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Abstract

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PRODUCT INNOVATION AND MARKET ACQUISITION OF FIRMS*

JEAN J. GABSZEWICZ** AND ORNELLA TAROLA***

ABSTRACT

The paper explores the incentives for an incumbent firm to acquire an entrant willing to sell a product innovation, rather than openly compete with this entrant and, in case of acquisition, the incentives to sell simultaneously both the existing products and the new one, rather than specializing on a single variant. We prove that, in some circumstances, an incumbent firm can find it profitable to make an acquisition proposal to the entrant in order to deter entry. Nevertheless, in this acquisition scenario, a product proliferation strategy is never observed at equilibrium. Rather, the incumbent restricts itself to offer either its own variant or the product innovation produced by the entrant, depending on the quality differential existing between them. It follows that, while being available for sale, sometimes the innovation simply remains unexploited[a].

1 Introduction

The paper explores *(i)* the incentives for an incumbent firm to acquire an entrant willing to sell a product innovation, rather than openly compete with this entrant, and *(ii),* in case of acquisition, the incentives to sell simultaneously both the existing products and the new one, rather than specializing on a single variant. These questions are intimately related to the problem of innovation. In the case of product innovation, who, from the incumbent or the inventor, has the stronger incentive to appropriate the benefits expected from its sales? Whatever the answer, will both variants survive in the market? While similar questions have been extensively considered for the case of *process* innovation (see Gallini (1984), Reinganum (1983), Gans and Stern(2000), Gilbert and Newbery (1982), among others), it seems to have been at least partially put aside in the case of *product* innovation. However, process and product innovations do generally go hand in hand, since a technological innovation is often viewed as successful either when it decreases substantially the production costs, and/or when it gives rise to a variant of the product which is unanimously ranked by consumers as being at the top of the existing quality ladder[b].

The questions raised in this paper arise because technological innovators are often different from those who commercialize the products coming out from their inventions. While the former are frequently research-oriented startup firms, the latter are rather market incumbents traditionally specialized in marketing the products and promoting their sales.This specialization of activities opens the door to potential competition between the incumbent(s) and the innovator since the latter can possibly threaten the former to commercialize itself its new variant if the incumbent firm(s) does not accept to pay a sufficiently high price for the right of selling it. The question is whether it would be more profitable for the incumbent to keep its own existing variant without launching in the market the new one, and incurring thereby the costs of competition, or to pay a significant acquisition price to the innovator to let him to delegate its power of launching it. The forces in presence are well described by Gans and Stern (2000): "when startup innovators and established firms cooperate at the commercialization stage, the bargaining power of each party (and thus the distribution of rents) depends, on the one hand, on the ability of the startup to threaten to enter the product market and impose competitive costs on the incumbent and, on the other hand, on the ability of the incumbent to threaten to expropriate the (improved variant resulting from) the startup's technology".

When the potential entrant decides to manufacture its new, higher quality, variant, we enter the world of competition in a *vertically differentiated market*: the incumbents sell the low quality variants while the entrant proposes to consumers a variant which is unanimously ranked by them at the top of the quality ladder. Then competition takes place among a larger number of variants than before entry[c]. By contrast, when the poten-

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[b]Several papers have explored the incidence of entry on competition in vertically differentiated market when competition takes place *via* price strategies; see in particular, Bonisseau J.M. and R. Lahmandi-Ayed (2006), Constantatos, C.and S. Perrakis (1997), Martinez-Giralt (1989) and Donnenfeld. S. and S. Weber (1979)

[c]This statement should be tempered by the fact that, in vertically differentiated markets, the sale of a higher quality product can en-

tial entrant is acquired at some price by an incumbent firm, several alternative scenarios can be observed. Either the acquirer introduces the new product and withdraws its existing variant from the market. Then, neither the number of variants nor the number of firms increase. This is observed in several differentiated markets. Significant examples are the products from the electronics and telecommunications industries, like new laptops or computer screens when faster processors, or screens with higher resolution, are discovered: the new versions enter the market, while their existing counterparts are withdrawn. Similarly, software with improved applications generally replace the old ones which are simply withdrawn[d]. Notice that, offering only the acquired variant enables the incumbent to escape from the erosion of its monopoly profits due to cannibalization.

Or the acquirer can decide to introduce the new acquired product innovation while keeping the existing one on sale. Then, a larger number of variants than before acquisition are sold while the number of firms operating in the market remains the same. This is observed for instance with the persistence of traditional cellular phones after the introduction of I-Pods, or the temporary survival of black and white TV-sets after the introduction of colour TV's in television manufacturing. Even if the number of firms has not increased, one should expect harsher competition in the market among existing variants simply because selling also the new variant increases for the acquirer the danger of cannibalizing its own existing product. Still, selling both the new and the old one could allow for price discrimination between consumers, selling the new, top quality, variant to consumers with higher willingness to pay or higher revenue, and the standard variant to poorer or less motivated consumers.

In order to analyze the incentives of incumbent firms to move along a specific scenario among those which were just evoked, we start focusing on a natural duopoly where a monopolist providing a single variant whose quality is exogenously given is threatened by a potential entrant. When entering the market, this rival offers a new variant whose quality is at the top of the quality ladder. With the aim of deterring entry, the incumbent firm can make an acquisition proposal to this entrant. The latter may either accept, or deny the acquisition proposal. When the acquisition proposal is accepted, the monopolist can decide either to market both its own variant and the product innovation produced by the rival, or restrict itself to offer only one of the two variants. In other words, he decides whether he privileges product proliferation, or product specialisation. As for the innovator, he/she can accept this proposal or turn it down and enter the market via *de novo* entry. We study under which conditions an acquisition agreement is more profitable than open competition to both parties. Would it be the case, we show however that *a product proliferation strategy is never observed at equilibrium.*

To examine how robust is the above conclusion, we then extend the analysis and assume now that the existing market initially consists of two incumbent firms, providing variants of different quality exogenously given, say, *high* and *low.* In line with the above, we assume that a potential entrant contemplates to enter this market with a quality which can be either at the top of the existing quality ladder, or at the intermediate level. In the first case, the new variant constitutes an *absolute product innovation* since it dominates in quality both the existing variants. In this first scenario, the game is as follows. At the first stage, the high quality incumbent firm makes an acquisition proposal to the innovator, who can accept or reject the proposal. In case of acquisition, the buyer decides between product proliferation and product specialization. Otherwise, *de novo* entry is observed. In the second scenario, the new variant constitutes a *relative product innovation* since it dominates in quality the existing low quality variant, but it is still dominated by the variant offered by the other high quality incumbent firm. The game we study in this alternative setting now develops along three stages as we contemplate the chance that even the low-quality incumbent can acquire the entrant, if no acquisition agreement has been reached by the innovator and the high quality incumbent at the first stage of the game. Accordingly,a *de novo* entry can only arise as a third-stage best strategy.

Our main conclusion is that, in some circumstances, an incumbent firm can find it profitable to make an acquisition proposal to the entrant in order to deter entry. Nevertheless, in this acquisition scenario, a product proliferation strategy is never observed at equilibrium. Rather, the incumbent restricts itself to offer either its own variant or the product innovation produced by the entrant, depending on the quality differential existing initially between them. Thus, a surprising corollary of this is that while being available for sale, sometimes the innovation simply remains unexploited!

tail the exit from the market of a lower quality one; see Gabszewicz and Thisse (1980).

[d]These examples are borrowed from Siebert (2002).

2 The case of one incumbent firm

Consider a market with an incumbent firm offering some variant i of a good to a population of consumers identified by the parameter $\theta \in [a,b]$, $0 < a < b$ and $2a < b < 4a$, and uniformly distributed with density equal to 1[e]. The demand model is directly inspired from traditional models of vertical product differentiation (see Mussa and Rosen, 1978; Gabszewicz and Thisse, 1979). Letting u_i denote the quality of variant i, the utility of consumer θ is given by

$$\theta u_i - p_i,$$

with p_i, denoting the price that customers pay for getting the variant provided by firm i. The average cost with respect to quality is assumed to be constant and, without loss of generality, we set it equal to zero. Accordingly, the profit function $\Pi_i(p_i)$ is given by

$$\Pi_i(p_i) = \left(b - \frac{p_i}{u_i}\right) p_i. \tag{1}$$

Maximization of (1) with respect to p_i gives the equilibrium price $\overset{\circ}{p_i} = \frac{bu_i}{2}$. Substituting this price in (1), we obtain the profit at the monopoly equilibrium $\overset{\circ}{\Pi_i}$, namely

$$\overset{\circ}{\Pi_i}\left(\overset{\circ}{p_i}\right) = \frac{u_i b^2}{4\,(b-a)}. \tag{2}$$

Now assume that a potential rival firm has discovered some alternative variant of the product which is of higher quality than the variant proposed by the incumbent: here we are not interested in the innovation process which has led to this product innovation, like in Gallini (1984), Reinganum (1983), Gans and Stern(2000), Gilbert and Newbery (1982), among others. Rather, we concentrate on the threat that this product innovation exerts in the market on the incumbent's profits. In order to bar entry, the incumbent monopolist can make an acquisition proposal to the rival and thereby avoid to openly compete with him/her while getting the innovation produced by the rival. Under acquisition, the incumbent can also decide whether offering both its own variant and the innovation produced by the rival, or destroys one variant, whathever it is. If the acquisition agreement is not reached, then a *de novo entry* takes place, and the two firms compete in price. Accordingly, in this scenario, the incumbent and the entrant get equilibrium profits $\overset{\circ}{\Pi_L}$ and $\overset{\circ}{\Pi_H}$, respectively, given by

$$\overset{\circ}{\Pi_H}\left(\overset{\circ}{p_H},\overset{\circ}{p_L}\right) = \frac{(u_H - u_L)\,(2b-a)^2}{9\,(b-a)} \tag{3}$$

$$\overset{\circ}{\Pi_L}\left(\overset{\circ}{p_H},\overset{\circ}{p_L}\right) = \frac{(u_H - u_L)\,(b-2a)^2}{9\,(b-a)}. \tag{4}$$

We study in a non cooperative game when acquisition is observed at equilibrium, and, if any, whether one or two variants are made available by the acquirer in the market. In other words, we examine when acquisition entails product proliferation.

Proposition 1 *In the case the incumbent decides to acquire the potential entrant, it is never profitable to sell both its own variant and the acquired one simultaneously: only the acquired variant is selected for sale.*

Proof. It suffices to notice that when the two variants H and L are sold, the maximization of monopoly profits $\Pi^{H+L}(p_H,p_L)$, $= (b - \frac{p_H - p_L}{u_H - u_L})p_H + (\frac{p_H - p_L}{u_H - u_L} - a)p_L$ with respect to p_H given that $p_L^* = au_L$, gives the equilibrium price $p_H^* = \frac{1}{2}(bu_H + 2au_L - bu_L)$. Thus, the corresponding equilibrium profits are $\Pi^{*H+L} = \frac{\left(4abu_L + b^2u_H - 4a^2u_L - b^2u_L\right)}{4(b-a)}$. As the difference between this value Π^{*H+L} and the monopoly profits $\overset{\circ}{\Pi_H}$ is equal to $\left(-\frac{1}{4}\right)u_L\,(b-2a)^2$, we conclude immediately that offering only the highest quality variant H is always more profitable than selling both the available variants H and L. **Q.E.D.** ■

Now, it remains to check when acquiring the innovator constitutes the best strategy for the monopolist and the intrant. To this end, it suffices to compare the profits under acquisition at some acquisition price with the profits realized under *de novo* entry. Let $\Pi_H^M = \frac{b^2u_H}{4(b-a)}$ be the monopoly revenues of the incumbent when offering the innovation with quality u_H, and $\overset{\circ}{\Pi_H}\left(\overset{\circ}{p_H},\overset{\circ}{p_L}\right) = \frac{(u_H-u_L)(2b-a)^2}{9(b-a)}$ and $\overset{\circ}{\Pi_L}\left(\overset{\circ}{p_H},\overset{\circ}{p_L}\right) = \frac{(u_H-u_L)(b-2a)^2}{9(b-a)}$ the duopoly revenues accruing to the high quality entrant and the henceforth low quality incumbent, respectively, in the case of *de novo* entry. From the viewpoint of the incumbent, an acquisition proposal turns out to be profitable if and only if the monopoly profits Π_H^M after paying an acquistion price P^a are still higher than the duopoly profits $\overset{\circ}{\Pi_L}$ derived from open competition with the high quality rival. On the entrant's perspective, the acquisition proposal is acceptable if the acquisition price P^a is at least equal to the profits it would obtain via entering the market and openly competing, namely $\overset{\circ}{\Pi_H}$. So let us assume that $P^a = \frac{(u_H-u_L)(2b-a)^2}{9(b-a)}$, which guarantees that the entrant would accept the

[e]This assumption guarantees that competition develops in a natural duopoly framework: exactly two firms, and no more, can make strictly positive profits at an interior price equilibrium.

deal. On the other hand, an acquisition agreement is preferred to a *de novo* entry strategy from the viewpoint of the incumbent iff: $\Pi_H^M - P^a \geqslant \Pi_L^{\circ}\left(p_H^{\circ}, p_L^{\circ}\right)$ or $f(u_H) = \frac{b^2 u_H}{4(b-a)} - \frac{(u_H-u_L)(2b-a)^2}{9(b-a)} - \frac{(u_H-u_L)(b-2a)^2}{9(b-a)} \geq 0$.

The function $f(u_H)$ is strictly positive when $u_H = u_L$. On the other hand, the left-hand term of this expression is a linear strictly decreasing function of u_H in the acceptable range of a and b-values. Consequently, it has a positive root u_H^* which is given by

$$u_H^* = \frac{4\left(5a^2 - 8ab + 5b^2\right)u_L}{(11b - 10a)(b - 2a)}.$$

Thus the function $f(u_H)$ is positive at the left of u_H^*, between u_L and u_H^*,and negative on its right. Accordingly, when $u_L < u_H < u_H^*$, the difference $f(u_H)$ is positive guaranteeing in particular that, when u_H is close to u_L, it is always better to acquire than openly compete. On the contrary, when the quality of the entrant's variant becomes by far larger than the quality proposed by the incumbent, profits under open competition become more important for both parties because they start to constitute "local monopolies", the incumbent specialising on the consumers with a lower willingness to pay and the entrant on the class of those customers with more intense preferences for the good. Also the acquisition price starts to be very high, discouraging the incumbent to acquire. Thus, we conclude that

Proposition 2 *There always exists a nonnull domain of u_H-values $[u_L, u_H^*]$ for which it is advantageous for the incumbent and the entrant to reach an acquisition agreement rather than openly compete. When $u_L < u_H < u_H^*$, acquisition is prefered to open competition while, when the reverse inequality holds, open competition is prefered to acquisition.*

An immediate corollary of the above proposition is that it is *always* better to acquire than to compete for all values of u_H satisfying the inequality $u_H < u_H^*$.

The two above propositions are quite in accordance with intuition. When the product innovation is "weak", in the sense that it does not constitute a substantial improvement with respect to the existing variant, competition could be very harmful to both firms because the two variants are almost homogeneous products and both profits are close to zero under price competition. So there exists a strong incentive both for the incumbent and the innovator to avoid open competition and prefer accordingly acquisition. On the contrary, when the intrant offers a variant which is substantially of much higher quality than the existing one offered by the incumbent, the former has a strong bargaining position with respect to the latter and can require such a high price from him that both firms find more advantageous to opt for a non cooperative behaviour and openly compete. This advantage is even reinforced because, due their local monopoly positions, competition becomes less harmful to the parties.

3 The case of two incumbent firms

It is important to examine the robustness of the above proposition, obtained in the case of a single incumbent. In particular, does this conclusion holds as well when the market is initially shared by *two* incumbent firms simultaneously threatened by the entry of a further competitor? To examine this problem, consider now a covered market with two incumbent firms, say firm H (high quality variant) and firm L (low quality variant), respectively. Profits accruing to these firms write as (3) and (4), respectively. As in the previous setting, we still assume that a potential entrant, firm F, contemplates to enter the market with a variant which can be *a priori* either at the top of the quality ladder (absolute product innovation) or in between the existing variants (relative product innovation). We analyse how the incumbent firms can react to this threat. We start analysing the case when the innovation is at the top of the quality ladder. In this scenario, when entering the market, the entrant can be acquired by the high quality incumbent firm H. Then, we consider the alternative case when the innovation lies in-between the high and the low quality variant. Accordingly, in the case of relative innovation, the innovator can be a *priori* acquired by both by the high quality incumbent and the low quality one.

Of course, equilibrium prices and profits are different functions of the qualities u_H, u_L and u_F (and of the parameters a and b) according as u_F is larger than u_H or in-between u_L and u_H. For sake of generality, we provide below the equilibrium profits for a general three firms-covered market case with three firms $1, 2$ and 3 such that $u_1 \leq u_2 \leq u_3$. This analysis embeds the two scenarios mentioned above, as it is sufficient to replace u_1, u_2 and u_3 with u_L, u_H, u_F (case i) and u_L, u_F, u_H (case ii), respectively, to cover the two possible cases.

It is easy to check that when the three firms compete against each other, given the natural duopoly setting,

revenues at equilibrium write as[f]

$$\Pi_3^*(p_3^*,p_2^*,p_1^*) = \frac{4b^2(u_3-u_1)^2(u_3-u_2)}{(b-a)\left(4u_3-u_2-3u_1\right)^2}$$
$$\Pi_2^*(p_3^*,p_2^*,p_1^*) = \frac{b^2(u_3-u_1)(u_3-u_2)(u_2-u_1)}{(b-a)\left(4u_3-u_2-3u_1\right)^2}$$
$$\Pi_1^*(p_3^*,p_2^*,p_1^*) = 0.$$

Accordingly, in the two cases when either $u_F > u_H > u_L$ or $u_H > u_F > u_L$, the equilibrium revenues accruing to the entrant are strictly positive and expressed by $\Pi_2^*(p_3^*,p_2^*,p_1^*) > 0$ or $\Pi_3^*(p_2^*,p_3^*,p_1^*) > 0$, respectively. Thus, it follows that, in both scenarios, the entrant can profitably enters the market. However, the low quality incumbent is no longer active in the market as $\Pi_1^*(p_3^*,p_2^*,p_1^*) = 0$.

3.1 Absolute product innovation: $u_F > u_H > u_L$

In order to derive the equilibrium path in this scenario, namely, $u_F > u_H > u_L$, we look for a subgame perfect Nash equilibrium in a non cooperative sequential entry/acquisition game which, in the case of absolute innovation, develops along two stages:

1. at the first stage, the incumbent firm H offers to buy the entrant at some price P_H (we set P_H equal to 0 if the firm does not want to acquire firm F) and, in case of acquisition, the buyer decides whether to sell both the variants u_H and u_F, or only one of them;
2. If firm F turns down its offer, at the second stage, it enters the market via *de novo* entry[g].

Solving the game backwards, we start from the second stage and thus consider first when a *de novo* entry can take place in the market at equilibrium. To this end we assume that firm H at the first stage has not reached an acquisition agreement with firm F, which accordingly enters the market if it is a profitable choice. We first check whether, in the case of acquisition, it is profitable for the incumbent to offer both the available variants u_H and u_F, or only one of them, clarifying afterwards under which circumstances acquisition is more profitable than open competition. For future reference, let us write revenues accruing to the three firms under de *novo* entry when the entrant's variant is at the top of the quality ladder:

$$\Pi_F^{entry} = \frac{4b^2\left(u_F-u_H\right)\left(u_F-u_L\right)^2}{(b-a)\left(4u_F-u_H-3u_L\right)^2};$$
$$\Pi_H^{entry} = \frac{b^2\left(u_F-u_H\right)\left(u_H-u_L\right)\left(u_F-u_L\right)}{(b-a)\left(4u_F-u_H-3u_L\right)^2};$$
$$\Pi_L^{entry} = 0.$$

The acquisition scenario: product proliferation vs product specialisation

Under acquisition, the buyer can offer both the variants u_F and u_H, or restrict itself to market only one variant. Of course, in the latter case, it always prefers to offer the entrant's variant u_F as, *ceteris paribus*, the wider the quality difference between variants, the higher the profits at equilibrium. Let us briefly consider profits accruing to the competing firms in both the above evoked cases.

If the incumbent provides both variants u_F and u_H, profit functions $\Pi_H^{F+H}(p_H,p_L,p_F)$ and $\Pi_L^{F+H}(p_H,p_L)$ are given by $\Pi_H^{F+H}(p_H,p_L,p_F) = \left(b-\frac{p_F-p_H}{u_F-u_H}\right)p_F + \left(\frac{p_F-p_H}{u_F-u_H}-\frac{p_H-p_L}{u_H-u_L}\right)p_H$ for the incumbent H and $\Pi_L^{F+H}(p_H,p_L) = (\frac{p_H-p_L}{u_H-u_L}-a)p_L$, for the other incumbent L. Maximization of these two expressions with respect to p_H and p_L, respectively, gives the corresponding equilibrium prices $\check{p}_F$ $\check{p}_H$ and $\check{p}_L$, namely,

$$\check{p}_F = \frac{(2b-a)}{3}(u_H-u_L) + \frac{b}{2}(u_F-u_H)$$
$$\check{p}_H = \frac{1}{3}(2b-a)(u_H-u_L)$$
$$\check{p}_L = \frac{1}{3}(b-2a)(u_H-u_L).$$

Finally, replacing these prices in $\Pi_H^{F+H}(p_H,p_L,p_F)$, we obtain the resulting equilibrium profits Π_H^{*F+H} for the incumbent firm H when introducing both variants in the market, namely,

$\Pi_H^{*F+H} = \frac{9b^2u_F+u_H\left(4a^2-16ab+7b^2\right)+u_L\left(16ab-4a^2-16b^2\right)}{36(b-a)}$. In the case when firm H decides to provide the market only with the entrant's variant u_F, then the profits accruing to the competing firms H and L write as $\Pi_F^+\left(p_F^+,p_L^+\right) = \frac{(u_F-u_L)(2b-a)^2}{9(b-a)}$ and $\Pi_L^+\left(p_F^+,p_L^+\right) =$

[f] See Appendix for details on computing these equilibrium values.

[g] Of course, in the alternative case of relative innovation, the sequential game develops along *three* stages, as a further intermediate stage where the acquisition proposal is made by the low quality incumbent takes place. Thus, if firm F turns down the acquisition offer by the high quality incumbent at the first stage, at the second stage firm L offers to buy firm F at some price P_L and, in the case of acquisition, the buyer decides whether to sell both variants u_L and u_F, or only one of them. Accordingly, a *de novo* entry, if any, can be observed only at the third stage of the game.

$\frac{(u_F-u_L)(b-2a)^2}{9(b-a)}$, respectively. From direct comparison between $\Pi_F^{*F+H}(p_H^*, p_L^*, p_F^*)$ and $\Pi_F^+(p_F^+, p_L^+)$, it immediately follows that:

Proposition 3 *In the case of acquisition, offering only the entrant's quality u_F is more profitable to the incumbent firm H than offering both variants.*

Let us now analyse the incentives to reach an acquisition agreement, with only the innovation u_F to be marketed.

Acquisition vs *de novo* entry

On the one hand, in order to be accepted, the acquisition proposal should yield the innovator a revenue R_H at least equal to the revenue when entering the market via *de novo* entry, namely Π_F^{entry}. On the other hand, it is convenient for the incumbent H to make such a proposal if, and only if, the duopoly profits $\Pi_F^+(p_F^+, p_L^+)$ obtained when acquiring the incumbent L after paying the acquisition price $R_H = \Pi_F^{entry}$ are larger than the profits it would get if the potential entrant would enter the market via a *de novo* entry strategy, namely Π_H^{entry}.

Accordingly, we can state that this acquisition takes place iff the profits that the two firms would get under acquisition are higher than the corresponding profits under a *de novo* entry scenario, or

$$\underbrace{\Pi_F^+ - R_H(=\Pi_F^{entry})}_{\text{Acquisition}} \geqslant \underbrace{\Pi_H^{entry}}_{\text{De novo entry}},$$

with $R_H = \Pi_F^{*entry}(p_H^*, p_L^*, p_F^*) = \frac{4b^2(u_F-u_H)(u_F-u_L)^2}{(b-a)(4u_F-u_H-3u_L)^2}$ and $\Pi_H^{*entry} = \frac{b^2(u_F-u_H)(u_H-u_L)(u_F-u_L)}{(b-a)(4u_F-u_H-3u_L)^2}$.

Let us denote by x the value b/a, l the value $u_H - u_L$, and f the value $u_F - u_H$. Thus, the sign of the difference between profits from acquisition and profits from *de novo* entry $\frac{(u_F-u_L)(2b-a)^2}{9(b-a)} - \frac{4b^2(u_F-u_H)(u_F-u_L)^2}{(b-a)(4u_F-u_H-3u_L)^2} - \frac{b^2(u_F-u_H)(u_H-u_L)(u_F-u_L)}{(b-a)(4u_F-u_H-3u_L)^2}$ has the same sign as the second degree polynomial $P(f,x)$ defined by

$$P(f,x) = \frac{(2x-1)^2}{9} - \frac{4x^2 f(f+l)}{(4f+3l)^2} - \frac{x^2 fl}{(4f+3l)^2},$$

under the assumption that $x \in [2,4]$. Notice that, $\frac{\partial P(f,x)}{\partial x} > 0$ for any $x \in [2,4]$. As $P(f,2) > 0$ we conclude that, the above difference is positive in the admissible range of x and thus acquiring is better than competing.

Proposition 4 *In the case of absolute innovation, the incentive for marketing the innovation by acquisition is stronger than the incentive for marketing it by entry.*

Notice that the above proposition justifies *ex-post* the reason why, in case of absolute product innovation, it is useless to introduce an intermediate stage allowing for acquisition of the entrant by the low incumbent: such an acquisition would never be part of the equilibrium path.

3.2 Relative product innovation: $u_H > u_F > u_L$

It is worth noting that the acquisition strategy turns out to belong to the path of a perfect subgame NE *also in the case when the quality provided by the entrant lies in the middle of the quality ladder.* Let us briefly examine the second and first stages of the sequential game corresponding to this case. First of all, we know that, at the third stage of the game, there is room for a *de novo entry* when the variant sold by the entrant lies in the middle of the quality ladder[h].

Accordingly, let us start by studying the second stage of the game. First, it is easy to prove that in the case of acquisition at the second stage, the low quality incumbent restricts its optimal selling strategy to its own variant u_L rather than to the entrant's variant[i]. Thus, it remains to clarify whether, under the assumption that $u_L < u_F < u_H$, the acquisition agreement between the low quality incumbent and the entrant with u_L only sold in the market is profitable. Notice that, in this scenario, the resulting acquisition price the low quality incumbent has to pay when acquiring the entrant, is lower than the one corresponding to the case when the variant u_F is at the top. Given this, it can be proved that the acquisition agreement turns out to be the second stage best strategy in some circumstances, precisely when the ratio $x = b/a$ is large and the entrant's quality u_F is not different enough from that of one of the two incumbents so as to ensure mild competition and substantial entrant's profits [j]. Indeed, in both the above described cases, one should expect competition after *de novo* entry to be very fierce and lead accordingly to low entrant's profits, privileging an acquisition strategy rather than open competition.

Now, we analyse whether acquisition can take place at the first stage of the game, in the case when it is not observed at the second stage. By applying the same

[h] this would be only prevented when the entrant would be at the bottom of the quality ladder.

[i] Indeed, the larger the gap between the variants in the market and the higher the resulting profits. Further, even in the case when both the variants u_F and u_L would be offered, the price of the low quality variant at equilibrium would still be equal to zero.

[j] See Appendix for details.

argument developed before, one could show that in the case of acquisition, at the first stage of the game offering the variant u_F is no longer profitable, the optimal selling strategy for the high quality incumbent being now to offer only variant u_H. Not even, a product proliferation strategy with both the variants u_F and u_H sold can be observed at equilibrium: in this case, the high quality incumbent would gain some further profits from consumers switching from the low quality variant u_L to the intermediate variant u_F. Nevertheless, it would suffer a loss in revenues from those consumers switching from the high quality variant u_H to the intermediate one. Finally, given this, it can be proved that an acquisition agreement is profitable at the first stage with u_H being sold only, when the ratio x is small or F's quality differs substantially from H's and L's so as to make entry a credible threat. By combining the above findings, we can state the following:

Proposition 5 *In the case of relative innovation, an acquisition agreement, either at the first stage or at the second stage of the game, is always prefered to a de novo entry.*

Proof. See Appendix. ■

Hence, whoever the acquiring incumbent, commercializing the innovation is detrimental for the incumbent's profit. Thus we conclude that:

Proposition 6 *In the case of relative innovation, whenever there is acquisition, the innovation is always left unexploited.*

The above proposition is interesting from a double viewpoint. The first is that, in the case of relative product innovation and acquisition, the improved variant is simply destroyed since neither incumbent is willing to commercialize it! The second is that, under absolute product innovation, the acquirer of the innovative firm decides to sell the product which it has acquired. Acquisition is motivated not only by the fear of harsher competition, but also by the desire to increase the profits by selling the innovated product. By contrast, in the case of relative product innovation, the only purpose for acquiring the entrant is to dampen competition since the acquirer never sells the improved variant!

4 Conclusion

Mergers' activities resulting from firms' specialization through innovative research on the one hand, and commerce and marketing on the other hand, are more and more frequently observed in the life of industries. These mergers seriously affect the trajectories of innovation which depend today on market competition among firms. As stated in the introduction, research and innovation which were performed before inside the firms, are often nowadays delegated to research-oriented startups. The traditional Schumpeterian view of "creative destruction"does not apply anymore in this context since the process of creative destruction supposes that innovation takes place inside the firm, and not be externalized to other economic agents.

On the other hand, by changing the number and characteristics of the variants supplied in the industry, or by reducing the number of competitors, these mergers'activities have substantially altered the nature of competition in the market, compared with the traditional effects of entry under open competition. Indeed, one should expect competition after *de novo* entry to be very fierce and lead to low entrant's profits, privileging accordingly an acquisition strategy rather than open competition. *A priori* preventing entry by acquisition of potential entrants seems to be a natural way for incumbent firms to protect the market against increased competition. Nevertheless, entry prevention has been mainly considered from the viewpoint of price strategies. The effects of these two types of *modus operandi* for preventing entry,-acquisition or price strategies-, should be differentiated. Using price strategies in order to bar entry generally affects competition because it reduces the number of variants, compared with the number of variants which would have existed if entry had been successful. Under acquisition, the outcome is *a priori* unclear. It depends whether the acquirer decides to sell simultaneously both its own product and the product it has acquired, or only a single variant, either its own one or the variant it has decided to acquire. In this paper, we have analyzed this problem under vertical product differentiation in the case of a single incumbent firm facing entry, then extending the analysis to the case of two incumbent firms. Our main conclusion is that an acquisition agreement between the incumbent firm(s) and the potential entrant can indeed be observed at equilibrium. Still, product proliferation is never observed at equilibrium: the acquirer always decides, in case of acquisition, to sell only a single variant. Of course the variant selected depends at which level of the quality ladder is located the quality proposed by the entrant, compared with the qualities previously offered by the incumbents.A surprising conclusion is that, in some circumstances (relative product innovation), the product innovation remains unexploited along the equi-

librium path due to price competition among firms, revealing thereby that market environments can influence innovation trajectories.

Economic theorists were always interested in analyzing how the number and the nature of firms affect competition among them. They started exploring this question in the framework of a homogeneous product market, moving later to the analysis of competition in a world of product differentiation. Nowadays, they notice that firms try to resist to the threat of entry by using strategies of acquisition. The use of such strategies may considerably affect both the competition in the market and the innovation paths, since firms are not necessarily constrained to commercialize the new improved variants of the products, or can delay their sale to benefit from alternative market conditions. This paper constitutes a tentative to explore rigorously some implications of this strategic renewal of firms.

5 Appendix

For sake of generality, we provide below the equilibrium profits for a general three firms-covered market case with three firms $1, 2$ and 3 such that $u_1 \leq u_2 \leq u_3$. Of course, this analysis embeds the two scenarios mentioned above, as it is sufficient to replace u_1, u_2 and u_3 with u_L, u_H, u_F case *(i)* and u_L, u_F, u_H (case *ii*), respectively, to cover the two cases of absolute and relative innovation.

The consumer θ^3 indifferent between being served by firm 3 or 2 at prices p_3 and p_2, respectively, writes as

$$\theta^3 = \frac{p_3 - p_2}{u_3 - u_2},$$

while the consumer θ^2 indifferent between buying variants provided by firm 1 or 2 at prices p_1 and p_2 is given by

$$\theta^2 = \frac{p_2 - p_1}{u_2 - u_1}.$$

Accordingly, the corresponding demand functions $D_3(p_3, p_2)$ and $D_2(p_3, p_2, p_1)$ for the firms $1, 2$ and 3, respectively, are

$$D_3(p_3, p_2) = b - \frac{p_3 - p_2}{u_3 - u_2}$$
$$D_2(p_3, p_2, p_1) = \frac{p_3 - p_2}{u_3 - u_2} - \frac{p_2 - p_1}{u_2 - u_1},$$

and

$$D_1(p_2, p_1) = \frac{p_2 - p_1}{u_2 - u_1} - a,$$

for firm 1. Thus, the respective profits functions write as

$$\Pi_3 = p_3(b - \frac{p_3 - p_2}{u_3 - u_2})$$
$$\Pi_2 = p_2(\frac{p_3 - p_2}{u_3 - u_2} - \frac{p_2 - p_1}{u_2 - u_1})$$
$$\Pi_1 = p_1(\frac{p_2 - p_1}{u_2 - u_1} - a)$$

From the first order conditions, it is easy to identify the following best reply functions

$$p_3 = \frac{1}{2}b\,(u_3 - u_2) + \frac{1}{2}p_2;$$
$$p_2 = \frac{(p_H(u_2 - u_1) + p_F(u_2 - u_2))}{2(u_3 - u_1)};$$
$$p_1 = \frac{(p_L + a\,(u_1 - u_2))}{2}.$$

Thus, solving the above system, we derive the candidate equilibrium prices $\tilde{p}_3$, $\tilde{p}_2$ and $\tilde{p}_1$, namely

$$\tilde{p}_3 = \frac{(u_3 - u_2)((a - 4b)u_1 + 3bu_3 + (b - a)u_2)}{6(u_3 - u_1)};$$
$$\tilde{p}_2 = \frac{(u_3 - u_2)(b - a)(u_2 - u_1)}{3(u_3 - u_1)};$$
$$\tilde{p}_1 = \frac{(u_2 - u_1)(3au_1 + (b - 4a)u_3 + (a - b)u_2)}{6(u_3 - u_1)}.$$

Notice however that due to the natural duopoly assumption $2a < b < 4a$, we must have

$$\frac{a}{b} \geq \frac{u_3 - u_2}{4u_3 - 3u_1 - u_2}, \tag{5}$$

which, in turn, implies that

$$\frac{(3au_1 + (b - 4a)u_3 + (a - b)u_2)}{6(u_3 - u_1)} \leq 0$$

or, equivalently, $\tilde{p}_1 \leq 0$. Accordingly, when the condition : $2a < b < 4a$ is satisfied, then the equilibrium value of p_1 is equal to 0. In that case, the value of best replies of firms 3 and 2 have to be computed against $p_1 = 0$, namely,

$$p_3 = \frac{1}{2}b\,(u_3 - u_2) + \frac{1}{2}p_2;$$
$$p_2 = \frac{(p_3(u_2 - u_1))}{2(u_3 - u_1)}.$$

Solving this system in p_3 and p_2, we get the equilibrium prices p_3^*, p_2^* and p_1^*, namely,

$$p_3^* = \frac{2b(u_3 - u_1)(u_3 - u_2)}{4u_3 - u_2 - 3u};$$
$$p_2^* = \frac{b(u_3 - u_2)(u_2 - u_1)}{4u_3 - u_2 - 3u_1};$$
$$p_1^* = 0.$$

Finally, profits at equilibrium write as

$$\Pi_3^*(p_3^*,p_2^*,p_1^*)=\frac{4b^2(u_3-u_1)^2(u_3-u_2)}{(b-a)\left(4u_3-u_2-3u_1\right)^2};$$
$$\Pi_2^*(p_3^*,p_2^*,p_1^*)=\frac{b^2(u_3-u_1)(u_3-u_2)(u_2-u_1)}{(b-a)\left(4u_3-u_2-3u_1\right)^2};$$
$$\Pi_F^*(p_3^*,p_2^*,p_1^*)=0.$$

Proof of Proposition 5. Let us start with the first part of the Proposition and focus on the second-stage best strategy. To this end, let us remark that at the second stage of the game, an acquisition agreement is preferred to a *de novo* entry strategy iff profits from *de novo* entry are lower than profits from acquisition after paying the acquisition price which is equal to zero (as the low quality firm can no longer be active in the market in the case of *de novo* entry, and thus its post-entry profits are equal to zero), namely:

$$\frac{b^2\left(u_H-u_L\right)\left(u_L-u_F\right)\left(u_F-u_H\right)}{(b-a)\left(3u_L-4u_H+u_F\right)^2}-\frac{\left(u_H-u_L\right)(b-2a)^2}{9\left(b-a\right)}<0.$$

Let us denote by Δ the value u_H-u_L, and by γ the value u_H-u_F. The sign of the above difference has the same sign as the expression

$$\frac{\gamma(\Delta-\gamma)}{(3\Delta+\gamma)^2}-\frac{1}{9}(x-2)^2, \tag{6}$$

given that we have assumed $x\in[2,4]$, in order to ensure that two, and only two, firms can make positive profits in this market. Denote by γ^- and γ^+ the roots of the second-order polynomial $P(\gamma)$ defined by

$$P(\gamma)=9\gamma(\Delta-\gamma)-(3\Delta+\gamma)^2(x-2)^2.$$

Notice that *(i)* the sign of $P(\gamma)$ is the sign of (6) , and that *(ii)* $P(\gamma)$ is strictly negative for all γ whenever $-61+64x-16x^2>0,\Leftrightarrow x\in[\frac{\sqrt{3}}{4}+2,4]$. The polynomial $P(\gamma)$ has two roots equal to, respectively : $\gamma^+=3\Delta\frac{\left(-\frac{5}{2}+4x-x^2+\frac{\sqrt{3}}{2}\sqrt{-61+64x-16x^2}\right)}{-4x+x^2+13}$ and $\gamma^-=3\Delta\frac{\left(-\frac{5}{2}+4x-x^2-\frac{\sqrt{3}}{2}\sqrt{-61+64x-16x^2}\right)}{-4x+x^2+13}$). Then, we can state the following:

(i) whenever $x\in[\frac{\sqrt{3}}{4}+2,4]$, it is more profitable to acquire, whatever the value of γ;

(ii) when $x\in[2,\frac{\sqrt{3}}{4}+2[$ it is more profitable to acquire whenever $\gamma\in[0,\gamma^-)$ or $\gamma\in(\gamma^+,\Delta]$, and to enter whenever $\gamma\in(\gamma^-,\gamma^+)$. The two options are indifferent to each other when $\gamma=\gamma^-$ or $\gamma=\gamma^+$.

Now, let us move to analyse the second part of the proposition and consider the first-stage best strategy. At the first stage of the game, an acquisition agreement is preferred to a de novo entry strategy if, and only if $\frac{4b^2(u_H-u_F)(u_L-u_H)^2}{(b-a)(3u_L-4u_H+u_F)^2}+\frac{b^2(u_H-u_L)(u_L-u_F)(u_F-u_H)}{(b-a)(3u_L-4u_H+u_F)^2}-\frac{(u_H-u_L)(2b-a)^2}{9(b-a)}<0$. The sign of this difference is the same as the sign of the expression

$$\frac{4\gamma x^2\Delta}{(3\Delta+\gamma)^2}+\frac{x^2\gamma(\Delta-\gamma)}{(3\Delta+\gamma)^2}-\frac{(2x-1)^2}{9}.$$

Simple calculations reveal that the above expression is always negative in the range of admissible values ($\frac{\gamma}{\Delta}\in[0,1]$, $x\in[2,4]$). **Q.E.D.**

References

1. Bonnisseau J.M. and R. Lahmandi-Ayed (2006). "Vertical differentiation: multiproduct strategy to face entry?", *Topics in Theoretical Economics*, 6 (1), 15.
2. Constantatos, C. and S. Perrakis (1997). Vertical differentiation: entry and market coverage with multiproduct firms, *International Journal of Industrial Organization*, 16(1), 81-103.
3. Donnenfeld S. and S. Weber (1995). Limit qualities and entry deterrence, *RAND Journal of Economics*, 26 (1), 113-130.
4. Gabszewicz, J.J. and J. Thisse (1979). "Price Competition, quality and income disparities", *Journal of Economic Theory* , 2 (3), 240-259.
5. Gallini, N.T. (1984). Deterrence by Market Sharing : a Strategic Incentive for Licensing. *American Economic Review*, 74 (5), 931-94.
6. Gans, J.S. and S. Stern (2000).Incumbency and R&D incentives : Licensing the "Gale of Creative Destruction". *Journal of Economics and Management Strategy,* (9) 485-511.
7. Gilbert, R. and D. Newbery (1982), Preempting Patenting and the Persistence of Monopoly. *American Economic Review*, 73 (4), 741-748.
8. Martinez-Giralt, X. (1989). On brand proliferation with vertical differentiation, *Economics Letters*, 30 (4), 279-286.
9. Mussa, M. and S. Rosen (1978). "Monopoly and product quality", *Journal of Economic Theory,*18, 301-317.
10. Siebert, S. (2003). The Introduction of New Product Qualities by Incumbent Firms : Market Proliferation versus Cannibalization, SP. 2003-11, Social Research Center, Berlin.

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