ASSESSMENT OF THE IN-PLANE CAPACITY OF MASONRY WALLS WITH THE HYBRID DISCRETE-FINITE ELEMENT METHOD

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Abstract *The most widespread numerical simulation method for structural response is undoubtedly the Finite Element Method (FEM). However, despite being powerful for modelling continuous structures, it is not fit for handling strong discontinuities. The Discrete Element Method (DEM) simulates interactions between rigid or deformable elements in contact, hence explicitly capturing the discontinuous nature of the structural response, particularly when subjected to extreme loadings. Nevertheless, it requires a time-stepping algorithm even for solving static or buckling problems. The Hybrid Discrete-Finite Element Method, shortly HybriDFEM, was recently introduced in the context of modelling one-dimensional beam-like members. Those members are divided along their longitudinal axis in a series of rectangular rigid blocks, and the deformation is concentrated at the interfaces between adjacent blocks, modelled as distributed nonlinear multidirectional springs. The method, developed within a FEM-like setting, allows for hybridisation with other finite elements (e.g., beam elements). Next to its ability to explicitly model pre-existing discontinuities along the member (e.g. masonry stereotomy), the method can be used for modelling continuous members with satisfactory accuracy by appropriately scaling the interface springs. As such, the HybriDFEM's formulation can accommodate hybrid discrete-continuous systems. In this paper, the HybriDFEM formulation is extended to 2D, with rectangular blocks in contact on all four faces. First, the algorithm to detect blocks in contact will be explained. Second, specific characteristics of the HybriDFEM applied to masonry modelling are presented. Then, the method is benchmarked against a two-dimensional problem from the literature where the in-plane capacity of walls made masonry blocks is investigated.*

1 Introduction

Accurately modelling the seismic behaviour of unreinforced masonry (URM) structures presents a significant challenge to civil engineers. Reliable numerical methods are crucial for assessing the structural integrity of such buildings, which also constitute a significant portion of the world's cultural heritage. One of the primary challenges when modelling URM blocky structures (e.g., brick or stone masonry) is their inherently discontinuous nature, which contradicts the fundamental assumptions of finite element methods (FEM). The FEM was originally developed for modelling continuous materials (de Borst, Crisfield et al. 2012, Rust 2015). To overcome this challenge, homogenization techniques are often employed, which come at the cost of reduced accuracy. When there is a need to explicitly model these discontinuities within the FEM framework, micro-modelling approaches become necessary, albeit at a higher computational expense (Munjiza and Latham 2002, Roca, Cervera et al. 2010).

However, alternative methods better suited for handling discontinuities exist. The Discrete Element Method (DEM) family represents structures as assemblies of rigid (or deformable) bodies connected through contact points or interfaces. The DEM employs numerical time-stepping algorithms to solve the system's equilibrium (Cundall and Strack 1979, Lemos 2007). Yet, for static or buckling problems, a quasi-static solving approach is required, resulting in high computational demands. Variations of the DEM, such as the Rigid Block (RB) model (Portioli, Godio et al. 2021), Rigid-Body-Spring Model (RBSM) (Casolo and Uva 2013), and the Applied Element Method (AEM) (Meguro and Tagel-Din 1998, Tagel-Din and Meguro 1999, Malomo, Pinho et al. 2019), share characteristics that make them suitable for modelling discontinuous structures like URM. The AEM, which models a structure as an assembly of rigid units in contact through distributed springs, is particularly interesting because it can be used to model both discontinuous structures, such as URM, and continuous structures like reinforced concrete (RC) walls (Meguro and Tagel-Din 1998), through appropriate scaling of the interface spring properties.

The method presented in this paper, named the Hybrid Discrete-Finite Element method (HybriDFEM), shares characteristics with AEM in that the structure is modelled as an assembly of rigid blocks in contact through distributed nonlinear springs, but the HybriDFEM introduces two significant distinctions with respect to the AEM. In both methods, contact interfaces between blocks are divided into a series of contact points, representing the system’s deformability. In the HybriDFEM, each contact point consists of two nonlinear springs in series, individually representing the deformability of the two elements in contact, whereas the AEM does not distinguish the deformability of the two contacting elements, even if they have different material properties. The second distinction is that the HybriDFEM's formulation closely resembles that of the classical FEM, allowing for seamless coupling of the two methods, as demonstrated in previous publications by the authors; for example, in (Bouckaert, Godio et al. 2023), a frame with vertical and lateral loads was successfully simulated with the following alternative approaches: (i) a full FEM beam model; (ii) a model fully composed of blocks with contact interfaces, and (iii) a combination of beam finite elements and blocks. The latter approach allows for an optimised blend between computational time and accuracy, which is relevant for problems with high levels of material and geometric nonlinear demands.

The current paper's primary focus is to show the extension of the HybriDFEM to 2D. The basic developments of the method are very briefly explained in Section 2, regarding the 1D case. For more theoretical details on the HybriDFEM formulation, readers can refer to (Bouckaert, Godio et al. 2024). One of the main challenges in this extension lies in developing an algorithm to detect contact between blocks, which is not as straightforward as in the 1D case. This will be discussed in detail in Section 3. Section 4 will highlight key aspects of the method in modelling URM structures, and finally Section 5 will present a benchmark validation of the model, assessing the in-plane capacity of unreinforced masonry walls with and without openings.

2 The HybriDFEM method

2.1 Block discretization

With the HybriDFEM, a planar structure is first discretized into a series of blocks, which define the number of Degrees of Freedom (DoFs) considered at the structural level (Bouckaert, Godio et al. 2024). The mechanical properties of the structure are concentrated at the interfaces of the blocks, called Contact Faces (CFs). The kinematics of each block is described by three nodal displacements, all stored in a vector of global displacements **U**. The kinematics of a CF is described by six DoFs, namely the displacements of its two adjacent blocks (Figure 2, left).

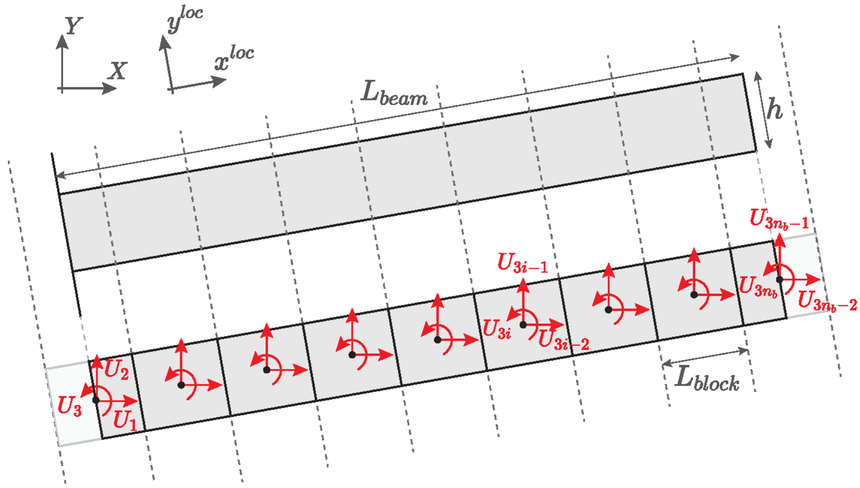

Figure 1. Block discretization of a HybriDFEM beam-like member (Bouckaert, Godio et al. 2024).

2.2 Contact pair discretization

After the block discretization, each CF is further discretized into a series of Contact Pairs (CPs), which are by default evenly distributed across the CF (Figure 2, middle). However, as will be discussed later, an even distribution of the CPs is not always most suited to the problem at hand, and this default distribution can be modified. Each CP consists of three points A, B and C, coinciding in the undeformed configuration. Points A and B are attached respectively to the first and second block, and they are connected to each other through two nonlinear bidimensional springs, connected at the hinged point C (Figure 2, right). The DoFs describing the kinematics of one such CP refer to the displacement and rotation of the contiguous blocks (first six DoFs of the CP) – which define the positions of points A and B, together with two DoFs determining the displacement of point C. The latter two DoFs are condensed for computations at the structural level, and a nonlinear solution procedure is required to determine them, as discussed below.

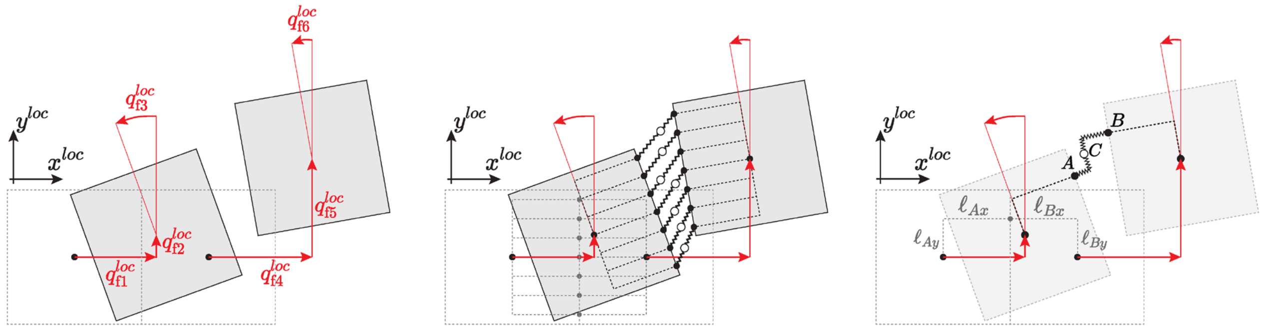

Figure 2. Kinematics of two blocks in contact (left); CP discretization of a contact face (middle); One contact pair composed of three points A, B and C and two bidirectional springs in series (right) (Bouckaert, Godio et al. 2024).

Since the position of A and B with respect to their respective block does not change, their kinematics, noted q_1^{loc} to q_6^{loc} can be derived solely from compatibility relations. The last two DoFs, q_7^{loc} and q_8^{loc}, however, depend on the relative stiffnesses of the two nonlinear springs in series. A linear compatibility equation and a possibly nonlinear equilibrium equation are necessary to compute those two degrees of freedom. The compatibility equation states that the sum of elongations of the two springs in series is equal to the relative displacement between point A and point B (Figure 3, left); the equilibrium relation states that the forces through the first bidirectional spring equals the force through the second bidirectional spring (Figure 3, right). This set of equations is solved by means of a Newton-Raphson solution procedure which is referred to as the 'internal' solution procedure, which stands in contrast with the 'external' solution procedure, corresponding to the classical solution procedure at the global level, common to nonlinear FEM. The internal (nonlinear) equilibrium equation allows to account for nonlinear constitutive relations. Nonlinear geometric effects are accounted for by isolating the CP deformation, excluding its rigid body motion, please refer to (Bouckaert, Godio et al. 2024) for more details.

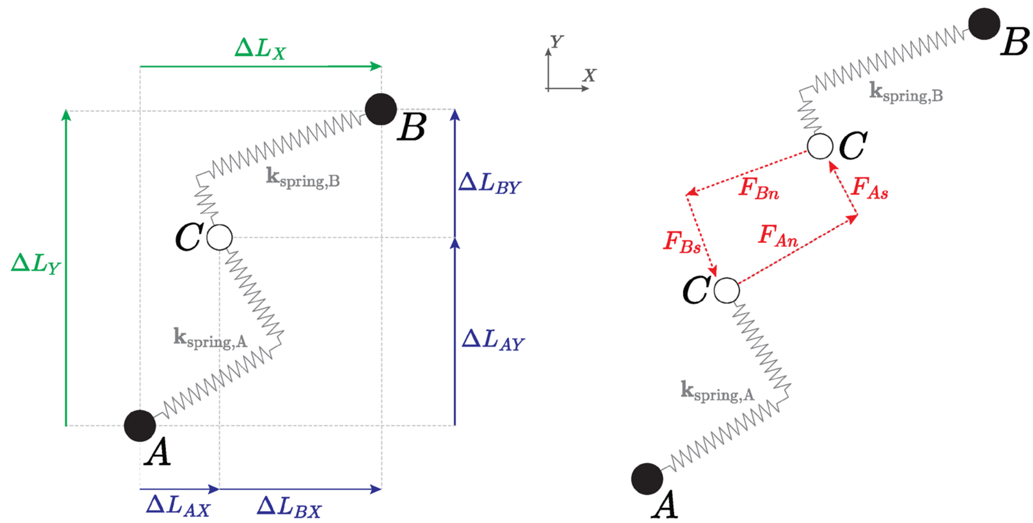

Figure 3. Linear compatibility relation (left) and possibly nonlinear equilibrium relation (right) for the internal solution procedure (Bouckaert, Godio et al. 2024).

Convergence of the internal solution procedure allows to find the position of point C and, more importantly, the bidirectional spring elongations and forces. Establishing equilibrium in the deformed configuration allows to compute the resisting forces of each CP while accounting for nonlinear geometric effects. From there, the resisting forces of each CF can be assembled from the resisting forces of all its CPs, and the structural resisting forces are computed by assembly of the CF resisting forces (Bouckaert, Godio et al. 2024).

The procedure described in this section allows to derive the resisting forces $\mathbf{P}_r$ of the structure for a given set of displacements $\mathbf{U}$. The same reasoning is applied to assemble the tangent stiffness matrix of the structure starting from the spring (or material) constitutive relations. When nonlinear geometric and material effects are considered, this tangent stiffness matrix can be used in an incremental solution procedure, commonly used in FEM, such as a load-control, displacement-control, or work-control procedure. More detail on this solution procedure has been provided by the authors in previous works (Bouckaert, Godio et al. 2024).

3 Contact face detection

When using the HybriDFEM to model elements with a predominant geometric direction (as shown in (Bouckaert, Godio et al. 2024)), the need for automatic detection and creation of CFs may not be immediately apparent, because they are all identical and evenly spaced across the beam. However, this simplification proves inadequate when CFs can potentially be created at all four edges of each block, and when the same block edge can come into contact with edges of more than one adjacent block. Consequently, the manual creation of contact interfaces becomes laborious. In the extension of the HybriDFEM to 2D structural elements, a simple contact face detection algorithm has been implemented, specifically for edge-to-edge contacts, which is, for the time being, the sole type of contact addressed with this method. It is worth noting that this algorithm is presently only employed at the start of a simulation for contact face creation, and no contact updates occur during a simulation.

The geometry of each block is defined by its four vertices, and potential blocks in contact are identified by computing the equation of the line formed by each pair of vertices that constitute an edge of a block. Once this calculation is performed for all blocks, potential block pairs in contact are determined by verifying whether two blocks share at least one edge line, within a user-specified tolerance. By default, all potential block pairs in contact are examined for shared edge lines. However, prior knowledge of the block disposition can significantly reduce the number of potential block pairs to be verified. For instance, if a square wall is discretized in both directions using *n x n* blocks, one can add a condition to search for shared edge lines with block *i* only at blocks *i-1, i+1, i-n*, and *i+n*, which correspond to the only four potential neighbours of block *i* if the blocks are perfectly aligned. However, if no contact update is performed, this simplification does not significantly change the simulation time.

Once the list of potential contact block pairs is created, the program iterates through all the pairs in this list to check if there is indeed a contact. If block 1 and block 2 are identified as potentially in contact through edge 1 and edge 2 respectively, the vertices of these two edges are sorted along the common line. This arrangement is used to determine whether the two edges overlap or not. Depending on the order of the sorted vertices, the three following scenarios are possible (see Figure 4):

- $n_{11} < n_{12} \leq n_{21} < n_{22}$: No overlap.
- $n_{11} \leq n_{21} \leq n_{12} \leq n_{22}$: Part or all of edge 2 is in contact with part or all of edge 1.
- $n_{11} \leq n_{21} < n_{22} \leq n_{12}$: Part or all of edge 2 is in contact with all of edge 1.
- $n_{21} \leq n_{11} < n_{12} \leq n_{22}$: Part or all of edge 1 is in contact with all of edge 2

where n_{ij} represents the *j*-th vertex of edge *i*. This systematic approach ensures efficient detection of contact interfaces between blocks (Figure 5), a crucial aspect of the extension of the HybriDFEM method for 2D elements.

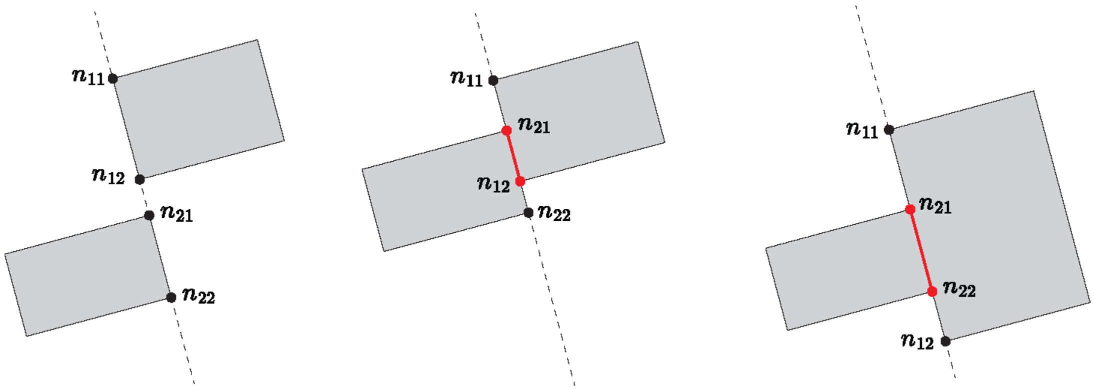

Figure 4. Possible scenarios for contact between two edges on the same line: no overlap (left), partial overlap (middle) or full overlap of one edge (right).

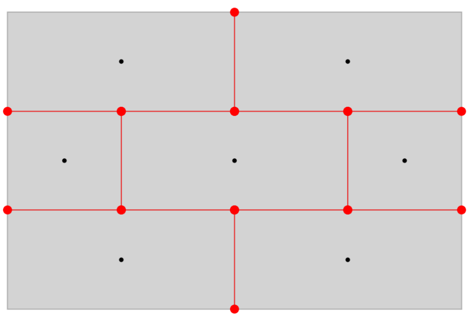

Figure 5. The 12 Contact Faces (CFs) obtained with the developed algorithm for a 7-block assembly.

4 HybriDFEM applied to URM

When a CF is identified between two blocks, the program automatically discretizes it into a series of contact pairs (CPs), composed of two bidirectional springs in series. When using the HybriDFEM as a proxy for continuous materials (i.e., RC walls), the mechanical properties of the springs are computed from a user-defined material law to represent deformability of a portion of the material. In that case, the user will use a constitutive law relating strains with stresses, for which the stiffness is expressed in Pa, and the program will automatically scale the spring properties depending on the user-defined CP discretization. This approach will be referred to as 'material law'-approach (Figure 6, left).

However, when the HybriDFEM is used to model blocky masonry walls, the block discretization is done such that it corresponds to the actual stereotomy of the wall. In that case, the springs are used to represent the deformability at the interface between two rigid units, and not the deformability of the units themselves, as is typically done in DEM. Therefore, when dealing with masonry walls in the HybriDFEM, the user will use a constitutive law relating spring elongations with spring forces, where the stiffness is expressed in N/m and does not depend on the dimensions of each block composing the structure. This approach will be referred to as 'contact law'-approach.

The contact law implemented in the HybriDFEM for modelling masonry structures is a Coulomb friction law, elastic in compression and with zero tensile strength and dilatancy. For the axial direction, it reads:

$$\begin{cases} F_n = k_n \cdot \Delta \mathrm{L_n} & if\ \Delta L_n \leq 0 \\ F_n = 0 & if\ \Delta L_n > 0 \end{cases} \tag{1}$$

where F_n, k_n and $\Delta\mathrm{L_n}$ represent respectively the spring force, stiffness, and elongation in the axial direction, positive in tension. From the axial spring force, the shear force in the spring is obtained. The contact law in the shear direction reads:

$$\begin{cases} F_s = k_s \cdot \Delta \mathrm{L_s} & if\ \mathrm{k_s} \cdot \Delta \mathrm{L_s} \leq \mu \cdot \mathrm{F_n} \\ F_s = \mu \cdot \mathrm{F_n} & if\ \mathrm{k_s} \cdot \Delta \mathrm{L_s} > \mu \cdot \mathrm{F_n} \end{cases} \tag{2}$$

where F_s, k_s and $\Delta\mathrm{L_s}$ represent respectively the spring force, stiffness and elongation in the shear direction, and μ the friction coefficient.

In the original formulation of the HybriDFEM presented in (Bouckaert, Godio et al. 2024), a CF is discretized into a series of contact pairs (CPs) uniformly distributed along the CF (Figure 6, left), and the stiffness of the CF was obtained by assembling the stiffnesses of all its CPs. Since this assembly operation happens at each iteration during the simulation, a reduced number of CPs can drastically improve the simulation time.

When modelling dry-stacked masonry, the response of the structure is mostly dominated by sliding and rocking of the blocks with respect to each other. Therefore, each contact face can be discretized without loss of information in only two CPs, located at both extremities of the CF, with a user-defined offset (Figure 6, right) that can represent rounding of the block edges. This configuration is sufficient to capture the rocking of a block on one of its corners and the relative sliding between two blocks, and reduces the assembly operation of each CF to a minimum.

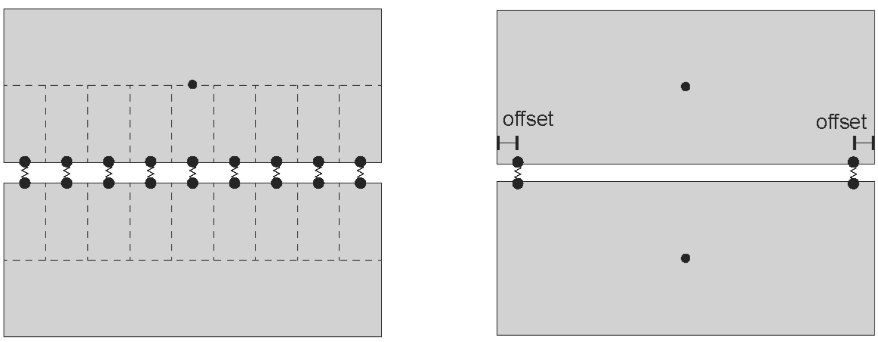

Figure 6. Evenly distributed CPs representing the deformability of their attributed portion of block (left). CPs located only at the extremities of the contact face, with a user-defined offset (right).

5 Validation benchmark

In the benchmark herein considered (Ferris and Tin-Loi 2001), the in-plane capacity of six walls with different configurations is investigated. Each of them is composed of 0.4m x 0.175m bricks, or half-bricks. Four of the walls include one or more openings. Each block is subjected to a constant vertical downwards force corresponding to its self-weight, and an increasing horizontal force which is made proportional to its self-weight by a load factor α to simulate seismic loading. The maximal value of α, corresponding to the maximal in-plane load that the wall can sustain, is called collapse load multiplier.

The goal of this simulation is to obtain the collapse load multiplier for the different geometries, along with the associated collapse mechanism. Throughout the simulation, the value of the load multiplier is progressively increased with a load-control procedure. The collapse load multiplier is the last load increment at which the program has converged to an equilibrium solution. The results obtained with the HybriDFEM model are compared with the results presented in (Ferris and Tin-Loi 2001), where the collapse load multiplier was obtained by means of a limit analysis problem, formulated as a mathematical optimization problem, by a rigid-no tension Coulomb frictional contact, zero dilatancy and a friction coefficient $\mu = 0.65$. Unlike limit analysis, where this coefficient is obtained with a kinematic approach and represents an upper bound of the collapse load multiplier, the solution provided here is obtained with a static approach and represents a lower bound of the actual collapse load multiplier. Therefore, one expects to obtain slightly lower values of the collapse load multiplier from the HybriDFEM as compared to limit analysis.

At its current state of development, the contact law used to model masonry in the HybriDFEM requires the user to set a value of axial stiffness k_n in compression – see eq. (1), and a value of shear stiffness k_s before sliding occurs – see eq. (2), and therefore it is not possible to simulate a perfectly rigid behaviour. For this benchmark, the contact law described hereabove is used with a coefficient of friction of $\mu = 0.65$ and stiffness parameters k_n and k_s both set to 10^7 N/m. The block's specific weight is set to 10 kN/m^2. All the contact faces are created with two CPs located at its extremities (Figure 6, right), and the offset parameter is set to 0 mm, corresponding to sharp corners.

The results obtained with the HybriDFEM in terms of collapse load multiplier are summarized in Table 1. The collapse mechanisms obtained for the six walls are shown in Figure 7, which can be compared with the collapse mechanisms from (Ferris and Tin-Loi 2001) shown in Figure 8. For most cases, the collapse mechanisms clearly display the same opening patterns, even if the effect of the introduction of compressive and shear stiffness is visible, mostly for the walls with openings, which display bending deformations in addition to the joint openings and sliding. The most extreme example of this phenomenon is visible for the third and sixth walls, where the collapse mechanism is different than the one obtained with limit analysis. The portion of the block that should be detaching from the rest of the wall stays in contact with the left part that is deforming in bending. This difference is in the origin of a significantly large difference in terms of collapse load multipliers for these two walls (Table 1).

Table 1. Comparison between collapse load multipliers obtained with limit analysis (Ferris and Tin-Loi 2001) and with the HybriDFEM.

	Limit analysis (Ferris and Tin-Loi 2001, Gilbert, Casapulla et al. 2006)	**HybriDFEM**	**Relative error [%]**
Wall 1	0.643	0.641	-0.3
Wall 2	0.564	0.556	-1.4
Wall 3	0.356	0.298	-16.2
Wall 4	0.264	0.263	-0.3
Wall 5	0.216	0.215	-0.4
Wall 6	0.297	0.174	-41.4

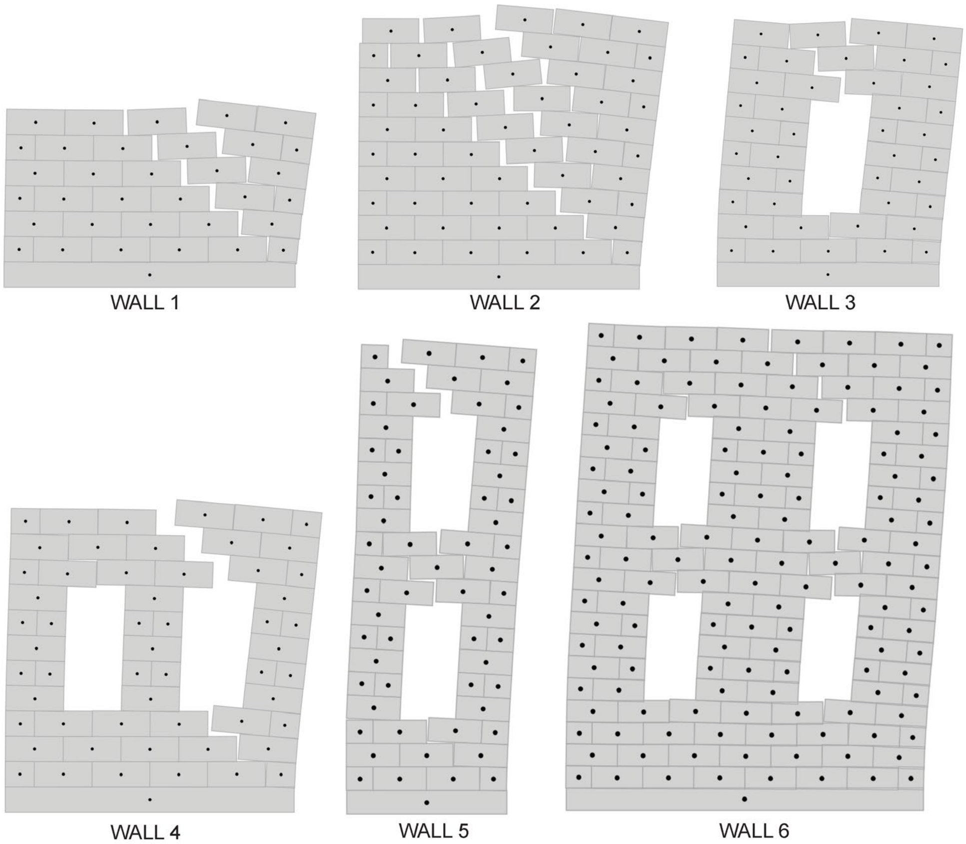

Figure 7. Collapse mechanisms of the six walls obtained with the HybriDFEM.

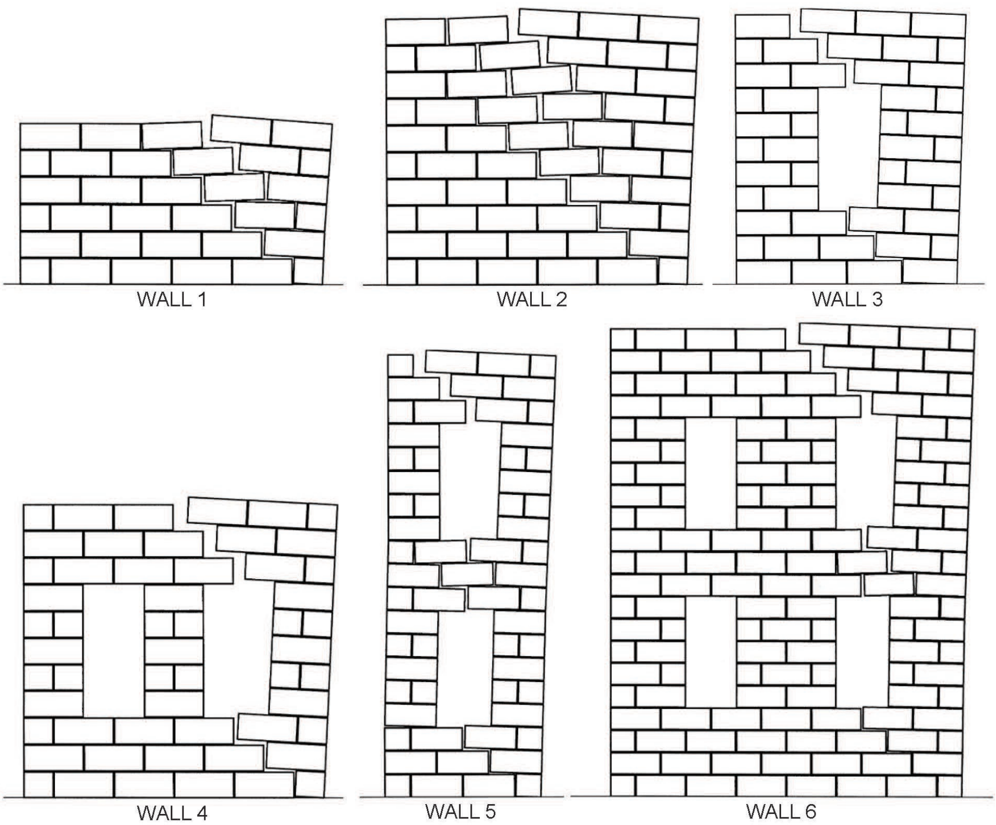

Figure 8. Collapse mechanisms of the six walls obtained with limit analysis (Ferris and Tin-Loi 2001).

6 Conclusions

In this paper, the HybriDFEM, previously presented by the authors to model beams or frames, has been broadened to structural elements extending in both in-plane geometric directions. Whilst the formulation itself, briefly presented in section 2, remains unchanged with respect to the formulation presented for 1D elements (Bouckaert, Godio et al. 2024), an unavoidable need for automatic detection of the contact interfaces arises when extending to 2D members. In Section 3, a simple contact detection algorithm is proposed for detecting edge-to-edge contacts, capturing both partial and full overlap of two block edges. This algorithm is applied on the structure in its undeformed configuration, and no contact update is performed throughout the simulation.

In Section 4, some specific adjustments were made to the method to make it more suited to model unreinforced masonry. The possibility to define a contact law at the interfaces rather than using a material law, as is done in classical DEM, allows for a better description of the interface deformability. Moreover, the response of dry-stacked masonry being mostly dominated by rocking and sliding, uniformly distributed contact pairs along each interface are not necessarily required. Since those imply a computationally heavy assembly operation at each iteration, CFs are modelled more efficiently with only two CPs located at the edges of the contact face.

Finally, a benchmark example was implemented with the HybriDFEM and compared with solutions from limit analysis (Ferris and Tin-Loi 2001). In this example, the in-plane capacity of dry-jointed masonry walls with and without openings was assessed through a load-control procedure. The results obtained with the HybriDFEM compared very well with the limit analysis for four out of six cases, where the difference in the collapse load multiplier was around 1%. However, since a perfectly rigid contact law in compression is not possible with the HybriDFEM, the response of the two remaining walls was influenced by flexural deformations of parts of the wall, resulting in drastically lower collapse load multipliers (-16% and -45%) and different collapse mechanisms than the ones obtained with limit analysis.

Despite the shortcomings of the method described hereabove, the HybriDFEM has proven effective to model the behaviour of strongly discontinuous structures such as dry-stacked masonry. The formulation resembles the formulation of the classical FEM, which allows to readily couple a discontinuous structural element with a continuous one. Contrary to DEM, no time-stepping algorithm is required for the analysis. Further developments of the method will include improvements on the contact laws that can be used to model masonry – e.g., to address the above limitation, the modelling of continuous 2D members such as reinforced concrete walls, and extension of the method to dynamic problems.

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