Moments of Entropy Production in Dissipative Devices

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We characterize the possible moments of entropy production for overdamped stationary Markov processes. We find a general formulation of the problem, and derive a new necessary condition relating the second and third moments. We determine all possible first, second, and third moments of entropy production for a white noise process. As a consequence, we obtain a lower bound for the skewness of the current fluctuations in dissipative devices such as transistors, thereby demonstrating that the Gaussianity assumption widely used, e.g., in electronic engineering is thermodynamically inconsistent.

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Introduction—Stochastic thermodynamics extends the laws of conventional thermodynamics and equilibrium statistical physics to mesoscopic systems in which random fluctuations are non negligible [1–3]. The theory is able to describe possibly strongly nonlinear systems operating far from equilibrium [4]. Modern nanoscale electronic devices, operating either in classical [5–8] or quantum regime [2,9–13], constitute one recent field of application of this theory. Recent contributions were dedicated to reliability assessment [14,15], or relation between noise and energy dissipation in digital CMOS circuits in non-stationary conditions [16–18].

The *local detailed balance (LDB) relation* [3,19] results from microscopic reversibility, and formulates the entropy production in terms of probabilities of direct and time-reversed trajectories. It holds in wide range of situations [1]. From the LDB one can derive a host of important results, such as the *fluctuation relation* [20], or the *thermodynamic uncertainty relations* (TUR) [21–27]. The TURs provide a fundamental lower bound for the variance of entropy production $\Delta\sigma$, and more generally observables antisymmetric under time reversal [21,28–30].

Beyond mean and variance, characterization of higher-order moments, like the *skewness* (the third central moment) quantifying the asymmetry of the fluctuations, provides finer information about the random physical process [31], especially far from equilibrium [6,30]. The topic is covered to a much lesser extent. Other theoretical works focus on special cases, notably noninteracting systems [32], unicyclic [29,32] and multicyclic [29] Markovian networks. As broadly reviewed in [7], skewness of electrical current fluctuations was experimentally reported in tunnel junctions [9,11], avalanche diodes [5], quantum devices [10], and metallic wire at cryogenic temperature [31].

In the present work, we characterize all possible values for moments of entropy production. Besides recovering the generalized TUR [33–35], it allows to find bounds on higher moments. As an illustration, we derive a novel, tight, bound between second and third moment. We also write the tightest relations that hold in the limit of low entropy production.

As the main result of this article, we find the relations that hold between the mean, variance and skewness of entropy production of any white noise that is thermodynamically consistent (in that it satisfies LDB). The bounds apply in particular to the flow (e.g., electric current) going through a purely dissipative device, in both equilibrium and far-from-equilibrium conditions. Our bound contains as particular cases several important special cases encountered in electronics, mechanics and chemistry: Johnson-Nyquist [36,37] or Einstein diffusion process (Brownian motion [38]); shot noise or any bidirectional Poisson process [6,8,39].

Problem statement—For a real random observable X, let $m_k = \langle X^k \rangle$ be its kth moment, for $k = 1, 2, 3, \ldots$ In this Letter, $\langle \cdot \rangle$ always denotes the expectation operator, i.e., ensemble average. A classic problem in probability theory is to characterize all the possible sequences of real numbers $m_1, m_2, m_3, \ldots$ that indeed emerge as the moments of some arbitrary random observable X. This problem was asked and solved by Stieltjes in the case of *nonnegative* real random variables [40]. He showed that these sequences are exactly generated by the convex combinations of sequences of the form $1, r, r^2, r^3, \ldots$ for some $r \geq 0$, i.e., $m_k = \sum_{\omega \in \Omega} p(\omega) r_\omega^k$, for some probability distribution p on some finite set Ω (in fact infinite or continuous convex combinations may also arise, but we write finite sums for notational simplicity). This characterization is in turn equivalent to a sequence of tight inequalities between the moments. For instance $\sqrt{m_2} \geq m_1 \geq 0$ is all there is to say about the m_1, m_2 alone: numbers satisfy those inequalities if and only if they are the moments of some non-negative real random observables over some space. Other nontrivial inequalities relate the higher-order moments.

In this Letter we ask which real numbers can possibly arise as the moments of entropy production of a (classical) system satisfying LDB, as defined below.

We also analyze the small entropy production case, and the case of stationary white noise. We characterize all possible cumulants of order one (mean), two (variance), and three (skewness) for thermodynamically consistent stationary white noise.

Moments of entropy production—Let Ω be a probability space with probability measure p and an involution $\omega \mapsto \bar{\omega}$ ("involution" means that $\bar{\bar{\omega}} = \omega$). This defines $\bar{p}(A) = p(\bar{A})$ for any event $A \subseteq \Omega$. Without loss of generality, and for the sake of simplicity of notations, we assume a discrete space, so that we can write, for an observable $f:\Omega \to \mathbb{R}$, the mean as $\langle f \rangle = \sum_{\omega\in\Omega} f(\omega)p(\omega)$ (even though in some cases the space Ω is continuous, and this sum should be implicitly understood as an integral).

Let $\Delta\sigma \equiv \ln(p/\bar{p})$. We call this observable the "entropy production," in reference to the situations where this observable, for now a purely mathematical definition, is indeed endowed with this physical meaning (up to Boltzmann's constant k_B). Those situations are discussed further below.

Our question is to characterize all the possible values taken by the moments of the entropy production, $m_k = \langle \Delta\sigma^k \rangle$ for $k = 1, 2, 3, \ldots$.

In the following we assume $\omega \neq \bar{\omega}$, for notational simplicity, and because $\omega = \bar{\omega}$ implies $\Delta\sigma(\omega) = 0$. This allows us to split Ω in pairs $\omega, \bar{\omega}$, each with probability $p(\{\omega, \bar{\omega}\}) = p(\omega) + p(\bar{\omega})$.

First observe that $\Delta\sigma^k(\omega) = \ln^k\{[p(\omega)]/[p(\bar{\omega})]\} = (-1)^k \Delta\sigma^k(\bar{\omega})$. For even k we have:

$$m_k = \sum_{\{\omega,\bar{\omega}\}} p(\{\omega,\bar{\omega}\}) \ln^k \frac{p(\omega)}{p(\bar{\omega})} \tag{1}$$

$$\equiv \sum_{\{\omega,\bar{\omega}\}} p(\{\omega,\bar{\omega}\}) s_\omega^k, \quad \text{with} \quad s_\omega \equiv \left| \ln \frac{p(\omega)}{p(\bar{\omega})} \right| \geq 0, \tag{2}$$

where the sum runs over all unordered pairs $\{\omega, \bar{\omega}\}$ (counted once). Remark that s_ω can assume any possible nonnegative value, regardless of $p(\{\omega, \bar{\omega}\})$, as the ratio $p(\omega)/p(\bar{\omega})$ provides no information upon the sum $p(\omega) + p(\bar{\omega})$ and conversely. Thus (1) implies that the sequence $(m_2, m_4, m_6, \ldots)$ is a convex combination of several (and possibly infinitely many) sequences of the form $(s^2, s^4, s^6, \ldots)$ for $s \geq 0$. Conversely, every convex combination $(\sum_{\omega\in\Omega_0} q(\omega)s_\omega^2, \sum_{\omega\in\Omega_0} q(\omega)s_\omega^4, \sum_{\omega\in\Omega_0} q(\omega)s_\omega^6, \ldots)$, for any nonnegative coefficients q summing to one over a set Ω_0, of such sequences may be seen as the moments $(m_2, m_4, m_6, \ldots)$ of some probability distribution p on some Ω. Indeed we may choose $\Omega = \Omega_0 \cup \bar{\Omega}_0 = \cup_{\omega\in\Omega_0} \{\omega, \bar{\omega}\}$ (with $\bar{\Omega}_0$ an identical copy of Ω_0), with probability distribution p so that $p(\{\omega, \bar{\omega}\}) = q(\omega)$ and $p(\omega)/p(\bar{\omega}) = e^{s_\omega}$ consistently with (2). In this way we have completely characterized the set of possible values for $(m_2, m_4, m_6, \ldots)$. We observe that this solution for the even moments of $\Delta\sigma$ coincides with Stieltjes's solution [40] for all the moments of the nonnegative random observable $\Delta\sigma^2$, with $r_\omega = s_\omega^2$. Consequently, the moments of $\Delta\sigma^2$ satisfy the same constraints binding those of any nonnegative random variable, and there is nothing else to say about their possible values.

The odd moments m_k satisfy

$$m_k = \sum_{\omega\in\Omega} p(\omega) \ln^k \frac{p(\omega)}{p(\bar{\omega})} \tag{3}$$

$$= \sum_{\{\omega,\bar{\omega}\}} (p(\omega) - p(\bar{\omega})) \ln^k \frac{p(\omega)}{p(\bar{\omega})} \tag{4}$$

$$= \sum_{\{\omega,\bar{\omega}\}} p(\{\omega,\bar{\omega}\}) \frac{p(\omega) - p(\bar{\omega})}{p(\omega) + p(\bar{\omega})} \ln^k \frac{p(\omega)}{p(\bar{\omega})} \tag{5}$$

$$= \sum_{\{\omega,\bar{\omega}\}} p(\{\omega,\bar{\omega}\}) \ln^k \frac{p(\omega)}{p(\bar{\omega})} \tanh \frac{1}{2} \ln \frac{p(\omega)}{p(\bar{\omega})} \tag{6}$$

$$= \sum_{\{\omega,\bar{\omega}\}} p(\{\omega,\bar{\omega}\}) s_\omega^k \tanh \frac{1}{2} s_\omega. \tag{7}$$

Thus, following the same reasoning, the possible values for *all* moments $(m_1, m_2, m_3, m_4, m_5, \ldots)$ are exactly the convex combinations of sequences $[s \tanh(s/2), s^2, s^3 \tanh(s/2), s^4, s^5 \tanh(s/2), \ldots]$.

Let us note here that odd moments are completely determined by even moments. Indeed the analytic expansion $\tanh x = x - x^3/3 + 2x^5/15 - \cdots$ comprises only odd powers. Thus for any odd k, we may write $s^k \tanh(s/2) = s^{k+1}/2 - s^{k+3}/8 + 2s^{k+5}/480 - \cdots$ with only even powers. Thus in the end we can express any odd m_k in terms of higher even moments:

$$m_k = m_{k+1}/2 - m_{k+3}/8 + 2m_{k+5}/480 - \ldots \tag{8}$$

This offers in principle a complete solution to the characterization of possible moments of entropy production: a sequence of real numbers $(m_1, m_2, m_3, m_4, \ldots)$ is the sequence of moments of an entropy production observable if and only if odd terms $(m_1, m_3, \ldots)$ each satisfy (8), and even terms $(m_2, m_4, \ldots)$ satisfy Stieltjes inequalities as mentioned above.

This solution characterizing all moments of entropy production is an important theoretical progress *per se*. However, in practical situations one often wants to bounds some specific moments, and find the specific relation that binds them.

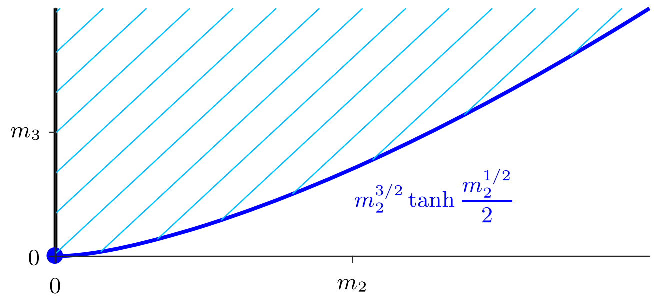

FIG. 1. The set of all possible pairs (m_2, m_3) for entropy production (hatched in sky blue), defined by (9), is obtained as the convex hull of the blue boundary curve (which is thus included in the set). The black boundary straight line $m_2 = 0$ [lower bound for m_2 in (9)] is *excluded*, save for the blue dot at the origin, which corresponds to the equilibrium (no entropy production).

As an important example, let us determine the set of all possible pairs (m_2, m_3) for entropy production. First, we construct the curve $(m_2, m_3) = [s^2, s^3 \tanh(s/2)]$ by letting the parameter s vary from 0 to $+\infty$ (in blue in Fig. 1). Then, the set is obtained by inspection of Fig. 1 as the convex hull of all the points of the curve (hatched area), thus described by

$$m_3 \geq m_2^{3/2} \tanh\frac{m_2^{1/2}}{2} > 0, \quad \text{or} \quad m_2 = m_3 = 0. \tag{9}$$

The second and third moments (m_2, m_3) of entropy production must satisfy (9). Conversely, from every pair (m_2, m_3) satisfying (9), we can build as above a probability space Ω with an involution that defines an entropy production whose second and third moments are m_2 and m_3.

The same methodology applied to (m_1, m_2) recovers the generalized thermodynamic uncertainty principle [33–35].

What we defined mathematically as entropy production $\Delta\sigma$ coincides with the usual physical entropy production (increase of entropy of the Universe) when Ω is the space of trajectories, over some time interval Δt, of a stationary Markov process modeling an overdamped physical process subject to a constant or time-symmetric protocol (the protocol referring to the transition probabilities characterizing the Markov process), and the involution is simply the time-reversal of the trajectory (i.e., the sequence of states traversed by the trajectory, read in reverse order), as a direct consequence of LDB [1,19,33–35]. "Stationary" here means that the probability distribution on the state of the Markov chain is identical at the beginning and at the end of the interval (and thus all intermediate times, in the case of a constant protocol). Notice, however, that our results in this section, for instance (9), hold mathematically for this observable whether or not it has the physical meaning of an entropy production. It may cover completely different meanings, where, for instance, the involution is not time reversal, but spin reversal in a spin system [33,41].

Small entropy production—Remember that if $m_1 = 0$, then all the moments are zero (as $p \equiv \bar{p}$). We now consider the possible moments of entropy production in the limit of small mean $m_1 = \langle \Delta\sigma \rangle$. This occurs, for instance, in the situation $\Delta t \to 0$ for any overdamped stationary Markov process.

Consider once again the convex set S of all possible moments $(m_1, m_2, m_3, \ldots)$. Consider the cone C of all half-lines issued from the origin $(0, 0, 0, \ldots)$ and meeting at least another point of S. Certainly this cone contains S. Moreover, it coincides with S in a vanishingly small neighborhood of the origin, i.e., for small m_1. The cone C can be seen as a "linearization" of S around the origin.

This cone is equivalently characterized by the convex set of all values taken by the "scaled coordinates" $(m_2/m_1, m_3/m_1, m_4/m_1, \ldots)$ (for all nonzero points in S) indicating the directions of the half-lines in C. In this representation, C is the convex hull of extremal points $(m_2/m_1, m_3/m_1, m_4/m_1, \ldots) = \{(s/[\tanh(s/2)]), s^2, (s^3/[\tanh(s/2)]), \ldots\}$, for $s \geq 0$. For the first three moments, following Fig. 2, we find

$$2 < \frac{m_2}{m_1} \leq \frac{\sqrt{\frac{m_3}{m_1}}}{\tanh\left(\frac{1}{2}\sqrt{\frac{m_3}{m_1}}\right)}, \tag{10}$$

or

$$\frac{m_2}{m_1} = 2 \quad \text{and} \quad \frac{m_3}{m_1} = 0. \tag{11}$$

Notice that (11) is only reached as a limiting case of (10) for $m_3 \ll m_1 \ll 1$. The (nonzero) moments (m_1, m_2, m_3) of entropy production necessarily satisfy (10) or (11). In the limit of small m_1, these inequalities are sufficient, in that they can be reached arbitrarily close to equality. The relation (10) may be relaxed to a simpler form by linearizing the rhs around the case (11), leading to

$$\frac{m_3}{m_1} \geq 6\left(\frac{m_2}{m_1} - 2\right) \geq 0. \tag{12}$$

As appears on Fig. 2 (dashed line), this bound is valid in general, equivalent to (10) near the limiting case (11), and otherwise conservative.

Stationary white noise—Pieces of trajectory of a white noise process on disjoint time intervals are, by definition, statistically independent. In stationary conditions, their statistics are also unaffected by translation of the time line.

For any such white noise, the entropy production $\Delta\sigma$ over a time interval Δt can be broken down as the sum of independent entropy productions over smaller intervals. This can be leveraged to show that the cumulants $c_1, c_2, c_3, \ldots$ of entropy production over time Δt are proportional to Δt, for any Δt (small or large), see, e.g., [6].

In the limit of short time intervals $\Delta t \to 0$, the cumulants c_k and the moments m_k coincide [6]: for instance,

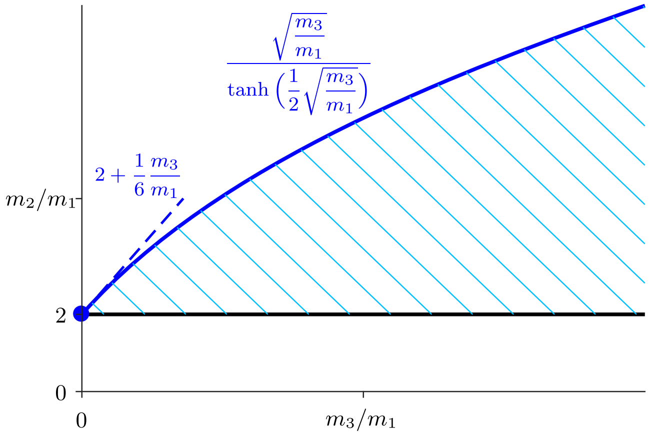

FIG. 2. The possible values for the moments m_1, m_2, and m_3 in the limit of small entropy production (hatched in sky blue), defined by (10) and (11), are generated as the convex hull of the blue boundary curve (which is thus included in the set). The linear approximation of this curve near the low skewness regime (0, 2) corresponds to the bound (12). The black boundary straight line $m_2/m_1 = 2$ is *excluded*. For the case of stationary white noise, i.e., (10) and (12) bis can be written in terms of cumulants and the axes label m_2/m_1 and m_3/m_1 can be substituted by c_2/c_1 (vertical axis) and c_3/c_1 (horizontal axis), respectively. Gaussian white noise is necessarily at the blue dot (0, 2), while the top blue curve is otherwise populated by bidirectional Poisson processes.

$c_2 = m_2 - m_1^2 = m_2 + \mathcal{O}(\Delta t^2)$, while c_2 is proportional to Δt. Similarly, $c_3 = m_3 - 3m_2 m_1 + 2m_1^3 = m_3 + \mathcal{O}(\Delta t^2)$, etc.

Thus, the relations (10) and (11), illustrated in Fig. 2 hold for cumulants of white noise, i.e., the ratios m_2/m_1 and m_3/m_1 being replaced with c_2/c_1 and c_3/c_1. For example (12) becomes for white noise

$$\frac{c_3}{c_1} \geq 6\left(\frac{c_2}{c_1} - 2\right) \geq 0. \tag{13}$$

Likewise, (10) and (11), with cumulants instead of moments, characterize the possible cumulants of thermodynamically consistent white noise, not only at short times (where they coincide with moments), but also at arbitrarily long times (by time proportionality of cumulants).

Notice that $c_2 \geq 2c_1$ is the expression of a particular case of the thermodynamic uncertainty relation [21]. Thus our relations suggest some converse of thermodynamic uncertainty relation, as providing an *upper* bound on the variance of the entropy production.

Let us also mention that the thermodynamic skewness relation (TSR) proposed by [30] predicts a non-negative skewness ($c_3 \geq 0$) when applied to a white noise process.

Dissipative devices—We consider a purely dissipative memoryless electronic device, like a homogeneous semiconductor or metallic bar (linear resistor) or a nonlinear diode or transistor, subject to a constant voltage V. In response to this external force, a random electrical current, which is adequately modeled as a (possibly nonzero mean) white noise process, flows through the resistor [6,7]. The total entropy production over a time interval Δt reads

$$\Delta\sigma = \frac{V}{k_B T}\Delta q = \frac{V}{k_B T}\int_{t_0}^{t_0+\Delta t} i(t)\mathrm{d}t, \tag{14}$$

where Δq is the random net charge through the device [integral of white noise current $i(t)$ over time Δt], k_B is Boltzmann's constant and T is the constant temperature of the environment, assimilated to a uniform thermal bath. Thus, entropy production and charge increment over a same time interval are related by (14), proportional through a constant factor, the "thermodynamic force" $V/k_B T$. Through (14), the moments of Δq relate to those of $\Delta\sigma$,

$$m_k = \left(\frac{V}{k_B T}\right)^k \langle \Delta q^k \rangle. \tag{15}$$

The cumulants of Δq are proportional to the cumulants c_k of $\Delta\sigma$ in the same way, because of the white noise property. Thus we may write that either

$$\frac{2k_B T}{V} < \frac{\langle(\Delta q - \langle\Delta q\rangle)^2\rangle}{\langle\Delta q\rangle} \leq \frac{\sqrt{\frac{\langle(\Delta q - \langle\Delta q\rangle)^3\rangle}{\langle\Delta q\rangle}}}{\tanh\left(\frac{1}{2}\frac{V}{k_B T}\sqrt{\frac{\langle(\Delta q - \langle\Delta q\rangle)^3\rangle}{\langle\Delta q\rangle}}\right)} \tag{16}$$

or

$$\langle(\Delta q - \langle\Delta q\rangle)^2\rangle = 2k_B T\frac{\langle\Delta q\rangle}{V} \quad \text{and} \quad \langle(\Delta q - \langle\Delta q\rangle)^3\rangle = 0. \tag{17}$$

The relation (17) is the famous Johnson-Nyquist relation [36,37], electrical equivalent to Einstein's diffusion law [38], assuming Gaussian-distributed fluctuations. It is a specific form of the fluctuation-dissipation theorem [42]. Note that our formulation does not assume linearity (where it is well known to hold, theoretically and empirically), and leaves the possibility for nonlinear resistors to obey a general Johnson-Nyquist law (17).

These relations reveal that a Gaussian noise model for the current in a resistor, or indeed any model with symmetric fluctuations around the mean, must obey (17). Conversely, any device showing fluctuations that exceed in variance the fluctuation-dissipation regime must exhibit positively skewed, thus non-Gaussian, fluctuations.

The rhs inequality in (16) holds with equality for a bidirectional Poisson random process, i.e., a white noise that is the difference of two Poisson processes with rates λ_+ and λ_-. Every arrival in the positive (negative) process generates an entropy of $\ln(\lambda_+/\lambda_-)$ (the opposite), following LDB. For instance in an electronic device, a charge carrier (electron, charge q_e) passing through the device subjected to a voltage V generates an entropy $\pm\ln(\lambda_+/\lambda_-) = \pm[(Vq_e)/(kT)]$. In this context, the bidirectional Poisson process is called *shot noise* [6,39]. Direct check up shows that all bidirectional Poisson random processes satisfy the relation with equality, and populate the top curve of the domain in Fig. 2. The interior of the domain is obtained by positive linear combinations of different Gaussian or bidirectional Poisson processes, consistently with Lévy-Khintchine theorem [43].

In many devices it is expected that for small thermodynamic force, $V \approx 0$, fluctuation-dissipation (Johnson-Nyquist) relation holds approximately. For increasing voltage, we may write a Taylor expansion (assuming differentiability) for the relative violation of fluctuation-dissipation relation:

$$\frac{\langle(\Delta q - \langle\Delta q\rangle)^2\rangle}{2k_B T \frac{\langle\Delta q\rangle}{V}} = \frac{1}{2}\frac{c_2}{c_1} \approx 1 + \frac{1}{2}\frac{V^2}{V_0^2}, \tag{18}$$

where the second order Taylor coefficient is denoted $1/V_0^2$. There is no first order coefficient since the expression must come to a minimum at $V = 0$. Here V_0 is interpreted as a characteristic voltage below which the violation to fluctuation-dissipation remains negligible. With this notation, we rewrite (13) as

$$\sqrt{\frac{\langle(\Delta q - \langle\Delta q\rangle)^3\rangle}{\langle\Delta q\rangle}} \geq \sqrt{6}\frac{k_B T}{V_0}. \tag{19}$$

This is valid for all purely resistive devices in all regimes, although conservative beyond the near-fluctuation-dissipation regime. It highlights that the fluctuations of the current are skewed in a nonvanishing manner, when compared to the current itself. This offers specific predictions on the third moments of fluctuations given the knowledge of mean and variance of currents are known as function of the voltage. The nonzero skewness of current fluctuations in nonlinear devices has been previously highlighted experimentally and theoretically in [6,7].

We formulate an empirically verifiable consequence of (16) on an important instance of nonlinear resistor: the channel of a so-called metal-oxide semiconductor (MOS) transistor. The mean and variance of Δq vary nonlinearly with the applied V; the equations [6, Eqs. (22)–(24)] are not reproduced here for brevity. As already reported in [6], $\langle(\Delta q - \langle\Delta q\rangle)^2\rangle/\langle\Delta q\rangle$ reaches the Johnson-Nyquist's lower bound ($2k_B T$) of (17) when $V \to 0$. This is consistent with the fact that the channel of the transistor behaves like a linear resistor for $V \to 0$.

For $V > 0$, Johnson-Nyquist's prediction (17) is exceeded and (16) must instead apply, inducing a lower bound on the third moment on charge flow, see dashed line in Fig. 3. The voltage V_0 appearing in (19) is valued at $V_0 = V_{\mathrm{sat}}/\sqrt{6}$, for the so-called saturation voltage V_{sat}.

We find instructive to consider realistic numerical values for the parameters involved. The value $k_B T \approx 4 \times 10^{-21}$ J $\approx$

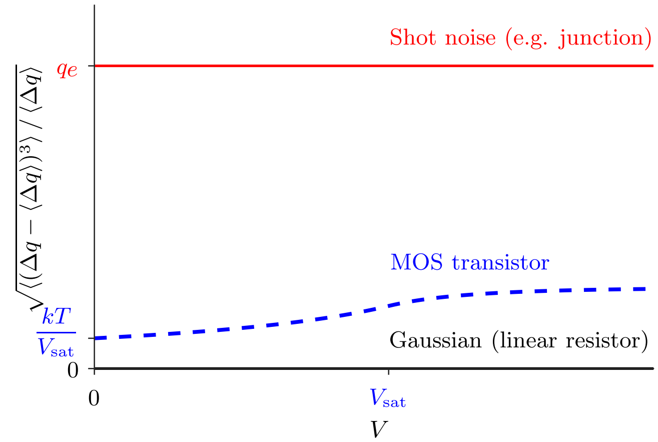

FIG. 3. Minimum skewness (third central moment) of current through a MOS transistor, predicted from (16) (dashed curve). Gaussian and shot noise skewness are also depicted for comparison.

25 meV at room temperature is well known. V_{sat} ranges from a few V in old μm MOS transistors and down to several hundreds of mV in the most advanced decananometer technologies. Taking $V_{\mathrm{sat}} = 250$ mV, as used in Fig. 3, the rhs of (19) results in $\sim q_e/10$. This value must be compared to the zero skewness of a pure Gaussian noise, and to q_e analytically computed for a shot noise, measured experimentally in a tunnel junction [9].

This theoretical result proves that the white noise in a MOS transistor is positively skewed and hence is not rigorously Gaussian, although this convenient assumption is widely used for noise modeling and circuit simulations [44]. It should be noted that the third moment of current of MOS transistors, despite the prominence of this device in the technology, have never been measured or modeled before, so that the results here form a prediction accessible to experiment.

Although we chose to illustrate our results on electronic resistive devices, equivalent situations and conclusions occur in various context where a purely dissipative system subjected to a constant "thermodynamic force," here $V/k_B T$, elsewhere mechanical force, difference of chemical potential, of temperatures, of concentration, generates a random "flow," respectively, speed, chemical flow, heat, or matter flow. In all these situations, the entropy production is the product of force with flow.

Discussion and conclusions—We have shown that the higher-order moments of entropy production satisfy nontrivial universal constraints with empirically measurable consequences. A natural continuation for those results would be the characterization of higher-order temporal correlations of nonwhite noise as produced by general Markov chains, in view to complement for example the recent results of autocorrelations and power spectral correlations [45–48]. Another continuation would be the extension to more general observables.

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