Fair Beamforming Design for MIMO ISAC Systems

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***Abstract*—In this paper, a fair beamforming scheme for multi-target multi-user Integrated Sensing and Communication (ISAC) applications is designed. In the considered scenarios, the ISAC system simultaneously receives radar echoes and communication signals from other ISAC systems. The considered optimisation objective is the Signal-to-Interference-plus-Noise Ratio (SINR) product, as the product metric naturally ensures fairness among the users and targets. This fairness, as well as the low complexity of the designed algorithm, makes it a good candidate for ISAC tracking applications and codebook design. Numerical analyses are performed, demonstrating the beamforming scheme behaviour in the presence of multiple targets and communication users. The benefit of the SINR-product metric over the SINR-sum is moreover emphasised, together with the robustness of the designed scheme against tracking inaccuracies and codebook discretisation.**

***Index Terms*—Beamforming, radar, communication, ISAC, DFRC, duplex, uplink**

I. Introduction

In traditional networks, the frequency bands of communication and sensing are separated. Yet, the frequencies considered for emerging networks make it possible to jointly design both functions. Such a paradigm is known as Integrated Sensing And Communication (ISAC), and it is viewed as a key enabler for emerging wireless applications [1]. Indeed, it potentially enables to improve the spectrum, hardware, and energy efficiency with respect to disjoint systems. These opportunities nevertheless come with challenges among others in the transmit and receive beamformer design. In multi-device ISAC systems, this design must fairly mitigate the interference occurring between communication users, between radar targets, and across users and targets.

In [2], [3], an uplink communication signal is received together with a radar echo, the beamformers being able to nullify the interference between both systems. Motivated by complexity reduction, [4] and [5] provide suboptimal beamformers, in a downlink scenario with a maximisation of one target Signal to Interference-plus-Noise Ratio (SINR), constrained by the SINR of communication and other targets. In [6], the radar Cramér-Rao Bound (CRB) is minimised with SINR constraints on the User Equipments (UEs). The beamformers are either obtained in closed-form in the single-user case, or the problem is relaxed and solved with numerical methods for multi-user systems. The radar CRB is also considered in [7] and minimised under a communication sum-rate constraint. In [8], an iterative algorithm optimises the radar power and communication rates while ensuring a low self-interference in a full-duplex scenario.

In order to optimise the performance related to all communication users and radar targets, one possible paradigm is to consider one metric as an objective with the others set as constraints [4], [6], [7]. Still, chosing adequate constraint thresholds is challenging and may require multiple iterations to ensure sufficient performance for both functions. Alternatively, a weighted-sum of the radar and communication objectives can be performed [8]. However, such formulation might totally neglect one objective in favour of the others. This issue is tackled in [9], in which a fair optimisation of the downlink communication and radar performances is performed. The proposed approach requires nonetheless to solve a sequence of convex optimisation problems and the computational burden this implies might be incompatible with the real-time tracking mechanisms usually associated with radar beamforming schemes. In summary, to our best knowledge, schemes able to fairly optimise the transmit and receive beamformers in a full-duplex multi-user multi-target scenario, with controlled numerical complexity, are currently lacking.

Based on the aforementioned works, the contributions of this paper are summarised as follows:

- The product of the radar and communication SINRs of all users and targets is selected as a performance metric to optimise multi-user multi-target ISAC systems. Such a metric is free from constraint thresholds and objective weights, and naturally promotes fair solutions as it is heavily impacted by the lowest SINRs.
- Beamformers are designed to optimise this SINR-product metric. Depending on the number of targets and users, the required assumptions, numerical complexity and optimality (global or local) of the algorithms are analysed.
- The developed procedures are numerically applied to an ISAC scenario, illustrating the beamforming design schemes when there are multiple targets and users. Additionally, a comparison between the SINR-sum and product metrics is performed, highlighting the superiority of the proposed metric in terms of fairness. The impact of an angle mismatch on the metric is also analysed to evaluate the robustness of the designed scheme against tracking inaccuracies and codebook discretisation.

To that aim, the system model is presented in Section II. Then, beamforming designs are detailed in Section III, for various

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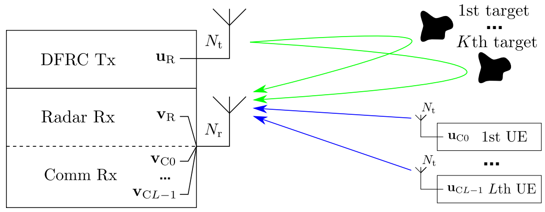

Fig. 1. ISAC uplink system model.

numbers of targets and UEs. Finally, the numerical analysis is described in Section IV.

II. System Model

The scenario of Figure 1 is considered. A $N_t \times N_r$ MIMO ISAC system simultaneously receives the radar echoes from K single-point targets, and uplink communication signals from L UEs, also equipped with N_t transmit antennas. The self-interference remaining after the analog suppression step is assumed to be negligible compared to the uplink signal and radar echoes power. Considering frequency-flat communication channels, the received signal thus reads as

$$\begin{aligned}\mathbf{r}(t) = &\sum_{k=0}^{K-1} \alpha_{\mathrm{R}k}\, e^{j2\pi f_{\mathrm{DR}k} t}\, \mathbf{a}_{\mathrm{Rr}k}\, \mathbf{a}_{\mathrm{Rt}k}^{\mathrm{H}}\, \mathbf{x}_\mathrm{R}(t-\tau_{\mathrm{R}k}) \\ &+ \sum_{l=0}^{L-1} \alpha_{\mathrm{C}l}\, e^{j2\pi f_{\mathrm{DC}l} t}\, \mathbf{a}_{\mathrm{Cr}l}\, \mathbf{a}_{\mathrm{Ct}l}^{\mathrm{H}}\, \mathbf{x}_{\mathrm{C}l}(t-\tau_{\mathrm{C}l}) + \mathbf{n}(t),\end{aligned} \tag{1}$$

with the R and C subscripts referring to the radar and communication functions, and $\cdot^{\mathrm{H}}$ the Hermitian operator. The channel coefficients are denoted by $\{\alpha_{\mathrm{R}k}, \alpha_{\mathrm{C}l}\}$, the Doppler frequencies by $\{f_{\mathrm{DR}k}, f_{\mathrm{DC}l}\}$, the transmit and receive steering vectors by $\{\mathbf{a}_{\mathrm{Rt}k}, \mathbf{a}_{\mathrm{Ct}l}\}$ and $\{\mathbf{a}_{\mathrm{Rr}k}, \mathbf{a}_{\mathrm{Cr}l}\}$, the radar signal at each transmit antenna by $\mathbf{x}_\mathrm{R}(t)$, the communication signals at the UE transmit antennas by $\{\mathbf{x}_{\mathrm{C}l}(t)\}$ and the noise at the receive antennas by $\mathbf{n}(t)$. When omitted, the spans of k and l in the notation $\{\cdot\}$ should be understood as $l = 0, \dots, L-1$, $k = 0, \dots, K-1$. The radar steering vectors are functions of the targets' angles of departure and arrival $\{\theta_{\mathrm{Rt}k}\}$ and $\{\theta_{\mathrm{Rr}k}\}$ and similarly for the communication ones. For readibility, the dependency of the steering vectors on the angles is omitted below. The transmitted signals are defined as $\mathbf{x}_\mathrm{R}(t) = \mathbf{u}_\mathrm{R}\, x_\mathrm{R}(t)$ and $\mathbf{x}_{\mathrm{C}l}(t) = \mathbf{u}_{\mathrm{C}l}\, x_{\mathrm{C}l}(t)$, $\forall l = 0, \dots L-1$, with x_R and $\{x_{\mathrm{C}l}\}$ the radar and communication signals. The $L+1$ complex transmit beamforming vectors which must be designed to optimise the ISAC performance are denoted by $\mathbf{u}_\mathrm{R}$ and $\{\mathbf{u}_{\mathrm{C}l}\}$. On the receive side, $\mathbf{r}(t)$ is processed differently for the radar echo and for each communication stream.

a) Radar processing: For the radar part, a unit-norm complex receive beamforming vector $\mathbf{v}_\mathrm{R}$, common to all targets, is applied, and the output is then processed by a matched filter. This leads to the following signal, from which the delays and Doppler frequencies of all targets can be estimated,

$$\begin{aligned} y_\mathrm{R}(\tau, f_\mathrm{D}) &= \int_t \mathbf{v}_\mathrm{R}^{\mathrm{H}} \mathbf{r}(t)\, x_\mathrm{R}^*(t-\tau)\, e^{-j2\pi f_\mathrm{D} t}\, \mathrm{d}t, \\ &= \sum_{k=0}^{K-1} \alpha_{\mathrm{R}k}\, \mathbf{v}_\mathrm{R}^{\mathrm{H}} \mathbf{a}_{\mathrm{Rr}k} \mathbf{a}_{\mathrm{Rt}k}^{\mathrm{H}} \mathbf{u}_\mathrm{R}\, \chi_{\mathrm{RR}k}(\tau, f_\mathrm{D}) \\ &\quad + \sum_{l=0}^{L-1} \alpha_{\mathrm{C}l}\, \mathbf{v}_\mathrm{R}^{\mathrm{H}} \mathbf{a}_{\mathrm{Cr}l} \mathbf{a}_{\mathrm{Ct}l}^{\mathrm{H}} \mathbf{u}_{\mathrm{C}l}\, \chi_{\mathrm{CR}l}(\tau, f_\mathrm{D}) + w_\mathrm{R}(\tau, f_\mathrm{D}), \end{aligned} \tag{2}$$

where $\chi_{\mathrm{RR}k}$ is the radar ambiguity function centered at $(\tau_{\mathrm{R}k}, f_{\mathrm{DR}k})$, $\chi_{\mathrm{CR}l}$ the correlation between the l^{th} communication waveform and the radar waveform, and w_R an additive white Gaussian noise with power σ_R^2. The radar detection probability for the k^{th} target is directly related to the SINR at the correct delay and Doppler frequency candidates [10]. Assuming independent channel coefficients, the k^{th} target SINR is computed as

$$\mathrm{SINR}_{\mathrm{R}k} = \frac{\beta_{\mathrm{RR}kk} \left|\mathbf{v}_\mathrm{R}^{\mathrm{H}} \mathbf{a}_{\mathrm{Rr}k}\right|^2 \left|\mathbf{u}_\mathrm{R}^{\mathrm{H}} \mathbf{a}_{\mathrm{Rt}k}\right|^2}{I_{\mathrm{RR}k} + I_{\mathrm{CR}k} + 1}, \tag{3}$$

with $I_{\mathrm{RR}k}$ and $I_{\mathrm{CR}k}$ the interferences arising respectively from the other targets and uplink signals:

$$I_{\mathrm{RR}k} = \sum_{\substack{k'=0 \\ k' \neq k}}^{K-1} \beta_{\mathrm{RR}k'k} \left|\mathbf{v}_\mathrm{R}^{\mathrm{H}} \mathbf{a}_{\mathrm{Rr}k'}\right|^2 \left|\mathbf{u}_\mathrm{R}^{\mathrm{H}} \mathbf{a}_{\mathrm{Rt}k'}\right|^2, \tag{4}$$

$$I_{\mathrm{CR}k} = \sum_{l=0}^{L-1} \beta_{\mathrm{CR}lk} \left|\mathbf{v}_\mathrm{R}^{\mathrm{H}} \mathbf{a}_{\mathrm{Cr}l}\right|^2 \left|\mathbf{u}_{\mathrm{C}l}^{\mathrm{H}} \mathbf{a}_{\mathrm{Ct}l}\right|^2, \tag{5}$$

where $\beta_{\mathrm{RR}k'k} \triangleq \mathbb{E}\left[|\alpha_{\mathrm{R}k'}|^2\right] |\chi_{\mathrm{RR}k'}(\tau_{\mathrm{R}k}, f_{\mathrm{DR}k})|^2 / \sigma_\mathrm{R}^2$, and $\beta_{\mathrm{CR}lk} \triangleq \mathbb{E}\left[|\alpha_{\mathrm{C}l}|^2\right] |\chi_{\mathrm{CR}l}(\tau_{\mathrm{R}k}, f_{\mathrm{DR}k})|^2 / \sigma_\mathrm{R}^2$.

b) Communication processing: For the communication part, each stream is processed separately. The l^{th} stream is a complex pulse train defined as $x_{\mathrm{C}l}(t) = \sum_n I_{nl}\, g(t-nT)$ with I_{nl}, the n^{th} transmitted symbol and g, the pulse shaping filter, assumed to be such that its autocorrelation fulfils the Nyquist Inter-Symbol Interference (ISI) criterion. A unit-norm complex receive beamforming vector $\mathbf{v}_{\mathrm{C}l}$ is applied for each stream, and the output is then processed by a matched filter, leading to the following signal, assuming a perfect synchronisation:

$$\begin{aligned} y_{\mathrm{C}l}[n] &= \int_t \mathbf{v}_{\mathrm{C}l}^{\mathrm{H}} \mathbf{r}(t+\tau_{\mathrm{C}l})\, e^{-j2\pi f_{\mathrm{DC}l} t}\, g^*(t-nT)\, \mathrm{d}t, \\ &= \sum_{k=0}^{K-1} \alpha_{\mathrm{R}k}\, \mathbf{v}_{\mathrm{C}l}^{\mathrm{H}} \mathbf{a}_{\mathrm{Rr}k} \mathbf{a}_{\mathrm{Rt}k}^{\mathrm{H}} \mathbf{u}_\mathrm{R}\, \chi_{\mathrm{RC}kl}[n] \\ &\quad + \sum_{l'=0}^{L-1} \alpha_{\mathrm{C}l'}\, \mathbf{v}_{\mathrm{C}l}^{\mathrm{H}} \mathbf{a}_{\mathrm{Cr}l'} \mathbf{a}_{\mathrm{Ct}l'}^{\mathrm{H}} \mathbf{u}_{\mathrm{C}l'}\, \chi_{\mathrm{CC}l'l}[n] + w_{\mathrm{C}l}[n]. \end{aligned} \tag{6}$$

Again, $\chi_{\mathrm{RC}kl}$ and $\chi_{\mathrm{CC}l'l}$ are the waveform correlations, and $w_{\mathrm{C}l}[n]$ is a white noise with power $\sigma_{\mathrm{C}l}^2$. It should be noted that since a perfect synchronisation is assumed, $\chi_{\mathrm{CC}ll}[n] \propto \delta[n]$, i.e., no ISI occurs. The SINR of the decision variable for the l^{th} user is therefore computed as

$$\mathrm{SINR}_{\mathrm{C}l}=\frac{\beta_{\mathrm{CC}ll}\,\left|\mathbf{v}_{\mathrm{C}l}^{\mathrm{H}}\mathbf{a}_{\mathrm{Cr}l}\right|^{2}\left|\mathbf{u}_{\mathrm{C}l}^{\mathrm{H}}\mathbf{a}_{\mathrm{Ct}l}\right|^{2}}{I_{\mathrm{RC}l}+I_{\mathrm{CC}l}+1},\tag{7}$$

with $I_{\mathrm{RC}l}$ and $I_{\mathrm{CC}l}$ the interference arising from the radar signals and other uplink signals:

$$I_{\mathrm{RC}l}=\sum_{k=0}^{K-1}\beta_{\mathrm{RC}kl}\,\left|\mathbf{v}_{\mathrm{C}l}^{\mathrm{H}}\mathbf{a}_{\mathrm{Rr}k}\right|^{2}\left|\mathbf{u}_{\mathrm{R}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Rt}k}\right|^{2},\tag{8}$$

$$I_{\mathrm{CC}l}=\sum_{\substack{l'=0\\l'\neq l}}^{L-1}\beta_{\mathrm{CC}l'l}\,\left|\mathbf{v}_{\mathrm{C}l}^{\mathrm{H}}\mathbf{a}_{\mathrm{Cr}l'}\right|^{2}\left|\mathbf{u}_{\mathrm{C}l'}^{\mathrm{H}}\mathbf{a}_{\mathrm{Ct}l'}\right|^{2},\tag{9}$$

where $\beta_{\mathrm{CC}l'l}=\mathbb{E}\left[\left|\alpha_{\mathrm{C}l'}\right|^{2}\right]\left|\chi_{\mathrm{CC}l'l}[n]\right|^{2}/\sigma_{\mathrm{C}l}^{2}$, and $\beta_{\mathrm{RC}kl}=\mathbb{E}\left[\left|\alpha_{\mathrm{R}k}\right|^{2}\right]\left|\chi_{\mathrm{RC}kl}[n]\right|^{2}/\sigma_{\mathrm{C}l}^{2}$.

III. Transmit and Receive Beamforming Design

In order to design the $L+1$ transmit beamforming vectors, and $L+1$ receive ones, we are interested in maximising the product of the radar and communication SINRs [11]:

$$\mathcal{M}\triangleq\prod_{k=0}^{K-1}\mathrm{SINR}_{\mathrm{R}k}\cdot\prod_{l=0}^{L-1}\mathrm{SINR}_{\mathrm{C}l},\tag{10}$$

the optimisation problem reading thus as

$$\begin{aligned}\max_{\mathbf{u}_{\mathrm{R}},\mathbf{v}_{\mathrm{R}},\{\mathbf{u}_{\mathrm{C}l},\mathbf{v}_{\mathrm{C}l}\}}\ &\mathcal{M}\\ \text{s.t. }&\|\mathbf{u}_{\mathrm{R}}\|_2=1,\ \|\mathbf{v}_{\mathrm{R}}\|_2=1,\\ &\|\mathbf{u}_{\mathrm{C}l}\|_2=1,\ \|\mathbf{v}_{\mathrm{C}l}\|_2=1,\quad\forall l=0,\dots L-1.\end{aligned}\tag{11}$$

Metric $\mathcal{M}$ offers the advantage of fairly considering all targets and communication users, since they all impact in the same manner the objective function. Furthermore, any low factor heavily penalises such a product, and consequently this formulation prevents the targets and UEs with low SINRs to be set aside without requiring to define any weight or threshold arbitrarily. At high SINR, it is moreover equivalent to a sum-rate maximisation [12]. The above optimisation problem is either performed with an ISAC system in tracking mode, or for a discrete parameter codebook. Thus, the target and communication system parameters are available for the optimisation. The impact of estimation errors in case of the tracking system, and discretisation errors in the codebook design, are analysed in Section IV. Table I describes the beamforming design schemes providing a solution to (11), for different numbers of targets and UEs. These schemes differ in terms of required assumption, achieved optimality, and numerical complexity, and they are detailed below.

a) Dimensionality reduction: As shown in [6], the optimal transmit beamforming vectors $\mathbf{u}_{\mathrm{R}}$ and $\{\mathbf{u}_{\mathrm{C}l}\}$ (resp. $\mathbf{v}_{\mathrm{R}}$ and $\{\mathbf{v}_{\mathrm{C}l}\}$) belong to the transmit (resp. receive) steering vectors space. The optimal solutions can thus be written in the form

$$\begin{aligned}\mathbf{u}_{\mathrm{R}}&=\mathbf{S}_{\mathrm{t}}\mathbf{y}_{\mathrm{R}}, & \mathbf{v}_{\mathrm{R}}&=\mathbf{S}_{\mathrm{r}}\mathbf{z}_{\mathrm{R}},\\ \mathbf{u}_{\mathrm{C}l}&=\mathbf{S}_{\mathrm{t}}\mathbf{y}_{\mathrm{C}l}, & \mathbf{v}_{\mathrm{C}l}&=\mathbf{S}_{\mathrm{r}}\mathbf{z}_{\mathrm{C}l},\end{aligned}\tag{12}$$

with $\mathbf{S}_{\mathrm{t}}\triangleq[\mathbf{a}_{\mathrm{Rt}0},\dots,\mathbf{a}_{\mathrm{Rt}K-1},\mathbf{a}_{\mathrm{Ct}0},\dots,\mathbf{a}_{\mathrm{Ct}L-1}]$ and $\mathbf{S}_{\mathrm{r}}\triangleq[\mathbf{a}_{\mathrm{Rr}0},\dots,\mathbf{a}_{\mathrm{Rr}K-1},\mathbf{a}_{\mathrm{Cr}0},\dots,\mathbf{a}_{\mathrm{Cr}L-1}]$; and with $\mathbf{y}_{\mathrm{R}},\mathbf{z}_{\mathrm{R}},\mathbf{y}_{\mathrm{C}l},\mathbf{y}_{\mathrm{C}l}\in\mathbb{C}^{K+L}$. This formulation reduces the dimension of the vectors to optimise from N_{t} or N_{r} elements to $K+L$, i.e. from the antenna space to the steering space. Below, the two formulations will be considered depending on the cases. For $\mathbf{u}_{\mathrm{R}}$ to be normalised to a unit power, the vector $\mathbf{y}_{\mathrm{R}}$ should be normalised to $\mathbf{y}_{\mathrm{R}}/\left\|\mathbf{S}_{\mathrm{t}}\mathbf{y}_{\mathrm{R}}\right\|$, and similarly for the others.

b) Single target and UE: Inspecting the expression of the metric $\mathcal{M}$ in the antenna space, it can be observed that it increases with $|\mathbf{u}_{\mathrm{C}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Ct}}|$ and $|\mathbf{u}_{\mathrm{R}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Rt}}|$, leading to $\mathbf{u}_{\mathrm{R}}^{\star}=\mathbf{a}_{\mathrm{Rt}}/\sqrt{N_{\mathrm{t}}}$ and $\mathbf{u}_{\mathrm{C}}^{\star}=\mathbf{a}_{\mathrm{Ct}}/\sqrt{N_{\mathrm{t}}}$. Plugging these optimal vectors into $\mathcal{M}$ leads to decoupled problems for the radar and communication receive beamforming vectors, whose solutions are the Minimum Variance Distortionless Response (MVDR) vectors [4] given in Table I. The above shows that even when the radar-communication interference is taken into account, the optimal transmit beamforming vectors are aligned with the transmit steering vectors, as in the interference-free case. Differently, the receive beamforming vectors are aligned with their steering vector, with a skewing depending on the level of interference and noise, and on the steering correlation $\gamma\triangleq\mathbf{a}_{\mathrm{Cr}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Rr}}/N_{\mathrm{r}}$.

c) Two targets only: Working in the steering space, the problem amounts to finding the size-two complex vectors $\mathbf{y}_{\mathrm{R}}$ and $\mathbf{z}_{\mathrm{R}}$, defined up to a phase shift and amplitude factor as $\mathbf{y}_{\mathrm{R}}=\left[\lambda,(1-\lambda)\,e^{j\phi}\right]^{\mathrm{T}}$, $\mathbf{z}_{\mathrm{R}}=\left[\mu,(1-\mu)\,e^{j\psi}\right]^{\mathrm{T}}$, with $\lambda,\mu\in[0,1]$. Plugging the normalised vectors $\mathbf{y}_{\mathrm{R}}/\left\|\mathbf{S}_{t}\mathbf{y}_{\mathrm{R}}\right\|$ and $\mathbf{z}_{\mathrm{R}}/\left\|\mathbf{S}_{r}\mathbf{z}_{\mathrm{R}}\right\|$ inside $\mathcal{M}$, the optimal phase shifts are obtained as $\phi^{*}=\arg\left\{\mathbf{a}_{\mathrm{Rt}1}^{\mathrm{H}}\mathbf{a}_{\mathrm{Rt}0}\right\}$ and $\psi^{*}=\arg\left\{\mathbf{a}_{\mathrm{Rr}1}^{\mathrm{H}}\mathbf{a}_{\mathrm{Rr}0}\right\}$, leading to the expressions of Table I. Based on this, a 2D-grid search on μ and λ can be performed. Instead, given μ (resp. λ), it can be checked that $-\log(\mathcal{M})$ is convex in λ (resp. μ), and thus it admits a unique extremum. Therefore, an Alternating Maximisation (AM) on μ and λ can be performed. To find the unique optimum at each step, efficient methods such as golden-section search [13] can be employed, with a complexity in $\mathcal{O}(\log(M))$ where M is the number of points in the $[0,1]$ interval.

d) Two targets and two UEs: For the sake of tractability, two assumptions are considered:

- the radar waveform is such that the targets do not interfere with each other, i.e., $\beta_{\mathrm{RR}k'k}=0\ \forall k'\neq k$, this being usually assumed for well-designed waveforms;
- for the UE transmit beamforming vectors, since the UEs are not collocated with the ISAC system, an uncoordinated design has to be carried out. Each UE thus optimises the power in the direction of the BS by setting $\mathbf{u}_{\mathrm{C}l}=\mathbf{a}_{\mathrm{Ct}l}/\sqrt{N_{\mathrm{t}}}$.

It remains to design $\mathbf{u}_{\mathrm{R}}$, $\mathbf{v}_{\mathrm{R}}$ and all $\{\mathbf{v}_{\mathrm{C}l}\}$ vectors. For the optimisation of $\mathbf{u}_{\mathrm{R}}$ (resp. $\mathbf{v}_{\mathrm{R}}$), the SINR product maximisation can be rewritten in the form of [14, Fact 7.4.9]

$$\mathbf{X}^{\star}=\underset{\substack{\mathbf{X}=\mathbf{X}^{\mathrm{H}}\\ \mathbf{X}\succeq\mathbf{0}\\ \operatorname{rank}(\mathbf{X})=1}}{\arg\max}\ \frac{\operatorname{vec}\{\mathbf{X}\}^{\mathrm{H}}\left(\mathbf{A}^{*}\otimes\mathbf{B}\right)\operatorname{vec}\{\mathbf{X}\}}{\operatorname{vec}\{\mathbf{X}\}^{\mathrm{H}}\left(\mathbf{C}^{*}\otimes\mathbf{D}\right)\operatorname{vec}\{\mathbf{X}\}},\tag{13}$$

TABLE I
TRANSMIT AND RECEIVE BEAMFORMING METHODS AGAINST THE NUMBER OF TARGETS AND USERS.

K	L	Assumption	Solution	Details	Optimality	Complexity $\mathcal{O}(\cdot)$
1	1	-	$\mathbf{u}_{\mathrm{R}}^{\star}=\mathbf{a}_{\mathrm{Rt}},\quad \mathbf{u}_{\mathrm{C}}^{\star}=\mathbf{a}_{\mathrm{Ct}},$ $\mathbf{v}_{\mathrm{R}}^{\star}=\mathbf{a}_{\mathrm{Rr}}-\frac{\mathbf{a}_{\mathrm{Cr}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Rr}}}{N_{\mathrm{r}}}\frac{N_{\mathrm{t}}N_{\mathrm{r}}}{1/\beta_{\mathrm{CR}}+N_{\mathrm{t}}N_{\mathrm{r}}}\mathbf{a}_{\mathrm{Cr}},$ $\mathbf{v}_{\mathrm{C}}^{\star}=\mathbf{a}_{\mathrm{Cr}}-\frac{\mathbf{a}_{\mathrm{Rr}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Cr}}}{N_{\mathrm{r}}}\frac{N_{\mathrm{t}}N_{\mathrm{r}}}{1/\beta_{\mathrm{RC}}+N_{\mathrm{t}}N_{\mathrm{r}}}\mathbf{a}_{\mathrm{Rr}},$	-	Global	N
2	0	-	$\mathbf{u}_{\mathrm{R}}^{\star}=\lambda\,\mathbf{a}_{\mathrm{Rt0}}+(1-\lambda)\,e^{j\arg\{\mathbf{a}_{\mathrm{Rt1}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Rt0}}\}}\mathbf{a}_{\mathrm{Rt1}}$ $\mathbf{v}_{\mathrm{R}}^{\star}=\mu\,\mathbf{a}_{\mathrm{Rr0}}+(1-\mu)\,e^{j\arg\{\mathbf{a}_{\mathrm{Rr1}},^{\mathrm{H}}\mathbf{a}_{\mathrm{Rr0}}\}}\mathbf{a}_{\mathrm{Rr1}}$	Grid search on (λ,μ)	Global	$N+M^2$
				AM on (λ,μ)	Local	$N+S_1\log(M)$
≤ 2	≤ 2	$\beta_{\mathrm{RR}k'k}=0$ $\mathbf{u}_{\mathrm{C}l}=\mathbf{a}_{\mathrm{Ct}l}$	Proposition 1 and (18)	AM on $(\mathbf{u}_{\mathrm{R}},\mathbf{v}_{\mathrm{C0}},\mathbf{v}_{\mathrm{C1}})$	Relaxed Local	S_2N^6
				AM on $(\mathbf{y}_{\mathrm{R}},\mathbf{z}_{\mathrm{C0}},\mathbf{z}_{\mathrm{C1}})$	Relaxed Local	$N+S_3$
K	L	-	$\mathbf{u}_{\mathrm{R}}^{\star},\mathbf{u}_{\mathrm{C}l}^{\star}\in\operatorname{span}\left\{\{\mathbf{a}_{\mathrm{Rt}k}\}_k,\{\mathbf{a}_{\mathrm{Ct}l}\}_l\right\}$ $\mathbf{v}_{\mathrm{R}}^{\star},\mathbf{v}_{\mathrm{C}l}^{\star}\in\operatorname{span}\left\{\{\mathbf{a}_{\mathrm{Rr}k}\}_k,\{\mathbf{a}_{\mathrm{Cr}l}\}_l\right\}$	AM	Local	$N+S_4M^2$

For the sake of readability, the normalisation constants of all beamformers have been omitted. Yet, they should be normalised to a unit norm. For the complexity analysis, the maximum number of antennas is denoted $N\triangleq\max(N_t,N_r)$. The letter M denotes the number of points selected to discretise the $[0;1]$ interval. The numbers S_1, S_2, S_3 and S_4 are the number of iterations performed in the alternating maximisation scheme, which depends on the variables on which the alternation takes place, and the desired accuracy. The scaling of the complexity with respect to K and L has not been included in the table, as these numbers are expected to remain small in practical scenarios.

with $\mathbf{A}=\mathbf{a}\mathbf{a}^{\mathrm{H}}$, $\mathbf{B}=\mathbf{b}\mathbf{b}^{\mathrm{H}}$, $\mathbf{C},\mathbf{D}$ positive definite matrices, and the optimal vector $\mathbf{x}^{\star}$ extracted as $\mathbf{X}^{\star}=\mathbf{x}^{\star}\mathbf{x}^{\star\mathrm{H}}$. On the one hand, when considering the vector $\mathbf{u}_{\mathrm{R}}$, the matrices $\mathbf{a}$, $\mathbf{b}$, $\mathbf{C}$ and $\mathbf{D}$ read as $\mathbf{a}\to\mathbf{a}_{\mathrm{Rt0}}$, $\mathbf{b}\to\mathbf{a}_{\mathrm{Rt1}}$,

$$\begin{aligned}\mathbf{C}&\to\sum_{k=0}^{1}\beta_{\mathrm{RC}k0}\,\left|\mathbf{v}_{\mathrm{C0}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Rr}k}\right|^2\mathbf{a}_{\mathrm{Rt}k}\mathbf{a}_{\mathrm{Rt}k}^{\mathrm{H}}\\&\qquad+\left(\beta_{\mathrm{CC10}}\left|\mathbf{u}_{\mathrm{C1}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Ct1}}\right|^2\left|\mathbf{v}_{\mathrm{C0}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Cr1}}\right|^2+1\right)\mathbf{I}_{N_{\mathrm{t}}},\\\mathbf{D}&\to\sum_{k=0}^{1}\beta_{\mathrm{RC}k1}\,\left|\mathbf{v}_{\mathrm{C1}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Rr}k}\right|^2\mathbf{a}_{\mathrm{Rt}k}\mathbf{a}_{\mathrm{Rt}k}^{\mathrm{H}}\\&\qquad+\left(\beta_{\mathrm{CC01}}\left|\mathbf{u}_{\mathrm{C0}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Ct0}}\right|^2\left|\mathbf{v}_{\mathrm{C1}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Cr0}}\right|^2+1\right)\mathbf{I}_{N_{\mathrm{t}}}.\end{aligned}\tag{14}$$

On the other hand, for $\mathbf{v}_{\mathrm{R}}$, they read as $\mathbf{a}\to\mathbf{a}_{\mathrm{Rr0}}$, $\mathbf{b}\to\mathbf{a}_{\mathrm{Rr1}}$,

$$\begin{aligned}\mathbf{C}&\to\sum_{l=0}^{1}\beta_{\mathrm{CR}l0}\,\left|\mathbf{u}_{\mathrm{C}l}^{\mathrm{H}}\mathbf{a}_{\mathrm{Ct}l}\right|^2\mathbf{a}_{\mathrm{Cr}l}\mathbf{a}_{\mathrm{Cr}l}^{\mathrm{H}}+\mathbf{I}_{N_{\mathrm{r}}},\\\mathbf{D}&\to\sum_{l=0}^{1}\beta_{\mathrm{CR}l1}\,\left|\mathbf{u}_{\mathrm{C}l}^{\mathrm{H}}\mathbf{a}_{\mathrm{Ct}l}\right|^2\mathbf{a}_{\mathrm{Cr}l}\mathbf{a}_{\mathrm{Cr}l}^{\mathrm{H}}+\mathbf{I}_{N_{\mathrm{r}}}.\end{aligned}\tag{15}$$

Both the objective function and the constraint set are nonconvex, and thus obtaining a solution of the above is challenging. An approximate solution is therefore obtained below, by relaxing the positive definite and rank-one constraint.

To that aim, given $\mathbf{U}\in\mathbb{C}^{N\times N}$, the matrix $\mathbf{M}_{\mathbf{U}}\in\mathbb{R}^{2N\times 2N}$ is defined as

$$\mathbf{M}_{\mathbf{U}}\triangleq\begin{bmatrix}\operatorname{Re}\{\mathbf{U}\} & -\operatorname{Im}\{\mathbf{U}\}\\ \operatorname{Im}\{\mathbf{U}\} & \operatorname{Re}\{\mathbf{U}\}\end{bmatrix}.\tag{16}$$

Given $\mathbf{E}\in\mathbb{C}^{N\times N}$, let us define $\operatorname{vec}_{\mathbb{C}}(\mathbf{E})\triangleq\left[\operatorname{vec}^{\mathrm{T}}(\operatorname{Re}\{\mathbf{E}\})\quad\operatorname{vec}^{\mathrm{T}}(\operatorname{Im}\{\mathbf{E}\})\right]^{\mathrm{T}}$, and $\operatorname{vec}_{\mathbb{C}}^{-1}(\cdot)$ as the inverse operator, with $\operatorname{vec}(\cdot)$ the vectorization operator. For Hermitian matrices, the half-vectorisation operator $\operatorname{vech}_{\mathbb{C}}(\mathbf{E})$ is defined similarly but with the duplicate entries excluded. That is, $\operatorname{vech}_{\mathbb{C}}(\mathbf{E})$ is the vector of size $N^2\times 1$ whose first $N(N+1)/2$ elements are the stacked columns of the upper triangular part of $\operatorname{Re}\{\mathbf{E}\}$, including the diagonal, and the last $N(N-1)/2$ elements are the stacked columns of the upper triangular part of $\operatorname{Im}\{\mathbf{E}\}$, excluding the diagonal. The duplication matrix $\mathbf{T}\in\mathbb{R}^{2N^2\times N^2}$ is then introduced as the unique matrix fulfilling $\mathbf{T}\operatorname{vech}_{\mathbb{C}}(\mathbf{E})=\operatorname{vec}_{\mathbb{C}}(\mathbf{E})$. The Kronecker product is written $\otimes$. With these elements, Proposition 1 gives the relaxed Hermitian solution of (13).

Proposition 1: *When considering the Hermitian constraint only, the solution of* (13) *is given by* $\mathbf{X}^{\star}=\operatorname{vec}_{\mathbb{C}}^{-1}(\mathbf{T}\mathbf{f})$, *with* $\mathbf{f}$ *a real principal eigenvector of*

$$\mathbf{K}=\left(\mathbf{T}^{\mathrm{T}}\mathbf{M}_{(\mathbf{C}^*\otimes\mathbf{D})}\mathbf{T}\right)^{-1}\mathbf{T}^{\mathrm{T}}\mathbf{M}_{(\mathbf{A}^*\otimes\mathbf{B})}\mathbf{T}.\tag{17}$$

Proof: This result follows from the observation that $\mathbf{x}^{\mathrm{H}}\mathbf{A}\mathbf{x}=\mathbf{e}^{\top}\mathbf{M}_{\mathbf{A}}\mathbf{e}$, with $\mathbf{e}^{\top}=\left[\operatorname{Re}\{\mathbf{x}\}^{\top},\operatorname{Im}\{\mathbf{x}\}^{\top}\right]$. The optimisation can thus be translated into the real problem

$$\arg\max_{\mathbf{f}\in\mathbb{R}^{N^2}}\frac{\mathbf{f}^{\top}\mathbf{T}^{\top}\mathbf{M}_{(\mathbf{A}^*\otimes\mathbf{B})}\mathbf{T}\mathbf{f}}{\mathbf{f}^{\top}\mathbf{T}^{\top}\mathbf{M}_{(\mathbf{C}^*\otimes\mathbf{D})}\mathbf{T}\mathbf{f}},$$

whose solution is given by the principal eigenvector of (17). This principal eigenvector can always be selected as real since if $\mathbf{v}$ is an eigenvector of $\mathbf{A}$, then $\operatorname{Re}\{\mathbf{v}\}$ is an eigenvector associated with the same eigenvalue. ■

Once the non-necessarily rank-one matrix $\mathbf{X}^{\star}$ is obtained, the optimal vector $\mathbf{x}^{\star}$ is extracted as the principal eigenvector of $\mathbf{X}^{\star}$. Depending on whether the transmit or receive matrix $\mathbf{X}^{\star}$ is considered, the above enables to obtain $\mathbf{u}_{\mathrm{R}}^{\star}$ and $\mathbf{v}_{\mathrm{R}}^{\star}$. As for the vectors $\{\mathbf{v}_{\mathrm{C}l}\}$, they are obtained as MVDR vectors reading as [4]

$$\mathbf{v}_{\mathrm{C}l}^{\star} = \eta_{\mathbf{v}_{\mathrm{C}l}} \Bigg(\sum_{k=0}^{1} \beta_{\mathrm{RC}kl} \left|\mathbf{u}_{\mathrm{R}}^{\mathrm{H}} \mathbf{a}_{\mathrm{Rt}k}\right|^2 \mathbf{a}_{\mathrm{Rr}k} \mathbf{a}_{\mathrm{Rr}k}^{\mathrm{H}} + \sum_{l' \neq l} \beta_{\mathrm{CC}l'l} \left|\mathbf{u}_{\mathrm{C}l}^{\mathrm{H}} \mathbf{a}_{\mathrm{Ct}l'}\right|^2 \mathbf{a}_{\mathrm{Cr}l'} \mathbf{a}_{\mathrm{Cr}l'}^{\mathrm{H}} + \mathbf{I}_{N_{\mathrm{r}}} \Bigg)^{-1} \mathbf{a}_{\mathrm{Cr}l}, \tag{18}$$

with $\eta_{\mathbf{v}_{\mathrm{C}l}}$ a normalisation factor. While $\mathbf{v}_{\mathrm{R}}^{\star}$ does not depend on the other beamformers, the optimal transmit beamforming vector $\mathbf{u}_{\mathrm{R}}^{\star}$ and the receive beamforming vectors $\mathbf{v}_{\mathrm{C}l}^{\star}$ are functions of each other. Therefore, AM can again be used until convergence to a point at which the SINR product decreases when performing the update. This behaviour may happen since the relaxed solution might not always increase the objective value, especially when the iterates approach an optimum.

A drawback of the above approach is that at each step of the AM, matrices of size $2N_t \times 2N_t$ and $2N_r \times 2N_r$ are manipulated, with a complexity scaling thus as N_t^6 and N_r^6 if naive matrix algorithms are employed for the inversion and eigenvector extraction. To circumvent this problem, for large numbers of antennas, the above computations can be performed in the steering space. In that space, the same development as above still hold, with the transmit and receive steering vectors $\mathbf{a}_{\mathrm{Rt}}$, $\mathbf{a}_{\mathrm{Ct}l}$, $\mathbf{a}_{\mathrm{Rr}}$ and $\mathbf{a}_{\mathrm{Cr}l}$ of (14) and (15) being replaced respectively by $\mathbf{S}_{\mathrm{t}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Rt}}$, $\mathbf{S}_{\mathrm{t}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Ct}l}$, $\mathbf{S}_{\mathrm{r}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Rr}}$ and $\mathbf{S}_{\mathrm{r}}^{\mathrm{H}}\mathbf{a}_{\mathrm{Cr}l}$, and the vectors $\mathbf{u}_{\mathrm{C}l}$ and $\mathbf{v}_{\mathrm{C}l}$, by $\mathbf{y}_{\mathrm{C}l}$ and $\mathbf{z}_{\mathrm{C}l}$. With these elements, an AM algorithm can be carried out in the steering space, with a constant complexity at each step since the number of user and targets is always smaller than 4 in this case.

e) Multiple targets and users: For arbitrary K and L, obtaining additional analytical results is extremely challenging. Locally optimal solutions can nevertheless be obtained by performing an AM between the transmit and receive beamforming vectors, in the steering space. For each vector, another AM on the complex elements of $\mathbf{y}_{\mathrm{R}}$, $\mathbf{z}_{\mathrm{R}}$, $\mathbf{y}_{\mathrm{C}l}$ and $\mathbf{z}_{\mathrm{C}l}$ must be carried out.

f) Method selection: Based on Table I, methods can be selected based on their optimality and complexity, depending on K and L. For $K = 1$, $L = 1$, the closed-form solutions have the lowest complexity while being globally optimal. For $K = 2$, $L = 0$, a global optimum can be found through the 2D-grid search. If this complexity is too high, then either the AM on (λ, μ) can be performed, or the methods for $K \leq 2$ and $L \leq 2$ can be applied in the steering space. For the other cases with $K \leq 2$, $L \leq 2$, the AM method should be operated in the steering space as it as a much lower complexity then in the antenna space. Finally, the complexity of the AM for general K and L makes it a relevant method only if $K \geq 3$ or $L \geq 3$, as otherwise the other methods are more efficient.

IV. Numerical Analysis

In this section, the developed ISAC metric and associated beamforming design schemes are analysed.

TABLE II
Scenarios' parameters

Targets:	$\boldsymbol{\theta}_{\mathrm{R}}$ = {12,43}°	$\beta_{\mathrm{RR}} = \begin{bmatrix} 1 & 0.5 \\ 0.1 & 1 \end{bmatrix}$	$\beta_{\mathrm{CR}} = \begin{bmatrix} 1 & 0.1 \\ 0.2 & 1 \end{bmatrix}$
UL signals:	$\boldsymbol{\theta}_{\mathrm{C}}$ = {21,-67}°	$\beta_{\mathrm{CC}} = \begin{bmatrix} 1 & 0.1 \\ 0.2 & 1 \end{bmatrix}$	$\beta_{\mathrm{RC}} = \begin{bmatrix} 0.5 & 0.5 \\ 0.1 & 0.1 \end{bmatrix}$

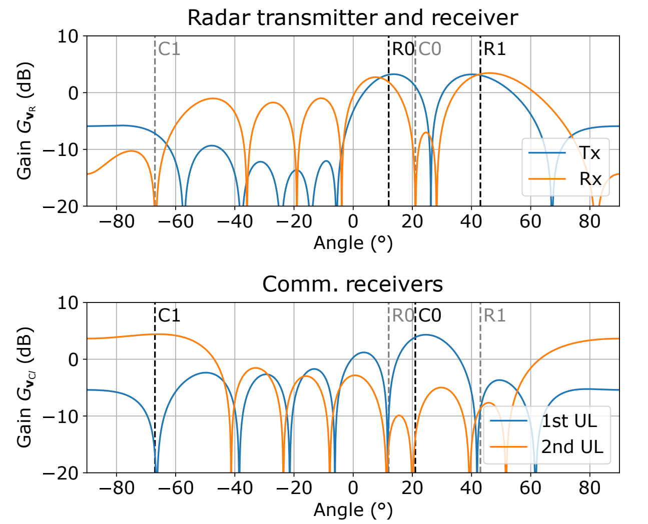

Fig. 2. Scenario with two targets and two UEs — communication receive (top) and radar transmit and receive beamforming gains (bottom).

A. Two-targets two-users scenario

The considered scenario, with two targets and two UEs, is described in Table II, with $N_{\mathrm{t}} = N_{\mathrm{r}} = 8$ and half-wavelength uniform linear arrays. Figure 2 first illustrates the communication and radar receiver beampatterns $G_{\mathbf{v}_{\mathrm{C},l}} \triangleq \left|\mathbf{v}_{\mathrm{C},l}^{\mathrm{H}}\mathbf{a}_{\mathrm{r}}(\theta)\right|$ and $G_{\mathbf{v}_{\mathrm{R}}} \triangleq \left|\mathbf{v}_{\mathrm{R}}^{\mathrm{H}}\mathbf{a}_{\mathrm{r}}(\theta)\right|$, with $\mathbf{a}_{\mathrm{r}}(\theta)$ the receive steering vector in direction θ. The beamforming vectors are those obtained with Proposition 1 and (18), when working in the steering space. As it can be observed in the upper graph, the receive communication beamforming vectors manage to provide low power gains in the direction of the radar targets and of the other communication users. In the bottom graph, the receive radar beamforming vector blinds the communication users, while providing high power gains for both targets simultaneously.

B. SINR-product metric

Next, the benefits of the SINR-product metric are investigated. Figure 3 illustrate a scenario with no communication user and two radar targets located at $\theta_{\mathrm{R0}} = 43°$ and $\theta_{\mathrm{R1}} = -43°$, with the interfering coefficients set to

$$\beta_{\mathrm{RR}} = \begin{bmatrix} \alpha & 0.1\alpha \\ 0.1 & 1 \end{bmatrix}.$$

The factor α models the path loss ratio between the first and second radar target, i.e. $\alpha < 1$ means that the first target is in less favorable conditions than the second. For a given α, the achievable Pareto-optimal SINR couples $(\mathrm{SINR}_{\mathrm{R0}}, \mathrm{SINR}_{\mathrm{R1}})$ are displayed, as well as the point selected by the SINR-product and SINR-sum metric. As it can be observed, the

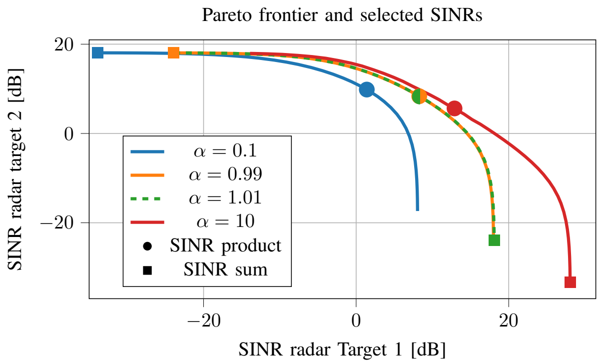

Fig. 3. Pareto frontier and selected point for the SINR product and sum metrics, in a scenario with two radar targets. The factor α is the ratio between the path loss of the first radar target and the second.

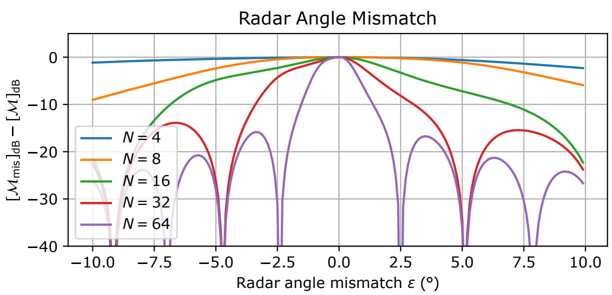

Fig. 4. Impact of a radar angle mismatch on the metric for different numbers of antennas.

product metric always selects a Pareto-optimal point, and fairly considers both targets, even if one suffers from severe channel degradation. In contrast, the SINR-sum metric drastically favours the target with the best channel condition, at the detriment of the other. As a result, the SINR-sum metric is extremely sensitive to channel changes, as it can be observed by comparing the $\alpha = 0.99$ and $\alpha = 1.01$ results: while the Pareto frontier remains almost unmodified, the SINR of the first target jumps from $-23\,\text{dB}$ to $18\,\text{dB}$.

C. *Impact of angle mismatch*

Finally, to evaluate the impact of an angle mismatch on the generated beamformers, we consider a 1-target 1-user scenario at angles $\theta_{\text{R}} = 43°$ and $\theta_{\text{C}} = -43°$, with $\beta_{\text{CR}} = \beta_{\text{RC}} = 0.1$. Assuming a mismatch ϵ in the radar angle θ_{R}, Figure 4 illustrates the impact of ϵ on the metric. The higher the number of antennas, the higher the sensitivity against the mismatch. Particularly large losses arise when the mismatch is sufficiently large to deviate the target outside the main lobe. In that scenario, the mismatch does not affect significantly the communication SINR, owing to the large difference between the target and user angles. In scenarios where the radar target and communication user are close to each other, the mismatch is even more detrimental, as it has a significant effect on the other function as well.

V. Conclusion

In this paper, joint radar and communication transmit and receive beamforming designs are developed for multi-target and uplink multi-user scenarios. A threshold-free fair beamformer design is ensured by choosing the product of all the SINRs as the optimisation objective. Depending on the number of targets and UEs, multiple algorithms are provided, differing in their complexity, optimality and required assumptions. The solutions obtained with these methods have been numerically illustrated and compared, demonstrating the fairness ensured by the SINR-product metric. The robustness of the schemes with respect to parameters uncertainty has also been assessed. In future works, the objective function could be adapted to introduce priority between the targets and communication signals.

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