further used for a visual understanding of the processes. From the initial guess $(\Delta z/b, \Delta y/b) = (-2, -2)$, the estimated wake position converges to the exact position, then oscillates around that position. As explained in Section 2.2.1, the high-fidelity simulations here incorporate ambient turbulence in order to accurately reproduce the environment around a flying aircraft. It explains the high frequency oscillations of the dynamics states (e.g., roll ϕ , pitch θ and yaw ψ) in Fig. 2.5 as well as the oscillations of the estimated position observed in Fig. 2.6. On the one hand, this turbulence can be seen as a model noise, as it is not reproduced in the aircraft surrogate model. On the other hand, the turbulence generates unsteadiness that might be beneficial for the detection, as it forces a movement of the follower and hence increases the observability of the process, as mentioned in [72] and [47].

Detection of oscillating wakes

We now investigate the estimation of moving wakes, produced by a leader that follows an oscillating trajectory, whose parameters are reported in the first column of Table 2.1. The follower is thus impacted by a wake moving up and down and from left to right; it acts so as to maintain a predefined reference position. The more unsteady the environment is, because of this strong oscillating wake, the more variations are observed in the dynamics measurements of the follower. Hypothetically, the larger the variations, the better the estimate. Indeed, large variations induced by the oscillating leader tend to overcome the measurement error or the model error. This means that more accurate estimations should be obtained when the follower is close to the wake. From the follower's perspective, the observability of the process might thus improve, when subjected to an unsteady wake. We investigate this assumption in the following example.

In this study case, the follower holds the same position as previously, $(y_F^{ref}, z_F^{ref}) = (1.1b, 0.0b)$, at a distance of $5b$ behind the leader. Periodic patterns similar to those of Fig. 1.13 are then observed on the follower's dynamics, reflecting the unsteady aerodynamic forces induced by the oscillating leader's wake. The EnKF estimations of the aerodynamic states q_{wake} and the attitude of the follower are displayed in Fig. 2.7. Two different filter configurations are also compared in this figure. While in the previous section, we evaluated the impact of the use of the measurement $z_{wake} = [\Gamma_{0,L}]$, we here highlight the impact of the measurement noise ϵ_i^{n+1} on the detection performances.

The first configuration in Fig. 2.7 uses the exact same parameters that are used in the *Filter 1* configuration of Fig. 2.5. In particular, the measurement noise levels ϵ_i^{n+1} are assumed to be high on every measurement, see Table 2.3 and Appendix C. In other terms, it means the filter does not have much confidence in the true measurements z that it receives from the flight environment. We observe that in such a situation, the aircraft is not able to discern the oscillatory movement of the wake of the leader. At the beginning of the detection procedure, the filter locks on to the initial steady position of the wake. The wake then starts to oscillate moving up and translating to the left, compared to the position of the follower. While the filter identifies the upward movement, it is not further able to sense the downward movement. This is because the wake also moves away to the left, which leads to a reduction of the aerodynamic influence of the wake on the aircraft. In practice, because the noise levels ϵ_i^{n+1} are high, the filter does not accurately estimate the dynamic states of the follower, which further translates into errors in estimating the wake position. Indeed, observe in Fig. 2.7 that,

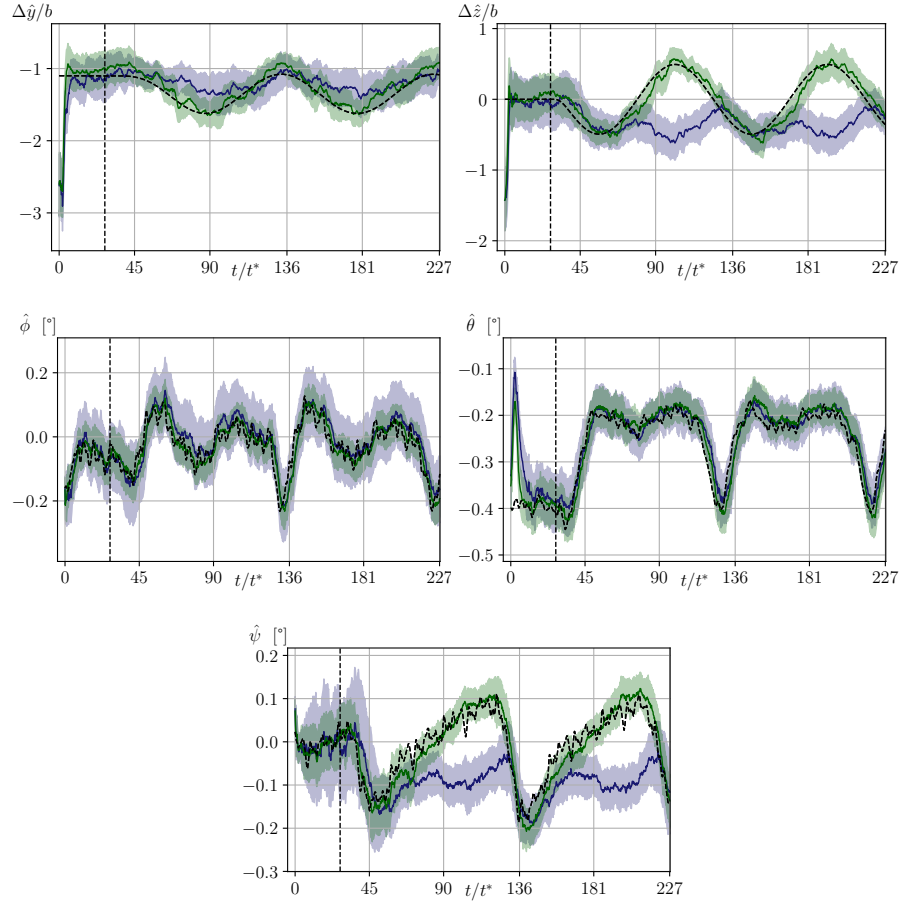
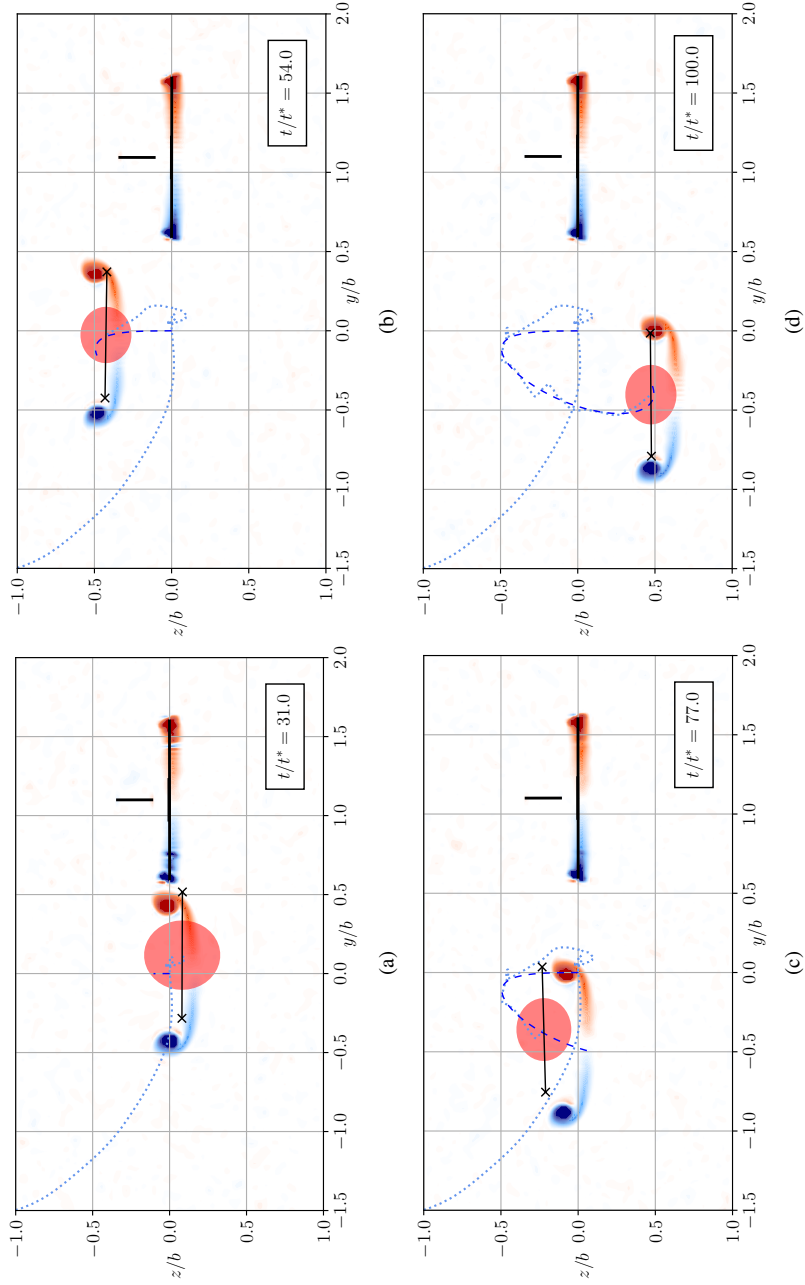


Figure 2.7: Detection of an oscillating wake. The black dashed line (- -) indicates the actual values of the states. The blue lines (—) (resp. green (—)) are the estimates with their *a posteriori* $1 - \sigma$ confidence interval produced by *Filter 1* (resp. *Filter 3*).

contrary to the roll and pitch angles, the yaw angle is not properly estimated. The need of a better estimation of the yaw angle is explained by the fact that the autopilot of the follower is designed to make it fly at 0° side slip angle, even under the influence of an external wake. When the leader's wake is moving, it modifies the flow field around the follower and therefore the angle of slip perceived by the follower. The autopilot reacts to this modification, which translates into a change in the yaw angle. An inaccurate estimation of the yaw angle leads therefore to incorrect wake position estimates. This



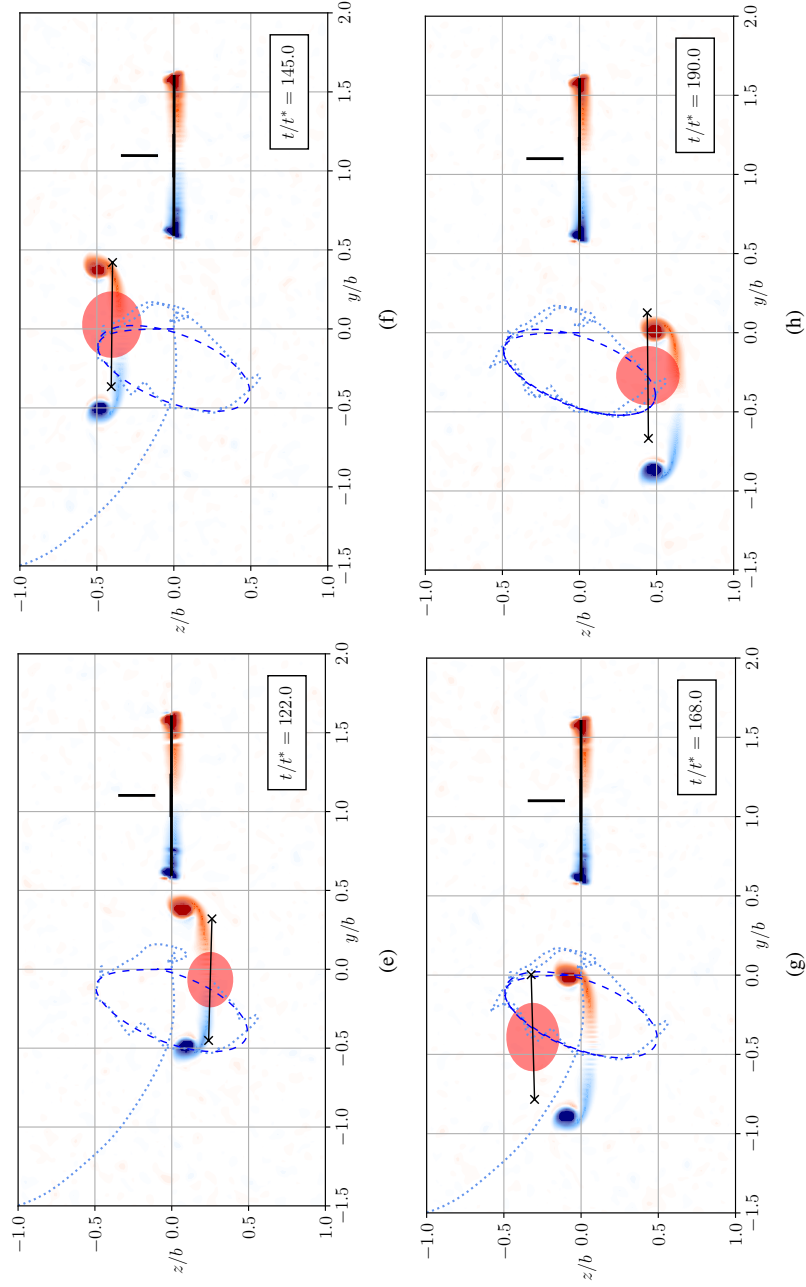


Figure 2.8: Vortex system estimated by the EnKF over time (Sensing mode). The background shows the streamwise vorticity behind the follower. The dotted line ($\cdots$) is the history of the estimated position with its $1 - \sigma$ confidence interval. The blue dashed line ($--$) is the actual trajectory of the leader's wake.

observation suggests that the lateral dynamics of the follower have a strong influence in the detection of the vertical relative position Δz .

We therefore consider a third filter configuration (*Filter 3*), in which we assume lower noise levels regarding the three attitude angles. Rather than the noise levels $R_\phi = R_\theta = R_\psi = 1^\circ$ of *Filter 1*, we decrease them tenfold, i.e., $R_\phi = R_\theta = R_\psi = 0.1^\circ$ for *Filter 3*, Table 2.3. Observe in Fig. 2.7 that this third filter estimates the yaw angle more accurately, which leads to a proper estimation of the position of the wake even when its position relative to the follower increases. Also observe that the standard deviation of the relative position estimates are smaller when using lower noise levels, because of improved confidence in the attitude measurements. In this simulation, the relative distance between the center of the wake and the follower barely exceeds $1.5b$. Accurate sensors and confidence in the measurements are required to obtain acceptable estimations. This will be even more the case at larger distances since the farther apart the wake and the follower, the weaker the aerodynamic interferences become. Therefore, even if the relative motion between the wake and the follower helps in the detection, a too large relative distance will degrade the sensitivity. These behaviors will depend on the accuracy of the measurements processed by the estimator as well as the ability of the surrogate model to represent the simulated flight environment.

Fig. 2.8 shows the estimated position of the vortices during the estimation process assuming low noise levels (i.e., using *Filter 3*), superimposed to the actual vorticity field. Even if the instantaneous estimation is not accurate, it is always located in the vicinity of the actual wake vortices and it reproduces the trajectory of the wake of the leader on average. The estimator thus shows robustness in the estimation process and is therefore a suitable tool to maintain an optimal position in the wake of a leader.

2.3.2 Wake tracking

Let us now investigate the ability of the follower to maintain in real time a constant position relative to a leader's wake. Taking advantage of the estimation of the position of the wake over time, the follower actively targets the optimal location in order to benefit from maximum fuel burn reduction at all times. In the investigated scenario, the leader also performs a sinusoidal oscillating trajectory. The period of oscillation was selected in order to be larger than the time response of the autopilot (see the second column of Table 2.1). Indeed, in particular for the lateral behavior, the tracking of the leader's wake by a too slow autopilot may lead to a wake encounter, which is a situation that a conventional autopilot is not able to deal with. In this section, we use *Filter 3* described in Section 2.3.1, i.e., we consider an EnKF that uses the complete wake

measurement vector ($\mathbf{z}_{\text{wake}} = [\Gamma_{0,L}, b_{0,L}]$) and assume lower noise levels regarding the three attitude angle measurements, see Table 2.3. The estimate of the position of the leader's wake $\hat{\mathbf{q}}_{\text{wake}}^n$ is provided to a wake analysis module (see Fig. 2.3) that filters it and provides an optimal target $\mathbf{x}_F^{\text{opt}}$ to be followed by the autopilot of the follower in the *wake tracking* mode (switch 2 in Fig. 2.3). A Double Exponential Moving Average (DEMA) is used to filter the relative position estimates Δy and Δz in order to provide a smooth target to the autopilot of the follower.

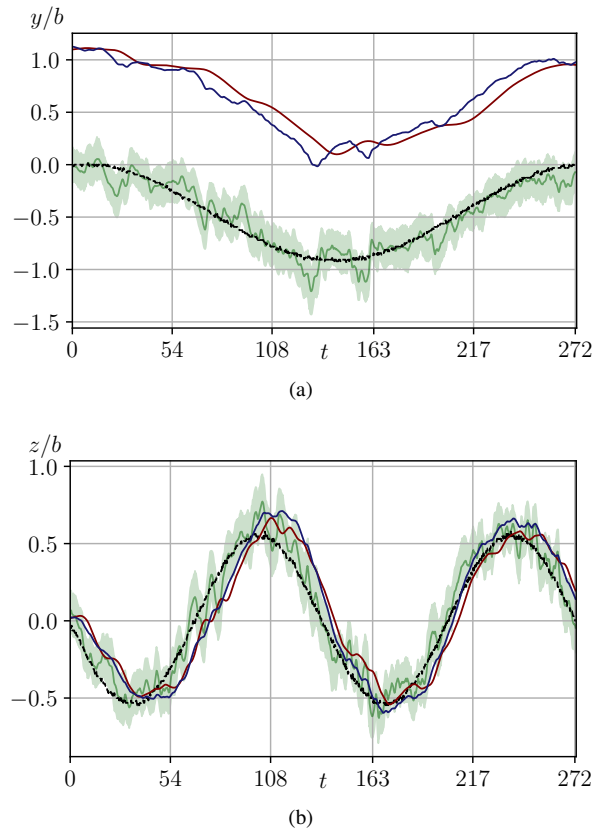


Figure 2.9: Performance of the EnKF in *wake tracking* mode. The green solid lines (—) (resp. black dashed (---)) are the estimates (resp. actual values) of the wake's position with their $1 - \sigma$ confidence interval. The blue solid line (—) is the target provided to the follower and the red solid line (—) is the actual trajectory of the follower.

The target has an offset $(\Delta y^{\text{ref}}, \Delta z^{\text{ref}}) = (-1.1b, 0.0b)$ from the wake estimated position which is close to the optimal position. Along the stream-wise direction, the

autopilot of the aircraft is instructed to maintain a constant speed U_∞ . The performance of the complete tracking strategy involving the EnKF and the filtered target provided to the follower is represented in Fig. 2.9.

The estimator shows similar performances as the one observed for the second filter in Section 2.3.1. The reference lateral and vertical positions are indeed accurately detected by the estimator, due to the use of a direct measurement of the wake intensity and low noise levels (as discussed in Section 2.3.1). When comparing the actual trajectory of the follower and the target provided to its autopilot, we observe that the follower accurately tracks this offset target. A time delay still remains between the trajectory of the follower and the position of the wake over time, which is explained by the delay induced by an autopilot that is not fast enough to track a target without delay. The target itself also presents a small delay as it results from a smoothing of the estimated position of the wake. This estimation is indeed delayed compared to the exact position of the wake because of the time response of the estimator.

Fig. 2.10 presents a visual interpretation of the tracking strategy, and shows instantaneous views of the flow in the cross section $x = x_F$. Even if the instantaneous estimated position of the wake is noisy and does not perfectly fit with the exact position of the wake, it is always located in the vicinity of the actual wake. Indeed, the estimation errors remain within 20% of the wing span. On average, the follower reproduces the same oscillating pattern as the one produced by the leader's wake.

By maintaining a nearly constant position relative to the wake, the follower always remains close to the leader's wake. It therefore benefits from a stronger influence, which leads to an improved estimation as already observed in Section 2.3.1. The use of the control commands as measurements becomes even more important in the detection, as it gives a direct information of the loads perceived by the follower while it is moving.

As already observed in Section 1.5.4, by staying in the close vicinity of the wake of the leader, the follower benefits from a decrease of the thrust required to follow a trajectory, because of an apparent drag reduction. We study the variation of the thrust command δ_{Th} applied during the wake tracking strategy compared to the reference thrust $\delta_{Th,0}$ of an isolated, steady aircraft. This wake tracking follower applies a time-varying thrust command $\delta_{Th}(t)$ as it adapts to the oscillating wake produced by the leader, that periodically oscillates at the frequency ω (given in Table 2.1). The thrust command therefore shows large oscillations: the increase in amplitude is due to the change in altitude of the follower that maintains a constant relative position with the leader. High thrust corresponds to climb phases and low thrust to the descending parts of the wake oscillations. To evaluate the real impact of the wake on the thrust required

by a wake tracking follower, we compute the average thrust command over time:

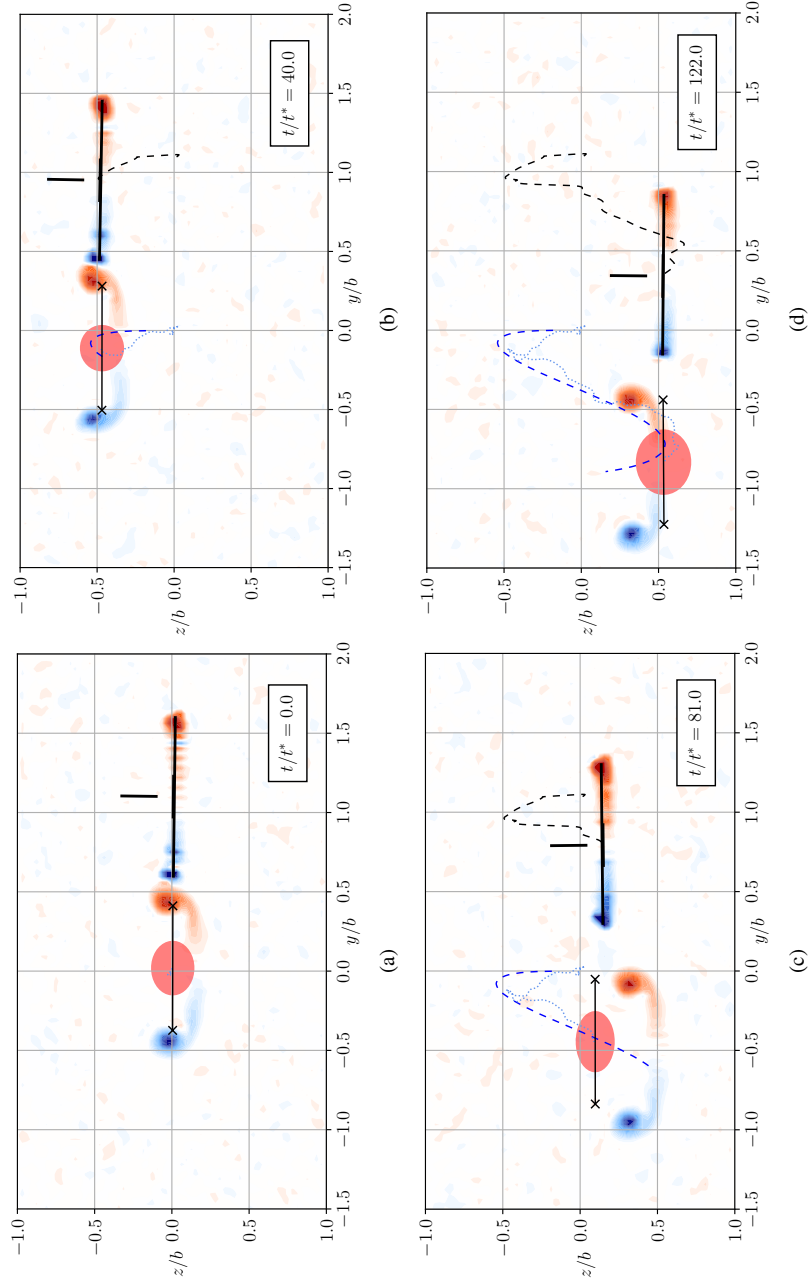
$$\langle \delta_{Th} \rangle = \frac{1}{T} \int_t^{t+T} \delta_{Th}(t) dt, \quad (2.20)$$

where $T = \frac{2\pi}{\omega}$ is the leader's wake oscillating period, see Table 2.1. This gives an average thrust reduction $(\langle \delta_{Th} \rangle - \delta_{Th,0}) / \delta_{Th,0} = -22\%$ compared to an aircraft flying outside any formation (i.e. $\delta_{Th,0} = 100\%$). This reduction is to be compared to the -24% thrust reduction performed by a similar aircraft maintaining the optimal position (i.e. $(y_F^{ref}, z_F^{ref}) = (1.1b, 0.0b)$) in the wake of a fixed leader $((y_L^{ref}, z_L^{ref}) = (0.0b, 0.0b))$, as described in Section 2.3.1, i.e. the *position hold* configuration. The lower thrust reduction observed during the *wake tracking* configuration is explained by the lag between the actual position of the wake and the position of the follower, as highlighted in Fig. 2.9. The follower is never positioned at the exact optimal position at the exact optimal time, as it tracks a moving position that it has to estimate first. However, we also compare the -22% thrust reduction of the wake tracking strategy to the -6% reduction performed by a similar aircraft maintaining a fixed position behind a moving leader, as described in Section 2.3.1. We conclude that even if we do not meet the maximal drag reduction possible, the wake tracking strategy enables the increase of the performance of formation flight compared to a passive positioning.

However, the energy savings predicted here above have to be considered prudently and with nuance. They are indeed larger than the instantaneous 18% savings measured by a F/A-18 airplane during experimental studies in close formation flight [9, 140] (military context), or the average 5% savings measured by a A350 aircraft in extended formation flight [2] (civil aviation).. The present setup will in fact lead to an over-prediction as it neglects any fuselage drag, uses idealized airfoil polars, and the aircraft are not flying at the maximum range operating point, i.e., $C_{d,parasitic} < C_{d,induced}$. Despite those differences, this setup proves that the active wake tracking strategy enables a significant improvement in drag reduction and hence increases the benefits in flying in formation.

2.4 Alternate wake tracking strategy

In this chapter, we have designed a tracking strategy based on an EnKF in order to predict *simultaneously* the dynamic states of the follower $\mathbf{q}_{dyn}$ and the wake state parameters $\mathbf{q}_{wake}$. While such an approach proved to be efficient, it does not exploit the full potential of dynamic measurements. For instance, it does not use measurements



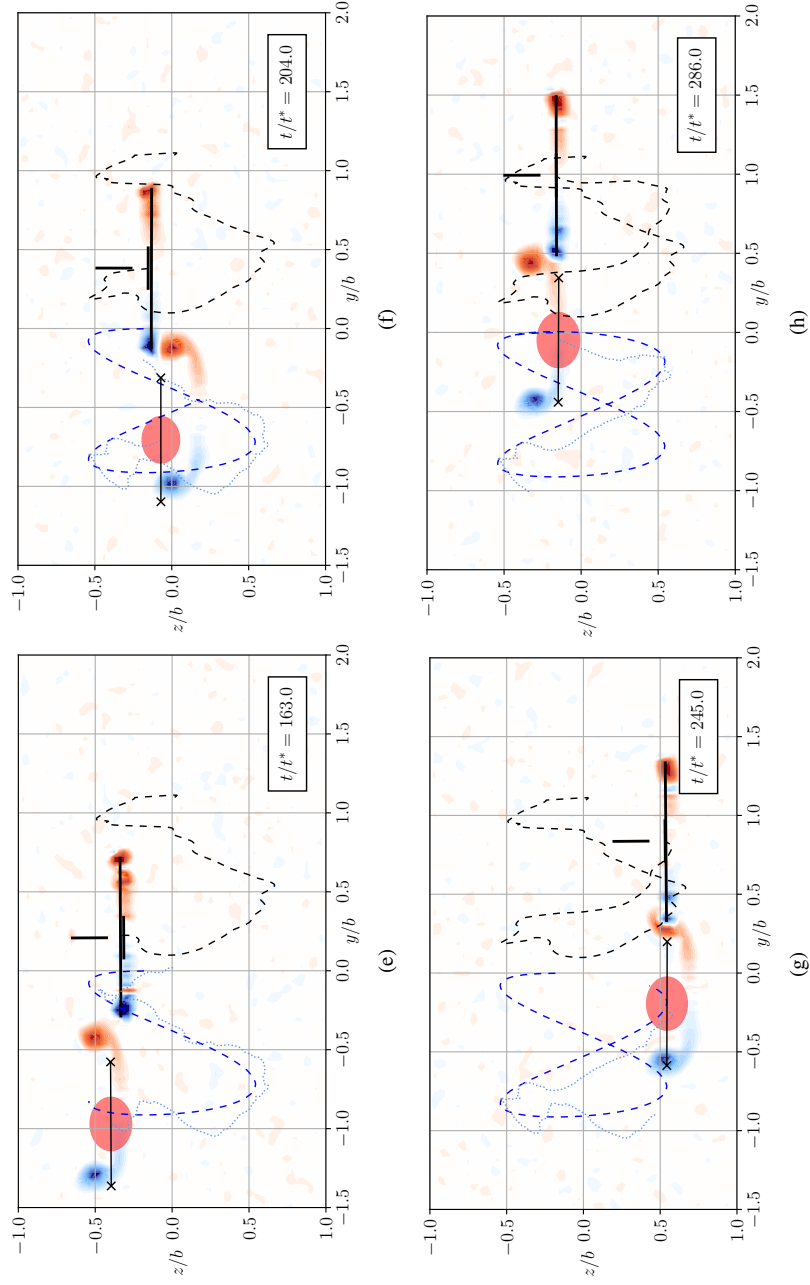


Figure 2.10: Vortex system estimated by the EnKF over time (*wake tracking mode*). The background shows the streamwise vorticity behind the follower. The dotted line (....) is the history of the estimated position with its $1 - \sigma$ confidence interval. The blue dashed line (--) is the actual trajectory of the leader's wake.

of the accelerations $\mathbf{a}_{K,F} = \dot{\mathbf{V}}_{K,F}$ and $\dot{\mathbf{\Omega}}_F = [\dot{p}, \dot{q}, \dot{r}]$ undergone by the follower, even if they are already available in modern aircraft. Also, the estimation of an aircraft dynamics $\mathbf{q}_{dyn}$ is already performed in modern aircraft through their Inertial Navigation System (INS) that uses IMU and GNSS to predict the translational and rotational position and velocity over time. The 6DOF dynamics of an aircraft can thus be assumed to be known with some uncertainty, rather than considered as states to be estimated. A similar approach can be imagined regarding the control commands of an aircraft that are estimated over time based on a well calibrated transfer function of their dynamics that links the command itself $\mathbf{u}_c = [\bar{\delta}_{Ail}, \bar{\delta}_{VT}, \bar{\delta}_{HT}, \bar{\delta}_{Th}]$ to the actual control surfaces deflection or thrust, i.e., $\mathbf{q}_{cont} = [\delta_{Ail}, \delta_{VT}, \delta_{HT}, \delta_{Th}]$. Alternate strategies are therefore possible to benefit even more from measurements offered by the aircraft dynamics.

Fig. 2.11 is a block scheme of an alternate version of the tracking strategy. The estimator is now split into three parts: the two first parts are based on linear Kalman Filters that are designed to predict the dynamics of the follower and the dynamics of the control commands, i.e., $\mathbf{q}_{dyn}$ and $\mathbf{q}_{cont}$, from a few measurements of the aircraft dynamics ($\mathbf{z}_{dyn}$, $\mathbf{z}_{acc}$, $\mathbf{u}_c$ and $\mathbf{z}_{cont}$ in Fig. 2.11). The last part is based on an EnKF that is designed to estimate the wake state parameters of a leader, i.e., $\mathbf{q}_{wake}$, from accelerations measurements. In the two first parts, we use linear state representations for the follower's 6DOF dynamics and for the control commands dynamics. The third part exploits the non-linear state model representation of the two-aircraft formation, previously referred as the surrogate model.

We refer to Appendix E for a detailed description of the equations behind the different blocks in Fig. 2.11, in particular for the two linear Kalman filters used. Here, we focus on the EnKF used to predict the wake state parameters $\mathbf{q}_{wake}$. It uses an estimate of the dynamics states and control states, $\hat{\mathbf{q}}_{dyn}$ and $\hat{\mathbf{q}}_{cont}$, as well as measurements of the accelerations $\mathbf{z}_{acc} = [\dot{U}_{K,F}, \dot{V}_{K,F}, \dot{W}_{K,F}, \dot{p}, \dot{q}, \dot{r}]$ undergone during the flight. In Fig. 2.11, the non-linear surrogate model is not used to propagate the dynamics of the follower (that are given as inputs). It is rather used to predicted the non-linear measurements $\mathbf{m}_{acc}$ that are compared to the true measurements $\mathbf{z}_{acc}$ in order to infer the impact of the leader on the follower, i.e., in order to deduce the wake state parameters $\mathbf{q}_{wake}$.

The use of acceleration measurements brings the EnKF methodology closer to other techniques described in the literature that exploits accelerations only [69, 28, 149]. In particular, it makes it possible to exploit the variations of vertical and roll accelerations (see Fig. 1.7) that are the quantities most affected by formation flight and the wake of a leader. We invite the curious reader to go through Appendix E, to

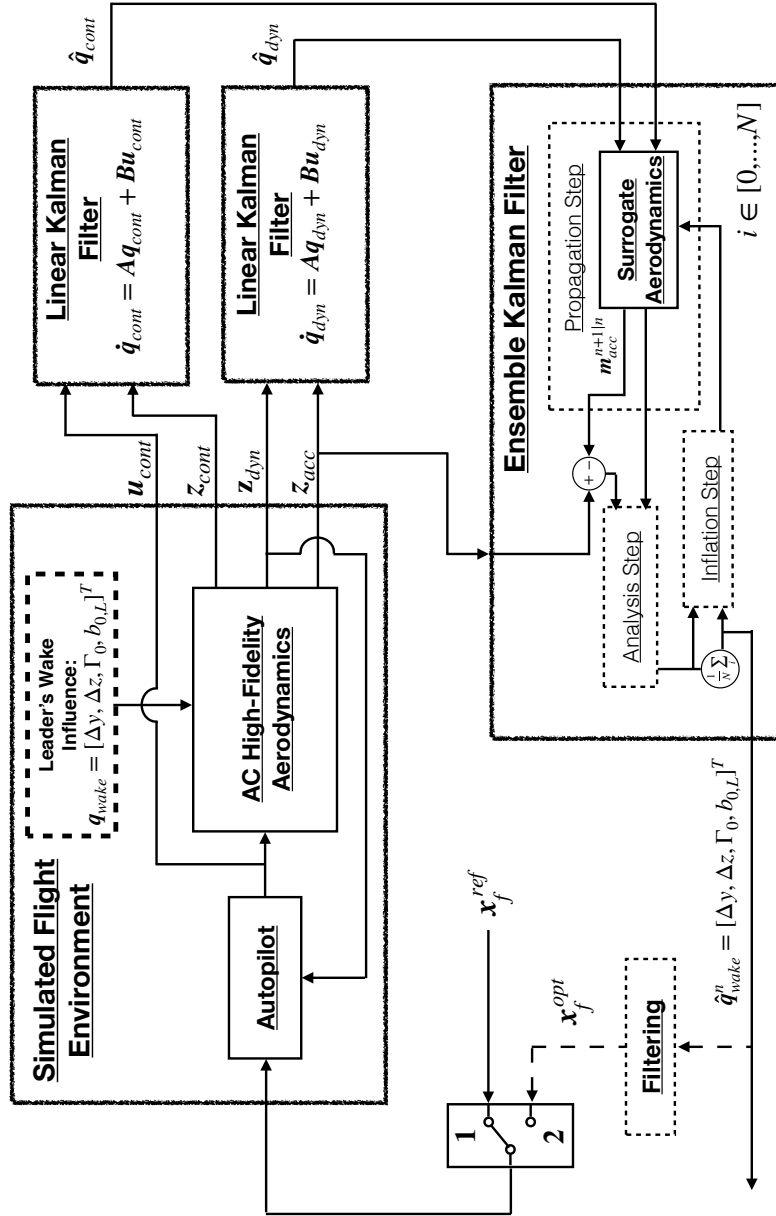


Figure 2.11: Block diagram of the alternate tracking strategy. The dynamics of the follower and of its control commands are estimated through linear Kalman Filters. This gives an estimation of the state of the follower that is exploited in the Ensemble Kalman filter in order to predict the state parameters of the leader's wake based on a surrogate model and accelerations measurements.

discover in details the alternate estimation strategy applied this time to the A350-like aircraft instead of the C172-like one in this chapter.

2.5 Conclusions

This work proposes a potential wake detection strategy based on aircraft flight dynamics measurements that a follower might adopt when flying in the vicinity of the wake of a leader in extended formation flight for energy savings. It extends the approach of Caprace et al. [21] based on an Ensemble Kalman Filter for the estimation of the position of the leader's wake relative to the position of the follower.

Large Eddy Simulations of the wake generated by two-aircraft formations in ambient turbulence produce a high-fidelity simulated flight environment. Aerodynamic loads are computed on each lifting surfaces of the aircraft. One can then integrate the 6DOF dynamics of each aircraft over time and control their trajectories through the use of a simple autopilot. The EnKF uses measurements of the follower's dynamics and estimates the relative position of the leader's wake in comparing them to a simplified yet accurate surrogate model of the two-aircraft formation dynamics.

Two flight configurations were investigated in this work. First, the follower is tasked with a *position hold*; this configuration is used to assess the performances of the estimator. The sole follower dynamics measurements were found to be sufficient to detect the relative vertical and lateral positions of the wake of the leader, as long as the follower is provided information regarding the mass and geometry of the leader.

The second configuration involved a *wake tracking* strategy. The estimator performed similar to that of the first configuration in terms of detection accuracy, while at the same time the follower was performing the desired tracking maneuvers in a turbulent environment. The current study has also proven the efficiency of a tracking strategy in terms of thrust reduction. To the authors' knowledge, this constitutes the first time a wake-tracking controller is shown to be effective in determining in real time the relative distance between the wake of a leader and the follower itself, in order to maintain a chosen relative position, without the need for dedicated hardware equipment.

For both configurations, the use of dynamics measurements augmented by an indirect measure of the leader's wake intensity proves sufficient to detect the position of a leader's wake when fed to the EnKF. Such a scenario was motivated by its relatively short-term feasibility: on current aircraft, an Inertial Measurements Unit, a Global Navigation Satellite System and an Automatic Dependent Surveillance-Broadcast receiver are readily available. Also, the need for an indirect measure of the leader's wake

intensity is explained by the mechanism behind the influence of the wake on the follower which is governed by the Bio-Savart law. Indeed, the latter implies an intrinsic correlation between the wake intensity and the relative position of the wake which leads to observability issues when using aircraft dynamics as unique measurements.

Therefore, the sole requirement is the design of an estimator surrogate model that represents accurately enough the aircraft dynamics and the influence of the wake of the leader. The effects of this accuracy on detection performance have been assessed. First, biases between the actual measurements (here provided by the simulated flight environment) and the simulated measurements provided by the surrogate model can lead to significant biases in the estimation of the position of the leader's wake. These errors seem to be amplified by the observability issues discussed above. On a related note, the lateral dynamics have been shown to govern the detection of the vertical relative position of the leader's wake, while the longitudinal dynamics, the lateral relative position. It is finally observed that the estimator struggles to detect the position of the wake, if the measurement noise levels are high. The estimator has shown improved performances when lower measurement noise levels were considered regarding the attitude angle measurements. In practice, this consists of assuming the use of accurate sensors aboard the aircraft.

Bio-inspired Wake Tracking for Aircraft Formation Flight Based on Reinforcement Learning

The content of this chapter is inspired by an article published in the proceedings of the AIAA Scitech Forum 2021 (remote conference), entitled "Bio-inspired Wake Tracking for Aircraft Formation Flight Based on Reinforcement Learning" [116], by I. Ransquin and P. Chatelain.

3.1 Introduction

Birds have been occupants of the sky long before mankind began to gradually appropriate it too. Thanks to the work of evolution, different species have been able to progressively develop and learn the most inventive techniques to optimize different flight objectives. For instance, birds of prey use soaring to maintain an elevated vantage point in their search for food. The formation flight technique performed by migratory birds in order to reduce their energy consumption is another example. In the latter, following birds in the formation indeed benefits from substantial apparent drag reduction when flying in the upwash region of the leaders' wake [94, 146].

To maintain such impressive V-shaped formations, birds have the ability to sense the upwash region induced by the wake produced by the leading birds. Then they minimize their energy expenditure by flying in the aerodynamically optimal position, as shown by Portugal et al. in [112]. In addition to sensing the flow, birds have learned over time how to exploit the sensed information in order to perform optimal actions.

Drawing inspiration from those examples characterized by animals that learn through experience, the present work focuses on the tracking of the optimal position for a follower aircraft in the wake of a leader through the use of a deep learning algorithm. Indeed, all the methods referenced in Chapter 2 or Appendix E [72, 21, 117] or developed in those chapters require the use of wake vortex models to deduce the optimal position in a wake. Learning how to fly in the wake of one or even several leaders offers an adaptive solution able to react to unpredicted behaviors or complex environments without the need for predetermined model of the vortices. The use of deep learning algorithms for flow problems is not new, and covers wide applications from flow classification [33, 3] and forecasting of turbulent environment [145, 105], to the load alleviation on wind turbines [35]. Among those algorithms, the Reinforcement Learning (RL) approach is used in [143] where fishes learn to school, leading to collective energy savings. The use of RL in order to teach gliders how to exploit thermal soaring, mimicking the behavior of birds of prey, has also been considered in the literature [153, 119, 102, 104, 127]. Therefore, the RL framework gives promising results for energy saving applications [65] in complex and fluctuating environment. In this chapter, we approach the wake tracking strategy as a problem of learning, which as never been tested before.

The learning process uses a model based on the PLL theory such as the one developed in Chapter 1. In this work, the follower uses measurements of its 6DOF dynamics to determine the velocity required to converge towards an optimal position in the wake of a leader. Two different strategies are envisioned during the learning process. The first one exploits the difference between accelerations undergone by a follower in the wake of a leader and accelerations perceived by a fictitious aircraft that would fly in the same dynamic state without any aerodynamic influence of the leader's wake. Such an approach helps understanding the intuition behind the perception of the wake flow from dynamics measurements. However, it requires to exploit a dynamic model of the follower in order to compute those differences. The second learning strategy removes the need for that model and exploits measurements of the entire dynamics, i.e., translational and rotational acceleration, velocity and position to learn to converge towards the optimal position.

We show that for both strategies, the follower is able to find the sweet spot in the wake of the leader. We then demonstrate the performance of the tracking strategy on a larger formation that follows the oscillating movement of a leader. We finally discuss the benefits and drawbacks of the proposed approach.

This chapter is organized as follow. In Section 3.2, we present the aerodynamic model characterizing the process and we describe the training process associated to

this framework. Section 3.3 describes the convergence of the process. The results are presented in Section 3.4 first by validating the learned tracking strategy in the case of a single aircraft flying in the wake of leader, second by assessing the performance of the strategy on a larger formation. We conclude this chapter with conclusions and perspectives in Section 3.5.

3.2 Methodology

The proposed tracking strategy for the follower's control is structured as a hierarchy of tasks, from high-level to low-level ones. It takes the form of a two-block architecture, as shown in Fig. 3.1. An autopilot computes the control surfaces (i.e. the aileron δ_{Ail} , the rudder δ_{VT} and the elevator δ_{HT} deflections) and the thrust command δ_{Th} of the follower in order to make it fly at a reference velocity $V_{K,F}^{ref}$. This reference is provided by a Neural Network (NN) that is trained by RL.

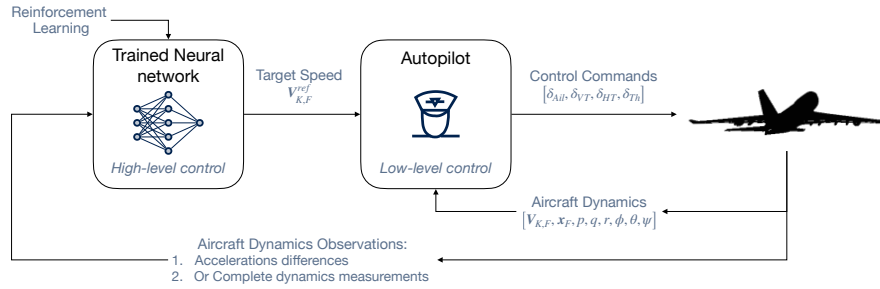


Figure 3.1: RL-based aircraft controller: the standard aircraft autopilot is fed with a flight path velocity target $V_{K,F}^{ref}$ that guides a follower towards the optimal position in the wake of a leader. This target is provided by a high-level control made of a neural network trained by RL.

RL is a process by which the agent (in this case, a follower) learns the best way to behave to achieve a goal by earning rewards through trial-and-error interactions with the environment, i.e., an atmosphere perturbed by a leader wake. It is a bio-inspired way of tackling control problems as it is not about defining control laws but learning from interactions in a goal-directed manner [136]. Offline learning is here used to design the formation flight controller in a model-free manner, thus without considering knowledge about the environment around the follower. This is in opposition with more classical approaches, such as the ones described in Chapter 2 that does initially make hypothesis on the environment dynamics. The trial-and-error procedure inherent to offline learning requires to perform a large set of simulations of the interactions of the

agent with its environment in order to get experience. While LES of a two-aircraft formation, as presented in [21, 117], accurately model those interactions, they are not computationally affordable when given the number of samples required to converge the learning process. Therefore, we leverage some of the tools developed in [117] and described in Chapter 1 for the learning procedure, namely a simplified and affordable 6DOF aerodynamic model of an aircraft. We briefly describe this model, then we present the high-level control based on RL. We finally outline the learning process.

3.2.1 Aircraft aerodynamics and dynamics

Identically to the simplified model described in Section 1.3 and exploited in Chapter 2, the PLL theory constitutes the core of the follower's aerodynamic model, while the wake of the leader is modeled as two counter-rotating vortex tubes of circulation $\Gamma_{0,L}$ and spacing $b_{0,L}$ inducing the upwash $V_W(x, t)$ on its environment. As depicted in Fig. 3.2 the follower aircraft is located at the position $\mathbf{x}_F = [x_F, y_F, z_F]^T$ in the inertial frame $(ox^i y^i z^i)$ and comprises three straight lifting lines representing the wing, the VTP and the HTP.

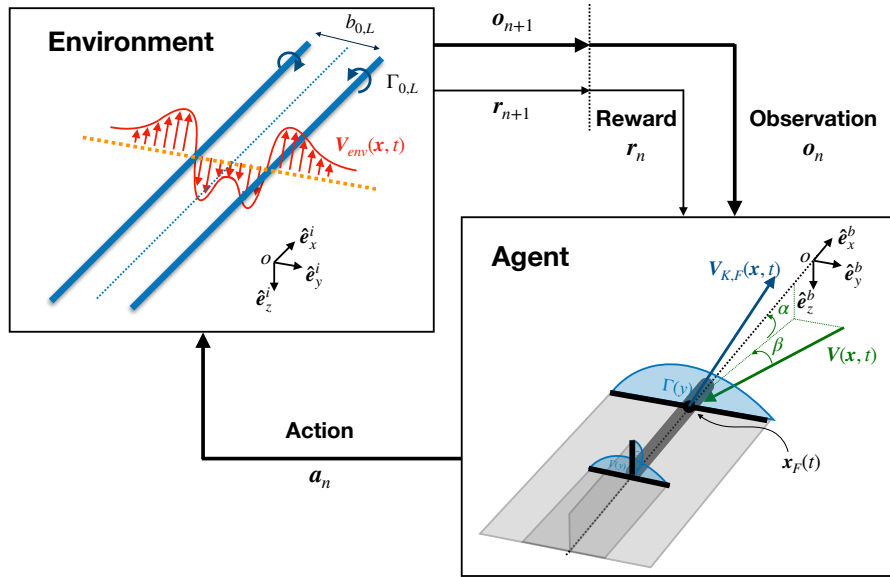


Figure 3.2: Reinforcement Learning (RL) framework made of the agent, the perturbed environment with its effect on the agent, and the RL iterations.

Using the PLL theory, one computes the circulation distribution along each one of the lifting lines and deduces the resulting forces $\mathbf{F}_F = [F_x, F_y, F_z]_i^T$ and moments $\mathbf{M}_F = [M_x, M_y, M_z]_b^T$ on the aircraft. The integration of the translational accelerations $\mathbf{a}_{K,F} = \dot{\mathbf{V}}_{K,F}$ and angular accelerations $\dot{\mathbf{\Omega}}_F = [\dot{p}, \dot{q}, \dot{r}]_b^T$ allows to compute the velocity $\mathbf{V}_{K,F}$, angular velocities $[p, q, r]$, position $\mathbf{x}_F$ and attitude $[\phi, \theta, \psi]$ over time, as detailed in Section 1.4.

To this complete 6DOF dynamics, we associate the autopilot described in Section 1.4.2 and designed to control the commands $\mathbf{u}_c$ of the aircraft (i.e., $\delta_{Ail}, \delta_{VT}, \delta_{HT}$ and δ_{Th}). The autopilot is here tuned to make the aircraft fly at a desired velocity $\mathbf{V}_{K,F}^{ref} = [U_{K,F}^{ref}, V_{K,F}^{ref}, W_{K,F}^{ref}]_i^T$ in its environment. In this study, the aircraft always target fixed longitudinal velocities, i.e., $U_K^{ref} = U_\infty$ remains constant, while the optimal targets for the vertical and lateral velocities are computed along the learning process. It is worth noticing that the streamwise velocity constraint $U_K^{ref} = U_\infty$ can make the required thrust vary abruptly. First, this is not appropriate in terms of passenger ride quality. More importantly, aircraft engines are not designed to react instantaneously to environment changes and rather adapt slowly. However, such an hypothesis simplifies the learning problem. Indeed, the wake surfing benefit is highly sensitive to the lateral and vertical relative position between the leader's wake and the follower which justifies to investigate a learning strategy considering the $\hat{e}_y^i - \hat{e}_z^i$ plane only. In this work, we use C172-like aircraft whose parameters are listed in Table 1.1

3.2.2 High-level control based on Reinforcement Learning

The high-level control of the wake tracking strategy consists in a deep NN. Its role is to learn the actions $\mathbf{a}_n$ that most increase the energy savings of the follower. A new action is taken every learning step n based on observations of the environment $\mathbf{o}_n$. The iterative procedure of the RL algorithm is presented in Fig. 3.2.

RL Algorithm

At each learning step, the agent gets the observation vector $\mathbf{o}_n$ from the environment based on the action $\mathbf{a}_n$ it chose to take. Through the interactions with this environment, the agent is given some reward $r_{n+1} = r(\mathbf{o}_n, \mathbf{a}_n)$ and ends up in a new state characterized by a new observation $\mathbf{o}_{n+1}$. A learning episode of an RL methodology is a succession of learning steps. In the formation flight context, an episode comes to an end when the observations have exceeded predefined limits or when a chosen number of learning steps has been reached, i.e., the size N_{ep} of the episode. The ultimate goal

of RL algorithms is to find the optimal mapping, called a policy $\pi(\mathbf{a}_n | \mathbf{o}_n)$, between the observations and the action given an objective function $J(\pi)$ built on the cumulated rewards obtained during an episode. A back-propagation algorithm is performed at the end of the episodes and is in charge of the updates of the parameters of the NN, i.e., the weights and biases.

In this work, we use an off-policy model-free reinforcement learning algorithm called Soft Actor Critic (SAC) [66] in order to train the neural network. SAC is part of the maximum entropy reinforcement learning methods. It therefore uses an entropy augmented objective function

$$J(\pi) = \mathbb{E}_{\pi} \left[\sum_{n=0}^{N_{ep}} \gamma^n r_n(\mathbf{o}_n, \mathbf{a}_n) - \zeta \log(\pi(\mathbf{a}_n | \mathbf{o}_n)) \right], \quad (3.1)$$

with γ the discount factor and ζ the positive entropy parameter. The parameter ζ explicitly controls the trade-off between exploitation of proven actions through return maximization, i.e., term $\sum_{n=0}^{\infty} \gamma^n r_n(\mathbf{o}_n, \mathbf{a}_n)$, and exploration through the entropy maximization, i.e., term $\log(\pi(\mathbf{a}_n | \mathbf{o}_n))$. The SAC algorithm provides sample-efficiency learning and a low sensitivity to hyper-parameters while retaining the benefits of entropy maximisation and stability. This makes it a state-of-the-art model-free deep RL method [66]. The NN is trained thanks to the open-source SAC implementation from STABLE-BASELINES3 [114]. We use PYTORCH [108] to build the fully connected NN used in the high-level controller. It is made of 3 hidden layers with 64 neurons with relu as activation function. The size of the input layer depends on the number of observations used (this is discussed in Section 3.3) while the output one comprises 2 neurons, one for each component of the action.

Observations, actions, reward

From the environment perspective, the agent is defined by the actions it performs. The follower is able to change its position within the perturbed environment, thanks to the autopilot and the modification of the target in the $\hat{e}_y^i - \hat{e}_z^i$ plane. The actions performed by the follower are therefore a modification of the target flight path velocities $V_{K,F}^{ref}$ and $W_{K,F}^{ref}$ for the lateral and vertical behaviors respectively. This is the action space

$$\mathcal{A} : \mathbf{a}_n = \begin{Bmatrix} V_{K,F}^{ref} \\ W_{K,F}^{ref} \end{Bmatrix}. \quad (3.2)$$

Another fundamental aspect of the RL problem for wake tracking consists in the observations the follower can exploit in its environment. This is the observation space

$\mathcal{O}$. There is no consensus on the sensorimotor cues used by birds to perceived their environment when exploiting thermal updrafts [119] or when surfing wakes [112]. However, it is likely that visual information is of prime importance because of the close vicinity between the birds that fly in close formations. Just like in Chapter 2, we here choose on purpose not to consider any direct information regarding the position of the wake of a leader and rather focus on dynamic observations only. We define two types of observation spaces. The first type is denoted $\mathcal{O}_1$ and is characterized by the use of acceleration differences. As we want to optimize the apparent drag reduction of the follower compared to a similar aircraft that flies in the similar unperturbed conditions, we compute the accelerations $\mathbf{a}_{K,F} = \dot{\mathbf{V}}_F$ and angular accelerations $\dot{\mathbf{\Omega}}_F = [\dot{p}, \dot{q}, \dot{r}]_b^T$ experienced by the follower when evolving in its environment. Then, we compare them to the accelerations $\mathbf{a}_{K,F}^{isol}$ and $\dot{\mathbf{\Omega}}_F^{isol}$ of an identical isolated aircraft, flying at the same speed, and subjected to the same control commands. However, it is not subjected to the influence of a perturbed environment, i.e. the follower is flying in a clean atmosphere. This allows to compute the acceleration differences $\Delta \mathbf{a}_{K,F} = \mathbf{a}_{K,F} - \mathbf{a}_{K,F}^{isol}$ and $\Delta \dot{\mathbf{\Omega}}_F = \dot{\mathbf{\Omega}}_F - \dot{\mathbf{\Omega}}_F^{isol}$ which are the potential observations provided by the learning follower in its environment, as depicted in Fig. 3.3. The use of acceleration differences initially allows to limit the number of observations to be provided to the learning algorithm. In addition, they allow to develop some intuition for formation flight training, as we show in Section 3.3.1. The main drawback of those differences is that they require a dynamic model of the aircraft in order to evaluate the accelerations $\mathbf{a}_{K,F}^{isol}$ and $\dot{\mathbf{\Omega}}_F^{isol}$ it would undergo if it were in the same configuration without the leader's influence. In addition to the acceleration differences, the actual velocities $[V_{K,F}, W_{K,F}]$ of the aircraft in the $\hat{e}_y^i - \hat{e}_z^i$ plane are also used as observations in $\mathcal{O}_1$. They provide information regarding the direction actually followed.

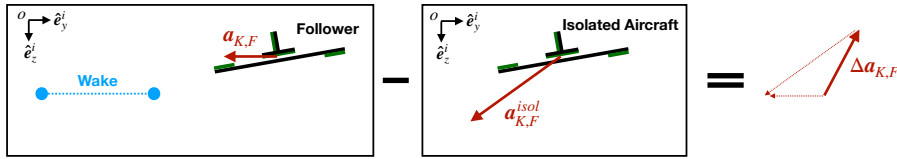


Figure 3.3: Computation of the difference of the accelerations perceived by the follower when flying in the wake of a leader.

The second observation space $\mathcal{O}_2$ uses the entire 6DOF dynamics of the follower as a measurement of the environment. This leads to a strategy that does not require the use of a background dynamic model. By learning to directly compare the dynamic states of the aircraft to the actual accelerations experienced by

the aircraft, the NN is trained to detect the presence of a wake and to lead the follower in its direction. In other words, the dynamic model used in background in the first strategy aims to be learned by the NN of the second one. This offers more flexibility for the NN due to the higher number of states. However, it *a priori* requires an increased learning effort. All the states of the follower are used, except for its position $\mathbf{x}_F = [x_F, y_F, z_F]$. This gives the observations spaces:

$$\mathcal{O}_1 : \mathbf{o}_n = \begin{cases} V_{K,F}, W_{K,F} \\ \Delta \dot{V}_{K,F} \\ \Delta \dot{p}, \Delta \dot{q}, \Delta \dot{r} \end{cases}, \quad (3.3) \quad \mathcal{O}_2 : \mathbf{o}_n = \begin{cases} V_{K,F}, W_{K,F} \\ p, q, r \\ \phi, \theta, \psi \\ \dot{V}_{K,F}, \dot{W}_{K,F} \\ \dot{p}, \dot{q}, \dot{r} \\ \delta_{Ail}, \delta_{VT}, \delta_{HT}, \delta_{Th} \end{cases}. \quad (3.4)$$

We do not use the longitudinal velocity $U_{K,F}$ as an observation of the flow environment, as the autopilot is designed to maintain a constant speed U_∞ . One also notes that we do not use the longitudinal acceleration $\Delta \dot{U}_{K,F}$, as it is directly related to the drag which is difficult to measure directly in practice. The difference in vertical acceleration $\Delta \dot{W}_{K,F}$ is not used as a potential observation in $\mathcal{O}_1$ either. Indeed, we use it as the reward r_n provided by the environment to the agent because the position of the maximum induced lift (and therefore minimum vertical acceleration $\Delta \dot{W}_{K,F}$) has been proven to be close to the optimal flying position in the wake of a leader [28], see Fig. 1.7a. Also, in order to fasten the convergence towards this optimal position, we get inspired by the work of Reddy [119] and we use a reward that linearly combines the difference in vertical acceleration $\Delta \dot{W}_{K,F}$ and the variation of this difference achieved over the subsequent time step. This is used for better learning performance. We have

$$\mathcal{R} : r_n = -\Delta \dot{W}_{K,F} + \tau \frac{d\Delta \dot{W}_{K,F}}{dt}, \quad (3.5)$$

where τ is a fine-tuned control parameter.

3.3 Learning process and convergence

The learning process implies going through two steps. The first one consists in training the NN so that it learns the optimal policy $\pi^*(\mathbf{a}_n | \mathbf{o}_n)$. For this first step, numerous learning episodes are performed using the aerodynamic model from Section 3.2.1. At the beginning of each episode, the wake of the leader is reset at the same initial position in the $\hat{\mathbf{e}}_y^i - \hat{\mathbf{e}}_z^i$ plane, i.e., $[y_L(0), z_L(0)] = [0, 0]$. It starts to follow an oscil-

latory trajectory similar to the one described in Eq. (1.57) from Section 1.5.4. The amplitudes and oscillating periods are chosen randomly for each learning episode, i.e., $A_y \sim \mathcal{U}(-0.5b, 0.5b)$, $A_z \sim \mathcal{U}(-0.75b, 0.75b)$, $T_y \sim \mathcal{U}(2T_{ph}, 4T_{ph})$ and $T_z \sim \mathcal{U}(1T_{ph}, 4T_{ph})$. $\mathcal{U}(x_1, x_2)$ denotes an uniform distribution between x_1 and x_2 and T_{ph} is the period of the phugoid mode of the aircraft. The vortex spacing $b_{0,L}^{ref} = \frac{\pi}{4}b$ is assumed to be constant along all the learning episodes. Regarding the wake intensity, we consider two different learning processes. In the first one, we assume the wake vortex intensity is constant $\Gamma_{0,L}^{ref} = \frac{mg}{\rho U_\infty b_{0,L}^{ref}}$. In the second one, we vary the leader wake intensity between different learning episodes and observe the impact of that variation on the performance of the NN controller. We have $\Gamma_{0,L} \sim \mathcal{U}(0.5\Gamma_{0,L}^{ref}, 1.5\Gamma_{0,L}^{ref})$. In other words, this second process evaluates the ability of the NN to detect the wake intensity in addition to its position.

Regarding the follower, it starts from a different randomly-chosen position at the start of every episode and performs the actions computed by the NN controller. The performance of the follower NN controller will slowly increase as it goes along the successive episodes. An episode ends when an observation $\mathbf{o}_n$ exceeds given boundaries, or when it has reached N_{ep} learning steps, each one characterized by a time interval $\Delta t_{update} = 1.5s$. During those intervals, the aerodynamic efforts are computed and the dynamics of the follower are integrated over time.

The second step consists in testing the trained policy $\pi(\mathbf{a}_n | \mathbf{o}_n)$ that has been learned in the form of the NN controller. The latter computes the velocity targets $\begin{bmatrix} V_{K,F}^{targ} \\ W_{K,F}^{targ} \end{bmatrix}$ and applies the appropriate thrust to maintain a cruise velocity $U_{K,F} = U_\infty$.

3.3.1 Training of neural network controllers based on $\mathcal{O}_1$

In this section, we study NN controllers trained using a part or all the observations from the space $\mathcal{O}_1$. We first focus on the acceleration differences $\Delta \mathbf{a}_{K,F}$ and $\Delta \dot{\mathbf{Q}}_F$ in order to investigate their relevance in the context of a formation flight wake tracking strategy. We then observe the convergence of the learning processes depending on the observations used to train the NN controllers.

Acceleration differences

In Fig. 3.4, we display the acceleration differences $\Delta \mathbf{a}_{K,F} = \mathbf{a}_{K,F} - \mathbf{a}_{K,F}^{isol}$ and $\Delta \dot{\mathbf{Q}}_F = \dot{\mathbf{Q}}_F - \dot{\mathbf{Q}}_F^{isol}$ for different positions of a follower to the right of the leader wake, located in $[y_L, z_L] = [0, 0]$ in the $\hat{\mathbf{e}}_y^i - \hat{\mathbf{e}}_z^i$ plane.

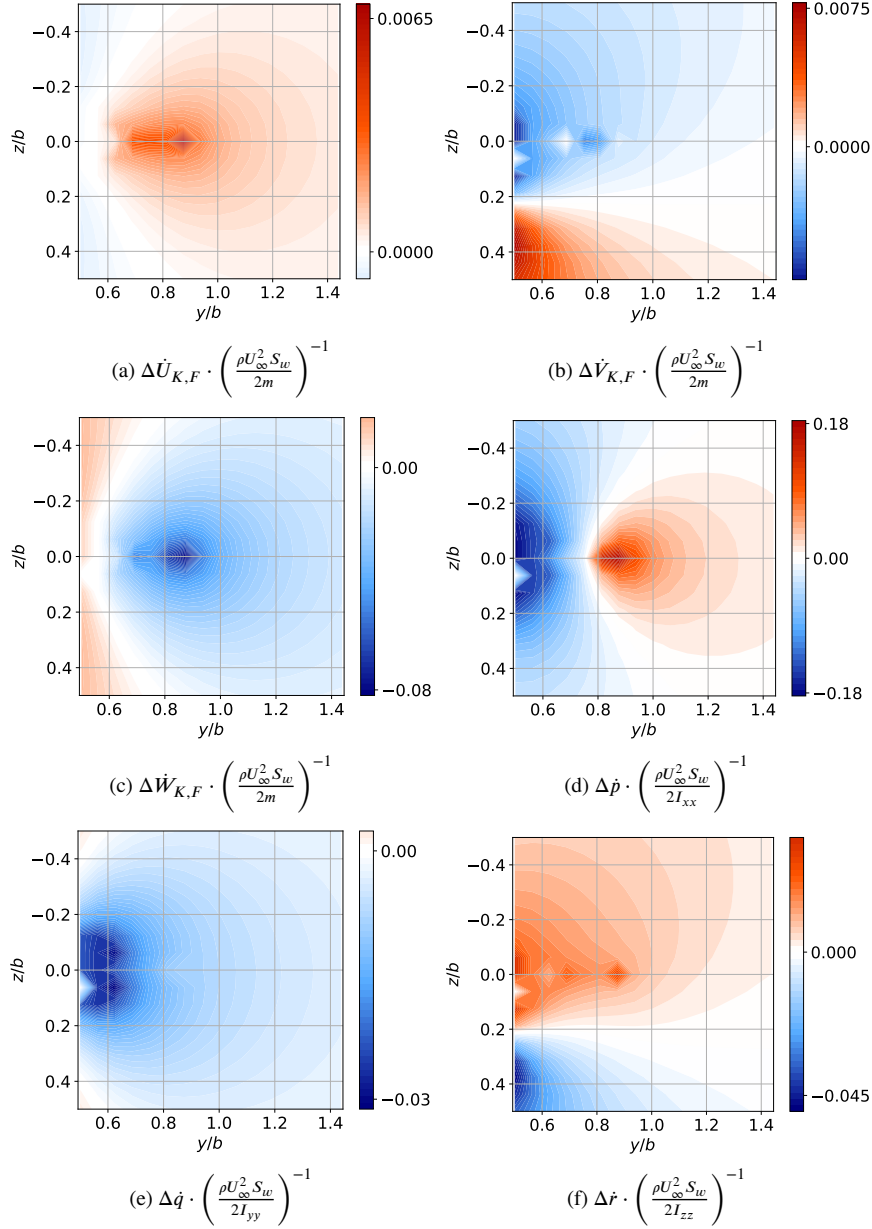


Figure 3.4: Contour plots of the dimensionless acceleration differences of the follower in the wake of the leader, with ρ the air density, S_w the wing surface, m the aircraft mass and $[I_{xx}, I_{yy}, I_{zz}]$ the moments of inertia.

The six graphs are obtained by trimming the control commands of the follower for each one of the positions. Because of the trimming, the follower maintains a steady configuration as the aerodynamic loads induced by the control commands balance the effect of the leader wake. This results in net zero accelerations for the 6DOF of the follower. In addition, a fictitious aircraft is put in the exact same configuration as the follower, i.e., same velocities $\mathbf{V}_{K,F}$ and $[p, q, r]$, same attitude $[\phi, \theta, \psi]$ and same control input $\mathbf{u}_c$. However, the external influence of the leader is removed for this fictitious aircraft. The latter experiences non-zero fictive accelerations in the aerodynamic model. This explains non-zeros acceleration differences in Fig. 3.4. As an example, let us study the difference $\Delta \dot{U}_{K,F}$ for a position close to the optimal one in $[y_F, z_F] = [1.1b, 0]$ in Fig. 3.4a. This acceleration difference is positive. When trimmed at that position, the follower benefits from an apparent drag reduction because positioned in the vicinity of the wake. The thrust required to maintain that position is therefore lower than the one required to maintain a position in an unperturbed atmosphere. An aircraft using that thrust would therefore be subjected to a negative acceleration along that axis which explains the positive difference. The asymmetry of $\Delta \dot{V}_{K,F}$ and $\Delta \dot{W}_{K,F}$ with respect to the horizontal axis $\hat{e}_y^i$ is explained by the asymmetry of the modeled aircraft whose VTP is positioned above the HTP. Let us note that the acceleration differences presented in Fig. 3.4 extend the results displayed in Fig. 1.7 regarding the aerodynamic influence of formation flight on the follower. Those results indicate that relevant information can be extracted from accelerations and aircraft dynamics in order to track the optimal position in a wake. Indeed, a combination of all the differences enables to map the environmen. Associated to the actual speed of the follower, the control strategy learned from the observation space $\mathcal{O}_1$ will be able to guide the latter towards an optimal position provided that an aerodynamic model of the aircraft is available.

It is interesting to observe that the acceleration difference fields show areas where those differences are negligible. This means that the accelerations perceived by the actual follower and the fictitious aircraft are equal. Those areas indicate that the impact of the wake vortices for an aircraft in this area is weak. This reflects a certain lack of observability, as already observed by Hemati et al. [72] and DeVries et al. [47]. However, those zero-value areas are not located in the same place for all the accelerations meaning that a follower is not completely blind with respect to the leader wake when crossing one region or another. We take a deeper look into observability in Chapter 4.

Convergence of the learning process

As mentioned in Section 3.2.2 the difference in vertical acceleration $\Delta \dot{W}_{K,F}$ is chosen as the reward r_n as the position of its minimum value in the wake is close to the optimal

one for energy saving. It is therefore not available in the observation space $\mathcal{O}_1$. We will not use the longitudinal acceleration $\Delta\dot{U}_{K,F}$ either. On the contrary, the rolling moment induced by a surrounding wake is known to be significant, as observed in Fig. 3.4. The acceleration difference in roll $\Delta\dot{p}$ is therefore necessarily used as an observation of the environment for the different NN controllers studied. In Fig. 3.5, we investigate the impact of the choice of the observations used on the convergence of the learning process. The figure presents the evolution of the mean reward obtained at different stages of the learning process for different observation combinations. In practice, the NN controllers are evaluated every 2500 learning steps by computing the cumulated rewards gathered along 20 different episodes. Those cumulated rewards are averaged to obtain a measure of the performance of the controllers over the learning processes.

We first investigate the impact of the use of one additional measurement in addition to the roll acceleration difference $\Delta\dot{p}$. In the first graph of Fig. 3.5, we observe that the two curves characterized by the use of the combinations $\Delta\dot{p} - \Delta\dot{V}_{K,F}$ and $\Delta\dot{p} - \Delta\dot{q}$ seem not to increase along the learning process. On the contrary, the learning curve that results from the use of the yaw acceleration difference $\Delta\dot{r}$ shows an important gradient at the beginning of the learning process. Therefore, the two first combinations are not appropriate for the learning of the tracking of a leader's wake. The good performance of the $\Delta\dot{p} - \Delta\dot{r}$ combination can be explained by the asymmetry of $\Delta\dot{r}$ with respect to the horizontal axis $\hat{e}_y^i$. This makes it possible to distinguish between an upper or a lower position of the leader with respect to the follower (see Fig. 3.4). Associated with $\Delta\dot{p}$, it allows a mapping of the environment behind the wake which leads to good tracking performances. This symmetry is not observed on the pitch acceleration difference $\Delta\dot{q}$ which explains its very low performance when used alone with $\Delta\dot{p}$. Finally, we might have expected good performances for the combination $\Delta\dot{p} - \Delta\dot{V}_{K,F}$ as $\Delta\dot{V}_{K,F}$ also presents a symmetry with respect to the horizontal axis $\hat{e}_y^i$. However, the variation of $\Delta\dot{V}_{K,F}$ with the relative position between the leader's wake and the follower might not be important enough to be detected during the learning phase. This is even more true as the wake intensity $\Gamma_{0,L}$ varies between two successive episodes. The variation of $\Delta\dot{V}_{K,F}$ is not appropriate compared to $\Delta\dot{r}$. In other terms, the combination $\Delta\dot{p} - \Delta\dot{r}$ seems to have a better observability with respect to the wake parameters, as it is able to lead the follower towards a better position, even when the wake intensity varies.

Now, we study the impact of the use of more than two acceleration differences during the learning process. As the pairwise $\Delta\dot{p} - \Delta\dot{V}_{K,F}$ and $\Delta\dot{p} - \Delta\dot{q}$ combinations do not lead to good performances, the combination $\Delta\dot{p} - \Delta\dot{q} - \Delta\dot{V}_{K,F}$ do not either. On the contrary, the combinations that use $\Delta\dot{p}$, $\Delta\dot{r}$ and other acceleration differences show

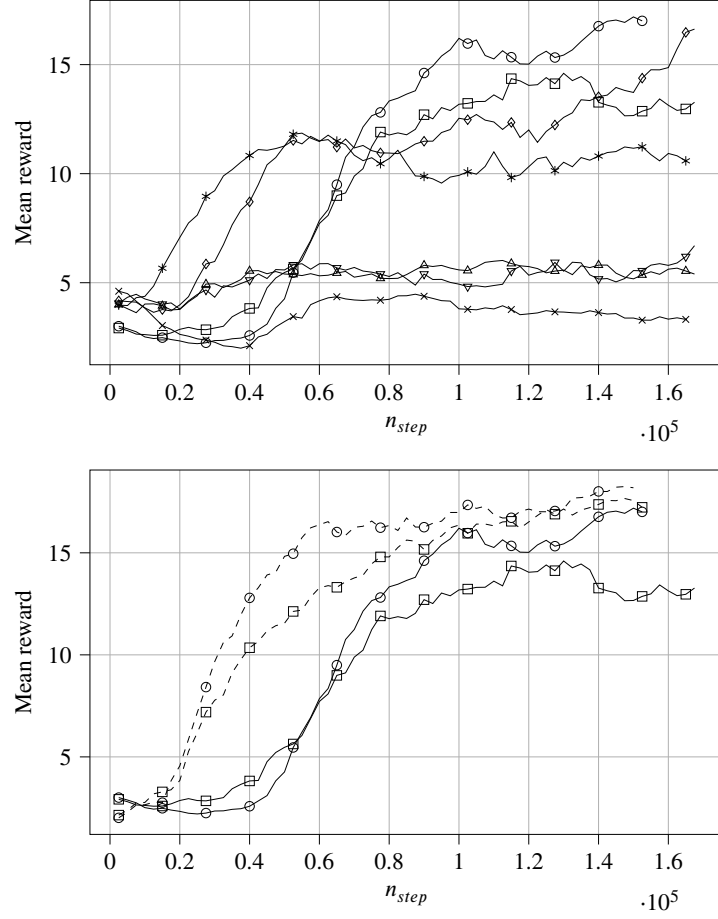


Figure 3.5: Training curves of the $\mathcal{O}_1$ -based RL algorithm for different combinations of observations. In addition to observation of the velocities $[V_{K,F}, W_{K,F}]$, the NN uses: $[\Delta\dot{p}, \Delta\dot{q}]$ (∇); $[\Delta\dot{p}, \Delta\dot{r}]$ ($*$); $[\Delta\dot{p}, \Delta\dot{V}_{K,F}]$ ($\times$); $[\Delta\dot{p}, \Delta\dot{q}, \Delta\dot{r}]$ ($\boxminus$); $[\Delta\dot{p}, \Delta\dot{q}, \Delta\dot{V}_{K,F}]$ ($\triangleleft$); $[\Delta\dot{p}, \Delta\dot{r}, \Delta\dot{V}_{K,F}]$ ($\diamond$); $[\Delta\dot{p}, \Delta\dot{q}, \Delta\dot{r}, \Delta\dot{V}_{K,F}]$ ($\ominus$); Solid lines (—) characterize learning process that involves a variation of $\Gamma_{0,L}$ during the learning process while dashed lines (--) do not.

the best learning performances. They are even better than the pair $\Delta\dot{p} - \Delta\dot{r}$. However, they require more learning steps to converge towards a *plateau*, as they need to deal with more information. We observe that the more observations used, the higher is the mean reward *plateau*. Nevertheless, the highest curve which is characterized by the

use of all the observations from $\mathcal{O}_1$, is not significantly higher than the others. This shows that information is redundant among all observations.

In the second graph of Fig. 3.5, we finally compare the impact of the variation of $\Gamma_{0,L}$ between the episodes of the learning process on the convergence curves of the NN controllers. We observe that the curves that result from a learning process during which $\Gamma_{0,L}$ varies (i.e., $\Gamma_{0,L} \sim \mathcal{U}(0.5\Gamma_{0,L}^{ref}, 1.5\Gamma_{0,L}^{ref})$) converges towards lower maximum mean rewards than the curves that result from a learning process with constant intensity, i.e., $\Gamma_{0,L} = \Gamma_{0,L}^{ref}$. In addition, the latter converge more rapidly towards their maximum values. This seems consistent as the first need to deal with an additional difficulty related to the variation of $\Gamma_{0,L}$. Finally, we observe that the combination of three observations converge towards the same maximum mean reward as the combination of four observations for the learning process that maintains $\Gamma_{0,L}$ constant, meaning that information is redundant. However, this is not observed when the wake intensity varies during the learning process. This means that the use of additional measurements can overcome the difficulty induced by the variation of $\Gamma_{0,L}$. In other terms, the observability associated to the wake intensity is improved with more observations.

3.3.2 Training of neural network controllers based on $\mathcal{O}_2$

In this section, we are interested in the learning curves that result from the training of NN controllers that use the entire dynamics of the aircraft as observations of the environment, i.e., the space $\mathcal{O}_2$. In Fig. 3.6 we compare two different learning processes that result from the same methodology as the one described in Section 3.3. During the first one, the circulation of the leader varies between two successive learning episodes while in the second, it remains constant.

For both learning strategies, we observe an increase of the mean of the cumulated rewards gathered during an episode when the number of learning steps increases. This proves the efficiency of the RL framework to learn the aerodynamic model from dynamics measurements in order to perform efficient formation flight. Each curve seems to reach a plateau which shows the convergence of the learning processes. Similarly to the convergence curves computed for the $\mathcal{O}_1$ -based RL algorithm, we observe that the learning process during which the circulation of the leader do not vary leads to a more performant NN controller. Nevertheless, the two learning processes seem to improve at the same rate during most of learning steps. This can be explained by their need, for both of them, to first learn the aerodynamic environment based on the follower's dynamic measurements before improving the tracking strategy. In addition, while the $\mathcal{O}_1$ -based RL algorithm needed at most 10^5 learning steps to converge, the $\mathcal{O}_2$ -based

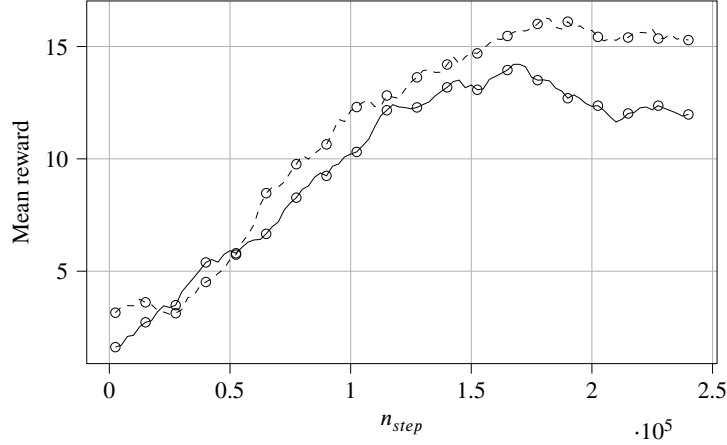


Figure 3.6: Training curves of the $\mathcal{O}_2$ -based RL algorithm. The solid line (—) characterizes the learning process that involves a variation of $\Gamma_{0,L}$ during the learning process, while the dashed line (---) does not.

RL algorithm requires at least $1.5 \cdot 10^5$ ones. However, this is not excessive considering the initial modeling task to be performed. Finally, we observe that the $\mathcal{O}_2$ -based RL algorithm is a bit less efficient in terms of mean reward gathering, compared to the best $\mathcal{O}_1$ -based RL algorithm. We explain this difference by the higher number of observations that the $\mathcal{O}_2$ -based RL algorithm is dealing with for the training of NN controllers characterized by the same amount of hidden layers and neurons. Further studies could focus on the influence of the size of the NN on the performance of the controller.

3.4 Results

In this section, we compute the trajectories of followers that use the RL-trained NN controllers designed in the previous sections. We focus on two controllers only: the first one uses all the observations from the space $\mathcal{O}_1$ while the second uses those of $\mathcal{O}_2$, see Eqs. 3.3 and 3.4. Both controllers have been trained following the methodology described in Section 3.3 with an oscillatory leader wake whose intensity was not the same between the learning episodes.

First, we observe the trajectories of followers that evolves in the wake of a fixed leader. For that case, we study the impact of $\Gamma_{0,L}$ on the resulting trajectories. In a second time, we observe the performances of the controllers when they are dealing with

moving leaders considering in that case a larger formation. Finally, the NN controllers are deployed within Large Eddy Simulations (LES) in order to simulate a RL-based wake tracking strategy in more realistic flow conditions. For all the subsequent simulations, we assume that all the aircraft (leaders or followers) are geometrically identical and maintains a cruise velocity $U_{K,F} = U_\infty$.

3.4.1 Tracking of a static leader

In this section, we exploit the two RL-trained NN controllers on a follower located behind a fixed leader. The follower always starts from the same initial position $[y_F(0), z_F(0)] = [1.5b, 1b]$ and we compute four different trajectories for each controllers. These are computed considering four different values for the wake intensity of the leader, i.e., $\Gamma_{0,L} = [0.5; 0.7; 1; 1.5] \cdot \Gamma_{0,L}^{ref}$. Those wake intensities induce different aerodynamic effects on the follower. The latter thus experiences different dynamics, which lead to different actions computed by the controllers, and different trajectories. Fig. 3.7 is a visual representation of one tracking trajectory: the follower starts from the initial position and is instructed target velocities. It then converges towards a position that should be close to the optimal one in the wake of the leader.

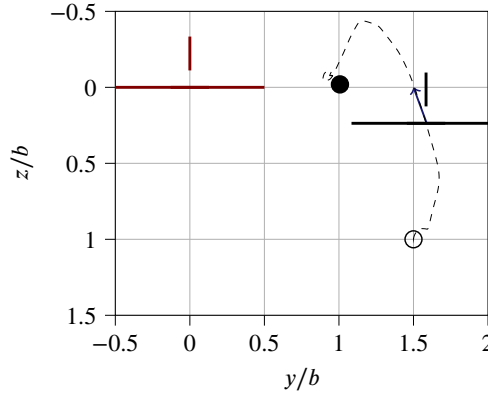


Figure 3.7: Trajectory of the follower () from its initial position () to the final one () located near the optimal position in the wake of a leader ().

Figs. 3.8 and 3.9 are time histories of the positions, reward and thrust observed along the trajectories followed by the follower under the influence of the different leaders.

Fig. 3.8 presents the time series that characterize the $\mathcal{O}_1$ -based NN controller. We observe that when subjected to the reference wake (i.e., $\Gamma_{0,L} = \Gamma_{0,L}^{ref}$), the follower con-

verges towards a steady configuration, reaching the position $[y_F(t_{\text{end}}), z_F(t_{\text{end}})] \simeq [1.b, 0.05b]$ after $t/t^* \simeq 150$. That position is close to the optimal one in terms of maximum energy savings (, i.e., $[y^{\text{opt}}(t_{\text{end}}), z^{\text{opt}}(t_{\text{end}})] \simeq [0.9b, 0.b]$, see Fig. 1.7). Before $t/t^* \simeq 150$, the follower's trajectory is oscillating, which can be explained by three different influences. First, the learning process might not have been completely converged, as observed in Fig. 3.5. In particular far away from the leader, the controller does not properly interpret the accelerations differences where their values are less important. Secondly, the reward provided to the follower during the learning process favors movement, through the term $\tau \frac{d\Delta\dot{W}_{K,F}}{dt}$ from Eq. 3.5. This term is used to encourage the follower to take action in favor with an increase of $\dot{W}_{K,F}$, which accelerates the learning convergence. However, this can explain some unsteadiness. The variation of the reward induced by the movement of the follower is clearly observed in the third graph in Fig. 3.8. Finally, because of the modification of the intensity $\Gamma_{0,L}$ between two successive episodes of the learning process, the accelerations differences presented in Fig. 3.4 and observed by the follower, are never the same since the wake intensity of the leader changes. This might bring some *confusion* to the follower while learning, except when it receives clearer information, i.e., important acceleration difference values, when close to the leader's wake.

This last reason seems to be the most plausible one. Indeed, one can see that when the wake intensity of the leader is more important (i.e., $\Gamma_{0,L} = 1.5\Gamma_{0,L}^{\text{ref}}$), the oscillations decrease. However, the convergence time towards a steady configuration is slower. On the other hand, when the assumed wake intensity is too low, the follower's trajectory is more oscillating, and it does not even seem to stabilize, e.g., when $\Gamma_{0,L} = 0.5\Gamma_{0,L}^{\text{ref}}$. Also, we observe that the positions towards which the follower converges are not the same when the assumed wake intensity varies. This confirms the difficulty that the NN controller experiences to deal with and to determine the intensity of the wake that impacts the follower.

In the last graph of Fig. 3.8 it is interesting to note that the thrust required by the follower to maintain a steady position in the wake of the leader ($\delta_{th} \simeq 83\%$) is lower than that required to fly alone ($\delta_{th} \simeq 100\%$). More surprisingly, we observe the no matter the intensity of the leader's wake, that thrust is nearly the same for the 3 steady configurations studied in Fig. 3.8. This is due to the fact that their position in the wake are not the same. This observation seems to show that the learned NN controller leads the follower towards positions with similar energy consumption and thus similar drag reduction, through the acceleration difference observations. This last graph also highlights the thrust oscillations observed during the movement of the follower. As explained in Section 3.2.1, those oscillations are explained by the constraint imposed

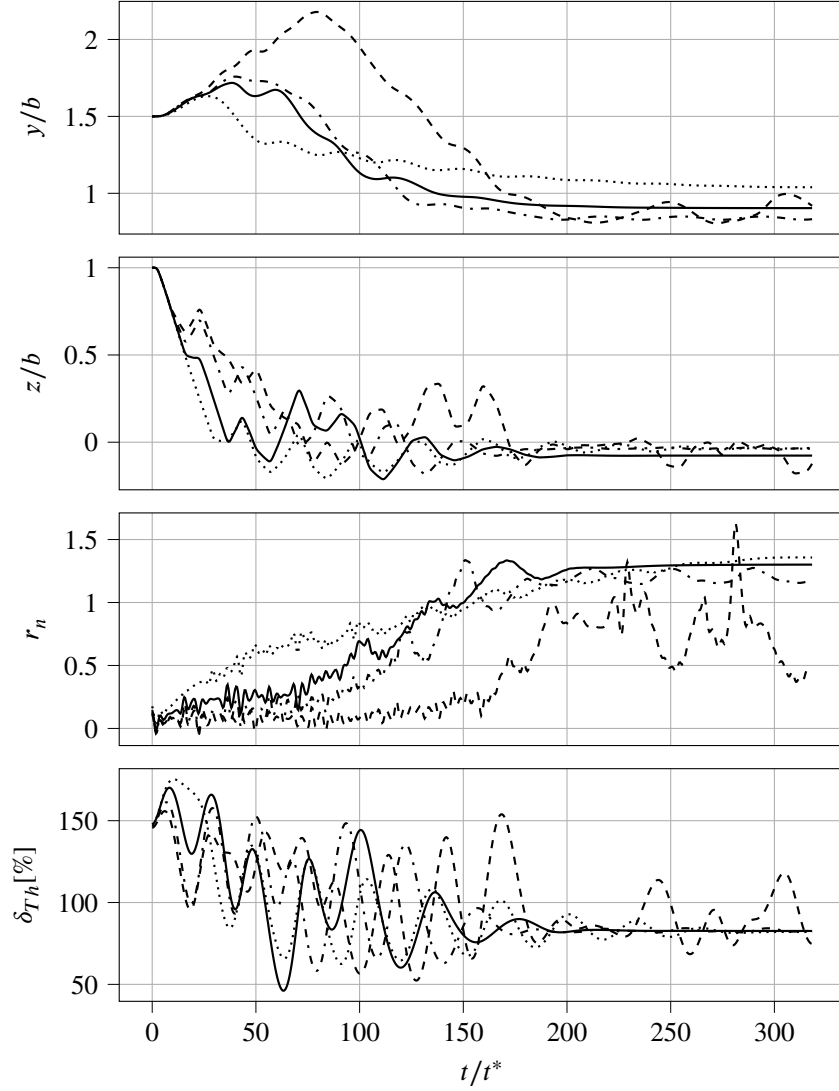


Figure 3.8: Time histories of the lateral position (top), vertical position (upper middle), instantaneous reward (lower middle) and instantaneous thrust (bottom) of a follower piloted by a RL-trained NN controllers that use observations from the space $\mathcal{O}_1$. The four trajectories are distinguished by the intensity of the wake that influences them; $1 \cdot \Gamma_{0,L}^{ref}$ (—); $0.5 \cdot \Gamma_{0,L}^{ref}$ (- -); $0.7 \cdot \Gamma_{0,L}^{ref}$ (- · -); $1.5 \cdot \Gamma_{0,L}^{ref}$ (····).

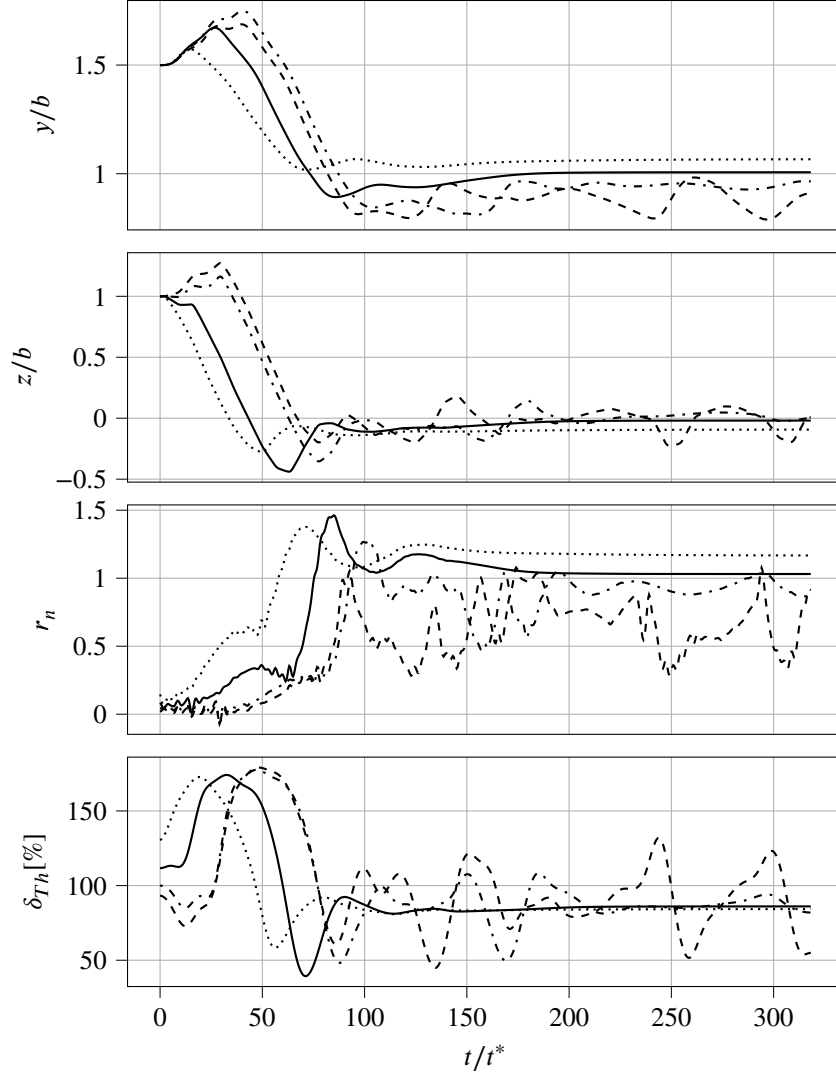


Figure 3.9: Time histories of the lateral position (top), vertical position (upper middle), instantaneous reward (lower middle) and instantaneous thrust (bottom) of a follower piloted by a RL-trained NN controllers that use observations from the space $\mathcal{O}_2$. The four trajectories are distinguished by the intensity of the wake that influences them; $1 \cdot \Gamma_{0,L}^{ref}$ (—); $0.5 \cdot \Gamma_{0,L}^{ref}$ (- -); $0.7 \cdot \Gamma_{0,L}^{ref}$ (- · -); $1.5 \cdot \Gamma_{0,L}^{ref}$ (····).

to the streamise velocity, i.e., $U_{K,F}^{ref} = U_{\infty}$. Such a constraint leads to non-realistic engines behaviors that are clearly not observed on actual aircraft engines that are driven by dynamics with slower time response.

Fig. 3.9 presents the same time series for the $\mathcal{O}_2$ -based NN controller. While one might have thought that using this second controller could have helped with the drawbacks of the first one, we observe that the $\mathcal{O}_2$ -based NN controller struggles with similar difficulties.

The approach phase to reach the vicinity of the leader is always about $t \simeq 100t^*$ long. When impacted by the reference wake (i.e., $\Gamma_{0,L} = \Gamma_{0,L}^{ref}$), the follower converges towards the position $[y_F(t_{end}), z_F(t_{end})] \simeq [1.0b, 0.0b]$. The latter is a bit further away than the position observed with the $\mathcal{O}_1$ -based NN controller. For a higher value of $\Gamma_{0,L}$, the convergence time is similar which seems to be an improvement compared to the the first controller. Also, where the first controller leads to strong oscillations during the approach phase towards the leader, the $\mathcal{O}_2$ -based NN controller produces smoother trajectories. This can be explained by the exploitation of the follower dynamics states (i.e., velocities $\mathbf{V}_{K,F}$ and $[p, q, r]$, attitude $[\phi, \theta, \psi]$) by the controller. This acts in a similar way as the state feedback in more conventional controllers. This helps to stabilize the system and therefore to reduce oscillations. Regarding lower wake intensities, the $\mathcal{O}_2$ -based NN controller also leads to unsteady positions. This is even true when the follower is close to leader's wake. This leads to oscillating rewards and thrust command of the engine. In particular, one observes high reductions of the thrust during the downward displacement of the follower that benefits from the gravity to maintain the desired velocity $\mathbf{V}_{K,F}$. Let us mention again that a realistic aircraft would not enforce such a stiff control of its nominal speed U_{∞} .

In general, we observe that the $\mathcal{O}_1$ -based controller seems more appropriate to maximize the reward and to reduce the energy consumption compared to the $\mathcal{O}_2$ -based controller. This is explained by a more direct exploitation of the information relative to the presence of the wake through the acceleration difference observations. The second controller indeed needs the learn an aerodynamic model that is considered to be known for the first one. However, we observe that the second controller drives the follower through more stable trajectories.

3.4.2 Tracking of an oscillating leader and formation flight

We now use the $\mathcal{O}_2$ -based controller on each follower of a four-aircraft formation. Except for the leader that follows an oscillating target, the followers track their predecessor wake and should therefore reproduce the oscillating maneuver performed by the

leader. The leader's follows a target described by Eq. (1.57), with $T_y = 4T_{ph}$, $T_z = 2T_{ph}$, $A_y = 0.4b$ and $A_z = 0.5b$. Fig. 3.10 presents snapshots of that formation over time.

We first observe that all the followers manage to stay in the vicinity of their predecessor wake. Moreover, the first follower manages to reproduce quite faithfully the trajectory produced by the leader, validating the performance of the trained controller for oscillating maneuvers. The trajectories of the second and third followers however divert from the initial one. Indeed, the trajectory of the aircraft preceding them is already different from the original one, leading to errors that propagate downstream in the formation. This behavior is related to the *string stability* of the formation, and as already been studied by Riehl et al. [122]. In their work, they design appropriate controllers able to limit the propagation of tracking errors within formation. The combination of their conclusions with our RL-based learning methodology could lead to a reduction of tracking errors. In addition to those propagation errors, the second and third followers are also under the influence of two and three wakes, instead of the single one that their NN controllers have been used to deal with during the learning process. Even if the influence of the wakes of the aircraft located further upstream is much weaker than the influence of the wake of the aircraft just in front, the dynamic observations are not exactly what the NN controllers expect. This can in turn lead to tracking errors and possible oscillations. On a global perspective, this RL-based tracking strategy seems to validate the initial intuition, which is that dynamic observations are able to reproduce and partially explain formation flight behaviors.

3.4.3 RL-based tracking controller in LES

The purpose in this section is to investigate the ability of the $\mathcal{O}_1$ and $\mathcal{O}_2$ -based controllers to exploit the knowledge acquired in the model described in Section 3.2.1 to realistic aircraft wakes computed by the VPM solver. The LES setup is represented in Fig. 3.11.

The leader performs an oscillating maneuver similar to the one described by Eq. (1.57) (with $T_y = 2T_{ph}$, $T_z = 1T_{ph}$, $A_y = -0.5b$ and $A_z = -0.5b$). The follower pursues the actions computed by the RL-trained controllers in order to track the leader's wake. The major differences here consist in both the introduction of turbulence and a different flight environment. The observations exploited by the controllers are thus doubly affected compared to the observations they were confronted with during the learning process. First, the aerodynamics are different, as the leader's wake now results from a vortex sheet that rolls up. The LOA wake model with which the

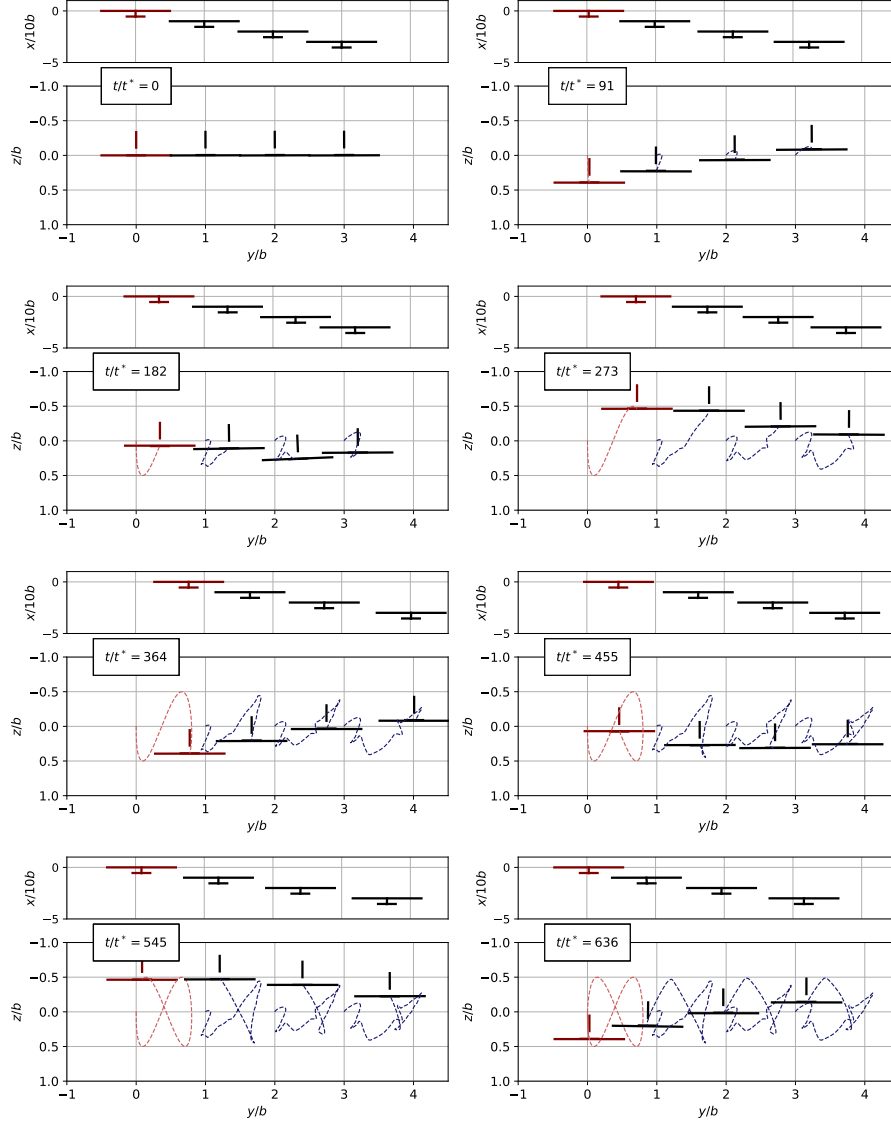


Figure 3.10: Trajectories computed by the $\mathcal{O}_2$ -based controller for four aircraft in formation flight. The leader (—) performed an oscillating maneuver (- -) while the followers (—) use the RL-based controller in order to track the optimal position in the wake of their predecessor in the formation (- -).

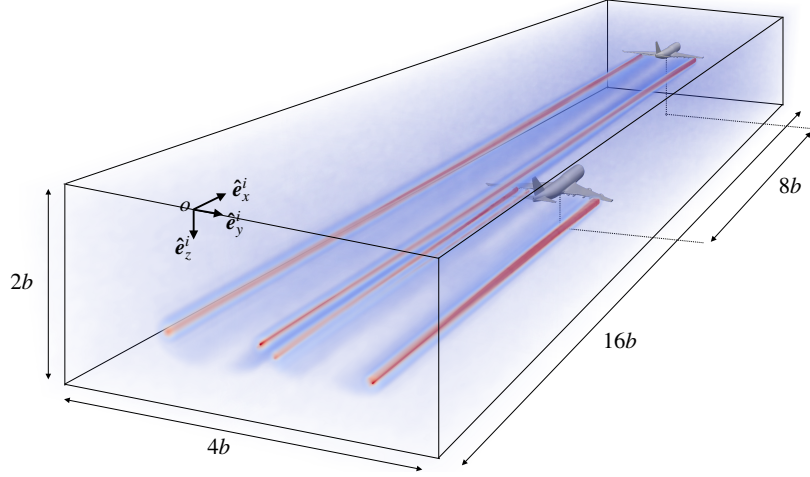


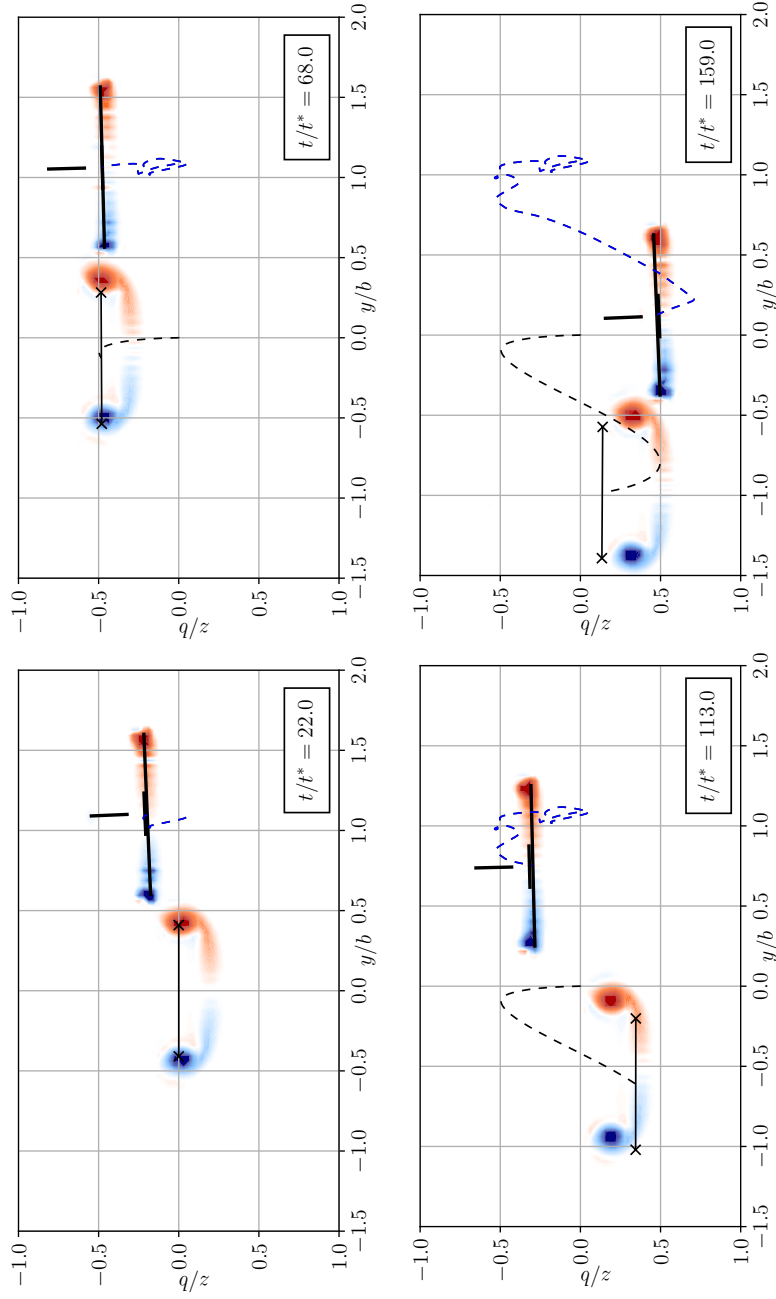
Figure 3.11: LES numerical setup: $16b \times 4b \times 2b$ domain, with the leader located $2b$ downstream the inlet, and the follower $8b$ downstream of the leader. The mesh size is such that $b/h = 32$. Boundary conditions are inflow-outflow in the streamwise direction, i.e., $\hat{e}_x^i$, unbounded flow in the two other directions, i.e., $\hat{e}_y^i$ and $\hat{e}_z^i$.

controllers were trained during the learning process does not accurately represent such vortices. Therefore, the aerodynamic loads undergone by the follower are not exactly the same as the ones computed by the simplified model described in Chapter 1. More importantly, synthetic turbulence is injected at the inlet and contributes to adding noise to the observations exploited by the controllers. The turbulence is characterized by a dimensionless Eddy Dissipation Rate (EDR) $\epsilon^* = \frac{\epsilon b_0}{V_0^3} \simeq 2 \cdot 10^{-4}$, corresponding to weak turbulence [44].

Figs. 3.12 and 3.13 present a visual interpretation of the RL-based tracking strategies and shows instantaneous views of the flow in the cross section at the follower's position $x = x_F$.

3.5 Conclusions

This study presents a bio-inspired wake tracking strategy that guides a follower to the optimal position in the wake of a leader in order to optimize the benefits of formation flight. To this end, trained neural networks provide velocity set-points to the autopilot of the follower that controls its 6 degrees-of-freedom dynamics. The resulting aerodynamic loads are computed thanks to a simplified, yet accurate aerodynamic model



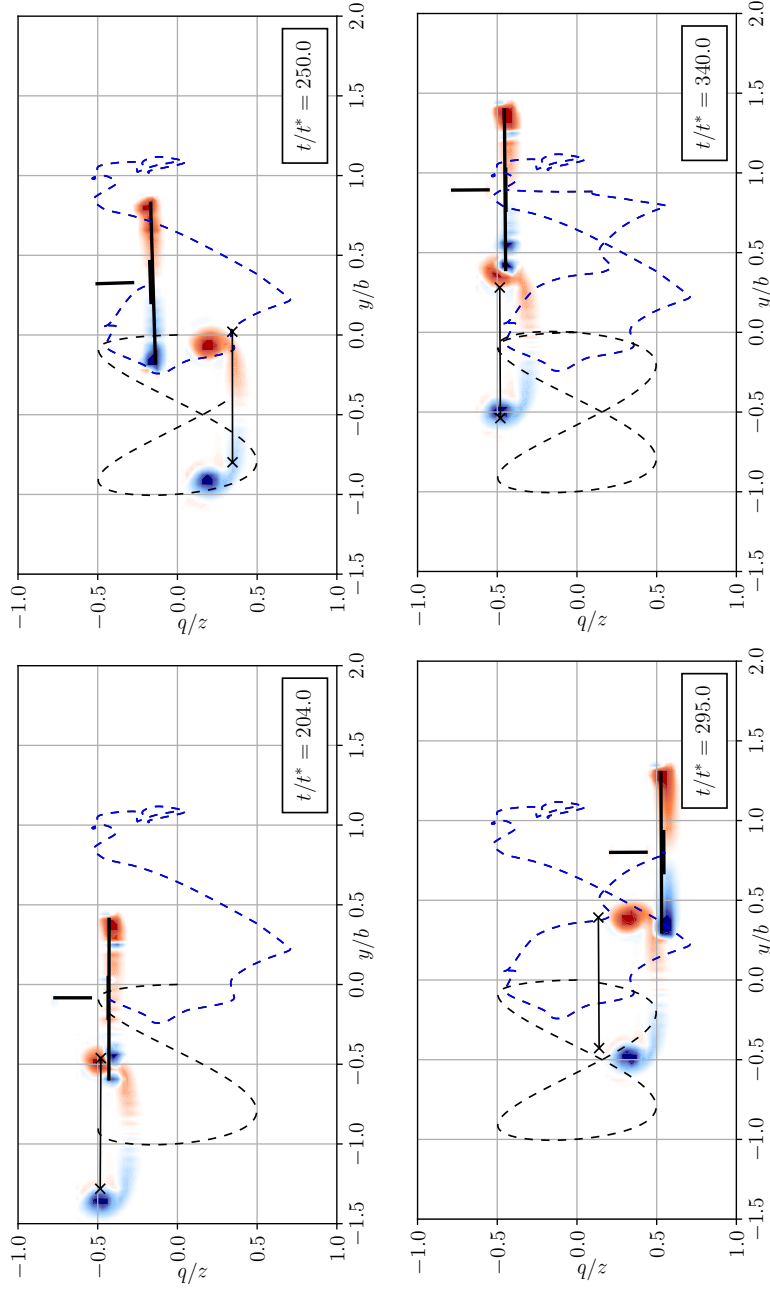
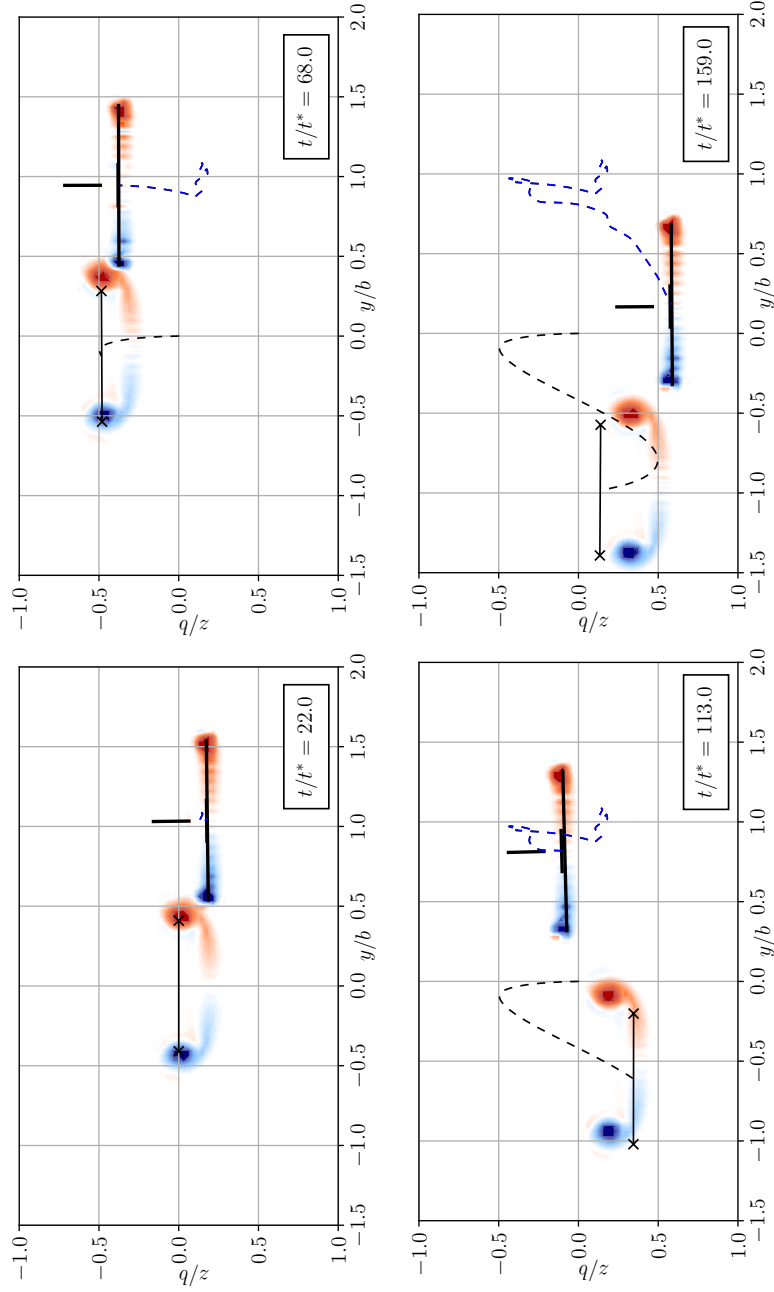


Figure 3.12: Follower's trajectory that results from the actions prescribed by the $\mathcal{O}_1$ -based controller. The background shows the streamwise vorticity behind the follower. The blue dashed line (---) is the history of the exact position of the leader (resp. of the follower).



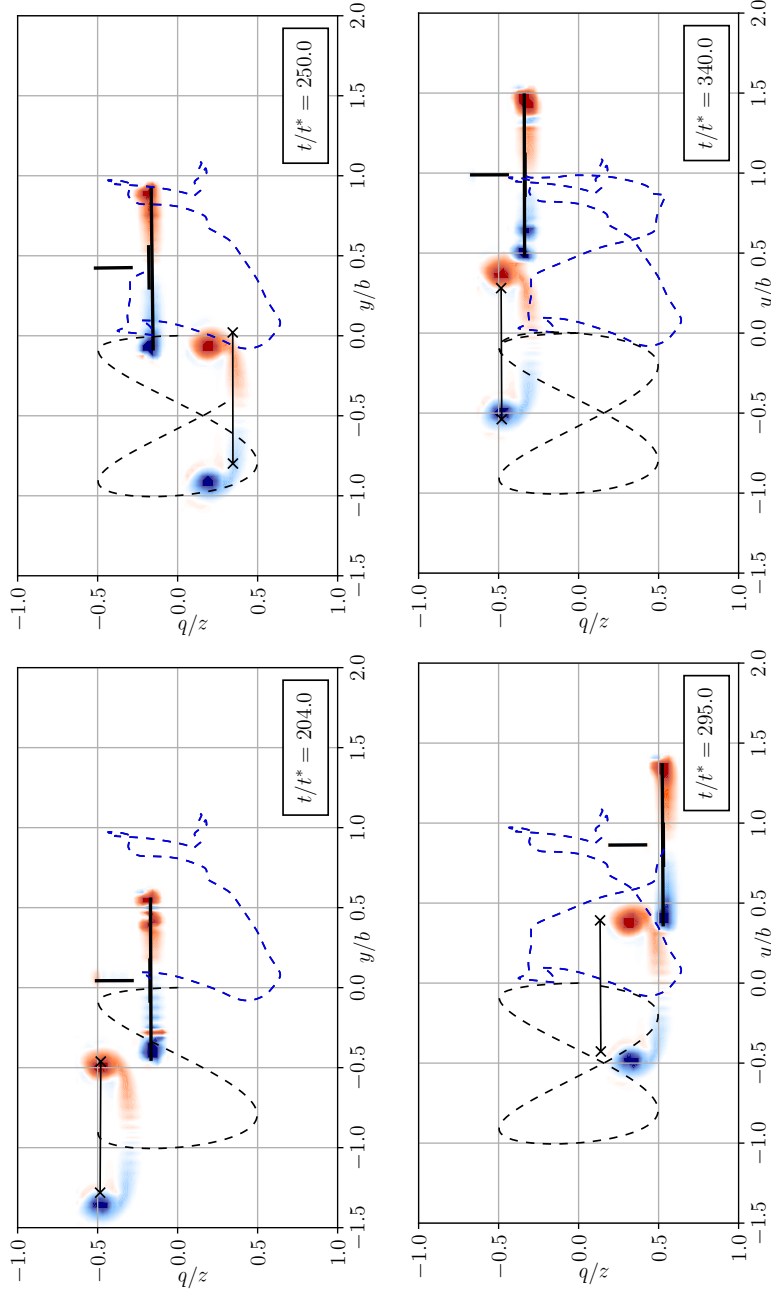


Figure 3.13: Follower's trajectory that results from the actions prescribed by the $\mathcal{O}_2$ -based controller. The background shows the streamwise vorticity behind the follower. The blue dashed line (---) (resp. the black (---)) is the history of the exact position of the leader (resp. of the follower).

based on Prandtl's lifting line theory. The reinforcement learning framework is used to train the neural networks based only on dynamic observations of the follower. This ensures a total independence of the tracking method from direct measurements of the leader wake position. In this work, we use two different observations sets in order to train two different neural network controllers. The first set consists in acceleration differences that compare the actual accelerations perceived by the follower to fictitious accelerations computed by a known aerodynamic model of the follower. The second one consists in measurements of the entire dynamics of the follower, i.e., its actual position, velocities, accelerations and control commands. The resulting neural network controller does not depend on a known aerodynamics model, but rather learns it during the training process. It can therefore be defined as a model-free-based tracking strategy.

In this work, we first studied the convergence of the learning processes towards the design of efficient wake tracking controllers. From the first observations set, we highlighted the relevant acceleration differences to be used in order to detect and track the wake of a leader. In particular, the roll and yaw acceleration differences were showed to be essential, denoting their importance in terms of the observability of the system. The use of other acceleration differences enables to improve the performance of the tracking strategy by giving information related to the wake intensity. We observed that the design of a neural network controller based on the second set of observations required a higher number of learning steps than the controller based on acceleration differences. This is explained by the higher number of observations to be dealt with by this second one, compared to the first.

We then compared the follower's trajectories resulting from the actions computed by the two RL-trained controllers. We observed that the second neural network controller led to more stable trajectories, with less oscillations. This is explained by the state feedback that this controller naturally involves, as it directly uses the dynamic states of the follower as observations from the environment. Whereas its performances in terms of energy savings were showed to be less than the first one, it was showed more reliable. Moreover, we tested the performances of the dynamics-based neural network controllers on more challenging test cases. The first one consisted in studying the trajectories of four aircraft one behind the other that were piloted by this RL-trained controller. We showed that the controller was able to pilot the aircraft in order to maintain the formation, even when the leader was performing oscillating maneuvers. However, we observed that aircraft located further downstream in the formation struggled in reproducing a trajectory similar to the one of the leader. Those trajectory errors highlight the seeming string-instability of the system. This would need to be further investigated

in order to clearly identify the origin of the errors, i.e., accumulation of errors only, or intrinsic string-instability of the dynamic system and associated controllers. Finally, we deployed the dynamics-based controller on aircraft evolving in the simulated flight environment based on Large Eddy Simulations. Despite the differences between the dynamic measurements computed by the LES and the ones the controllers were trained with, the RL-trained controllers were able to track the wake of a leader, even in turbulent environment. This shows the controllers ability to adapt to their environment, thanks to the wide conditions in which it has been trained.

We have shown through the examples described in this work the ability of the RL framework to manage with various formation flight configurations. However, we did not focus on the management and propagation of the uncertainties that remain the biggest challenges to tackle. Indeed, measurement errors or the uncertainty related the model were for instance not taken into account during the training process. Thanks to the flexibility of the RL framework, and the promising results described in this work, such improvement can clearly be envisioned in future work. Multi-objective optimizations can also be envisioned in order to exploit even more that flexibility. Finally, while offline learning as been implemented in this chapter through the exploitation of a surrogate model, online learning could be clearly envisioned in order to make the resulting formation flight controller even more adaptable to new configurations or perturbed environment.

Observability in a Formation Flight Context

4.1 Introduction

Birds are forerunners in exploiting their environment to perform efficient flight strategies, e.g., wake surfing by migratory birds or thermal updraft exploitation by the soaring birds. Both techniques require to determine the features of the flow to be harvested, from well-chosen measurements of the environment. In order to apply these flight techniques to aircraft, the choice of the measurements is thus important since aircraft have to be able to *observe* a parametrization of these features, e.g., position and intensity of the wake of a leader or of the thermal updraft. In other terms, the measurements have to be directly or indirectly impacted by a variation of the flow parameters, or the other way around, those parameters can be inferred from the measurements. This is defined as the *observability* of those parameters by a defined measurement set and it may not be constant or optimal over time and space. The characterization of this observability during an estimation process is therefore crucial. In this chapter, we are interested in the quantification of observability in the context of formation flight.

Close formation aerodynamic modeling, control and experiments have shown the benefits of formation flight [7, 9, 71, 57]. Other studies go a step further which consists in estimating the leader's wake state parameters without having the prior knowledge of the position of the leader itself [28, 12, 72]. In [47], DeVries et al. associate the observability aspect to the estimation problem. They use the Empirical Observability Gramian in order to develop optimal control strategies that steer the follower to a desired position while accounting for the observability, and not considering the energy

saving objective only [142]. In this chapter, we pursue that effort in order to quantify the wake observability given predefined measurements. While distributed pressure measurements along the wing are used in [47], we rather use measurements of the dynamics of the follower. This is motivated by their direct availability on modern aircraft without requiring any additional hardware equipment as opposed to the pressure-based scheme. We therefore build on the developments made in Chapter 2 and we introduce the observability formalism.

Although the concept of observability is frequently used in flow estimation applications [48, 88], it is little studied in the context of formation flight except for the aforementioned work [47]. In this work, we have highlighted some issues related to the concept in Chapters 2 and 3. We observed the importance of some measurements compared to others during estimation or learning processes. We have also highlighted the difficulty to predict all the wake state parameters simultaneously. Observability is also well received in the world of thermal soaring [103] which is very close to the problem of formation flight in some aspects. Indeed, in that context, an aircraft needs to be able to localize the updraft and to exploit its energy while controlling the dynamics of the glider [82]. Moreover, the methods used to characterize the thermals are similar to the problem of interest. They range from model-based estimation strategies [81, 106] to model free localization methods [46, 153, 104] involving, for instance, reinforcement learning. In [147], Wettengl et al. use the Fisher Information Matrix in order to derive an observability criterion that quantify the achievable accuracy of an estimator through the Cramér-Rao lower bound.

In this chapter, we employ observability criteria derived from the Fisher Information Matrix to quantify spatial regions prone to a degraded estimation performance by a wake tracking strategy. The focal idea is to enhance the accuracy achieved by a follower in the prediction of the leader's wake characteristics. For that purpose, we use optimal control framework in order to exploit a simplified dynamic model of two aircraft in formation and to highlight tracking trajectories that maximize the observability. We first described in Section 4.2 the dynamic model and the criteria defined for the quantification of observability of non-linear systems. In Section 4.3, we present the envisioned optimal control strategies and the optimal trajectories computed for several formation flight configurations. We conclude with some conclusions and perspectives in Section 4.4.

4.2 Two-aircraft formation and wake observability

On the contrary to the methodology described in Chapter 3 where we used reinforcement learning techniques, we here focus on optimal control tools in order to optimize the observability of the wake parameters along trajectories. Similarly to RL techniques, such an approach also depends on cost functions that are minimized along trajectories. However, contrary to RL, by defining a simpler aircraft dynamic model than the one defined in Chapter 3, it is possible to evaluate different cost functions, possibly associating different objectives, without having to go through a learning process for each cost function.

We first review in Section 4.2.1 the simplified dynamic model of a two-aircraft formation used in this chapter. We then describe in Section 4.2.2 the tools from non-linear observability that enables the definition of observability criteria of the leader's wake parameters. We also link those criteria to the filtering problem through the Cramer-Rao lower bound.

4.2.1 Two degrees-of-freedom dynamics

A simplified aerodynamic model of an aircraft has been described in Chapter 1, i.e., the surrogate model. It allows the computation of both the aerodynamic efforts undergone by the 3 lifting surfaces of an aircraft and its resulting 6DOF dynamics over time. In this chapter, we use the same model to compute the aerodynamic loads undergone by the follower. However, we restrict the movement of the aircraft to the two translational degrees-of-freedom (2DOF) dynamics in the $\hat{e}_y^i - \hat{e}_z^i$ plane in order to reduce the complexity of the optimal control problem we solve in Section 4.3. We therefore decouple the dynamics of the follower from its kinematics. The follower is able to control its vertical and lateral kinematics (, i.e., $V_{K,F}$ and $W_{K,F}$), while its axial velocity is fixed, i.e., $U_{K,F} = U_\infty$. Therefore, the states of the follower are characterized by its position and velocity in the $\hat{e}_y^i - \hat{e}_z^i$ plane, i.e.,

$$\mathbf{q}_{dyn} = [V_{K,F}, W_{K,F}, y_F, z_F]^T. \quad (4.1)$$

We control these simple dynamics by means of velocity commands following first order laws, i.e.,

$$\begin{cases} \dot{V}_{K,F} &= -\frac{1}{\tau} \left(V_{K,F} - V_{K,F}^{\text{targ}} \right) \\ \dot{W}_{K,F} &= -\frac{1}{\tau} \left(W_{K,F} - W_{K,F}^{\text{targ}} \right) \\ \dot{y}_F &= V_{K,F} \\ \dot{z}_F &= W_{K,F} \end{cases}, \quad (4.2)$$

where $\mathbf{u}_{dyn} = [V_{K,F}^{\text{targ}}, W_{K,F}^{\text{targ}}]^T$ is a target velocity for the follower and τ is the time response of the system.

The leader is in steady level flight far upstream of the follower such that its wake is assumed to be rolled-up when it reaches the follower. Similarly to the surrogate model described in the Chapter 2, it is represented as two straight counter-rotating vortex tubes of circulation $\Gamma_{0,L}$ separated by a distance $b_{0,L}$. The vortices are aligned with the inertial $\hat{\mathbf{e}}_x^i$ axis and are regularized using a LOA wake vortex model (see Appendix A), with a core radius $r_c/b = 0.05$ [14]. We use the quantities $[y_{wake}, z_{wake}]$ to represent the position of the leader's wake. The schematic representation of the 2D problem described above is shown in Fig. 4.1. In this chapter, we use the geometrical parameters of a A350-like aircraft defined in Table 1.1.

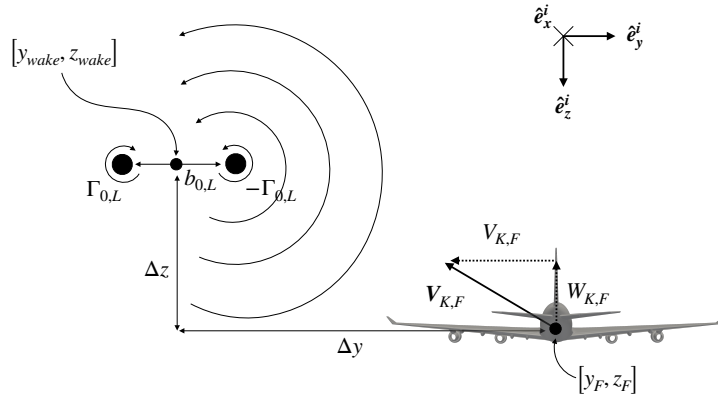


Figure 4.1: Simplified 2D representation of a follower in the wake of a leader. The wake produces a velocity signature on its environment that impacts the aerodynamics of the follower.

Following that representation, the state parameters of the leader's wake are defined by the vector $\mathbf{q}_{\text{wake}} = [\Gamma_{0,L}, b_{0,L}, y_{\text{wake}}, z_{\text{wake}}]^T$ of size $M = 4$. We assume that those states do not vary over time and are parameters of the system dynamics.

We finally define the measurements that the follower has at his disposal to observe the leader's wake parameters. Although we only consider the 2D dynamics of the aircraft, we use measurements of fictitious aerodynamic accelerations according to the 6DOF that the follower would undergo if its dynamics were left free. Indeed, the three lifting surfaces of the aircraft are subjected to aerodynamic and gravitational forces and torques under the influence of the leader's wake. Those efforts are not taken into account in the kinematic equations from Eq. (4.2) but we use the accelerations they would produce as measurements in addition to measurements of the angle of attack and slip angle. We have the measurement vector

$$\mathbf{m} = \left[\frac{F_x^i}{m}, \frac{F_y^i}{m}, \frac{F_z^i}{m}, \frac{M_x^b}{I_{xx}}, \frac{M_y^b}{I_{yy}}, \frac{M_z^b}{I_{zz}}, \alpha, \beta \right]^T, \quad (4.3)$$

where $\mathbf{F} = [F_x^i, F_y^i, F_z^i]^T$ and $\mathbf{M} = [M_x^b, M_y^b, M_z^b]^T$ are the total forces and torques experienced by the follower, as described in Eqs. 1.35 and 1.36. m is the mass of the aircraft and $[I_{xx}, I_{yy}, I_{zz}]$ are the moment of inertia about the principal axes. We finally associate a Gaussian noise to each measurement which gives us the state-space representation

$$\begin{cases} \dot{\mathbf{q}}_{\text{dyn}}(t) &= \mathbf{f}(\mathbf{q}_{\text{dyn}}, \mathbf{u}_{\text{dyn}}) \\ \mathbf{m}(t) &= \mathbf{h}(\mathbf{q}_{\text{dyn}}, \mathbf{q}_{\text{wake}}) + \boldsymbol{\epsilon} \end{cases}, \quad (4.4)$$

where $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$ is the zero-mean Gaussian measurement noise with covariance $\mathbf{R}$. That model is used in Section 4.2.2 to quantify the observability of the wake states parameters $\mathbf{q}_{\text{wake}}$ with the measurements $\mathbf{m}(t)$ in order to design an optimal trajectory controller for the follower.

Two remarks need to be made concerning this model. First, we note that the *fictitious* accelerations measured in these conditions have higher magnitude than what would experience an aircraft whose 6DOF dynamics are integrated over time. Indeed, its control commands (i.e., control surfaces and thrust) trim the efforts as does the aircraft defined in Chapter 1. However, we show in the next section that it is rather the variation of these acceleration measurements with respect to the leader state parameters that need to be computed. Moreover, these variations computed from the 2DOF aircraft dynamics (i.e., described through Eqs. 4.2 and 4.3) are mathematical functions whose spatial evolution (i.e., in terms of amplitude or gradients) in the $\hat{e}_y^i - \hat{e}_z^i$ plane

is similar to the one observed on the variations computed from the 6DOF trimmed aircraft dynamics (, i.e., from Chapter 1). This justifies the present simplification and the use of those acceleration measurements, as it enables the computation of similar effects (, i.e., accelerations variations with respect to the leader state parameters) at a lower cost (, i.e., 2DOF dynamics *versus* 6DOF dynamics). Secondly, one notices that the measurement vector $\mathbf{m}$ does not include the dynamic states $\mathbf{q}_{dyn}$ of the follower while these clearly influence the fictitious accelerations perceived. Their knowledge would therefore clearly be necessary for the estimation of the wake state parameters. However, as it is explained in Chapter 2 and Appendix E, we here decouple the dynamics of the states of the follower and the dynamics of the wake state parameters. Therefore, by assuming that the dynamics of the follower is known, thanks to a dedicated estimator for instance, we can now focus on the observability of the wake state parameters by the accelerations measurements $\mathbf{m}$ only. Let us note that such an assumption places the following study in an idealized context and that the observability of the vortex effects will be all the more impacted as little is known about the follower's dynamics.

4.2.2 Criteria for observability quantification

The observability framework hinges upon the fact that the initial state $\mathbf{q}(t_0) = \mathbf{q}_0$ of a dynamical system can be determined from the time history of the measurement $\mathbf{m}(t)$ and the time history of the command $\mathbf{u}(t)$, for $t \in [t_0, t_0 + T_h]$ and where T_h is the horizon of the simulated dynamics [73, 130]. In this chapter, rather than the study of the observability of the dynamic states $\mathbf{q}_{dyn}$ that we assume to be known, we quantify the observability of the leader's state parameters $\mathbf{q}_{wake}$ with the dynamic measurements $\mathbf{m}$. The system is thus observable if the wake state parameter vector $\mathbf{q}_{wake}$ can be determined from the time history of the measurements vector $\mathbf{m}(t)$ and of the aircraft dynamics vector $\mathbf{q}_{dyn}(t)$ that depend on the command $\mathbf{u}_{dyn}(t)$, for $t \in [t_0, t_0 + T_h]$. Through the measurement function $\mathbf{h}(\mathbf{q}_{dyn}, \mathbf{q}_{wake})$, this is obviously true. It remains to quantify the observability according to the position of the follower in the wake of the leader.

In the case of a linear system, tests for state observability are easily deduced from the linear matrices that define the system and in particular from criteria on the singular values of the observability Gramian matrix [130]. Indeed, large singular values imply that a system is easily invertible while very small singular values, close to zero, imply it is not. In the case of non-linear dynamics, in particular non-linear measurement functions such as the one obtained in Eq. (4.4), the observability is more complex to analyze as it requires a differential geometry approach. The reader is referred to [74]

for a brief review of the relevant key elements of nonlinear observability and to [99] for more complete discussion.

Instead of linearizing the system that could degrade the input/output relationship of the system from Eq. (4.4), an alternate method referred to as the Empirical Observability Gramian (EOG) of non-linearized systems has been developed by Krener et al. in [86] and Batista et al. in [8]. As described in [86] and reproduced in [47], the components of the $M \times M$ EOG matrix applied to our model are given by

$$W^{i,j}(t_0, T_h) = \frac{1}{4\epsilon_i\epsilon_j} \int_{t_0}^{t_0+T_h} (\mathbf{m}^{+i}(\tau) - \mathbf{m}^{-i}(\tau))^T (\mathbf{m}^{+j}(\tau) - \mathbf{m}^{-j}(\tau)) d\tau, \quad (4.5)$$

where a variation ϵ_i of the i^{th} wake state parameter, i.e., $\mathbf{q}_{\text{wake}}^{\pm i} = \mathbf{q}_{\text{wake}} \pm \epsilon_i \mathbf{e}_i$ for $i = 1 \dots M$ gives the perturbed measurement vector $\mathbf{m}^{\pm i}(t) = \mathbf{h}(\mathbf{q}_{\text{dyn}}(t), \mathbf{q}_{\text{wake}}^{\pm i})$, and where $\mathbf{e}_i$ is the i^{th} basis vector of $\mathbb{R}^M$ where $\mathbf{q}_{\text{wake}}$ belongs to.

One notes in Eq. (4.5) that the EOG does not take the accuracy of each measurement into account, meaning that the relevance of one compared to the other in the quantification of the observability is not considered. In other terms, the measurement covariance noise $\mathbf{R}$ is not taken into account in the observability quantification. Each measurement is assumed to be as accurate as the others. Therefore, another relevant mathematical tool for observability quantification is used, i.e., the Steady Fisher Information Matrix (SFIM), whose relation to the EOG is described in [115] and [103]. We compute the components of the $M \times M$ of SFIM $\mathbf{I}_S(t_0, T_h)$ using

$$\mathbf{I}_S^{i,j}(t_0, T_h) = \frac{1}{4\epsilon_i\epsilon_j} \int_{t_0}^{t_0+T_h} (\mathbf{m}^{+i}(\tau) - \mathbf{m}^{-i}(\tau))^T \mathbf{R}^{-1} (\mathbf{m}^{+j}(\tau) - \mathbf{m}^{-j}(\tau)) d\tau. \quad (4.6)$$

It should be noted that SFIM is a special case of the Fisher Information Matrix $\mathbf{I}$ (FIM in [103] or Posterior Fisher Information Matrix PFIM in [115]) that is related to the attainable error covariance of an estimator through the Cramér-Rao inequality [138]. Indeed, if an estimation $\hat{\mathbf{q}}_{\text{wake}}$ of the wake state parameters $\mathbf{q}_{\text{wake}}$ was predicted from the measurements $\mathbf{m}$ using one chosen estimator (, i.e., such as the one presented in the previous Chapter 2), the covariance $\mathbf{P}$ associated to this estimate can not be lower than the inverse of the matrix $\mathbf{I}$. In other words, we have

$$\mathbf{P} = E((\hat{\mathbf{q}}_{\text{wake}} - \mathbf{q}_{\text{wake}})(\hat{\mathbf{q}}_{\text{wake}} - \mathbf{q}_{\text{wake}})^T) \geq \mathbf{I}^{-1}, \quad (4.7)$$

where the matrix inequality $\mathbf{P} \geq \mathbf{I}^{-1}$ is understood to mean that the matrix $\mathbf{P} - \mathbf{I}^{-1}$ is positive semidefinite. This relation thus directly relates the concept of observability to the estimation problem as the one described in Chapter 2, and provides a lower bound for the estimation error covariance $\mathbf{P}$. By maximizing the observability along

trajectories, one can therefore improve the performance of an estimator of the wake parameters that would be work simultaneously.

Scalar observability criteria can be deduced from the aforementioned matrices. Following the methodology described in [86], DeVries et al. use in [47] the *unobservability index* ξ computed as the reciprocal of the smallest singular value of the EOG and the *estimation condition number* ζ defined as the ratio of the largest and smallest singular values of EOG, for the quantification of aircraft wake observability, i.e.,

$$\xi_h = \frac{1}{\sigma_{\min}}, \quad (4.8)$$

$$\zeta_h = \frac{\sigma_{\max}}{\sigma_{\min}}, \quad (4.9)$$

where $\sigma_{\max}$ and $\sigma_{\min}$ are the extrema among the singular values of the EOG. In [103], the scalar observability criterion proposed for the quantification of the observability of thermal updraft in a gliding context is given by the trace of the inverse of FIM, or similarly

$$\Lambda_h = \text{tr} \left(\mathbf{I}_S^{-1} (t_0, T_h) \right) = \sum_{i=1}^M \left(\frac{1}{\lambda_i} \right), \quad (4.10)$$

where the λ_i are the eigenvalues of the FIM.

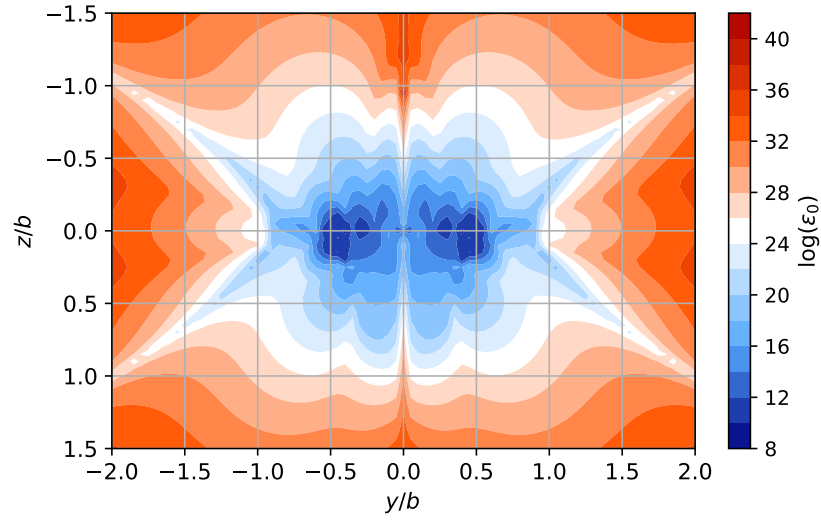
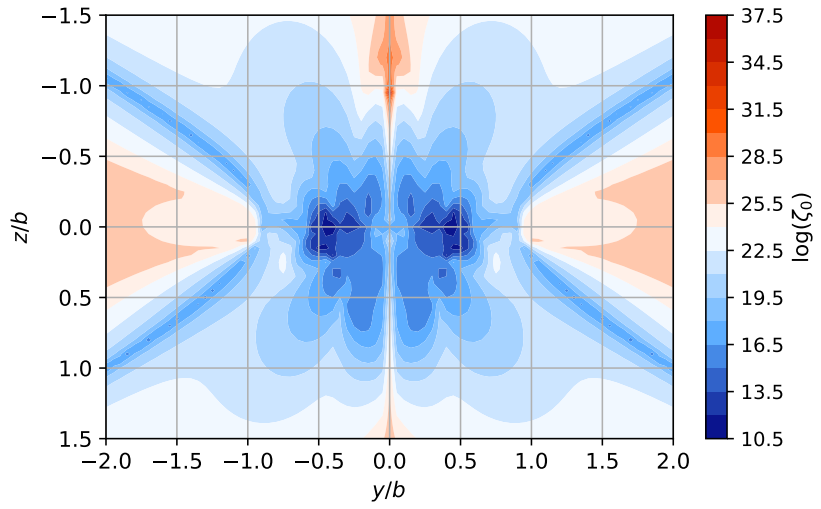
4.2.3 Visualisation of the observability

In this work, we focus on the SFIM to evaluate the observability of the leader's state parameters $\mathbf{q}_{\text{wake}}$ along trajectories of the follower in order to take measurement noise into account. We also make the distinction between *integral* and *instantaneous* observability criteria. Indeed, Eq. (4.6) computes the SFIM by considering the measurements $\mathbf{m}(t)$ observed over the time interval $t \in [t_0, t_0 + T_h]$. It provides integral observability information over that interval, i.e., ξ_h, ζ_h and Λ_h . On the other hand, we define the instantaneous observability SFIM by considering the *instantaneous* measurement vector $\mathbf{m}(t_0)$ only which gives

$$\mathbf{I}_{S,0}^{i,j} (t_0) = \frac{1}{4\epsilon_i \epsilon_j} \left(\mathbf{m}^{+i} (t_0) - \mathbf{m}^{-i} (t_0) \right)^T \mathbf{R}^{-1} \left(\mathbf{m}^{+j} (t_0) - \mathbf{m}^{-j} (t_0) \right), \quad (4.11)$$

from which one computes the resulting *instantaneous* observability criteria, e.g. ξ_0, ζ_0 and Λ_0

The instantaneous definition enables the observation of the dependency of the observability criteria on the position of a static follower with respect to the position of the leader. Indeed, Figs. 4.2a and 4.2b show the unobservability index ξ_0 and estimation

(a) Unobservability index ξ_0 .(b) Estimation condition number ζ_0 .

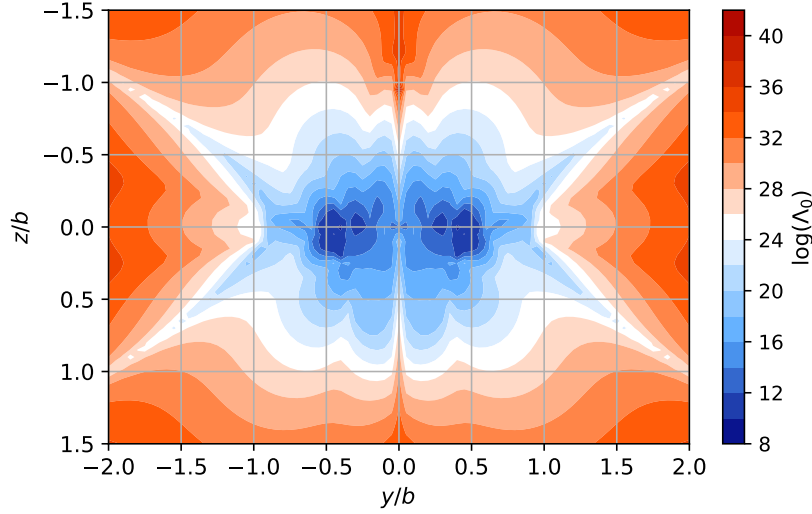
(c) Trace of the inverse SFIM Λ_0 .

Figure 4.1: Observability criteria deduced from the instantaneous SFIM $I_{S,0}(t_0)$ computed at different positions of the follower in the wake of the leader.

condition number ζ_0 computed using the singular values of the instantaneous SFIM from Eq. (4.11). Fig. 4.1c shows the trace of the inverse of that same matrix, i.e., Λ_0 .

We first observe that while it exists a left-right symmetry for all the criteria, the bottom-up symmetry does not exist, even if there are similarities. This means there is a difference in the accelerations experienced by the follower depending on the positions below and above the wake of the leader. This difference is explained by the position of the VTP of the aircraft, which is located above HTP as described in Chapter 1. This induces, among others, differences in the lateral force experienced by an aircraft located above or below the horizontal axis $z = 0$ where the leader is located.

From the three previous figures, we also observe that the criteria tends to increase steeply when the distance between the follower and the wake increases signifying a decrease in observability. In addition to this decrease with distance, we also observe a great inhomogeneity in observability as a function of positions. This shows that some of them seem to be more suitable than others when the follower wishes to observe (or estimate) the characteristics of the wake.

We then observe that the unobservability index ξ_0 and the trace of the inverse of the SFIM Λ_0 are nearly identical. This is in fact consistent with their definition described in Eqs. 4.8 and 4.10. Indeed, the SFIM is symmetric and positive definite. Therefore,

its eigenvalues and singular values are identical. Then, by observing that the smallest eigenvalue $\lambda_{\min}$ is significantly smaller than the others, we have $\Lambda_0 = \sum_i^M \frac{1}{\lambda_i} \simeq \frac{1}{\lambda_{\min}} = \xi_0$. We also compute the eigenvector $\mathbf{v}_{\min}$ associated to the smallest eigenvalue $\lambda_{\min}$, and we observe that almost everywhere in the wake of the aircraft, the eigenvector $\mathbf{v}_{\min}$ can be associated to the wake intensity $\Gamma_{0,L}$. In other words, $\mathbf{v}_{\min}$ is closed to a unit vector whose only non-zero component is attributed to $\Gamma_{0,L}$. As already highlighted in [47], the state parameter $\Gamma_{0,L}$ is therefore the least observable among the four states $\mathbf{q}_{\text{wake}}$. Although they were dealing with pressure measurements in their case, it is worth mentioning that we come to the same conclusion as DeVries et al. in [47] while we are dealing with acceleration measurements in ours.

When one uses the whole SFIM matrix and in particular uses all the eigenvalues of the SFIM matrix to determine observability parameters (e.g., Λ_0), quantities with various dimensions and orders of magnitude are combined together. In the present case, the positions $[y_{\text{wake}}, z_{\text{wake}}]$, the wake intensity $\Gamma_{0,L}$ and the spacing between the wake vortices $b_{0,L}$ have obviously different dimensional properties and various ranges. Hence, whilst being reasonable scalar measures of the observability, ξ_0 , ζ_0 and Λ_0 are most likely dominated by a subset of the eigenvalues of the SFIM, i.e., eigenvalue associated with $\Gamma_{0,L}$ as mentioned earlier. Furthermore, we have highlighted in Chapter 2 that the estimation of all the wake state parameters based on dynamic measurements was challenging. This has led us to consider the wake circulation $\Gamma_{0,L}$ and spacing $b_{0,L}$ as known states of the system. More precisely, we used direct measurements of these two quantities during the estimation process, similarly to what is done during the estimation strategies described in Chapter 2. In terms of observability, this translates into considering the 2×2 -sized sub-matrix of $\mathbf{I}_{S,0}$ that involves the variation of the measurements with respect to the wake position parameters $[y_{\text{wake}}, z_{\text{wake}}]$ only, i.e., $\mathbf{I}_{S,0}^{yz}$. Following the methodology described in [103], we propose to treat the observability of the wake position separately from the observability of the wake circulation $\Gamma_{0,L}$ and the spacing $b_{0,L}$. The latter might be tracked by evaluating the corresponding elements of the main diagonal of the inverse of $\mathbf{I}_{S,0}$ while the observability with respect to the wake position $[y_{\text{wake}}, z_{\text{wake}}]$ is assessed by evaluating the eigenvalues of the sub-matrix $\mathbf{I}_{S,0}^{yz}$. Fig. 4.2 presents the corresponding Λ_0^{yz} criterion computed from that sub-matrix.

The simplification of the observability problem to a two-position parameters problem allows to go further in the characterization of the instantaneous observability fields, and in particular of Λ_0^{yz} . Indeed, in addition to the study of the eigenvalues and in particular of the weakest one $\lambda_{\min}$, we now have a closer look at the associated

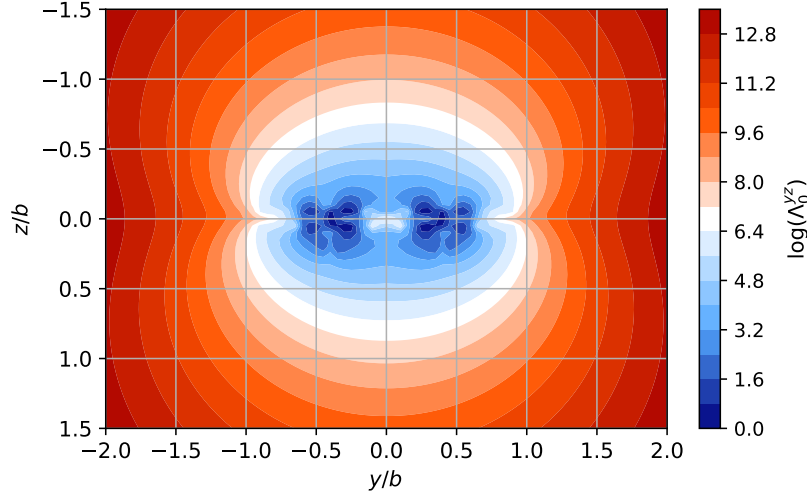


Figure 4.2: Observability criterion Λ_0^{yz} deduced from the SFIM sub-matrix $\mathbf{I}_{S,0}^{yz}$ at different static positions of the follower in the wake of the leader.

eigenvector

$$\mathbf{v}_{\min} = [v_{\min}^1, v_{\min}^2] = [v_{\min}^y, v_{\min}^z], \quad (4.12)$$

which indicates the orientation in the state space that does improve the observability the least. When dealing with the parameters $[y_{wake}, z_{wake}]$ only, this orientation directly translates into a spatial orientation. In precise terms, a small variation of the position of the leader following the orientation of the vector $\mathbf{v}_{\min}$ in the $\hat{\mathbf{e}}_y - \hat{\mathbf{e}}_z$ plane would modify the measurement the least. Or, in other terms, a displacement of the follower along that orientation would improve the observability of the states the least. From the components of eigenvector $\mathbf{v}_{\min}$, we compute the angle κ_0 that characterizes that least observable orientation, i.e.,

$$\kappa_0 = \arctan \left(\frac{v_{\min}^z}{|v_{\min}^y|} \right), \quad (4.13)$$

where $|\cdot|$ denotes the absolute value. The extrema for κ_0 are 0 and $\pi/2$. The former indicates that a variation along the horizontal axis (, i.e., parallel to $\hat{\mathbf{e}}_y$, meaning a variation of y_{wake}) is the least observable; the latter indicates that a variation along the vertical axis (, i.e., parallel to $\hat{\mathbf{e}}_z$, meaning a variation of z_{wake}) is the least observable. Fig. 4.3 presents contours of the angle κ_0 , as well as segments orthogonal to the least observable orientation.

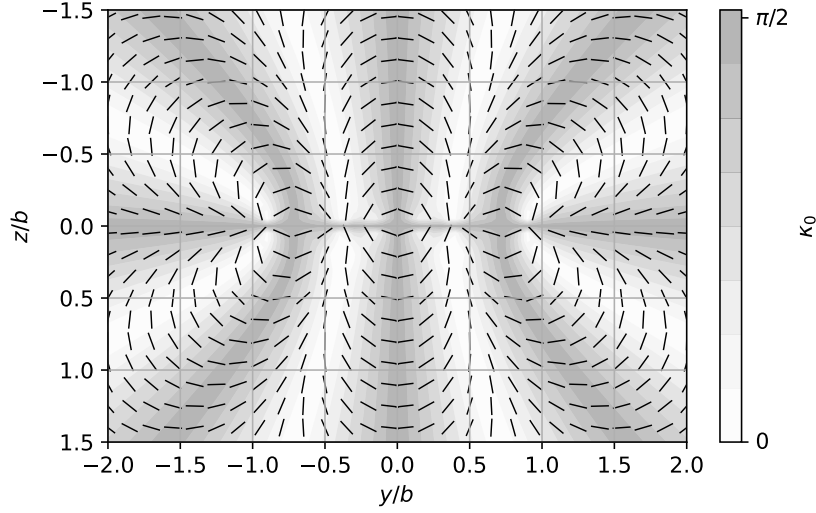


Figure 4.3: Angle κ_0 indicating the least observable spatial orientation. The segments (—) indicate the observability-favorable orientation to follow, i.e., horizontal ones are located in $\kappa_0 = \pi/2$ areas and vertical ones are located in $\kappa_0 = 0$ areas.

This figure shows that by excluding $\Gamma_{0,L}$ and $b_{0,L}$, we unveil an interesting spatial distribution that indicates that areas are more favorable to the observability of one or the other wake coordinate. More importantly, Fig. 4.3 gives information about the orientation that the follower has to follow to ensure the maximum observability increase. This orientation is perpendicular to the least observable direction.

To the orientation, one can associate a direction given by the observability criterion Λ_0^{xy} displayed in Fig. 4.2 and reproduced in Fig. 4.4. The segments directions is chosen so as to maximize the observability gain, i.e., through a decrease of Λ_0^{yz} .

We insist on the fact that the observability criteria such as the ones described here above are based on *instantaneous* positions, i.e., they are based on the eigenvalues of a SFIM that only exploits a vector of measurements $\mathbf{m}(t_0)$ measured in one single position of the follower in the wake. The impact of the measurement vector $\mathbf{m}(t)$ along a horizon trajectory $t \in [t_0; t_0 + T_h]$ on the observability criteria is likely to modify the previous observability fields. Nevertheless, these instantaneous results are quite instructive as to which directions one should follow to optimise observability.

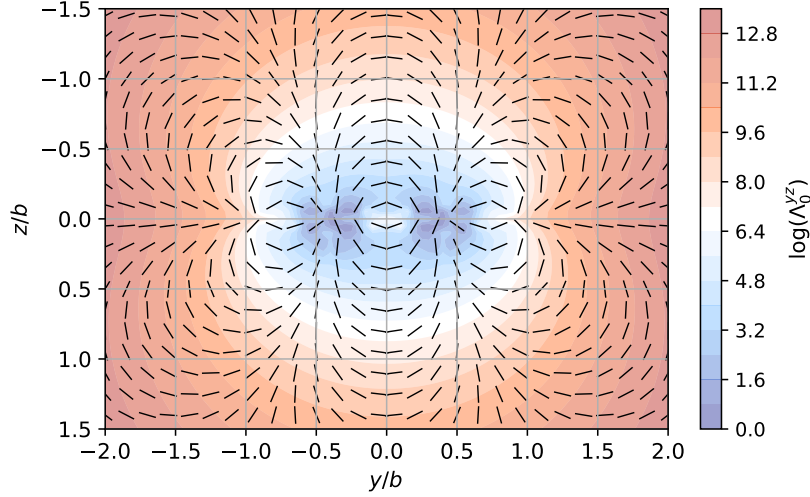


Figure 4.4: Observability criterion Λ_0^{yz} with segments (—) indicating the observability-favorable orientation to follow. The direction is deduced from the gradient of Λ_0^{yz} .

4.3 Optimal control for observability maximization

In this section, we focus on an optimal control problem that consists in determining trajectories that maximize observability (or conversely minimize unobservability) from a given initial dynamic state $\mathbf{q}_{dyn}(t_0)$ along a predefined horizon $t \in [t_0, t_0 + T_h]$. Therefore, at first, we neglect the objective of maximizing the energy savings that was pursued in Chapters 2 and 3. We will observe that observability and energy savings maximization can be antagonistic objectives. For this reason, we try in a second time to reconcile them through the development of an alternate optimal control strategy.

A cost function J needs to be defined along the time horizon in order to determine the optimal control sequence $\mathbf{U}(t_0, T_h) = [\mathbf{u}_{dyn}(t_0), \dots, \mathbf{u}_{dyn}(t), \dots, \mathbf{u}_{dyn}(t_0 + T_h)]$ that minimizes the cost function, i.e.,

$$\min_{\mathbf{U}} J(\mathbf{U}). \quad (4.14)$$

The cost function is subjected to equality constraints that guarantees the dynamics described in Eq. (4.2) are respected over time. Constraints on the inputs are also imposed, i.e.,

$$\mathbf{u}_{\min} \leq \mathbf{u}_{dyn}(t) \leq \mathbf{u}_{\max}. \quad (4.15)$$

4.3.1 Observability-based cost functions

We define two types of cost functions based on the SFIM sub-matrix I_S^{yz} in order to evaluate the observability of the wake position. First, as shown in the previous section, it is possible to determine *instantaneous* observability criteria associated to each one of the relative position between the leader and the follower. Similarly to the strategy described in [47], we use the instantaneous observability map Λ_0^{yz} as a metric to guide the follower towards a desired position in the wake of an aircraft. We define the first cost function

$$J_1(\mathbf{U}) = \int_{t_0}^{t_0+T_h} \exp(\Lambda_0^{yz}(\mathbf{q}_{dyn}(\tau))) d\tau, \quad (4.16)$$

that integrates the *instantaneous* observability criterion Λ_0^{yz} along the optimisation horizon $t \in [t_0, t_0 + T_h]$.

We define a second cost function that rather uses the observability criterion Λ_h^{yz} computed from the sub-matrix of the *integral* SFIM $I_S^{yz}(t_0, T_h)$. The cost function thus writes

$$J_2(\mathbf{U}) = \exp(\Lambda_h^{yz}(\mathbf{Q}_{dyn}(t_0, T_h))), \quad (4.17)$$

where $\mathbf{Q}_{dyn} = [\mathbf{q}_{dyn}(t_0), \dots, \mathbf{q}_{dyn}(t), \dots, \mathbf{q}_{dyn}(t_0 + T_h)]$ is the dynamic state sequence that results from the command sequence $\mathbf{U}$ in the interval $t \in [t_0, t_0 + T_h]$. This cost function therefore consists in maximizing the eigenvalues of the sub-matrix $I_S^{yz}(t_0, T_h)$ that gathers all the measurements observed along the entire horizon interval.

With the help of Fig. 4.2 the cost function J_1 is easy to interpret as it consists in following the trajectory on the Λ_0^{yz} map such that it minimizes its integral. As the horizon T_h gets shorter, the minimization of J_1 tends towards following the gradient of Λ_0^{yz} .

As for the cost function J_2 , it directly reflects the expectation of the estimation error for an efficient estimation filter through the Cramér-Rao inequality presented in Eq. (4.7). Indeed, we have that the trace of the lower bound for the estimation error covariance $\mathbf{P}$ equals the sum of the reciprocal values of the eigenvalues of I_S^{yz} , i.e.,

$$\Lambda_h^{yz} = \text{Tr}(I_S^{yz}) \leq \text{Tr}(\mathbf{P}) = E\left((\hat{\mathbf{q}}_{wake}^{yz} - \mathbf{q}_{wake}^{yz})^T (\hat{\mathbf{q}}_{wake}^{yz} - \mathbf{q}_{wake}^{yz})\right), \quad (4.18)$$

where $\hat{\mathbf{q}}_{wake}^{yz}$ would be the position of the wake as estimated by an appropriate estimator. Λ_h^{yz} is an observability measure that is directly related to the estimation problem such as the one described in Chapter 2. Therefore, by minimizing J_2 , one determines the trajectory over a defined horizon T_h that gathers the measurements $\mathbf{M}(t_0, t_0 + T_h) = [\mathbf{m}(t_0), \dots, \mathbf{m}(t), \dots, \mathbf{q}(t_0 + T_h)]$ that provide *all together* the most

relevant information for the estimation of the state parameters q_{wake}^{yz} over that horizon, as it minimise the covariance of the estimate. On the other hand, J_1 finds the trajectory that gathers the measurements $\mathbf{M}(t_0, t_0 + T_h)$ such that the integral of their *instantaneous* information is maximum. In fact, J_1 ignores any redundancy in the measurements accumulated over the trajectory. For instance, one might use a trajectory that maximises the observability related to the lateral position y_{wake} but never gathers information related to the vertical position z_{wake} , see Fig. 4.3 to observe the specific areas. The cost function J_1 might provide a low cost for that trajectory as each position along that trajectory gives a lot of information on the position y_{wake} , *individually*. However, J_2 will not compute an optimal cost for that trajectory as it provides few information related to z_{wake} . Any trajectory that gives more information on z_{wake} than the first considered will result in a lower cost J_2 .

4.3.2 Aircraft optimal trajectories

In this section, we compute trajectories of an aircraft attempting to optimize the observability of the leader's wake position for several scenarios characterized by different initial positions or cost functions. We rely on CASADI [4] in order to compute the control sequences $\mathbf{U}$ that minimize the cost functions. This open-source tool is used for nonlinear optimization and algorithmic differentiation. It facilitates rapid implementation of different methods for numerical optimal control in an offline context and for nonlinear predictive control, based on a syntax borrowed from computer algebra systems (or symbolic algebra systems). We use the direct multiple shooting method for the resolution of the optimal control problem defined by J_1 or J_2 under the dynamics defined by Eq. (4.4).

Trajectory optimisation without final state

We are first interested in the comparison of the trajectories computed by the two cost functions J_1 and J_2 . Fig. 4.5 shows the trajectories obtained for several initial conditions without any constrain on the final state. The follower therefore reaches a final position as computed by the optimal controller. We simulate the follower trajectories over a period $T_{\text{tot}} = 30$ s, with an integration time step $\Delta t = 0.25$ s. The horizon time is $T_h = 4$ s and the control optimization is updated at every second, i.e., $\Delta t_{\text{up}} = 1$ s. This means we recompute the optimal control sequence $\mathbf{U}(t_0, t_0 + T_h)$ at every second of the simulation using as initial condition for the optimization the current state of the aircraft.

It should be noted that solving optimal control problems is not obvious, nor is the proof of the uniqueness of the solution. In this work, the cost functions depends on the observability criteria that are *a priori* regular (, i.e., continuity and derivability) , leading to the convergence of the resolution. The major drawback in the use of optimal control lies in the possible large size of the nonlinear problem, in particular when using the multiple shooting method. This limits among other things the horizon time T_h . Nevertheless, because of the simple dynamics used in this chapter, this drawback is limited, and different cost functions can be easily tested. The observability maximization is also an opportunity to apply another optimization tool in the context of formation flight.

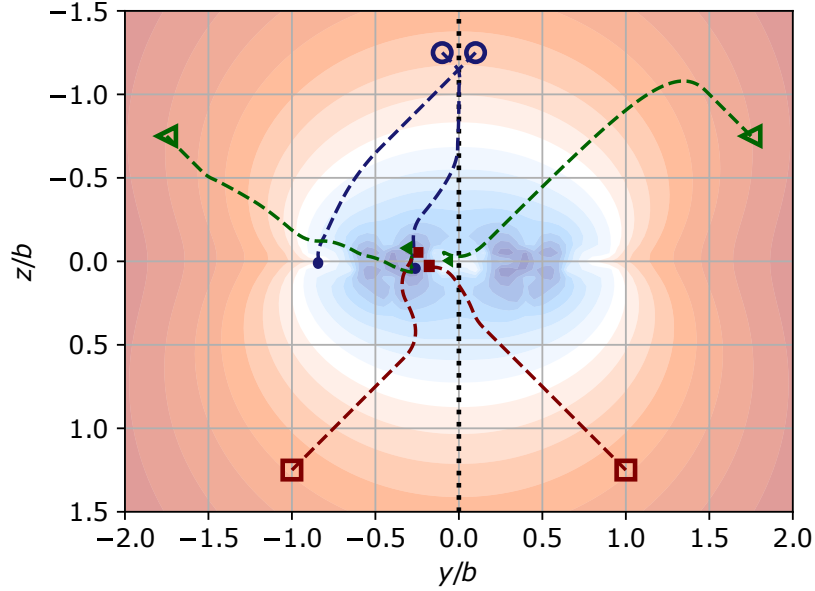


Figure 4.5: Trajectories of the aircraft from different initial conditions ($\square$, $\circ$, Δ) to the final state as computed by the optimal controller ($\blacksquare$, $\bullet$, $\blacktriangle$). Trajectories starting on the left-hand side are the result of the optimisation of the J_1 cost function. Trajectories starting on the right-hand side are the result of the optimisation of the J_2 cost function.

On the left-hand side of Fig. 4.5, we observe that all the trajectories computed by J_1 converge towards a similar position which is the position in the wake where Λ_0^{yz} is minimum. This is consistent with the anticipated trajectory consisting in following the gradient of Λ_0^{yz} . The optimization over an horizon T_h helps to converge towards

the global minimum of the observability field Λ_0^{yz} . This horizon also explains that the method does not exactly follow the gradient of the Λ_0^{yz} as it anticipates the future states and observability from the current time t up to $t+T_h$. Moreover, the Λ_0^{yz} field displayed on Fig. 4.5 represents the observability computed for a static follower on each relative position which is not the case when the follower evolves with a defined velocity along the computed trajectory.

The right-hand side of Fig. 4.5 shows different behaviors. We observe that the final positions vary with the initial condition without necessarily reaching the position where Λ_0^{yz} is minimum. The cost-function indeed does not optimize the *instantaneous* Λ_0^{yz} but a measure of observability characterized by the accumulation of observed local SFIM matrices, see Eq. (4.6). A trajectory does therefore not follow the instantaneous gradient of Λ_0^{yz} and reaches successive positions such that the *integral* criterion Λ_h^{yz} decrease over the computation horizon. Where the convergence of the solution was guaranteed with J_1 (it follows the gradient of the observability criterion), this is more difficult to show or verify in the case of J_2 , which also might explain the different final positions.

We observe that both optimisation strategies do not lead the follower towards appropriate positions in terms of energy consumption optimization. Indeed, instead of leading it on either side of the wake vortices, i.e., $\begin{bmatrix} y_F^{opt}, z_F^{opt} \end{bmatrix} \simeq [0.9b, 0.0b]$, the strategies rather lead them to the center of the wake or to the center of one of the vortices. These positions are not desired and are even dangerous because of the downwash wake-induced velocities and the important rolling moments that the aircraft undergoes. It is precisely these important efforts that are detected by the optimization strategies to lead the follower those positions. Indeed, the observability is increased thanks to these efforts. The observability and the positioning minimizing the consumption of the aircraft are thus antagonistic objectives that need to be balanced in a tracking strategy.

Trajectory optimisation with final state constraint

In the previous section, the cost functions only focus on the optimization of an observability criterion. However, in a formation flight context, the follower first desires to reach a position in the wake of leader that guarantees a reduction in its energy consumption. This desired position must also be constrained by safety and comfort considerations. We thus here modify the cost functions defined in Section 4.3.1 with the addition of a penalization term on the final state and control command, through a

Linear-Quadratic-Regulator (LQR). We write

$$\begin{aligned}
 J'_1(\mathbf{U}) &= \rho_1 \cdot \int_{t_0}^{t_0+T_h} \exp(\Lambda_0^{yz}(\mathbf{q}_{dyn}(\tau))) d\tau \\
 &+ \int_{t_0}^{t_0+T_h} \left[\mathbf{u}(\tau) \mathbf{R}_1 \mathbf{u}(\tau)^T + \left(\mathbf{q}_{dyn}(\tau) - \mathbf{q}_{dyn}^{\text{targ}}(\tau) \right) \mathbf{R}_2 \left(\mathbf{q}_{dyn}(\tau) - \mathbf{q}_{dyn}^{\text{targ}}(\tau) \right)^T \right] d\tau,
 \end{aligned} \tag{4.19}$$

$$\begin{aligned}
 J'_2(\mathbf{U}) &= \rho_2 \cdot \exp(\Lambda_h^{yz}(\mathbf{Q}_{dyn}(t_0, T_h))) \\
 &+ \int_{t_0}^{t_0+T_h} \left[\mathbf{u}(\tau) \mathbf{R}_1 \mathbf{u}(\tau)^T + \left(\mathbf{q}_{dyn}(\tau) - \mathbf{q}_{dyn}^{\text{targ}}(\tau) \right) \mathbf{R}_2 \left(\mathbf{q}_{dyn}(\tau) - \mathbf{q}_{dyn}^{\text{targ}}(\tau) \right)^T \right] d\tau,
 \end{aligned} \tag{4.20}$$

where the constant weights $\mathbf{R}_1 \in \mathbb{R}^{2 \times 2}$, $\mathbf{R}_2 \in \mathbb{R}^{4 \times 4}$, ρ_1 and ρ_2 are positive definite. We constraint the control command with the identity matrix $\mathbf{R}_1 = \mathbb{I}^{2 \times 2}$. The elements of matrix $\mathbf{R}_2$ are set to zero, except for the diagonal elements associated to the position, i.e., $\mathbf{R}_2^{y,y} = \mathbf{R}_2^{z,z} = 1$. This consists in constraining the follower's position only (and not its velocity), through the target state $\mathbf{q}_{dyn}^{\text{targ}}$ that contains the information of the final desired position. The coefficients ρ_1 and ρ_2 are trim parameters. Figs. 4.6 and 4.7 present the trajectories computed by J'_1 and J'_2 for different values of the trim parameters and different targets.

The optimal control strategies defined by the cost function J'_1 and J'_2 do not strictly impose the final state of the trajectory but rather rely on the LQR to converge towards it. Indeed, a long distance between the initial and the final position, e.g., in Fig. 4.6, requires a long and possibly variable time horizon T_h . Because of the strong nonlinearity of the problem and the iterative solver used (i.e., multiple shooting method), the resolution of the receding-horizon optimal control is costly and does not enable the imposition of a hard constraint on the last state either along the horizon T_h . This has a direct impact on the trajectories computed.

In fact, the cost functions reflect a multi-objective optimization. Therefore, their optimization is a compromise between the minimization of observability and the convergence towards the target position. We observe that low values of ρ_1 or ρ_2 lead to trajectories that approach the desired position more directly but with low observability. On the other hand, higher values allow to increase the observability along the route, with the risk of not reaching the target position. This is particularly true for J'_1 in the upper side of Fig. 4.6 or the left-hand side of Fig. 4.7. Indeed, its resulting trajectories never attain the target $[y^{\text{targ}}, z^{\text{targ}}] = [1.75b, 0.75b]$ or $[y^{\text{targ}}, z^{\text{targ}}] = [-1.1b, 0b]$ when $\rho_1 > 0$ and converge towards different steady positions. J'_2 on the other hand is

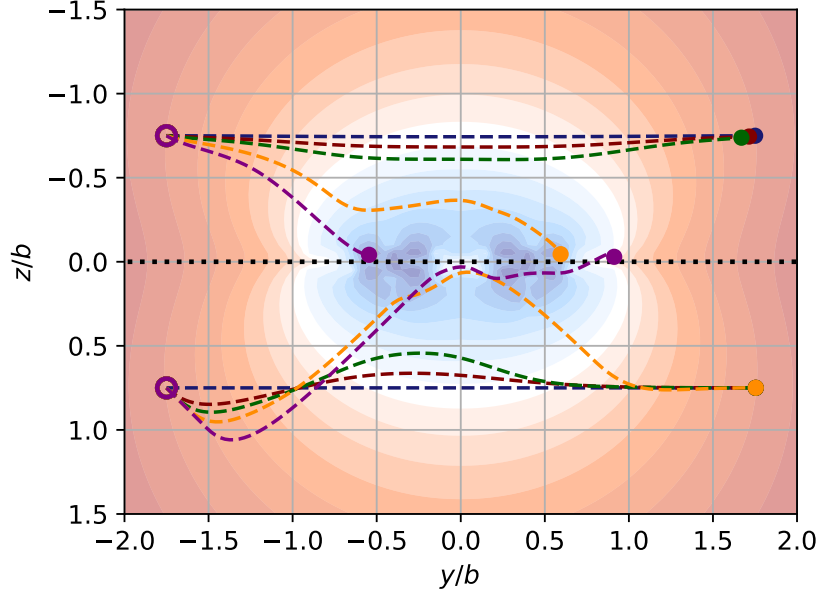


Figure 4.6: Optimal trajectories of the aircraft. The upper side shows trajectories computed by J'_1 , with $\rho_1 = [0, 100, 200, 500, 1000]$, i.e., (---), (---), (---), (---), (---). The lower side shows trajectories computed by J'_2 , with $\rho_2 = [0, 1000, 2000, 5000, 50000]$, i.e., (---), (---), (---), (---), (---).

able to converge towards the desired position provided that the weight ρ_2 associated to the observability is not too important. This explain by the fact that in J'_2 , the observability cost is computed by a single value for the whole trajectory, much akin to a penalization on the final state. As long as this penalization does not completely erase the LQR controller influence, the desired position is reached. However, by comparing the lower side of Fig. 4.6 and the right-hand side of Fig. 4.7, we observe that for a similar value $\rho_2 = 5000$, one initial condition can reach the desired position, while another does not. What's more, we observe in Fig. 4.7 that the optimization of J'_2 can lead to oscillating trajectories. The trim of the cost function is therefore an essential prerequisite for the design of a controller allowing the maximization of the observability while ensuring to reach a desired position.

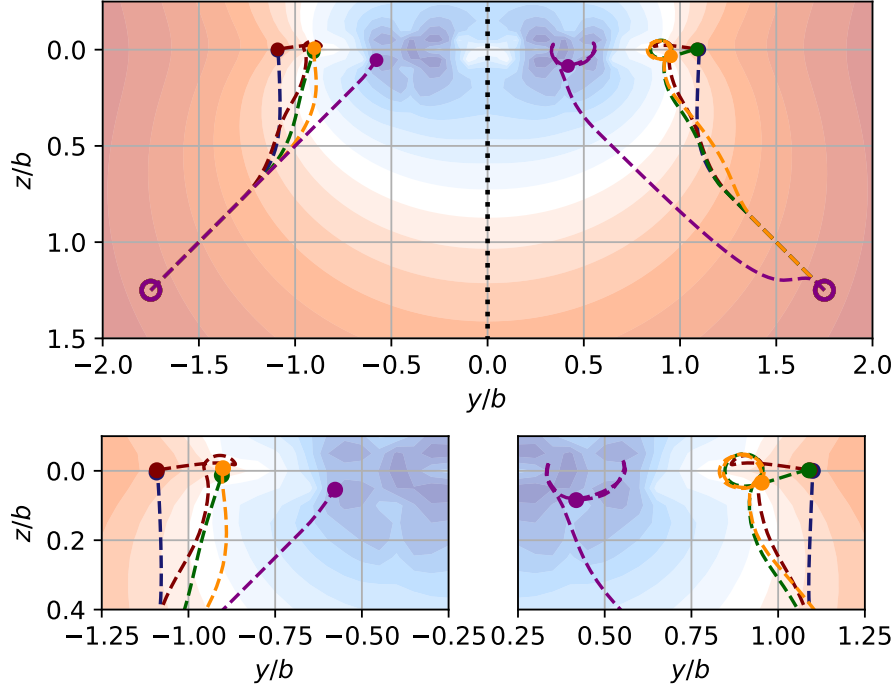


Figure 4.7: Optimal trajectories of the aircraft. The left-hand side shows trajectories computed by J'_1 , with $\rho_1 = [0, 100, 200, 500, 1000]$, i.e., (---), (---), (---), (---), (---). The right-hand side shows trajectories computed by J'_2 , with $\rho_2 = [0, 1000, 2000, 5000, 50000]$, i.e., (---), (---), (---), (---), (---).

4.4 Conclusions

In this chapter, rather than the optimization of the final position, we have focused on the optimization of the entire trajectory that an aircraft have to follow in order to converge towards that position in the wake of leader. Using the observability framework, we have employed criteria deduced from the Steady Fisher Information Matrix to quantify spatial regions in a wake prone to a degraded estimation performance by a wake sensing strategy. While this work had already been initiated in [47] using the Empirical Observability Gramian, we have been interested in the use of alternate measurements based on the aircraft dynamics rather than distributed pressure sensors along the wing. The former are indeed readily available in today's conventional aircraft flight computers.

We use in this chapter a simplified aerodynamic model that enables the computation of the aerodynamic efforts, i.e., forces and torques, undergone by an aircraft along its 6 degrees-of-freedom axes. However, we restrict its motion to a simplified two-degrees-of-freedom kinematics in the plane perpendicular to the streamwise direction. This enables the reduction of the complexity of the optimization problem. Confirming other previous studies, we have highlighted that the wake intensity was the least observable parameter of the system. This was here showed using acceleration measurements perceived by the follower. Therefore, we have restricted our attention to the observability of the wake position and we have clearly identified an alternation between areas prone to the observability of the lateral position and areas prone to the observability of the vertical position. This showed the need for a variation of the relative position of the follower with respect to the wake in order to improve the observability of the system. Finally, we have presented two optimal controllers designed to optimize aircraft trajectories. The first one used instantaneous values of the observability criteria while the second was based on integral values of the criteria. We showed that the latter was able to deal with the redundancy of the information gathered in the measurement and was therefore more prone to an improvement of the performance of a tracking strategy over time. Nevertheless, we have shown that maximizing the observability only along trajectories does not lead the follower to an optimal position in terms of energy savings. On the contrary, an energetically unfavorable final position is reached. A control strategy that associates observability and convergence towards a desired position have therefore be investigated.

Because of the complexity and the computational cost associated with the use of the optimal control framework, we have limited the problem to the two dimensional kinematics of the aircraft in this work. Taking into account all the dynamics can be considered in future work by looking at different optimization strategies. Also, by taking the Unsteady Fisher Information Matrix rather than the Steady one into account, an optimized strategy will be able to adapt to the loss of information related to the measurement noise and to the uncertainty related to the wake model itself. Finally, the approach presented in this chapter is based on the use of additional control tools. As already mentioned in Chapter 3, the use of other frameworks, such as the efficient RL one, can be considered in order to optimize the multi-objective problem of wake tracking under the observability constraint.

Aircraft Formation Wake dynamics

The content of this chapter is partially inspired by an article published in the proceedings of the AIAA Scitech Forum 2022 (San Diego), entitled "Dynamics of the Wake Vortices of a Two-aircraft Formation, Hazard Assessment at Large Distances and Sensitivity Analysis" [118], by I. Ransquin, M. Duponcheel, and P. Chatelain.

5.1 Introduction

The flow physics of wake vortex systems emanating from fixed wing aircraft have been intensively investigated in the past 30 years, resulting in a vast literature. This is explained on the one hand by all the fundamental aspects inherent to the dynamics of such fixed wing wake vortices, from their emission to their far-wake evolution. On the other hand, aircraft wake dynamics are related to much more practical issues as they have a direct impact on the Air Traffic Management (ATM), both at the airport level (i.e., aircraft spacing enforced at takeoff [29] or landing [61]) and at the air corridor level (i.e., wake vortex encounters [11]). Each of these perspectives has motivated numerous works, among others the study of the wake vortices formation and geometry, their decay and eventual demise, their interactions with each other and with their environment. The reviews of Spalart [132] and Breitsamter [14] provide general overviews of all these topics for the curious reader.

As described in Chapter 1, the vortex sheet emanating from a single wing undergoes a roll-up process which leads to the formation of a two-vortex system (2-VS). Several models have been developed to describe the resulting vortices, essentially by

modeling the circulation profiles, as summarized in [60]. Related research efforts have also focused on understanding the numerous instabilities occurring in the 2-VS. Those range from the small scale elliptic instability [92, 90] to the large scale motions of the vortices referred to as the *Crow instability* [38], which lead to the demise of the wake through reconnections of the wake vortices (see [126] for experimental and [59] for numerical investigations).

In this chapter, we are interested in the problems of wake vortices generation and interactions in the context of formation flight. For a two-aircraft formation, the wake indeed results in a four vortex system (4-VS), made of the vortex pairs emanating from the leader and the follower. The leader vortex pair includes the Leader Outboard Vortex (LOV) and the Leader Inboard Vortex (LIV), while the follower vortex pair includes the Follower Inboard Vortex (FIV) (i.e., the vortex closest to the LIV) and the Follower Outboard Vortex (FOV). This 4-VS is to be differentiated from the 4-VS that emanates from a single aircraft during certain flight phases, during which secondary vortices are produced by the horizontal tail or the deflection of flaps. Such configurations have been investigated both theoretically [56, 14] and numerically [26] because of their impact on the decay of the primary vortices.

Civilian applications target a concept known as extended formation flight [100] it better addresses safety concerns. In that case, the follower is flying at several tens of wingspans downstream of the leader, which leads to an asymmetric 4-VS. Indeed, the vortex pair produced by the leader affects the vortex sheet that emanates from the follower right upon its conception. In [139], Unterstrasser et al. model the wake of a two-aircraft formation as two pairs of rolled-up vortices. Using Large-Eddy Simulations (LES), they study the movement of each vortex individually and investigate the resulting far-field wake, and in particular the plume dimensions and the contrail formation. They also show the important sensitivity of the resulting 4-VS dynamics to the relative position of the two vortex pairs that made the 4-VS, compared to the relatively simple dynamics of a 2-VS characterized by the sink of the vortices due to their self-induced velocity.

Some crucial questions remain. In a formation flight context, this sensitivity to relative position leads to even more complex behaviors, as the position between the wake of the leader and the follower is not constant over time. Indeed, wake tracking strategies such as the one described in Chapter 2, aim to maintain a constant relative position between the follower and the leader's wake. However, it is very likely that the relative distance will be noisy and uncertain because of the inaccuracy of the chosen tracking equipment, the leader's motion or turbulence-induced perturbations of the leader's wake. The uncertainty on this relative position, associated with the

non-linear character of the interaction between vortices, leads to an uncertainty on the position of the wake vortices, from their emission to the far-wake. In this chapter, we introduce a methodology to estimate in a probabilistic sense the characteristics of a wake produced by two aircraft in formation flight whose relative position is uncertain. For that purpose, we employ LES in order to capture the roll-up of the vortex sheets originating from the leader and follower, their mutual interactions, and their evolution within ambient turbulence into a fully developed 4-VS. The numerical tool is based on the state-of-the-art Vortex Particle-Mesh (VPM) flow solver already described previously in Chapter 1. We propose a calibrated combination of 3D Space Developing (SD) and 3D Time Developing (TD) VPM simulations to capture the full phenomenological chain of such wakes. To our knowledge, the combination of both SD and TD simulations has never been performed in the context of formation flight. Simulations are performed for several formation flight configurations around a reference in order to quantify the impact of the imprecision of follower's relative position on wake propagation and on the resulting consequences and constraints for the air traffic management, e.g., in terms of Rolling Moment Coefficient sensed in the wake of the formation.

This chapter is structured as follows. We present in Section 5.2 the methodology that involves the use of both SD and TD simulations and we describe the aircraft model and the formation flight configurations studied. In particular, we justify the need for two aircraft configurations, one characterized by a high load of the wing and the second characterized by a more classical load, in order to validate the simulation methodology developed. Opening the results section, we first bring a particular attention to the validation of the simulation methodology in Section 5.3. We then describe the sensitivity observed in the wake vortices evolution, and the wake vortex characteristics with respect to the positioning of the follower relative to the leader's wake (Section 5.4). This leads to the description of a probabilistic analysis of the wake evolution; we finally describe and compare the global characteristics of the wake with those of a single aircraft wake (Section 5.4.3).

5.2 Methodology

5.2.1 Combination of Time Developing and Space Developing simulations

In this work, we use the VPM solver with the Immersed Lifting and Dragging Line [23] method to simulate the wake of a two-aircraft formation. We use SD simulations to accurately reproduce the 3D interactions phenomena occurring during the roll-up of

the vortex sheets emanating from the leader and the follower. From these simulations, we generate an initial condition for a TD simulation, which then allows to study the large scale and long-term evolution of the asymmetric wake resulting from the two-aircraft formation.

In order to generate the initial condition for the TD simulation, one could think of simply extracting at one specific time, a volume of the SD simulation corresponding to the simulation domain of the TD simulation, as presented in Fig. 5.1a. However, it must be realized that all the slices in the extracted volume are located at different distances downstream of the follower and therefore correspond to different wake ages. The wake in the volume is thus not statistically homogeneous in the axial direction and, in particular, its vertical position is (slightly) different at both extremities of the volume. This leads to problems when periodic boundary conditions are applied in the TD simulation. The longer the TD domain, the more significant this issue is. This is thus not the proper way to initialise the TD simulation.

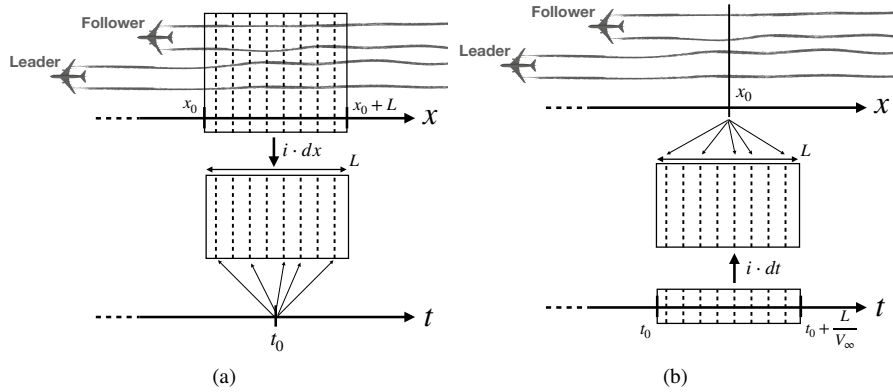


Figure 5.1: Generation of the initial condition of the TD simulation from the SD simulation. The incorrect way consists in selecting a volume in the SD simulation to generate an initial volume for the TD simulation (5.1a). The appropriate way consists in copying the same slice of the SD simulation, at different times (5.1b).

In order to properly initialise it, one has to realize that the SD and TD simulations reverse the effects of space and time. On the one hand, the age τ of the vortex is related to the streamwise distance x in SD simulations whereas it is related to time t in TD simulations. On the other hand, different occurrences i of the wake are related to a temporal discretization $i \cdot dt$ in a SD simulation whereas they are related to different streamwise positions $i \cdot dx$ in a TD simulation. The average wake is thus obtained

by time-averaging in a SD simulation whereas it is obtained as space-averaging (in the axial direction) in the TD situation. Consequently, the initial condition is obtained by considering that the time sampling $i \cdot \Delta t$ at a fixed station in the SD simulation can be translated into a spatial sampling $i \cdot \Delta x$ in the TD simulation, while respecting constraints of spatial and temporal equivalence, see Fig. 5.1b. All the extracted slices correspond to the same vortex age and the TD initial volume is thus statistically homogeneous in the axial direction. This equivalence between time and space is akin to Taylor's frozen turbulence hypothesis [137]. Indeed, if the turbulent fluctuation field is frozen (i.e., does not evolve with time) and is simply advected through the sampling station, then the space-time equivalence is exact. For real turbulence, the turbulence is not frozen, yet the Taylor's hypothesis is reasonable for sufficiently large scales. Since the space-time equivalence is not exact, the procedure introduces some small-amplitude non-physical small-scale structures in the TD initial condition. However, those scales quickly evolve into proper small-scale turbulence that only weakly influence the longterm evolution. More importantly, the spatial spectrum of the fluctuations in the TD correctly reproduces the frequency spectrum in the sampling plane. This is thus the best initial condition in order to perform TD simulations of the development of wake vortex instabilities which lead to the fast decay of the wake.

This two-step methodology is justified by the geometrical parameters envisioned for the aircraft of the formation. We study a flight scenario that involves two identical aircraft with geometrical properties similar to an A350. Assuming such aircraft have a wingspan $b = 64m$, a wing aspect ration $AR = 9.75$, a mass $m = m_L = m_F = 250T$ and fly in steady flight at $U_\infty = 250$ m/s at FL330, we approximate the circulation of the wake vortices Γ_0 as it was already described in Chapter 1 using

$$mg = \rho U_\infty \Gamma_0 b_0, \quad (5.1)$$

where g is the gravity constant, ρ the ambient air density and $b_0 \simeq b \left(\frac{\pi}{4} \right)$ the vortex spacing distance between the two vortices. We obtain $\Gamma_0 \simeq 485$ m²/s. We then compute the characteristic quantities

$$V_0 = \frac{\Gamma_0}{2\pi b_0} \simeq 1.55 \text{ m/s} \quad (5.2)$$

$$t_0 = \frac{b_0}{V_0} \simeq 32.6 \text{ s}, \quad (5.3)$$

where V_0 is the vertical self-induced velocity of the wake, and t_0 is the characteristic time. Under those conditions, the simulation of the wake propagation over one characteristic time t_0 requires to simulate the wake over a distance $U_\infty \cdot t_0 = 8150$ m $\simeq 130$ b.

Computing the propagation of the wake of a formation over several characteristic times t_0 , as it would be required for an analysis of the impact of such a wake on ATM, would lead to prohibitive computation costs which justifies the combined use of SD and TD simulations.

The aircraft geometrical parameters we have just described are associated to a first aircraft configuration that we call the *Normally Loaded Aircraft* (NLA). In this work, the two-step simulation methodology is also applied to a second aircraft configuration that we call *Heavily Loaded Aircraft* (HLA), because it is flying at a lower speed, for an equivalent weight, and therefore has a higher lift coefficient C_L . Indeed, for an aircraft in vertical equilibrium (i.e., the lift force compensates the weight) and from the definition of the lift coefficient we have

$$mg = \frac{1}{2}\rho (U_{\infty}^{NLA})^2 \frac{C_L^{NLA}}{\mathcal{R}} b^2, \quad (5.4)$$

$$= \frac{1}{2}\rho (U_{\infty}^{HLA})^2 \frac{C_L^{HLA}}{\mathcal{R}} b^2. \quad (5.5)$$

For HLA, we use a flying velocity $U_{\infty}^{HLA} = 0.5U_{\infty}^{NLA}$ and we deduce $C_L^{HLA} = 4C_L^{NLA}$, every other parameters remaining the same. This theoretical HLA is used to validate the SD-TD simulation methodology, as it enables to compute with a SD simulation the same wake propagation as the NLA on a four times shorter computational domain. Indeed, from Eqs. 5.1, 5.2 and 5.3 one deduces that the HLA generates a wake twice as strong as the NLA (i.e., $\Gamma_0^{HLA} = 2\Gamma_0^{NLA}$), with a self-induced velocity twice as high (i.e., $V_0^{HLA} = 2V_0^{NLA}$); the HLA wake time scale is therefore two times shorter (i.e., $t_0^{HLA} = t_0^{NLA}/2$). We now explicitly define the dimensionless age of a vortex in the wake of the SD simulation τ_{SD} at a distance x downstream of the follower

$$\tau_{SD} = \frac{x_{SD}}{U_{\infty}t_0}. \quad (5.6)$$

Therefore, at the same distance x_{SD} , we have $\tau_{SD}^{HLA} = 4\tau_{SD}^{NLA}$, meaning that at that same distance, the wake of the HLA is four times older or, equivalently, at the same age, the wake of the HLA is four times closer to the aircraft. With the HLA, we are thus able to compute the full wake phenomenological chain (i.e., from the sheet rollup, to the far wake Crow-type oscillation) on a four times shorter SD computational domain. This makes possible a comparison of the wake produced by the SD and the combined SD-TD methodology; it is presented in Section 5.3. The age of the wake in a TD simulation τ_{TD} is defined by the time t_{TD} of the simulation, due to the reverse effects

of time and space in SD and TD simulations, i.e.,

$$\tau_{TD} = \frac{t_{TD}}{t_0}. \quad (5.7)$$

A summary of the NLA and HLA geometrical characteristics is provided in Table 5.1.

	NLA	HLA
Span	$b = 64$ m	$b = 64$ m
Aspect ratio	$\mathcal{R} = 9.75$	$\mathcal{R} = 9.75$
Chord distribution	Elliptic	Elliptic
Sectional lift slope	$a_0 = 2\pi$	$a_0 = 2\pi$
Main wing lift factor	$C_L = \mathbf{0.47}$	$C_L = \mathbf{1.87}$
Mass	$m = 250000$ kg	$m = 250000$ kg
Speed	$U_\infty = \mathbf{250m/s}$	$U_\infty = \mathbf{125m/s}$
Air density	$\rho = 0.4$ kg/m ³	$\rho = 0.4$ kg/m ³

Table 5.1: Geometrical parameters of the aircraft used in the simulations.

5.2.2 Aircraft model

While the simulation of the wake of a single aircraft does not necessarily require to take into account the vehicle dynamics, the simulation of the wake of a formation requires to be able to trim the dynamics of the follower. Indeed, in a formation flight context, the follower aircraft benefits from the upwash generated by the leader's wake, which ultimately results in excess lift if we were to set the follower at the same pitch angle as the leader's. Moreover, the follower undergoes a rolling moment, which has to be compensated by an action of the ailerons. Those control actions ultimately impact the complete 6DOF dynamics of the aircraft, as described in Chapter 1. For the sake of simplicity though, we adopt an aircraft configuration brought to its essential components for both the NLA and HLA. In this study, we represent the aircraft as a single wing whose pitch angle θ can be modified and which is equipped with outboard ailerons, as depicted in Fig. 5.2. The ILDL method implemented in the VPM solver as described in Chapter 1 is used to simulate the wake of such a wing. Following that method, the aileron deflection is modeled as a local modification of the polar in order to reproduce the corresponding change in aerodynamic loads. The controller presented in Fig. 5.3 uses a Proportional-Integral-Derivative (PID) feedback to adapt the pitch angle θ and the deflection of the ailerons δ_{Ail} in order to compensate exactly the weight of the plane by its lift (i.e. to cancel the resulting vertical force F_z), and to cancel the

rolling moment M_x . For each simulation presented in this work, the inertial position of the aircraft is fixed over time, as well as the roll and yaw angles of the aircraft which are set to 0. In spite of these simplifications, the force and moment balances ensure that the vortices shed by the follower are captured accurately.

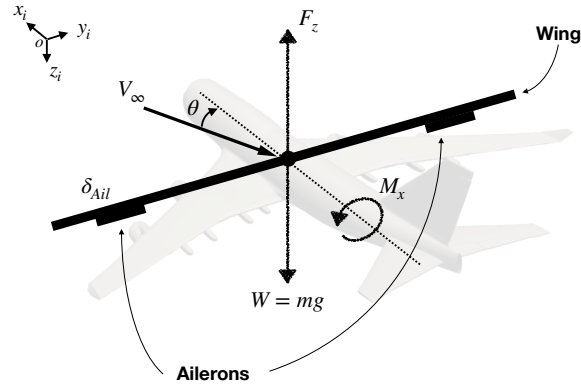


Figure 5.2: Simplified aerodynamic model of the aircraft represented as a single ILDL equipped with ailerons, the deflection of which, δ_{Ail} , is used to compensate the rolling moment M_x . The pitch angle is modified to compensate the weight W with the appropriate vertical force F_z .

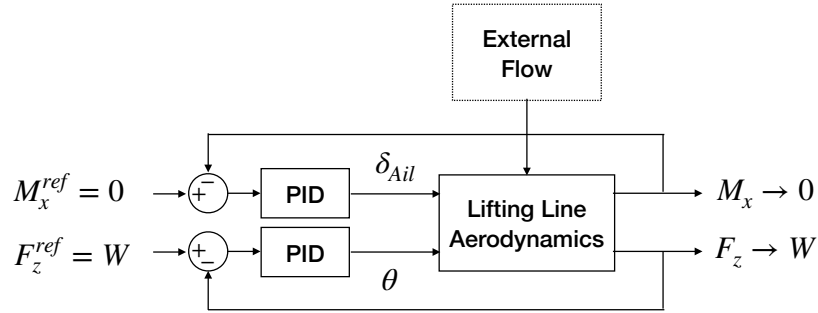


Figure 5.3: Block diagram of the PID controllers designed to adapt the ailerons deflection δ_{Ail} and the pitch angle θ of the aircraft in level flight in order to trim the aircraft under the influence of external flows (i.e., turbulence or wake vortices).

5.2.3 Simulation setup

Two different simulation setups are used for the SD and TD simulations, one associated to the HLA configuration and the other to the NLA configuration. As mentioned in Section 5.2.1, the HLA setup is designed to validate the combined SD-TD methodology. The associated SD simulation setup therefore requires to simulate the wake far downstream of the formation. The follower is located ten spans behind the leader in the streamwise direction, and the wake is propagated up to 58 spans downstream of the formation. This distance ensures that we capture the initial large scale and long-term evolutions of the vortices in the wake. In particular, one might expect to observe strong interactions between the two inner vortices of the 4-VS.

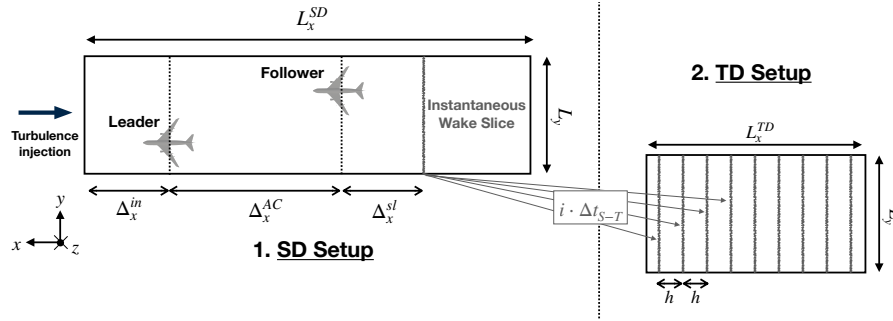


Figure 5.4: Description of the SD and TD simulation setups.

The SD simulation of the HLA configuration is also used to extract an initial condition for the TD simulation by using the instantaneous vorticity fields observed on 2D slices normal to the streamwise direction, as described in Fig. 5.4. We select a slice located $\Delta_x^{sl} = 2b$ behind the follower and generate a computational domain of size $L_x^{TD} \times L_y \times L_z = 16b \times 4b \times 4b$ into which we copy the 2D vorticity field computed by the SD simulation every time step such that $\Delta t_{S-T} = \frac{h}{U_\infty}$, where h is grid resolution of the mesh used in the TD simulation ($h = b/64$). The temporal sampling of the slice observed from the SD simulation associated with the streamwise extent of a TD simulation guarantees to capture the instability modes up to the theoretical fastest-growing Crow mode [38] of the equivalent pair. This explains the choice for a $16b$ -long TD computational domain.

The NLA setup is used to generate various initial conditions in order to quantify the impact of the relative position of the aircraft on the vortex propagation. It is not used to simulate the wake far downstream of the formation, but it has to adequately represent the formation itself. The follower is therefore located twenty spans behind the leader in

the streamwise direction. This distance ensures the roll-up of the leader's wake without increasing disproportionately the computational domain size, here at $L_x^{SD} \times L_y \times L_z = 40b \times 4b \times 4b$. The building of the initial condition for the TD simulation of the NLA configuration follows the same methodology as the HLA configuration, and we select the slice located $\Delta_x^{sl} = 4b$ behind the follower to extract the initial condition. We show thereby in Section 5.3 that beyond two to three spans behind the follower, the 3D interaction phenomena are no longer governing the flow which becomes mainly 2D, which assures the $4b$ slice to be an appropriate choice.

For the SD simulations of both the HLA and NLA, synthetic turbulence reproducing the spectrum of Homogenous Isotropic Turbulence (HIT), and generated using the Mann algorithm [96], is smoothly injected at the inlet of the computational domain to reproduce as closely as possible actual flight and wake conditions. This affects the wake generation mechanism itself and seeds the flow with perturbations that may trigger instabilities. The turbulence is characterized by an integral length scale $L = 530$ m and a dimensionless Eddy Dissipation Rate (EDR) $\epsilon^* = \frac{\epsilon b_0}{V_0^3} \simeq 2 \cdot 10^{-4}$, corresponding to weak turbulence [44], which is a reasonable assumption for a formation flight application. According to the flow perceived (i.e., turbulence and wake vortices), the leader and follower adapt their pitch and aileron deflections in order to achieve vertical and roll axis equilibrium as described in Section 5.2.2. The parameters used for all the simulations are gathered in Table 5.2.

	NLA	HLA
Space developing		
Domain size ($L_x^{SD} \times L_y \times L_z$)	$40b \times 4b \times 4b$	$72b \times 4b \times 4b$
Grid resolution	$h/b = 1/64$	$h/b = 1/64$
Inlet buffer	$\Delta_x^{in} = 4b$	$\Delta_x^{in} = 4b$
Aircraft spacing	$\Delta_x^{AC} = 20b$	$\Delta_x^{AC} = 10b$
Extrusion slice	$\Delta_x^{sl} = 4b$	$\Delta_x^{sl} = 4b$
Turbulence integral length scale	$L = 530m$	$L = 530m$
Eddy Dissipation Rate (EDR)	$\epsilon^* \simeq 0.0002$	$\epsilon^* \simeq 0.0002$
Time developing		
Domain size ($L_x^{TD} \times L_y \times L_z$)	$16b \times 4b \times 4b$	$16b \times 4b \times 4b$
Grid resolution	$h/b = 1/64$	$h/b = 1/64$

Table 5.2: Numerical parameters of the VPM simulations.

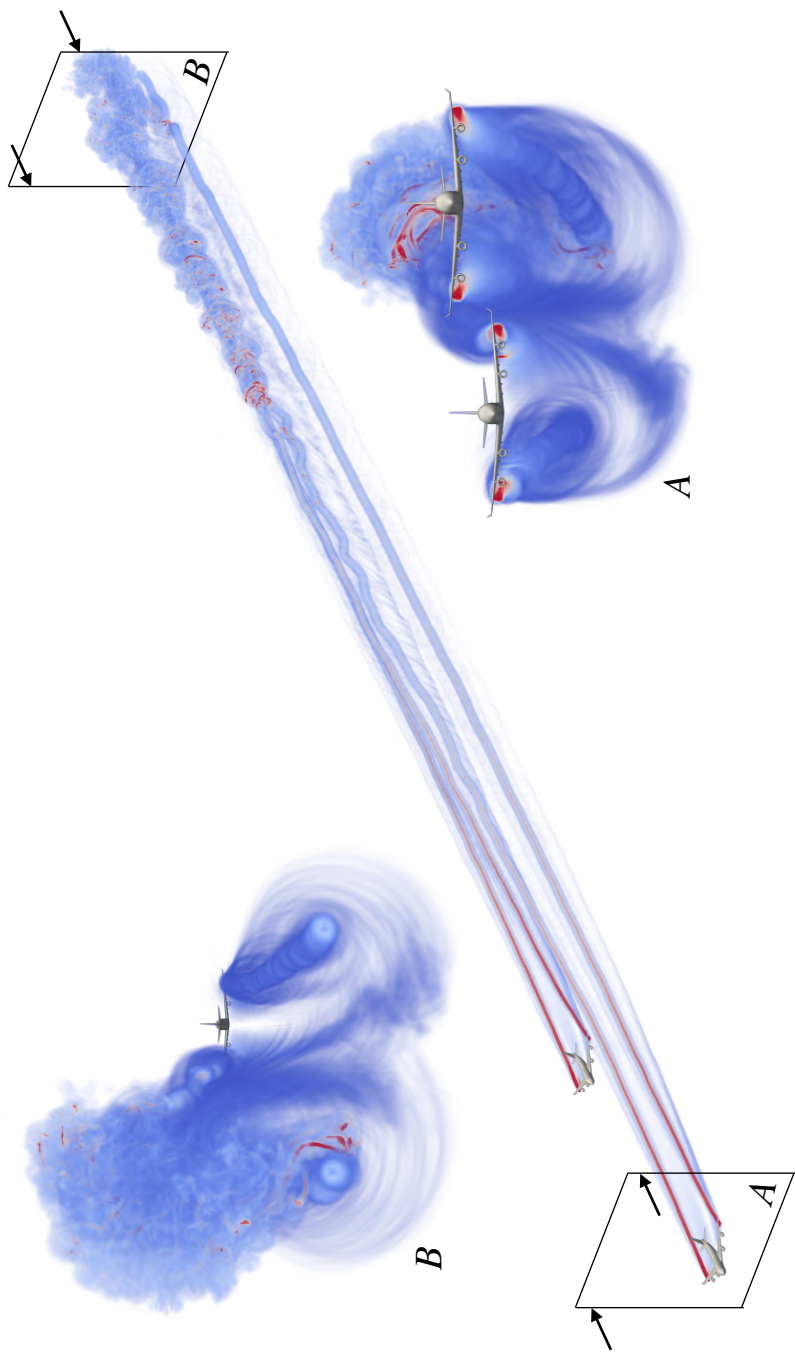
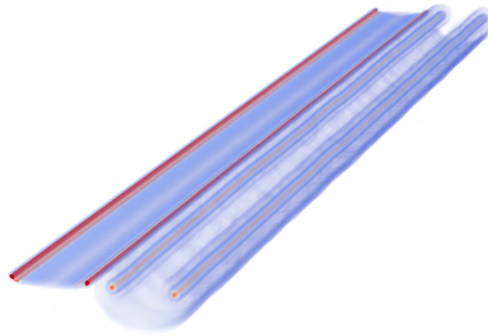
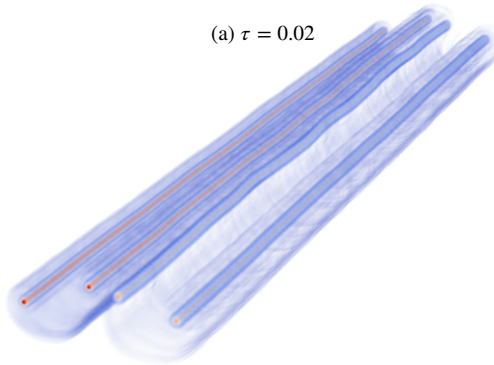
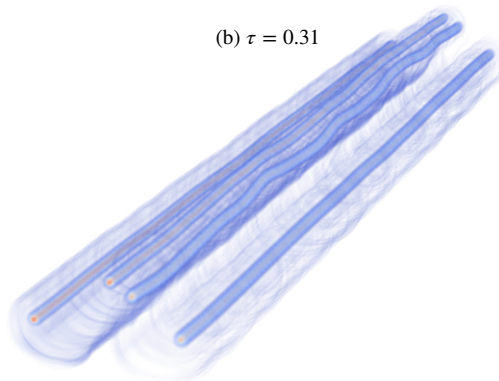


Figure 5.5: Volume rendering of the vorticity shed in the wake of the two-aircraft formation (HLA setup, SD simulation).

(a) $\tau = 0.02$ (b) $\tau = 0.31$ (c) $\tau = 0.62$

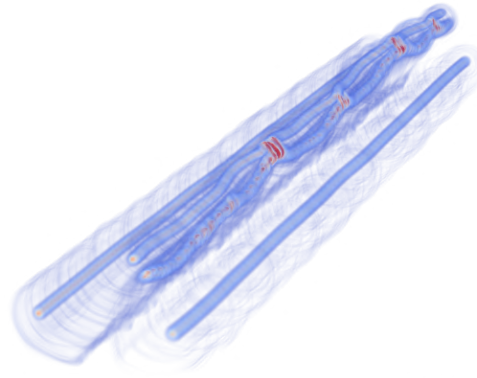
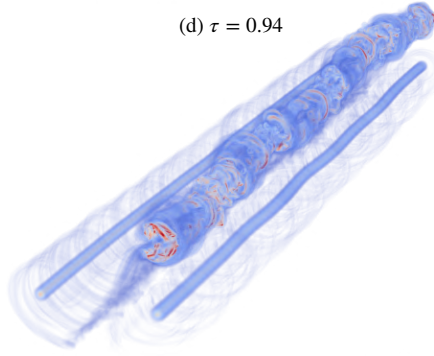
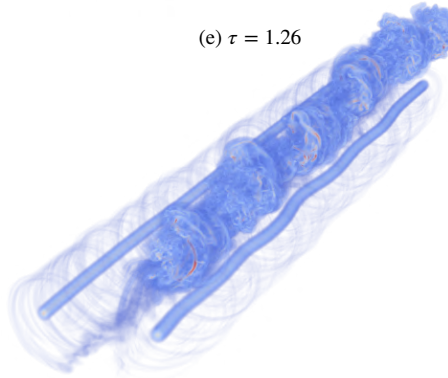
(d) $\tau = 0.94$ (e) $\tau = 1.26$ (f) $\tau = 1.57$

Figure 5.6: Volume rendering of the vorticity at six different wake ages. The initial condition $\tau = 0$ of this TD simulation is extracted from the SD simulation presented in Fig. 5.5.

5.3 Validation of the SD-TD methodology

In this section, we validate the methodology that involves the combination of TD and SD simulations for the simulation of an aircraft formation wake. We use the HLA configuration described in Section 5.2.3 in order to compute wake characteristics over long distances downstream the formation using both the SD and TD simulations. The wake of a two-HLA formation as computed by the SD simulation setup described in Table 5.2 is visualized in Fig. 5.5.

In the near wake of each aircraft, the vortex sheet starts to roll up about the locations with the most intense vorticity, in wing tip regions. The vorticity hence concentrates into the tip vortices, which results in the formation of the aforementioned 4-VS system. In weak turbulence and in the presence of low profile drag, the propagation of aircraft vortices usually tends to be smooth, leading to straight and steady vortices. This smooth propagation is observed for the FOV, i.e., the right-most vortex observed from the rear view in Fig. 5.5. The inboard vortices FIV and LIV (, i.e., the middle vortices) exhibit drastically different behaviors because of their close vicinity. The surrounding turbulence and wake velocity deficit inherent to wing profile drag triggers instabilities and the oscillation of the inboard vortices, i.e., Crow type oscillation. The amplitude of the oscillations grows and finally leads to the reconnection of vortices and the development of small scale structures. Those structures undergo stretching and instabilities of their own, leading to the dissipation of the inboard pair. In the formation displayed in Fig. 5.5 the LOV (i.e., the left-most vortex observed from the rear view in the figure) has a similar behavior to the FOV in most of the simulation. However, because of its biased propagation, the inboard pair gets closer to the LOV which leads to initial interactions that might eventually lead to a larger structure made of the LOV, LIV and FIV. A more in-depth analysis of the process just described will allow us to highlight the importance of the follower's position relative to the leader's wake in the global wake propagation and in the impact on the air traffic management.

The same wake, computed by the TD simulation setup this time, is visualized in Fig. 5.6. Following the methodology described in Section 5.2.3, the initial vorticity field at $\tau = 0$ is built from successive instantaneous vorticity fields extracted at one defined slice downstream of the follower from the simulation displayed in Fig. 5.5.

At early ages of the flow, one observes that the vortices of the formation are mainly straight, with a weak oscillation of the LOV and LIV betraying the impact of the ambient turbulence during the SD simulation. The LIV and LOV are almost totally rolled up and turbulent, unlike the FIV and FOV, between which we still observe a part of the vortex sheet of the follower. The LIV and FIV then move up and towards the left-hand

side of the computational domain. This movement is explained by the self-induced velocity of the vortices, and by the asymmetry in terms of vortex roll-up and intensity that leads to the leftward displacement. Similar to the SD simulation, a Crow-type oscillation appears between the LIV and FIV. However, as they have slightly different circulations, due to different flow conditions and the younger age of the FIV, the reconnection events do not lead to typical vortex rings. One rather observes the generation of secondary structures, akin to the omega loops observed in [107]. Again, these structures undergo stretching and instabilities of their own, leading to the dissipation of the inboard vortex pair.

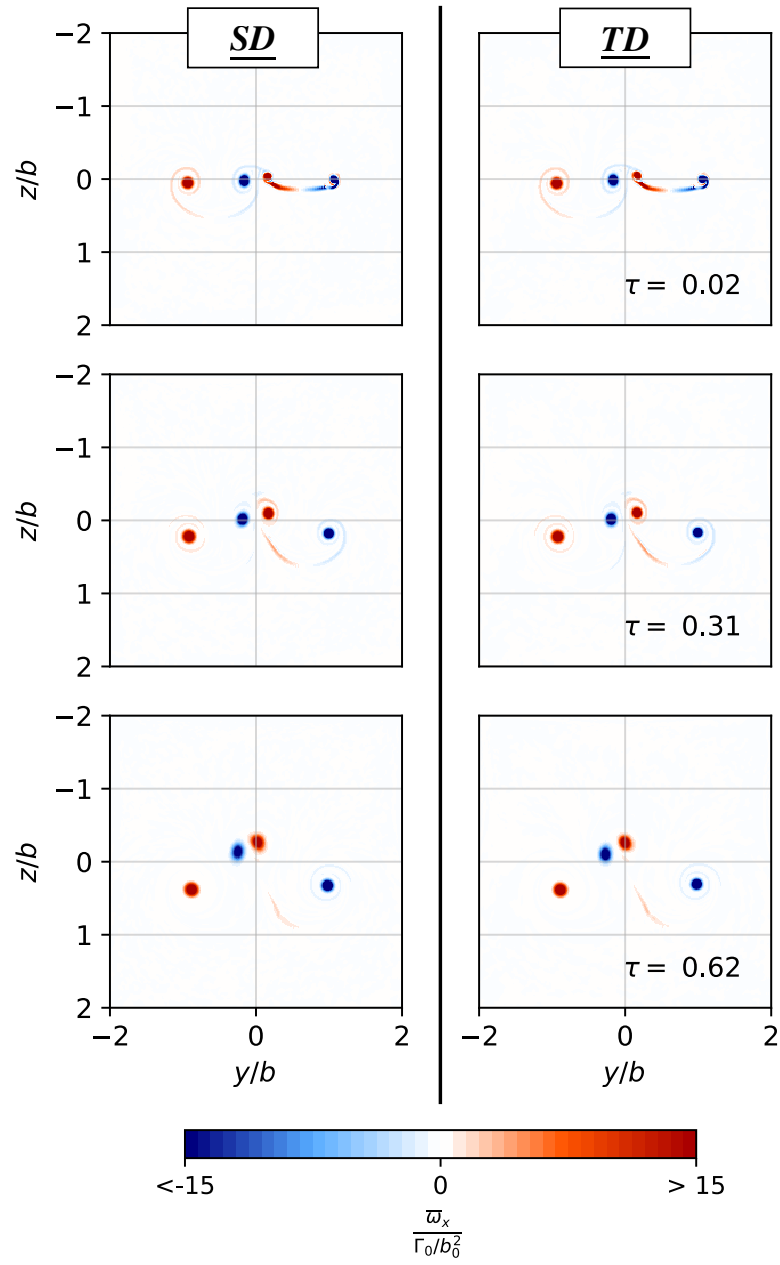
5.3.1 Wake mean flow and vortex position

The comparison between the simulations is performed through the analysis of statistics in the wake gathered in several $\hat{e}_y^i - \hat{e}_z^i$ planes (i.e., slices). In the SD simulation, each slice gathers several temporal occurrences $i \cdot \Delta t$ of the wake at the same age $\tau_{SD} = \frac{x_{SD}}{U_\infty t_0}$. The mean fields are obtained by averaging these temporal occurrences $i \cdot \Delta t$, which is similar to the temporal averaging of the field. In particular, the average axial vorticity field is computed using

$$\begin{aligned} \bar{\omega}_x(\tau_{SD}, y, z) &= \frac{1}{T_h} \int_t^{t+T_h} \omega_x(\tau_{SD}, y, z, t'_{SD}) dt'_{SD} \\ &\simeq \frac{1}{N} \sum_{i=0}^N \omega_x(\tau_{SD}, y, z, t_{SD}^i) \\ &= \frac{1}{N} \sum_{i=0}^N \omega_x^i(\tau_{SD}, y, z) = \langle \omega_x^i(\tau_{SD}, y, z) \rangle, \end{aligned} \quad (5.8)$$

where T_h is a time horizon that integrates all the temporal scales of the wake, N is the number of selected time occurrences in that horizon, and $\langle \cdot \rangle$ is the average of all the occurrences.

In the TD simulation, different occurrences of the wake at a chosen age $\tau_{TD} = \frac{t_{TD}}{t_0}$ are obtained among different slices $i \cdot \Delta x$ spread along the streamwise direction of the simulation. The mean fields at a given age τ_{TD} are thus obtained by averaging over all the slices $i \cdot \Delta x$, which is similar to a longitudinal averaging of the field. The average



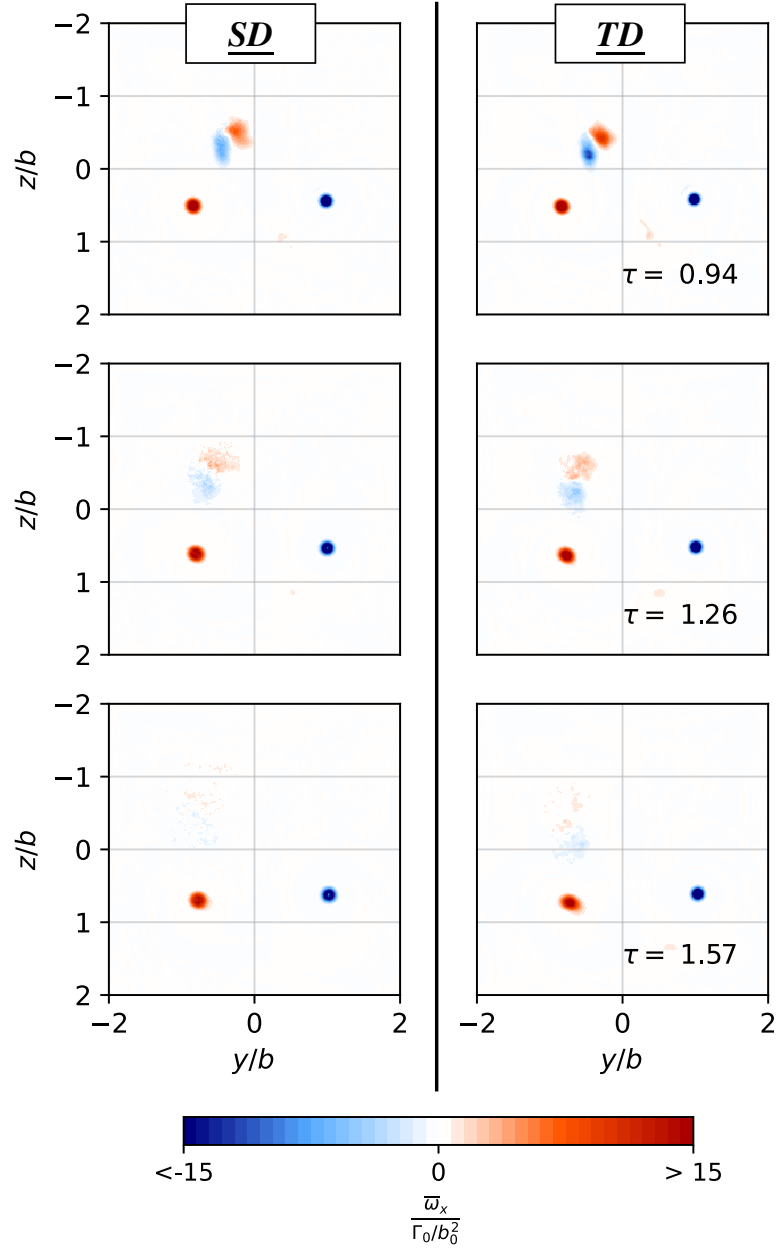


Figure 5.7: Volume rendering of the magnitude of the mean vorticity shed in the wake of the two formation (HLA setup) for the SD (left columns) and TD (right columns) simulations.

axial vorticity field is thus:

$$\begin{aligned}
 \bar{\omega}_x(y, z, \tau_{TD}) &= \frac{1}{L_x^{TD}} \int_0^{L_x^{TD}} \omega_x(x_{TD}, y, z, \tau_{TD}) dx_{SD} \\
 &\simeq \frac{1}{N} \sum_{i=0}^N \omega_x(x_{TD}^i, y, z, \tau_{TD}) \\
 &= \frac{1}{N} \sum_{i=0}^N \omega_x^i(y, z, \tau_{TD}) = \langle \omega_x^i(y, z, \tau_{TD}) \rangle . \quad (5.9)
 \end{aligned}$$

In the following results, we take care to respect these differences in the computation of the mean fields, and we represent the statistics in terms of the non-dimension age $\tau = \tau_{SD} = \tau_{TD}$ that we now interchangeably use to define both the axial position in a SD simulation (i.e., $x_{SD} \rightarrow \frac{x_{SD}}{U_\infty t_0} \rightarrow \tau$) and the simulation time of the TD simulation (i.e., $t_{TD} \rightarrow \frac{t_{TD}}{t_0} \rightarrow \tau$). Throughout this chapter, we thus drop the subscripts $\cdot_{SD}$ and $\cdot_{TD}$ from physical symbols. We also replace both the time dependence $i \cdot \Delta t$ of a field computed by an SD simulation and the axial dependence $i \cdot \Delta x$ of a field computed by a TD simulation by the superscript $\cdot^i$, because of their equivalence. With those conventions, we can now use the same notations for both the SD and TD simulations.

The averaged axial vorticity fields computed by SD and TD simulations are presented in Fig. 5.7 for several formation wake ages. We observe that the fields remain very similar between the two simulations, even far downstream of the formation. Differences become noticeable at $\tau = 1.26$; from then on, the inboard pair seems to dissipate faster in the SD simulation.

The mean vortex cores are easily recognizable until the age $\tau = 1.57$, where the inboard pair seems to have completely disappeared. As mentioned during the analysis of the 3D instantaneous vorticity fields, we also observe in $\tau = 0.02$ that the leader's vortex pair is nearly fully rolled-up when it reaches the follower. In contrast, the follower's vortex sheet is still distinguishable in the interval $\tau \in [0, 0.4]$. Furthermore, the vortex sheet of the FIV appears to last longer than its counterpart of the FOV, and rolls up into a secondary vortex. This is explained by the control actions performed by the follower that adapts the deflection of its ailerons in order to compensate the rolling moment induced by the leader's wake. The ailerons deflection induces a modification of the circulation distribution along the line which results in the modified vorticity shedding.

From the average axial vorticity fields, one might want to compare the trajectories of the wake vortices. In order to measure the mean displacement of the vortices, we

compute the position of the vortex centroid using a Gaussian mask, leading to

$$\bar{\omega}_{x,R}(\tau) = \int_{\Omega} \frac{1}{\pi R^2} \exp \left(-\frac{\left((y - y_c^a(\tau))^2 + (z - z_c^a(\tau))^2 \right)}{R^2} \right) \bar{\omega}_x(y, z, \tau) d\Omega, \quad (5.10)$$

$$y_c^a(\tau) = \frac{1}{\bar{\omega}_{x,R}} \int_{\Omega} \frac{y}{\pi R^2} \exp \left(-\frac{\left((y - y_c^a(\tau))^2 + (z - z_c^a(\tau))^2 \right)}{R^2} \right) \bar{\omega}_x(y, z, \tau) d\Omega, \quad (5.11)$$

$$z_c^a(\tau) = \frac{1}{\bar{\omega}_{x,R}} \int_{\Omega} \frac{z}{\pi R^2} \exp \left(-\frac{\left((y - y_c^a(\tau))^2 + (z - z_c^a(\tau))^2 \right)}{R^2} \right) \bar{\omega}_x(y, z, \tau) d\Omega, \quad (5.12)$$

where Ω is the area covered by the slice (i.e., the $4b$ square), R is a smoothing parameter and $\bar{\omega}_{x,R}$ is the Gaussian-smoothed average vorticity around the vortex centroid in $(y_c^a(\tau), z_c^a(\tau))$. This results in an iterative procedure performed for each one the LOV, LIV, FIV and FOV. The superscript $\cdot^a$ is used to stress that the vortex centroid is obtained relatively to the *average* vorticity field. One observes that this procedure can also be applied to instantaneous vorticity field $\omega_x^i(x, y, \tau)$ rather than average vorticity field $\bar{\omega}_x(y, z, \tau)$ in order to deduce the position of the instantaneous vortex centroid $(y_c^i(\tau), z_c^i(\tau))$.

Fig. 5.8 compares the trajectories of the vortex centroids computed from the average axial vorticity. It shows a very good correspondence between the SD and TD trajectories, which validates the methodology in this average perspective.

5.3.2 Vortex intensity and circulation distribution

In practice, vortices are unsteady and a more detailed analysis can be done by looking at temporal statistics. A characterization of the wake vortices is performed through the computation of their global circulation. When dealing with one single aircraft, this circulation is usually measured by computing the wake circulation in the half plane, i.e.,

$$\Gamma_{HP} = \int_0^\infty \int_{-\infty}^\infty \omega_x dz dy. \quad (5.13)$$

This definition is not applicable in the context of formation flight though. Indeed, the wake now involves the vortices of two aircraft. We adopt an alternative definition,

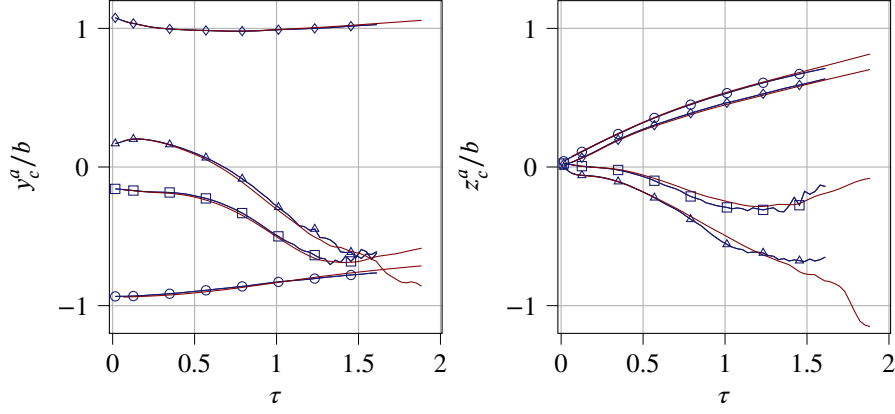


Figure 5.8: Comparison of the position of the two-aircraft formation vortices, computed from the average vorticity field $\bar{\omega}_x(\tau)$. (—) is the SD simulation, (—) is the TD simulation, ($\circ$) is the LOV, ($\square$) is the LIV, (Δ) is the FIV, and ($\diamond$) is the FOV

through the computation of the *radial circulation distribution* of each one of the vortices. Considering occurrences ω_x^i of the axial vorticity field, the distribution is computed by integration of the axial vorticity component on a disk of radius r centered on the instantaneous vortex position $(y_c^i(\tau), z_c^i(\tau))$:

$$\Gamma^i(\tau, r) = \int_0^r \int_0^{2\pi} \omega_x^i(r', \theta, \tau) r' d\theta dr'. \quad (5.14)$$

The circulation of each vortex is defined as the maximum of the radial circulation distribution, i.e.,

$$\Gamma_v^i(\tau) = \max_r \Gamma^i(\tau, r). \quad (5.15)$$

From Eq. (5.14), we compute the vortex 2D equivalent tangential velocity profiles

$$u_\theta^i(\tau, r) = \frac{\Gamma^i(\tau, r)}{2\pi r}, \quad (5.16)$$

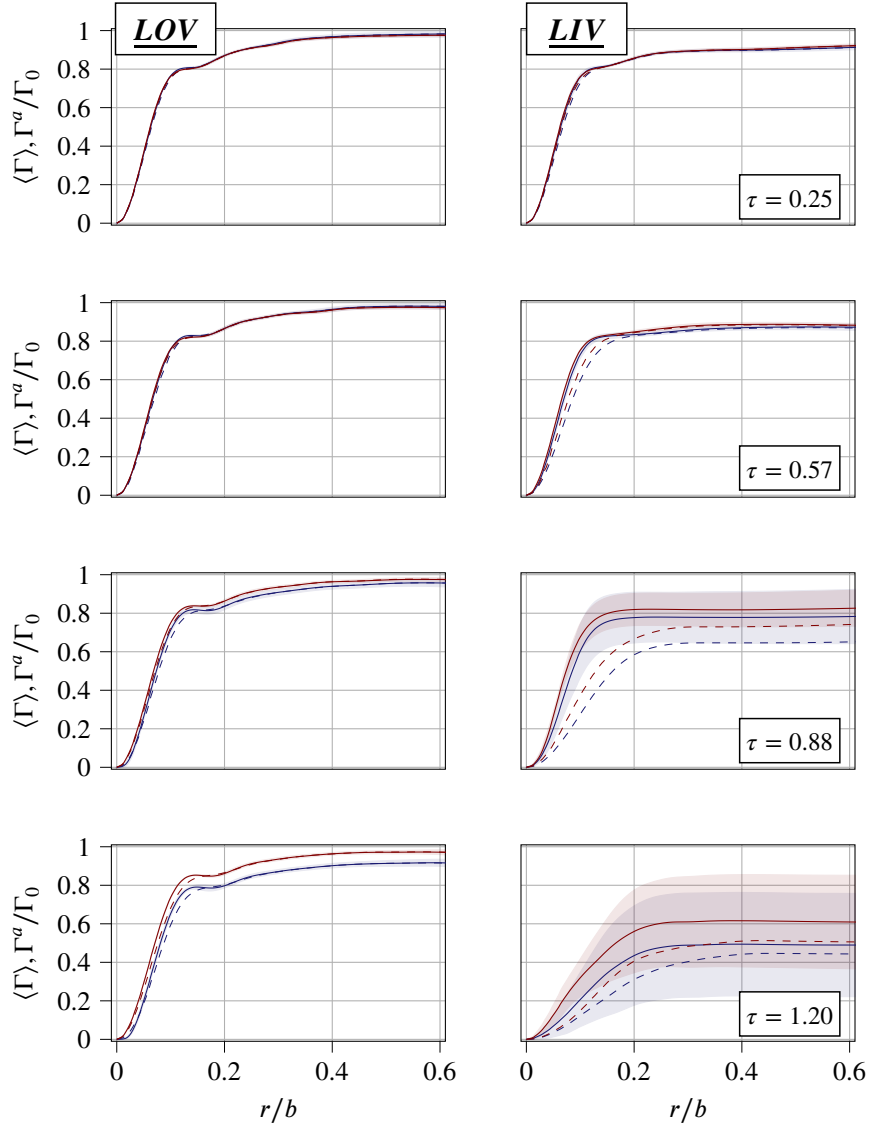
which give access to the core radius of the vortex $r_c^i(\tau) = \arg \max_r u_\theta^i(\tau, r)$. By processing a set of occurrences at several ages τ of the wake, and identifying the position of each vortex, we compute distributions centered on the vortex. We first compute the averaged radial circulation distribution $\langle \Gamma^i(\tau, r) \rangle$ that we present in Fig. 5.11 for the vortices of the wake computed from the SD and TD simulations. We also present the radial circulation distribution $\Gamma^a(\tau, r)$ computed from the average vorticity field $\bar{\omega}_x(y, z, \tau)$ at different ages.

For the four vortices, at early age τ , we observe a region close to the vortex core r_c where the circulation distribution stops increasing. This region is dominated by viscous effects in their core [109]. Beyond this region, the circulation evolves gently towards the maximum circulation. The outboard vortices follow this trend at every ages, due to the weak interaction they have with each other or with the inboard vortices. It is relevant to mention that the two outboard vortices are not at the same stage of roll-up, since one is emitted by the leader ten spans before the follower. The initial circulation distributions of the leader and the follower vortices are also different, due to the ailerons deflection that is significant for the follower (to compensate for the induced rolling moment) and negligible for the leader. The SD and TD simulations provide similar trends for the LOV and FOV, except for older ages where the SD simulation predicts weaker vortex intensities.

The inboard vortices (i.e., LIV and FIV) have a different behavior which is explained by their close vicinity and strong interactions. These interactions explain the variability in the circulation distribution that increases with the age of the vortices. The variability is essentially observed outside of the core and comes from the time varying vorticity field. Indeed, we observe that the variability increase corresponds to a circulation distribution computed from the average vorticity field $\Gamma^a(\tau, r)$ that is markedly lower than the average circulation distribution $\langle \Gamma^i(\tau, r) \rangle$.

Fig. 5.10 presents the evolution of the vortex circulations, as computed by Eq. (5.15). We clearly observe two phases in the decay of the inboard vortices. During the first phase, a very slow decay is observed in the maximum circulation, even for the circulation measured from the average vorticity $\Gamma^a(\tau, r)$ and for the average circulation distribution $\langle \Gamma^i(\tau, r) \rangle$. This decrease is to be attributed to the presence of turbulence. It is explained in [45] that the ambient turbulence indeed favors the exchange of circulation between primary vortices and turbulence. Later on, a second phase of rapid decay appears. This strong decay phase is explained by the 3-D interactions between the two inboard vortices when they come close to each other. This two-phase decay is not observed on the outboard vortices because of their distance from each other. Also, as the inboard pair first moves up, it does not interact with the outboard pair. We observe that for all the four vortices, the TD simulations seem to overestimate the circulation of the vortices over time. Indeed, the strong decay of the inboard vortices occurs at the same age for both simulations, but appears to be slower in the SD simulation. Also, while the circulation of the outboard vortices remains constant in the TD simulation, a decrease is observed in the SD simulation.

From the mean flow field, and from all the flow field occurrences, one extracts the tangential velocity profiles $u_\theta^a(\tau, r)$ and $u_\theta^i(\tau, r)$. In Fig. 5.11 we observe the evolu-



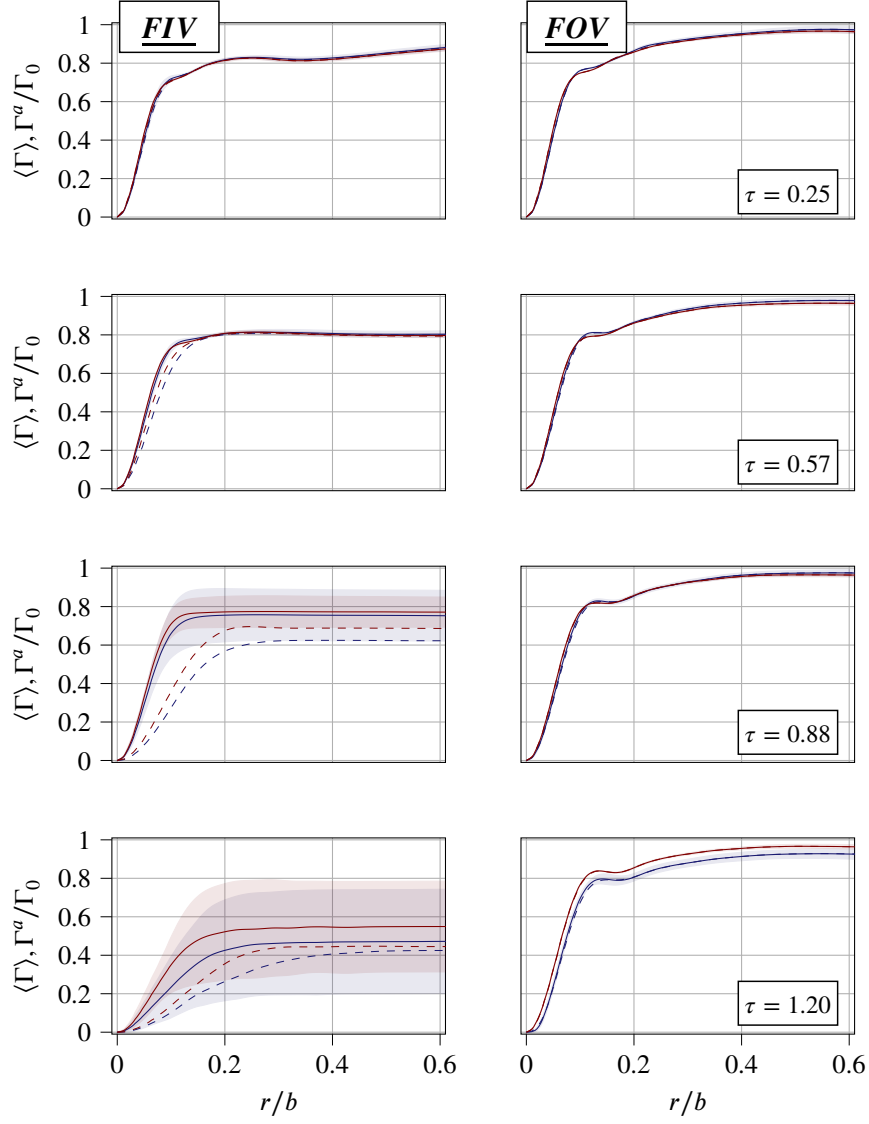


Figure 5.9: Radial circulation distribution of the wake vortices: LOV, LIV, FIV and FOV (from left to right). (—) is the SD simulation, (---) is the TD simulation. A solid line (—) is $\langle \Gamma^l(\tau, r) \rangle$ while a dashed line (---) is $\Gamma^a(\tau, r)$. Colored areas represent the $1 - \sigma$ envelope.

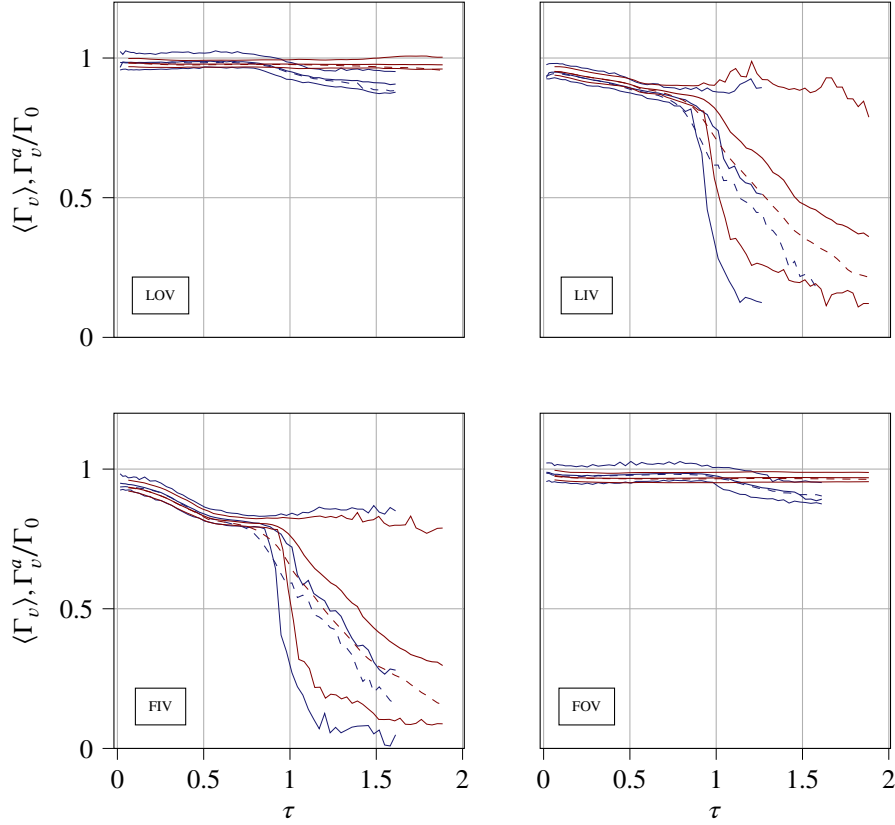


Figure 5.10: Vortices circulation evolution. (—) is the SD simulation, (—) is the TD simulation. A solid line (—) is $\langle \Gamma_v^i(\tau) \rangle$ while a dashed line (--) is $\Gamma_v^a(\tau)$. Thick lines (—) represents the average while thin lines (—) represents the 95% envelope.

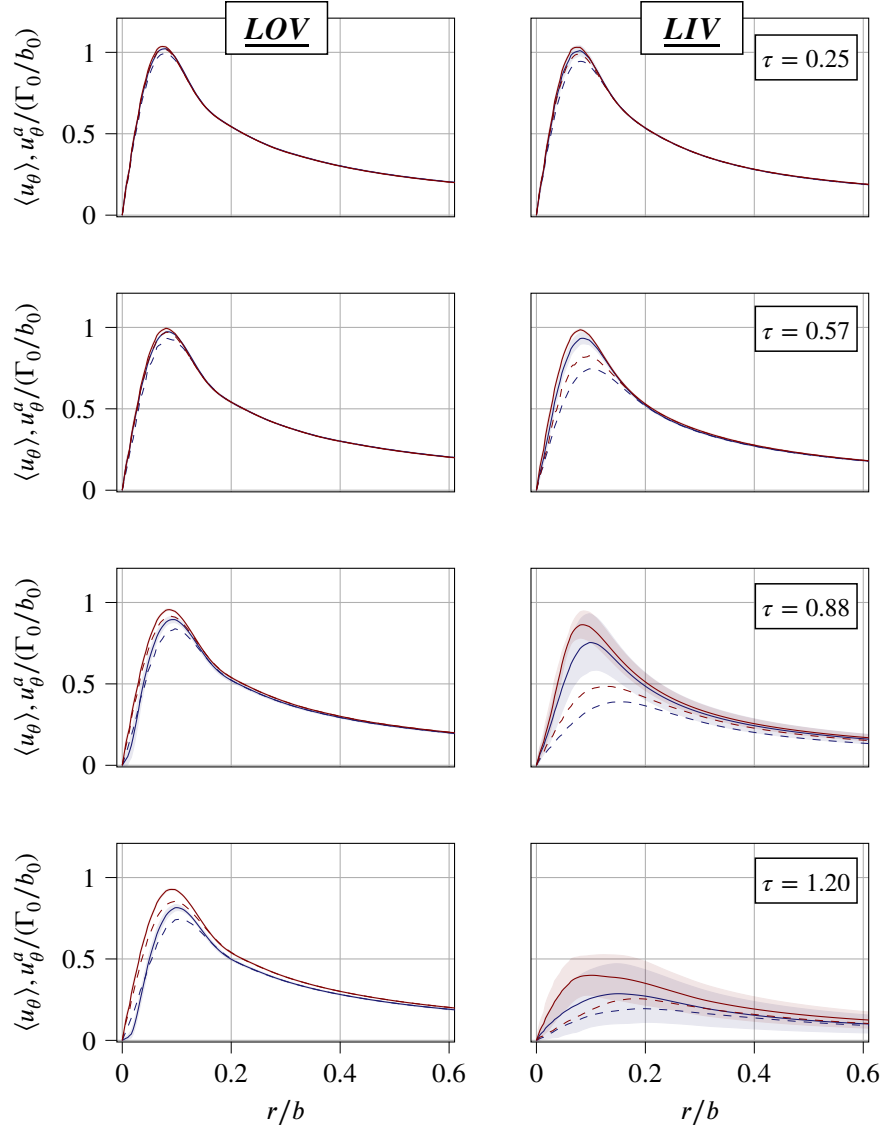
tion of those profiles with the age τ . We then observe the size of the core $r_c^a(\tau)$ and $r_c^i(\tau)$ of each of the vortices. The core of the vortices of the leader are more important than those of the vortices of the follower, due to their emission ten spans upstream. The cores of the leader (resp. the follower) amount to roughly 7% (resp. 8%) of b at $\tau \simeq 0.25$, which is more important than the classical 4–5% value. This is explained by the strong vortex intensity inherent to the HLA used in the simulations and the numerical resolution. The core sizes increase while the vortex sheets of the wing continue rolling up. Also, the interactions between the inboard vortices lead to a decrease of the tangential velocity, an increase of the core size and a variability on these quantities.

Note that the core size based on the average flow $r_c^a(\tau)$ is similar to $r_c(\tau, t)$ as long as the 3D interactions (between the inboard vortices) are not dominant.

5.3.3 Energy evolution

As observed in the 3D visualizations of the flow in Figs. 5.5 and 5.6, vortices evolving in a turbulent environment are subjected to long- and short-wavelength instabilities. Among them, we mention the Crow instability. In order to visualize the development of those instabilities, we present in Fig. 5.12 the evolution of the axial energy modes. The energy is computed from the 3D vorticity fields extracted from the TD simulation. We observe a two phase decrease of the mode 0 . First, it remains nearly constant in the wake before decreasing more abruptly at $\tau \simeq 0.8$. This drop is explained again by the interactions of the inboard vortices that interacts and dissipate their energy due to the onset of more effective instabilities. Indeed, we observe the increase of some modes and in particular, the ones associated to the wavelength $\lambda = 1b$ and $\lambda = 2b$ which become the two most energetic while not dominant at early ages. Those modes have to be related to the dominant medium- to long-wavelength oscillations observed between the inboard vortices that are separated by a distance of approximatively $\Delta_y^{in} = 0.25 - 0.3b$. This then amounts to an oscillation with $\lambda \simeq 2b \simeq 8\Delta_y^{in}$, and lead to so-called Omega loops, due to the unequal circulation between the two vortices.

The comparison between the statistics measured on the reference SD simulation and the TD simulation are here presented in order to validate the two-step methodology we use in the next section to simulate the wake of various aircraft configuration over long distances. Even if the TD simulation does not perfectly reproduces the characteristics of the wake, in particular in terms of vortex circulation or energy dissipation over the very long distances, we observe that the TD simulation is more conservative then the SD simulation. This means the TD will not dissipate the wake fast enough over the wake age, and the vortices circulation and energy measured in the TD simulation will be overestimated compared to results coming from a SD simulation. Such an overestimation will not be detrimental to an analysis of the impact of aircraft formation wake on air traffic management. Indeed, in the worst case, it would lead to more drastic safety margins.



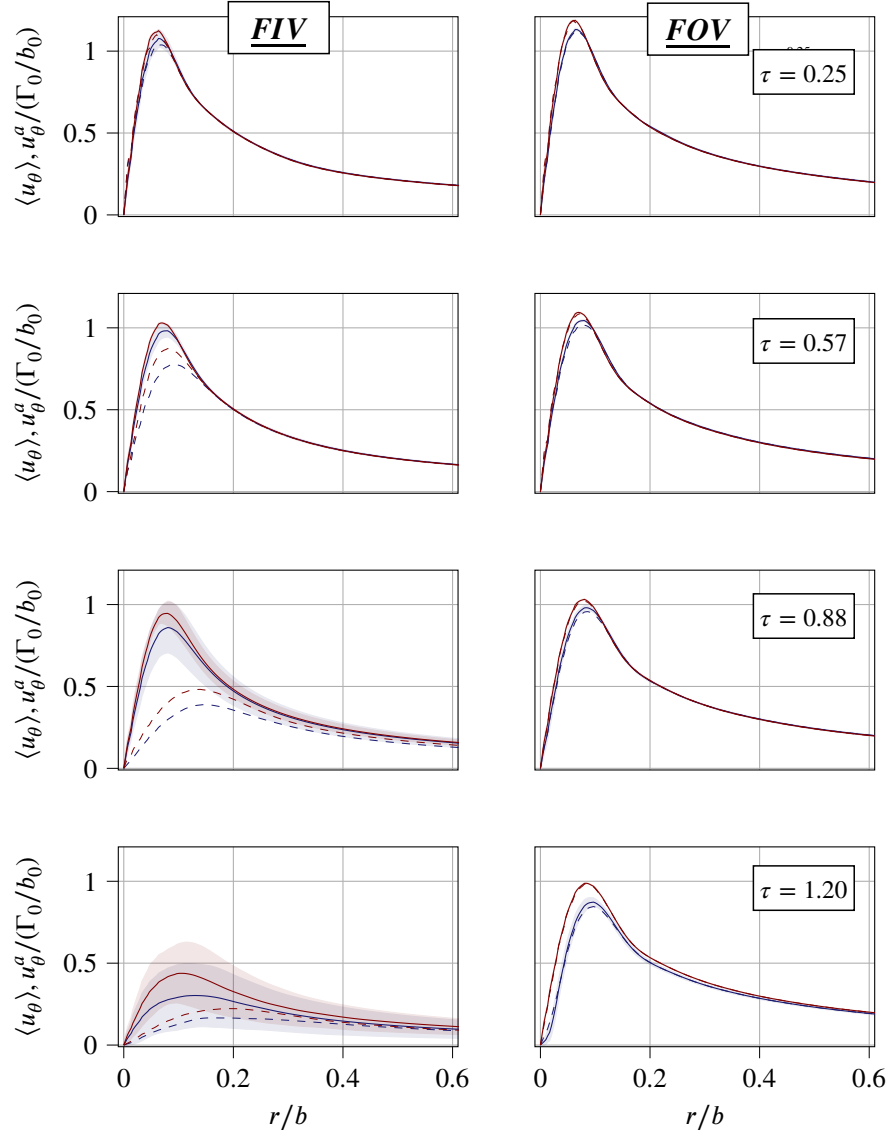


Figure 5.11: Tangential velocity induced by the wake vortices: LOV, LIV, FIV and FOV (from left to right). (—) is the SD simulation, (—) is the TD simulation. A solid line (—) is $\langle u_\theta(\tau, t, r) \rangle$ while a dashed line (--) is $u_\theta^a(\tau, r)$. Colored areas represent the $1 - \sigma$ envelope.

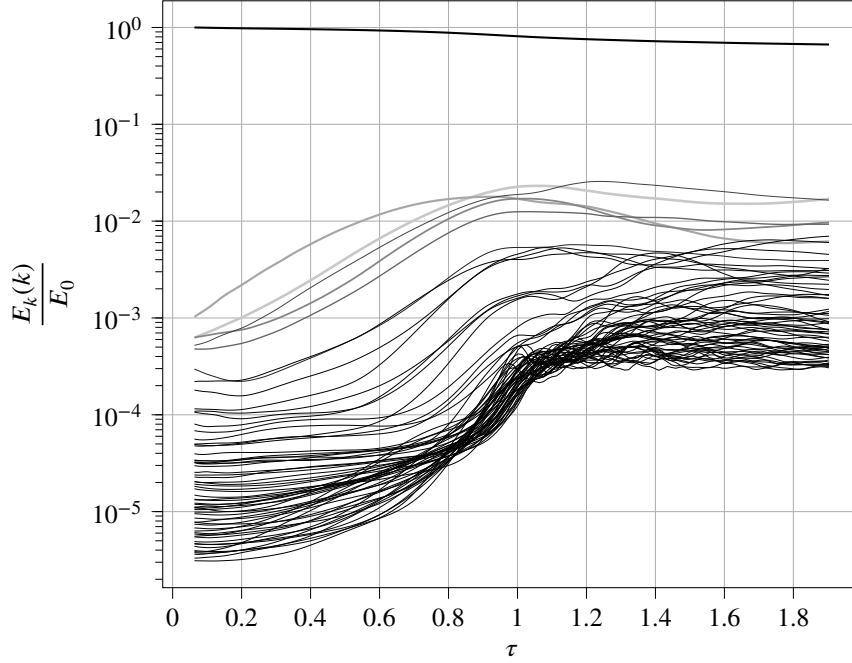


Figure 5.12: Longitudinal modal energy evolution computed from the instantaneous energy $E^i(\tau)$ measured TD simulations. Modes $\lambda = [0.25b, 0.5b, 1b, 2b, 4b, 8b, 16b]$ are represented (from — to —). The displayed energy are dimensionless, using the initial rigid mode energy E_0 .

5.4 Formation wake propagation under relative position uncertainty

In a formation flight context, the uncertainty on the relative position between the leader's wake and the follower leads to a complex behavior in terms of wake propagation, as already highlighted by [139]. In addition to the non-linear character of the interactions between vortices, this leads to an uncertainty on the position of the wake vortices, from their emission to the far-wake evolution. In this section, we model the relative position uncertainty by several formation configurations, all located around a reference one. We simulate the wakes and vortex propagation of each of these configurations. By associating a weight to each configuration according to a Gaussian distribution centered on the reference configuration, it is possible to determine statistics of the wake properties, such as the vortices position or intensity. This is much akin

a *Monte Carlo* approach, with a limited number of formation configurations because of their simulation cost. We use the methodology described in Section 5.2.3 using the NLA, and we chose the relative position $(\Delta y, \Delta z) = (1.15b, 0.0b)$ as the reference configuration, considering it as optimal in terms of energy savings, while guaranteeing a safety margin with respect to the position of the leader's vortices. The wakes of sixteen other formation configurations around the reference are computed, as presented on Fig. 5.13.

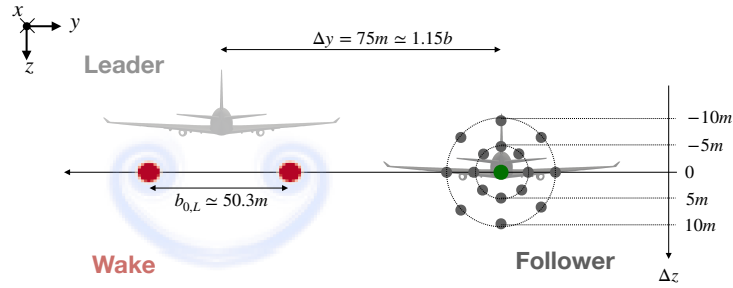


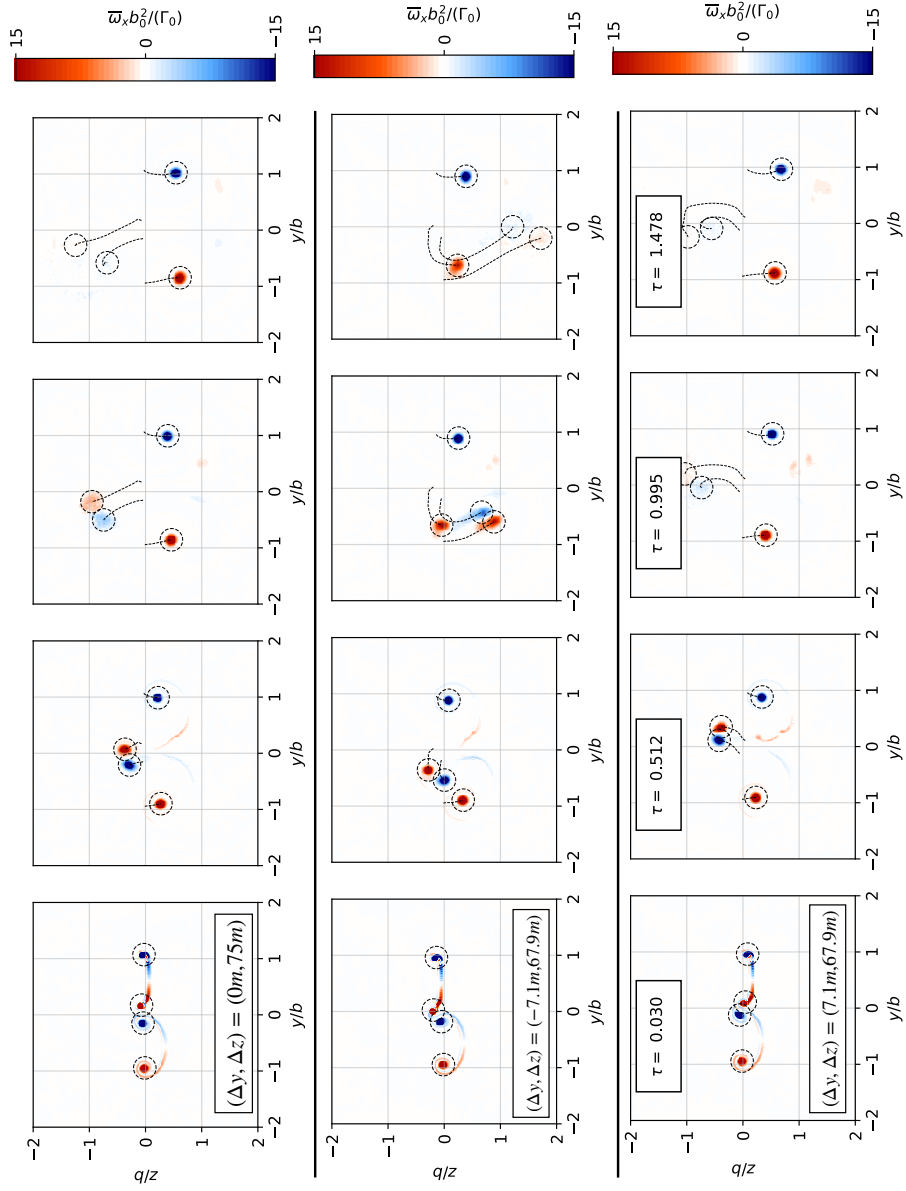
Figure 5.13: Sketch of the positions of the follower used for the NLA simulations: the green dot represents the reference position of the follower while the sixteen grey dots indicate the positions used to represent the relative position uncertainty. Each of these relative positions might be reached when the follower is tracking the leader's wake.

In the following subsections, we first investigate the sensitivity of the vortex propagation over time to the relative initial position based on the average axial vorticity field. We then investigate the characteristics of the instantaneous field in order to provide a statistical picture of the propagation of the wake. Finally, we close the result section by comparing the operational characteristics of the wake of a formation with those of the wake of an isolated aircraft.

5.4.1 Sensitivity of the wake propagation to the relative positioning

In this section, we focus on the longitudinally averaged vorticity field $\overline{\omega}_x$ as computed using Eq. (5.9). By averaging the vorticity along the x axis, we remove the secondary vortical structures and we study the mean evolution of the wake over time. In particular, this enables to determine the trajectory of the vortices for the different configurations using Eq. (5.10). The iterative procedure is performed for each one the LOV, LIV, FIV and FOV.

Fig. 5.13 shows the average vorticity fields with the vortex trajectories at different ages of the wake and for five formation flight configurations, including the reference



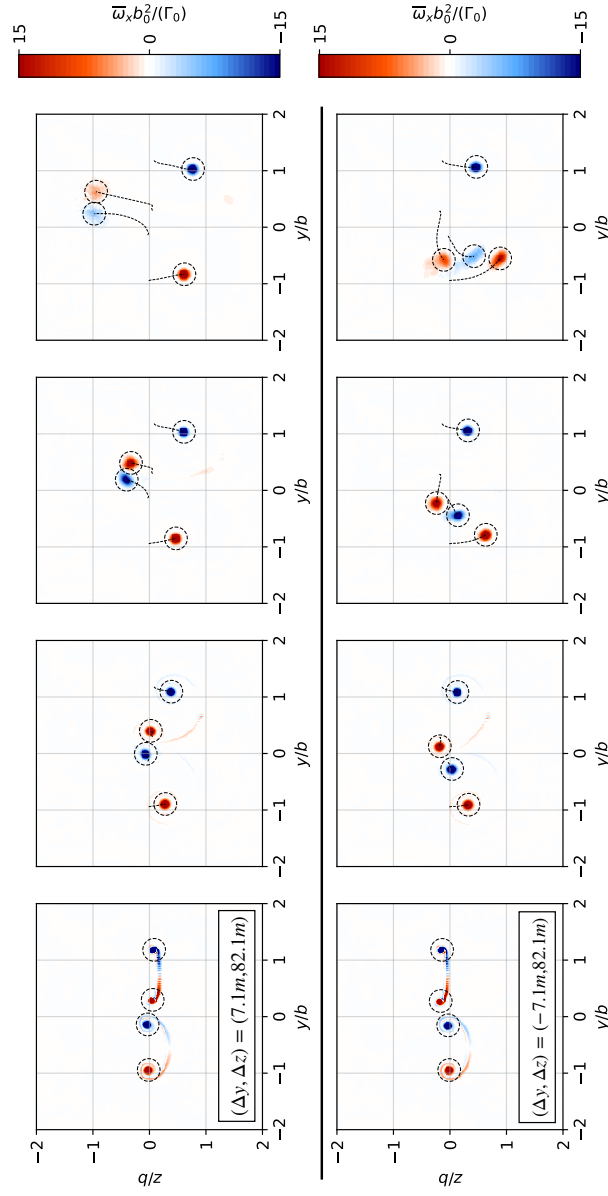


Figure 5.13: Streamwise averaged vorticity field $\bar{\omega}_x$ at different ages of the wake behind the two-aircraft formation for several relative position configurations. The dashed circle indicates the instantaneous position of each vortex, while the dashed line is the trajectory. From top to bottom: $(\Delta y, \Delta z) = (0m, 75m)$, $(\Delta y, \Delta z) = (-7.1m, 67.9m)$, $(\Delta y, \Delta z) = (7.1m, 82.1m)$, $(\Delta y, \Delta z) = (-7.1m, 82.1m)$

i.e., $(\Delta y, \Delta z) = (75m, 0m)$. The four other configurations are selected to demonstrate the diversity and complexity of possible vortex evolutions. On the first row of Fig. 5.13, we clearly observe an asymmetry of the wake propagation for the reference characterized by the alignment of the four vortices of the formation on the $y = 0$ horizontal ligne. The inboard vortex pair indeed shows a left- and upward bended trajectory that is explained by the different ages and intensities of the two vortices, as explained in Section 5.3.1. The outboard vortices follow a downward displacement, as they are less and less influenced by the inboard pair and they induce a downward velocity on each other, in the fashion of two vortices in a 2VS system.

When the follower is located above the leader's vortices, as in the second or fifth row in Fig. 5.13, the inboard pair initially moves leftward, and enters the influence area of the leader outboard vortex (LOV). The interactions with this vortex might either separate the inboard pair (like in the second row), or generate a system made of three vortex that is more difficult to define.

When the follower is located below the leader's vortices, as in the third and fourth row in Fig. 5.13, the inboard pair initially moves rightward and rapidly moves upward, without entering the influence area of the follower outboard vortex (FOV). Indeed, as in the reference configuration, the circulation difference and asymmetry between the vortices of the inboard pair ($\Gamma_0^{LIV} > \Gamma_0^{FIV}$) will cause their trajectory to bend towards the left, in opposition to the initial movement. The FOV is therefore more difficult to reach by the inboard pair. From a global point of view, we observe that a change in the relative position completely modifies the wake behavior over time, and in particular the vortex trajectories. This first analysis has been made on average vorticity fields, which clearly hint at even higher complexity for the instantaneous fields.

5.4.2 Probabilistic wake propagation

In this section, we perform a statistical analysis of the wake propagation around the reference two-aircraft formation configuration, in order to take into account the non-stationarity of the vorticity field and the vortex characteristics. For this purpose, we associate weights $\bar{w}_j$ to each one of the 17 configurations presented in Fig. 5.13. They correspond to the probability for the formation to be in the configuration j during a formation flight tracking procedure. These weights are computed using the Gaussian

distribution

$$w_j = \frac{1}{\pi\sigma_t^2} \exp \left(-\frac{\left((\Delta y^j - \Delta y^{ref})^2 + (\Delta z^j - \Delta z^{ref})^2 \right)}{\sigma_t^2} \right) \quad (5.17)$$

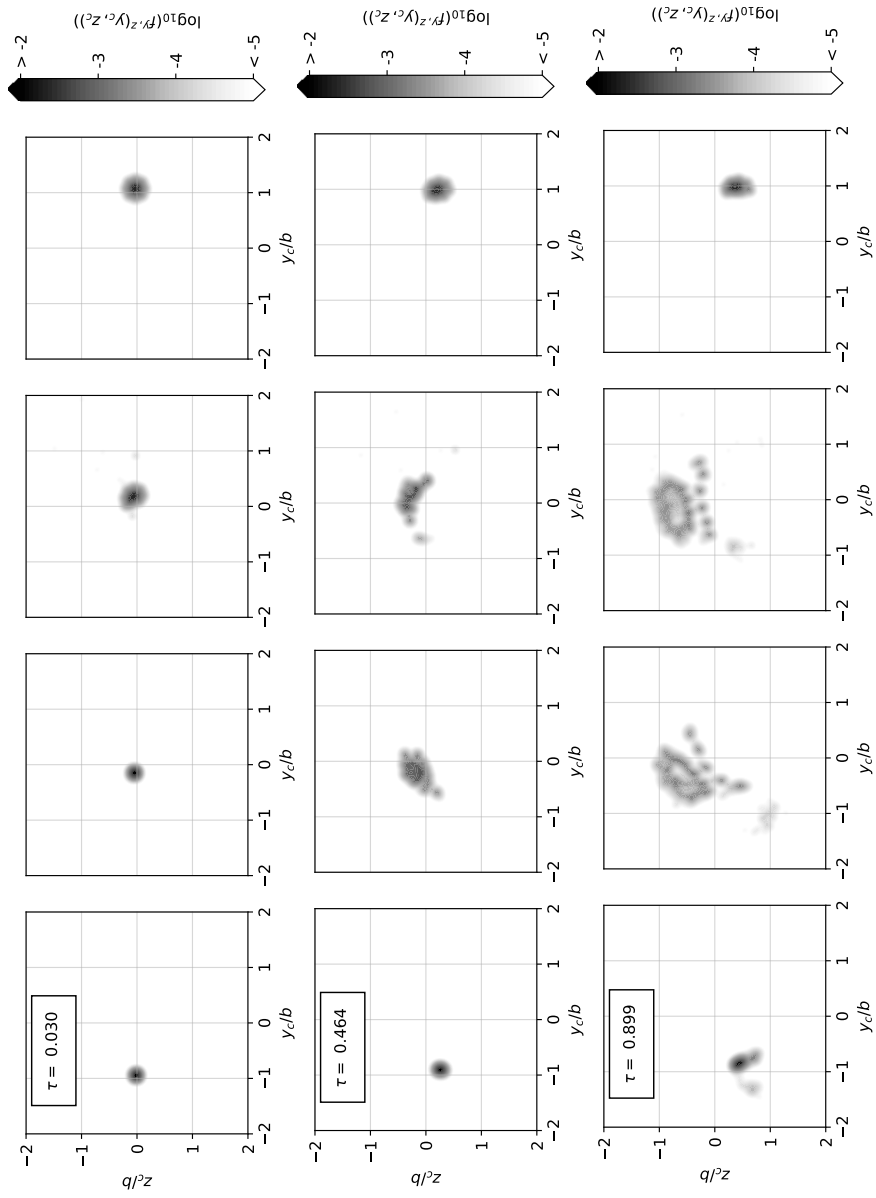
$$\bar{w}_j = \frac{w_j}{\sum_i^N w_i}, \quad (5.18)$$

where $(\Delta y^{ref}, \Delta z^{ref})$ and $(\Delta y^i, \Delta z^i)$ are the relative positions used for the reference configuration and the configuration i , $\sigma_t = 10m$ is the tracking variance and $N = 17$ is the total number of configurations simulated. We then select 32 occurrences i of the wake for each configuration j among the seventeen. For each one of the $17 \times 32 = 544$ *snapshot* of the wake, we compute the instantaneous position and circulation associated with each of the four vortices over the wake age τ . Each vortex k of each *snapshot* (i, j) is thus a Dirac-like image of the wake characteristics. By gathering the characteristics of each one of those Dirac-like images, we can determine a statistical description of the vortices wake propagation and their intensity over the wake age.

To do so, we regularize the Probabilistic Density Function (PDF) associated to a single vortex k of a *snapshot* (i, j) by multiplying its Dirac-like distribution with a Gaussian. This allows the description of the joint PDF of the positions and wake intensity over the wake age, for each one of the vortex k . We have

$$f_k(\Gamma_v, y_c, z_c, \tau) = \sum_{j=1}^{N=17} \bar{w}_j \sum_{i=1}^{32} \frac{1}{32} \frac{1}{\pi\sigma_v^2} \exp \left(-\frac{\left((y_c - y_c^{i,j,k}(\tau))^2 + (z_c - \right)}{\sigma_v^2} \right) \cdot \frac{1}{\sqrt{\pi}\sigma_\Gamma} \exp \left(-\frac{\left(\Gamma_v - \Gamma_v^{i,j,k}(\tau) \right)^2}{\sigma_\Gamma^2} \right), \quad (5.19)$$

where $[y_c^{i,j,k}(\tau), z_c^{i,j,k}(\tau)]$ and $\Gamma_v^{i,j,k}(\tau)$ are the positions and maximum intensity of the vortex k computed in the occurrence i of simulation j at age τ . σ_v and σ_Γ are mollification parameters associated to the position and intensity of the vortex. The positions $[y_c^{i,j,k}(\tau), z_c^{i,j,k}(\tau)]$ and wake intensity $\Gamma_v^{i,j,k}(\tau)$ are computed using the same methodology as the one described in Section 5.4.1 in considering the 32 instantaneous occurrences from the seventeen configurations, i.e., Eqs. 5.10 and 5.15. This function is computed in assuming that each one of the 32 occurrences extracted in one configuration is equiprobable with respect to the others from that same configuration.



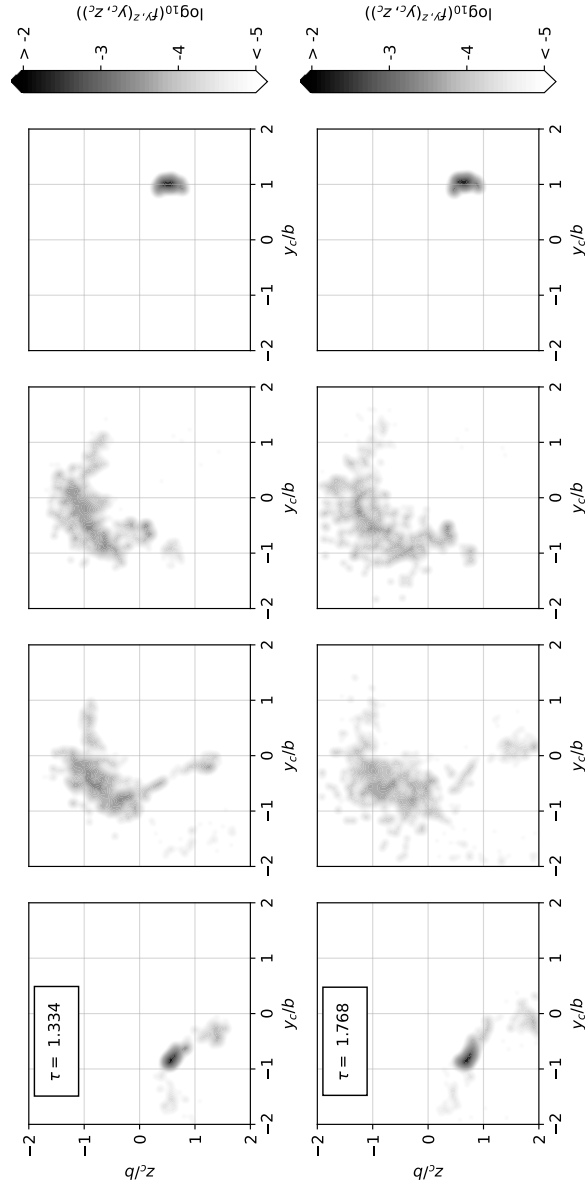


Figure 5.13: PDF $f_k^{y,z}(y_c, z_c, \tau)$ of the position of each one of the four wake vortices (i.e., $k = [\text{LOV}; \text{LIV}; \text{FIV}; \text{FOV}]$ from left to right) at five different wake ages.

From Eq. (5.19), one can compute the marginal distribution for the positions (y_c, z_c) of each one of the vortices, by integrating over the variable Γ_v , i.e.,

$$f_k^{y,z}(y_c, z_c, \tau) = \int_0^\infty f_k(\Gamma_v, y_c, z_c, \tau) d\Gamma_v. \quad (5.20)$$

Fig. 5.13 presents that PDF for each one of the vortices (i.e., LOV, LIV, FIV and FOV) at five different wake ages.

We first observe the characteristics of the FOV (i.e., right column in Fig. 5.13), the distribution of which remains very concentrated over time. This is due to the absence of interactions with other vortices during the period considered. It only undergoes a downward movement that appears as if induced essentially by the LOV. The FIV and LIV forming the inboard pair present much wider distributions which can be explained by their strong mutual interactions in all of the flight configurations studied. Also, these two vortices potentially interact with the LOV, which stretches their distribution even more. Finally, the LOV presents a distribution as concentrated as the FOV. Nevertheless, we observe the appearance of secondary clusters, explained by possible interactions with the inboard pair, as the one observed on the second row in Fig. 5.13.

In a similar way, we can derive the marginal distribution for the circulation of each one of the vortices $f_k^\Gamma(\Gamma_c, \tau)$, by integrating over the positions (y_c, z_c) , i.e.,

$$f_k^\Gamma(\Gamma_v, \tau) = \int_{-\infty}^\infty \int_{-\infty}^\infty f_k(\Gamma_v, y_c, z_c, \tau) dy_c dz_c. \quad (5.21)$$

Fig. 5.14 presents the evolution over time of that PDF, for each one of the vortices.

As expected, we observe that the PDF of the circulation of the FOV is concentrated close to Γ_0 and does not change with the wake age, showing once again the absence of interactions with the other vortices. The inboard vortices LIV and FIV are the most interacting ones, and their PDFs tend to expand over a larger interval. For $\tau > 1$, the PDFs draw two distinct clusters, one concentrated around $\Gamma_v \sim 0.9\Gamma_0$, and a second, wider, around $\Gamma_v \sim 0.2\Gamma_0$. The first one indicates that the instantaneous vortices circulation of the LIV and FIV can still be intense despite their interactions. The second one precisely reflects these interactions which lead to their dissipation. Finally, the circulation of the LOV also shows a concentrated PDF close to Γ_0 . However, from $\tau > 0.5$, it exists a non-zero probability to observe lower vortex wake intensity, due to the possible interactions with the inboard pair, as already highlighted before.

These descriptions of the joint PDF show that although the general tendency leads to the dissipation of the inboard vortex pair and to the formation of an extend 2VS made of the LOV and FOV, very different configurations can appear punctually. The

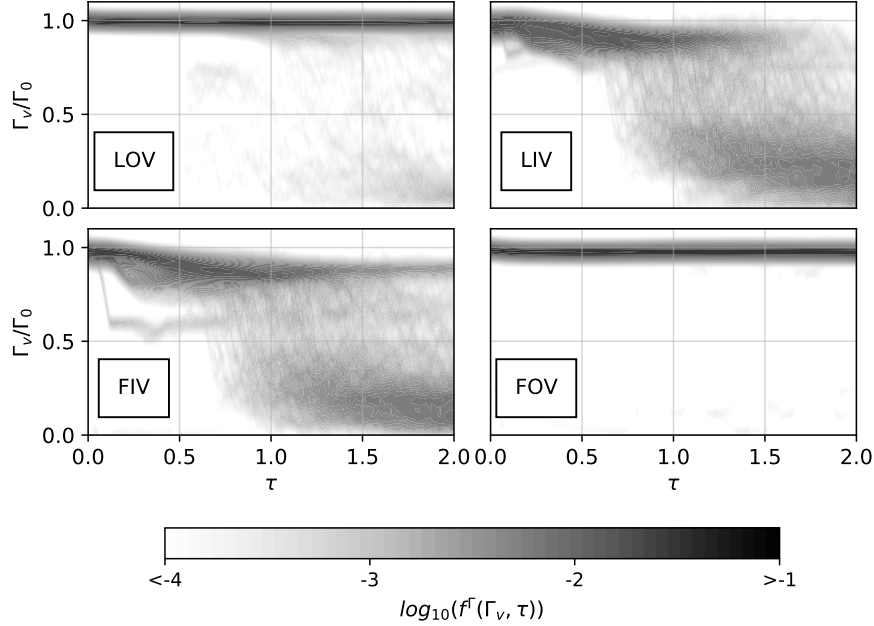


Figure 5.14: Evolution of the PDF $f_k^\Gamma(\Gamma_v, \tau)$ of the circulation of the four wake vortices (i.e., LOV, LIV, FIV and FOV) with the wake age.

only real certainty comes from the FOV which maintains a very constant circulation and a very predictable trajectory over the wake age. This can be a benefit for a third aircraft that could surf it, but can also be a constraint for the ATM whose role is to contain the hazard posed by dangerous long lasting structures in the sky.

5.4.3 Operational analysis

The propagation of the wake of an isolated aircraft has long been studied and led to numerous studies, some of which in operational and air traffic management context [95, 133]. Specifically, the hazard posed to a following aircraft has been studied and quantified through the rolling moment induced by the vortices on this hypothetical follower [89]. In the context of formation flight, this induced rolling moment is of course controlled by an action on the aircraft dynamics (e.g., aileron deflections), the follower willing to stay in the vicinity of the leader's vortices. However, the wake of a two-aircraft formation exhibits complex behaviors, which would therefore lead to a different rolling moment induced on a third aircraft that would cross that wake. In this

section, we use simple metrics to quantify the impact of a two-aircraft formation wake on this third aircraft.

We denote the vertical velocity field of an existing wake encountered by this aircraft at a location y' along its wingspan by $W_F(y', \tau)$, i.e., the vertical component of the airspeed on the aircraft. We assume that this velocity is small compared to the flight velocity U_∞ of the aircraft. This allows to consider the effect of the wake as a small perturbation on the wing loading of the aircraft. An occurrence i of the induced rolling moment $M_l^i(y, z, \tau)$ on the aircraft located in a τ -old wake, at position (y, z) is then obtained by

$$M_l^i(y, z, \tau, t) = \frac{1}{2} \rho U_\infty^2 \int_{-b/2}^{b/2} C_{l,\alpha}(y') c(y') \frac{W_F^i(y', \tau)}{U_\infty} y' dy', \quad (5.22)$$

where $C_{l,\alpha}(y') = \frac{\partial C_l}{\partial \alpha}$ is the lift slope coefficient of the aerodynamic profile at location y' and $c(y')$ is the chord at that profile. The Rolling Moment Coefficient (RMC) is then defined as

$$\text{RMC}^i(y, z, \tau) = \frac{M_l^i}{\frac{1}{2} \rho U_\infty^2 S b}, \quad (5.23)$$

where S is the surface of the wing of the aircraft.

The RMC is thus a scalar field that is computed from occurrences of the velocity field that results from the propagation of the wake of a formation. We thus computed the RMC for all the formation configurations described in Fig. 5.13 and we compute the maximum of the absolute value of RMC measured in the wake as well as its position, i.e.,

$$|\text{RMC}_{Max}|^{i,j}(\tau) = \max_{y,z} |\text{RMC}^{i,j}(y, z, \tau)| \quad (5.24)$$

$$(y_{Max}^{i,j}(\tau), z_{Max}^{i,j}(\tau)) = \arg \max_{y,z} |\text{RMC}^{i,j}(y, z, \tau)| \quad (5.25)$$

where $|\cdot|$ denotes the absolute value and $(y_{Max}^{i,j}(\tau), z_{Max}^{i,j}(\tau))$ is the position of the maximum. As in Section 5.4.2, we associate weights $\bar{w}_j$ from Eq. (5.18) to each j of the 17 configurations presented in Fig. 5.13 and for which we dispose of 32 occurrences i . Using weighted average and weighted quantiles, we deduced statistics of the maximum of the absolute value of RMC, presented in Fig. 5.15 as well as a PDF of its position $f_{\text{RMC}}(x, y, \tau)$ over the wake age, displayed in Fig. 5.16.

We observe in Fig. 5.15 that the maximum RMC experienced in the wake of a formation can be nearly 50% higher than the one experienced in the wake of an isolated aircraft. Those high values are observed at the beginning of the wake propagation, for

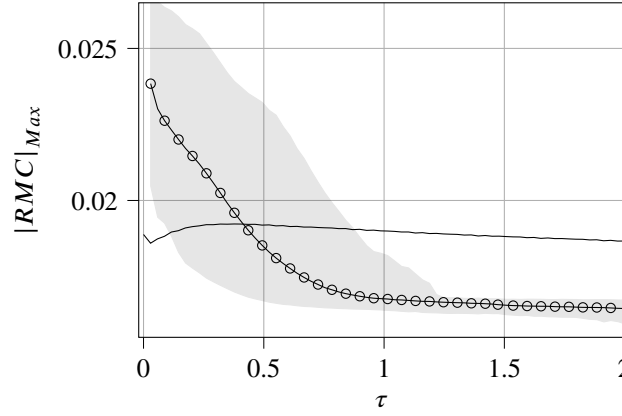


Figure 5.15: Evolution of the weighted average of the maximum absolute value of RMC (—○—) and 90% envelope experienced (shaded area) in the wake of a two-aircraft formation. The solid line (—) is the evolution of the RMC experienced in the wake of an isolated aircraft.

$\tau \in [0, 0.5]$. We also observe in Fig. 5.16 that for that period, the PDF of the position of the maximum RMC shows the highest probability at the vicinity of the inboard vortex pair. Indeed, as the LIV and FIV are close to each other, but not dissipated yet, they induce strong velocity gradients and therefore strong rolling moments.

When the wake gets older, the inboard pair dissipates, which explains the reduction of the maximum RMC, until it becomes weaker than the one experienced in the wake of a single aircraft. When the inboard pair has entirely dissipated, the PDF of the position of the maximum RMC shows that the maximum RMC is more likely to appear in the vicinity of one of the outboard vortices, and in particular the LOV on the left. Indeed, we have observed in the previous section that this vortex was initially slightly stronger than the FOV. Due to a wider separation between the LOV and FOV (nearly two spans), the velocity gradients are weaker than in the wake of a single aircraft (for which the distance between the vortices is lower than one span), thence explaining the lower value of the maximum RMC. Moreover, the latter remains very constant, as the outboard vortices interact little with each other. The wake of the formation thus appears to be less hazardous than the wake of a single aircraft for the time considered. However, one should realize that the wake vortices of a leader will dissipate at an earlier age than the ones off the two-aircraft formation as proven by the slope of the curves displayed on Fig. 5.15. This is due to the smaller distance, i.e., b_0 that separates them compared to the FOV and LOV separated by a distance $\simeq 2b_0$. As a consequence,

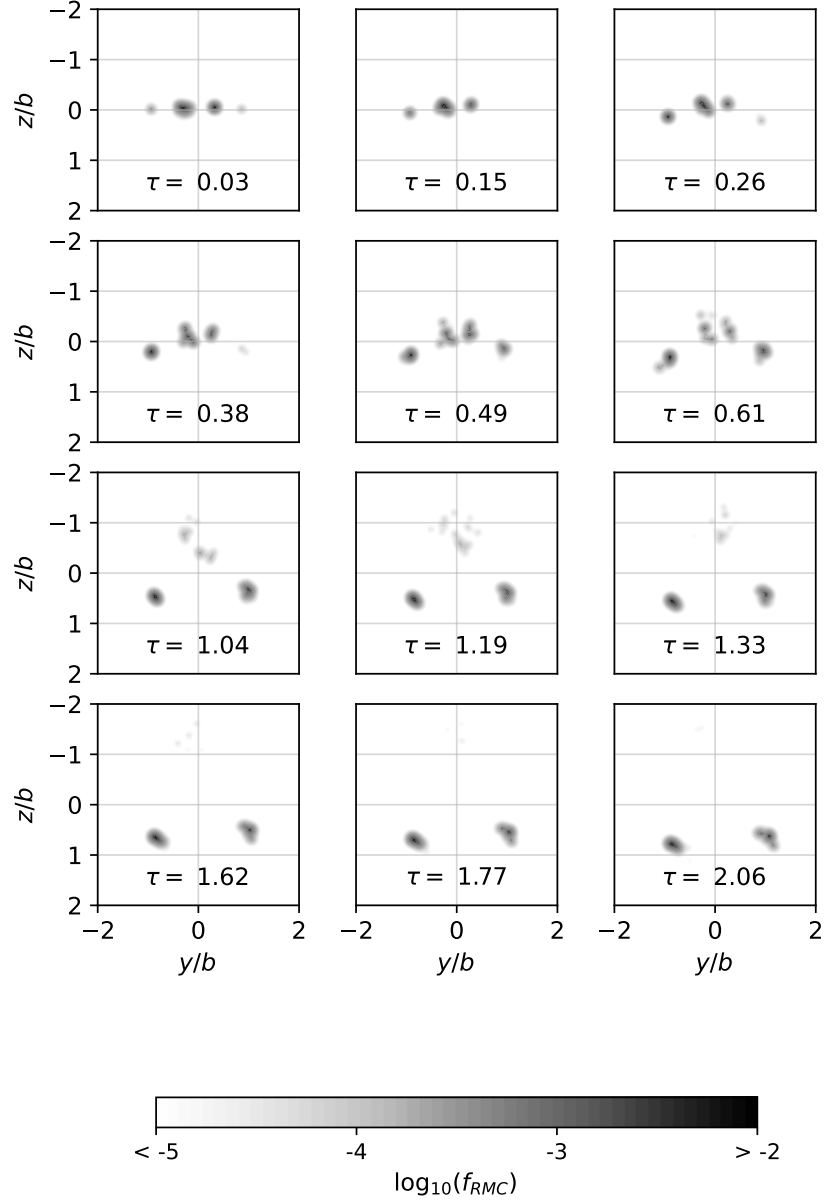


Figure 5.16: PDF $f_{RMC}(y, z, \tau)$ of the position of the RMC critic at different wake ages.

the remaining wake vortices of the two aircraft formation move downward two times slower. Those observations also need to be taken into consideration by the Air Traffic Management in addition to the evolution of the RMC over time.

In the context of formation flight, we get interested in a second wake global quantity that we name the Vertical Force Coefficient (VFC) that characterizes the increase in vertical force experienced by an aircraft flying in a wake because of the upwash W_F . An important VFC characterizes the "free lift" that the wake induces on following flying devices. We compute the vertical force F_z and the resulting VFC using the equations

$$F_z^i(y, z, \tau) = \frac{1}{2} \rho U_\infty^2 \int_{y-b/2}^{y+b/2} C_{l,\alpha}(y') c(y') \frac{W_F^i(y', z, \tau)}{U_\infty} dy' \quad (5.26)$$

$$\text{VFC}^i(y, z, \tau) = - \frac{F_z^i(y, z, \tau)}{\frac{1}{2} \rho U_\infty^2 S} . \quad (5.27)$$

Again, the VFC is a scalar field computed for different occurrences i of the wake at age τ . Similarly to the RMC, we compute statistics of the maximum of VFC, without the absolute value function this time. Indeed, while a positive or negative rolling moment has the same unfavorable effect, a positive vertical force $F_z > 0$ (i.e., pointing down, $\text{VFC} < 0$) is undesired and a negative force (i.e., pointing up, $\text{VFC} > 0$) is beneficial. We get interested in the latter, i.e., $\text{VFC}_{Max} = \max_{y,z} \text{VFC}$. Following the same methodology as before, we deduced the statistics of VFC_{Max} , presented in Fig. 5.17 and the probability density function of its position $f_{\text{VFC}}(y, z, \tau)$ over the wake age, displayed in Fig. 5.18.

Again, we compare VFC_{Max} computed in the wake of the two-aircraft formation with VFC_{Max} computed in the wake of an isolated aircraft. We observe this time that the maximum VFC is at least 25% greater in the wake of a formation. Unlike the RMC, the variability of the maximum VFC is initially weak for early wake ages $\tau \in [0, 0.75]$. In that period, it is more likely to observe the maximum VFC only in the vicinity of the outboard vortices, in particular close to the FOV on the right. When the inboard pair has really started to interact, there is a non-zero probability to cross the maximum VFC at the vicinity of that pair, as observed on Fig. 5.18 for $\tau > 1$. This age corresponds to the increase in the variability of the maximum VFC in Fig. 5.17. Sporadic events related to the inboard pair interaction may induce strong local upwash velocities. On average, it is more likely to find the maximum VFC close to the FOV which persists for an even longer period of time, and its value being greater than the one measured behind a single aircraft. One next step would be to extend the two-aircraft formation

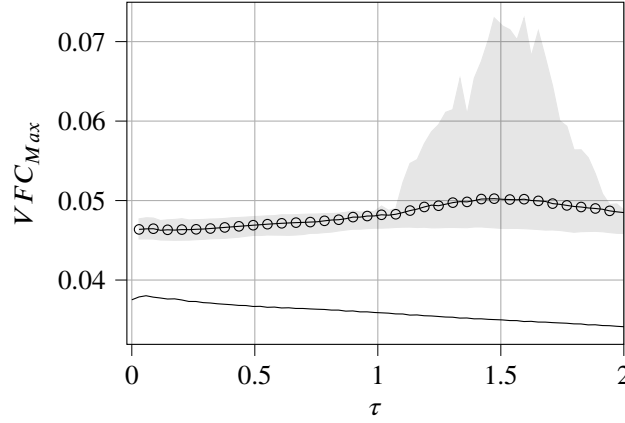


Figure 5.17: Evolution of the weighted average of the maximum absolute value of VFC ($\circ$) and 90% envelope experienced (shaded area) in the wake of a two-aircraft formation. The solid line ($-$) is the evolution of the VFC experienced in the wake of an isolated aircraft.

in adding a third aircraft that would benefit from an additional vertical force than the second one in the formation.

5.5 Conclusions

In this chapter, we presented a methodology used to quantify in a probabilistic sense the characteristics of the wake produced by two aircraft in formation flight whose relative position is uncertain. We employed Large Eddy Simulations performed using the Vortex Particle-Mesh method with Immersed Lifting Lines to simulate several formation flight configurations around a reference. The combination of Space Developing and Time Developing simulations enabled to accurately reproduce the interactions phenomena occurring during the roll-up of the vortex sheets emanating from the aircraft, as well as the large scale and long time evolution of the asymmetric wake resulting from each one of the configurations. In particular, we show that such an approach is necessary to accurately model the intrinsic asymmetry of the wake of a formation coming from the age difference between the leader's vortices and the follower's vortex sheet. The simplified aircraft dynamics that were taken into account in the simulations also participate to the wake asymmetry. Indeed, controllers adapt the ailerons deflections to compensate the induced rolling moment of the follower which in return locally modifies the vorticity produced by the lifting lines.

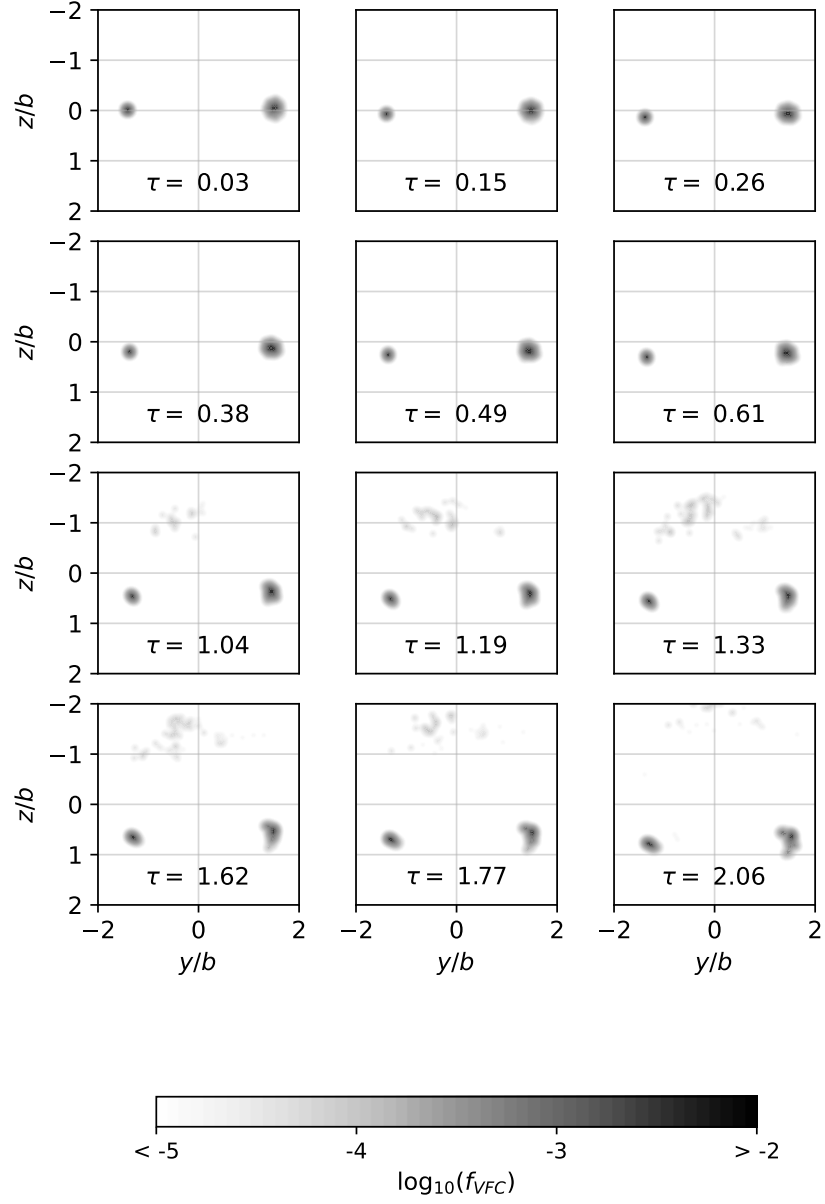


Figure 5.18: PDF $f_{VFC}(y, z, \tau)$ of the position of the maximum VFC at different wake ages.

We first showed that the methodology that combines Space Developing and Time Developing simulations is appropriate for the study of the impact of formation flight on the Air Traffic Management. The wake properties are indeed comparable to those computed by purely Space Developing simulations, considered as the reference. The methodology indeed computes similar wake characteristics, although being a little less dissipative in terms of wake circulation and energy. However, this guarantees to provide conservative results in studies concerned with aircraft safety.

The high fidelity of the simulations allowed to highlight the sensitivity of the propagation of the wake vortices to the relative position between the follower and the leader's wake. We described the trajectories of the vortices for the different configurations using time average vorticity fields. We then combined the results produced for all the formation configurations studied, and we deduced a statistical model of the wake propagation of the formations, thanks to the use of instantaneous vorticity fields. We showed that the outboard vortex of the follower had a very concentrated statistical distribution both in terms of position and intensity. The latter was therefore very little impacted by the rest of the wake. The inner vortices showed a very large variability in terms of position and intensity which was explained by their very strong mutual interaction in the form of Crow-type oscillations. Due to the asymmetry of the wake, we observed that the inboard pair tended to move towards the leader's outboard vortex, which increased the probability of interactions between that pair and the leader outboard vortex. The latter thus showed a concentrated position distribution associated to a wide distribution in terms of vortex intensity.

The aforementioned probabilistic analysis led us to observe the impact of a formation wake in an operational context, and to compare it to the wake of an isolated aircraft. By computing a Rolling Moment Coefficient in the wake of both the formations and the isolated aircraft, we observed that the wake of a formation was less constraining in terms of Rolling Moment Coefficient intensity. However, it did not decrease as fast as the one computed in the wake of an isolated aircraft and moves slower downward. We also defined a Vertical Force Coefficient that characterized the increase in vertical force experienced by an aircraft flying in a wake because of upwash winds. That Vertical Force Coefficient was shown more intense in the wake of a formation, compared to the wake of an isolated aircraft. This observation would justify the deployment of larger aircraft formations, as a third aircraft would enjoy an even greater benefit.

The simulations carried out in this paper allowed us to highlight the complexity and diversity of the dynamics of vortices in the wake of a two-aircraft formation. They allowed to evaluate the medium-term behaviors after the formations, and only need to be pursued on even longer propagation periods to bring a real light on the impact of the

formation flight on the air traffic management. With the same purpose, the simulation of more varied formation configurations should also be considered in order to quantify in a more global way the impact that formation flight could have. The present probabilistic approach, in fact, lays the foundations for the design of an operational model of the propagation of aircraft formation wakes.

Conclusions

This thesis has been dedicated to the simulation and study of formation flight for energy savings, a flying strategy inspired by birds. We have directed our attention toward the concept of extended formation flight that is characterized by much safer separation distances between the aircraft compared to close formation flight usually performed for tactical reasons by military aircraft. In this context, it is essential for the follower to continuously locate the wake vortices of the leader rather than the leader itself. At large distances, the wake motion is indeed an important factor in properly positioning the trailing aircraft.

A large part of this work was devoted to the development of wake tracking strategies based on dynamic measurements of the aircraft. For this purpose, an important effort has been put on the development of simulation environments able to predict and control the 6 degree-of-freedom dynamics of an aircraft under the influence of the wakes of preceding ones. This was carried out for two different environments characterized by two different levels of fidelity and accuracy.

Exploiting these two, we have been able to explore different strategies to be pursued by a follower for the estimation of the wake characteristics of a leader and for the learning of optimal actions to perform in that environment. From the dynamic measurements that such strategies are handling with, we have highlighted the observability issues related to the estimation of the wake of a leader.

Finally, we have taken advantage of the ability of the high-fidelity simulation environment to accurately propagate aircraft wakes in order to study the impact of formation flight on the air traffic management.

To conclude this thesis, we provide a summary of the most significant contributions identified in this work and we highlight the potential improvements and perspectives.

Contributions

Formation flight modeling

Chapter 1 of this thesis was dedicated to the modeling of the 6 degrees-of-freedom (6DOF) dynamics of an aircraft and its implementation in the Vortex-Particle-Mesh (VPM) flow solver to perform Large Eddy Simulations (LES) of auto-piloted aircraft in formation. Based on an Immersed Lifting and Dragging Line (ILDL) method developed in other studies, we have modeled the effects of the three important lifting surfaces of an aircraft (i.e., wing, horizontal tail plane and vertical tail plane) on the flow and we have computed the resulting aerodynamic loads on the aircraft. This has allowed to take a major part of the aerodynamic interferences that arise from the wake generated by the leader on the follower into account, i.e., wake induced forces and moments. From those efforts, we have computed the accelerations undergone by the aircraft and deduced the equations of motion. Therefore, the 6DOF dynamic states can be integrated over time and feedback control laws enable to control them. While formation flight analysis usually considers a decoupling of the aircraft dynamics with respect to the aerodynamic interferences of the leader's wake, we have been able to control the entire 6DOF dynamics of an aircraft within LES leading to a *simulated flight environment* for formation flight.

In parallel to this first model, we have also developed in Chapter 1 a simplified formation flight model that uses the Prandtl's Lifting Line (PLL) theory to account for that same aerodynamic influence of a leader's wake on the follower. We have used this so-called *surrogate model* to help in the design of the control laws exploited in the simulated flight environment. We have highlighted the similarities between the models in terms of wing loading and resulting dynamics. The development of those models opened the door for their combined use in model-based wake estimation strategies.

Wake vortex estimation

We have developed in Chapter 2 a wake sensing strategy for the estimation of a leader's wake parameters (primarily position and intensity) by a follower aircraft. Its implementation is based on an Ensemble Kalman Filter (EnKF) that was developed in previous studies. While sensing strategies developed in previous works exploited direct measurements of the flow or distributed pressure measurements along the wing, we have proved in this work the ability to predict wake characteristics based on the use of dynamic measurements of the follower. Although the estimation of the wake position as well as its intensity is theoretically possible by using those measurements only,

we have highlighted the need for an indirect measure of the leader's wake intensity. Indeed, due to the intrinsic correlation between the wake intensity and the relative position of the wake, we have observed observability issues when using aircraft dynamics as unique measurements. These issues have been all the more highlighted as the measurement noise of some specific dynamic measurements were important. This has reflected the need for sufficiently accurate measurement equipments.

However, thanks to the modeling of its 6DOF dynamics and based on the aforementioned strategy, the follower was proved to be able to track in real time the estimated wake position. This allows the follower to maintain a constant position relative to the leader's wake, without the need for a permanent communication with the leader, nor dedicated hardware equipment. We showed promising results in terms of formation maintenance and energy savings even under realistic flight conditions, i.e., in a turbulent environment.

In addition to the need for quality dynamic measurements, the main drawback of this first tracking strategy lies in the exploitation of an aerodynamic model of the real flight environment. This model is propagated over time by the EnKF. In Chapter 3 we have developed an alternate strategy based on the Reinforcement Learning (RL) framework. The latter was used to train neural networks from dynamic observations of the follower. We have shown that it was possible for the the follower to learn and act in the wake of a leader in order to reduce energy consumption without having to initially predict the position of the wake. The preliminary series of tests conducted in this work showed that a follower aircraft could converge and follow an optimal position in the wake of an oscillating leader. We have also highlighted the importance of some specific observations compared to others in order to achieve such a performance.

From both the Kalman filter-based and the RL-based strategies, we have highlighted observability issues related to the prediction or learning of leader's wake characteristics. Therefore, in Chapter 4, we have been interested in the observability formalism applied to the wake estimation and sensing problem. By exploiting scalar observability criteria introduced in the literature, we have quantified spatial regions in the wake of a leader that are prone to degraded prediction performance when using dynamic measurements of the follower only. We did not go so far as to quantify the relevance of one measurement compared to another. However, we were able to use optimal control tools in order to compute trajectories that led the follower towards desired positions while optimizing the observability related to dynamic measurements. An ultimate step that could not be realized in this work would have been to simultaneously estimate the position of a wake using the strategy described in Chapter 2 while optimizing the trajectory in terms of observability maximization thanks to optimal control.

Formation flight wake propagation

In addition to the modeling of aircraft dynamics in LES, we have been interested in Chapter 5 in the long-term propagation of the wake generated by two identical aircraft flying in formation and whose relative position is uncertain. To that end, we have employed LES in order to capture the roll-up of the vortex sheets originating from the leader and follower, their mutual interactions and their evolution within ambient turbulence into a fully developed four vortex system (4VS). We have first proposed a calibrated combination of 3D Space Developing (SD) and 3D Time Developing (TD) VPM simulations to capture the full phenomenological chain of such wakes without the need for rolled-up wake models. The methodology was verified by comparing the wake statistics computed in both TD and SD simulations over a period of simulations of about two wake characteristic times. We showed that such an approach was necessary to accurately model the intrinsic asymmetry of the wake of a formation coming from the age difference between the leader's vortices and the follower's vortex sheet.

We have then highlighted the sensitivity of the formation wake propagation to the relative position between follower and leader's wake. By combining the results produced for different formation flight configurations, we deduced a statistical model of the wake propagation of a two-aircraft formation. We explained the behaviors of the four wake vortices involved, and highlighted the complexity and diversity of the vortices dynamics. In particular, we have shown that aircraft formation wakes differ markedly from those of a single aircraft. We have shown that the global characteristics of the wake such as the Rolling Moment Coefficient or the Vertical Force Coefficient experienced in the wake were favorable to the implementation of larger formation in terms of energy savings for instance. However, the generated wake seemed to persist over longer periods of time, which constitutes an important decisional factor for the air traffic management.

Perspectives

The thesis contributed in to development of innovative sensing strategies and led to numerical investigations in the context of formation flight. Various topics have been studied in a theoretical context and constitute starting points for further improvements and applications.

On the sensing of wake flows

The wake sensing and tracking strategy developed in Chapter 2 demonstrated its relevance and performances for the tracking of aircraft wakes in order to reduce energy consumption. The next step in the demonstration of the effectiveness of such an approach would consist in an experimental and operational implementation. While in our study we have considered formations of civilian aircraft, it would be wise to initially deploy the strategy with a pair of fixed wing Unmanned Aerial Vehicle (UAV) in formation. This opens the door to several challenges to be tackled. First, we highlighted the need for measurements of the dynamics of an aircraft to perform an accurate wake estimation. Onboard equipments will need to measure those in an appropriate way, i.e., measurements accuracy and availability. Secondly, we have developed in Chapter 1 a very "academic" surrogate model of the aerodynamics of an aircraft based on the Prandtl Lifting Line which is exploited by an EnKF. It offered good agreement in terms of measurements with our simulated flight environment. However, it would need to be adapted to the more realistic conditions for an operational setup. The development of a model based on aerodynamic derivatives deduced from actual measurements observed during dedicated test campaigns is a prerequisite for an operational deployment of the EnKF-based approach. The RL-based tracking strategy developed in Chapter 3 is an alternative solution that could overcome this difficulty by learning such a model in real time from the aircraft within the formation.

We have been focused on the sole use of dynamic measurements only for the wake estimation in Chapters 2 and 3; the performance of this approach should be compared to that of others already presented in the literature. In particular, one could be interested in comparing our strategy to methods based on a direct measurement of the wake flow, e.g., using a Light Detection And Ranging (LIDAR) sensor for instance [97]. The LES performed in this work already provide a way to simulate wake flow data. They could be used to test the performance of a LIDAR sensor that would be used in a strategy based on the direct measurement of the flow to track the leader's wake vortices. By varying the measurement noise associated to one strategy or the other, one might compare the performance of the tracking strategies and check the relevance of one method rather than the other. An even better wake tracking strategy could be designed by combining both approaches. Indeed, redundancy is never a bad thing if one seek to the certification of flight procedure.

In this thesis, we have focused on the estimation of airplanes wake only. However, literature shows that the wakes of helicopters and other Vertical Take-off and Landing aircraft (VTOL) also transition to a system dominated by two parallel vortices [22, 19]. The latter could be exploited in a way similar to airplane wakes for energy

savings provided with the appropriate aerodynamic models and simulation tools. The extension of the models and simulation tools developed in this thesis to these contexts opens the door to new formation flight perspectives. Moreover, the formation flight has been inspired from the synchronized flight of migratory birds. Yet, which sensorimotor cues of the flow are available to birds and how they are exploited to drive the birds in optimal configurations remain open problems. Based on LES of the wake of a bird flapping flight [32], the sensing and learning tools developed in this thesis can offer the opportunity to learn more about the measurements required to achieve this instinctive performance.

The observability of the wake parameters presented in Chapter 4 also deserves further study and, more interestingly, is related to the identification of sensorimotor cues for bird formation flight. While we have only highlighted areas characterized by higher or lower observability in the wake of a leader based on dynamics measurements, one might want to evaluate the relevance of one observation compared to the others in the wake estimation context. In other terms, the observability formalism should be used to highlight what are those best sensorimotor cues. Finally, in this thesis we have considered observability and energy consumption optimization separately. Further work would involve to develop strategies that help converging towards energy optimal positions while maximizing the observability of the flow. This leads in fact to a multiple objective strategy that could also be envisioned within RL framework through the definition of appropriate rewards.

On formation flight simulation

In Chapter 5, we have focused on the propagation of two-aircraft formation wakes and we have revealed the long-term uncertainty of the wake vortices characteristics with respect to the uncertainty associated with the relative position between two identical aircraft in the formation. For this specific case, we have already highlighted a large spectrum of wake behaviors. However, this example is only one among the many that could occur in the formation flight context. In particular, the use of aircraft with different masses will lead to even more complex behaviors. Indeed we have highlighted the strong impact of the asymmetry between the wake vortices of the leader and the vortex sheet of the follower during the initial interactions. This asymmetry will be amplified when considering aircraft with markedly different wakes, i.e., large and small airplanes, helicopters, etc.

The simulation of the asymmetry has also been possible thanks to the modeling of the aircraft dynamics. The latter was proven useful in Chapter 5 as well as in Chap-

ters 2 and 3. The addition of aeroelastic effects and fluid structure interactions will further improve the modeling. First, it will add process noise to the sensing strategies developed in Chapter 2. Then, it will also impact the wake propagation of a formation due to the asymmetric deformation of the follower in response to the asymmetric wing loading.

The strong impact of the initial physics on the wake propagation reveals the relevance of the simulation methodology that has been developed in this work and that combines SD and TD simulations in order to simulate the full phenomenological chain of the wake propagation. As already highlighted previously, the simulation of formation is not limited to airplanes. The simulation methodology based on SD and TD simulations offers opportunities for the simulations of rotorcraft, as already performed in [19] and might be tested for the simulation of the wake of flapping wing birds.

The wake propagation of formation flight as it has been tested in Chapter 5 is a first step in the design development of operational model of formation flight wake. It will be interesting to enrich the simulations performed in this work in order to develop a more appropriate tool that could be exploited by the Air Traffic Management.

There are of course many other additional topics related to formation flight that have not been addressed in this work. Understanding the impact of unsteady loading caused by flying in a dynamic wake environment is one of them. This dynamic behavior is due not only to the fluid mechanics, but also to the unsteadiness in maintaining the aircraft positioning with respect to the wake. Structural fatigue, vibration, as well as passenger ride quality need to be assessed in aeroelastic analyses and flight tests. Another potential topic of future work is mission analyses and routing. Trade-offs in terms of cruise speed, altitude or total drag have already been studied in the literature but would need to be further pursued for full mission analysis and routing studies [13, 155, 84]. Those are some among the wide variety of topics that need to be explored before formation flight to become an operational flight strategy.

Bibliography

- [1] A. R. Ahmadi and S. E. Widnall, “Unsteady lifting-line theory as a singular perturbation problem,” *Journal of Fluid Mechanics*, vol. 153, pp. 59–81, 1985.
- [2] Airbus. (2021, November) Airbus and its partners demonstrate how sharing the skies can save airlines fuel and reduce co2 emissions. [Online]. Available: <https://www.airbus.com/en/newsroom/press-releases/2021-11-airbus-and-its-partners-demonstrate-how-sharing-the-skies-can-save>
- [3] M. Alsalman, B. Colvert, and E. Kanso, “Training bioinspired sensors to classify flows,” *Bioinspiration & biomimetics*, vol. 14, no. 1, p. 016009, 2018.
- [4] J. A. E. Andersson, J. Gillis, G. Horn, J. B. Rawlings, and M. Diehl, “CasADi – A software framework for nonlinear optimization and optimal control,” *Mathematical Programming Computation*, vol. 11, no. 1, pp. 1–36, 2019.
- [5] M. Asch, M. Bocquet, and M. Nodet, *Data assimilation: methods, algorithms, and applications*. SIAM, 2016.
- [6] P. Balty, D.-G. Caprace, J. Waucquez, M. Coquelet, and P. Chatelain, “Multi-physics simulations of the dynamic and wakes of a floating vertical axis wind turbine,” in *TORQUE*, vol. 1618. Delft: IOP Publishing, 2020, p. 062053.
- [7] Z. A. Bangash, R. P. Sanchez, A. Ahmed, and M. Khan, “Aerodynamics of formation flight,” *Journal of aircraft*, vol. 43, no. 4, pp. 907–912, 2006.
- [8] P. Batista, C. Silvestre, and P. Oliveira, “Single range aided navigation and source localization: Observability and filter design,” *Systems & Control Letters*, vol. 60, no. 8, pp. 665–673, 2011.

- [9] M. Beukenberg and D. Hummel, "Aerodynamics, performance and control of airplanes in formation flight," in *ICAS, Congress, 17 th, Stockholm, Sweden, Proceedings.*, no. ICAS-90-5.9.3, 1990, pp. 1777–1794.
- [10] S. R. Bieniawski, S. Rosenzweig, and W. B. Blake, "Summary of Flight Testing and Results for the Formation Flight for Aerodynamic Benefit Program," in *52nd Aerospace Sciences Meeting*, no. 1457. AIAA, 2014, pp. 1–26.
- [11] D. Bieniek, R. Luckner, I. De Visscher, and G. Winckelmans, "Simulation Methods for Aircraft Encounters with Deformed Wake Vortices," *Journal of Aircraft*, 2016.
- [12] P. Binetti, K. B. Ariyur, M. Krstic, and F. Bernelli, "Formation Flight Optimization Using Extremum Seeking Feedback," *Journal of Guidance, Control, and Dynamics*, vol. 26, no. 1, pp. 132–142, 2015/12/10 2003.
- [13] G. C. Bower, T. C. Flanzer, and I. M. Kroo, "Formation geometries and route optimization for commercial formation flight," in *27th AIAA Applied Aerodynamics Conference*, 2009, p. 3615.
- [14] C. Breitsamter, "Wake vortex characteristics of transport aircraft," *Progress in Aerospace Sciences*, vol. 47, no. 2, pp. 89 – 134, 2011. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0376042110000576>
- [15] M. Brodecki and K. Subbarao, "Autonomous formation flight control system using in-flight sweet-spot estimation," *Journal of Guidance, Control, and Dynamics*, vol. 38, no. 6, pp. 1083–1096, 2014.
- [16] S. L. Brunton, B. R. Noack, and P. Koumoutsakos, "Machine learning for fluid mechanics," *Annual Review of Fluid Mechanics*, vol. 52, no. 1, pp. 477–508, 2020. [Online]. Available: <https://doi.org/10.1146/annurev-fluid-010719-060214>
- [17] G. H. Bryan, *Stability in aviation: an introduction to dynamical stability as applied to the motions of aeroplanes.* Macmillan and Company, limited, 1911.
- [18] G. Burgers, P. Jan van Leeuwen, and G. Evensen, "Analysis scheme in the ensemble kalman filter," *Monthly weather review*, vol. 126, no. 6, pp. 1719–1724, 1998.
- [19] D.-G. Caprache and A. Ning, "Large eddy simulation of the wakes of three urban air mobility vehicles," in *Vertical Flight Society's 78th Annual Forum*, 2022.

- [20] D.-G. Caprace, P. Chatelain, and G. Winckelmans, “Lifting line with various mollifications: Theory and application to an elliptical wing,” *AIAA Journal*, vol. 57, no. 1, pp. 17–28, 2019.
- [21] D.-G. Caprace, G. Winckelmans, P. Chatelain, and J. Eldredge, “Wake vortex detection and tracking for aircraft formation flight,” in *AIAA Aviation 2019 Forum*, AIAA, Ed., June 2019. [Online]. Available: <https://arc.aiaa.org/doi/abs/10.2514/6.2019-3329>
- [22] D.-G. Caprace, P. Chatelain, and G. Winckelmans, “Wakes of rotorcraft in advancing flight: A large eddy simulation study,” *Physics of Fluids*, vol. 32, no. 8, p. 087107, 2020.
- [23] D.-G. Caprace, G. Winckelmans, and P. Chatelain, “An immersed lifting and dragging line model for the vortex particle-mesh method,” *Theoretical and Computational Fluid Dynamics*, vol. 34, pp. 21–48, 2020.
- [24] D.-G. Caprace, G. Winckelmans, and P. Chatelain, “Assessment of the vortex particle-mesh method for efficient LES of hovering rotors and their wakes,” in *AIAA Scitech 2021 Forum*, 2021, p. 0738.
- [25] L. D. Chambers, O. Akanyeti, R. Venturelli, J. Ježov, J. Brown, M. Kruusmaa, P. Fiorini, and W. Megill, “A fish perspective: detecting flow features while moving using an artificial lateral line in steady and unsteady flow,” *Journal of the Royal Society Interface*, vol. 11, no. 99, p. 20140467, 2014.
- [26] P. Chatelain, A. Curioni, M. Bergdorf, D. Rossinelli, W. Andreoni, and P. Koumoutsakos, “Billion vortex particle Direct Numerical Simulations of aircraft wakes,” *Computer Methods in Applied Mechanics and Engineering*, vol. 197, no. 13, pp. 1296–1304, Feb 2008.
- [27] P. Chatelain, S. Backaert, G. Winckelmans, and S. Kern, “Large eddy simulation of wind turbine wakes,” *Flow, Turbulence and Combustion*, vol. 91, no. 3, pp. 587–605, 2013.
- [28] D. F. Chichka, J. L. Speyer, C. Fanti, and C. G. Park, “Peak-seeking control for drag reduction in formation flight,” *Journal of Guidance, Control, and Dynamics*, vol. 29, no. 5, pp. 1221–1230, 2006. [Online]. Available: <https://doi.org/10.2514/1.15424>

- [29] J. Cho, B. J. Lee, T. Misaka, and K. Yee, “Study on decay characteristics of vertical four-vortex system for increasing airport capacity,” *Aerospace Science and Technology*, vol. 105, p. 106017, 2020.
- [30] R. Cocle, G. Winckelmans, and G. Daeninck, “Combining the vortex-in-cell and parallel fast multipole methods for efficient domain decomposition simulations,” *Journal of Computational Physics*, vol. 227, no. 21, pp. 9091–9120, Nov 2008.
- [31] R. Cocle, L. Bricteux, and G. Winckelmans, “Scale dependence and asymptotic very high Reynolds number spectral behavior of multiscale subgrid models,” *Physics of Fluids*, vol. 21, no. 8, p. 085101, Aug 2009.
- [32] V. Colognesi, R. Ronsse, and P. Chatelain, “Model coupling biomechanics and fluid dynamics for the simulation of controlled flapping flight,” *Bioinspiration & Biomimetics*, vol. 16, no. 2, jan 2021.
- [33] B. Colvert, M. Alsalman, and E. Kanso, “Classifying vortex wakes using neural networks,” *arXiv preprint*, 2017.
- [34] M. V. Cook, *Flight dynamics principles: a linear systems approach to aircraft stability and control*. Butterworth-Heinemann, 2012.
- [35] M. Coquelet, L. Bricteux, M. Moens, and P. Chatelain, “A reinforcement-learning approach for individual pitch control,” *Wind Energy*, 2022.
- [36] G.-H. Cottet and P. Koumoutsakos, *Vortex Methods, Theory and Practice*. Cambridge University Press, 2000.
- [37] J. Crouch, “Airplane trailing vortices and their control,” *Comptes Rendus Physique*, vol. 6, no. 4-5, pp. 487–499, 2005/6// 2005.
- [38] S. C. Crow, “Stability theory for a pair of trailing vortices,” *AIAA Journal*, vol. 8, no. 12, pp. 2172–2179, 1970.
- [39] A. F. da Silva and T. Colonius, “A bias-aware enkf estimator for aerodynamic flows,” in *2018 Fluid Dynamics Conference*, 2018, p. 3225.
- [40] T. Daniels, W. Smith, and S. Kireev, “Recent developments on airborne forward looking interferometer for the detection of wake vortices,” in *4th AIAA Atmospheric and Space Environments Conference*, 2012, p. 2791.

- [41] D. Darakananda, J. Eldredge, A. da Silva, T. Colonius, and D. R. Williams, “Enkf-based dynamic estimation of separated flows with a low-order vortex model,” in *2018 AIAA Aerospace Sciences Meeting*, 2018, p. 0811.
- [42] A. de Bruin, “Synthesis of eu far-wake project results: Final synthesis report,” FAR-Wake, Tech. Rep. D 4.2-3, 2008. [Online]. Available: far-wake.irphe.univ-mrs.fr/IMG/deliverables/deliverables.html
- [43] A. de Bruin and G. Winckelmans, “Cross-flow kinetic energy and core size growth of analytically defined wake vortex pairs,” National Aerospace Laboratory NLR, Tech. Rep. NLR-CR-2005-412, 2005.
- [44] I. De Visscher, “Interaction of wake vortices with the atmosphere and the ground : advanced numerical simulation and operational modeling,” Ph.D. dissertation, Université catholique de Louvain, 2012.
- [45] I. De Visscher, L. Bricteux, and G. Winckelmans, “Aircraft vortices in stably stratified and weakly turbulent atmospheres: simulation and modeling,” *AIAA Journal*, vol. 51, no. 3, pp. 551–566, 2013.
- [46] N. Depenbusch and J. Langelaan, “Coordinated mapping and exploration for autonomous soaring,” in *Infotech@ Aerospace 2011*, 2011, p. 1436.
- [47] L. DeVries and D. A. Paley, “Wake Sensing and Estimation for Control of Autonomous Aircraft in Formation Flight,” *Journal of Guidance, Control, and Dynamics*, pp. 1–10, 2015/12/09 2015.
- [48] L. DeVries, S. J. Majumdar, and D. A. Paley, “Observability-based optimization of coordinated sampling trajectories for recursive estimation of a strong, spatially varying flowfield,” *Journal of intelligent & robotic systems*, vol. 70, no. 1, pp. 527–544, 2013.
- [49] N. Docquier, A. Poncelet, and P. Fisette, “Robotran: a powerful symbolic generator of multibody models,” *Mechanical Sciences*, vol. 4, no. 1, pp. 199–219, 2013.
- [50] A. Dogan, S. Sato, and W. Blake, “Flight control and simulation for aerial refueling,” in *AIAA guidance, navigation, and control conference and exhibit*, 2005, p. 6264.
- [51] M. Drela, “Integrated simulation model for preliminary aerodynamic, structural, and control-law design of aircraft,” *AIAA Paper*, vol. 99, p. 1394, 1999.

- [52] A. M. Edstrand, T. B. Davis, P. J. Schmid, K. Taira, and L. N. Cattafesta, "On the mechanism of trailing vortex wandering," *Journal of Fluid Mechanics*, vol. 801, 2016.
- [53] Eurocontrol, "Challenges of Growth 2013: Task 4: European Air Traffic in 2035," Eurocontrol, Tech. Rep., June 2013.
- [54] G. Evensen, "Sequential data assimilation with a nonlinear quasi-geostrophic model using monte carlo methods to forecast error statistics," *Journal of Geophysical Research: Oceans*, vol. 99, no. C5, pp. 10 143–10 162, 1994.
- [55] G. Evensen, *Data assimilation: the ensemble Kalman filter*. Springer Science & Business Media, 2009.
- [56] D. Fabre and L. Jacquin, "Stability of a four-vortex aircraft wake model," *Physics of Fluids*, vol. 12, no. 10, pp. 2438–2443, 2000.
- [57] T. C. Flanzer, S. R. Bieniawski, and J. A. Brown, "Advances in cooperative trajectories for commercial applications," in *AIAA Scitech 2020 Forum*, 2020. [Online]. Available: <https://arc.aiaa.org/doi/abs/10.2514/6.2020-1257>
- [58] B. Friedland, "Treatment of bias in recursive filtering," *IEEE Transactions on Automatic Control*, vol. 14, no. 4, pp. 359–367, 1969.
- [59] J. F. Garten, J. Werne, D. C. Fritts, and S. Arendt, "Direct numerical simulations of the Crow instability and subsequent vortex reconnection in a stratified fluid," *Journal of Fluid Mechanics*, vol. 426, pp. 1–45, 2001.
- [60] T. Gerz, F. Holzäpfel, and D. Darracq, "Commercial aircraft wake vortices," *Progress in Aerospace Sciences*, vol. 38, no. 3, pp. 181–208, 2002.
- [61] T. Gerz, F. Holzäpfel, W. Bryant, F. Köpp, M. Frech, A. Tafferner, and G. Winckelmans, "Research towards a wake-vortex advisory system for optimal aircraft spacing," *Comptes Rendus Physique*, vol. 6, no. 4-5, pp. 501–523, 2005/0.
- [62] H. Glauert, *The Elements of Aerofoil and Airscrew Theory*. The University Press, 1926.
- [63] D. Goshen-Meskin and I. Y. Bar-Itzhack, "Unified approach to inertial navigation system error modeling," *Journal of Guidance, Control, and Dynamics*, vol. 15, no. 3, pp. 648–653, 1992.

- [64] J.-L. Guermond, "A generalized lifting-line theory for curved and swept wings," *Journal of Fluid Mechanics*, vol. 211, pp. 497–513, 1990.
- [65] P. Gunnarson, I. Mandralis, G. Novati, P. Koumoutsakos, and J. O. Dabiri, "Learning efficient navigation in vortical flow fields," *arXiv preprint arXiv:2102.10536*, 2021.
- [66] T. Haarnoja, A. Zhou, P. Abbeel, and S. Levine, "Soft actor-critic: Off-policy maximum entropy deep reinforcement learning with a stochastic actor," *Haarnoja, Tuomas and Zhou, Aurick and Abbeel, Pieter and Levine, Sergey*, 2018.
- [67] J. N. Hallock and F. Holzäpfel, "A review of recent wake vortex research for increasing airport capacity," *Progress in Aerospace Sciences*, vol. 98, pp. 27–36, 2018.
- [68] J. Hansen and B. Cobleigh, "Induced moment effects of formation flight using two f/a-18 aircraft," in *AIAA Atmospheric Flight Mechanics Conference and Exhibit*, 2002, p. 4489.
- [69] C. E. Hanson, *A Fuzzy Technique for Performing Lateral-Axis Formation Flight Navigation Using Wingtip Vortices*. National Aeronautics and Space Administration, Dryden Flight Research Center, 2003.
- [70] C. E. Hanson, S. Andrade, and J. Pahle, "Experimental measurements of passenger ride quality during aircraft wake surfing," in *2018 Atmospheric Flight Mechanics Conference*, 2018, p. 3561.
- [71] C. E. Hanson, J. Pahle, J. R. Reynolds, and S. Andrade, "Experimental measurements of fuel savings during aircraft wake surfing," in *2018 Atmospheric Flight Mechanics Conference*, 2018, p. 3560.
- [72] M. S. Hemati, J. D. Eldredge, and J. L. Speyer, "Wake Sensing for Aircraft Formation Flight," *Journal of Guidance, Control, and Dynamics*, vol. 37, no. 2, pp. 513–524, 2016/01/25 2014.
- [73] R. Hermann and A. Krener, "Nonlinear controllability and observability," *IEEE Transactions on automatic control*, vol. 22, no. 5, pp. 728–740, 1977.
- [74] B. T. Hinson, M. K. Binder, and K. A. Morgansen, "Path planning to optimize observability in a planar uniform flow field," in *2013 American Control Conference*. IEEE, 2013, pp. 1392–1399.

- [75] F. Holzäpfel, “Effects of environmental and aircraft parameters on wake vortex behavior,” *Journal of Aircraft*, vol. 51, no. 5, pp. 1490–1500, 2014.
- [76] F. Holzäpfel, T. Hofbauer, D. Darracq, H. Moet, F. Garnier, and C. F. Gago, “Analysis of wake vortex decay mechanisms in the atmosphere,” *Aerospace Science and Technology*, vol. 7, no. 4, pp. 263 – 275, 2003. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S1270963803000269>
- [77] D. Hummel, “The use of aircraft wakes to achieve power reductions in formation flight,” in *The Characterization & Modification of Wakes from Lifting Vehicles in Fluid*. NATO, 1996, vol. AGARD-CP-584, no. 36, pp. 1–13.
- [78] H. Jeanmart and G. Winckelmans, “Investigation of eddy-viscosity models modified using discrete filters: A simplified regularized variational multiscale model and an enhanced field model,” *Physics of Fluids*, vol. 19, no. 5, p. 055110, May 2007.
- [79] N. Joukowski, “Sur les tourbillons adjoints,” *Transcript of the Physical Section, Imperial Society of Friends of Natural Sciences, Moscow*, vol. 23, 1906.
- [80] H. Kaden, “Aufwicklung einer unstabilen unstetigkeitsfläche,” *Ingenieur-Archiv*, vol. 2, no. 2, pp. 140–168, 1931.
- [81] A. D. Kahn, “Atmospheric thermal location estimation,” *Journal of Guidance, Control, and Dynamics*, vol. 40, no. 9, pp. 2363–2369, 2017.
- [82] A. D. Kahn and D. J. Edwards, “Navigation, guidance, and control of a micro unmanned aerial glider,” *Journal of Guidance, Control, and Dynamics*, vol. 42, no. 11, pp. 2474–2484, 2019.
- [83] J. Katz and A. Plotkin, *Low-speed aerodynamics*. Cambridge university press, 2001, vol. 13.
- [84] T. E. Kent and A. G. Richards, “Potential of formation flight for commercial aviation: Three case studies,” *Journal of Aircraft*, vol. 0, no. 0, pp. 1–14, 2020. [Online]. Available: <https://doi.org/10.2514/1.C035954>
- [85] M. Koifman and I. Y. Bar-Itzhack, “Inertial navigation system aided by aircraft dynamics,” *IEEE transactions on control systems technology*, vol. 7, no. 4, pp. 487–493, 1999.

- [86] A. J. Krener and K. Ide, "Measures of unobservability," in *Proceedings of the 48th IEEE Conference on Decision and Control (CDC) held jointly with 2009 28th Chinese Control Conference*. IEEE, 2009, pp. 6401–6406.
- [87] W. Kutta, "Auftriebskräfte in strömenden flüssigkeiten," *Illustrierte Aeronautische Mitteilungen*, vol. 6, no. 133, pp. 133–135, 1902.
- [88] F. D. Lagor, K. Ide, and D. A. Paley, "Incorporating prior knowledge in observability-based path planning for ocean sampling," *Systems & Control Letters*, vol. 97, pp. 169–175, 2016.
- [89] S. Lang, J. Tittsworth, W. Bryant, P. Wilson, C. Lepadatu, D. Delisi, D. Lai, and G. Greene, "Progress on an icao wake turbulence re-categorization effort," *AIAA Atmospheric and Space Environments Conference*, p. 7682, 2010.
- [90] F. Laporte and A. Corjon, "Direct numerical simulations of the elliptic instability of a vortex pair," *Physics of Fluids*, vol. 12, no. 5, pp. 1016–1031, 2020/01/03 2000. [Online]. Available: <https://doi.org/10.1063/1.870357>
- [91] N. R. Lawrance and S. Sukkariéh, "Autonomous exploration of a wind field with a gliding aircraft," *Journal of guidance, control, and dynamics*, vol. 34, no. 3, pp. 719–733, 2011.
- [92] S. Le Dizès and F. Laporte, "Theoretical predictions for the elliptical instability in a two-vortex flow," *Journal of Fluid Mechanics*, vol. 471, pp. 169–201, 2002.
- [93] J. G. Lee, C. G. Park, and H. W. Park, "Multiposition alignment of strapdown inertial navigation system," *IEEE Transactions on Aerospace and Electronic systems*, vol. 29, no. 4, pp. 1323–1328, 1993.
- [94] P. Lissaman and C. A. Shollenberger, "Formation flight of birds," *Science*, vol. 168, no. 3934, pp. 1003–1005, 1970.
- [95] R. Luckner, G. Höhne, and M. Fuhrmann, "Hazard criteria for wake vortex encounters during approach," *Aerospace Science and Technology*, vol. 8, no. 8, pp. 673–687, 2004.
- [96] J. Mann, "Wind field simulation," *Probabilistic Engineering Mechanics*, vol. 13, no. 4, pp. 269–282, 10 1998.
- [97] D. T. Michel, A. Dolfi-Bouteyre, D. Goular, B. Augère, C. Planchat, D. Fleury, L. Lombard, M. Valla, and C. Besson, "Onboard wake vortex localization with

- a coherent 1.5 μm doppler lidar for aircraft in formation flight configuration,” *Optics Express*, vol. 28, no. 10, pp. 14 374–14 385, May 2020. [Online]. Available: <http://www.opticsexpress.org/abstract.cfm?URI=oe-28-10-14374>
- [98] C. Mimeau and I. Mortazavi, “A review of vortex methods and their applications: From creation to recent advances,” *Fluids*, vol. 6, no. 2, 2021. [Online]. Available: <https://www.mdpi.com/2311-5521/6/2/68>
- [99] H. Nijmeijer and A. J. Van der Schaft, *Nonlinear dynamical control systems*. Springer, 1990, vol. 175.
- [100] A. Ning, T. C. Flanzer, and I. M. Kroo, “Aerodynamic performance of extended formation flight,” *Journal of Aircraft*, vol. 48, no. 3, pp. 855–865, 2011.
- [101] S. A. Ning, I. Kroo, J. J. Alonso, and S. K. Lele, *Aircraft drag reduction through extended formation flight*. Stanford University, 2011.
- [102] S. Notter, M. Zürn, P. Groß, and W. Fichter, “Reinforced learning to cross-country soar in the vertical plane of motion,” in *AIAA Scitech 2019 Forum*, 2019, p. 1420.
- [103] S. Notter, P. Groß, P. Schrapel, and W. Fichter, “Multiple thermal updraft estimation and observability analysis,” *Journal of Guidance, Control, and Dynamics*, vol. 43, no. 3, pp. 490–503, 2020.
- [104] S. Notter, F. Schimpf, and W. Fichter, “Hierarchical reinforcement learning approach towards autonomous cross-country soaring,” in *AIAA Scitech 2021 Forum*, 2021, p. 2010.
- [105] G. Novati, H. L. de Laroussilhe, and P. Koumoutsakos, “Automating turbulence modelling by multi-agent reinforcement learning,” *Nature Machine Intelligence*, vol. 3, no. 1, pp. 87–96, 2021.
- [106] P. Oettershagen, T. Stastny, T. Hinzmann, K. Rudin, T. Mantel, A. Melzer, B. Wawrzacz, G. Hitz, and R. Siegwart, “Robotic technologies for solar-powered uavs: Fully autonomous updraft-aware aerial sensing for multiday search-and-rescue missions,” *Journal of Field Robotics*, vol. 35, no. 4, pp. 612–640, 2018.
- [107] J. M. Ortega, R. L. Bristol, and Ö. Savas, “Experimental study of the instability of unequal-strength counter-rotating vortex pairs,” *Journal of Fluid Mechanics*, vol. 474, pp. 35–84, Jan 2003.

- [108] A. Paszke, S. Gross, F. Massa, A. Lerer, J. Bradbury, G. Chanan, T. Killeen, Z. Lin, N. Gimelshein, L. Antiga *et al.*, “Pytorch: An imperative style, high-performance deep learning library,” *Advances in neural information processing systems*, vol. 32, 2019.
- [109] W. R. C. Phillips, “The turbulent trailing vortex during roll-up,” *Journal of Fluid Mechanics*, vol. 105, pp. 451–467, 1981. [Online]. Available: <https://www.cambridge.org/core/article/turbulent-trailing-vortex-during-rollup/FF92FB5C373CE261C66645FD6BF8B89C>
- [110] W. Phillips and D. Snyder, “Modern adaptation of Prandtl’s classic lifting-line theory,” *Journal of Aircraft*, vol. 37, no. 4, pp. 662–670, 2000.
- [111] P. Phlips, R. East, and N. Pratt, “An unsteady lifting line theory of flapping wings with application to the forward flight of birds,” *Journal of fluid mechanics*, vol. 112, pp. 97–125, 1981.
- [112] S. J. Portugal, T. Y. Hubel, J. Fritz, S. Heese, D. Trobe, B. Voelkl, S. Hailes, A. M. Wilson, and J. R. Usherwood, “Upwash exploitation and downwash avoidance by flap phasing in ibis formation flight,” *Nature*, vol. 505, no. 7483, pp. 399–402, 01 2014.
- [113] L. Prandtl, *Tragflügeltheorie*. Göttinger Nachrichten, Mathematisch-Physikalische Klasse, Germany, 1918, pp. 451–477.
- [114] A. Raffin, A. Hill, A. Gleave, A. Kanervisto, M. Ernestus, and N. Dormann, “Stable-baselines3: Reliable reinforcement learning implementations,” *Journal of Machine Learning Research*, 2021.
- [115] M. Rafieisakhaei, S. Chakravorty, and P. Kumar, “On the use of the observability gramian for partially observed robotic path planning problems,” *arXiv preprint arXiv:1801.09877*, 2018.
- [116] I. Ransquin and P. Chatelain, “Bio-inspired wake tracking for aircraft formation flight based on reinforcement learning,” in *AIAA Scitech 2021 Forum*, AIAA, Ed., 2021.
- [117] I. Ransquin, D.-G. Caprace, M. Duponcheel, and P. Chatelain, “Wake vortex detection and tracking for aircraft formation flight,” *Journal of Guidance, Control, and Dynamics*, vol. 44, no. 12, pp. 2225–2243, August 2021.

- [118] I. Ransquin, M. Duponcheel, and P. Chatelain, “Dynamics of the wake vortices of a two-aircraft formation, hazard assessment at large distances and sensitivity analysis,” in *AIAA SCITECH 2022 Forum*, 2022, p. 1198.
- [119] G. Reddy, A. Celani, T. J. Sejnowski, and M. Vergassola, “Learning to soar in turbulent environments,” *Proceedings of the National Academy of Sciences*, vol. 113, no. 33, pp. E4877–E4884, 2016.
- [120] J. T. Reid and D. F. Hunsaker, “A general approach to lifting-line theory, applied to wings with sweep,” in *AIAA Scitech 2020 Forum*, 2020. [Online]. Available: <https://arc.aiaa.org/doi/abs/10.2514/6.2020-1287>
- [121] A. Reinke, S. Kauertz, T. Bauer, D. Fischenberg, F. Holzäpfel, D. Niedermeier, D. Vechtel, and C. Wolkensinger, “Aircraft Wake Vortex State-of-the-Art & Research Needs,” in *WakeNet3 Europe*, 2015.
- [122] J. R. Riehl, E. A. Hufstedler, P. Chatelain, and J. M. Hendrickx, “String stability of energy-saving aircraft formations,” *Journal of Guidance, Control, and Dynamics*, vol. 45, no. 5, pp. 935–943, 2022.
- [123] R. J. Rodenhiser, W. W. Durgin, and H. Johari, “Ultrasonic method for aircraft wake vortex detection,” *Journal of aircraft*, vol. 44, no. 3, pp. 726–732, 2007.
- [124] C. Roy and T. Leweke, “Experiments on vortex meandering,” FAR-Wake, Tech. Rep. TR 1.1.1-4, April 2008.
- [125] J. J. Ryan, “Formation flight system extremum-seeking-control using blended performance parameters,” patent, Jan. 9 2018, uS9864380B1.
- [126] T. Sarpkaya, “Decay of wake vortices of large aircraft,” *AIAA journal*, vol. 36, no. 9, pp. 1671–1679, 1998.
- [127] F. Schimpf, S. Notter, P. Groß, and W. Fichter, “Multi-agent reinforcement learning for thermalling in updrafts,” in *AIAA Scitech 2021 Forum*, 2021, p. 0864.
- [128] H. Schlichting, “Leistungsersparnis im ver-bandsflug,” *Mitt. Dt. Akad. Luftforschung*, vol. H, no. 2, pp. 97–135, 1942.
- [129] P. Sclavounos, “An unsteady lifting-line theory,” *Journal of engineering mathematics*, vol. 21, no. 3, pp. 201–226, 1987.

- [130] S. Skogestad and I. Postlethwaite, *Multivariable feedback control: analysis and design*. Citeseer, 2002.
- [131] J. N. Sørensen, R. F. Mikkelsen, D. S. Henningson, S. Ivanell, S. Sarmast, and S. J. Andersen, “Simulation of wind turbine wakes using the actuator line technique,” *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, vol. 373, no. 2035, 2015.
- [132] P. R. Spalart, “Airplane Trailing Vortices,” *Annual Review of Fluid Mechanics*, vol. 30, no. 1, pp. 107–138, 1998.
- [133] L. Speijker, A. Vidal, F. Barbaresco, T. Gerz, H. Barny, and G. Winckelmans, “Atc-wake-integrated wake vortex safety and capacity system.” National Aerospace Laboratory NLR, 2006.
- [134] J. L. Speyer and J. D. Eldredge, “Automated operation of aircraft system in inverted-v formations,” May 11 2017, uS Patent 2017/0131726 A1.
- [135] M. Steen and P. Hecker, “Investigating the potential future usage of combined airborne wake vortex prediction models and measurement sensors,” in *2018 Atmospheric and Space Environments Conference*, 2018, p. 3022.
- [136] R. S. Sutton and A. G. Barto, *Reinforcement Learning: An Introduction*. MIT Press, 1998.
- [137] G. I. Taylor, “The spectrum of turbulence,” *Proceedings of the Royal Society of London. Series A-Mathematical and Physical Sciences*, vol. 164, no. 919, pp. 476–490, 1938.
- [138] P. Tichavsky, C. H. Muravchik, and A. Nehorai, “Posterior cramér-rao bounds for discrete-time nonlinear filtering,” *IEEE Transactions on signal processing*, vol. 46, no. 5, pp. 1386–1396, 1998.
- [139] S. Unterstrasser and A. Stephan, “Far field wake vortex evolution of two aircraft formation flight and implications on young contrails,” *The Aeronautical Journal*, vol. 124, no. 1275, pp. 667–702, 2020.
- [140] M. Vachon, R. Ray, K. Walsh, and K. Ennix, “F/a-18 aircraft performance benefits measured during the autonomous formation flight project,” NASA, Tech. Rep. TM-2003-210734, 2002.

- [141] P. J. Van Leeuwen, “Comment on “data assimilation using an ensemble kalman filter technique”,” *Monthly Weather Review*, vol. 127, no. 6, pp. 1374–1377, 1999.
- [142] F. Van Wyk, “Optimal control for minimum thrust demand in extended formation flight,” Ph.D. dissertation, Stellenbosch: Stellenbosch University, 2016.
- [143] S. Verma, G. Novati, and P. Koumoutsakos, “Efficient collective swimming by harnessing vortices through deep reinforcement learning,” *Proceedings of the National Academy of Sciences*, vol. 115, no. 23, pp. 5849–5854, 2018. [Online]. Available: <http://www.pnas.org/content/115/23/5849>
- [144] S. Verma, C. Papadimitriou, N. Lüthen, G. Arampatzis, and P. Koumoutsakos, “Optimal sensor placement for artificial swimmers,” *Journal of Fluid Mechanics*, vol. 884, 2020.
- [145] P. R. Vlachas, W. Byeon, Z. Y. Wan, T. P. Sapsis, and P. Koumoutsakos, “Data-driven forecasting of high-dimensional chaotic systems with long short-term memory networks,” *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 474, no. 2213, p. 20170844, 2018.
- [146] H. Weimerskirch, J. Martin, Y. Clerquin, P. Alexandre, and S. Jiraskova, “Energy saving in flight formation,” *Nature*, vol. 413, no. 6857, pp. 697–698, 2001.
- [147] N. Wettengl, S. Notter, and W. Fichter, “Enhancing updraft observability by optimal path planning,” in *AIAA SCITECH 2022 Forum*, 2022, p. 2216.
- [148] J. S. Whitaker and T. M. Hamill, “Evaluating methods to account for system errors in ensemble data assimilation,” *Monthly Weather Review*, vol. 140, no. 9, pp. 3078–3089, 2012.
- [149] B. T. Whitehead, S. R. Bieniawski, D. Halaas, and E. Lavretsky, “Formation flight control,” Feb. 3 2015, uS Patent 8,949,090.
- [150] C. Wieselsberger, “Beitrag zur erklärung deswinkelfluges einiger zugvogel,” *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, vol. V, no. 15, pp. 225–229, 1914.
- [151] G. Winckelmans, R. Cocle, L. Dufresne, and R. Capart, “Vortex methods and their application to trailing wake vortex simulations,” *C. R. Phys.*, vol. 6, no. 4-5, pp. 467–486, 2005.

- [152] G. S. Winckelmans, *Encyclopedia of Computational Mechanics*. John Wiley & Sons, Ltd, 2004, ch. Vortex Methods, pp. 129–153.
- [153] T. D. Woodbury, C. Dunn, and J. Valasek, “Autonomous soaring using reinforcement learning for trajectory generation,” in *52nd Aerospace Sciences Meeting*, 2014, p. 0990.
- [154] Y. Wu, X. Hu, D. Hu, T. Li, and J. Lian, “Strapdown inertial navigation system algorithms based on dual quaternions,” *IEEE transactions on aerospace and electronic systems*, vol. 41, no. 1, pp. 110–132, 2005.
- [155] J. Xu, S. Andrew Ning, G. Bower, and I. Kroo, “Aircraft route optimization for formation flight,” *Journal of Aircraft*, vol. 51, no. 2, pp. 490–501, 2014.
- [156] R. Zanetti and R. H. Bishop, “Kalman filters with uncompensated biases,” *Journal of Guidance, Control, and Dynamics*, vol. 35, no. 1, pp. 327–335, 2012.
- [157] Y. Zhai, X. Zheng, and G. Xie, “Fish lateral line inspired flow sensors and flow-aided control: A review,” *Journal of Bionic Engineering*, vol. 18, no. 2, pp. 264–291, 2021.

Aircraft wake vortex models

The characterization of an aircraft wake vortex primarily starts with the definition of its radial circulation distribution $\Gamma(r)$, where r is the distance to the center of the vortex. That distribution is exactly measured using Eq. (1.9) which is recalled here,

$$\Gamma(r) = \int_0^{2\pi} \mathbf{u}(r, \theta) \cdot \hat{\mathbf{e}}_\theta^i r d\theta \quad (\text{A.1})$$

$$= \int_0^{2\pi} \int_0^r \boldsymbol{\omega}(r, \theta) \cdot \hat{\mathbf{e}}_x^i r' dr' d\theta, \quad (\text{A.2})$$

where $\mathbf{u}(r, \theta)$ and $\boldsymbol{\omega}(r, \theta)$ are the velocity and vorticity profiles induced by the vortex on the environment. The circulation distribution drives the vortex dynamics, and thus the dynamics of the vortex system that characterize the wake (e.g., 2 vortex-system for the wake of an aircraft, 4 vortex-system for the wake of a two-aircraft formation, etc). For an axisymmetric vortex, one easily defines the distribution of the azimuthal velocity and axial vorticity distributions

$$u_\theta(r) = \frac{\Gamma(r)}{2\pi r}, \quad (\text{A.3})$$

$$\omega_x(r) = \frac{1}{2\pi r} \frac{d\Gamma(r)}{dr}. \quad (\text{A.4})$$

From the tangential velocity profile, we introduce the definition for vortex core radius, r_c , defined as the location where $u_\theta(r)$ is maximum, i.e., $r_c = \arg \max_r u_\theta(r)$. Let's note that the exact value of the core size is somewhat undetermined, as it is very difficult to capture the vortex core details of real aircraft wake, either numerically or experimentally. In the present stage of knowledge of aircraft wake vortices, it is believed that aircraft wake vortices have a core radius ranging from 3% to 5% of the aircraft wingspan b [60, 14].

Vortex models

Different models exist to represent a rolled-up aircraft wake vortex. A first and most commonly used class of models is characterized by one degree-of-freedom, i.e., one single *inner* length scale which plays the role of tuning parameter. Those models are called "one-scale" vortex models and the most popular are:

- the *Rankine* (or *top-hat*) model,

$$\frac{\Gamma(r)}{\Gamma_0} = \begin{cases} \left(\frac{r}{r_c}\right)^2 & \text{for } |r| < r_c \\ 1 & \text{elsewhere.} \end{cases} \quad (\text{A.5})$$

Note that this function is not differentiable, and therefore not appropriate for the definition of the axial vorticity ω_x ;

- the *Moore-Rosenhead* (or *Burnham-Hallock*, or *Low Order Algebraic* (LOA)) model,

$$\frac{\Gamma(r)}{\Gamma_0} = \frac{r^2}{r^2 + r_c^2}; \quad (\text{A.6})$$

- the *High Order Algebraic* (HOA) model,

$$\frac{\Gamma(r)}{\Gamma_0} = \frac{r^2 (r^2 + 2\sigma^2)}{(r^2 + r_c^2)^2}, \quad (\text{A.7})$$

with $\sigma = 1.3344 \cdot r_c$;

- the *Lamb-Oseen* (or *Gaussian*) model,

$$\frac{\Gamma(r)}{\Gamma_0} = 1 - \exp\left(-\frac{r^2}{\sigma^2}\right), \quad (\text{A.8})$$

with $\sigma = 0.8921 \cdot r_c$.

An alternative to the use of an inner scale consist in using an *outer* length scale such as the spacing between the vortices b_0 to define the model. An example of the kind of model is the *Kaden-Winckelmans* model. From the derivation of the roll-up at the tip of a parabolically loaded vortex sheet by Kaden [80], Winckelmans indeed proposed an extended model that approximates the distribution of a rolled-up thin vortex sheet

shed by a complete elliptically loaded wing, $\Gamma(y) \sim \left(1 - \left(\frac{y}{b/2}\right)^2\right)^{1/2}$. This leads to

$$\frac{\Gamma(r)}{\Gamma_0} = \frac{\alpha \left(\frac{r}{b_0/2}\right)^{1/2}}{\left(1 + \alpha \left(\frac{r}{b_0/2}\right)\right)}, \quad (\text{A.9})$$

where $\alpha = \left(\frac{\pi}{2}\right)^2 - \log 2 = 1.7743$. This model can be extended for the wake vortices of hyper-elliptically loaded wing, i.e., characterized by the spanwise circulation distribution $\Gamma(y) \sim \left(1 - \left(\frac{y}{b/2}\right)^p\right)^{1/p}$,

$$\frac{\Gamma(r)}{\Gamma_0} = \frac{\alpha \left(\frac{r}{b_0/2}\right)^{1/p}}{\left(1 + (\alpha - 1) \left(\frac{r}{b_0/2}\right)\right)}. \quad (\text{A.10})$$

For an improved representation of wake vortices, one can use two-scale models characterized by two degrees-of-freedom, e.g., r_c and b_0 in the same model:

- the *regularized Kaden Winckelmans* model:

$$\frac{\Gamma(r)}{\Gamma_0} = \left[1 - \exp\left(-\left(\frac{r}{\sigma}\right)^{3/2}\right)\right] \frac{\alpha \left(\frac{r}{b_0/2}\right)^{1/2}}{\left(1 + \alpha \left(\frac{r}{b_0/2}\right)\right)}, \quad (\text{A.11})$$

introduces the parameter σ to account for a finite viscous core size;

- the *Proctor-Winckelmans* model

$$\frac{\Gamma(r)}{\Gamma_0} = 1 - \exp\left(-\frac{\left(\frac{r}{\sigma}\right)^2}{\left(1 + \left(\frac{\left(\frac{r}{\sigma}\right)^2}{\left(\frac{r}{a_2}\right)^{3/4}}\right)^q\right)^{1/q}}\right), \quad (\text{A.12})$$

where σ and a_2 are functions of r_c and b . This model is a regularized extension of the Proctor model in order to assure a continuity of the derivatives of $\Gamma(r)$, which is not guaranteed in the Proctor model. We invite the reader to consult Appendix A. of [44] for a more detailed description of the Proctor-Winckelmans model.

Other models exist, and summaries can be found in [60] or [43]. Data assimilation applications in the context of formation flight as the one described in Chapter 2 requires simplified aircraft wake representation, such as those models. The estimation tools being mainly to determine global parameters of aircraft wake, even a simple model such as the LOA model may be sufficient.

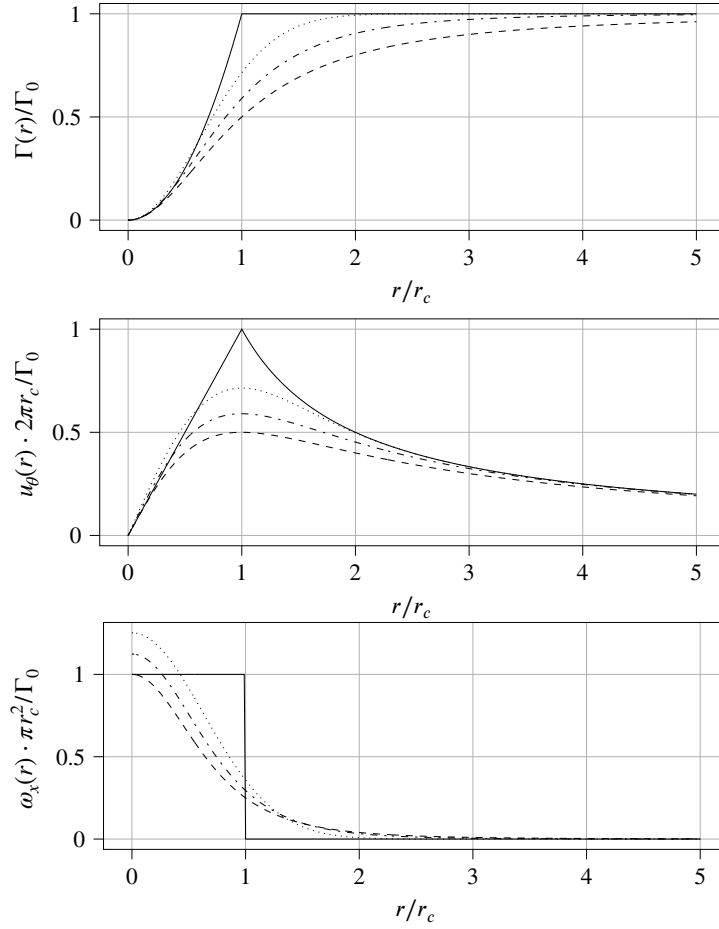


Figure A.1: Profiles of circulation (top), tangential velocity (middle), and axial vorticity (bottom) for usual one-scale vortex models: *Rankine* model (—), *Gaussian* model (····), *Low-order algebraic* model (---), and *High-order algebraic* model (-·-·).

Linearized aircraft dynamics

This section presents the linearized dynamics of the C172-like and A350-like aircraft whose aerodynamics and non-linear dynamics has been described in Chapter 1. The design of an autopilot and mostly the computation of the feedback gains that it implies is far more easily performed based on a linear representation of the aircraft dynamics. Indeed, the evaluation of stability derivatives computed around a trimmed level flight enables to determine the linear dynamics of the aircraft around that level flight. This leads to a representation of the form

$$\Delta \dot{q}(t) = \mathbf{A} \Delta q(t) + \mathbf{B} \Delta u_c(t), \quad (\text{B.1})$$

where $\mathbf{A}$ and $\mathbf{B}$ are the linearized matrices of the state space representation and are computed from the aerodynamic model using central differences. In order to write the linearized dynamics of an aircraft, we use the dynamic states defined in Eq. (1.41). Moreover, in addition to use the components of the flight velocity vector defined in the inertial reference frame $(ox^i y^i z^i)$ as states (i.e., $V_K = [U_K, V_K, W_K]^i$), we also use the components of the flight velocity vector defined in the body frame attached to the aircraft $(ox^b y^b z^b)$ (i.e., $V_K = [U_K, V_K, W_K]^i = [u_K, v_K, w_K]^b$) in order to facilitate the derivation of the linearized equations, see [34]. In the linearized representation, the behavior of simple fixed-wing aircraft can be separated into longitudinal and lateral dynamics, which are linearly uncoupled [34], simplifying the control problem.

B.1 C172-like aircraft

For the C172-like aircraft, we assume an instantaneous response time from the command $u_c = [\bar{\delta}_{Ail}, \bar{\delta}_{VT}, \bar{\delta}_{HT}, \bar{\delta}_{Th}]$ to the effective control surface deflections or the

effective thrust, i.e., $\mathbf{q}_{cont} = [\delta_{Ail}, \delta_{VT}, \delta_{HT}, \delta_{Th}]$. In other words, we have

$$\mathbf{q}_{cont} = \mathbf{u}_c. \quad (\text{B.2})$$

The control state vector $\mathbf{q}_{cont}$ is therefore not included in the dynamic equation but is rather considered as the input $\mathbf{u}_c$. The longitudinal states and measurements are

$$\begin{aligned} \mathbf{q}_{long} &= \begin{pmatrix} U_K & W_K & q & \theta & x & z \end{pmatrix}^T, \quad \mathbf{u}_{long} = \begin{pmatrix} \bar{\delta}_{HT} & \bar{\delta}_{Th} \end{pmatrix}^T, \\ \mathbf{m}_{long} &= \begin{pmatrix} U_K & W_K & q & \theta & x & z & \alpha & \gamma & u_K & w_K \end{pmatrix}^T, \end{aligned}$$

and the lateral states and measurements are

$$\begin{aligned} \mathbf{q}_{lat} &= \begin{pmatrix} V_K & p & r & \phi & \psi & y_F \end{pmatrix}^T, \quad \mathbf{u}_{lat} = \begin{pmatrix} \bar{\delta}_{Ail} & \bar{\delta}_{VT} \end{pmatrix}^T, \\ \mathbf{m}_{lat} &= \begin{pmatrix} V_K & p & r & \phi & \psi & y & \beta & v_K \end{pmatrix}^T. \end{aligned}$$

Linearized dynamics

$$\begin{aligned} \mathbf{A}_{long} &= \begin{pmatrix} -0.005238 & 0.1233 & 0 & -10 & 0 & 0 \\ -0.4071 & -3.1058 & 48.23 & 0 & 0 & 0 \\ 0.006310 & -0.3968 & -4.111 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -50 & 0 & 0 \end{pmatrix}, \\ \mathbf{B}_{long} &= \begin{pmatrix} -0.04022 & -1.468 & -3.426 & 0 & 0 & 0 \\ 0.001429 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^T, \\ \mathbf{C}_{long} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0.02 & 0 & 0 & 0 & 0 \\ 0 & -0.02 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -50.00 & 0 & 0 \end{pmatrix}, \end{aligned}$$

$$\begin{aligned}
\mathbf{A}_{\text{lat}} &= \begin{pmatrix} -0.2150 & -0.5311 & -48.70 & 10 & 0 & 0 \\ -0.2844 & -11.42 & 3.315 & 0 & 0 & 0 \\ 0.3396 & 0.6015 & -2.050 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\
\mathbf{B}_{\text{lat}} &= \begin{pmatrix} 0 & -3.910 & 0.06832 & 0 & 0 & 0 \\ -1.075 & -1.422 & 1.698 & 0 & 0 & 0 \end{pmatrix}, \\
\mathbf{C}_{\text{lat}} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0.02 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 50 & 0 \end{pmatrix}.
\end{aligned}$$

Controller parameters

The state-space velocity-tracking part of the autopilot is made up of the state proportional gain matrices:

$$\begin{aligned}
\mathbf{K}_{q_{\text{long}}} &= \begin{pmatrix} -0.04317 & 0.5569 & -2.305 & -49.03 & 0 & 0 \\ 3728 & 29.26 & 439 & 7291 & 0 & 0 \end{pmatrix}, \\
\mathbf{K}_{q_{\text{lat}}} &= \begin{pmatrix} -2.963 & -1.845 & -3.371 & -25.85 & -153.2 & 0 \\ -0.7370 & 0.07704 & 4.583 & -2.825 & -6.093 & 0 \end{pmatrix},
\end{aligned}$$

and of the integrative part whose gain matrices are given by

$$\begin{aligned}
\mathbf{K}_{v_{x,i}} &= \begin{pmatrix} 0.03760 \\ 9998 \end{pmatrix}, \quad \mathbf{K}_{v_{y,i}} = \begin{pmatrix} -1.742 \\ -0.09760 \end{pmatrix}, \\
\mathbf{K}_{v_{z,i}} &= \begin{pmatrix} 1.7450 \\ -215.4 \end{pmatrix}, \quad \mathbf{K}_{\beta} = \begin{pmatrix} 5.592 \\ -99.83 \end{pmatrix}.
\end{aligned}$$

As explained in Section 1.4.2, the feedback gains are designed using a Linear Quadratic Regulator (LQR), much akin to what is done in [12]. We initialize the parameters of the LQR controller so that the resulting gains allow a high-speed tracking

and responsive control commands. Under these conditions, the obtained gains are therefore important compared to the ones observed in classical control loops.

The inertial position-tracking part of the autopilot is made up of the PD compensators,

$$\begin{aligned} k_{Px_i} &= 0.8, & k_{Py_i} &= 0.35, & k_{Pz_i} &= 0.75, \\ k_{Dx_i} &= 0.1, & k_{Dy_i} &= -0.00025, & k_{Dz_i} &= 0, \end{aligned}$$

and of the rate limiters

$$\frac{|U_i|_{\max}}{U_\infty} = 0.1, \quad \frac{|V_i|_{\max}}{U_\infty} = 0.02, \quad \frac{|W_i|_{\max}}{U_\infty} = 0.06.$$

B.2 A350-like aircraft

For the A350-like aircraft, we assume first order actuation dynamics for all the commands. The control states $\mathbf{q}_{\text{cont}} = [\delta_{Ail}, \delta_{VT}, \delta_{HT}, \delta_{Th}]$ thus undergo a phase shift compare to the control inputs $\mathbf{u}_c = [\bar{\delta}_{Ail}, \bar{\delta}_{VT}, \bar{\delta}_{HT}, \bar{\delta}_{Th}]$, i.e.,

$$\begin{cases} \dot{\delta}_{Ail} = -\frac{1}{\tau_{Ail}} (\delta_{Ail} - \bar{\delta}_{Ail}) \\ \dot{\delta}_{VT} = -\frac{1}{\tau_{VT}} (\delta_{VT} - \bar{\delta}_{VT}) \\ \dot{\delta}_{HT} = -\frac{1}{\tau_{HT}} (\delta_{HT} - \bar{\delta}_{HT}) \\ \dot{\delta}_{Th} = -\frac{1}{\tau_{Th}} (\delta_{Th} - \bar{\delta}_{Th}) \end{cases} \quad \text{..} \quad (\text{B.3})$$

Those dynamics are thus included in the linearized dynamics of the A350-like aircraft. The longitudinal states and measurements are

$$\begin{aligned} \mathbf{q}_{\text{long}} &= \left(U_K \quad W_K \quad q \quad \theta \quad x \quad z \quad \delta_{HT} \quad \delta_{Th} \right)^T, & \mathbf{u}_{\text{long}} &= \left(\bar{\delta}_{HT} \quad \bar{\delta}_{Th} \right)^T, \\ \mathbf{m}_{\text{long}} &= \left(U_K \quad W_K \quad q \quad \theta \quad x \quad z \quad \alpha \quad \gamma \quad u_K \quad w_K \right)^T, \end{aligned}$$

and the lateral states and measurements are

$$\begin{aligned} \mathbf{q}_{\text{lat}} &= \left(V_K \quad p \quad r \quad \phi \quad \psi \quad y \quad \delta_{Ail} \quad \delta_{VT} \right)^T, & \mathbf{u}_{\text{lat}} &= \left(\bar{\delta}_{Ail} \quad \bar{\delta}_{VT} \right)^T, \\ \mathbf{m}_{\text{lat}} &= \left(V_K \quad p \quad r \quad \phi \quad \psi \quad y \quad \beta \quad v_K \right)^T. \end{aligned}$$

Linearized dynamics

$$\mathbf{A}_{\text{long}} = \begin{pmatrix} -0.00402 & 0.0208 & 0 & -10 & 0 & 0 & -0.0858 & 0.000033 \\ -0.0798 & -0.584 & 245.7 & 0 & 0 & 0 & -2.653 & 0 \\ 0.00110 & -0.0330 & -2.243 & 0 & 0 & 0 & -1.402 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -250 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3.333 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix},$$

$$\mathbf{B}_{\text{long}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 3.333 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. \end{pmatrix}^T,$$

$$\mathbf{C}_{\text{long}} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0.02 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.02 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -250.00 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\mathbf{A}_{\text{lat}} = \begin{pmatrix} -0.0264 & -0.2446 & -248.9 & 10 & 0 & 0 & 0 & -0.6598 \\ -0.00638 & -4.645 & 1.0288 & 0 & 0 & 0 & -2.300 & -0.1595 \\ 0.01081 & 0.4714 & -0.4430 & 0 & 0 & 0 & 0.0292 & 0.2702 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 250 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3.333 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3.333 \end{pmatrix},$$

$$\mathbf{B}_{\text{lat}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 3.333 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.333 \end{pmatrix}^T,$$

$$\mathbf{C}_{\text{lat}} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0.02 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 250 & 0 & 0 & 0 \end{pmatrix}$$

Controller parameters

The state-space velocity-tracking part of the autopilot is made up of the stateproportional gain matrices:

$$\mathbf{K}_{q_{\text{long}}} = \begin{pmatrix} -0.00695 & 0.0602 & -0.979 & -17.64 & 0 & 0 & 0 & 0 \\ 169715 & 999.1 & 104071 & -2110596 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\mathbf{K}_{q_{\text{lat}}} = \begin{pmatrix} -0.1786 & -0.5533 & 0.8329 & -3.096 & -40.66 & 0 & 0 \\ -0.1064 & 0.02997 & 14.62 & -0.2831 & 4.752 & 0 & 0 \end{pmatrix},$$

and of the integrative part whose gain matrices are given by

$$\mathbf{K}_{V_{x,i}} = \begin{pmatrix} -0.000813 \\ 49945 \end{pmatrix}, \quad \mathbf{K}_{V_{y,i}} = \begin{pmatrix} -0.0433 \\ 0.00490 \end{pmatrix},$$

$$\mathbf{K}_{V_{z,i}} = \begin{pmatrix} 0.0348 \\ 4659 \end{pmatrix}, \quad \mathbf{K}_{\beta} = \begin{pmatrix} -5.619 \\ -49.68 \end{pmatrix}.$$

Similarly to the C172-like aircraft, we initialize the parameters of the LQR controller so that the resulting gains allow a high-speed tracking and responsive control commands. The inertial position-tracking part of the autopilot is made up of the PD compensators,

$$k_{Px_i} = 0.2, \quad k_{Py_i} = 0.04, \quad k_{Pz_i} = 0.1,$$

$$k_{Dx_i} = 0, \quad k_{Dy_i} = 0, \quad k_{Dz_i} = 0,$$

and of the rate limiters

$$\frac{|U_i|_{\text{max}}}{U_{\infty}} = 0.04, \quad \frac{|V_i|_{\text{max}}}{U_{\infty}} = 0.04, \quad \frac{|W_i|_{\text{max}}}{U_{\infty}} = 0.04.$$

Ensemble Kalman filter parameters

The measurement noise ϵ_i and the state inflation α_i are random variables that follow zero-mean normal distributions,

$$\begin{cases} \epsilon_i \sim \mathcal{N}(0, \mathbf{R}) \\ \alpha_i \sim \mathcal{N}(0, \mathbf{Q}_\alpha) \end{cases}, \quad (\text{C.1})$$

as it is described in Chapter 2. The covariance matrices $\mathbf{R}$ and $\mathbf{Q}_\alpha$ are diagonal and constant over the estimation process. The inflation variance associated to each one of the states are fine tuned from the analysis of the dynamics of an aircraft. Adjustments were performed as we went along a larger number of simulations, and on a case-by-case basis for each one of the states or measurements. The values used in the simulations are given in Table C.1.

Table C.1: Additive inflation variance associated to each one of the states (i.e. diagonal of the matrix $\mathbf{Q}_\alpha$).

x_F/b	0.00003	ϕ	0.0004°
y_F/b	0.0005	θ	0.0003°
z_F/b	0.0003	ψ	0.0004°
$U_{K,F}/U_\infty$	0.000006	pb/U_∞	0.0001°
$V_{K,F}/U_\infty$	0.00006	qb/U_∞	0.00006°
$W_{K,F}/U_\infty$	0.00006	rb/U_∞	0.00008°
$\Delta y/b$	0.04	$\Gamma_{0,L}/\bar{\Gamma}_{0,L}$	0.05
$\Delta z/b$	0.04	$b_{0,L}/b$	0.05

The noise level variances associated to each one of the measurements are related to the accuracy of the sensors used in a real application, and to the accuracy of the surrogate model used in the filter. Two sets of measurement variances are used in Chapter 2. The first one refers to the "high noise levels" (i.e. *Filter 1* and *Filter 2*). The second one refers to "low noise levels" (i.e. *Filter 3*) and differs from the first one with a reduction of the noise levels associated to attitude angles only. The values used in the simulations are given in Table C.2.

Table C.2: Noise level variances associated to each measurement (i.e. diagonal of the matrix $\mathbf{R}$) for the two sets envisioned in this study.

	High levels	Low levels
x_F/b	0.1	0.1
y_F/b	0.1	0.1
z_F/b	0.1	0.1
$U_{K,F}/U_\infty$	0.002	0.002
$V_{K,F}/U_\infty$	0.002	0.002
$W_{K,F}/U_\infty$	0.002	0.002
ϕ	1°	0.1°
θ	1°	0.1°
ψ	1°	0.1°
pb/U_∞	0.02°	0.02°
qb/U_∞	0.02°	0.02°
rb/U_∞	0.02°	0.02°
δ_{Ail}	1°	1°
δ_{VT}	1°	1°
δ_{HT}	1°	1°
$\delta_{Th}/\delta_{Th,0}$	0.1	0.1
α	1°	1°
β	1°	1°
$\Gamma_{0,L}/\bar{\Gamma}_{0,L}$	0.3	0.3
$b_{0,L}/\bar{b}_{0,L}$	0.2	0.2

Upwash sensitivity to the wake parameters

As described in Appendix A, aircraft wake vortices are represented by models that reproduce the velocities induced by those vortices on their environment. Considering the *Low Order Algebraic* (LOA) model (or *Moore-Rosenhaed* model, or *Burham-Hallock* model), one has the circulation, the azimuthal velocity and axial vorticity distributions of a vortex, i.e., $\Gamma(r)$, $u_\theta(r)$ and $\omega_x(r)$ respectively:

$$\begin{cases} \Gamma(r) &= \Gamma_0 \frac{r^2}{r^2+r_c^2} \\ u_\theta(r) &= \frac{\Gamma_0}{2\pi} \frac{r}{r^2+r_c^2} \\ \omega_x(r) &= \frac{\Gamma_0}{\pi} \frac{r_c^2}{(r^2+r_c^2)^2} \end{cases}, \quad (\text{D.1})$$

where Γ_0 is the wake intensity, r_c is the core radius and $r = \sqrt{y^2 + z^2}$ is the distance to the center of the vortex, with y and z the components of the radial vector in a cartesian frame, i.e., $\mathbf{r} = r\hat{\mathbf{e}}_r = y\hat{\mathbf{e}}_y + z\hat{\mathbf{e}}_z$.

In this chapter, we study the sensitivity of the upwash velocity $w = u_\theta \frac{y}{\sqrt{y^2+z^2}}$ generated by a two vortex system (2VS) with respect to the wake intensity Γ_0 and the lateral position y . We focus on the upwash generated along a line passing through the two vortices, i.e., $z = 0$ in the inertial frame, and we consider that the distance b_0 between the vortices is constant, with $r_c = 0.05b$ [14]. Fig. D.1 is a representation of the problem studied.

In the context of formation flight, we assume that the lateral distance between the center of the 2VS and a follower flying in that 2VS is never lower than $y_{min} = \frac{3b_0}{4}$,

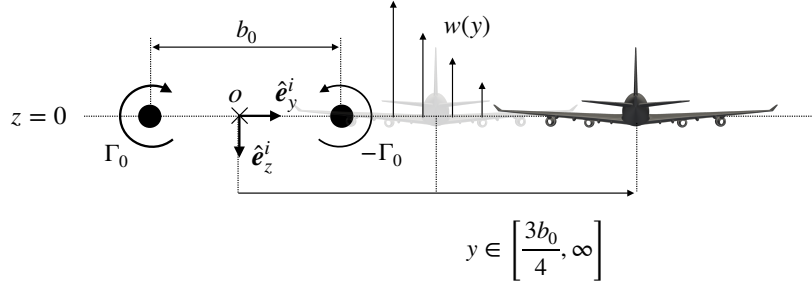


Figure D.1: Representation of the sensitivity analysis. It is restricted to the study of the influence of the wake intensity Γ_0 and the lateral position y on the upwash $w(y)$ generated by a 2VS, assuming b_0 is constant, and the vertical position is $z = 0$.

i.e., the follower never enters the 2VS, for safety and energy efficiency reasons. For all position greater than that position, i.e., $y \in \left[\frac{3b_0}{4}, \infty\right]$, we observe that each vortex of the 2VS can be considered as a singular potential vortex. Indeed, we have for the two vortices

$$\Gamma_{left} \left(y = \frac{3b_0}{4}, z = 0 \right) = \Gamma_0 \frac{r^2}{r^2 + r_c^2} = 0.9901\Gamma_0 \simeq \Gamma_0 \quad (D.2)$$

$$\Gamma_{right} \left(y = \frac{3b_0}{4}, z = 0 \right) = -\Gamma_0 \frac{r^2}{r^2 + r_c^2} = -0.9996\Gamma_0 \simeq -\Gamma_0 . \quad (D.3)$$

Under that hypothesis, we compute the upwash velocity induced by the 2VS:

$$w(y) = \frac{\Gamma_0}{2\pi} \left(\frac{1}{y + \frac{b_0}{2}} - \frac{1}{y - \frac{b_0}{2}} \right) \quad (D.4)$$

$$= -\frac{\Gamma_0 b_0}{2\pi} \frac{1}{y^2 - \left(\frac{b_0}{2}\right)^2} . \quad (D.5)$$

From that definition, we first derive an expression for the sensitivity of the upwash velocity with respect to a variation of the vortices intensity Γ_0 , i.e.,

$$\frac{\partial w(y)}{\partial \Gamma_0} = -\frac{b_0}{2\pi} \frac{1}{y^2 - \left(\frac{b_0}{2}\right)^2} \quad (\text{D.6})$$

$$\Gamma_0 \frac{\partial w(y)}{\partial \Gamma_0} = w(y) \quad (\text{D.7})$$

$$\boxed{\frac{\partial w(y)}{w(y)} = \frac{\partial \Gamma_0}{\Gamma_0}}. \quad (\text{D.8})$$

Also, we derive an expression for the sensitivity of the upwash velocity with respect to a variation of the lateral position y , i.e.,

$$\frac{\partial w(y)}{\partial y} = \frac{\Gamma_0 b_0}{2\pi} \frac{2y}{\left(y^2 - \left(\frac{b_0}{2}\right)^2\right)^2} \quad (\text{D.9})$$

$$b_0 \frac{\partial w(y)}{\partial y} = -\frac{\Gamma_0 b_0}{2\pi} \frac{1}{y^2 - \left(\frac{b_0}{2}\right)^2} \cdot \frac{-b_0^2}{y^2 - \left(\frac{b_0}{2}\right)^2} \left(\frac{y}{b_0/2}\right) \quad (\text{D.10})$$

$$= w(y) \cdot \frac{-4}{\left(\frac{y}{b_0/2}\right)^2 - 1} \left(\frac{y}{b_0/2}\right) \quad (\text{D.11})$$

We define $\eta = \frac{y}{b_0/2}$ with $\eta \in \left[\frac{3}{2}, \infty\right]$ and we obtain

$$\frac{\partial w(y)}{\partial y} = \frac{w(y)}{b_0} \cdot \frac{-4\eta}{\eta^2 - 1} \quad (\text{D.12})$$

$$\frac{\partial w(y)}{w(y)} = \frac{-2\eta}{\eta^2 - 1} d\eta \quad (\text{D.13})$$

$$\boxed{\frac{\partial w(y)}{\partial y} = \frac{-2}{1 - \frac{1}{\eta^2}} \frac{d\eta}{\eta}}. \quad (\text{D.14})$$

Eqs. D.8 and D.14 provide direct information on the influence of the intensity Γ_0 and the position y on the upwash wind and therefore on the follower dynamics. For instance, Eq. (D.8) demonstrates that a 5% increase in terms of wake intensity (, i.e.,

$\frac{\Delta\Gamma_0}{\Gamma_{0,i}} = 0.05$) lead to a 5% increase in terms of upwash velocity, i.e.,

$$\frac{\Delta w}{w_i} = \frac{\Delta\Gamma_0}{\Gamma_{0,i}} \quad (\text{D.15})$$

$$= 0.05, \quad (\text{D.16})$$

whatever the wake intensity. Regarding the lateral position y , Eq. (D.14) illustrates a more complex behavior. Let's consider that the follower is located at an initial position $y_i = 2b_0$, i.e., $\eta_i = 4$. A 5% increase in lateral position (, i.e., $\frac{\Delta\eta}{\eta_i} = 0.05$) lead to

$$\frac{\Delta w}{w_i} = \frac{-2}{1 - \frac{1}{\eta_i^2}} \frac{\Delta\eta}{\eta_i} \quad (\text{D.17})$$

$$= -2.133 \cdot 0.05 \quad (\text{D.18})$$

$$= 0.1066. \quad (\text{D.19})$$

Therefore, a 11% decrease in upwash velocity is computed. The upwash velocity is thus more sensitive to the lateral position than the wake intensity, at the position $y = 2b_0$. What's more, that sensitivity depends on the lateral distance itself, as it can be very high close to the 2VS and at the limit, converges to a factor -2 .

The analysis can be continued by computing the sensitivity of integrated quantities along a wing with respect to the wake intensity or the relative position. In particular, the sensitivity of the vertical force of the rolling moment would provide a direct info on the influence of the wake parameters on the dynamics of the follower.

INS-based wake tracking strategy for formation flight

E.1 Introduction

In Chapter 2, we have designed a tracking strategy based on an Ensemble Kalman Filter (EnKF) in order to estimate the state parameters of a leader's wake from measurements of the six degrees-of-freedom (6DOF) dynamics of a follower. Simultaneously, the estimator predicts the dynamic states of the follower (i.e., position and velocity, in translation and in rotation along the three spatial directions) and the wake state parameters. This then allows the follower to track a desired position relative to the estimated wake position in order to optimize its energy consumption over time.

Such a methodology allows, among other things, to integrate model uncertainties, by introducing process noise through the inflation step that characterized the EnKF. These model uncertainties are, for instance, associated to the poor knowledge of the aerodynamic parameters of the aircraft or the flow in some flight configurations, or to the absence of an accurate aero-elastic model of the aircraft.

However, due to the non-linearity of the entire system, this coupled estimation comes at the cost of the propagation of a non-linear state model along a set or *ensemble* of trajectories, as required by the EnKF. The latter indeed follows a Monte Carlo approach to determine the statistical characteristics of the estimate. Each set is characterized by hypotheses (or *guess*) of the leader's wake state parameters, and hypotheses of the follower dynamic states. While the propagation of the first follows a random walk dynamics (through the inflation step of the EnKF), the propagation of each hypotheses of the follower's states requires to integrate the entire 6DOF dynamics. This

leads to a set of spatial trajectories propagated over time. This requires possibly important computational resources. But mostly, it does not exploit dynamic measurements in an optimal way.

Indeed, the estimation of an aircraft dynamics is already performed in modern aircraft through their Inertial Navigation System (INS) that uses Inertial Measurements Units (IMU) and Global Navigation Satellite Systems (GNSS) to predict the translational and rotational position and velocity over time. The 6DOF dynamics of an aircraft can thus be assumed to be known, with some uncertainty. A similar approach can be imagined regarding the control commands of an aircraft that are estimated over time based on a well calibrated transfer function of their dynamics that links the command itself to the actual control surfaces deflection or thrust.

In this appendix, we thus focus on an alternate estimation procedure that still aims at predicting the leader's wake state parameters using an EnKF. However, it now considers a simplified filtering method for the estimation of the follower's dynamics. The latter are now used as known parameters of the EnKF, rather than states to be estimated. In the strategy designed here, the nonlinear surrogate model described in Chapter 1 is not used to propagate the dynamics of the follower. It is rather used to have access to the nonlinear measurements that reflect the impact of the leader on the follower. The dynamics of the follower are still propagated over time. However, we use simplified linear dynamics that do not directly involve the dynamics of the leader's wake state parameters.

We first present Section E.2 the estimation methodology and related dynamic equations. We then present in Section E.3 results produced by this alternate estimation method.

E.2 Use of a set of Kalman filters for the wake parameters estimation

Similarly to Chapter 2, we use the high-fidelity simulated flight environment of a two-aircraft formation described in Chapter 1 instead of considering actual flight tests data. Thanks to Large Eddy Simulations (LES) that are based on the Vortex-Particle-Mesh (VPM) method, this environment simulates the unsteady wake flow that is emitted by the system of two aircraft in formation and produces the reference measurements used to feed the estimator. In particular, the 6DOF dynamics of the aircraft are computed over time, and measurements of those dynamics are thus available. See Chapters 1 and 2 for a more detailed description.

In this alternate version of the tracking strategy, the estimator is now split into three parts: two estimators are designed to predict the dynamics of the follower and the dynamics of the control commands, i.e., $\mathbf{q}_{dyn}$ and $\mathbf{q}_{cont}$. The last estimator is designed to estimate the wake state parameters of a leader over time, i.e., $\mathbf{q}_{wake}$. In the two first ones, we use linear state representations for the follower's 6DOF dynamics and for the control commands dynamics. The third estimator exploits the non-linear state model representation of the two-aircraft formation, previously referred as the surrogate model. Similarly to Fig. 2.1, Fig. E.1 is a block scheme of the complete estimation methodology. This section aims at describing the equations behind that alternate model before showing its performance.

First, we describe the two linear Kalman filters. Then, we briefly describe the EnKF used to exploit acceleration measurements in order to predict the leader's wake state parameters.

E.2.1 Follower dynamics estimation

Contrary to Chapter 2 that is based on the C172-like aircraft to exemplify the initial wake tracking strategy, we here use a A350-like aircraft whose geometrical properties are presented in Table 1.1.

On such aircraft, an INS is used to define its attitude, position and dynamics over time. The design of such systems is the result of many research efforts because of their strategic importance in navigation for the space and aviation industry [85, 154]. In particular, the modeling and reduction of errors [63, 93] are of prime importance in the design and development of INS. In this work, we do not aim at designing an INS that includes all the state-of-the-art improvements. For the sake of simplicity, we reduce INS to its simplest form, i.e., the use of well-chosen measurements that allow to estimate the whole dynamics states by exploiting a linear state model through a linear Kalman filter. We assume the INS is equipped with GPS and IMU that provide measurements of angular and translational accelerations, measurement of angular velocities, as well as measurements of the positions of the aircraft in the inertial frame. We use the same inertial and body frames as the ones described in Chapter 1 and use the same aircraft states. In this section, we consider an extremely simplified state model in background of the INS that simply integrates the accelerations and velocities over time to model the behavior of the follower. Similarly to the model described in Chapter 1, the dynamic states of the aircraft reduce to the vector

$$\mathbf{q}_{dyn} = [U_{K,F}, V_{K,F}, W_{K,F}, x_F, y_F, z_F, p, q, r, \phi, \theta, \psi], \quad (\text{E.1})$$

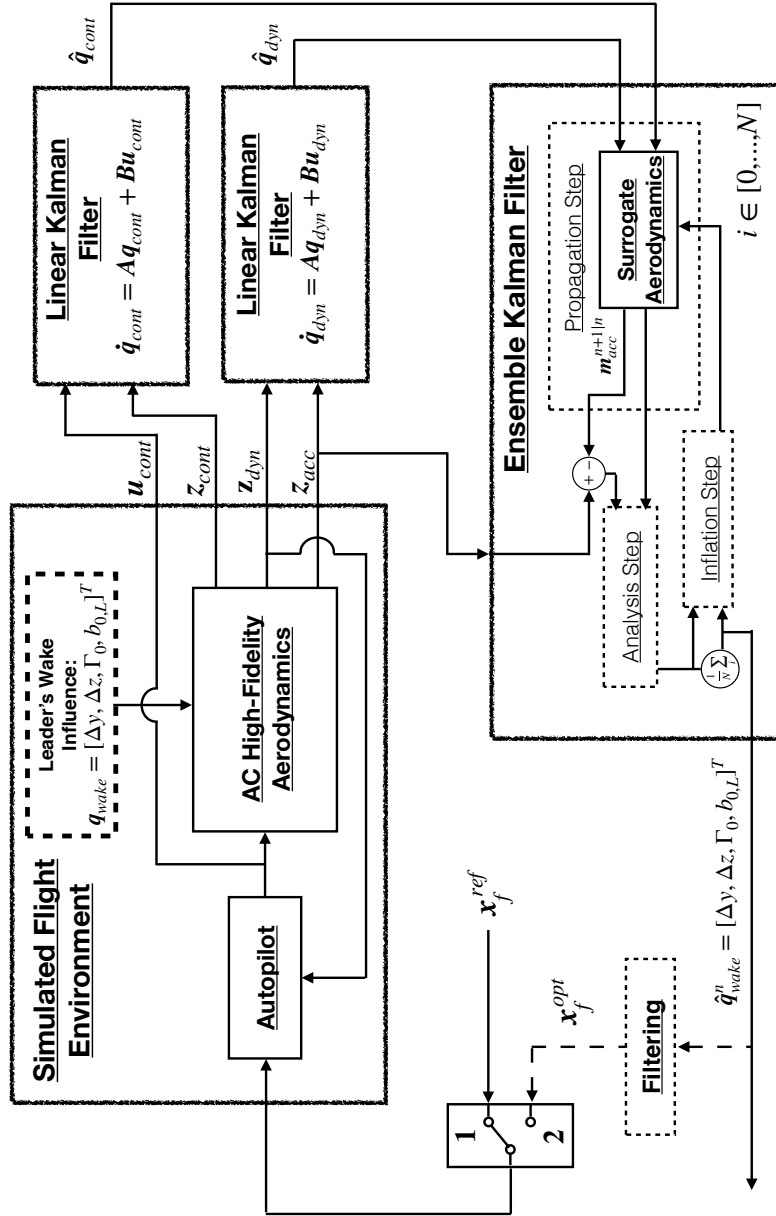


Figure E.1: Block diagram of the tracking strategy including the simulated flight environment, the estimator and the filtering tool to provide a smooth target.

where the subscript $\cdot_F$ denotes the follower, $V_{K,F} = [U_{K,F}, V_{K,F}, W_{K,F}]_i$ is the flight path velocity, $\mathbf{x}_F = [x_F, y_F, z_F]$ is the inertial position, $[p, q, r]$ are the angular velocities and $[\phi, \theta, \psi]$ is the attitude. The inputs of the linear model exploited by the linear Kalman filter are the translational and angular accelerations $\mathbf{a}_{K,F}(t) = [\dot{U}_{K,F}, \dot{V}_{K,F}, \dot{W}_{K,F}]$ and $\dot{\mathbf{\Omega}}_F = [\dot{p}, \dot{q}, \dot{r}]$ measured by the IMU. They are integrated over time to compute the velocities and positions. The discrete-time propagation equations are thus

$$\begin{cases} \hat{\mathbf{V}}_{K,F}^{n+1|n} &= \hat{\mathbf{V}}_{K,F}^n + \Delta t \mathbf{a}_{K,F}^n + \boldsymbol{\sigma}_v^n \\ \hat{\mathbf{x}}_F^{n+1|n} &= \hat{\mathbf{x}}_F^n + \Delta t \hat{\mathbf{V}}_{K,F}^n + \frac{\Delta t^2}{2} \mathbf{a}_{K,F}^n + \boldsymbol{\sigma}_x^n \end{cases}, \quad (\text{E.2})$$

$$\begin{cases} \hat{p}^{n+1|n} &= \hat{p}^n + \Delta t \dot{p}^n + \sigma_p^n \\ \hat{q}^{n+1|n} &= \hat{q}^n + \Delta t \dot{q}^n + \sigma_q^n, \\ \hat{r}^{n+1|n} &= \hat{r}^n + \Delta t \dot{r}^n + \sigma_r^n \end{cases}, \quad (\text{E.3})$$

$$\begin{cases} \hat{\phi}^{n+1|n} &= \hat{\phi}^n + \Delta t \dot{\phi}^n + \frac{\Delta t^2}{2} \ddot{\phi}^n + \sigma_\phi^n \\ \hat{\theta}^{n+1|n} &= \hat{\theta}^n + \Delta t \dot{\theta}^n + \frac{\Delta t^2}{2} \ddot{\theta}^n + \sigma_\theta^n, \\ \hat{\psi}^{n+1|n} &= \hat{\psi}^n + \Delta t \dot{\psi}^n + \frac{\Delta t^2}{2} \ddot{\psi}^n + \sigma_\psi^n \end{cases}, \quad (\text{E.4})$$

where $\hat{\cdot}$ denotes the estimate, the superscript n denotes the n^{th} time step at the time instance $t^n = n\Delta t$ with the sampling interval Δt , and the superscript $n+1|n$ denotes the propagation of the estimated state through the dynamic model, before the measurement update step of the Kalman filter formulation. $\boldsymbol{\sigma}_{dyn}^n$ is the process noise which is assumed to be zero mean Gaussian noise with covariance $\mathbf{Q}_{dyn}$, i.e., $\boldsymbol{\sigma}_{dyn}^n \sim \mathcal{N}(0, \mathbf{Q}_{dyn})$. The dynamic state covariance matrix is also propagated, following the usual linear Kalman formulation, i.e.,

$$\mathbf{P}_{dyn}^{n+1|n} = \mathbf{F}_{dyn}^n \mathbf{P}_{dyn}^n \left(\mathbf{F}_{dyn}^n \right)^T + \mathbf{Q}_{dyn}, \quad (\text{E.5})$$

where $\mathbf{F}_{dyn}^n$ is the state transition matrix deduced from Eqs. E.2, E.3 and E.4, i.e., $\mathbf{q}_{dyn}^{n+1|n} = \mathbf{F}_{dyn}^n \mathbf{q}_{dyn}^n + \boldsymbol{\sigma}_{dyn}^n$. The measurement update for the dynamic states estimation is done exploiting true measurements of accelerations, inertial position and angular velocity, i.e.,

$$\mathbf{z}_{dyn} = [\mathbf{a}_{K,F}, \dot{\mathbf{\Omega}}_F, \mathbf{x}_F, p, q, r], \quad (\text{E.6})$$

where $\mathbf{z}_{acc} = [\mathbf{a}_{K,F}, \dot{\mathbf{\Omega}}_F]$ and $[p, q, r]$ are measured by an IMU, and $\mathbf{x}_F$ is measured by a GPS. At time n , a predicted measurement $\mathbf{m}_{dyn}^n$ of the estimated state $\hat{\mathbf{q}}_{dyn}^n$ is made according to

$$\mathbf{m}_{dyn}^{n+1|n} = \mathbf{H}_{dyn}^n \hat{\mathbf{q}}_{dyn}^{n+1|n} + \boldsymbol{\epsilon}_{dyn}^n, \quad (\text{E.7})$$

where $\mathbf{H}_{dyn}^n$ is the observation model which maps the observation to the state space and ϵ_{dyn}^n is the observation noise which is assumed to be zero mean Gaussian noise with covariance $\mathbf{R}_{dyn}$, i.e., $\epsilon_{dyn}^n \sim \mathcal{N}(0, \mathbf{R}_{dyn})$. The Kalman update is performed using

$$\begin{cases} \mathbf{K}^n &= \mathbf{P}_{dyn}^{n+1|n} \left(\mathbf{H}_{dyn}^n \right)^T \left(\mathbf{H}_{dyn}^n \mathbf{P}_{dyn}^{n+1|n} \left(\mathbf{H}_{dyn}^n \right)^T + \mathbf{R}_{dyn} \right)^{-1} \\ \hat{\mathbf{q}}_{dyn}^{n+1|n+1} &= \hat{\mathbf{q}}_{dyn}^{n+1|n} + \mathbf{K}^n \left(\mathbf{z}_{dyn} - \mathbf{m}_{dyn}^{n+1|n} \right) \\ \mathbf{P}_{dyn}^{n+1|n+1} &= \left(\mathbf{I} - \mathbf{K}^n \mathbf{H}_{dyn}^n \right) \mathbf{P}_{dyn}^{n+1|n} \end{cases}, \quad (\text{E.8})$$

and the procedure is iterated over time, i.e., $\mathbf{P}_{dyn}^{n+1|n+1} \rightarrow \mathbf{P}_{dyn}^n$ and $\hat{\mathbf{q}}_{dyn}^{n+1|n+1} \rightarrow \hat{\mathbf{q}}_{dyn}^n$

E.2.2 Control dynamics estimation

We follow a similar approach for the modeling of the control dynamics, from the aircraft control command inputs $\mathbf{u}_{cont} = [\bar{\delta}_{Ail}, \bar{\delta}_{VT}, \bar{\delta}_{HT}, \bar{\delta}_{Th}]$ to the effective control surface deflections or the effective thrust, i.e., the control states $\mathbf{q}_{cont} = [\delta_{Ail}, \delta_{VT}, \delta_{HT}, \delta_{Th}]$. We use a linear propagation model (see Chapter 1) that is exploited by a second linear Kalman filter. In this work, we assume the control states dynamics are modeled as first order transfer functions. This translates into the following discrete time propagation equations:

$$\begin{cases} \hat{\delta}_{Ail}^{n+1|n} &= \hat{\delta}_{Ail}^n - \frac{\Delta t}{\tau_{Ail}} (\hat{\delta}_{Ail}^n - \bar{\delta}_{Ail}^n) + \sigma_{Ail}^n \\ \hat{\delta}_{VT}^{n+1|n} &= \hat{\delta}_{VT}^n - \frac{\Delta t}{\tau_{VT}} (\hat{\delta}_{VT}^n - \bar{\delta}_{VT}^n) + \sigma_{VT}^n \\ \hat{\delta}_{HT}^{n+1|n} &= \hat{\delta}_{HT}^n - \frac{\Delta t}{\tau_{HT}} (\hat{\delta}_{HT}^n - \bar{\delta}_{HT}^n) + \sigma_{HT}^n \\ \hat{\delta}_{Th}^{n+1|n} &= \hat{\delta}_{Th}^n - \frac{\Delta t}{\tau_{Th}} (\hat{\delta}_{Th}^n - \bar{\delta}_{Th}^n) + \sigma_{Th}^n \end{cases}, \quad (\text{E.9})$$

where $\tau_{Ail}, \tau_{HT}, \tau_{VT}, \tau_{Th}$ are the time constants of the first order dynamics, and σ_{cont}^n is the zero mean Gaussian process noise with covariance $\mathbf{Q}_{cont}$, i.e., $\sigma_{cont}^n \sim \mathcal{N}(0, \mathbf{Q}_{cont})$. Similarly to the aircraft 6DOF dynamics, the control states are estimated following a linear Kalman filter, exploiting true measurements of the control surfaces deflections and thrust $\mathbf{z}_{cont} = [\delta_{Ail}, \delta_{VT}, \delta_{HT}, \delta_{Th}]$. Those measurements are considered noisy, with a zero-mean Gaussian measurement noise with covariance $\mathbf{R}_{cont}$, i.e., $\epsilon_{cont}^n \sim \mathcal{N}(0, \mathbf{R}_{cont})$. By applying Eqs. E.5 and E.8 to the control state dynamic equations, the estimated control states are propagated over time as well as their covariance, i.e., $\hat{\mathbf{q}}_{cont}^{n+1|n+1} \rightarrow \hat{\mathbf{q}}_{cont}^n$ and $\mathbf{P}_{cont}^{n+1|n+1} \rightarrow \mathbf{P}_{cont}^n$

The two linear Kalman estimators are thus used to obtain an estimation of the state of the aircraft, i.e., dynamics $\hat{\mathbf{q}}_{dyn}$ and control $\hat{\mathbf{q}}_{cont}$, in order to feed the EnKF whose objective is to estimate the position of the leader.

E.2.3 Leader wake state parameters estimation

From the point of view of the EnKF designed in this appendix, the states $\hat{\mathbf{q}}_{dyn}$ and $\hat{\mathbf{q}}_{cont}$ are considered as known inputs that do not need to be estimated by the EnKF, as depicted in Fig. E.1. Each estimation step is characterized by a propagation step through a state space model, followed by an update step that consists in the analysis and inflation steps for the EnKF. In this section, we adapt the equations of the EnKF described in Section 2.2.2 to the estimation problem of this appendix, that deals with a system with a fewer number of states, other measurements and therefore other dynamics.

First, the state vector that is propagated through the EnKF reduces to the leader's wake state parameters $\mathbf{q} = \mathbf{q}_{wake} = [\Gamma_{0,L}, b_{0,L}, \Delta y, \Delta z]$ only. As the EnKF uses a Monte-Carlo approach to propagate the probability density function of those states through the system, the wake state vector $\mathbf{q} = \mathbf{q}_{wake}$ is represented by a discrete ensemble of size N . Each member of the ensemble is denoted $\mathbf{q}_{i, wake}$, with $i \in [1, N]$. The propagation step consists in the forward evolution of each member of the ensemble through the surrogate model described in Section 2.2.3. In that model, the dynamics associated to the wake states are considered to be constant over time, meaning that except for any process noise, those states do not change during the propagation step of the EnKF. In other words, we have the discrete-time propagation

$$\mathbf{q}_{i, wake}^{n+1|n} = \mathbf{q}_{i, wake}^n \quad (\text{E.10})$$

We recall that the process noise is not explicitly represented in the propagation step of the EnKF, as observed in the previous equation. Indeed, that noise is implicitly added to the ensemble members because of the Monte Carlo formulation of the EnKF, see Section 2.2.2. Nevertheless, covariance state inflation is implemented and alters the wake state vector of each ensemble at each estimation step in a similar form to Gaussian random walk dynamics, see Eq. (2.9) in Section 2.2.2.

The prediction step also consists in the production of measurements $\mathbf{m}_i^{n+1|n}$ intended to be compared to true measurements $\mathbf{z}_i$ observed from the simulated flight environment. The vector $\mathbf{m}_i^{n+1|n}$ is computed by the surrogate model through the observation function $\mathbf{h}$, i.e.,

$$\mathbf{m}_i^{n+1|n} = \mathbf{h} \left(\mathbf{q}_{i, wake}^n; \hat{\mathbf{q}}_{dyn}, \hat{\mathbf{q}}_{cont} \right), \quad (\text{E.11})$$

where the ";" in $\mathbf{h}$ denotes the fact that $\hat{\mathbf{q}}_{dyn}$ and $\hat{\mathbf{q}}_{cont}$ have already been computed by the linear Kalman Filters when the EnKF starts to proceed. In this version of the EnKF, the measurement vector has to involve quantities such that they are modified when changes in the leader's wake states $\mathbf{q}_{i,wake}^n$ occur.

The measurements that are thus considered in this filter are no longer positions and velocities in translation and rotation, but rather translational and rotational accelerations, i.e., $\mathbf{m} = \mathbf{m}_{acc}$ with

$$\mathbf{m}_{acc} = [\mathbf{a}_{K,F}, \dot{\mathbf{\Omega}}] . \quad (\text{E.12})$$

Intuitively, one easily understands that a modification of any of the wake states leads to a modification of the velocity field induced by that wake on the follower. The surrogate model is designed to compute aerodynamic efforts on the follower that result from those velocities. Therefore, it computes accelerations that depend on the wake state parameters. As a result, during the prediction step, an N -sized ensemble of measurement vectors $\mathbf{m}_{i,acc}^{n+1|n}$ is computed through the propagation of each member $\mathbf{q}_{i,wake}^n$ of the state ensemble in the observation function $\mathbf{h}$.

During the update step of the EnKF, the innovation first consists in comparing true acceleration measurements $\mathbf{z}_{acc}$ provided by the simulated flight environment to each one of the measurement ensemble vector $\mathbf{m}_{i,acc}^{n+1|n}$, i.e.,

$$\mathbf{y}_{i,acc}^{n+1} = \left(\mathbf{z}_{acc}^{n+1} + \boldsymbol{\epsilon}_{i,acc}^{n+1} \right) - \mathbf{m}_{i,acc}^{n+1|n} , \quad (\text{E.13})$$

where the random vector $\boldsymbol{\epsilon}_{i,acc}^{n+1} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{acc})$ is the zero-mean Gaussian noise with covariance $\mathbf{R}_{acc}$ associated to the acceleration measurements provided by the simulated flight environment. This noise has a very specific physical meaning, as it is directly related to the precision of the accelerations sensors used in a real application, i.e., the IMU.

The analysis and the inflation are then performed at each iteration of the EnKF in order to update the *a priori* state ensemble members $\mathbf{q}_{i,wake}^{n+1|n}$ to the *a posteriori* state ensemble members $\mathbf{q}_{i,wake}^{n+1}$. The *a posteriori* state estimate and associated covariance matrices are deduced by taking the mean $\hat{\mathbf{q}}_{wake}^{n+1}$ and covariance $\mathbf{P}_{wake}^{n+1}$ of those ensemble members, i.e.,

$$\hat{\mathbf{q}}_{wake}^{n+1} = \frac{1}{N} \sum_{i=1}^N \mathbf{q}_{i,wake}^{n+1} , \quad (\text{E.14})$$

$$\mathbf{P}_{wake}^{n+1} = \frac{1}{N-1} \sum_{i=1}^N \left(\mathbf{q}_{i,wake}^{n+1} - \hat{\mathbf{q}}_{wake}^{n+1} \right) \left(\mathbf{q}_{i,wake}^{n+1} - \hat{\mathbf{q}}_{wake}^{n+1} \right)^T . \quad (\text{E.15})$$

We refer to Section 2.2.2 and Eqs. 2.1 to 2.9 for a more detailed description of the general methodology behind the EnKF that is shared by this appendix and Chapter 2.

E.2.4 Summary of the detection strategy

The entire estimator is made of three Kalman filters used to predict the 20 states of the two-aircraft formation system. The twelve dynamic states $\mathbf{q}_{dyn}$ of the follower and the four control states $\mathbf{q}_{cont}$ of its command are propagated using the linear models formally described by Eqs. E.2, E.3, E.4 and E.9. The four states $\mathbf{q}_{wake}$ of the leader's wake are propagated through constant dynamics, see Eq. (E.10). In order to preserve the intrinsic non-linearity influence of the wake on the follower, the propagated acceleration measurements $\mathbf{m}_{acc}$ used during the update step of the EnKF are obtained thanks to the non-linear aerodynamic model formally described by the function $\mathbf{h}$ in Eq. (E.11). That last function now considers the follower state estimates $\hat{\mathbf{q}}_{dyn}$ and $\hat{\mathbf{q}}_{cont}$ as parameters, thanks to the estimation provided by the first two linear estimators.

The temporal evolution of some of the dynamic and control states of the follower as well as the temporal evolution of the accelerations experienced by the follower are measured to a certain precision using the inboard IMU and GPS. These measurements are gathered in the vector

$$\mathbf{z} = \mathbf{z}_{dyn} \cup \mathbf{z}_{cont} \cup \mathbf{z}_{acc}.$$

The latter is compared to an *a priori* estimate of the measurement vector $\mathbf{m}^{n+1|n}$. It gathers the measurements computed by the linear Kalman Filter through the propagations of the estimated dynamic and control states $\hat{\mathbf{q}}_{dyn}^{n+1|n}$ and $\hat{\mathbf{q}}_{cont}^{n+1|n}$. Using the classical linear Kalman formulation, *a posteriori* estimates of the 16 states $\hat{\mathbf{q}}_{dyn}^{n+1}$ and $\hat{\mathbf{q}}_{cont}^{n+1}$ of the follower are thus computed.

The EnKF exploits the temporal evolution of the accelerations $\mathbf{z}_{acc}$ undergone by the follower only. They are compared to *a priori* estimates of those accelerations $\mathbf{m}_{acc}^{n+1|n}$ propagated through the non-linear aerodynamic model $\mathbf{h}$ that already benefits from the follower estimated states $\hat{\mathbf{q}}_{dyn}$ and $\hat{\mathbf{q}}_{cont}$. In the EnKF formulation, the probability density function of the estimated states and measurements are deduced using a Monte-Carlo approach.

E.3 Results

Compared to Chapter 2 where we have focused on different filter configurations characterized by different tuning parameters, we here study one filter design only. We show its performance for three different formation flight configurations. All of them consists

in a two-aircraft formation, whose wake propagation and aircraft dynamics are computed in the virtual flight environment presented in Chapter 1. The latter is based on the VPM method to perform LES of the wake flow behind the aircraft. The reference parameters of the VPM simulations performed are reported in Table E.1. In the different scenarios, the leader is instructed to follow a sinusoidal target trajectory similar to the one described in Eq. (1.57). The parameters of the oscillations (i.e. amplitudes A_y, A_z and periods T_y, T_z) are reported in Table E.2. The frequencies are expressed in terms of the period of the phugoid mode $T_{ph} \simeq 125s$ of the A350-like aircraft here used. We use a longitudinal distance of $4b$ between the leader and the follower.

Domain size ($(L_x \times L_y \times L_z)$)	$8b \times 4b \times 2b$
Size of the cubic mesh cells	$h/b = 1/32$

Table E.1: Numerical parameters used in the VPM simulations.

	<i>Config. A</i>	<i>Config. B</i>	<i>Config. C</i>
A_y/b	0.5	0.5	0.5
A_z/b	0.5	0.5	0.5
T_y/T_{ph}	1	0.5	1
T_z/T_{ph}	1	0.5	1
Eddy Dissipation Rate (EDR)	0	0.002	0.002
Autopilot mode	1	1	2

Table E.2: Parameters of the oscillating maneuvers performed by the leader, and parameters of the follower's autopilot mode, for the 3 formation flight configurations studied.

In the first two configurations presented in Section E.3.1, the follower maintains a fixed position $\mathbf{x}_F^{ref}$ in the wake of an oscillating leader. In the same time, it predicts the position of the wake of the leader over time, using the estimation strategy presented in the previous sections. This is referred to as the *position hold* mode, i.e., switch 1 in Fig. E.1. In a first simulation, the leader performs a slow oscillating maneuver in an unperturbed environment, i.e., characterized by the absence of turbulence. In the second simulation, the leader performs a faster oscillating maneuver in a turbulent environment characterized by a dimensionless Eddy Dissipation Rate (EDR) $\epsilon^* = \frac{\epsilon b_0}{V_0^3} \simeq 0.002$. This is defined as a moderate turbulence intensity in the literature [44].

In the third configuration presented in Section E.3.2, the autopilot of the follower actively targets a constant relative position $\mathbf{x}_F^{opt}$ compared to the wake position that

is estimated by the estimator. The latter mode is called *wake tracking*, i.e., switch 2 in Fig. E.1. In this third configuration, the leader performs an oscillating maneuver as well, in a turbulent environment characterized by an $\text{EDR} = 0.002$. In the next sections, we use a dimensionless time definition, i.e., using the convection time $t^* = b/U_\infty$.

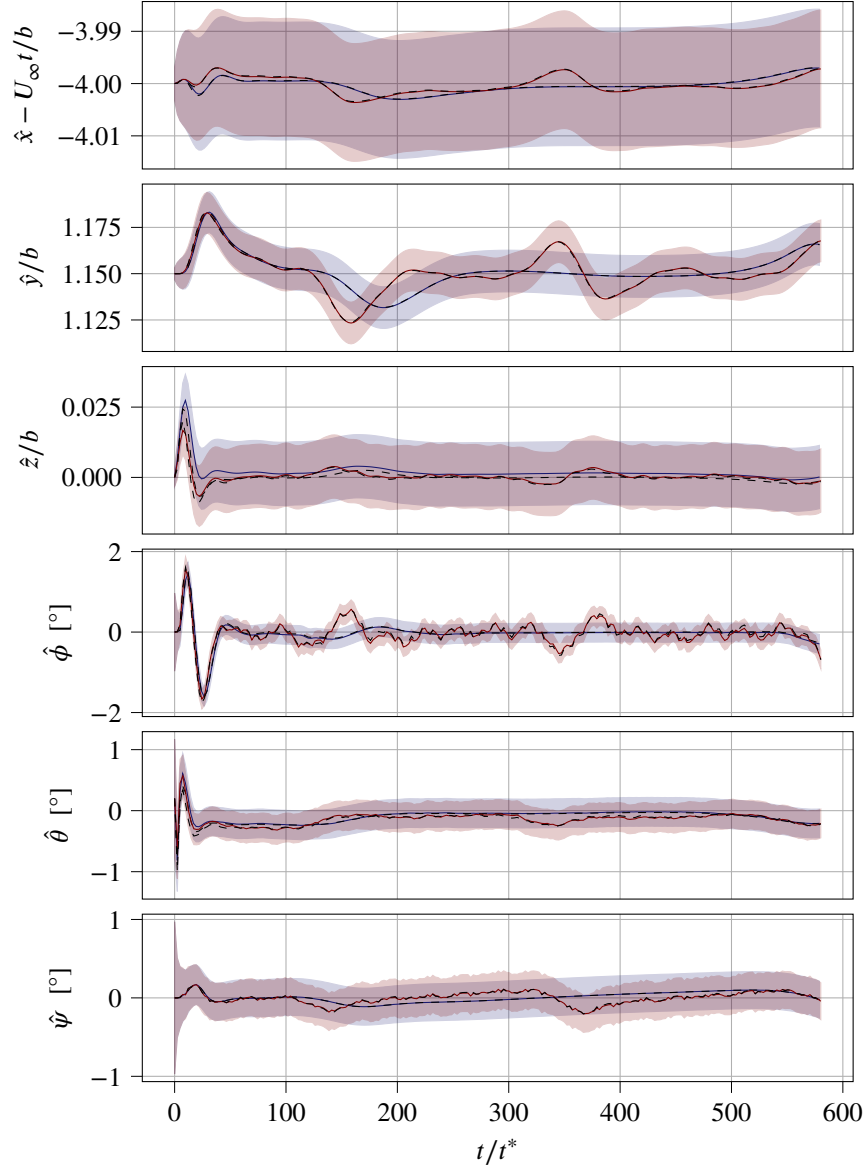
E.3.1 Wake detection

In this section, we focus on the *Config A.* and *Config B.* described in Table E.2. The follower maintains a fixed position in the wake of an oscillating leader. While its dynamics are affected by the wake aerodynamic interferences, the EnKF predicts the characteristics of the wake.

First, we observe in Figs. E.2 and E.3 the estimated dynamic states $\mathbf{q}_{dyn}$ and control states $\mathbf{q}_{cont}$ of the follower, as predicted by the linear Kalman filters over time. We compare them to the dynamics of the follower that are computed by the simulated flight environment.

We observe that for both configurations, the linear estimators are able to accurately estimate all the states of the aircraft. This was predictable, given that we assume in our model that the linear Kalman filters, in addition to the model they propagate, exploit measurements of the actual accelerations, angular velocities and effective deflections of the command, i.e., $\mathbf{z} = \mathbf{z}_{accel} \cup \mathbf{z}_{dyn} \cup \mathbf{z}_{cont}$. Due to the noise associated to each one of the measurements, and the process noise related to the model, we observe on the graphs the uncertainty of each one of the estimated states. The linear Kalman filters indeed compute the Probability Density Function (PDF) of those states.

The PDFs computed in our simulations directly result from the covariance matrices chosen for the measurement noise and the model noise, i.e., $\mathbf{R}_{dyn}$, $\mathbf{R}_{cont}$, $\mathbf{Q}_{dyn}$ and $\mathbf{Q}_{cont}$ in Eqs. E.5 and E.8. In the simulations presented here, these covariance matrices have been fine tuned in such a way that they effectively enable to predict the position of a wake over time. Indeed, measurement noises are important as they are directly related to the accuracy of the measuring instruments used on board the aircraft. In modern ones, IMUs provide quality information in terms of translational and angular accelerations, and angular velocities. GPS allows to measure the position. The Kalman filter presented here exploits these measurements to deduce the translational velocities and attitude of the aircraft in addition to refining the measurements thanks to the model they exploit. We also assume that the Kalman filter exploit a direct measurement of the effective deflection of the control surfaces as well as the effective thrust of the engine.



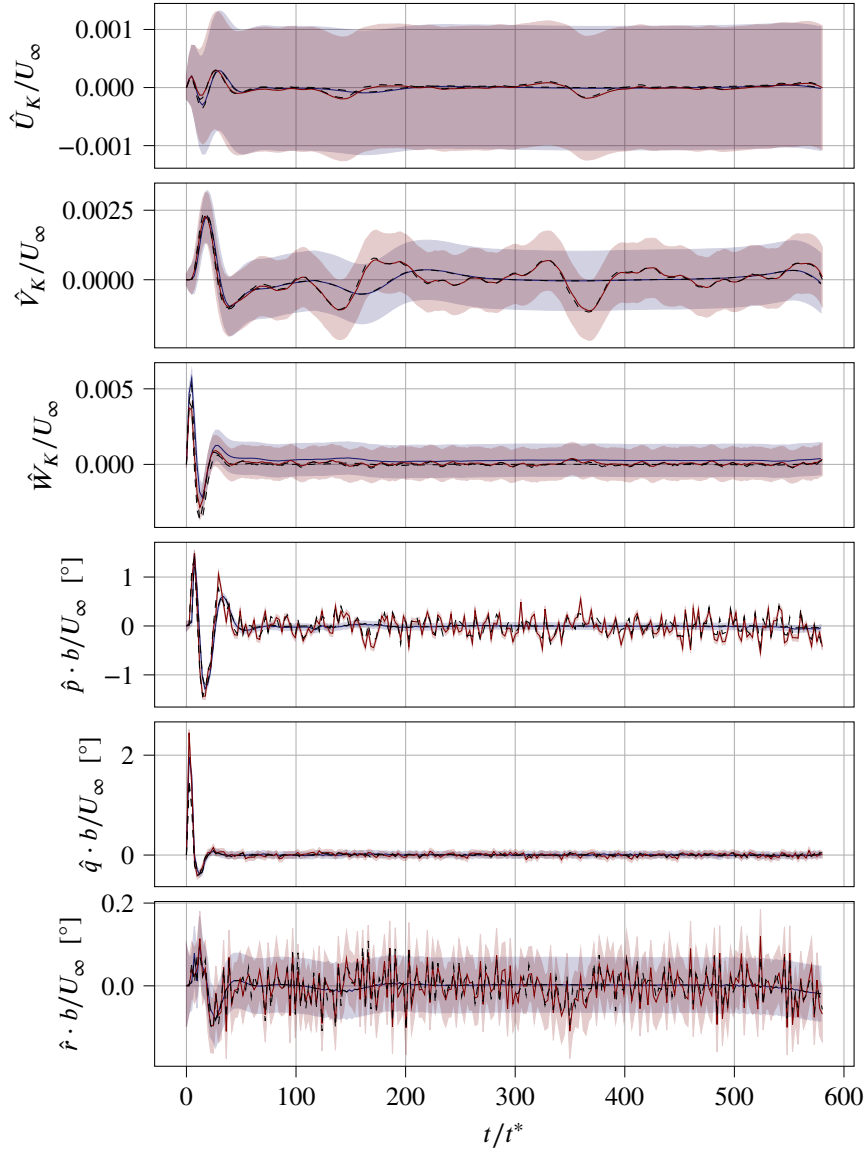


Figure E.2: Estimation of the follower's dynamic states. The black dashed lines (--) indicates the actual values of the states. The blue lines (—) (resp. red (—)) are the estimates with their *a posteriori* $1 - \sigma$ confidence interval for *Config. A* (resp. *Config. B*).

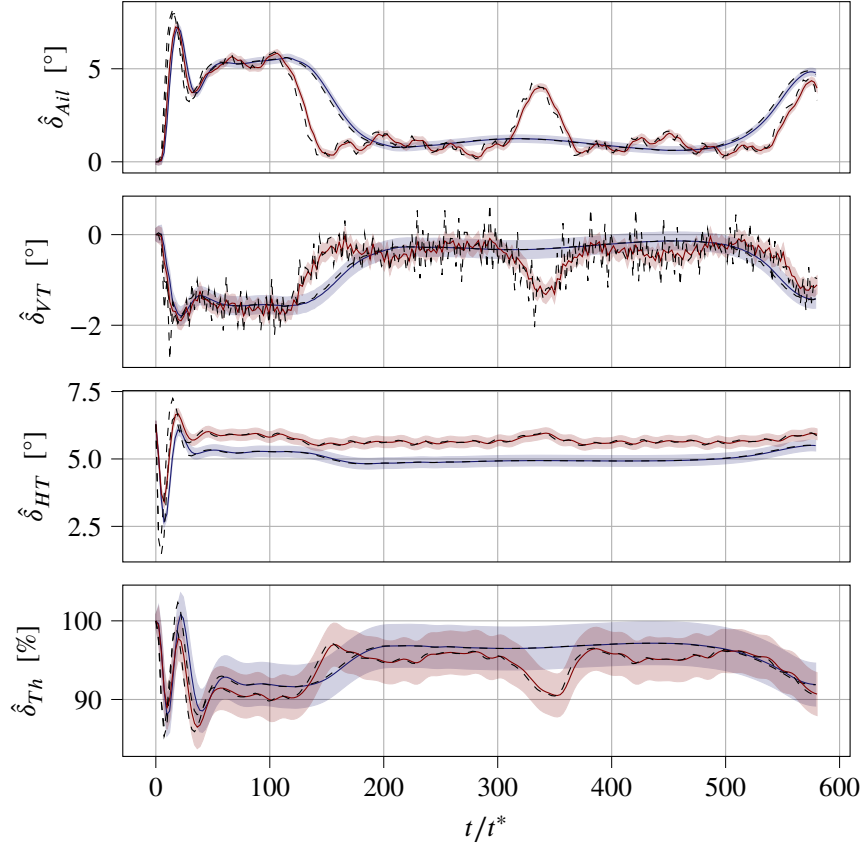


Figure E.3: Estimation of the follower's controller dynamic states. The black dashed lines (--) indicates the actual values of the states. The blue lines (—) (resp. red (—)) are the estimates with their *a posteriori* $1 - \sigma$ confidence interval for *Config. A* (resp. *Config. B*).

This is of course a strong hypothesis, but it was showed to be necessary to be able to track the wake of the leader over time.

In these figures, it is also important to note the slight temporal offsets that exists between the true dynamic states and the estimated states. This is particularly noticeable on the control states in Fig. E.3. These offsets are due to trade-off that the Kalman filters have to manage between the use of the measurement or the use of the propagated model.

In Fig. E.5, we present the leader's wake state parameters, as they are predicted by the EnKF over time.

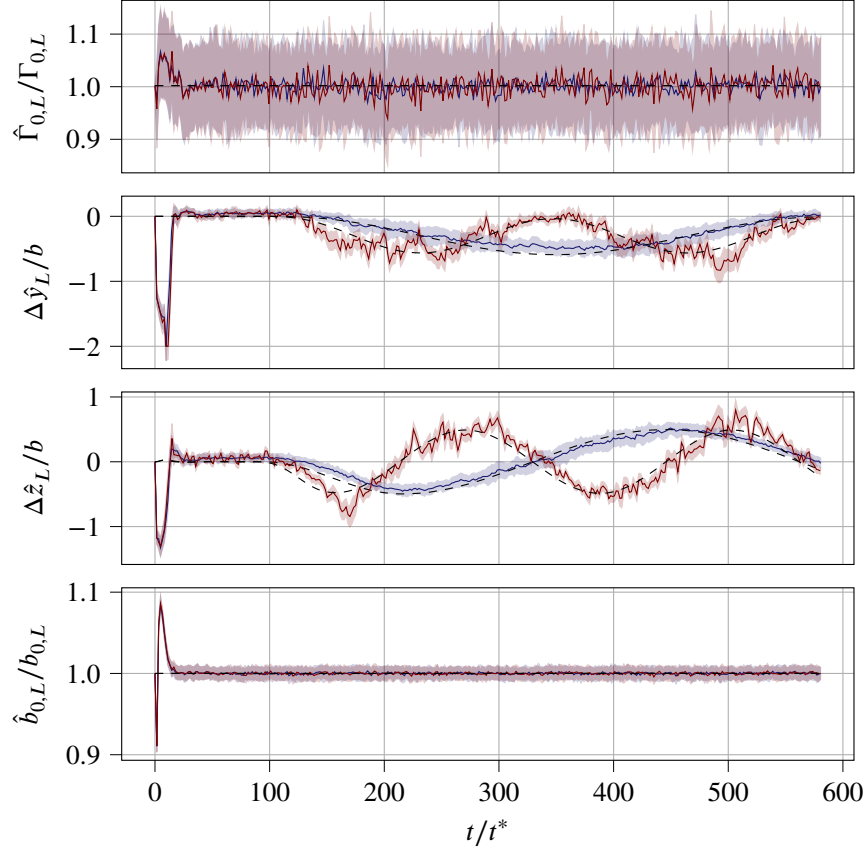


Figure E.4: Estimation of the leader's wake state parameters over time. The black dashed lines (---) indicates the actual values of the states. The blue lines (—) (resp. red (—)) are the estimates with their *a posteriori* $1 - \sigma$ confidence interval for *Config. A* (resp. *Config. B*).

We first observe that the wake intensity $\Gamma_{0,L}$ as well as the spacing $b_{0,L}$ are nearly perfectly estimated for both configurations. This is due to the fact that the EnKF exploits measurement of those parameters. They are provided by a reasonable extension of the Automatic Dependent Surveillance-Broadcast (ADS-B), that episodically broadcasts information related to the leader, in particular its mass m , similarly to the estimator designed in Chapter 2. In this work, this hypothesis translates in our assumption that the leader's properties are initially known, and vary little within the time scales considered.

Under these conditions, we observe that the estimator is able to predict the leader's wake position over time, for the two configurations studied. Where the estimated position is particularly smooth in *Config. A*, we observe oscillations in *Config. B*. They are due to the turbulence which directly impacts the measured accelerations and therefore the entire dynamics as observed in Figs. E.2 and Fig. E.3. Nevertheless, this results in a good estimation, which demonstrates the appropriateness of the methodology developed in this appendix.

E.3.2 Wake tracking

In this section, we now investigate the ability of the follower to maintain in real time a constant position relative to a leader's wake. Taking advantage of the estimation of the position of the wake over time, the follower actively targets the optimal location in order to benefit from maximum fuel burn reduction at all times. The leader performs a sinusoidal oscillating trajectory whose parameters are presented in Table E.2 under *Config. C*. We present the leader's wake state parameters in Fig. E.5.

In this third configuration, the follower also uses information provided by an extended ADS-B, and therefore exploits measurements of $\Gamma_{0,L}$ and $b_{0,L}$. This explains the accurate estimation of those states on Fig. E.5. We also observe that the follower is able to predict the position of the wake during the majority of the simulation, while following a trajectory which is itself oscillating. However, we observe an important error at the end of the trajectory. It is explained by the strong interactions between the wake of the leader and the follower at $t \simeq 500t^*$. Indeed, for the period of time $t \in [450t^*, 550t^*]$, the leader's wake gets closer to the follower, instead of moving away from it during the rest of the simulation. What's more, the estimator predicts the wake position with a small delay which can be explained by the offset induced by the estimation of the follower dynamics states by the linear Kalman filters described previously. Therefore, the follower does not anticipate the convergence of the trajectories quickly enough. The control laws that governs the lateral dynamics of the follower being slower, the follower is finally subjected to a strong rolling moment that needs to be compensated. This leads to a disturbed follower dynamics and estimation errors of the wake itself.

Fig. E.5 presents a visual interpretation of the tracking strategy, and shows instantaneous views of the flow in the cross section $x = x_F$. Even if the instantaneous estimated position of the wake is noisy and does not perfectly fit with the exact position of the wake, it is always located in the vicinity of the actual wake. For the period $t \in [450t^*, 550t^*]$ however, the important estimation error is obvious, as well as the

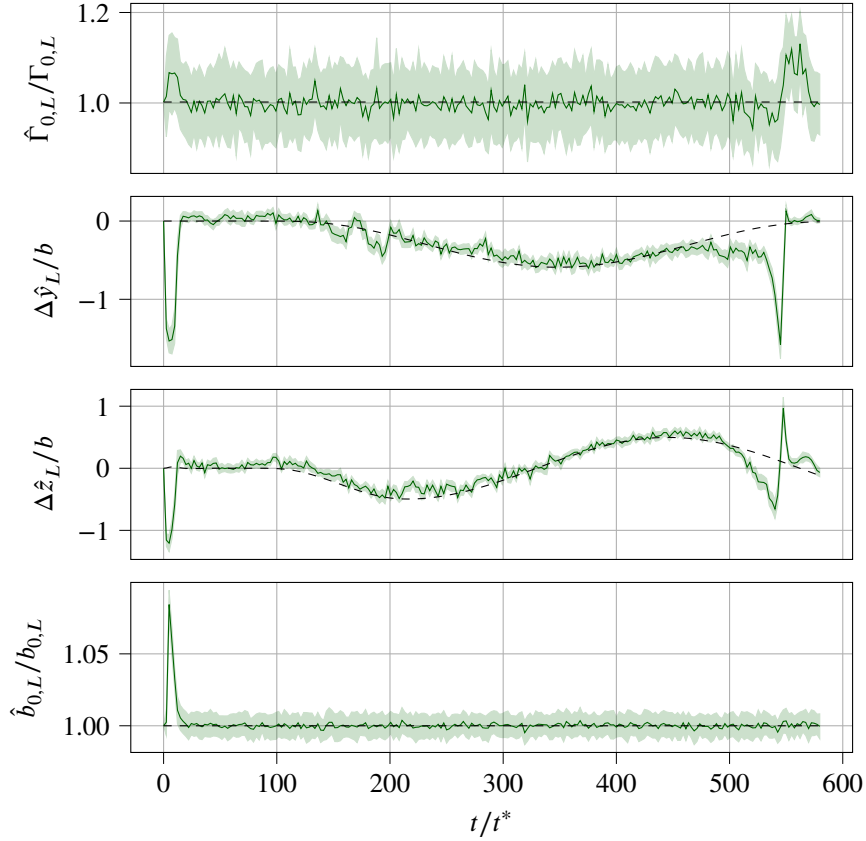
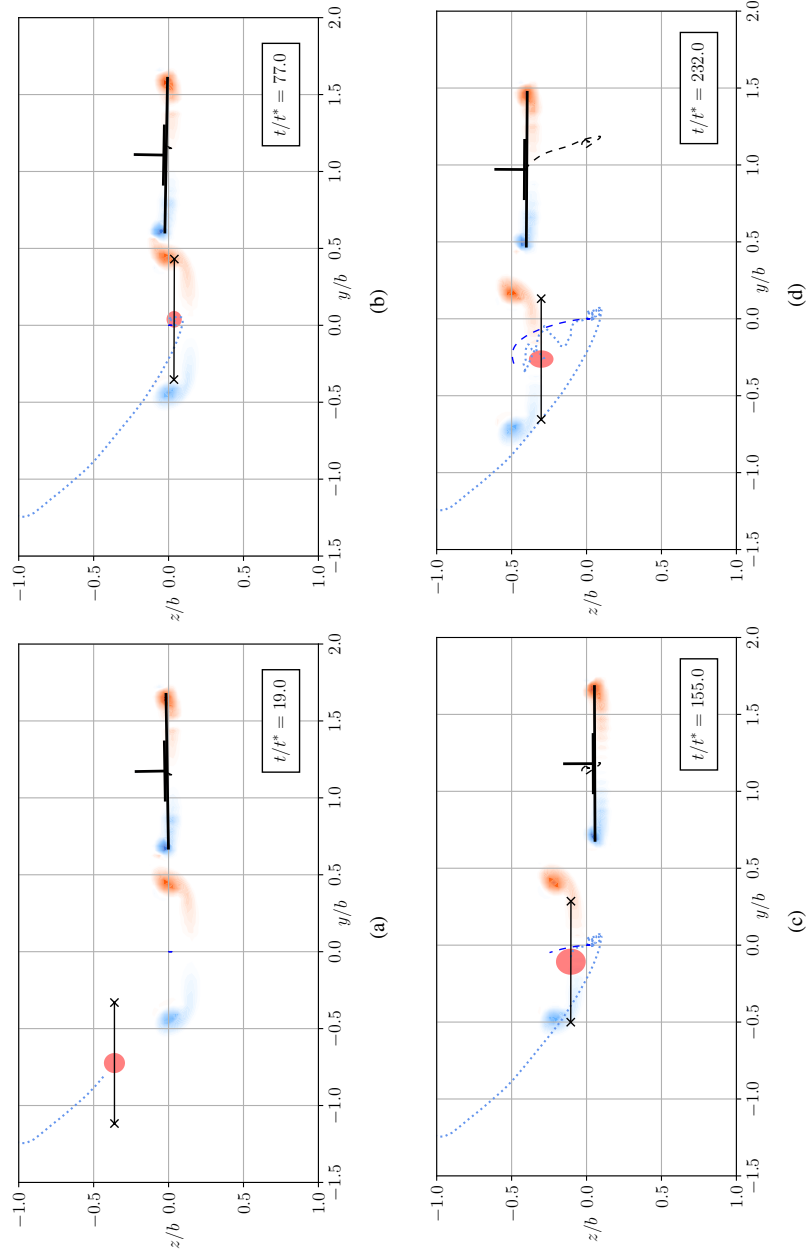


Figure E.5: Estimation of the leader's wake state parameters over time. The black dashed lines (--) indicates the actual values of the states. The green lune lines (—) are the estimates with their *a posteriori* $1 - \sigma$ confidence interval for *Config. C*.

close proximity of the follower with the leader's wake which can explain the error. This is in fact the main drawback of the methodology developed. The dynamics used to measure the wake can be strongly disturbed by the wake itself, which can lead to a wrong estimation, which might worsen the estimation, etc.

E.4 Conclusions

In this work, we have presented an alternate wake tracking strategy to the one described in Chapter 2. It also uses the dynamics of the follower in order to predict the charac-



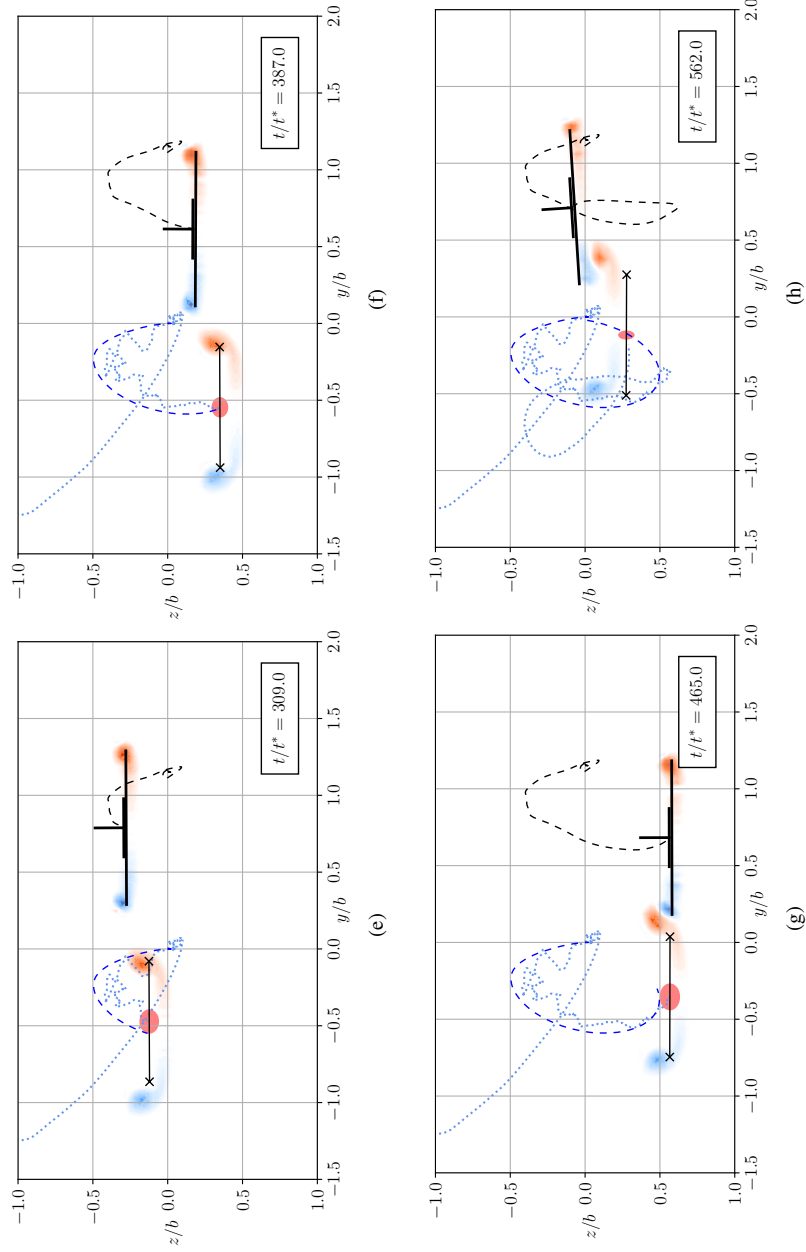


Figure E.5: Vortex system estimated by the EnKF over time. The background shows the vorticity behind the follower. The blue dotted line (.....) us the history of the estimated position with its $1 - \sigma$ confidence interval (●). The blue dashed line (---) is the actual trajectory of the leader's wake. The black dashed line (---) is the actual trajectory of the follower.

teristics of a leader's wake. However, where both the aircraft dynamics of the follower and the dynamics of the leader's wake were predicted simultaneously in Chapter 2, we decouple the latter the strategy described in this appendix. Indeed, the follower's dynamics are estimated thanks to a simplified version of an INS that uses linear Kalman filters. The leader's state parameters are deduced using a similar EnKF as the one described in Chapter 2. Nevertheless, the measurements exploited by the EnKF in this new strategy now reduce to accelerations measurements and aerodynamic angles as the rest of the dynamics is considered to be known.

We show in this chapter that this strategy is also able to perform wake tracking in challenging environment, i.e., turbulent environment computed in Large Eddy Simulations. Also, we used in this appendix the A350-like aircraft described in Chapter 1. Further work would consist in comparing the two approach. In particular, the impact of the measurement noise and the uncertainty associated to the dynamic states would have to be evaluate.

Kalman filters parameters

Linear Kalman filters

The linear Kalman filters presented in Section E.2.1 exploit both dynamic measurements and dynamic models in order to estimate the states of the systems. Measurement noises σ_{dyn} and σ_{cont} , and process noises ϵ_{dyn} and ϵ_{cont} are associated to each one of those. They follow zero-mean normal distributions, i.e.,

$$\begin{cases} \sigma_{dyn} \sim \mathcal{N}(0, \mathbf{Q}_{dyn}) \\ \sigma_{cont} \sim \mathcal{N}(0, \mathbf{Q}_{cont}) \end{cases} \quad (\text{E.16}) \quad \begin{cases} \epsilon_{dyn} \sim \mathcal{N}(0, \mathbf{R}_{dyn}) \\ \epsilon_{cont} \sim \mathcal{N}(0, \mathbf{R}_{cont}) \end{cases} \quad (\text{E.17})$$

The covariance matrices $\mathbf{R}$ and $\mathbf{Q}$ are diagonal and constant over the estimation process. The values used in the simulations are given in Tables E.3 and E.4.

Ensemble Kalman filter

The ensemble Kalman filter presented in Section E.2.3 exploits accelerations measurements in order to estimate the leader's wake state parameters. Measurement noises ϵ_{acc} are associated to each one of them. Also, rather than process noise, the EnKF inflates the wake states through additive state inflation α . Both the measurement noises and

the state inflation follow zero-mean normal distributions, i.e.,

$$\begin{cases} \epsilon_{acc} \sim \mathcal{N}(0, \mathbf{R}_{acc}) \\ \alpha \sim \mathcal{N}(0, \mathbf{Q}_{cont}) \end{cases} \quad (\text{E.18})$$

The values used in the simulations are given in Tables E.5 and E.6.

x_F	$2m$	$\dot{U}_{K,F}$	0.01 m/s^2
y_F	$2m$	$\dot{V}_{K,F}$	0.01 m/s^2
z_F	$2m$	$\dot{W}_{K,F}$	0.01 m/s^2
$\dot{p}$	$0.1^\circ/\text{s}^2$	p	$0.1^\circ/\text{s}$
$\dot{q}$	$0.1^\circ/\text{s}^2$	q	$0.1^\circ/\text{s}$
$\dot{r}$	$0.1^\circ/\text{s}^2$	r	$0.1^\circ/\text{s}$
δ_{Ail}	0.5°	δ_{VT}	0.5°
δ_{HT}	0.5°	δ_{Th}	$20kN$

Table E.3: Square root of the measurement noise level variances associated to the linear Kalman filter measurements (i.e. diagonal of the matrices $\mathbf{R}_{dyn}$ and $\mathbf{R}_{cont}$).

x_F	$0.2m$	$U_{K,F}$	$0.05m/s$
y_F	$0.2m$	$V_{K,F}$	$0.05m/s$
z_F	$0.2m$	$W_{K,F}$	$0.05m/s$
ϕ	0.05°	p	$0.05^\circ/s$
θ	0.05°	q	$0.05^\circ/s$
ψ	0.05°	r	$0.05^\circ/s$
δ_{Ail}	0.1°	δ_{VT}	0.1°
δ_{HT}	0.1°	δ_{Th}	$2kN$

Table E.4: Square root of the process noise level variances associated to the linear Kalman filter states (i.e. diagonal of the matrices $\mathbf{Q}_{dyn}$ and $\mathbf{Q}_{cont}$).

$\dot{U}_{K,F}$	5 m/s^2	$\dot{p}$	$1^\circ/\text{s}^2$
$\dot{V}_{K,F}$	5 m/s^2	$\dot{q}$	$1^\circ/\text{s}^2$
$\dot{W}_{K,F}$	5 m/s^2	$\dot{r}$	$0.1^\circ/\text{s}^2$

Table E.5: Square root of the measurement noise level variances associated to the EnKF measurements (i.e. diagonal of the matrix $\mathbf{R}_{acc}$).

$\Gamma_{0,L}$	$290m^2/s$	$b_{0,L}$	$1m$
Δy_L	$4m$	Δz_L	$4m$

Table E.6: Square root of the additive inflation variances associated to the EnKF states (i.e. diagonal of the matrices $\mathbf{Q}_\alpha$, see Section 2.2.2 for more details).