

Florian Marion*

Change and Location: A New and Old Case against Functionality

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Abstract: In this paper, I shall discuss the question whether a concrete object can be multi-located while it is moving or not. I shall say nothing on the vexed issue of multi-location in and for itself. Instead, my discussion will support a ‘might’-conditional claim: ‘if multi-location were possible, then change might imply multi-location’. To do this, after a very short clarification of the various meanings of ‘to be located’, I will first present and discuss Diodorus’ arguments against the reality of motion, since they focus on the question of what the location of the moving item is, and then scrutinize Hegel’s reply to Diodorus’ reasonings, insofar as his answer consists in claiming that an object in motion is in many locations at once. This paper aims to explore the various metaphysical possibilities concerning the logic of location underlying change.

Keywords: metaphysics; theories of location; metaphysics of change; logic; change; metaphysics of space

In this paper, I shall discuss the question whether a concrete object can be multi-located while it is moving or not.

I shall say nothing on the vexed issue of multi-location in and for itself. Indeed, the debate is subtle and difficult (see Kleinschmidt 2011; Costa and Calosi 2020): there are plenty arguments in favour or against the metaphysical possibility of multi-location, but I believe that none is decisive enough to win the race. Therefore, my discussion will support a ‘might’-conditional claim: ‘if multi-location were possible, then change might imply multi-location’. In other words, I wish to offer a case-study for multi-location.

Compliance with Ethical Standards: I would like to inform you that the research presented in this paper is free of any conflict of interest and did not involve human or animal participants.

***Corresponding author: Florian Marion**, Institut Supérieur de Philosophie/Centre De Wulf-Mansion, Université catholique de Louvain, Place du cardinal Mercier 14, bte L3.06.01, 1348 Louvain-la-Neuve, Belgium, E-mail: florian.marion@uclouvain.be. <https://orcid.org/0000-0002-0234-4435>

To do this, after a very short clarification of the various meanings of ‘to be located’ (§1), I will first present and discuss Diodorus Cronus’ arguments against the reality of the state-of-being-in-motion, since they focus on the question of what the location of the moving item is (§2), and then scrutinize Hegel’s reply to Diodorus’ reasonings, insofar as his answer consists in claiming that an object in motion is in many locations at once (§3). Although many philosophers of the past are referred to, this paper does not aim to be a piece of scholarship, but to explore the various metaphysical possibilities concerning the logic of location underlying change.

1 Logics of Location and the Meanings of ‘Being Localized’

Logics of location are extensions of mereotopology which is itself an extension of mereology: while mereology studies the relation of parthood ‘*x* is a part of *y*’ (see Cotnoir and Varzi 2021), mereotopology adds the relation of connection ‘*x* is connected to *y*’ (see Casati and Varzi 1999: 51–115), and logics of location add the relation of localization ‘*x* is located in *y*’ (see Perzanowski 1993, Casati and Varzi 1999: 117–167, Hudson 2006: 97–122, Varzi 2007: 1012–1030 and Parsons 2007).

What does ‘being localized at/in’ mean? There are many sorts of location: exact location, whole location, partial location, weak location, and so on. An item is exactly located somewhere iff the item has exactly the same shape and size as this location, that is, the item exactly fills up its exact location and shares with it the same locative relations to other items; an item is wholly located somewhere iff each part of the item is located in this region (for instance, Brocéliande Forest is wholly located in Brittany); an item is partly located somewhere iff at least one part of the item is exactly located in this region (Brittany is partly located in Brocéliande Forest); and an item is weakly located iff at least one part of the item is located in a part of this region (in other words: iff the location is not completely free of the item, see Parsons 2007: 203; e.g., the Vilaine river is weakly located in Brittany, its source being in Mayenne).

Amongst the most disputed issues in theories of location is the question whether it is metaphysically possible for an object to have many exact locations or not.¹ The usual examples of putatively multi-located objects in the literature focus, first, on abstract universals or properties that are simultaneously instantiated in many concrete objects and thus possess many spatial locations, and second, on tri-dimensional persisting objects that fully exist at various temporal locations (viz.

¹ The other hot spot is the question whether a mereological atom can be exactly located in a non-zero extended region – a view held in ancient times by Diodorus Cronus whom we speak to soon.

enduring items that persist by being wholly present at many times).² I will not offer any insight about these cases. Rather, I will propose another hitherto neglected case in which multi-location seems to be a live option.

2 Diodorus Cronus' Arguments and the Location of the Changing Thing

Ancient dialecticians, from Zeno to Aristotle, have worried a lot about the existence and the nature of motion (there are not alone: in the East, Gōngsūn Lóng, Nāgārjuna and Sēngzhào too have dialecticalized a lot about motion³). I will not discuss the well-known Zeno's propaganda against the reality of change, but another unfortunately less-known argument from Diodorus Cronus who was a distinguished dialectician of the Megaric Circle and an heir of Eubulides the famous inventor of both the Liar and the Sorites (on Diodorus Cronus, see Sedley 1977). Diodorus' reasonings are quite different from Zeno's ones insofar as they deal only with the topic of the *location* of the changing thing ('where is the moving item?') and say nothing at all about *time* or *measure* as does the famous Zeno's Arrow for instance.

The two arguments of Diodorus share the same valid structure, they are *modi tollens* in which the consequent of the conditional premiss is a disjunction (that is: $A \rightarrow (B \vee C)$, $\neg B, \neg C \vdash \neg A$). Here the textual testimonies of the two arguments (for historical and philosophical in-depth discussions on Diodorus' arguments against motion, see Marion 2023: 93–325 and Duncombe 2023):

Diodorus-1 argument:

The fact that motion will not exist, as Diodorus showed when threatening of indivisible places and bodies. For the partless body contained in the first indivisible place does not move; for it is contained in the partless place and fills it up. And again: the body situated in the second place does not move, for it has moved already. But if the moving object neither moves in the first place – inasmuch as it exists in the first – nor yet in the second, and besides these no third place is conceived, then that which is said to move does not move. (Sextus Empiricus, *AM* 10.143, translation from Bury 1936: 283)

² Other items that are sometimes taken to be multi-located are the followings: omnipresent beings as God, 'Parfitian' fruits of a fission or a teletransportation, time travellers who meet themselves young or old, transworld objects, etc.

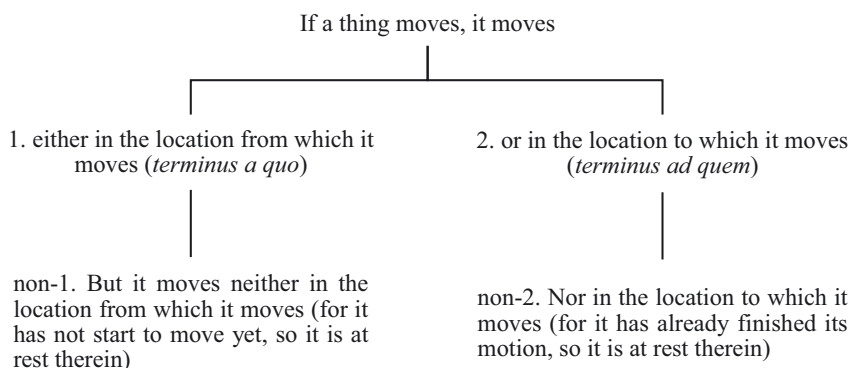
³ For Gōngsūn Lóng, see Chan 1963: 235 and Fung 2009: 170–182. For Nāgārjuna, see Garfield 1995: 6–9, 35–36, Ganeri 2001: 63–66 and Westerhoff 2009: 129–152. For Sēngzhào, see Chan 1963: 344–350.

Diodorus' strengthened argument or Diodorus-2 argument:

If a thing moves, it moves either in the place where it is or in that where it is not; but it moves neither in the place where it is (for it remains therein) nor in that where it is not (for it does not exist therein); therefore nothing moves. (Sextus Empiricus, *AM* 10.87, translation from Bury 1936: 253)

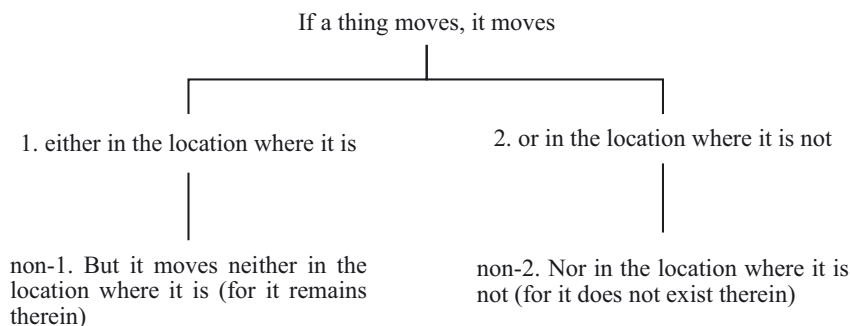
These arguments can be represented by means of the following diagrams.

- *Diodorus-1* argument (*modulo* the atomist/indivibilist background that is not relevant for our discussion – in fact, Diodorus' reasoning only presupposes that any item of any size occupies an exact location):



Therefore nothing moves

- *Diodorus' strengthened argument (Diodorus-2):*



Therefore nothing moves

Let summarize the ancient-but-not-old debate between Diodorus, Plato and Aristotle about the location of the moving item.

All protagonists agree with Diodorus' claim that if an item is exactly located somewhere, it does not move therein. The usual <bad> objection to this thesis consists in pointing out that a *rotating* object (like Kripke's spinning disk, see Lewis 1986: xiii n.5, Casati and Varzi 1999: 176–177, Sider 2001: 226–236 and Kleinschmidt 2017: 196–197) moves in its exact location, but then Diodorus' arguments successfully apply to the parts of the rotating object: they do not move in their exact locations, see *PH* 3.72, *AM* 10.51–52, 93, 103–104 (cf. Casati and Varzi 1999: 176). The claim that nothing moves in its exact location might be reformulated as follows: if an item and each of its parts are exactly located somewhere, neither the whole nor the parts move. I shall ignore such a refinement thereafter, given that the rotating items lay themselves open to Diodorus' reasonings and, therefore, are not counter-examples but innocuous poorly complexified cases.

Diodorus-1 argument (*PH* 3.73, *AM* 10.106–107, 120, 143): a moving item has an exact location, either it is exactly in its *terminus a quo* or exactly in its *terminus ad quem*, but it moves neither in the location from which it moves (for it has not start to move yet) nor in the location to which it moves (for it has already finished its motion), therefore it does not move. It cannot be nowhere (rejection of the excluded-middle), nor can it be exactly in two locations at once (because that implies a contradiction, see Leibniz's *Pacidius Philalethi*, A VI.3 545.6–24 in which multi-location is taken to imply a contradiction and *Theoria Motus Abstracti*, A VI.2 265.24–266.4 in which multi-location does not imply any contradiction – I come back to this issue soon). Thus, since there remains no viable option, nothing can move.

Plato's and Aristotle's answer (respectively from *Parmenides*, 138b7–139b3 and *Physics*, VI.4 234b10–20, 10 240b20–241a6: they basically offer the same argument as Diodorus-1 argument but develop a mereological alternative, so I do not quote these texts on which see White 1992: 69–72 and Marion 2023: 210–225): rather, a moving item has two partial locations, it is partially in its *terminus a quo* and partially in its *terminus ad quem*, and therefore it moves across them. The moving item is *partially* in the two neighbouring places. (A simple way to object to Diodorus-1 argument and Plato's and Aristotle's answer is to state that, according to Standard Analysis and Cantor-Dedekind axiom, there are no neighbouring locations or directly successive locations in a densely ordered continuum; therefore, there is no motion from a location to the next adjacent one. This objection does not rule out Diodorus-2 argument however).

Strengthened or extended Diodorus' argument (in fact, the *original* argument of Diodorus from *PH* 2.242–243, 245, 3.71–75, *AM* 1.309–312, 10.48, 85–120, 344–347, see Priest 2017: 225–226 for a similar reasoning): a moving item has an exact location, but nothing moves in its exact location ('where it is'), and therefore it does not move

(given that it is quite obvious that it cannot move where it is not, in a location where it does not exist). Yet, if a moving item is partially in its *terminus a quo* and partially in its *terminus ad quem*, then its exact location is the union or fusion of its two partial locations, therefore a moving item has an exact location, and so it does not move.

Plato and Aristotle: <awkward silence>

(Aristotle's second answer (from *Physics*, VIII.8): a moving thing is *actually* nowhere, but it is *virtually* somewhere. In other words, a moving thing is neither nowhere nor somewhere: its exact location is only a *virtual* one.

I shall not discuss *Aristotle-2* because the issue becomes modal, and I have not in this paper the space to *modalize* the discussion. For a thorough study of Diodorus' argument and the replies of Plato and Aristotle, see Marion 2023: 93–325 + White 1992; Duncombe 2023).

A subtlety in Diodorus' thought is that, while he denies that any item can *be moving*, he allows that an item can *have moved* however (*PH* 3.71-75, *AM* 10.85-120). Diodorus is a good and gifted dialectician: while he carefully argues for some theses ('nothing is moving because there is no location where an item could be moving'), he does not go beyond what is the right and proper conclusion of his arguments (and so, he does not follow Zeno's strong and unmotivated immobilism in claiming that 'nothing was moved, nothing is moving, and nothing will move', see Russell 1903: §327 for the right 'Diodorean' conclusion of Zeno's propaganda).

For Diodorus, and given Greek language, the verb 'to move' can be true at the 'complete/achieved' or 'perfective' aspect, but never at the 'incomplete/unachieved' or 'continuative/durative' one (*mutatis mutandis*: our gerundive). Thus, while the Eleatic followers of Parmenides and Zeno support that change and motion do not exist whatever the meaning of 'exist' is, Diodorus only argues for the weaker claim that there is no *real* transition from a state to another, but he does not contest the fact that there are many variations: the same item which was formerly here is elsewhere now, so, although there is no time at which it was moving between them, it has moved from the first location to the second however; in other words the fact that the object's position varies with time indicates that there has been a motion, even if there has not been any real transitional process from a position to another (in Diodorus' words: "it must *have moved* since what had earlier been seen to be in one place is now later seen to be in a different one, which would not have happened if it had not moved" *AM* 10.86).

In fact, Diodorus' theory of motion is very similar⁴ to the now orthodox account called 'at-at theory of motion' or 'cinematographic theory of motion' developed by

4 'Very similar' not 'identical', Diodorus' underdetermined theory draws a large spectrum of conceptual possibilities from Epicurus' atomism to Stoic and al-Nazzām's leaps, from Damascius' shifts to Ockham's and Russell's at-at theory, see Marion 2023: 191–201.

Russell 1903: §327, §§332–333, §§442–447 (on the relation between Diodorus' kinetics and Russell's theory, see Marion 2023: 191–201). Nowadays, everybody who agrees with Diodorus' propaganda stands with those who believe that change is nothing but the following description (the now received view): an object x has changed if and only if there is some predicate Φ that is true of x at a time but not true of x at some later time, and there is no transition (i.e. no state of *changing*) from Φ to non- Φ ; or this other definition of physical motion: for anything x to move, at least some parts of x have to be exactly present at different locations at different times. Change is mere variation, nothing more (*contra* Aristotle). Diodorus' arguments are not sophisms as some have believed but genuine puzzles for the processualist metaphysician who thinks that there is a real transitional process in-between the *terminus a quo* and the next *terminus ad quem*, i.e., who defends that there is some intrinsic state/property that distinguishes a moving object from an unmoved one: if so, Diodorus asks 'where is the moving item while it is moving?'.

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Sketchy formalization of the dialectical discussion

There are many kinds of location relations (in general, I borrow the notation from Casati and Varzi 1999). To simplify notation, I shall often drop universal quantifiers at the beginning of formulas, they are to be understood as universally closed however.

- *Parthood relation*. ' $x \leq y$ ' means ' x is a part of y '. The parthood relation is reflexive, transitive (and so, parthood is a preorder) and antisymmetric,
- *Exact location*. ' Lxy ' means ' x is exactly located in y ',
- *Whole location*. ' $WLxy$ ' means ' x is wholly located in y ',
- *Weak location*. ' $GLxy$ ' means ' x is weakly located in y '
- *Partial location*. ' $PLxy$ ' means ' x is partly located in y ',
- *Motion*. ' Mxy ' means ' x is moving in y ',

' $Mx \oplus yz$ ' could be intuitively understood as ' x is moving from y to the next location z ', but strictly speaking it means ' x is moving in the fusion of y and z '. The mereological fusion operator $\oplus$ is the usual binary, commutative, associative and idempotent sum-operator.

For convenience, I will take exact location as a primitive. I cover the technical issues involved by such a choice with the cloak of charity, especially the fact that if exact location is taken as a primitive, then, obviously, *Exactness* – the idea that every located object has an exact location – logically follows (see Parsons 2007: 205–210 and Costa and Calosi 2020). This fact should not bother us insofar as this paper will not discuss *Exactness* in length, but rather another principle called

Functionality (which is often itself a logical truth when only weak location is taken as primitive in the theory, see Parsons 2007: 205 and the discussion of Costa and Calosi 2020).

With L as a primitive, WL , GL and PL can be defined as follows:

- $WLxy =_{df.} \forall z[z \leq x \rightarrow \exists u(u \leq y \wedge Lzu)]$
- $GLxy =_{df.} \exists z\exists u(z \leq x \wedge u \leq y \wedge Lzu)$
- $PLxy =_{df.} \exists z(z \leq x \wedge Lzy)$

I do not sketch any axiomatization of the logic of location, because, first, almost all usual axioms from Casati and Varzi 1999 or Parsons 2007 give rise to fine-grained controversy (in a way, the formal discussions about location are reminiscent of Carnap's pluralist motto: "in logic, there is no morals" Carnap 1937: xiv-xv, 51–52),⁵ and second, everything I am about to say is compatible with a wide range of axiomatizations (for a general logic of location that is *neutral*, at least, with respect to the disputed axioms of *Exactness* and *Functionality*, see Correia 2022).

The main assumption shared both by Diodorus, Plato and Aristotle is that if an item x is exactly located in y then it does not move in y , i.e., $Lxy \rightarrow \neg Mxy$. All the burden of Diodorus' arguments is carried by this theorem.

Diodorus' two arguments, formalized in the framework of logics of location, are as follows:

- Diodorus-1* $Mx \oplus yz \rightarrow (Lxy \vee Lxz), \neg Lxy, \neg Lxz \vdash \neg Mx \oplus yz$ (provided that y and z are two distinct immediately adjacent locations of the same size of x)
- Diodorus-2* $\exists y Mxy \rightarrow [\exists z(Lxz \wedge Mxz) \vee \exists z(\neg GLxz \wedge Mxz)], \neg \exists z(Lxz \wedge Mxz), \neg \exists z(\neg GLxz \wedge Mxz) \vdash \neg \exists y Mxy$

The solution that Plato and Aristotle bring to *Diodorus-1* is formulated as follows:

- Plato & Aristotle* $Mx \oplus yz \rightarrow [(PLxy \wedge PLxz) \wedge \forall u(PLxu \rightarrow (u \leq y \vee u \leq z))]$ (provided that y and z are two distinct immediately adjacent locations of the same size of x)

And the reply of Diodorus to *Plato & Aristotle*:

- Diodorus' reply* $[(PLxy \wedge PLxz) \wedge \forall u(PLxu \rightarrow (u \leq y \vee u \leq z))] \rightarrow Lx \oplus yz, Lxy \rightarrow \neg Mxy \vdash [(PLxy \wedge PLxz) \wedge \forall u(PLxu \rightarrow (u \leq y \vee u \leq z))] \rightarrow \neg Mx \oplus yz$

(For the record, here is the modal solution that Aristotle finally offers to the challenge of the location of the moving item:

⁵ *Weak Expansivity*, namely $\forall x\forall y\forall z\forall u[(x \leq y \wedge Lxz \wedge Lyu) \rightarrow z \leq u]$, seems to be one of the rare uncontroversial axioms, see Parsons 2007: 224 however.

Aristotle-2 $Mx \oplus yz \rightarrow \exists u[u \leq \oplus yz \wedge (VLxu \wedge \neg ALxu)]$ where ‘VLxy’ means ‘x is virtually exactly located in y’ and ‘ALxy’ means ‘x is actually exactly located in y’)

3 Multi-Location and Change: Hegel’s Reply to Diodorus

Despite the fact that they argue in the framework of theories of location, no ancient dialectician had seriously discussed the possibility of multi-location in the case of change. Surely, it was a philosophical prejudice (but the eyes of Greek thinkers were darkened by a lot of philosophical *a priori* blind-spots, from the mundane area of non-morality of slavery to the heavenly sector of the conceivability of post-Cantorian actual infinite). It was not until Hegel’s discussion of Zeno’s aporias that this possibility was finally briefly examined and vindicated by some.

Before going any further into this, it should be noted that *being multi-located* need not be identical to *being in motion*. At most, I would like to argue that *being multi-located in a certain way* might be the same as *being in motion*. Indeed, the qualification ‘in a certain way’ (without further specification) is required: not all multi-located objects are in motion (as, for the dialetheist physicist, not every contradictory object is moving), for instance, the property of redness is multi-located in various concrete objects without being in motion, a time-traveller can be at rest in various spatiotemporal locations (see Kleinschmidt 2017),⁶ and, to give a last example, Porphyry told us that Pythagoras was once seen simultaneously at different places without moving at either one of them (namely: Metapontum in Italy and Tauromenium in Sicily, see *Life of Pythagoras*, §27). For someone who defends that motion entails multi-location is not committed at all to the converse implication that multi-location entails motion (as, for the dialetheist who supports that motion entails being in a contradictory state, but not the converse implication that being in a contradictory state entails being in motion, for he believes, for instance, that the Liar-sentence ‘this sentence is false’ is in an alethic contradictory state – both true and false – without believing that this sentence is moving). To sum up, albeit there are many ways to be multi-located (one for universal properties, one for

⁶ Kleinschmidt 2017 argues that, in some special conditions, the case of a multi-located time-traveller who is exactly located and at rest in many distinct regions poses a threat to the ‘at-at’ account of motion famously defended in Russell 1903. Her point is that neither to be in a different location in different times, nor to be multi-located is a sufficient condition for being in motion. But, in fact, everyone agree. The point is rather the reverse, that is, whether being in motion is a sufficient condition for being multi-located or not.

transtemporal entities, etc.), only one kind of multi-location is specific to change, viz., that of the same item (*proviso* relevant specifications and fine-grained characterizations: for instance when both the *objective* and the *subjective* times are taken into account, cf. Kleinschmidt 2017: 192–193) which has one and only one exact location at the beginning and at the end of the process, but many in the *on-going* of it.

There are some ‘intuitive’ axioms in the theories of location that are assumed by some logicians and rejected by others, amongst them the principles of *Functionality* (Casati and Varzi 1999: 121, Parsons 2007: 205–206, 219–220, 228–229, Cotnoir and Varzi 2021), *Exactness* (Parsons 2007: 205–210) and Locke’s Principle of Unicity of Occupation (*EHU*, II.27, §1, axiom sometimes called *Injectivity of Location*). The first says that an item has only one exact location, the second that if an item is weakly located it has an exact location, the last that an exact location can have only one occupant. *Functionality* prevents multi-location, and *Locke’s Principle* precludes co-location.

- *Functionality*. If x is exactly located in y and x is exactly located in z , then y is identical to z , i.e. $\forall x \forall y \forall z [(Lxy \wedge Lxz) \rightarrow y = z]$
- *Locke’s Principle*. If x is exactly located in y and z is exactly located in y , then x is identical to z , i.e. $\forall x \forall y \forall z [(Lxy \wedge Lzy) \rightarrow x = z]$
- *Exactness*. If there is some y such that x is weakly located in y , then there is some z such that x is exactly located in z , i.e. $\forall x (\exists y GLxy \rightarrow \exists z Lxz)$
- *Part-Whole Exactness*. If there is some y such that x is wholly located in y , then there is some z such that x is exactly located in z , i.e. $\forall x (\exists y WLxy \rightarrow \exists z Lxz)$

As we will see, Hegel’s view could be interpreted as to imply the rejection of one or another of these axioms, in such a way that the Hegelian account of motion might be appraised as an indirect rationale for favouring one or the other logic of location.

Hegel was one of the rare philosophers to have taken Diodorus’ puzzle at face value. In his *Lectures on the History of Philosophy*, while discussing Zeno’s antimobilist propaganda, he writes in a style that reminds us of Diodorus’ prose:

“If we wish to make motion clear to ourselves, we say that the body is in one place and then it goes to another; because it moves, it is no longer in the first, but yet not in the second; were it in either it would be at rest [= *Diodorus-1*].

Where then is it? If we say that it is between both, this is to convey nothing at all, for were it between both, it would be in a place, and this presents the same difficulty [= *Diodorus’ reply to Plato & Aristotle*].

But movement means to be in this place and not to be in it, and thus to be in both alike [= dialetheist answer to Diodorus' two aporias]." (Hegel 1833: 313–314, translation from Haldane 1892: 273–274)

In the *Greater Logic*, Hegel summarizes his view as follows:⁷

External sensuous motion itself is contradiction's immediate existence. Something moves, not because at one moment it is here and at another there, but because at one and the same moment it is here and at another here, because in this 'here', it at once is and is not. The ancient dialecticians must be granted the contradictions that they pointed out in motion; but it does not follow that therefore there is no motion, but on the contrary, that motion is *existent* contradiction itself. (Hegel 1831b: 76, translation from Miller 1969: 440)

Hegel's words can be interpreted in many ways (see Marion 2023: 162–163, 190n.4). Each of these readings is a way of neutralizing *Diodorus-2* by supporting an acceptable reading for $Mxy \rightarrow \neg \exists [(z \in y \wedge Lxz) \wedge \neg u(Lxu \rightarrow u = z)]$, namely by arguing for (at least) one of the disjunct of $Mxy \rightarrow \neg \exists [(z \in y \wedge Lxz) \wedge \neg u \wedge \neg v [(u \in v \wedge v \in u) \wedge (Lxu \wedge Lxv)]]$.

- either the moving item is vaguely or fuzzy located, it has a weak or whole location but no exact location (i.e. $Mxy \rightarrow [WLxy \wedge \forall z(z \leq y \rightarrow \neg Lxz)]$), that entails the rejection of *Exactness* (if so, exact location cannot be the sole primitive in the logical theory, WL or GL needs to be a primitive too) and the metaphysical claim that there is some irreducible ontological vagueness in the world (for instance at Planck scale). This position is advocated by some infinitesimalist metaphysicians as White 1985: 271–275, 1992: 284–326 (who interprets Stoic kinetics with Non-Standard Analysis) and Reeder 2015 (who uses Smooth Infinitesimal Analysis to answer to Zeno's Arrow), and, perhaps, by some dialetheists as Priest 1985, 2006: 172–181, 213–220;
- or the moving item has more than one exact location (i.e. $Mxy \rightarrow \exists u \exists v [(u \leq y \wedge v \leq y) \wedge (Lxu \wedge Lxv) \wedge (u \not\leq v \wedge v \not\leq u)]$), viz. it is multi-located, that rules out *Functionality*. This position is held by Bradley 1883: I, ch.2 §11 n.19 ("if the parts be discrete, then not only will motion imply that a thing is in several places in one time (and this is a fact), but also...") and Priest 1985, 2006: 175–181. Those who support this claim also believe that if an item is multi-located then it is moving, and thus *being in motion* and *being multi-located (in a certain way)* are taken to be synonymous (or, in Carnapian words, they are L-equivalent);
- or both, the moving item is vaguely multi-located. Such a case can be intuitively illustrated by the blurred streaking of a moving object in a photograph (the so-called 'motion blur' effect): first, the object in motion appears to be located in

⁷ Some parts of these two texts from Hegel are also quoted in Priest 2006: 176–177.

many regions; second, no region is clear enough or sufficiently well determined to be an exact location of the moving object (see Correia 2022: 571–574 for a brief logical account of such an *inexact* multi-location).

Both exegeses can be read as implying a dialetheist stance according to which a moving item occupies a contradictory location (the orthodox reading of Hegel's words according to Hegel himself, Engels 1878: 111–112 and McTaggart 1896: 84 + Routley et al. 1982: 62, Priest and Routley 1989a: 77, 1989b: 522) provided that:

- either *being fuzzy* implies *being in a contradictory state* (Priest 2010, 2013 who argue that some cases of vagueness share the same deep structure with the self-referential paradoxes as the Liar, namely the *Inclosure Schema*, and therefore imply inconsistency), i.e. $\forall z[(WLxy \wedge (z \leq y)) \rightarrow (Lxz \wedge \neg Lxz)]$, that entails the recovery of *Exactness* (in virtue of Conjunctive Simplification $Lxz \wedge \neg Lxz \vdash Lxz$) but the rejection of *Functionality* (moreover, it can be noted that, since *Exactness* is a logical truth when exact location is taken as the only primitive location-relation, any logician who wishes to take exact location as a unique primitive and to allow the possibility of fuzzy location need to take a paraconsistent stance beforehand);
- or *being here and elsewhere* at once is the same either as, on the one hand, *being here and not-being here*, and, on the other hand, *being elsewhere and not-being elsewhere*, or as *being somewhere and being nowhere* (Hegel himself, Leibniz A VI.3 545.6–24, Łukasiewicz 1910: 159–162, Priest 1982, 1985, 2006, Routley & Hyde 1992: 279), i.e. $\forall y\forall z[(Lxy \wedge Lxz \wedge z \leq y \wedge y \leq z) \rightarrow [(Lxy \wedge \neg Lxy) \wedge (Lxz \wedge \neg Lxz)]]$. The basic idea of Hegel seems to be that *passing over x* implies *not-not-being at x* and so *being at x*, while *not-resting at x* implies *not-being at x* (it would be elsewhere, at an arbitrary z); therefore, for all y, if y passes throughout x, y is at x and y is not at x at once.

Hegel's dialetheism about motion (a view greatly expanded and argued for by Priest 1985, 2006: 159–181, 213–220, 295–299, 2017) is not surprising at all given the well-known fact that, at the very beginning of Hegel's *Logic* (Hegel 1830a: 188–195, 1831a: 83–115), Becoming (of which physical motion is a concretion) puts away (*aufheben*) both Being and Nothing – that is, Becoming is the unity that cancels-and-preserves the opposite Being and Nothing. And so, every process is in itself the expression of the original contradiction between being and not-being.

Of course, the inference from vague or multi-location to dialetheism can be easily resisted, for neither $\forall z[(WLxy \wedge (z \leq y)) \rightarrow (Lxz \wedge \neg Lxz)]$ nor $\forall y\forall z[(Lxy \wedge Lxz \wedge z \leq y \wedge y \leq z) \rightarrow [(Lxy \wedge \neg Lxy) \wedge (Lxz \wedge \neg Lxz)]]$ are mandatory for axiomatizing a credible theory of location (for instance, a logician can avoid *Exactness* by taking weak location as a primitive instead of exact location). And

so, the Hegelian account of motion needs not to be a dialetheist one (*contra* Leibniz, Hegel, Łukasiewicz and Priest themselves⁸): a body can be *spread* out over a non-null interval or be in many locations at once without instantiating any contradiction. To borrow a familiar and most-used example, while walking through the door and momentary standing in the doorway, you're not indoor and not indoor at once (which would be instantiating a contradictory location), but both indoor and outdoor, that is, you're in many locations at the same time: multi-location needs not entail inconsistency (*pace* Priest 1982: 252, 2006: 161, 2017: 5 and Weber 2021: 285).

Hegel does not argue for multi-location. His reasoning is that, by Diodorean lights, the moving item cannot be either fully in its *terminus a quo* nor fully in its *terminus ad quem*, and it cannot be in the *in-between* these two places either (because, still by Diodorean lights, it would exactly be where it is and, so, would not move). Therefore, the only option that remains is that the moving item is fully in both places at once. If such a reasoning forms its main justification, the rationale for multi-location would be rather weak. Priest offers a stronger and more convincing motive for multi-location in case of change: the multi-location of the moving item is the sole idea that satisfactorily explains its *progress* through all the intermediary places of the cinematic trajectory (Priest 1985, 2006: 180, 2017: 225). Indeed, at each instant of its journey, the moving item occupies an interval of space greater than itself (more precisely: it occupies many spaces of the same size as itself, the sum of which forms a space greater than itself) which can be paired with a non-zero progress rather than only one space strictly equal to itself that is associated with a null-sized progress (as it is the case in the cinematographic theory called 'at-at theory of motion' from Russell 1903: §327, §§332–333, §§442–447). Thus, the moving item makes an *advance* or *progress* during each single instant, in such a way that multi-location answers to Zeno's Arrow.

Indeed, Zeno argues that nothing can move because a moving item cannot make any progress at any step of its so-called motion (at each step, it occupies only one exact location *that has the same size as itself*⁹ and so, according to $Lxy \rightarrow \neg Mxy$ and *Functionality*, it does not move). In other words, Zeno points out that if there were any motion, such a so-called *going somewhere* would absurdly be a sum of *going nowhere*. From the standpoint of measure-theory, Zeno's Arrow states that, for a non-degenerate period $[t_0, t_n]$ (with either $n \in \mathbb{N}$ or $n \in \mathbb{R}$) during which an

⁸ To be fair with Priest, the dialetheic stance about motion is argued for in Priest 1982, 2006: 159–171 in which Priest is above all concerned with the puzzle of the instant of change and the Leibniz Continuity Condition (LCC), rather than in his development of the 'Spread Hypothesis'.

⁹ κατὰ τὸ ἴσον, see *Physics*, VI.9 239b5–7 and Themistius, in *Phys.* 199.4–11. The fact that at every step of its journey, the arrow is *in front of a space that has a size equal to itself* is the main point in historical Zeno's Paradox (see Marion 2023: 125–149 for a discussion and bibliographical notes).

item is moving, the measure of the *union* of the degenerate intervals of the cinematic path (i.e. $\mu(\{t_0, \dots, t_n\})$ where μ is a function that measures the length of the processual progress) is not equal to the *sum* of the measures of these degenerate intervals (since $\forall t_k \in \{t_0, \dots, t_n\}, \mu(\{t_k\}) = 0$ because at each instant $t \in [t_0, t_n]$ the moving item occupies only one exact location that has the same size as itself, and so no progress is made). Thus, Zeno's Arrow claims that motion does not satisfy a principle of additivity (for instance, the σ -additivity if $n \in \mathbb{N}$, see Skyrms 1983, White 1992: 8–14, 199–201, Priest 2006: 174–175):

$$\mu\left(\bigcup_{k=0}^n \{t_k\}\right) \neq \sum_{k=0}^n \mu(\{t_k\})$$

since

$$\mu\left(\bigcup_{k=0}^n \{t_k\}\right) > 0 \quad \text{and} \quad \sum_{k=0}^n \mu(\{t_k\}) = 0$$

Such an additivity principle is recovered with multi-location, since at each step of its journey, the moving item would be located in many locations in such a way that it is in a *locus* greater than itself, i.e. $\forall t_k \in \{t_0, \dots, t_n\}, \mu(\{t_k\}) > 0$ since $\mu(\{t_k\}) = \alpha \sigma(a)$ where $\sigma(a)$ is the size of the item a and α a numerical *positive* constant (perhaps, intuitively, the positive integer of locations where the item is exactly located at while it is moving). The successful reply to Zeno's Arrow vindicates the Hegelian account of motion and the multi-location hypothesis.

It should be remarked however that, though multi-location offers a clever solution to both Zeno's Arrow and *Diodorus-1* argument, multi-location fails to answer to some strengthened Diodorus' argument (since Diodorus' arguments use only $Lxy \rightarrow \neg Mxy$ and do not allude to *Functionality* at all). Indeed, multi-location entails an important subtlety, viz. a fine-grained distinction between *being entirely located at* and *being wholly located at* (see Parsons 2007: 211–212): for an arbitrary item x , to be *entirely* in y means that everywhere disjoint from y is free of x (i.e. $ELxy =_{df.} \forall z[z \not\leq y \rightarrow \neg \exists u(u \leq x \wedge Luz)]$), but to be *wholly* in y means that none of x is missing from y (i.e. $WLxy =_{df.} \forall z[z \leq x \rightarrow \exists u(u \leq y \wedge Lzu)]$). Therefore, unlike uniquely-located objects for which whole and entire locations coincide, a multi-located item is *wholly* located in each of its exact locations, but *entirely* located only in the fusion of its exact locations. Yet, Diodorus claims that nothing moves *where it is*, and, of course, a multi-located object is where it is, namely it is located at each of its exact locations of which the mereological sum forms its entire location. Since that, Diodorus can react to Hegel and the supporters of multi-location by endorsing the thesis that if an item has many exact locations, then it is entirely located in the fusion of them,

i.e. $\exists y \exists z [((Lxy \wedge Lxz) \wedge (y \not\leq z \wedge z \not\leq y) \wedge \forall u (Lxu \rightarrow (u = y \vee u = z))) \rightarrow ELx \oplus yz]$, and by adding the claim that nothing moves in its entire location, i.e. $ELxy \rightarrow \neg Mxy$.¹⁰

Of course, a partisan of multi-location could reply that, precisely, *moving* and *changing* are intensionally identical to *being multi-located in a certain way* (that is, *being multi-located in a certain way* is the instantaneous intrinsic property that allows to distinguish a moving item from an unmoved one; perhaps, following the suggestion from Priest 1985: 345, 2006: 178, the ‘length’ of the multi-location might be tied to another property of the item, namely its velocity). In doing so, the supporter of multi-location is in position to reject the universal validity of $ELxy \rightarrow \neg Mxy$ and, therefore, is able to block Diodorus’ machinery (see Priest 2017: 225–226 who meets a similar Diodorean/Zenonian objection by drawing the L-equivalence for concrete items between *changing* and *being in a contradictory state in a certain way*, in order to rule out $(Lxy \wedge \neg Lxy) \rightarrow \neg Mxy$).

4 Conclusion

In this paper, I have tried to show, first, that, to challenge Diodorus’ arguments, any account of change needs to clarify its underlying logic of location, and, second, that a plausible analysis of change might involve, as Hegel once suggested, that motion entails multi-location.

Abbreviations

DIOGÈNE LAËRCE

DL

De vitis, dogmatis et apophthegmatis eorum qui in philosophia claruerunt

LEIBNIZ, GOTTFRIED WILHELM

A

Deutsche Akademie der Wissenschaften zu Berlin (ed.). 1923–?. *Sämtliche Schriften und Briefe*, Berlin.

LOCKE, JOHN

EHU

An Essay Concerning Human Understanding

SEXTUS EMPIRICUS

AM

Adversus Mathematicos

PH

Pyrrhoniae hypotyposes

¹⁰ To remain neutral on the question whether multi-location is a conceptual possibility or not (amongst other aims), Correia 2022 develops a general theory of location which uses a slightly modified variant of entire location as primitive. Needless to say, this general theory has neither *Exactness* nor *Functionality* as axioms.

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