Chapter 1

Continuous model

Continuous Mechanics as well as Thermodynamics can principally be justified by statistical tools. For example, the exact description of a gas mole is very difficult because of the high number of interacting molecules ($\mathcal{O}(10^{23})$!). Therefore, thermodynamical variables and parameters are statistical objects that characterize the global effect of the set of the concerned molecules. The microscopic level is then the molecule scale and the macroscopic one is defined by statistical operations. Besides, it is precisely the choice of a statistical approach which allows the variables and parameters to **smoothly** and **continuously** vary in time and space. This feature results in the use of classical mathematical tools to solve the equations. Hence, this finally leads to a so-called **continuous description**.

In flows in porous media, the Navier-Stokes equations could more or less easily be solved for a single pore. However, the problem description and solution are considerably more complicated when the whole medium is to be taken into account. Besides this, experimental comparison is also a hard task.

It is precisely the porous medium complexity which incites us to use a continuous model. This approach is applied at a larger scale that summarizes all the porous medium details. The so-called microscopic scale is then Continuum Mechanics, in which each individual pore is separately considered and in which the inside flowing fluid can be observed. The macroscopic scale is defined by an analogous method with Thermodynamics and Continuum Mechanics.

To include microscopic effects into a macroscopic description, it is necessary to define

the concept of **Representative Elementary Volume** (REV) (Fig. 1.1). This volume represents the observation scale, in other words the measurement window. It is determined by considering all the specific REVs corresponding to each state variable and parameter of the problem. If all these individual REVs are of the same order of magnitude, it is possible to define at each location a unique problem REV, common to all variables and parameters. The REV must (i) be small enough in such a way that the spatial distribution of the considered variable is almost constant in the REV and (ii) be large enough to provide a continuous description of the problem (1; 8; 9).

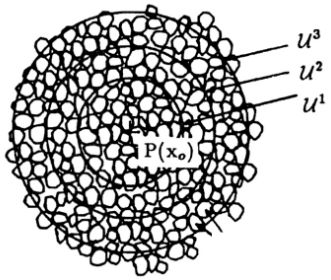

Figure 1.1: $\mathscr{U}^1$, $\mathscr{U}^2$, $\mathscr{U}^3$, different examples of REV for a porous medium (1).

The porosity representation (void volume/total volume) is addressed in Fig. 1.2. On the curve, three main regions can be distinguished : (1) a convergence zone in which the fluctuations decrease, this smooth effect being linked to inhomogeneities and boundary influence, (2) a quite flat and constant level zone, this being the searched scale which can lead to a continuous medium description, and finally (3) a zone where the volume is very small and the oscillations grow up in frequency and amplitude, since the volume can consist of solid or fluid only (with the porosity equal to 0 or 1).

The medium description provided by Thermodynamics and Continuum Mechanics can also be obtained by means of statistical considerations. Indeed, another approach to describe the porous medium behaviour consists in considering numerous measurements of a property at any given point and given time and then calculating the statistical expected value of this measured property, thereby performing a so-called ensemble averaging. Of course an experimental device has to be designed in order to measure the considered property and the experiments must be performed several times in order to provide a statistically meaningful average. In this sense, the expected value can be practically determined. This approach is besides physically coherent in the sense that it is based on experimental measurements.

Therefore it is highly interesting to here introduce the ergodic principle (10). This

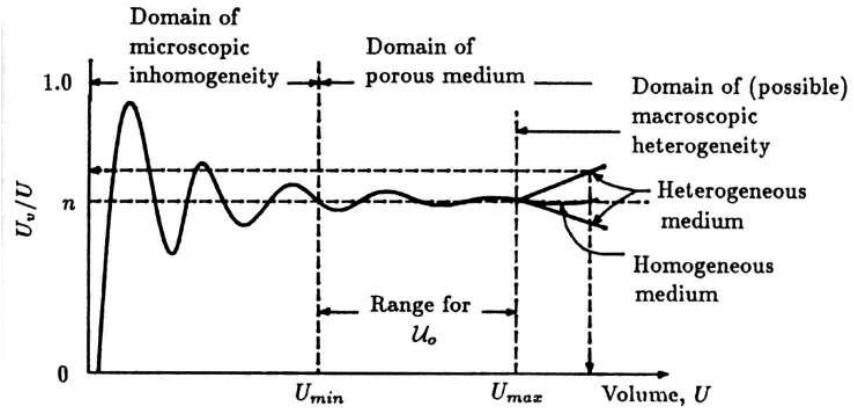

Figure 1.2: REV scale determination for the porosity (cf. Fig.1.1). Three main regions are distinguished. A correct REV lies in the middle zone, where the calculated porosity is quite constant when the sample volume varies (1).

principle indeed consists in linking the statistical approach with the spatial averaging approach by stating a non-obvious relationship between the detailed spatial distribution of a variable and its statistical expected value. Using the ergodic principle when this is allowed then provides a mathematically coherent meaning to the spatial averaged value of the field. Also this implies a kind of stationarity or statistical homogeneity of the field.

Finally, it should be noted that this link can be helpful to determine the REV scale, which appears to be in general an uneasy task. Nonetheless, as it will be shown later, the knowledge of the REV scale is not always necessary to model the porous medium behaviour.

An important consequence of introducing the REV concept in a porous medium study is that each problem phase is defined everywhere inside the medium. Indeed, considering a liquid saturated medium, any **mathematical point** still contains both phases thanks to the definition of the REV, even if the **real point** is only located inside a single phase.

To conclude this introduction, the advantages of using such Continuum Mechanics approach are summed up herebelow (1) :

- The interface description between the different phases, which is usually highly complex, is not necessary.
- The involved phenomena are described by differentiable variables. Classical mathematical methods can then be used to solve the problem.
- The calculated quantities have a macroscopical meaning and can then be linked to experimental measurements. A comparison of the developed model with experiments can be performed.
- Besides this, the equation parameters have also a macroscopic meaning and can themselves be measured.

This chapter develops the equations of flows in porous media with an approach based on continuous modelling. For this purpose, different intermediate objectives are suggested. The first goal is to provide a way to obtain a final equation set in which all the primary variables are macroscopic expressions. The second objective pursued is to use the described method to effectively get the macroscopic equation set. Finally, a third issue is to present and analyze some typical effects occurring in porous media. These developments will bring forth some new physical objects such as the **permeability tensor** and the **dispersion tensor**.

In order to achieve these goals, some preliminaries are firstly required and the averaging process is then presented. The microscopic scale equations are recalled and the averaging tools are applied to the mass, motion, energy and species conservation laws. Constitutive laws are finally proposed and justified in order to develop the final equation set.

This chapter has mainly been inspired by references (1) and (8).

1.1 Preliminaries and notations

Let $\mathscr{U}$ denote the domain of volume U occupied by the porous medium, $\mathscr{U}_s$ the domain of volume U_s occupied by the solid matrix and $\mathscr{U}_v$ the domain of volume U_v occupied by the pores (which is filled by gases and fluids) :

$$\mathscr{U} = \mathscr{U}_s \cup \mathscr{U}_v. \tag{1.1.1}$$

The volume filled by the fluid is denoted by U_f, and its domain is denoted by $\mathscr{U}_f$. The fluid occupies a part of the pores, $\mathscr{U}_f \subseteq \mathscr{U}_v$. In the case of a saturated medium, the

domains are totally and exactly overlapping :

$$\mathscr{U}_f = \mathscr{U}_v. \tag{1.1.2}$$

Subscript *s* labels the solid matrix, while *v* stands for the void and f for the fluid.

In our first developments, it is important to be able to accurately describe the microscopic medium. The definition of the characteristic function of the pores is then useful, $\underline{x}$ being the current position :

$$\chi_v(\underline{x}) = \begin{cases} 1 & \text{if } \underline{x} \in \mathscr{U}_v \text{ (= lies in the pores);} \\ 0 & \text{if } \underline{x} \in \mathscr{U}_s \text{ (= lies in the matrix).} \end{cases} \tag{1.1.3}$$

Considering a multi-phase medium, a characteristic function can be defined for each phase α :

$$\chi_\alpha(\underline{x}) = \begin{cases} 1 & \text{if } \underline{x} \in \mathscr{U}_\alpha \text{ (= lies in phase } \alpha); \\ 0 & \text{if } \underline{x} \in \mathscr{U} / \mathscr{U}_\alpha \text{ (= lies elsewhere).} \end{cases} \tag{1.1.4}$$

After determination of the appropriate REV, the characteristic function enables us to define different kinds of averages. For this purpose, let $\mathscr{U}_0$ be the REV spatial domain of volume U_0. Besides, it is also convenient to differentiate macroscopic coordinates, noted as $\underline{x}$ and microscopic coordinates, noted as $\underline{x}'$. For any field *B*, we can define :

- the **volume average** of *B* :

$$\langle B(\underline{x},t)\rangle = \frac{1}{U_0(\underline{x},t)} \int_{\mathscr{U}_0(\underline{x},t)} B(\underline{x}',t)\, dU(\underline{x}'), \tag{1.1.5}$$

- the **volumetric phase average** of *B* for the phase α :

$$\begin{array}{ll} \langle B_\alpha(\underline{x},t)\rangle & = \frac{1}{U_0(\underline{x},t)} \int_{\mathscr{U}_0(\underline{x},t)} B(\underline{x}',t)\, \chi_\alpha(\underline{x}',t)\, dU(\underline{x}') \\ & \equiv \frac{1}{U_0(\underline{x},t)} \int_{\mathscr{U}_{0\alpha}(\underline{x},t)} B(\underline{x}',t)\, dU_\alpha(\underline{x}'), \end{array} \tag{1.1.6}$$

- the **volumetric intrinsic phase average** of *B* :

$$\begin{array}{ll} \langle B_\alpha(\underline{x},t)\rangle^\alpha & = \frac{1}{U_{0\alpha}(\underline{x},t)} \int_{\mathscr{U}_{0\alpha}(\underline{x},t)} B(\underline{x}',t)\, \chi_\alpha(\underline{x}',t)\, dU(\underline{x}') \\ & \equiv \frac{1}{U_{0\alpha}(\underline{x},t)} \int_{\mathscr{U}_{0\alpha}(\underline{x},t)} B(\underline{x}',t)\, dU_\alpha(\underline{x}') . \end{array} \tag{1.1.7}$$

Attention must be paid to the spatial domain of $\underline{x}'$, $\mathscr{U}_{0(\alpha)}(\underline{x},t)$. This indicates that the microscopic point $\underline{x}'$ belongs at time *t* to (the α-phase of) the REV centered on the

macroscopic point $\underline{x}$ (Fig. 1.3). The characteristic function of the α-phase, noted χ_α, enables us to only take into account the points lying in the considered phase. Besides, the average may be calculated on the total REV or only on the REV part belonging to α-phase. The respective meanings of these averages will appear in the following lines. Finally, it is important to note that all averages are exclusively function of the time and the macroscopic position vector. This means that the averaging processes bring forth **macroscopic objects from microscopic objects**. For the sake of simplifying the notations, the field dependence with respect to $\underline{x}$, $\underline{x}'$ and t are omitted in the following developments.

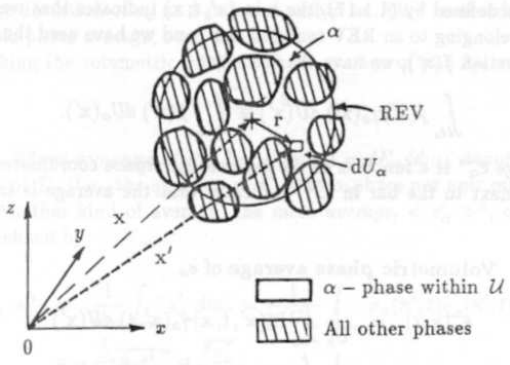

Figure 1.3: Definition sketch for volume averaging. Difference between microscopic and macroscopic coordinates (1).

In order to illustrate the meaning of the different definitions, the particular case of mass density is considered :

- the **mass density volume average**

$$\langle \rho \rangle = \frac{1}{U_0} \int_{\mathscr{U}_0} \rho \, dU = \frac{M_{tot}}{U_0}, \tag{1.1.8}$$

  represents the **apparent mass density** of the porous medium ;

- the **mass density volumetric phase average** for phase α

$$\langle \rho_\alpha \rangle = \frac{1}{U_0} \int_{\mathscr{U}_0} \rho \, \chi_\alpha \, dU = \frac{M_\alpha}{U_0}, \tag{1.1.9}$$

  is the **apparent mass density of the α-phase** ;

- the **volumetric intrinsic phase average of the mass density** for phase α

$$\langle \rho_\alpha \rangle^\alpha = \frac{1}{U_{0\alpha}} \int_{\mathscr{U}_{0\alpha}} \rho \ \chi_\alpha \ dU = \frac{M_\alpha}{U_{0\alpha}}, \tag{1.1.10}$$

  is the **mass density mean average value of the α-phase**.

Another important case to consider is the volumetric average of characteristic phase functions. With the characteristic **void** function, the local **porosity** can be defined :

$$\langle \chi_v \rangle = \frac{1}{U_0} \int_{\mathscr{U}_0} \chi_v \ dU = \frac{U_{0v}}{U_0} = \varepsilon_v. \tag{1.1.11}$$

It represents the local ratio of the pore volume to the total volume (pores and matrix). The porosity thus indicates the liquid volume fraction inside a saturated medium. Such a ratio can be determined for any α-phase and is called fraction volume of this phase. It is noted ε_α and defined as :

$$\langle \chi_\alpha \rangle = \frac{1}{U_0} \int_{\mathscr{U}_0} \chi_\alpha \ dU = \frac{U_{0\alpha}}{U_0} = \varepsilon_\alpha. \tag{1.1.12}$$

In particular, in hydrology, this concept defines the humidity $\theta_w = U_{0w}/U_0$ as the ratio between the water volume and the porous medium volume containing this fluid. Finally, let us note that this definition provides an obvious link between phase and intrinsic phase averages :

$$\langle B_\alpha \rangle = \varepsilon_\alpha \ \langle B_\alpha \rangle^\alpha \ . \tag{1.1.13}$$

1.2 Averaging process and mathematical results

This process consists in averaging the microscopic equations in order to obtain the macroscopic equations. The unknowns are the **macroscopic variables**. The advantage of this approach is to provide a clear understanding of the microscopic causes of the macroscopic effects. Another method would consist in directly work with macroscopic variables. The risk is to only rely on intuition and to forget some microscopic phenomena which could however result in non-negligible macroscopic effects. Because of the use of precise mathematical tools, no microscopic effects will be forgotten with the averaging method.

Before developing a porous medium model, let us start by presenting some useful mathematical propositions :

Gradient theorem : let B be a scalar field on a volume U, S_{vs} be the interface inside this volume between the pores and the matrix, and $\underline{n}_{vs}$ stand for the unit normal from pores to matrix. Then, the averaged gradient can be expressed as a function of the mean value gradient :

$$\langle \underline{\nabla} B_\alpha \rangle = \underline{\nabla} \langle B_\alpha \rangle + \frac{1}{U} \int_{S_{vs}} B_\alpha \, \underline{n}_{vs} \, dS. \tag{1.2.14}$$

Demonstrations can be found in (8; 11). Extensions also deal with a first- or second-order tensor field $\underline{\underline{S}}$:

$$\langle \underline{\nabla} \cdot \underline{\underline{S}}_\alpha \rangle = \underline{\nabla} \cdot \langle \underline{\underline{S}}_\alpha \rangle + \frac{1}{U} \int_{S_{vs}} \underline{\underline{S}}_\alpha \cdot \underline{n}_{vs} \, dS. \tag{1.2.15}$$

Average phase gradient decomposition : let $S_{\alpha s}$ be the interfacial area between the α-phase and the matrix and $\underline{n}_{\alpha s}$ denote its normal from the phase α to the matrix. The average α-phase gradient of the field B can be rewritten as :

$$\underline{\nabla} \langle B_\alpha \rangle = \varepsilon_\alpha \underline{\nabla} \langle B_\alpha \rangle^\alpha + \langle B_\alpha \rangle^\alpha \underline{\nabla} \varepsilon_\alpha \ . \tag{1.2.16}$$

This property is easily retrieved by using equation (1.1.13) and the product derivation rule.

Porosity gradient : finally, the porosity gradient can be rewritten as a function of microscopic details :

$$\underline{\nabla} \varepsilon_v = -\frac{1}{U} \int_{S_{vs}} \underline{n}_{vs} \, dS \ . \tag{1.2.17}$$

It is demonstrated in (8) by applying the gradient theorem to the pore characteristic function.

Note that the first and the third result can be applied between the matrix and the fluid in case of saturated medium. 'pore' is then replaced by 'fluid' and 'v' by 'f' in the formulas.

A more precise framework is needed to develop the model. Therefore, our main hypotheses are developed in the following subsection.

1.3 Basic hypotheses

In a first step, the focus is put on the **flow through the medium** and the equations are developed from an Eulerian viewpoint. In some applications, Solid Mechanics will nevertheless be investigated to establish a non-linear elastic model of porous media. Our basic assumptions mostly pertain to Fluid Mechanics. They are listed herebelow :

(i) the solid phase is stationary or at least a quasi-static behaviour is assumed ;

(ii) there is no mass exchange between the solid and the fluid ;

(iii) the solid and (assumed incompressible) fluid have constant and uniform density;

(iv) the fluid viscosity is independent of shear rate (the fluid is Newtonian) ;

(v) the moving front is sharply defined; ahead of the front there is no fluid, and behind the front the pores are fully saturated with fluid ;

(vi) there is no consolidation. Since the matrix can consist of fibers (as in the case of Resin Transfer Molding (RTM) or Structural Reaction Injection Molding (SRIM) for example), consolidation would concern fluid penetration inside fiber bundles. For our purpose the fiber bundles are considered as impermeable and the fluid cannot flow inside these bundles ;

(vii) the capillary action is negligible ; it can be shown that this is the case for RTM or SRIM processes (12), contrarily to what happens in case of soil filtration (13). Nevertheless, this assumption must be considered with very high attention in each case. Its formal expression involves the dimensionlesss Bond[1] and capillary[2] numbers, which relate the gravity force or the viscous force to the capillary force, respectively;

(viii) the only body force is gravity.

At this point, the physical framework is set, the mathematical tools have been presented and the model derivation can proceed. Microscopic equations are firstly recalled for each conservation law for each phase. The averaging process then follows before eventual introduction of appropriate constitutive laws in order to close the system. Boundary conditions are discussed. Attention is drawn when the basic assumptions are used while additional hypotheses pertaining to a specific model are clearly stated.

1.4 Mass conservation

The mass balance equation is also called the **continuity equation**. Two phases are taken into account, viz. the solid and the fluid phase.

[1] Bond number = $N_B = \rho g R^2/\sigma$, where ρ is the specific mass of the liquid, R is the pore radius and σ is the air-liquid surface tension.

[2] Capillary number = $N_{Ca} = \eta V/\sigma$, where η and V are the viscosity and the velocity of the liquid and σ is the air-liquid surface tension.

The first case is very simply treated in our framework. Since the solid is stationary (H.i) and its mass density is constant (H.iii), this phase automatically satisfies continuity principle.

In classical Fluids Mechanics, mass balance is written as :

$$\frac{\partial \rho_f}{\partial t} + \underline{\nabla} \cdot \left(\rho_f \underline{v}_f \right) = 0 \, , \tag{1.4.18}$$

where ρ_f denotes the fluid mass density and $\underline{v}_f$ its velocity. Let us here recall that the Fluids Mechanics scale is the microscopic scale. Applying the averaging process, the latter equation transforms into :

$$\langle \frac{\partial \rho_f}{\partial t} \rangle + \langle \underline{\nabla} \cdot \left(\rho_f \underline{v}_f \right) \rangle = 0 \, . \tag{1.4.19}$$

Since the REV is assumed not to change with time, the integration order can be inverted in the first term while in the second term, the gradient theorem is applied :

$$\frac{\partial \langle \rho_f \rangle}{\partial t} + \underline{\nabla} \cdot \langle \rho_f \underline{v}_f \rangle \; + \; \frac{1}{V} \int_{S_{fs}} \rho_f \underline{v}_f \cdot \underline{n}_{fs} \, dS = 0 \, . \tag{1.4.20}$$

As there is no consolidation (H.vi) and the fluid is Newtonian (H.iv), its velocity vanishes on the interfacial area. The last term is then equal to zero. Finally, since the density is constant (H.iii), it can be factored out from the remaining terms and simplified to provide the equation :

$$\frac{\partial \varepsilon_f}{\partial t} + \underline{\nabla} \cdot \langle \underline{v}_f \rangle \; = 0 \, . \tag{1.4.21}$$

In a saturated medium (H.v) with no consolidation (H.vi), the first term is equal to zero and the equation finally writes as :

$$\boxed{\underline{\nabla} \cdot \langle \underline{v}_f \rangle \; = 0} \, . \tag{1.4.22}$$

This equation has the same form as the classical fluid incompressibility equation. A difference lies nevertheless in the fact that Eq. (1.4.22) applies on the **phase average** velocity. This equation (1.4.22) is the final **macroscopic** equation for mass balance. It expresses a relationship between macroscopic variables (here the single Darcy velocity, $\langle \underline{v}_f \rangle$).

At this point it is important to recall the physical meaning of averaged velocities. For this purpose, the unidirectional flow represented on Fig.1.4 is considered.

Definitions (1.1.6) and (1.1.7) are then applied on the entire volume U_0 :

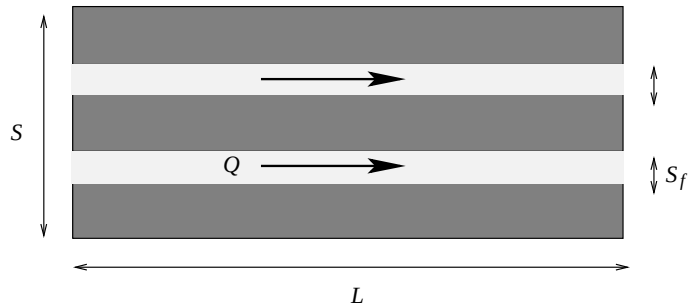

Figure 1.4: Meaning of the phase averaged and the intrinsic phase averaged velocities ; v_r is defined such that $v_r S_f = Q$, S_f being the surface crossed by the fluid and Q the flow rate $[\mathrm{m}^3/\mathrm{s}]$; $U_0 = SL$ is the total volume.

- The **velocity phase average**

$$\langle v_f \rangle = \frac{1}{U_0} \int_{\mathscr{U}_0} v\, \chi_f \, dU = \frac{v_r S_f L}{SL} = v_r \, \frac{S_f}{S} = v_r \varepsilon_f, \tag{1.4.23}$$

  is the **filtration velocity** or the so called **Darcy velocity**. Multiplying $\langle v_f \rangle$ by the real area S, the flow rate Q is retrieved.

- The **velocity intrinsic phase average**

$$\langle v_f \rangle^f = \frac{1}{U_f} \int_{\mathscr{U}_0} v\, \chi_f \, dU = \frac{v_r S_f L}{S_f L} = v_r, \tag{1.4.24}$$

  is the **mean real velocity**. In order to retrieve the flow rate, this velocity has to be multiplied by the **effective area** S_f (the area crossed by the fluid). Note that relation (1.1.13) is also retrieved.

Determination of the effective area S_f is usually not an easy task, contrarily to the total surface S. This clearly shows that the Darcy velocity is easier to measure experimentally and should better be considered as a primary variable in the model. Then intrinsic phase averaged velocity is obtained afterwards by multiplying Darcy velocity by porosity.

1.5 Momentum balance

The classical local momentum equation is written as :

$$\rho\frac{\partial \underline{v}}{\partial t}+\rho\underline{v}\cdot\underline{\nabla}\underline{v}=\underline{\nabla}\cdot\underline{\underline{\sigma}}+\rho\underline{g}, \tag{1.5.25}$$

where $\underline{g}$ stands for the body force and $\underline{\underline{\sigma}}$ is the stress tensor. With the use of the continuity equation, this equation transforms into :

$$\frac{\partial(\rho\underline{v})}{\partial t}+\underline{\nabla}\cdot(\rho\underline{v}\underline{v})=\underline{\nabla}\cdot\underline{\underline{\sigma}}+\rho\underline{g}. \tag{1.5.26}$$

This equation represents the local microscopic expression of the momentum balance principle. It is valid for the two phases. The liquid is firstly considered by taking the phase average of Eq.(1.5.26) :

$$\langle\frac{\partial\rho_f\underline{v}_f}{\partial t}\rangle+\langle\underline{\nabla}\cdot\rho_f\underline{v}_f\underline{v}_f\rangle=\langle\underline{\nabla}\cdot\underline{\underline{\sigma}}_f\rangle+\langle\rho_f\underline{g}\rangle \tag{1.5.27}$$

Considering a constant mass density (H.iii), a stationary solid phase (H.i), and applying the gradient theorem and inverting the integration order, the latter equation becomes :

$$\rho_f\frac{\partial\langle\underline{v}_f\rangle}{\partial t}+\rho_f\langle\underline{\nabla}\cdot(\underline{v}_f\underline{v}_f)\rangle=\underline{\nabla}\cdot\langle\underline{\underline{\sigma}}_f\rangle+\frac{1}{V}\int_{S_{fs}}\underline{\underline{\sigma}}_f\cdot\underline{n}_{fs}\,dS+\langle\rho_f\underline{g}\rangle. \tag{1.5.28}$$

The stress tensor can be rewritten by isolating a isotropic contribution p_f and an extra-stress tensor $\underline{\underline{\tau}}_f$:

$$\underline{\underline{\sigma}}_f=-p_f\underline{\underline{\delta}}+\underline{\underline{\tau}}_f. \tag{1.5.29}$$

In this study, the body force is solely due to the gravity (H.viii) and hence derives from a potential :

$$\underline{g}=-\underline{\nabla}(gh). \tag{1.5.30}$$

With these assumptions and again applying the gradient theorem, the averaged momentum equation becomes :

$$\begin{aligned}\rho_f\tfrac{\partial\langle\underline{v}_f\rangle}{\partial t}+\rho_f\langle\underline{\nabla}\cdot(\underline{v}_f\underline{v}_f)\rangle=\quad&\underline{\nabla}\cdot\langle\tau_f\rangle-\underline{\nabla}\langle p_f+\rho_f gh\rangle+\tfrac{1}{V}\textstyle\int_{S_{fs}}\underline{\underline{\sigma}}_f\cdot\underline{n}_{fs}\,dS\\&-\tfrac{1}{V}\textstyle\int_{S_{fs}}\rho_f gh\,\underline{n}_{fs}\,dS\,.\end{aligned} \tag{1.5.31}$$

In the second term of the right-hand side (r.h.s.), $p_f+\rho_f gh$ is usually called the **modified pressure** and is noted as P_f. Pressure being an intensive variable, it is clear that during an experiment the measured pressure is usually the **intrinsic** pressure phase

average $\langle p_f \rangle^f$, also called the **pore pressure**,. Eq. (1.1.13) is used in order to introduce the intrinsic variable $\langle P_f \rangle^f$ by decomposing the phase averaged gradient. With the porosity gradient law Eq. (1.2.17), we finally get :

$$\rho_f \frac{\partial \langle \underline{v}_f \rangle}{\partial t} + \rho_f \langle \nabla \cdot (\underline{v}_f \underline{v}_f) \rangle = \quad -\varepsilon_f \nabla \langle P_f \rangle^f + \nabla \cdot \langle \tau_f \rangle$$

$$\underbrace{\underbrace{+\frac{1}{V}\int_{S_{fs}} \underline{\sigma}_f \cdot \underline{n}_{fs}\, dS}_{-\underline{f}_T} \underbrace{-\frac{1}{V}\int_{S_{fs}} \rho_f g h\, \underline{n}_{fs}\, dS}_{-\underline{f}_{gf}} \underbrace{+\langle P_f \rangle^f \frac{1}{V}\int_{S_{fs}} \underline{n}_{fs}\, dS}_{\underline{f}_P}}_{-\underline{f}_d} \tag{1.5.32}$$

The last term collects all the interfacial interactions between solid and fluid. Therefore $\underline{f}_d$ represents the **drag force** of the fluid acting on the solid along their common interface S_{fs}. Several microscopic features contribute to this macroscopic effect : (i) the stress distribution $\underline{f}_T$, (ii) the solid contribution to the fluid buoyancy $\underline{f}_{gf}$, and (iii) the effect of the porosity gradient $\underline{f}_P$ (Fig. 1.5). All these terms represent the effects of the different acting forces : viscous stress, gravity and pressure.

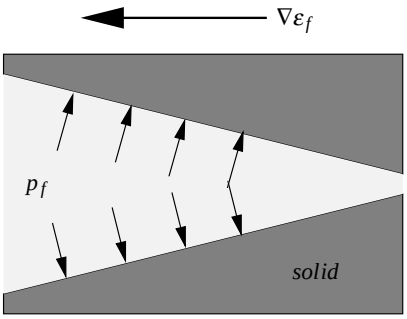

Figure 1.5: Example of a porous medium with a fluid volume fraction gradient. The hydrostatic pressure in the fluid generates a force on the internal surface which acts in the direction opposite to $\nabla \varepsilon_f = -\frac{1}{V}\int_{S_{fs}} n_{fs}\, dS$.

Equation (1.5.32) includes all the macroscopic effects. Except the first term in each side, the equation must nevertheless be transformed in such a way that each of its terms is expressed with primary variables. From the last developments, it becomes clear that these primary variables will be selected as the phase velocity $\langle v_f \rangle$ and the intrinsic modified phase pressure $\langle P_f \rangle^f$.

1.5.1 Inertial term

The so-called inertial term is the second one of the left-hand side (l.h.s.) of Eq. (1.5.32), $\rho_f \langle \underline{\nabla} \cdot \underline{v}_f \underline{v}_f \rangle$. It represents the macroscopic transport of momentum through the medium. Defining here the pointwise deviation of $\underline{v}_f$ from its intrinsic average value as :

$$\hat{\underline{v}}_f = \underline{v}_f - \langle \underline{v}_f \rangle^f, \tag{1.5.33}$$

it can be easily shown that, for a saturated porous medium, the averaged product $\langle \underline{v}_f \underline{v}_f \rangle$ can be rewritten as :

$$\langle \underline{v}_f \underline{v}_f \rangle = \frac{1}{\varepsilon_f} \langle \underline{v}_f \rangle \langle \underline{v}_f \rangle + \langle \hat{\underline{v}}_f \hat{\underline{v}}_f \rangle. \tag{1.5.34}$$

Applying the gradient theorem, the inertial term then becomes :

$$\rho_f \langle \underline{\nabla} \cdot (\underline{v}_f \underline{v}_f) \rangle = \rho_f \underline{\nabla} \cdot \left[\frac{1}{\varepsilon_f} \langle \underline{v}_f \rangle \langle \underline{v}_f \rangle + \langle \hat{\underline{v}}_f \hat{\underline{v}}_f \rangle \right] + \rho_f \int_{S_{fs}} \underline{v}_f \underline{v}_f \cdot \underline{n}_{fs} \, dS. \tag{1.5.35}$$

As previously, the last term vanishes because the fluid is viscous (H.iv) and there is no consolidation (H.vi). The first term is called the **bulk inertia term** and represents the convection of momentum by the average velocity. The second term is called **inertial dispersion** term. It takes into account the momentum transport due to microscopic velocity fluctuations around the average velocity. The average velocity indeed does not capture the whole momentum transport effect. There are in addition microscopic velocity fluctuations which physically transport momentum and results in a macroscopic mixing effect, which is proportional to the velocity norm, as it will be seen later. This effect is analogous to the Reynolds stress in turbulence. However, in the present case, momentum mixing takes place even in laminar flows and is due to the heterogeneous and geometric character of the porous medium. These remarks suggest to express momentum mixing in the following form :

$$\rho_f \underline{\nabla} \cdot \langle \hat{\underline{v}}_f \hat{\underline{v}}_f \rangle = \rho_f \underline{\nabla} \cdot \left[\underline{\underline{M}}^D : \left(\underline{\nabla} \langle \underline{v}_f \rangle + \underline{\nabla} \langle \underline{v}_f \rangle^T \right) \right], \tag{1.5.36}$$

where subscript T indicates the transpose operator and where $\underline{\underline{M}}^D$ is a fourth order tensor, which is a function of $\langle \underline{v}_f \rangle$ and mostly depends on the medium geometric properties. The use of a fourth order tensor should enable us to take into account the geometry and the velocity dependence of momentum mixing from a tensorial view point. This issue is further adressed in Section 1.6.2.

Finally expression (1.5.36) leads us to considering inertial dispersion as an additional viscosity effect. This approach is in agreement with Brinkman's theory (Section 1.5.4) and observations where an **apparent viscosity** seems to act in fluids flowing in porous media (14).

1.5.2 Stress term

In the decomposition of Eq. (1.5.29), the extra-stress tensor $\underline{\underline{\tau}}_f$ must be developed by means of an appropriate constitutive equation. Considering a Newtonian fluid (H.iv), we have :

$$\underline{\underline{\tau}}_f = \eta \left(\underline{\nabla} \underline{v}_f + \underline{\nabla} \underline{v}_f^T \right). \tag{1.5.37}$$

In order to develop Eq. (1.5.32), the velocity gradient phase average is taken :

$$\langle \underline{\nabla} \underline{v}_f \rangle = \underline{\nabla} \langle \underline{v}_f \rangle + \frac{1}{V} \int_{S_{fs}} \underline{v}_f \, \underline{n}_{fs} \, dS. \tag{1.5.38}$$

As previously, the last term is equal to zero. With use of Eq. (1.5.37), the extra-stress term in Eq. (1.5.32) can be rewritten as a relation between the primary variables :

$$\underline{\nabla} \cdot \langle \tau_f \rangle = \underline{\nabla} \cdot \left[\eta \left(\underline{\nabla} \langle \underline{v}_f \rangle + \underline{\nabla} \langle \underline{v}_f \rangle^T \right) \right] \tag{1.5.39}$$

where η is the dynamic viscosity (in $[\mathrm{kg/ms}]$). If the viscosity is constant, the extra-term relative to the gradient transpose plays no role and can be eliminated, as in the Navier-Stokes equations. RTM and SRIM processes are studied in followings sections. In these processes, the resin experiences non negligible viscosity changes during curing. Therefore, the complete Eq. (1.5.39) is considered and the gradient transpose is kept in the final formulation as the viscosity is kept inside the divergence expression, its value being discussed later.

1.5.3 Drag force. Darcy's and Forchheimer's models

This theory has not been yet the object of a general agreement in the scientific community. Therefore, different models exist and will be discussed, among which best-known are Darcy's and Forchheimer's models. The isotropic and anisotropic cases will be addressed.

The objective of this subsection is to propose different models for the drag force $\underline{f}_d$, as based on specific assumptions. The physical effects acting in this force have been previously presented and are used in order to construct appropriate constitutive equations which solely include the parameters and primary variables of the problem. Some porous medium characteristic fields (mainly the permeability tensor) are also introduced.

The first attempts to establish a flow law in porous media were simply performed by experimental fitting. Explanations were further searched in order to understand the

physics of this behaviour. The present approach is close to this latter approach. Some very simple assumptions are proposed and theory is developed. It is shown that the two basic Darcy and Forchheimer laws can be retrieved on this basis.

An interesting historical report on this issue has been given by Lage (15), who describes how these theories were in fact due to several improvements brought across the time by several researchers (Hazen, Poisseuille, Kozeny, Dupuit...). For example, the original Darcy theory included no viscosity dependence and is slightly different from what is nowadays his name.

Darcy's model

The first flow law in porous media was proposed by Darcy, then *Inspecteur Général des Ponts et Chaussées*, who related his analysis in a report on the public fountains of Dijon (1856). An annexe of this report described the filtering of water through a packed sand column. From his experiments Darcy deduced the following law :

$$k = -U\frac{e}{\Delta p} \tag{1.5.40}$$

where U stands for the fluid average speed, Δp for the total pressure difference across the sand, e for the height of the sand layer and k is defined as the **hydraulic conductivity**.

Hazen then observed a temperature dependence of the hydraulic conductivity in 1893. Subsequently, Kozeny showed that it is in fact the thermal dependence of the viscosity which is hidden behind these observations (1927). The Darcy law for isotropic media was then written in the following form :

$$U = -\frac{K}{\eta}\frac{\Delta p}{e} \tag{1.5.41}$$

where K is a specific porous medium parameter, which is called **permeability** $[\mathrm{m}^2]$.

Nevertheless these equations are global in the sense that they take into account the entire medium. No local law, in the macroscopic sense, is here proposed. Moreover these laws only deal with isotropic media.

In order to average equations, let us turn back our attention to macroscopic developments and the expression of f_d in Eq. (1.5.32). We firstly focus on finding the drag force f_d for isotropic media. We will assume that this force is due to (a) the velocity

difference between the fluid and solid phase, (b) the hypothesis of negligible inertial effects. With these assumptions, the following expression is proposed :

$$\underline{f_d} = R(\eta, l_0)[\langle \underline{v}_f \rangle^f - \langle \underline{v}_s \rangle^s] = R(\eta, l_0)\langle \underline{v}_f \rangle^f, \tag{1.5.42}$$

since the solid is rigid. Dimensional analysis provides the expression $R = \eta / k' l_0^2$ where l_0 stands for a local length scale of the porous medium (typically the pore average diameter) and k' is a dimensionless constant. As inertial effects are negligible, no density dependence appears in the expression of R. Defining the permeability K as

$$K = \varepsilon_f^2 l_0^2 k', \tag{1.5.43}$$

Eq. (1.5.42) becomes :

$$\underline{f_d} = \varepsilon_f^2 \frac{\eta}{K} \langle \underline{v}_f \rangle^f = \varepsilon_f \frac{\eta}{K} \langle \underline{v}_f \rangle. \tag{1.5.44}$$

At this point, it is very important to remark that the above law sets the solid and fluid effects apart via the permeability and viscosity respectively, indeed these properties are exclusively function of their respective phase. Besides the knowledge of permeability allows us to define a local porous medium scale l_0 since Eq. (1.5.43) gives :

$$l_0 \approx \sqrt{K}\,. \tag{1.5.45}$$

Finally, the expression (1.5.44) can be inserted in Eq. (1.5.32). If inertial effect are negligible (b), the l.h.s. is neglected and if the viscous stress divergence (the so-called bulk viscosity effect) is also neglected, the following equation is finally obtained :

$$0 = -\varepsilon_f \underline{\nabla} \langle P_f \rangle^f + 0 - \varepsilon_f \frac{\eta}{K} \langle \underline{v}_f \rangle. \tag{1.5.46}$$

Algebraic manipulations finally give :

$$\langle \underline{v}_f \rangle = -\frac{K}{\eta} \underline{\nabla} \langle P_f \rangle^f. \tag{1.5.47}$$

This equation is similar to Eq. (1.5.41), the main difference lying in its local form. Therefore classical mathematical tools can be used.

The meaning of this law can be understood by a comparison with Poiseuille's law, which indeed establishes a link between the pressure gradient ∇p and the flow rate Q in a rectilinear pipe ($Q = -\frac{\pi R^4}{8\eta} \nabla p$) and expresses the effects of the fluid viscosity and the geometry on the flow rate. Similarly, expression (1.5.47) also involves viscous effects (via η) and geometry influence (via K).

Generalization to the non-isotropic cases is very easy under the same assumptions as before, expect isotropy. Similar arguments provide the expression :

$$\underline{f_d} = \frac{\eta}{k' l_0^2} \underline{\underline{r}} \cdot [\langle \underline{v}_f \rangle^f - \langle \underline{v}_s \rangle^s] = \frac{\eta}{k' l_0^2} \underline{\underline{r}} \cdot \langle \underline{v}_f \rangle^f, \tag{1.5.48}$$

where $\underline{\underline{r}}$ is a dimensionless second-order resistance tensor. The second-order permeability tensor is then defined as :

$$\underline{\underline{K}} = \varepsilon_f^2 k' l_0^2 \, \underline{\underline{r}}^{-1}, \tag{1.5.49}$$

with the same meaning as before except that the medium anisotropy is taken into account. Since a vanishing fluid-solid drag implies a vanishing fluid velocity, this tensor is invertible. Moreover it is commonly assumed to be symmetric. This property has been demonstrated as being valid for an **orthotropic medium** but not for any porous medium (8). Nevertheless none of the equations which follow rely on $\underline{\underline{K}}$ being symmetric.

Finally, the non-isotropic expression of Eq. (1.5.47) gives the generalized Darcy equation in the form :

$$\boxed{\langle \underline{v}_f \rangle = -\tfrac{1}{\eta} \underline{\underline{K}} \cdot \underline{\nabla} \langle P_f \rangle^f} \, . \tag{1.5.50}$$

This equation only contains primary variables and represents the momentum balance when **inertial and bulk viscous effects are neglected**. More precisely, neglecting the bulk viscous effects means that the divergence of the averaged viscous stress is negligible in comparison with the viscous drag.

Forchheimer's model

Analyzing Darcy's equation in the form (1.5.46) shows that the pressure gradient can be seen as the driving force while the viscous term is the drag force (as opposed to fluid motion). From a synthetic view point, the Forchheimer's model consists in introducing an additional term into the drag force :

$$\underline{f}_d = \underbrace{\varepsilon_f \frac{\eta}{K} \langle \underline{v}_f \rangle}_{\textit{viscous drag}} + \underbrace{\varepsilon_f C \rho \, \langle \underline{v}_f \rangle \, \| \langle \underline{v}_f \rangle \|}_{\textit{form drag}} . \tag{1.5.51}$$

As previously explained, the first term results from fluid friction on the solid boundary. The second term expresses the form effect of the solid phase on the flow. Hence, the physical phenomenon responsible for the quadratic term in the Forchheimer equation is the **form stress** exerted on the fluid by any solid surface obstructing the flow path. The parameter C [m^{-1}] is related to the geometry of the solid permeable medium.

Under the same conditions as for Eq. (1.5.46) **(inertial and bulk viscous terms neglected, except that the form effect is not negligible)**, the Forchheimer's equation

writes as :

$$\boxed{0 = -\nabla\langle P_f\rangle^f - \frac{\eta}{K}\langle \underline{v}_f\rangle - C\rho\ \langle \underline{v}_f\rangle\ \|\langle \underline{v}_f\rangle\|}\ . \tag{1.5.52}$$

This macroscopic equation is similar to the expression historically obtained by experimental fitting. As shown by Lage (15), the quadratic law was erroneously attributed to Forchheimer. The model was proposed because some observations in that time suggested that better fitting so achieved with a quadratic equation rather than with the linear Darcy law. The quadratic term was however proposed by Dupuit (1863) in the second edition of his report on the *theoretical and practical studies of water flow in uncovered channels and through permeable soil*. Hence, naming Eq. (1.5.52) as the Forchheimer equation seems to be unwarranted and historically unfair to Dupuit.

It is now important to quantify the onset of the non-linear flow regime in Forchheimer model. The ratio between the form force D_f and the viscous force D_v is physically meaningful and seems to be appropriate for this purpose :

$$\frac{D_f}{D_v} = \frac{C\rho V^2}{\frac{\eta}{K}V} = \frac{C\rho K}{\eta}V, \tag{1.5.53}$$

where a unidirectional problem with fluid velocity V is considered for the sake of simplicity. This ratio depends on the properties of both the solid and the liquid phase. The influence of the velocity is also highlighted. However this law has not received general agreement yet. Commonly, the onset of non-linear effects is defined by means of the pore sized Reynolds number :

$$Re_p = \frac{\rho V\sqrt{K}}{\eta}. \tag{1.5.54}$$

The switch to Forchheimer's regime then occurs when $Re_p \geq 0.1$. This rule is mostly based on observations on flows experimentally performed in media with particles of similar shape, but has no physical meaning related to the ratio between form and viscous effects. It can besides be shown that when porous media with particles of similar shape are used, the form coefficient C is relatively uniform and of the order of $\mathcal{O}\left(K^{-1/2}\right)$ (15). Re_p is then proportional to the ratio (1.5.53) and it appears that the Reynolds based rule is rather coincidental to determine the onset of non-linear effects while a physical rule based on (1.5.53) with an appropriate factor should be more correct.

Eq. (1.5.52) is developed for isotropic media. There are very few theoretical developments relating to the extension of Forchheimer's law to the anisotropic case. Nevertheless Knupp et al. (16) have constructed a law based on a variational principle. The authors minimize an expression similar to the power required to maintain steady

flow. Anisotropy results from the presence of both the permeability ($\underline{\underline{K}}$) and form ($\underline{\underline{C}}$) tensors. Imagine indeed a geometrically isotropic medium made of identical spheres with a small patch of sand roughness attached to their surfaces. As the viscous drag will be isotropic (permeability), the presence of the patch will induce flow separation at different surface locations for different flow directions. The model proposed is then roughly :

$$0 = -\underline{\nabla}\langle P_f\rangle^f - \eta\left[1+\frac{\rho\Gamma}{\eta}\kappa^{3/2}\left(\langle \underline{v}_f\rangle\cdot\underline{\underline{K}}^{-1}\langle \underline{v}_f\rangle\right)^{1/2}\right]\underline{\underline{K}}^{-1}\langle \underline{v}_f\rangle \tag{1.5.55}$$

with

$$\kappa = (\det\underline{\underline{K}})^{1/3} \text{ et } \Gamma = (\det\underline{\underline{C}})^{1/3}. \tag{1.5.56}$$

Although, the anisotropy due to the form effect seems to be taken into account by the presence of the form tensor $\underline{\underline{C}}$, the determinant (det) calculus acts as an averaging process and seems to eliminate the directional effects. Moreover this theoretical law has not yet received an experimental validation. Despite this, the reliable variational principle on which it is based encourages further developments of this theory.

In this subsection the focus was mainly given to providing a physical meaning. Nevertheless the interested reader will find all the required mathematical details in (1; 17; 18; 19). In these references averaging procedure has indeed been carried out to obtain macroscopic equations containing the quadratic Forchheimer term.

Other models

From our investigations, it turns out that general agreement on the momentum law is not reached yet in the literature. For example some authors propose a cubic head loss law at low flow rate (20; 21). No physical argument leads to this law, which is obtained either by experimental fitting (21) or by a homogenization technique (20). Besides this, at high velocity, the Forchheimer regime has also been discussed in (22) where a cubic law is proposed for high flow rates too. Nevertheless, the same author abandons this idea in the same article and it will no longer be discussed here. In order to gather all these informations, the best in our mind is thus to construct a function exhibiting a cubic behaviour at low velocity and a quadratic behaviour at high flow rate. Therefore, we propose the following complete model :

$$0 = -\underline{\nabla}\langle P_f\rangle^f - \frac{\eta}{K}\langle \underline{v}_f\rangle\left(1+\left(\frac{\rho KC}{\eta}\|\langle \underline{v}_f\rangle\|\right)^2\right)^{1/2}. \tag{1.5.57}$$

It can indeed be observed that :

$$\begin{aligned}
&\text{- at low flow rate, } \|\langle \underline{v}_f \rangle\| \ll 1 : \quad 0 \approx -\underline{\nabla}\langle P_f \rangle^f - \frac{\eta}{K}\,\langle \underline{v}_f \rangle \left(1 + \frac{1}{2}\left(\frac{K\rho C}{\eta}\|\langle \underline{v}_f \rangle\|\right)^2\right) \\
&\text{- at high flow rate, } \|\langle \underline{v}_f \rangle\| \gg 1 : \quad 0 \approx -\underline{\nabla}\langle P_f \rangle^f - \rho C\,\langle \underline{v}_f \rangle \|\langle \underline{v}_f \rangle\|
\end{aligned} \tag{1.5.58}$$

These asymptotic expressions indicate (i) that the typical order of magnitude of the cubic coefficient is $\mathscr{O}\left(\rho^2/\eta\right)$ for a porous medium with particules of similar shape (which corroborates the results of (20)) and (ii) that the quadratic coefficient is as in Forchheimer's model. Hence, the asymptotic behaviour of model (1.5.57) respects both extreme observations. Moreover it eliminates the presence of the norm operator which is not analytical.

However, it should be noted that the complete model considers isotropic media only. In addition, it is a purely mathematical model, without any physical meaning. Finally, it should be kept in mind that few agreements has been observed on the cubic loss of head law.

Hence, in this work, we will focus on the Darcy and Forchheimer equations only. The chapter of Lage (15) seems to bring some understanding in these equations by gathering all existing informations and justifying the choice of these laws from physically correct assumptions.

The Darcy and Forchheimer models have the drawback to consider only slip boundary conditions at the border. The mathematical equation type indeed requires a single pressure or normal velocity component to be imposed on the entire domain boundary in order to provide a well-posed problem. Nevertheless, in problems where an interfacial treatment is important, it may be crucial to impose both the normal and tangential velocity components on determined boundaries. Hence, a last interesting simplified model has to be examined before summarizing the fluid phase equation. This model, which is called Brinkman's model, is able to treat no-slip boundary conditions.

1.5.4 Brinkman's model

When **the form force is negligible and when bulk viscosity effects are important with respect to inertial term**, the Brinkman equation is applicable and is written as :

$$0 = \varepsilon_f \underline{\nabla}\langle P_f \rangle^f + \varepsilon_f \eta \underline{\underline{K}}^{-1} \cdot \langle \underline{v}_f \rangle + \underbrace{\underline{\nabla} \cdot \left[\hat{\eta}\left(\underline{\nabla}\langle \underline{v}_f \rangle + \underline{\nabla}\langle \underline{v}_f \rangle^T\right)\right]}_{\textit{Brinkman term}}, \tag{1.5.59}$$

where $\hat{\eta}$ is the **modified viscosity**. Several authors consider that this viscosity is the same as the real viscosity. However, on the one hand, some experimental results have

shown that the "fitting" viscosity is in fact different from the molecular viscosity (14) ($\hat{\eta} = 7.5\eta$). On the other hand, previous developments on the inertial term have highlighted the existence of a so-called **inertial dispersion** which could be responsible for the observed increased viscosity. All this effect can be collected in a fourth-order tensor $\underline{\underline{M}}^D$ which takes into account the geometry of the porous medium and the velocity influence. Let us yet note that Nield has shown that for highly porous media ($\varepsilon_f \geq 0.6$) this effect is negligible (23).

Another important observation is that the presence of divergence term gives to equation (1.5.59) the adequate mathematical type in such a way that no-slip boundary conditions can be imposed at the border. The equation type indeed requires that stress or velocity be imposed on the domain boundary. This results in the existence of a boundary layer whose thickness is determined by letting the drag force be of the same order of magnitude as the Brinkman term, whose order is $\mathscr{O}\left((\hat{\eta}K/\varepsilon_f\eta)^{1/2}\right)$. The latter expression is usually very small since for common porous materials the permeability is about $10^{-5}\text{m}^2 \sim 10^{-8}\text{m}^2$ (23) and since $\hat{\eta}/\eta$ is moderately high (14). Hence, the influence of this parameter could be very local in the flow solution. Nevertheless, it may have a higher influence on heat transfer.

1.5.5 General equations for the fluid phase

Collecting all the previous informations, a general law can be written for the fluid phase in the isotropic case :

$$\boxed{\begin{aligned}\rho_f \tfrac{\partial \langle \underline{v}_f \rangle}{\partial t} + \rho_f \underline{\nabla} \cdot \left[\tfrac{1}{\varepsilon_f} \langle \underline{v}_f \rangle \langle \underline{v}_f \rangle \right] = \quad & -\varepsilon_f \underline{\nabla} \langle P_f \rangle^f - \varepsilon_f \tfrac{\eta}{K} \langle \underline{v}_f \rangle - \varepsilon_f C \rho \, \langle \underline{v}_f \rangle \, \| \langle \underline{v}_f \rangle \| \\ & + \underline{\nabla} \cdot \left[\hat{\eta} \left(\underline{\nabla} \langle \underline{v}_f \rangle + \underline{\nabla} \langle \underline{v}_f \rangle^T \right) \right]\end{aligned}} \; . \tag{1.5.60}$$

For the anisotropic case, the Forchheimer terms will be neglected :

$$\boxed{\begin{aligned}\rho_f \tfrac{\partial \langle \underline{v}_f \rangle}{\partial t} + \rho_f \underline{\nabla} \cdot \left[\tfrac{1}{\varepsilon_f} \langle \underline{v}_f \rangle \langle \underline{v}_f \rangle \right] = \quad & -\varepsilon_f \underline{\nabla} \langle P_f \rangle^f - \varepsilon_f \eta \underline{\underline{K}}^{-1} \cdot \langle \underline{v}_f \rangle \\ & + \underline{\nabla} \cdot \left[\left(\eta \underline{\underline{I}} - \rho_f \underline{\underline{M}}^D \right) : \left(\underline{\nabla} \langle \underline{v}_f \rangle + \underline{\nabla} \langle \underline{v}_f \rangle^T \right) \right]\end{aligned}} \; . \tag{1.5.61}$$

These laws are valid under the very general assumptions that flow can be transient, while inertia effects are not neglected and momentum dispersion is taken into account.

Before considering the energy equation, the equation governing the solid domain must be developed. This issue is the objective of the following subsection.

1.5.6 Solid phase

The solid phase momentum equation provides the associated stresses in relation with the ambient conditions. Since this phase is stationary (H.i), the microscopic equation is written as follows :

$$\underline{0} = \underline{\nabla} \cdot \underline{\underline{\sigma}}_s + \rho_s \underline{g}. \tag{1.5.62}$$

This is in fact an equilibrium equation. Taking the volume average of this equation, with the assumption that gravity is the only body force (H.viii), one obtains :

$$\underline{0} = \underline{\nabla} \cdot \langle \underline{\underline{\sigma}}_s \rangle + \underbrace{\int_{S_{fs}} \underline{\underline{\sigma}}_s \cdot \underline{n}_{sf} \, dS}_{\underline{f}_T} - \underline{\nabla} \langle \rho_s g h \rangle - \underbrace{\int_{S_{fs}} \rho_s g h \, \underline{n}_{sf} \, dS}_{\underline{f}_{gs}}, \tag{1.5.63}$$

where

- with neglected capillarity (H.vii), the normal stress components match on the solid-liquid interface S_{fs}, $\underline{\underline{\sigma}}_s \cdot \underline{n}_{sf} + \underline{\underline{\sigma}}_f \cdot \underline{n}_{fs} = \underline{0}$. The term $\underline{f}_T$ is exactly the same as in Eq. (1.5.32) and represents the total fluid-solid interaction ;
- the gravity integral $\underline{f}_{gs}$ can be interpreted as the fluid contribution to solid buoyancy (via Archimede's force).

Finally, considering a homogeneous saturated porous medium with no consolidation (H.vi), the solid volume fraction ε_s is constant and the last equation can be rewritten in the form :

$$\underline{0} = \underline{\nabla} \cdot \langle \underline{\underline{\sigma}}_s \rangle + \varepsilon_s \rho_s \underline{g} + \underline{f}_T - \underline{f}_{gs} \, . \tag{1.5.64}$$

At this point, a constitutive equation for the solid stress $\langle \underline{\underline{\sigma}}_s \rangle$ is required, together with a description of the corresponding boundary conditions. In the major part of our work, we only focus on the behaviour of the fluid phase when studying the application of the theory to RTM and SRIM processes. In order to completely describe these processes, a description of the friction stress between the solid mat and the mold could additionally be required.

In order to conclude this section, Table 1.1 summarizes the equations and their particular applicability conditions.

1.6 Energy equation

This section describes the establishment of the energy equation in order to predict how the average temperature evolves in space and time. Starting from the same assumptions as before (see Section 1.3), we can consider a saturated porous medium. Two options exist for setting up the energy equation. Either the solid and fluid phases are separately treated or the solid and fluid are assumed to share the same local average temperature (**local equilibrium model**). Since this assumption seems to be applicable to the modeling of RTM and SRIM processes (8), for the sake of simplicity, only local equilibrium model will be discussed in this work.

The local equilibrium hypothesis in fact means that, even if the fluid and the solid exhibit different temperatures, this difference is negligible in such a way that one can assume that their respective intrinsic phase averages are equal. These temperatures are noted as follows :

$$\langle T_s \rangle^s = \langle T_f \rangle^f = \langle T \rangle. \tag{1.6.65}$$

Such as pressure, temperature is an intensive variable. Hence, its intrinsic average is physically meaningful and can be used as a primary variable. Therefore both the fluid and solid phase equations are developed as a function of their respective intrinsic phase averages, before being put together in a single balance equation by use of the local equilibrium assumption.

The starting point is again the microscopic energy conservation equation. For an incompressible fluid it is written as :

$$\rho_f C_f \frac{\partial T_f}{\partial t} + \rho_f C_f \underline{\nabla} \cdot \left(T_f \underline{v}_f \right) = -\underline{\nabla} \cdot \underline{q}_f + \Phi_f \tag{1.6.66}$$

where C_f stands for the specific heat at constant pressure [J/kg°K], $\underline{q}_f$ for the heat flux [J/m^{2}s] and Φ_f for the rate of heat dissipation or supply per unit volume [J/m^{3}s]. Taking the phase average of this equation yields

$$\langle \rho_f C_f \frac{\partial T_f}{\partial t} \rangle + \langle \rho_f C_f \underline{\nabla} \cdot \left(T_f \underline{v}_f \right) \rangle = -\langle \underline{\nabla} \cdot \underline{q}_f \rangle + \langle \Phi_f \rangle. \tag{1.6.67}$$

The first term of l.h.s. easily transforms as previously into :

$$\langle \rho_f C_f \frac{\partial T_f}{\partial t} \rangle = \rho_f C_f \frac{\partial \langle T_f \rangle}{\partial t} = \varepsilon_f \rho_f C_f \frac{\partial \langle T_f \rangle^f}{\partial t} \tag{1.6.68}$$

where specific heat variations have been neglected and considering a constant fluid density.

Applying the gradient theorem, considering a no-slip viscous boundary condition and keeping in mind the previous remarks on specific heat and density, the second term becomes :

$$\langle \rho_f C_f \underline{\nabla} \cdot \left(T_f \underline{v}_f \right) \rangle = \rho_f C_f \underline{\nabla} \cdot \langle T_f \underline{v}_f \rangle + \underbrace{\int_{S_{fs}} T_f \underline{v}_f \cdot \underline{n}_{fs} \, dS}_{=0}. \tag{1.6.69}$$

As shown in Section 1.5.1, pointwise fields can be decomposed in average and deviations :

$$\underline{v} = \langle \underline{v}_f \rangle^f + \hat{\underline{v}}_f \quad \text{and} \quad T_f = \langle T_f \rangle^f + \hat{T}_f. \tag{1.6.70}$$

The product average is then rewritten in a similar way as in Eq. (1.5.34) :

$$\langle T_f \underline{v}_f \rangle = \langle T_f \rangle^f \langle \underline{v}_f \rangle + \langle \hat{T}_f \hat{\underline{v}}_f \rangle. \tag{1.6.71}$$

Applying the continuity equation (1.4.22), the divergence term becomes :

$$\langle \rho_f C_f \underline{\nabla} \cdot \left(T_f \underline{v}_f \right) \rangle = \rho_f C_f \langle \underline{v}_f \rangle \cdot \underline{\nabla} \langle T_f \rangle^f + \rho_f C_f \underline{\nabla} \cdot \langle \hat{T}_f \hat{\underline{v}}_f \rangle. \tag{1.6.72}$$

The last term of this equation is similar to the one appearing in Eq. (1.5.35) and is thus responsible for the **dispersion effect**. It is important to already note that this formulation does not allow any dispersion in case of homogeneous temperature field $\langle \hat{T}_f \hat{\underline{v}}_f \rangle = \underline{0}$, which seems to be physically pertinent. In order to develop the r.h.s. of Eq. (1.6.67) as a function of $\langle T_f \rangle^f$, additional assumptions and hence constitutive equations have to be introduced.

Firstly the isotropic Fourier law is assumed in both phases. The heat flux in the fluid is then written as :

$$\underline{q}_f = -k_f \underline{\nabla} T_f, \tag{1.6.73}$$

where k_f is the fluid conductivity coefficient [J/ms°K]. Applying two times the gradient theorem, the flux term of Eq. (1.6.67) becomes :

$$-\langle \underline{\nabla} \cdot \underline{q}_f \rangle = \underline{\nabla} \cdot \left[k_f \underline{\nabla} (\varepsilon_f \langle T_f \rangle^f) + \frac{k_f}{V} \int_{S_{fs}} T_f \, \underline{n}_{fs} \, dS \right] + \frac{k_f}{V} \int_{S_{fs}} \underline{\nabla} T_f \cdot \underline{n}_{fs} \, dS \, . \tag{1.6.74}$$

As the radius of the averaging volume is much smaller than the macroscopic lengths of the domain of interest, the second term of the r.h.s. of Eq. (1.6.74) can be transformed by Eq. (1.2.17) into :

$$\frac{k_f}{V} \int_{S_{fs}} T_f \, \underline{n}_{fs} \, dS = \frac{k_f}{V} \int_{S_{fs}} (\langle T_f \rangle^f + \hat{T}_f) \, \underline{n}_{fs} \, dS = \frac{k_f}{V} \int_{S_{fs}} \hat{T}_f \, \underline{n}_{fs} \, dS - k_f \langle T_f \rangle^f \underline{\nabla} \varepsilon_f \, . \tag{1.6.75}$$

Taking this latter equation into account and developing the first term of Eq. (1.6.74) leads to neglecting the term in $\underline{\nabla} \varepsilon_f$ in Eq. (1.6.75). Finally Eq. (1.6.74) becomes :

$$-\langle \underline{\nabla} \cdot \underline{q}_f \rangle = \underline{\nabla} \cdot \left[\varepsilon_f k_f \underline{\nabla} \langle T_f \rangle^f + \frac{k_f}{V} \int_{S_{fs}} \hat{T}_f \, \underline{n}_{fs} \, dS \right] + \frac{k_f}{V} \int_{S_{fs}} \underline{\nabla} T_f \cdot \underline{n}_{fs} \, dS \, . \tag{1.6.76}$$

Table 1.1: Momentum equation summary for the fluid phase

Equation	Applicability conditions	Medium type	Boundary conditions (BC)	Drawbacks
Darcy Eq. (1.5.50) $\underline{0} = -\underline{\nabla}\langle P_f\rangle^f - \frac{1}{\eta}\underline{\underline{K}}^{-1}\cdot\langle \underline{v}_f\rangle$	Inertial forces and bulk viscous effects negligible.	Anisotropic	Normal velocity or pressure	- Non-linear inertial effects missing - Low BC possibilities
Forchheimer Eq. (1.5.52) $0 = -\underline{\nabla}\langle P_f\rangle^f - \frac{\eta}{K}\langle \underline{v}_f\rangle - C\rho\ \langle \underline{v}_f\rangle\ \|\langle \underline{v}_f\rangle\|$	Inertial forces and bulk viscous effects negligible but form forces present	Isotropic	Normal velocity or pressure	- Low BC possibilities, - Anisotropic ext. missing
Transition model Eq. (1.5.57) $0 = -\underline{\nabla}\langle P_f\rangle^f - \frac{\eta}{K}\langle \underline{v}_f\rangle\left(1+\left(\frac{\rho KC}{\eta}\|\langle \underline{v}_f\rangle\|\right)^2\right)^{1/2}$	Inertial forces and bulk viscous effects negligible but form forces present	Isotropic	Normal velocity or pressure	- Low BC possibilities, - Anisotropic ext. missing, - No experimental comparison.
Brinkman Eq. (1.5.59) $0 = \varepsilon_f\underline{\nabla}\langle P_f\rangle^f + \varepsilon_f\eta\underline{\underline{K}}^{-1}\cdot\langle \underline{v}_f\rangle$ $+\underline{\nabla}\cdot\left[\hat{\eta}\left(\underline{\nabla}\langle \underline{v}_f\rangle + \underline{\nabla}\langle \underline{v}_f\rangle^T\right)\right]$	Forchheimer's form forces negligible	Anisotropic	Velocity or stress	Determination of a model for the modified viscosity and momentum the dispersion tensor
General Eq. (1.5.60) $\rho_f\frac{\partial\langle \underline{v}_f\rangle}{\partial t} + \rho_f\underline{\nabla}\cdot\left[\frac{1}{\varepsilon_f}\langle \underline{v}_f\rangle\langle \underline{v}_f\rangle\right] =$ $-\varepsilon_f\underline{\nabla}\langle P_f\rangle^f - \varepsilon_f\frac{\eta}{K}\langle \underline{v}_f\rangle - \varepsilon_f C\rho\ \langle \underline{v}_f\rangle\ \|\langle \underline{v}_f\rangle\|$ $+\underline{\nabla}\cdot\left[\hat{\eta}\left(\underline{\nabla}\langle \underline{v}_f\rangle + \underline{\nabla}\langle \underline{v}_f\rangle^T\right)\right]$		Isotropic	Velocity or stress	- Highly (too) detailed, - Anisotropic extension missing

Secondly, the dissipation term can involve chemical reaction and viscosity effects. In case of RTM and SRIM processes, a resin is injected into a mold and undergoes a curing reaction activated either by heating or by chemical mixing. The reaction can generate or absorb heat and can be characterized by an **extent of conversion** c_f whose range is from 0 (no reaction) to 1 (full reaction). As explained in Section 5.3, this variable concentrates all the chemical effects involved in the transformation reaction, which is then characterized by a single kinetic function, itself depending on temperature and on the extent of conversion :

$$\frac{dc_f}{dt} = f_c(c_f, T_f). \tag{1.6.77}$$

If the heat of reaction per unit mass is denoted by H_R [J/kg], the rate of heat released per unit fluid volume is then written as :

$$\left[\Phi_f\right]_{reaction} = \rho_f H_R \frac{dc_f}{dt} = \rho_f H_R f_c(c_f, T_f). \tag{1.6.78}$$

This expression is easily averaged to provide the expression :

$$\left[\langle\Phi_f\rangle^f\right]_{reaction} = \rho_f H_R \langle f_c(c_f, T_f)\rangle^f. \tag{1.6.79}$$

The developments associated with viscous dissipation are detailed in Annexe A. The result can generally be written for a Darcy flow as :

$$\left[\langle\Phi_f\rangle\right]_{viscous} = \eta\ \langle\underline{v}_f\rangle \cdot \underline{\underline{K}}^{-1} \cdot \langle\underline{v}_f\rangle. \tag{1.6.80}$$

Collecting the results of Eqs.(1.6.68), (1.6.72), (1.6.76), (1.6.79) and (1.6.80), Eq. (1.6.67) becomes :

$$\begin{aligned}
&\varepsilon_f \rho_f C_f \tfrac{\partial \langle T_f\rangle^f}{\partial t} + \rho_f C_f \langle\underline{v}_f\rangle \cdot \underline{\nabla}\langle T_f\rangle^f = \\
&\underline{\nabla} \cdot \left[\varepsilon_f k_f \underline{\nabla}\langle T_f\rangle^f + \tfrac{k_f}{V}\int_{S_{fs}} \hat{T}_f\ \underline{n}_{fs}\ dS\right] + \tfrac{k_f}{V}\int_{S_{fs}} \underline{\nabla} T_f \cdot \underline{n}_{fs}\ dS \\
&+\rho_f H_R \langle f_c(c_f, T_f)\rangle^f + \eta\ \langle\underline{v}_f\rangle \cdot \underline{\underline{K}}^{-1} \cdot \langle\underline{v}_f\rangle - \rho_f C_f \underline{\nabla} \cdot (\hat{T}_f \hat{\underline{v}}_f).
\end{aligned} \tag{1.6.81}$$

The corresponding equation for the solid phase can be generated by exchanging the subscripts s and f and observing that both $\underline{v}_s$ and Φ_s vanish (stationary solid and no reaction in the solid) :

$$\varepsilon_s \rho_f C_s \frac{\partial \langle T_s\rangle^s}{\partial t} = \underline{\nabla} \cdot \left[\varepsilon_s k_s \underline{\nabla}\langle T_s\rangle^s + \frac{k_s}{V}\int_{S_{fs}} \hat{T}_s\ \underline{n}_{sf}\ dS\right] + \frac{k_f}{V}\int_{S_{fs}} \underline{\nabla} T_s \cdot \underline{n}_{sf}\ dS \tag{1.6.82}$$

Therefore, these equations represent the energy balance per unit of total volume for the fluid and the solid phases. They exhibit the usual transient, convection, conduction

and dissipation terms. However, as previously shown, a particular term appears in the fluid phase, viz. the dispersion term. This energy term is better understood and accepted than its inertial counterpart (see Sections 1.5.1 and 1.5.4) since it is easier to observe (1; 8; 9; 17; 18; 24; 2; 25; 26). Hence it must be kept in the developments and should be modelled as a function of the primary variables.

Noting that $\underline{n}_{fs} = -\underline{n}_{sf}$, the total energy balance is obtained by adding the two phase equations :

$$\begin{aligned}
&\varepsilon_f \rho_f C_f \tfrac{\partial \langle T_f \rangle^f}{\partial t} + \varepsilon_s \rho_f C_s \tfrac{\partial \langle T_s \rangle^s}{\partial t} + \rho_f C_f \langle \underline{v}_f \rangle \cdot \underline{\nabla} \langle T_f \rangle^f = \\
&\underline{\nabla} \cdot \left[\varepsilon_f k_f \underline{\nabla} \langle T_f \rangle^f + \varepsilon_s k_s \underline{\nabla} \langle T_s \rangle^s + \tfrac{k_f}{V} \int_{S_{fs}} \hat{T}_f \, \underline{n}_{fs} \, dS + \tfrac{k_s}{V} \int_{S_{fs}} \hat{T}_s \, \underline{n}_{sf} \, dS \right] \\
&\underbrace{+ \frac{1}{V} \int_{S_{fs}} (k_f \underline{\nabla} T_f - k_s \underline{\nabla} T_s) \cdot \underline{n}_{fs} \, dS}_{(*)} \\
&+ \rho_f H_R \langle f_c(c_f, T_f) \rangle^f + \eta \, \langle \underline{v}_f \rangle \cdot \underline{\underline{K}}^{-1} \cdot \langle \underline{v}_f \rangle - \rho_f C_f \underline{\nabla} \cdot \langle \hat{T}_f \hat{\underline{v}}_f \rangle \, .
\end{aligned} \tag{1.6.83}$$

The **local equilibrium** assumption is here used. Indeed, the strong interphase heat exchange (expressed by the marked term (*)) prohibits any significant temperature difference between the solid and the fluid. Definition (1.6.65) can then be assumed, where $\langle T \rangle$ is the volume-average temperature for both phases. Using this hypothesis and requiring temperature continuity on the interfacial area S_{fs}, it can besides be asserted that :

$$\hat{T}_f = \hat{T}_s. \tag{1.6.84}$$

In order to keep heat flux continuity across the interphase area, the flux exchange term is assumed to vanish :

$$\frac{1}{V} \int_{S_{fs}} (k_f \underline{\nabla} T_f - k_s \underline{\nabla} T_s) \cdot \underline{n}_{fs} \, dS = 0. \tag{1.6.85}$$

These considerations imply that Eq. (1.6.83) transforms into :

$$\begin{aligned}
&(\varepsilon_f \rho_f C_f + \varepsilon_s \rho_f C_s) \tfrac{\partial \langle T \rangle}{\partial t} + \rho_f C_f \langle \underline{v}_f \rangle \cdot \underline{\nabla} \langle T \rangle = \\
&\underline{\nabla} \cdot \left[(\varepsilon_f k_f + \varepsilon_s k_s) \underline{\nabla} \langle T \rangle + \tfrac{k_f - k_s}{V} \int_{S_{fs}} \hat{T}_f \, \underline{n}_{fs} \, dS \right] \\
&+ \rho_f H_R \langle f_c(c_f, T_f) \rangle^f + \eta \, \langle \underline{v}_f \rangle \cdot \underline{\underline{K}}^{-1} \cdot \langle \underline{v}_f \rangle - \rho_f C_f \underline{\nabla} \cdot \langle \hat{T}_f \hat{\underline{v}}_f \rangle \, .
\end{aligned} \tag{1.6.86}$$

Expressions are still needed for the temperature deviation and for the dispersion effect.

1.6.1 Effective conduction tensor

Whitaker (27) as well as Nozad et al. (28) suggest to express $\hat{T}_f$ as a the Taylor expansion of $\underline{\nabla}\langle T\rangle$ about the point $\underline{\nabla}\langle T\rangle = 0$. This idea corresponds to the fact that $\underline{\nabla}\langle T\rangle$ experiences reduced changes over the averaging volume. They find :

$$\hat{T}_f = \underline{b}(\underline{x}') \cdot \underline{\nabla}\langle T\rangle, \tag{1.6.87}$$

where $\underline{b}$ is a vector depending on the REV local space coordinates only and is determined by solving a PDE described for periodic media in (25; 28). This mathematical feature allows us to define the **effective conductivity tensor** as :

$$\underline{\underline{k}}_e = (\varepsilon_f k_f + \varepsilon_s k_s)\ \underline{\underline{\delta}} + \frac{k_f - k_s}{V} \int_{S_{fs}} \underline{b}\ \underline{n}_{fs}\ dS. \tag{1.6.88}$$

Nevertheless, in applications such as RTM processes, the porous medium cannot be assumed as periodic and it is seldom feasible to calculate temperature deviations from micro-mechanical arguments. Therefore a theoretical determination of the effective conductivity tensor is not always possible. In most cases, only experimental measurements can provide good estimates, even if it is not so easy to separate the conduction from the dispersion effect.

1.6.2 Dispersion modelling

As previously explained, dispersion term takes its origin from the combined effects of the pore tortuosity and the medium network. On the one hand, the porous medium is composed of tortuous pores in which the fluid flows and transports various physical properties such as energy as well as momentum. The pointwise velocity and these quantities are then usually not equal to their average counterparts. Hence these different quantities are transported at a different rate. On the other hand, the porous medium network is such that pores meet and separate within short distances. The transported quantity can then locally move at a velocity differing from to the average velocity. These local transport phenomena then result in a macroscopic mixing effect which is called **hydrodynamic dispersion**. This is a quite same observation as in Thermodynamics, where the molecule motion (Brownian noise) is responsible for the diffusion effect. The main difference however lies in the fact that thermodynamical diffusion is independent of the matter velocity, which is not the case with hydrodynamic dispersion. Moreover other pore scale phenomena can increase the macroscopic mixing such as recirculations, obstructions or eddy effects. All these effects should nevertheless be included in the model (2). In conclusion, hydrodynamic mixing is expected to depend on the medium geometry and the velocity. This assumption corroborates several observations, where increasing the velocity induces an increase of the total mixing too

(Fig. 1.6). This is not the case for usual flows, where the conduction phenomenon is independent of the local velocity.

Using Eq. (1.6.87) and assuming that the average gradient is constant on the REV, the dispersion term can be written as :

$$\rho_f C_f \underline{\nabla} \cdot \langle \hat{T}_f \underline{\hat{v}}_f \rangle = \rho_f C_f \underline{\nabla} \cdot \left(\varepsilon_f \langle \underline{\hat{v}}_f \underline{b} \rangle^f \underline{\nabla} \langle T \rangle \right). \tag{1.6.89}$$

This equation leads us to define the effective thermal dispersion tensor as :

$$\underline{\underline{K}}^D = -\varepsilon_f \rho_f C_f \langle \underline{\hat{v}}_f \underline{b} \rangle^f = \varepsilon_f \rho_f C_f \underline{\underline{D}}^D. \tag{1.6.90}$$

where the dispersion tensor $\underline{\underline{D}}^D$ is defined as :

$$\underline{\underline{D}}^D = -\langle \underline{\hat{v}}_f \underline{b} \rangle^f. \tag{1.6.91}$$

If the vector $\underline{b}$ depends on the medium geometry, the dispersion tensor $\underline{\underline{D}}^D$ is expected to depend on the velocity. However, although dispersion is a key-effect for the flow in a porous medium, there is a serious lack of complete models. Nevertheless Mal et al. (26) suggest to model the dispersion tensor as :

$$\underline{\underline{D}}^D = \underline{\underline{\underline{L}}} : \frac{\langle \underline{v}_f \rangle \langle \underline{v}_f \rangle}{\| \langle \underline{v}_f \rangle \|}, \tag{1.6.92}$$

where $\underline{\underline{\underline{L}}}$ is called the **fourth-order dispersion tensor**, whose coefficients depend on the medium geometry. This model seems to be convenient because it can take into account the interactions of the fluid velocity and the internal porous network.

The same idea can be used to represent **inertial dispersion** in Eq. (1.5.36), where an appropriate expression of $\underline{\underline{\underline{M}}}_D$ as a function of $\underline{\underline{\underline{L}}}$ and $\langle \underline{v}_f \rangle$ should be inserted. By analogy with temperature dispersion, momentum dispersion could then be modelled by a six-order tensor $\underline{\underline{\underline{B}}}^D$. Momentum dispersion should then be rewritten as :

$$\begin{aligned} \rho_f \underline{\nabla} \cdot \langle \underline{\hat{v}}_f \underline{\hat{v}}_f \rangle &= \rho_f \underline{\nabla} \cdot \left[\underline{\underline{\underline{M}}}^D \left(\underline{\underline{\underline{L}}}, \langle \underline{v}_f \rangle \right) : \left(\underline{\nabla} \langle \underline{v}_f \rangle + \underline{\nabla} \langle \underline{v}_f \rangle^T \right) \right] \\ &= \rho_f \underline{\nabla} \cdot \left[\left(\underline{\underline{\underline{B}}}^D \left(\underline{\underline{\underline{L}}} \right) : \frac{\langle \underline{v}_f \rangle \langle \underline{v}_f \rangle}{\| \langle \underline{v}_f \rangle \|} \right) : \left(\underline{\nabla} \langle \underline{v}_f \rangle + \underline{\nabla} \langle \underline{v}_f \rangle^T \right) \right]. \end{aligned} \tag{1.6.93}$$

However, this model has not received any validation yet. It is only proposed from the observation that dispersion has the same origin in both cases although this effect acts either on momentum or on temperature.

Particular cases such as isotropic and transversely isotropic media are deeper investigated in Chapter 5. The related models are then justified and developed.

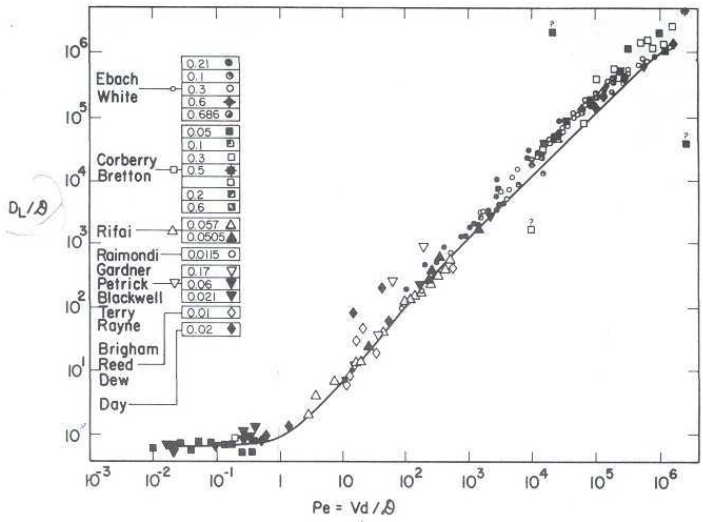

Figure 1.6: Relationship between molecular diffusion (see Section 1.7) and convective dispersion (2). $\mathscr{D}$ is the molecular diffusivity (equivalent to the conductivity tensor k_e) $V\ d$ is the Darcy velocity times the particle diameter and thus $Pe = V\ d/\mathscr{D}$ is a non-dimensional number, D_L is the sum of the molecular diffusivity and of the dispersion diffusivity (equivalent to $k_e + K_D$).

A main problem for flows in porous media is to measure the parameters of the governing equations (typically the dispersion tensor as well as the conductivity tensor coefficients). The difficulty here results from the number of unknown coefficients and from the fact that it is very difficult to isolate the influence of each individual phenomenon and to elaborate a sufficient number of pertinent experiments. In this sense, only the total effect can be measured whatever the microscopical phenomena are.

Taking these considerations into account, Eq. (1.6.87) to Eq. (1.6.90) can be inserted in Eq. (1.6.86) to provide the final energy balance :

$$\boxed{\begin{aligned}&(\varepsilon_f \rho_f C_f + \varepsilon_s \rho_f C_s)\frac{\partial \langle T\rangle}{\partial t} + \rho_f C_f \langle \underline{v}_f\rangle \cdot \underline{\nabla}\langle T\rangle = \\ &\underline{\nabla}\cdot\left[\left(\underline{k}_e + \underline{K}^D\right)\underline{\nabla}\langle T\rangle\right] + \rho_f H_R f_c(\langle c_f\rangle^f, \langle T\rangle) + \eta\ \langle \underline{v}_f\rangle \cdot \underline{K}^{-1} \cdot \langle \underline{v}_f\rangle\ ,\end{aligned}} \tag{1.6.94}$$

where we have considered that $\langle f_c(c_f, T_f)\rangle^f \approx f_c(\langle c_f\rangle^f, \langle T\rangle)$ is a good approximation.

Again, following the same approach as for temperature, the pointwise concentration can be expressed as a sum of intrinsic average and deviation :

$$c_f = \langle c_f \rangle^f + \hat{c}_f. \tag{1.7.96}$$

Assuming that Fick's law is valid, the diffusive flux can be written as :

$$\underline{J}_f = -D_f \underline{\nabla} c_f. \tag{1.7.97}$$

The flux term of Eq. (1.7.95) can then be rewritten similarly to Eq. (1.6.74) :

$$-\underline{\nabla} \cdot \langle \underline{J}_f \rangle = \underline{\nabla} \cdot \left[\varepsilon_f D_f \underline{\nabla} \langle c_f \rangle^f + \frac{1}{V} \int_{S_{fs}} \hat{c}_f \, \underline{n}_{fs} \, dS \right], \tag{1.7.98}$$

the interphase flux being zero because there is no interphase mass transfer. As for temperature, the deviation concentration can be expressed as a function of the intrinsic concentration gradient and a vector $\underline{f}$ only depending on the local space coordinates :

$$\hat{c}_f = \underline{f}(\underline{x}') \cdot \underline{\nabla} \langle c_f \rangle^f. \tag{1.7.99}$$

Considering that $\underline{\nabla} \langle c_f \rangle^f$ is constant on the REV, the **effective diffusivity tensor** $\underline{\underline{D}}_e$ is defined as :

$$\underline{\underline{D}}_e = D_f \underline{\underline{\delta}} + \frac{1}{V_f} \int_{S_{fs}} \underline{f} \, \underline{n}_{fs} \, dS = D_f \underline{\underline{\delta}} + \underline{\underline{\tau}}, \tag{1.7.100}$$

where the **tortuosity tensor** $\underline{\underline{\tau}}$ is expressed as :

$$\underline{\underline{\tau}} = \frac{1}{V_f} \int_{S_{fs}} \underline{f} \, \underline{n}_{fs} \, dS. \tag{1.7.101}$$

The dispersive diffusivity tensor $\underline{\underline{D}}^D$ is defined as :

$$\underline{\underline{D}}^D = -\frac{1}{V_f} \int_{V_f} \hat{\underline{v}}_f \, \underline{f} \, dV = -\langle \hat{\underline{v}}_f \, \underline{f} \rangle^f. \tag{1.7.102}$$

Introducing these definitions in Eq. (1.7.95), the balance equation becomes :

$$\boxed{\varepsilon_f \frac{\partial \langle c_f \rangle^f}{\partial t} + \langle \underline{v}_f \rangle \cdot \underline{\nabla} \langle c_f \rangle^f = \underline{\nabla} \cdot \left[\varepsilon_f \left(\underline{\underline{D}}_e + \underline{\underline{D}}^D \right) \cdot \underline{\nabla} \langle c_f \rangle^f \right] + \varepsilon_f f_c(\langle c_f \rangle^f, \langle T_f \rangle^f),} \tag{1.7.103}$$

where we have again introduced the approximation $\langle f_c(c_f, T_f) \rangle^f \approx f_c(\langle c_f \rangle^f, \langle T_f \rangle^f)$, whose accuracy relies on the local variations c_f and T_f which are assumed to be very small compared to the non-linearities of the source function f_c.

The mathematical type of the governing equation for species conservation (1.7.103) is the same as for temperature. The boundary conditions are similar and consist in imposing either the concentration or the species flux along the boundary.

1.8 Constitutive equations

Only two constitutive equations are required to close the model : one for the viscosity and one for the source reaction kinetics term in the energy and species conservation equations. For RTM and SRIM processes, this study is limited to the dependence of both functions upon temperature and cure factor :

$$\eta = f_\eta(c_f, T_f) \quad \text{and} \quad \Phi_f = f_c(c_f, T_f). \tag{1.8.104}$$

In moulding processes, the viscosity undergoes large changes so that its pointwise value can be very different from its average. This phenomenon rises new difficulties in the establishment of the governing equations. Indeed the viscosity was assumed constant in all previous developments. Nevertheless, on the one hand we can assume as for curing that deviations are small compared to the nonlinearities of the constitutive law. On the other hand, an approximation in the averaging process is necessary in order to progress toward a closed equation set. Hence, the viscosity will just be replaced by its intrinsic averaged value :

$$\eta \approx \langle \eta_f \rangle^f = \langle f_{\eta\, f}(c, T) \rangle^f \approx f_\eta(\langle c_f, \rangle^f, \langle T_f \rangle^f) \tag{1.8.105}$$

Example are illustrated in Chapter 5 for certain resine types used in RTM and SRIM processes.

1.9 Conclusion

In this first chapter, the averaging process has been used to provide a closed set of equations governing possibly reacting flows in porous media. It has appeared that such an approach has the main advantage of taking all relevant effects into account. In this way hydrodynamic dispersion has been shown as playing a major role in heat and species transport. Main hypotheses have been posed and different models for the momentum equation have been discussed together with the corresponding applicability conditions. The permeability tensor and its meaning have been analyzed, as well as the modified viscosity and its origin. The energy equation has been developed on the basis of the local equilibrium model, which assumes that solid and fluid average temperatures are equal. Introducing the effective conductivity tensor together with the dispersion tensor has been needed. Species conservation equations for reacting flows have been introduced in a similar way as for the energy equation. In each case, the appropriate boundary conditions have been detailed and isotropic and non-isotropic models have also been presented. Finally, constitutive laws and their associated theory have been developed.

The final set of appropriate governing equations is summarized herebelow for the fluid phase :

Mass balance

$$\underline{\nabla} \cdot \langle \underline{v}_f \rangle = 0$$

Momentum balance (isotropic case)

$$\rho_f \frac{\partial \langle \underline{v}_f \rangle}{\partial t} + \rho_f \underline{\nabla} \cdot \left[\frac{1}{\varepsilon_f} \langle \underline{v}_f \rangle \langle \underline{v}_f \rangle \right] =$$

$$-\varepsilon_f \underline{\nabla} \langle P_f \rangle^f - \varepsilon_f \frac{\langle \eta_f \rangle^f}{K} \langle \underline{v}_f \rangle - \varepsilon_f C \rho \, \langle \underline{v}_f \rangle \, \| \langle \underline{v}_f \rangle \| + \underline{\nabla} \cdot \left[\langle \hat{\eta}_f \rangle^f \left(\underline{\nabla} \langle \underline{v}_f \rangle + \underline{\nabla} \langle \underline{v}_f \rangle^T \right) \right]$$

Motion balance (anisotropic case)

$$\rho_f \frac{\partial \langle \underline{v}_f \rangle}{\partial t} + \rho_f \underline{\nabla} \cdot \left[\frac{1}{\varepsilon_f} \langle \underline{v}_f \rangle \langle \underline{v}_f \rangle \right] =$$

$$-\varepsilon_f \underline{\nabla} \langle P_f \rangle^f - \varepsilon_f \langle \eta_f \rangle^f \underline{\underline{K}}^{-1} \cdot \langle \underline{v}_f \rangle + \underline{\nabla} \cdot \left[\left(\langle \eta_f \rangle^f \underline{\underline{I}} - \rho_f \underline{\underline{M}}^D \right) : \left(\underline{\nabla} \langle \underline{v}_f \rangle + \underline{\nabla} \langle \underline{v}_f \rangle^T \right) \right]$$

Energy balance

$$(\varepsilon_f \rho_f C_f + \varepsilon_s \rho_f C_s) \frac{\partial \langle T \rangle}{\partial t} + \rho_f C_f \langle \underline{v}_f \rangle \cdot \underline{\nabla} \langle T \rangle =$$

$$\underline{\nabla} \cdot \left[\left(\underline{\underline{k}}_e + \underline{\underline{K}}^D \right) \underline{\nabla} \langle T \rangle \right] + \rho_f H_R f_c(\langle c_f \rangle^f, \langle T \rangle) + \eta \, \langle \underline{v}_f \rangle \cdot \underline{\underline{K}}^{-1} \cdot \langle \underline{v}_f \rangle$$

Species concentration balance

$$\varepsilon_f \frac{\partial \langle c_f \rangle^f}{\partial t} + \langle \underline{v}_f \rangle \cdot \underline{\nabla} \langle c_f \rangle^f = \underline{\nabla} \cdot \left[\varepsilon_f \left(\underline{\underline{D}}_e + \underline{\underline{D}}^D \right) \cdot \underline{\nabla} \langle c_f \rangle^f \right] + \varepsilon_f f_c(\langle c_f \rangle^f, \langle T_f \rangle^f)$$

Constitutive laws

$$\eta = f_\eta(c_f, T_f) \quad \text{and} \quad \Phi_f = f_c(c_f, T_f)$$

(1.9.106)

The solution of this equation set provides the velocity, pressure, temperature and curing state mean fields in the fluid phase as a function of time and space. This equation set must obviously be adapted to the application concerned. In general , dimensional analysis allows us to neglect some effects and thus to simplify the equations. Examples for RTM and SRIM processes can be found in (8).

In order to handle the solid phase system, constitutive equations must be added. This problem is addressed in Chapter 6 where the behaviour of cartilage as a porous medium is studied. Indeed, at the present stage, we have two equations at our disposal, viz. the energy equation (assuming local equilibrium) and the stress equilibrium equation. Hence at this point, the solid phase equation set is :

Stress equilibrium

$$\underline{0} = \underline{\nabla} \cdot \langle \underline{\underline{\sigma}}_s \rangle$$

Energy balance

cf. the fluid phase

Constitutive laws

cf. Chapter 6

(1.9.107)

The following chapter investigates another technique to tackle the study of flows in porous media, viz. the micro-macro approach.