User-Defined Gestures for Tangible Interaction with Cubes for Smart Home Control

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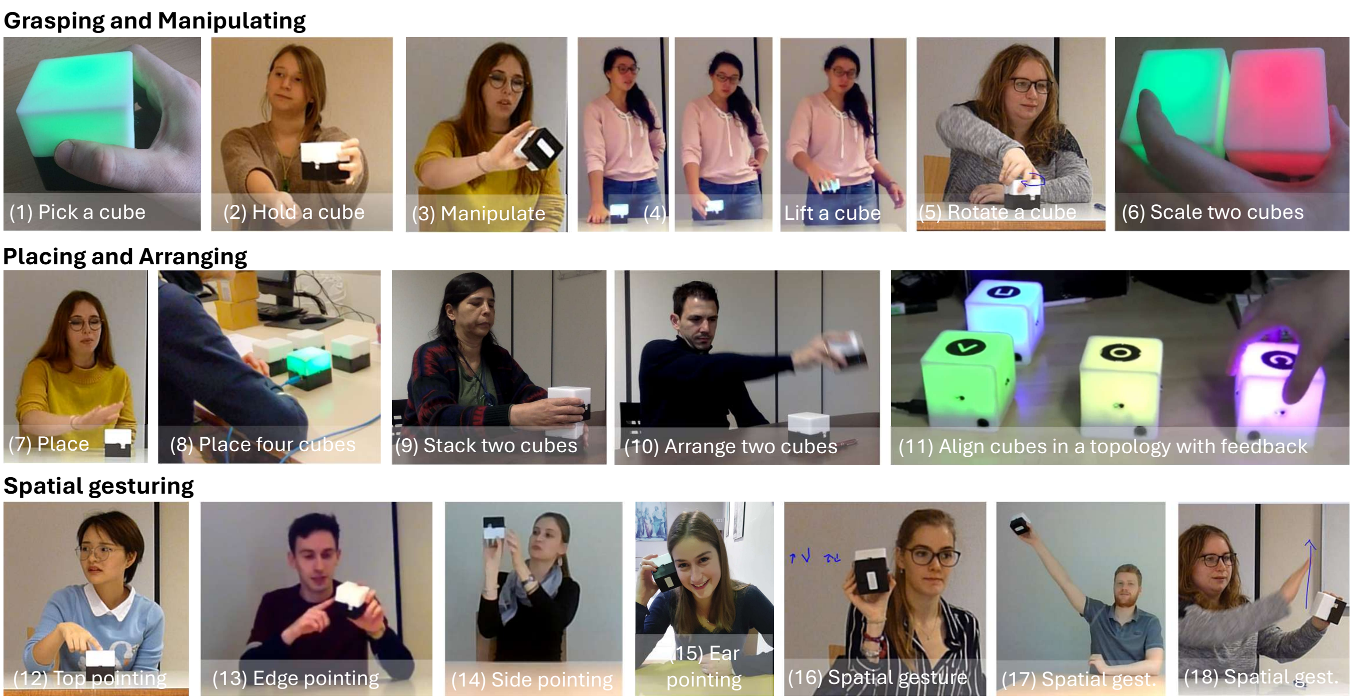

Figure 1: Three families of user-defined gesture categories for tangible interaction with AudioCubes for home control.

Abstract

The cube is an object often used for tangible interaction across many fields of application, notably by manipulating cubes or performing gestures with or on them. Although some system- or designer-defined gestures exist for tangible cubes, user-defined gestures for them remain unexplored. To bridge this gap, we conducted three gesture elicitation studies, each using an identical experimental protocol on a different sample of 30 participants proposing a gesture for 13, 16, and 20 referents controlling a smart home appliance, respectively. Based on the total of 1,440 gesture proposals, we established a classification of user-defined gestures for designing tangible interaction with cubes in smart home control structured into seven major categories, which are further broken down into 55 subcategories. This classification is abstracted from properties such as the number of fingers and hands used to manipulate a cube, the repetition of gestures, their combination, and the time of articulation. A Poisson law estimates the number of participants to be recruited to obtain a desired number of gesture categories.

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CCS Concepts

• **Hardware** → **Sensors and actuators**; **Tactile and hand-based interfaces**; • **Human-centered computing** → *Interaction devices*; *Graphical user interfaces*; **Gestural input**; **User interface management systems**; User interface design.

Keywords

Cube tangible object, Gesture elicitation study, Gestural input, Gesture-based interaction, Gesture design, Input devices and technology, Tangible user interaction.

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1 Introduction

Tangible User Interfaces (TUIs) [24, 63, 72] enable end users to interact with digital information by manipulating physical objects connected to this information [25, 71]. Among the various form factors used for TUIs, cubes have become very popular and are frequently used for several reasons: end users can perform a wide variety of tasks with cubes, such as 1D, 2D or 3D data selection tasks [52], 3D spatio-temporal data visualization [30], cubes benefit of an intuitive physical affordance [66] that can be augmented [38], including many degrees of freedom available, many manipulations [59, 69] and many mappings [87] possible, cubes favor engagement of real-world knowledge [54] and playfulness [38].

Users are familiar with everyday cubes, as they are easy to recognize, understand and manipulate [17], thus making them suitable for rich properties of embodied interaction [55] and cognition [24]. In particular, cubes support spatial mapping [32] of digital data where each face can represent different functions [3], states [11], or data dimensions [87]. Cubes benefit from rich mathematical properties [54], such as symmetry axes (*e.g.,* for a tight coupling between real-world objects and the digital world [72], being the simplest geometric polyhedron [66]. Cubes are easy to grasp with human hands [72] with different spans [13] and to manipulate [19], suggesting a large set of affordances [66]. A cube evokes a natural manipulation related to human habits since childhood, intuitively [33], cognitively [65], ecologically [61], and physically [54].

Gestural input with tangible objects [37], such as cubes, enhances interaction by combining [42] the intuitiveness of physical manipulation of objects with the expressiveness of gesture-based control [55] and input [39]. Tangible cubes offer a tactile, spatially rich interface that supports natural interactions, making digital information and functions accessible [35] and engaging, especially in educational [97], learning [69], creative [40], recreational [3], and collaborative [39] or distributed contexts of use [43]. Cubes combined with gestures are a powerful means of enabling users to perform actions such as 2D/3D positioning, rotation, scaling, or navigation [30], with reduced cognitive load [65].

The gestures considered in studies, whether for a cube or another object, are mostly designed specifically for the tasks of a particular application area. This can vary widely from one area to another, questioning their reusability and/or generalizability. There is a need to investigate gestures beyond the boundaries of these areas, exploring potential gestures with a tangible cube as widely and systematically as possible. This should expand our overall understanding of gestures through a tangible cube, particularly by separating the gestures preferred by people from those that are theoretically possible. This should also inform the design of gesture sets for tangible cubes from a more open perspective, not necessarily restricted to a specific area. To identify gestures defined by users, rather than those promoted by a designer or system [92], we also need to know how many participants should be recruited for elicitation studies, raising the question of how many gesture categories should be discovered as a function of the number of participants. Our research question can thus be formulated as follows:

RQ = What are the user-defined gestures to interact with tangible cubes for smart home control?

This question also subsumes the sub-question: *SQ = how many participants are required to obtain a predefined number of user-defined gestures?* Estimating how many participants are needed in one or multiple gesture elicitation studies (GESs) [90] is important because it determines whether the resulting gesture set is diverse, representative, meaningful, and methodologically sound, while ensuring the efficient use of research resources. User-defined gestures in GESs reflect how end users interact naturally and intuitively by gesture. If too few participants contribute gestures, the resulting set may be biased by idiosyncratic habits, cultural background, or prior experience. If too many participants contribute gestures, there may be too many repetitions that are not meaningful.

GESs typically rely on agreement metrics (*e.g.,* agreement scores [90], agreement rates [78], or maximum consensus [44]) to identify which gestures feel natural or intuitive to participants. To compute meaningful agreement, we need multiple participants to suggest gestures for the same referents in **multiple studies** to investigate various replications. If too few participants contribute gestures, the agreement values become unstable or meaningless. If too many participants contribute, the agreement values tend to decrease [78].

To address the main research question and its sub-question, our work focuses on user-defined gestures to be proposed for controlling an appliance in a smart home, a context of use that is popular [77, 80] in ambient intelligence [7] This paper contributes to gestural interaction with tangible cubes by conducting three GESs in this specific context, each using an identical experimental protocol, with AudioCubes, a representative example of tangible cubes. These results are consolidated into a classification of user-defined gestures for tangible interaction with cubes in a smart home, organized into seven major categories. We also provide a mathematical model that estimates the number of participants to be recruited to achieve a desired number of gesture categories in a study.

The remainder of this paper is structured as follows. Section 2 reviews some work on tangible cubes and gestural input. Section 3 presents AudioCubes, a set of wireless cubes used in the experiments reported in Section 4. Section 5 provides our classification and mathematical model. Section 6 discusses the virtues and limitations of this work. Section 7 concludes the paper by summarizing the findings and presenting some future avenues of this work.

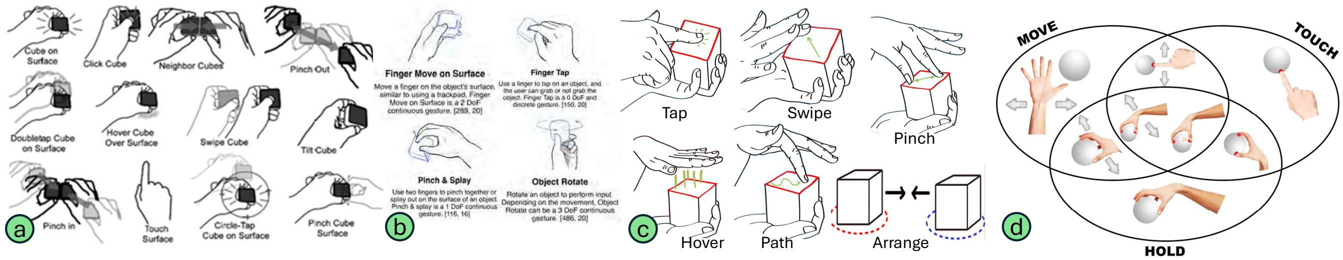


Figure 2: Excerpts from ActiveTokens [73] (a), spatial TUI [29] (b), TangibleTouch [55] (c), and Move-Hold-Touch [5] (d).

2 Related Work

This research touches on three research topics, *i.e.*,, tangible interaction with cubes, gesture elicitation studies, and classification aspects, which will be discussed in the following three subsections.

2.1 Tangible Interaction with Cubes

Cubic TUIs are suitable for tangible interaction, as they combine maneuverability (they are easy to manipulate and move around), stability (they remain in place during interactions), and scalability (they scale up/down to represent different data dimensions) [30, 63]. From a Distributed Cognition [22] perspective, cubes can be manipulated by the two hands of a single person (two-handed input [72]) or multiple persons (multi-user input [68]) in a distributed fashion [43] in different physical setups, such as constructive assemblies [40]. A cubic shape is generally perceived more positively compared to a rounded one [57]. For these reasons, research on gestural input with tangible cubes has explored various approaches to enhance interaction in several domains of human activity, such as music creation [3], sound design [61], navigational tasks [75] in large datasets [33], shopping [59], virtual reality [26], eLearning [38], and storytelling [96]. We therefore discuss related work to some of these areas. Since a significant portion of the work related to tangible interaction with a cube falls within the domain of audio/sound, we devote an entire subsection to it, while other domains are covered in the next subsection.

2.1.1 Tangible Cubes in Music. Music creation and sound design are creative tasks in which the musician or artist directly manipulates a musical instrument. They interact directly with synthesizers and keyboards through physical controls such as sliders, knobs, buttons, and displays. Replacement of these hardware interfaces and equipment with interactive software has been a trend [17] until a whole recording studio can fit on a computer. However, most musicians still use electronic instruments, as they do not always remember which parameter is controlled by which widget. For these reasons, cubic TUIs have been extensively investigated in music and sound design [21]. For example, Poupyrev [56] combines cubic TUIs with augmented reality and 3DUI to allow users to play music with or without any musical instrument. AudioPad [50, 51] and reacTable [34] were developed to support the expressiveness, legibility, and visibility of musicians by allowing them to create sound by tracking the positions of TUIs on a tabletop. The TUIs control the sound parameters. reacTable uses modular synthesis, whereas AudioPad uses loop-based synthesis, where electromagnetic pucks are associated with different functions: once the musician has turned these objects on a tabletop, this action is transferred to the musical instrument that replicates the functions of the objects. MusicCube [3] recognizes three gestures to shuffle music in playlists, regulate volume and rhythm, and navigate with multicolored lights on cubes, thus making digital music tangible on a tabletop by controlling its access through cubic TUIs, such as MusicTable [9], BlockJam [46], and reacTable [34]. A DisplayCube with gesture recognition and 6 displays is introduced as a TUI in [38], leading to GestureCube [38], a digitally augmented cube for gestural control of media appliances. AudioCubes [61] address some problems of sound manipulation by replicating their counterpart devices [32]. They enable sound creators to move, arrange, and rotate tangible cubes that can be wirelessly coupled by positioning them within each other's interaction range on a tabletop to explore a sound design space. In summary, while many initiatives use cubic TUIs for music, the actions performed on them are mainly to grasp, position, and arrange them, not to explore a large set of gestures.

2.1.2 Tangible Cubes in Other Domains of Application. Tangible cubes also pursue purposes outside of music, ranging from spatial 2D/3D data navigation [30], 3D modelling [4] to pedagogical [37, 38] and therapeutic aspects [10, 86]. For example, the "Put-That-There" system [12] tracks the arm 3D position to navigate through space in a multimodal way. CubicMouse [26] is an input device for specifying a location in space by controlling its position and orientation in a virtual reality model. Reski and Alissandrakis [58] describes an augmented reality interface using a virtual cube that can be interacted with through 3D gestural input to browse and select data. With InterCube [59], end users navigate a 3D shopping menu by rotating cubes in any direction. Beyond simple navigation, Navigational Blocks [16] are spatially arranged based on their position, orientation, and movement to produce narrative storytelling in a museum. With DisplayCubes [37, 38], developers can deploy learning applications based on tangible cubes, which are reported to be intuitive and playful for quizzes, multiple choices, and tests for children aged 6 to 12. MediCubes [86] embeds non-invasive sensors measuring pulse, temperature, blood oxygen saturation, and lung capacity of a child while playing. Data are collected and imported into a storytelling game that guides the child through a series of physiological measurements perceived as pleasant. Each MediCube senses one body parameter, but gestures are neither attached nor recognized. Tangible props coupled with a digital cube have been used to illustrate the farm-to-table process and food knowledge with WooGu [94]. GLiMT [35] is a tangible wooden cube to assist older adults who struggle with traditional screen-based solutions in

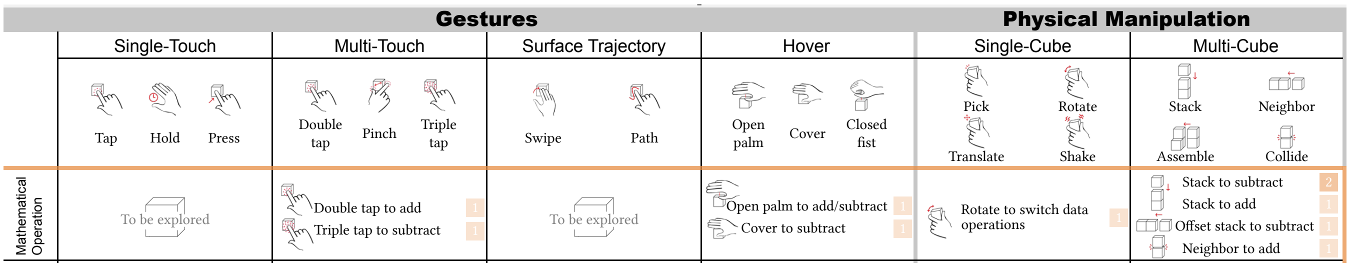


Figure 3: Gestures performed on a tangible cube for data visualization in mixed reality [30] (used with authors' permission).

a care home. Any-cubes [60] use one cube to train recognition of physical objects that activate another cube that controls an actuator. Similarly to the previous subsection, the cube actions are performed mainly to grasp, position, and arrange them, and not for input.

He et al. [30] identified an interesting set of 19 gestures performed on cubes for data manipulation by people in mixed reality (Fig. 3): tap, hold, and press for single-touching a cube; double/triple tap and pinch for multi-touching a cube; swipe and path for surface trajectory; open/close palm and cover for hovering a cube; pick, translate, rotate, and shake for single manipulating a single cube; and stack, assemble, collide, and neighbor for manipulating several cubes. Although these gestures were performed on tangible cubes for data visualization in mixed reality, they remain relevant to our studies. These gestures are precisely mapped to functions, such as "stack to add" or "stack to subtract", but they remain expressed in a very general manner, such as "tap", "hold", which we want to further detail. These passive cubes are equipped with markers for computer vision-based gesture recognition. Instead, we are considering active cubes equipped with sensors that capture gestural input directly. Thus, the locus of control is the cube itself, not a remote one.

2.2 Gesture Elicitation Studies for Tangible Interaction

Designers should iterate [92] several times, using the choice of gesture-object pairs to test the designed interaction. Although the first iteration could be achieved with a mockup, further steps should involve working prototypes that recognize user gestures. An alternative approach is to ask users which gestures they would perform or prefer for the envisioned commands using Gesture Elicitation Studies (GESs) [90]. They have often been used to derive gesture taxonomies and improve the user experience of particular interfaces (see [84] for a survey). Hoff et al. [31] tested Morris et al. [44]'s increased production of gestures and examined kinesthetic priming to reduce legacy bias; the practical effectiveness of these strategies might be limited, given a medium effect size and wide variance between participants.

Cafaro et al. [15] introduced *Framed Guessability*, a methodology for conducting a GES that relyies on Embodied Allegories to create a unifying context that spans across the locus of interaction (*e.g.,* the body for deictic gestures) and the focus of interaction (*e.g.,* the graphical interface receiving the command). This method increases the chance that users select the same gesture or movement to produce an effect within the virtual context. The consensus gesture set is also expressed as more intuitive by end users.

Among the 216 GESs analyzed in a systematic literature review [85], the 267 studies examined in a subsequent survey [84], and the comprehensive review of GES methodologies [18], only a single study was identified as one with tangible interaction, specifically involving the use of cubes. To date, no other studies have been reported that address tangible interaction or the use of other tangible objects in this context.

Valdes et al. [73] conducted a GES asking participants to propose gestures to execute eight commands (*e.g.,* define and modify a range) to query big data by manipulating Sifteo cubes on a Microsoft PixelSense tabletop. They came up with twelve gestures (Fig. 2-a): cube on the surface, click cube, neighbor cubes, pinch in/out, double tap cube on the surface, hover cube on the surface, swipe cube, tilt cube, touch cube on the surface, circle tap cube on the surface, and pinch cube on the surface. These gestures were classified according to three properties: *space* (*i.e.,* in air, on a surface, and hybrid), *flow* (*i.e.,* discrete *vs.* continuous), and *cardinality* (*e.g.,* atomic, compound, and parallel). This study is closest to our work because it also conducts a GES to derive a consensus set of cube gestures for tangible interaction, but only for ActiveToken, not in general. Battistoni et al. [7] *et al.* conducted a GES asking participants to propose TactCube-based gestures for 12 commands executed on three smart devices, *i.e.,* a smart speaker, a smart bulb, and a smart curtain.

Gong et al. [29] also conducted a GES asking participants which tangible objects would optimize their affordance for tangible actions. Although this study did not involve any cube among the stimuli, their table of gestures represents a few cube gestures (Fig. 2-b), but in a limited number of cases with a focus on affordances. Unlike this study, our work aims to conduct multiple GESs to provide a comprehensive exploration of user-defined gestures in a consistent context (*i.e.,* smart home control) and achieve classification.

When several cubes are involved in a setup, cultural and social aspects arise [38]: Computer-Supported Collaborative Work and Learning (CSCW and CSCL, resp.) are promising areas of application by enabling end users to collaboratively manipulate cubes for a shared goal, like in ActiveCube [87], for writing an algorithm in AlgoBlocks [68], for learning in CubeToLearn [69] or assessing cognitive abilities for different tasks, like spatial organization [49], thanks to a correlation with the cube [65]. Other areas of application also include, but are not limited to: monitoring large-scale systems [33], producing a 3D cube structure with a bijection between the TUI 3D world and its representation into a GUI 2D world [36], and architectural education with CubeExplorer [67].

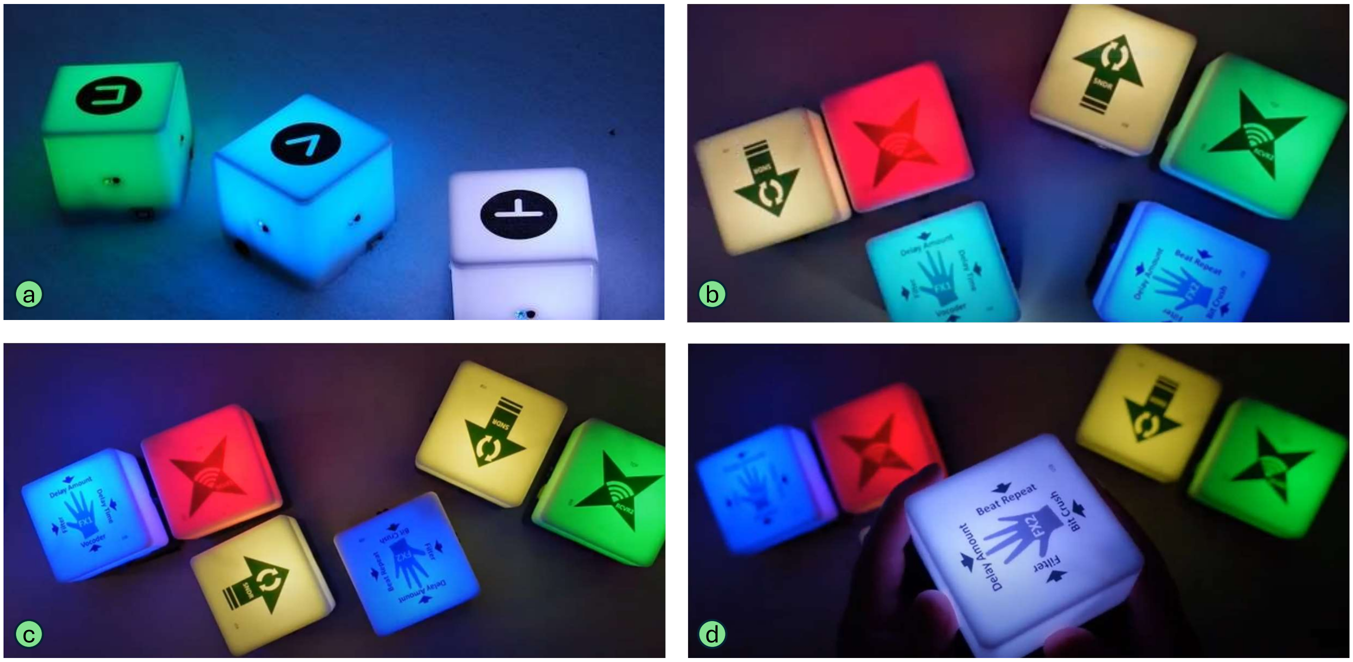


Figure 4: Various configurations of AudioCubes: three independent cubes (a), an initial topology of AudioCubes (b), rotating a cube in a topology (c), and lifting the cube in the air (d).

2.3 Classification Frameworks for Tangible Interaction

Sheridan et al. [66] identified 17 gripping manipulations (not gestures) on a fictive wooden cube: turning (86%), rolling (58%), shaking (49%), throwing (35%), and shifting (30%). Participants rated these manipulations by importance: turning (87%), moving (54%), positioning (36%), orientation (32%), and shaking (28%). A first classification emerged from this study according to three parameters: action (manipulation), description of the action, and events (*i.e.,* the type of action particular to that manipulation). The Tangible Gesture Interaction Framework (TGIF) [5] classifies gestures for tangible interaction according to three relationships with objects (Fig. 2-d): moving the hand, holding a tangible object such as a sphere, or touching it. Move, hold, and touch can then be combined to obtain a wide variety of tangible gestures. This framework is applicable to any tangible object, including cubes. Its generalizability provides a good starting point for investigating user-defined gestures tailored to cubes. Other taxonomies are also inspiring.

Fernaeus et al. [23] discusses four ways in which tangible objects work as resources for action: for physical manipulation; for referential, social, and contextually oriented action; for perception and sensory experience; and for digitally mediated action. Fishkin [24] structures tangible objects in a taxonomy that uses metaphor and embodiment as its two axes. Leong et al. [40] presents a descriptive conceptual framework that facilitates the systematic exploration and critical consideration of constructive assemblies, such as the cube arrangement. Shaer et al. [64] propose the Token and Constraints (TAC) paradigm to capture the core components of TUIs. Ullmer and Ishii [71] presents the MCRpd model for tangible interaction, which relates the role of physical and digital representations of the control. More recently, the TangibleTouch toolkit introduced a method for designing surface-based gestures on interactive tangible cubes using capacitive sensing, applicable in many domains [55]. This toolkit supports the recognition of six gestures and their combination for this purpose (Fig. 2-*c*): tap, swipe, pinch, hover, path, and arrange. This is precisely the type of gesture that we want to investigate more broadly. Unlike this toolkit, we will conduct GESs to determine them.

3 AudioCubes used for Gestural Input in Tangible Interaction

3.1 Summary

Most of the works reviewed do not exactly use gesture interaction as a means of input on a tangible cube. Rather, they used the physical position of the cubes [5] and their arrangement [40, 49] to deduce commands. Very few of them actually use such gestures, or even mention them as a means of input. When they do, the number of gestures remains small (*e.g.,* 2 [11], 3 [3], 6 [55], or 7 [66]), their types are not very varied or differentiated (*e.g.,* flipping only [95], rotation only [11]), and their expressiveness is limited to combine gestural and tangible interaction [42]. TouchTangible [55] makes explicit its cube gestures (Fig. 2-*c*), but does not aim at designing such gestures. To our knowledge, no study has systematically explored how gestures with tangible cubes could be designed. Some GESs [90] exist [29, 73], but they do not pursue this objective, remaining specific to their field of application, such as for virtual [20], augmented [53], or mixed reality [30], and not with active cubes. In

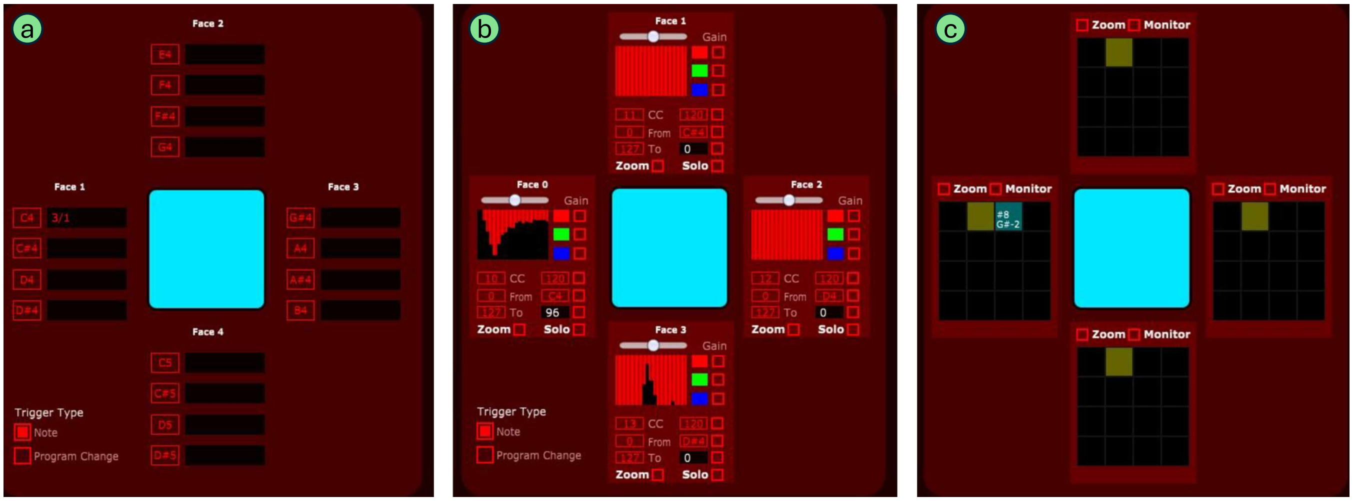


Figure 5: The three AudioCube configuration modes: Sender-Receiver (a), Sensor (b), and Topology (c).

response to these shortcomings, this work aims to explore possible gestures as widely and systematically as possible. Classifications of cube gestures are not explored with fine granularity. The cubes in [30] are expressed in a generic way (Fig. 3).

We need to distinguish between gestures that are theoretically possible and gestures that are actually desired or preferred by end-users [83], which justifies the use of one or more elicitation studies to identify gestures defined by users (and not uniquely by the system or designers [92]) as an appropriate method [18] to arrive at a broad classification [15, 90]. In addition, knowing how many participants could contribute helps determine how many categories of gestures could ultimately be obtained for a consensus set and balance the number of participants with the number of gestures. In the remainder of this paper, we hypothesize that the tangible cubes used for the experiments (AudioCubes) are fixed polyhedra to preserve the properties mentioned in Section 1. Because their form factor is stable, fixed cubes provide consistent affordances: users always know how to grasp, rotate, or arrange cubes, which reduces cognitive load.

To perform our GESs, we present AudioCubes (Fig. 4), a cubic TUI that senses its position, movement in space and rotation, and detects its physical topology with others at run-time. A AudioCube consists of a wireless, battery-powered computer embedded in a plastic cube with built-in sound and general-purpose signal-generating and processing capabilities. Each face emits some colored light for continuous visual feedback (Fig. 4-a) and can be attached with a symbol representing its functions (*e.g.,* a digital sound effect in Fig. 4-d). See Appendix A for a description and Appendix B for positioning AudioCubes with respect to some frameworks.

3.2 How the AudioCubes Work

Each AudioCube senses the location and orientation of the other, as well as the distance to the hands, fingers, and other objects. Each face, except the bottom and top faces, has a communication port capable of transmitting and receiving digital or analog signals, using infrared light, to and from other AudioCube nearby. They can be arranged together to form a physical topology that can be detected and recognized. AudioCubes communicate with each other through this port and can exchange sound or data signals, depending on the software running for this purpose.

3.3 AudioCubes Operating Modes

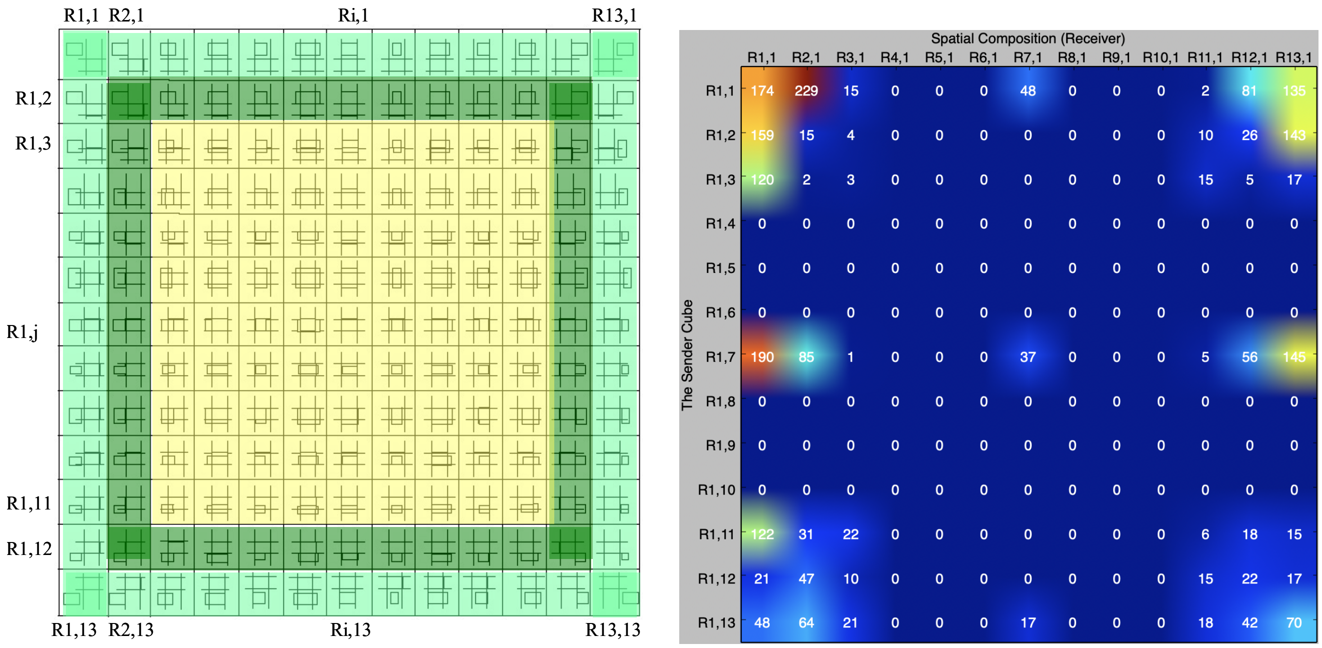
AudioCubes operate in one of three modes (Fig. 5):

(a) Sender-Receiver Mode: one cube plays the Sender and the other plays the Receiver, which transmits to the software the number on the face of the Sender and the number of the face with which it perceives the Sender. When a Sender is positioned next to a Receiver and the sensors line up, the Receiver detects the Sender, and the Receiver will output a message through that port when it detects the Sender.
(b) Sensor mode: a cube will work by itself, it does not detect any other cube, it just measures distance to the hands or other objects nearby using its sensors. Each face has a graphical display that shows this distance, which is sensitive to lighting conditions[1]. The Sender-Receiver mode uses one wireless cube as Sender and one wired cube as Receiver) to generate messages. In contrast, the Sensor mode generates MIDI continuous controllers.
(c) Topology mode: when the MIDI Bridge starts, the cubes are configured in this mode by default. This mode offers the broadest range of possibilities of all modes, allowing the generation of MIDI trigger notes when two cubes detect each other while also providing distance data between them.

3.4 Positioning AudioCubes in a Topology

While other TUIs detect their relative position in terms of angles based on the interaction range [61], AudioCubes characterize their topology in terms of cube compositions. There are thirteen types of interval relationships, classified as either *sequential* (before) and *simultaneous* (equal, meets, overlaps, during, starts, or finishes) [2].

[1]Lighting variations can be compensated using the Gain slider on top of the graphical display of the distance for each face.



(a) The theoretical space based on Allen [2]'s relationships. **(b) The corresponding heatmap observed in our studies.**

Figure 6: Spatial compositions of two cubes: the theoretical space (*a*) shows how two cubes can be arranged, separated (light green), tiled (dark green), or superimposed (yellow); the corresponding heatmap of these compositions found in our studies (b). For example, two cubes of the same size placed exactly one above the other (cell $R_7, 7$ in (a)) were observed 37 times (b).

Each relationship, except for the symmetric equal relation, has an inverse. These unidimensional temporal intervals can be generalized to n dimensions [81], for example, to $2D$ [70]. In this framework, a composition of two cubes within a topology is represented as SpatialComposition $(C1, C2)=(R_i, R_j)$, where $\forall i, j \in \{1, \ldots, 13\}$, R_i denotes the spatial relationship as the identifier of the spatial relationship between $C1$ and $C2$ along the X axis, and R_j along the Y axis. Fig. 6a illustrates the 13^2=169 compositions of two cubes where the first two rows (*i.e.,* $R_{1,1}$–$R_{13,1}$ and $R_{1,2}$–$R_{13,2}$), the first two columns (*i.e.,* $R_{1,1}$–$R_{1,13}$ and $R_{2,1}$–$R_{2,13}$), and the last two rows represent compositions where cubes are not stacked. In Fig. 6b, these cases, shown in light green, indicate that the cubes are completely separated. Cubes that touch or share a part are depicted in dark green, while the yellow area represents spatial compositions involving overlap. A *topology* is thus defined as a set of all spatial compositions between cubes.

3.5 Rotating an AudioCube

To characterize how a AudioCube can be rotated in the software, we need to identify the 13 symmetry axes of a cube (Fig. 7): four diagonals, three axes joining the centers of opposite faces, and six axes joining the midpoints of two opposite edges of the cube. There are two rotations for each diagonal (by an angle of 120° or 240°), different from a non-rotation, bringing the cube into coincidence with itself. There are eight rotations around the diagonals. For each axis joining the centers of two opposite faces, there are three different rotations from the non-rotation (by an angle of 90°, 180°, or 270°) [54]. Thus, three axes give nine rotations. Finally, the cube can be rotated 180° around any axis joining the midpoints of two opposite edges, leading to six rotations. Hence, $(8 + 9 + 6) = 23$ rotations with identity give 24 possible rotations (see Appendix D). The cube group is a mathematical group of $O(24)$, which is the largest order in mathematics (see our demonstration in Appendix C). If a cube rotation can be mapped to one command, users can make 24 different commands by rotating a AudioCube. Now that AudioCubes can be characterized with spatial compositions in their topology and axes of symmetry in their rotation, we are ready to perform some studies with them.

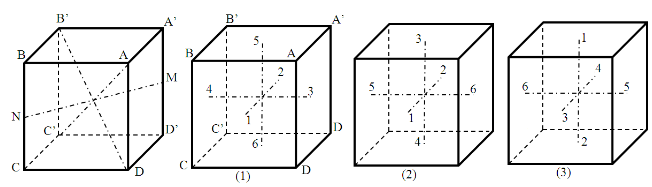
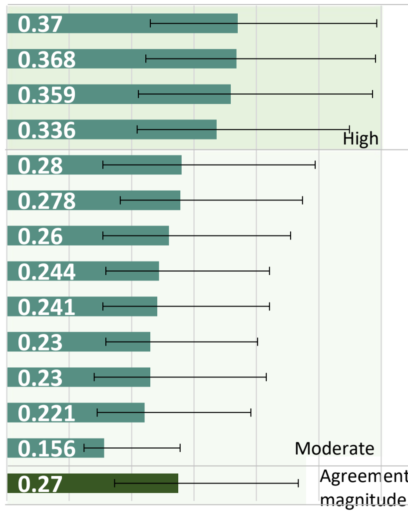


Figure 7: Thirteen symmetry axes of a cube.

4 Gesture Elicitation Studies

To address our research question, we conduct three GESs to explore user-defined gestures useful for designing tangible interaction with AudioCubes, a representative examples of cubic TUIs. This section presents methodological aspects common to the GESs. The specific details of each GES and its results are given afterward.



Referent (r)	Agreement rate AR(r) in [0..1]	Thinking-Time	Goodness-of-Fit	Consensus gesture
Navigate the platform	0.37	7.49 ± 2.23	△ 6.80 ± 2.02	Slide the cube on the table
Play/pause	0.368	▽ 9.76 ± 1.18	6.73 ± 2.29	Moving in the air
Exit the film	0.359	6.48 ± 1.50	▽ 5.63 ± 2.21	Turn the cube face down
Skip the recap	0.336	6.88 ± 3.83	5.73 ± 2.16	Lift the cube
Mute	0.28	7.53 ± 1.72	6.10 ± 2.47	Tap a face once with a hand
Start again	0.278	7.14 ± 2.33	6.57 ± 2.36	Tap a face once with a finger
Display subtitles	0.26	6.59 ± 3.11	6.57 ± 2.20	Slide fingers on a face
Move forward/backward in...	0.244	△ 3.26 ± 0.85	△ 7.30 ± 2.52	Moving forward/backward
Add to My list	0.241	6.01 ± 2.47	6.53 ± 2.00	Tap a face once with a hand
Turn up/down Volume	0.23	8.35 ± 2.16	6.00 ± 2.71	Slide fingers on a face
Next episode	0.23	7.28 ± 2.47	6.53 ± 2.64	Rotate the cube on itself
Turn TV on/off	0.221	△ 6.14 ± 2.40	6.70 ± 2.34	Tap a face once with a hand
Adjust Reading Speed	0.156	▽ 8.23 ± 2.17	▽ 5.77 ± 2.36	Slide fingers on a face
Average	0.27	**7.01 ± 2.19**	**6.38 ± 2.33**	

Figure 8: Consensus set of gestures resulting from the gesture elicitation study #1: referent, Agreement-Rate and magnitude, Thinking-Time, Goodness-of-Fit, and consensus gesture. The two best and worst values of the two variables are depicted by a top-pointing arrow and a bottom-pointing arrow, respectively. The error bars represent a confidence interval of 95%.

4.1 Method

4.1.1 Participants. Thirty[2] independent and voluntary participants were recruited for each GES through a contact list in different organizations and via social networks (GES1: 15 women, 15 men, Male/Female ratio=1, aged between 19 and 58 years, M=40.97, SD=19.89 – GES2: 14 women, 16 men, Male/Female ratio= 1.14, aged between 17 and 64 years, M=36.93, SD=17.64, – GES3: 13 women, 17 men, Male/Female ratio= 1.31, aged between 19 and 77 years, M=45.74, SD=21.49). Their occupations and professions varied. For example, GES1 included 11 employees, 5 independent workers, 4 retired individuals, 4 students, 3 executives, and 3 unemployed individuals. All participants reported frequent use of computers and smartphones, less use of tablets, no dexterity problems, no vision problems, and no prior experience with AudioCubes.

4.1.2 Stimuli. We identified the most frequently executed functions across various use contexts. Thirteen referents were selected for interacting with a smart TV in GES #1 [41, 77, 80]: "Turn the TV on/off", "Turn the volume up/down", "Mute", "Display subtitles", "Play/pause", "Move forward/backward in a film", "Exit the film", "Go to next episode in a series", "Navigate the seasons of a series", "Adjust the reading speed", "Add a film to my list", "Skip the recap", "Start again". We created a Microsoft PowerPoint presentation showing a *referent* [90] for each function through a before/after AudioCube action on a single slide, providing a simplified view of the interaction between cause and effect.

4.1.3 Setup and Apparatus. The experiment was carried out in a quiet usability lab. One laptop randomly displayed the referents to the participants, and another was used for up to four AudioCubes in Sender-Receiver mode. All AudioCube gestures were recorded using an application through the MIDI Bridge and a camera placed in the back to capture their hands and fingers without their faces.

4.1.4 Procedure and Task. The participants were welcomed to the setup by an experimenter. They were asked to sign an informed, GDPR-compliant consent form that had been formally approved by the Ethics Committee in Psychology of our organization. Then, they were provided with information about the GES and its general procedure. They were told they could leave at any time, without justification or consequences. No compensation was offered. Each session implemented the original procedure of GES [78, 90]: participants were presented with the *referents*, *i.e.,* actions to ensure the function in each context, for which they proposed a AudioCube gesture to execute those referents, *i.e.,* gestures that fit referents well, are easy to produce and remember. The participants were instructed to remain as natural as possible. The global order of referents was randomized per participant based on a real random generator based on weather temperatures. Each session took approximately 25 minutes per participant. One researcher welcomed participants and checked the form filling, and another experimenter presented referents, ensuring minimal interference from and among the researchers to mitigate the Hawthorne effect. The dependent variables for our within-subjects design are:

(1) Agreement-Rate (AR): a real variable representing the agreement among participants to propose a gesture for a referent, calculated by AGATe [78] along with its magnitude (AR ≤0.1=low, 0.1< AR ≤0.3= moderate, 0.3< AR ≤0.5= high, 0.5< AR ≤1= very high).
(2) Thinking-Time (TT): a real variable defined as the average time elapsed between the first showing of the referent and

[2] 30 participants is a common benchmark from the Central Limit Theorem for certain statistical analyses [80].

the moment when the participant knew which gesture would be preferred, in seconds [27].

(3) Goodness-of-Fit (GoF): a real variable defined as the average rating from 1 to 10 expressing to what extent the participants thought their gesture was appropriate [27].

4.2 Results and Discussion of GES #1- Smart TV with one AudioCube

We collected 30 participants × 13 referents = 390 gesture proposals. Fig. 8 shows the consensus set of gestures resulting from the study, decomposed into the name of the gesture most commonly agreed on (col. 5) by maximizing Agreement-Rate and its magnitude (col. 2), its corresponding Thinking-Time (col. 3), and Goodness-of-Fit (col. 4). The referents (col. 1) are sorted in decreasing order of their Agreement-Rate.

The Agreement-Rates are overall moderate to high in magnitude (M=0.27, SD=0.20), ranging between 0.156 (moderate) for "Adjust Reading Speed" and 0.37 (high) for "Navigate the platform". In global sampling, $\frac{9}{13}$ = 69% of the rates belong to the moderate consensus category and $\frac{4}{13}$ = 31% of them belong to the high range, making them appropriate and eligible for gesture classification. Most gestures received agreement rates slightly higher than, or close to, those reported in the GES literature (*e.g.,* [78], which summarized agreement rates in 18 GESs).

The Goodness-of-Fit received generally a high value that indicates subjective satisfaction of the participants with the gestures they proposed (*e.g.,* M=7.01, SD=2.19), which is again aligned with positive results from the literature [27, 85]. They mostly appreciated their gesture proposals for "Move forward/backward in the film" (7.30 ±2.52 - which also received the fastest thinking time) and "Navigate the platform" (6.80 ±2.02), and less for unfamiliar commands such as "Adjust reading speed" (5.57 ±2.36 - also the slowest thinking time) and "Exit the film" (5.63 ±2.21). The Thinking-Time received reasonable values (M=7.01 s., SD=2.19 s. - which is a narrower deviation than usually), ranging from very short times (*e.g.,* "Move in the film" in 3.26 s and "Turn TV on/off" in 6.14 s) to longer times, often associated with less familiar or frequent commands (*e.g.,* "Adjust reading speed" in 8.23 s). The Thinking-Time correlates negatively with the Goodness-of-Fit (Pearson's $r_{n=13}= -.59$, R^2=.64): referents with a higher thinking time have a lower Goodness-of-Fit, thereby suggesting that participants were less satisfied with their gesture proposal when they took more time to decide on them. Although several categories have emerged from this study, they remain insufficient for generalization. For this purpose and for research reproducibility [76, 88, 89]), we are conducting a second study of type-5 [28] (same method, same population, new participants, different team and setup, same protocol).

4.3 Results and Discussion of GES #2- Smart TV with two AudioCubes

This subsection discusses the results of the second GES performed in a living room (instead of a usability lab in GES #1) equipped with a smart TV, two AudioCubes configured in Topology mode, and a revised list of 16 referents for smart TV. We collected 30 participants × 15 referents = 450 gesture proposals. Fig. 9 shows the consensus set of gestures resulting from the study, decomposed into the name of the gesture mostly agreed on (col. 5) by maximizing the Agreement-Rate and its magnitude (col. 2), its corresponding Thinking-Time (col. 3), and Goodness-of-Fit (col. 4). The referents (col. 1) are sorted in decreasing order of their Agreement-Rate.

The Agreement-Rates are all low in magnitude (M=0.10, SD=0.23), ranging from 0.03 for "Child mode" and 0.127 for "Turn up/down the volume", except for "Pause the music" (moderate, 0.161). In this study, most gestures received an agreement rate lower than or close to those reported in the GES literature [78]. The average agreement rate in this study was about half that of the previous study, for two reasons: (1) participants were able to propose gestures with two cubes in a living room set up to resemble a domestic environment, which relaxed them, they were given the option of using one or two cubes, and (2) the number of referents increased to 16. This greatly enlarged the design space for gesture possibilities, reducing the agreement rate, but within acceptable limits. This is also reflected in the next measure.

The Thinking-Time received reasonable values (M=12.46 s., SD=9.50 s. - which is about 78% more than in the previous study), ranging from very short times, as usually the case (*e.g.,* "Play the music" in 6.88 s. and "Turn up/down volume" in 8.01 s.) to much longer times (*e.g.,* "Go to the album of the music playing" in 18.98 s. and "Start rooming" in 17.32 s). The Goodness-of-Fit did not change much on average with respect to the previous study (M=6.93, SD=2.07), which is again aligned with positive results from the literature [27, 85]. They mostly appreciated their gesture proposals for "Pause the music" (7.77 ± 2.04) *ex aequo* with "Go to the next/previous music" (7.77 ±1.67) - two very familiar functions-, and less for unfamiliar commands such as "Play the music again" (5.73 ± 2.16). In this study, the team identified 20 gesture classes, compared with 11 in the previous survey. To continue the effort towards extension and generalization, we conducted a third study of type-7 [28] (same method, different population, team, and setup).

4.4 Results and Discussion of GES #3- Smart Living Room with two AudioCubes

This subsection discusses the results of the second GES performed in a living room (instead of a usability lab in GES #1) equipped with a smart TV, two AudioCubes configured in Sender-Receiver mode, and a revised list of 20 referents for a smart living room equipped with different devices, such as an air conditioning system. Some referents were common with GES #2 (*e.g.,* "Turn TV On", "Go to next item") to compare them in two contexts. In contrast, some other referents were different to introduce some variability (*e.g.,* "Brighten lights" and "Dim lights"). We collected 30 participants × 20 referents = 600 gesture proposals. Fig. 10 shows the consensus set of gestures resulting from the study GES #3, decomposed into the name of the gesture mostly agreed on (col. 5) by maximizing the Agreement-Rate and its magnitude (col. 2), its corresponding Thinking-Time (col. 3), Goodness-of-Fit (col. 4). The referents (col. 1) are sorted in decreasing order of their Agreement-Rate.

The Agreement-Rates range from low ($\frac{8}{20}$ = 40%) to high ($\frac{3}{20}$ = 15%) in magnitude (M=0.181, SD=0.36), ranging between 0.099 (low) for "Turn light off" and 0.356 (high) for "Turn down the volume". $\frac{12}{20}$ = 60% of proposals have a magnitude higher than 'low', which is a significant part of the proposals belonging to an

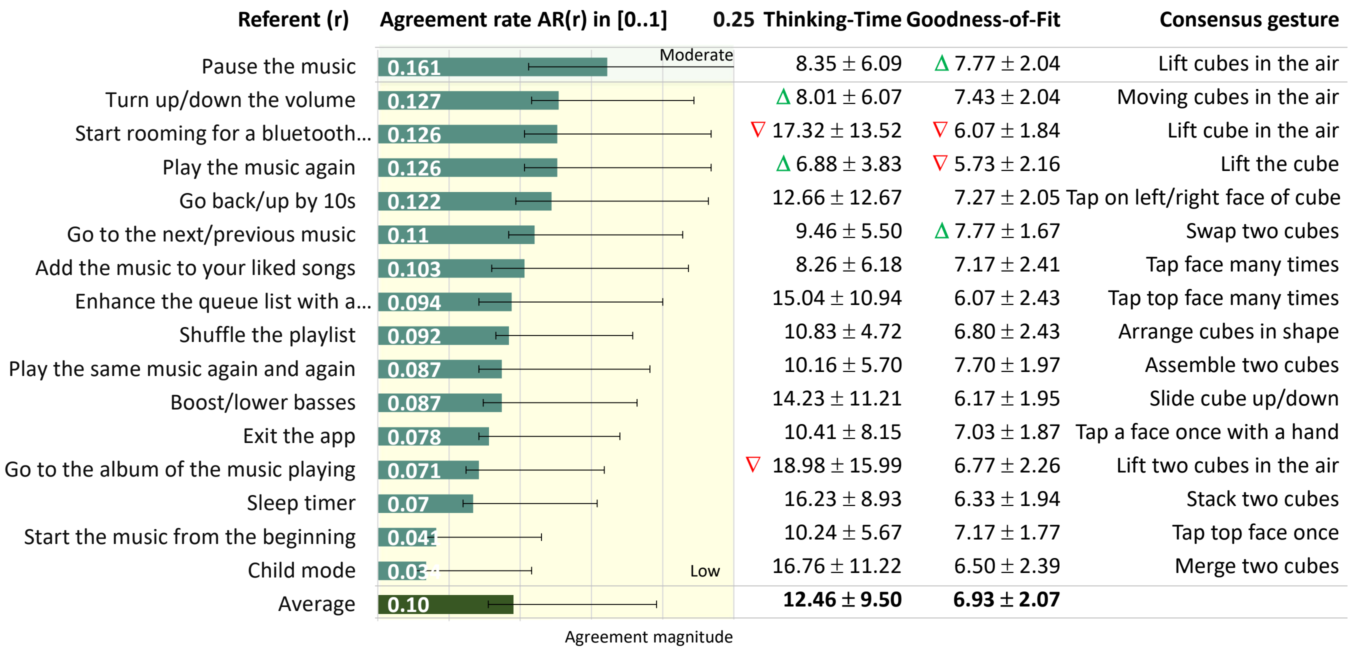


Figure 9: Consensus set of gestures resulting from the gesture elicitation study #2: referent, Agreement-Rate and magnitude, Thinking-Time, Goodness-of-Fit, and consensus gesture. The two best and worst values of the two variables are depicted by a top-pointing arrow and a bottom-pointing arrow, respectively. The error bars represent a confidence interval of 95%.

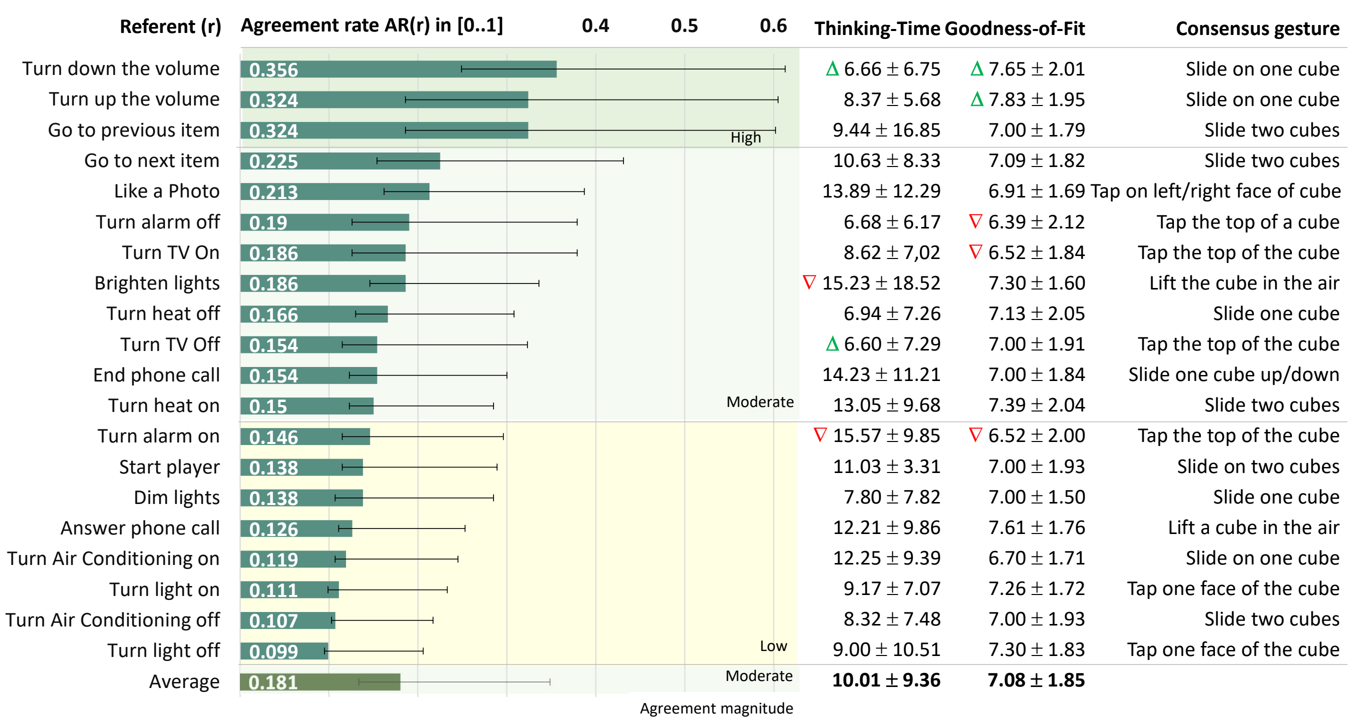


Figure 10: Consensus set of gestures resulting from the gesture elicitation study #3: referent, Agreement-Rate and magnitude, Thinking-Time, Goodness-of-Fit, and consensus gesture. The two best and worst values of the two variables are depicted by a top-pointing arrow and a bottom-pointing arrow, respectively. The error bars represent a confidence interval of 95%.

acceptable range. Most of the agreement rates are very similar to those in GES #1 in both magnitude and proportion. "Turn down the volume" and "Turn up the volume" received the highest agreement among participants, as they are the most frequent and familiar.

The Thinking-Time received values (*M*=10.01s, *SD*=9.36s) that are aligned with GES #2 ranging from very short times (*e.g.,* "Turn TV Off" in 6.60 s and "Turn down volume" in 6.66 s) to much longer times (*e.g.,* "Brighten lights" in 15.23 s and "Turn alarm on" in 15.57 s). The Goodness-of-Fit did not change much on average with respect to the previous study (*M*=7.08, *SD*=1.85), which is again aligned with positive results from the literature [27, 85]. Despite the different reproducibility types in this study compared to the two previous ones, we can see that the value intervals for the three measurements remain relatively constant and well-adapted to the environmental conditions, both in terms of their average and standard deviation. This relative similarity of value distributions encourages us to retain the results of these studies for classification purposes, which we address in the next section.

5 Designing User-Defined Gestures with Tangible Cubes

5.1 Classification

In total, we collected 390 + 450 + 600 = 1,440 gesture proposals from 3 × 30 = 90 participants for 13+16+20 = 49 referents. Each gesture proposal was identified, characterized according to *a descriptive labeling* [62] using Nielsen et al. [48] *et al.*'s procedure, and assigned to a unique gesture category. In this procedure, the gestures are based on their motions and actions rather than their semantic meaning or manipulation, such as in Aigner et al. [1]'s taxonomy.

Since different experimenters were involved in each study to characterize each gesture proposal, some labeling inconsistencies may have been introduced. Therefore, an independent researcher applied a *decomposition-extension* to each gesture proposal (Table 1): the labeling of each gesture proposed in a GES was first captured in freeform in a gesture sheet (Step 1), then decomposed (Step 2) into Action verb, object, parameter(s) inspired by Angelini et al. [5] and detailed in terms of cube part [20], finger(s), hand(s) [53], motion, and spatial composition (see Fig. 6a for all theoretical compositions and Fig. 6b for the corresponding heatmap found in our studies). The decomposed labels (Step 2) were then extended to obtain gesture categories (Step 3), establishing independence from the cube parts, fingers(s), and hand(s). For this purpose, the independent researcher created a table mapping each gesture proposal to a category in our classification and applied it uniformly. The resulting categories are therefore structured as follows. Category-1 gathers one-hand or two-hand contactless gesture with a cube positioned on a surface: the hands move in the air without touching the cube which remains stable, thus avoiding any physical touch. After this category, all remaining gestures are contact-based. Category-2 gathers one-hand contact-based gestures with a cube on a surface: a single-hand contact enables a simple physical interaction with the cube. This category contains many sub-categories that exceed the number of ten in Category-2. Thus, a numbering skip is intentionally introduced: there is no Category-3. Category-4 gathers two-hand contact-based gestures with a cube on a surface: in contrast to the previous category, both hands manipulate the cube. Category-5 gathers one-hand grasping gestures of a cube in mid-air: the movement and control are done with one-hand grasp. Category-6 does the same with two hands instead of one: it gathers two-hand grasping gestures of a cube in mid-air, as it requires both hands to hold and transfer the cube. Category-7 gathers gestures to arrange multiple cubes simultaneously, including placing, stacking, and patterning.

To further address our research question, the resulting classification of user-defined gestures (Table 2 continued in Table 3) has been built to satisfy several abstraction properties.

5.2 Abstraction Properties of the Classification

The gesture categories are abstracted from the **number of fingers or hands** used to perform them. They are independent of the number of fingers or hands used for issuing gestures. Another study could investigate how fingers are placed on the cube to manipulate it, similar to what [20] did for a virtual cube. We observed no difference in meaning when the participant performed cube gestures with one or more fingers of the same hand, or with both hands. This observation is similar for the hands. For example, touch gestures in Category-1 can be performed with one or many fingers, whether they belong to the same hand or not. However, touching is sometimes substituted for a larger surface than the fingertip, such as one or more fingerprints, a hand palm, or the entire hand surface, which may convey a special meaning.

The gesture categories are abstracted from their **repetition**. They are independent of the number of times the same cube gesture has been performed: two successive repetitions of the same gesture, performed within an average time interval, are considered as two occurrences of the same gesture. Now, if two such repetitions occurred within a very short time interval, this repetition would make sense, but we have rarely observed it. For example, touch gestures can be repeated at will. Two consecutive occurrences of other gestures belonging to two categories are considered separate. For example, two successive swipes are considered two distinct gestures. Consequently, the abstraction from repetition also abstracts the categories from their rhythm. Very few cube gestures were repeated with a rhythmic pattern, such as up and down several times, based on a rhythmic pattern on a face or Morse code on a face, which requires understanding the code.

The gesture categories are abstracted from their **articulation time**. They are independent of the gesture speed: two executions of the same gesture, performed at different speeds, are considered to belong to the same category. If two such executions were to occur frequently, a distinction could make sense based on their speed, but we have rarely observed this. For example, some gestures induce some meaning to their speed: a fast gesture in front of a cube face could be interpreted differently from a slow gesture of the same type. Very few occurrences were found.

The gesture categories are abstracted from their **persistence time**. They are independent of the persistence in time with which the gesture was performed: two executions of the same gesture, one instantaneous and the other prolonged in time, thus having different persistence in time, are considered to belong to the same category. Now, if two such executions were to occur frequently, differentiation might make sense, but we have rarely observed this.

Table 1: The three steps for mapping a gesture proposal to a gesture category (Participant P2).

Step 1: Initial Capture of the Proposed Cube Gesture

Referent	Initial labeling	Referent	Initial labeling
Turn TV on	Return the current cube down and push on its top face with 2 fingers	**Turn TV off**	Separate the control cube horizontally from the source cube with the dominant hand until the setting is off
Thinking time	9.6 sec	Thinking time	15.1 sec
Goodness of Fit	7	Goodness of Fit	7

Step 2: Decomposition of Gesture Proposals

Referent	Initial labeling	Referent	Initial labeling
Turn TV on	Turn the cube on another face Touch one cube face with 2 fingers	**Turn TV off**	Move a cube horizontally by pushing it right

Step 3: Extension to Gesture Categories

Referent	Initial labeling	Referent	Initial labeling
Turn TV on	Spill the cube on another face (36), Touch one or many cube faces with one or more fingers (22)	**Turn TV off**	Move a cube horizontally by pushing/pulling left/right (32)

For example, a gesture of touching the face can be instantaneous (the participant touches the face and then withdraws his hand) or sustained for a particular time (as if to prolong the action). Persistence over time seems to be a function of the contact surface: the greater the contact surface, the greater the possibilities for different persistence. Participants seem to be more inclined to perform shorter persistence on the corners and edges of the cube than on the faces, as if the faces were acting as a contactor. For example, touching a cube corner with one or two hands can be done once or held for a few seconds, but is rarely used.

The gesture categories are abstracted from their **pressure**. They are independent of the pressure with which the gesture is exerted on a contact surface: two executions of the same gesture, performed with different pressures, are considered to belong to the same category. If now two or more such executions were to occur frequently, a differentiation could make sense according to their pressure: *e.g.,* regular touch, light pressure, firm pressure. We do not distinguish gestures with different pressures, not because the AudioCube used in the experiment is not pressure-sensitive (other tangible objects exist for this purpose in the Buxton taxonomy [14]), but mainly because very few occurrences were observed.

Category-6 cube gestures are abstracted from their main **operation**, such as move, swipe, or touch. This category is very brief, as it mainly reproduces a gesture from another category (*e.g.,* 1, 2, or 5) performed with the cube in one hand, then with the other hand, whether the hand is moving or not. Consequently, the corresponding gesture of the related category is made abstract and not determined.

The Category-1 scope is restricted to the simplest cube gestures in terms of articulation. These gestures could, in theory, be any one-handed mid-air or surface-constrained gesture performed without touching a cube (see existing classifications of mid-air gestures). In practice, little or no variation in such gestures was observed, not because the sensor could not recognize them, especially subtle ones, but mainly because people did not propose them. Such gestures were limited to moving a hand from a cube or to a cube, with some exceptions, such as drawing a symbol or a particular movement (*e.g.,* a swing, finer snapping).

Cube gestures are preferred according to their contact surface. The greater the surface area in contact with the cube, the more easily and precisely the cube gesture will be performed. For this reason, participants preferred cube gestures in decreasing order of contact surface: first to one or more faces, then to the edges, and finally to the corners.

Cube gestures are preferred according to their accessibility or visibility. The more accessible or visible the face, the edge, or the corner of the cube, the easier it is to perform the cube gesture. For this reason, participants preferred cube gestures in decreasing order of accessibility or visibility: top face, front face, left and right faces, and then back face. For example, Category-1 covers moving gestures performed preferably on the top face, on the face in front of the participant, and then on the lateral faces. Since such gestures were performed preferably on the top face, a unique category (16) is created. Similarly to this category, Category-2 covers touch, swipe, and moving gestures performed preferably on the top face, the front face, and on the lateral face, rarely on the back face. What applies to cube faces also applies to edges and corners, although the latter are much less preferred. For example, the three edges (*i.e.,* top, left, and right) and the two corners (*i.e.,* top left and top right) closest to the person are preferred to those at the back of the cube.

Category-1 contactless gestures prefer to be executed with one hand only. All sub-categories of contactless gestures, except one (19), involve one hand only because the vast majority of proposed gestures do so. Reproducing the same gesture sub-categories with two hands is feasible, but we did not observe many variations; the most frequent was moving the two hands to a cube or departing from a cube, most often in a symmetric manner, aiming to hide or reveal two opposite faces. For this reason, we did not create other sub-categories for two-handed contactless gestures.

Table 2: Classification of user-defined gestures with tangible cubes (Part 1/2).

Code	Definition
Cat. 1	One/two-handed contactless gesture with a cube on a surface
10	Move one hand horizontally (left/right) from/to a cube face
11	" vertically (up/down) from/to a cube face
12	" forward/backward from/to a cube face
13	" circularly (rotate) in front of a cube face (clockwise or counter-clockwise)
14	Draw a symbol in front of a cube face
15	Perform another contactless gesture in front of a cube face (*e.g.,* snap fingers)
16	Move one hand vertically on top of the cube
17	Swing one hand on top of the cube
18	Perform any contactless gesture on top of the cube
19	Move two hands horizontally (left/right) from/to two faces of a cube
Cat. 2	One-handed contact with a cube on a surface
20	Touch one or many cube corners with one or many fingers, one or many times
21	" edges with one or many fingers, one or many times
22	" faces with one or many fingers, one or many times
23	" faces with more than a fingertip: one or more fingerprints (*e.g.,* thumb and index), a palm hand, or the entire hand surface
24	Touch two or more cube corners with one or many fingers/hands, one or many times
25	" edges with one or many fingers/hands, one or many times
26	" faces with one or many fingers/hands, one or many times
27	Swipe on a cube edge (left/right, up/down, forward/backward): horizontal, vertical, lateral
28	" horizontally on a cube face (left/right)
29	" vertically on a cube face (up/down)
30	" circularly on a cube face (forward/backward)
31	Draw a symbol on a cube face with one or many fingers
32	Move a cube horizontally by pushing/pulling left/right
33	Move a cube forward/backward by pushing/pulling in front of the cube or in the rear
34	Rotate the cube clockwise/counter-clockwise by grasping it
35	Twirl the cube clockwise/counter-clockwise
36	Spill the cube on another face
Cat. 4	Two-handed contact with a cube on a surface
40	Take a cube in one hand on a surface and perform any Category-2 gesture with the other hand
41	Touch two or more cube corners with fingers of two hands, one or many times
42	" edges with fingers of two hands, one or many times
43	" faces with fingers of two hands, one or many times

Category-6 gestures could be expanded with specific gestures conveying a particular meaning, such as swapping or flipping [95]. These actions are high-level. For example, moving a cube in front of another may convey an ordering. Arranging cubes in 2D may have some meaning and introduce a new category. Two-cube gestures go beyond replicating one-cube gestures, for example, stacking, unstacking, and arranging cubes in 3D (*e.g.,* in a stair).

5.3 Mathematical Model

To address our sub-question *SQ*= how many participants are required to obtain a predefined number of user-defined gestures, we examined the number of gesture categories proposed by each participant. Fig. 11-a presents the cumulative number of categories introduced for the thirteen referents in GES #1 by the first twelve participants. At the beginning, participants propose gestures that fall into different categories. As new referents are presented, their creative ideation, which draws on working memory, cognitive resources, executive control, and the space of potential gestures, diminishes. The further the participant advances in the experiment, the fewer new categories are introduced (Fig. 11-b). Consequently, the ability to propose new categories declines (Fig. 11-b), but is internally consistent for each participant. The participants exhibit a relatively stable overall yield, typically contributing between five and ten new categories across thirteen referents (Fig. 11-c).

Nielsen and Landauer [47] identified a mathematical model for estimating the number of evaluators needed to find usability problems of graphical user interfaces in various interactive systems.

Table 3: Classification of user-defined gestures with tangible cubes (Part 2/2).

Code	Definition
Cat. 5	One-handed grasping of a cube in the air
50	Lift a cube in the air, move it horizontally, release it on the surface
51	" , move it vertically, release it on the surface
52	" , move it forward/backward, release it on the surface
53	" , move it circularly (clockwise/counter-clockwise)
54	" , move it to a body part (body deictic, eg ear) and release it on the surface
55	" , rotate it horizontally, and release it
56	" , rotate it laterally on another face, and release it
57	" , shake it, and release it on the surface
58	" , roll it on a table, and release it
59	" , throw it
Cat. 6	Two-handed grasping of a cube in the air
60	Lift a cube with one hand in the air and perform any Category-1 gesture with the other hand, maybe while moving, without touching
61	Lift a cube with one hand in the air and perform any Category-2 gesture with the other hand, maybe while moving
62	Lift a cube with two hands in the air and perform any Category-7 gesture, maybe while moving
63	Swap a cube from one hand to another, and vice versa
Cat. 7	Multi-Cube Arrangement Gestures, including placing, stacking, and patterning
70	Align two or more cubes side-by-side on a flat surface, so that adjacent faces detect each other via proximity (*e.g.,* Cube A->face 1 detects Cube B-> face 3)
71	Stack one cube on another, detected by proximity
72	Arrange cubes in different patterns (*e.g.,* triangle, square, cross)
73	Arrange cubes in a stepwise pattern, increasing height (*e.g.,* place one cube directly on top of another)
74	Align two or more cubes in a straight horizontal or vertical line
75	Stack cubes while offsetting each one to form a spiral or angled tower (*e.g.,* alternating stacks)
76	Place cubes symmetrically on both sides of a center point (*i.e.,* mirror placement)
77	Arrange cubes in a grid (*e.g.,* a 2×2 or 3×3 matrix)
78	Place cubes around another cube to simulate enclosing (Enclosure)
79	Drop cubes one by one to form a timed sequence pattern (Cascade)

Drawing on Nielsen and Landauer [47]'s modeling of usability-problem discovery, an analogous model estimates the number of participants required to propose a target number of gesture categories. We treat the appearance of new gesture categories as a Poisson process, wherein the number of new gesture categories within the time interval of an experiment follows a Poisson distribution. The Poisson law for the number of arrivals (here, the number of new gesture categories) states that the number of random, independent events (here, an event represents a new gesture category) occurring in a fixed interval of time (here, the duration of an experiment for a participant) follows a Poisson distribution that can be properly estimated [45]. Under a Poisson distribution with the expectation of λ events in a given interval, the probability of k events in the same interval is [93]:

$$p(k) = \mathbb{P}(X = k) = \frac{\lambda^k e^{-\lambda}}{k!} \tag{1}$$

The main hypothesis for applying a Poisson law is that the probability of proposing a gesture category in any GES is independent of the gesture categories that have been previously proposed, either by the same participant or not. We believe that this assumption is realistic for the same participant as each participant was instructed to propose a gesture independently of what has been proposed before, since the referents were presented randomly. We also believe that this assumption holds across participants since they never met and they were not aware of what has been done by others. Thus, the probability that a participant proposes a specific gesture category would be independent of whether the other participants had proposed the same category.

Therefore, key hypotheses of this process include a constant average arrival rate (each participant always proposes one gesture for each referent and the Thinking-Times are limited), independent arrivals in non-overlapping intervals (gesture categories are independent and the participants do not know others' proposals), and a very low probability of multiple arrivals in an small time interval (this would not happen since we limited one gesture per referent). The number of arrivals in an interval of length τ follows a Poisson distribution with parameter λ, where λ is the average

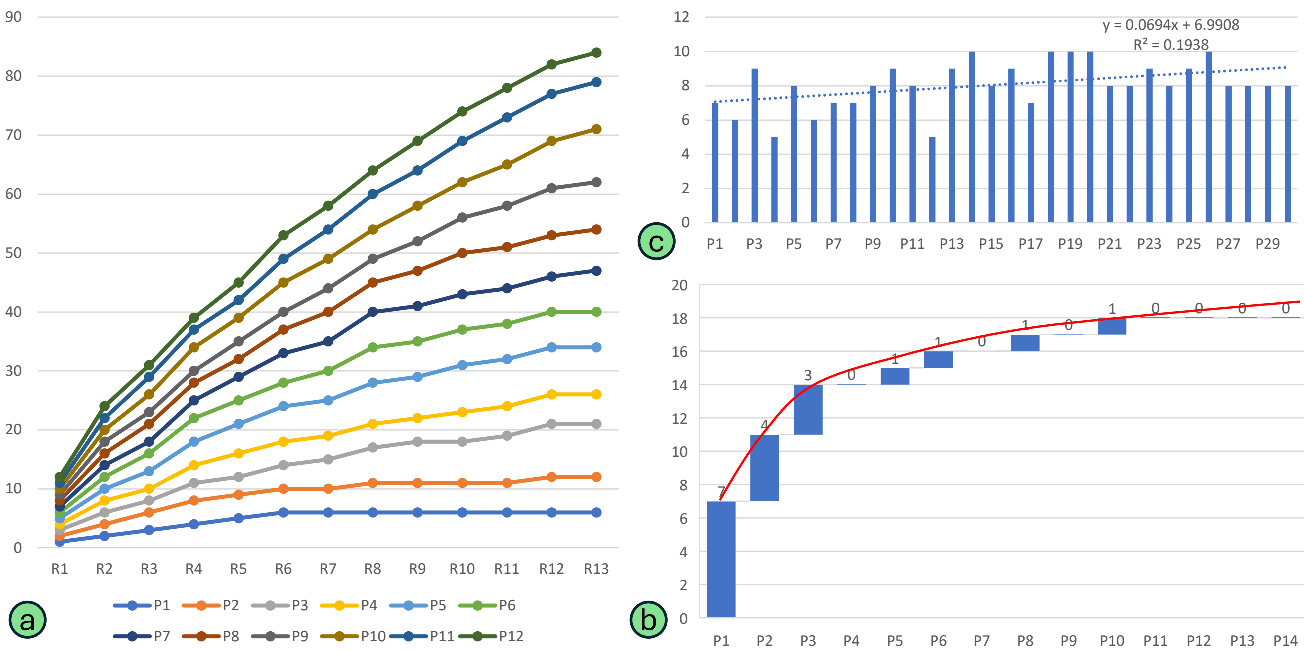


Figure 11: Distribution of gesture categories: cumulative number of gesture categories per participant (a), number of new gesture categories by participant (b), and number of categories per participant (c).

arrival rate. We can therefore apply a Poisson law to the production of gesture proposals. The red curve in Fig. 11-c indicates the fitted curve according to equation [47]:

$$\text{NewCategories}(i) = N(1 - (1 - \lambda)^i) \tag{2}$$

where N is the total number of gesture categories desired, i is the number of participants in the study, and λ is the average probability that a participant introduces a new gesture category.

Let us now explain the formula 2. Since λ is the probability of receiving a new gesture category by a participant, then $(1 - \lambda)$ corresponds to the probability of not receiving a new gesture category by this participant. Having i participants proposing gestures belonging to categories, then the probability that none of the participants will propose a gesture category is $(1 - \lambda)^i$. Thus, the probability that a gesture category would be proposed by at least one participant is the inverse: $1 - (1 - \lambda)^i$. Since N is the total number of gesture categories, we multiply this proportion of categories found by N to obtain the total number of distinct categories proposed: $N \times (1 - (1 - \lambda)^i)$, which is eq. 2. Solving for i, equation 2 gives

$$i = \frac{\ln\left(1 - \frac{\text{NewCategories}(i)}{N}\right)}{\ln(1 - \lambda)} \tag{3}$$

If we desire to reach 90% (p=0.9) of all gesture categories, *i.e.*, NewCategories(i)=0.9 N, then:

$$i = \frac{\ln(1 - p)}{\ln(1 - \lambda)} \tag{4}$$

This shows that the required number of participants of a GES depends logarithmically on the remaining proportion of undiscovered gesture categories and on λ. For example, equation 4 with λ=0.5 and λ=0.2 gives $i=\frac{ln(0.1)}{ln(0.5)}\approx 3.32$ and $i=\frac{ln(0.1)}{ln(0.8)}\approx 10.32$, respectively. This means that, in practice, at least 4 and 11 participants are required to reach 90% of the expected gesture categories, respectively.

The participant $P10$ was the last to propose a new gesture category (Fig. 11-c, N=20). Ten participants proposed 90% of desired categories. Nielsen and Landauer [47] acknowledge two simplifications: (#1: The average). The model assumes that all participants are "average" and propose the same number of gesture categories. In reality, some participants are better than others or may propose more gesture categories. This simplification causes the model to slightly underestimate the true probability of proposing a gesture (λ) and, consequently, slightly overestimate the total number of gesture categories (N). (#2: Problem independence). The model assumes that the proposition of one gesture category does not influence the discovery of another. However, in reality, some gesture categories are related (if we propose one, it may help us propose another). This simplification has the opposite effect on the estimation of N.

An interesting observation regarding the number of participants N is that the values of the agreement rates AR between them are discrete, not continuous, in the [0,1] interval as they depend on N. Consequently, for N=10 participants, AR can take values in the [0,1] interval that are sampled at 1/45 (where $\frac{10\times 9}{2}$=45). If N=20, values are sampled at 1/190 (where $\frac{20\times 19}{2}$=190, a much finer unit. So, even if the gesture set identified with N=10 participants is very similar to that of N=20 participants, the numerical consensus reflected in the agreement rate may be different due to the different units [79]. One could guess that a low N value may be enough to identify a representative set of gestures, but at the expense of over- or underestimating the true agreement in the sample, which is used anyway to select the gestures with high consensus.

6 Discussion

6.1 Descriptive, Comparative, and Generative Virtues of the Classification

A quality interaction model must possess three virtues [8]: a *descriptive virtue* (*i.e.,* the model ability to describe a wide range of interfaces according to a consistent set of properties), a *comparative virtue* (*i.e.,* the model ability to compare different interfaces of the same category against the same properties, and to assess design alternatives), and a *generative virtue* (*i.e.,* the model ability to generate designs or ideas for new interfaces). We propose a discussion of our classification according to these three virtues.

6.1.1 Descriptive Virtue. Our classification of cube gestures for designing tangible interaction (Section 5) was analyzed according to their syntax and semantics to abstract it from several attributes (Section 5.2). We provided several examples (Fig. 1) that show how such gestures for tangible interaction can be described in terms of independent categories, regardless of properties such as pressure, speed, and direction. The classification presented seven global categories that can be further refined into sub-categories. Most gestures can be described using the proposed sub-categories.

However, some gestures that focus on a particular motion rather than tangible interaction with the object may require a more complex description. The wide variety of possible gestures with cubes suggests that a simple description of the full spectrum of tangible interaction is not always possible. A trade-off between the simplicity of the description and representativity of the (sub-)categories was pursued to enable a systematic classification of cube gestures. In principle, each cube gesture for tangible interaction can be mapped to a category in the same way, thereby allowing the comparison of gestures on the same basis, as addressed in the next virtue.

6.1.2 Comparative Virtue. As soon as cube gestures are mapped to a category in the classification, we can compare them on the same basis. Comparison of alternative gestures would be challenging without considering the specific application domain of the tangible interaction that the designer would like to build [92]. By exploring the classification, it is possible to choose from a wide range of gestures based on the application domain or the affordances of everyday objects [5]. More explorations are needed for each application area, *e.g.,* by comparing different gesture categories or different gesture sets by forming user evaluations.

GESs are a compelling approach to maximize user acceptance and guessability of gestures [90]. A wider evaluation of tangible gestures with interactive systems could be conducted according to the qualities of tangible interaction [74]. Anyway, our classification of user-defined gestures with tangible cubes can be compared with other references, such as Sheridan et al. [66]'s classification (Table 4), Potts et al. [55]'s (Table 5), and He et al. [30]'s (Table 6), to observe two types of correspondence: (1) *bijective correspondences*, in which an initial gesture belongs to a single category in our classification, and (2) *surjective correspondences*, in which an initial gesture can be related to several categories in our classification, mainly due to the precise definition of our classification compared to the initial one. Based on this comparison, we did not identify any missing gestures. Due the context of smart home use, we cannot generalize it to any other areas of activity.

6.1.3 Generative Virtue. As soon as cube gestures can be compared with each other against the same properties, we can identify new gestures or families of gestures that were not yet discovered or imagined, therefore supporting the "generation" of new gestures that were not considered before. There are still a variety of new cube gestures that can be explored, especially for conveying some meaning that is domain-dependent, that can be combined, that can be arranged in a meaningful topology (as characterized in Fig. 6). The same gesture can probably be used to interact with tangible objects other than a cube, having the same effect in the application.

The generative virtue should also address the question whether we can generate new design ideas, based on the results of the three GESs, which subsumes the question of transferability of our cube-based gesture classification from smart home control to other domains of activity. Since not all aspects of our classification generalize equally well beyond the specific context of use, we need to distinguish highly transferable, conditionally transferable, and context-specific aspects, based on their dependence on domain semantics, physical affordances, and interaction goals.

Highly Transferable Aspects. The motor primitive gestures, such as rotation, translation, lifting, tapping, and swiping, demonstrate strong transferability across domains of activity. These gestures are grounded in everyday physical experience and reflect what has been described in prior work as embodied interaction, where users leverage pre-existing motor skills to interact with digital systems. Because these gestures are independent of any specific domain, they form a stable and reusable part of the interaction vocabulary. Similarly, single-cube manipulations (*e.g.,* rotate, pick up, shake) are broadly applicable. These interactions align with object-centric paradigms common in TUIs. Some gesture–command mappings, such as tap to select, rotate to change view, cover to hide, also exhibit a degree of transferability, as they rely on widely understood metaphors. These mappings are not inherently tied to smart home control but instead reflect generalizable design conventions.

Conditionally Transferable Aspects. Multi-cube gestures, such as stacking, aligning, grouping, and assembling, are conditionally transferable as they depend on whether the target domain benefits from spatial or relational representations. In smart home control, multi-cube gestures may represent some spatial actions that also exist in other domains, such as grouping or composing, but applied on different domain elements. Where spatial or relational representations exist, multi-cube configurations can provide a powerful and expressive interaction mechanism. Where they do not, these gestures risk becoming arbitrary.

Context-specific, non Transferable Aspects. The least transferable aspects of our gesture classification are those tied to smart home-specific semantics. Some gestures proposed by participants in this context rely on metaphors grounded in physical environments, such as rooms, appliances, and spatial zones. For example discrete, command-based interactions (*e.g.,* Turn a device on/off, Increase/decrease a value) may not translate well to domains of activity that require continuous exploration or open-ended interaction. These mappings do not generalize easily because they depend on domain-specific interpretations rather than universal interaction principles.

Table 4: Correspondence between Sheridan et al. [66]'s classification and ours.

Gesture	Definition	Our categories
Hold	To have or maintain cube in the grasp	5, 50-54
Place	To put a cube in or as if in a particular place or position	32, 33, 60
Pick up	To take hold of and lift up	50-53
Rotate	To turn the cube about an axis or center in a continuous, fluid motion exposing three sides of the cube at one time	34, 55, 56
Turn	To cause the cube to move around an axis or a center, exposing one side at a time	53, 55, 56
Flip	One fluid movement to cause the cube to turn over to expose the opposite side of the cube	36, 56
Twist	To rotate the cube while taking a curving path or direction	35, 73

Table 5: Correspondence between Potts et al. [55]'s classification and ours.

Gesture	Definition	Our categories
Tap	Touch a cube face with one finger	22, 23
Swipe	Swipe on a cube face to produce a rectilinear trajectory with one finger	27, 28, 29
Pinch	Slide two fingers toward each other on one face of a cube	53, 55, 56
Hover	Move an open palm vertically above a cube face	16
Path	Draw freeform on a cube face to produce any surface trajectory	31
Arrange	Move two cubes toward each other horizontally on a surface	70

Table 6: Correspondence between He et al. [30]'s classification and ours.

Gesture	Definition	Our categories
Tap	Single touch on a cube face	22, 23
Hold	Move a cube with single touch	32, 33, 60
Press	Push on a cube face with a single touch	22, 23
Double tap	Double-touch on a cube face	22, 23
Pinch	To cause the cube to move around an axis or a center, exposing one side at a time	53, 55, 56
Triple tap	Triple-touch on a cube face	22, 23
Swipe	Swipe on a cube face to produce a rectilinear trajectory	28, 29
Path	Draw freeform on a cube face to produce any surface trajectory	31
Open palm	Open a hand palm above a cube	16
Cover	Cover a cube with the hand	11
Closed fist	Close a hand palm above a cube	16
Pick	Lift a single cube	50-54
Translate	Move a cube horizontally	10
Rotate	Rotate a cube on a surface	34
Shake	Shake a cube above a surface	58
Stack	Arrange multiple cubes in a vertical stack	75
Assemble	Arrange cubes in a matrix	70, 77
Neighbor	Merge two cubes together	74
Collide	Split two cubes from a vertical stack	79

6.2 Limitations of this Work

Several threats [82, 91] have influenced the results of our studies and classification, which we discuss below.

6.2.1 External Validity. Our GESs were performed at three locations (*i.e.*,, a usability lab, a living room, and a smart house) to ensure a varying level of control. Despite this coverage, there are significant differences between them. Thus, our empirical results should be interpreted with the understanding that they reflect a partial representation of real-world domains. The representativeness of the results can affect their generalizability. The gestures analyzed in our GESs, as abstracted in our classification, can be considered a baseline. They cannot represent the whole design space of cube gestures. We plan to conduct more GEss to confirm or refute our results. The number of referents and their complexity were determined to enable participants to complete the experimental session within a limited time to avoid fatigue. Replications are envisioned with different setups, perhaps with other reproducibility types [28] or contextual dimensions [76].

6.2.2 Internal Validity. The learning effect, the understandability of the protocol, and the representativeness of AudioCubes must be considered. The learning effect was mitigated by randomizing the order of presentation of the referents. The understandability of the elicitation was assessed throughout the studies. To avoid possible sources of bias, the materials were evaluated by an experienced researcher. We mitigated the instrumentation validity by using AudioCubes, which are specific, tangible cubes that represent only a fraction of the potential tangible objects. The internal procedure followed for performing a GES and its usage in the mathematical model depend on two factors:

(1) The participant must be exposed to the referent and think about a gesture proposal each time independently. Participants can probably be considered independent of each other. While referents were also presented randomly, some of them are easier to generate a gesture proposal than others due to familiarity with the expected action (*e.g.,* US participants are probably more accustomed to controlling an air conditioning system than EU participants in northern countries) and due to the legacy bias [44].
(2) The experimenter has to correctly perceive and report each gesture proposal independently. Since we hired different experimenters in each GES, they can be considered independent, and their ability to perceive and recognize a gesture proposal is also independent. Some experimenters may recognize a gesture proposal in the experimental scene with varying accuracy and interpretation. The probability that an experimenter recognizes a gesture proposal may also depend on whether the same proposal has been seen before in the experiment but not recognized. Each experimenter had no prior exposure to any GES procedure, and all were trained according to the same material and method to preserve independence.

Nielsen and Landauer [47]' model includes a role for different application scenarios supported by different types of systems, such as an integrated spreadsheet, a word processor, and a bibliography database. This is not covered by this article, which kept the scenario constant throughout the three studies. Application to another field of activity involves determining the value of λ in Eq. 2 specific to the scenario, *e.g.,* ranging from .12 to .58 in [47]. The main limitations of the Poisson law, as we have applied it in this work, are its reliance on several assumptions which, if violated, could make the model less accurate, by underestimation or overestimation:

- *Independence of gesture proposals*: The Poisson law assumes that the occurrence of one gesture category does not affect the probability of another category happening. This assumption may be violated for gestures proposed to clustered or related referents (*e.g.,* "Turn TV On" and "Turn TV off"), where the gesture proposed for the first referent may impact the likelihood of the gesture proposed for the second, even when randomized.
- *Constant rate of gesture proposals* (λ): The model assumes that the average rate at which gestures are proposed is constant throughout the study. This assumption may be violated for gestures proposed with very different Thinking-Times.
- *No simultaneous gesture proposals*: The Poisson law assumes that two different gestures cannot be proposed at the same instant. This assumption is unlikely to be violated since the participant is always instructed to propose one gesture at a time. If a participant asks for withdrawing a proposal, a new single proposal can replace the withdrawn one.
- *Overdispersion*: When gestures are not independent or the rate is not constant, it can lead to a situation called *overdispersion*, where the variance is greater than the mean. Suppose we combine the gestures proposed by different population samplings with different rates (each sampling being a Poisson law within each GES). In that case, the overall model will underestimate the true error, leading to poor decision-making, which is why we computed the number of new categories (Eq. 2) only within a GES, not across GESs.

6.2.3 Construct Validity. The within-subject design adopted in the GESs could be restructured as a between-subject design to mitigate potential learning effects that influence the elicitation process and the measurement of its results. The metrics and the participants' apprehension about being evaluated must be considered. We mitigated the threat to metrics by using three metrics, keeping them consistent across all studies. The participants' possible apprehension about being evaluated was mitigated as they were informed that AudioCubes were assessed.

6.2.4 Conclusion Validity. Data collection and the validity of the calculations must be considered. To mitigate threats to data collection, we systematically applied exact measurement and data extraction procedures across studies. The most appropriate calculations were performed by selecting them based on the experimental design and the nature of the variables and their hypotheses.

7 Conclusion and Future Work

This paper presented a family of three gesture elicitation studies conducted each with a different setup, but with the same method using AudioCubes, a set of wireless battery-powered cubic TUIs with built-in processing capabilities that can detect and recognize their topology at run-time. AudioCubes were considered representative examples of cubic TUIs to carry out studies and better understand how end users interact with them. Observations and metrics obtained across the three gesture elicitation studies led to a classification of cube gestures for designing tangible interaction, abstracted from specific properties. We then presented a model for estimating the number of participants to reach a desired number of gesture categories.

Our future work would explore avenues of gesture elicitation studies in various directions as follows: studies involving uncovered gestures (*e.g.,* spatial motion gestures, shaking, accelerating, and decelerating, producing a 3D assembly, drawing symbols), requiring more rotation along the symmetry axes, now that we know all possibilities. Conducting other studies, with other replication types [28], to confirm or refute the (sub-)categories of our classification.

As soon as the experiment involves more than one cube to manipulate, it opens up new avenues of study: for a single user, how many cubes are they willing to handle at the same time, given that their two hands cannot handle more than two cubes at once, unless they are stacked; for multiple users, how many cubes are they willing to handle in order to coordinate their actions. It is often observed that, in this scenario, each participant prefers to handle a single cube at a time, which allows us to determine the individual effect on the group arrangement. When the cubes are interdependent, their arrangement must reflect any relationships between them, which requires further study.

Therefore, both exploring collaborative multi-user cube gestures and investigating cultural or demographic variation in gesture preference would directly address the generalizability of cube gestures reported in our classification. Three contextual variations could induce replications that would help establish which aspects of the classification are context-dependent or not [76]: a cross-cohort replication, with cross-cultural variations, would help identify which cube gestures represent more universal patterns of manipulation; a cross-platform replication would help identify which gestures could be specific to a particular device, such as the one used in our studies; a cross-environment replication, would help identify how the location and the environmental conditions could affect the gesture manipulation. A follow-up study could provide a more concrete guidance for how designers might use the gesture classification in practice, thereby validating its utility in real system development.

Finally, other polyhedra could be subject to a similar work to determine how our classification of user-defined gestures for designing tangible interaction with cubes would remain stable and could be expanded.

Open Science. Our website presenting AudioCubes https://percussa.com/what-are-audiocubes/ provides the reader with resources dedicated to using AudioCubes, such as setup, manual, and firmware. Demonstration videos are accessible on our web site at https://percussa.com/audiocubes-video-gallery/#ss8 and on YouTube at https://www.youtube.com/watch?v=ofkGzwZV088.

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References

[1] Roland Aigner, Daniel Wigdor, Hrvoje Benko, Michael Haller, David Lindbauer, Alexandra Ion, Shengdong Zhao, and Jeffrey Tzu Kwan Valino Koh. 2012. *Understanding Mid-Air Hand Gestures: A Study of Human Preferences in Usage of Gesture Types for HCI.* Technical Report MSR-TR-2012-111. Microsoft. https://www.microsoft.com/en-us/research/publication/understanding-mid-air-hand-gestures-a-study-of-human-preferences-in-usage-of-gesture-types-for-hci/

[2] James F. Allen. 1983. Maintaining Knowledge About Temporal Intervals. *Commun. ACM* 26, 11 (1983), 832–843. https://doi.org/10.1145/182.358434

[3] Miguel Bruns Alonso and David V. Keyson. 2005. MusicCube: making digital music tangible. In *Proceedings of ACM International Conference on Human Factors in Computing Systems, Extended Abstracts* (Portland, OR, USA) *(CHI EA '05)*. Association for Computing Machinery, New York, NY, USA, 1176–1179. https://doi.org/10.1145/1056808.1056870

[4] David Anderson, James L. Frankel, Joe Marks, Aseem Agarwala, Paul Beardsley, Jessica Hodgins, Darren Leigh, Kathy Ryall, Eddie Sullivan, and Jonathan S. Yedidia. 2000. Tangible interaction + graphical interpretation: a new approach to 3D modeling. In *Proceedings of the 27th Annual Conference on Computer Graphics and Interactive Techniques (SIGGRAPH '00)*. ACM Press/Addison-Wesley Publishing Co., USA, 393–402. https://doi.org/10.1145/344779.344960

[5] Leonardo Angelini, Denis Lalanne, Elise Van den Hoven, Omar Abou Khaled, and Elena Mugellini. 2015. Move, Hold and Touch: A Framework for Tangible Gesture Interactive Systems. *Machines* 3, 3 (2015), 173–207. https://doi.org/10.3390/machines3030173

[6] Michael Artin. August 13, 2010. *Algebra* (2nd ed.). Pearson, London, UK.

[7] Pietro Battistoni, Marianna Di Gregorio, Marco Romano, Monica Sebillo, and Giuliana Vitiello. 2021. TactCube: Designing Mobile Interactions with Ambient Intelligence. In *Human-Computer Interaction – INTERACT 2021*, Carmelo Ardito, Rosa Lanzilotti, Alessio Malizia, Helen Petrie, Antonio Piccinno, Giuseppe Desolda, and Kori Inkpen (Eds.). Springer International Publishing, Cham, 599–609.

[8] Michel Beaudouin-Lafon. 2004. Designing interaction, not interfaces. In *Proceedings of the Working Conference on Advanced Visual Interfaces* (Gallipoli, Italy) *(AVI '04)*. Association for Computing Machinery, New York, NY, USA, 15–22. https://doi.org/10.1145/989863.989865

[9] R. Berry, M. Makino, N. Hikawa, and M. Suzuki. 2003. The augmented composer project: the music table. In *Proceedings of the Second IEEE and ACM International Symposium on Mixed and Augmented Reality* (Tokyo, Japan) *(ISMAR '03)*. IEEE Press, Piscataway, NJ, USA, 338–339. https://doi.org/10.1109/ISMAR.2003.1240749

[10] David Bertolo, Stéphanie Fleck, Camille Lemiere, and Isabelle Pecci. 2025. Tangible User Interface in Health: A Scoping Review. In *Proceedings of the Nineteenth International Conference on Tangible, Embedded, and Embodied Interaction (TEI '25)*. Association for Computing Machinery, New York, NY, USA, Article 32, 25 pages. https://doi.org/10.1145/3689050.3704951

[11] Florian Block, Albrecht Schmidt, Nicolas Villar, and Hans W. Gellersen. 2004. Towards a Playful User Interface for Home Entertainment Systems. In *Ambient Intelligence*, P. Markopoulos, B. Eggen, E. Aarts, and J.L. Crowley (Eds.). Springer Berlin Heidelberg, Berlin, Heidelberg, 207–217.

[12] Richard A. Bolt. 1980. &Ldquo;Put-that-there&Rdquo;: Voice and Gesture at the Graphics Interface. *SIGGRAPH Comput. Graph.* 14, 3 (July 1980), 262–270. https://doi.org/10.1145/965105.807503

[13] Rhonda Boyle, Robin Boyle, and Erica Booker. 2015. Pianist Hand Spans: Gender and Ethnic Differences and Implications for Piano Playing. In *Proceedings of the 12th Australasian Piano Pedagogy Conference* (Victorian College of the Arts Secondary School, Melbourne, Australia) *(APCC '15)*. Australasian Piano Pedagogy Conference Association (APPCA), Victoria Park, WA 6979, Australia, 1–79. http://www.appca.com.au/pdf/papers2015/part1/2015-APPC-Boyles-Booker---Pianist-Hand-Spans-Gender-and-Ethnic-Differences.pdf

[14] William Buxton. 1983. Lexical and pragmatic considerations of input structures. *SIGGRAPH Comput. Graph.* 17, 1 (Jan. 1983), 31–37. https://doi.org/10.1145/988584.988586

[15] Francesco Cafaro, Leilah Lyons, Raymond Kang, Josh Radinsky, Jessica Roberts, and Kristen Vogt. 2014. Framed guessability: using embodied allegories to increase user agreement on gesture sets. In *Proceedings of the 8th International Conference on Tangible, Embedded and Embodied Interaction* (Munich, Germany) *(TEI '14)*. Association for Computing Machinery, New York, NY, USA, 197–204. https://doi.org/10.1145/2540930.2540944

[16] Ken Camarata, Ellen Yi-Luen Do, Brian R. Johnson, and Mark D. Gross. 2002. Navigational Blocks: Navigating Information Space with Tangible Media. In *Proceedings of the 7th International Conference on Intelligent User Interfaces* (San Francisco, California, USA) *(IUI '02)*. ACM, New York, NY, USA, 31–38. https://doi.org/10.1145/502716.502725

[17] Joel Chadabe. 2002. The limitations of mapping as a structural descriptive in electronic instruments. In *Proceedings of the 2002 Conference on New Interfaces for Musical Expression* (Dublin, Ireland) *(NIME '02)*. National University of Singapore, SGP, 1–5.

[18] Yuting Cheng, Zhanwei Wu, and Ruowei Xiao. 2024. Exploring Methods to Optimize Gesture Elicitation Studies: A Systematic Literature Review. *IEEE Access* 12 (2024), 64958–64979. https://doi.org/10.1109/ACCESS.2024.3387269

[19] Salvador Cobos, Manuel Ferre, M. Ángel Sánchez-Urán, Javier Ortego, and Rafael Aracil. 2010. Human hand descriptions and gesture recognition for

object manipulation. *Computer Methods in Biomechanics and Biomedical Engineering* 13, 3 (2010), 305–317. https://doi.org/10.1080/10255840903208171 arXiv:https://doi.org/10.1080/10255840903208171 PMID: 20146129.

[20] Aurélie Cohé and Martin Hachet. 2012. Understanding User Gestures for Manipulating 3D Objects from Touchscreen Inputs. In *Proceedings of Graphics Interface 2012* (Toronto, Ontario, Canada) *(GI '12)*. Canadian Information Processing Society, Toronto, Ont., Canada, Canada, 157–164. http://dl.acm.org/citation.cfm?id=2305276.2305303

[21] Arfib Daniel and Dudon Jacques. 2002. A digital emulator of the photosonic instrument. In *Proceedings of the 2002 Conference on New Interfaces for Musical Expression* (Dublin, Ireland) *(NIME '02)*. National University of Singapore, SGP, 1–4.

[22] Hutchins Edwin. February 1995. *Cognition in the Wild*. MIT Press, New York, NY, USA.

[23] Ylva Fernaeus, Jakob Tholander, and Martin Jonsson. 2008. Beyond representations: towards an action-centric perspective on tangible interaction. *Int. J. Arts Technol.* 1, 3/4 (2008), 249–267. https://doi.org/10.1504/IJART.2008.022362

[24] Kenneth P. Fishkin. 2004. A taxonomy for and analysis of tangible interfaces. *Personal and Ubiquitous Computing* 8, 5 (01 Sep 2004), 347–358. https://doi.org/10.1007/s00779-004-0297-4

[25] George W. Fitzmaurice, Hiroshi Ishii, and William A. S. Buxton. 1995. Bricks: laying the foundations for graspable user interfaces. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems* (Denver, Colorado, USA) *(CHI '95)*. ACM Press/Addison-Wesley Publishing Co., USA, 442–449. https://doi.org/10.1145/223904.223964

[26] Bernd Fröhlich and John Plate. 2000. The Cubic Mouse: A New Device for Three-dimensional Input. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems* (The Hague, The Netherlands) *(CHI '00)*. ACM, New York, NY, USA, 526–531. https://doi.org/10.1145/332040.332491

[27] Bogdan-Florin Gheran, Jean Vanderdonckt, and Radu-Daniel Vatavu. 2018. Gestures for Smart Rings: Empirical Results, Insights, and Design Implications. In *Proceedings of the 2018 Designing Interactive Systems Conference* (Hong Kong, China) *(DIS '18)*. Association for Computing Machinery, New York, NY, USA, 623–635. https://doi.org/10.1145/3196709.3196741

[28] Bogdan-Florin Gheran, Santiago Villarreal-Narvaez, Radu-Daniel Vatavu, and Jean Vanderdonckt. 2022. RepliGES and GEStory: Visual Tools for Systematizing and Consolidating Knowledge on User-Defined Gestures. In *Proceedings of the 2022 International Conference on Advanced Visual Interfaces* (Frascati, Rome, Italy) *(AVI '22)*. Association for Computing Machinery, New York, NY, USA, Article 5, 9 pages. https://doi.org/10.1145/3531073.3531112

[29] Weilun Gong, Stephanie Santosa, Tovi Grossman, Michael Glueck, Daniel Clarke, and Frances Lai. 2023. Affordance-Based and User-Defined Gestures for Spatial Tangible Interaction. In *Proceedings of the 2023 ACM Designing Interactive Systems Conference* (Pittsburgh, PA, USA) *(DIS '23)*. Association for Computing Machinery, New York, NY, USA, 1500–1514. https://doi.org/10.1145/3563657.3596032

[30] Shuqi He, Haonan Yao, Luyan Jiang, Kaiwen Li, Nan Xiang, Yue Li, Hai-Ning Liang, and Lingyun Yu. 2024. Data Cubes in Hand: A Design Space of Tangible Cubes for Visualizing 3D Spatio-Temporal Data in Mixed Reality. In *Proceedings of the CHI Conference on Human Factors in Computing Systems* (Honolulu, HI, USA) *(CHI '24)*. Association for Computing Machinery, New York, NY, USA, Article 209, 21 pages. https://doi.org/10.1145/3613904.3642740

[31] Lynn Hoff, Eva Hornecker, and Sven Bertel. 2016. Modifying Gesture Elicitation: Do Kinaesthetic Priming and Increased Production Reduce Legacy Bias?. In *Proceedings of the TEI '16: Tenth International Conference on Tangible, Embedded, and Embodied Interaction* (Eindhoven, Netherlands) *(TEI '16)*. Association for Computing Machinery, New York, NY, USA, 86–91. https://doi.org/10.1145/2839462.2839472

[32] Andy Hunt, Marcelo M. Wanderley, and Matthew Paradis. 2003. The Importance of Parameter Mapping in Electronic Instrument Design. *Journal of New Music Research* 32, 4 (2003), 429–440. https://doi.org/10.1076/jnmr.32.4.429.18853 arXiv:https://www.tandfonline.com/doi/pdf/10.1076/jnmr.32.4.429.18853

[33] Elliot Jaffe, Aviva Dayan, and Amnon Dekel. 2007. Cube Management System: A Tangible Interface for Monitoring Large Scale Systems. In *Proceedings of the 2007 Symposium on Computer Human Interaction for the Management of Information Technology* (Cambridge, Massachusetts) *(CHIMIT '07)*. ACM, New York, NY, USA, Article 4, 6 pages. https://doi.org/10.1145/1234772.1234778

[34] Sergi Jordà, Günter Geiger, Marcos Alonso, and Martin Kaltenbrunner. 2007. The reacTable: exploring the synergy between live music performance and tabletop tangible interfaces. In *Proceedings of the 1st International Conference on Tangible and Embedded Interaction* (Baton Rouge, Louisiana) *(TEI '07)*. Association for Computing Machinery, New York, NY, USA, 139–146. https://doi.org/10.1145/1226969.1226998

[35] Suhas Govind Joshi and Heidi Bråthen. 2016. Lowering the Threshold: Reconnecting Elderly Users with Assistive Technology Through Tangible Interfaces. In *Human Aspects of IT for the Aged Population. Design for Aging*, J. Zhou and G. Salvendy (Eds.). Springer Int. Publishing, Cham, 52–63.

[36] Yoshifumi Kitamura, Yuichi Itoh, and Fumio Kishino. 2001. Real-time 3D Interaction with ActiveCube. In *CHI '01 Extended Abstracts on Human Factors in Computing Systems* (Seattle, Washington) *(CHI EA '01)*. ACM, New York, NY, USA, 355–356. https://doi.org/10.1145/634067.634277

[37] Matthias Kranz, Stefan Freund, Paul Holleis, Albrecht Schmidt, and Holger Arndt. 2006. Developing Gestural Input. In *Proceedings of 26th International Conference on Distributed Computing Systems Workshops* (Lisboa, Portugal) *(ICDCS '06 Workshops)*. IEEE Computer Society, Piscataway, NJ, USA, 63. https://doi.org/10.1109/ICDCSW.2006.41

[38] Matthias Kranz, Dominik Schmidt, Paul Holleis, and Albrecht Schmidt. 2005. A Display Cube as a Tangible User Interface. Adj. In *Adjunct Proceedings of 7th International Conference on Ubiquitous Computing (UbiComp'2005)*. Lancaster University Library, Lancaster, UK, 1–2. http://eprints.lancs.ac.uk/41523/

[39] Doo Young Kwon, Stephan Würmlin, and Markus Gross. 2007. mCube – Towards a Versatile Gesture Input Device for Ubiquitous Computing Environments. In *Ubiquitous Computing Systems*, Haruhisa Ichikawa, We-Duke Cho, Ichiro Satoh, and Hee Yong Youn (Eds.). Springer Berlin Heidelberg, Berlin, Heidelberg, 228–239.

[40] Joanne Leong, Florian Perteneder, Hans-Christian Jetter, and Michael Haller. 2017. What a Life!: Building a Framework for Constructive Assemblies. In *Proceedings of the Eleventh International Conference on Tangible, Embedded, and Embodied Interaction* (Yokohama, Japan) *(TEI '17)*. ACM, New York, NY, USA, 57–66. https://doi.org/10.1145/3024969.3024985

[41] Ramon Lobato, Alexa Scarlata, and Bruno Schivinski. 2024. Smart TV Users and Interfaces: Who's in Control? *International Journal of Communication* 18, 0 (2024), 21. https://ijoc.org/index.php/ijoc/article/view/22469

[42] Mary Lou Maher and Lina Lee. 2017. *Designing for Gesture and Tangible Interaction*. Morgan & Claypool Publishers, San Rafael, CA 94903, USA. https://doi.org/10.2200/S00758ED1V01Y201702HCI036

[43] Nicolai Marquardt and Saul Greenberg. 2007. Distributed Physical Interfaces with Shared Phidgets. In *Proceedings of the 1st International Conference on Tangible and Embedded Interaction* (Baton Rouge, Louisiana) *(TEI '07)*. ACM, New York, NY, USA, 13–20. https://doi.org/10.1145/1226969.1226973

[44] Meredith Ringel Morris, Andreea Danielescu, Steven Drucker, Danyel Fisher, Bongshin Lee, m. c. schraefel, and Jacob O. Wobbrock. 2014. Reducing legacy bias in gesture elicitation studies. *Interactions* 21, 3 (May 2014), 40–45. https://doi.org/10.1145/2591689

[45] Kaddour Najim, Enso Ikonen, and Ait-Kadi Daoud. 2004. *Chapter 2 - Estimation of Probability Densities*. Kogan Page Science, Oxford, 93–166. https://doi.org/10.1016/B978-190399655-3/50011-6

[46] Henry Newton-Dunn, Hiroaki Nakano, and James Gibson. 2003. Block Jam: A Tangible Interface for Interactive Music. *Journal of New Music Research* 32, 4 (2003), 383–393. https://doi.org/10.1076/jnmr.32.4.383.18852 arXiv:https://www.tandfonline.com/doi/pdf/10.1076/jnmr.32.4.383.18852

[47] Jakob Nielsen and Thomas K. Landauer. 1993. A mathematical model of the finding of usability problems. In *Proceedings of the INTERACT '93 and CHI '93 Conference on Human Factors in Computing Systems* (Amsterdam, The Netherlands) *(CHI '93)*. Association for Computing Machinery, New York, NY, USA, 206–213. https://doi.org/10.1145/169059.169166

[48] Michael Nielsen, Moritz Störring, Thomas B. Moeslund, and Erik Granum. 2004. A Procedure for Developing Intuitive and Ergonomic Gesture Interfaces for HCI. In *Gesture-Based Communication in Human-Computer Interaction*, Antonio Camurri and Gualtiero Volpe (Eds.). Springer, Heidelberg, 409–420.

[49] James Patten and Hiroshi Ishii. 2000. A Comparison of Spatial Organization Strategies in Graphical and Tangible User Interfaces. In *Proceedings of DARE 2000 on Designing Augmented Reality Environments* (Elsinore, Denmark) *(DARE '00)*. ACM, New York, NY, USA, 41–50. https://doi.org/10.1145/354666.354671

[50] James Patten, Ben Recht, and Hiroshi Ishii. 2002. Audiopad: a tag-based interface for musical performance. In *Proceedings of the International Conference on New Interfaces for Musical Expression* (Dublin, Ireland) *(NIME '02)*. National University of Singapore, SGP, 1–6. https://doi.org/10.5555/1085171.1085175

[51] James Patten, Ben Recht, and Hiroshi Ishii. 2006. Interaction techniques for musical performance with tabletop tangible interfaces. In *Proceedings of the ACM SIGCHI International Conference on Advances in Computer Entertainment Technology* (Hollywood, California, USA) *(ACE '06)*. Association for Computing Machinery, New York, NY, USA, 27–es. https://doi.org/10.1145/1178823.1178856

[52] Gary Perelman, Marcos Serrano, and Emmanuel Dubois. 2024. Exploiting Physical Referent Features as Input for Multidimensional Data Selection in Augmented Reality. *ACM Trans. Comput.-Hum. Interact.* 31, 4, Article 50 (Sept. 2024), 40 pages. https://doi.org/10.1145/3648613

[53] Thammathip Piumsomboon, Adrian Clark, Mark Billinghurst, and Andy Cockburn. 2013. User-Defined Gestures for Augmented Reality. In *Human-Computer Interaction – INTERACT 2013*, Paula Kotzé, Gary Marsden, Gitte Lindgaard, Janet Wesson, and Marco Winckler (Eds.). Springer Berlin Heidelberg, Berlin, Heidelberg, 282–299.

[54] G. Michael Poor, Samuel D. Jaffee, Laura Marie Leventhal, Jordan Ringenberg, Dale S. Klopfer, Guy Zimmerman, and Brandi A. Klein. 2016. Applying the Norman 1986 User-Centered Model to Post-WIMP UIs: Theoretical Predictions and Empirical Outcomes. *ACM Trans. Comput.-Hum. Interact.* 23, 5, Article 30 (October 2016), 33 pages. https://doi.org/10.1145/2983531

[55] Dominic Potts, Martynas Dabravalskis, and Steven Houben. 2022. TangibleTouch: A Toolkit for Designing Surface-based Gestures for Tangible Interfaces. In *Proceedings of the 16th International Conference on Tangible, Embedded, and Embodied Interaction* (Daejeon Republic of Korea) *(TEI '22)*. Association for Computing Machinery, New York, NY, USA, Article 39, 14 pages. https://doi.org/10.1145/3490149.3502263

[56] Ivan Poupyrev. 2000. Augmented Groove: Collaborative Jamming in Augmented Reality. In *Proceedings of ACM SIGGRAPH'2000 Conference - Abstracts and Applications*. ACM, New York, NY, USA, 77. https://ci.nii.ac.jp/naid/10022426818/en/

[57] Laura Pruszko, Hongri Gu, Julien Bourgeois, Yann Laurillau, and Céline Coutrix. 2023. Modular Tangible User Interfaces: Impact of Module Shape and Bonding Strength on Interaction. In *Proceedings of the Seventeenth International Conference on Tangible, Embedded, and Embodied Interaction* (Warsaw, Poland) *(TEI '23)*. Association for Computing Machinery, New York, NY, USA, Article 1, 15 pages. https://doi.org/10.1145/3569009.3572731

[58] Nico Reski and Aris Alissandrakis. 2018. Using an Augmented Reality Cube-Like Interface and 3D Gesture-Based Interaction to Navigate and Manipulate Data. In *Proceedings of the 11th International Symposium on Visual Information Communication and Interaction* (Växjö, Sweden) *(VINCI '18)*. Association for Computing Machinery, New York, NY, USA, 92–96. https://doi.org/10.1145/3231622.3231625

[59] Benjamin Salem and Harold Peeters. 2007. InterCUBE: A Study into Merging Action and Interaction Spaces. In *Proceedings of IFIP TC 13 International Conference on Human-Computer Interaction, INTERACT 2007 (Lecture Notes in Computer Science)*, Cécilia Baranauskas, Philippe Palanque, Julio Abascal, and Simone Diniz Junqueira Barbosa (Eds.). Springer, Berlin, Heidelberg, 57–70. https://doi.org/10.1007/978-3-540-74800-7_5

[60] Alexander Scheidt and Tim Pulver. 2019. Any-Cubes: A Children's Toy for Learning AI: Enhanced Play with Deep Learning and MQTT. In *Proceedings of Mensch Und Computer 2019* (Hamburg, Germany) *(MuC '19)*. Association for Computing Machinery, New York, NY, USA, 893–895. https://doi.org/10.1145/3340764.3345375

[61] Bert Schiettecatte and Jean Vanderdonckt. 2008. AudioCubes: A Distributed Cube Tangible Interface Based on Interaction Range for Sound Design. In *Proceedings of the 2Nd International Conference on Tangible and Embedded Interaction* (Bonn, Germany) *(TEI '08)*. ACM, New York, NY, USA, 3–10. https://doi.org/10.1145/1347390.1347394

[62] Teddy Seyed, Chris Burns, Mario Costa Sousa, Frank Maurer, and Anthony Tang. 2012. Eliciting Usable Gestures for Multi-display Environments. In *Proc. of the 2012 ACM International Conference on Interactive Tabletops and Surfaces* (Cambridge, Massachusetts, USA) *(ITS '12)*. ACM, New York, NY, USA, 41–50. https://doi.org/10.1145/2396636.2396643

[63] Orit Shaer and Eva Hornecker. 2010. Tangible User Interfaces: Past, Present, and Future Directions. *Foundations and Trends in Human–Computer Interaction* 3, 1–2 (2010), 4–137. https://doi.org/10.1561/1100000026

[64] Orit Shaer, Nancy Leland, Eduardo H. Calvillo Gámez, and Robert J. K. Jacob. 2004. The TAC paradigm: specifying tangible user interfaces. *Pers. Ubiquitous Comput.* 8, 5 (2004), 359–369. https://doi.org/10.1007/S00779-004-0298-3

[65] Ehud Sharlin, Yuichi Itoh, Benjamin Watson, Yoshifumi Kitamura, Steve Sutphen, and Lili Liu. 2002. Cognitive Cubes: A Tangible User Interface for Cognitive Assessment. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems* (Minneapolis, Minnesota, USA) *(CHI '02)*. ACM, New York, NY, USA, 347–354. https://doi.org/10.1145/503376.503438

[66] J. G. Sheridan, B. W. Short, K. Van Laerhoven, N. Villar, and G. Kortuem. 2003. Exploring cube affordance: towards a classification of non-verbal dynamics of physical interfaces for wearable computing. In *Proceedings of the IEE Eurowearable Int. Conf.* (Birmingham, UK). Institution of Engineering and Technology, Stevenage, SG1 2UA, United Kingdom, 113–118. https://doi.org/10.1049/ic:20030156

[67] Hyunyoung Song, François Guimbretière, Michael A. Ambrose, and Carl Lostritto. 2007. CubeExplorer: An Evaluation of Interaction Techniques in Architectural Education. In *Human-Computer Interaction – INTERACT 2007*, Cécilia Baranauskas, Philippe Palanque, Julio Abascal, and Simone Diniz Junqueira Barbosa (Eds.). Springer Berlin Heidelberg, Berlin, Heidelberg, 43–56.

[68] Hideyuki Suzuki and Hiroshi Kato. 1995. Interaction-level Support for Collaborative Learning: AlgoBlock&Mdash;an Open Programming Language. In *The First International Conference on Computer Support for Collaborative Learning* (Indiana Univ., Bloomington, Indiana, USA) *(CSCL '95)*. L. Erlbaum Associates Inc., Hillsdale, NJ, USA, 349–355. https://doi.org/10.3115/222020.222828

[69] Lucia Terrenghi, Matthias Kranz, Paul Holleis, and Albrecht Schmidt. 2006. A Cube to Learn: A Tangible User Interface for the Design of a Learning Appliance. *Personal Ubiquitous Comput.* 10, 2-3 (January 2006), 153–158. https://doi.org/10.1007/s00779-005-0025-8

[70] Daniela Gorski Trevisan, Jean Vanderdonckt, and Benoît Macq. 2004. Conceptualising mixed spaces of interaction for designing continuous interaction. *Virtual Reality* 8, 2 (01 Jun 2004), 83–95. https://doi.org/10.1007/s10055-004-0140-2

[71] B. Ullmer and H. Ishii. 2000. Emerging frameworks for tangible user interfaces. *IBM Sys. J.* 39, 3.4 (2000), 915–931. https://doi.org/10.1147/sj.393.0915

[72] Brygg Ullmer, Hiroshi Ishii, and Robert J. K. Jacob. 2005. Token+Constraint Systems for Tangible Interaction with Digital Information. *ACM Trans. Comput.-Hum. Interact.* 12, 1 (March 2005), 81–118. https://doi.org/10.1145/1057237.1057242

[73] Consuelo Valdes, Diana Eastman, Casey Grote, Shantanu Thatte, Orit Shaer, Ali Mazalek, Brygg Ullmer, and Miriam K. Konkel. 2014. Exploring the Design Space of Gestural Interaction with Active Tokens Through User-defined Gestures. In *Proceedings of the ACM Conference on Human Factors in Computing Systems* (Toronto, Ontario, Canada) *(CHI '14)*. Association for Computing Machinery, New York, NY, USA, 4107–4116. https://doi.org/10.1145/2556288.2557373

[74] Elise van den Hoven, Joep Frens, Dima Aliakseyeu, Jean-Bernard Martens, Kees Overbeeke, and Peter Peters. 2007. Design research & tangible interaction. In *Proceedings of the 1st International Conference on Tangible and Embedded Interaction* (Baton Rouge, Louisiana) *(TEI '07)*. Association for Computing Machinery, New York, NY, USA, 109–115. https://doi.org/10.1145/1226969.1226993

[75] Kristof Van Laerhoven, Nicolas Villar, Albrecht Schmidt, Gerd Kortuem, and Hans Gellersen. 2003. Using an Autonomous Cube for Basic Navigation and Input. In *Proceedings of the 5th International Conference on Multimodal Interfaces* (Vancouver, British Columbia, Canada) *(ICMI '03)*. ACM, New York, NY, USA, 203–210. https://doi.org/10.1145/958432.958472

[76] Jean Vanderdonckt and Radu-Daniel Vatavu. 2025. Context is Key for Reproducibility of Empirical Studies in Human-Computer Interaction. In *Proceedings of the 3rd ACM Conference on Reproducibility and Replicability (ACM REP '25)*. Association for Computing Machinery, New York, NY, USA, 41–50. https://doi.org/10.1145/3736731.3746142

[77] Radu-Daniel Vatavu. 2012. User-defined gestures for free-hand TV control. In *Proceedings of the 10th European Conference on Interactive TV and Video* (Berlin, Germany) *(EuroITV '12)*. Association for Computing Machinery, New York, NY, USA, 45–48. https://doi.org/10.1145/2325616.2325626

[78] Radu-Daniel Vatavu and Jacob O. Wobbrock. 2015. Formalizing Agreement Analysis for Elicitation Studies: New Measures, Significance Test, and Toolkit. In *Proceedings of the 33rd Annual ACM Conference on Human Factors in Computing Systems* (Seoul, Republic of Korea) *(CHI '15)*. Association for Computing Machinery, New York, NY, USA, 1325–1334. https://doi.org/10.1145/2702123.2702223

[79] Radu-Daniel Vatavu and Jacob O. Wobbrock. 2022. Clarifying Agreement Calculations and Analysis for End-User Elicitation Studies. *ACM Trans. Comput.-Hum. Interact.* 29, 1, Article 5 (Jan. 2022), 70 pages. https://doi.org/10.1145/3476101

[80] Radu-Daniel Vatavu and Ionut-Alexandru Zaiti. 2014. Leap gestures for TV: insights from an elicitation study. In *Proceedings of the ACM International Conference on Interactive Experiences for TV and Online Video* (Newcastle Upon Tyne, United Kingdom) *(TVX '14)*. Association for Computing Machinery, New York, NY, USA, 131–138. https://doi.org/10.1145/2602299.2602316

[81] Michael Vazirgiannis, Yannis Theodoridis, and Timos Sellis. 1998. Spatio-temporal composition and indexing for large multimedia applications. *Multimedia Systems* 6, 4 (01 Jul 1998), 284–298. https://doi.org/10.1007/s005300050094

[82] Roberto Verdecchia, Emelie Engström, Patricia Lago, Per Runeson, and Qunying Song. 2023. Threats to validity in software engineering research: A critical reflection. *Information and Software Technology* 164 (2023), 107329. https://doi.org/10.1016/j.infsof.2023.107329

[83] Santiago Villarreal-Narvaez, Arthur Sluÿters, Jean Vanderdonckt, and Efrem Mbaki Luzayisu. 2022. Theoretically-Defined vs. User-Defined Squeeze Gestures. *Proc. ACM Hum. Comput. Interact.* 6, ISS (2022), 73–102. https://doi.org/10.1145/3567805

[84] Santiago Villarreal-Narvaez, Arthur Sluÿters, Jean Vanderdonckt, and Radu-Daniel Vatavu. 2024. Brave New GES World: A Systematic Literature Review of Gestures and Referents in Gesture Elicitation Studies. *ACM Comput. Surv.* 56, 5 (2024), 128:1–128:55. https://doi.org/10.1145/3636458

[85] Santiago Villarreal-Narvaez, Jean Vanderdonckt, Radu-Daniel Vatavu, and Jacob O. Wobbrock. 2020. A Systematic Review of Gesture Elicitation Studies: What Can We Learn from 216 Studies?. In *Proceedings of the ACM Int. COnf. on Designing Interactive Systems* (Eindhoven, Netherlands) *(DIS '20)*. Association for Computing Machinery, New York, NY, USA, 855–872. https://doi.org/10.1145/3357236.3395511

[86] Emanuel Vonach, Marianne Ternek, Georg Gerstweiler, and Hannes Kaufmann. 2016. Design of a Health Monitoring Toy for Children. In *Proceedings of the The 15th International Conference on Interaction Design and Children* (Manchester, United Kingdom) *(IDC '16)*. Association for Computing Machinery, New York, NY, USA, 58–67. https://doi.org/10.1145/2930674.2930694

[87] Ryoichi Watanabe, Yuichi Itoh, Masatsugu Asai, Yoshifumi Kitamura, Fumio Kishino, and Hideo Kikuchi. 2004. The Soul of ActiveCube: Implementing a Flexible, Multimodal, Three-dimensional Spatial Tangible Interface. In *Proceedings of the 2004 ACM SIGCHI International Conference on Advances in Computer Entertainment Technology* (Singapore) *(ACE '04)*. ACM, New York, NY, USA, 173–180. https://doi.org/10.1145/1067343.1067364

[88] Max L. Wilson, Ed H. Chi, Stuart Reeves, and David Coyle. 2014. RepliCHI: the workshop II. In *Proceedings of ACM Conference on Human Factors in Computing Systems* (Toronto, ON, Canada) *(CHI'14)*, Matt Jones, Philippe A. Palanque, Albrecht Schmidt, and Tovi Grossman (Eds.). ACM, New York, NY, USA, 33–36. https://doi.org/10.1145/2559206.2559233

[89] Max L. Wilson, Wendy Mackay, Ed Chi, Michael Bernstein, Dan Russell, and Harold Thimbleby. 2011. RepliCHI - CHI Should Be Replicating and Validating Results More: Discuss. In *CHI EA'11*. ACM, New York, NY, USA, 463–466.

[90] Jacob O. Wobbrock, Meredith Ringel Morris, and Andrew D. Wilson. 2009. User-defined Gestures for Surface Computing. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems* (Boston, MA, USA) *(CHI '09)*. ACM, New York, NY, USA, 1083–1092. https://doi.org/10.1145/1518701.1518866

[91] Claes Wohlin, Per Runeson, Martin Höst, Magnus C. Ohlsson, Björn Regnell, and Anders Wesslén. 2012. *Empirical Strategies*. Springer Berlin Heidelberg, Berlin, Heidelberg, 9–36. https://doi.org/10.1007/978-3-642-29044-2_2

[92] Haijun Xia, Michael Glueck, Michelle Annett, Michael Wang, and Daniel Wigdor. 2022. Iteratively Designing Gesture Vocabularies: A Survey and Analysis of Best Practices in the HCI Literature. *ACM Trans. Comput. Hum. Interact.* 29, 4 (2022), 37:1–37:54. https://doi.org/10.1145/3503537

[93] Roy D. Yates and David J. Goodman. 2014. *A Probability and Stochastic Processes: A Friendly Introduction for Electrical and Computer Engineers*. John Wiley, Hoboken, NJ, USA. https://www.wiley.com/en-us/Probability+and+Stochastic+Processes%3A+A+Friendly+Introduction+for+Electrical+and+Computer+Engineers%2C+3rd+Edition-p-9781118324561

[94] Hongni Ye, Tong Wu, Lawrence H Kim, Min Fan, and Xin Tong. 2023. WooGu: Exploring an Embodied Tangible User Interface for Supporting Children to Learn Farm-to-Table Food Knowledge. In *Proceedings of the 22nd Annual ACM Interaction Design and Children Conference* (Chicago, IL, USA) *(IDC '23)*. Association for Computing Machinery, New York, NY, USA, 681–687. https://doi.org/10.1145/3585088.3593876

[95] J. Yin and H. Liu. 2011. The Empirical Study of Input Modalities Sensed from Flipping a Cube. In *Proceedings of the 4th International Conference on Intelligent Networks and Intelligent Systems* (Kuming, China) *(ICINIS '11)*. IEEE Press, Piscataway, NJ, USA, 5–8. https://doi.org/10.1109/ICINIS.2011.61

[96] ZhiYing Zhou, Adrian David Cheok, and JiunHorng Pan. 2004. 3D Story Cube: An Interactive Tangible User Interface for Storytelling with 3D Graphics and Audio. *Personal Ubiquitous Comput.* 8, 5 (Septermber 2004), 374–376. https://doi.org/10.1007/s00779-004-0300-0

[97] Oren Zuckerman, Saeed Arida, and Mitchel Resnick. 2005. Extending tangible interfaces for education: digital montessori-inspired manipulatives. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems* (Portland, Oregon, USA) *(CHI '05)*. Association for Computing Machinery, New York, NY, USA, 859–868. https://doi.org/10.1145/1054972.1055093

A Technical Description of AudioCubes

A AudioCube is a 2.95 inches^3 (75 mm^3) [3] acrylic-made cube TUI, weighting 800 g and containing an Inertial Measurement Unit (IMU) to capture the cube's linear acceleration, angular velocity, and orientation, a Digital Signal Processor (DSP) with optical sensors and emitters (*i.e.,* infrared, red, green, and blue LEDs). Each cube is powered by a Li-Poly rechargeable battery that can be charged via a special port (Fig. 12-c) and can be configured by software via a USB 2.0-compatible cable (Fig. 12-a). The maximum operating time before running out of power is about 2.5 hours. Each lateral face is equipped with a sensor and an emitter (Fig. 12-b). The sensors and emitters receive and send signals, which are generated or processed by the DSP in the cube. A custom infrared sensing and communication (1 kHz/10 bit) is also included (see ports #1 to #4 in Fig. 12-b). By positioning the cubes relative to each other and moving them around, *e.g.,* on a tabletop or another surface, a signal-processing network can be created that forms a cube topology (*i.e.,* a physical arrangement of cubes interconnected).

The minimum displacement that a AudioCube can detect is approximately 5 mm (0.19 inches) at a frequency of 60 Hz, thereby offering high speed. The response time falls within [0,01 s.; 0,1 s.], making the cube appear to respond in real time from the end user's perspective with low latency. The minimum and maximum frequency for emitting or receiving signals (*e.g.,* sounds) is [140 Hz,280 Hz]. The minimum distance between two cubes to interconnect them is about 0.5 cm (1.96 inches), and the maximum distance between them is about 50 cm (19.68 inches). Each AudioCube provides the following functions:

- *Continuous tracking of cube distance* with respect to a hand, angle after moving, rotating, and permuting faces. An internal description of all 24 rotations (Appendix D) is embedded in the cube.
- *Continuous topology detection*: by detecting the presence or absence of a cube in the range of another cube on any lateral face (not the top or the bottom), multiple cubes can be detected in the way they are arranged. For example, two cubes can be arranged according to any of the compositions examined in Fig. 6.
- *Real-time identification*: by detection of whether the cube is an emitter (*e.g.,* play a sound), a receiver (*e.g.,* record a sound), or a transceiver which combines an emitter and a receiver (*e.g.,* a special effect).

When AudioCubes are arranged in a topology, they could form a modular electronic musical instrument initially introduced for creating music and visuals, but not restricted to this field of application. AudioCubes were made from scratch by Bert Schiettecatte and developed by Percussa, a brand of Noisetron LLC. Each AudioCube provides a Max/MSP/Jitter software development kit consisting of a collection of example patches. The AudioCubes have built-in RGB lighting, which is used for example by DJs, VJs, producers for live performance, music education, and interactive projects around the electronic music world. AudioCubes work with Midi software to create possibilities with a DAW.

The USB 2.0 port can also be used for firmware upgrades and communication, as well as for custom analog/digital signal processing electronics. It provides compatibility with MS Windows and Mac OS X operating systems. No drivers need to be installed on Mac or Windows, as AudioCubes use class drivers: we plug the cubes in via USB, and they are automatically detected. 200 Mb of disk space and 1 Gb of RAM are required. AudioCubes are also equipped with LED light sources. The color can be set via the free software, and attenuated by hand position in certain modes. We can also send RGB commands to cubes via MIDI continuous controller messages. Unlike reacTable [34], a AudioCube is a self-contained computer with sensors, thereby eliminating the need for cameras, projectors, tables, and computers to run the tangible interaction.

B Positioning of AudioCubes with respect to other TUIs

AudioCubes have been developed to identify some conceptual elements. In some TUIs, the TUI objects are active, contain electronics, and process data independently. In other cases, they are passive objects associated with data [49] or programming code stored on the computer system. Therefore, the computer system can be separated or contained within the TUI itself (*i.e.,* a processor embedded in each object). Some TUIs combine these interaction modes and properties to form distributed systems [43]. Therefore, the location of the functional core of a TUI is said to be separate, contained (or embedded), or distributed, depending on where semantic functions attached to TUI objects are coded: outside TUI objects, inside, or simultaneously inside and outside. AudioCubes are embedded TUIs, as their accelerometer and gyroscope capture their relative position in real time and transmit it via the MIDI Bridge. In general, TUIs can be subdivided into three classes [71]: spatial, relational, and constructive systems. We can observe that combinations of properties from each of these classes (*e.g.,* mixed constructive/relational) are also possible. AudioCubes are TUIs that can be classified as a mixture of a constructive assembly system with an interface that benefits from spatial compositions, employing properties of each of the classes mentioned earlier. The interface fits the MCRpd model [71] as follows:

- Physical representations are *computationally coupled* to the underlying digital information. Our TUI does not have such visible bindings. Perhaps one such binding can be seen between the geometry of the cubes and the effect this might have on the sound processing in certain situations.
- Physical representations *embody mechanisms for interactive control.* In our interface, there are multiple bindings between the position and orientation of the cubes and the parameters of the processing algorithm, determined by the cubes, each of which processes a different part of the algorithm.
- Physical representations are *perceptually coupled* to actively mediated digital representations. The physical elements play a central role in our interaction. Since there are no complex graphical user interfaces displayed on the cubes (only one full-color LED for each cube and some representations, as in

[3]This edge length belongs to the category of *medium cubes* [30], with an length between 5 cm and 10 cm, which can still be held with one hand, while requiring the other hand to designate and orient areas of interest. This is aligned with the analysis of cube sizes reported by Perelman et al. [52].

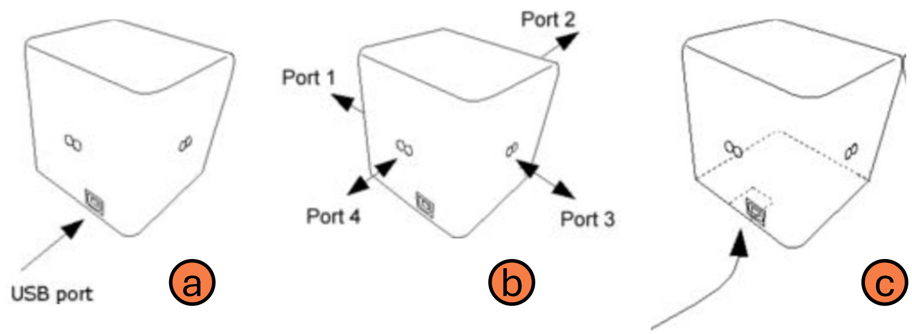


Figure 12: The ports of a AudioCube: USB port (a), the four infrared lateral ports (b), and the charging port (c).

Fig. 4 and Fig. 1-11), the only real display we can think of is an auditory display.

- The physical state of interface artifacts partially reflects the system's digital state. The state of the system is directly coupled to the physical state of the cubes: the parameters of the sound-processing algorithms depend on the cubes' locations and orientations.

We can further classify our TUI according to Ullmer et al. [72] as reproduced in Table 7 and according to Fitzmaurice et al. [25]. If we apply the TAC paradigm [64] to our TUI, the results are comparable to those of Shaer for Computational Building Blocks [4]: any block can be a token or a constraint depending on the TAC we consider. A cube is a token when it is coupled with parameters in the sound processing algorithm and is constrained by its neighbors. This constraint is implicit: when the user does not want to move the cube because the resulting sound is inappropriate, this constrains the cube. The same holds for each cube, since all processing cubes are identical. The special microphone or audio jack cubes in our system are similarly constrained by their neighbors: for example, the processing or the sound stops when the user moves the cubes out of the field of view of the other cubes. Table 8 positions cubes in the TAC paradigm [63].

Why Cubes? A cube gives 24 possible rotations (Appendix D). We assume that one rotation of the cube corresponds to one user command. Then, users can make 24 different commands by rotating the cube. Moreover, it is very easy to manipulate the cube; even a child can do that. So we can see that cubes can be considered for many user types. In addition to this, there are many reasons to choose a cube for tangible interaction:

- We were interested in whether the system's geometry would affect the patterns that users would create.
- Cubes are easy to transport and arrange.
- It would be easier to physically design and build the system if it were a collection of cubes.
- Designing the electronics and lighting to fit inside the enclosure would be much harder if the geometry were not box-like.
- It would make the system more expensive and time-consuming to build.
- Cubes, if they have the correct dimensions, are very easy to grasp and manipulate, even for children. This assumption likely holds, as most bricks in DUPLO (LEGO) toys have simple rectangular or cubic shapes.
- It would make our system more robust, less likely to suffer damage if it were accidentally dropped (in fact, this happened several times and proved to be true during our studies).

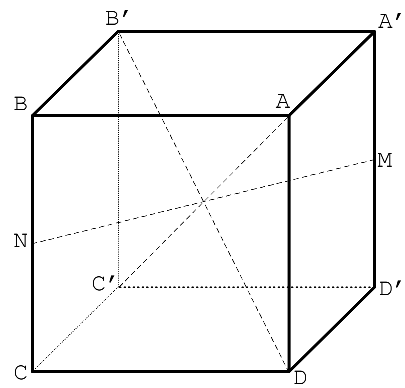


Figure 13: The rotation diagonals of a cube.

C Demonstration of the Order of the Cube Rotation Group

If an isometry of a given geometric figure F is defined as a transformation in which the transformed figure F' is equal to a figure of F, then all isometries of the cube must be determined to compute its rotation order. A cube includes 13 axes of symmetry: four diagonals, three axes joining the centers of the opposite faces, and six axes joining the midpoints of two opposite edges of the cube. There are two potential rotations for each diagonal (by 120° or 240°), which differ from a non-rotation and bring the cube back to its initial position. When we consider all possible rotations around the diagonals, we find that there are 8 in total. For each axis joining the centers of two opposite faces, there are three different rotations from the non-rotation concerning the cube (by an angle of 90°, 180°, or 270°). Thus, three axes give nine rotations. Finally, the cube can be rotated by 180° around an axis that joins the midpoints of two opposite edges, leading to six such rotations. Consequently, there are 8+9+6 = 23 rotations, different from the non-rotation, that transform the cube to its initial state. If the identity rotation is added, we obtain

Table 7: Positioning of AudioCubes according to TUI framework of Ullmer et al. [72].

Classifier	Location
Brick's internal ability	Smart
Input / Output	Both properties sensed and dis-played
Spatially aware	Mutual awareness
Communication	Wireless between blocks, cable for connecting to the system
Interaction time space	Quick
Bricks in use at the same time	$\tilde{1}0$
Function assignment	Programmable, can be transient, mostly space multiplexed
Interaction representation	Mix, but physical dominates
Physical and Virtual Layers	Layers superimposed
Bond between Physical	Virtual Layers Tightly coupled, but no real tracking as in spatial interfaces
Operating granularity	Room, inch accuracy
Operating surface type	Between static and dynamic, cubes change color
Operating surface texture	Continuous, but some cubes connect to audio amplifier using a cable

Table 8: Positioning of AudioCubes with respect to the TAC paradigm.

TAC	Representation		Behavior		
	Token	Constraint	Variable	Action	Observed feedback
1	Cube	Nearby cubes	Cube	Add/Remove to neighbor cubes	Sound changes, cube colours change

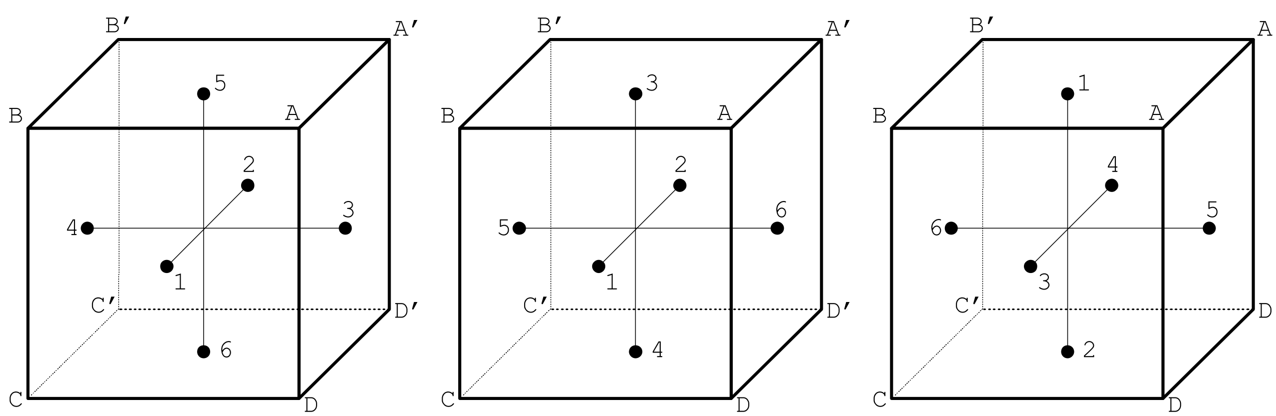


Figure 14: Three cube rotations of angles (π, $\pi/2$, $\pi/2$).

a total of 24 different rotations. We have the following result: the isometries of the cube are rotations around its axes of symmetry. Before we continue studying the rotation group of the cube, we will prove the following lemma:

Lemma 1. Every rotation around the four diagonals of a cube gives a permutation of them. Only identity rotation transforms each diagonal of a cube into itself.

Proof. Let us define a cube by the edges of two equal rectangles (A, B, C, D) and (A', B', C', D') such that $BN+NC = BC$ and $A'MD = A'm + MD'$. Any rotation of the cube that preserves two diagonals, for example AC' and DB', also preserves the diagonal plane $ADC'B'$ (Figure 13). Any non-null rotation (different from a non-rotation) conserving a plane is operated around an axis that is situated in this plane or perpendicular to this plane. In case the axis is located in the plane, the angle of rotation is equal to π. A rotation of angle π around an axis located in its own plane conserves only the lines included in the plane that are perpendicular to the axis of rotation.

The rectangle $ADC'B'$ is not a square: its diagonals are not perpendicular and cannot be transformed into themselves by rotation of an axis located in the plane of the rectangle. Thus, both diagonals AC' and DB' can be converted only into themselves by rotating the cube around an axis that is perpendicular to the plane $ADC'B'$. The axis MN that joins the midpoints of two opposite edges $A'D'$ and BC is perpendicular to the plane $ADC'B'$.

The diagonals AC' and DB' are only transformed into themselves since the cube is rotated around the axis MN by an angle of π. In this case, two other diagonals BD' and CA' are permuted so that there does not exist any non-null rotation of the cube that can transform each of the four diagonals of this cube into itself.

Thus, every rotation of the cube gives a permutation of its diagonals. Only the identity rotation transforms each diagonal into itself. As a result, if the rotations a and b are different, then the diagonals undergo different permutations. Therefore, two rotations a and b engender the same substitution of diagonals, and substitution $a - b$ preserves four diagonals. Thus, it would be an identical substitution and, as a consequence, the rotations a and b should not be different. A set of 24 rotations of a cube is its group of rotational symmetries, in which these rotations generate the substitutions of diagonals. We know that four elements give $1 \times 2 \times 3 \times 4 = 24$ permutations. ∎

As a result, there is a one-to-one correspondence between the rotation group of the cube and the substitution group of the four diagonals. Based on this correspondence, the addition of two rotations corresponds to the addition of two substitutions. The theorem is established as follows:

Theorem of the cube rotation group. The rotation group of the cube is isomorphic to the substitution group concerning four elements[4].

Proof. Let us assume that the diagonals AC' is 1, BD' is 2, CA' is 3 and DB' is 4 (Figure 13). The edge A of the diagonal 1 is an intersection point of faces 1, 3, 5. The edge B of the diagonal 2 is an intersection point of faces 1, 4, 5. The edge C of the diagonal 3 is an intersection point of faces 1, 4, and 6. The edge B of the diagonal 4 is an intersection point of faces 1, 3, and 6.

For example, the rotation of an angle $(0, \pi/2, 0)$ sends the diagonal 1 to the second place, the diagonal 2 to the third place, the diagonal 3 to the fourth place, and the diagonal 4 to the first place. This rotation gives an odd permutation (Fig. 14):

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$$

The rotation of angles (π, $\pi/2$, $\pi/2$) gives an even permutation (Fig. 14):

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 2 & 3 \end{pmatrix}$$

∎

[4]Based on Theorem 6.12.1- A finite subgroup of SO_3 is O, the octahedral group of 24 rotational symmetries of a cube or an octahedron, [6], p. 183.

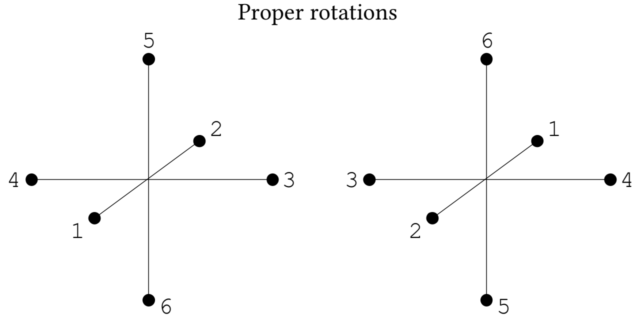
D The Twenty-four Cube Rotations

The following figures show the 24 elements of the rotation group of the cube[5] with the corresponding angles α, β, and γ, which are called *proper rotations* of the cube. Reversal of the cube with respect to the center of symmetry gives, again, 24 elements, and they are called *improper rotations* of the cube.

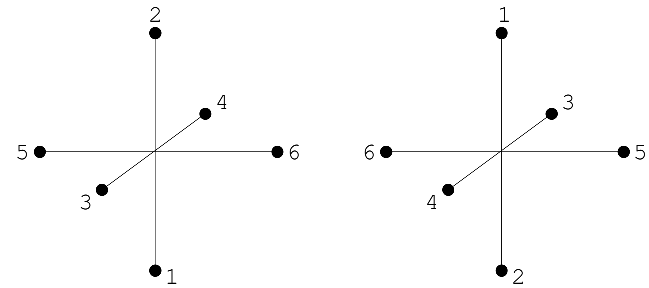
[5] See for example Mark Ronan, The rotations of the cube.

Proper rotations

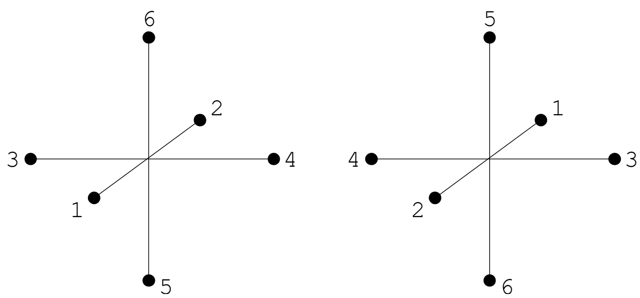
Improper rotations



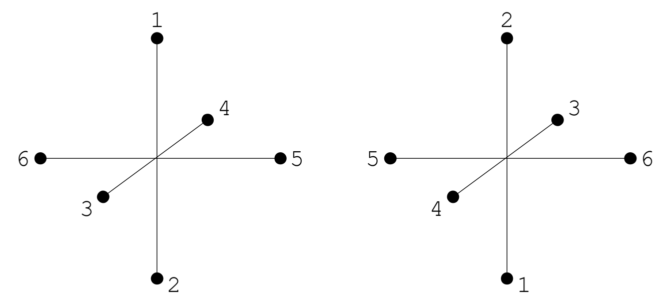
(a) $\alpha(0), \beta(0), \gamma(0)$



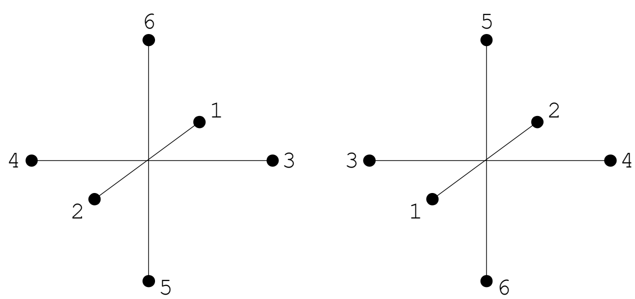
(b) $\alpha(0), \beta(\pi), \gamma(\pi)$



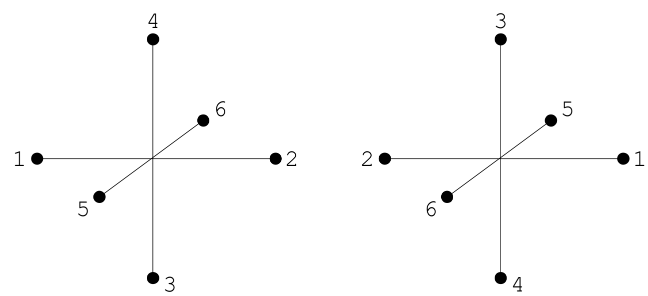
(c) $\alpha(0), \beta(\pi), \gamma(\pi)$



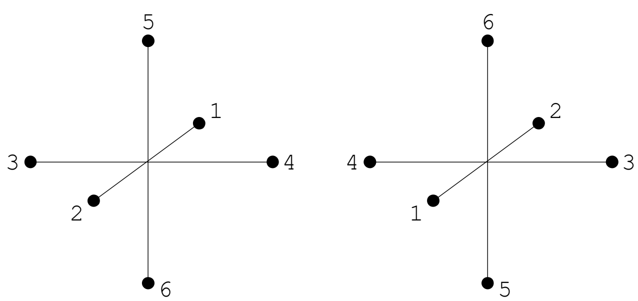
(d) $\alpha(0), \beta(0), \gamma(\pi)$



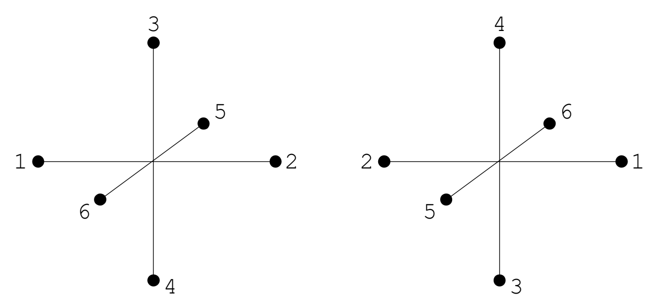
(e) $\alpha(0), \beta(\pi/2), \gamma(\pi/2)$



(f) $\alpha(\pi), \beta(\pi/2), \gamma(3\pi/2)$



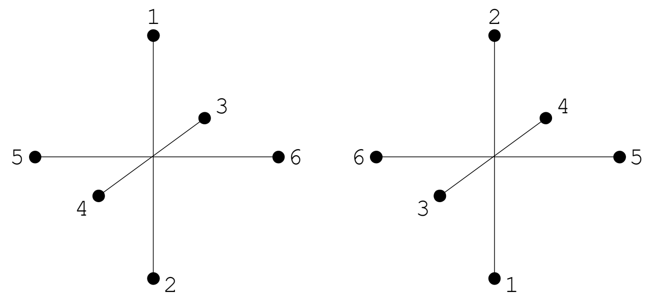
(g) $\alpha(0), \beta(\pi/2), \gamma(3\pi/2)$



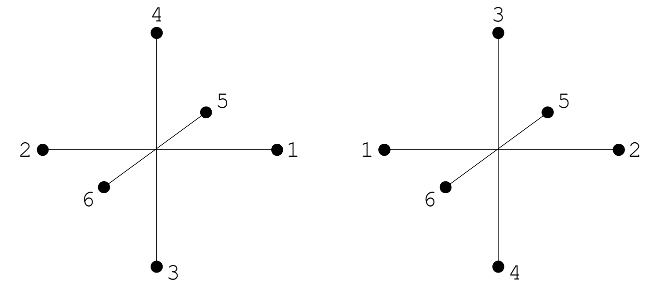
(h) $\alpha(\pi), \beta(\pi/2), \gamma(\pi/2)$

Proper rotations

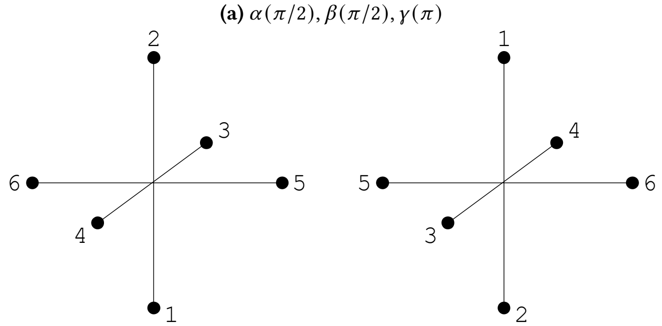
Improper rotations



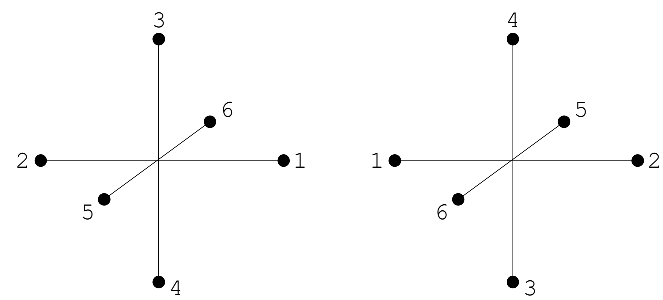
(a) $\alpha(\pi/2), \beta(\pi/2), \gamma(\pi)$



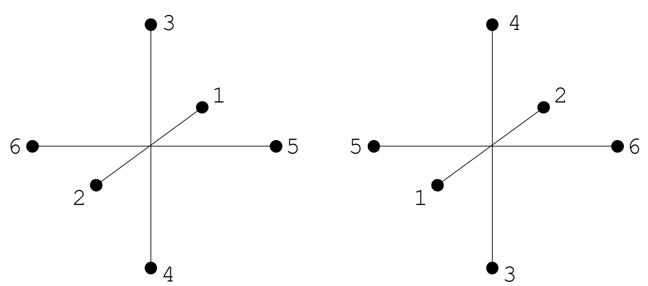
(b) $\alpha(3\pi/2), \beta(\pi/2), \gamma(0)$



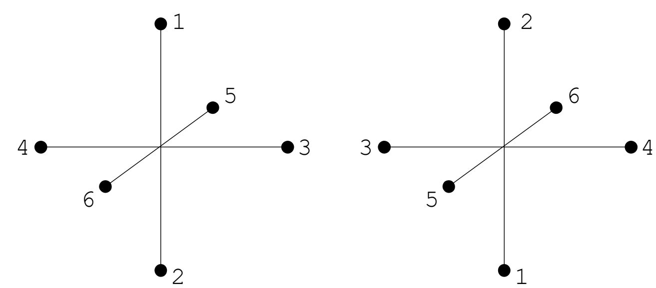
(c) $\alpha(3\pi/2), \beta(\pi/2), \gamma(\pi)$



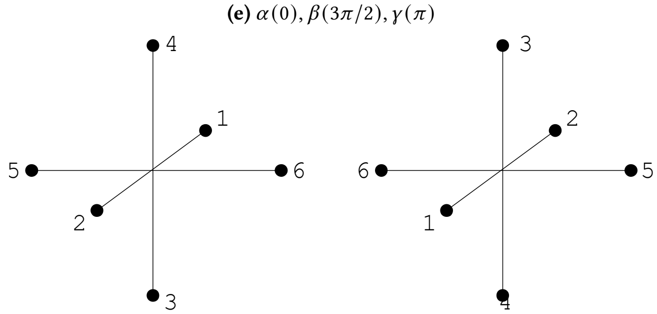
(d) $\alpha(\pi/2), \beta(\pi/2), \gamma(0)$



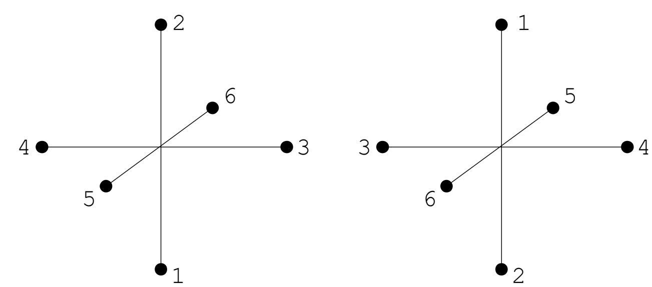
(e) $\alpha(0), \beta(3\pi/2), \gamma(\pi)$



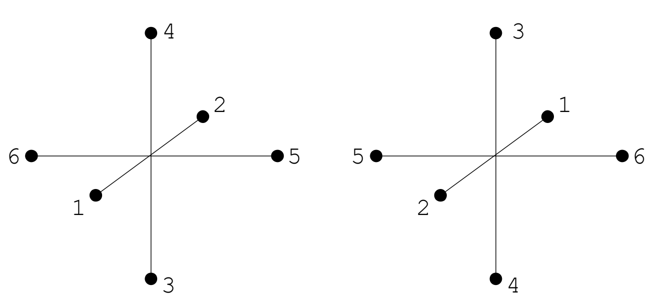
(f) $\alpha(0), \beta(\pi/2), \gamma(\pi)$



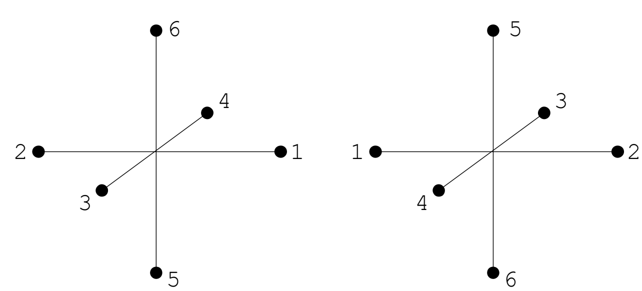
(g) $\alpha(0), \beta(3\pi/2), \gamma(0)$



(h) $\alpha(0), \beta(\pi/2), \gamma(0)$



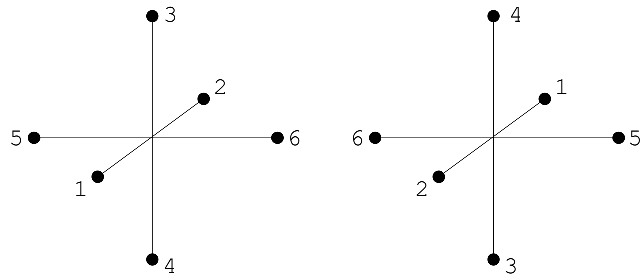
(i) $\alpha(\pi/2), \beta(\pi/2), \gamma(\pi/2)$



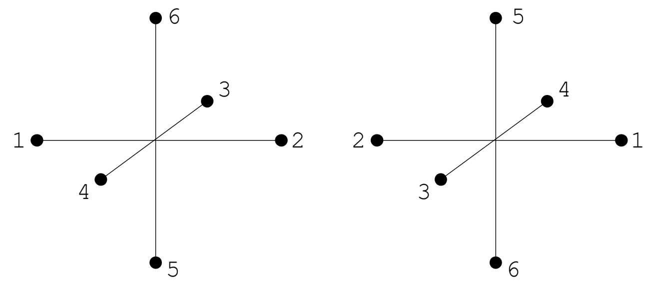
(j) $\alpha(3\pi/2), \beta(\pi/2), \gamma(3\pi/2)$

Proper rotations

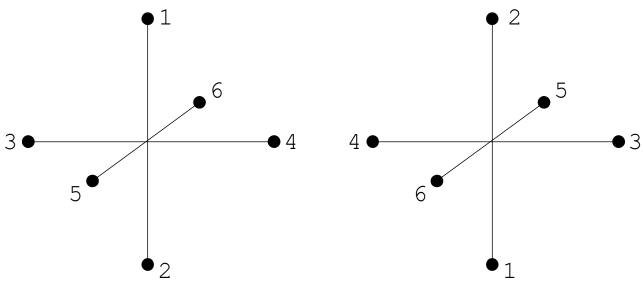
Improper rotations



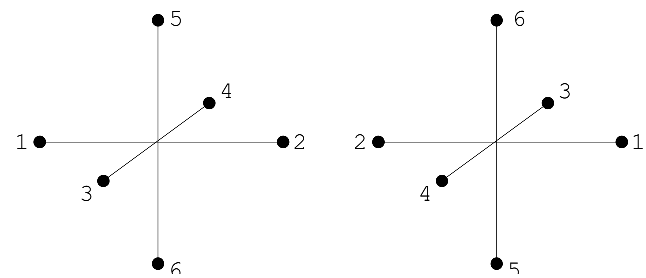
(a) $\alpha(3\pi/2), \beta(\pi/2), \gamma(3\pi/2)$



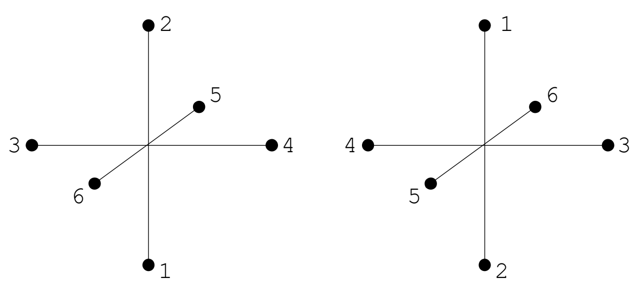
(b) $\alpha(\pi/2), \beta(\pi/2), \gamma(3\pi/2)$



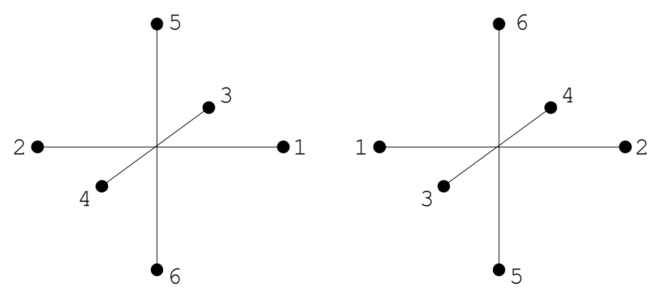
(c) $\alpha(\pi/2), \beta(\pi), \gamma(0)$



(d) $\alpha(3\pi/2), \beta(\pi), \gamma(0)$



(e) $\alpha(0), \beta(0), \gamma(3\pi/2)$



(f) $\alpha(0), \beta(0), \gamma(\pi/2)$